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Consistent Flexibility: Enforcement of Fiscal Rules Through Political Incentives

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Abstract

We study a fiscal policy model in which the government is present-biased, leading to an excessive public deficit. An optimally designed fiscal rule needs to trade off the benefit of committing the government to not overspend against the benefit of granting it flexibility to react to shocks to tax revenues. Unlike prior work, we characterize a rule that is enforced through political incentives: the punishment for a violation of the rule consists in a reduction of the politician's payoff from being in office during the following period. We show that the optimal fiscal rule prescribes a zero structural deficit and only partially accounts for revenue shocks. Moreover - and somewhat surprising - a government with a stronger ex ante deficit bias should be granted a higher degree of flexibility. Flexibility leads to more rather than less fiscal discipline because the punishment for a rule violation is less driven by luck and more dependent on actual policy choices. Thus a trade-off between fiscal discipline and fiscal rule flexibility, as often claimed in the context of the EU's Stability and Growth Pact, does not typically exist in our model.

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1 Introduction

Fiscal rules are widely used to constrain a government's fiscal policy and aim for moderate levels of budget deficits, debt or expenditure levels. They have become increasingly prevalent, in place in 97 countries in 2013, compared to only seven countries in 1990¹. Governments however do not always respect these rules. For example, in 2003, the governments of France and Germany violated the terms of the European Union's Stability and Growth Pact by running deficits above the allowed limit, without facing any formal sanction (see Schuknecht et al. 2011). Out of 28 EU countries 25 have been subject to an Excessive Deficit Procedure (EDP) at some point in the past, an indication that compliance with fiscal rules in the EU cannot be taken for granted. The issue of compliance is not confined to the EU however. Eyraud et al. (2018) report that lack of compliance is a worldwide problem. Therefore, credible enforcement mechanisms are critical to the institution of fiscal rules, for governments only abide to rules if the ensuing penalties for breaching them are severe enough. Nevertheless, a monetary punishment is often politically unfeasible, as the case of the European Union's Stability and Growth Pact has shown so far.²

In addition to the issue about enforcement of fiscal rules, there is an ongoing political debate regarding the design of fiscal rules, which has centered on the *degree of flexibility* of such rules with respect to macroeconomic shocks. On the one hand, there is support among academics and policy makers that fiscal rules should be (more) flexible in order to ensure smooth provision of governmental services and to avoid welfare losses in case of large negative shocks. For example, the European Commission has recently introduced more flexible interpretations in the handling of the Stability and Growth Pact (European Commission, 2015). A prominent group of French and German economists (Benassy-Quere et al. 2018) has advocated further flexibility. On the other hand, others are concerned that more flexible rules would be less effective in disciplining politicians that are biased towards excessive spending, and may lead to larger sovereign debt and higher risks of default (see for instance Burret and Schnellenbach (2013) and Deutsche Bundesbank (2017)).

In light of these debates, we ask how an optimal fiscal rule should look like when the political process leads to a deficit bias and monetary punishment mechanisms are absent. More specifically, we ask how restrictive in terms of a maximum deficit limit and how flexible in terms of accommodating shocks to public finances a fiscal rule should be. We are not the first to discuss the optimal design of fiscal rules. Our analysis of fiscal rules shares several similarities with the approach used in Amador, Werning, and

¹See IMF Fiscal Rules Data Set, 2013 and Budina et al. (2012), Yared (2019).

²In the preventive arm of the Stability and Growth Pact a deposit of 0.2% of GDP for Euro area countries is mandated in case of violation, but it has never been implemented, despite of multiple and repeated violations occurred in several EU member countries. In addition, in the corrective arm financial sanctions regarding in the European Structural and Investment Fund are foreseen. For details see European Commission (2018).

Angeletos (2006) and Halac and Yared (2014), in particular the role of a government that is present-biased towards public spending because of the overlapping-generation nature of the voter’s problem. Jackson and Yariv (2015, 2014) propose a model that exhibits similar properties. Other papers obtain an analogue result as a consequence of political turnover (e.g., Aguiar and Amador, 2011).³ Our approach differs in some important elements from the previous literature, however, and has the following two key features.

First, a shock to tax revenues makes compliance with a fiscal rule uncertain because the shock is realized after policymakers have submitted their budgetary plans and elections have taken place. Uncertainty about government revenues and expenditures is a central feature of budgetary planning and forecasting. We assume that the realization of the shock is observable to all agents in the economy including the designer of the fiscal rule. The latter assumption is a reasonable approximation in many countries in which independent fiscal institutions (“fiscal watchdogs”) either need to endorse government budget projections or do their own projections, and in addition assess compliance with fiscal objectives *ex post* (Beetsma and Debrun, 2018, Horvath, 2017). Our framework differs from Halac and Yared (2014, 2019), who assume that a shock to the value of public spending is observable to the government but not to the public and fiscal policy is chosen by the government after observing the realization of the shock. If in their framework the shock was observable and contractible, the first best allocation could be implemented. By contrast, in our setup the symmetric information does not guarantee optimality because present-biased policymakers draw up their fiscal policy plans prior to elections and the resolution of the budgetary uncertainty. An implication of our setup is that, consistent with the above evidence from existing fiscal rules, compliance with fiscal rule is a stochastic outcome and depends on the realization of the shock. In our setup we assume full commitment by policymakers to their policy platforms, who face an election that is held before the shock is realized. Adjusting fiscal policy after observing the fiscal shock might not always be possible because the time to response may be too short (e.g., the large inflow of refugees into the EU in the fall of 2015) and would imply that voters took this into account in the voting process, leading to less than full commitment.

Second, we assume that monetary punishments to rule violations are absent. A number of reasons motivate this assumption. In particular, such mechanism may not be credible, as the punishment would be wasteful *ex post*. It is either simply wasting resources or in the context of supranational institution like the EU involves a pure transfer of resources from countries with high marginal utility of the public good to countries which have a comparatively low marginal utility. In the latter case there is also the danger

³See also Persson and Svensson (1989), Alesina and Tabellini (1990), Alesina and Perotti (1995), Lizzeri (1999), Tornell and Lane (1999), Battaglini and Coate (2008), and Caballero and Yared (2010).

of exit in the sense that a country facing (monetary) punishments from the other countries or from a supranational institution may threaten to exit the union. In addition, the violation of a non-conditional deficit rule is typically more likely during a recession. Thus, a monetary punishment tends to have pro-cyclical effects on the government budget, and to reduce in turn the ability of policy-makers of smoothing public consumption over time. This may generate credibility issues, because the fiscal authority may step back on the commitment to punish violations during a recession. Instead we assume that the violation of a fiscal rule leads to a loss in the rent of holding office in the next period, which (partially) disciplines politicians. Such enforcement mechanism has the advantage - relative to the one based on monetary punishments - that the problem of the *credibility of commitment* to punish is typically less severe, at least as long as the enforcer of the fiscal rule is independent of the government. Moreover, because the punishment affects politicians rather than citizens, this mechanism is less prone to induce disruptive or normatively worrying consequences, e.g. it does not have direct pro-cyclical fiscal effects.⁴ A loss in the rent of holding office for violation of a fiscal rule requires that ultimately voters value compliance of fiscal rules, and that the media or other institutions like fiscal councils make non-compliance public.

Making these two key assumptions, we analyze whether an optimally designed fiscal rule can reach the outcome a social planner would choose, and we characterize the optimal fiscal rule. A fiscal rule consists in a function that maps the values of (1) the output and (2) the ratio of the tax shock to output into a maximum level of deficit to output. In order to characterize a deficit rule we define two measures. The first is *tightness*, i.e. the level of the maximum (structural) deficit. That is, the highest deficit level allowed under the rule if the tax shock takes its expected value of zero. The second is *flexibility*, i.e. the degree to which the tax shock modifies the maximum deficit level. The latter captures the degree to which fiscal rules should accommodate macroeconomic circumstances, one offs and other temporary measures. While the first generation fiscal rules like the Maastricht criteria (inter alia: headline deficit no more than 3% of GDP) did not account for these circumstances, second generation rules like the Fiscal Compact do exactly this (medium term objective for structural deficit 0.5% of GDP, see European Commission, 2018), and are more flexible.

To derive the optimal design of the fiscal rule, we consider an environment with two periods in which the government is present-biased towards public spending. Current generations of voters do not fully internalize the harm that public debt imposes on future generations. This preference structure results

⁴See Beetsma et al. 2017, Horvath 2017. Reputation effects are likely to play a role in the context of the Greek fiscal crisis. The enforcement of a delegation of EU, ECB, and IMF experts - the so-called Troika - to verify the implementation of certain reforms in Greece starting in 2010 can be seen as an example of non-monetary punishment. If a violation of a fiscal rule is considered a plus in the view of voters of the violating government, as may have been the case in the recent struggle between the Italian government and the European Commission, then fiscal rules may become ineffective in disciplining politicians. In such a case compliance with fiscal rules may be impossible to achieve.

naturally from the aggregation of heterogeneous, time-consistent citizen's preferences in an overlapping-generation setting. At the beginning of each period, a politician is chosen in a two-candidate contest. Politicians are purely office seekers and voting is probabilistic. The government in office chooses in each period a linear tax rate on labor income and the level of expected public debt. Together these policies jointly determine the amount of public spending on a public good. An i.i.d. shock affecting tax revenues is observed in period 1 after the policy choice and is public information. This feature implies that compliance with a fiscal rule is a stochastic outcome, as the government sets fiscal policy that includes a planned debt/deficit level, but the actual deficit/debt level emerges only after the shock is realized. Unlike most of the existing literature that either assumes perfect enforcement of fiscal rules or self-enforcing rules, our setup allows for occasional violation, which seems better in line with the evidence reported above.

We derive the following four main results. First, the benchmark policy of the social planner can be always implemented via an optimally designed fiscal rule (Prop. 3). A fiscal rule is therefore sufficient to deal with the joint issues of the political distortion and the stochastic nature of the budget process.

Second, we characterize the class of fiscal rules that implements the benchmark and satisfy some minimal conditions. We find that any of such optimal rules prescribes a *zero structural deficit* (Prop. 4i). The intuition that underpins this result is simple. Politicians' tax choices affect output by distorting labor supply decisions. The fiscal rule is in the form of a threshold on the deficit/output ratio. This implies that the maximum level of deficit allowed by the rule is increasing in output, at a rate equal to the value of the threshold itself. Thus, if the latter is set to zero, then there is no impact of output on the probability that a violation of the rule occurs, and therefore imposing a zero structural deficit is sufficient to ensure that tax choices are not distorted.

Moreover, we show that typically the *optimal rule accounts only partially for the tax shock*, that is, the maximum deficit under the rule is the target level minus a fraction less than one of the tax shock relative to GDP (Prop. 4ii). A full consideration of tax shocks under the target of a balanced structural budget is typically not optimal because either the marginal cost of increasing public debt in terms of expected cost of rule violation becomes too large and hence the rule induces a debt level that is too small, or the probability of punishment approaches 1, implying that the politician faces a fixed expected cost of rule violation, which does not affect their optimal choices. While we acknowledge the simplicity of our theoretical framework, we find it interesting to note that the result on the optimality of a zero structural deficit is close to the requirement of a (nearly) balanced budget laid down in Art. 3(1) of the Fiscal Compact, as well as the requirements in the German and Swiss debt brakes.

Third, any optimal fiscal rule prescribes *more flexibility* to governments that have - *ceteris paribus* -

stronger incentives to run excessive deficit in the first period, as measured by the political present bias due to the neglect of the interest of future generations in the current political process (Prop. 5). The intuition is the following: because the shock is not observed in the moment in which the fiscal policy is chosen in the first period, the policy maker faces a probability of being punished in the next period (if she gets reelected). The more flexible the rule is, the larger is the *marginal effect* of increasing the planned deficit on the probability of being punished. In other words, a more flexible rule is more effective in disciplining the politician because it implies a stronger link between current fiscal policy and the probability of a future punishment. At the extreme opposite of the spectrum, under a very inflexible rule the *marginal effect* of increasing expected deficit on the probability of being punished is very small, because the probability of being punished depends heavily on the realization of the macroeconomic shock, i.e. on luck rather than on the chosen fiscal policies.

Fourth, we analyze the case of a linear fiscal rule, under which the deficit target is a linear function of the tightness parameter and the parameter that captures the flexibility to the tax revenue shock. Such a rule is more in line with actual fiscal rules. Since current fiscal rules are often considered to be too complex, the study of simple rules is politically highly relevant. We show that the optimal policy can be implemented even under a linear fiscal rule, provided that the variance of the tax shock is sufficiently large (Prop. 6). In that case the properties of the optimal linear rule mimic those of the optimal general fiscal rule, that is, a zero structural deficit, optimal flexibility less than full, and flexibility is increasing in the political present bias (Corollary 7).

Our analysis is in some way complementary to Halac and Yared (2019), who also examine the design of optimal fiscal rules. Their framework differs from ours in a number of ways. Besides the above mentioned difference in timing of events, Halac and Yared look at the properties of the optimal punishment by society when a fiscal rule is violated. They show that incentives are high-powered in the sense that the continuation value of the government when the rule is complied with and when not is maximal (given exogenous limits on continuation values), which is in line with other work on optimal contracts in the presence of adverse selection. Importantly, our setup differs also from theirs by assuming that a distortionary labor income tax is used to fund public good spending, whereas in their setup the government is only concerned about shifting government resources from nondistortionary sources across time periods. In our framework compliance with a fiscal rule is thus linked via the taxation decision to the efficiency of the market outcome.

Our results relate to the design and use of fiscal rules in practice. First, the zero structural deficit is in line with those fiscal rules that require a (structurally) balanced budget or targets a balance near to

that, such as balanced budget rules in the US (for an analysis see, for example, Asatryan et al. 2018) or the German debt brake. The reason for using balanced budget rules in practice lies probably in their simplicity and intuitive appeal to policymakers and citizens, whereas in our model it is the interaction of the violation of rules and the distortionary effects of taxation that drive the result.

Second, as noted earlier, so called first generation fiscal rules like the Maastricht criteria do no account for business cycle effects and thus tend to have an undesirable procyclical effect. The second generation of fiscal rules therefore accounts for cyclical fluctuations. In practice, these rules are considered advantageous from an economic perspective, but are often criticized on practical matters, because the output gap is hard to estimate in real time and subject to substantial revision over short time periods. Our results indicate that full flexibility is not optimal even when the output gap estimation itself is not an issue. We discuss these and further policy aspects in section 5.

The remainder of the paper is organized as follows. In section 2 we describe the model and solve for the first best rule in the absence of political economy considerations. In section 3 we then introduce voting over candidates which leads to a present bias in government spending. The existence and features of the optimal (linear and nonlinear) fiscal rule are considered in section 4. In section 5 we discuss our theoretical results in light of the current debate in the EU on the design and flexibility of fiscal rules. Section 6 concludes.

2 Model

We study a small open economy that lasts for two periods $b = 1, 2$. The population of consumers-voters is a continuum of size 1 in each period. A share θ_1 of the population is of type $T = Y$ and cares both about the current period and about the next period, while a share $(1 - \theta_1)$ is of type $T = O$ and only cares about the current period. One can think about the two types to be “young” vs. “old” voters (an alternative interpretation could be “forward looking” and “myopic” voters). A young voter survives to period 2 with probability equal to π . Thus, a share $\pi\theta_1$ of the population lives for two periods. The political present bias that we introduce later into the model and drives our results is directly linked to this share. The individuals born at the beginning of period 2 represent a share $\theta_2 = 1 - \pi\theta_1$ of the total population in that period.

All individuals work and consume a consumption good in both periods. There are no savings.⁵ The government collects taxes on labor income and provides public goods in periods 1 and 2. Tax revenues

⁵Results qualitatively hold for an economy with savings.

are stochastic in period 1.

At the beginning of period $b = 1$ two candidates run for elections. Each of them fully commits to a policy platform consisting of a linear income tax rate on labor income and a level of planned debt. The actual level of debt is determined after a shock to tax revenues is realized given the policy package (t_1, g_1) implemented by the winner of the election. At the beginning of period $b = 2$ the same two candidates run for elections. Each of them fully commits to a policy platform consisting of a linear income tax rate on labor income. There is no default, thus all debt must be repaid in period 2, and the public good level follows as a residuum.⁶ An elected candidate always implements the platform he/she proposes before the elections.

A deficit rule can be imposed in period 1, whose violation carries cost for the government in period 2. The stochastic nature of tax revenues makes compliance with the deficit rule uncertain *ex ante*.

2.1 Private sector

Consumers in each period $b \in \{1, 2\}$ derive utility from consumption of a private good c_b , which is produced using labor as only input with a linear technology, and of a public good g_b . In each period $b \in \{1, 2\}$ individuals supply labor $l_b \in [0, \bar{l}]$ and are compensated at wage rate w_b (equal to their productivity). They face a strictly convex cost of labor $v(l_b)$. The wage at time 2 is assumed to be $w_2 \geq v'(\bar{l})$, which implies that the labor supply in period 2 is fully inelastic.

Income is taxed at a linear rate t_b , such that $c_b = (1 - t_b)w_b l_b$. Thus, the within-period utility of any type of consumer for $b \in \{1, 2\}$ is given by $U(c_b, l_b, g_b) = c_b - v(l_b) + u(g_b)$, where u is strictly concave and satisfies $\lim_{g_b \rightarrow 0} u'(g_b) = +\infty$. The lifetime utility of a young household born in period 1 is therefore

$$U(c_1, l_1, g_1) + \beta \pi E[U(c_2, l_2, g_2)], \quad (1)$$

where β is the discount factor. Individuals born in period 2 live for one period only. Thus, the young generation born in period 2 enjoys utility $U(c_2, l_2, g_2) = c_2 - v(l_2) + u(g_2)$.

Notice that the wage rate w_b and the utility cost of labor $v(\cdot)$ are identical across young and old citizens in any given period, as is the quasi-linear utility function. Thus, the two types face the same trade-off between utility from consumption and cost of labor. As a result, the optimal labor supply is the same across types. Because of that, for ease of notation we denote with l_b the labor supply of a citizen of any type in period b .

⁶These simplifying assumptions about public finances are imposed only for ease of exposition. All the results go through in a closed economy with endogenous interest rate and default.

