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Abstract

In the past sixty years, transistor sizes and weights have decreased by 50 percent every eighteen months, following Moore's Law. Smaller and lighter electronics have increased productivity in virtually every industry and spurred the creation of entirely new sectors of the economy. However, while the effect of the increasing quality of computers and electronics on GDP has been widely studied, the question of how electronic miniaturization affects economic growth has been unexplored. To quantify the effect of electronic miniaturization on GDP, this paper builds an economic growth model that incorporates physical constraints on firms' production sets. This model allows for new types of productivity spillovers that are driven by products' physical characteristics. Not only are there spillovers from changes in industry productivity, but also, there can be "size spillovers," where the miniaturization of one industry's product leads to miniaturization of products that are downstream in the supply chain, reflecting how transistor miniaturization has led to the decrease in size of a large variety of electronic products. Using a new data set of product weights and sizes, we test the predictions of the model and show that Moore's Law accounts for approximately 3.5 percent of all productivity growth in the 1982-2007 period, and for 37.5 percent of the productivity growth in heavy manufacturing industries. The results are robust under multiple specifications, and increase in strength during the 1997-2007 subperiod.

Key words: economic growth, productivity, electronic miniaturization

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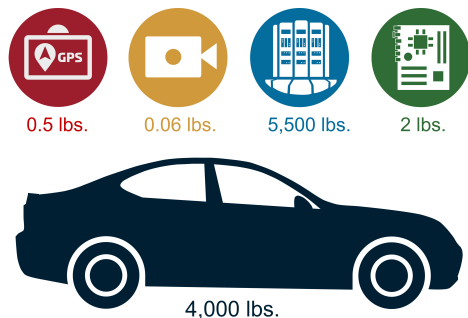
1 Introduction

Developing new combinations of existing materials is a key driver of economic growth. The earliest results on endogenous technical change had an implicit assumption that, as ideas arrived to the market, they could be combined with each other to yield exponential growth in the number of existing products (Romer 1993). Weitzman (1998) made this intuition formal by proposing a model of recombinant growth, where new ideas are generated by researchers who combine existing ideas. Acemoglu and Azar (2020) and Jones (2021) built on Weitzman’s work by developing models where growth is driven by the ability of firms to choose from an exponentially increasing number of sets of input suppliers. Each combination is a “recipe” with a different level of productivity, and firms choose the cost-minimizing combination. As new inputs arrive, the number of recipes increases exponentially, and economic output grows.

The discovery of these new combinations is often driven by improvements in the physical properties of the materials being combined. However, most papers in the combinatorial growth literature and in the economic growth literature more generally, do not take into account improvements in materials’ physical properties when modeling technological progress. For example, James Watt worked on steam engines for 15 years before the invention of high-precision machining tools made steam engines energy-efficient and commercially feasible. Aviation would not be possible without light-weight and strong aluminum, and aluminum could be mass produced only after improvements in electrical engineering. And modern computers and electronics, which have permeated every aspect of contemporary firms and households, would not have been possible without the development of ever-shrinking semi-conductors.

To capture how improvements in physical properties lead to new technological innovations, this paper builds a model where firms have physical constraints on their production sets, and where these constraints dictate which inputs can and cannot be combined. This feature captures real-world examples of physical limitations in production, such as engines needing to be above a certain efficiency threshold, airplanes having to satisfy strict weight limits, and electronics needing to be small enough to fit into office equipment, cars and industrial machines.

My model builds upon Acemoglu and Azar (2020), while adding feasibility constraints limiting which input combinations can be attempted by firms. Each input material has a vector of physical properties, such as its size, electrical conductivity, or melting point. The set of materials used has to satisfy a combinatorial constraint, such as a Knapsack constraint that limits the sum of sizes of input materials, or a graph matching constraint determining which materials can be safely used together. Both of these types of constraints are illustrated in Figure 1. In a broader context, we can think of other examples of combinatorial constraints,



(a) Knapsack Constraints in Cars

	Base Resin	Weak Acids	Strong Acids	Weak Alkalis	Strong Alkalis	Organic Solvents	Alcohols	Hydro Carbons	Fuels	Gamma Radiation
Acrylic	(PMMA)	P	G	F	P	P	P	F	G	G
Polycarbonate	(PC)	F1	F	P	P3	G	P	P	G	F
Polycarbonate/Acrylic Alloy	(PC/PMMA)	G	G	G	P	F	P	F	F	F

