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Application of the CLEAN Algorithm to Three Dimensional Coded Aperture Imaging

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Economics Working Paper Series

Faculty of Business, Economics and Law, AUT

**Application of the CLEAN Algorithm to Three
Dimensional Coded Aperture Imaging**

Kevin Byard

2020/14

Application of the CLEAN Algorithm to Three Dimensional Coded Aperture Imaging

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Abstract

An iterative technique for the removal of artifacts caused by the near field effects of a coded aperture imaging system is presented. The technique, which we call z-Clean, first locates high energy sources within a three dimensional field of view using a least squares method and then removes the artifacts using a method similar to that of the CLEAN algorithm used in radio astronomy, but instead operating in the detector shadowgram domain rather than the final image domain. Computer simulations were performed of observations of four point sources of different intensities and at different depths from the detector. Both a continuous detector of 1cm FWHM detection capability and a pixellated detector with 0.2cm square pixels were investigated using a Modified Uniformly Redundant Array coded aperture of element size 0.6cm. The efficacy of the z-Clean technique for artifact removal is demonstrated for both detector types for the three strongest sources of 100kBq, 50kBq and 10kBq using plane separations of 2cm, 1cm, 0.5cm and 0.1cm, to leave only small ghosts lying up to around 2cm from the reconstructed source depth. For twenty trials of each observation, the three strongest sources are reconstructed no further than 0.7cm from the closest plane with many being from 0cm to 0.5cm for both detector types. The depth location for all three strongest sources using both detector types is no worse than 0.5cm from the actual source depth and is in most cases much better, being closer than 0.1cm for the strongest source at plane separations of 1cm, 0.5cm and 0.1cm. z-Clean was not able to remove the artifacts nor determine accurately the depth of the weakest source of 5kBq and in general sources that experience a phasing error are less accurately located although still better than 0.5cm from the actual source depth for all such cases. The artifact removal and very good depth location come at the expense of an impact on the signal to noise

ratio (SNR) of the sources. For the strongest source and using the continuous detector the SNR increases unexpectedly to give values higher than that for observations made only in the critical plane due to the ghosting of this source in other planes at different depths. For all other cases there is a decrease in SNR which is more marked for finer plane separations and for weaker sources.

Keywords: coded aperture, three dimensional imaging, tomography, gamma ray imaging, image processing

1. Introduction

Coded aperture imaging has become the major technique for forming images in the high energy domain [1, 2, 3, 4]. In this technique, an aperture consisting of opaque and transparent elements is placed between a photon emitting source and a position sensitive detector. The result is a shadowgram on the detector, which in some applications can be saved on computer as a dataset consisting of a set of Cartesian coordinates for each count on the detector - a form known as *list mode* - and in others may instead be stored as a matrix of counts in the form of pixels. We hereafter refer to a detector operating in list mode as *continuous* and a detector consisting of pixels as *pixellated*. The detector shadowgram needs to be subsequently decoded to produce a reconstructed image of the source distribution. The coded aperture technique was originally proposed for high energy astronomical imaging [4] including X-rays [5] and gamma-rays [6] where the incoming radiation effectively comes from infinity and therefore rays from a point source that reach the detector are parallel to each other. This means that the object distribution is essentially a flat two-dimensional (2D) field with the decoded image being a similarly flat reconstruction of the source distribution.

However other applications of coded apertures have also been proposed, many using near field optics including medical imaging [7, 8] and land mine detection [9]. In the near field the object is placed close to the detector, and so the rays from a point in the object diverge giving rise to different problems compared to the far field. The near field itself may be thought of as a stack of 2D *planes*, each parallel to the aperture and detector but lying at different distances from the detector [8]. We hereafter refer to the perpendicular distance of a point or a plane from the detector as its *depth*. In tomography an attempt is made to ascertain the source distribution in each plane from the detector shadowgram and hence produce an overall three-dimensional

29 (3D) image. To this end a number of studies into tomographical imaging
30 have been conducted [8, 10, 11, 12, 13]. While it is possible to create images
31 in different planes by discretising the detector data accordingly and decoding
32 each plane, artifacts in the decoded image of one plane typically arise due to
33 the presence of sources from other planes. For example Kazachkov et al. [8]
34 were able to ascertain the different depths of three point sources by ‘focusing’
35 on each plane although the non-focussed sources from the other planes are
36 still visible in the form of blurred artifacts. Similarly Mu and Liu were able
37 to determine in which of nine planes, each separated by a centimetre, lay two
38 extended objects in the shape of a ‘V’ and ‘H’, although again, large artifacts
39 remained in the other planes [13].

40 The results in [8] and [13] indicate that further processing of their 3D
41 images is possible. Therefore in this article we attempt to achieve this with
42 an iterative source removal similar to the CLEAN algorithm used in radio
43 astronomy [14].

44 2. Coded Aperture Imaging

45 For many practical coded aperture imaging applications, the system often
46 chosen is what we term a *perfect* system, namely one where the correlation
47 function of the aperture has perfectly flat sidelobes and the shadowgram is
48 congruent to a *unit pattern* (also sometimes called the basic pattern) of the
49 aperture [3]. Such systems include the uniformly redundant arrays (URA) [3]
50 and modified uniformly redundant arrays (MURA) [15]. Note that in far field
51 imaging, such as in astronomy, when using a perfect aperture with a detector
52 that is the same physical size as the aperture unit pattern, all point sources
53 lying in the fully coded field of view (where they project a shadowgram over
54 the entire detection plane) will generate a shadowgram that is congruent to
55 the aperture unit pattern and so such a system is always perfect. However,
56 forming images in the near field will only be perfect for URAs and MURAs
57 if all sources being imaged lie in the exact plane that enables these sources
58 each to cast a shadow of exactly one unit pattern or its cyclic repetition onto
59 the detector. Here we refer to such a plane of a perfect system as the *critical*
60 plane. It is evident that only one critical plane exists for a given perfect
61 coded aperture system (which for astronomical applications lies at infinity)
62 and that these coded aperture systems lose their perfect imaging capability
63 when used to observe a scene in which the sources lie at different depths
64 because the aperture portions cast by such sources cannot all simultaneously

65 be congruent to the aperture unit pattern. Therefore forming images using
 66 a coded aperture in the near field presents different problems compared to a
 67 flat image from sources at large distances.

68 Consider a coded aperture imaging system based on a square geometry
 69 with an aperture to detector separation of s . Also consider two point sources
 70 α and β positioned respectively at depths z_α and z_β from the detector as
 71 shown in Fig. 1. Source α lies at a smaller depth and casts a shadow of
 72 a portion of the aperture onto the detector. This portion is shown as a
 73 small square on the aperture and the shadow that it casts is shown on the
 74 detector. Source β is situated at a larger depth than source α and it casts
 75 a shadow of different portion of the aperture, shown as a larger square,
 76 onto the detector. To avoid confusion, the shadow of β is not shown. The
 77 closer source casts a smaller aperture portion while at the same time casting
 78 larger shadows of the individual aperture elements than the further source.
 79 While the relative sizes of the shadows of the apertures and their elements
 80 from the different sources allow some depth information to be ascertained,
 81 in the reconstruction of the final image, sources from one plane are often
 82 seen as blurred artifacts in the other plane and vice-versa [8]. To decode the
 83 shadowgram and create a reconstructed image of a plane at depth z , we can
 84 adopt the following reasoning. Consider a coded aperture system based on
 85 a rectangular geometry consisting of a $(0, 1)$ aperture $A(i, j)$ of dimensions
 86 $V \times W$ where $A(i, j) = 1$ for open (transparent) elements and $A(i, j) = 0$
 87 for closed (opaque) elements and where $0 \leq i \leq V - 1$ and $0 \leq j \leq W - 1$.
 88 A number of different aperture configurations exist but for a general system
 89 using the *balanced decoding* algorithm [3], we have a decoding array $G(i, j)$
 90 of the same dimensions as A where:

$$G(i, j) = 2A(i, j) - 1 \quad (1)$$

91 although slight modifications to Eq. (1) exist for certain types of aperture
 92 such as the Pseudo-Noise Product Array (PNP) [16] or the MURA arrays
 93 [15].

94 Assume that we are using a continuous detector, namely one that gives the
 95 individual detected (x, y) positions of each photon in list mode. Also consider
 96 the reconstruction of the image of a 3D field of view (FOV) for a plane at
 97 depth z using this detector. Since reconstructing the image requires a cross-
 98 correlation of the shadowgram with the decoding array [3], it is necessary to
 99 discretise the detector shadowgram into squares called *bins*, such that each

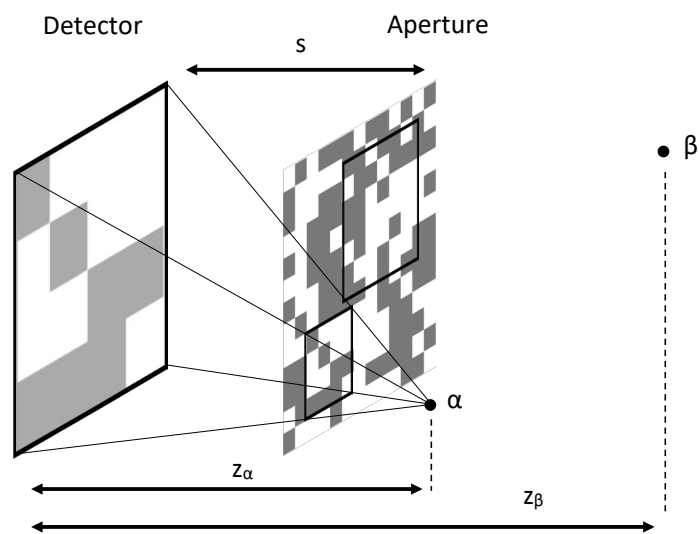


Figure 1: Coded aperture imaging of near field objects.

100 bin size is the same as that of an aperture element when projected from
 101 a point source in the plane at depth z onto the detector. Therefore the
 102 shadowgram is divided into $v_z \times w_z$ bins and the detected counts within each
 103 bin are summed to give a detector array $P_z(i_z, j_z)$ where $0 \leq i_z \leq v_z - 1$
 104 and $0 \leq j_z \leq w_z - 1$. The z subscripts are used here to remind us that the
 105 various parameters all depend on the plane depth, for example the number
 106 of bins v_z and w_z both increase with increasing z .

107 The decoded image $I_z(x_z, y_z)$ for the plane at depth z is calculated as
 108 follows:

$$I_z(x_z, y_z) = \sum_{x_z=0}^{v_z-1} \sum_{y_z=0}^{w_z-1} P_z(i_z, j_z) G(i_z + x_z, j_z + y_z) \quad (2)$$

109 [3]. Each (x_z, y_z, z) in Eq. (2) can be thought of as a 3D reconstructed
 110 object pixel, sometimes also called a *voxel*. Note that since the number of
 111 reconstructed voxels at depth z is given by $(V - v_z + 1, W - w_z + 1)$ then
 112 we have $0 \leq x_z \leq V - v_z$ and $0 \leq y_z \leq W - w_z$. Reconstruction of the
 113 image at a given plane at depth z using Eq. (2) effectively gives an image
 114 that is ‘focused’ on that plane. While Eq. (2) provides a method to focus
 115 on the different planes of a 3D FOV, sources from one plane typically cause
 116 artifacts on other planes in the image leading to overall image degradation.
 117 The purpose of this article is to attempt to remove such near field artifacts
 118 produced in 3D coded aperture imaging and also determine the actual depth
 119 of the source.