2.2 Government sector

The government faces different decisions over time. In period 1 tax revenue has two components:

$$T_1 = t_1 w_1 l_1 + \epsilon, \quad (2)$$

where $t_1 \in [0, 1]$ is the tax rate, w_1 is the wage rate, and l_1 is the (endogenous) labor supply. The second component ϵ is the realization of a i.i.d. shock with support $\epsilon \in [-a, a]$, and such that $E[\epsilon] = 0$. Specifically, we assume that the shock on tax revenues ϵ is distributed as a two-sided symmetrically truncated normal. Its c.d.f. $F(\epsilon)$ looks as follows

$$F(\epsilon) = \begin{cases} 0 & \epsilon < -a \\ \Phi\left(\frac{\epsilon}{\sigma_\epsilon^2}\right) / \left[\Phi\left(\frac{a}{\sigma_\epsilon^2}\right) - \Phi\left(-\frac{a}{\sigma_\epsilon^2}\right)\right] & -a \leq \epsilon \leq a \\ 1 & \epsilon > a \end{cases} \quad (3)$$

where $\Phi(\cdot)$ is the c.d.f. of the standard normal distribution. The truncation is imposed to avoid problems such as negative public good supplies due to excessively large negative tax shocks.

The government can borrow from abroad at a fixed interest rate r . Let D_1^{act} denote the stock of debt at the end of period 1, after the tax shock has realized. The intended debt level D_1 is the one planned prior to the realization of the tax shock. Thus, $D_1^{act} = D_1 - \epsilon$, as ϵ is a positive tax revenue shock. In period 1, by assumption the government repays its existing debt inherited from the past D_0 . Before the shock is realized, the planned government budget in period 1 must satisfy

$$g_1 \leq t_1 w_1 l_1 - D_0(1 + r) + D_1. \quad (4)$$

We assume in the following that the budget constraint holds with equality and write public consumption good as function of the tax rate and the intended debt level $g_1(t_1, D_1)$.⁷

The government budget constraint in period 2 has formula:

$$g_2 \leq t_2 w_2 l_2 - (D_1 - \epsilon)(1 + r) = t_2 w_2 l_2 - D_1^{act}(1 + r) \quad (5)$$

Similarly to period 1, we construct $g_2(t_2, \epsilon)$, using the budget constraint in period 2.

⁷We impose the constraint $g_1(t_1, D_1) \geq 0$ to avoid negative public consumption, and set $g_1 = 0$ for all (t_1, D_1) such that $g_1(t_1, D_1) \leq 0$, if any exists. The assumption $u'(0) = +\infty$ ensures that this constraint is never binding.

We assume that the value of productivity w_2 is large enough to ensure that the repayment of debt in period 2 can be always fully satisfied. Specifically, we impose $\bar{D}_1 < w_2\bar{l}(1+r) - a$, where \bar{D}_1 represents the maximum value of the intended debt in period 1. Moreover, we assume that the choice of planned debt level D_1 lies in the range $[\underline{D}_1, \bar{D}_1]$. Lastly, the bounds $\underline{D}_1, \bar{D}_1$ satisfy $\beta(1+r) \geq u'(g_1(0, \bar{D}_1)) \geq 1$ and $\underline{D}_1 \leq D_0(1+r) - w_1\bar{l}_1$.⁸

2.3 Normative Benchmark: Social Planner's Problem

For this analysis we introduce a benevolent social planner who can set D_1 and t_1 optimally in period 1, from which the public good level in period 1 follows immediately from (4). Thus, the planner in period 1 chooses a policy $(t_1, D_1) \in X$, where $X \equiv \{(t_1, D_1) \in [0, 1] \times [\underline{D}_1, \bar{D}_1] \mid g_1(t_1, D_1) \geq 0\}$. In period 2, based on the actual debt level of period 1, the planner chooses the labor tax $t_2 \in [0, 1]$ and the public good level. Recall that labor supply is exogenous in period 2, which implies in conjunction with the separable utility function that g_2 is implicitly defined by the planner's first order condition for utility maximization in period 2, $u'(g_2) = 1$, and is independent of D_1 . Therefore, for a given actual inherited public debt level from period 1, D_1^{act} , the planner's optimal tax rate in period 2 follows from the government budget constraint (5). These considerations allow us to move to the analysis of the planner's period 1 optimization problem (while anticipating the period 2 outcome).⁹

Denote with $u^Y(t_1, D_1)$ the indirect expected lifetime utility enjoyed by a young voter in period 1 under policy (t_1, D_1) , and with $u^O(t_1, D_1)$ the one enjoyed by a old voter. The former writes:

$$u_1^Y(t_1, D_1) = (1 - t_1)w_1l_1(t_1) - v(l_1(t_1)) + u(g_1(t_1, D_1)) + \beta\pi E[(1 - t_2)w_2l_2 - v(l_2) + u(g_2(t_2, D_1, \epsilon)) \mid t_1, D_1] \quad (6)$$

where expectation are rational given history. The latter is given by:

$$u_1^O(t_1, D_1) = (1 - t_1)w_1l_1(t_1) - v(l_1(t_1)) + u(g_1(t_1, D_1)) \quad (7)$$

Lastly, the indirect utility of a young or old individual individual in period 2 writes:¹⁰

$$u_2^Y(t_2, D_1, \epsilon) = u_2^O(t_2, D_1, \epsilon) = (1 - t_2)w_2l_2 - v(l_2) + u(g_2(t_2, D_1, \epsilon)) \quad (8)$$

⁸These assumptions ensures that the socially optimal planned debt level and tax rate in period 1 are an interior solution, as illustrated in section 2.3.

⁹The assumption of exogenous labor supply in period 2 is solely a matter of convenience. If labor supply in period 2 is not fully inelastic, the equilibrium conditions illustrating the optimal intertemporal allocation of resources change slightly, but the trade-offs underpinning the social planner's choice are qualitatively unchanged.

¹⁰Notice that the indirect utility of a young individual in period 2 is identical to the one of an old individual in the same period.

The social planner maximizes the sum of the utilities of all individuals over both periods. He/she discounts the utility of the future generation at rate β .¹¹ Thus, her objective function writes

$$\theta_1 u_1^Y(t_1, D_1) + (1 - \theta_1) u_1^O(t_1, D_1) + \beta \theta_2 E[u_2^Y(t_2, D_1, \epsilon) | t_1, D_1], \quad (9)$$

where $\theta_2 = 1 - \pi\theta_1$ is the share of young individuals in period 2, as introduced above. It is easy to show that the social planner's objective function is strictly concave in (t_1, D_1) . Substituting the formulas from (6)-(8) for $u^Y(t_1, D_1)$, $u^O(t_1, D_1)$, and $u_2^Y(t_2, \epsilon)$ into the above, we derive the planner's problem

$$\begin{aligned} & (1 - t_1)w_1l_1(t_1) - v(l_1) + u(g_1(t_1, D_1)) + \\ \max_{(t_1, D_1) \in X} & + \beta E[(1 - t_2)w_2l_2(t_2) - v(l_2) + u(g_2(t_2, D_1, \epsilon)) | t_1, D_1] \end{aligned} \quad (10)$$

The solution to (10), denoted by (t_1^*, D_1^*) , is called the *optimal policy*. Notice that the social planner's objective function is independent of θ_1 , θ_2 , and π . Rational expectations imply that in period t_2 is chosen optimally given D_1 and ϵ . Thus, the first order conditions are:

$$[t_1] := w_1l_1(t_1)[u'(g_1(t_1, D_1^*)) (1 + \eta_1(t_1)) - 1] = 0 \quad (11)$$

$$[D_1] := u'(g_1(t_1, D_1^*)) - \beta(1 + r) = 0 \quad (12)$$

where $\eta_1(t_1)$ is the tax elasticity of labor supply at tax rate t_1 . The assumptions on the function u ensure that solution of the planner problem is interior. Condition (12) shows that social planner's optimal level of g_2 is independent of D_1 (due to exogenous labor supply). Hence the social cost of an increase in D_1 by one unit is the discounted value of the the repayment of debt and the interest on it.

2.4 Deficit rule

In section 3 we assume that fiscal policy in any given period is not chosen by a social planner but by a policymaker who won the election in that period. Because policymakers focus on current voters, the well-being of future generations is ignored. This generates a present bias and leads to excessive deficit, against which a deficit rule may be put in place. In the remainder of section 2 we describe the structure of the fiscal rule that will be considered in section 3.

For this purpose, let s_1 denote the tax shock to output ratio, i.e. $s_1 = \frac{\text{shock}_1}{\text{output}_1} = \frac{\epsilon}{y_1}$, and y_1 denote

¹¹Alternatively, one could assume that the social planner discounts utility of future generation at a different rate relative to the ones of young individuals in period 1.

the output, with $s_1 \in S$. Let $Y := [\underline{y}_1, \bar{y}_1]$ denote the range of admitted values of y_1 , where $\underline{y}_1 = w_1 l_1(\bar{t}_1)$ and $\bar{y}_1 = w_1 l_1(0)$, and with S the range of values for s_1 , i.e. $S := [-a/\underline{y}_1, a/\bar{y}_1]$.

A deficit rule R is in place, defined by the C^2 function $R : S \times Y \rightarrow \mathbb{R}$. The government is compliant with the rule after the realization of the tax shock if and only if

$$\frac{\text{deficit}_1}{\text{output}_1} = \frac{D_1^{\text{act}} - D_0}{y_1} = \frac{g_1 + rD_0 - t_1 y_1 - \epsilon}{y_1} \leq R(s_1, y_1), \quad (13)$$

where we have used (4) and the relationship between actual and planned deficit. Given a rule R , we define a threshold of the shock on tax revenues $\tilde{\epsilon}(t_1, D_1, y_1 \mid R)$, based on (13), below which the politician gets punished as the one that solves:

$$\frac{D_1 - D_0}{y_1} - \frac{\tilde{\epsilon}(t_1, D_1, y_1 \mid R)}{y_1} = R(\tilde{\epsilon}(t_1, D_1, y_1 \mid R) / y_1, y_1) \quad (14)$$

Our setup presumes that the realization of the tax shock is fully observable to voters and the designer of the fiscal rule. In practice, policymakers may have some room for manipulating the data if they have superior information. However, the presence of fiscal councils and the widespread use of government's fiscal and economic projections to be endorsed or verified by other, independent institutions make systematic manipulation unlikely. More generally, we believe that a noisy signal of the realization of the tax shock would not change our subsequent results, as long as voters and the rule designer take the signals rationally into account.

For any given rule R we define, the following concepts:

1. The *tightness* is the level of the rule R at $s_1 = 0$, that is, in a “normal” situation where the shock is zero,

$$K(y_1 \mid R) = R(0, y_1) \quad (15)$$

2. The *flexibility* is the marginal effect of a decrease in the shock-to-output ratio s_1 on the level of the rule R evaluated at $s_1 = \tilde{s}_1 \equiv \tilde{\epsilon}_1(t_1, D_1, y_1 \mid R) / y_1$,¹² i.e.

$$\Delta(t_1, D_1, y_1 \mid R) = - \frac{\partial R(s_1, y_1)}{\partial s_1} \Big|_{s_1 = \tilde{s}_1} \quad (16)$$

An interesting case, also considered below in detail, is the one represented by a *linear rule* in the form

¹²A more general definition of flexibility should consider the value of $-\frac{\partial R(s_1, y_1)}{\partial s_1}$ at all possible values of the shock to output ratio s_1 . In order to pin down a unique measure, it is natural to evaluate the derivative at \tilde{s}_1 , which is the value of s_1 at which flexibility does matter to determine the principal's punishment decision, and in turn the agents' choices.

$R(s_1, y_1) = k - \delta s_1$ for parameters $k \in [0, \bar{k}]$ and $\delta \in (-\infty, +\infty)$. In such case the government is compliant with the rule if and only if:

$$\frac{deficit_1}{output_1} \leq \underbrace{k}_{level} + \underbrace{\delta}_{flexibility} \times \left(-\frac{shock}{output_1} \right) \quad (17)$$

Notice that in the case of a linear rule tightness and flexibility are equal to the values of the parameters k and δ , respectively. Specifically, $K(y_1 | R) = k$ and $\Delta(t_1, D_1, y_1 | R) = \delta$.

3 Political Equilibrium

We now turn to a positive model of fiscal policy choices. In each period two candidates compete for the support of voters, and the elected winner implements her preferred choice. We use a probabilistic voting model in the tradition of Lindbeck and Weibull (1987) and Banks and Duggan (2005). The equilibrium concept is Subgame-Perfect Nash Equilibrium.

3.1 Timing of events and choices

At the beginning of period 1 two candidates denoted by superscript $I \in \{A, B\}$ run for elections. Each of them fully commits to a policy platform (t_1^I, D_1^I) consisting of a linear income tax rate and a level of planned debt. The (planned) level of public good follows from this policy proposal via the government budget constraint (4). The winner of the election implements her proposed platform. Voters observe the policy and choose their labor supply l_1 and consumption c_1 . The government collects labor taxes and provides a public good g_1 . At the end of period 1 a shock on tax revenues is realized and it is publicly observable. Such realization determines the actual level of debt accumulated D_1^{act} .

At the beginning of the second period a new election takes place between the same two candidates. Each of them fully commits to a policy platform consisting solely of a linear income tax rate t_2^I , which via the government budget constraint defines public consumption, as there is by assumption no tax shock and no new borrowing in the second period. Then - if a deficit rule is in place - a supranatural authority or an independent fiscal institutions verifies if a violation of the rule has occurred in period 1 and, if so, imposes a punishment to the politician in power. The punishment is thus a cost to the policy maker regardless of who was in power in the previous period. The winner of the elections implements her proposed platform. The government collects taxes, provides a public good g_2 , and repays debt.

As mentioned in the introduction, our setup is one of full commitment. Politicians are elected in

period 1 on the basis of their policy platform, that is implemented once a person is voted into office. This is a reasonable assumption if the time to respond after the shock is too short. Moreover, if policy adjustments were feasible ex post voters would need to take this into account ex ante. The motivation behind our assumption of punishment in case of rule violation can be understood in the context of the fiscal framework in the EU. Independent fiscal institutions at the national level, the EU Commission, and the general public notice violations of the fiscal rule, which in turn leads to a loss of reputation or other type of loss of the government in period 2. If the public or voters prefer a violation of the fiscal rule, the rule cannot be enforced. We come back to this issue when we discuss the results and possible extensions of our paper.

The politician that holds the office in period $b \in \{1, 2\}$ enjoys an exogenous rent W_b . If a violation of the fiscal rule has occurred in period 1, then the rent enjoyed by the politician that holds the office in period 2 - whether incumbent or not - is reduced by an exogenous amount $C < W_2$.¹³

Each politician wishes to maximize the weighted expected return of being in office in the two periods. In period 2 politicians take as given the actual debt level inherited from period 1, and chooses the tax rate. For example, politician A in period 1 maximizes

$$\begin{aligned} \Pi_1^A = & Pr(win_1^A | t_1^A, D_1^A, t_1^B, D_1^B) \times W_1 + Pr(win_2^A | t_1^A, D_1^A, t_1^B, D_1^B) \times W_2 \\ & - Pr(win_2^A, nc | t_1^A, D_1^A, t_1^B, D_1^B) \times C \end{aligned} \quad (18)$$

where win_s^A denotes the event corresponding to a victory of candidate A in the election at time b , and nc denotes the event of non-compliance with the fiscal rule in period 2.

The outcome of elections is probabilistic and shaped by voters' preferences.

In each period, each voter - given the platform proposed by both candidates - casts her vote for candidate A if the utility difference from electing A vs. B is positive. The utility difference depends upon a deterministic and a stochastic component. Recall that $u^T(t_1, D_1)$ represents the indirect expected lifetime utility enjoyed by a type T voter under policy (t_1, D_1) . The deterministic part consists of the difference between the utility induced by the policy platforms that each politician has proposed, i.e. $u^T(t_1^A, D_1^A) - u^T(t_1^B, D_1^B)$. The stochastic part is simply a common preference shock ν_1 , which is assumed to be i.i.d. across time independent of the tax shock ϵ , and normally distributed with mean μ and variance σ_ν^2 . Thus,

¹³Because we assume that the independent fiscal institution cares solely about voters' welfare and not about politicians' payoffs, this punishment strategy is always (weakly) ex-post incentive compatible for the institution.

a voter of type $T \in \{Y, O\}$ casts her vote for candidate A in period 1 if and only if

$$u_1^T(t_1^A, D_1^A) - u_1^T(t_1^B, D_1^B) + \nu_1 \geq 0 \quad (19)$$

Similarly, in period 2 a voter of type $T \in \{Y, O\}$ casts her vote for candidate A if and only if $u_2^T(t_2^A, D_1, \epsilon) - u_2^T(t_2^B, D_1, \epsilon) + \nu_2 \geq 0$.

3.2 Voting Equilibrium and Equivalent Problem

It is well known that in a large class of probabilistic voting model the equilibrium policy outcome corresponds to the platform that maximizes a weighted average of the voters' expected utilities (Lindbeck and Weibull, 1987; Banks and Duggan, 2005). In our setting, we can prove a similar result. Namely, under some technical restrictions¹⁴, there exists a symmetric equilibrium in which both politicians propose the same platform.¹⁵ Such a platform maximizes a weighted average of the expected utility of period 1's voters, *corrected* by a factor $C^e Pr(nc | (t_1, D_1), R)$, where the *equivalent reputational cost* C^e captures the expected reputational cost that the politician must face in period 2 as a consequence of the punishment that is imposed if a violation of the deficit rule R occurs, and $Pr(nc | (t_1, D_1), R)$ is the probability that the rule is violated ("nc" stands for non-compliance) given the policy implemented in period 1. The equivalent reputational cost C^e is itself a function of the exogenous rent loss C , and of the endogenous probability of reelection faced by each politician.