(b) Matching Constraints in Materials

Figure 1: This figure shows two examples of combinatorial constraints on which input materials can be combined. Subfigure (a) illustrates a Knapsack constraint in car manufacturing: the total sum of weights of car components cannot exceed the weight of the car, which in this figure is 4000 lbs. Small electronic components, such as modern GPS devices (0.5 lbs), cameras (0.06 lbs) and Engine Control Units (2 lbs) all fit comfortably inside the car. However, state-of-the-art supercomputers (5500 lbs) cannot be incorporated into the car without violating the Knapsack constraint. Subfigure (b) illustrates a Matching constraints in materials science, where different types of plastic resins cannot be combined with certain types of materials. The table’s rows correspond to acrylic, polycarbonate and a polycarbonate/acrylic alloy. The table’s columns correspond to different types of chemicals and radiation sources, including acids, alkalis, solvents, fuels and gamma radiation. Each entry in the table indicates whether a given plastic resin can be combined with a given chemical or radiation source. Entries in blue represent good performance, in red represent poor performance, and in grey represent fair performance.

such as logistics firms matching items to warehouses, ride-sharing companies matching drivers to passenger requests, and airlines buying the rights to use runways in different airports.

As the physical properties of materials improve, industries’ constraints are relaxed and new productivity-enhancing combinations are developed. I use this observation to develop a dynamic version of the model in which the joint arrival of new materials and new applications drives growth. Without new materials, no new combinations are possible, and there is no growth. Without applications, the new materials are not useful, and again there is no growth. It is the combined development of materials and applications at the same time that leads to sustained growth. This is similar to models of Schumpeterian Waves (Helpman and Trajtenberg, 1998, Violante, 2002, Aghion, Howitt and Violante, 2002 and Hornstein, Krusell and Violante, 2005), where the arrival of a new technology—such as a material with improved physical properties—does not directly lead to productivity growth. Instead, productivity growth occurs only after the new technology has been sufficiently adapted to be commercialized. These models can explain, for example, how the arrival of General Purpose Technologies (GPTs) can lead to productivity slowdowns and increased wage inequality, thus matching empirical facts documented by Jovanovic and Rousseau (2005).

As in Acemoglu and Azar (2020) and Jones (2021), growth in the model is driven by the combination of a small number of materials into exponentially many different “recipes,” a force absent from workhorse models of economic growth. Growth is not driven by the exponential arrival of new products (as it is in Romer 1990; Grossman and Helpman 1992; Jones 1995; Eaton and Kortum 2001; or Klette and Kortum 2004), nor by proportional improvements in quality (as in the quality-ladder models of Aghion and Howitt 1992; and Grossman and Helpman 1991), nor by thick-tailed productivity draws continuously improving technology and spreading in the economy via a diffusion process (as in Akcigit, Celik and Greenwood 2016; Lucas 2009; Lucas and Moll 2014; and Perla and Tonetti 2014).

The closest point of comparison for this paper is Weitzman (1998) together with the follow-up work of Auerswald, Kauffman, Lobo, and Shell (2000) and Ghiglino (2012). The main difference with these models of recombinant growth is that they assume that there is an idea-generating function that grows exponentially when new ideas are obtained by combining existing ideas. In contrast, I assume that materials arrive more slowly, linearly with time instead of exponentially. Firms can use combinations of products in production, with each combination yielding a different productivity draw. As there is an exponentially growing number of combinations—under the right distributional assumptions—productivity grows exponentially as new materials and new combinations arrive.

In contrast with Acemoglu and Azar (2020) and other models that study firm productivity in networks, this model allows for new types of productivity spillovers that are driven by products’ physical characteristics. Not only are there productivity spillovers from changes in industry TFP, but there can also be “physical spillovers” from changes in the physical properties of one material that lead to changes in materials that are downstream in the supply chain. These physical spillovers reflect, for example, how transistor miniaturization has led to the decrease in size of a large variety of electronic products.

Applying the predictions of this model to the data, I estimate the effect of electronic miniaturization on economic growth using a reduced form approach. I focus on the special case where the combinatorial constraints are Knapsack constraints limiting the size of materials that can be used in any given product such as a car or an industrial machine. The model predicts that, when new, more powerful generations of large computers arrive in the market, industries with larger products, such as the aerospace industry, can use them as inputs in a way that cars and medical equipment cannot.

