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## Face Masks, Yeast, and Toilet Paper: Panic Purchases and Stockpiling

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# Face Masks, Yeast, and Toilet Paper: Panic Purchases and Stockpiling\*

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## Abstract

During the COVID-19 pandemic, some goods suddenly became scarce due to panic purchases and stockpiling. The decision to hoard is influenced by higher-order beliefs. If an agent believes that other agents think that a good will become scarce, she concludes that these other agents will hoard, and thus tries to preempt them by hoarding herself. To capture such behaviour, we construct a model with a global game. Agents receive noisy information about some variable that influences supply or demand. They then form higher-order beliefs, and possibly panic and hoard the product.

We analyze determinants of such panics. Endogenizing prices and quantities, we show that producers with market power set prices strategically to induce panics and thus boost demand. Absent market power, strong competition leads to low prices and thus little overproduction, also causing panics. We also consider firms who need the scarce good as an intermediate product in their production chain, and show that a regulator would require such firms to stockpile the product, thus increasing resilience.

**Keywords:** Stockpiling, hoarding, panic purchases, scarce goods, price stickiness, COVID-19, resilience.

**JEL-Codes:** L15, L11, I11.

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# 1 Introduction

During the outbreak of the COVID-19 pandemic, some goods suddenly became scarce. The U.S. Food & Drug Administration releases current updates about the scarcity of medical devices such as face masks and medical gloves (FDA, 2021). Prominent non-medical goods were flour, baking yeast, canned products, PlayStations, jigsaw puzzles, toilet paper, tissues, guns and ammunition, coins, and gold bars.<sup>1</sup> As a potential reason, households were stockpiling these goods exactly because they thought that they would become scarce. The purchasing decision can be interpreted as a game with strategic complementarities. Such games often have multiple equilibria, one of which is the panic equilibrium in which households hoard goods. The global games approach uses a slightly perturbed game to select an equilibrium and analyze the prevalence of such panics.

In our model, there is a continuum of agents that gets sick with some probability, in which case they need a specific drug. The drug stands for any good with (in the short run) fixed quantity, rigid prices, and an illiquid secondary market. To be on the safe side, agents can already hoard the drug before they know whether they will become sick. If an agent buys the drug but remains healthy, he cannot re-sell the drug. We then assume that the utility from taking the drug (or its supply, or the probability to become sick) is random, and agents receive the information with an infinitesimal bit of noise. This infinitesimal perturbation of the model leads to unique equilibrium selection. On this basis, one can calculate the probability of panic purchases. Unsurprisingly, we find that the panic equilibrium occurs when supply is low, expected demand is high, prices are low and the utility of the good is high.

Calculating expected revenues and profits of producers, one can go on and endogenize the quantity and price of the product. We find that, in comparison to the welfare optimum, firms produce too little in order to induce panics that thus upward jumps in demand. In comparison to the monopoly price absent panics, the profit-maximizing price is lower, also to induce panics and demand increases. Competition between multiple producers has an ambiguous effect on the prevalence of panics. It typically leads to a reduction in prices, inducing panics, but may also lead to an increase in supply, preventing panics.

In an extension of the model, we interpret the good as an intermediate good. For example, face masks are used on construction sites, in the furniture industry, and in hospitals. Production chains may stall if one essential ingredient is missing. In the pandemic, hospitals were caught by surprise; many had stored too few face masks. During and after the

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<sup>1</sup>[en.wikipedia.org/wiki/Shortages\\_related\\_to\\_the\\_COVID-19\\_pandemic#Consumer\\_goods](https://en.wikipedia.org/wiki/Shortages_related_to_the_COVID-19_pandemic#Consumer_goods)

shortages, firms attempted to redesign supply chains (Shih, 2020; Choi, Rogers, and Vakil, 2020; OECD, 2020; Deloitte, 2020). Antràs (2020); Meier and Pinto (2020); Kazunobu and Kohei (2021); Simola (2021) document the negative effects of international supply chain disruptions. There were calls for public interventions, for example regulations for firms or hospitals to stockpile critical goods, or the government building stockpiles (Kaliya-Perumal, Kharlukhi, and Omar, 2020; European Parliament, 2020). An extension of the model is suitable to analyze the equilibrium effect of stockpiles in a supply chain. We show that, in a model with endogenous prices and quantities, stockpiles have a positive externality on other firms with similar supply chains. There is also a positive externality on households, the original source of the panic shortages. As a consequence, a regulator may set minimum quantities for stockpiling. It may also subsidize stockpiles, or stockpile the critical goods itself. This substantiates the argument of Fabra, Motta, and Peitz (2020). Producers are worse off, however.

**Literature.** We add to two strands of literature. *First* and foremost, we apply the global games methodology. Introduced by Carlsson and van Damme (1993), there have been lots of applications (Morris and Shin, 2003, see). Our model builds on the global games models on bank runs, see Goldstein and Pauzner (2005) and especially Allen, Carletti, Goldstein, and Leonello (2018), who also let the variance of the noise term converge to zero. The differences between our model and a bank run game à la Diamond and Dybvig (1983) are minor. In a bank run model, the invested money earns a return when not withdrawn, face masks do not. In a bank run model, risk aversion plays an important role. Our households are risk neutral. On the other hands, we make the assumption that there is no secondary market for face masks. Note that our contribution is an application of the global games framework, not a theoretical contribution to it.

*Second*, the paper delivers a theoretical framework to decipher the shortages in the COVID-19 pandemic, especially at the start. These shortages have been visible in the pandemic, but they can occur whenever there are demand or supply shocks, when prices are rigid.

On the theoretical side, Bénabou (1989) shows that sticky prices lead to incentives for stockpiling. Lim and Tang (2013) present a model where a monopolist tries to price-discriminate by selling in two periods, in the face of arbitrageurs on a secondary market. Of course, the literature on dynamic pricing strategies and customers' reaction is huge, both in marketing and in industrial economics. Our paper is the first, however, to construct a theory model for hoarding panics. As mentioned, we benefit from the structural similarities to banking models.

There is a lot of evidence on hoarding. Gráda (2009) gives a historical overview over the role of shortages in famines. Gönül and Srinivasan (1996); Erdem, Imai, and Keane (2003); Sun, Neslin, and Srinivasan (2003); Sun (2005) find that consumers stockpile when they expect rising prices. Hansman, Hong, De Paula, and Singh (2020) find that sticky prices exacerbate hoarding. Mandel and Veetil (2020) estimate price reactions due to the first COVID-19 lockdown in the United States. Clemens, Matkovic, Faasse, and Geersa (2020) documents the determinants for purchasing health-related products at the beginning of the pandemic.

## 2 The Basic Model

Consider an economy with a continuum of agents of mass 1. Agents fall sick with probability  $m \in (0, 1)$ . If an agent falls sick and takes the drug, her utility is  $u$ . If she does not take the drug, her utility is normalized to zero. The price of the drug is  $p < u$ , the available quantity is  $q < 1$ . There is no secondary market; if an agent has bought the drug, she cannot sell it to anyone else, and it cannot be used at some later date. Some of these variables will be made to depend on a macroeconomic, exogenous state of the world later, and some will be endogenized in an extended version of the game. In the model, we use the word “drugs”, but one can also think of face masks, storable food, and much more.

The time and information structure is as follows. First, in the early stage, each agent decides whether to buy the drug at an early stage. Then she learns whether she will fall sick. Then at the later stage, if the drug is not yet sold out, she can again try to buy it. Then, if she has bought the drug, she consumes it. If she buys the drug before knowing whether she will need it, she “hoards” the drug. Alternatively, she can just “wait.”