The probabilistic nature of the voting process, together with the presence of the fiscal rule, imply that the candidates' equilibrium platforms are identical to the policy that a partially benevolent social planner would choose, who differs from a social planner's problem (see 9) due to the possible cost of violation of the fiscal rule and the lack of accounting for future young generations. We will refer to this fictive agent as the *representative politician* or, more simply, the *politician*. The proof to this equivalence result is provided in Appendix A. In the rest of the paper, we use the objective function of the representative politician to characterize the policy choices in equilibrium. The politician's problem in period 1 writes:

$$\max_{(t_1, D_1) \in X} \theta_1 u_1^Y(t_1, D_1) + (1 - \theta_1) u_1^O(t_1, D_1) - \beta C^e Pr(nc | (t_1, D_1), R) \quad (20)$$

¹⁴The requirements are that both the variance σ_v^2 of the distribution of the voters' common taste shock and the rent W_1 from being in office in period 1 are sufficiently large. If these two conditions are satisfied, then the equivalence between the outcome of the electoral game and the choice of the representative politician holds true as long as the objective function of the representative politician to be strictly concave. Details in Appendix A.

¹⁵When no rule is in place such equilibrium is also the unique equilibrium of the electoral game. Otherwise, multiplicity is possible. Details in the online appendix.

Similarly, in period 2, the equilibrium platform maximizes the weighted expected utility of period 2's voters. Formal proofs of these results are provided in Appendix A.

Using the formulas for $u_1^Y(t_1, D_1)$, $u_1^O(t_1, D_1)$, and abstracting from the parts that do not affect the optimal outcome, one can rewrite the politician's problem as follows:

$$\begin{aligned} \max_{(t_1, D_1) \in X} & (1 - t_1)w_1l_1(t_1) - v(l_1) + u(g_1(t_1, D_1)) - \beta C^e Pr(nc \mid (t_1, D_1), R) + \\ & + \beta\pi\theta_1 E[(1 - t_2)w_2l_2(t_2) - v(l_2) + u(g_2(t_2, \epsilon)) \mid t_1, D_1] \end{aligned} \quad (21)$$

Comparing the above with the social planner's problem in formula (10), it is immediately evident that the two objective functions are identical, except for two aspects. First, the politician discounts future utility at rate $\beta\pi\theta_1$, while the social planner does so at rate β only. Because of that, we call $B_1 = 1 - \pi\theta_1$ the *political present bias*, which is the larger the smaller are the probability of survival and the size of the young cohort in period 1. Second, the politician's objective function includes a cost C^e to be paid if the fiscal rule is violated.

Before we turn to the characterization of the main results in section 4 on the optimal design of a fiscal rule it is useful to understand the effect of the political bias on fiscal policy even when a fiscal rule is absent. The first result states that a politician facing a more present-biased electorate tends to run a larger government deficit. This result holds both in the presence of a fiscal rule and with no fiscal rule.

Proposition 1. *The expected deficit is increasing in the political present bias $B_1 = (1 - \pi\theta_1)$.*

Proof. See Appendix B.

The result is not surprising given the nature of the political process. It is largely in line with political economic models of public debt in a context of intergenerational redistribution, as reviewed by Alesina and Passalacqua (2016).

The next result follows from Proposition 1 and implies that without a fiscal rule the political process leads to an inefficient outcome, because the voters, on average, do not care about the future as much as a benevolent social planner does.

Proposition 2. *In the absence of a fiscal rule the equilibrium level of deficit in period 1 is weakly larger than the optimal level.*

Proof. See Appendix B.

The deficit is strictly larger than the optimal level whenever there is a political present bias, that is, $\pi\theta_1 < 1$. An implication of Proposition 2 is that in the presence of a present bias the period 1 tax rate is lower than the socially optimal one.

4 The Design of the Fiscal Rule

In this section we characterize the optimal fiscal rule when fiscal policy is chosen via the political process described in the previous section. Proposition 2 gives room for a fiscal rule to improve the outcome. However, it is far from clear whether a fiscal rule can implement the optimal allocation that would be induced by a social planner who chooses the tax rate and the debt level in period 1 directly (which we denoted by t_1^*, D_1^*). In period 2 there is no political bias, thus the politician's policy choice is the same as the one of the social planner. But the period 2 choice is affected by the level of debt accumulated in period 1, thus it is typically different from the one that would prevail if the social planner had chosen the policy in period 1. In this sense, the equilibrium policy in period 1 spills over into period 2, even though there is no further shock in that period, a fiscal rule does not need to be satisfied, and taxation is lump sum as labor supply is exogenous.

Before moving to the main results, we first define implementation via a deficit rule and then define a desirable property of a deficit rule R .

In section 2.4 we have defined a deficit rule. In section 3.1 we have described the game played by the two candidates, and defined the equilibrium concept. Now we use these two concepts to define implementation. From the perspective of the principal a fiscal rule is fully optimal if it induces the agents to implement the same fiscal policy that the planner would choose if he could dictate his/her preferred policy in period 1. This concept is formally defined as follows.

Definition 1. A rule R is said to implement the optimal policy (t_1^*, D_1^*) for a given level of political present bias B_1 if there exists an SPNE of the electoral game such that the unique policy platform optimally chosen by both candidates in period 1 in the presence of such a rule is (t_1^*, D_1^*) .

Definition 1 clarifies that we adopt a weak concept of implementation which allows for the possibility of multiple equilibria of the electoral game.

Definition 2. A deficit rule R satisfies *tightness constant in output (TCO)* if $\frac{\partial R(0, y_1)}{\partial y_1} = 0$.

Condition *(TCO)* states that the level of structural deficit to output prescribed by the fiscal rule should not vary with the per capita income level. This is equivalent to requiring the tightness of the rule to be constant in y_1 , i.e. to rule out rules whose level is output-dependent. *(TCO)* is trivially satisfied by any rule that is output-independent, i.e. such that $\frac{\partial R(s_1, y_1)}{\partial y_1} = 0$ for all s_1 .

This property is deemed as desirable whenever output is a variable that can potentially be misrepresented or manipulated by the politician. Moreover, it restricts the attention to a class of rules that is arguably superior in terms of ease of adoption and implementation. For instance, output typically exhibits a positive time trend. Thus, a rule whose level is dependent on output is going to prescribe a different level of structural deficit over time. Lastly, *(TCO)* is satisfied by a large class of widely adopted deficit rules. For example, the linear rule described in section 2.4 trivially satisfies this condition because $R(0, y_1) = k$, which is constant in y_1 . This type of rule is further analyzed in sections 4.2 - 4.3.

4.1 Characterization of the Optimal Deficit Rule

Let $R_{B_1}^*$ denote a rule – if it exists – that implements the optimal policy (t_1^*, D_1^*) for a level of bias B_1 . Our first main result establishes that the optimal policy is implementable.

Proposition 3. *A deficit rule R that implements the optimal policy (t_1^*, D_1^*) and satisfies condition *(TCO)* always exists.*

Proof. See Appendix B.

Proposition 3 ensures implementability. Now we characterize the family of rules that are optimal in the sense of Definition 1. In this case we focus our attention on the class of rules that satisfy the property stated in Definition 2.

Proposition 4. *If a deficit rule R satisfies *(TCO)* and implements the optimal policy (t_1^*, D_1^*) , then (i) the tightness of the rule $K(y_1 | R)$ is zero, i.e. the rule prescribes zero structural deficit, and (ii) the flexibility of the rule $\Delta(t_1^*, D_1^*, y_1 | R)$ is lower than 1.*

Proof. See Appendix B.

Proposition 4 is a key result of our paper and describes the qualitative properties of the optimal fiscal rule. Proposition 4 (i) states that any output-invariant optimal rule prescribes *zero structural deficit*, i.e. zero deficit in the presence of a null shock $\epsilon = 0$. Notice that this does not necessarily mean that the actual expected deficit induced by the rule is going to be equal to zero, as the realization of the tax shock determines the actual deficit.

To understand Proposition 4 intuitively, it is helpful to recall the threshold level of the tax shock that just leads to fiscal rule compliance (14). If the tightness of the fiscal rule $K(y_1 \mid R)$ is non-zero, the tax rate influences the threshold via the effect on labor supply, that is $d\tilde{\varepsilon}/dt_1 \neq 0$. By contrast, $K(y_1 \mid R) = 0$ induces a tax setting by the policy maker that is optimal in the sense of balancing the marginal cost and benefits of taxation in that period given D_1 . From this it becomes clear that the flexibility $\Delta(t_1, D_1, y_1 \mid R)$ is driving the debt decision of the policy maker.

The results in Proposition 4 typically hold true even if a slightly different theoretical environment is assumed. For instance, in section 4.2 we show that the main insights of Proposition 4 hold true even if one restricts the attention to the family of linear deficit rules.

The next step consists in studying the comparative statics of the optimal rule. Specifically, we are interested in studying how the optimal degree of flexibility of the deficit rule responds to a marginal increase in the political present bias. In order to perform this exercise we need to impose additional structure because the optimal deficit rule is typically not unique for any given level of political present bias B_1 . Thus, we need a notion of monotonicity to account for the multiplicity. That is, we need to establish a criterium to compare the flexibility of any of the (possibly many) rules that are optimal at bias $B_1 = B'_1$ with that of any rule that is optimal at bias $B''_1 > B'_1$ for $|B''_1 - B'_1|$ arbitrarily small. Let R' denote a rule that is optimal at bias $B_1 = B'_1$. Informally, our approach consists in constructing all the possible parametric families that include R' and that possess one (or more) members that implement the optimal policy at bias level B''_1 . Then we evaluate the flexibility of any rule that is optimal at B''_1 and that is a member of one of those families. If the flexibility of all such rules is weakly higher than that of R' , and this result hold true for all rules R' that are optimal at $B_1 = B'$, then we say that the flexibility of the optimal rule is weakly increasing in the political present bias B_1 in a neighborhood of $B_1 = B'_1$. Formally, consider a family of rules ρ_r defined by the C^2 function $r : S \times Y \times Z_r$ with $Z_r = [\zeta_r, \bar{\zeta}_r]$.

A rule R is said to be a member of family ρ_r (and writes $R \in \rho_r$) if there exists $\zeta \in Z$ such that $R(\cdot, \cdot) = r(\cdot, \cdot; \zeta)$. Lastly, let $\zeta^*(B_1)$ denote the value of ζ such that a rule R with $R(\cdot, \cdot) = r(\cdot, \cdot; \zeta^*(B_1))$ implements the optimal policy (t_1^*, D_1^*) given bias B_1 . It is easy to show that a family ρ_r such that $R \in \rho_r$ can be constructed for any possible rule R .¹⁶ Moreover, it can be shown that for any family ρ_r such that $r(\cdot, \cdot; \zeta^*(B_1'))$ implements the optimal policy at $B_1 = B_1'$, then $r(\cdot, \cdot; \zeta^*(B_1))$ also implements the optimal policy in a neighborhood of $B_1 = B_1'$ under mild restrictions.¹⁷ Let $\mathcal{R}_r(B_1')$ denote the set of all families of rules ρ_r such that $R \in \rho_r$ and such that for any value of B_1 in a neighborhood of $B_1 = B_1'$ there exists $\zeta \in Z$ such that $r(\cdot, \cdot; \zeta)$ implements the optimal policy (t_1^*, D_1^*) .

Definition 3. (Monotonicity). Suppose R implements the optimal policy (t_1^*, D_1^*) for a given bias $B_1' \in (0, 1)$. Then the flexibility $\Delta(t_1^*, D_1^*, y_1 | R)$ of the optimal rule is weakly increasing in the political present bias B_1 in a neighborhood of $B_1 = B_1'$ if $\left. \frac{\partial \Delta(t_1^*, D_1^*, y_1 | r(\cdot, \cdot; \zeta^*(B_1)))}{\partial B_1} \right|_{B_1=B_1'} \geq 0$ for all possible families $\rho_r \in \mathcal{R}_r(B_1')$.

This definition delivers a very general notion of monotonicity. Namely, it applies to all possible families of rules that include R , and that admit a representation in the form $r(\cdot, \cdot; \zeta)$. Using this notion of monotonicity, we can state the next main result of this paper.

Proposition 5. *There exists finite $\varsigma > 0$ such that if the variance of the tax shock is sufficient $\sigma_\epsilon \geq \varsigma$, then the flexibility of the optimal rule $\Delta(t_1^*, D_1^*, y_1 | R_{B_1}^*)$ is weakly increasing in the political present bias B_1 .*

Proof. See Appendix B.

Notice that a more present-biased government requires, *ceteris paribus*, a larger reduction in the level of intended deficit in order to achieve the socially desirable outcome. Proposition 5 implies that such larger deficit reduction can be achieved through a more flexible deficit rule. Thus, the result in Proposition 5 suggests that flexibility may actually *encourage* fiscal discipline, rather than jeopardizing it. The

¹⁶For instance, setting $Z = \mathbb{R}_+$, the rule R is part of family ρ_r for $r(\cdot, \cdot; \zeta) = \zeta R(\cdot, \cdot)$ at $\zeta = 1$.

¹⁷In particular, one needs $\zeta_r < \zeta^*(B_1') < \bar{\zeta}_r$, and $r_{s\zeta}(1 + r_s) \neq r_{ss}r_\zeta$ at the optimal policy and at the threshold $\tilde{\epsilon}_1$. The latter condition ensures that the marginal probability of non-compliance with respect to D_1 is not invariant in ζ at $B_1 = B_1'$.

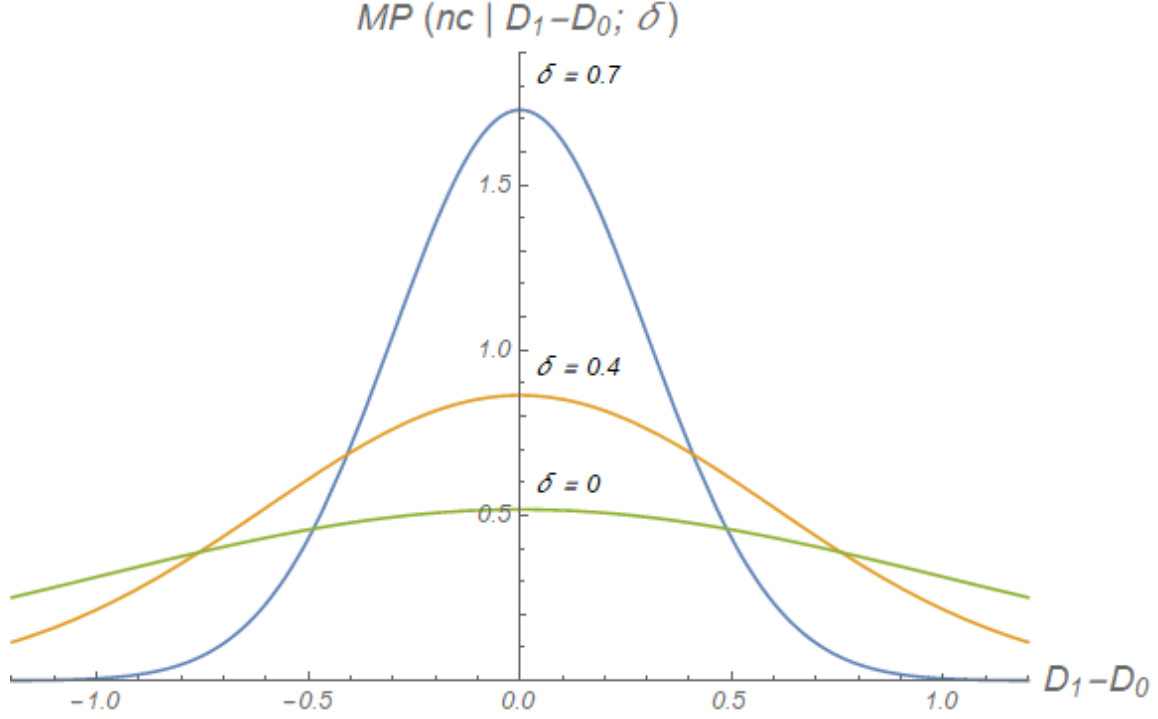


Figure 1: Marginal probability of non-compliance vs. deficit at three different flexibility levels.

intuition is the following. A more flexible fiscal rule reduces the weight of the shock on tax revenues in determining the probability of punishment, and increases the weight of the actual policy choices made by the politician. Therefore the *marginal effect* of running a larger expected deficit on the probability of punishment typically increases with the degree of flexibility. Figure 1 illustrates this point. The statement is true whenever the distribution of the tax shock is sufficiently “flat”, i.e. if σ_ϵ is large enough. As a result, a more flexible fiscal rule tends to be more effective in disciplining the politician. Therefore a trade-off between *fiscal discipline* and *flexibility* - often discussed among EU policy makers in recent times - may not always exist.

While Proposition 5 is reassuring in this context, one might be concerned that rules in practice are not complex enough to implement the socially optimal solution, with the possible consequence that the result on flexibility may no longer hold. We therefore turn to the case of a linear rule that appears to be much closer to actual fiscal rules.

4.2 Optimal Linear Rule

Consider a linear rule in the form $R(s_1, y_1) = k - \delta s_1$, with $k \in [0, \bar{k}]$, $\delta \in \mathbb{R}$, as introduced in section 2.4. Given this rule, the government is compliant if $\frac{\text{deficit}_1}{\text{output}_1} \leq k - \delta s_1$. Proposition 2 gives room for a fiscal rule to improve the outcome compared to no rule. However, a linear fiscal rule may not always be able to implement the optimal allocation (t_1^*, D_1^*) . Two problems may arise: Firstly, a linear rule may cause the politician's objective function to be non-concave, even if the social planner's objective function is concave.¹⁸ Secondly, even if the rule can improve the outcome, it may not be able to achieve the optimal policy within the range of admissible parameter values. Nevertheless, under certain conditions on parameters of the model both these problems can be resolved. Specifically, we can prove that the optimal policy is implementable as long as the variance of the shock on tax revenue ϵ is large enough. This finding is formalized in the following statement.

Proposition 6. *The linear rule implements the optimal policy if the tax shock has enough variance: $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$ for some $\bar{\sigma}_\epsilon \in (0, \infty)$.*

Proof. See Appendix C.

Proposition 6 delivers some important intuition. First, the implementation of the optimal policy through a linear rule is possible if the politician's marginal expected cost of violating the rule is large enough. This is ensured if the distribution of the shock on tax revenues has enough variance.

Our next result shows the equivalent of Propositions 4 and 5 in the context of a linear rule. For this purpose, define a threshold $\tilde{\delta} = 1 - \frac{|D_1^* - D_0|}{\sigma_\epsilon}$ for $D_1^* - D_0 \neq 0$ and $\tilde{\delta} = 1 - \frac{C^e \phi(0)}{2\Phi(a/\sigma_\epsilon) - 1} \frac{1}{\sigma_\epsilon} \frac{1}{B_1(1+r)}$ for $D_1^* - D_0 = 0$.

