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# Risky Financial Collateral, Firm Heterogeneity, and the Impact of Eligibility Requirements <sup>\*</sup>

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## Abstract

We study the eligibility of corporate bonds as collateral for central bank operations and its effect on interbank and corporate bond markets. While money market functionality increases in the amount of eligible assets, a thorough assessment of collateral policies must also account for endogenous responses of the corporate sector: banks increase demand for corporate bonds if they are eligible as collateral, and firms increase their leverage and default risk in response. This has adverse effects on the money market due to costs associated with deteriorating collateral quality. To jointly analyze the dynamics of collateral supply and collateral quality, we construct a heterogeneous firm model with defaultable bonds which banks use to collateralize money market borrowing. In this setting, eligible firms pay lower spreads and have higher leverage, consistent with empirical evidence. The central bank faces a trade-off between fostering collateral supply and increased risk-taking on the corporate bond market, which deteriorates collateral quality in equilibrium. Calibrating the model to Euro Area data, we find that reducing eligibility requirements from A- to BBB- increases collateral supply by 33%, while collateral default risk increases by 53%. Under an adverse shock to firm fundamentals, these numbers increase to 32% and 63%, i.e. collateral quality deteriorates disproportionately. Ultimately, firm fundamentals place restrictions on the efficacy of central bank collateral policy.

*Keywords:* Money Markets, Financial Collateral, Eligibility Premia, Firm Heterogeneity, Credit Risk, Corporate Capital Structure

*JEL Classification:* E23, E44, G11, G32

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# 1 Introduction

Central bank collateral frameworks are of paramount importance to the financial architecture in most advanced economies, both due to the sheer size of central bank facilities and due to the adoption of the central bank framework on the interbank market. In this paper, we study the eligibility of corporate bonds as collateral and its effect on interbank and corporate bond markets. While interbank market functionality increases in the amount of eligible assets, a thorough assessment of collateral policies must also account for endogenous responses of the corporate sector. To the best of our knowledge, this paper is the first to study the impact of eligibility requirements on interbank and corporate bond markets in a joint framework. This allows us to derive a trade-off between fostering collateral supply and subsidizing firm risk-taking for central bank eligibility requirements, and to study different counterfactual collateral policies.

As we show in table 1, eligibility of corporate bonds as collateral in central bank operations varies across countries and over time. For example, the Federal Reserve only admits government bonds, while the European Central Bank allows a plethora of risky assets to be pledged. In addition, several central banks, the ECB among them, drastically reduced eligibility requirements during the financial crisis of 2008, giving rise to the question how relaxing eligibility requirements affects interbank and corporate bond markets in normal times, and in episodes of elevated financial and fundamental risk.

Table 1: Non-Financial Corporate Bonds in Various Collateral Frameworks

Central Bank	Eligible Pre GFC (Min. Rating)	Eligible Post GFC (Min. Rating)
Australia	No	Yes (AAA)
Canada	No	No
Eurosystem	Yes (A-)	Yes (BBB)
Japan	Yes (A)	Yes (BBB) <sup>†</sup>
Sweden	No	No*
Switzerland	Yes (AA-)**	Yes (AA-)**
United Kingdom	No	No
United States <sup>††</sup>	No	No

Notes: <sup>†</sup>: multiple changes after Financial Crisis; \*: Temporarily allowed; \*\*: only applicable to small number of bonds; <sup>††</sup>: Only allowed in the discount window at a minimum rating of AAA. Source: Bank for International Settlements (2013) & national CBs. Note that we only consider frameworks before any changes due to the Covid-19 pandemic.

To answer these questions, we build a model with heterogeneous firms that can default on their bonds and banks which use these bonds to collateralize borrowing among each other and with the central bank. Firms have an incentive to issue bonds because these shield dividends from taxation. Issuing bonds introduces default incentives if the firm is hit by an adverse shock that makes debt roll-over prohibitively costly. The capital structure is determined by the standard trade-off between tax advantages and bankruptcy costs. Bonds are held and priced by banks. We assume that banking activity is subject to uninsurable liquidity shocks as in Bianchi and Bigio (2021), which banks settle on a collateralized interbank market or by using central bank standing facilities. We interpret this market for cash as the *money market* and assume that all money market borrowing is subject to eligibility requirements for collateral.

In the spirit of Piazzesi and Schneider (2018), banks incur costs from money market borrowing that are decreasing in the amount of eligible collateral, implying that banks are willing to pay premia for eligible assets. At the same time, accepting low-quality collateral is costly for money market lenders due to risk management expenses or counterparty risk.<sup>1</sup> Bonds, therefore, have a dual role as investment object and as financial collateral, and both, their quantity and quality, is relevant for money market outcomes.

Making corporate bonds eligible as collateral affects corporate bond markets in two ways: first, investors price the eligibility benefits provided by bonds, i.e. they are willing to pay an eligibility premium, shifting the pricing schedule for corporate bonds outwards, *ceteris paribus*. Second, this shift affects the optimal leverage decision. On the one hand, firms can take advantage of banks' high valuation of corporate bonds and issue more bonds to increase dividend payouts. On the other hand, firms can sustain the same dividend-payout by issuing a smaller face value of bonds. We show in an analytically tractable version of the model with short-lived firms that, under a monotone hazard rate assumption on firm revenues, the former effect dominates: firms weakly increase their borrowing, consistent with empirical evidence.<sup>2</sup> We show that the associated increase of default risk dampens the shift of the price schedule due to compensation for elevated default risk.

On an aggregate level, the economy is affected through two distinct transmission channels. First, collateral supply increases in response to a relaxation of eligibility requirements not only because of the mechanical effect of accepting a larger number of bonds but also because of the endogenous adjustment of the firm debt choice. This reduces costs of money market borrowing. We label this the *collateral supply channel*. Second, elevated default risk deteriorates collateral quality, which we then call the *collateral quality channel*. Characterizing aggregate implications is challenging, since the impact of eligibility requirements on firms differs along the cross-section. High-risk firms are not affected by eligibility at all, while debt and default incentives of low-risk firms are distorted by eligibility. This makes a heterogeneous firm model essential to adequately study aggregate implications.

A key firm characteristic in this setup is its *eligible debt capacity* which is defined as the maximum leverage a firm can choose without losing eligibility. Eligible debt capacity is increasing in current firm revenues: when idiosyncratic revenues are persistent, a higher revenue draw in the current period implies that default in the near future is unlikely, such that it can issue larger amounts of bonds without losing eligibility. Collateral supply is directly affected by eligibility requirements through the eligible debt capacity, since it specifies the maximum default risk a bond can carry while still being accepted as collateral. This applies to eligibility requirements on the interbank market and to central bank collateral policies.

The effect on collateral supply can be further decomposed into an *intensive margin* (already eligible firms increase their collateral supply) and an *extensive margin* (new firms become eligible). This distinction is important since the intensive margin is necessarily associated with elevated default risk, while the extensive margin can have crowding-in effect as it incentivizes intermediate-risk borrowers to remain at moderate risk levels. Their relative importance is shaped by the aggregate state as we outline below.

<sup>1</sup>The role of (costly) counterparty risk for interbank market outcomes is made explicit in Heider et al. (2015). For a practical review on the role of counterparty risk, see also Pirrong (2011). Corradin et al. (2017) provide a broader discussion regarding the role of collateral on interbank markets.

<sup>2</sup>Eligibility premia increase the revenues a firm can raise by issuing bonds with the same face value. This can never make firms worse off, since the old bond position is still feasible after the relaxation and firms never decrease their bond position.

To analyze debt and default dynamics along the firm distribution and their impact on collateral supply and quality, we solve the model using fully global methods and subsequently simulate long panels of firms. We use these panels to calibrate the model to Euro area data by using a merged dataset of corporate bonds from *IHS Markit* and corporate balance sheet data from *Compustat Global*. We focus on a sample from 2004Q1-2009Q4, including a relatively stable period of low fundamental risk, and the financial crisis. All calibrated parameters are informed based on the pre-crisis period 2004Q1-2008Q3. The calibrated model is able to replicate the main features of the cross-sectional firm distribution over leverage, corporate bond spreads, and eligibility premia.

In this setting, we study two different policies: our benchmark scenario are tight eligibility requirements, corresponding to the ECB practice before the 2008 crisis, which only accepted bonds rated A- or higher. Second, we consider lenient eligibility requirements, under which all bonds rated BBB- or higher are eligible at the central bank. To study the impact of these policies during normal and crisis times, our model features aggregate risk in the form of Markov switching. The only difference between the two states (boom and crisis) are firm fundamentals. We follow the literature on risk shocks (Christiano et al., 2014), and assume that the variance of idiosyncratic firm revenues increases in the crisis state. A larger revenue variance significantly increases default risk. Holding central bank collateral policy constant, the crisis state is characterized by a substantial reduction of collateral supply, since the fundamental shock reduces eligible debt capacity.

Eligibility requirements introduce a discontinuity in bond price schedules: firms with low leverage and sufficient profitability receive a premium on their bond prices up to their eligible debt capacity. A lenient policy shifts the location of the discontinuity into regions of higher leverage and default risk. Since shocks are persistent, a lenient policy increases the probability of a firm to be eligible in the future, thereby lowering spreads in anticipation. Hence, collateral policy is not only relevant for firms that are near their eligible debt capacity in the current period, but affects all firms via the continuation value.

This observation is only valid in partial equilibrium, where firm policies are kept constant. The simulated firm panel reveals that firms respond to a lenient policy by increasing their leverage, which in turn leads to higher default risk. As a result, the spread and leverage distributions shift to the right in the case of lenient eligibility requirements. Hence, collateral supply increases at the expense of collateral quality.

Firm responses to collateral policy differ between boom and crisis periods. While the latter feature high default incentives, they are usually relatively short. Lenient eligibility requirements in a crisis have therefore a crowding-in effect via the extensive margin, since they increase bond prices and, thereby, repayment incentives. At the same time, these policies subsidize leverage and risk-taking in boom periods mainly through the intensive margin. The opposite is true for the tight policy, which features low risk-taking in booms but fails to provide repayment incentives during crisis periods. While lenient eligibility requirements always increase collateral supply, the total effect on collateral quality is therefore ambiguous a priori because of the persistence of aggregate shocks.

Using our calibrated model, however, we find that on an aggregate level, the lenient policy is characterised by a 33% increase in collateral supply and a 53% increase in collateral default risk during boom periods relative to a tight policy. During a crisis period, lenient eligibility requirements lead to significantly higher cost for money market lenders. The increase in collateral default risk amounts to 63%, while the increase in collateral supply is only 32%, relative to the tight policy. The quality of collateral

plays a crucial role for this result. Recall that a lenient policy subsidizes leverage and risk-taking, such that the deterioration of firm fundamentals is reinforced during crisis periods, while increased repayment incentives play a minor role in our calibrated model. Moreover, the intensive margins accounts for 37% of the increase in collateral supply during boom periods, while the share declines to 20% during crisis.

As shown by Mancini et al. (2015) and Pérignon et al. (2018), the European interbank market proved to be remarkably resilient during the financial crisis and the European sovereign debt crisis, Gorton and Metrick (2012) and Krishnamurthy et al. (2014) provide evidence of substantial disruptions on the US interbank repo market. To evaluate the effectiveness of central bank eligibility requirements in the presence interbank market tightening, we repeat our policy experiments in a setting with interbank market disruptions. In this case, the reduction in collateral supply is more severe than in our baseline calibration, since interbank lenders do not accept collateral of medium quality. To restore the same degree of money market functionality, the share central bank lending has to be larger than in the baseline scenario.

This observation follows from the fact that the central bank in its role as lender of last resort can in principle substitute wholesale funding markets. At the same time, it faces a similar restriction as the interbank market does in the availability of collateral. Our simulations show that this policy has also larger adverse effects on collateral quality. Specifically relaxing central bank eligibility from A to BBB during booms increases collateral supply by 37% and collateral default risk by 57%. In a crisis, the numbers increase to 42% and 105%. At some degree of leniency, these adverse effects become so large that firm fundamentals place restrictions on the efficacy of central bank policy through the availability potentially eligible bonds.

In this context, the allocation of costs between interbank lenders and the central banks is of independent interest. In our model with endogenous firm responses, taking on a higher share of money market lending implies taking on a larger share of collateral default costs. Specifically, lenient eligibility requirements are characterised by an increase from 33 to 51% of collateral default costs borne by the central bank. During crisis periods this number even increases to 82%. Costs borne by private market participants, such as central clearing counterparties (CCP), might negatively affect financial stability (Biais et al. (2016)), if the solvency of CCPs themselves is contested. In contrast, costs borne by the central bank may lead to central bank losses and thus inflationary pressure (Chapman et al. (2010)). This has to be taken into account in the design of central bank collateral frameworks, but is beyond the scope of this paper.

**Related Literature** There is extensive literature on central bank interventions on the money market. Our paper relates to Allen et al. (2009), who stress the role of aggregate liquidity risk for interbank market outcomes. Open market operations can address liquidity shortages by stabilizing the short-term funding rate in their model. In our model, the central bank can address aggregate shocks to the interbank market by adjusting eligibility requirements downwards, if this is not an option, for example if the zero lower bound binds. Acharya et al. (2011) study the effect of asset fundamentals on interbank market stability. In their setup, even a small shock to the fundamentals of collateral can cause the interbank market to dry up entirely. Gorton and Ordoñez (2014) make a similar argument based on the information insensitivity of collateralized borrowing arrangements during credit booms. Once perceived collateral quality drops, interbank market participants are incentivized to recover information about collateral fundamentals and



cease to accept a potentially large share of eligible securities. By taking into account the effect of eligibility premia on collateral default risk, our model provides complementary rationales for the build-up of leverage and fragility during booms which in turn amplifies shocks during crisis periods.

A group of papers explicitly considers the heterogeneity of eligible assets and implications for central bank policy. Koulischer and Struyven (2014) employ an argument of cash lenders being the second-best user of collateral in a model where banks prefer to pledge high-quality assets on the private money market segment. Choi et al. (2021) take a macroprudential approach to central bank collateral requirements and point out that lending against low-quality collateral can be beneficial, if banks can use high-quality collateral on the interbank market instead. The distinction of collateral between private and public is discussed by Infante and Ordoñez (2020). They argue that bad fundamentals, such as high volatilities, impair risk-sharing, through a collateral channel. This logic is similar to the contraction of money market activity due to deteriorating fundamentals that we also find in our model. Empirical support for these approaches is provided by Mota (2021), who analyzes the private production of quasi-safe assets using US corporate bond data. It should be stressed that eligible corporate bonds are at most *quasi*-safe, i.e. their price contains default risk compensation and eligibility premia. Indeed, Kacperzyk et al. (2020) show that only banks are able to produce safe assets in the form of certificates of deposits.

These theories are backed by empirical studies. Ashcraft et al. (2011) find a sizeable impact of haircuts on asset prices using an event study around announcement and implementation of the Term Asset-Backed Securities Loan Facility (TALF), combined with survey evidence and unpredictable bond rejections. Casola and Koulischer (2019) show that banks respond to collateral policy by adjusting the mix of assets they pledge in OMOs using European data between 2009 and 2011. A group of papers documents effects of collateral policy changes on the non-financial sector. Several papers use Longer Term Refinancing Operations (LTRO) and the corresponding surprise decrease of corporate loan eligibility requirements (referred to as Additional Credit Claims, ACC) in their identification strategy: Bekkum et al. (2018) examine a decrease in eligibility requirements in December 2011 for residential mortgage-backed securities in the Netherlands and show that banks increased loan supply significantly. Mésonnier et al. (2020) use bank level data from France and find that banks lowered the loan rate for newly eligible firms by 7bp compared to the untreated group with slightly worse credit rating, accounting for half their pre-treatment spread. On a similar dataset, Cahn et al. (2018) show that the amount of loans to these firms rose significantly by 8.5 %. What is more, also issuers of previously eligible debt increased their borrowing by 12 %. Pelizzon et al. (2020) document empirically supply effects for corporate debt using security-level data. Their identification relies on ECB-discretion when formally eligible bonds are actually put on the list of eligible assets and find eligibility premia of similar magnitude as studies using the LTRO and CSPP policies for identification. This highlights the presence of a collateral channel also during a conventional policy regime. Todorov (2020) finds that issuers of targeted bonds increase their dividend payouts by four times, relative to pre-treatment levels, but do not increase investment. This highlights potentially unintended behavior on the firm side.

The implications for corporate financial policy are of independent interest. Most macro models of corporate investment in the presence idiosyncratic shocks and debt constraints imply (counterfactually) that the overwhelming majority of firms does not pay dividends. This follows from the shadow value of retained earnings exceeding the marginal value of paying out dividends as long as there is a posi-

tive probability of being debt-constrained in the future. Since only a very small fraction of firms has outgrown these debt constraints permanently, this results in a permanent zero-dividend policy of most firms. By taking a dynamic capital structure approach from the corporate finance literature (Hennessy and Whited (2007) and Strebulaev and Whited (2012) among others), we overcome this problem: firms do not outgrow financial frictions, since default always yields a positive continuation value. Therefore, firms will regularly choose positive dividends and willingly take the risk of costly debt restructuring. While these mechanics are standard in corporate finance, these papers typically consider the firm problem without studying the aggregate response of the firm sector. Our paper, hence, contributes to filling the gap between corporate finance and macroeconomic approaches to capital structure dynamics.

**Outline** The paper is structured as follows. Section 2 introduces eligibility premia into a simple model of corporate capital structure and presents the main policy trade-off. We present our full model in section 3 and the calibration to Euro Area data in section 4. In section 5, we evaluate several counterfactuals and policy experiments with respect to real and interbank shocks. Section 6 concludes.

## 2 Eligibility Requirements and the Non-Financial Sector: An Illustration

This section introduces a simplified version of our model to highlight the relevant transmission mechanism of collateral policy to firm decisions. Time is discrete and indexed by  $t = 1, 2, \dots$  and there is no aggregate risk. Each period is divided into two sub-periods between which there is no discounting. The model features three groups of agents: a production sector (*firms*), financial intermediaries (*banks*), and investors. Firms live for two periods and produce the all-purpose good. Throughout the paper, we do not endogeneize the investment decision. Following recent findings by Lian and Ma (2020), firms borrow against their future cash flow. Investors consume and own both firms and banks, which in turn invest into corporate bonds.

Bonds provide a tax advantage to firms, since interest expenses are deductible from taxes. In the first sub-period, banks are subject to idiosyncratic liquidity shocks, but firms do not issue additional bonds until the second sub-period. Banks experiencing net cash outflows can, therefore, only settle their liquidity deficit with the central bank against eligible collateral, which is in fixed supply at this stage. At the beginning of the second sub-period, idiosyncratic revenue shocks realize. Given their shock realization and financial position, firms either default or repay. A set of new firms enters the economy, asset markets clear and portfolios are fixed until the next period. We close the model by adding a fiscal authority taxing corporate profits and paying lump-sum transfers to investors. Figure 1 provides an overview.

**Investors** There is a representative investor owning all firms and banks in the economy.<sup>3</sup> For simplicity we assume that the investor is risk-neutral and does not discount the future, such that the maximization

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<sup>3</sup>Investors can either be interpreted as households or as competitive equity fund manager, but this distinction is not relevant in the remainder of the model.



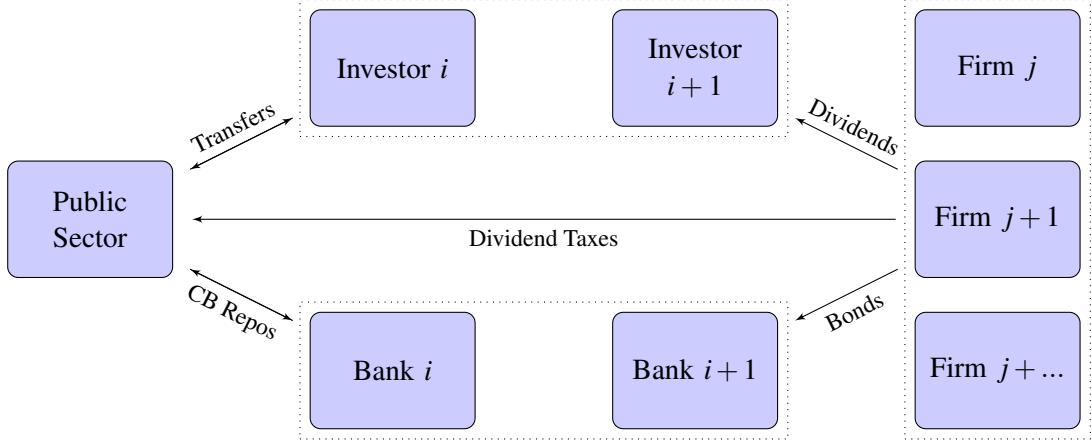


Figure 1: Model Overview

problem can be written in recursive form,

$$\begin{aligned}
 V(e_t^j) &= \max_{e_{t+1}^j} C_t + \mathbb{E}_t [V(e_{t+1}^j)] \\
 \text{s.t. } C_t + \int_j e_{t+1}^j p_t^j dj &= \\
 &\int_j e_t^j \left( \delta_t^j (p_t^{def,j} + d_t^{def,j} - \tau(d_t^{def,j})) + (1 - \delta_t^j) (p_t^{rep,j} + d_t^{rep,j} - \tau(d_t^{rep,j})) \right) dj + d_t^B + T_t,
 \end{aligned}$$

where consumption is denoted by  $C_t$  and equity of firm  $j$  is denoted by  $e_t^j$ . Dividend payments  $d_t^j$  and equity prices  $p_t^j$  depend on whether the firm defaults ( $\delta_t^j = 1$ ) or not, which we denote by the superscript *def* and *rep*. Dividends are taxed according to the tax schedule  $\tau(d_t^j)$ , which we specify below. Investors also own banks, from which they get dividends  $d_t^B$ , and receive transfers from the public sector  $T_t$ .