**Equilibrium.** There are three types of equilibria; one in which all agents hoard the drug, one in which all agents wait, and a mixed strategy equilibrium. First, consider the case that all agents hoard. Because  $q < 1$ , the drug will be sold out in the early stage, and the probability that an agent is allocated the drug is  $q$ . In expectations, she pays  $qp$ . If with probability  $(1 - q)$  she is not allocated the drug at the early stage, she will not be able to buy it at the later stage. She falls sick with probability  $m$ , unable to buy the drug. Her expected utility is thus

$$U_{\text{hoard}} = q(mu - p). \quad (1)$$

If, out of equilibrium, she waits, she will always be too late to buy the drug. Her expected utility is then

$$U_{\text{wait}} = 0. \quad (2)$$

Comparing these expected utilities, the hoarding equilibrium exists if and only if

$$u \geq \bar{u}_0 = \frac{p}{m}. \quad (3)$$

Second, consider the case that all agents wait. The drug will not be sold out in the early stage. If, out of equilibrium, the agent buys the drug at the early stage, she gets it for sure, and her expected utility is

$$U_{\text{hoard}} = m u - p. \quad (4)$$

If the agent waits until the later stage, she will buy only if she falls sick with probability  $m$ . If  $m \leq q$ , she will get the drug for sure, and her expected utility is

$$U_{\text{wait}} = m(u - p). \quad (5)$$

In this case, the utility from waiting is always strictly higher than that from hoarding, so waiting is always an equilibrium. If  $q < m$ , she will get the drug only with probability  $q/m$ , thus her expected utility is

$$U_{\text{wait}} = m \frac{q}{m} (u - p) = q(u - p). \quad (6)$$

In this case, waiting is an equilibrium strategy if and only if

$$u \leq \bar{u}_1 = p \frac{1 - q}{m - q} \quad (7)$$

which is larger than  $\bar{u}_0$  if  $m > q$ , and has no meaningful solution for  $m \leq q$ . The equilibrium in mixed strategies is unstable, so we do not analyze it. Summing up, if  $u < \bar{u}_0$ , then waiting is the only equilibrium strategy. If  $u > \bar{u}_1$ , then hoarding is the only equilibrium strategy. If  $u \in [\bar{u}_0, \bar{u}_1]$ , there are multiple equilibria.

**Welfare.** In the first best, the drug is allocated only to those who need it. The price does not enter welfare, it is only a transfer. The expected utility is  $q u$  if  $q < m$ , or  $m u$  if  $m < q$ , like in the waiting equilibrium. The expected utility in the hoarding equilibrium equals  $q m u$  and is thus strictly smaller. If agents could coordinate on the welfare-dominant waiting equilibrium, where it exists, they could reach the first-best.

### 3 Noisy Information and Panics

In addition to the above model, assume there is a uniformly distributed macro variable  $\vartheta \in [0, 1]$ . A high  $\vartheta$  stands for a good state of the economy. Agents cannot directly observe  $\vartheta$ , but instead observe  $\vartheta + \varepsilon_i$ , where  $\varepsilon_i$  is a noise term that is stochastically independent between agents, and with the macro variable. It is uniformly distributed on  $[0, \varepsilon]$ , where  $\varepsilon > 0$  is assumed to be small. To be precise, we consider the limit of  $\varepsilon \rightarrow 0$ .

The time and information structure is as follows. First, agents learn their private signal  $\vartheta + \varepsilon_i$ . Then, in the early stage, each agent decides whether to buy the drug at an early stage. Then she learns whether she falls sick. Then in the later stage, if the drug is not yet sold out, she can again try to buy the drug, and then take it.

We will consider three specifications. In the first specification, agents are uncertain about the utility of the drug,. Here,  $u$  is a function of  $\vartheta$ , with  $u'(\vartheta) > 0$ . In the second specification, agents are uncertain about demand for the drug, hence  $m$  is a function of  $\vartheta$ , with  $m'(\vartheta) < 0$ . In a third specification, agents are uncertain about supply of the drug, hence  $q$  is a function of  $\vartheta$ , with  $q'(\vartheta) > 0$ .

In the first specification, the utility from taking the drug is a function  $u(\vartheta)$ . Because  $\vartheta$  is a uniformly distributed random variable, it can just be interpreted as the percentile of the state of nature that realizes. For  $\vartheta = 1$ , the best possible state of nature is drawn, the drug is as effective as one could ever think of, and its utility is at its maximum. For  $\vartheta = 0$ , the worst possible state of nature is drawn, the drug is as *ineffective* as one could ever think of, and its utility is at its minimum. As a final example, for  $\vartheta = 1/2$ , one is in the median case. In half of the conceivable cases, the drug could have had a higher utility. Note that the distribution of  $u$  can take any shape. Let  $F(u)$  denote the distribution function of  $u$ , and  $f(u)$  its density function.

Agents do not know *ex ante* how useful the drug is, but they get a private signal. Take one specific agent  $i$  at the early stage, considering whether to hoard or to wait after receiving her private signal  $\vartheta + \varepsilon_i$ . Other agents may be more optimistic than her, in the sense that they have received a signal with a higher  $\varepsilon_j$ . Of course, other agents might also have received a lower signal, and be more pessimistic. Let  $n$  denote the number of agents that are more *pessimistic* than agent  $i$ . There are two consequences. First, from the perspective of agent  $i$ , this  $n$  follows a uniform distribution. It could be any number between 0 and 100 percent, with equal probability. Second, if agent  $i$  is indifferent between hoarding and waiting, then  $n$  agents will hoard because they are more pessimistic, and  $1 - n$  agents will wait. We are looking for the critical  $\bar{\vartheta}$ , that is, the critical  $\bar{u} = u(\bar{\vartheta})$ , such that agents will

hoard whenever they expect a utility below this  $\bar{u}$ . The probability of a run on the drugs is thus  $\Pr\{\vartheta \leq \bar{\vartheta}\} = \bar{\vartheta} = u^{-1}(\bar{u}) = F(\bar{u})$ , due to the uniform distribution of  $\vartheta$ .

We will see that the algebraic equations differ, depending on whether there are more drugs than sick agents ( $m \leq q$ ), hence sufficiently many drugs to potentially supply all sick agents. Of course, if agents panic, there can still be a shortage even in this case. Alternatively, if  $q < m$ , there will always be a shortage, either already at the early date, or later.

**The case of  $m \leq q$ .** If  $m \leq q \leq 1$ , there are *enough* drugs in principle to serve all sick agents. Three things can happen. If  $n > q$ , the drug will be sold out already at the early stage. If  $n \leq q$ , the  $n$  hoarding agents will demand  $n$  drugs at the early stage, so  $q - n$  will remain for the second stage. The demand at that stage will be  $m(1 - n)$ . So at that stage, there will be sufficient supply if and only if

$$q - n \geq m(1 - n) \quad \Longleftrightarrow \quad n < \bar{n} = \frac{m - q}{m - 1}. \quad (8)$$

Otherwise, the drug will be sold out at the late stage, and there will be rationing. For the agent, the consequences in the three cases differ. If  $n > q$ , the drug will be rationed even for hoarding agents. Their expected utility is  $q/n(mu - p)$ . The expected utility for waiting agents is 0.