Corollary 7. *If the optimal policy (t_1^*, D_1^*) is implementable by a linear rule $R = k - \delta s_1$ for all B_1 within a range $[B_1', B_1'']$, then:*

- (i) *the implementation occurs at $k^* = 0$ and $\delta^* \leq \tilde{\delta}$;*
- (ii) *the optimal degree of flexibility δ^* is weakly increasing in the political present bias B_1 within such range.*

¹⁸The conditions for concavity to hold in the presence of a linear rule are non-trivial. If the tax shock is truncated-normally distributed, they imply the variance of the distribution to be sufficiently large relatively to the maximum deficit $\bar{D}_1 - D_0$. See Appendix A.2 for details.

Proof. See Appendix C.

The main insight of Corollary 7 is that under the optimal linear rule tax shocks are not fully, but only partially accounted for, as $\tilde{\delta}$ is always strictly below one.¹⁹ To see that a full consideration of tax shocks ($\delta = 1$) under the assumption $k = 0$ is never optimal, note that the marginal cost of increasing public debt in terms of expected cost of rule violation reaches a peak in the interior of the interval $(0, 1)$, and is decreasing in δ if the latter parameter is close enough to 1. This is shown in Figure 2, where the marginal probability of non-compliance as function of δ is plotted for three levels of the deficit $D_1 - D_0$. Hence, increasing the flexibility δ in the downward sloping part of the curve improves rather than worsens the incentives for deficit making, contrary to what is needed. In addition, Corollary 7 (ii) states that the optimal flexibility δ^* is weakly increasing in B_1 . Intuitively, an increase in the bias B_1 does not affect the policy choice of the politician if it is just compensated by an increase in the marginal expected cost of rule violation. It follows that the optimal flexibility δ^* must lie within a range of values such that the expected cost of rule violation is increasing in δ . Thus it cannot lie in a neighborhood of $\delta = 1$. This result is consistent with the general result in Proposition 4 (ii), which states that optimal flexibility is lower than 1.

Figure 2

In summary, the results in Corollary 7 are in line with Proposition 4 and 5. The optimal linear rule prescribes *zero structural deficit*, i.e. zero deficit in expectation, and shows that flexibility may actually *encourage fiscal discipline*, rather than jeopardizing it.

4.3 Linear rule: Non-Implementable Case

Suppose that the optimal policy described in Corollary 7 is not implementable through a linear fiscal rule $(0, \delta^*)$, because the condition $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$ that ensures implementability is not satisfied. In this case, we cannot rule out that the best linear rule that is consistent with the permissible parameter space $k \in [0, \bar{k}]$ could be worse than not having a linear fiscal rule at all, and hence would not be optimal. In the following, however, we characterize the *optimal linear rule* that maximizes the social planner's utility,

¹⁹Note that the optimal deficit rule may feature negative flexibility, i.e. $\delta^* < 0$. This outcome corresponds to the case in which the cost of violating the rule C^e is very large, such that the representative politician's optimal policy features a sub-optimally low level of deficit for any rule with $k = 0$ and $\delta \in [0, \tilde{\delta})$. In words, the rule provides too much fiscal discipline. As such, this case is unlikely to occur within the range of empirically relevant values of the parameters of the model. Moreover, we can show that $\delta^* \in [0, \tilde{\delta})$ if W_1 is sufficiently large, i.e. $W_1 \geq \bar{W}_1$ for some threshold $\bar{W}_1 > 0$. The proof to this result is provided in the online appendix.

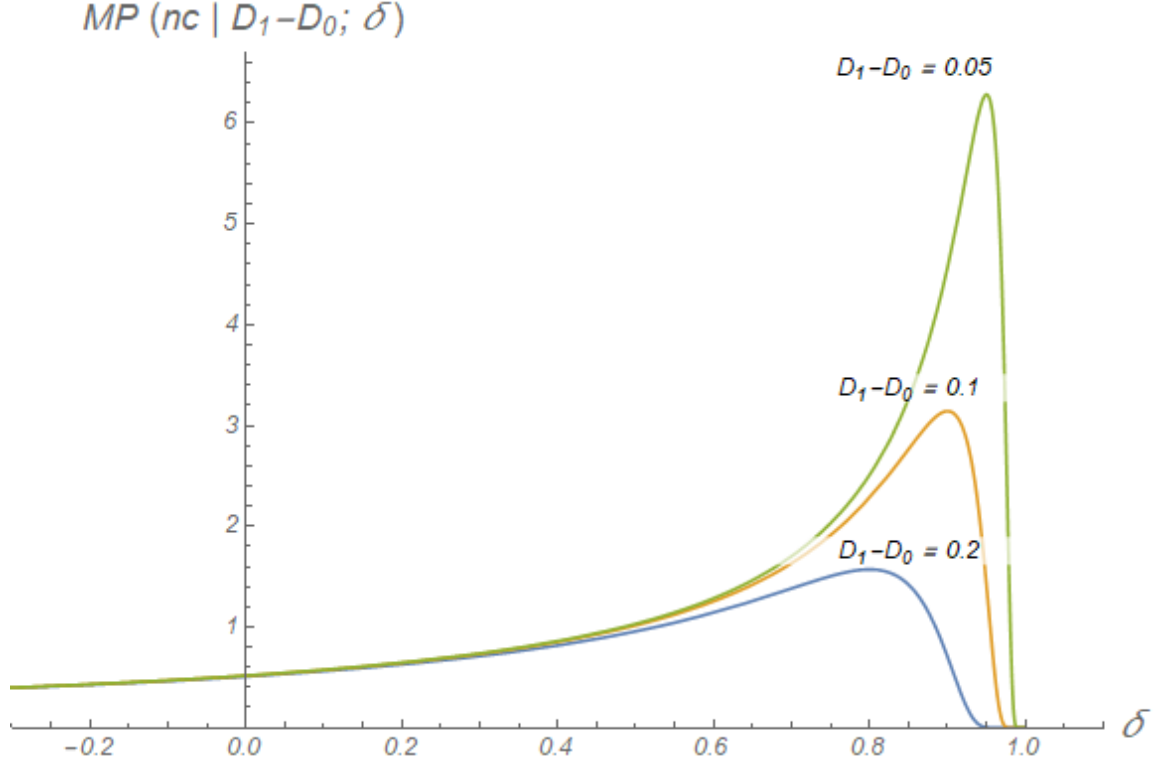


Figure 2: Marginal probability of non-compliance vs. flexibility at three different deficit levels.

assuming that it strictly improves with respect to the case in which no rule is in place. This allows us to get insights about the direction of the tightness and flexibility parameters when the optimal policy cannot be implemented but still a fiscal rule is beneficial. In that sense, we characterize a second-best rather than a third-best environment.

Let (t_1^{**}, D_1^{**}) be the policy chosen by the representative politician given a rule (k, δ) . Define $\delta^{max}(t_1, D_1) \in (-\infty, 1)$, if it exists, to be the level of δ that maximizes the expected marginal cost of punishment for the politician at $k = 0$, i.e.

$$\hat{\delta}(D_1) \equiv \arg \max_{\delta \in (-\infty, 1)} \left\{ \frac{\partial}{\partial D_1} \left[\frac{1}{1-\delta} f\left(\frac{D_1 - D_0}{1-\delta}\right) \right] \right\} \quad (22)$$

Using the truncated-normal distribution it is easy to show that $\hat{\delta}(D_1) = 1 - \frac{|D_1 - D_0|}{\sigma_\epsilon} \leq 1$. Lastly, we recursively define δ^{max} as $\delta^{max} \equiv \hat{\delta}(D_1^{**})$, which satisfies $\delta^{max} < 1$. Using this definition we can state the following result.

Proposition 8. *Assume that the optimal policy is not implementable by a linear rule. (i) Whenever*

the optimal linear rule requires a zero structural deficit level, $k^ = 0$, the optimal linear rule makes use of the maximum flexibility in the sense $\delta^{opt} = \delta^{max}$. (ii) If the tax elasticity of labor supply is small, i.e. $|\eta_1(t_1^{**})| < \bar{\eta}$ for some $\bar{\eta} > 0$, the optimal linear rule is characterized by a strictly positive deficit maximum, $k^* > 0$.*

Proof. See Appendix C.

Proposition 8 provides a characterization that is far from complete. Yet, it delivers an important insight regarding the intuition that underpins the main results of this paper. The regulator's key trade-off when setting a deficit rule is between the benefit of disciplining present-biased politicians and the cost of generating distortions in the choice of tax rates affecting labor supply decisions. Corollary 7 (i) suggests that the authority can avoid intra-temporal distortions by choosing a rule that prescribes zero structural deficit ($k = 0$), but such choice reduces the set of instruments that the regulator can use to discipline the politician to the sole flexibility parameter δ .

Under the conditions in Corollary 7 flexibility alone is sufficient to provide the optimal amount of discipline, and therefore the socially optimal policy is implementable. However, if the conditions in Corollary 7 are not satisfied it is not possible to induce sufficient fiscal discipline using flexibility only. Thus, the regulator can improve social welfare by manipulating the tightness of the fiscal rule k . Whether this is optimal or not depends on the trade off between the benefit of reducing inter-temporal distortions and the cost of generating intra-temporal inefficiencies in labor supply decisions. Thus, the optimal tightness in this case depends upon the tax elasticity of labor supply. If η_1 is large in magnitude, then the distortions on labor supply are substantial, thus the optimal deficit rule prescribes $k^* = 0$, even if this implies a sub-optimal inter-temporal allocation of resources. Conversely, if the labor supply is sufficiently inelastic the regulator optimally allows for some intra-temporal inefficiency in order to achieve stronger fiscal discipline.

5 Discussion of model and policy implications

Model Extensions

Our results depend on a number of simplifying assumptions, which we now briefly discuss. Firstly, our analysis abstracts from the possibility of asymmetric information between the regulatory authority, the politicians, and the voters. Our results imply that the degree of flexibility of the optimal deficit rule depends upon the level of average present bias within the population of voters. Thus, if such level is not

perfectly observable by the principal, then agents may have an incentive to misrepresent the extent of the bias in order to obtain a more favorable deficit rule. While this is a concrete theoretical possibility, we believe that is not likely to be a key issue for the purposes of our application. The reason is that regulatory authorities are typically independent from the executive power.²⁰ Thus, it is reasonable to assume that the principal infers the level of B_1 directly from the citizens' observable characteristics and behavior, such as sociodemographic composition and past electoral choices, rather than from information reported by the ruling politicians. Voters are less likely to manipulate the design of the rule than politicians because the assumption of a very large number of voters implies that each of them has no strict individual incentive to misrepresent his/her preferences. Moreover, a collective action carried out by a large number of voters aiming to distort the principal's design of the deficit rule requires a substantial degree of coordination and sophistication in voters' choices, which seems hardly plausible.

Secondly, we assume that the voters' taste shock are independently distributed over time. Allowing for serial correlation does not qualitatively change our result, but may have consequences on the optimal degree of flexibility. Specifically, in the presence of positive serial correlation every rule tends to be – *ceteris paribus* – more effective in disciplining the politician, while the opposite is true if the correlation is negative. Because higher flexibility corresponds to stronger fiscal discipline in our model, the obvious consequence is that the optimal deficit rule prescribes less flexibility relative to the baseline result in the former case, and more flexibility in the latter. The intuition is simple. Positive serial correlation translates into an “incumbent effect” in period 2, because a candidate's electoral success today implies a higher probability that the same candidate will win the elections tomorrow. Whenever the incumbent is more likely to be reelected, he/she assigns a higher weight to the payoff from being in office in period 2. Thus, each politician in period 1 is less “present biased” if the taste shock exhibits positive serial correlation.

Thirdly, we assume that voters and the regulatory authority possess perfect information regarding politicians' competence, preferences, and moral standards. As a result, voters' choices in period 2 are independent of the politicians' behavior in period 1. That is, we rule out the possibility of *retrospective voting* aiming to punish incompetent or dishonest politicians. While this is admittedly a strong restriction, it does not drive the key tradeoffs that underpin our results. Nevertheless, it has consequences for the optimality of a deficit rule. For instance, if economic performance and tax revenues are a function of the ruling politician's unobservable ability, then each politician may have an incentive to propose an excessively generous fiscal policy in period 1 in order to signal high competence to voters. As a result, the

²⁰Also notice that politicians do not necessarily benefit from a more flexible rule ex-ante.

flexibility of the deficit rule should be adjusted in order to induce the optimal degree of fiscal discipline, but the optimally designed rule may no longer be capable of implementing the socially optimal outcome.

Policy Relevance

Our results speak to the actual design and use of fiscal rules. First, the zero structural deficit is in line with those fiscal rules that require a (structurally) balanced budget or targets a balance near to that. For example, many countries and states in the US have balanced budget rules (for an analysis of balanced budget rules see, for example, Asatryan et al. 2018). The German debt brake requires the federal government to run a deficit of no more than 0.35% of GDP, and imposes a balanced budget from states (Länder). The reason for (near) balanced budget rules in practice lies probably in its simplicity and intuitive appeal to policymakers and citizens, rather than in our more sophisticated argument that is based on the interaction of the violation of rules and the distortionary effects of taxation. The intention of such rules is to lower deficits or debt levels, and the evidence in Asatryan et al. (2018) shows that the rules have such an effect.

Second, as noted earlier, so called first generation fiscal rules like the Maastricht criteria do no account for business cycle effects and thus tend to have an undesirable procyclical effect. The second generation of fiscal rules, such as the German debt brake or the Fiscal Compact, have been designed to account for cyclical fluctuations. Given the definition of a cyclical effect, often measured by the difference between potential and actual output (output gap), the two rules mentioned fully adjust for the business cycle. In practice, these rules are considered advantageous from an economic/conceptual perspective, but are often criticized on practical matters, because the output gap is hard to estimate in real time and subject to substantial revision over short time periods, and still appear to lead too often to procyclical fiscal policies. Our results indicate that full flexibility is not optimal even when the output gap estimation itself is not an issue.

Recent interpretation of the Stability and Growth Pact by the European Commission (2015) introduces further flexibility regarding the required fiscal adjustment towards the medium term objective (MTO), a country-specific deficit target (often around 0.5-1%), when the MTO has not been reached. In particular, the European Commission demands lower fiscal adjustment the worse is the current output gap. Our model speaks indirectly to this issue because the Commission is concerned with the *adjustment* to the MTO when the fiscal target has not been reached, while our model concerns the *level* of the deficit target. In the Commission's framework, a lower adjustment speed in case of a bad shock can be interpreted in our framework as a looser deficit target however. Since the EU fiscal rules do account for the business cycle, the additional flexibility seems to suggest more than full responsiveness to shocks, which is in contrast to

our results. Interestingly, the European Fiscal Board (EFB) in its recent report (2019) calls for discarding the flexibility interpretation because the rules have “failed to generate differentiated recommendations that reconcile sustainability and stabilization objectives” (p. 74).

Finally, our results suggest a novel link between flexibility of rules and fiscal discipline, and recommends more flexible rules for countries with a stronger deficit bias. It is hard to analyze this relationship empirically for a number of reasons: lack of data (over time) and identification challenges, in part due to endogenous and time varying enforcement. A very tentative look at the number of violations of the fiscal rule under the Stability and Growth Pact before and after the introduction of the 2015 flexibility clause does not provide clear evidence, probably because of confounding factors and the small number of observations (see European Fiscal Board 2019, Table 2.7). The compliance with the preventive arm of the SGP matches well with the planned deficit in our model. We also note that the present bias in practice may be to some extent endogenous, while in our theoretical model it is exogenous due to the generational structure. Hence, recommending more flexibility of rules could be problematic from a normative perspective if the present bias can be manipulated, for example, through changes in institutions that lead to less stable governments (bigger common pool problems). Nevertheless, our result is useful because it sheds new light on the nature of the trade-off between flexibility and fiscal discipline.

6 Conclusion

The role played by the adoption of fiscal rules in disciplining present-biased governments and reducing excessive public spending is at the core of the political debate in the European Union and beyond. Specifically, politicians and policy makers have been discussing how to optimally trade off the benefit of committing the government to not overspend against the benefit of granting it flexibility to react to fiscal shocks. In a setting that incorporates some of the features that characterize the regulatory and political environment of the European Union, such as the use of rules that target the deficit-to-GDP ratio and the induced government reputation loss for violation of fiscal rules, we show that this trade off may not always exist. Specifically, we show that a larger degree of fiscal flexibility typically encourages fiscal discipline, rather than jeopardizing it, and in the case the first best can be implemented the optimal degree of flexibility is partial and increasing in the political bias.

Our main result holds true both for general deficit rules, and if the regulator is constrained to use linear rules. We show that a deficit rule that induces the government to implement the ex-ante socially optimal fiscal policy always exists. Moreover, we show that, under mild restrictions, the regulator can

achieve the most preferred outcome by using a simple linear rule. Lastly, we show that the optimal deficit rule always prescribes zero structural deficit, i.e. the rule should punish any deviation from balanced budget unless a negative tax shock has occurred.

Appendix

Appendix A Political Process

Description of the two-candidate electoral competition. Voters have preferences in period 1 given by formulas (6) and (7), and in period 2 by formula (8). Let ϑ_1 and ϑ_2 denote the share of young individuals in period 1 and 2, respectively.²¹

We assume an exogenous birth process and positive probability of survival from period 1 to period 2 denoted by π . Namely, suppose that an additional generation of size ϑ_2 is born in period 2 and that a share π of the past period young generation survives and becomes the old generation in the following period. This implies that in the second period there are $\pi\vartheta_1$ old voters and ϑ_2 young voters. Lastly, assume that the total size of the population remains constant in the two periods, which implies $\vartheta_2 = 1 - \pi\vartheta_1$. This means that the share of elderly voters in the economy increases between the two periods if $\pi \geq \frac{1-\vartheta_1}{\vartheta_1}$, and decreases otherwise.

We adopt a modified version of Lindbeck and Weibull's (1987) and Banks and Duggan's (2005) probabilistic voting model.