120 Note that in far field imaging such as high energy astronomy, there is
 121 only one image plane, lying at infinity, and so there is no image degradation
 122 from sources in other planes.

123 3. Image Processing Technique: z-Clean

124 For the purpose of demonstrating the image processing technique we sim-
 125 ulate a prototype coded aperture system having a number of idealised pa-
 126 rameters. We assume a continuous square detector of width 30cm having
 127 100% detection efficiency and perfect photon location detection. We employ
 128 a non-repeating random pattern for the prototype system aperture and de-
 129 fine the square central region of $v \times v$ elements as the *core* of the aperture
 130 with a full aperture of dimension $2v - 1 \times 2v - 1$ that is not a repeat of
 131 the core but instead an array of fully randomised open and closed elements.
 132 Note that for a perfect aperture, the core is the unit pattern of the aperture

133 with size v and the full aperture is a $2v - 1 \times 2v - 1$ cyclic repeat of the
 134 core. The aperture used for the prototype is the square 13×13 element
 135 random configuration with a core of $v = 7$ shown in Fig. 2(a) where the dark
 136 and light areas represent the opaque and transparent aperture elements re-
 137 spectively. The aperture elements used in the simulations are of size 2cm, of
 138 negligible thickness and with a closed element opacity of 100%. The aperture
 139 to detector separation is 30cm. For these parameters the depth of the plane
 140 that casts a shadow of the central 7×7 aperture elements onto the detector
 141 (the equivalent of the critical plane of a perfect system) is 56.25cm from the
 142 detector. For this discussion we number the planes in order of z , with plane
 143 1 being the plane closest to the detector.

144 For the prototype observation we take the planes as being at 56.25cm
 145 (plane 1, also the critical plane), 61.25cm (plane 2) and 66.25cm (plane 3)
 146 from the detector and thus we have a total of three planes with a plane
 147 separation of 5cm. We assume a single point source situated in plane 2
 148 (61.25cm from the detector) in the centre of the FOV, emitting photons with
 149 a detection rate of $0.01 \text{ cm}^{-2} \text{ s}^{-1}$. A background rate of $0.001 \text{ counts cm}^{-2}$
 150 s^{-1} is assumed and the observation time is 600s.

151 The initial shadowgram showing the distribution of counts on the contin-
 152 uous detector is shown in Fig. 2(b) where each point represents the detected
 153 position of a photon count. Such location of the individual counts, albeit in
 154 practice containing statistical errors, is crucial to the image processing tech-
 155 nique. Therefore it is necessary for the data to be in list mode, and note that
 156 the shadowgram image in Fig. 2(b) is an unbinned representation of the data
 157 in list mode, since individual detected photon positions can be discerned.
 158 The central portion of the aperture creating the shadowgram is clearly evi-
 159 dent in the distribution of counts shown, with photons from the strong source
 160 forming the central pattern of the aperture along with a weaker background.
 161 To commence image reconstruction we take each plane and we superimpose
 162 the projected bins from these onto the shadowgram. We hereafter refer to
 163 the superimposition of bins onto the shadowgram as a *map*. The maps for
 164 planes 1 and 2 are shown in the top images in Fig. 3. Note that the map for
 165 plane 2 is a good fit with the shadowgram since this plane is where the source
 166 is actually located, while the map for plane 1 represents a less good fit with
 167 this nearer plane having larger bin sizes. To demonstrate this point more
 168 clearly the black square shows in negative the counts corresponding to those
 169 photons that passed through a particular open aperture element, namely two
 170 elements up and two elements left from the central element. Note that the

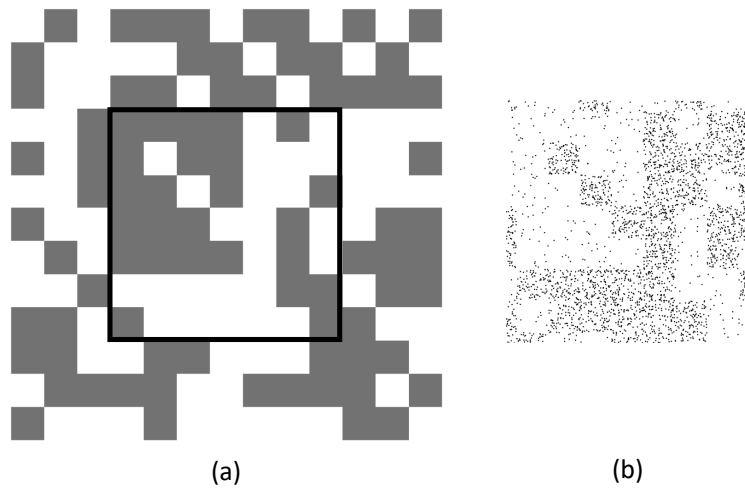


Figure 2: Prototype coded aperture system with (a) random 13×13 element coded aperture with the $v = 7$ element core shown in the central square, (b) continuous detector shadowgram of a single point source in plane 2 at distance $z = 61.25\text{cm}$ from the detector.

171 black square corresponds exactly with a bin on the map for plane 2 but not
 172 exactly with a bin of the map for plane 1. A similar situation exists for
 173 plane 3, but again the fit is not as good as for plane 2, due this time to the
 174 bins being smaller for plane 3. Binning the shadowgram gives the matrices
 175 in the lower part of Fig. 3. Using these matrices and decoding each plane
 176 according to the binning and cross-correlation method outlined in Section 2,
 177 Eq. (2) we obtain the isometrically projected images in Fig. 4(a). Note that
 178 the source, which is clearly visible with high intensity at the centre of plane
 179 2, also appears with somewhat lower intensity at the centres of the other
 180 planes. Thus we have results similar to those of Kazachkov et al. [8] and
 181 Mu and Liu [13] inasmuch that sources are ‘blurred’ between planes causing
 182 artifacts. We now attempt to remove these artifacts in our example.

183 We begin by determining the most likely position of a point source in
 184 each plane. We do this by performing a simple least-squares fitting of the
 185 shadowgram with all possible source positions within each plane of the 3D
 186 FOV, in the form of a χ^2 minimisation, similar to the method proposed for
 187 astronomy by Ducros and Ducros [17]. Consider a given plane at depth z .
 188 For this analysis we dispense with the z subscripts, recognising that we are
 189 working in a given plane at depth z . For each possible source position (x, y)
 190 at depth z we calculate a value of χ^2 as follows:

$$\chi^2(x, y) = \sum_{i=0}^{V-v} \sum_{j=0}^{W-w} (1/\sigma_{ij}) [P(i, j) - B - S_{xy}A(i + x, j + y)]^2 \quad (3)$$

191 [17, p. 49-50] where $P(i, j)$ represents the bin counts, examples being the
 192 matrices in Fig. 3, σ_{xy} is the variance of the counting statistics of P and A
 193 is the binary (0, 1) aperture function. The quantities S_{xy} and B are numerically
 194 modelled values, where S_{xy} represents the intensity per detector bin corre-
 195 sponding to the open aperture elements at a distance z from the detector of
 196 a source situated at a lateral (i.e. the perpendicular direction to z) source
 197 position (x, y) and B represents the background noise per detector bin. Note
 198 that we are here assuming a uniform background of B which is independent
 199 of both the source location and detector location. For each possible lateral
 200 source position (x, y) we set the partial derivatives of χ^2 in Eq. (3) with
 201 respect to B and S_{xy} to zero, calculate B and S_{xy} from the resulting pair
 202 of simultaneous equations and then calculate χ^2 by substituting for B and
 203 S_{xy} back into Eq. (3), giving χ^2 values for each lateral source position (x, y) .
 204 Repeating this procedure for all planes gives a 3D matrix of χ^2 values for

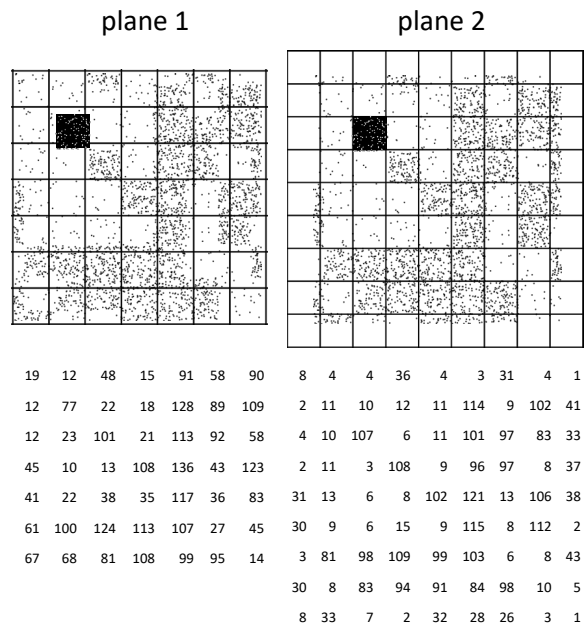


Figure 3: Maps for planes 1 and 2, consisting of bins superimposed on the detector shadowgram. Also shown are the detector image matrices after the counts have been binned.

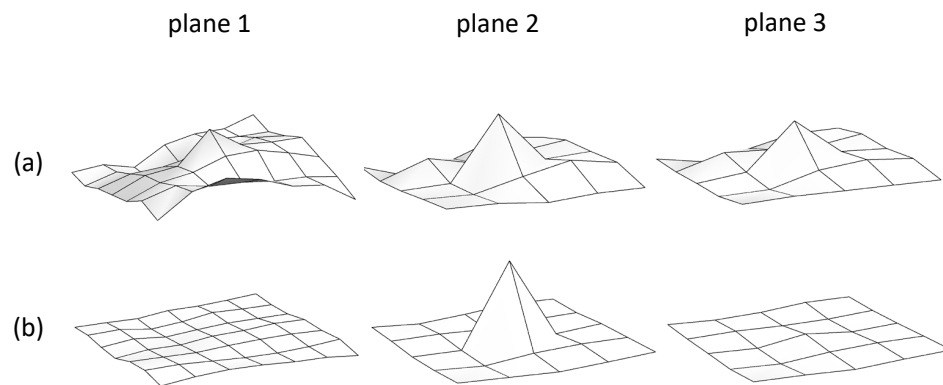


Figure 4: Decoded images for the prototype system for each plane: (a) without z-Clean image processing (b) after the use of z-Clean.