If  $n \leq q$ , the expected utility for hoarding agents is  $mu - p$ . If  $n < \bar{n}$ , there is no rationing at the later stage, so the utility for waiting agents is simply  $u - p$ . If  $n \in (\bar{n}, q)$ , there is rationing at the later stage, and waiting agents' utility is  $(u - p)$  times the probability of getting the drug,

$$\frac{q - n}{(1 - n)m}.$$

Summing up, the expected utility from hoarding is

$$\begin{aligned} U_{\text{enough}}^{\text{hoard}} &= \int_0^{\bar{n}} (mu - p) dn + \int_{\bar{n}}^q (mu - p) dn + \int_q^1 \frac{q}{n} (mu - p) dn \\ &= q(mu - p) - q(mu - p) \ln q \\ &= q(mu - p)(1 - \ln q). \end{aligned} \quad (9)$$

Remember that  $q < 1$ , such that the last bracket exceeds 1. The expected utility from



waiting is

$$\begin{aligned}
U_{\text{enough}}^{\text{wait}} &= \int_0^{\bar{n}} m(u-p) dn + \int_{\bar{n}}^q \frac{q-n}{(1-n)m} m(u-p) dn + \int_q^1 0 dn \\
&= \bar{n} m(u-p) + \left( (q-\bar{n}) + (1-q) \ln \frac{1-q}{1-\bar{n}} \right) (u-p) \\
&= \left( q - (1-m)\bar{n} + (1-q) \ln \frac{1-q}{1-\bar{n}} \right) (u-p).
\end{aligned} \tag{10}$$

For the critical  $\bar{u}$ , agent  $i$  must be indifferent between hoarding and waiting,  $U_{\text{enough}}^{\text{wait}} = U_{\text{enough}}^{\text{hoard}}$ . Both (9) and (10) are linear in  $u$ , so solving for the critical  $\bar{u}$  yields

$$\bar{u}_{\text{enough}} = p \frac{m-q + (1-q) \ln(1-m) + q \ln q}{(1-q)(m + \ln(1-m)) + m q \ln q}. \tag{11}$$

What is the interpretation of this value? An agent that observes a private signal  $u(\vartheta + \varepsilon_i) > \bar{u}_{\text{enough}}$  will hoard the drug. Now remember that  $\varepsilon_i$  is small, it ranges in the interval  $[0, \varepsilon]$ . Hence if  $u(\vartheta + \varepsilon) > \bar{u}_{\text{enough}}$ , all agents will hoard the drug. If  $u(\vartheta) < \bar{u}_{\text{enough}}$ , all agents will wait patiently. In the limit of  $\varepsilon \rightarrow 0$ , all agents hoard if  $u(\vartheta) > \bar{u}_{\text{enough}}$ , and wait if  $u(\vartheta) < \bar{u}_{\text{enough}}$ . Only in the zero-probability event of  $u(\vartheta) = \bar{u}_{\text{enough}}$ , a fraction of agents hoards.

**The case of  $q < m$ .** If  $q < m$ , the drug will always be *scarce* at some point. Only two things can happen. Either there is rationing already at the early stage, or at the latest there is rationing in the later stage. Formally, the  $\bar{n}$  from equation (8) would turn (weakly) negative for  $q < m$ . Consequently, the first integral in equations (9) and (10) is dropped, and the second integral starts from zero instead of  $\bar{n}$ . We get

$$\begin{aligned}
U_{\text{scarce}}^{\text{hoard}} &= \int_0^q (m u - p) dn + \int_q^1 \frac{q}{n} (m u - p) dn \\
&= q(m u - p) - q(m u - p) \ln q \\
&= q(m u - p)(1 - \ln q),
\end{aligned} \tag{12}$$

the same term as before. The utility from waiting is now

$$\begin{aligned}
U_{\text{scarce}}^{\text{wait}} &= \int_0^q \frac{q-n}{(1-n)m} m(u-p) dn + \int_q^1 0 dn \\
&= (u-p) (q + (1-q) \ln(1-q)).
\end{aligned} \tag{13}$$

For the critical  $\bar{u}$ , agent  $i$  must be indifferent between hoarding and waiting,  $U_{\text{scarce}}^{\text{wait}} = U_{\text{scarce}}^{\text{hoard}}$ , yielding

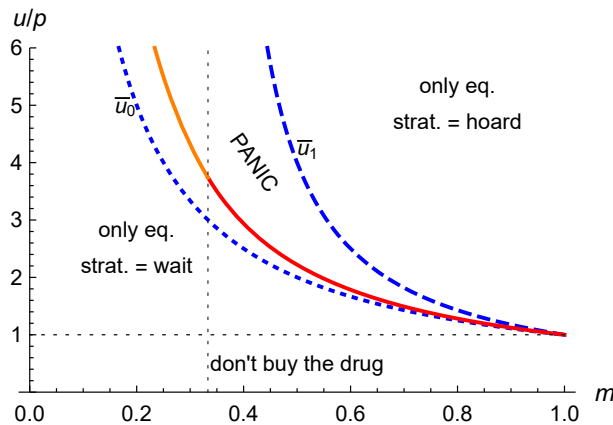
$$\bar{u}_{\text{scarce}} = p \frac{(1-q) \ln(1-q) + q \ln q}{(1-q) \ln(1-q) + q(1-m + m \ln q)}. \tag{14}$$

The interpretation of  $\bar{u}_{\text{scarce}}$  is the same as above. An agent that observes a private signal  $u(\vartheta + \varepsilon_i) > \bar{u}_{\text{scarce}}$  will hoard the drug. Because  $\varepsilon_i \in [0, \varepsilon]$  and we consider the limiting  $\varepsilon \rightarrow 0$ , *all* agents hoard if  $u(\vartheta) > \bar{u}_{\text{scarce}}$ , and wait if  $u(\vartheta) < \bar{u}_{\text{scarce}}$ .

**Dominance Regions.** As is known from the literature on global games, the equilibrium is unique whenever upper and lower dominance regions exist. For  $m > q$ , there are not enough drugs for all sick agents. For  $u < \bar{u}_0$ , there is only the waiting equilibrium. This is the lower dominance region. If  $u > \bar{u}_1$ , there is only the hoarding equilibrium. This is the upper dominance region. For  $m \leq q$ , there is no  $\bar{u}_1$ , and thus no upper dominance region. From a theoretical perspective, one can choose one of two approaches. Either one assumes that for  $\vartheta$  close to 1, other parameters (not only  $u$ ) also change their value, such that an upper dominance region is assumed into existence. Another alternative is to apply an additional equilibrium selection criterium. We choose the first approach, such that the equilibrium endogenously becomes unique.

**Interpretation and Comparative Statics.** The model is very stylized and contains few parameters. There are two parameters with a monetary dimension,  $u$  and  $p$ . We can either set  $p = 1$  without loss of generality, or look at the fraction  $u/p$  instead. There are two more variables, the probability of getting sick,  $m$ , and the supply of drugs,  $q$ . For an illustrative (but representative) example, we set  $q = 1/3$ ; the drugs suffice for a third of the population. The following Figure 1 shows the critical utilities as a function of the supply of drugs  $q$ .

Figure 1: Hoarding and Waiting Equilibria



Here, the blue dashed curve shows  $\bar{u}_1$ . Above this curve, the utility of the drug is so large that agents want to be on the safe side and hoard. The blue dotted curve shows  $\bar{u}_0$ . Below this curve, the utility of the drug is so low that agents just wait until they see whether they become sick. Obviously, if  $m < q = 1/3$  (vertical dotted line), there are potentially enough drugs for all agents, so there is a waiting equilibrium even for very high values of  $u/p$ . Below the value of  $u/p < 1$ , the utility of the drug is so low that agents do not buy it, even if they know they will become sick. Between the two blue curves, in the *absence* of the  $\varepsilon$ -noise in the information, there are multiple equilibria. If all other agents hoard, it is optimal to also hoard. If all other agents wait, it is optimal to also wait.