Period 2. Define the share of type $T \in \{Y, O\}$ voters that vote for candidate A in period 2 as:

$$\Pi_2^{A,T}(t_2^A, t_2^B, D_1, \epsilon) = H_1^T(u_2^T(t_2^A, D_1, \epsilon) - u_2^T(t_2^B, D_1, \epsilon) + \nu_2), \quad (23)$$

where H_1^T is increasing, $\frac{\partial H_1^T(x)}{\partial x} = h_1^T(x)$ and $h_1^T(0) = 0.5$ for any $T \in \{Y, O\}$.²² The first assumption

²¹Note that the notation ϑ_b differs from θ_b used in the main body of the paper. Later in this section it will become clear that our theoretical framework implies $\theta_b = \vartheta_b$ for $b = 1, 2$.

²²Notice that, as in Banks and Duggan 2005, $H_2^T(\bar{x}_2 + \nu_2)$ can be interpreted also as the probability of a voter of type T to vote for politician A conditional on ν_2 and $x = \bar{x}_2$. This is true because, if the number of voters is arbitrarily large, then the law of large numbers implies that the share of votes for candidate B denoted with $H_2^T(\bar{x}_2 + \nu_2)$ becomes *exactly* equal to such probability. To see this, define an i.i.d. random variable ξ_j with c.d.f. $H_2^T(\cdot)$ and p.d.f. $h_2^T(\cdot)$. Say an individual j vote for B if $-\bar{x}_2 - \nu_2 + \xi_j \geq 0$, which means that $Pr(\text{vote } B | \bar{x}_2 + \nu_2) = \int_{\bar{x}_2 + \nu_2}^{\infty} h_2^T(\xi) d\xi = 1 - H_2^T(\bar{x}_2 + \nu_2)$ and therefore $Pr(\text{vote } A | \bar{x}_2 + \nu_2) = H_2^T(\bar{x}_2 + \nu_2)$. The share of votes for candidate A for a voting population of size n is given by $\frac{\sum_{j=1}^n \mathbf{1}(\xi_j \leq \bar{x}_2 + \nu_2)}{n}$. The law of large numbers implies that $\lim_{n \rightarrow \infty} \frac{\sum_{j=1}^n \mathbf{1}(\xi_j \leq \bar{x}_2 + \nu_2)}{n} = E_\xi[\mathbf{1}(\xi_j \leq \bar{x}_2 + \nu_2)] = \int_{-\infty}^{\bar{x}_2 + \nu_2} h_2^T(\xi) d\xi = H_2^T(\bar{x}_2 + \nu_2)$, implying that, for an arbitrarily large number of voters, $H_2^T(\bar{x}_2 + \nu_2)$ is both the share of votes for A and the probability of A winning given ν_2 . As a consequence, the uncertainty in the electoral outcome is entirely due to the common shock ν_2 . This implies in turn that for a large electorate the presence of a common shock ν_2 is necessary to have probabilistic voting. Without ν_2 the electoral outcome would be deterministic for all values of x_2 , except the exact point in which $H_2^T(\bar{x}_2) = .5$ and the two candidates win with equal probability.

implies that the share of votes for candidate A is weakly increasing in the utility difference induced by the policies proposed by candidate A and candidate B (standard). The second assumption states the two types of voters do not have ex-ante asymmetric preferences for the two candidates and that if the two candidates propose the same platform, then the expected share of votes for each candidate is .5. Moreover, ν_b is a continuous i.i.d. random variable with c.d.f. $G_b(a)$. The random variable ν_b represents a random realization of some shift in voters' behavior due to circumstances that cannot be foreseen by the candidates and it is common to all voters. Thus, the share of vote for candidate A in the whole population of voters in period 2 is

$$\Pi_2^A(t_2^A, t_2^B, \vartheta_2) = \frac{\vartheta_2 H_2^Y(u_2^Y(t_2^A, D_1, \epsilon) - u_2^T(t_2^B, D_1, \epsilon) + \nu_2)}{+(1 - \vartheta_2) H_2^O(u_2^O(t_2^A, D_1, \epsilon) - u_2^O(t_2^B, D_1, \epsilon) + \nu_2)}. \quad (24)$$

The probability of victory for candidate A vs B in period 2 is:

$$\pi_2^A(t_2^A, t_2^B, D_1, \epsilon, \vartheta_1) = \frac{Pr[\vartheta_2 H_2^Y(u_2^Y(t_2^A, D_1, \epsilon) - u_2^T(t_2^B, D_1, \epsilon) + \nu_2)}{+(1 - \vartheta_2) H_2^O(u_2^O(t_2^A, D_1, \epsilon) - u_2^O(t_2^B, D_1, \epsilon) + \nu_2) \geq .5]}, \quad (25)$$

while the probability of victory for candidate B in period 2 is simply $\pi_2^B(t_2^A, t_2^B, D_1, \epsilon, \vartheta_1) = 1 - \pi_2^A(t_2^A, t_2^B, D_1, \epsilon, \vartheta_1)$. Politician A in period 2 maximizes her expected payoff, which is given by:

$$P(t_2^A, t_2^B, D_1, \epsilon, \vartheta_1, W_2, C, ph) = [W_2 - C \times \mathbf{1}(ph = nc)] \pi_2^A(t_2^A, t_2^B, D_1, \epsilon, \vartheta_1) \quad (26)$$

where $ph \in \{nc, c\}$ indicates whether the government was compliant to the fiscal rule in period 1. One can easily derive the expected payoff of candidate B by noticing that the share of voters of type T that vote for candidate B is $1 - H_2^T$. Lastly, define the weight θ_b for $b \in \{1, 2\}$ as follows:

$$\theta_b = \frac{\vartheta_b h_b^Y(0)}{\vartheta_b h_b^Y(0) + (1 - \vartheta_b) h_b^O(0)} \quad (27)$$

which implies $\theta_b = \vartheta_b$ if $h_b^Y(0) = h_b^O(0)$.

We omit the description of optimal candidates' behavior in period 2 because it is a standard outcome of the Lindbeck and Weibull's (1987)'s framework, namely, in period 2 both candidates solve a standard two-candidates probabilistic voting symmetric zero-sum game. Standard results in Banks and Duggan (2005) and Lindbeck and Weibull (1987) apply. Specifically, if the distribution of the voters' taste shock

ν_2 has large enough variance,²³ then there exists a unique Nash equilibrium, which is in pure strategies, and such that both candidates propose the same platform and win the elections with equal probability. Lastly, the equilibrium platform of both candidates is the policy that maximizes the expected utility voters in period 2. Details of the proofs are provided in the online appendix.

Period 1. The electoral game in period 1 is slightly different from the standard Lindbeck and Weibull's (1987)'s framework. Candidates in period 1 possess perfect foresight regarding future outcomes. Thus, the problem in period 1 is solved by backward induction and results in a SPNE of the electoral game.

Similarly to period 2, the probability of victory for candidate A vs B in period 1 is:

$$\pi_1^A(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta_1) = \frac{\Pr [\vartheta H_1^Y (u_1^Y(t_1^A, D_1^A) - u_1^Y(t_1^B, D_1^B) + \nu_1) + (1 - \vartheta) H_1^O (u_1^O(t_1^A, D_1^A) - u_1^O(t_1^B, D_1^B) + \nu_1) \geq .5]}{1} \quad (28)$$

where H_1^Y, H_1^O represent the probability that voters of type Y, O vote for candidate A . The probability of victory for candidate B in period 1 is simply $\pi_1^B(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta) = 1 - \pi_1^A(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta)$. Politician A in period 1 maximizes her expected payoff, which using the assumption that ϵ and ν_1 are independently distributed, has formula:

$$\begin{aligned} P(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta_1, W_1, W_2, C) = \\ \{W_1 + \beta E_1 [\pi_2^A(\hat{t}_2^A, \hat{t}_2^B, D_1, \epsilon, \vartheta_1) (W_2 - C \Pr(nc | (t_1, D_1), R)) | t_1, D_1]\} \pi_1^A(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta) \\ + \beta E_1 [\pi_2^A(\hat{t}_2^A, \hat{t}_2^B, D_1, \epsilon, \vartheta_1) (W_2 - C \Pr(nc | (\bar{t}_1^B, \bar{D}_1^B), R)) | t_1, D_1] [1 - \pi_1^A(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta)], \end{aligned} \quad (29)$$

where \hat{t}_2^A, \hat{t}_2^B are the rational expectation values of t_2^A, t_2^B conditional on t_1, D_1 . One can easily derive the expected payoff of candidate B by noticing that the share of voters of type T that vote for candidate C is $1 - H^T$. Let $\tilde{\nu}_1(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta_1)$ solve

$$\vartheta_1 H_1^Y (u^Y(t_1^A, D_1^A) - u^Y(t_1^B, D_1^B) + \tilde{\nu}_1) + (1 - \vartheta_1) H_1^O (u^O(t_1^A, D_1^A) - u^O(t_1^B, D_1^B) + \tilde{\nu}_1) - .5 = 0 \quad (30)$$

i.e. $\tilde{\nu}_1$ is the level of common taste shock ν_1 such that each of the two candidates obtain exactly half of the votes given policy platforms $(t_1^A, D_1^A), (t_1^B, D_1^B)$. Define $\tilde{\nu}_1^A(t_1, D_1) \equiv \tilde{\nu}_1(t_1, D_1, \bar{t}_1^B, \bar{D}_1^B, \vartheta_1)$ and $\tilde{\nu}_1^B(t_1, D_1) \equiv \tilde{\nu}_1(\bar{t}_1^A, \bar{D}_1^A, t_1, D_1, \vartheta_1)$

Both politicians and voters can fully anticipate the outcome in period 2 conditional on the choices made

²³This condition corresponds to the restriction on $g'_2(\nu_2)/g_2(\nu_2)$ in Lindbeck and Weibull (1987).

in period 1 and the realization of the shock. Using the optimal proposals in period 2 (conditional on the realization of the tax shock ϵ), the objective function of a politician simplifies and the problem of candidate A is:

$$\max_{\{t_1, D_1\} \in X} \begin{aligned} & \{1 - G_1 [\tilde{\nu}_1(t_1, D_1, \bar{t}_1^B, \bar{D}_1^B, \vartheta)]\} [W_1 - 0.5\beta CPr(nc | (t_1, D_1), R)] \\ & - G_1 [\tilde{\nu}_1(t_1, D_1, \bar{t}_1^B, \bar{D}_1^B, \vartheta)] 0.5\beta CPr(nc | (\bar{t}_1^B, \bar{D}_1^B), R) + 0.5\beta W_2 \end{aligned} \quad (31)$$

and similarly for candidate B :

$$\max_{\{t_1, D_1\} \in X} \begin{aligned} & G_1 [\tilde{\nu}_1(\bar{t}_1^A, \bar{D}_1^A, t_1, D_1, \vartheta)] [W_1 - 0.5\beta CPr(nc | (t_1, D_1), R)] \\ & - \{1 - G_1 [\tilde{\nu}_1(\bar{t}_1^A, \bar{D}_1^A, t_1, D_1, \vartheta)]\} CPr(nc | (\bar{t}_1^A, \bar{D}_1^A), R) + 0.5\beta W_2 \end{aligned} \quad (32)$$

We define $\bar{u}(t_1, D_1; \theta_1)$ to be the weighted average of voters' utilities. i.e.:

$$\bar{u}(t_1, D_1; \theta_1) = \theta_1 u^Y(t_1, D_1) + (1 - \theta_1) u^O(t_1, D_1), \quad (33)$$

and for $I \in \{A, B\}$ the following adjusted cost term for rule violation:

$$C^I(W_1, t_1, D_1, t_1^{-I}, D_1^{-I}) \equiv \frac{0.5C}{[W_1 - 0.5C\beta [Pr(nc | (t_1, D_1), R) - Pr(nc | (t_1^{-I}, D_1^{-I}), R)]] g_1(\tilde{\nu}_1^I)} > 0 \quad (34)$$

if $W_1 > 0.5C\beta$. Furthermore, we consider the matrix:

$$M_O^I \equiv \begin{bmatrix} O_{tt}^I(t_1, D_1, t_1^{-I}, D_1^{-I}) & O_{Dt}^I(t_1, D_1, t_1^{-I}, D_1^{-I}) \\ O_{tD}^I(t_1, D_1, t_1^{-I}, D_1^{-I}) & O_{DD}^I(t_1, D_1, t_1^{-I}, D_1^{-I}) \end{bmatrix} \quad (35)$$

whose entries are defined as follows:

$$\begin{aligned} O_{tt}(t_1, D_1, t_1^{-I}, D_1^{-I}) &\equiv \frac{\partial^2 \bar{u}(t_1, D_1; \theta_1)}{\partial t_1^2} - \beta C^I(W_1, t_1, D_1, t_1^{-I}, D_1^{-I}) \frac{\partial}{\partial t_1^2} [Pr(nc | (t_1, D_1), R)] \\ O_{DD}(t_1, D_1, t_1^{-I}, D_1^{-I}) &\equiv \frac{\partial^2 \bar{u}(t_1, D_1; \theta_1)}{\partial D_1^2} - \beta C^I(W_1, t_1, D_1, t_1^{-I}, D_1^{-I}) \frac{\partial}{\partial D_1^2} [Pr(nc | (t_1, D_1), R)] \\ O_{tD}(t_1, D_1, t_1^{-I}, D_1^{-I}) &\equiv \frac{\partial^2 \bar{u}(t_1, D_1; \theta_1)}{\partial t_1 \partial D_1} - \beta C^I(W_1, t_1, D_1, t_1^{-I}, D_1^{-I}) \frac{\partial}{\partial t_1 \partial D_1} [Pr(nc | (t_1, D_1), R)] \end{aligned} \quad (36)$$

We can state the following result.

Proposition A.1. *If M_O^A, M_O^B are negative definite for all $(t_1^A, D_1^A, t_1^B, D_1^B)$ in $X \times X$ and W_1, σ_ν^2 are both sufficiently large, then (i) in period 1 there exists a symmetric Nash equilibrium in pure strategies; (ii) the equilibrium platform $(t_1^A, D_1^A) = (t_1^B, D_1^B)$ maximizes the weighted expected utility of period 1*

voters - with weight θ_1 to voters of type Y and $(1 - \theta_1)$ to voters of type O , minus the expected cost $\beta C^e \Pr(nc \mid (t_1, D_1), R)$ of violating the deficit rule in the following period; (iii) if $h_1^Y(0) = h_1^O(0) = \bar{h}_1$, then $\theta_1 = \vartheta_1$, i.e. the policy proposed in equilibrium is that chosen by a social planner that maximizes the expected utility of country i 's period 1 voters facing a cost C^e in the event in which a deficit rule is violated in the following period.

Proof. Part (i). Because the distribution of $\tilde{\nu}_1$ is symmetric about zero, the optimization problems in (31) and (32) show that the game is symmetric. Nevertheless, the presence of the cost of punishment implies that the game – differently from most traditional probabilistic voting electoral games in the literature such as Banks and Duggan (2005) and Lindbeck and Weibull (1987) – is not a zero-sum game. Thus, the proof of existence requires additional restrictions relative to that in those papers.

Existence. Define for $I \in \{A, B\}$ the following:

$$w^I(W_1, t_1, D_1, t_1^{-I}, D_1^{-I}) \equiv W_1 - 0.5\beta C [\Pr(nc \mid (t_1, D_1), R) - \Pr(nc \mid (t_1^{-I}, D_1^{-I}), R)] \quad (37)$$

and we assume that $W_1 > 0.5\beta C$ to ensure that $w^I(W_1, t_1, D_1, t_1^{-I}, D_1^{-I}) > 0$ for all $t_1, D_1, t_1^{-I}, D_1^{-I}$. The FOCs of candidate A write:

$$[t_1] : \quad \begin{aligned} & w^A(W_1, t_1, D_1, t_1^B, D_1^B) g_1(\tilde{\nu}_1^A) \left\{ \frac{\partial}{\partial t_1} [\bar{u}(t_1, D_1; \theta_1)] \right\} \\ & - 0.5\beta C [1 - G_1(\tilde{\nu}_1^A)] \frac{\partial}{\partial t_1} [\Pr(nc \mid (t_1, D_1), R)] \Big\} = 0 \end{aligned} \quad (38)$$

and:

$$[D_1] : \quad \begin{aligned} & w^A(W_1, t_1, D_1, t_1^B, D_1^B) g_1(\tilde{\nu}_1^A) \left\{ \frac{\partial}{\partial D_1} [\bar{u}(t_1, D_1; \theta_1)] \right\} \\ & - 0.5\beta C [1 - G_1(\tilde{\nu}_1^A)] \frac{\partial}{\partial D_1} [\Pr(nc \mid (t_1, D_1), R)] \Big\} = 0 \end{aligned} \quad (39)$$

where the above conditions are binding for interior solutions, i.e. whenever the implicit constraints $0 \leq t_1 \leq 1$ and $D_0 \leq D_1 \leq \bar{D}_1$ are not binding. Candidate B solves a problem that mirrors that of candidate A . Notice that in any equilibrium it must be that $\tilde{\nu}_1^A(t_1, D_1) = \tilde{\nu}_1^B(t_1, D_1) = \tilde{\nu}_1(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta_1)$. The proof of existence consists in four steps.

1. Candidates' objective functions are strictly concave in own actions. *Proof.* Let $V^I(t_1, D_1, t_1^{-I}, D_1^{-I})$ denote the objective function of candidate $I \in \{A, B\}$ and $V_{xy}^I = \frac{\partial^2 V^I}{\partial x \partial y}$. For strict concavity it is

sufficient to show that the Jacobian matrix:

$$\begin{bmatrix} V_{tt}^I & V_{tD}^I \\ V_{Dt}^I & V_{DD}^I \end{bmatrix} \quad (40)$$

is negative definite for each $I \in \{A, B\}$ and for all possible $(t_1^{-I}, D_1^{-I}) \in X$. Thus, we need $V_{tt}^I < 0$, $V_{DD}^I < 0$, and $V_{tt}^I V_{DD}^I - (V_{tD}^I)^2 > 0$ for $I \in \{A, B\}$. These conditions are satisfied if (a) the matrices M_O^A, M_O^B are negative definite for all $(t_1^A, D_1^A, t_1^B, D_1^B)$ in $X \times X$ and if (b) the period 1 rent W_1 and the variance of the taste shock σ_ν^2 are both sufficiently large. Sufficient conditions for (a) to hold true is in Lemma A.2.²⁴ Specifically, if (a) is satisfied, then there exists a (possibly not unique) pair of thresholds $(\widetilde{W}_1, \widetilde{\sigma}_\nu^2)$ with finite $\widetilde{W}_1, \widetilde{\sigma}_\nu^2$ such that the matrix is negative definite for all $(t_1^{-I}, D_1^{-I}) \in X$ if $W_1 > \widetilde{W}_1$ and $\sigma_\nu^2 > \widetilde{\sigma}_\nu^2$. The detailed proof to this result is presented in the online appendix.