205 all possible voxel positions (x, y, z) within the 3D FOV. For example the χ^2
 206 values for the maps in Fig. 3, for a source in the centre of the FOV of plane
 207 2 are 9.51 for plane 1 and 1.29 for plane 2. The calculation for plane 3 is
 208 2.85. All other possible source positions in the 3D FOV give larger χ^2 values
 209 than these three. The 3D position having the lowest value of χ^2 is deemed
 210 to be the most likely position to contain a point source. We call this point
 211 a *candidate* which is in position (x_1, y_1, z_1) of the 3D reconstructed image.
 212 In our example therefore, the map that produces the first candidate is that
 213 for the central reconstructed voxel of plane 2. This is because the fit of this
 214 particular map is the best. We now process this candidate position by deter-
 215 mining for the candidate its value of S_{xy} , which we now denote by S_1 , and
 216 removing from the detector shadowgram S_1 randomly chosen counts from
 217 each of the bins of the map for plane 2 in Fig. 3 that correspond to an open
 218 aperture element at candidate position (x_1, y_1, z_1) to give a new shadowgram
 219 with fewer counts. An initial 3D matrix $T(x, y, z)$ with zero-valued elements
 220 is defined and the total number of removed counts are then stored and added
 221 to cell (x_1, y_1, z_1) of T to be used later. The number of counts subtracted
 222 from each bin not at the boundary of the map needs to be equal for each
 223 corresponding open aperture element, which is S_1 . For an open element bin
 224 at the map boundary we subtract the same number of counts but multiplied
 225 by the fraction of the bin areas ‘occupied’ by the detector shadowgram in
 226 that bin. After subtracting and storing counts, we perform a second iteration
 227 of the whole process but this time on the new shadowgram to determine a
 228 second candidate with position (x_2, y_2, z_2) and $S_{xy} = S_2$. The corresponding
 229 counts are subtracted from the relevant bins of the new shadowgram and
 230 added to T . Note that in different iterations, candidates may coincide but
 231 the procedure continues the same. Note also that if there are no counts to
 232 subtract from a particular bin then no subtraction takes place from that bin
 233 although subtractions from other bins continue. However it is worth noting
 234 at this point that the large source and background fluxes used in this study
 235 give high detector counts so this situation does not eventuate.

236 We continue the process until eventually a candidate returns a source
 237 intensity S_{xy} of a negative value which indicates that all possible positive
 238 sources have been exhausted. We then perform a cross-correlation of the
 239 remaining shadowgrams with the decoding function as per Eq. (2) for each
 240 plane and finally we add the data from the matrix T to the corresponding
 241 voxels from the planes of I_z to produce the final decoded images. Note that
 242 this is similar to the technique used to remove artifacts from coded aperture

243 systems having imperfect detectors in [18] and is similar to the CLEAN
244 algorithm used in radio astronomy [14] but we here operate in the detector
245 shadowgram domain rather than the final reconstructed image domain. For
246 ease of discussion we hereafter refer to the whole image processing technique
247 as *z-Clean*, with processed images being referred to as having been *z-Cleaned*.

248 In our prototype example the *z-Cleaned* images are shown in Fig. 4(b).
249 We can see that the reconstructed source position is clearly visible in plane
250 2, the actual plane containing the source, while the artifacts in the other
251 planes have been largely removed although some minor ‘ghosting’ can be
252 seen in the central pixel of plane 3. Note also that the *z-Clean* technique has
253 the added benefit of removing much of the noise produced by the random
254 nature of the coded aperture, with the processed images in Fig. 4(b) having
255 flatter sidelobes than those in Fig. 4(a). This is to be expected, since the
256 CLEAN algorithm is capable of removing a range of significant artifacts,
257 which includes those caused by the random noise produced when using a
258 coded aperture that does not have perfect imaging capability, such as the
259 random aperture used in the prototype.

260 We conclude this section by commenting on the point that in coded aper-
261 ture imaging, cleaning can be accomplished either by operating in the 2D
262 shadowgram domain, and subtracting individual photon counts from the de-
263 tector, or operating in the 3D reconstructed image domain and subtracting
264 spurious peaks from each of the planes being studied. We here adopt the
265 approach of subtracting from the 2D detector domain as it is much simpler
266 than the complex computation when operating in the 3D domain.

267 4. Computer Simulations

268 Computer simulations were conducted to test the *z-Clean* technique de-
269 scribed in Section 3 when using two types of high energy photon detector: a
270 continuous detector and a pixellated detector. It is evident that a continu-
271 ous detector is a certain type of idealised detector inasmuch that data can
272 be acquired in list mode and hence affords the user the opportunity to define
273 the bin size according to a particular plane depth being studied. However in
274 practice, detectors are often not continuous but are instead pixellated. For
275 example 0.2cm pixels are typical for cadmium zinc telluride detectors. There-
276 fore for most applications it is also necessary to assess the *z-Clean* technique
277 when used with a pixellated detector.

278 For both types of detector, a number of assumptions are made when per-
279 forming the simulations, including a few idealisations. For the continuous
280 detector, staff at Auckland Hospital were consulted as to the expected per-
281 formance of such a detector system when using a coded aperture. As a result
282 of these discussions, the continuous detector simulated is a square 35cm by
283 35cm plate, possessing a photon location accuracy that has a Gaussian pro-
284 file with a full width at half maximum (FWHM) of 1cm. For the pixellated
285 detector we employ 0.2cm pixels which, assuming a 35cm \times 35cm detector
286 (or an array of smaller detectors with a total size of 35cm \times 35cm) gives a
287 detector with 175 \times 175 pixels. The pixellated detector is assumed to have
288 the capability of rejecting simultaneous multi-site events and hence able, for
289 example, to reject any photons that undergo Compton scattering that de-
290 posits energy in more than one pixel. For both detectors a photon detection
291 efficiency of 70% is assumed.

292 For both the continuous and pixellated detectors systems, the aperture
293 pattern chosen is a square MURA with unit pattern (or core) of size $v = 31$,
294 cyclically repeated to give an overall aperture of size 61 \times 61 elements (namely
295 $2v - 1 \times 2v - 1$) and 50% throughput shown in Fig. 5. Past research by
296 Fenimore [19] and by in't Zand et al. [20] into optimum aperture throughput
297 show that, while values other than 50% may be best for certain source fields,
298 a 50% throughput still gives very good results for point source observations.
299 Furthermore, as the purpose of this study is to demonstrate the efficacy of
300 z-Clean, we here use the more well-known MURA aperture for simplicity
301 and familiarity while recognising that testing z-Clean with different aperture
302 types is a possible area for further research that is beyond the scope of this
303 study. The aperture to detector separation is set at 30cm and aperture
304 elements of size 0.6cm are chosen giving an overall aperture size of 36.6cm.
305 The reason for choosing this aperture element size is that it means that the
306 projected aperture elements from sources in the FOV onto the detector are of
307 the order of the FWHM of the continuous detector. Perfectly square aperture
308 elements of negligible thickness are assumed with a closed element opacity
309 of 99%. A uniform detector background of one count $\text{cm}^{-2} \text{s}^{-1}$ is assumed.
310 Observation time is 600s.

311 For all simulated observations, a field with four point sources of different
312 intensities is chosen. The first is of activity 100kBq situated at a depth of
313 72cm from the detector and lying in the centre of the FOV, the second is
314 of activity 50kBq at a depth of 74.7cm and lying to the left of the central
315 source, the third is 10kBq at 69.5cm, lying to the right of the central source

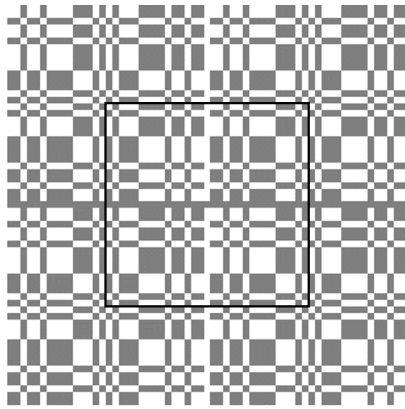


Figure 5: MURA coded aperture of $v = 31$ and overall size 61×61 used in the simulations of Sections 5 and 6. The central $v \times v$ unit pattern, or core, is also shown inside the black edged square.

316 and the fourth is 5kBq at 71cm lying roughly halfway between the centre and
317 the lower left corner of the FOV (see also Fig. 6). These source parameters
318 are chosen to demonstrate the efficacy of the z-Clean technique when both
319 weak and strong sources are present, lying at depths representing exactly
320 those in a plane to be decoded and also between planes to be decoded. All
321 sources are assumed to be in the very centre of a voxel in the lateral (x, y)
322 direction, which for data in list mode is always possible as we can define our
323 own detector bins without any great loss of generality. For each observation
324 twenty trials are conducted to give an indication of the spread of the signal
325 to noise ratio (SNR) values of the individual sources. We present results of
326 four point sources only, since results for fewer sources are at least as good
327 and in many cases marginally better than the results for four sources. The
328 quality of the resulting z-Cleaned images is also compared to those produced
329 by twenty trials of a perfect coded aperture system operating only in its
330 critical plane and hence not affected by depth effects.

331 Once the detector data is collected the z-Clean technique is applied be-
332 tween the minimum and maximum depths of 66cm and 78cm respectively.
333 For each observation the shadowgram is decoded for a range of plane separa-
334 tions with planes being equally separated in each case. We test the technique
335 for plane separations of 2cm (giving 7 planes from 66cm to 78cm), 1cm (13
336 planes), 0.5cm (25 planes) and 0.1cm (121 planes). In all cases the central
337 plane lies at a depth of 72cm.

338 As noted, the results and SNR values of the sources are compared to
339 those expected for a perfect imaging system operating only in its critical
340 plane. For this we use the same 31×31 element MURA. Because perfect
341 imaging for MURA arrays requires the source to lie in the critical plane,
342 we adopt the following approach to obtain correct SNR values for a point
343 source at a given depth. Employing the 31×31 element MURA aperture
344 we use the same system parameters as per the z-Cleaned image observations
345 except that we adjust the aperture to detector separation so as to place the
346 point source in the critical plane, where the source now casts a shadow of
347 exactly a full unit pattern of the aperture onto the detector. Then we make
348 an observation of this single source, ensuring that we adjust the background
349 level to take account of the extra statistics produced by the presence of the
350 other sources at other depths but not modulated by the aperture, and decode
351 using Eq. (2) (note that we do not process this image any further as we
352 wish to compare our z-Cleaned images with a single decoded image of a
353 perfect system that suffers no degradation from depth effects). Thus we are

354 comparing our simulated observation results to a genuinely perfect coded
355 aperture system that is observing a single source, with corrected statistics
356 for other sources unmodulated by the aperture. Again we perform twenty
357 repeated trials of each critical plane observation.

358 5. Continuous Detector Results

359 Images for a typical observation using the continuous detector are shown
360 in Fig. 6. Presented in this figure are the unprocessed and z-Cleaned images
361 for a plane separation of 2cm, and z-Cleaned images for 1cm plane separation.
362 Although more planes than these were processed, only those planes from
363 depths 70cm to 78cm are shown as there were no noticeable artifacts in the
364 z-Cleaned images for planes outside this range.