I test this prediction using a two-stage least-squares approach and a new dataset on product weights and sizes. The data shows that, after controlling for industry characteristics and time, heavy-manufacturing industries with larger products are more likely to purchase

computers and electronic materials.¹ The regression results shows that these purchases have a significant effect on productivity. I use this two-stage least-square regression to show that electronic miniaturization accounts for 37.5% of all TFP growth in heavy manufacturing and 3.5% of all TFP growth during the 1982-2007 period. These results are robust to multiple specifications, and grow stronger over time.

1.1 Related Work

Production Networks and Distortions This paper belongs within the input-output network literature started by Leontief. In particular, it relies on the models of input-output linkages proposed by Long and Plosser (1983) and analyzed further by Ciccone(2002), Gabaix (2011), Jones (2011), Acemoglu et. al. (2012), Acemoglu et. al (2017), Bartleme and Gorodnichenko (2015), Biglio and La’o (2017), Baqaee (2018), Fadinger, Ghigolino and Teteryatnikova (2018), Liu (2017), Baqaee and Farhi (2019a, 2019b), Caliendo et. al (2018) and Liu and Tsyvinski (2021).

The main modeling contribution of this paper is to augment the Long and Plosser model by having physical constraints which distort firms’ production sets, leading to endogenous adoption of suppliers. The main empirical contribution is to use this augmented model to estimate how changes in the physical characteristics of computers and electronic devices increased manufacturing productivity in the United States.

Because the input-output weights in the model are endogenous, this paper is closely related to work by Jones (2013), Biglio and La’O (2017), Liu (2018), Fadinger et. al. (2018), and Caliendo et. al. (2018) who analyze models of production networks with distortions. This paper is also related to work by Carvalho et. al. (2016) and Baqaee and Farhi (2019a), who study models of endogenous input-output networks where firms’ production functions are not Cobb-Douglas. In all of these models, the input-output structure of the economy is endogenous. The economic intuition is that—in the presence of distortions such as markups, taxes and subsidies—the allocation of goods in the economy may be inefficient. Furthermore, when the production function is not Cobb-Douglas, increases in productivity in one sector may not increase aggregate TFP as much as they would in a non-distorted economy.

In my model, distortions arise from the combinatorial constraints on production, and affect allocative efficiency and the propagation of shocks. For example, if industry j has a weight constraint that binds from above, it will substitute away from heavy inputs into lighter inputs. If a heavy machine in industry i becomes more productive, this increase in

¹Heavy-manufacturing industries are those with 2-digit Standard Industry Classification (SIC) codes ranging from 34 to 39 and include industrial machinery, appliances and precision instruments, but exclude industries in which electronics cannot be embedded in the final product, such as food, apparel, and glass manufacturing.

productivity may not propagate to industry j because industry j 's weight constraint will prevent it from demanding more units of that machine.

Endogenous Production Networks The literature on production networks and distortions considers changes in the input-output network at the intensive margin. However, there is a recent and growing literature that analyzes changes in the production network at the extensive margin. Atalay et. al. (2011) and Carvalho and Voigtlander (2015) give rule-based² models of network formation. More recently, Oberfield (2018), Acemoglu and Azar (2020), Acemoglu and Tahbaz-Salehi (2020) and Taschereau-Dumouchel (2020) consider models where the structure of the network is determined by individual firms maximizing profit.

In Oberfield's (2018) model, each firm bargains with a set of suppliers and chooses one input, leading suppliers with very high productivity to become "superstars", supplying many firms at once. Acemoglu and Azar (2020) present a model where firms choose arbitrary sets of suppliers, where each possible set of suppliers induces a different production technology. They use this framework to construct an endogenous growth model, where the source of growth is the exponentially increasing number of combinations of suppliers that a firm can use in its production function. Acemoglu and Tahbaz-Salehi (2020) develop a framework where firms purchase inputs from "customized" suppliers and use bargaining to reach agreement on how to split the surplus generated by their relationship. They apply this model to understand how firm failures can cascade throughout the supply chain and amplify recessions. Taschereau-Dumouchel's (2020) model studies the cascading effect of firms deciding to enter or exit the market. An exiting firm will have negative spillovers on its clients and suppliers, leading multiple firms to exit the network simultaneously.

In my model, changes in the parameters of the model can lead to a change in the input-output network at the extensive margin. A sudden change in the size of a product (say, because of technological advancement) can lead to a change in the structure of the input-output network. For example, desktop computers replaced mainframes in the 1980s, laptops replaced desktops in the 1990s and 2000s, and tablets replaced laptops in the 2010s. In all these changes, the shift in demand toward lower-weight products was made possible by technological improvements.

