

A Service of

ZBU

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Byard, Kevin

Working Paper Application of the CLEAN Algorithm to Three Dimensional Coded Aperture Imaging

Economics Working Paper Series, No. 2020/14

Provided in Cooperation with: Faculty of Business, Economics and Law, Auckland University of Technology (AUT)

Suggested Citation: Byard, Kevin (2020) : Application of the CLEAN Algorithm to Three Dimensional Coded Aperture Imaging, Economics Working Paper Series, No. 2020/14, Auckland University of Technology (AUT), Faculty of Business, Economics and Law, Auckland

This Version is available at: https://hdl.handle.net/10419/242586

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



WWW.ECONSTOR.EU



Economics Working Paper Series *Faculty of Business, Economics and Law, AUT*

Application of the CLEAN Algorithm to Three Dimensional Coded Aperture Imaging

Kevin Byard

2020/14

Application of the CLEAN Algorithm to Three Dimensional Coded Aperture Imaging

Kevin Byard

School of Economics, Faculty of Business, Economics and Law, Auckland University of Technology, Auckland 1142, New Zealand

Abstract

An iterative technique for the removal of artifacts caused by the near field effects of a coded aperture imaging system is presented. The technique, which we call z-Clean, first locates high energy sources within a three dimensional field of view using a least squares method and then removes the artifacts using a method similar to that of the CLEAN algorithm used in radio astronomy, but instead operating in the detector shadowgram domain rather than the final image domain. Computer simulations were performed of observations of four point sources of different intensities and at different depths from the detector. Both a continuous detector of 1cm FWHM detection capability and a pixellated detector with 0.2cm square pixels were investigated using a Modified Uniformly Redundant Array coded aperture of element size 0.6cm. The efficacy of the z-Clean technique for artifact removal is demonstrated for both detector types for the three strongest sources of 100kBq, 50kBq and 10kBq using plane separations of 2cm, 1cm, 0.5cm and 0.1cm, to leave only small ghosts lying up to around 2cm from the reconstructed source depth. For twenty trials of each observation, the three strongest sources are reconstructed no further than 0.7 cm from the closest plane with many being from 0cm to 0.5cm for both detector types. The depth location for all three strongest sources using both detector types is no worse than 0.5cm from the actual source depth and is in most cases much better, being closer than 0.1cm for the strongest source at plane separations of 1cm, 0.5cm and 0.1cm. z-Clean was not able to remove the artifacts nor determine accurately the depth of the weakest source of 5kBq and in general sources that experience a phasing error are less accurately located although still better than 0.5cm from the actual source depth for all such cases. The artifact removal and very good depth location come at the expense of an impact on the signal to noise

Preprint submitted to Elsevier

September 30, 2020

ratio (SNR) of the sources. For the strongest source and using the continuous detector the SNR increases unexpectedly to give values higher than that for observations made only in the critical plane due to the ghosting of this source in other planes at different depths. For all other cases there is a decrease in SNR which is more marked for finer plane separations and for weaker sources.

Keywords: coded aperture, three dimensional imaging, tomography, gamma ray imaging, image processing

1 1. Introduction

Coded aperture imaging has become the major technique for forming im-2 ages in the high energy domain [1, 2, 3, 4]. In this technique, an aperture 3 consisting of opaque and transparent elements is placed between a photon emitting source and a position sensitive detector. The result is a shadow-5 gram on the detector, which in some applications can be saved on computer 6 as a dataset consisting of a set of Cartesian coordinates for each count on the 7 detector - a form known as *list mode* - and in others may instead be stored 8 as a matrix of counts in the form of pixels. We hereafter refer to a detector 9 operating in list mode as *continuous* and a detector consisting of pixels as 10 *pixellated.* The detector shadowgram needs to be subsequently decoded to 11 produce a reconstructed image of the source distribution. The coded aper-12 ture technique was originally proposed for high energy astronomical imaging 13 [4] including X-rays [5] and gamma-rays [6] where the incoming radiation 14 effectively comes from infinity and therefore rays from a point source that 15 reach the detector are parallel to each other. This means that the object 16 distribution is essentially a flat two-dimensional (2D) field with the decoded 17 image being a similarly flat reconstruction of the source distribution. 18

However other applications of coded apertures have also been proposed, 19 many using near field optics including medical imaging [7, 8] and land mine 20 detection [9]. In the near field the object is placed close to the detector, and 21 so the rays from a point in the object diverge giving rise to different problems 22 compared to the far field. The near field itself may be thought of as a stack 23 of 2D *planes*, each parallel to the aperture and detector but lying at different 24 distances from the detector [8]. We hereafter refer to the perpendicular 25 distance of a point or a plane from the detector as its *depth*. In tomography 26 an attempt is made to ascertain the source distribution in each plane from 27 the detector shadowgram and hence produce an overall three-dimensional 28

(3D) image. To this end a number of studies into tomographical imaging 29 have been conducted [8, 10, 11, 12, 13]. While it is possible to create images 30 in different planes by discretising the detector data accordingly and decoding 31 each plane, artifacts in the decoded image of one plane typically arise due to 32 the presence of sources from other planes. For example Kazachkov et al. [8] 33 were able to ascertain the different depths of three point sources by 'focusing' 34 on each plane although the non-focussed sources from the other planes are 35 still visible in the form of blurred artifacts. Similarly Mu and Liu were able 36 to determine in which of nine planes, each separated by a centimetre, lay two 37 extended objects in the shape of a 'V' and 'H', although again, large artifacts 38 remained in the other planes [13]. 39

The results in [8] and [13] indicate that further processing of their 3D images is possible. Therefore in this article we attempt to achieve this with an iterative source removal similar to the CLEAN algorithm used in radio astronomy [14].

44 2. Coded Aperture Imaging

For many practical coded aperture imaging applications, the system often 45 chosen is what we term a *perfect* system, namely one where the correlation 46 function of the aperture has perfectly flat sidelobes and the shadowgram is 47 congruent to a *unit pattern* (also sometimes called the basic pattern) of the 48 aperture [3]. Such systems include the uniformly redundant arrays (URA) [3] 49 and modified uniformly redundant arrays (MURA) [15]. Note that in far field 50 imaging, such as in astronomy, when using a perfect aperture with a detector 51 that is the same physical size as the aperture unit pattern, all point sources 52 lying in the fully coded field of view (where they project a shadowgram over 53 the entire detection plane) will generate a shadowgram that is congruent to 54 the aperture unit pattern and so such a system is always perfect. However, 55 forming images in the near field will only be perfect for URAs and MURAs 56 if all sources being imaged lie in the exact plane that enables these sources 57 each to cast a shadow of exactly one unit pattern or its cyclic repetition onto 58 the detector. Here we refer to such a plane of a perfect system as the *critical* 59 plane. It is evident that only one critical plane exists for a given perfect 60 coded aperture system (which for astronomical applications lies at infinity) 61 and that these coded aperture systems lose their perfect imaging capability 62 when used to observe a scene in which the sources lie at different depths 63 because the aperture portions cast by such sources cannot all simultaneously 64

⁶⁵ be congruent to the aperture unit pattern. Therefore forming images using
⁶⁶ a coded aperture in the near field presents different problems compared to a
⁶⁷ flat image from sources at large distances.

Consider a coded aperture imaging system based on a square geometry 68 with an aperture to detector separation of s. Also consider two point sources 69 α and β positioned respectively at depths z_{α} and z_{β} from the detector as 70 shown in Fig. 1. Source α lies at a smaller depth and casts a shadow of 71 a portion of the aperture onto the detector. This portion is shown as a 72 small square on the aperture and the shadow that it casts is shown on the 73 detector. Source β is situated at a larger depth than source α and it casts 74 a shadow of different portion of the aperture, shown as a larger square, 75 onto the detector. To avoid confusion, the shadow of β is not shown. The 76 closer source casts a smaller aperture portion while at the same time casting 77 larger shadows of the individual aperture elements than the further source. 78 While the relative sizes of the shadows of the apertures and their elements 79 from the different sources allow some depth information to be ascertained, 80 in the reconstruction of the final image, sources from one plane are often 81 seen as blurred artifacts in the other plane and vice-versa [8]. To decode the 82 shadowgram and create a reconstructed image of a plane at depth z, we can 83 adopt the following reasoning. Consider a coded aperture system based on 84 a rectangular geometry consisting of a (0,1) aperture A(i,j) of dimensions 85 $V \times W$ where A(i,j) = 1 for open (transparent) elements and A(i,j) = 086 for closed (opaque) elements and where $0 \le i \le V - 1$ and $0 \le j \le W - 1$. 87 A number of different aperture configurations exist but for a general system 88 using the balanced decoding algorithm [3], we have a decoding array G(i, j)89 of the same dimensions as A where:

$$G(i,j) = 2A(i,j) - 1$$
 (1)

although slight modifications to Eq. (1) exist for certain types of aperture
such as the Pseudo-Noise Product Array (PNP) [16] or the MURA arrays
[15].

Assume that we are using a continuous detector, namely one that gives the individual detected (x, y) positions of each photon in list mode. Also consider the reconstruction of the image of a 3D field of view (FOV) for a plane at depth z using this detector. Since reconstructing the image requires a crosscorrelation of the shadowgram with the decoding array [3], it is necessary to discretise the detector shadowgram into squares called *bins*, such that each



Figure 1: Coded aperture imaging of near field objects.