365 For the unprocessed images at 2cm plane separation large artifacts, sim-
366 ilar to those in Fig. 4(a), are present in all planes in the form of repetitions
367 of the 100kBq source at the centre of the FOV and of the 50kBq source left
368 of the centre of the FOV. The 10kBq source and repeated artifacts are just
369 visible to the right of the centre of the FOV at depths of 70cm, 72cm and
370 74cm. As a result of the large artifacts it is difficult to ascertain the true
371 depths of these sources. However, the z-Cleaned images for a plane separa-
372 tion of 2cm demonstrate the efficacy of the z-Clean technique, inasmuch that
373 the large artifacts that recur in all planes of the unprocessed images have
374 been largely removed from the planes where the sources are not present,
375 to leave the true sources clearly visible with only minor ghosting in other
376 planes. We here make the distinction that an artifact is a repeat of a source
377 appearing in a different plane to the actual source for an unprocessed image
378 and a ghost is such a repeat but in an image that has been processed by
379 the use of z-Clean. Typically the ghosts are much smaller than the artifacts.
380 The central 100kBq source is clearly reconstructed at its correct depth of
381 72cm and has also been successfully z-Cleaned, with only minor ghosting of
382 this source in the adjacent planes at 70cm and 74cm, as would be expected
383 given the imperfect photon position location capability of the detector. The
384 50kBq source has also been successfully z-Cleaned and the reconstructed flux
385 of this source, which is actually positioned at a depth of 74.7cm is shared
386 in roughly correct proportions between the planes either side of this depth,
387 namely there is a large peak in the closest plane at 74cm and a smaller peak
388 in the more distant plane at 76cm. The phenomenon of a single source being
389 shared over more than a single pixel, or in this case a voxel, is also known as

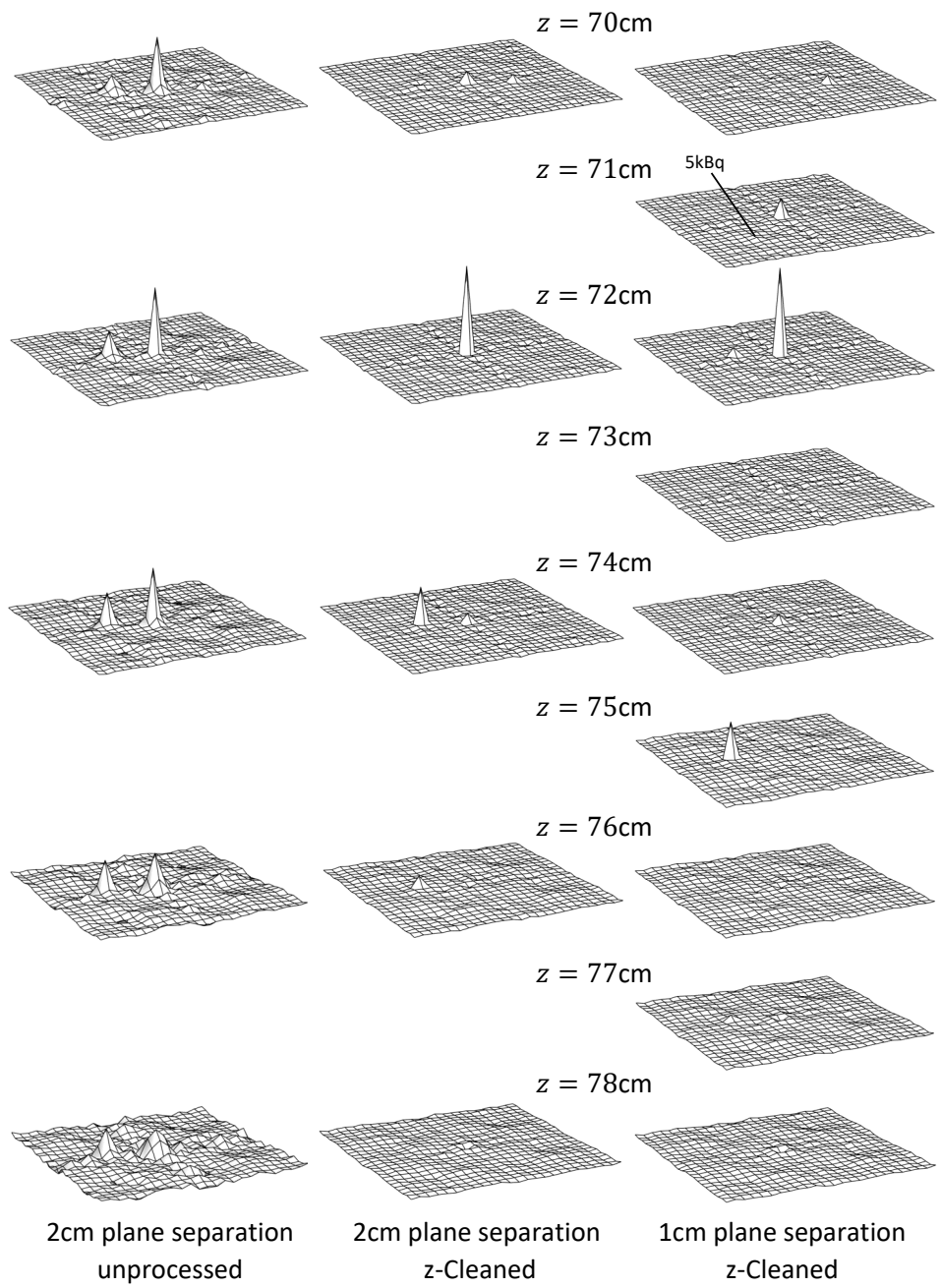


Figure 6: Typical images for simulations using z-Clean for 2cm and 1cm plane separations for the continuous detector.

390 a *phasing error* [21] and has been discussed in the literature for sources lying
391 close to object pixel boundaries in the (x, y) directions [22]. In this case we
392 have similar phasing error occurring but this time over more than one voxel
393 in the z direction. We hereafter refer to this phenomenon, when it occurs, as
394 *phasing*. The 10kBq source lying at 69.5cm is reconstructed and successfully
395 z-Cleaned and appears in its closest plane at 70cm although there was no
396 apparent phasing of this source in the 68cm plane. The 5kBq source lying
397 at a depth of 71cm (indicated in Fig. 6 at 1cm plane separation) is barely
398 visible at 2cm plane separation at depths of 70cm and 72cm.

399 For 1cm plane separation, the central 100kBq source has been successfully
400 z-Cleaned although there is ghosting in the adjacent plane at 71cm and also
401 two planes away at 74cm, the 50kBq source is reconstructed entirely at its
402 closest plane of 75cm with no significant phasing of the source at 74cm, but
403 with ghosting at 77cm, and the 10kBq source at 69.5cm depth is visible in
404 the plane at 70cm but there was no ghosting or phasing of this source in any
405 of the other planes. The 5kBq source that lies at 71cm depth is indicated
406 and is only just visible in the plane at this depth. However, close inspection
407 of the images reveals artifacts of this source of approximately the same size
408 also appearing in the planes at depths of 70cm and 72cm, indicating that no
409 z-Cleaning of this source has taken place.

410 The results in Fig. 6 represent a single typical observation for plane separa-
411 tions of 2cm and 1cm. However for each observation twenty trials were car-
412 ried out and more detailed results from these for the three strongest sources
413 are shown in Fig. 7 and Tables 1 and 2. Data for the 5kBq source is not in-
414 cluded in the tables because there was no successful z-Cleaning of this source
415 and hence no method of determining the parameters for the tables. Fig. 7
416 shows some SNR depth profiles for the three strongest reconstructed sources
417 at plane separations of 2cm and 1cm, as well as for the finer depth resolutions
418 of 0.5cm and 0.1cm. Here, and for all later observations to follow, each graph
419 shows profiles judiciously chosen to demonstrate a typical range of outcomes
420 for each case. In some cases either two or three of each of the twenty trials
421 are shown while for those cases where all trials, or a large majority of trials
422 (eighteen or nineteen) follow a particular profile, the mean profile is plotted
423 and marked with disks at the graph vertices. The number of such trials is
424 labelled appropriately on the relevant graphs. In these cases the standard
425 error in the mean was also calculated, although it is worth stating at this
426 point that for all such profiles shown, the resulting error bars are smaller
427 than the disks on the graphs.

Table 1: SNR, reconstructed depth (z) and PSLA_z results obtained for twenty trials of the continuous detector for the three strongest sources. For those values without errors quoted, the error is less than 0.001.

	100kBq at 72cm	50kBq at 74.7cm	10kBq at 69.5cm
Critical plane SNR	111.7 ± 0.3	78.8 ± 0.4	22.0 ± 0.2
<hr/>			
2cm plane separation			
SNR	131.7 ± 0.5	56.0 ± 0.5	11.3 ± 0.1
z (cm)	71.93 ± 0.01	74.52 ± 0.02	69.87 ± 0.02
PSLA_z (cm)	0.021	0.051	0.250 ± 0.003
<hr/>			
1cm plane separation			
SNR	125.5 ± 0.5	50.3 ± 0.5	11.6 ± 0.2
z (cm)	71.99 ± 0.02	74.81 ± 0.07	69.45 ± 0.10
PSLA_z (cm)	0.011	0.028	0.122 ± 0.002
<hr/>			
0.5cm plane separation			
SNR	125.9 ± 1.0	50.0 ± 0.6	11.6 ± 0.2
z (cm)	72.02 ± 0.01	74.46 ± 0.04	69.47 ± 0.06
PSLA_z (cm)	0.006	0.014	0.061 ± 0.001
<hr/>			
0.1cm plane separation			
SNR	112.7 ± 1.3	49.0 ± 0.5	11.3 ± 0.2
z (cm)	71.95 ± 0.02	74.60 ± 0.04	69.47 ± 0.05
PSLA_z (cm)	0.001	0.003	0.012

428 Table 1 presents the reconstructed SNR values for the three strongest
429 sources, as well as the mean SNR for twenty trials of the same source oper-
430 ating in the critical plane. The table also shows depth calculations and the
431 point source location accuracies (PSLA) in the depth direction, PSLA_z . All
432 quantities are shown with standard errors in the means, unless the errors are
433 very small in which case no errors are reported. For sources that are shared
434 between planes with a phasing error, the SNR for each trial is calculated in
435 quadrature from the two planes containing the source, with statistics calcu-
436 lated from the twenty different SNR values from all trials. Reconstructed
437 source depths are calculated separately for each trial by Gaussian fitting to
438 these individual profiles, and then mean depths with statistics are obtained
439 from the twenty trials.

440 PSLA_z has also been estimated from the results and is also presented in
441 Table 1. The PS�A of a coded aperture imaging system is dependent upon
442 the source SNR. For a perfect coded aperture system operating only in its

Table 2: Continuous detector: furthest distances of main reconstructed peaks from the actual source depth of twenty trials for each observation of the three strongest sources. All values are in cm. Asterisks denote actual source not lying at a plane to be decoded.