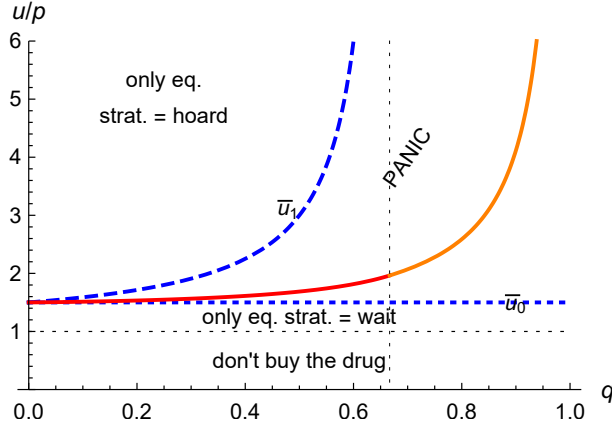
Now turn to the full game with the  $\varepsilon$ -noise. Left of the vertical dotted line, we are in the regime with  $m \leq q$ , so the critical utility is  $\bar{u}_{\text{enough}}$  as given in (11) (orange part of the curve). Right of that line, we are in the regime with  $q < m$ , so the critical utility is  $\bar{u}_{\text{scarce}}$  as given in (14) (red part of the curve). One can see that the behavior of the curve is not very different in the red or orange part. Importantly, above this joint  $\bar{u}$ -curve, there is only the hoarding equilibrium in the game with the  $\varepsilon$ -noise. Between this  $\bar{u}$ -curve and the blue dashed curve, agents panic. In the absence of the noise, there would be a waiting equilibrium that agents could coordinate upon, but instead agents panic and stockpile. The reason is that due to the noise, agents are now ordered by their degree of optimism, but they do not know where they are positioned in that ordering. Maybe they are on the optimistic side, while more pessimistic agents are already hoarding. Taking this into account, agents rationally become more nervous. Such higher-order beliefs lead to rational panics.

Note that  $\bar{u}_{\text{enough}}$  is defined for all positive values of  $m$ , and the limit for  $m \rightarrow 0$  is infinite. This means that, no matter how small the supply of drugs is, there is always a critical utility  $\bar{u}$  below which agents will choose to wait. On the other hand, for  $m \rightarrow 1$ , agents get sick with certainty, so the only equilibrium is to buy the drug as soon as possible. The properties of the figure are general, see proposition 1 in the appendix.

Let us now set  $m = 2/3$  and vary  $q$ . Not surprisingly, the ensuing Figure 2 looks like a horizontal mirroring. In addition, there is now a utility below which waiting is the only equilibrium strategy, independent of  $q$ .

**Stochastic Demand or Supply.** Up to here, we have assumed that  $u$  is stochastic, in that it depends on a stochastic macro variable  $\vartheta$ . What happens if instead  $m$  is stochastic? For example in a pandemic, the measure of agents that will need the drug may be unknown. One could model this simply by again having a state variable  $\vartheta$ , but now letting  $m$  depend

Figure 2: Equilibria Depending on  $q$  and  $u/p$



on  $\vartheta$ . As another alternative,  $q$  may be stochastic. Ex ante, it may be unclear how many drugs will be delivered by producers. In that case,  $q$  would be a function of  $\vartheta$ .

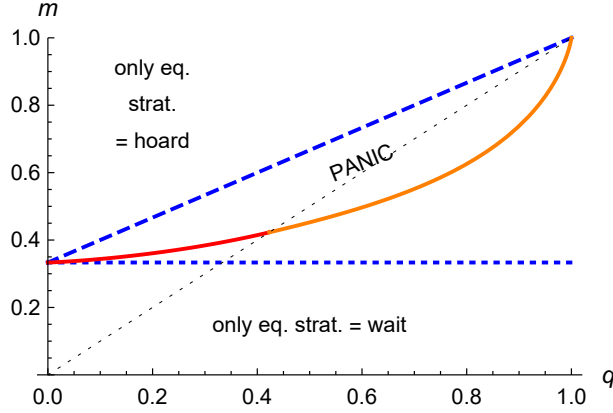
Surprisingly, in these specifications, surprisingly little changes. The integrals for agents utilities (9) and (10), as well as (12) and (13) remain unchanged. As a consequence, also the indifference conditions remain unchanged. However, we must now solve for some critical  $\bar{m}$  (or some  $\bar{q}$ , respectively). Mathematically, solving for  $m$  involves special functions (the productlog), and there is not closed-form solution for  $q$ .

The following Figure 3 shows the different regimes for varying  $q$  and  $m$ , based on a numerical simulation with  $u/p = 3$ . The blue lines show the regimes in the absence of noise in the information. For small  $m$ , there is only the waiting equilibrium. For large  $m$ , especially in combination with low  $q$ , there is only the hoarding equilibrium. In the triangle between the blue lines, both the hoarding equilibrium and the waiting equilibrium exist.

The red-orange curve shows the border between regimes in the presence of  $\varepsilon$ -noise. Again, orange shows the case of  $q > m$ , and red shows  $q < m$ . The algebraic equation for the two parts differ, but apparently, there is no structural break in the properties.

Of the triangle with initially multiple equilibria (absent the  $\varepsilon$ -noise), different chunks turn into the panic-region when introducing the  $\varepsilon$ -noise. The slope of the red curve is zero at  $q = 0$ . This means that for low quantities, due to the noise, agents panic almost always when before they could have coordinated on waiting. For  $q = 1$ , it is the other way round. Where there were multiple equilibria in the absence of  $\varepsilon$ -noise, agents now just wait and

Figure 3: Equilibria Depending on  $q$  and  $m$



see. One can also look at the same question in the opposite direction. The slope of the orange curve converges to infinity for  $q \rightarrow 1$ . This means that, for large  $m$ , for parameter constellations with multiple equilibria only the hoarding equilibrium survives. On the other hand, for  $m \rightarrow k/u$ , only the waiting equilibrium survives. This is not surprising, because for  $m < k/u$ , there is only the waiting equilibrium. For a formal statement, see Proposition 3.

**Expected Revenue.** Instead of the agents, let us now put the focus on the firms that produce the drug, or the product that we have in mind. We can calculate the expected revenue without introducing additional assumptions. For a monopolist with zero production costs, this is identical to the profit function. In an oligopoly, or for a monopolist with positive marginal costs, one would have to add additional assumptions on the structure of competition, or on the cost function. We can calculate the expected revenue for given price  $p$  and supply  $q$ . Again, start with the assumption that  $u$  is stochastic, it depends on the macro-state  $\vartheta$  and has the density function  $f(u)$ .