¹⁰⁰ bin size is the same as that of an aperture element when projected from ¹⁰¹ a point source in the plane at depth z onto the detector. Therefore the ¹⁰² shadowgram is divided into $v_z \times w_z$ bins and the detected counts within each ¹⁰³ bin are summed to give a detector array $P_z(i_z, j_z)$ where $0 \le i_z \le v_z - 1$ ¹⁰⁴ and $0 \le j_z \le w_z - 1$. The z subscripts are used here to remind us that the ¹⁰⁵ various parameters all depend on the plane depth, for example the number ¹⁰⁶ of bins v_z and w_z both increase with increasing z.

¹⁰⁷ The decoded image $I_z(x_z, y_z)$ for the plane at depth z is calculated as ¹⁰⁸ follows:

$$I_z(x_z, y_z) = \sum_{x_z=0}^{v_z-1} \sum_{y_z=0}^{w_z-1} P_z(i_z, j_z) G(i_z + x_z, j_z + y_z)$$
(2)

[3]. Each (x_z, y_z, z) in Eq. (2) can be thought of as a 3D reconstructed 109 object pixel, sometimes also called a *voxel*. Note that since the number of 110 reconstructed voxels at depth z is given by $(V - v_z + 1, W - w_z + 1)$ then 111 we have $0 \le x_z \le V - v_z$ and $0 \le y_z \le W - w_z$. Reconstruction of the 112 image at a given plane at depth z using Eq. (2) effectively gives an image 113 that is 'focused' on that plane. While Eq. (2) provides a method to focus 114 on the different planes of a 3D FOV, sources from one plane typically cause 115 artifacts on other planes in the image leading to overall image degradation. 116 The purpose of this article is to attempt to remove such near field artifacts 117 produced in 3D coded aperture imaging and also determine the actual depth 118 of the source. 119

Note that in far field imaging such as high energy astronomy, there is only one image plane, lying at infinity, and so there is no image degradation from sources in other planes.

¹²³ 3. Image Processing Technique: z-Clean

For the purpose of demonstrating the image processing technique we sim-124 ulate a prototype coded aperture system having a number of idealised pa-125 rameters. We assume a continuous square detector of width 30cm having 126 100% detection efficiency and perfect photon location detection. We employ 127 a non-repeating random pattern for the prototype system aperture and de-128 fine the square central region of $v \times v$ elements as the *core* of the aperture 129 with a full aperture of dimension $2v - 1 \times 2v - 1$ that is not a repeat of 130 the core but instead an array of fully randomised open and closed elements. 131 Note that for a perfect aperture, the core is the unit pattern of the aperture 132

with size v and the full aperture is a $2v - 1 \times 2v - 1$ cyclic repeat of the 133 core. The aperture used for the prototype is the square 13×13 element 134 random configuration with a core of v = 7 shown in Fig. 2(a) where the dark 135 and light areas represent the opaque and transparent aperture elements re-136 spectively. The aperture elements used in the simulations are of size 2cm, of 137 negligible thickness and with a closed element opacity of 100%. The aperture 138 to detector separation is 30cm. For these parameters the depth of the plane 139 that casts a shadow of the central 7×7 aperture elements onto the detector 140 (the equivalent of the critical plane of a perfect system) is 56.25cm from the 141 detector. For this discussion we number the planes in order of z, with plane 142 1 being the plane closest to the detector. 143

For the prototype observation we take the planes as being at 56.25cm (plane 1, also the critical plane), 61.25cm (plane 2) and 66.25cm (plane 3) from the detector and thus we have a total of three planes with a plane separation of 5cm. We assume a single point source situated in plane 2 (61.25cm from the detector) in the centre of the FOV, emitting photons with a detection rate of 0.01 cm⁻² s⁻¹. A background rate of 0.001 counts cm⁻² s⁻¹ is assumed and the observation time is 600s.

The initial shadowgram showing the distribution of counts on the contin-151 uous detector is shown in Fig. 2(b) where each point represents the detected 152 position of a photon count. Such location of the individual counts, albeit in 153 practice containing statistical errors, is crucial to the image processing tech-154 nique. Therefore it is necessary for the data to be in list mode, and note that 155 the shadowgram image in Fig. 2(b) is an unbinned representation of the data 156 in list mode, since individual detected photon positions can be discerned. 157 The central portion of the aperture creating the shadowgram is clearly evi-158 dent in the distribution of counts shown, with photons from the strong source 159 forming the central pattern of the aperture along with a weaker background. 160 To commence image reconstruction we take each plane and we superimpose 161 the projected bins from these onto the shadowgram. We hereafter refer to 162 the superimposition of bins onto the shadowgram as a map. The maps for 163 planes 1 and 2 are shown in the top images in Fig. 3. Note that the map for 164 plane 2 is a good fit with the shadowgram since this plane is where the source 165 is actually located, while the map for plane 1 represents a less good fit with 166 this nearer plane having larger bin sizes. To demonstrate this point more 167 clearly the black square shows in negative the counts corresponding to those 168 photons that passed through a particular open aperture element, namely two 169 elements up and two elements left from the central element. Note that the 170



Figure 2: Prototype coded aperture system with (a) random 13×13 element coded aperture with the v = 7 element core shown in the central square, (b) continuous detector shadowgram of a single point source in plane 2 at distance z = 61.25cm from the detector.

black square corresponds exactly with a bin on the map for plane 2 but not 171 exactly with a bin of the map for plane 1. A similar situation exists for 172 plane 3, but again the fit is not as good as for plane 2, due this time to the 173 bins being smaller for plane 3. Binning the shadowgram gives the matrices 174 in the lower part of Fig. 3. Using these matrices and decoding each plane 175 according to the binning and cross-correlation method outlined in Section 2, 176 Eq. (2) we obtain the isometrically projected images in Fig. 4(a). Note that 177 the source, which is clearly visible with high intensity at the centre of plane 178 2, also appears with somewhat lower intensity at the centres of the other 170 planes. Thus we have results similar to those of Kazachkov et al. [8] and 180 Mu and Liu [13] inasmuch that sources are 'blurred' between planes causing 181 artifacts. We now attempt to remove these artifacts in our example. 182

We begin by determining the most likely position of a point source in 183 each plane. We do this by performing a simple least-squares fitting of the 184 shadowgram with all possible source positions within each plane of the 3D 185 FOV, in the form of a χ^2 minimisation, similar to the method proposed for 186 astronomy by Ducros and Ducros [17]. Consider a given plane at depth z. 187 For this analysis we dispense with the z subscripts, recognising that we are 188 working in a given plane at depth z. For each possible source position (x, y)189 at depth z we calculate a value of χ^2 as follows: 190

$$\chi^{2}(x,y) = \sum_{i=0}^{V-v} \sum_{j=0}^{W-w} (1/\sigma_{ij}) [P(i,j) - B - S_{xy}A(i+x,j+y)]^{2}$$
(3)

[17, p. 49-50] where P(i, j) represents the bin counts, examples being the 191 matrices in Fig. 3, σ_{xy} is the variance of the counting statistics of P and A is 192 the binary (0,1) aperture function. The quantities S_{xy} and B are numerically 193 modelled values, where S_{xy} represents the intensity per detector bin corre-194 sponding to the open aperture elements at a distance z from the detector of 195 a source situated at a lateral (i.e. the perpendicular direction to z) source 196 position (x, y) and B represents the background noise per detector bin. Note 197 that we are here assuming a uniform background of B which is independent 198 of both the source location and detector location. For each possible lateral 190 source position (x, y) we set the partial derivatives of χ^2 in Eq. (3) with 200 respect to B and S_{xy} to zero, calculate B and S_{xy} from the resulting pair 201 of simultaneous equations and then calculate χ^2 by substituting for B and 202 S_{xy} back into Eq. (3), giving χ^2 values for each lateral source position (x, y). 203 Repeating this procedure for all planes gives a 3D matrix of χ^2 values for 204



Figure 3: Maps for planes 1 and 2, consisting of bins superimposed on the detector shadowgram. Also shown are the detector image matrices after the counts have been binned.