Plane separation	Source		
	100kBq at 72cm	50kBq at 74.7cm	10kBq at 69.5cm
2	0	0.7*	0.5*
1	0	0.7*	0.5*
0.5	0	0.7*	0.5
0.1	0.2	0.4	0.5

443 critical plane, the 2D PSLA in the (x, y) plane can be calculated using the
 444 results of Skinner [23] and is given by $\text{PSLA} = s \times d\alpha$ where s is the aperture
 445 to detector separation and $d\alpha$ is the PSLA in radians. Using [23] Eq. (31)
 446 and assuming aperture elements and detector pixels of size m we have

$$\text{PSLA} = \frac{s}{\text{SNR}} k [2(m/s)^2]^{1/2} \quad (4)$$

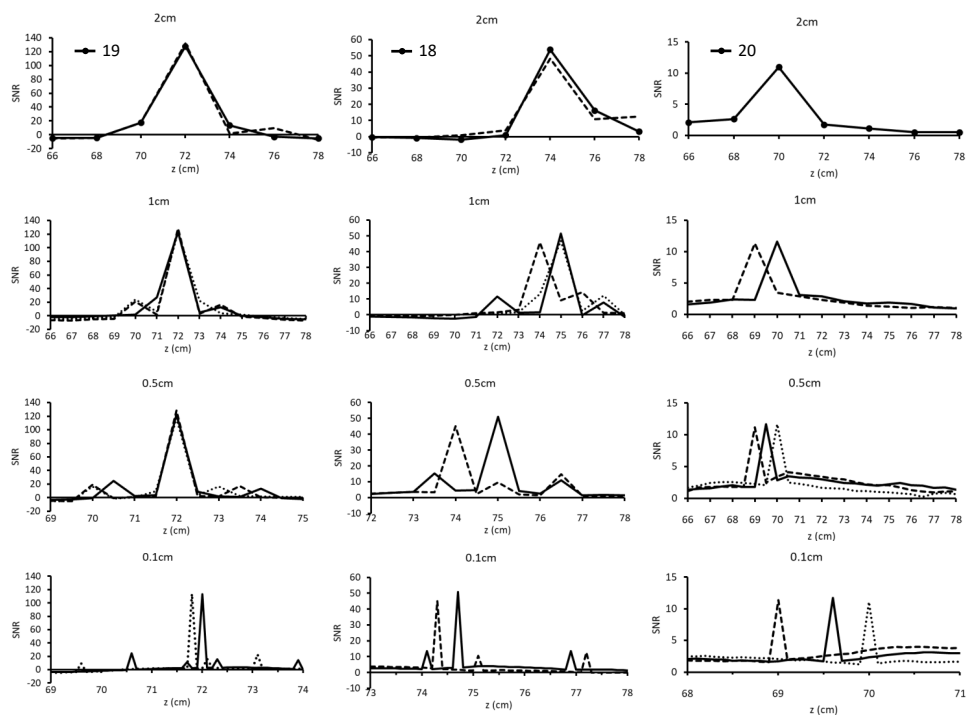
447 where $k \simeq 1$ is Skinner's constant. Now, in the z direction, calculating the
 448 PSLA, which we denote PSLA_z , is directly analogous to Skinner's analysis.
 449 Instead of a pixel of size m in the (x, y) plane we have a voxel of size d in the
 450 z direction. Substituting this with $k = 1$ into Eq. (4) and simplifying gives

$$\text{PSLA}_z = \frac{1.4d}{\text{SNR}}. \quad (5)$$

451 The results for the continuous detector are now discussed.

452 5.1. Continuous Detector - Depth Profiles

453 For the 100kBq source at 2cm plane separation (top graph in Fig. 7(a))
 454 two SNR depth profiles are shown. The dark line represents the mean profile
 455 of nineteen trials (denoted by the key in the corner of the graph) that all
 456 follow this same general profile. As stated above, the error bars to represent
 457 standard errors in the means are smaller than the disks at the profile vertices.
 458 This mean profile shows a clear peak at the actual source depth of 72cm, but
 459 with ghosts appearing in the adjacent planes at depths of 70cm and 74cm.
 460 The remaining trial gives the dashed profile on the graph, which shows the
 461 reconstructed source peak correctly positioned at 72cm and with a ghost
 462 in the adjacent plane at 70cm, but also a ghost situated on the opposite

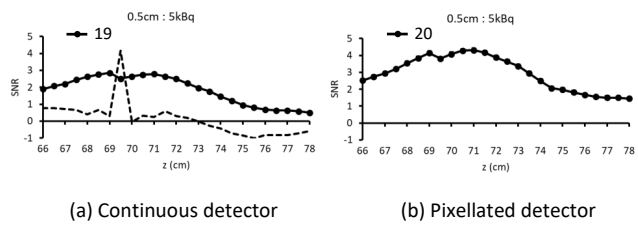


(a) 100kBq at 72cm

(b) 50kBq at 74.7cm

(c) 10kBq at 69.5cm

Figure 7: Selected examples of SNR depth profiles of the three strongest sources using a continuous detector. The graph titles show the plane separations.



23
 Figure 8: Profiles for the 5kBq source at 71cm depth using 0.5cm plane separation for both detector types.

463 side of the peak at 76cm depth, which is two planes away from the main
 464 peak. Therefore for all twenty trials the main reconstructed peak appears at
 465 the correct depth of the source, namely 72cm, although there is a very small
 466 amount of variation in the positions of ghosts. For the 100kBq source at both
 467 1cm and 0.5cm plane separations, Fig. 7(a) shows three profiles in each case,
 468 one for each of three selected trials, and all producing reconstructed source
 469 peaks at the correct depth of 72cm. However, ghosts of differing sizes appear
 470 at varying depths, being separated from the main peak by a combination of
 471 one and/or two planes for 1cm separation and up to four planes for 0.5cm
 472 separation, to give a range of differing profiles. The appearance of ghosts
 473 at different depths for different trials indicates that their occurrence is not
 474 purely systematic but contains a random component. For both of these
 475 plane separations, the results represent a blurring of up to around 2cm from
 476 the main reconstructed peak. As is the case for 2cm plane separation, all
 477 trials reconstruct the main peak in the correct position at 72cm depth. For
 478 0.1cm plane separation, Fig. 6 shows profiles for two trials, where the main
 479 reconstructed peaks appear at different depths for the different trials, one
 480 in the correct position at 72cm and the other at 71.8cm. It is because not
 481 all of the reconstructed main peaks appear in the same plane that no finer
 482 plane separation than 0.1cm was studied for the continuous detector. Both of
 483 the profiles for 0.1cm plane separation present non-systematic ghosts either
 484 side of and up to approximately 2cm away from the main peaks. Again,
 485 the variation in the positions of the ghosts for different trials indicates that
 486 their appearance in the z -Clean process is not systematic. Table 2 shows the
 487 furthest distance in the z direction for the twenty trials of each observation
 488 of any of the main reconstructed peaks from the actual source depth. The
 489 results for all three plane separations of 2cm, 1cm and 0.5cm have all twenty
 490 trials reconstructing the main peak of the 100kBq source at 72cm, and hence
 491 a furthest distance of 0cm from the actual source depth, as per Table 2. For
 492 0.1cm plane separation the furthest distance of a main peak from the actual
 493 source position is 0.2cm.

494 Profiles for the 50kBq source located at 74.7cm depth are shown in Fig.
 495 7(b). For 2cm plane separation, eighteen trials give a profile similar to the
 496 dark line (top graph). The phasing is evident in the sharing of the recon-
 497 structed source flux over two planes, namely 74cm and 76cm, as to be ex-
 498 pected from the source depth of 74.7cm. The remaining two trials give profiles
 499 similar to the dashed line. Unlike the case for the 100kBq source, for the
 500 finer plane separations of 1cm, 0.5cm and 0.1cm the reconstructed peaks for

501 different trials do not all appear at the same depth, as shown in the profiles,
502 three of which are shown for 1cm plane separation and two each for 0.5cm
503 and 0.1cm plane separations. However, for all trials of this source at 0.5cm
504 and 0.1cm plane separations, the main reconstructed peaks appear no further
505 than 0.7cm and 0.4cm from the actual source depth respectively (Table 2).
506 Once again, the appearance of ghosts either side of each main peak is not
507 systematic and seems to represent an overall blurring of around 2cm to 3cm
508 either side of the main peaks.

509 For the twenty trials of the 10kBq source for 2cm plane separation, every
510 trial follows the same profile as the top graph in Fig. 7(c). The main peak
511 is reconstructed at 70cm, which is 0.5cm from the actual source depth. The
512 voxels either side of the main peak are below the 3σ threshold so there is no
513 obvious phasing of this source from the main reconstructed peak. For 1cm
514 plane separation for this source, the twenty trials follow either one of the
515 two profiles shown in the second graph in Fig. 7(c), with peaks appearing
516 at either 69cm or 70cm depth, thus being no further than 0.5cm from the
517 actual source depth. For the finer plane separations of 0.5cm and 0.1cm,
518 the reconstructed peaks also do not all appear at the same depth, although
519 for all trials of the 10kBq source, these peaks appear no further than 0.5cm
520 from the actual source depth for all plane separations. Furthermore, unlike
521 the two stronger sources, no ghosts appear in the profiles. This is due to the
522 fact that no candidates at depths other than the main peak are obtained,
523 probably as a result of the weak nature of the 10kBq source which is less likely
524 to contribute candidates in more than one iteration of the z-Clean process
525 while in the presence of other stronger sources and a high background level.
526 The z-Cleaning of the three strongest sources to remove large artifacts and
527 leave only a small level of minor ghosting demonstrates the efficacy of the
528 z-Clean technique for these three sources.

529 As mentioned earlier, the 5kBq source is barely visible and does not
530 appear to have been z-Cleaned. In Fig. 8(a) the dark line shows the mean
531 profile of nineteen trials for the 5kBq source at 0.5cm plane separation using
532 the continuous detector, indicating that for all of these trials the SNR barely
533 reaches 3 at around the actual source depth. There is no reconstructed peak,
534 and hence no z-Cleaning of the images for these trials. Note that this result
535 is also consistent with the 1cm plane separation images of Fig. 6 for which z-
536 Clean also did not remove the artifacts. However, one trial gives the dashed
537 profile, with a clearly reconstructed peak at 69.5cm depth, namely 1.5cm
538 from the actual source depth of 71cm, and lower sidelobes, indicating that z-

539 Clean has taken place but giving a less accurate reconstructed peak location
540 than for the three stronger sources. For the other plane separations we do
541 not present results since the profiles obtained for all trials of the 5kBq source
542 were similar to those of the dark line in Fig. 6, indicating no z-Cleaning for
543 these cases. This would suggest that for the parameters studied, successful
544 z-Cleaning of a 5kBq source for a given observation would only occur very
545 rarely.