If  $m > q$  (scarcity), all drugs will be bought sooner or later, at least if the utility  $u$  exceeds the price  $p$ . The expected revenue is

$$\begin{aligned} ER_{\text{scarce}} &= \underbrace{\int_0^p 0 \cdot f(u) du}_{\text{no drugs bought}} + \int_p^\infty p q \cdot f(u) du \\ &= p (1 - F(p)) q \end{aligned} \tag{15}$$

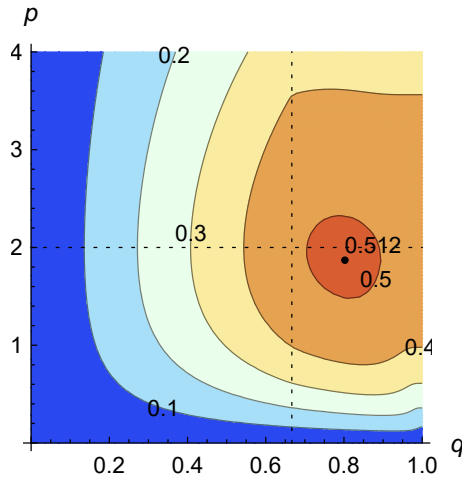
If  $m < q$  (enough drugs), the aggregate demand will be  $q$  only in a panic. In the waiting

equilibrium, it will be only  $m$ . The expected revenue is hence

$$\begin{aligned} ER_{\text{enough}} &= \underbrace{\int_0^p 0 \cdot f(u) du}_{\text{no drugs bought}} + \underbrace{\int_p^{\bar{u}_{\text{enough}}} p m \cdot f(u) du}_{\text{waiting eq.}} + \underbrace{\int_{\bar{u}_{\text{enough}}}^{\infty} p q \cdot f(u) du}_{\text{hoarding eq.}} \\ &= p \left( (F(\bar{u}_{\text{enough}}) - F(p)) m + (1 - F(\bar{u}_{\text{enough}})) q \right). \end{aligned} \quad (16)$$

For a concrete example, let  $f(u)$  follow an exponential distribution with mean  $\mu = 2$ , and set  $m = 2/3$ . The following Figure 4 shows the expected revenue in a contour plot.

Figure 4: Expected Revenues for Firms



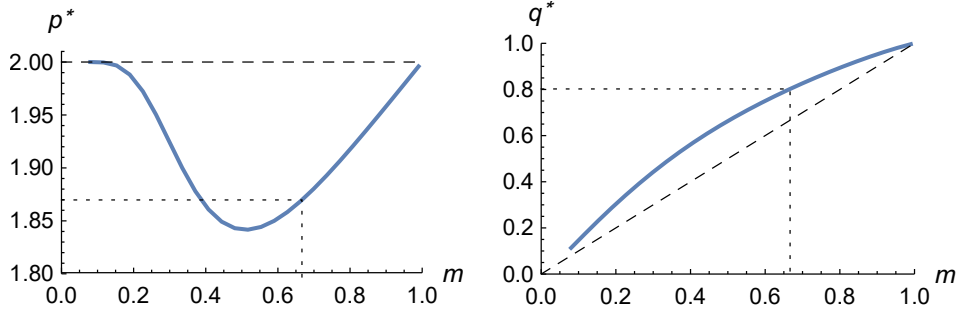
Left of the dotted line, we are in the regime with scarce drugs,  $q < m$ . The revenue-function increases linearly in  $q$ , and for each  $q$  has its maximum at  $p = \mu = 2$ . Right of the dotted line, there are enough drugs ( $m \leq q$ ), but there can still be panics. There are a number of interesting facts. First, the revenue function takes its maximum in the second regime with enough drugs. This property is general, because the revenue function is increasing in  $q$  for  $q < m$ , it is decreasing in  $q$  at  $q = 1$ , it is increasing in  $p$  at  $p = 0$ , and decreasing in  $p$  for  $p \rightarrow \infty$ , at least if the distribution of  $u$  is thin enough that its variance exists, which is a rather weak assumption. For a formal statement, see Proposition 2.

In this numerical example, the maximum is at  $q^* = 0.8023$  and  $p^* = 1.8698$ , reaching a value of  $ER = 0.512$ . This means that a monopolist with zero costs would choose a quantity such that there are enough drugs for all sick agents, but yet few enough to cause a panic with some probability. In fact, the monopolist appreciates panics: in a panic, he will sell his complete supply; absent a panic, he will not. Because he appreciates panics, and because the probability of a panic decreases in the price of drugs, he lowers the

price. This causes more panics and thus higher sales. At the revenue-maximizing point,  $F(\bar{u}) = 91.25\%$ , so the probability of a panic is  $1 - F(\bar{u}) = 8.75\%$ .

We can look at comparative statics, again for the example of  $u$  exponentially distributed. One can show that an increase in the mean  $\mu$  leads to a proportional increase of the optimal price  $p^*$  and leaves the optimal quantity  $q^*$  unchanged. The only interesting parameter is thus  $m$ . A numerical simulation, again with  $\mu = 2$  and  $m = 2/3$ , yields the following plots.

Figure 5: Revenue-Maximizing Price and Quantity



If  $m \rightarrow 1$ , there is no benefit from causing a panic, because sales are already close to their maximum. If  $m \rightarrow 0$ , it is very difficult to cause a panic, because the fraction of sick agents is so small. Hence, in both cases, the optimal price  $p^*$  is not much lower than the optimum with scarce drugs (marked by the dashed line). The parameter constellation with  $m = 2/3$  from the contour plot in Figure 4 is marked by the dotted line. Consistently, the optimal price is  $p^* = 1.8698$ .

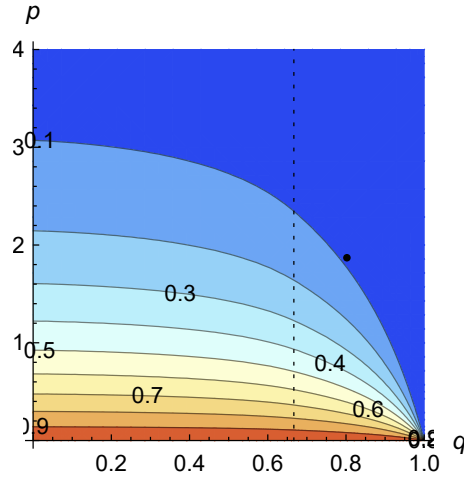
The second plot in Figure 5 shows the optimal quantity. The dashed diagonal shows the locus where there are just enough drugs,  $m = q$ . We see that the firm wants to produce more than enough drugs especially if it has lowered the price in order to cause panic purchases. Importantly, the firm does not want to set  $q = 1$ , even if the cost of production is zero. With  $q = 1$ , in a panic, the firm could sell drugs to all agents. However, with  $q = 1$ , the probability of a panic would be zero. Hence, the firm chooses  $q^*$  carefully; not too low such it can sell more in case of a panic, but not too high in order to cause panics in the first place.

**Competition.** Competition typically leads to lower prices  $p$ . How exactly the price depends on the number of firms, or on the level of competition, may depend on many specific factors, and on the type of competition. The effect of competition on quantity

$q$  is ambiguous. On the one hand, oligopolists tend to keep quantities small in order to achieve higher prices. A higher level of competition would then induce a higher aggregate quantity  $q$ . In this model, however, quantities above  $m$  can be sold only in a panic. With positive probability there will be remainders. These constitute an expected cost in the profit function of firms. If the pressure on prices is large, they can compete only by reducing possible remainders. This leads to a reduction in the aggregate quantity  $q$ .

The probability of a panic depends only indirectly on competition, through the equilibrium price  $p$  and quantity  $q$ . Figure 6 shows the iso-panic curves as a function of price and quantity. Parameters are as before. As an example, the revenue-maximizing point from Figure 4 is marked. As we already know, it has a panic probability of 8.75%, thus slightly below the 10%-curve.

Figure 6: Panic Probabilities



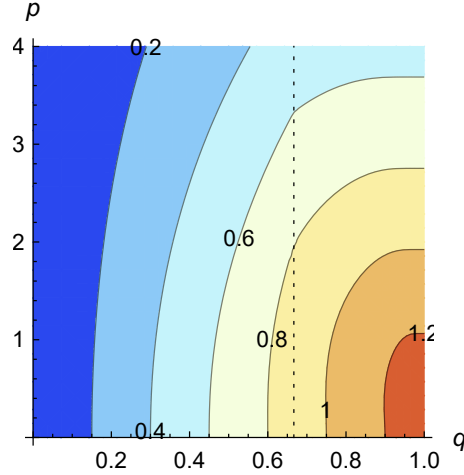
We see that the effect of competition on the propensity of panics is ambiguous. When the initial equilibrium quantity is low, iso-panic curves are nearly horizontal. As a consequence, an increase in competition that would bring down prices would lead to more panics. When the initial equilibrium quantity is high, iso-panic curves can be relatively steep. If an increase in competition leads to an increase in quantity  $q$ , is likely to reduce the probability.