Figure 4: Decoded images for the prototype system for each plane: (a) without z-Clean image processing (b) after the use of z-Clean.

all possible voxel positions (x, y, z) within the 3D FOV. For example the χ^2 205 values for the maps in Fig. 3, for a source in the centre of the FOV of plane 206 2 are 9.51 for plane 1 and 1.29 for plane 2. The calculation for plane 3 is 207 2.85. All other possible source positions in the 3D FOV give larger χ^2 values 208 than these three. The 3D position having the lowest value of χ^2 is deemed 209 to be the most likely position to contain a point source. We call this point 210 a candidate which is in position (x_1, y_1, z_1) of the 3D reconstructed image. 211 In our example therefore, the map that produces the first candidate is that 212 for the central reconstructed voxel of plane 2. This is because the fit of this 213 particular map is the best. We now process this candidate position by deter-214 mining for the candidate its value of S_{xy} , which we now denote by S_1 , and 215 removing from the detector shadowgram S_1 randomly chosen counts from 216 each of the bins of the map for plane 2 in Fig. 3 that correspond to an open 217 aperture element at candidate position (x_1, y_1, z_1) to give a new shadowgram 218 with fewer counts. An initial 3D matrix T(x, y, z) with zero-valued elements 219 is defined and the total number of removed counts are then stored and added 220 to cell (x_1, y_1, z_1) of T to be used later. The number of counts subtracted 221 from each bin not at the boundary of the map needs to be equal for each 222 corresponding open aperture element, which is S_1 . For an open element bin 223 at the map boundary we subtract the same number of counts but multiplied 224 by the fraction of the bin areas 'occupied' by the detector shadowgram in 225 that bin. After subtracting and storing counts, we perform a second iteration 226 of the whole process but this time on the new shadowgram to determine a 227 second candidate with position (x_2, y_2, z_2) and $S_{xy} = S_2$. The corresponding 228 counts are subtracted from the relevant bins of the new shadowgram and 229 added to T. Note that in different iterations, candidates may coincide but 230 the procedure continues the same. Note also that if there are no counts to 231 subtract from a particular bin then no subtraction takes place from that bin 232 although subtractions from other bins continue. However it is worth noting 233 at this point that the large source and background fluxes used in this study 234 give high detector counts so this situation does not eventuate. 235

We continue the process until eventually a candidate returns a source intensity S_{xy} of a negative value which indicates that all possible positive sources have been exhausted. We then perform a cross-correlation of the remaining shadowgrams with the decoding function as per Eq. (2) for each plane and finally we add the data from the matrix T to the corresponding voxels from the planes of I_z to produce the final decoded images. Note that this is similar to the technique used to remove artifacts from coded aperture

systems having imperfect detectors in [18] and is similar to the CLEAN 243 algorithm used in radio astronomy [14] but we here operate in the detector 244 shadowgram domain rather than the final reconstructed image domain. For 245 ease of discussion we hereafter refer to the whole image processing technique 246 as z-Clean, with processed images being referred to as having been z-Cleaned. 247 In our prototype example the z-Cleaned images are shown in Fig. 4(b). 248 We can see that the reconstructed source position is clearly visible in plane 249 2, the actual plane containing the source, while the artifacts in the other 250 planes have been largely removed although some minor 'ghosting' can be 251 seen in the central pixel of plane 3. Note also that the z-Clean technique has 252 the added benefit of removing much of the noise produced by the random 253 nature of the coded aperture, with the processed images in Fig. 4(b) having 254 flatter sidelobes than those in Fig. 4(a). This is to be expected, since the 255 CLEAN algorithm is capable of removing a range of significant artifacts, 256 which includes those caused by the random noise produced when using a 257 coded aperture that does not have perfect imaging capability, such as the 258 random aperture used in the prototype. 259

We conclude this section by commenting on the point that in coded aperture imaging, cleaning can be accomplished either by operating in the 2D shadowgram domain, and subtracting individual photon counts from the detector, or operating in the 3D reconstructed image domain and subtracting spurious peaks from each of the planes being studied. We here adopt the approach of subtracting from the 2D detector domain as it is much simpler than the complex computation when operating in the 3D domain.

²⁶⁷ 4. Computer Simulations

Computer simulations were conducted to test the z-Clean technique de-268 scribed in Section 3 when using two types of high energy photon detector: a 269 continuous detector and a pixellated detector. It is evident that a continu-270 ous detector is a certain type of idealised detector inasmuch that data can 271 be acquired in list mode and hence affords the user the opportunity to define 272 the bin size according to a particular plane depth being studied. However in 273 practice, detectors are often not continuous but are instead pixellated. For 274 example 0.2cm pixels are typical for cadmium zinc telluride detectors. There-275 fore for most applications it is also necessary to assess the z-Clean technique 276 when used with a pixellated detector. 277

For both types of detector, a number of assumptions are made when per-278 forming the simulations, including a few idealisations. For the continuous 279 detector, staff at Auckland Hospital were consulted as to the expected per-280 formance of such a detector system when using a coded aperture. As a result 281 of these discussions, the continuous detector simulated is a square 35cm by 282 35cm plate, possessing a photon location accuracy that has a Gaussian pro-283 file with a full width at half maximum (FWHM) of 1cm. For the pixellated 284 detector we employ 0.2 cm pixels which, assuming a $35 \text{cm} \times 35 \text{cm}$ detector 285 (or an array of smaller detectors with a total size of $35 \text{cm} \times 35 \text{cm}$) gives a 286 detector with 175×175 pixels. The pixellated detector is assumed to have 287 the capability of rejecting simultaneous multi-site events and hence able, for 288 example, to reject any photons that undergo Compton scattering that de-289 posits energy in more than one pixel. For both detectors a photon detection 290 efficiency of 70% is assumed. 291

For both the continuous and pixellated detectors systems, the aperture 292 pattern chosen is a square MURA with unit pattern (or core) of size v = 31, 293 cyclically repeated to give an overall aperture of size 61×61 elements (namely 294 $2v - 1 \times 2v - 1$) and 50% throughput shown in Fig. 5. Past research by 295 Fenimore [19] and by in't Zand et al. [20] into optimum aperture throughput 296 show that, while values other than 50% may be best for certain source fields, 297 a 50% throughput still gives very good results for point source observations. 298 Furthermore, as the purpose of this study is to demonstrate the efficacy of 299 z-Clean, we here use the more well-known MURA aperture for simplicity 300 and familiarity while recognising that testing z-Clean with different aperture 301 types is a possible area for further research that is beyond the scope of this 302 The aperture to detector separation is set at 30cm and aperture study. 303 elements of size 0.6cm are chosen giving an overall aperture size of 36.6cm. 304 The reason for choosing this aperture element size is that it means that the 305 projected aperture elements from sources in the FOV onto the detector are of 306 the order of the FWHM of the continuous detector. Perfectly square aperture 307 elements of negligible thickness are assumed with a closed element opacity 308 of 99%. A uniform detector background of one count $\rm cm^{-2} \ s^{-1}$ is assumed. 309 Observation time is 600s. 310

For all simulated observations, a field with four point sources of different intensities is chosen. The first is of activity 100kBq situated at a depth of 72cm from the detector and lying in the centre of the FOV, the second is of activity 50kBq at a depth of 74.7cm and lying to the left of the central source, the third is 10kBq at 69.5cm, lying to the right of the central source



Figure 5: MURA coded aperture of v = 31 and overall size 61×61 used in the simulations of Sections 5 and 6. The central $v \times v$ unit pattern, or core, is also shown inside the black edged square.

and the fourth is 5kBq at 71cm lying roughly halfway between the centre and 316 the lower left corner of the FOV (see also Fig. 6). These source parameters 317 are chosen to demonstrate the efficacy of the z-Clean technique when both 318 weak and strong sources are present, lying at depths representing exactly 319 those in a plane to be decoded and also between planes to be decoded. All 320 sources are assumed to be in the very centre of a voxel in the lateral (x, y)321 direction, which for data in list mode is always possible as we can define our 322 own detector bins without any great loss of generality. For each observation 323 twenty trials are conducted to give an indication of the spread of the signal 324 to noise ratio (SNR) values of the individual sources. We present results of 325 four point sources only, since results for fewer sources are at least as good 326 and in many cases marginally better than the results for four sources. The 327 quality of the resulting z-Cleaned images is also compared to those produced 328 by twenty trials of a perfect coded aperture system operating only in its 320 critical plane and hence not affected by depth effects. 330

Once the detector data is collected the z-Clean technique is applied between the minimum and maximum depths of 66cm and 78cm respectively. For each observation the shadowgram is decoded for a range of plane separations with planes being equally separated in each case. We test the technique for plane separations of 2cm (giving 7 planes from 66cm to 78cm), 1cm (13 planes), 0.5cm (25 planes) and 0.1cm (121 planes). In all cases the central plane lies at a depth of 72cm.

As noted, the results and SNR values of the sources are compared to 338 those expected for a perfect imaging system operating only in its critical 339 plane. For this we use the same 31×31 element MURA. Because perfect 340 imaging for MURA arrays requires the source to lie in the critical plane, 341 we adopt the following approach to obtain correct SNR values for a point 342 source at a given depth. Employing the 31×31 element MURA aperture 343 we use the same system parameters as per the z-Cleaned image observations 344 except that we adjust the aperture to detector separation so as to place the 345 point source in the critical plane, where the source now casts a shadow of 346 exactly a full unit pattern of the aperture onto the detector. Then we make 347 an observation of this single source, ensuring that we adjust the background 348 level to take account of the extra statistics produced by the presence of the 349 other sources at other depths but not modulated by the aperture, and decode 350 (2) (note that we do not process this image any further as we using Eq. 351 wish to compare our z-Cleaned images with a single decoded image of a 352 perfect system that suffers no degradation from depth effects). Thus we are 353

³⁵⁴ comparing our simulated observation results to a genuinely perfect coded
³⁵⁵ aperture system that is observing a single source, with corrected statistics
³⁵⁶ for other sources unmodulated by the aperture. Again we perform twenty
³⁵⁷ repeated trials of each critical plane observation.

5. Continuous Detector Results

Images for a typical observation using the continuous detector are shown in Fig. 6. Presented in this figure are the unprocessed and z-Cleaned images for a plane separation of 2cm, and z-Cleaned images for 1cm plane separation. Although more planes than these were processed, only those planes from depths 70cm to 78cm are shown as there were no noticeable artifacts in the z-Cleaned images for planes outside this range.