Effect of Information Technology on GDP Jorgenson and Stiroh (1999,2000) and Jorgenson (2005) show that information technology (including computers, software and communications equipment) accounted for a majority of TFP growth in the period 1973-2002.³

²For example, Atalay et. al. (2011) use a preferential attachment model.

³Table 8 in Jorgenson (2005) shows that information technology contributed 68% of all TFP growth in the 1973-1989 period, 88% of all TFP growth in the 1989-1995 period, and around 66% of all TFP growth

More recent work by the Bureau of Economic Analysis (Barefoot et al. 2018) showed that, for the 2006-2016 period, the digital economy accounted for 28% of real GDP Growth.

In contrast with previous work—which focuses on the price and quality of computers—this paper focuses on the effect of electronic miniaturization on the productivity of heavy manufacturing industries. The results in Section 6 show that expenditures in computers and electronics account for almost all of TFP growth in heavy manufacturing and that electronic miniaturization accounts for 37.5% of this growth. These results are coherent with previous results in the literature that show how information technology accounts for the vast majority of TFP growth in the economy. However, I highlight that the identification strategy applies only to industries whose products can have computers and electronics embedded in them and does not generalize to other sectors such as food or glass manufacturing. Thus, I do not account for any effect that computers and electronics may have on TFP through their direct application in non-heavy manufacturing sectors such as the service industry.⁴

Physical Properties and International Trade In the international trade literature, Hummels and Skiba (2004) show that products’ physical features, such as their weight, affect trading costs. They use this observation to argue for the existence of the Alchian-Allen effect, where high quality varieties within an industry are more likely to be exported than low quality varieties. Evans and Harrigan (2003), Harrigan (2005), Baldwin and Harrigan (2007) and Harrigan and Deng (2010) expand on this observation and show that heavier goods are traded between physically close countries, while lighter goods are traded by physically distant countries. Similarly, lighter goods are much more likely to be traded by air than by sea.

Roadmap The rest of the paper is organized as follows. Section 2 shows the baseline static model of production with combinatorial constraints. Section 3 shows that a generically unique equilibrium exists, and characterizes the equilibrium under standard assumptions on productivity distributions. Section 4 derives comparative statics on the effect that changes in product’s physical characteristics have on aggregate GDP. Section 5 develops a dynamic model where improvements in products’ physical characteristics drives GDP growth. Section 6 contains empirical evidence on the effect of electronic miniaturization on productivity. Section 7, discusses potential future work, and concludes. The Appendices contain proofs for all Propositions and Theorems along with additional robustness results.

in the 1995-2002 period.

⁴I account for indirect effects arising from the use of heavy manufacturing products in other industries, through the use of Domar weights when computing the effect of changes in heavy manufacturing productivity on aggregate TFP.

2 A Static Model

This section presents a tractable general equilibrium model that captures how changes in the physical properties of primary goods allows them to be combined in new ways, expanding the production possibilities frontier.

In the model, there is a finite number of primary industries, which one can think of as producing materials or components that have physical properties. Changes in the physical properties of one primary good have effects on the physical properties of other primary goods as well. In particular, this type of spillover effect captures the notion that smaller semiconductors have led to smaller electronic components and smaller electronic machines in general.

The second key feature of the model is that a large number of intermediate industries—which one can think of as producing fixed machines, vehicles, or appliances—can choose the subset of primary industries’ goods that they use as inputs. Different subsets S of primary inputs represent the many different ways that an intermediate good can be produced. Each subset S further corresponds to an independent random productivity draw $A(S)$, so that new combinations of inputs—such as replacing mechanical buttons with touchscreens—may lead to potentially more productive machines.

A novel aspect of the model is that intermediate industries cannot choose arbitrary sets S of inputs. Rather, S is restricted by constraints that may depend on the physical properties of the primary goods contained in S . I show in Section 5, that as the physical properties of primary goods change, new input sets S are unlocked, and productivity increases.

The model is closed by adding a final industry, which aggregates the output of all the intermediate industries into a final good. This allows us to use the law of large numbers to treat aggregate productivity as a deterministic function of the number of feasible combinations of primary inputs available to intermediate industries.