**Banks** There is a unit mass of perfectly competitive banks, which are specialized in fixed income investment. They price bonds risk-neutrally without discounting. Corporate bonds are one-period discount bonds that promise to pay one unit of the all-purpose in  $t + 1$ , the corporate bond portfolio is denoted  $(b_{t+1}^j)_{j \in (0,1)}$ . In the first sub-period, with probability  $\frac{1}{2}$ , banks experience a liquidity deficit  $w$  that requires immediate settlement, such as net deposit inflows and withdrawals (see for example Bianchi and Bigio, 2021). To settle these flows, banks need additional liquidity. We assume that in the first sub-period no trade with other private agents is possible and liquidity deficits have to be settled using fully collateralized short-term loans.<sup>4</sup> Since idiosyncratic firm revenues are stochastic, bond repayment is uncertain when repos are negotiated and the central bank accepts bonds only if they are sufficiently safe, i.e. if

<sup>4</sup>In the quantitative analysis in section 3, we add a private repo market in addition to central bank repos. As documented by De Fiore et al. (2019), the secured segment of the European Money Market is of far greater importance than the unsecured segment with a market share of 95 % in 2017.

their default probability  $\Lambda_t^j$  is below an eligibility threshold  $\Lambda^{CB}$ ,

$$\Phi^{CB}(\Lambda_t^j) = \begin{cases} 1 & \text{if } \Lambda_t^j \leq \Lambda^{CB} \\ 0 & \text{else} \end{cases}. \quad (1)$$

where  $\Lambda_t^j \equiv \mathbb{E}_t[\delta(b_{t+1}^j)]$  is the *default probability*, described below. Banks can obtain one unit of liquidity ("cash") for each eligible bond. Note that the collateral supply is fixed when banks approach the central bank. We impose that money market borrowing is costly and represent these costs by

$$\mathcal{L}(\bar{B}_{t+1}, w) = w \cdot \max \left\{ \bar{l} - \frac{l_0}{l_1} \left( \frac{\bar{B}_{t+1}}{w} \right)^{l_1}, 0 \right\},$$

where  $l_0 > 0$  is a scaling parameter and  $0 < l_1 < 1$  governs the curvature. Here, we define total available collateral as  $\bar{B}_{t+1} \equiv \int_j \Phi^{CB}(\Lambda_t^j) \Upsilon_t^j b_{t+1}^j dj$ , where  $\Upsilon_t^j = \mathbb{E}_t[1 - \delta(b_{t+1}^j)]$  is the *expected payoff* of bond  $j$  in sub-period 2 of period  $t$ , which depends on the firm default decision.<sup>5</sup> These costs are increasing in the liquidity deficit  $w$  and decreasing in total available collateral  $\bar{B}_{t+1}$ . The parameter  $\bar{l}$  defines a satiation level of collateral.<sup>6</sup> The period  $t$  maximization problem of bank  $i$  depends on expected payoffs via the realized payoffs  $k_t^j$  in the budget constraint of period  $t + 1$  and can be written recursively as

$$V^B(b_t^{j,i}) = \max_{b_{t+1}^{j,i}} d_t^B + \mathbb{E}_t[V^B(b_{t+1}^{j,i})] \quad \text{s.t.} \quad d_t^B = \int_j k_t^j b_t^{j,i} dj - \int_j q_t^j b_{t+1}^{j,i} dj - \mathcal{L}(\bar{B}_t, w),$$

yields a zero-profit condition for the bond price

$$q(b_{t+1}^j) = \Upsilon_t \left( 1 + \Phi^{CB}(\Lambda_t^j) \cdot \frac{l_0}{2} \bar{B}_{t+1}^{l_1-1} w^{1-l_1} \right). \quad (2)$$

In this expression, the eligibility premium is given by  $L_1 \equiv \frac{l_0}{2} \bar{B}_{t+1}^{l_1-1} w^{1-l_1}$ , which is increasing in the liquidity deficit  $w$  and decreasing in collateral supply  $\bar{B}_{t+1}$ .

**Firms** Each period, a continuum of two period-lived firms, indexed by  $j$ , enters the economy. Their equity is fixed and normalized to  $e_t^j = e_{t+1}^j = 1$ . Firms need to finance the fixed investment  $x$  into their production technology in  $t$  and produce stochastic revenues  $\mu_{t+1}$  in the second period of their lifetime ( $t + 1$ ). We assume that  $\mu_{t+1}$  is independent across firms, and denote its pdf and cdf by  $f(\mu_{t+1}|s)$  and  $F(\mu_{t+1}|s)$ , respectively. Firms are heterogeneous in their probability distribution over revenues. The distribution is assumed to be continuous and to satisfy a monotone hazard rate condition of the form  $\frac{\partial \mu_{t+1} h(\mu_{t+1}|s)}{\partial \mu_{t+1}} > 0$ , where  $h(\mu_{t+1}|s) \equiv \frac{f(\mu_{t+1}|s)}{1-F(\mu_{t+1}|s)}$ . Moreover, the parameter  $s$  shifts the probability mass according to  $F(\mu_{t+1}|s) = F(\mu_{t+1} - s)$ , which implies that  $\frac{\partial \mu_{t+1} h(\mu_{t+1}|s)}{\partial s} < 0$ . We assume that  $s$  follows some continuous distribution over the open interval  $[s^-, \infty]$ . Firms have an incentive to issue bonds,

<sup>5</sup>The expected payoff differs from the market value of bonds at issuance  $q_t$ , since the latter is discounted and contains an eligibility premium, while  $\Upsilon_t$  does not.

<sup>6</sup>If there were no collateral available, this expression would reflect the cost of borrowing on the unsecured segment. This level never binds in the quantitative exercise.

because interest payments are tax-deductible, and the tax-schedule is specified as

$$\tau(d_t^j) = \begin{cases} \tau d_t^j & \text{if } d_t^j > 0 \\ 0 & \text{if } d_t^j \leq 0, \end{cases}$$

i.e. dividends  $d_t$  are taxed at flat rate  $\tau$ , while negative dividends ( $d_t < 0$ ) are not taxed and can be interpreted as equity issuance. For a purely equity-financed firms, the dividend stream then is given by

$$d_t = -x \quad \text{and} \quad d_{t+1} = \max\{\mu_{t+1}, \mu^{def}\}(1 - \tau).$$

The firm has an incentive to issue bonds  $b_{t+1} > 0$  in the first period, since it reduces the tax burden over the firm life-cycle. The bond price schedule will be identical across firms due to the i.i.d. nature of the shock. However, leverage also introduces costs, because firms may default on their corporate bonds, introducing the standard trade-off between tax benefits and bankruptcy costs. In case of default, bondholders receive nothing and firm-owner utility is reduced to  $F(\mu^{def})$ .<sup>7</sup> We assume that  $s^-$  is sufficiently low, such that some firms are not eligible even when choosing  $b_{t+1} = 0$ , i.e.  $F(\mu^{def} - s^-) > \Lambda^{CB}$ . The default problem in period  $t + 1$ , conditional on beginning of period debt  $b_{t+1}$  can be written as

$$p(b_{t+1}) = \max_{\delta_{t+1}} \delta_{t+1}(1 - \tau(\mu^{def}))\mu^{def} + (1 - \delta_{t+1})(\mu_{t+1} - b_{t+1} - \tau(\mu_{t+1} - b_{t+1})), \quad (3)$$

which yields the default rule

$$\delta_{t+1} = \mathbb{1}(\mu^{def} > \mu_{t+1} - b_{t+1}),$$

such that  $F(\mu^{def} + b_{t+1}|s)$  represents the default probability  $\Lambda_t^j$ . Substituting into (2), the bond price can be expressed in terms of the cdf of  $\mu_{t+1}$ , the default threshold  $\mu^{def} + b_{t+1}$ , and the eligibility premium:

$$q(b_{t+1}|s) = (1 - F(\mu^{def} + b_{t+1}|s)) \left( 1 + \mathbb{1}\{F(\mu^{def} + b_{t+1}|s) \leq \Lambda^{CB}\} L_1 \right).$$

Consider next the firm maximization problem in period  $t$ , where the objective is given by the expected, discounted dividend stream of a levered firm,

$$\max_{b_{t+1}} q(b_{t+1}|s)b_{t+1} - x + \mathbb{E}[p(b_{t+1})], \quad (4)$$

where  $q(b_{t+1}|s)$  and  $p(b_{t+1})$  can be written explicitly using (3) and (4)

$$\begin{aligned} p(s) = \max_{b_{t+1}} & (1 - F(\mu^{def} + b_{t+1}|s))(1 + \mathbb{1}\{F(\mu^{def} + b_{t+1}|s) \leq \Lambda^{CB}\} L_1) \cdot b_{t+1} - x \\ & + (1 - \tau) \left( \int_{\underline{\mu}}^{\mu^{def} + b_{t+1}} \mu^{def} dF(\mu_{t+1}|s) + \int_{\mu^{def} + b_{t+1}}^{\bar{\mu}} \mu_{t+1} - b_{t+1} dF(\mu_{t+1}|s) \right). \end{aligned}$$

<sup>7</sup>The  $\mu^{def}$  can be interpreted as fraction of funds that firm-owners can divert during a default. They can also be interpreted as linear utility costs of default  $\mu_{t+1}^j - \mu^{def}$ . In the quantitative analysis, we use a concave specification of utility costs.

The first-order condition associated with problem (4) depends on the endogenously chosen default risk:

$$f(\mu^{def} + b_{t+1}|s) \cdot b_{t+1} = (1 - F(\mu^{def} + b_{t+1}|s)) \cdot \tau, \quad \text{if } F(\mu^{def} + b_{t+1}|s) > \Lambda^{CB}, \quad (5)$$

$$f(\mu^{def} + b_{t+1}|s) \cdot b_{t+1} \cdot (1 + L_1) = (1 - F(\mu^{def} + b_{t+1}|s)) \cdot (\tau + L_1), \quad \text{if } F(\mu^{def} + b_{t+1}|s) \leq \Lambda^{CB}. \quad (6)$$

For a non-eligible firm, the condition dictates that the marginal tax benefit in repayment states  $(1 - F(\mu^{def} + b_{t+1}))\tau$  equals marginal borrowing costs induced by additional default states  $f(\mu^{def} + b_{t+1}) \cdot b_{t+1}$ . Eligibility premia distort this trade-off by making debt issuance more attractive over and above tax benefits on the one hand, but make borrowing more costly on the margin, since banks lose pecuniary and non-pecuniary benefits, if fundamentals deteriorate. Expressing the first-order conditions in terms of hazard rates, we have

$$h(\mu^{def} + b_{t+1}|s) \cdot b_{t+1} = \tau, \quad \text{if } F(\mu^{def} + b_{t+1}|s) > \Lambda^{CB}, \quad (7)$$

$$h(\mu^{def} + b_{t+1}|s) \cdot b_{t+1} = \frac{\tau + L_1}{1 + L_1} \quad \text{if } F(\mu^{def} + b_{t+1}|s) \leq \Lambda^{CB}. \quad (8)$$

It follows from these conditions, that optimal leverage for an eligible firm exceeds that of an otherwise identical, non-eligible firm, consistent with empirical evidence. We define the *eligible debt capacity*  $\tilde{b}_{t+1}(s) \equiv F^{-1}(\Lambda^{CB}|s) - \mu^{def}$  as highest possible debt choice for which the default probability does not exceed the threshold. Also, we denote by  $b_{t+1}^1(s)$  the debt level satisfying (7) and by  $b_{t+1}^2(s)$  the debt level satisfying (8). How does a firm select itself into eligible and non-eligible regions, taken the eligibility threshold  $\Lambda$  as given? Recall that firms are heterogeneous with respect to the support of their revenue distribution. Intuitively, firm profitability matters for its debt choice. In Proposition 1, we partition firms into three groups.

**Proposition 1.** There are two cut-off values  $s^0$  and  $s^2$  for the shifting parameters, such that

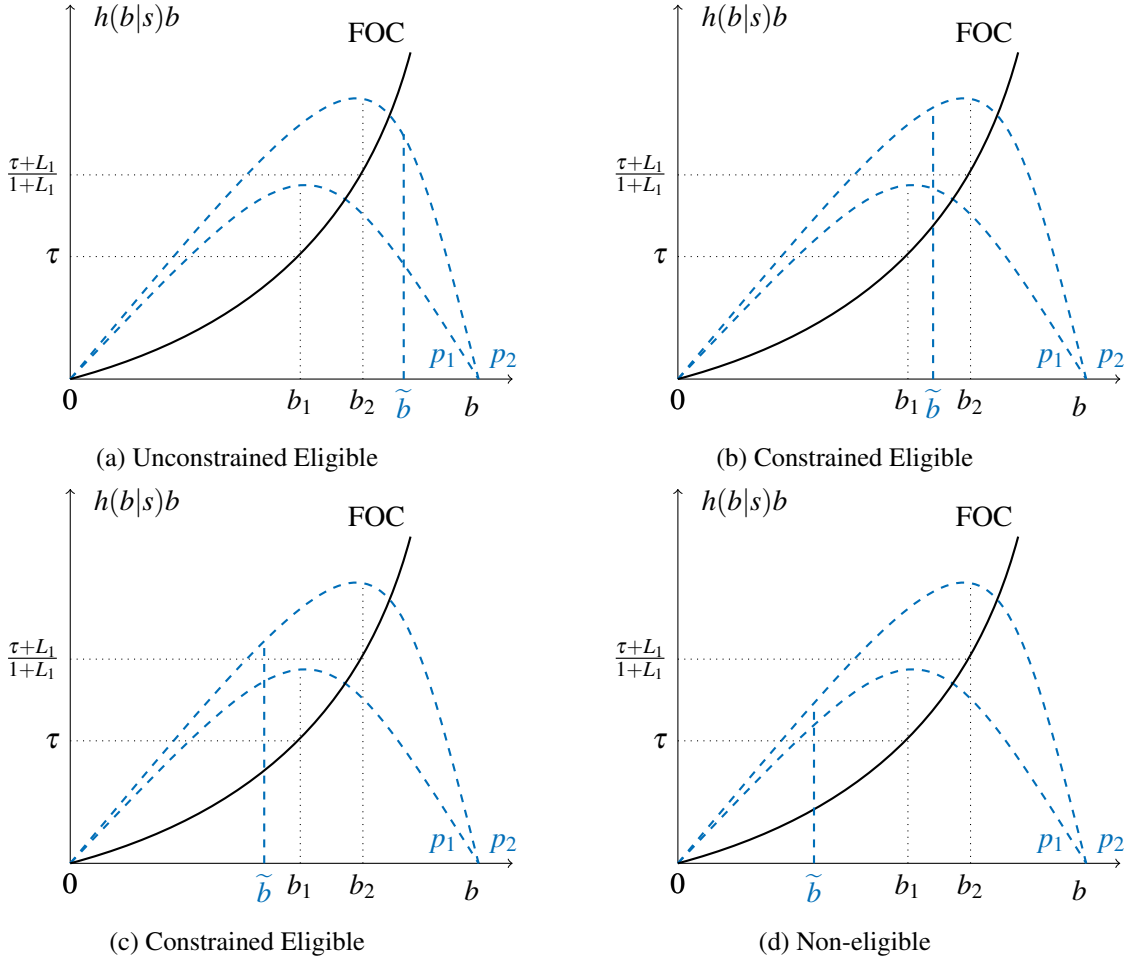
- Firms with  $s > s^2$  are *unconstrained eligible* in the sense that they choose  $b_{t+1}$  according to (7).
- Firms with  $s^0 < s < s^2$  are *constrained eligible* in the sense that borrow up to their eligible debt capacity  $\tilde{b}_{t+1}(s)$ .
- Firms with  $s < s^0$  are *non-eligible* and choose  $b_{t+1}$  according to (8).

*Proof:* See appendix A.1.

In Figure 2, we provide an illustration by plotting the first-order conditions (8) and (7) in solid black lines. Objective functions, i.e. the equity value, for the case of eligibility and non-eligibility are denoted as  $p_2$  and  $p_1$  (blue dashed lines). There are four possible combinations of  $b_{t+1}(s)^1$ ,  $b_{t+1}^2(s)$ , and  $\tilde{b}_{t+1}(s)$ . Figure 2a shows the case of  $\tilde{b}_{t+1} > b_{t+1}^2$ . The eligible debt capacity of an *unconstrained eligible* firm is sufficiently high, such that it can satisfy (8). Figure 2b shows a firm with insufficient debt capacity to satisfy (8), i.e.  $b_{t+1}^2(s)$  is not feasible, whereas satisfying (7) would be possible,  $b_{t+1}^1 < \tilde{b}_{t+1} < b_{t+1}^2$ . However, the value of the objective  $p_2(\tilde{b}_{t+1}(s)|s)$  exceeds the value at  $p_2(b_{t+1}^1(s)|s)$  because  $p_2$  is upward

sloping for all  $b < b_{t+1}^2(s)$  so that the firm chooses to be just eligible at a debt level  $\tilde{b}_{t+1}(s)$ : such a firm is *constrained eligible*. Within the case of  $\tilde{b}_{t+1}(s) < b_{t+1}^1(s)$ , there are two sub-cases: first, choosing  $b_{t+1}^1(s)$  is still feasible, but the firm can be better off by choosing  $\tilde{b}_{t+1}(s)$ , since  $p_2(\tilde{b}_{t+1}(s)|s) > p_1(b_{t+1}^1(s)|s)$ , as in figure 2c. Such a firm chooses to be just eligible and is also classified as *constrained eligible*. Second, firms with a sufficiently low  $s$  optimally choose  $b_{t+1}^1(s)$  and forgo eligibility, since the debt reduction required for eligibility is too large, as in figure 2d. These firms are *non-eligible*.

Figure 2: Firm Debt Choice along Revenue States  $s$



How do eligibility requirements affect welfare-relevant aggregates? We analyze these effects by holding the marginal benefit of collateral  $L_1$  constant, i.e. we provide an approximate comparative static argument. As a starting point, we characterize aggregate collateral  $\bar{B}_{t+1}$  in terms of the cut-off values  $s^0$  and  $s^2$ , which determine the partitioning of firms into eligibility regions:

$$\bar{B}_{t+1} = \int_{s_0}^{s^2} \left(1 - F\left(\tilde{b}_{t+1}(s) + \mu^{def}\right)\right) \tilde{b}_{t+1}(s) ds + \int_{s^2}^{\infty} \left(1 - F\left(b_{t+1}^2(s) + \mu^{def}\right)\right) b_{t+1}^2(s) ds. \quad (9)$$

Changes to  $\bar{B}_{t+1}$  directly represent the collateral supply channel. Similarly, the aggregate default rate  $\mathcal{M}_t$

can be expressed in terms of the profitability cut-offs, which represents the collateral quality channel:

$$\mathcal{M}_t = \int_{s^-}^{s^0} F(b_{t+1}(s) + \mu^{def}) ds + \int_{s_0}^{s^2} F(\tilde{b}_{t+1}(s) + \mu^{def}) ds + \int_{s_2}^{\infty} F(b_{t+1}(s) + \mu^{def}) ds. \quad (10)$$

A change in  $\Lambda^{CB}$  affects both cut-off levels and has heterogeneous effects on firms in the different regions. Changes in  $s^0$  and  $s^2$  reflect the extensive margin of eligibility requirements, since some firms will become unconstrained eligible and can choose according to the first-order condition (8), while others move from non-eligible to constrained eligible. As we shall see, only the latter movement has an effect on aggregate collateral. Firm reactions within each region constitute the intensive margin that be determined separately. Firms with  $s > s^2$  remain unconstrained eligible and do not change their debt and default policies. The same argument applies to non-eligible firms with  $s < s^0$ . Constrained eligible firms are directly effected: while they still choose  $\tilde{b}_{t+1}(s)$ , the size of their eligible debt capacity changes. In particular, an increase in  $\Lambda^{CB}$  increases  $\tilde{b}_{t+1}(s)$  for all  $s$ . This can be summarized in the following Lemma.

**Lemma 1.** Holding the eligibility premium  $L_1$  constant, a relaxation of collateral standards, i.e. increasing  $\Lambda^{CB}$ , decreases liquidity costs  $\mathcal{L}(\bar{B}_{t+1}, w)$  by increasing collateral supply  $\bar{B}_{t+1}$ , and increases the aggregate default rate  $\mathcal{M}_t$ .

*Proof:* See appendix A.2.

When changing eligibility requirements, the central bank faces a trade-off between fostering collateral supply (i.e. through lower eligibility thresholds or the response of firms) and deteriorating collateral quality by subsidizing firm risk-taking. While lemma 1 is not specific about the nature of costs associated with low collateral quality, we relate them directly to money market lending costs in the quantitative analysis. Also, the simplified model does not account for debt rollover, aggregate risk, and changes to the marginal benefit of collateral. The evaluation of these effects is only feasible numerically, which we turn to in the following sections.

### 3 Model

In this section, we extend our model from section 2 along multiple dimensions. We divide investors into firm-owners and bank-owners that differ in their discount factor to introduce an additional borrowing motive. Firms are infinitely lived and can issue long-term bonds. If a firm defaults, it enters a renegotiation process with banks that takes a stochastic number of periods. Repaying firms produce and reimburse the maturing share of their bonds outstanding. Each period an exogenous fraction of defaulters successfully restructures its debt and re-enters the corporate bond market. Firms transfer dividends to equity investors and those firms with financial market access decide on additional bond issuance.

In addition to entering repurchase agreements with the central bank, banks trade on private repo markets. The central bank sets an eligibility requirement on the public segment, while private eligibility requirements are determined endogenously. To model the dynamics of a segmented money market without losing tractability, we divide each period into three sub-periods. Investors and firms are only active in the third sub-period, while the money market is open in sub-periods 1 and 2. Finally, our model fea-



tures aggregate risk. The aggregate state is denoted by  $z_t$  and is assumed to follow a two-state Markov chain with transition matrix  $\Pi_z$ . The states will be referred to as boom ( $z_t = b$ ) and crisis ( $z_t = c$ ).

### 3.1 Bank-owners and Firm-owners

Our model features bank-owners (fixed-income investors), and firm-owners (equity investors), which are both risk-neutral but differ in their subjective time-discount factors. Bank-owners are passive in our model: they simply consume bank dividends  $d_t^B$  and lump-sum transfers  $T_t$  from the fiscal authority

$$C_t^B = d_t^B + T_t . \quad (11)$$

The discount factor of bank-owners is  $1/(1+i^{rf})$ , where  $i^{rf}$  is the time-invariant risk-free rate. Firm-owners discount the future with factor  $\beta < 1/(1+i^{rf})$ . Their maximization problem is given by

$$\begin{aligned} V^E(e_t(\cdot), z_t) &= \max_{e_{t+1}(\cdot)} C_t^E + \beta \mathbb{E}_t [V^E(e_{t+1}(\cdot), z_{t+1})] \\ \text{s.t. } C_t^E + \int_j e_{t+1}^j p_t^j dj &= \\ \int_j e_t^j \Big( h_t^j (p_t^{def,j} + d_t^{def,j} - \tau(d_t^{def,j})) + (1 - h_t^j) (p_t^{rep,j} + d_t^{rep,j} - \tau(d_t^{rep,j})) \Big) dj , \end{aligned} \quad (12)$$

where  $\tau(d)$  are dividend taxes,  $d_t^j$  are dividends paid out by firm  $j$ ,  $\delta_t^j$  the default indicator for firm  $j$ , and  $\int_j p_t^j e_{t+1}^j dj$  denotes investment into firm equity. Firm owners can be given the alternative interpretation as fund managers in the equity market.