546 *5.2. Continuous Detector - Reconstructed Source Depths*

547 While Table 2 gives the furthest reconstruction of a peak from the ac-
548 tual source depth, Gaussian fitting of the data allows more accurate source
549 depth information to be determined. For each trial, the data was fitted to
550 a Gaussian distribution for each of the three strongest sources to determine
551 source depth for each trial and the statistical data was then calculated from
552 the twenty trials. The results are given in Table 1.

553 The depth location is very good in all cases for the three strongest sources
554 for all plane separations, with the values all being compatible with the ac-
555 tual depths of the three sources in all cases. In general the sources that
556 suffer a phasing error are located with lower precision, namely the 50kBq
557 source using 2cm, 1cm and 0.5cm plane separations for which reconstructed
558 source depths were 0.18cm, 0.11cm and 0.24cm from the actual source depth
559 respectively, and the 10kBq source at 2cm and 1cm plane separations for
560 which reconstructed source depths are respectively 0.37cm and 0.05cm from
561 the actual source depths. For all other source observations at other plane
562 separations, where there is no phasing error, the reconstructed source depths
563 are generally better, ranging from around 0.1cm from the actual source depth
564 for many cases to only 0.01cm from the actual source depth in the case of the
565 100kBq source at 1cm plane separation. For all cases of the three strongest
566 sources the depth location is very good although, outside of a phasing error
567 discrepancy, there is no clear systematic pattern in the ability to reconstruct
568 a source at the correct depth.

569 *5.3. Continuous Detector - SNR and $PSLA_z$*

570 The SNR data for the reconstructed 100kBq, 50kBq and 10kBq sources
571 using all plane separations is given in Table 1, along with values for the
572 critical plane observations. For the 100kBq source the reconstructed SNR
573 for all plane separations unexpectedly exceeds the value for the critical plane
574 observations. The reason for this is explained as follows. For observations

575 at the critical plane, a reconstructed source peak comes with a number of
 576 smaller ghosts either side and adjacent to the central peak, typically four to
 577 eight in a cross or a square formation, due to a blurring that is typical of the
 578 coded aperture imaging technique. These artifacts increase the variability of
 579 the sidelobes around the reconstructed source peak and lead to a reduction
 580 in the reconstructed source SNR. Now, in the iterative z-Clean process, the
 581 first candidate for the strong 100kBq source is correctly chosen in the central
 582 (x, y) voxel and at a depth of 72cm for which counts are then subtracted and
 583 stored appropriately for later use. However, the second candidate chosen for
 584 this source is typically not located in a voxel at the same depth and adjacent
 585 to the main peak, but instead is located in the same (x, y) position but at a
 586 different depth, an example being at 70cm depth for the 2cm plane separation
 587 images and appearing as a ghost in Fig. 6. When counts are removed for this
 588 incorrectly-located ghost candidate, it also singularly removes counts that
 589 would otherwise have been allocated to a number of sidelobe candidates
 590 that would have appeared at 72cm depth and adjacent to the main peak,
 591 had the second candidate not already have been assigned these counts, thus
 592 suppressing the sidelobes and reducing the overall variability of the image,
 593 and hence artificially increasing the SNR of the main reconstructed peak.
 594 Note for the particular case of Fig. 6, the effect is compounded by a further
 595 ghost located in the same (x, y) position but at 74cm depth. For 2cm plane
 596 separation the effect here is to reduce the noise by approximately 18% and
 597 increase the SNR from 117 to 131. Note that for weaker sources, this effect
 598 is less marked as here the detector background has the more dominant effect
 599 than the location of incorrect candidates on the overall variability. The SNR
 600 of the 100kBq source reduces slightly as the plane separation becomes finer,
 601 from 131.7 ± 0.5 at 2cm plane separation to 112.7 ± 1.3 at 0.1cm plane
 602 separation. As a result of the unexpected increase in the SNR for this strong
 603 source, the PSLA_z values for this source need to be taken with some level of
 604 caution, and in reality the quantities are probably more accurately reflected
 605 by using the critical plane SNR value of 111.7 in Eq. (5). Using this value
 606 gives PSLA_z values of 0.025cm, 0.013cm, 0.006cm and 0.001cm for plane
 607 separations of 2cm, 1cm, 0.5cm and 0.1cm respectively.

608 For the 50kBq source, there is a clear phasing for 2cm plane separation
 609 over the two planes at depths of 74cm and 76cm. Therefore the SNR values
 610 for this source are calculated in quadrature over these two planes for each of
 611 the twenty trials individually and the results are combined to give the mean
 612 and standard error in the mean in Table 1. The results indicate a reduction

613 in SNR compared to that of the critical plane observation that increases with
614 decreasing plane separation, ranging from 29% for 2cm plane separation, up
615 to 38% for 0.1cm plane separation.

616 For the 10kBq source, the SNR similarly suffers a reduction compared
617 to the critical plane, which is more severe compared to that for the 50kBq
618 source, being around 47% to 49%. However, the reconstructed SNR is much
619 more consistent for the 10kBq source across different planes, with very little
620 variation.

621 As already noted, $PSLA_z$ depends on SNR and plane separation, gen-
622 erally improving with increased SNR and with finer plane separation. For
623 the 100kBq source $PSLA_z$ ranges from 0.021cm for 2cm plane separation
624 to 0.001cm for 0.1cm plane separation, for the 50kBq source from 0.051cm
625 at 2cm plane separation to 0.003cm at 0.1cm plane separation, and for the
626 10kBq from 0.250cm at 2cm plane separation to 0.012cm at 0.1cm plane
627 separation.

628 **6. Pixellated Detector Results**

629 As noted in Section 4, the use of the chosen pixellated detector means
630 that the data collected comes as a 175×175 matrix of counts. However,
631 the z-Clean technique requires data to be in list mode, namely having actual
632 positions of individual detected photons, so that the counts can be binned
633 and z-Cleaned according to which plane is being studied at any particular
634 time. Therefore to replicate a list mode output, each detected count within a
635 detector pixel is randomly allocated an (x, y) coordinate within that pixel on
636 the detector so that rather than having a matrix of counts, we instead have
637 a list of (x, y) detector coordinates as though the data were in list mode. We
638 then proceed with the z-Clean technique as explained in Section 3. In the
639 following we present results of simulations described in Section 4.

640 The z-Cleaned images of a typical observation for each of the plane sepa-
641 rations of 2cm and 1cm are shown in Fig. 9. As for the continuous detector
642 images, only planes from depths 70cm to 78cm are shown as there were no
643 noticeable ghosts in the planes outside this range. The images in Fig. 9
644 are similar to those for the continuous detector in that the three strongest
645 sources are all clearly visible and have largely been successfully z-Cleaned,
646 with only minor ghosting in some nearby planes.

647 At 2cm plane separation, the 100kBq source is reconstructed at the actual
648 source depth of 72cm with a ghost at 70cm although, unlike the case for the

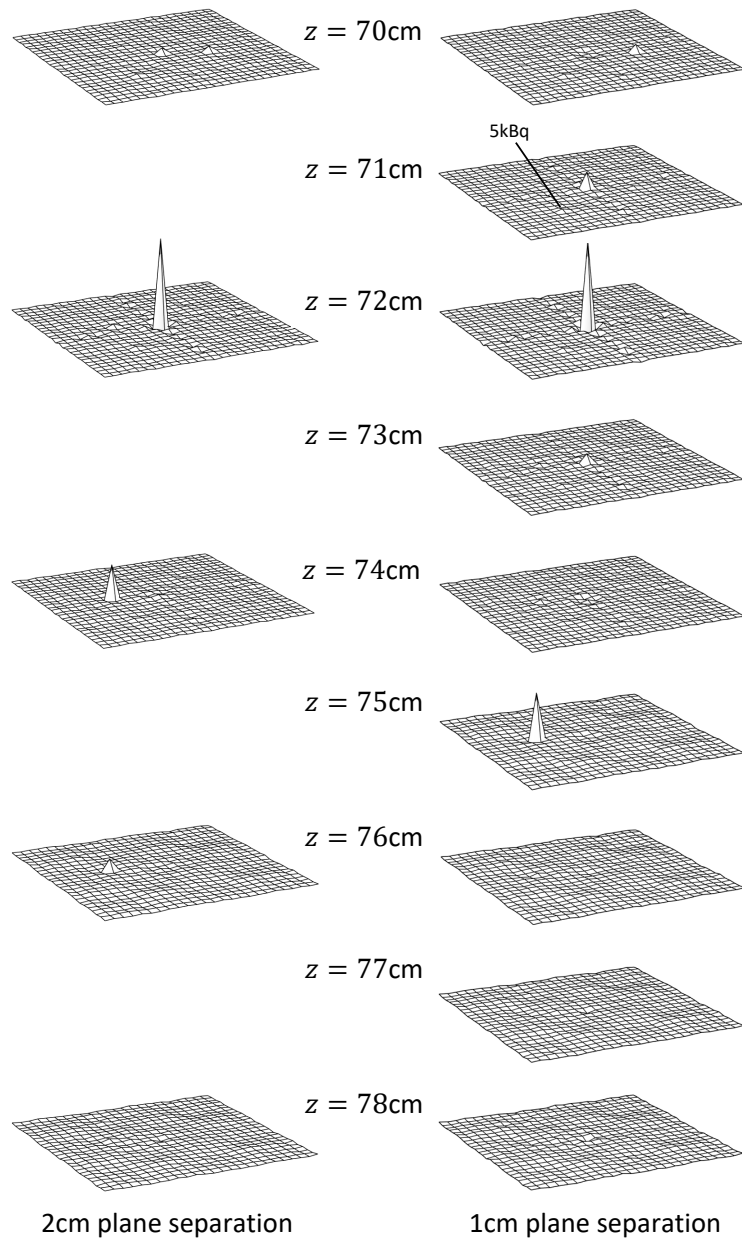


Figure 9: Typical images for simulations using z-Clean for 2cm and 1cm plane separations for the pixellated detector.

649 continuous detector, there is no significant ghosting at 74cm. The 50kBq
650 source at 74.7cm depth is again proportionately shared between two planes
651 as a phasing error at depths of 74cm and 76cm, with no ghosting in any
652 other plane, and the 10kBq source at 69.5cm depth is visible at 70cm with
653 no phasing or ghosting evident in any other plane. The 5kBq source at 71cm
654 depth is just discernable in the planes at 70cm, 72cm and possibly even at
655 74cm, with no apparent z-Cleaning of this source having taken place.

656 At 1cm plane separation the 100kBq source is clearly reconstructed at
657 its correct depth of 72cm depth with minor ghosting at 71cm and 73cm,
658 the 50kBq source situated at 74.7cm depth is reconstructed entirely at the
659 closest depth of 75cm with no phasing in the 74cm plane and no ghosting,
660 and the 10kBq source situated at 69.5cm is reconstructed entirely at 70cm
661 with no phasing of this source having occurred at 69cm. The 5kBq source is
662 just visible at its correct depth in the plane at 71cm but also with the same
663 approximate reconstructed height at 70cm, 72cm, 73cm and even possibly
664 74cm. Such repeated occurrences indicates that no z-Cleaning of this source
665 has taken place.