For the unprocessed images at 2cm plane separation large artifacts, sim-365 ilar to those in Fig. 4(a), are present in all planes in the form of repetitions 366 of the 100kBq source at the centre of the FOV and of the 50kBq source left 367 of the centre of the FOV. The 10kBq source and repeated artifacts are just 368 visible to the right of the centre of the FOV at depths of 70cm, 72cm and 369 74cm. As a result of the large artifacts it is difficult to ascertain the true 370 depths of these sources. However, the z-Cleaned images for a plane separa-371 tion of 2cm demonstrate the efficacy of the z-Clean technique, inasmuch that 372 the large artifacts that recur in all planes of the unprocessed images have 373 been largely removed from the planes where the sources are not present. 374 to leave the true sources clearly visible with only minor ghosting in other 375 planes. We here make the distinction that an artifact is a repeat of a source 376 appearing in a different plane to the actual source for an unprocessed image 377 and a ghost is such a repeat but in an image that has been processed by 378 the use of z-Clean. Typically the ghosts are much smaller than the artifacts. 379 The central 100kBq source is clearly reconstructed at its correct depth of 380 72cm and has also been successfully z-Cleaned, with only minor ghosting of 381 this source in the adjacent planes at 70cm and 74cm, as would be expected 382 given the imperfect photon position location capability of the detector. The 383 50kBq source has also been successfully z-Cleaned and the reconstructed flux 384 of this source, which is actually positioned at a depth of 74.7 cm is shared 385 in roughly correct proportions between the planes either side of this depth, 386 namely there is a large peak in the closest plane at 74cm and a smaller peak 387 in the more distant plane at 76cm. The phenomenon of a single source being 388 shared over more than a single pixel, or in this case a voxel, is also known as 389



Figure 6: Typical images for simulations using z-Clean for 2cm and 1cm plane separations for the continuous detector.

18

a *phasing error* [21] and has been discussed in the literature for sources lying 390 close to object pixel boundaries in the (x, y) directions [22]. In this case we 391 have similar phasing error occurring but this time over more than one voxel 392 in the z direction. We hereafter refer to this phenomenon, when it occurs, as 393 phasing. The 10kBq source lying at 69.5cm is reconstructed and successfully 394 z-Cleaned and appears in its closest plane at 70cm although there was no 395 apparent phasing of this source in the 68cm plane. The 5kBq source lying 396 at a depth of 71cm (indicated in Fig. 6 at 1cm plane separation) is barely 397 visible at 2cm plane separation at depths of 70cm and 72cm. 398

For 1cm plane separation, the central 100kBq source has been successfully 399 z-Cleaned although there is ghosting in the adjacent plane at 71cm and also 400 two planes away at 74cm, the 50kBq source is reconstructed entirely at its 401 closest plane of 75cm with no significant phasing of the source at 74cm, but 402 with ghosting at 77cm, and the 10kBq source at 69.5cm depth is visible in 403 the plane at 70cm but there was no ghosting or phasing of this source in any 404 of the other planes. The 5kBq source that lies at 71cm depth is indicated 405 and is only just visible in the plane at this depth. However, close inspection 406 of the images reveals artifacts of this source of approximately the same size 407 also appearing in the planes at depths of 70cm and 72cm, indicating that no 408 z-Cleaning of this source has taken place. 409

The results in Fig. 6 represent a single typical observation for plane sepa-410 rations of 2cm and 1cm. However for each observation twenty trials were car-411 ried out and more detailed results from these for the three strongest sources 412 are shown in Fig. 7 and Tables 1 and 2. Data for the 5kBq source is not in-413 cluded in the tables because there was no successful z-Cleaning of this source 414 and hence no method of determining the parameters for the tables. Fig. 7 415 shows some SNR depth profiles for the three strongest reconstructed sources 416 at plane separations of 2cm and 1cm, as well as for the finer depth resolutions 417 of 0.5cm and 0.1cm. Here, and for all later observations to follow, each graph 418 shows profiles judiciously chosen to demonstrate a typical range of outcomes 419 for each case. In some cases either two or three of each of the twenty trials 420 are shown while for those cases where all trials, or a large majority of trials 421 (eighteen or nineteen) follow a particular profile, the mean profile is plotted 422 and marked with disks at the graph vertices. The number of such trials is 423 labelled appropriately on the relevant graphs. In these cases the standard 424 error in the mean was also calculated, although it is worth stating at this 425 point that for all such profiles shown, the resulting error bars are smaller 426 than the disks on the graphs. 427

| | 100kBq at 72 cm | 50kBq at 74.7 cm | 10kBq at 69.5 cm |
|------------------------|-------------------|--------------------|--------------------|
| Critical plane SNR | 111.7 ± 0.3 | 78.8 ± 0.4 | 22.0 ± 0.2 |
| 2cm plane separation | | | |
| SNR | 131.7 ± 0.5 | 56.0 ± 0.5 | 11.3 ± 0.1 |
| $z~({ m cm})$ | 71.93 ± 0.01 | 74.52 ± 0.02 | 69.87 ± 0.02 |
| PSLA_z (cm) | 0.021 | 0.051 | 0.250 ± 0.003 |
| 1cm plane separation | | | |
| SNR | 125.5 ± 0.5 | 50.3 ± 0.5 | 11.6 ± 0.2 |
| $z~({ m cm})$ | 71.99 ± 0.02 | 74.81 ± 0.07 | 69.45 ± 0.10 |
| $PSLA_z$ (cm) | 0.011 | 0.028 | 0.122 ± 0.002 |
| 0.5cm plane separation | | | |
| SNR | 125.9 ± 1.0 | 50.0 ± 0.6 | 11.6 ± 0.2 |
| $z~({ m cm})$ | 72.02 ± 0.01 | 74.46 ± 0.04 | 69.47 ± 0.06 |
| $PSLA_z$ (cm) | 0.006 | 0.014 | 0.061 ± 0.001 |
| 0.1cm plane separation | | | |
| SNR | 112.7 ± 1.3 | 49.0 ± 0.5 | 11.3 ± 0.2 |
| $z~({ m cm})$ | 71.95 ± 0.02 | 74.60 ± 0.04 | 69.47 ± 0.05 |
| PSLA_z (cm) | 0.001 | 0.003 | 0.012 |

Table 1: SNR, reconstructed depth (z) and PSLA_z results obtained for twenty trials of the continuous detector for the three strongest sources. For those values without errors quoted, the error is less than 0.001.

Table 1 presents the reconstructed SNR values for the three strongest 428 sources, as well as the mean SNR for twenty trials of the same source oper-429 ating in the critical plane. The table also shows depth calculations and the 430 point source location accuracies (PSLA) in the depth direction, $PSLA_z$. All 431 quantities are shown with standard errors in the means, unless the errors are 432 very small in which case no errors are reported. For sources that are shared 433 between planes with a phasing error, the SNR for each trial is calculated in 434 quadrature from the two planes containing the source, with statistics calcu-435 lated from the twenty different SNR values from all trials. Reconstructed 436 source depths are calculated separately for each trial by Gaussian fitting to 437 these individual profiles, and then mean depths with statistics are obtained 438 from the twenty trials. 439

PSLA_z has also been estimated from the results and is also presented in Table 1. The PSLA of a coded aperture imaging system is dependent upon the source SNR. For a perfect coded aperture system operating only in its

Table 2: Continuous detector: furthest distances of main reconstructed peaks from the actual source depth of twenty trials for each observation of the three strongest sources. All values are in cm. Asterisks denote actual source not lying at a plane to be decoded.

| | Source | | |
|------------------|----------------|--------------------|--------------------|
| Plane separation | 100kBq at 72cm | 50kBq at 74.7 cm | 10kBq at 69.5 cm |
| 2 | 0 | 0.7^{*} | 0.5^{*} |
| 1 | 0 | 0.7^{*} | 0.5^{*} |
| 0.5 | 0 | 0.7^{*} | 0.5 |
| 0.1 | 0.2 | 0.4 | 0.5 |

critical plane, the 2D PSLA in the (x, y) plane can be calculated using the results of Skinner [23] and is given by PSLA = $s \times d\alpha$ where s is the aperture to detector separation and $d\alpha$ is the PSLA in radians. Using [23] Eq. (31) and assuming aperture elements and detector pixels of size m we have

$$PSLA = \frac{s}{SNR} k [2(m/s)^2]^{1/2}$$
(4)

where $k \simeq 1$ is Skinner's constant. Now, in the z direction, calculating the PSLA, which we denote PSLA_z, is directly analogous to Skinner's analysis. Instead of a pixel of size m in the (x, y) plane we have a voxel of size d in the z direction. Substituting this with k = 1 into Eq. (4) and simplifying gives

$$PSLA_z = \frac{1.4d}{SNR}.$$
(5)

⁴⁵¹ The results for the continuous detector are now discussed.