### 3.2 Firms

There is a continuum of competitive firms, indexed by  $j$ . Their equity is fixed and normalized to  $e_t^j = 1$  for all  $t$ . We assume that managers choose bonds and dividends to maximize shareholder value, corresponding to the market capitalization of firm  $j$ . Firms produce the homogeneous all-purpose good with stochastic technology,

$$y_t^j = \frac{\mu_t^j}{\alpha} - x , \quad (13)$$

where  $x, \alpha > 1$  and  $\mu_t$  follows an AR(1) process in logs

$$\log \mu_{t+1}^j = \rho_\mu \log \mu_t^j + \sigma_{\mu,t+1} \varepsilon_{t+1}^j \quad \text{with } \varepsilon_t^j \sim N(0, 1) . \quad (14)$$

The revenue shock can reflect stochastic input costs, a TFP-shock, or demand fluctuations. The parameter  $\alpha$  is related to the asset-to-revenue ratio of firm  $j$  and will be directly taken from the data in the quantitative analysis. To introduce benefits of scale, firms incur fixed cost of production  $x$  in each period. The volatility of idiosyncratic revenues  $\sigma_{\mu,t}$  is time-varying and part of the aggregate state. Firm operations are financed by issuing bonds or by adjusting dividend payouts. Corporate bonds  $b_{t+1}^j$ , which mature in the subsequent period with probability  $\pi$ , pay coupon  $\kappa$  and are valued - according to the law of one price

- like new issues at price  $q_t$ .<sup>8</sup> Making bonds long-term enables us to generate realistic leverage ratios, which is important for our analysis as the aggregate level of bonds outstanding is a payoff-relevant part of the aggregate state. As second source of financing, firms can choose to adjust their dividend payments  $d_t^j$ . We assume piecewise-linear taxation of dividends

$$\tau(d_t^j) = \begin{cases} \tau d_t^j & \text{if } d_t^j > 0 \\ 0 & \text{if } d_t^j \leq 0, \end{cases}$$

where  $\tau$  is the tax rate. After the revenue shock realizes, firms decide whether or not to repay their bonds. There are two consequences of default: debt is restructured and firm-owners incur utility costs during the restructuring process. Restructuring takes a stochastic number of periods, a successful restructuring is denoted by the indicator  $\xi_t$ . The probability of leaving autarky  $\mathbb{E}_t[\xi_{t+1}]$  is denoted  $\eta$ , such that the average duration of a recovery is  $1/\eta$ . The law of motion for the credit status  $h_t^j$  can be written recursively

$$h_t^j = \xi_t^j(1 - \delta_t^j)(1 - h_{t-1}^j) + (1 - \delta_t^j)h_{t-1}^j. \quad (15)$$

In this notation  $h_t^j = 0$  indicates a restructuring period, i.e. a bad credit status, and  $h_t^j = 1$  indicates good credit status, i.e. access to the corporate bond market. Whenever  $h_t^j = 0$ , the firm can not adjust its bond position and incurs utility costs

$$m(\mu_t^j) = \max\{m_0 + m_1 \log(\mu_t^j), 0\}. \quad (16)$$

The functional form captures utility losses from direct bankruptcy costs such as legal fees and indirect losses from damaged reputation or supply chains disruptions. While the former can plausibly assumed to be constant, it is reasonable to assume that the latter depend on the potential revenue draw. We choose the convenient log-specification to account for this feature. More generally, utility costs of default have been used in the sovereign default literature (Bianchi et al., 2018) as they provide sufficient flexibility to match default risk and leverage dynamics. Since our policy analysis in section 5 is organized around costs due to decreased collateral quality incurred by interbank market participants, the use of utility costs of default also conveniently circumvents a discussion of welfare losses from default incurred by other agents. After successful restructuring, the firm re-enters capital markets with a reduced debt level of  $v^{rec}b_t$  with  $0 < v^{rec} < 1$ . The default problem can be written

$$p(b_t^j, \mu_t^j, z_t) = \max_{\delta_t^j \in \{0,1\}} \delta_t^j p^{def}(b_t^j, \mu_t^j, z_t) + (1 - \delta_t^j) p^{rep}(b_t^j, \mu_t^j, z_t), \quad (17)$$

<sup>8</sup>The yield to redemption  $\tilde{r}$  for corporate bonds is determined by the internal rate of return of a perpetuity with constant decay:

$$\begin{aligned} q &= \sum_{t=1}^{\infty} \frac{CF_t}{(1+\tilde{r})^t} = \sum_{t=1}^{\infty} \frac{(\pi + \kappa)(1-\pi)^{t-1}}{(1+\tilde{r})^t} = \frac{\pi + \kappa}{1-\pi} \sum_{t=1}^{\infty} \left(\frac{1-\pi}{1+\tilde{r}}\right)^t = \frac{\pi + \kappa}{1-\pi} \left(\sum_{t=0}^{\infty} \left(\frac{1-\pi}{1+\tilde{r}}\right)^t - 1\right) \\ &= \frac{\pi + \kappa}{1-\pi} \left(\frac{1}{1 - \frac{1-\pi}{1+\tilde{r}}} - 1\right) = \frac{\pi + \kappa}{1-\pi} \frac{-\tilde{r} - \pi + 1 + \tilde{r}}{\tilde{r} + \pi} = \frac{\pi + \kappa}{\pi + \tilde{r}} \end{aligned}$$

For the yield-to-redemption it follows that  $\tilde{r} = \frac{\pi + \kappa}{q} - \pi$ . The corporate bond spread  $s$  is defined as  $\tilde{r} - i^{rf}$ .

The value of default is given by

$$p^{def}(b_t^j, \mu_t^j, z_t) = d^{def}(\mu_t^j) - \tau(d^{def}(\mu_t^j)) + \beta \mathbb{E}_t \left[ \eta p(v^{rec} b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) + (1 - \eta) p^{def}(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \right],$$

$$\text{with } d^{def}(\mu_t^j) = \frac{\mu_t^j - \max\{m_0 + m_1 \log(\mu_t^j), 0\}}{\alpha} - x. \quad (18)$$

If the firm does not default, it sells its output, makes coupon payments, and repays the redemption share of bonds. It then issues new bonds, such that its maximization problem conditional on repayment can be represented by the Bellman equation

$$p^{rep}(b_t^j, \mu_t^j, z_t) = \max_{d_t^j, b_{t+1}^j} d_t^j - \tau(d_t^j) + \beta \mathbb{E}_t \left[ p(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \right]$$

$$\text{s.t. } d_t^j = \frac{\mu_t^j}{\alpha} - x - (\pi + \kappa) b_t^j + q(b_{t+1}^j, \mu_t^j, z_t) (b_{t+1}^j - (1 - \pi) b_t^j). \quad (19)$$

We denote the bond  $b(b_t^j, \mu_t^j, z_t)$  and dividend  $d(b_t^j, \mu_t^j, z_t)$  policies as the maximizers of (19). Dividend and default policies by firms are linked to firm-owner optimization via

$$p(b_{t+1}^j, \mu_{t+1}^j, z_t) = \beta \mathbb{E}_t \left[ \delta(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \left( d^{def}(\mu_{t+1}^j) - \tau(d^{def}(\mu_{t+1}^j)) + p^{def}(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \right) + \right. \\ \left. (1 - \delta(b_{t+1}^j, \mu_{t+1}^j, z_{t+1})) \left( d(b_{t+1}^j, \mu_{t+1}^j, z_t) - \tau(d(b_{t+1}^j, \mu_{t+1}^j, z_t)) + p(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \right) \right], \quad (20)$$

so that the price of equity equals its expected return.

### 3.3 Banks

This section introduces a segmented money market. As in section 2, at the beginning of sub-period 1, banks draw an idiosyncratic liquidity shock: with probability  $\frac{1}{2}$ , banks have a liquidity deficit  $w > 0$ , which can only be settled on the money market. Those banks will be referred to as *cash borrowers*. Different to section 2, with probability  $\frac{1}{2}$ , a bank receives a liquidity surplus of the same size and will become a *cash lender*. There is no aggregate liquidity shock. Since banks are the only active agents in sub-period 1 and 2, liquidity shocks are settled on a fully collateralized money market, which consists of two segments. Borrowers can approach the *central bank* (the public segment) or *trade with other banks* (the private segment) that experienced positive liquidity shocks.<sup>9</sup>

We abstract from creation of collateral *within* the financial sector.<sup>10</sup> By our assumption on the market structure, there is also no trade with other sectors in the first and second sub-periods. This has two implications: first, intermediaries hit by a negative shock can only settle their transactions with other

<sup>9</sup>In the Euro area, the total volume of repos collateralized with corporate bonds is rather small. Nyborg and Roesler (2019) document that only 1 % of total collateral on the interbank is corporate bonds, while increases to 3-5 % in the case of central bank operations. However, corporate bond investors can easily engage in so called collateral upgrades on the securities lending market. Specifically, market participants can exchange high-quality corporate bonds easily for government bonds and use them on the interbank repo market. As this securities exchange arrangements have not been subject to major disruptions or explicitly targeted by central bank interventions, we omit this layer of complexity in the following.

<sup>10</sup>Nyborg (2017) provides evidence on collateral own-use. To adequately address these issues, one would need to extend the financial sector building block along several dimensions, which is beyond the scope of our paper.

intermediaries, or with the central bank. Second, collateral is in fixed supply on the money market. In the following, we call all trades on this market *repurchase agreements* (repos). Money market repos are initiated in the first sub-period of period  $t$  and settled at the beginning of the third sub-period, when banks receive new cash inflows from their assets. Since firm productivities (and, hence, corporate bond pay-offs) are unknown when repos are negotiated, the value of collateral at maturity of the repo is also uncertain. Thus, bonds are only accepted as collateral if they satisfy a specific eligibility requirement.

The private money market segment is modelled after several salient features of the Euro interbank repo market, which is predominantly cleared by CCPs. The largest of these CCPs is Eurex, which offers clearing, collateral management, and settlement of repos. For more details on market structure, we refer to Mancini et al. (2015), who provide an anatomy of the Euro interbank market. The relevant empirical regularities for our setup can be summarized as follows:

1. Repo market trading is anonymous.
2. Cash lenders can re-use their collateral with the ECB.
3. Eligibility requirements are set by Eurex and are tighter than ECB requirements.
4. Conditional on eligibility, haircuts are identical on both segments.

We discuss them in more detail in appendix B. Anonymity of trade allows us to abstract from explicitly modelling bilateral trade. Due to the possibility of collateral re-use, we assume that cash lenders can pledge securities accepted as collateral on the private market at the central bank. To rationalize the third observation, we assume that cash lending entail costs, for example from counterparty risk.<sup>11</sup> We assume that trading on the private money market takes place *before* the central bank becomes active. Cash lenders and borrowers have stochastic access to the central bank and can pledge securities with probability  $\varsigma$ .<sup>12</sup> Together this will imply that cash lenders are only able to offload lending costs with probability  $\varsigma$ , such that the eligibility requirement is stricter on the interbank market. It will also imply co-existence of both segments in equilibrium. Figure 3 describes our timing assumption and the different stages of the bank problem. The fourth point motivates our representation of collateral frameworks solely by the eligibility requirement. All bonds issued by firms with a default probability exceeding the eligibility thresholds are non-eligible, while eligible bonds can be pledged without a haircut. In practice, eligible bonds have collateral values less than 100 % due to other risk factors, which are not present in our setup. In the following, we introduce each segment in turn.

**Money Market Segments** We begin with describing the public segment. The auxiliary variable

$$\Lambda_t^j \equiv \mathbb{E}_t \left[ \delta(b_t^j, \mu_{t-1}^j, z_{t-1}) \right], \quad (21)$$

<sup>11</sup>Boissel et al. (2017) show that these costs played an important role for CCPs during the European debt crisis. Afonso et al. (2011) find for US money market data that counterparty risk was an important driver during the financial crisis of 2008-09. Capponi and Cheng (2018) discuss a trade-off between fees and collateral requirements in the context of derivatives clearing. Heider et al. (2015) assume counterparty risk in form of a fraction of borrowing banks defaulting on their interbank market obligations. From a macro perspective, it is not relevant whether costs are borne by cash lenders directly or by the CCP who sets eligibility requirements on behalf of cash lenders and distributes its profits. See also the discussion in Pirrong (2011), chapter IX.

<sup>12</sup>Note that  $\varsigma$  is security-specific and not bank-specific, reflecting the idea that some securities are locked in dedicated portfolios or with subsidiaries that can not participate in central bank operations directly.

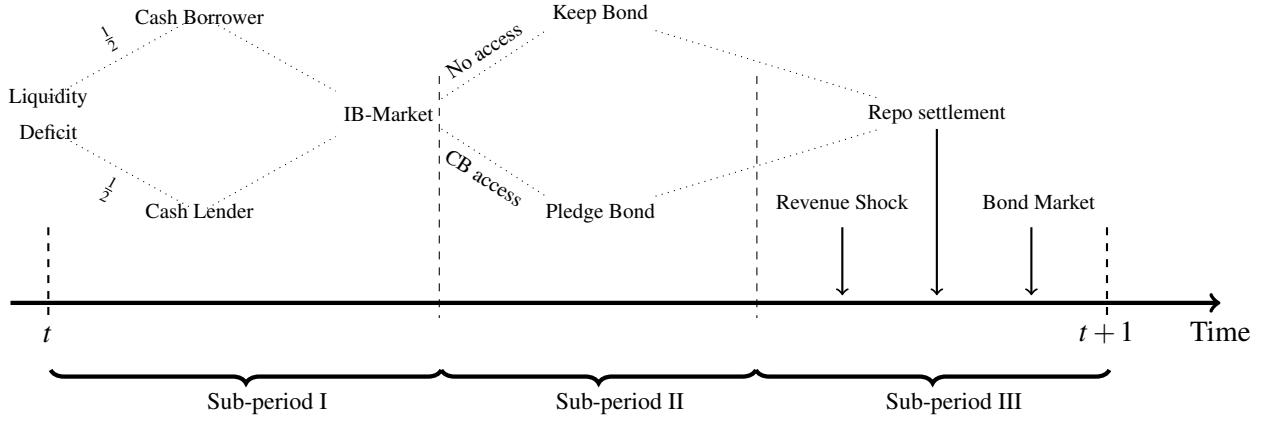


Figure 3: Timing Assumption

denotes the default probability of firm  $j$  in sub-period 2. Therefore, we describe central bank policy only in terms of the eligibility threshold:

$$\Phi^{CB}(\Lambda_t^j, z_t) = \begin{cases} 1 & \text{if } \Lambda_t^j \leq \Lambda^{CB}(z_t) \\ 0 & \text{else} \end{cases}.$$

Since the central bank eligibility requirement  $\Lambda^{CB}(z_t)$  is the key policy variable in our model, we allow it to be state-dependent. On the private segment, eligibility is determined endogenously, taking central bank policy and firm behaviour as given. We assume that all interbank lending is channelled through a central clearing counterparty or via a custodian in a tri-party repo, such that we can express the benefits of market participation independently of the contract counterparty.<sup>13</sup> Interbank market trades are characterized by an (endogenously determined) eligibility requirement  $\Phi^{IB}(\Lambda_t^j, z_t)$  and an (exogenously set) fee  $f$ . As will become clear shortly, the interbank eligibility requirement will depend on the aggregate state as well. We rely on the fourth finding regarding interbank markets and impose that either  $\Phi^{IB}(\Lambda_t^j, z_t) = \Phi^{CB}(\Lambda_t^j, z_t)$  or  $\Phi^{IB}(\Lambda_t^j, z_t) = 0$ .

**Cash Lenders** Lending banks distribute all fees as dividends to shareholders in the subsequent centralized market. Note that the fee does not depend on the collateral pledged, consistent with our assumption that borrowers keep a margin accounts with the central counterparty. Lender  $i$  accepting corporate bond  $j$  incur costs that can be represented by the function

$$K(\Lambda_t^{j,-i}, z_t) = (1 - \varsigma) \cdot k^{MM}(z_t) \cdot \Lambda_t^{j,-i} \cdot \Phi^{IB}(\Lambda_t^{j,-i}, z_t) \Upsilon_t^{j,-i} b_t^{j,-i} + \varsigma \cdot k^{MM}(z_t) \cdot \Lambda_t^{j,-i} \cdot \max\{\Phi^{IB}(\Lambda_t^{j,-i}, z_t) - \Phi^{CB}(\Lambda_t^{j,-i}, z_t), 0\} \Upsilon_t^{j,-i} b_t^{j,-i}, \quad (22)$$

where  $k^{MM}(z_t) > 0$  is a cost parameter and  $\Upsilon_t^j$  is the *expected cum-coupon payoff* in the second sub-period. This object is closely tied to, but different from the market price - which is only defined in the second sub-period - since there is no discounting between sub-periods. Also note that costs are

<sup>13</sup>This reflects the notion that central counterparties specify pools of eligible collateral and aggregate different positions of a single market participant into a margin account.

expressed in terms of *accepted* bonds  $b_t^{j,-i}$  and does not depend on lender  $i$ 's corporate bond portfolio. Equation (22) summarizes all expenses associated with money market activity carried by lenders. If bond  $j$  is accepted on the interbank market ( $\Phi^{IB}(\Lambda_t^{j,-i}, z_t) = 1$ ), but can not be pledged at the central bank, cash lenders have a long position  $\Upsilon_t^{j,-i} b_t^{j,-i}$  subject to counterparty risk. Conversely, if the bond can be pledged at the central bank, only the uncovered long position  $\max\{\Phi^{IB}(\Lambda_t^{j,-i}, z_t) - \Phi^{CB}(\Lambda_t^{j,-i}, z_t), 0\} \cdot \Upsilon_t^{j,-i} b_t^{j,-i}$  is subject to counterparty risk. In all parametrizations we consider, cash lenders only accept bonds, if they can be re-used to borrow from the central bank, i.e. costs are zero in case of central bank access. The hypothetical where bonds are accepted on the interbank market, but not by the central bank is therefore irrelevant in the following.

The long positions subject to counterparty risk are weighted by collateral quality  $\Lambda_t^j$  and the (potentially state-dependent) parameter  $k^{MM}(z_t)$ . This parameter reflects all costs associated with low-quality collateral, such as risk management expenses related and counterparty risk, which we discussed above. We assume that the central bank (described below) is also subject to these costs.

**Cash Borrowers** As in section 2, we assume that liquidity costs of cash borrowers depend on the *total collateral value* of bank  $i$ 's corporate bond portfolio, which is given by

$$\bar{B}(z_t) = \int_j \left( \theta(\Lambda_t^j) \Phi^{IB}(\Lambda_t^j, z_t) + (1 - \theta(\Lambda_t^j)) \zeta \Phi^{CB}(\Lambda_t^j, z_t) \right) \Upsilon_t^j b_t^j dj, \quad (23)$$

where  $\theta(\Lambda_t^j) = 1$  if a corporate is pledged on the private segment and  $\theta(\Lambda_t^j) = 0$  if borrowers retain the bond and wait for potential access to central bank facilities. We characterize the optimal  $\theta$  below. As in section 2, we specify a functional form for liquidity costs

$$L(\bar{B}, w) = w \cdot \max \left\{ \bar{l} - \frac{l_0}{l_1} \left( \frac{\bar{B}}{w} \right)^{l_1}, 0 \right\}, \quad l_0, w > 0, \quad 0 < l_1 < 1. \quad (24)$$

All costs are paid in the third sub-period when production takes place and profits are transferred to bank owners. This ensures that money market positions do not enter as additional state variable when banks make their portfolio choice in the third sub-period. We solve the bank problem by backward induction.

**Bank Problem in Sub-Period 2: Central Bank Facilities** Once interbank repos are negotiated, banks learn which securities they can pledge with the central bank. Since retaining bonds incurs costs from counterparty risk for cash lenders, banks will re-use all bonds with the central bank to offset their long positions. Likewise, cash borrowers use all eligible bonds that were not pledged on the interbank market to collateralize a repo with the central bank, since this reduces their liquidity costs.

**Cash Lenders in Sub-Period 1** We first determine the interbank eligibility requirement from the maximization of cash lenders. Interbank credit supply is characterized by a continuum of banks competing for accepting corporate bond  $b_t^{j,-i}$  against lending cash. Revenues from cash lending are given by the fee multiplied with the total lending volume minus *expected* lending costs. The maximization problem for



accepting ( $a_t^j = 1$ ) corporate bond  $b_t^{j,-i}$  is given by

$$\max_{a_t^j \in \{0,1\}} a_t^j \left( f \Phi^{IB}(\Lambda_t^{j,-i}, z_t) - (1 - \varsigma) k^{MM}(z_t) \cdot \Lambda_t^{j,-i} \cdot \Phi^{IB}(\Lambda_t^{j,-i}, z_t) \right. \\ \left. - \varsigma k^{MM}(z_t) \cdot \Lambda_t^{j,-i} \cdot \max\{\Phi^{IB}(\Lambda_t^{j,-i}, z_t) - \Phi^{CB}(\Lambda_t^{j,-i}, z_t), 0\} \right) \Upsilon_t^{j,-i} b_t^{j,-i}.$$

When  $\Phi^{IB}(\Lambda_t^{j,-i}, z_t) = \Phi^{CB}(\Lambda_t^{j,-i}, z_t) > 0$ , the FOC defines a threshold  $\tilde{\Lambda}(z_t)$  above which lenders do not accept corporate bond  $j$  as collateral

$$\tilde{\Lambda}(z_t) = \frac{f}{(1 - \varsigma) k^{MM}(z_t)}, \quad (25)$$

which is independent of  $\Phi^{CB}(\Lambda_t^{j,-i}, z_t)$ . In the (hypothetical) case of  $\Phi^{CB}(\Lambda_t^{j,-i}, z_t) = 0$ , central bank access becomes irrelevant and the eligibility threshold is given by

$$\hat{\Lambda}(z_t) = \frac{f}{k^{MM}(z_t)}, \quad (26)$$

with  $\hat{\Lambda}(z_t) < \tilde{\Lambda}(z_t)$ . Intuitively, if the central bank does not accept any corporate bonds, interbank lenders only accept very high rated bonds since they are exposed to counterparty risk with probability one. This setup also implies that standing facilities are not used at all, if  $\Lambda^{CB}(z_t) \leq \tilde{\Lambda}(z_t)$ . In the baseline calibration, we choose parameters such that  $\Lambda^{CB}(z_t = b) = \Lambda^{CB}(z_t = c) = \Lambda^A$  and  $\hat{\Lambda}(z_t = b) = \Lambda^{AA}$ , i.e. only bonds rated A or higher are accepted on the private segment in boom periods. Crisis periods are associated with a tightening of interbank market lending standards, which we model by an elevated cost parameter  $k_b^{IB} < k_c^{IB}$ , such that only very high quality collateral is accepted. Aggregating over cash lenders  $i$  and corporate bonds  $j$  yields a decomposition of lending costs into the share borne by cash lenders and by the central bank.

$$\mathcal{K}^{CB}(z_t) = \frac{\varsigma}{2} k^{MM}(z_t) \cdot \int_j \Phi^{CB}(\Lambda_t^j, z_t) \Lambda_t^{j,-i} b_t^{j,-i} \Upsilon_t^{j,-i} d j, \quad (27)$$

$$\mathcal{K}^{IB}(z_t) = \frac{1 - \varsigma}{2} k^{MM}(z_t) \cdot \int_j \Phi^{IB}(\Lambda_t^j, z_t) \theta(\Lambda_t^j, z_t) \Lambda_t^{j,-i} b_t^{j,-i} \Upsilon_t^{j,-i} d j. \quad (28)$$

Adding up these expressions, we obtain

$$\mathcal{K}^{MM}(z_t) = \mathcal{K}^{CB}(z_t) + \mathcal{K}^{IB}(z_t) \quad (29) \\ = \frac{k^{MM}(z_t)}{2} \cdot \int_j \left( \underbrace{\varsigma(1 - \theta(\cdot)) \Phi^{CB}(\cdot)}_{\text{CB Borrowing}} + \underbrace{(1 - \varsigma) \theta(\cdot) \Phi^{IB}(\cdot)}_{\text{IB Borrowing}} + \underbrace{\varsigma \theta(\cdot) \Phi^{CB}(\cdot)}_{\text{Collateral Re-Use}} \right) \Lambda_t^{j,-i} b_t^{j,-i} \Upsilon_t^{j,-i} d j$$

Inspecting (29), we note that interbank lenders do not bear costs for the fraction  $\varsigma$  of bonds pledged with the central bank, due to collateral re-use. Total costs borne by interbank lenders are given by the fraction  $1 - \varsigma$  of bonds that are accepted from interbank borrowers, but can not be pledged by the central bank to neutralize the long position in interbank repos. The share of direct central bank lending to banks with a liquidity deficit is given by the fraction  $\varsigma$  of retained bonds. This decomposition implicitly defines

bounds on the share of lending costs borne by the central bank, if it accepts corporate bonds as collateral. If  $\theta_t^j = 1$  for all  $j$ , only private lenders use the public segment to pledge the collateral they received. The share borne by the central bank is then given by the probability of access  $\frac{\frac{1}{2}\varsigma}{\frac{1}{2}\varsigma + \frac{1}{2}(1-\varsigma)} = \varsigma$ . Conversely, if  $\theta_t^j = 0$  for all  $j$ , all money market borrowing takes place via the central bank facilities, while lending costs borne by banks are zero. Instead, all lending costs are borne by banks when the central bank has stricter eligibility requirements than the private segment or does not accept corporate bonds.