666 More detailed results for the three strongest sources are presented as
667 profiles in Fig. 10 and in Tables 3 and 4, with SNR and depth calculations
668 presented in Table 3.

669 *6.1. Pixellated Detector - Depth Profiles*

670 For the 100kBq source at plane separations of 2cm, 1cm and 0.5cm, all
671 twenty trials for each give similar profiles to the mean profiles shown in the
672 top three graphs in Fig. 10(a), with very little variation in each case, and
673 the main peak being reconstructed at exactly the source depth of 72cm. At
674 2cm plane separation there is a single significant, but minor, ghost at 70cm,
675 reflecting the example of the images in Fig. 9, and at 1cm plane separation,
676 minor ghosting takes place in the voxels either side of the reconstructed main
677 peak. The ghosting is relatively more marked for 1cm plane separation than
678 for 2cm plane separation. For 0.5cm plane separation, the ghosts appear two
679 voxels either side of the main peak, with high consistency. At 0.1cm plane
680 separation all twenty trials reconstruct the main peak at the exact source
681 depth of 72cm but not all trials follow the same profile. The bottom graph
682 in Fig. 10(a) shows two profiles that represent the images having the most
683 extreme positions of the ghosts which lie either side of and approximately
684 1cm from the main peak. The 0.1cm plane separation therefore represents

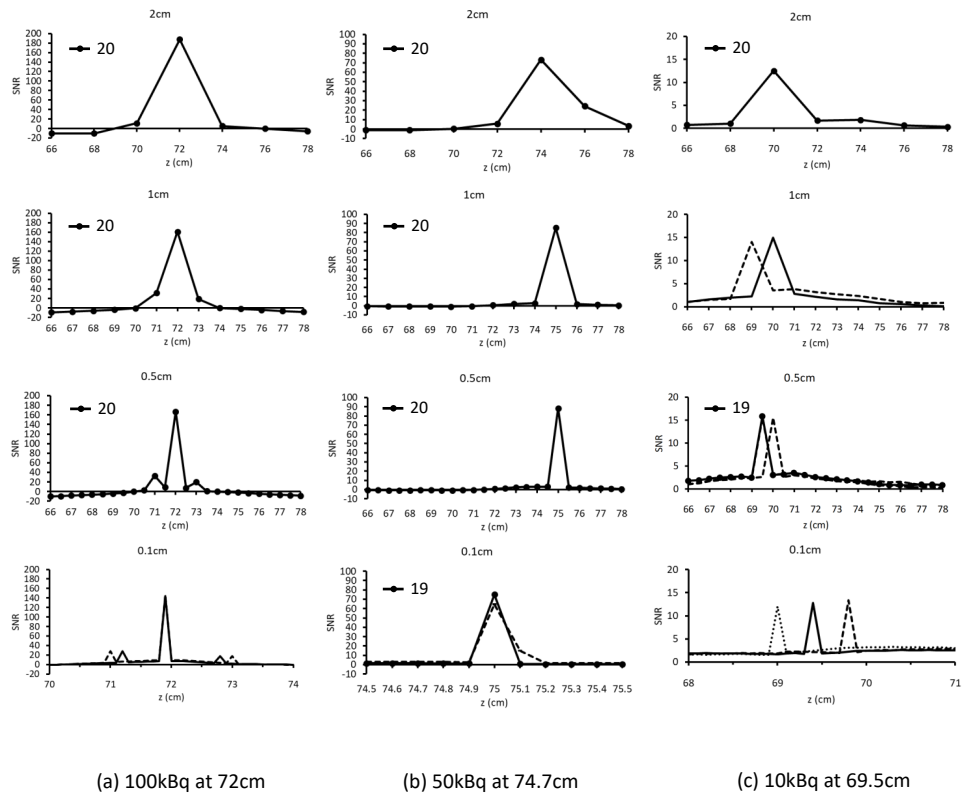


Figure 10: SNR depth profiles of the three sources using the pixellated detector.

Table 3: SNR, reconstructed depth (z) and $PSLA_z$ results obtained for twenty trials of the pixellated detector for the three strongest sources. For those values without errors quoted, the error is less than 0.001.

	100kBq at 72cm	50kBq at 74.7cm	10kBq at 69.5cm
Critical plane SNR	247.9 ± 1.2	158.1 ± 0.8	42.6 ± 0.4
<hr/>			
2cm plane separation			
SNR	187.9 ± 0.6	77.1 ± 0.3	12.7 ± 0.1
z (cm)	71.879 ± 0.002	74.394 ± 0.003	69.978 ± 0.016
$PSLA_z$ (cm)	0.015	0.037	0.224 ± 0.001
<hr/>			
1cm plane separation			
SNR	164.2 ± 0.5	85.3 ± 0.3	14.9 ± 0.2
z (cm)	71.931 ± 0.001	74.965 ± 0.003	69.663 ± 0.101
$PSLA_z$ (cm)	0.009	0.017	0.095 ± 0.001
<hr/>			
0.5cm plane separation			
SNR	167.0 ± 0.5	88.5 ± 0.3	15.5 ± 0.1
z (cm)	71.993	74.990 ± 0.001	69.547 ± 0.024
$PSLA_z$ (cm)	0.004	0.008	0.046
<hr/>			
0.1cm plane separation			
SNR	144.5 ± 0.4	74.7 ± 0.6	13.2 ± 0.1
z (cm)	71.900	74.997 ± 0.002	69.496 ± 0.064
$PSLA_z$ (cm)	0.001	0.002	0.011

685 the only one studied so far for the 100kBq source using a pixellated detector
686 where there is any variation in the positions of the ghosts.

687 For the 50kBq source at 74.7cm depth all twenty trials for plane separa-
688 tions of 2cm, 1cm and 0.5cm follow the same profiles as each of their
689 respective graphs in Fig. 10(b) with little variation, while at 0.1cm plane
690 separation only one of the trials (dashed line) deviates from the dark line
691 profile shown in the bottom graph of Fig. 10(b). At 2cm plane separation
692 there is clear phasing between 74 and 76cm with the higher peak being closest
693 to the actual source depth and the values shared in roughly the correct pro-
694 portions for this source. For the finer plane separations there is no phasing
695 evident on any of the graphs but in all cases the main peak is reconstructed
696 in the closest plane to the actual source depth, namely at 75cm.

697 For the 10kBq source at 69.5cm depth and at 2cm plane separation, all
698 twenty trials reconstruct the source in the closest plane at 70cm depth with no
699 evident phasing or ghosting in any other plane. The finer plane separations

Table 4: Pixellated detector: furthest distances of main reconstructed peaks from the actual source depth of twenty trials for each observation of the three strongest sources. All values are in cm. Asterisks denote actual source not lying at a plane to be decoded.

Plane separation	Source		
	100kBq at 72cm	50kBq at 74.7cm	10kBq at 69.5cm
2	0	0.7*	0.5*
1	0	0.3*	0.5*
0.5	0	0.3*	0.5
0.1	0	0.3	0.5
0.02	0.12		

700 give profiles that exhibit a single main peak with no phasing or ghosting
701 evident. For 1cm plane separation all trials are similar to one of the two
702 profiles shown, where the peak is reconstructed at either 69cm or 70cm,
703 namely in one of the two planes closest to the actual source depth. For
704 0.5cm plane separation nineteen trials reconstruct the peak at the actual
705 source depth of 69.5cm with the remaining trial being only one plane and
706 0.5cm away from the actual source depth. For 0.1cm plane separation peaks
707 are reconstructed at various depths, three examples being given which include
708 the two extremes at depths of 69cm and 69.8cm and a typical intervening
709 example. Again the efficacy of the z-Clean technique is demonstrated for the
710 three strongest sources.

711 For the 5kBq source all twenty trials for 0.5cm plane separation follow
712 the same general profile shown in Fig. 8(b), with the profile itself being the
713 mean for all the trials. The highest part of the graph reaches an SNR value
714 of approximately 4 at around the actual source depth of 71cm, but there
715 is no clear peak which indicates that no z-Cleaning has occurred and that
716 significant artifacts in the form of repetitions of the reconstructed source are
717 present in many of the nearby planes. Profiles for the other plane separations
718 followed similar profiles to that in Fig. 8(b), indicating no z-Cleaning of this
719 source, so these are not individually reported here.

720 6.2. Pixellated Detector - Reconstructed Source Depths

721 The depth location using the pixellated detector is very good for the
722 three strongest sources for all plane separations, with all reconstructed depths
723 being compatible with the actual source depths for all cases. As is the case for
724 the continuous detector, the calculated depths of the reconstructed sources

725 that suffer a phasing error are generally slightly less accurately determined
726 than those for which no phasing occurs. In particular the 50kBq source
727 at 74.7cm is reconstructed between 0.265cm and 0.306cm from the actual
728 depth although this result is still very good. Even in the worst case, namely
729 the 10kBq source at 2cm plane separation, the reconstructed depth is only
730 0.478cm from the actual source depth. At plane separations of 1cm and finer
731 the 10kBq source is able to be reconstructed to within 0.2cm of its actual
732 depth, and the 100kBq source to within better than 0.1cm of its actual depth.

733 *6.3. Pixellated Detector - SNR and $PSLA_z$*

734 Table 3 shows that the SNR for the 100kBq reconstructed source is re-
735 duced by the implementation of z-Clean compared to critical plane obser-
736 vations for all plane separations, being generally more severe as the plane
737 separation decreases, from 24% for 2cm plane separation to 42% for 0.1cm
738 plane separation. Whereas the continuous detector experiences an unex-
739 pected SNR increase caused by the generation of incorrect candidates at
740 different depths which remove detector counts that would otherwise have
741 been assigned to potential ghosts adjacent to the main peak at 72cm depth,
742 the use of a pixellated detector, which bins the counts in precisely defined
743 pixels with no Gaussian spread, and hence no FWHM of detection, means
744 that there would be no ghosting in the adjacent planes at 72cm and hence
745 the detection of incorrect candidates at other depths and at the same lateral
746 position has no effect on the overall variability, and hence no SNR increase
747 occurs. Instead the use of z-Clean here comes at the expense of a reduction
748 in the SNR compared to those of the critical plane observations.

749 For the 50kBq reconstructed source the SNR reduction is more severe,
750 being between 44% for 0.5cm plane separation to 51% for 2cm plane separa-
751 tion. The SNR for 2cm plane separation is higher than that for 0.1cm plane
752 separation. However the plane separations of 1cm and 0.5cm suffer less SNR
753 degradation than 2cm plane separation, due probably to the source being
754 located very close to the actual planes to be decoded and hence suffering
755 a less severe phasing error. The SNR for the 10kBq reconstructed source,
756 while remaining sufficiently high to render the source clearly visible, is quite
757 severely impacted by the implementation of z-Clean, falling by 70% for 2cm
758 plane separation and by the smaller amount of 64% for the best case for
759 this source of 0.5cm plane separation, again probably due to the source ly-
760 ing at an actual plane to be studied for this particular plane separation. At

761 0.1cm plane separation the SNR decrease is more severe with a 69% decrease
762 compared to the critical plane observations.