452 5.1. Continuous Detector - Depth Profiles

For the 100kBq source at 2 cm plane separation (top graph in Fig. 7(a)) 453 two SNR depth profiles are shown. The dark line represents the mean profile 454 of nineteen trials (denoted by the key in the corner of the graph) that all 455 follow this same general profile. As stated above, the error bars to represent 456 standard errors in the means are smaller than the disks at the profile vertices. 457 This mean profile shows a clear peak at the actual source depth of 72cm, but 458 with ghosts appearing in the adjacent planes at depths of 70cm and 74cm. 459 The remaining trial gives the dashed profile on the graph, which shows the 460 reconstructed source peak correctly positioned at 72cm and with a ghost 461 in the adjacent plane at 70cm, but also a ghost situated on the opposite 462



Figure 7: Selected examples of SNR depth profiles of the three strongest sources using a continuous detector. The graph titles show the plane separations.



Figure 8: Profiles for the 5kBq source at 71cm depth using 0.5cm plane separation for both detector types.

side of the peak at 76cm depth, which is two planes away from the main 463 peak. Therefore for all twenty trials the main reconstructed peak appears at 464 the correct depth of the source, namely 72cm, although there is a very small 465 amount of variation in the positions of ghosts. For the 100kBq source at both 466 1 cm and 0.5 cm plane separations, Fig. 7(a) shows three profiles in each case, 467 one for each of three selected trials, and all producing reconstructed source 468 peaks at the correct depth of 72cm. However, ghosts of differing sizes appear 469 at varying depths, being separated from the main peak by a combination of 470 one and/or two planes for 1cm separation and up to four planes for 0.5cm 471 separation, to give a range of differing profiles. The appearance of ghosts 472 at different depths for different trials indicates that their occurrence is not 473 purely systematic but contains a random component. For both of these 474 plane separations, the results represent a blurring of up to around 2cm from 475 the main reconstructed peak. As is the case for 2cm plane separation, all 476 trials reconstruct the main peak in the correct position at 72cm depth. For 477 0.1cm plane separation, Fig. 6 shows profiles for two trials, where the main 478 reconstructed peaks appear at different depths for the different trials, one 479 in the correct position at 72cm and the other at 71.8cm. It is because not 480 all of the reconstructed main peaks appear in the same plane that no finer 481 plane separation than 0.1cm was studied for the continuous detector. Both of 482 the profiles for 0.1cm plane separation present non-systematic ghosts either 483 side of and up to approximately 2cm away from the main peaks. Again, 484 the variation in the positions of the ghosts for different trials indicates that 485 their appearance in the z-Clean process is not systematic. Table 2 shows the 486 furthest distance in the z direction for the twenty trials of each observation 487 of any of the main reconstructed peaks from the actual source depth. The 488 results for all three plane separations of 2cm, 1cm and 0.5cm have all twenty 489 trials reconstructing the main peak of the 100kBq source at 72cm, and hence 490 a furthest distance of 0cm from the actual source depth, as per Table 2. For 491 0.1cm plane separation the furthest distance of a main peak from the actual 492 source position is 0.2cm. 493

Profiles for the 50kBq source located at 74.7cm depth are shown in Fig. 7(b). For 2cm plane separation, eighteen trials give a profile similar to the dark line (top graph). The phasing is evident in the sharing of the reconstructed source flux over two planes, namely 74cm and 76cm, as to be expected from the source depth of 74.7cm. The remaining two trials give profiles similar to the dashed line. Unlike the case for the 100kBq source, for the finer plane separations of 1cm, 0.5cm and 0.1cm the reconstructed peaks for

different trials do not all appear at the same depth, as shown in the profiles, 501 three of which are shown for 1cm plane separation and two each for 0.5cm 502 and 0.1cm plane separations. However, for all trials of this source at 0.5cm 503 and 0.1 cm plane separations, the main reconstructed peaks appear no further 504 than 0.7cm and 0.4cm from the actual source depth respectively (Table 2). 505 Once again, the appearance of ghosts either side of each main peak is not 506 systematic and seems to represent an overall blurring of around 2cm to 3cm 507 either side of the main peaks. 508

For the twenty trials of the 10kBg source for 2cm plane separation, every 500 trial follows the same profile as the top graph in Fig. 7(c). The main peak 510 is reconstructed at 70cm, which is 0.5cm from the actual source depth. The 511 voxels either side of the main peak are below the 3σ threshold so there is no 512 obvious phasing of this source from the main reconstructed peak. For 1cm 513 plane separation for this source, the twenty trials follow either one of the 514 two profiles shown in the second graph in Fig. 7(c), with peaks appearing 515 at either 69cm or 70cm depth, thus being no further than 0.5cm from the 516 actual source depth. For the finer plane separations of 0.5cm and 0.1cm, 517 the reconstructed peaks also do not all appear at the same depth, although 518 for all trials of the 10kBq source, these peaks appear no further than 0.5cm 519 from the actual source depth for all plane separations. Furthermore, unlike 520 the two stronger sources, no ghosts appear in the profiles. This is due to the 521 fact that no candidates at depths other than the main peak are obtained, 522 probably as a result of the weak nature of the 10kBq source which is less likely 523 to contribute candidates in more than one iteration of the z-Clean process 524 while in the presence of other stronger sources and a high background level. 525 The z-Cleaning of the three strongest sources to remove large artifacts and 526 leave only a small level of minor ghosting demonstrates the efficacy of the 527 z-Clean technique for these three sources. 528

As mentioned earlier, the 5kBq source is barely visible and does not 529 appear to have been z-Cleaned. In Fig. 8(a) the dark line shows the mean 530 profile of nineteen trials for the 5kBg source at 0.5cm plane separation using 531 the continuous detector, indicating that for all of these trials the SNR barely 532 reaches 3 at around the actual source depth. There is no reconstructed peak, 533 and hence no z-Cleaning of the images for these trials. Note that this result 534 is also consistent with the 1cm plane separation images of Fig. 6 for which z-535 Clean also did not remove the artifacts. However, one trial gives the dashed 536 profile, with a clearly reconstructed peak at 69.5cm depth, namely 1.5cm 537 from the actual source depth of 71cm, and lower sidelobes, indicating that z-538

Clean has taken place but giving a less accurate reconstructed peak location than for the three stronger sources. For the other plane separations we do not present results since the profiles obtained for all trials of the 5kBq source were similar to those of the dark line in Fig. 6, indicating no z-Cleaning for these cases. This would suggest that for the parameters studied, successful z-Cleaning of a 5kBq source for a given observation would only occur very rarely.

546 5.2. Continuous Detector - Reconstructed Source Depths

While Table 2 gives the furthest reconstruction of a peak from the actual source depth, Gaussian fitting of the data allows more accurate source depth information to be determined. For each trial, the data was fitted to a Gaussian distribution for each of the three strongest sources to determine source depth for each trial and the statistical data was then calculated from the twenty trials. The results are given in Table 1.

The depth location is very good in all cases for the three strongest sources 553 for all plane separations, with the values all being compatible with the ac-554 tual depths of the three sources in all cases. In general the sources that 555 suffer a phasing error are located with lower precision, namely the 50kBq 556 source using 2cm, 1cm and 0.5cm plane separations for which reconstructed 557 source depths were 0.18cm, 0.11cm and 0.24cm from the actual source depth 558 respectively, and the 10kBq source at 2cm and 1cm plane separations for 559 which reconstructed source depths are respectively 0.37cm and 0.05cm from 560 the actual source depths. For all other source observations at other plane 561 separations, where there is no phasing error, the reconstructed source depths 562 are generally better, ranging from around 0.1cm from the actual source depth 563 for many cases to only 0.01cm from the actual source depth in the case of the 564 100kBq source at 1cm plane separation. For all cases of the three strongest 565 sources the depth location is very good although, outside of a phasing error 566 discrepancy, there is no clear systematic pattern in the ability to reconstruct 567 a source at the correct depth. 568

569 5.3. Continuous Detector - SNR and $PSLA_z$

The SNR data for the reconstructed 100kBq, 50kBq and 10kBq sources using all plane separations is given in Table 1, along with values for the critical plane observations. For the 100kBq source the reconstructed SNR for all plane separations unexpectedly exceeds the value for the critical plane observations. The reason for this is explained as follows. For observations

at the critical plane, a reconstructed source peak comes with a number of 575 smaller ghosts either side and adjacent to the central peak, typically four to 576 eight in a cross or a square formation, due to a blurring that is typical of the 577 coded aperture imaging technique. These artifacts increase the variability of 578 the sidelobes around the reconstructed source peak and lead to a reduction 579 in the reconstructed source SNR. Now, in the iterative z-Clean process, the 580 first candidate for the strong 100kBq source is correctly chosen in the central 581 (x, y) voxel and at a depth of 72cm for which counts are then subtracted and 582 stored appropriately for later use. However, the second candidate chosen for 583 this source is typically not located in a voxel at the same depth and adjacent 584 to the main peak, but instead is located in the same (x, y) position but at a 585 different depth, an example being at 70cm depth for the 2cm plane separation 586 images and appearing as a ghost in Fig. 6. When counts are removed for this 587 incorrectly-located ghost candidate, it also singularly removes counts that 588 would otherwise have been allocated to a number of sidelobe candidates 589 that would have appeared at 72cm depth and adjacent to the main peak, 590 had the second candidate not already have been assigned these counts, thus 591 suppressing the sidelobes and reducing the overall variability of the image, 592 and hence artificially increasing the SNR of the main reconstructed peak. 593 Note for the particular case of Fig. 6, the effect is compounded by a further 594 ghost located in the same (x, y) position but at 74cm depth. For 2cm plane 595 separation the effect here is to reduce the noise by approximately 18% and 596 increase the SNR from 117 to 131. Note that for weaker sources, this effect 597 is less marked as here the detector background has the more dominant effect 598 than the location of incorrect candidates on the overall variability. The SNR 599 of the 100kBq source reduces slightly as the plane separation becomes finer, 600 from 131.7 ± 0.5 at 2cm plane separation to 112.7 ± 1.3 at 0.1cm plane 601 separation. As a result of the unexpected increase in the SNR for this strong 602 source, the $PSLA_z$ values for this source need to be taken with some level of 603 caution, and in reality the quantities are probably more accurately reflected 604 by using the critical plane SNR value of 111.7 in Eq. (5). Using this value 605 gives $PSLA_z$ values of 0.025cm, 0.013cm, 0.006cm and 0.001cm for plane 606 separations of 2cm, 1cm, 0.5cm and 0.1cm respectively. 607

For the 50kBq source, there is a clear phasing for 2cm plane separation over the two planes at depths of 74cm and 76cm. Therefore the SNR values for this source are calculated in quadrature over these two planes for each of the twenty trials individually and the results are combined to give the mean and standard error in the mean in Table 1. The results indicate a reduction in SNR compared to that of the critical plane observation that increases with
 decreasing plane separation, ranging from 29% for 2cm plane separation, up
 to 38% for 0.1cm plane separation.