2. A symmetric NE exists. *Proof.* Lemma 7 in Dasgupta and Maskin (1986) implies that a symmetric game possesses a symmetric mixed strategy equilibrium if (i) the set of players' actions is non-empty and compact, and the objective function V^I for $I = A, B$ satisfies the following conditions: (ii) $V^A + V^B$ is upper semi-continuous, and (iii) V^I is bounded and weakly lower semi-continuous in (t_1^I, D_1^I) . In our application the set of players' actions $[0, 1]^2 \times [D_0, \bar{D}_1]^2$ is non-empty and compact. Conditions (ii) and (ii) are satisfied because it is jointly continuous in $t_1^A, D_1^A, t_1^B, D_1^B$. Thus, the game satisfies all the properties of Lemma 7 in Dasgupta and Maskin (1986), which implies that the game possesses a symmetric mixed strategy equilibrium. Details are provided in the online appendix.
3. All NE are in pure strategies. *Proof.* If each candidate I 's objective function is strictly concave in (t_1^I, D_1^I) , then all best responses to mixed strategies, and therefore all electoral equilibria, are in pure strategies (as in Banks and Duggan, 2005, proof to Theorem 2).
4. There exists at least one symmetric pure strategy Nash equilibrium. *Proof.* Straightforward from results 1., 2., and 3.

Part (ii) (*Equivalent problem*). Consider the equilibrium conditions of each candidate in a symmetric pure strategies Nash equilibrium. In such type of equilibrium it must be true that $\tilde{\nu}_1^A = \tilde{\nu}_1^B = \bar{\nu}_1 = 0.5$

²⁴The condition on σ_ν^2 corresponds to the restriction on $g'_2(\nu_2)/g_2(\nu_2)$ that ensures concavity in Lindbeck and Weibull (1987). The additional condition on W_1 is needed because of the interaction between the probability of winning the elections and the probability of entering a punishment phase in period 2 in each candidate's objective function, which is not an issue in traditional probabilistic voting models.

and $w^I(W_1, t_1, D_1, t_1^{-I}, D_1^{-I}) = W_1$. Then, the FOCs in (38) and (39) for candidate $I \in \{A, B\}$ can be rewritten as follows:

$$[t_1] : \left[\theta_1 \frac{\partial}{\partial t_1} [u^Y(t_1, D_1)] + (1 - \theta_1) \frac{\partial}{\partial t_1} [u^O(t_1, D_1)] \right] - \beta C^e \frac{\partial}{\partial t_1} [Pr(nc | (t_1, D_1), R)] = 0 \quad (41)$$

and

$$[D_1] : \left[\theta_1 \frac{\partial}{\partial D_1} [u^Y(t_1, D_1)] + (1 - \theta_1) \frac{\partial}{\partial D_1} [u^O(t_1, D_1)] \right] - \beta C^e \frac{\partial}{\partial D_1} [Pr(nc | (t_1, D_1), R)] = 0 \quad (42)$$

where $C^e = \frac{C}{4W_{1g1}(0.5)}$. Notice that the FOCs above are the same as those of a partially benevolent social planner solving:

$$\max_{\{t_1, D_1\} \in X} \underbrace{\theta_1 u^Y(t_1, D_1) + (1 - \theta_1) u^O(t_1, D_1)}_{\bar{u}(t_1, D_1; \theta_1)} - \beta C^e Pr(nc | (t_1, D_1), R) \quad (43)$$

Because the maximization problem in (43) is characterized by a strictly concave objective function and a convex feasible set, and the equilibrium conditions of the electoral game in (41) and (42) are identical to the FOCs of the planner problem in (43), then the optimal solution (t_1^*, D_1^*) satisfies $(t_1^*, D_1^*) = (t_1^A, D_1^A) = (t_1^B, D_1^B)$. Q.E.D.

Part (iii). If $h_1^Y(0) = h_1^O(0)$, which would be the case for instance if $H_1^Y(\cdot) = H_1^O(\cdot)$, then $\theta_1 = \vartheta_1$ and the problem in (43) is the same as that of a social planner that maximizes the expected utility of country i 's period 1 voters facing a cost c in the event in which a deficit rule is violated in the following period, i.e. the politician maximize the expected utility of period 1-voters corrected for a cost associated to the probability of violating the rule. Q.E.D.

Lemma A.2. *For finite σ_ν , W_1 and strictly concave $\bar{u}(t_1, D_1; \theta_1)$ the matrices M_O^A , M_O^B are negative definite for all $(t_1^A, D_1^A, t_1^B, D_1^B)$ in $X \times X$ if either of the following conditions hold:*

1. $Pr(nc | (t_1, D_1), R)$ is weakly convex;
2. $\tilde{\epsilon}(t_1, D_1)$ is weakly convex and σ_ϵ is large enough, i.e $\sigma_\epsilon \geq \check{\sigma}_\epsilon$ for some threshold $\check{\sigma}_\epsilon > 0$.
3. $\tilde{\epsilon}(t_1, D_1)$ is weakly convex and σ_ϵ is small enough, i.e $\sigma_\epsilon \leq \hat{\sigma}_\epsilon$ for some threshold $\hat{\sigma}_\epsilon > 0$.

Proof. Because $\bar{u}(t_1, D_1; \theta_1)$ is strictly concave, in case (1) $Pr(nc | (t_1, D_1), R)$ is weakly convex and therefore $\bar{u}(t_1, D_1; \theta_1) - \beta C^e Pr(nc | (t_1, D_1), R)$ is strictly concave. In case (2) - (3) the cross derivatives of

$Pr(nc \mid (t_1, D_1), R)$ become small enough in magnitude, such that $\bar{u}(t_1, D_1; \theta_1) - \beta C^e Pr(nc \mid (t_1, D_1), R)$ is strictly concave. Thus, in both cases M_O^A, M_O^B must be negative definite. Details are provided in the online appendix. Q.E.D.

Appendix B

In this section we maintain the assumption that the conditions for equivalence between the outcome of the electoral game and that of the modified social planned problem stated in Proposition A.1 are satisfied. Thus, all the proofs make use of the latter.

Proposition 1. *The expected deficit of each country i is increasing in the political present bias $B_1 = (1 - \pi\theta_1)$.*

Proof. First we need to consider the problem of the elected politician in period 2. From the previous section we know that in period 2, the problem is equivalent to the one of social planner that maximizes voters' expected utility. The problem is:

$$\begin{aligned} \max_{t_2 \in [0,1]} & [(1 - t_2)w_2\bar{l}_2 - v(\bar{l}_2) + u(g_2(t_2, \epsilon))] \\ \text{s.t. } & g_2(t_2, \epsilon) = t_2w_2\bar{l}_2 - (D_1 - \epsilon)(1 + r) \end{aligned} \quad (44)$$

where

$$g_2(t_2, \epsilon) = t_2w_2\bar{l}_2 - (D_1 - \epsilon)(1 + r) \quad (45)$$

Define the tax elasticity of labor supply as:

$$\eta_b(t_b) = \frac{\partial l_b^*}{\partial t_b} \frac{t_b}{l_b^*} = \begin{cases} -\frac{w_b t_b}{v''(l_b^*) l_b^*} & \text{if } 0 < l_b^* < \bar{l}_b \\ 0 & \text{otherwise} \end{cases} \quad (46)$$

The FOC implies $[t_2] : w_2\bar{l}_2 \{-1 + u'(g_2)\} = 0$. Notice that this equation implies:

$$g_2 = u'^{-1}(1) = \bar{g}_2 \quad (47)$$

which implies that g_2 is independent of D_1 (this is a consequence of linearity).

And therefore the problem in period 1 can be rewritten as follows:

$$\max_{(t_1, D_1) \in X} (1 - t_1)w_1l_1^* - v(l_1^*) + u(g_1(t_1, D_1) - \beta C^e Pr(nc | (t_1, D_1), R) + \beta\pi\theta_1 \{w_2l_2^* - \bar{g}_2 - v(l_2^*) + u(\bar{g}_2) - D_1(1 + r)\}) \quad (48)$$

First, notice that because $g_1(t_1, D_1)$ is a concave function the set X is a convex set. Moreover, the assumption $u(0) = \infty$ implies that constraint $g_1(t_1, D_1) \geq 0$ embedded in the definition of X is never binding. Thus, we can ignore it in deriving the optimality conditions of each agent. Calculate the FOCs w.r.t. t_1 and D_1 :

$$[t_1] : -w_1l_1 + u'(g_1)w_1l_1[1 + \eta_1(t_1)] - \beta C^e \frac{\partial Pr(nc | (t_1, D_1), R)}{\partial t_1} = 0 \quad (49)$$

$$[D_1] : u'(g_1(t_1, D_1)) + \beta B_1(1 + r) - \beta(1 + r) - \beta C^e \frac{\partial Pr(nc | (t_1, D_1), R)}{\partial D_1} = 0 \quad (50)$$

To ensure that the conditions of Proposition A.1 are satisfied we need to ensure that M_O^A , M_O^B are negative definite. From Lemma A.2, a necessary condition is that $\bar{u}(t_1, D_1; \theta_1)$ is strictly concave. For that to hold true we need the Hessian matrix of \bar{u} to be negative definite. The conditions are:

$$\begin{aligned} \bar{u}_{DD} &= u''(g_1(t_1, D_1)) < 0 \\ \bar{u}_{tt} &= y_1^2 u''(g_1(t_1, D_1))(1 + \eta(t_1))^2 + y_1 u'(g_1(t_1, D_1)) \frac{d\eta}{dt_1} < 0 \\ \bar{u}_{DD}\bar{u}_{tt} - \bar{u}_{tD}^2 &= u''(g_1(t_1, D_1))y_1 u'(g_1(t_1, D_1)) \frac{d\eta}{dt_1} > 0 \end{aligned} \quad (51)$$

Thus, the strict concavity of u implies all the conditions are satisfied as long as $\frac{d\eta}{dt_1} < 0$. This is true if $v'''(l_1) \geq 0$ for all $l_1 \in [0, \bar{l}]$. Thus, all the conditions of Proposition A.1 are satisfied as long as either condition (1) or (2) of Lemma A.2 is satisfied. Also notice that M_O^I negative definite also implies that the objective function of the representative politician is strictly concave. Thus, the optimal policy solves the FOCs and the sign of the comparative statics of interest can be obtained by differentiating the FOCs. Specifically, we calculate the cross derivatives V_{xy} of the politician's objective function V , noticing that $Pr(nc | (t_1, D_1), R)$ is a function of t_1, D_1, R , but it is invariant in B_1 . Differentiating (49) and (50) we obtain:

$$V_{tB} = 0 \quad (52)$$

$$V_{DB} = \beta(1 + r) > 0 \quad (53)$$

$$V_{tD} = u''(g_1(t_1, D_1))w_1l_1[1 + \eta_1(t_1)] - \beta C^e \frac{\partial^2 Pr(nc | (t_1, D_1), R)}{\partial D_1 \partial t_1} \quad (54)$$

Recall that by assumption u'' and $\frac{\partial^2 Pr(nc | (t_1, D_1), R)}{\partial D_1 \partial t_1}$ possess finite values. There are three possible cases. (i) The solution to the politician's optimization problem is a corner with respect to D_1 , which implies that D_1 is constant in B_1 , or (ii) it is interior with respect to D_1 and a corner for t_1 , which implies $\frac{dD_1^*}{dB_1} = -\frac{V_{DB}}{V_{DD}} > 0$, or (iii) it is interior with respect to both D_1 and t_1 , which implies $\frac{dD_1^*}{dB_1} = -\frac{V_{DB}V_{tt} - V_{tB}V_{tD}}{V_{DD}V_{tt} - V_{tD}^2} = -\frac{V_{DB}V_{tt}}{V_{DD}V_{tt} - V_{tD}^2} > 0$. In all three cases $\frac{dD_1^*}{dB_1} \geq 0$, i.e. D_1 is weakly increasing in B_1 . Q.E.D.

Proposition 2. *In the absence of a fiscal rule the equilibrium level of deficit in period 1 is weakly larger than the optimal level.*

Proof. First we must derive the condition for the optimal choice of the social planner. This planner can decide D_1 and t_1 optimally (no need of the deficit rule). The social planner problem is stated in formula (10). In period 2, the platform chosen is the same as the one of the politician, which corresponds to the one of a planner that maximizes the sum of voters utilities. Thus, the problem in period 1 can be rewritten as follows:

$$\max_{(t_1, D_1) \in X} (1 - t_1)w_1l_1(t_1) - v(l_1) + u(g_1(t_1, D_1)) + \beta \{w_2l_2 - \bar{g}_2 - v(l_2) + u(\bar{g}_2) - (1 + r)E_1[D_1 - \epsilon]\} \quad (55)$$

where voters possess perfect foresight of the values of all variables in period 2. Calculate the FOCs:

$$[t_1^{SP}] : -w_1l_1 + u'(g_1)w_1l_1[1 + \eta_1(t_1)] = 0 \quad (56)$$

$$[D_1^{SP}] : u'(g_1(t_1, D_1)) - \beta(1 + r) = 0 \quad (57)$$

Recall that under the assumptions on u both the social planner and the representative politician face a strictly convex maximization problem. Thus, it is sufficient to compare the FOCs in this section with the FOCs of the politician in (49) and (50). For $C^e = 0$ (i.e. no fiscal rule), we obtain $V_t - V_t^{SP} = 0$, and $V_D - V_D^{SP} = \beta(1 - \pi\theta_1)(1 + r) \geq 0$, thus it must be true that $D_1^* \geq D_1^{SP}$. Q.E.D.

Lemma 1. *The solution to the social planner's problem (t_1^*, D_1^*) is interior.*

Proof. Consider V_t^{SP}, V_D^{SP} in (56) and (57) and suppose the solution is not interior. First, the assumption $\underline{D}_1 \leq D_0(1+r) - w_1\bar{l}$ ensures that the constraint $g(t_1, D_1) \geq 0$ is never satisfied at $D_1 = \underline{D}_1$. Thus, the boundary of X is defined by a threshold $\underline{D}_1(t_1) = t_1 w_1 l_1(t_1) - D_0(1+r)$. Then if the solution is not interior there must be some $(t_1, D_1) \in X$ such that either (A) $V_D^{SP} < 0$ at $D_1 = \underline{D}_1(t_1)$, or (B) $V_D^{SP} > 0$ at $D_1 = \bar{D}_1$, or (C) $V_t^{SP} < 0$ at $t_1 = 0$, or (D) $V_t^{SP} > 0$ at $t_1 = 1$. Part (a). Recall that $\underline{D}_1(t_1)$ is the minimum feasible D_1 given t_1 , and that $g_1(t_1, \underline{D}_1(t_1)) = 0$. The condition $u'(0) = +\infty$ ensures that the constraint $g(t_1, D_1) \geq 0$ is never binding. Thus, $V_D^{SP} > 0$ at $D_1 = \underline{D}_1(t_1)$ for all $t \in [0, 1]$. Part (b). The assumption $u'(\bar{D}_1 - D_0(1+r)) \leq \beta(1+r)$ ensures that $V_D^{SP} \leq 0$ for all $t \in [0, 1]$. Part (c). At $t_1 = 0$ one gets $V_t^{SP} = -w_1 l_1(0) [1 - u'(D_1 - D_0(1+r))][1 + \eta_1(0)]$. Recall that $\eta_1(t_1) = -\frac{w_1 t_1}{v''(l_1(t_1))l_1(t_1)}$. Because v is strictly increasing and strictly convex we get $v''(l_1(0))l_1(0) > 0$, which implies $\eta_1(0) = 0$. Using this result into the formula for V_t^{SP} at $t_1 = 0$ we obtain $-w_1 l_1(0) [1 - u'(D_1 - D_0(1+r))]$. Thus, the assumption $u'(\bar{D}_1 - D_0(1+r)) \geq 1$ ensures that $V_t^{SP} \geq 0$ at $t_1 = 0$ for all $(0, D_1)$ in X . Part (d). Because v is strictly increasing and strictly convex we get $v''(l_1(1))l_1(1) = 0$, which implies $\eta_1(1) = -\infty$. Thus, $V_t^{SP} < 0$ at $t_1 = 1$. Parts (a), (b), (c), and (d) together imply that an element $(t_1, D_1) \in X$ that satisfies either conditions (A), (B), (C), or (D) does not exist. This leads to a contradiction. Q.E.D.

Proposition 3. *(i) A fiscal rule R that implements the optimal policy (t_1^*, D_1^*) and that satisfies conditions (TCO) always exists.*

Proof. Consider the rule $R(s_1, y_1) = \zeta \frac{F(s_1 y_1)}{y_1} - \zeta \frac{F(0)}{y_1} - s_1$. It is easy to show that this rule satisfies condition (1) for $F(0) = 0$. A violation of the rule occurs iff $\frac{D_1 - D_0}{y_1} - s_1 > \zeta \frac{F(s_1 y_1)}{y_1} - \zeta \frac{F(0)}{y_1} - s_1$, which rewrites:

$$\frac{D_1 - D_0}{\zeta} - F(0) > F(\epsilon) \quad (58)$$

Because F is strictly increasing over $[-a, a]$, it is invertible. Thus we can rewrite condition (58) as follows:

$$F^{-1} \left(\frac{D_1 - D_0}{\zeta} - F(0) \right) > \epsilon \quad (59)$$

Recall that the probability of non-compliance given policy (t_1, D_1) and rule $R(s_1, y_1)$ writes:

$$\begin{aligned} Pr(nc \mid R(s_1, y_1), (t_1, D_1)) &= Pr\left(\epsilon < F^{-1}\left(\frac{D_1 - D_0}{\zeta} - F(0)\right)\right) \\ &= F\left(F^{-1}\left(\frac{D_1 - D_0}{\zeta} - F(0)\right)\right) = \frac{D_1 - D_0}{\zeta} - F(0) \end{aligned} \quad (60)$$

Thus, the expected cost of punishment becomes $\frac{C^e}{\zeta} (D_1 - D_0) - C^e F(0)$. Thus, this rule trivially satisfies the condition of Lemma A.1 (1) that ensures that the solution to the electoral game is the same as that to the problem of the representative politician, and that the objective function is strictly concave. Thus, we can use the FOCs of the politician (see proof to Proposition 1) which write:

$$[D_1] := u'(g_t) - \beta\pi\theta_1(1+r) - \beta\frac{C^e}{\zeta} = 0 \quad (61)$$

$$[t_1] := -y_1 \{1 - u'(g_1)[1 + \eta_1(t_1)]\} = 0 \quad (62)$$

Secondly, the objective function of the social planner is strictly concave given the assumptions on u and v . Thus, sufficient conditions for the solution to the politician's problem to be socially optimal are:

$$V_D - V_D^{SP} = \beta \left((1 - \pi\theta_1)(1+r) - \frac{C^e}{\zeta} \right) = 0 \quad (63)$$

$$V_t - V_t^{SP} = 0 \quad (64)$$

Thus, setting

$$\zeta^* = \frac{C^e}{(1 - \pi\theta_1)(1+r)} \quad (65)$$

solves the equation in (63), implying that the principal and the politician's FOCs are made equal to each other. Lastly, both the objective function of the principal and the one of the politician are strictly concave in (t_1, D_1) . Thus, the result above implies $(t_1^{**}, D_1^{**}) = (t_1^*, D_1^*)$. Q.E.D.