**Welfare.** Welfare is a related, but slightly different statistic. If no drugs are sold, welfare is zero. In the presence of a panic, drugs are allocated randomly to  $q$  agents, of which  $m$  will become sick, so welfare is  $qmu$ . In the absence of a panic, if  $q < m$ , all  $q$  drugs are



allocated to sick agents, to welfare is  $qu$ . If  $m < q$ , only the  $m$  sick agents will buy the drug, so welfare is  $mu$ . The iso-welfare curves are depicted in the following Figure 7.

Figure 7: Welfare



We see that welfare increases in  $q$ . This result hinges on the implicit assumption that the drug can be produced at zero cost. Then, having a higher  $q$  increases the availability of the drug and reduces the probability of a panic, both raise welfare. We also see that welfare decreases in the price  $p$ . This result hinges on the assumption of an exponential distribution. The thickest part of the distribution of  $u$  is at the lower end, and a lower price means that the drug will be bought with a higher probability. This more-than-outweighs the higher probability of a panic.<sup>2</sup>

## 4 The Hoarding Good as an Intermediate Product

In the crisis, masks became scarce, so carpenters and construction workers had to stop working. As a consequence, complete construction sites came to a halt. In hospitals, face masks can be interpreted as intermediate products in a production chain. To produce surgery, one needs one surgeon, one face mask, and much more. When face masks were

<sup>2</sup>It makes sense to choose a density function with support  $[0, \infty)$  for the distribution of  $u$ . Here, the exponential distribution is tractable, but the class of exponential distributions is embedded in the class of gamma distributions. In that class, distributions with a shape parameters above 1 have lower weight on low outcomes. Log-normal distributions would be another natural choice. For these classes of distribution functions, one can get the result that a higher price increases welfare because it reduces the probability of panics.

bought up by private households, hospitals ran into problems. In such a setting, we want to analyze the prevalence of panics, and how can the ensuing problems be fixed. For consistency, we still call the product a “drug”. One can think of a pain killer needed for a certain treatment. But bear in mind that the model applies to a large variety of (potentially hoardable) products. The critical assumptions are that the prices are sticky in the short run, quantities do not adjust immediately, there is some exogenous shock, and in addition now that the product is critical for another industry.

Assume that there is a continuum of hospitals, needing an aggregate volume  $h$  of the drug. Each unit yields a utility (or profit) of  $u_h$  for the hospital. We concentrate on the case  $u_h > p$ , such that hospitals want to buy the drug. There are a number of conceivable time and information structures. Let us assume for now that agents move first, decide whether to preempt the hospitals (and one-another) and buy the drug at the early stage. Then hospitals decide whether to buy their  $h$  drugs if possible. Agents then realize whether they become sick and, like above, get a second chance to buy the drug. For now, we also assume that firms cannot price differentiate between agents and hospitals.

**The Model.** Let us jump right into the analysis of the indifference point for agents with  $\varepsilon$ -noise in the information about the utility of the drug. First, consider the case that there are potentially enough drugs for hospitals and agents,  $q \geq m + h$ . Take an agent  $i$  at her indifference point, with information such that her expected utility is  $\bar{u}$ . Again, the number of agents that are more pessimistic is denoted with  $n$ , where  $n$  is uniformly distributed on  $[0, 1]$ . If  $n > q$ , there will be a shortage of drugs already at the early stage. If  $n \leq q$ , there are enough drugs at the early stage, and  $q - n$  remain for later. If  $h > q - n$ , these remaining drugs do not even suffice to satisfy hospitals’ demand. No drugs will be left for the later stage. If  $h \leq q - n$ , there are  $q - n - h$  drugs left for the second period. The demand in the second period is  $(1 - n)m$ . Drugs will suffice if  $q - n - h \geq (1 - n)m$ , thus if  $n \leq (q - m - h)/(1 - m)$ .

The expected utility with the hoarding strategy is always  $mu - p$ . Only if  $n > q$ , they get the drug only with probability  $q/n$ , hence the expected utility is  $q/n(mu - p)$ ,

$$\begin{aligned}
U^{\text{hoard}} = (mu - p) & \left( \underbrace{\int_0^{\frac{q-m-h}{1-m}} 1 \, dn}_{\text{no rationing at all}} + \underbrace{\int_{\frac{q-m-h}{1-m}}^{q-h} 1 \, dn}_{\text{rationing at late stage}} \right. \\
& + \underbrace{\int_{q-h}^q 1 \, dn}_{\text{rationing for hospitals}} + \underbrace{\int_q^1 \frac{q}{n} \, dn}_{\text{rationing at early stage}} \left. \right). \tag{17}
\end{aligned}$$

For the waiting strategy, the utility is  $m(u - p)$  if there are enough drugs even at the later stage, thus if  $n \in (0, \frac{q-m-h}{1-m}]$ . If there is rationing at the later stage, waiting agents want the drug with probability  $m$  but get it (and thus the utility  $u - p$ ) only with probability  $\frac{q-n-h}{(1-n)m}$ , this is the case if  $n \in (\frac{q-m-h}{1-m}, q-h]$ . If  $n \in (q-h, q]$ , drugs are rationed even for hospitals, hence waiting agents will not obtain it. If  $n \in (q, 1]$ , drugs are rationed even at the early stage, so waiting agents will not get it. Summing up, the expected utility is

$$U^{\text{wait}} = m(u - p) \left( \underbrace{\int_0^{\frac{q-m-h}{1-m}} 1 \, dn}_{\text{no rationing at all}} + \underbrace{\int_{\frac{q-m-h}{1-m}}^{q-h} \frac{q-n-h}{(1-n)m} \, dn}_{\text{rationing at late stage}} + \underbrace{\int_{q-h}^q 0 \, dn}_{\text{rationing for hospitals}} + \underbrace{\int_q^1 0 \, dn}_{\text{rationing at early stage}} \right). \quad (18)$$

If  $h \leq q < m + h$ , the first integral in (17) and (18) vanishes, and the second integral starts at zero. If even  $q < h$ , then both the first and second integral vanish, and the third integral starts at zero. However, in that case, there are never any drugs left for the late period, and the waiting strategy is dominated. Again, both expected utilities are linear in  $u$ , so it is messy but straightforward to solve for the indifference point  $\bar{u}$ .

Figure 8: Hoarding and Waiting Equilibria with Hospitals

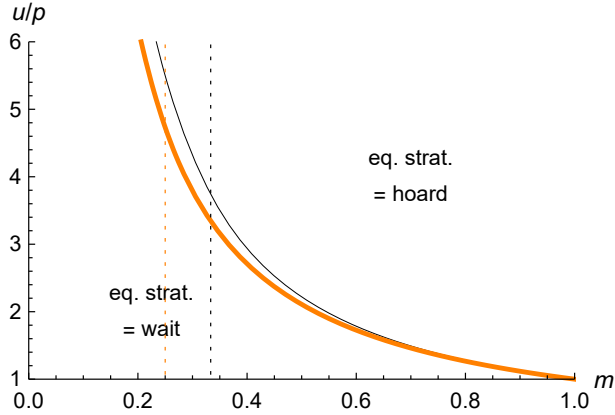


Figure 8 shows the critical utility  $\bar{u}$  as a multiple of  $p$ . As before,  $q = 1/3$ . The thin black curve has  $h = 0$ , it is identical to the colored curve in Figure 1. The thick orange curve has  $h = 1/12$ . The dotted lines show the place where  $q = m + h$ . We see that the additional demand from hospitals leads to a higher prevalence of the hoarding equilibrium, that is, panic purchases. This is not surprising, as the purchases from hospitals make the waiting strategy more risky, but leave the returns from the hoarding strategy unchanged.