763 PSLA_z for all sources improves with finer plane separations from 2cm to
764 0.1cm, ranging from 0.015cm to 0.001cm respectively for the 100kBq source,
765 from 0.037cm to 0.002cm for the 50kBq source and from 0.224cm to 0.011cm
766 for the 10kBq source.

767 *6.4. Pixellated Detector - 100kBq source only*

768 As stated in Section 6, when using the pixellated detector all twenty trials
769 for each plane separation successfully reconstructs the main source peak at
770 the correct depth of 72cm, even down to 0.1cm plane separation (Fig. 10(a)).
771 In view of this point, a further set of twenty trials was conducted of an
772 observation of only a single 100kBq source at 72cm in the absence of the
773 other three sources to ascertain whether z-Clean can reconstruct the main
774 peak of this source at the same depth of 72cm but at the even finer depth
775 resolution of 0.02cm. The reason that only this single source is observed is
776 due to the infeasibly long run times that the z-Clean process requires for such
777 fine resolution processing of fields containing multiple sources. Therefore we
778 observe just this single source with the same background level of one count
779 cm⁻² s⁻¹. Fig. 11 shows portions of the profiles of three of the twenty trials,
780 from depths of 71.4cm to 72.4cm. The main peaks of the example three trials
781 are separated and small ghosts are visible for some of the trials at around
782 71.5cm and 71.6cm depth although for these and all the remaining twenty
783 trials, similar sized ghosts to those in the figure were visible from around
784 70.5cm up to around 73.5cm beyond the range of the graph in the figure.
785 For the twenty trials at this plane separation, the source depth is calculated
786 at 72.088 ± 0.004 cm, which is comparable to the results for the other plane
787 separations as given in Table 3. Table 4 shows that the furthest plane that
788 a reconstructed source appears for the twenty trials is only 0.12cm from the
789 actual source depth.

790 **7. Point Spread Function**

791 The point spread function (PSF) of a coded aperture system can often be
792 calculated theoretically. However, theoretical calculation of the PSF in the
793 depth direction for images processed using the z-Clean technique, which we
794 denote as PSF_z, is extremely difficult. Therefore we attempt to determine a
795 best estimate for PSF_z by simulating an observation of an extremely strong

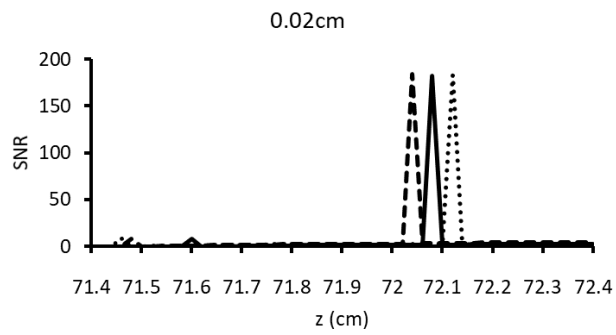


Figure 11: Portions of SNR depth profiles for a single 100kBq source using a pixellated detector.

796 single source in the absence of any background counts and performing z-
797 Clean on the data. This has been done for both the continuous detector and
798 pixellated detector, observing a 5MBq source in the centre of the FOV at
799 72cm depth (i.e. in the same position as the 100kBq source in the previous
800 sections) and using the same system parameters as in Section 4 but with no
801 background counts. This is done for each of the plane separations studied in
802 Section 4. In each case, only one trial is conducted because the extremely long
803 run times for the programs using such huge count numbers in conjunction
804 with z-Clean renders multiple trials infeasible.

805 The depth profiles for the two detector types are shown in Fig. 12. For
806 both detector types at all plane separations the main peak is reconstructed
807 at the correct source depth of 72cm. In all cases ghosts are present either
808 side of the main peak. The ghosting is roughly symmetrical when using
809 the continuous detector for plane separations of 2cm and 1cm, and for the
810 pixellated detector for 2cm plane separation. However for the remaining cases
811 the ghosting is not symmetrically distributed and in some cases consists of
812 three or more statistically significant ghosts. Furthermore it must be noted
813 that the profiles are for a single source lying in a specific voxel and in the
814 absence of the influence of other sources in the FOV. It is therefore to be
815 expected that for other source positions in the FOV, PSF_z would take on a
816 very large range of profiles with hugely varying numbers of and positions of
817 ghosts.

818 8. Conclusions

819 This article presents an image processing technique, which we call z-
820 Clean, that removes the repeated artifacts associated with image reconstruc-
821 tion in the 3D FOV when using a coded aperture imaging system. The tech-
822 nique includes determining the lateral positions and depths of point sources
823 and removing artifacts caused on some planes by sources from another plane.

824 For a continuous detector with a 1cm FWHM detection capability at
825 plane separations of 2cm, 1cm, 0.5cm and 0.1cm, the z-Clean technique is
826 able to resolve three (100kBq, 50kBq and 10kBq) or four (also 5kBq) point
827 sources very well, while at the same time significantly reducing to the level
828 of minor ghosting the large artifacts caused by sources in other planes. The
829 efficacy of z-Clean is thus demonstrated for the three stronger sources al-
830 though there is some ghosting of the 100kBq and 50kBq sources, consisting
831 of smaller peaks appearing in planes other than the one containing the source.

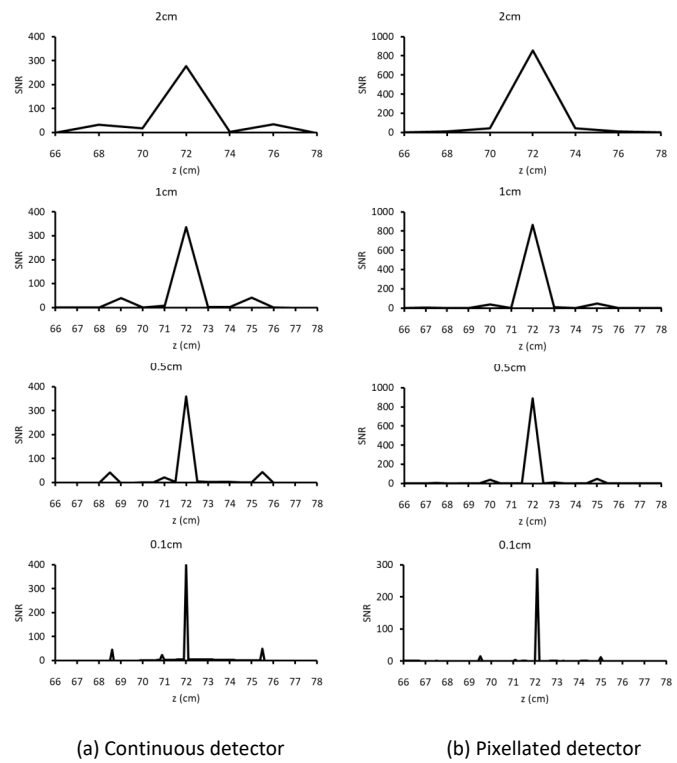


Figure 12: Estimated PSF_z profiles for a single 5MBq source with no background.

832 The distributions of the ghosts show some small variability for the 100kBq
833 and 50kBq sources at 2cm plane separation and larger variability for finer
834 plane separations. The variability in the positions of the ghosts indicates
835 their presence as being non-systematic and containing a random component.
836 Ghosts typically appear up to around 2cm from the actual source depth.
837 Reconstructed images of the 10kBq source show a clear source peak for all
838 plane separations but with no ghosting evident, due to there being no can-
839 didates for this source in the z-Clean process other than the first one for
840 which a main peak is reconstructed. For the 5kBq source, z-Cleaning would
841 appear to occur only very rarely and even then to offer less accurate depth
842 location than for the stronger sources. The depth location and $PSLA_z$ for all
843 plane separations are very good for the three strongest sources and individual
844 trials are able to reconstruct a source peak to 0.2cm or better from the ac-
845 tual source depth for the 100kBq source, with a worst performance of 0.7cm
846 from the actual source depth for the 50kBq source, which is still very good.
847 Sources that suffer a phasing error are generally less accurately located. The
848 excellent depth location and $PSLA_z$ come at the expense of an impact on
849 the SNR, which for the strongest source increases unexpectedly due to the
850 incorrect detection of candidates lying at the same lateral position as the
851 actual source but at different depths, which suppresses ghosts that would
852 otherwise appear adjacent to the actual source in the correct plane. For the
853 weaker sources, SNR is reduced, in some cases quite severely, so the excellent
854 depth resolution made possible by z-Clean is traded off with an impact on
855 SNR.

856 For a pixellated detector with 0.2cm pixels, the efficacy of z-Clean is also
857 demonstrated, with the large repeated artifacts being removed and leaving
858 the three strongest sources clearly visible with only minor ghosting in some
859 other planes. The reconstruction of the source peaks and the ghosting is
860 quite consistent for the 100kBq and 50kBq sources at 2cm, 1cm and 0.5cm
861 plane separations, although there is some small variability in the ghosting
862 at 0.1cm plane separation. The reconstructed 10kBq source experienced no
863 ghosting but the reconstructed peaks appear at different depths for different
864 trials, although no further than 0.5cm from the actual source depth. As
865 was the case for the continuous detector, the depth location for the three
866 strongest sources is very good, being better than 0.5cm in all cases and
867 better than 0.1cm for the 100kBq source at plane separations of 1cm, 0.5cm
868 and 0.1cm. $PSLA_z$ improves with SNR and finer plane separation, being as
869 good as 0.001cm for the 100kBq source at 0.1cm plane separation. Again the

870 very good depth location and $PSLA_z$ come at the expense of a significant
871 reduction in SNR, whose severity is higher for weaker sources.

872 As a technique and for certain situations, z-Clean is an efficacious method
873 of removing the artifacts that typically appear in the planes of a 3D coded
874 aperture imaging system when observing a source in a given plane and sug-
875 gests that the results in earlier work by other authors, including Kazachkov
876 et al. [8] and by Mu and Liu [13] might be improved upon. However this
877 paper represents just a starting point in the idea of removing artifacts in
878 3D coded aperture imaging, and further work in this area is possible which
879 is beyond the scope of this article. Perhaps most importantly, such work
880 could include observing different types of source distribution, in particular
881 extended sources. This could be useful in medical imaging, for instance,
882 where details of body organ structure are often required and hence good
883 quality images of such extended objects are needed. Related to this could
884 also be the study of the effect of different aperture throughput values, as well
885 as what benefit, if any, different aperture throughput has when used to pro-
886 cess images of extended sources using z-Clean [20]. Other future work could
887 include investigating the use of z-Clean in the 3D image domain, rather than
888 the 2D detector domain and a more detailed study into the use of detectors
889 with different pixel size to bin size ratios, which would extend the work to
890 include a wider range of detector parameters. Of particular interest to this
891 entire field is the possibility of conducting experimental laboratory tests to
892 observe real high energy sources using a physical position sensitive photon
893 detector and applying z-Clean to the data.

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