Lemma 2. *A rule R that satisfies (TCO) implements the optimal policy only if it can be written in the form $R(s_1, y_1) = H(s_1 y_1)/y_1$.*

Proof. First, notice that the implementation of the optimal policy (t_1^*, D_1^*) occurs only if (t_1^*, D_1^*) is a global maximum of the politician's objective function. Second, Lemma 1 ensures that (t_1^*, D_1^*) is an interior solution for both policy dimensions. Thus, a necessary condition for implementability is that the politician's objective function is locally concave at $(t_1, D_1) = (t_1^*, D_1^*)$, else it cannot be an interior global maximum. Thus, implementation occurs if and only if the FOCs are satisfied at (t_1^*, D_1^*) . This is

the case if and only if $V_t - V_t^{SP} = 0$ and $V_D - V_D^{SP} = 0$ at (t_1^*, D_1^*) . Recall a violation of the rule occurs if the inequality in (13) is not satisfied. This is equivalent to:

$$\frac{D_1 - D_0}{y_1} > R(s_1, y_1) + s_1 \quad (66)$$

Thus, the probability of a violation of the rule is

$$Pr(y_1 R(s_1, y_1) + s_1 - (D_1 - D_0) \leq 0) \quad (67)$$

Take the derivative with respect to t_1 .

$$\frac{\partial Pr(y_1 R(s_1, y_1) + s_1 y_1 - (D_1 - D_0) \leq 0)}{\partial t_1} = 0 \quad \forall s_1 \in \left[-\frac{a}{y_1}, \frac{a}{y_1} \right] \quad (68)$$

Define $L(s_1, y_1) = y_1 R(s_1, y_1) + s_1 y_1$. The above becomes

$$\frac{\partial Pr(L(s_1, y_1) - D_1 + D_0 < 0)}{\partial t_1} = \frac{\partial Pr(L(s_1, y_1) - D_1 + D_0 < 0)}{\partial L} \frac{\partial L(\epsilon/y_1, y_1)}{\partial t_1} = 0 \quad (69)$$

Notice that because D_1^* is interior by Lemma 1, any rule that implements the social optimum must be such that $V_D - V_D^{SP} = 0$ at (t_1^*, D_1^*) . This implies that it must be true:

$$-C^e \frac{\partial Pr(L(\epsilon/y_1, y_1) - D_1 + D_0 < 0)}{\partial D_1} \neq 0 \quad (70)$$

because otherwise the rule cannot induce $V_D - V_D^{SP} = 0$ and $V_t - V_t^{SP} = 0$ at (t_1^*, D_1^*) . Also notice that

$$-C^e \frac{\partial Pr(L(\epsilon/y_1, y_1) - D_1 + D_0 < 0)}{\partial D_1} = C^e \frac{\partial Pr(L(\epsilon/y_1, y_1) - D_1 + D_0 < 0)}{\partial L} \neq 0 \quad (71)$$

Thus, for the condition $V_t - V_t^{SP} = 0$ to hold, we need to ensure that $\frac{\partial L(\epsilon/y_1, y_1)}{\partial t_1} = 0$. This solves

$$\begin{aligned} \frac{\partial L(\epsilon/y_1, y_1)}{\partial t_1} &= \left[R(\epsilon/y_1, y_1) + y_1 \frac{\partial R(\epsilon/y_1, y_1)}{\partial y_1} \right] \frac{dy_1}{dt_1} \\ &= R(\epsilon/y_1, y_1) [1 + \xi(\epsilon/y_1, y_1)] \frac{dy_1}{dt_1} = 0 \quad \forall s_1 \in \left[-\frac{a}{y_1}, \frac{a}{y_1} \right], \forall y_1 \in (0, +\infty) \end{aligned} \quad (72)$$

This is true only if either $R(\epsilon/y_1, y_1) = 0$ for all ϵ, y_1 , or if $\xi(\epsilon/y_1, y_1) = -1$. The latter is true for all

$\forall y_1 \in (0, +\infty)$ if and only if $R(s_1, y_1) = \frac{H(s_1 y_1)}{y_1}$ for some function H . In fact, in this case one gets:

$$\frac{\partial R(\epsilon/y_1, y_1)}{\partial y_1} \frac{y_1}{R(\epsilon/y_1, y_1)} = -\frac{H_1(\epsilon)}{y_1^2} \frac{y_1}{H(\epsilon)} y_1 = -1 \quad (73)$$

Thus, either (i) $R(s_1, y_1) = 0$ for all s_1, y_1 , or (ii) $R(s_1, y_1; \zeta) = \frac{H(s_1 y_1)}{y_1}$ for some function H . But case (i) is also a rule in the form $R(s_1, y_1) = \frac{H(s_1 y_1)}{y_1}$, specifically the case in which $H(\epsilon) = 0$ for all $\epsilon \in [-a, a]$. Thus, a rule R implements the optimal policy only if it can be written in the form $R(s_1, y_1) = H(s_1 y_1)/y_1$. Q.E.D.

Proposition 4. *If a deficit rule R satisfies (TCO) and implements the optimal policy (t_1^*, D_1^*) , then (i) the tightness of the rule $K(y_1 | R)$ is zero, i.e. the rule prescribes zero structural deficit, and (ii) the flexibility of the rule $\Delta(t_1^*, D_1^*, y_1 | R)$ is lower than 1.*

Proof. Part (i). Using the result in Lemma 2, if R satisfies (TCO) and implements the optimal policy, tightness is given by:

$$K(y_1 | R_{B_1}^*) = R_{B_1}^*(0, y_1) = \frac{H(0)}{y_1} \quad (74)$$

For some function H . Tightness constant in output implies:

$$\frac{\partial R_{B_1}^*(0, y_1)}{\partial y_1} = -\frac{H(0)}{(y_1)^2} = 0 \quad (75)$$

for all $y_1 \in [0, +\infty)$. This condition is satisfied if and only if $H(0) = 0$. Plug this result into the formula for $K(y_1 | R_{B_1}^*)$ to get

$$K(y_1 | R_{B_1}^*) = 0/y_1 = 0 \quad (76)$$

i.e. the optimal tightness of any rule that implements the optimal policy and satisfies (TCO) is equal to zero.

Part (ii). First, notice that the optimality condition for D_1 leads to the necessary condition for implementation of the social optimum:

$$B_1(1+r) - C^e \frac{f(\tilde{\epsilon}(t_1^*, D_1^*; \zeta))}{1 - \Delta^*} = 0 \quad (77)$$

which is never satisfied for any $\Delta^* > 1$. Thus, any rule that implements the social optimum such

that $-\frac{\partial R(s_1, y_1)}{\partial s_1} \leq 1$ at $\epsilon = \tilde{\epsilon}(t_1^*, D_1^*)$. Second, notice that $-\frac{\partial R(s_1, y_1)}{\partial s_1} = \Delta(t_1^*, D_1^*, y_1 | R) \rightarrow 1^-$ implies either (a) $\nexists \epsilon \in [-a, a]$ that satisfies (14), which implies $\frac{\partial Pr(L(\epsilon/y_1, y_1) - D_1 + D_0 < 0)}{\partial D_1} = 0$, or (b) $\lim_{\Delta \rightarrow 1^-} \frac{\partial Pr(L(\epsilon/y_1, y_1) - D_1 + D_0 < 0)}{\partial D_1} = +\infty$ and therefore condition (77) is not satisfied. Both cases imply that the rule cannot implement the optimal policy. Thus, $-\frac{\partial R(s_1, y_1)}{\partial s_1} \neq 1$ must be true in a neighborhood of D_1^* . Lastly, $-\frac{\partial R(s_1, y_1)}{\partial s_1} \leq 1$ and $-\frac{\partial R(s_1, y_1)}{\partial s_1} \neq 1$ imply $-\frac{\partial R(s_1, y_1)}{\partial s_1} < 1$. Q.E.D.

Proposition 5. *There exists finite $\varsigma > 0$ such that if $\sigma_\epsilon \geq \varsigma$, then the flexibility of the optimal rule $\Delta(t_1^*, D_1^*, y_1 | R_{B_1}^*)$ is weakly increasing in the political present bias B_1 .*

Proof. From Lemma 2 we know that a rule that implements the optimal policy and satisfies (TCO) must have functional form:

$$R(s_1, y_1) = \frac{H(s_1 y_1)}{y_1} \quad (78)$$

Define $A(s_1 y_1; \zeta)$ for some parameter $\zeta \in [\underline{\zeta}, \bar{\zeta}]$, and such that at $\zeta = \zeta^*$ for some $\zeta^* \in [\underline{\zeta}, \bar{\zeta}]$, as follows:

$$A(s_1 y_1; \zeta^*) = H(s_1 y_1) + s_1 y_1 \quad (79)$$

This implies that all the possible families ρ_r that satisfy the required conditions that can be constructed such that $R(s_1, y_1) \in \rho_r$ have form:

$$r(s_1 y_1; \zeta) = \frac{A(s_1 y_1; \zeta)}{y_1} - s_1 \quad (80)$$

Proposition 4 (ii) implies $H'(s_1 y_1) > -1$ in a neighborhood of D_1^* , and therefore $A_1(s_1 y_1; \zeta^*) = [H'(s_1 y_1) + 1] > 0$, i.e. the function A is locally strictly increasing in $s_1 y_1$. Suppose the distribution of the shock ϵ is such that $\sigma_\epsilon \rightarrow +\infty$ (i.e., $\epsilon \sim Unif[-a, a]$). A punishment occur if:

$$\frac{deficit_1}{output_1} = \frac{D_1^* - D_0 - \epsilon}{y_1} > \frac{A(s_1 y_1; \zeta)}{y_1} - s_1 \quad (81)$$

We define threshold $\tilde{\epsilon}(t_1^*, D_1^*; \zeta^*)$ as follows:

$$D_1^* - D_0 = A(\tilde{\epsilon}(t_1^*, D_1^*; \zeta); \zeta) \quad (82)$$

Thus, we derive

$$\frac{\partial \tilde{\epsilon}(t_1^*, D_1^*; \zeta)}{\partial \zeta} = -\frac{A_2(\tilde{\epsilon}(t_1^*, D_1^*; \zeta); \zeta)}{A_1(\tilde{\epsilon}(t_1^*, D_1^*; \zeta); \zeta)}$$

Therefore the probability of a punishment given t_1, D_1, ζ is:

$$Pr(nc \mid R, (t_1, D_1)) = F(\tilde{\epsilon}(t_1, D_1; \zeta)) \quad (83)$$

the optimality condition becomes

$$B_1(1+r) - C^e \frac{f(\tilde{\epsilon}(t_1^*, D_1^*; \zeta^*))}{A_1(\tilde{\epsilon}(t_1^*, D_1^*; \zeta^*); \zeta^*)} = 0 \quad (84)$$

where ζ^* is a value $\zeta \in Z$ that solves the equation above (and that exists by construction). Notice that differentiating with respect to the bias B_1 one gets

$$\frac{\partial \zeta^*}{\partial B_1} = -\frac{A_1(\tilde{\epsilon}(t_1^*, D_1^*; \zeta^*); \zeta^*)}{A_2(\tilde{\epsilon}(t_1^*, D_1^*; \zeta^*); \zeta^*) B_1} \left/ \left[\frac{f'(\tilde{\epsilon})}{f(\tilde{\epsilon})} + \frac{A_{12}A_1}{A_2} - A_{11} \right] \right. \quad (85)$$

Firstly, recall that optimal flexibility writes $\Delta^* = -R_s(\tilde{\epsilon}(t_1^*, D_1^*; \zeta^*)/y_1, y_1) = 1 - A_1(\tilde{\epsilon}(t_1^*, D_1^*; \zeta^*); \zeta^*)$.

Thus, using the formula for $\frac{\partial \tilde{\epsilon}(t_1^*, D_1^*; \zeta)}{\partial \zeta}$ we get

$$\frac{\partial \Delta^*}{\partial B_1} = -\frac{A_2(\tilde{\epsilon}(t_1^*, D_1^*; \zeta^*); \zeta^*)}{A_1} \left[\frac{A_{12}A_1}{A_2} - A_{11} \right] \frac{\partial \zeta^*}{\partial B_1} \quad (86)$$

Secondly, using the formula for $\frac{\partial \zeta^*}{\partial B_1}$, and the assumption that ϵ possesses a truncated normal distribution, which implies $f'(\tilde{\epsilon})/f(\tilde{\epsilon}) = -\tilde{\epsilon}/\sigma_\epsilon^2$, we get

$$\frac{\partial \Delta^*}{\partial B_1} = \frac{1}{B_1} \frac{A_{12}A_1 - A_{11}A_2}{A_{12}A_1 - A_{11}A_2 - A_2\tilde{\epsilon}/\sigma_\epsilon^2} \quad (87)$$

Lastly, recall that we started the proof by assuming $\sigma_\epsilon \rightarrow +\infty$, and that A satisfies $A_1 > 0$ at $\tilde{\epsilon}(t_1^*, D_1^*; \zeta^*); \zeta^*$ (see above), and possesses finite first and second derivatives. Notice that from Definition 3 monotonicity is defined over families $\rho_r \in \mathcal{R}_r$ such that $r(\cdot, \cdot; \zeta^*(B'_1))$ implements the optimal policy for all the values of B_1 in a neighborhood of $B_1 = B'_1$. This implies that r is such that at $B_1 = B'_1$, it is true that $A_{12}A_1 - A_{11}A_2 \neq 0$. It is easy to show that if $A_{12}A_1 - A_{11}A_2 = 0$ at $B_1 = B'_1$, then the rule $r(\cdot, \cdot; \zeta^*)$ cannot implement the optimal policy for all values of B_1 in a neighborhood of $B_1 = B'_1$, because in that case $\frac{\partial^2 Pr(nc|r)}{\partial D_1 \partial \zeta} \Big|_{\zeta=\zeta^*} = 0$, which implies in turn that the optimality condition $B_1(1+r) - c \frac{\partial Pr(nc|r)}{\partial D_1} \Big|_{\zeta=\zeta^*} = 0$ is not satisfied by any value of B_1 in the neighborhood of $B_1 = B'_1$, other than at the exact point $B_1 = B'_1$. Thus, it must be true that $A_{12}A_1 - A_{11}A_2 \neq 0$. Using the formula for

$\frac{\partial \Delta^*}{\partial B_1}$, $A_{12}A_1 - A_{11}A_2 \neq 0$ implies that:

$$\lim_{\sigma_\epsilon \rightarrow +\infty} \frac{\partial \Delta^*}{\partial B_1} = \frac{1}{B_1} > 0 \quad (88)$$

The above states that if ϵ is uniformly distributed over $[-a, a]$, then $\frac{\partial \Delta^*}{\partial B_1}$ is strictly positive. Because $\frac{\partial \Delta^*}{\partial B_1}$ is continuous in σ_ϵ , the intermediate value theorem implies that either $\frac{\partial \Delta^*}{\partial B_1} \geq 0$ for all values of σ_ϵ at which the family ρ_r can implement the optimal policy for some $\zeta \in Z$ – in such case set $\varsigma = 0$ –, or there exists $+\infty > \varsigma_r > 0$ such that if $\sigma_\epsilon \geq \varsigma_r$ then $\frac{\partial \Delta^*}{\partial B_1} \geq 0$. Lastly, because this is true for all possible families $\rho_r \in \mathcal{R}_r$, there exists finite $\varsigma < +\infty$ such that if $\sigma_\epsilon \geq \varsigma$ then $\frac{\partial \Delta^*}{\partial B_1} \geq 0$. Q.E.D.

Appendix C

This section presents the proof of the results about linear rules.

Proposition 6. *The linear rule implements the optimal policy if the tax shock has enough variance: $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$ for some $\bar{\sigma}_\epsilon \in (0, \infty)$.*

Proof. We maintain the assumption that $\sigma_\nu > \tilde{\sigma}_\nu$ and $W_1 > \widetilde{W}_1$ from Proposition A.1. Lemma A.2 (2) implies that for $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$ the matrices M_O^A , M_O^B are negative definite. Thus, Proposition A.1 implies that (a) the equilibrium policy outcome of the electoral game is the same at that of the equivalent problem of the representative politician; (b) the objective function of the representative politician is strictly concave.