**Cash Borrowers in Sub-Period 1** Taking as given all eligibility requirements, borrowers choose which share of each bond  $j$  to pledge on the interbank market *before* knowing whether the bond can be used to collateralize central bank borrowing. The pledging decision will be denoted by the function  $\theta(\Lambda_t^j, z_t)$  which depends on firm default probabilities through eligibility requirements. Here,  $\theta(\Lambda_t^j, z_t)$  is the share pledged on the interbank market. While in principle it is possible to only pledge a share on the private market segment and retain the rest for central bank borrowing, the discrete nature of eligibility standards will reduce the problem to a binary choice, as we shall see shortly. The maximization problem of cash borrowers depends on the aggregate state and can be written as

$$\mathcal{L}(z_t) \equiv \max_{\theta(\cdot)} -L(\bar{B}(z_t), w) - f \int_j \theta(\Lambda_t^j, z_t) \Phi^{IB}(\Lambda_t^j, z_t) \Upsilon_t^j b_t^j dj \quad \text{s.t.} \quad \theta(\Lambda_t^j, z_t) \in [0, 1], \quad (30)$$

where  $\mathcal{L}(z_t)$  is defined as the effective collateral value of bank  $i$ 's corporate bond portfolio under the optimal pledging choice. Note that borrowers only incur fees for those bonds that are pledged on the private market segment. In the following, we restrict the analysis to the interval  $\Lambda_t^j \in [0, \Lambda^A]$ , because bonds rated lower than A are not accepted on the private segment. The first order condition w.r.t.  $\theta(\cdot)$  is given by the point-wise derivative of (30)

$$0 = \left( l_0 \bar{B}(z_t)^{l_1-1} w_t^{1-l_1} \left( -\varsigma \Phi^{CB}(\Lambda_t^j, z_t) + \Phi^{IB}(\Lambda_t^j, z_t) \right) - f \Phi^{IB}(\Lambda_t^j, z_t) \right) \Upsilon_t^j b_t^j + \zeta_t^- - \zeta_t^+, \quad (31)$$

where  $\zeta_t^+$  denotes the multiplier on  $\theta(\cdot) \leq 1$  and  $\zeta_t^-$  the multiplier on  $\theta(\cdot) \geq 0$ . By our assumption on  $L$ , the first-order condition requires  $\theta(\cdot) \in \{0, 1\}$  as long as  $\Phi^{CB}(\Lambda_t^j, z_t) = \Phi^{IB}(\Lambda_t^j, z_t) > 0$  and  $\Upsilon_t^j b_t^j > 0$  holds. Otherwise both multipliers would be zero and eq. (31) has no interior solution. Since at most one constraint binds by definition, either  $\zeta_t^- > 0$  or  $\zeta_t^+ > 0$ , while the multiplier on the other constraint will be zero, such that the pledging decision may be written

$$\theta_t^j(\Lambda_t^j, z_t) = \mathbb{1} \left\{ l_0 \bar{B}(z_t)^{l_1-1} w_t^{1-l_1} \left( \Phi^{IB}(\Lambda_t^j, z_t) - \varsigma \Phi^{CB}(\Lambda_t^j, z_t) \right) > f \Phi^{IB}(\Lambda_t^j, z_t) \right\}. \quad (32)$$

If  $\Phi^{IB}(\Lambda_t^j, z_t) = 0$  while at the same time  $\Phi^{CB}(\Lambda_t^j, z_t) = 1$ , the bond will be retained ( $\theta_t^j = 0$ ) in the first sub-period and pledged at the central bank with probability  $\varsigma$  in the second sub-period. If  $\Phi^{IB}(\Lambda_t^j, z_t) \geq \Phi^{CB}(\Lambda_t^j, z_t)$ , the pledging decision depends on access probabilities, fees, and eligibility premia: pledging on the interbank market eliminates uncertainty about central bank access, but is costly due to the fee  $f$ . Equation (32) reveals that borrowers prefer to keep the bond and wait for access to central bank borrowing, if private market fees are expensive or the probability of CB access is very high. Having determined money market outcomes, we now turn to the pass-through of money market outcomes to the

corporate bond market.

**Bank Problem in Sub-Period 3: Corporate Bond Market** Combining the previous elements, the Bellman equation associated with the maximization problem of bank  $i$  in any third sub-period is,

$$\begin{aligned}
V^B(b_t^i(\cdot), z_t) = & \max_{b_{t+1}^{j,i}} d_t^B + \frac{1}{1 + i^r f} \mathbb{E}_t [V^B(b_{t+1}^i(\cdot), z_{t+1})] \\
\text{s.t. } d_t^B = & \int_j k_t^j b_t^{j,i} dj - \int_j q(b_{t+1}^j, \mu_t^j, z_t) b_{t+1}^{j,i} dj \\
& - \frac{1}{2} (1 - \varsigma) k^{MM}(z_t) \cdot \int_j \Phi^{IB}(\Lambda_t^j, z_t) \theta(\Lambda_t^j, z_t) b_t^{j,-i} \Upsilon_t^{j,-i} dj - \frac{L(\bar{B}^i(z_t))}{2}.
\end{aligned} \tag{33}$$

Note that the costs bank  $i$  incurs in the first sub-period of  $t + 1$ , conditional on being a money market *lender*, do not depend on bank  $i$ 's portfolio choice, but on the collateral accepted  $\Upsilon_t^{-i} b_t^{-i}$  from other borrowers. Also, observe that lending fees  $f \cdot \Phi^{IB}(\Lambda_t^j, z_t)$  cancel out, as banks are interbank lenders and borrowers with equal probability. The per-unit price schedule of for corporate bonds can thus be written

$$\begin{aligned}
q(b_{t+1}^j, \mu_t^j, z_t) = & \frac{1}{1 + i^r f} \cdot \mathbb{E}_t \left[ \delta(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) q^{def}(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) + \right. \\
& \left. \left( 1 - \delta(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \right) \left( \pi + \kappa + (1 - \pi) q(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \right) \right] \cdot \\
& \left( 1 + \frac{1}{2} \left( l_0 \bar{B}(z_t)^{l_1 - 1} w_t^{1 - l_1} \left( \varsigma (1 - \theta(\Lambda_t^j, z_t)) \Phi^{CB}(\Lambda_t^j, z_t) + \theta(\Lambda_t^j, z_t) \Phi^{IB}(\Lambda_t^j, z_t) - \theta(\Lambda_t^j, z_t) \cdot f \right) \right) \right).
\end{aligned} \tag{34}$$

The total payoff contains a pecuniary part and an eligibility premium. The pecuniary part depends on default in  $t + 1$ : if the firm repays, bonds pay the coupon  $\kappa$ , the maturing fraction  $\pi$  is redeemed, and the remainder  $1 - \pi$  is rolled over at the next period's market price, which is obtained from evaluating next period's pricing schedule at optimal firm policies. In case of default, bonds are priced according to

$$q^{def}(b_t^j, \mu_t^j, z_t) = \frac{1}{1 + i^r f} \mathbb{E}_t \left[ (1 - \eta) q^{def}(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) + \eta v^{rec} q(b_{t+1}^{rec}, \mu_{t+1}^j, z_{t+1}) \right]. \tag{35}$$

In this model, eligibility premia are driven by liquidity costs of cash borrowers, which directly affect the pricing of corporate bonds via the contribution of bond  $j$  to aggregate collateral. In contrast, interbank lending costs incurred by bank  $i$  are independent of bank  $i$ 's portfolio choice and only depend on the collateral pledged by cash borrowers, such that they do not show up directly in (34). However, they affect the eligibility premium through the interbank market eligibility requirements. Our detailed money market specification allows us to consider also money market disruptions, such as a smaller counterparty risk tolerance by lenders (governed by the parameter  $k^{MM}$ ) in a counterfactual.

### 3.4 Public Sector and Equilibrium

The fiscal authority receives dividend taxes and makes lump-sum transfers  $T(z_t)$  to investors. Since the central bank becomes an active counterparty on the money market, seigniorage revenues or losses appear in the public sector budget constraint, which is given by

$$T_t = \int_j (1 - h(b_t^j, \mu_t^j, z_t)) \tau(d(b_t^j, \mu_t^j, z_t)) + h(b_t^j, \mu_t^j, z_t) \tau(d^{def}(\mu_t^j)) dj - \frac{1}{2} \varsigma^{MM}(z_t) \int_j \Lambda_t^j \Phi^{CB}(\Lambda_t^j, z_t) b_t^{j,-CB} \Upsilon_t^{j,-CB} dj. \quad (36)$$

Note that the central bank accepts bonds from a share  $\varsigma$  of interbank borrowers *and* lenders, since both types will approach the central bank each period after the private money market closes. To close the model, we impose that the government adjusts  $T_t$  such that (36) holds and define the market clearing condition for the all-purpose good:

$$\int_j \frac{\mu_t^j}{\alpha} dj - x = C(z_t) + \mathcal{K}^{MM}(z_t) + \frac{\mathcal{L}(\bar{B}(z_t))}{2}. \quad (37)$$

Equation (37) collects resources spent on money market borrowing and lending. The public sector budget (36) shows that the cost share borne by the public sector increases in  $\varsigma$  and  $\Phi^{CB}$ . While from a resource efficiency perspective it does not matter whether the central bank or private banks are paying lending costs, the allocation of costs between private and public sector may put effective constraints on central bank policy that are beyond the scope of our paper.

**Recursive Competitive Equilibrium** A competitive equilibrium is given by bond price schedules  $q(b_{t+1}^j, \mu_t^j, z_t)$ , equity price schedules  $p(b_{t+1}^j, \mu_t^j, z_t)$ , default policy  $\delta(b_t^j, \mu_t^j, z_t)$ , dividend policy  $d(b_t^j, \mu_t^j, z_t)$ , and debt policy  $b(b_t^j, \mu_t^j, z_t)$ , pledging decisions  $\theta(b_t^j, \mu_t^j, z_t)$ , and aggregate quantities  $\bar{B}(z_t)$ ,  $C(z_t)$ ,  $K(z_t)$ ,  $L(z_t)$ ,  $T(z_t)$ , such that

- Given the pricing schedules for bonds and their individual states, the default policy solves (17). Bond and dividend policies solve (19).
- Taken firm policies as given, the equity price is consistent with household optimization.
- Taken firm policies as given, the bond price schedule is consistent with banks' optimization.
- Bank profits are zero.
- Equity, corporate bond, and goods markets clear.
- The government budget constraint holds.
- The aggregate shock follows  $z_{t+1}^l = \pi_{kl} z_t^k + (1 - \pi_{kk}) z_t^l \quad l \in \{b, c\}$
- The law of motion for the distribution of firms over credit status, bond holdings and firm-specific revenues follows

$$\begin{aligned}
& g_{t+1} \left( h_{t+1}, b_{t+1}, \mu_{t+1}, z_{t+1}^l \right) \\
&= \sum_{k \in \{b, c\}} \int \int \mathbb{1} \{ \rho_\mu \mu_t + \sigma_\mu \varepsilon_{t+1}^\mu = \mu_{t+1} \} \times \\
& \quad \left[ (1 - h_t) \left[ \eta \mathbb{1} \left\{ b(\mathbf{v}^{rec} b_t, \mu_t, z_t^k) = b_{t+1} \right\} \mathbb{1} \{ 1 = h_{t+1} \} + (1 - \eta) \mathbb{1} \{ b_t = b_{t+1} \} \mathbb{1} \{ 0 = h_{t+1} \} \right] \right. \\
& \quad \left. + h_t \left[ (1 - \delta(b_t, \mu_t, z_t^k)) \mathbb{1} \left\{ b(b_t, \mu_t, z_t^k) = b_{t+1} \right\} \mathbb{1} \{ 1 = h_{t+1} \} + \delta(b_t, \mu_t, z_t^k) \mathbb{1} \{ b_t = b_{t+1} \} \mathbb{1} \{ 0 = h_{t+1} \} \right] \right] \times \\
& \quad g_t(h_t, b_t, \mu_t) f(\varepsilon_{t+1}^\mu) d\varepsilon_{t+1}^\mu db_{t+1} \pi_{kl}, \quad l \in \{b, c\}
\end{aligned}$$

## 4 Calibration

We solve our model using fully global solution methods, building on Strebulaev and Whited (2012). The algorithm essentially contains three steps: first, we solve firm policies given bond price schedules on a discrete grid by iterating on the value function of firm managers, which corresponds to shareholder value. Second, given these policies, we compute the distribution of firms over individual states. The third step consists of computing aggregate quantities and updating bond price schedules. For a detailed description of the algorithm and the parameters governing our numerical approximation we refer to appendix D.2.

**Empirical Strategy** Our quantitative analysis is based on Euro Area data between 2004Q1, the earliest data with reliable interbank and corporate bond data, and 2009Q4. By choosing this truncation point, we have a sample that can be subset into a boom period from 2004Q1-2008Q3 and a crisis from 2008Q4-2009Q4.<sup>14</sup> Each period corresponds to one quarter. From the length of both sub-samples we infer the crisis probability  $\pi_{bc} = 0.05$ , i.e. one crisis in 20 years, while the crisis duration of 5 quarters implies  $\pi_{cb} = 0.2$ . All calibrated parameters that are not explicitly linked to the aggregate state are informed by the pre-crisis sub-sample. Our calibration is based on an equilibrium with tight eligibility requirements in boom and crisis periods. We adopt this strategy, because relaxation of eligibility requirements set a precedent: changing the policy parameter in the baseline calibration would imply that bond prices during the pre-crisis sub-sample reflect this policy change, which appears to be an unreasonable assumption.

**Corporate Bond Market** The first anchor of our quantitative exercise consists of the firm distribution, measured by corporate bond spreads and leverage. We obtain data from *IHS Markit* on the total fixed income securities universe in Europe which we subset for non-financial corporate bonds and from company data available through *Compustat Global*. A detailed description of the construction of our dataset is given in appendix C. We characterize the bond spread distribution by the 25 %, 50 %, and 75 %-quantiles. The boom period is represented by four relatively calm years at the beginning of the sample period. The crisis sample is given by 5 quarters following the Lehman insolvency, which is characterized by elevated spread levels and a significant increase in cross-sectional dispersion. Regarding the financial side, we set the recovery rate of debt in default to  $\mathbf{v}^{rec} = 0.67$ , which is a standard value in the pricing

<sup>14</sup>At the same time, we exclude periods where the central bank conducted major asset purchase programmes. Since our model does not explicitly model central bank interventions, we restrict our analysis to effects of collateral policy in a conventional monetary policy regime.

of fixed income securities. To inform maturity  $\pi$  and coupon  $\kappa$  parameters, we take the market-value-weighted average of the corresponding values of the iBoxx EUR Non-Financials and the iBoxx EUR High Yield core Non-Financials ex crossover LC for each month in our sample.

**Money Market** The second set of parameters is related to the money market and eligibility requirements. *First*, we use the ECB collateral framework before and after the 2008 crisis to inform the eligibility requirements for central bank repos. A crucial ingredient of the ECB and Eurex collateral frameworks is the reliance on *ratings*, which are difficult to model parsimoniously. Therefore, we adopt an indirect approach and map rating categories into expected default risk using default probabilities implied in iBoxx bond prices. We follow the methodology lined out in Hull (2006), which derives implied default probabilities from price differentials to a risk-free bond with the same cashflow profile. Bond prices are corrected for the estimated eligibility premium of 7bp (Pelizzon et al., 2020) when computing implied default probabilities (IDP).<sup>15</sup>

Default probability thresholds for the central bank collateral framework for the baseline calibration and the main counterfactuals follow from the IDPs in the *A* and *BBB*-rating buckets, which correspond to the ECB eligibility requirements before and after 2008 (Nyborg, 2017). The list of eligible bonds can be obtained from the ECB directly. Based on our matching with the IHS Markit database, we subset the list by ratings, and truncate each bucket at the 95 % quantile of IDPs. This procedure delivers cut-off values at  $\Lambda^{AA} = 0.01$ ,  $\Lambda^A = 0.015$ , and  $\Lambda^{BBB} = 0.035$ . Further descriptive statistics are summarized in appendix C.<sup>16</sup>

Following the Eurex extended collateral pool, we impose that banks are willing to lend against A-rated bonds ( $\tilde{\Lambda}_b = \tilde{\Lambda}_c = 0.015$ ), if these can be pledged with the central bank. Since the problem of cash lenders is, however, characterized by two parameters,  $k_b^{MM}$  and  $\varsigma$ , we need a second condition to pin them down jointly. Following the composition of the Eurex core basket, table 7 in appendix C, which consist of 97 % AA or higher rated bonds, it is reasonable to assume that cash lenders are willing to always lend against these assets, irrespective of central bank policy. We set  $\hat{\Lambda}_b = 0.01$  and can therefore pin down  $k_b^{MM} = \frac{f}{0.01}$  in boom periods via (26). Then, we use (25) to back out  $\varsigma$  via the expression  $1 - \varsigma = \frac{f}{0.015k_b^{MM}}$ .

*Second*, we use empirical findings from Pelizzon et al. (2020) to calibrate  $l_0$ . They make use of the ECB having discretion in including bonds that formally satisfy eligibility requirements in the list of actually eligible assets. This discretion generates a randomly selected control group of bonds that eventually become eligible, but are not yet accepted by the ECB. They estimate a yield reaction to surprise eligibility of 7bp. Our structural model permits explicit calculation of a surprise inclusion by setting  $\Phi^{CB} = 0$  when pricing the bond (holding firm policies fixed) and compare this hypothetical price to the equilibrium price. In contrast to a randomly selected control group, our structural model allows us to compute the hypothetical period  $t$ -price of corporate bond  $j$  that is not eligible in period  $t$ , but becomes

<sup>15</sup>The model-implied counterpart for the implied default probability can be constructed as follows: The present value of losses given default can be approximated by the perpetuity structure  $\sum_{t=1}^{\infty} \left( \frac{1-\pi}{1+i^f} \right)^t (1 - v^{rec}) q^{rf} = \frac{1-\pi}{i^f + \pi} (1 - v^{rec}) q^{rf}$ , where the risk-free bond has the time-invariant price  $\frac{\pi + \kappa}{\pi + i^f}$ . Given the price difference between the risk-free bond and the corporate bond, the IDP follows.

<sup>16</sup>Results using the Eurex list of eligible collateral instead delivers similar cut-offs.



eligible in period  $t + 1$ . This price

$$q^0(b_{t+1}^j, \mu_t^j, z_t) = \frac{1}{1 + irf} \mathbb{E}_t \left[ \delta(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) q^{def}(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \right. \\ \left. + \left( 1 - \delta(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \right) \left( \pi + \kappa + (1 - \pi) q \left( b(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}), \mu_{t+1}^j, z_{t+1} \right) \right) \right], \quad (38)$$

contains eligibility premia from  $t + 1$  onwards via the continuation value  $q \left( b(b_{t+1}^j, \mu_{t+1}^j), \mu_{t+1}^j \right)$  and is therefore distinct from a CDS-type security, which *never* becomes eligible. The premium is then simply given by  $s_t^j - s_t^{j,0}$  where the spreads are computed from the stochastic maturity structure. A (synthetic) CDS can be priced according to

$$q^{CDS}(b_{t+1}^j, \mu_t^j, z_t) = \frac{1}{1 + irf} \mathbb{E}_t \left[ \delta(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) q^{CDS,def}(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \right. \\ \left. + \left( 1 - \delta(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}) \right) \left( \pi + \kappa + (1 - \pi) q^{CDS} \left( b(b_{t+1}^j, \mu_{t+1}^j, z_{t+1}), \mu_{t+1}^j, z_{t+1} \right) \right) \right]. \quad (39)$$

*Third*, we construct a proxy for the size of liquidity deficits  $w$ . Following De Fiore et al. (2019), we take annual ECB Money Market Surveys and compute average borrowing on the collateralized interbank market segment, relative to balance sheet size (for a details on the data see appendix C).

**Calibrated Parameters and Moment Selection** In order to inform the remaining parameters, we target all quartiles of leverage  $b_t^j$  and bond spreads  $s_t^j$  in the pre 2008Q4-subsample, i.e. before the Great Financial Crisis. To calibrate the extent of the fundamental shock, we target the increase of leverage during crisis periods, as measured by the 75%-quantile. Together with the eligibility premium, this gives us 8 moments. In table 2 we summarize all parameters for our baseline calibration.

Table 2: Parameter Choices

Parameter	Value	Source
Crisis probability $\pi_{bc}$	0.05	One crisis in 20 quarters
Probability of leaving crisis $\pi_{cb}$	0.2	Crisis duration 5 quarters
Bank discount rate $i^{rf}$	0.003	EURIBOR-EU-HCPI
Revenues/Assets $\alpha$	4.69	Compustat Global
Corporate + Dividend Tax Rate $\tau$	0.5	Eurostat
Recovery Rate in default $\nu^{rec}$	0.66	Hackbarth et al. (2006)
Re-entry Probability $\eta$	0.125	Bris et al. (2006)
Coupon Rate $\kappa$	0.01	Markit iBoxx
Maturity Parameter $\pi$	0.0625	Markit iBoxx
Interbank Lending fee $f$	0.73bp	Pelizzon et al. (2020)
Interbank Lending Costs $k_b$	0.0073	EUREX and ECB List
Interbank Lending Costs $k_c$	0.00109	EUREX and ECB List
Liquidity Cost Intercept $\bar{l}$	0.005	Marginal Lending Facility
Liquidity Cost Curvature $l_1$	0.1	Schmitz et al. (2017)
Probability of CB access $\varsigma$	0.33	EUREX and ECB List
Liquidity Deficit $w$	3	European Money Market Survey
Cost function parameter $l_0$	2.5e-04	Calibrated
Borrower discount factor $\tilde{\beta}$	0.989	Calibrated
Fixed cost $x$	0.175	Calibrated
Bankruptcy costs level $m_0$	0.055	Calibrated
Bankruptcy costs slope $m_1$	0.25	Calibrated
Idiosyncratic revenues $\rho_\mu$	0.81	Calibrated
Idiosyncratic revenues $\sigma_{\mu,b}^2$	0.01	Calibrated
Idiosyncratic revenues $\sigma_{\mu,c}^2$	0.04	Calibrated

**Model Fit** In table 3 we show targeted moments in boldface. All data moments are *time-series averages* over the periods 2004Q1-2008Q3 (boom), 2008Q4-2009Q4 (crisis) or both, the quantiles correspond to the cross-sectional distribution. The model counterpart to leverage (at book values) is simply  $b_t^j$ . In order to compute the model-implied eligibility premium, we directly exploit the identification strategy of Pelizzon et al. (2020). To compute cross-sectional moments, we simulate a panel of 1000 firms for 11000 periods and discard the first 1000 periods as burn-in. To compute the eligibility premium, we compare the 'treated' group of traded bonds, i.e. all eligible bonds, with the control group of synthetic bonds, i.e. we evaluate (39) at the equilibrium firm policies. Since these synthetic bonds are issued by the same firm, the model-implied regression controls for firm level characteristics by design.