For the 10kBq source, the SNR similarly suffers a reduction compared to the critical plane, which is more severe compared to that for the 50kBq source, being around 47% to 49%. However, the reconstructed SNR is much more consistent for the 10kBq source across different planes, with very little variation.

As already noted, $PSLA_z$ depends on SNR and plane separation, generally improving with increased SNR and with finer plane separation. For the 100kBq source $PSLA_z$ ranges from 0.021cm for 2cm plane separation to 0.001cm for 0.1cm plane separation, for the 50kBq source from 0.051cm at 2cm plane separation to 0.003cm at 0.1cm plane separation, and for the 10kBq from 0.250cm at 2cm plane separation to 0.012cm at 0.1cm plane separation.

628 6. Pixellated Detector Results

As noted in Section 4, the use of the chosen pixellated detector means 629 that the data collected comes as a 175×175 matrix of counts. However, 630 the z-Clean technique requires data to be in list mode, namely having actual 631 positions of individual detected photons, so that the counts can be binned 632 and z-Cleaned according to which plane is being studied at any particular 633 time. Therefore to replicate a list mode output, each detected count within a 634 detector pixel is randomly allocated an (x, y) coordinate within that pixel on 635 the detector so that rather than having a matrix of counts, we instead have 636 a list of (x, y) detector coordinates as though the data were in list mode. We 637 then proceed with the z-Clean technique as explained in Section 3. In the 638 following we present results of simulations described in Section 4. 639

The z-Cleaned images of a typical observation for each of the plane separations of 2cm and 1cm are shown in Fig. 9. As for the continuous detector images, only planes from depths 70cm to 78cm are shown as there were no noticeable ghosts in the planes outside this range. The images in Fig. 9 are similar to those for the continuous detector in that the three strongest sources are all clearly visible and have largely been successfully z-Cleaned, with only minor ghosting in some nearby planes.

At 2cm plane separation, the 100kBq source is reconstructed at the actual source depth of 72cm with a ghost at 70cm although, unlike the case for the



29

Figure 9: Typical images for simulations using z-Clean for 2cm and 1cm plane separations for the pixellated detector.

continuous detector, there is no significant ghosting at 74cm. The 50kBq source at 74.7cm depth is again proportionately shared between two planes as a phasing error at depths of 74cm and 76cm, with no ghosting in any other plane, and the 10kBq source at 69.5cm depth is visible at 70cm with no phasing or ghosting evident in any other plane. The 5kBq source at 71cm depth is just discernable in the planes at 70cm, 72cm and possibly even at 74cm, with no apparent z-Cleaning of this source having taken place.

At 1cm plane separation the 100kBq source is clearly reconstructed at 656 its correct depth of 72cm depth with minor ghosting at 71cm and 73cm, 657 the 50kBq source situated at 74.7cm depth is reconstructed entirely at the 658 closest depth of 75cm with no phasing in the 74cm plane and no ghosting, 659 and the 10 kBq source situated at 69.5 cm is reconstructed entirely at 70 cm660 with no phasing of this source having occurred at 69cm. The 5kBq source is 661 just visible at its correct depth in the plane at 71cm but also with the same 662 approximate reconstructed height at 70cm, 72cm, 73cm and even possibly 663 74cm. Such repeated occurrences indicates that no z-Cleaning of this source 664 has taken place. 665

More detailed results for the three strongest sources are presented as profiles in Fig. 10 and in Tables 3 and 4, with SNR and depth calculations presented in Table 3.

669 6.1. Pixellated Detector - Depth Profiles

For the 100kBg source at plane separations of 2cm, 1cm and 0.5cm, all 670 twenty trials for each give similar profiles to the mean profiles shown in the 671 top three graphs in Fig. 10(a), with very little variation in each case, and 672 the main peak being reconstructed at exactly the source depth of 72cm. At 673 2cm plane separation there is a single significant, but minor, ghost at 70cm, 674 reflecting the example of the images in Fig. 9, and at 1cm plane separation. 675 minor ghosting takes place in the voxels either side of the reconstructed main 676 peak. The ghosting is relatively more marked for 1cm plane separation than 677 for 2cm plane separation. For 0.5cm plane separation, the ghosts appear two 678 voxels either side of the main peak, with high consistency. At 0.1cm plane 679 separation all twenty trials reconstruct the main peak at the exact source 680 depth of 72cm but not all trials follow the same profile. The bottom graph 681 in Fig. 10(a) shows two profiles that represent the images having the most 682 extreme positions of the ghosts which lie either side of and approximately 683 1cm from the main peak. The 0.1cm plane separation therefore represents 684



31 Figure 10: SNR depth profiles of the three sources using the pixellated detector.

| | 100kBq at 72 cm | 50kBq at 74.7 cm | 10kBq at 69.5 cm |
|------------------------|--------------------|--------------------|--------------------|
| Critical plane SNR | 247.9 ± 1.2 | 158.1 ± 0.8 | 42.6 ± 0.4 |
| 2cm plane separation | | | |
| SNR | 187.9 ± 0.6 | 77.1 ± 0.3 | 12.7 ± 0.1 |
| $z~({ m cm})$ | 71.879 ± 0.002 | 74.394 ± 0.003 | 69.978 ± 0.016 |
| PSLA_z (cm) | 0.015 | 0.037 | 0.224 ± 0.001 |
| 1cm plane separation | | | |
| SNR | 164.2 ± 0.5 | 85.3 ± 0.3 | 14.9 ± 0.2 |
| $z~({ m cm})$ | 71.931 ± 0.001 | 74.965 ± 0.003 | 69.663 ± 0.101 |
| PSLA_z (cm) | 0.009 | 0.017 | 0.095 ± 0.001 |
| 0.5cm plane separation | | | |
| SNR | 167.0 ± 0.5 | 88.5 ± 0.3 | 15.5 ± 0.1 |
| $z~({\rm cm})$ | 71.993 | 74.990 ± 0.001 | 69.547 ± 0.024 |
| PSLA_z (cm) | 0.004 | 0.008 | 0.046 |
| 0.1cm plane separation | | | |
| SNR | 144.5 ± 0.4 | 74.7 ± 0.6 | 13.2 ± 0.1 |
| $z~({ m cm})$ | 71.900 | 74.997 ± 0.002 | 69.496 ± 0.064 |
| PSLA_z (cm) | 0.001 | 0.002 | 0.011 |

Table 3: SNR, reconstructed depth (z) and PSLA_z results obtained for twenty trials of the pixellated detector for the three strongest sources. For those values without errors quoted, the error is less than 0.001.

the only one studied so far for the 100kBq source using a pixellated detector where there is any variation in the positions of the ghosts.

For the 50kBq source at 74.7cm depth all twenty trials for plane sep-687 arations of 2cm, 1cm and 0.5cm follow the same profiles as each of their 688 respective graphs in Fig. 10(b) with little variation, while at 0.1cm plane 689 separation only one of the trials (dashed line) deviates from the dark line 690 profile shown in the bottom graph of Fig. 10(b). At 2cm plane separation 691 there is clear phasing between 74 and 76cm with the higher peak being closest 692 to the actual source depth and the values shared in roughly the correct pro-693 portions for this source. For the finer plane separations there is no phasing 694 evident on any of the graphs but in all cases the main peak is reconstructed 695 in the closest plane to the actual source depth, namely at 75cm. 696

For the 10kBq source at 69.5cm depth and at 2cm plane separation, all twenty trials reconstruct the source in the closest plane at 70cm depth with no evident phasing or ghosting in any other plane. The finer plane separations

| | | Source | |
|------------------|-------------------|--------------------|--------------------|
| Plane separation | 100kBq at 72 cm | 50kBq at 74.7 cm | 10kBq at 69.5 cm |
| 2 | 0 | 0.7^{*} | 0.5^{*} |
| 1 | 0 | 0.3^{*} | 0.5^{*} |
| 0.5 | 0 | 0.3^{*} | 0.5 |
| 0.1 | 0 | 0.3 | 0.5 |
| 0.02 | 0.12 | | |
| | | | |

Table 4: Pixellated detector: furthest distances of main reconstructed peaks from the actual source depth of twenty trials for each observation of the three strongest sources. All values are in cm. Asterisks denote actual source not lying at a plane to be decoded.