Suppose the linear rule does not implement the optimal policy. The difference between FOCs of the representative politician and those of the social planner at (t_1^*, D_1^*) writes:

$$V_D - V_D^{SP} = 0 = \beta(1 - \pi\theta_1)(1 + r) - \frac{\beta C^e}{1 - \delta} f\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1 - \delta}\right) = 0 \quad (89)$$

$$V_t - V_t^{SP} = 0 = \beta \frac{C^e kw_1 l_1}{(1 - \delta)t_1} \eta_1(t_1) f\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1 - \delta}\right) = 0 \quad (90)$$

First, set $k = 0$ to ensure that equation (90) is satisfied at (t_1^*, D_1^*) . For equation (89), recall that the truncated-normal distribution implies:

$$f\left(\frac{D_1 - D_0}{1 - \delta}\right) = \frac{1}{2\Phi(a/\sigma_\epsilon) - 1} \frac{1}{\sqrt{2\pi}\sigma_\epsilon} \exp\left(-\frac{1}{2\sigma_\epsilon^2} \left(\frac{D_1 - D_0}{1 - \delta}\right)^2\right) \quad (91)$$

where Φ denotes the cdf of the standard normal distribution. Implementability requires the existence

of a $\delta^* \in (-\infty, 1)$ such that equation (89) is satisfied at (t_1^*, D_1^*) . Recall that Proposition 4 (ii) implies that any rule that implements the social optimum must be such that $\delta < 1$. Because $V_D - V_D^{SP}$ is continuous in δ over $(-\infty, 1)$, by the intermediate value theorem it is sufficient to show that there exists $\delta^a, \delta^b \in (-\infty, 1)$ such that:

$$\frac{\beta C^e}{1-\delta^a} f\left(\frac{D_1^*-D_0}{1-\delta^a}\right) - \beta(1-\pi\theta_1)(1+r) \leq 0 \quad (a) \quad (92)$$

$$\frac{\beta C^e}{1-\delta^b} f\left(\frac{D_1^*-D_0}{1-\delta^b}\right) - \beta(1-\pi\theta_1)(1+r) \geq 0 \quad (b)$$

Consider any $C^e > 0$. For condition (92) (b) first suppose (1) $|D_1^* - D_0| > 0$. Define $\delta^b = 1 - \frac{|D_1^* - D_0|}{\alpha\sigma_\epsilon}$ for some finite $\alpha > 0$. Then $-\frac{1}{2\sigma_\epsilon^2} \left(\frac{D_1^* - D_0}{1-\delta^b}\right)^2 = -\frac{\alpha^2}{2}$ and $\frac{D_1^* - D_0}{1-\delta^b} = \alpha\sigma_\epsilon$ for $D_1^* - D_0 > 0$ and $\frac{D_1^* - D_0}{1-\delta^b} = -\alpha\sigma_\epsilon$ for $D_1^* - D_0 < 0$. Lastly, we need to ensure that $\frac{D_1^* - D_0}{1-\delta^b} \in [-a, a]$. For any finite $\sigma_\epsilon > 0$ it is sufficient to choose α such that $\alpha \leq a/\sigma_\epsilon$. Notice that $\delta^b \in (-\infty, 1)$. Set $\delta = \delta^b$. Then using de l'Hopital rule we get:

$$\lim_{\sigma_\epsilon \rightarrow +\infty} \frac{\beta C^e}{1-\delta^b} f\left(\frac{D_1^* - D_0}{1-\delta^b}\right) = \frac{1}{0} \frac{\alpha}{\sqrt{2\pi} |D_1^* - D_0|} \exp\left(-\frac{\alpha^2}{2}\right) = +\infty \quad (93)$$

If instead (2) $D_1^* - D_0 = 0$, define $\delta^{b2} = 1 - \frac{1}{2\Phi(a/\sigma_\epsilon)-1} \frac{1}{\sqrt{2\pi}\sigma_\epsilon} \frac{1}{\beta(1-\pi\theta_1)(1+r)}$. We get:

$$\frac{\beta C^e}{1-\delta^{b2}} f(0) = \beta(1-\pi\theta_1)(1+r) \quad (94)$$

which ensures that condition (92) (b) is satisfied with equality. Results (1) and (2) ensure the existence for any $C^e > 0$ of a finite threshold $\dot{\sigma}_\epsilon$ such that the condition (92) (b) is satisfied for any finite $\sigma_\epsilon \geq \dot{\sigma}_\epsilon$. For condition (92) (a) notice that:

$$\lim_{\delta^a \rightarrow -\infty} \frac{\beta C^e}{1-\delta^a} f\left(\frac{D_1^* - D_0}{1-\delta^a}\right) - \beta(1-\pi\theta_1)(1+r) = -\beta(1-\pi\theta_1)(1+r) < 0 \quad (95)$$

Thus, condition (92) (a) is always satisfied. Lastly, notice that the LHS of (92) (a) and (b) are continuous functions of δ in the range $(-\infty, 1)$. Thus, by the intermediate value theorem there exists $\delta^* \in (-\infty, 1)$ such that $V_D - V_D^{SP} = 0$ at (t_1^*, D_1^*) . Because the problems of the social planner and of the politician are both (strictly) convex problems, and the FOCs are made identical at (t_1^*, D_1^*) for $(k, \delta) = (0, \delta^*)$, then (t_1^*, D_1^*) is the unique global maximum of the politicians' objective function if $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$. Thus, the linear rule does implement the optimal policy if $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$. This leads to a contradiction. Q.E.D.

Corollary 7. *If the optimal policy (t_1^*, D_1^*) is implementable by a linear rule $R = k - \delta s_1$ for all B_1 within a range $[B'_1, B''_1]$, then:*

- (i) *the implementation occurs at $k^* = 0$ and $\delta^* \leq \tilde{\delta}$;*
- (ii) *the optimal degree of flexibility δ^* is weakly increasing in the political present bias B_1 within such range.*

Proof. Part (i). The rule $R = k - \delta s_1$ satisfies (TCO). Thus, result (i) is straightforward from Proposition 4 parts (i) and (ii).

Part (ii). From part (i) it must be true that $k^* = 0$ and $\delta^* \in (-\infty, 1)$. The linear rule implies $A(\tilde{s}_1 y_1; \zeta) = \tilde{\epsilon}(t_1^*, D_1^*; \zeta) \cdot \zeta = \tilde{\epsilon}(t_1^*, D_1^*)(1 - \delta)$ and therefore $A_1 = (1 - \delta)$, $A_2 = \tilde{\epsilon}(t_1^*, D_1^*)$. The formula for $\frac{\partial \Delta^*}{\partial B_1}$ for the case in which the optimal policy is implementable is that in 87. Apply it to the rule $R = k - \delta s_1$ to get:

$$\frac{\partial \Delta^*}{\partial B_1} = \frac{1}{B_1} \left[1 + \frac{\tilde{\epsilon}(t_1^*, D_1^*)^2}{\sigma_\epsilon^2(1 - \delta^*)} \right]^{-1} \quad (96)$$

which is strictly positive for any value of $\tilde{\epsilon}(t_1^*, D_1^*)$ given $\delta^* \in (-\infty, 1)$. Q.E.D.

Proposition 8. *Assume that the optimal policy is not implementable by a linear rule. (i) Whenever the optimal linear rule requires a zero structural deficit level, $k^* = 0$, the optimal linear rule makes use of the maximum flexibility in the sense $\delta^{opt} = \delta^{max}$. (ii) If the tax elasticity of labor supply is small, i.e. $|\eta_1(t_1^{**})| < \bar{\eta}$ for some $\bar{\eta} > 0$, the optimal linear rule is characterized by a strictly positive deficit maximum, $k^* > 0$.*

Proof. Part (ii). Step 1. We show that even if the optimal policy is not implementable, the conditions of Proposition A.1 are still satisfied. Conditions $W_1 \geq \widetilde{W}_1$ and $\sigma_\nu > \tilde{\sigma}_\nu$ are assumed to hold true. Thus, we need to verify whether the matrices M_O^A , M_O^B can be negative definite for small σ_ϵ . Recall that Proposition 6 implies that if $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$ then the socially optimal policy is implementable. Thus, in this case it must be true that $\sigma_\epsilon < \bar{\sigma}_\epsilon$. Lemma A.2 (3) ensures the existence of a finite threshold $\hat{\sigma}_\epsilon > 0$ such that if $\sigma_\epsilon \leq \hat{\sigma}_\epsilon$ then the required condition is satisfied.

Step 2. We show that if $k = 0$ at (t_1^{**}, D_1^{**}) and D_1^{**} is interior it must be true that $V_D^{SP} \neq 0$. Suppose $V_D^{SP} = 0$. $k = 0$ implies $V_t = V_t^{SP}$ for all $(t_1, D_1) \in X$. Thus, if $V_D^{SP} = 0$ then (t_1^{**}, D_1^{**}) satisfies the FOCs of the social planner, and given the strict concavity of the objective function this implies in turn that the socially optimal policy is implementable, leading to a contradiction. Thus, it must be $V_D^{SP} \neq 0$.

Step 3. We show that if the optimal rule has $k^* = 0$, then it must have $\delta^* = \delta^{max}$. Set $k = 0$ and recall $V_D^{SP} \neq 0$ at (t_1^{**}, D_1^{**}) from step 2. Suppose $(0, \delta^*)$ is optimal and $\delta^* \neq \delta^{max}$. If $\delta \neq \delta^{max}$ and D_1^{**} is interior then $\frac{dD_1^{**}}{d\delta} \neq 0$. Recall that $k = 0$ implies $V_t^{SP} = V_t$, which in turn implies that either (a) t_1^{**} is an interior solution and $V_t^{SP} = V_t = 0$, or (b) t_1^{**} is a corner solution and $\frac{dt_1^{**}}{d\delta} = 0$. Thus, a marginal change in δ implies:

$$\left. \frac{dV^{SP}}{d\delta} \right|_{(t_1^{**}, D_1^{**})} = V_D^{SP} \frac{dD_1^{**}}{d\delta} + \underbrace{V_t^{SP} \frac{dt_1^{**}}{d\delta}}_{=0} = V_D^{SP} \frac{dD_1^{**}}{d\delta} \quad (97)$$

If D_1^{**} is interior $V_t^{SP} \neq 0$ from step 2, then $\frac{dD_1^{**}}{d\delta} \neq 0$ implies that either $\frac{dV^{SP}}{d\delta} > 0$ or $\frac{dV^{SP}}{d\delta} < 0$. In both cases, δ^* is not optimal because a marginal change in δ increases social welfare. Thus, δ^* can be optimal only if $\delta^* = \delta^{max}$. If D_1^{**} is a corner solution, then $\left. \frac{dV^{SP}}{d\delta} \right|_{(t_1^{**}, D_1^{**})} = 0$ and any δ in a neighborhood of $\delta^* = \delta^{max}$ is optimal.

Step 4. We show that if $(0, \delta^*)$ is optimal and D_1^{**} is interior, then $V_t^{SP} < 0$ at (t_1^{**}, D_1^{**}) . We have shown in step 2 that $V_D^{SP} \neq 0$. Step 3 states that δ^* can be optimal only if $\delta^* = \delta^{max}$. Suppose $V_t^{SP} > 0$ at (t_1^{**}, D_1^{**}) . It is easy to show that $\frac{dD_1^{**}}{d\delta} > 0$ for $\delta > \delta^{max}$ and $\frac{dD_1^{**}}{d\delta} < 0$ for $\delta < \delta^{max}$ in a neighborhood of $\delta = \delta^{max}$. This implies $\frac{dV^{SP}}{d\delta} = V_t^{SP} \times \frac{dD_1^{**}}{d\delta} < 0$ for $\delta^* < \delta^{max}$ and $\frac{dV^{SP}}{d\delta} = V_t^{SP} \times \frac{dD_1^{**}}{d\delta} > 0$ for $\delta^* > \delta^{max}$, i.e. δ^* is a local minimum of the planner's objective function, thus it cannot be optimal. Thus, if D_1^{**} is interior it must be $V_t^{SP} < 0$ at (t_1^{**}, D_1^{**}) .

Step 5. We prove that if $|\eta_1(t_1^{**})| \leq \bar{\eta}$ then the optimal rule cannot feature $k^* = 0$. Suppose $k^* = 0$. Then either D_1^{**} is a corner or $\delta^* = \delta^{max}$ by step 3 and $V_D^{SP} < 0$ at (t_1^{**}, D_1^{**}) by step 4. We study the effect of a marginal change in k at $k = 0$ on the social planner's utility in a neighborhood of (t_1^{**}, D_1^{**}) . Recall that $k = 0$ implies $V_t^{SP} = V_t$, which in turn implies that either (a) t_1^{**} is an interior solution and $V_t^{SP} = V_t = 0$, or (b) t_1^{**} is a corner solution and $\frac{dt_1^{**}}{dk} = 0$.

Now we study the effect of a marginal change in k at $k = 0$ on the social planner's utility in a neighborhood of (t_1^{**}, D_1^{**}) . Recall that $\frac{\partial V^{SP}}{\partial t_1} \frac{dt_1^{**}}{dk} = 0$ and $\frac{\partial V^{SP}}{\partial D_1} < 0$ from step 1. Thus, $\left. \frac{dV^{SP}}{dk} \right|_{(t_1^{**}, D_1^{**})}$ writes:

$$\left. \frac{dV^{SP}}{dk} \right|_{(t_1^{**}, D_1^{**})} = V_D^{SP} \frac{dD_1^{**}}{dk} + \underbrace{V_t^{SP} \frac{dt_1^{**}}{dk}}_{=0} = V_D^{SP} \frac{dD_1^{**}}{dk} \quad (98)$$

Thus, the sign of (98) is equal to $\text{sign}\left(-\frac{dD_1^{**}}{dk}\right)$. Because of the strict concavity of V we can use traditional comparative statics methods, that lead to the following results. (a) For interior D_1^{**} and interior t_1^{**} (a) we get

$$\frac{dD_1^{**}}{dk} = -\frac{V_{Dk} \times V_{tt} - V_{Dt} \times V_{tk}}{V_{DD}V_{tt} - V_{Dt}^2} \quad (99)$$

(b) for interior D_1^{**} and corner t_1^{**} we get $\frac{dD_1^{**}}{dk} = -\frac{V_{Dk}}{V_{DD}} < 0$, and (c) for corner D_1^{**} trivially $\frac{dD_1^{**}}{dk} = 0$. In the latter case, condition (95) implies that it must be $D_1^{**} > D_1^*$ which implies $D_1^{**} = \bar{D}_1$. But if the optimal rule induces $D_1^{**} = \bar{D}_1$, then no strictly welfare improving deficit rule exists, which implies that an optimal linear rule does not exist, and the statement does not apply. Thus, we must analyze only case (a) and (b). The formulas (a) and (b) imply that to study the sign of $\frac{dD_1^{**}}{dk}$ we must derive the cross derivatives of V evaluated at (t_1^{**}, D_1^{**}) . Setting $k = 0$, such derivatives have formula:

$$V_{tk} = \beta \frac{C^e w_1 l_1}{(1-\delta)t_1^{**}} \eta_1(t_1^{**}) f\left(\frac{D_1^{**} - D_0}{1-\delta}\right) < 0 \quad (100)$$

$$V_{Dk} = \frac{\beta C^e y_1}{(1-\delta)^2} f'\left(\frac{D_1^{**} - D_0}{1-\delta}\right) < 0 \quad (101)$$

$$V_{Dt} = u''(g_1(t_1^{**}, D_1^{**})) y_1 [1 + \eta_1(t_1^{**})] < 0 \quad (102)$$

$$V_{tt} = y_1 \left\{ u''(g_1(t_1^{**}, D_1^{**})) [1 + \eta_1(t_1^{**})]^2 y_1 + u'(g_1(t_1^{**}, D_1^{**})) \frac{\partial \eta_1(t_1)}{\partial t_1} \right\} < 0 \quad (103)$$

Case (a). Notice that $D_1^{**} - D_0 \neq 0$ because at $D_1^{**} - D_0 = 0$ always implies $\left. \frac{dV^{SP}}{d\delta} \right|_{(t_1^{**}, D_1^{**})} > 0$, thus δ^* cannot be optimal. Moreover, $V_{DD}V_{tt} - V_{Dt}^2 > 0$ and $V_{DD} < 0$ because the objective function of the politician is strictly concave. Using the formulas above, and $\frac{(D_1^{**} - D_0)^2}{\sigma_\epsilon^2} = (1 - \delta^{max})^2$ we can construct the formula for $-(V_{Dk} \times V_{tt} - V_{Dt} \times V_{tk})$, which writes:

$$\begin{aligned} & u''(g_1(t_1^{**}, D_1^{**})) [1 + \eta_1(t_1^{**})] \beta \frac{C^e y_1^2}{(1-\delta^{max})} f\left(-\frac{\sigma_\epsilon^2}{2}\right) \frac{1+t_1^{**}}{t_1^{**}} \\ & \times \left\{ \eta_1(t_1) + \frac{t_1^{**}}{1+t_1^{**}} + \frac{u'(g_1(t_1, D_1))}{u''(g_1(t_1, D_1)) [1 + \eta_1(t_1)] y_1} \frac{\partial \eta_1(t_1)}{\partial t_1} \right\} \end{aligned} \quad (104)$$

Define $\bar{\eta} \equiv \frac{t_1^{**}}{1+t_1^{**}} + \frac{u'(g_1(t_1^{**}, D_1^{**}))}{u''(g_1(t_1^{**}, D_1^{**})) [1 + \eta_1(t_1^{**})] y_1} \frac{\partial \eta_1(t_1^{**})}{\partial t_1}$ and notice that the assumptions on v imply $\bar{\eta} > 0$. The cross derivatives above imply that if $|\eta_1(t_1^{**})| < \bar{\eta}$, then $-(V_{Dk} \times V_{tt} - V_{Dt} \times V_{tk}) > 0$, which $\left. \frac{dV^{SP}}{dk} \right|_{(t_1^{**}, D_1^{**})} > 0$ at $k = 0$. Thus, $(0, \delta^*)$ is not optimal. Case (b). If t_1^{**} is a corner solution, then $\frac{dD_1^{**}}{dk} = -\frac{V_{Dk}}{V_{DD}} < 0$ which implies that $k = 0$ cannot be optimal for any value of $\eta_1(t_1^{**})$. This leads to a contradiction.

Part (i). If $|\eta_1(t_1^{**})| \geq \bar{\eta}$ then $-(V_{Dk} \times V_{tt} - V_{Dt} \times V_{tk}) > 0$. In such case $\left. \frac{dV^{SP}}{dk} \right|_{(t_1^{**}, D_1^{**})} \leq 0$ at $k = 0$, thus $k^* = 0$ is locally a maximum given $\delta = \delta^{max}$. From part (ii) step 3 we know that at $k = 0$ the welfare-maximizing δ is $\delta = \delta^{max}$. Thus, $(0, \delta^{max})$ is locally optimal. Q.E.D.

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