Table 3: Targeted Moments

Moment	Boom		Crisis		Overall	
	Data	Model	Data	Model	Data	Model
Bond Spread, 1st Quartile	<b>28</b>	<b>26</b>	103	22	44	25
Bond Spread, Median	<b>45</b>	<b>35</b>	160	37	72	36
Bond Spread, 3rd Quartile	<b>70</b>	<b>77</b>	248	119	107	84
Leverage, 1st Quartile	<b>9.6</b>	<b>13.4</b>	10.8	12.6	9.8	13.0
Leverage, Median	<b>23.3</b>	<b>21.9</b>	25.3	23.0	23.7	22.0
Leverage, 3rd Quartile	<b>36.9</b>	<b>34.4</b>	<b>39.7</b>	<b>38.6</b>	37.4	35.3
Eligibility Premium	-	-	-	-	<b>7</b>	<b>7</b>

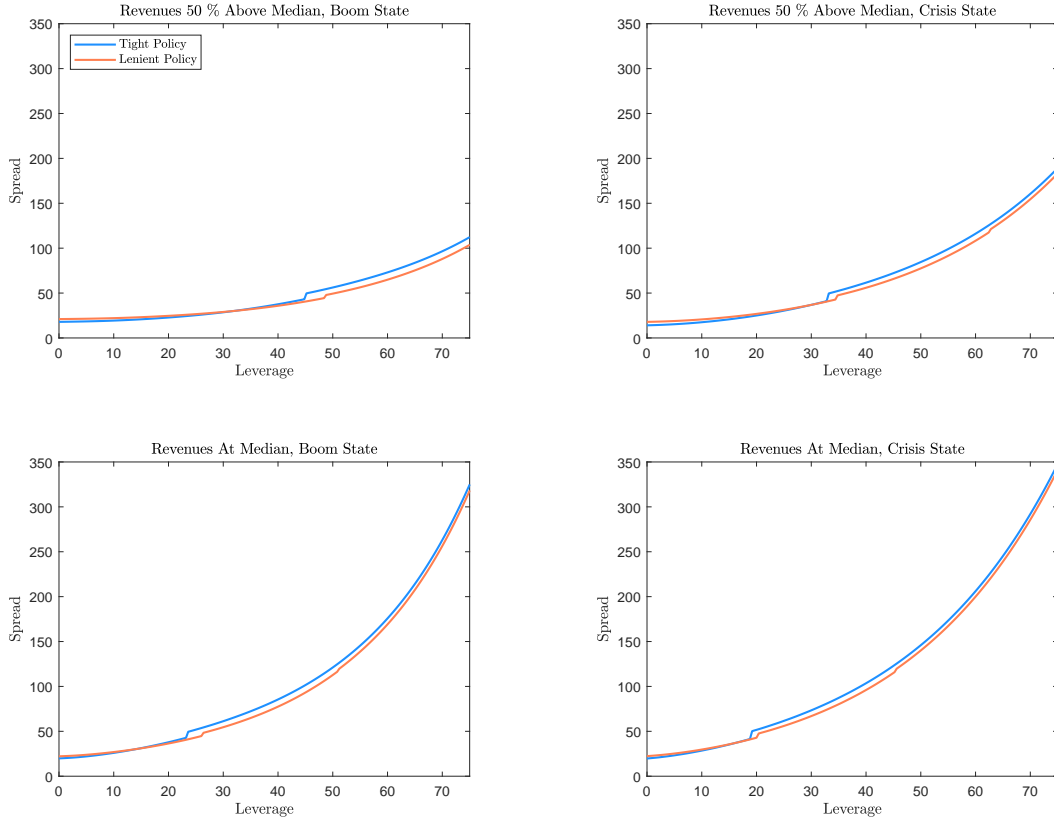
Notes: Spreads are annualized and in basis points, leverage in percentage points. Targeted moments are boldfaced.

The simulated firm panel exhibits a cross-sectional distribution of leverage closely aligned with the data. Bond spreads are mostly consistent with the data, although the median spread is slightly too small. The eligibility premium closely matches the data moment. Comparing time-series means across different states, data and model show a broadly consistent pattern: spreads reduce in boom periods and rise in crisis states. A characteristic of our framework is that bond with small default risk remain the only ones eligible in crises episodes so that their spread is even smaller than during booms due to increasing eligibility premia. In contrast, the model-implied distribution of leverage picks up the increase observed in the data during crisis. Finally, recall that our target for the eligibility premium is taken from Pelizzon et al. (2020). The authors do not perform calculations for boom (crisis) sub-samples, such that we can only target the premium in the overall sample. Quantitatively the increase of the spread distribution in crisis episodes falls short in the model, compared to the data. This is an expected result, since bonds are priced risk-neutrally and there are no frictions regarding net-worth constraints for banks, which were identified as an important driver of non-financial bond spreads in Gilchrist and Zakrajšek (2012).

## 5 Quantitative Analysis

With the parametrized model in hand, this section numerically examines the impact of eligibility requirements on corporate bond spreads, the firm distribution, and macroeconomic aggregates. We then study the role of interbank market disruptions for the transmission of central bank eligibility. The discussion is centered around two different policy regimes. The first (blue) corresponds to tight eligibility requirements from the baseline calibration with  $\Lambda_b^{CB} = \Lambda_c^{CB} = \Lambda^A$ . The second (orange) shows the case of lenient eligibility requirements  $\Lambda_b^{CB} = \Lambda_c^{CB} = \Lambda^{BBB}$  in all states. The notion of tight and lenient eligibility requirements is based on the ECB policy before and after the financial crisis of 2008. Appendix E.2 provides additional results for a larger range of policy parameters.

Figure 4: Spreads Implied by Equilibrium Bond Price Schedules



Notes: Conditional on the revenue draw we display the ensuing bond spread for different levels of firm leverage. The blue (orange) line denotes a tight (lenient) central bank collateral policy.

## 5.1 Eligibility and Bond Spreads

We start with a closer inspection of corporate bond spreads in boom and crisis episodes and show spreads implied by equilibrium bond-price *schedules* in figure 4. These schedules are consistent with the zero profit condition of banks, but are not directly informative about the firm distribution. They nevertheless provide intuition about the effect of leverage, revenue shocks, and eligibility on default risk. We plot schedules for a corporate revenue level 50% above and directly at the median revenue realization. The left panel represents the boom state, while the right panel correspond to the crisis state.

For each policy, the spreads are higher for firms in a lower revenue state and increase in leverage. Furthermore, the spread level in the crisis state is generally higher. Note that the spread is more elastic with respect to leverage for low-revenue firms which can be related to the shape of the revenue shock distribution. To see this, consider two firms with identical leverage but different revenue states. Since revenues are persistent, a low-revenue firm will likely receive a low revenue draw in the next period, such that increasing leverage is adding relatively many default states, leading to a marked increase in spreads. In contrast, a high-revenue firm can increase leverage without a strong increase in default risk, leading to a relatively modest increase in spreads.

The discontinuities in the bond price schedule represent the eligibility threshold and, equivalently,

eligible debt capacity. Several features are noteworthy. First, for each policy, the jump is located further to the left (at a lower leverage  $b_{t+1}^j$ ) in the crisis state, compared to the boom state. The jump is shifted to the left for riskier firms in the lower panel. Second, we observe two distinct jumps under lenient policies, which is due to the difference between private and public eligibility requirements. Note that there is now second jump indicating central bank eligible debt capacity for high-quality borrowers in the boom state, since they are always eligible under lenient policy.

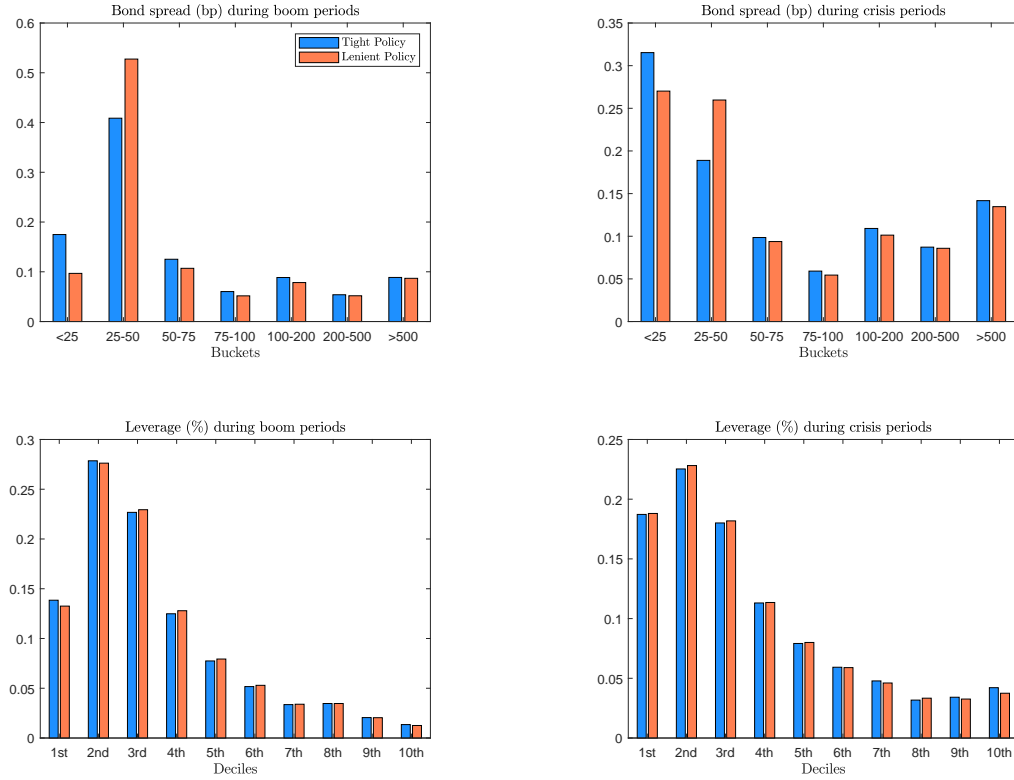
Holding idiosyncratic and aggregate revenues constant, the effect of different eligibility requirements can be inferred from the location of the discontinuities. In all cases, lenient eligibility requirements (a higher  $\Lambda^{CB}$ ) increase eligible debt capacity: the jump in the orange schedule is shifted significantly to the right. Moreover, this shift is larger in the crisis state. Furthermore, one feature stands out in the case of tight policy: for very low leverage (and thus low default risk, holding the revenue state fixed) the bond spread in crisis periods is smaller than during boom periods. This is due to the substantial decline in collateral supply, such that bonds of remaining eligible firms carry an exceptionally large eligibility premium, which even exceeds the elevated default risk premium.

To sum up, the introduction of eligibility benefits from corporate collateral affects firms in different ways. While firms at the eligibility threshold directly benefit from a reduction in the spread, also firms that are not at their eligible debt capacity benefit from a shift in their spread schedule, since their bonds may be eligible in a future period (captured by the continuation value of a bond). Moreover, more generous eligibility thresholds, in principle, would incentivize firms to taken on more risk through higher leverage choices. Finally, our results illustrate that under lenient eligibility requirements, bonds can be divided into those that are only publicly eligible and into those that are additionally eligible for private markets.

## 5.2 Eligibility Requirements and the Firm Distribution

Since the price schedules are only one part of the firm problem, we now show the effects of eligibility requirements using the cross-sectional firm distributions in equilibrium. Therefore we plot histograms of bond-spreads, and leverage in figure 5, which illustrate the impact of eligibility requirements via macroeconomic aggregates on the firm distribution. Of course, macroeconomic aggregates are endogenous, and we discuss how these depend on central bank policy and the cross-sectional distribution in figures 6 and 7.

Figure 5: Cross-Sectional Distribution



Notes: Histograms represent the distribution of all firms that are not in default. Differently-colored bars denote policies.

Starting with bond spreads during boom periods in the upper left panel, we observe a significantly higher share of bonds in the second lowest risk bucket (25-50bp) for the lenient requirements compared to the tight case. This on the one hand follows from the fact that more bonds carry an eligibility premium under this policy. On the other hand, there are less bonds in the lowest risk bucket, which is a first evidence of increased risk taking. Consistent with the shape of the spread distribution, we observe lower leverage is more prevalent under tight eligibility requirements (lower left panel).

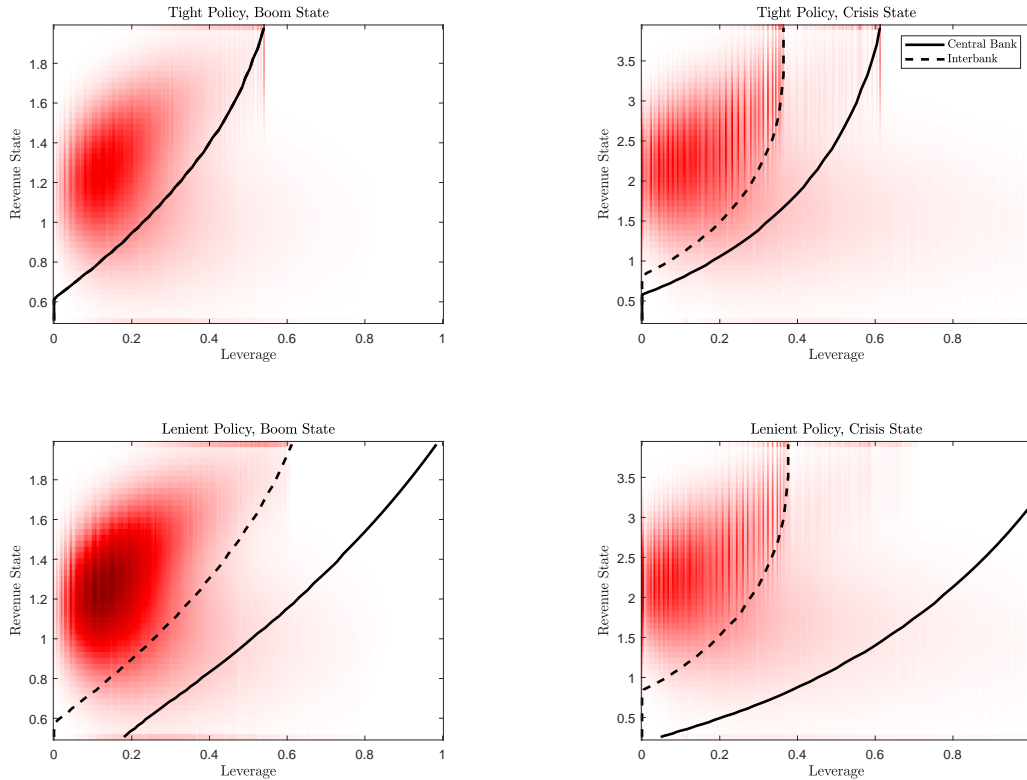
During a crisis period, the spread distribution is shifted to the right for all policies, compared to boom periods. This mainly follows from an increase in fundamental risk. The shift also affects the shape of the distribution for lenient and tight eligibility standards, which have their mode now in the lowest risk bucket. At the same time the share of firms in the highest risk bucket (and also the share of defaulters) increases significantly, albeit for different reasons. To see this, note that the bond price can be decomposed into the present value of cash flows and the present value of collateral service flows via the continuation value. Under tight eligibility requirements, the right tail is more pronounced, since many bonds with intermediate risk lose their eligibility premium in the crisis state, such that the default option becomes more attractive. At the same time, the heavy left tail follows from the exceptionally large eligibility premium carried by low-risk bonds, due to small collateral supply in the crisis state, which could already be seen from figure 4. Together, this leads to a pronounced low spread bucket, and relatively few firms in the second lowest bucket. Under lenient eligibility requirements, firms enter the



crisis state with higher levels of leverage, such that debt rollover is prohibitively costly for many firms due to bad fundamentals. Thus, the right tail is relatively pronounced in this case, too. At the same time, collateral supply is large, such that current collateral service leads only to a modest eligibility premium. However, since the aggregate state turns into a boom with high probability, future collateral service is relatively high, such that spreads are low and both left-most spread buckets have high mass.

The description of aggregate and cross-sectional dynamics is completed by the effect of private and public eligibility requirements on the cross-sectional distribution and, ultimately, collateral supply. In figure 6, we illustrate eligible debt capacity of firms, i.e. the maximum leverage firms can choose in a given revenue state without losing eligibility for both aggregate states in different policy regimes. Fixing a revenue draw on the y-axis, the solid black line marks the leverage choice  $b_{t+1}^j$  such that  $\Lambda(b_{t+1}^j, \mu_t^j, z_t) < \Lambda^{CB}(z_t)$ , while the dashed black line indicates  $b_{t+1}^j$  such that  $\Lambda(b_{t+1}^j, \mu_t^j, z_t) < \Lambda^{IB}(z_t)$ . To relate eligible debt capacity to macroeconomic aggregates, we also show the firm distribution over idiosyncratic states  $(b_{t+1}^j, \mu_t^j)$  in form of a heatmap. Here, idiosyncratic states with high mass are marked in brighter colors. Eligible firms are located on the north-west of the black lines, while non-eligible firms lie in the south-east.

Figure 6: Eligibility Requirements along the Cross-Section



Notes: The shaded area reflects the cross-sectional distribution over leverage and revenues for both aggregate states, respectively. Firms in restructuring are excluded. The black lines represent the eligible debt capacities on the private (dashed) and public segments (solid).

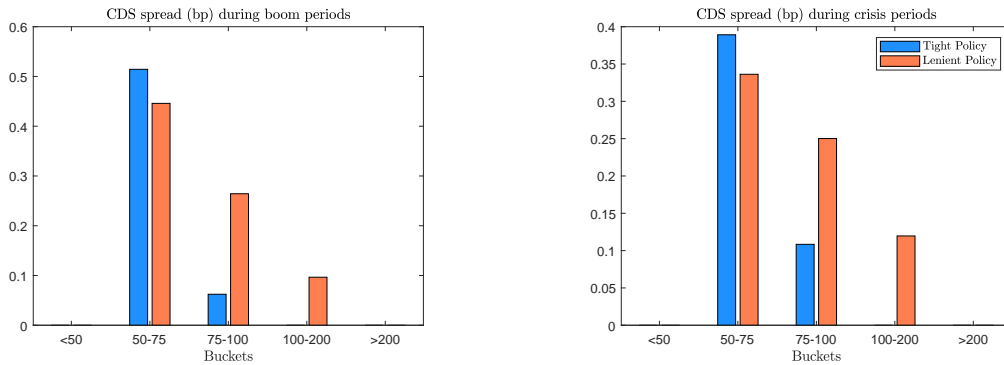
The majority of firms is located around the median revenue state and leverage values slightly below 20 %, consistent with the quantiles used in the calibration (table 3). In all cases, eligible debt capacity

is increasing in  $\mu_t^j$  and decreasing in  $b_{t+1}^j$ . The top left panel shows that in the baseline scenario with tight policy a large share of bonds is eligible on both market segments, i.e. the solid and dashed lines are identical. Note that the y-axis has a considerably larger range in the crisis state due to the risk shock (top right panel). In the crisis state, eligibility on the private segment shifts to the left due to the interbank market freeze. For example a firm at median revenues  $\mu_t^j = 1$  can now only choose leverage up to  $b_{t+1}^j = 0.1$  without losing eligibility on the private market compared to around  $b_{t+1}^j = 0.25$  in the boom state. At the same time, the eligibility threshold on the public segment is higher. However, the mass of firms choosing leverage slightly below the private market eligibility threshold is large, while almost no firms choose leverage slightly above the threshold. These firms can be interpreted as constrained eligible with respect to the interbank market in the terminology of section 2.

Under lenient eligibility requirements (middle and lower panels), the firm mass shifts slightly to the right, associated with elevated risk-taking. At the same time, the central bank eligibility debt capacity shifts drastically to the south-east since requirements are lowered. As the bottom right panel reveals, its outward shift during the crisis state is similar under lenient eligibility requirements, while interbank eligible debt capacity hardly changes. Due to the large discrepancy of the dashed and solid black lines in these cases, it can be seen that lowering eligibility requirements during crisis periods has a substantial effect on aggregate collateral, while the mass of additionally eligible firms in the boom state is rather modest.

To relate the preceding analysis to welfare-relevant aggregate cost terms from (37), note that the quantity of collateral is represented by the weighted integral over all firms north-west of the central bank eligibility frontier (solid black line). The quality of collateral, which is the second determinant of lending cost  $\mathcal{K}$ , can be represented by the cross-section of synthetic CDS-spreads over eligible firms (see figure 7). CDS spreads serve as a measure of default risk and, hence, do not contain eligible premia.

Figure 7: Cross-Sectional Distribution of Collateral Quality



Notes: Number of eligible bonds in a given bucket relative to the mass of all bonds (excluding those in restructuring).

In boom periods, collateral portfolios are riskier under the tight relative to the lenient policy, thereby implying a lower quality of collateral. The collateral portfolio under lenient policies contains a larger number of securities, which are more risky. In crisis periods, the collateral portfolio is smaller and has higher default risk for all policies. Comparing different policies, it turns out that under lenient require-

ments the collateral portfolio are of similar size, but the quality is much lower under lenient policy. This directly represents the collateral quality channel operating through debt and default incentives of the non-financial sector.

In summary, the cross section of firms reveals that a lenient policy increases the amount of eligible securities but lowers its quality as it incentivize firms to take on more risk. We now turn to a discussion of how the relative strength of both channels plays out in the aggregate.

### 5.3 Macroeconomic Aggregates

The preceding results regarding the cross-section can be summarized using the aggregate cost terms during boom and crisis states in table 4. These aggregates are related to investor consumption via market clearing for the all-purpose good (37). Collateral supply in the boom state is similar to the observed value in the Euro Area (11 % of GDP), which puts our analysis in a reasonable macroeconomic perspective. Since our model is not a general equilibrium characterization, we only compare cost terms across policies and aggregate states, but still report absolute values of each cost term for completeness.<sup>17</sup>

Table 4: Macroeconomic Aggregates for Different Policies

State Policy	Boom		Crisis	
	Tight	Lenient	Tight	Lenient
Collateral Supply $\bar{B}$	11.58	15.44	8.63	11.35
- Relative to Tight Policy	1.000	1.334	1.000	1.316
Lending Costs $\mathcal{K}$	5e-04	7e-04	4e-04	6e-04
- Relative to Tight Policy	1.000	1.526	1.000	1.628
- Share Borne by CB	33%	51%	38%	59%
Liquidity Costs $\mathcal{L}$	0.479	0.471	0.487	0.480
- Relative to Tight Policy	1.000	0.984	1.000	0.985

*Notes:* All values except share borne by central bank  $\mathcal{K}^{CB}/\mathcal{K}^{MM}$  are expressed relative to the baseline calibration with tight eligibility requirements. Since  $\mathcal{K}^{CB}$  is zero in the baseline calibration, we express the share of central bank lending costs relative to total lending costs in the respective parameterization.