Come

give profiles that exhibit a single main peak with no phasing or ghosting 700 evident. For 1cm plane separation all trials are similar to one of the two 701 profiles shown, where the peak is reconstructed at either 69cm or 70cm, 702 namely in one of the two planes closest to the actual source depth. For 703 0.5cm plane separation nineteen trials reconstruct the peak at the actual 704 source depth of 69.5cm with the remaining trial being only one plane and 705 0.5cm away from the actual source depth. For 0.1cm plane separation peaks 706 are reconstructed at various depths, three examples being given which include 707 the two extremes at depths of 69cm and 69.8cm and a typical intervening 708 example. Again the efficacy of the z-Clean technique is demonstrated for the 709 three strongest sources. 710

For the 5kBq source all twenty trials for 0.5cm plane separation follow 711 the same general profile shown in Fig. 8(b), with the profile itself being the 712 mean for all the trials. The highest part of the graph reaches an SNR value 713 of approximately 4 at around the actual source depth of 71cm, but there 714 is no clear peak which indicates that no z-Cleaning has occurred and that 715 significant artifacts in the form of repetitions of the reconstructed source are 716 present in many of the nearby planes. Profiles for the other plane separations 717 followed similar profiles to that in Fig. 8(b), indicating no z-Cleaning of this 718 source, so the these are not individually reported here. 710

720 6.2. Pixellated Detector - Reconstructed Source Depths

The depth location using the pixellated detector is very good for the three strongest sources for all plane separations, with all reconstructed depths being compatible with the actual source depths for all cases. As is the case for the continuous detector, the calculated depths of the reconstructed sources

that suffer a phasing error are generally slightly less accurately determined 725 than those for which no phasing occurs. In particular the 50kBq source 726 at 74.7cm is reconstructed between 0.265cm and 0.306cm from the actual 727 depth although this result is still very good. Even in the worst case, namely 728 the 10kBq source at 2cm plane separation, the reconstructed depth is only 729 0.478 cm from the actual source depth. At plane separations of 1 cm and finer 730 the 10 kBq source is able to be reconstructed to within 0.2cm of its actual 731 depth, and the 100kBq source to within better than 0.1cm of its actual depth. 732

$_{733}$ 6.3. Pixellated Detector - SNR and PSLA_z

Table 3 shows that the SNR for the 100kBq reconstructed source is re-734 duced by the implementation of z-Clean compared to critical plane obser-735 vations for all plane separations, being generally more severe as the plane 736 separation decreases, from 24% for 2cm plane separation to 42% for 0.1cm 737 plane separation. Whereas the continuous detector experiences an unex-738 pected SNR increase caused by the generation of incorrect candidates at 739 different depths which remove detector counts that would otherwise have 740 been assigned to potential ghosts adjacent to the main peak at 72cm depth, 741 the use of a pixellated detector, which bins the counts in precisely defined 742 pixels with no Gaussian spread, and hence no FWHM of detection, means 743 that there would be no ghosting in the adjacent planes at 72cm and hence 744 the detection of incorrect candidates at other depths and at the same lateral 745 position has no effect on the overall variability, and hence no SNR increase 746 occurs. Instead the use of z-Clean here comes at the expense of a reduction 747 in the SNR compared to those of the critical plane observations. 748

For the 50kBg reconstructed source the SNR reduction is more severe, 749 being between 44% for 0.5cm plane separation to 51% for 2cm plane separa-750 tion. The SNR for 2cm plane separation is higher than that for 0.1cm plane 751 separation. However the plane separations of 1cm and 0.5cm suffer less SNR 752 degradation than 2cm plane separation, due probably to the source being 753 located very close to the actual planes to be decoded and hence suffering 754 a less severe phasing error. The SNR for the 10kBq reconstructed source, 755 while remaining sufficiently high to render the source clearly visible, is quite 756 severely impacted by the implementation of z-Clean, falling by 70% for 2cm 757 plane separation and by the smaller amount of 64% for the best case for 758 this source of 0.5cm plane separation, again probably due to the source ly-759 ing at an actual plane to be studied for this particular plane separation. At 760

0.1cm plane separation the SNR decrease is more severe with a 69% decrease
compared to the critical plane observations.

PSLA_z for all sources improves with finer plane separations from 2cm to 0.1cm, ranging from 0.015cm to 0.001cm respectively for the 100kBq source, from 0.037cm to 0.002cm for the 50kBq source and from 0.224cm to 0.011cm for the 10kBq source.

767 6.4. Pixellated Detector - 100kBq source only

As stated in Section 6, when using the pixellated detector all twenty trials 768 for each plane separation successfully reconstructs the main source peak at 769 the correct depth of 72cm, even down to 0.1cm plane separation (Fig. 10(a)). 770 In view of this point, a further set of twenty trials was conducted of an 771 observation of only a single 100kBq source at 72cm in the absence of the 772 other three sources to ascertain whether z-Clean can reconstruct the main 773 peak of this source at the same depth of 72cm but at the even finer depth 774 resolution of 0.02cm. The reason that only this single source is observed is 775 due to the infeasibly long run times that the z-Clean process requires for such 776 fine resolution processing of fields containing multiple sources. Therefore we 777 observe just this single source with the same background level of one count 778 $\rm cm^{-2} \ s^{-1}$. Fig. 11 shows portions of the profiles of three of the twenty trials, 779 from depths of 71.4cm to 72.4cm. The main peaks of the example three trials 780 are separated and small ghosts are visible for some of the trials at around 781 71.5cm and 71.6cm depth although for these and all the remaining twenty 782 trials, similar sized ghosts to those in the figure were visible from around 783 70.5cm up to around 73.5cm beyond the range of the graph in the figure. 784 For the twenty trials at this plane separation, the source depth is calculated 785 at 72.088 ± 0.004 cm, which is comparable to the results for the other plane 786 separations as given in Table 3. Table 4 shows that the furthest plane that 787 a reconstructed source appears for the twenty trials is only 0.12cm from the 788 actual source depth. 789

790 7. Point Spread Function

The point spread function (PSF) of a coded aperture system can often be calculated theoretically. However, theoretical calculation of the PSF in the depth direction for images processed using the z-Clean technique, which we denote as PSF_z , is extremely difficult. Therefore we attempt to determine a best estimate for PSF_z by simulating an observation of an extremely strong



Figure 11: Portions of SNR depth profiles for a single 100kBq source using a pixellated detector.

single source in the absence of any background counts and performing z-796 Clean on the data. This has been done for both the continuous detector and 797 pixellated detector, observing a 5MBq source in the centre of the FOV at 798 72cm depth (i.e. in the same position as the 100kBq source in the previous 799 sections) and using the same system parameters as in Section 4 but with no 800 background counts. This is done for each of the plane separations studied in 801 Section 4. In each case, only one trial is conducted because the extremely long 802 run times for the programs using such huge count numbers in conjunction 803 with z-Clean renders multiple trials infeasible. 804

The depth profiles for the two detector types are shown in Fig. 12. For 805 both detector types at all plane separations the main peak is reconstructed 806 at the correct source depth of 72cm. In all cases ghosts are present either 807 side of the main peak. The ghosting is roughly symmetrical when using 808 the continuous detector for plane separations of 2cm and 1cm, and for the 800 pixellated detector for 2cm plane separation. However for the remaining cases 810 the ghosting is not symmetrically distributed and in some cases consists of 811 three or more statistically significant ghosts. Furthermore it must be noted 812 that the profiles are for a single source lying in a specific voxel and in the 813 absence of the influence of other sources in the FOV. It is therefore to be 814 expected that for other source positions in the FOV, PSF_z would take on a 815 very large range of profiles with hugely varying numbers of and positions of 816 ghosts. 817

818 8. Conclusions

This article presents an image processing technique, which we call z-819 Clean, that removes the repeated artifacts associated with image reconstruc-820 tion in the 3D FOV when using a coded aperture imaging system. The tech-821 nique includes determining the lateral positions and depths of point sources 822 and removing artifacts caused on some planes by sources from another plane. 823 For a continuous detector with a 1cm FWHM detection capability at 824 plane separations of 2cm, 1cm, 0.5cm and 0.1cm, the z-Clean technique is 825 able to resolve three (100 kBq, 50 kBq and 10 kBq) of four (also 5 kBq) point 826 sources very well, while at the same time significantly reducing to the level 827 of minor ghosting the large artifacts caused by sources in other planes. The 828 efficacy of z-Clean is thus demonstrated for the three stronger sources al-829 though there is some ghosting of the 100kBq and 50kBq sources, consisting 830 of smaller peaks appearing in planes other than the one containing the source. 831



Figure 12: Estimated PSF_z profiles for a single 5MBq source with no background.