There are new questions to be answered. First, if hospital demand enters the stage, how will the revenue-maximizing strategy change? Second, what are the effects if hospitals already order some drugs in advance and stockpile them?

If  $q \leq h + m$ , drugs are *scarce*. If they are cheap enough ( $u > p$ ), agents want to buy, demand is  $h + m$ , and firms can sell the complete stock  $q$ . If drugs are too expensive ( $p > u$ ), only hospitals will buy, and demand is only  $h$ . We do not discuss the limiting cases like  $p = u$ , because they are irrelevant in the integral.

$$\begin{aligned} ER_{\text{scarce}} &= \underbrace{\int_0^p p h \cdot f(u) du}_{\text{only hospitals buy}} + \underbrace{\int_p^\infty p q \cdot f(u) du}_{\text{hospitals and agents buy}} \\ &= p h F(p) + p q (1 - F(p)). \end{aligned} \quad (19)$$

If  $q > h + m$ , there are potentially *enough* drugs for hospitals and agents. Firms can sell  $h$  drugs if  $u < p$ , they sell  $h + m$  drugs if  $u \in (p, \bar{u})$ , and the complete stock  $q$  if  $u > \bar{u}$ . The expected revenue is

$$\begin{aligned} ER_{\text{enough}} &= \underbrace{\int_0^p p h \cdot f(u) du}_{\text{only hospitals buy}} + \underbrace{\int_p^{\bar{u}_{\text{enough}}} p (h + m) \cdot f(u) du}_{\text{waiting eq.}} + \underbrace{\int_{\bar{u}_{\text{enough}}}^\infty p q \cdot f(u) du}_{\text{hoarding eq.}} \\ &= p h F(p) + p (h + m) (F(\bar{u}_{\text{enough}}) - F(p)) + p q (1 - F(\bar{u}_{\text{enough}})). \end{aligned} \quad (20)$$

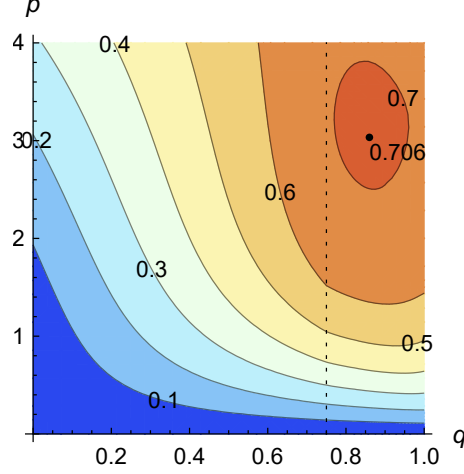
With the same parameter choices as before,  $m = 2/3$ ,  $h = 1/12$  and  $u$  being exponentially distributed with mean  $\mu = 2$ , we arrive at Figure 9.

We can now discuss the effect of an increase in  $h$ . There is an immediate effect on the agents' propensity to panic and hoard. *Ceteris paribus*, the hospitals' demand is irrelevant for hoarding agents, but reduces the utility of waiting agents. As Figure 8 has already shown, the critical  $\bar{u}$  falls; agents panic already for lower utility.

In equilibrium, firms will adapt prices and quantities to a increase in hospital demand  $h$ . Figure 9 shows the expected revenue. The revenue-maximizing quantity is at  $q^* = 0.8599$  (up from  $q^* = 0.8023$  for  $h = 0$ ). The welfare-optimum, however, is now at  $q = 1 + h = 1.0833$ . Here, an increase in  $h$  lets the welfare optimum move away from the revenue-maximizing quantity.

The revenue-maximizing price is now at  $p^* = 3.0335$  (up from  $p^* = 1.8697$ ), with the following rationale. We have assumed that the hospitals' demand is price-inelastic. They demand  $h = 1/12$  units as long as  $p < u_h$ . In the figure, we have  $u_h \geq 4$ , that is, somewhere above the upper plotting range. (For  $u_h < 4$ , there would be a structural break at  $p = u_h$

Figure 9: Expected Revenues for Firms



visible in the figure.) As a direct consequence, the monopolist has a strong incentive to demand higher prices. As an indirect consequence, because low prices fuel panics, there will now be fewer panics. This effect is rather mechanical. The monopolist now supplies to a second market, hospitals, which is relatively price-inelastic. He will therefore raise prices. But because higher prices inhibit panics, there will be fewer panics. The aggregate effect of hospitals' demand on the probability of panics is ambiguous. The direct effect leads to more panics, the indirect effect through the reaction of prices leads to fewer panics. In our numerical example, we have  $F(\bar{u}) = 97.30\%$ , hence a panic probability of  $1 - F(\bar{u}) = 2.70\%$  (down from  $8.75\%$  for  $h = 0$ ). Apparently, the indirect effect through the price reaction dominates here. Proposition 4 states that also in the presence of hospitals, the revenue-maximizing quantity is below the welfare-optimum, and the revenue-maximizing price is still low in order to induce panics.

**Competition between Producers.** We do not want to overemphasize the above numerical results, as it depends heavily on specific assumptions. For example, in a situation with competition between producing firms, the equilibrium price would be lower right away. Then, when hospitals enter the scene, the price would not react as much. The indirect effect on panics would thus be smaller, the direct effect would prevail. A thorough discussion would require a more structured, and therefore specific, model. We choose to only conjecture that the effect of additional demand from hospitals on the probability of panics is ambiguous, and it tends to be positive (more panics) when there is competition between producers.

**Inventory Regulation for Hospitals.** Assume that hospitals can be required to stockpile drugs, in order to withstand the potential shortage in a panic, thus to increase their resilience. To be consistent, we must also assume that stockpiling the drug comes at a cost for hospitals. Otherwise, it would be optimal for hospitals to maximize their inventory, to stock exactly the quantity that they know they will need. We still assume that the price is sticky, that hospitals and agents pay the same price at all dates. This implies that an increase in the aggregate stock implies a reduction in the demand between the agents's early and late buying dates. The propensity to the hoarding equilibrium is reduced. Hence if one hospital hoards, there is a positive externality on all other hospitals. Due to this externality, taking the hospitals' perspective, leaving the stockpiling decision to the hospitals would lead to a sub-optimal level of stocks.

A welfare-maximizing regulator would also take the agents' and the firms' utility into consideration. If hospitals stock the drug, agents profit from a lower probability of panics, and thus from a better allocation of the drug to the sick. The firms suffer from the regulation. Firms profit from panics, because they can then sell more than  $m + h$ . The welfare improvement from a regulation for the level of inventories is not a Pareto improvement.

Note that a regulator can use multiple instruments to increase resilience, and thus welfare. One option is the introduction of minimum inventories for hospitals. Alternatively, the government could itself build up an inventory as a buffer for shortages. Or, it could just subsidize the creation of inventories, for example through their tax treatment. The optimal mix of regulation depends on the information that the regulator can access, and on cost differences between firms and the public to build up stocks.

## 5 Conclusion

We have constructed a simple model to analyze the propensity of hoarding panics and shortages. One possible application are shortages as in the initial phase the COVID-19 pandemic. The model applies whenever prices are rigid, at least to some degree, and there are sudden shocks to supply or demand, or the parameters that influence supply or demand.