In the baseline calibration with tight eligibility requirements, the crisis state is characterized by significant contractions in collateral supply and a simultaneous deterioration of collateral quality compared to a boom:  $\bar{B}$  (and therefore money market lending) drops by around 50%, while lending costs drop by 25 %, which is a direct consequence of the riskier collateral portfolio (see figure 7). Note that this increase is primarily caused by the shock to firm fundamentals, since during crisis periods, collateral requirements are actually *stricter* on the interbank market. Under lenient eligibility requirements, the drop in  $\bar{B}$  relative to boom periods is similar to the baseline calibration, but lending costs decrease by even less (around 15 %), since the central bank lends against BBB-rated bonds. Table 4 also establishes

<sup>17</sup>To put the rather small size of lending costs into perspective, note that we are not considering other adverse effects of corporate default, such as real costs of restructuring and costs associated with bank balance sheets, both of which is beyond the scope of our paper, but of potential relevance in a general equilibrium characterization.

the potential trade-off that arises when setting central bank eligibility requirements. On the one hand, a lenient policy increases collateral supply and reduces liquidity cost. On the other hand, it raises lending cost and the risk taken by the central bank.

We next decompose the effect of eligibility requirements into the intensive and extensive margin. This decomposition is approximate since the persistent shocks and long-term bonds make a clear-cut distinction between firm policies at a specific idiosyncratic state across two equilibria impossible. Instead, we compute the bond issuance of *newly eligible bonds* for each policy in boom and crisis states separately. In comparison to a tight policy under lenient requirements central bank eligibility but not private bond eligibility changes, so that newly eligible bonds are those that bank borrowers use on the public market (beyond those already pledged). Since interbank eligibility requirements do not depend on central bank requirements, we believe that this approximation is sufficiently accurate, although it does not take into account that firms may gain or lose private market eligibility in response to central bank policy.<sup>18</sup>

During boom periods, accepting riskier bonds under a lenient policy increases money market lending approximately by 37%, compared to the tight policy.<sup>19</sup> 37% of this increase can be attributed to the intensive margin. In the crisis period, the ratio of increased money market lending to the costs of deteriorating collateral quality is less favourable: while collateral supply increases by 42%, lending costs almost double, compared to tight eligibility requirements. Here, the intensive margin makes up only 20% of the increase.

While the movements in  $\mathcal{L}$  closely mirror those in  $\bar{B}_t$ , the preceding decomposition is important for lending costs. By construction, cash borrowers do not tap the public segment under tight policy - the 33% lending costs borne by the central bank are due to collateral re-use of cash lenders. This share increases to 44% under lenient policy, associated with the gap between private and public eligible debt capacity in figure 11. During the crisis state, the share of lending costs borne by the central bank increases, and it increases more under lenient requirements, i.e. the central bank plays a much larger role as money market lender.

## 5.4 The Role of Interbank Market Tightening

This section discusses the role of central bank eligibility requirement in the presence of tightening of interbank eligibility requirements. While such a tightening, sometimes also called a freeze or dry-up, was largely absent in the Euro area, Krishnamurthy et al. (2014) and Gorton and Metrick (2012) document such a tightening for the US repo market. We model interbank market tightening as an increase in the lending cost parameter such that cash lenders are only willing to lend against high-quality collateral. Specifically, we set  $k_c^{MM}$  during crisis periods such that only AA bonds are accepted on the interbank market. Using the Eurex data, we obtain  $\tilde{\Lambda}_c = 0.01$ . By employing (25), we get  $k_c^{MM} = \frac{f}{0.01(1-\zeta)}$ .

<sup>18</sup>Firms that are eligible at the central bank might for example forego the large premia associated with interbank eligibility and lever up, a similar logic to the eligible/non-eligible margin.

<sup>19</sup>Note that, since banks only borrow from the central bank with probability  $\zeta$ , this understates the increase in potentially eligible collateral, which increases by 110%.

Table 5: Macroeconomic Aggregates for Different Policies with Interbank Tightening

State Policy	Boom		Crisis	
	Tight	Lenient	Tight	Lenient
Collateral Supply $\bar{B}$	11.20	15.30	5.38	7.66
- Relative to Tight Policy	1.000	1.367	1.000	1.423
Lending Costs $\mathcal{K}$	5e-04	7e-04	4e-04	6e-04
- Relative to Tight Policy	1.000	1.567	1.000	2.051
- Share Borne by CB	33%	51%	61%	82%
Liquidity Costs $\mathcal{L}$	0.480	0.472	0.499	0.490
- Relative to Tight Policy	1.000	0.982	1.000	0.982

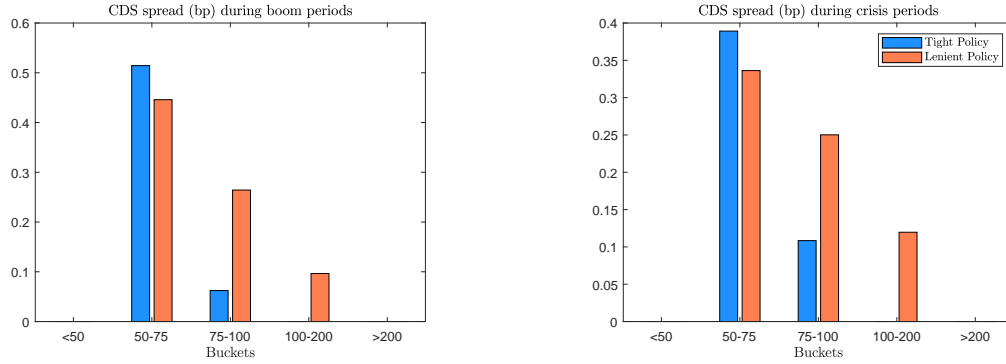
Notes: All values except share borne by central bank  $\mathcal{K}^{CB}/\mathcal{K}$  are expressed relative to the baseline calibration with tight eligibility requirements. Since  $\mathcal{K}^{CB}$  is zero in the baseline calibration, we express the share of central bank lending costs relative to total lending costs in the respective parameterization.

In table 5, we display macroeconomic aggregates under the same collateral policies as before. While the results for boom periods are similar to table 5, the crisis state exhibits some differences. Aggregate collateral in the boom state is only slightly higher than in the full model, irrespective of eligibility requirements. Under a tight policy for example, collateral drops to 8.62 during the crisis state, while in the full model collateral supply contracts to 5.38. Since there is no further tightening of private eligibility requirements, the drop in collateral supply is solely caused by the fundamental shock and the effect of eligibility requirements on lending costs and the share borne by the central bank is smaller in magnitude. Similarly, the relatively small effect on aggregate collateral translates into a more modest increase of liquidity costs.

Since the crisis and boom states are more similar in the risk-shocks only case, the impact of eligibility requirements is correspondingly smaller as well. The lenient policies still increase collateral supply and thereby reduce liquidity cost. During boom periods, the lenient policy increases  $\bar{B}$  by 33%, which is accompanied by an 53%-increase in lending costs. Adverse effects on collateral quality are also smaller during crisis periods, compared to the model with interbank market disruptions:  $\bar{B}$  increases by 32%, costs by 63%.

The intensive margin is slightly more prominent during boom periods, which makes up 63% of additionally eligible bonds. During crisis periods the intensive margin is responsible for 81% of the increase, which is still larger than in the boom state, but less prominent than in the full model with interbank shocks. Since there is no further tightening of interbank lending standards in this situation and few bonds cease being eligible on the private market. This makes our approximate decomposition more accurate. Naturally, without the interbank market disruption the central bank takes on a smaller share of the interbank market (60 % in the crisis state).

Figure 8: Cross-Sectional Distribution of Collateral Quality with Interbank Tightening



Notes: Number of eligible bonds in a given bucket relative to the mass of all bonds (excluding those in restructuring).

Taken together, the simulation experiments suggest that reducing the eligibility threshold can foster interbank market lending by increasing collateral supply, where the costs of doing so are larger during a crisis state. This raises the question whether supplementary policy response margins are required, such as easing access to central bank lending. These more unconventional central bank measures are beyond the scope of our work. Based on the findings of our numerical policy analysis, the clout of these measures is likely to be restricted by fundamentals of the underlying collateral at some point.

## 6 Conclusion

This paper evaluated the effects of central bank eligibility requirements on money market outcomes (the users of collateral) and firm dynamics (the issuers of collateral) in a joint framework. Combining approaches from the literature on interbank markets with a dynamic corporate capital structure model helps disentangling the response of banks and firms to changes to the central bank's eligibility requirements. Our numerical findings suggest that relaxing eligibility requirements in periods of elevated fundamental risk increases collateral supply at the cost of deteriorating collateral quality. While such policies can address interbank market disruptions by taking on a larger share of money market lending, firm fundamentals eventually place restrictions on their efficacy. This policy ultimately shifts costs from the private to the public sector losses, which relates to the role of central bank facilities as lender of last resort.

A general equilibrium analysis requires a more differentiated analysis of the distribution of lending costs among private and public lenders. It should also be stressed that we take investment opportunities as exogenous. A model with endogenous investment would allow to study real effects of eligibility requirements using a richer trade-off between distributing cashflows as dividends and investment. We also did not account for bank loans as alternative source of financing, which is also a margin affected by eligibility requirements, as those make bond financing more attractive. Both extensions add layers of complexity to our present framework, which we leave to future research.

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## A Proofs

This section contains the proofs of Proposition 1 and lemma 1. Most steps of the latter follow directly from different steps of the former.

### A.1 Proof of Proposition 1

The partitioning of firms into different groups (unconstrained eligible, constrained eligible, and ineligible) uses the fact that there are three *potentially* optimal debt choices for every  $s$ . The first possibility is to issue bonds  $\tilde{b}_{t+1}(s)$  to be exactly at the eligibility threshold. By the strict monotonicity of  $F(\mu^{def} + b_{t+1}|s)$  in  $b_{t+1}$ , there is a unique  $\tilde{b}_{t+1}(s) \equiv F^{-1}(\Lambda^{CB}|s) - \mu^{def}$  where the corporate bond is just eligible. Second, there is a debt level  $b_{t+1}^1$  satisfying the first-order condition (7) for the case of ineligibility. Third, the level  $b_{t+1}^2$  solves (8), the first-order condition in the eligibility case. Under the monotonicity assumption on  $h(b_{t+1}) \cdot b_{t+1}$ , both conditions are satisfied by a unique  $b_{t+1}^1$  and  $b_{t+1}^2$ , respectively. Moreover, since  $\frac{\tau+L_1}{1+L_1} > \tau$  for every  $L_1 > 0$  and  $0 < \tau < 1$ , it holds that  $b_{t+1}^1 < b_{t+1}^2$ , which reflects that the outward shift of the bond price schedule due to eligibility incentivizes the firm to choose a higher leverage. The remainder of the proof characterizes which of these three debt levels is optimal, given the type parameter  $s$ .

**Existence of Type Space Partitions** There is a positive mass of unproductive firms, such that  $\tilde{b}_{t+1}(s) = 0 < b_{t+1}^1(s) < b_{t+1}^2(s)$ , which holds at least for  $s = s^-$  by assumption. These firms are not able to issue any bonds without exceeding the minimum quality requirement  $\Lambda^{CB}$ , i.e. their eligible debt capacity is zero. On the other hand there are firms with positive eligible debt capacity. This can be shown by finding values  $s^1$  and  $s^2$  such that  $b_{t+1}^1(s^1) = \tilde{b}_{t+1}(s^1)$  and  $b_{t+1}^2(s^2) = \tilde{b}_{t+1}(s^2)$ , i.e. firms are able to issue debt according to (7) and (8) without loosing eligibility. Moreover, these partitions can be shown to be monotonic in  $s$ , since  $\frac{\partial b_{t+1}^1(s)}{\partial s}$  and  $\frac{\partial b_{t+1}^2(s)}{\partial s}$  are smaller than  $\frac{\partial \tilde{b}_{t+1}(s)}{\partial s}$  for all  $s$ . From the mass-shifting property of  $s$ , we can express the eligible debt capacity as

$$\tilde{b}_{t+1}(s) = F^{-1}(\Lambda^{CB}) - \mu^{def} + s \quad (\text{A.1})$$

Plugging this into the derivative of the objective functions  $p_1$  and  $p_2$ , we get

$$\left. \frac{\partial p_1(s)}{\partial b} \right|_{\tilde{b}_{t+1}(s)} = \tau(1 - \Lambda^{CB}) - \left( F^{-1}(\Lambda^{CB}) - \mu^{def} + s \right) f(F^{-1}(\Lambda^{CB})) \quad (\text{A.2})$$

$$\left. \frac{\partial p_2(s)}{\partial b} \right|_{\tilde{b}_{t+1}(s)} = \frac{\tau + L_1}{1 + L_1} (1 - \Lambda^{CB}) - \left( F^{-1}(\Lambda^{CB}) - \mu^{def} + s \right) f(F^{-1}(\Lambda^{CB})). \quad (\text{A.3})$$

For a sufficiently profitable firm, i.e. firms with a large  $s$ , eligible debt capacity  $\tilde{b}_{t+1}(s)$  lies on the downward sloping part of the objective function. Since the objective is concave by the monotone hazard rate assumption,  $\tilde{b}_{t+1}(s)$  is not optimal and such a firm issues less debt than in could without loosing eligibility. Moreover, by continuity of  $s$ ,  $s^1$  and  $s^2$  are unique, and, since  $\tau < \frac{\tau+L_1}{1+L_1}$ , it holds that  $s^1 < s^2$ . To ensure that  $s^- < s^1$ , we exploit monotonicity of the first-order conditions in  $s$  and the fact that  $\frac{\partial \tilde{b}_{t+1}(s)}{\partial s} = 1$ .

Implicitly differentiating (5) and (6) with respect to  $s$ , we get

$$\begin{aligned}\frac{\partial b_{t+1}^1(s)}{\partial s} &= \frac{(1 - F(b_{t+1}^1 + \mu^{def} - s)) f'(b_{t+1}^1 + \mu^{def} - s) b_{t+1}^1 + f(b_{t+1}^1 + \mu^{def} - s)^2 b_{t+1}^1}{(1 - F(b_{t+1}^1 + \mu^{def} - s)) (f'(b_{t+1}^1 + \mu^{def} - s) b_{t+1}^1 + f(b_{t+1}^1 + \mu^{def} - s)) + f(b_{t+1}^1 + \mu^{def} - s)^2 b_{t+1}^1} \\ \frac{\partial b_{t+1}^2(s)}{\partial s} &= \frac{(1 - F(b_{t+1}^2 + \mu^{def} - s)) f'(b_{t+1}^2 + \mu^{def} - s) b_{t+1}^2 + f(b_{t+1}^2 + \mu^{def} - s)^2 b_{t+1}^2}{(1 - F(b_{t+1}^2 + \mu^{def} - s)) (f'(b_{t+1}^2 + \mu^{def} - s) b_{t+1}^2 + f(b_{t+1}^2 + \mu^{def} - s)) + f(b_{t+1}^2 + \mu^{def} - s)^2 b_{t+1}^2}.\end{aligned}$$

Since the (transformed) first-order conditions (5) and (6) imply that firms are risky, i.e.  $f(b_{t+1}^1 + \mu^{def}|s) > 0$  and  $f(b_{t+1}^2 + \mu^{def}|s) > 0$  such that the partial derivatives  $\frac{\partial b_{t+1}^1(s)}{\partial s}$  and  $\frac{\partial b_{t+1}^2(s)}{\partial s}$  are strictly smaller than 1.

**Characterizing Debt Choices** For every  $s > s^2$ , firms issue less debt than it could issue without losing eligibility. All firms with  $s > s^2$  choose leverage according to their first-order condition and are called *unconstrained eligible*. Consider next firms, that can not choose their optimal borrowing without losing eligibility, i.e. firms with  $s < s^2$ . Define the hypothetical value functions for a never eligible firm  $p^1(b_{t+1}|s)$  and an always eligible firm as  $p^2(b_{t+1}|s)$ . All firms between  $s^1$  and  $s^2$  choose to be just eligible and lever up until  $\tilde{b}_{t+1}(s)$ , since for them  $p^2(b_{t+1}^2(s)|s)$  is not feasible and  $p^1(b_{t+1}^1(s)) < p^2(b_{t+1}^1(s)) < p^2(\tilde{b}_{t+1}(s))$ . The first inequality follows from  $p^2(b_{t+1}|s) > p^1(b_{t+1}|s)$  for all  $b_{t+1}$ , holding  $s$  constant. The second inequality follows from the fact that  $p^2$  is increasing between  $b_{t+1}^1(s)$  and  $\tilde{b}_{t+1}(s)$ . Finally, there is a threshold  $s^0 < s^1$ , below which firms choose  $b_{t+1}^1(s)$  and are not eligible. All firms between  $s^0$  and  $s^1$  also choose  $\tilde{b}_{t+1}(s)$ . The value  $s^0$  is implicitly defined through the indifference condition  $p^2(\tilde{b}_{t+1}|s^0) = p^1(b_{t+1}^1|s^0)$ . The assumptions on the revenue distribution will imply the existence of exactly one  $s^0$  by the intermediate value theorem. To see this, consider their difference

$$\Delta(s) \equiv p^2(\tilde{b}_{t+1}(s)|s) - p^1(b_{t+1}^1(s)|s). \quad (\text{A.4})$$

Obviously  $\Delta(s^1) > 0$ , because  $b_{t+1}^1(s^1) = \tilde{b}_{t+1}(s^1)$  and  $p^2(\tilde{b}_{t+1}(s^1)|s^1) > p^1(\tilde{b}_{t+1}(s^1)|s^1)$ . In addition, there exists a level  $s^-$  where  $F(\mu^{def}|s^-) > \Lambda^{CB}$  by assumption. At this level  $p^2(\tilde{b}_{t+1}(s^-)|s^-) - p^1(b_{t+1}^1(s^-)|s^-) < 0$ , because  $\tilde{b}_{t+1}(s^-) = 0$  and  $p^2(\tilde{b}_{t+1}(s^-)|s^-)$  is the value of the unlevered firm. Choosing  $b_{t+1} = 0$  however violates (7) and therefore  $p^1(b_{t+1}^1(s)|s)$  exceeds the value of an unlevered firm for every  $s$ . Together with continuity of  $s$ , this already implies existence of at least one  $s^0$  by the intermediate value theorem. To establish uniqueness, we differentiate  $\Delta$  with respect to  $s$ . For the first part, which can be written explicitly as

$$\begin{aligned}p^2(\tilde{b}_{t+1}(s)|s) &= (1 - \Lambda^{CB})(1 + L_1)\tilde{b}_{t+1} - x \\ &\quad + (1 - \tau) \left( \int_{\underline{\mu}}^{\mu^{def} + \tilde{b}_{t+1}(s)} \mu^{def} dF(\mu_{t+1}|s) + \int_{\mu^{def} + \tilde{b}_{t+1}(s)}^{\bar{\mu}} \mu_{t+1} - \tilde{b}_{t+1}(s) dF(\mu_{t+1}|s) \right),\end{aligned}$$

the derivative is given by

$$\begin{aligned}
\frac{\partial p^2(\tilde{b}_{t+1}(s)|s)}{\partial s} &= (1 - \Lambda^{CB})(1 + L_1) + (1 - \tau) \left( \int_{\underline{\mu}}^{\mu^{def} + \tilde{b}_{t+1}(s)} -\mu^{def} f'(\mu_{t+1} - s) d\mu_{t+1} \right. \\
&\quad \left. + \int_{\mu^{def} + \tilde{b}_{t+1}(s)}^{\bar{\mu}} -(\mu_{t+1} - \tilde{b}_{t+1}(s)) f'(\mu_{t+1} - s) - f(\mu_{t+1} - s) d\mu_{t+1} \right) \\
&= (1 - \Lambda^{CB})(1 + L_1) + (1 - \tau) \left( -\mu^{def} f(\mu^{def} + \tilde{b}_{t+1}(s)|s) \right. \\
&\quad \left. - \tilde{b}_{t+1}(s) f(\mu^{def} + \tilde{b}_{t+1}(s)|s) - \left( 1 - F(\mu^{def} + \tilde{b}_{t+1}(s)|s) \right) + \int_{\mu^{def} + \tilde{b}_{t+1}(s)}^{\bar{\mu}} -\mu_{t+1} f'(\mu_{t+1}|s) d\mu_{t+1} \right) \\
&= (1 - \Lambda^{CB})(1 + L_1)
\end{aligned} \tag{A.5}$$

Here we used again that  $\frac{\partial \tilde{b}_{t+1}}{\partial s} = 1$ . The second part is given by  $\frac{\partial p^1(b_{t+1}(s), s)}{\partial s}$ , since  $\frac{\partial p^1(b_{t+1}(s), s)}{\partial b_{t+1}} = 0$  by optimality, when totally differentiating  $p^1(b_{t+1}(s)|s)$  with respect to  $s$ . Specifically,

$$\begin{aligned}
\frac{\partial p^1(b_{t+1}^1(s)|s)}{\partial s} &= f(\mu^{def} + b_{t+1}^1(s) - s) \cdot b_{t+1}^1(s) + (1 - \tau) \left( \int_{\underline{\mu}}^{\mu^{def} + b_{t+1}^1(s)} -\mu^{def} f'(\mu_{t+1} - s) d\mu_{t+1} \right. \\
&\quad \left. + \int_{\mu^{def} + b_{t+1}^1(s)}^{\bar{\mu}} -(\mu_{t+1} - b_{t+1}^1(s)) f'(\mu_{t+1} - s) d\mu_{t+1} \right) \\
&= \tau \left( 1 - F(\mu^{def} + b_{t+1}^1(s)|s) \right) + (1 - \tau) \left( 1 - F(\mu^{def} + b_{t+1}^1(s)|s) \right) \\
&= 1 - F(\mu^{def} + b_{t+1}^1(s)|s)
\end{aligned} \tag{A.6}$$

In the second line, we directly used the first-order condition (7). Putting both parts together, we have

$$\frac{\partial \Delta(s)}{\partial s} = \frac{\partial p^2(\tilde{b}_{t+1}(s)|s)}{\partial s} - \frac{\partial p^1(b_{t+1}^1(s)|s)}{\partial s} = (1 - \Lambda^{CB})(1 + L_1) - \left( 1 - F(\mu^{def} + b_{t+1}^1(s)|s) \right) > 0.$$