The distributions of the ghosts show some small variability for the 100kBq 832 and 50kBq sources at 2cm plane separation and larger variability for finer 833 plane separations. The variability in the positions of the ghosts indicates 834 their presence as being non-systematic and containing a random component. 835 Ghosts typically appear up to around 2cm from the actual source depth. 836 Reconstructed images of the 10kBq source show a clear source peak for all 837 plane separations but with no ghosting evident, due to there being no can-838 didates for this source in the z-Clean process other than the first one for 839 which a main peak is reconstructed. For the 5kBq source, z-Cleaning would 840 appear to occur only very rarely and even then to offer less accurate depth 841 location than for the stronger sources. The depth location and $PSLA_z$ for all 842 plane separations are very good for the three strongest sources and individual 843 trials are able to reconstruct a source peak to 0.2cm or better from the ac-844 tual source depth for the 100kBq source, with a worst performance of 0.7cm 845 from the actual source depth for the 50kBq source, which is still very good. 846 Sources that suffer a phasing error are generally less accurately located. The 847 excellent depth location and $PSLA_z$ come at the expense of an impact on 848 the SNR, which for the strongest source increases unexpectedly due to the 849 incorrect detection of candidates lying at the same lateral position as the 850 actual source but at different depths, which suppresses ghosts that would 851 otherwise appear adjacent to the actual source in the correct plane. For the 852 weaker sources, SNR is reduced, in some cases quite severely, so the excellent 853 depth resolution made possible by z-Clean is traded off with an impact on 854 SNR. 855

For a pixellated detector with 0.2cm pixels, the efficacy of z-Clean is also 856 demonstrated, with the large repeated artifacts being removed and leaving 857 the three strongest sources clearly visible with only minor ghosting in some 858 other planes. The reconstruction of the source peaks and the ghosting is 859 quite consistent for the 100kBq and 50kBq sources at 2cm, 1cm and 0.5cm 860 plane separations, although there is some small variability in the ghosting 861 at 0.1cm plane separation. The reconstructed 10kBq source experienced no 862 ghosting but the reconstructed peaks appear at different depths for different 863 trials, although no further than 0.5cm from the actual source depth. As 864 was the case for the continuous detector, the depth location for the three 865 strongest sources is very good, being better than 0.5cm in all cases and 866 better than 0.1cm for the 100kBq source at plane separations of 1cm, 0.5cm 867 and 0.1cm. $PSLA_z$ improves with SNR and finer plane separation, being as 868 good as 0.001cm for the 100kBq source at 0.1cm plane separation. Again the 869

very good depth location and $PSLA_z$ come at the expense of a significant reduction in SNR, whose severity is higher for weaker sources.

As a technique and for certain situations, z-Clean is an efficacious method 872 of removing the artifacts that typically appear in the planes of a 3D coded 873 aperture imaging system when observing a source in a given plane and sug-874 gests that the results in earlier work by other authors, including Kazachkov 875 et al. [8] and by Mu and Liu [13] might be improved upon. However this 876 paper represents just a starting point in the idea of removing artifacts in 877 3D coded aperture imaging, and further work in this area is possible which 878 is beyond the scope of this article. Perhaps most importantly, such work 879 could include observing different types of source distribution, in particular 880 extended sources. This could be useful in medical imaging, for instance, 881 where details of body organ structure are often required and hence good 882 quality images of such extended objects are needed. Related to this could 883 also be the study of the effect of different aperture throughput values, as well 884 as what benefit, if any, different aperture throughput has when used to pro-885 cess images of extended sources using z-Clean [20]. Other future work could 886 include investigating the use of z-Clean in the 3D image domain, rather than 887 the 2D detector domain and a more detailed study into the use of detectors 888 with different pixel size to bin size ratios, which would extend the work to 889 include a wider range of detector parameters. Of particular interest to this 890 entire field is the possibility of conducting experimental laboratory tests to 891 observe real high energy sources using a physical position sensitive photon 892 detector and applying z-Clean to the data. 893

⁸⁹⁴ 9. Acknowledgments

The author would like to thank Drs. Mike Rutland and Ibrahim Hassan of Auckland Hospital, Dr. Berry Allen of the Waikato District Health Board and Drs. Paul Shutler and Stuart Springham of Nanyang Technological University, Singapore for very useful discussions during this work. Also the author would like to thank the two referees for their splendid feedback.

900 References

[1] L. Mertz and N.O. Young, "Fresnel transformations of images," in *Proceedings of the International Conference on Optical Instruments and Techniques*, K. J. Habell, ed. (Chapman and Hall, 1961), pp. 305–310.

- [2] R.H. Dicke, "Scatter-hole cameras for X-rays and gamma rays," Astrophys. J. Lett. 153, L101–106 (1968).
- [3] E.E. Fenimore and T.M. Cannon, "Coded aperture imaging with uniformly redundant arrays," Appl. Opt. 17, 337–347 (1978).
- [4] E. Caroli, J.B. Stephen, G. Di Cocco, L. Natalucci and A. Spizzichino,
 "Coded aperture imaging in X- and gamma-ray astronomy," Spa. Sci.
 Rev. 45, 349–403, (1987).
- [5] G.K. Skinner, C.J. Eyles, A.P. Willmore, D. Bertram, M.J. Church,
 J.R.H. Herring, J. Ponman and M.P. Watt, "The Spacelab 2 X-ray telescope coded mask imaging in orbit," Astrophys. Lett. Comm. 27, 199–
 209 (1988).
- [6] C. Winkler, T. J.-L. Courvoisier, G. Di Cocco, N. Gehrels, A. Giménez,
 S. Grebenev, W. Hermsen, J. M. Mas-Hesse, F. Lebrun, N. Lund, G.
 G. C. Palumbo, J. Paul, J.-P. Roques, H. Schnopper, V. Schönfelder,
 R. Sunyaev, 16, B. Teegarden, P. Ubertini, G. Vedrenne and A. J. Dean,
 "The INTEGRAL mission," Astron. Astrophys. 411, L1–6 (2003).
- [7] W.L. Rogers, K.F. Koral, R. Mayans, P.F. Leonard, J.H. Thrall, T.J.
 Brady and J.W. Keyes, "Coded aperture imaging of the heart," J. Nucl.
 Med. 21, 371–378 (1979).
- [8] Y.P. Kazachkov, D.S. Semenov and N.P. Goryacheva, "Application of coded apertures in medical gamma-ray cameras," Instr. Exper. Tech.
 50, 267–274 (2007).
- [9] A.A. Faust, R.E. Rothschild, P. Leblanc and J.E. McFee, "Development of a coded aperture X-Ray backscatter imager for explosive device detection," IEEE Trans. Nucl. Sci. 56, 299–307 (2009).
- [10] K.F. Koral, W.L. Rogers and G.F. Knoll, "Digital tomographic imaging
 with a time-modulated pseudorandom coded aperture and anger camera," J. Nucl. Med. 16, 402–413 (1974).
- [11] T.M. Cannon and E.E. Fenimore, "Tomographical imaging using uniformly redundant arrays," Appl. Opt. 18, 1052–1057 (1979).

- J.S. Fleming and B.A. Goddard, "An evaluation of techniques for stationary coded aperture three-dimensional imaging in nuclear medicine,"
 Nucl. Instrum. Meth. Phys. Res. 221, 242–246 (1984).
- ⁹³⁷ [13] Z. Mu and Y. Liu, "Aperture collimation correction and maximumlikelihood image reconstruction for near-field coded aperture imaging of single photon emission computerized tomography," IEEE Trans. Med. Imaging 25, 701–711 (2006).
- [14] B.G. Clark, "An efficient implementation of the algorithm CLEAN,"
 Astron. Astrophys. 89, 377–378 (1980).
- ⁹⁴³ [15] S.R. Gottesman and E.E Fenimore, "New family of binary arrays for ⁹⁴⁴ coded aperture imaging," Appl. Opt. **28**, 4344–4352 (1989).
- S.R. Gottesman and E.J. Schneid, "PNP a new class of coded aperture arrays," IEEE Trans. Nucl. Sci. 33, 745–749 (1986).
- ⁹⁴⁷ [17] G. Ducros and R. Ducros, "Statistical analysis for coded-aperture gamma-ray telescope," Nucl. Instrum. Meth. Phys. Res. A221, 49–53 (1984).
- [18] K. Byard and D. Ramsden, "Coded aperture imaging using imperfect detector systems," Nucl. Instrum. Meth. Phys. Res. A342, 600-608 (1994).
- ⁹⁵² [19] E.E. Fenimore, "Coded aperture imaging: predicted performance of uni⁹⁵³ formly redundant arrays," Appl. Opt. **17**, 3562–3570 (1978).
- J.J.M in't Zand, J. Heise and R. Jager, "The optimum open fraction of coded apertures. With an application to the wide field X-ray cameras of SAX," Astron. Astrophys. 288, 665–3674 (1994).
- ⁹⁵⁷ [21] E.E. Fenimore and G.S. Weston, "Fast delta Hadamard transform,"
 ⁹⁵⁸ Appl. Opt. **20**, 3058–3067 (1981).
- ⁹⁵⁹ [22] I.D. Jupp, K. Byard and A.J. Dean, "An improved sampling configura⁹⁶⁰ tion for a coded aperture telescope," Nucl. Instrum. Meth. Phys. Res.
 ⁹⁶¹ A345, 576-584 (1994).
- ⁹⁶² [23] G.K. Skinner, "Sensitivity of coded mask telescopes," Appl. Opt. 47, 2739–2749 (2008).