During the shortages during the pandemic, disruptions were especially large when scarce products were needed in the industry, for example as intermediate products. The necessity of regulation, however, is not obvious. A regulator might force the industry to stockpile more of the (potentially scarce) product in advance. But firms in the industry themselves suffered from the shortages, so they would decide to stock more of that product in the

future, also absent regulation. We have extended the basic model by additional firms (in our wording, hospitals) that need the product in their production chain. We find that there is a positive externality if firms decide to procure some of the product early and stockpile it. As a consequence, the equilibrium size of the inventory is too low. A prudential regulation on inventory for is thus rationalized.

Due to the simplicity of the basic model, one can think of more extensions. Especially in times of scarcity, secondary markets might emerge, potentially black markets. In our model, we have ruled out such secondary markets and black markets. It would be a straightforward exercise to implement them into the model. One could interpret the agents of our model as different economies that are linked by trade, or as agents in different economies linked by trade. In such a setting, one can analyze the consequence of trade barriers to the occurrence of panics.

Another simplifying assumption in the model is the time structure. Agents need the product only at one date, and can decide to hoard at one earlier date. They do not see shelves emptier than normal, and then decide to hoard products. This could be changed, e.g., in a continuous time model in the spirit of He and Xiong (2012), in which agents could observe the shelves and make inferences when shelves become emptier. A third assumption is that the propagation of the pandemic is not explicitly modelled. Many models on the COVID-19 pandemic use ordinary differential equations, for example SIR models, to describe the propagation of the infection (Acemoglu, Chernozhukov, Werning, and Whinston, 2020; Acharya, Jiang, Richmond, and von Thadden, 2020). On this basis, one could also develop prediction that could be tested with recent data.

## A Appendix

**Proposition 1** *Figures 1, 2 and 3 are general in the sense that*

- $\bar{u}$  as a function of  $m$  is differentiable and decreasing, and has  $\lim_{m \rightarrow 0} \bar{u}(m) = \infty$  and  $\bar{u}(1) = p$  (Figure 1),
- $\bar{u}$  as a function of  $q$  is differentiable and increasing, and has  $\lim_{q \rightarrow 0} \bar{u}(q) = p/m$  and  $\lim_{q \rightarrow 1} \bar{u}(q) = \infty$ , (Figure 2),
- $\bar{m}$  as a function of  $q$  is differentiable and increasing, and has  $\lim_{q \rightarrow 0} \bar{m}(q) = p/u$  and  $\lim_{q \rightarrow 1} \bar{m}(q) = 1$ , (Figure 3).

**Proof of Proposition 1.** For  $m \leq q$ ,  $\bar{u}$  is given by  $\bar{u}_{\text{enough}}$  as in (11). For  $m \rightarrow 0$ , the numerator converges to  $p q (1 - \ln q)$ , and the denominator goes to zero. Hence,  $\lim_{m \rightarrow 0} \bar{u} = \infty$ . Along the same line, for  $q \rightarrow 1$ , the numerator converges to  $p(1 - m)$ , and the denominator goes to zero. Hence,  $\lim_{q \rightarrow 1} \bar{u} = \infty$ .

For  $m \leq q$ ,  $\bar{u}$  is given by  $\bar{u}_{\text{scarce}}$  as in (14). Entering  $m = 1$  yields  $\bar{u} = p$ . For the limit  $q \rightarrow 0$ , both the numerator and denominator in (14) go to infinity. Using de L'Hospital yields  $\lim_{q \rightarrow 0} \bar{u} = p/m$ .

For  $m = q$ ,  $\bar{u}_{\text{enough}}$  and  $\bar{u}_{\text{scarce}}$  coincide. To show that the function  $\bar{u}$  is differentiable at the point  $m = q$ , look at the implicit definitions in order to use the implicit function theorem,

$$\begin{aligned} G_{\text{enough}} &:= \bar{u}_{\text{enough}} ((1 - q)(m + \ln(1 - m)) + m q \ln q) \\ &\quad - p (m - q + (1 - q) \ln(1 - m) + q \ln q) = 0 \quad \text{and} \end{aligned} \quad (21)$$

$$\begin{aligned} G_{\text{scarce}} &:= \bar{u}_{\text{scarce}} ((1 - q) \ln(1 - q) + q(1 - m + m \ln q)) \\ &\quad - p ((1 - q) \ln(1 - q) + q \ln q) = 0. \end{aligned} \quad (22)$$

For  $q = m$ , all possible partial derivatives coincide,  $\partial G_{\text{enough}} / \partial \bar{u} = \partial G_{\text{scarce}} / \partial \bar{u}$ ,  $\partial G_{\text{enough}} / \partial \bar{u} = \partial G_{\text{scarce}} / \partial \bar{p}$ ,  $\partial G_{\text{enough}} / \partial \bar{u} = \partial G_{\text{scarce}} / \partial \bar{m}$  and  $\partial G_{\text{enough}} / \partial \bar{u} = \partial G_{\text{scarce}} / \partial \bar{q}$ . Hence  $\bar{u}$  as a function of  $m$  (Figure 1),  $\bar{u}$  as a function of  $q$  (Figure 2), and  $\bar{m}$  as a function of  $q$  (Figure 3) are differentiable. Remainder of the proof: see online appendix. ■

**Proposition 2** *The revenue-maximizing quantity  $q^*$  fulfils  $q^* > m$  but  $q^* < 1$ . The revenue-maximizing price  $p^*$  fulfils  $p^* < \bar{p}$ , with  $\bar{p} = \arg\max_{\{p\}} p F(p)$ .*

**Proof of Proposition 2.** See online appendix. ■

**Proposition 3**  $\partial \bar{u} / \partial h < 0$ ,  $\partial \bar{m} / \partial h < 0$ , and  $\partial \bar{q} / \partial h > 0$ .

**Proof of Proposition 3.** For  $q \geq m + h$ , the critical  $\bar{u}$  is given by  $U^{\text{hoard}} = U^{\text{wait}}$ , as defined in (17) and (18). We can rewrite as

$$U^{\text{hoard}} = (m \bar{u} - p) \text{Int}^{\text{hoard}} = U^{\text{wait}} = m (\bar{u} - p) \text{Int}^{\text{wait}},$$

where  $\text{Int}^{\text{hoard}}$  and  $\text{Int}^{\text{wait}}$  comprise the four integrals in (17) and (18). Rearranging yields

$$\bar{u} = p \frac{1/m - \text{Int}^{\text{wait}} / \text{Int}^{\text{hoard}}}{1 - \text{Int}^{\text{wait}} / \text{Int}^{\text{hoard}}}.$$

We have  $\partial \text{Int}^{\text{hoard}} / \partial h = 0$  but  $\partial \text{Int}^{\text{wait}} / \partial h < 0$ . Together with the above equation for  $\bar{u}$ , this yields  $\partial \bar{u} / \partial h < 0$ . The proof for  $q < m + h$  is exactly the same. The proofs for the derivatives of the critical  $\bar{m}$  and  $\bar{q}$  are similar. ■



**Proposition 4** *In the economy with hospitals, the revenue-maximizing quantity  $q^*$  fulfils  $q^* > m + h$  but  $q^* < 1 + h$ . The revenue-maximizing price  $p^*$  fulfils  $p^* < \bar{p}$ , with  $\bar{p} = \operatorname{argmax}_{\{p \leq u_n\}} p(h + m(1 - F(p)))$ .*

**Proof of Proposition 4.** See online appendix. ■

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