The sign follows from the fact that  $\tilde{b}_{t+1}(s) < b_{t+1}^1(s)$  holds in the region of interest. This implies that the default probability at  $b_{t+1}^1(s)$  exceeds the eligibility threshold, i.e.  $F(\mu^{def} + b_{t+1}^1(s)|s) > \Lambda^{CB}$ . The inequality follows from  $(1 - F(\mu^{def} + b_{t+1}^1(s)|s)) < 1 - \Lambda^{CB}$  and  $L_1 > 0$ . Since  $\Delta(s)$  is continuous and monotonically increasing, there exists a unique  $s^0$  where the firm is indifferent between constrained eligibility and non-eligibility by the intermediate value theorem. All firms between  $s^0$  and  $s^2$  are called *constrained eligible*, firms below  $s^0$  are *non-eligible*.  $\square$

## A.2 Proof of Lemma 1

The derivative of total collateral with respect to eligibility requirements is given by

$$\begin{aligned} \frac{\partial \bar{B}_t}{\partial \Lambda^{CB}} &= \left(1 - F\left(\tilde{b}_{t+1}(s_2) + \mu^{def}\right)\right) \tilde{b}_{t+1}(s_2) \frac{\partial s_2}{\partial \Lambda^{CB}} - \left(1 - F\left(\tilde{b}_{t+1}(s_0) + \mu^{def}\right)\right) \tilde{b}_{t+1}(s_0) \frac{\partial s_0}{\partial \Lambda^{CB}} \\ &\quad + \int_{s_0}^{s_2} \frac{\partial \tilde{b}}{\partial \Lambda^{CB}} \left[ \left(1 - F\left(\tilde{b}_{t+1}(s) + \mu^{def}\right)\right) - \tilde{b}_{t+1}(s) \left(f\left(\tilde{b}_{t+1}(s) + \mu^{def}\right)\right) \right] ds \\ &\quad - \left(1 - F\left(b_{t+1}^2(s_2) + \mu^{def}\right)\right) b_{t+1}^2(s_2) \frac{\partial s_2}{\partial \Lambda^{CB}} \end{aligned}$$

By the implicit definition of  $s^2$  as the firm where  $\tilde{b}_{t+1}(s_2) = b_{t+1}^2(s_2)$  holds exactly, the first and last term in this expression cancel. We can rearrange for the change in market value of debt outstanding

$$\frac{\partial \bar{B}_t}{\partial \Lambda^{CB}} = - \left(1 - F\left(\tilde{b}_{t+1}(s_0) + \mu^{def}\right)\right) \tilde{b}_{t+1}(s_0) \frac{\partial s_0}{\partial \Lambda^{CB}} + \int_{s_0}^{s_2} \frac{\partial \tilde{b}}{\partial \Lambda^{CB}} \frac{1 - \tilde{b}_{t+1}(s) h\left(\tilde{b}_{t+1}(s) + \mu^{def}\right)}{1 - F\left(\tilde{b}_{t+1}(s) + \mu^{def}\right)} ds.$$

The first part reflects the extensive margin as formerly ineligible firms are now able to issue eligible collateral. The extensive margin has a positive effect, since  $\frac{\partial s_0}{\partial \Lambda^{CB}} < 0$ . This can be seen from the indifference condition (A.4). While the value of being ineligible  $p^1(b_{t+1}^1(s)|s)$  does not depend on the eligibility threshold, the value of being constrained eligible  $p^2(\tilde{b}_{t+1}(s)|s)$  increases in  $\Lambda^{CB}$ . Differentiating eligible debt capacity  $\tilde{b}_{t+1}(s)$  with respect to the eligibility threshold yields

$$\frac{\partial \tilde{b}_{t+1}(s)}{\partial \Lambda^{CB}} = \frac{\partial F^{-1}(\Lambda^{CB}|s)}{\partial \Lambda^{CB}} = \frac{1}{f(F^{-1}(\Lambda^{CB}|s))}, \quad (\text{A.7})$$

where the last step follows from the inverse function theorem. Relaxing eligibility requirements increases the eligible debt capacity and a constrained firm will always be better off after a relaxation of eligibility requirements. The argument is completed by observing the value of a constrained eligible firm is increasing in the shifting parameter, as we showed in (A.5). Then, denoting eligible debt capacity and cut-off values before the policy change by  $(s^{0,old}, \tilde{b}_{t+1}^{0,old})$ , the effect can be summarized as

$$p^2(\tilde{b}_{t+1}^{old}|s^{0,old}) < p^2(\tilde{b}_{t+1}^{new}|s^{0,old}) < p^2(\tilde{b}_{t+1}^{new}|s^{0,new}),$$

such that the indifference point shifts to the left, i.e.  $s^{0,new} < s^{0,old}$ .

Regarding the intensive margin, all constrained eligible firms increase their debt outstanding. This again follows from (A.7). Moreover, the market value of debt outstanding increases, which follows from the monotone hazard rate condition, since for any  $s < s^2$  it holds that  $\tilde{b}_{t+1}(s) h\left(\tilde{b}_{t+1}(s) + \mu^{def}\right) < \frac{\tau + L_1}{1 + L_1} < 1$ . Intuitively, these firms issue less debt than optimal under eligibility such that they directly benefit from relaxed eligibility requirements.

Differentiating (10) with respect to  $\Lambda^{CB}$ , we have

$$\begin{aligned} \frac{\partial \mathcal{M}_t}{\partial \Lambda^{CB}} = & F\left(b_{t+1}(s_0) + \mu^{def}\right) \frac{\partial s_0}{\partial \Lambda^{CB}} + F\left(\tilde{b}_{t+1}(s_2 + \mu^{def})\right) \frac{\partial s_2}{\partial \Lambda^{CB}} - F\left(\tilde{b}_{t+1}(s_0) + \mu^{def}\right) \frac{\partial s_0}{\partial \Lambda^{CB}} \\ & + \int_{s_0}^{s_2} f\left(\tilde{b}_{t+1}(s) + \mu^{def}\right) \frac{\partial \tilde{b}}{\partial \Lambda^{CB}} ds - F\left(b_{t+1}(s_2) + \mu^{def}\right) \frac{\partial s_2}{\partial \Lambda^{CB}}. \end{aligned}$$

Again, since the integral bounds  $s_0$  and  $s_2$  are indifference points, both extensive margins cancel. When using eq. (A.1) and the definition of  $\tilde{b}_{t+1}(s)$ , the entire expression simplifies to  $\frac{\partial \tilde{D}_t}{\partial \Lambda^{CB}} = \int_{s_0}^{s_2} \mathbf{1} ds > 0$ .  $\square$

## B Eligibility on Private and Public Market Segments

This section review the eligibility of corporate bonds in ECB operations and on the interbank market. Table 6 gives an overview of changes in the ECB collateral framework since 2007 (see Nyborg (2017)). Corporate bonds were eligible prior to the 2008-09 crises at a comparatively low minimum requirement of A-. In response to the financial crises, the minimum requirements were reduced from A- to BBB-, which extended the amount of eligible assets and thereby broadened financial intermediaries' access to central bank liquidity. The smaller changes in 2011 and 2013 suggest that some fine-tuning was necessary after the initial relaxation. Nevertheless, the reduction of the minimum rating requirement was by far the largest adjustment, which motivates our choice of modelling collateral policy as a step function.

Table 6: Corporate Bonds in the ECB Collateral Framework

Timespan	Regime	Haircut: A- or higher	Haircut: BBB- to BBB+
01 Jan 2007 - 24 Oct 2008	Fitch, S&P and Moody's are accepted ECAI, minimum requirement A-	4.5 %	100 %
25 Oct 2008 - 31 Dec 2010	DBRS legally and practically accepted as ECAI, minimum requirement BBB-	4.5 %	9.5 %
01 Jan 2011 - 30 Sep 2013	Tightening of haircuts	5 %	25.5 %
01 Oct 2013 - 01 Dec 2019	Relaxation of haircuts	3 %	22.5 %

Notes: Corporate bond with fixed coupon and maturity of 3 to 5 years; DBRS: Dominion Bond Rating Service, ECAI: external credit assessment institutions

Further, we document some empirical regularities to motivate our modelling choices regarding the money market in section 3. The Euro money market is dominated by relatively few marketplaces, Eurex being the largest of those. They play an important role in specifying collateral adequacy criteria, applying haircuts, and collateral pooling. Specifically, when entering repos settled by Eurex, participants need to maintain a pool of collateral. Eurex offers to trade based on different baskets of collateral. Of particular relevance to us is the *General Collateral Pooling Market*, which is based on ECB-eligible securities.<sup>20</sup>

<sup>20</sup>Eurex offers different collateral baskets. The GC pooling segment is among the biggest.

Importantly, collateral received through transactions on this market is viable for re-use in other money market transaction, e.g. in the ECB’s standing facilities.

A further relevant feature of the Eurex repo market is the open order book. In principle, trades can be agreed bilaterally and settled through the Eurex system. However, the compelling attribute of CCPs are anonymous trades. Eurex maintains a quote book in which participating institutions enter collateral and volume specifications. Traders can then hit on quotes. This feature allows us to formulate the interbank market as a centrally cleared market in which we can abstract from Nash-bargaining between market participants that would be necessary to capture bilateral agreements.

To provide justification for eligibility requirements on the private segment, we compare the list of eligible collateral in August 2019 published on the ECB website to the Eurex core and extended baskets.<sup>21</sup> The latter two differ in terms of the number of assets included. The general collateral basket includes around 3,000 securities that mostly fulfil a LCR level 1 criterion, indicating high liquidity. The extended basket covers around 14,000 securities in total. In order to compare bond characteristics for private and public collateral baskets, we match these lists with the *IHS Markit* iBoxx database that we use in the quantitative analysis. We exclude all zero-coupon bonds and restrict the sample to bonds with a maturity between 3 and 7 years.<sup>22</sup> We subset the matched and filtered lists by rating in table 7. None of the lists contains corporate bonds rated BB or lower, but the Eurex lists are much more skewed towards highly rated bonds (mostly the general collateral list), while more than half of the ECB-eligible bonds are rated BBB.

Table 7: Share of Eligible Corporate Bonds by Rating Category

Rating	ECB	EUREX Ext.	EUREX core
AAA	0	38.17	65.60
AA	6.68	26.70	31.75
A	40.34	17.88	2.65
BBB	52.98	14.25	0

Notes: Based on matched EUREX-iBoxx constituent lists. Values in Percentage Points.

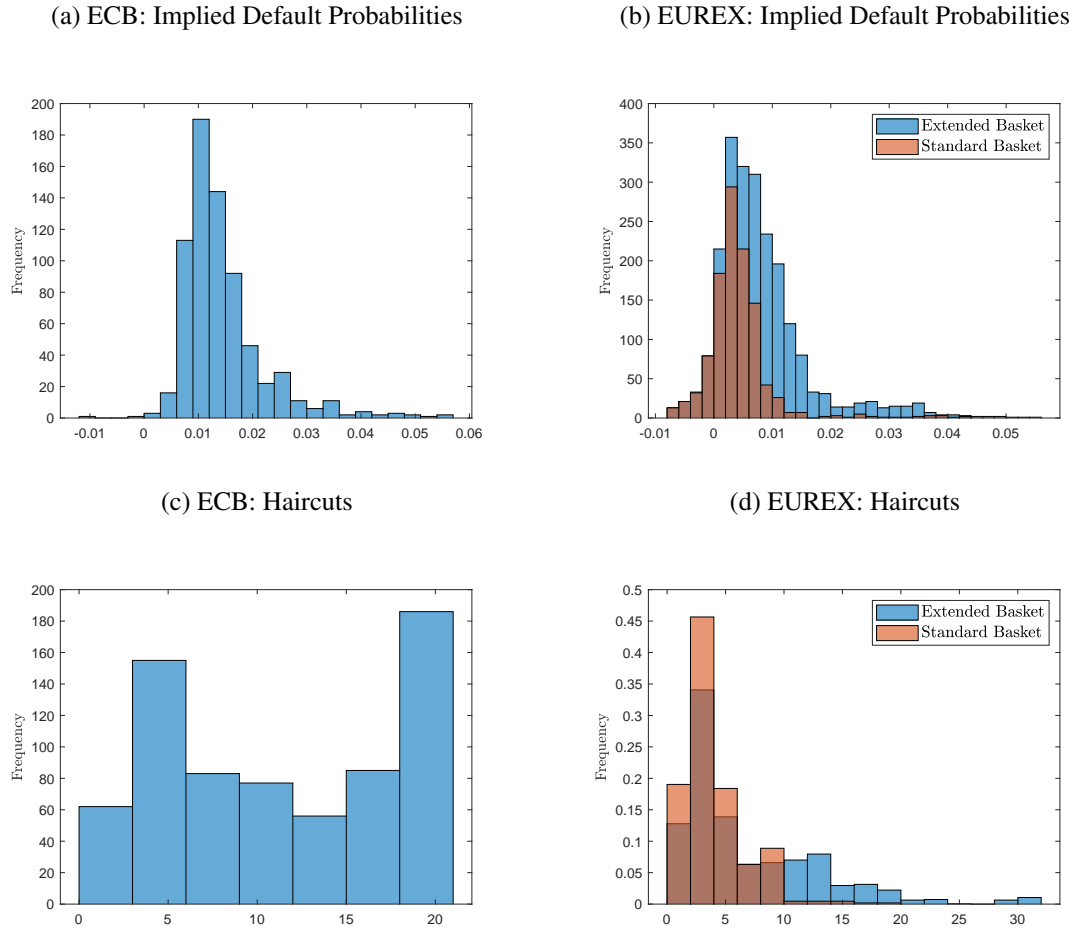
Consequently, the distribution of implied default probabilities (which is derived based on bond characteristics and the risk-free rate) and haircuts of eligible bonds is shifted to the right for the ECB-basket, as demonstrated in figure 9.<sup>23</sup> Unsurprisingly, among the Eurex lists the extended basket includes securities with, on average, higher implied default probability and, consequently, higher haircuts.

<sup>21</sup>The core basket is officially called *GC Pooling ECB Basket* while the extended basket *GC Pooling ECB EXTended Basket*

<sup>22</sup>This matching and the filtering lead to a under-representation of very high rated bonds, since these often do not satisfy the liquidity criteria to be included in iBoxx indices. In fact, there are several eligible AAA-corporate bonds on the ECB list.

<sup>23</sup>See appendix C for more details.

Figure 9: Distribution of Eligible Corporate Bonds



Notes: Histograms refer to matched samples of corporate bonds listed on the ECB and Eurex lists with Markit iBoxx Index constituents.

As mentioned in Mancini et al. (2015), the matched list reveals that *all* securities receive the same haircut on the ECB list and the Eurex GCP baskets. The discrete haircut brackets are stemming from different maturity and rating buckets in both haircut frameworks. These findings suggest that there is an imperfect pass-through from central bank eligibility to the private segment, where the imperfection stems from the inclusion of securities rather than haircuts conditional on inclusion. We will construct our model of segmented interbank money markets along these observations.

## C Data

### C.1 Corporate Bond Data

We merge monthly data on the corporate bond universe in Europe from the iBoxx High Yield and Investment Grade Index families, provided by *IHS Markit*. We apply the following inclusion criteria:

1. Bond issuers are head-quartered in countries that were member of the Euro Area from 2003-2012.
2. Issuers are non-financial firms.



3. The bond is denominated in Euro, senior, not callable, not collateralized, and pays a fixed coupon.
4. The issuer is part of the constituent list for at least 48 Months, corresponding to 41.7 % of our sample period.

Bond issuers are provided by *Markit* and we consider only the parent company level (identified by Bloomberg Ticker), since it can be reasonably assumed that dedicated financial management subsidiaries are identical from an economic perspective to the respective parent company.

## C.2 Company Data

Next, we match company names to their unique *Compustat* identifier (*gvkey*) and drop all companies which are not represented in the *Compustat Global* database. For the remaining firms we query *Compustat* for earnings, balance sheet, and securities data.

Table 8: Data Sources and Ticker: Compustat Global

<i>Series</i>	<i>Database</i>	<i>Mnemonic</i>	<i>Unit</i>
Firm Total Assets	firmq	AT	Millions of Euros
Firm Long-Term Liabilities	firmq	DLTT	Millions of Euros
Equity Price	secd	PRCC _ F	Euros
Shares Outstanding	secd	CSHO	-

## C.3 Money Market Data

To obtain stylized facts on how to model the money market, we need data on private European interbank repo markets. The Eurex Repo trading platform is one of the biggest provider in Europe and offers different baskets of eligible securities it accepts when acting as CCP. In particular, we focus on the GCP ECB EXTENDED basket which consists of a subset of eligible securities from the official ECB list (mainly the safest ones). Unfortunately, Eurex does not offer historical data so that we base our analysis on the list of the April 11<sup>th</sup> 2019.<sup>24</sup> For the same day we download the list of eligible ECB securities against which we compare.

First, we restrict the comparison to corporate securities and delete all other entries. We further match the information on securities from both datasets based on ISINs. We repeat this step based on the Markit iBoxx constituent list of that month to obtain a broader dataset.

Second, based on price data from the Markit lists we compute yield to maturities and implied default probabilities (IDP) for each bond. The computation of the IDPs proceeds as in chapter 23 of Hull (2006):

- Given the bonds principal, coupon and maturity as well as the risk-free interest rate, we calculate the price of the bond before each coupon payment (assuming no default in between).
- Given our choice of the recovery rate (66%) one can calculate the loss given default for a given period. Using the riskless rate, calculate its present value (LGD).

<sup>24</sup>Choosing a different day does not significantly change results.

- Adding up the LGD for each point in time, one can back out the default probability implied by market prices: assuming the corporate bond has price  $X$  and the comparable riskless bond price is  $Y$ , their difference implies what market participants expect to lose,  $X - Y = IDP \cdot LGD$ . The implied default probability follows as  $IDP = \frac{X-Y}{LGD}$ .

## D Supplement to Model and its Computation

This section contains an auxiliary derivation of the market clearing condition for the all-purpose good that directly related to potentially welfare relevant cost terms.

### D.1 Derivation of the Resource Constraint

In this section we derive the resource constraint which contains potentially welfare relevant cost terms, associated with money market borrowing and lending. Since the resource constraint must hold for arbitrary bank and firm policies, we omit the dependency of firm variables on the idiosyncratic state. To ease exposition, we also do not make the dependency of policy variables and macro aggregates on the aggregate state  $z_t$  explicit. The derivation starts by aggregating the budgets of bank-owners and firm-owners. The joint budget reads

$$C_t + \int_j e_{t+1}^j p_t^j dj = d_t^B + T_t + \int_j e_t^j \left( h_t^j (p_t^{def,j} + d_t^{def,j} - \tau(d_t^{def,j})) + (1 - h_t^j) (p_t^{rep,j} + d_t^{rep,j} - \tau(d_t^{rep,j})) \right) dj.$$

Recall that in equilibrium  $e_t^j = e_{t+1}^j = 1$ . Plugging in the government budget the definition of  $p_t^j$ , this can be written

$$\begin{aligned} C_t + \int_j p_t^j dj &= d_t^B \\ &+ \int_j \left( h_t^j (p_t^{def,j} + d_t^{def,j} - \tau(d_t^{def,j})) + (1 - h_t^j) (p_t^{rep,j} + d_t^{rep,j} - \tau(d_t^{rep,j})) \right) dj \\ &+ \int_j (1 - h_t^j) \tau(d_t^{rep,j}) + h_t^j \tau(d_t^{def,j}) dj - \frac{\zeta_t^{kMM}}{2} \int_j \Lambda_t^j \Phi^{CB}(\Lambda_t^j) b_t^j \Upsilon_t^j dj, \end{aligned}$$

Inserting bank profits yields

$$\begin{aligned}
C_t = & \int_j \left( h_t^j (d_t^{def,j}) + (1 - h_t^j) (d_t^{rep,j}) \right) dj - \frac{\varsigma k_t^{MM}}{2} \int_j \Lambda_t^j \Phi^{CB}(\Lambda_t^j) b_t^j \Upsilon_t^j dj \\
& + \int_j \left( (1 - h_t^j) \left( \pi + \kappa + (1 - \pi) q(\cdot) \right) + h_t^j q^{def}(\cdot) \right) b_t^j dj - \int_j q(\cdot) b_{t+1}^j dj \\
& - \frac{(1 - \varsigma) k_t^{MM}}{2} \cdot \int_j \Lambda_t^j \Phi^{IB}(\Lambda_t^j) \theta_t^j b_t^j \Upsilon_t^j dj - \frac{\mathcal{L}(\bar{B}_{t+1})}{2} .
\end{aligned}$$

Note that the central bank facility is used by cash borrowers that choose not to borrow on the private market (and therefore use the central bank to obtain funds) and by cash borrowers re-using collateral at the central bank. Hence:

$$\frac{\varsigma k_t^{MM}}{2} \int_j \Lambda_t^j \Phi^{CB}(\Lambda_t^j) b_t^j \Upsilon_t^j dj = \frac{\varsigma k_t^{MM}}{2} \int_j \Lambda_t^j (1 - \theta_t^j) \Phi^{CB}(\Lambda_t^j) b_t^j \Upsilon_t^j + \Lambda_t^j \theta_t^j \Phi^{CB}(\Lambda_t^j) b_t^j \Upsilon_t^j dj .$$

Note further that for the case that collateral  $j$  can be pledged at the private market ( $\theta_t^j = 1$ ), by assumption we must have  $\Phi^{CB} = \Phi^{LM}$ . We can then rewrite the expression as

$$\begin{aligned}
C_t = & \int_j \left( h_t^j d_t^{def,j} + (1 - \delta_t^j) d_t^{rep,j} \right) dj - \frac{k_t^{MM}}{2} \int_j \Lambda_t^j (1 - \theta_t^j) \Phi^{CB}(\Lambda_t^j) b_t^j \Upsilon_t^j dj \\
& + \int_j \left( (1 - h_t^j) \left( \pi + \kappa + (1 - \pi) q(\cdot) \right) + \delta_t^j q^{def}(\cdot) \right) b_t^j dj - \int_j q(\cdot) b_{t+1}^j dj \\
& - \frac{k_t^{MM}}{2} \int_j \Lambda_t^j \Phi^{IB}(\Lambda_t^j) \theta_t^j b_t^j \Upsilon_t^j dj - \frac{\mathcal{L}(\bar{B}_{t+1})}{2} .
\end{aligned}$$

Using the definition of total lending costs (22) we get:

$$\begin{aligned}
C_t = & \int_j \left( h_t^j d_t^{def,j} + (1 - h_t^j) d_t^{rep,j} \right) dj - \mathcal{K}_t^{MM} \\
& + \int_j \left( (1 - h_t^j) \left( \pi + \kappa + (1 - \pi) q(\cdot) \right) + h_t^j q^{def}(\cdot) \right) b_t^j dj - \int_j q(\cdot) b_{t+1}^j dj .
\end{aligned}$$

Next, consider firms. Their production is given by

$$\begin{aligned}
d_t^{rep}(b_t^j, \mu_t^j) &= \frac{\mu_t^j}{\alpha} - x - \left( \pi + \kappa \right) b_t^j + q(\cdot) \left( b_{t+1}^j - (1 - \pi) b_t^j \right) \\
d_t^{def}(\mu_t^j) &= \frac{\mu_t^j}{\alpha} - x .
\end{aligned}$$

Because we assumed utility costs of default, there are no resource losses incurred by defaulting firms at this stage. Plugging in these definitions,

$$C_t = \int_j \left( h_t^j \left( \frac{\mu_t^j}{\alpha} - x \right) + (1 - h_t^j) \left( \frac{\mu_t^j}{\alpha} - x - (\pi + \kappa) b_t^j + q(b_{t+1}^j, \mu_t^j) (b_{t+1}^j - (1 - \pi) b_t^j) \right) \right) dj \\ - \mathcal{K}_t^{MM} + \int_j \left( (1 - h_t^j) (\pi + \kappa + (1 - \pi) q(\cdot)) + h_t^j q^{def}(\cdot) \right) b_t^j dj - \int_j q(\cdot) b_{t+1}^j dj,$$

we can rewrite the equation as,

$$C_t + \mathcal{K}_t^{MM} + \frac{\mathcal{L}(\bar{B}_{t+1})}{2} = \int_j h_t^j \frac{\mu_t^j}{\alpha} + (1 - h_t^j) \frac{\mu_t^j}{\alpha} dj - x.$$

## D.2 Computational Algorithm

We solve the individual firm problem by iterating on the bond price schedule and shareholder value functions over a discrete grid. Idiosyncratic revenues are discretized over the grid  $\mathfrak{z}$  of length  $n_\mu$ , we denote the corresponding transition matrix  $\Pi_\mu$ . Leverage is discretized over a grid  $\mathfrak{b}$  of length  $n_b$ . Since the restructuring status  $h_t$  is also an endogenous state, there are  $2n_b$  endogenous states in total. To overcome the typical convergence issues in models with long-term debt and endogenous default, we use taste shocks as proposed by Gordon (2018). The mass shifter for endogenous states follows immediately from debt and default policy functions and is denoted  $\Pi_s$ . This matrix maps the current idiosyncratic state  $(\mu_t, h_t, b_t, z_t)$ , into next periods endogenous state, i.e. has dimension  $2n_\mu \cdot 2n_b \times 2n_b$ .

Together with the transition matrix of idiosyncratic revenues and the aggregate state, the combined mass shifter  $\Pi_z = \Pi_s \otimes \Pi_\mu \otimes 1_{2 \times 2}$ , where  $1_{2 \times 2}$  denotes the two-dimensional identity matrix.  $\Pi_z$  object is block-diagonal (one block for each aggregate state) and implicitly defines the firm distribution  $g_z$  via  $g_z' = g_z' \Pi_g^{(z)}$ . In this notation  $g_z$  denotes the firm distribution in state  $z$  and  $\Pi_g^{(z)}$  refers to the respective block on the diagonal of  $\Pi_g$ . Extracting the distribution thus boils down to computing the right Eigenvalue to  $\Pi_g^{(z)'}.$  This is numerically feasible since  $\Pi_g$  is sparse. Once  $g$  is computed for  $z \in \{b, c\}$ , aggregation over firms and computing aggregate prices and quantities is straightforward.

1. At iteration step  $\iota = 0$ , guess aggregate quantities, firm policies, and pricing schedules from the finite horizon limit of the firm problem.
2. Given aggregate quantities of the previous iteration
  - (a) solve the firm problem taken as given the bond price schedule and value function (shareholder value) from the previous iteration.
  - (b) compute the corresponding endogenous mass shifter  $\Pi_s$ ,
  - (c) obtain the combined shifter  $\Pi_g$  from the endogenous mass shifter  $\Pi_s$  and transition matrix for revenue shock  $\Pi_\mu$  and aggregate state  $\Pi_z$ . Note that  $\Pi_g$  is sparse.
  - (d) update the distribution  $g_z$  by iterating on  $g_z = g_z' \Pi_g^{(z)}$  for both blocks on the diagonal.

- (e) compute implied default probabilities, based on  $q^0$  and compute eligibility on both money market segments.
- (f) given the firm distribution and firm policies of the current iterations,
  - i. compute the pledging choice  $\theta(\mu_t, b_{t+1}, z_t)$ , given aggregate collateral
  - ii. aggregate over firms to update  $\bar{B}(z_t)$ , given the pledging choice.
  - iii. if  $\theta(\mu_t, b_{t+1}, z_t)$  converges PROCEED, else, go back to (i)
- (g) Update price schedules. If these converge, STOP, else go back to (a).

Leverage is discretized on an equispaced grid with 251 points over the unit interval. The revenue shock is discretized using the method of Tauchen on an equispaced grid with 51 points over the interval  $[-4\sigma_\mu, +4\sigma_\mu]$ . The standard derivation of the taste shock is set to 1e-03 to ensure convergence in the baseline calibration. This is typically achieved within 500 iterations.

## E Supplementary Policy Analysis

This section provides supplementary results regarding central bank eligibility requirements. The first subsection presents an analysis of the cross-section for a model without interbank shocks, which we omitted in section 5.4. The second subsection contains supplementary results on macroeconomic aggregates for a larger policy space.

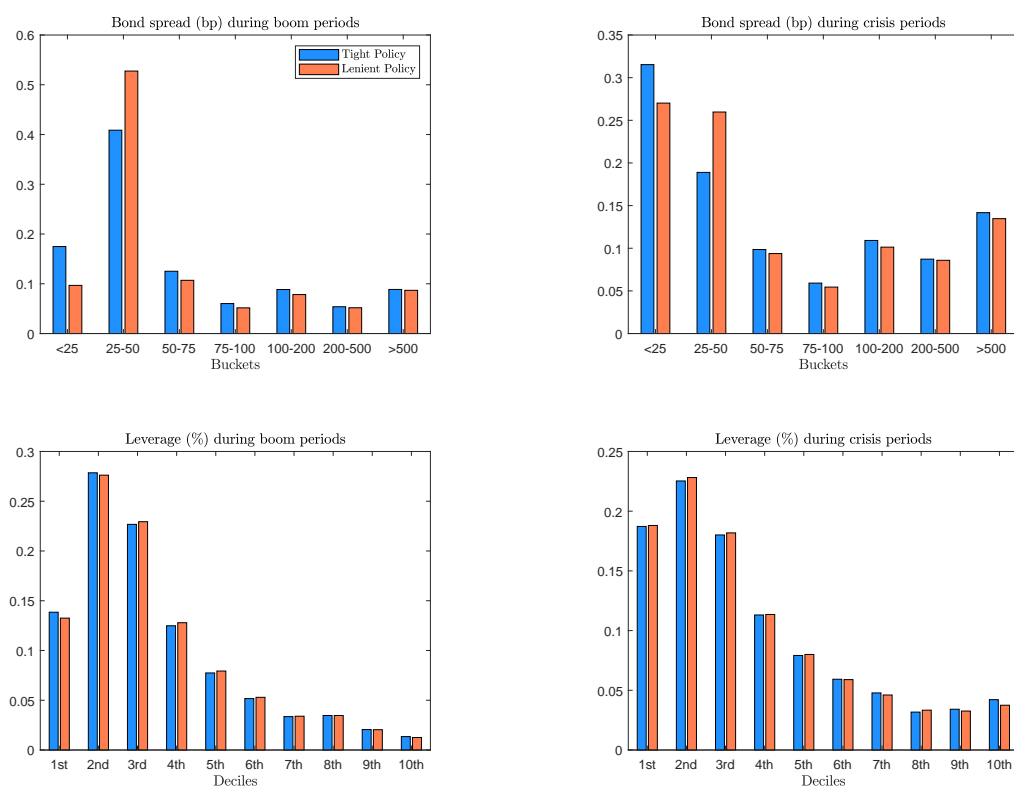
### E.1 Cross-Sectional Distribution with Interbank Market Tightening

Supplementary to the baseline results in figure 5, this section provides further results for the situation where the crisis state features an interbank market tightening in addition to a deterioration of firm fundamentals. As expected, the differences between different collateral policies are notably larger. If there is no shock on the interbank market in crisis periods, the difference between crises and boom states is larger, especially concerning eligible debt capacity and, thereby, collateral supply. Since collateral policies directly target the money market, central bank eligibility requirements have larger effects on collateral supply.

Still, the cross section looks broadly similar to the baseline parameterization, in particular during booms. The lenient policy features a high share of bonds in the second lowest risk bucket, but fewer in the lowest one, compared to tight policies. While a high share of bonds is eligible, increased risk-taking also reduces the number of firms with very low spreads. As in the baseline case, the spread and leverage distributions exhibit more mass in the right tail during crisis periods, since default risk still increases as fundamentals deteriorate. The tight policy, again, displays high mass in the lowest spread bucket, which indicates an exceptionally high eligible premium for remaining eligible firms.

Next, consider private and public eligible debt capacity. Again, the picture in figure 11 is broadly consistent with the results from the baseline model without interbank market tightening in figure 6. In addition to a deterioration of firm fundamentals, the dashed black line shifts north-west. In figure 6 these effects are missing and the dashed black line remains closer to the one denoting central bank eligible debt capacity (solid) in a crisis period. Hence, the total mass of eligible firms in a crisis is higher (mass

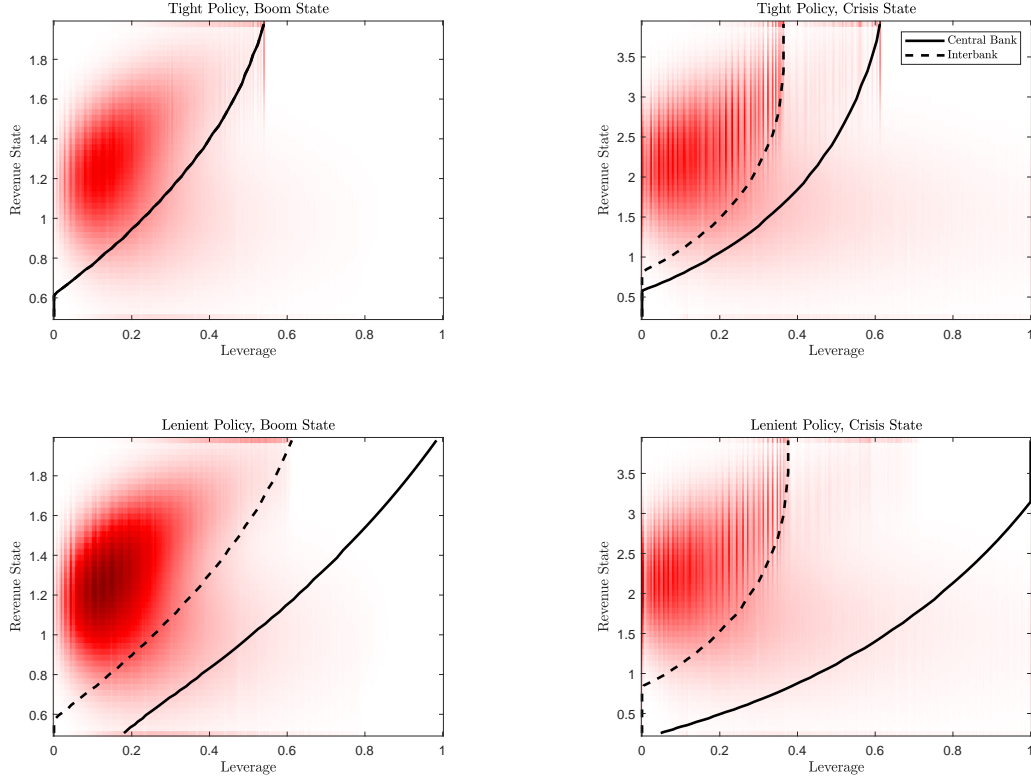
Figure 10: Cross-Sectional Distribution with Interbank Tightening



Notes: Histograms represent the distribution of all firms that are not in default. Differently-colored bars denote policies.

left of the black lines). In the scenarios in which a difference between public and private eligible debt capacity arises, we again observe constrained eligible firms that choose leverage just below their capacity threshold.

Figure 11: Eligibility Requirements over Cross-Sectional Distribution with Interbank Tightening



Notes: The shaded area reflects the cross-sectional distribution over leverage and revenues for both aggregate states, respectively. Firms in restructuring are excluded. The black line represents the eligible debt capacity on the private (dashed) and public segment (solid).

## E.2 Macroeconomic Aggregates across the Policy Space

To put the results presented in the main paper into a broader context, we compute macroeconomic aggregates over a grid of eligibility requirements  $(\Lambda_b^{CB}, \Lambda_c^{CB})$  in figure 12, which is an approximation of the central bank policy space. The off-diagonal cases correspond to time-varying eligibility requirements, where  $\Lambda_b^{CB} < \Lambda_c^{CB}$  reflects a countercyclical and  $\Lambda_b^{CB} > \Lambda_c^{CB}$  a procyclical policy, respectively. Results are displayed relative to the baseline scenario of uniformly tight eligibility requirements, i.e.  $\Lambda_b^{CB} = \Lambda_c^{CB} = 0.015$ . Recall that a lower value for  $\Lambda^{CB}$  corresponds to a tighter policy. In general, these surfaces show only limited slant, i.e. macroeconomic aggregates respond mostly to variations of the eligibility requirement that applies to the current state.

We begin with lending costs, which are proportional to collateral supply and collateral quality by construction. In both states, lenient eligibility requirements in state  $z$  foster money market lending and thereby costs in state  $z$ . They also increase lending in the respective other state, although to a lesser extent. The upper panel of figure 12 consequently shows lending cost surfaces with positive, albeit small slant. Generally, there is little scope for reducing costs compared to the tight policy baseline, since usage of the central bank facilities is rather low over and above collateral re-use under the tight policy. We observe that lending costs are relative more responsive to eligibility requirements in the crisis state, i.e. the surfaces are steeper. This is due to the relatively strong simultaneous decline in collateral quality in

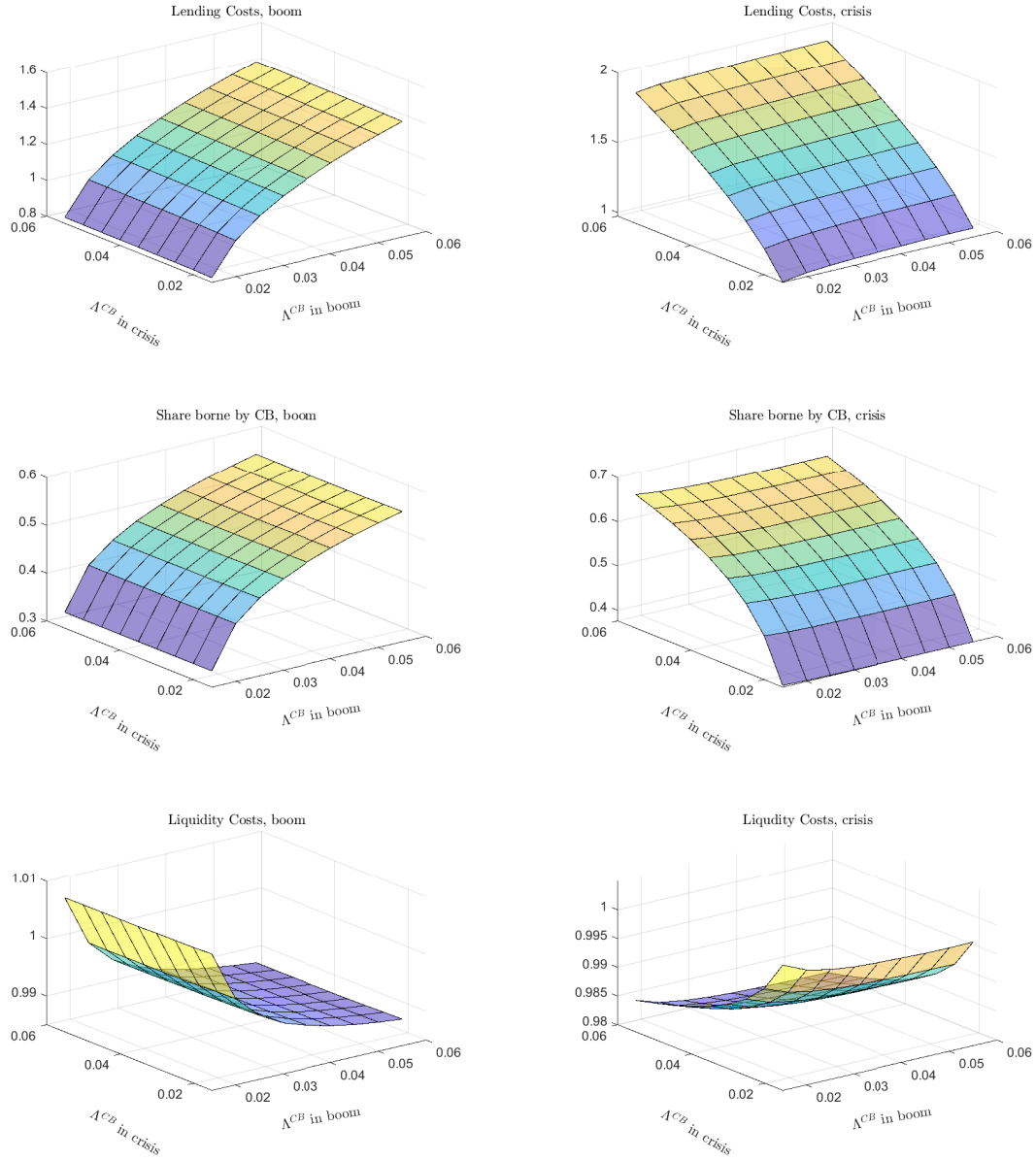
the crisis state.

Second, consider the share of lending cost borne by the public sector. For central bank requirements stricter than interbank requirements, the share decreases below  $\varsigma$ . The remaining share derives from the small but positive amount of bonds that are still eligible at the central bank re-used by cash lenders. On the other hand, under lenient eligibility requirements borrowers increase their reliance on central bank borrowing, while collateral re-use remains constant. In the crises scenario we observe, a similar pattern. Yet, money market borrowers heavily rely on the public segment as interbank lending standards tighten. This is visible as a steep increase in the share borne for low eligibility requirements in the middle panel of figure 12.

Finally, liquidity cost incurred by cash borrowers fall as collateral supply increases. The surfaces in the lower panel of figure 12 exhibit almost no slant, since liquidity costs only depend on collateral supply and the adverse effect of collateral quality is largely absent. While the relative change of liquidity costs in the range of eligibility thresholds considered is rather small (up to -2.5 %) the relative increase in lending costs is much larger. To put things into perspective, recall that liquidity costs  $\mathcal{L}$  exceed lending costs  $\mathcal{K}$  by a factor 100 (see table 5).



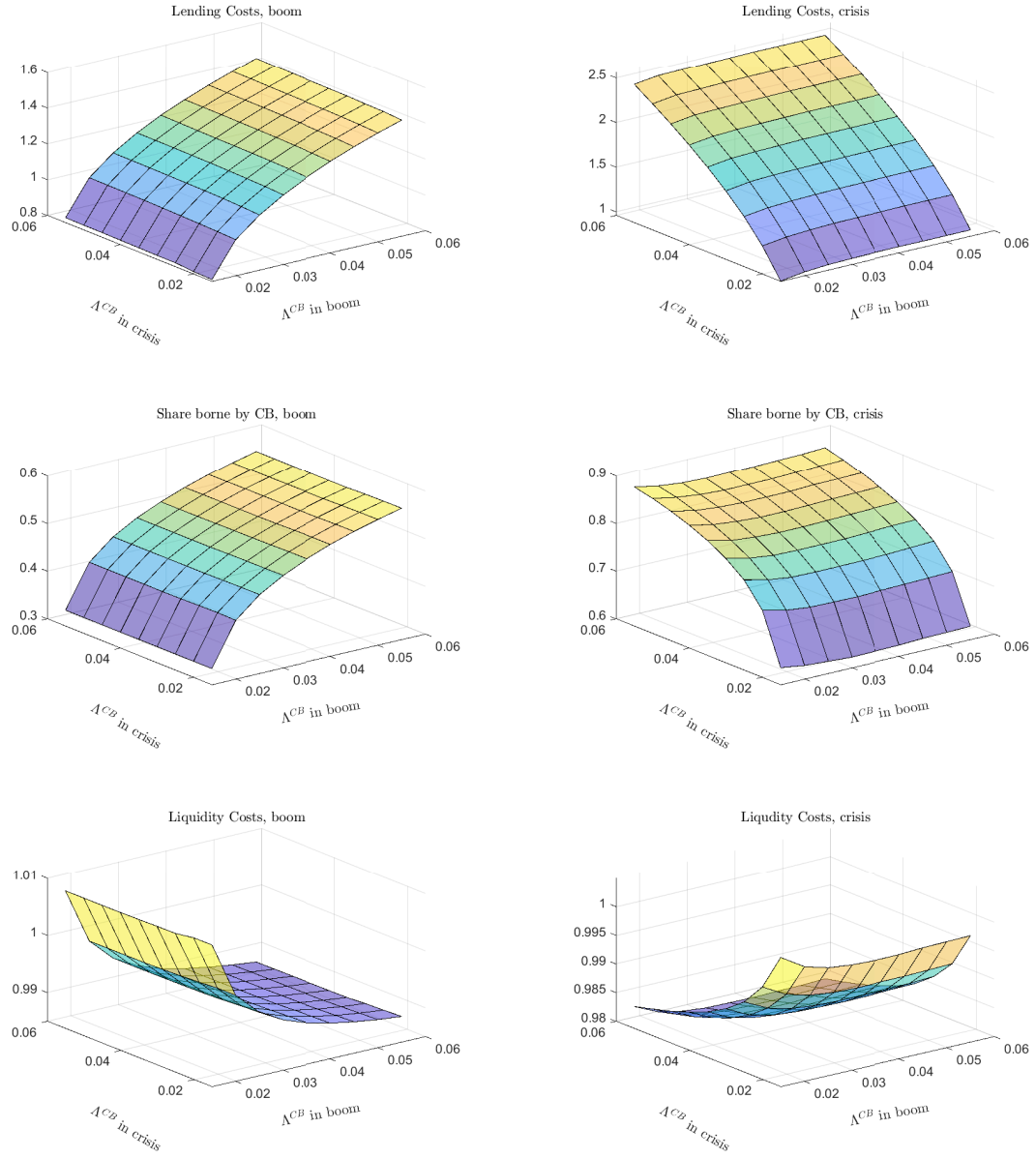
Figure 12: Policy Analysis



Notes: We vary the respective eligibility thresholds in booms (x-axis) and crisis (z-axis) in both states. Results are displayed along the z-axis relative to a tight central bank policy,  $\Lambda_b^{CB} = \Lambda_c^{CB} = 0.015$ .

Results for the case with interbank disruptions are obtained accordingly and presented in figure 13. While the shape of cost functions is similar to figure 12, the relative effects are of higher magnitude. The most remarkable differences are the relatively pronounced increase in total lending costs and in the share borne by the central bank. Since the lending cost parameter is now lower in the crisis compared to the boom, the dynamics of  $\mathcal{K}$  are not only driven by aggregate collateral  $\bar{B}$  but also by changes in private money market eligibility. Since the interbank market is disrupted in the crisis state, there is a sharper increase in the share of lending costs borne by the central bank.

Figure 13: Policy Analysis with Interbank Tightening



Notes: We vary the respective eligibility thresholds in booms (x-axis) and crisis (z-axis) in both states. Results are displayed along the z-axis relative to a tight central bank policy,  $\Lambda_b^{CB} = \Lambda_c^{CB} = 0.015$ .