2.1 Market Structure

There are three types of industries:

1. Primary industries, which are indexed by $j \in \{1, \dots, J\}$. Each primary industry j produces one good using labor and other primary industries’ goods as inputs. Good j has a vector of properties $\theta_j \in \mathbb{R}^p$. I denote the set of all primary industries by $\mathcal{J} = \{1, \dots, J\}$.
2. Intermediate industries, which are indexed by $i = (\tau, \iota)$. Here, $\tau \in (0, +\infty)$ is a threshold which specifies which sets S of primary goods can be combined as inputs to the

intermediate industry's production function, and $\iota \in [0, 1]$ is an index that distinguishes different industries with the same threshold τ .⁵ Intermediate industries are distributed with a density $\nu(i)di$. Throughout the paper, I assume that ν depends only on τ , so that $\nu(i)di = \nu(\tau)d\iota d\tau$. Without loss of generality, I assume that $\int \nu(i)di = 1$. Finally, I assume that each intermediate industry i produces one good using primary goods as inputs. For convenience of notation, I write τ_i when I refer to the threshold of industry i , and denote the set of all intermediate industries by $\mathcal{I} = (0, +\infty) \times [0, 1]$.

3. A final industry, denoted by f . The final industry produces one final good, using an aggregate of the intermediate industries' goods.

Households There is a representative household which consumes C_f units of the final good, and has utility $U(C_f)$. The household supplies one unit of labor inelastically, and has a budget constraint $P_f C_f \leq W$, where P_f is the price of the final good, and W is the wage paid to labor. Throughout, I assume that the wage is the numeraire, so that $W = 1$.

Primary Industries Each primary industry j has a strictly quasi-concave, continuous and increasing production function with constant returns to scale

$$Y_j = F_j((X_{jk})_{k=1}^J, L_j)$$

where X_{jk} is industry j 's demand for good k and L_j is industry j 's demand for labor. Good j has a vector of physical characteristics $\theta_j \in \mathbb{R}^p$, which is determined by a continuously differentiable function $\theta_j = \theta_j(\zeta)$. The input ζ is a vector of fundamental properties which can affect any primary industry.

In addition to the standard assumptions of quasi-concavity, continuity, and monotonicity, I make the assumption that labor is essential in the production of every primary good, so that $F_j(X, 0) = 0$ for all X . This assumption prevents equilibria where production is fully automated.

Example 1 (Leontief Production Functions) *Consider an economy where primary in-*

⁵While this may seem at first an excess of notation, the 2-dimensional indexing of intermediate industries will be helpful to make the results more tractable. In particular, having a continuum of thresholds τ will allow us to define and compute $\frac{\partial \log Y}{\partial \theta_j}$, the effect of a change in a primary good's physical properties on aggregate log-output. Having a unit mass of industries $\iota \in [0, 1]$ for every threshold τ will allow us to use the law of large numbers and have deterministic aggregate output and productivity, even when individual firm productivities are drawn from random distributions.

dustry j has a Leontief production function

$$Y_j = \min\{A_j L_j, \min\{\frac{X_{jk}}{\alpha_{jk}}\}\}.$$

With some abuse of notation, we can write $\alpha_{jk} = 0$ when industry j does not use industry k 's good as an input.

Each primary good has only one physical property, its size, denoted by θ_j . Since producing one unit of good j requires α_{jk} units of good k , the size of one unit of good j will be given by $\theta_j = \sum_{k=0}^J \alpha_{jk} \theta_k + \zeta_j$, where $\zeta_j \geq 0$ is an idiosyncratic factor affecting the size of good j . In matrix form, this equation is given by

$$\theta = \alpha \theta + \zeta \tag{1}$$

As long as $\mathcal{L} = (I - \alpha)^{-1}$ exists, we can write $\theta = \theta(\zeta) =_{\text{def}} (I - \alpha)^{-1} \zeta$, which is a continuously differentiable function. The intuition behind this specification is that if the idiosyncratic size ζ_k of good k decreases by $\Delta \zeta_k$, then the size θ_j of industry j 's good will decrease, through network spillover effects, by $\mathcal{L}_{jk} \Delta \zeta_k$.

Intermediate Industries Each intermediate industry i produces a single good using labor and primary goods as inputs. The production technology is given as a menu of production functions, indexed by the subset $S_i \subset \mathcal{J}$ of primary inputs used in production. More concretely, given a set of inputs S_i , industry (τ, i) has a strictly quasi-concave, continuous and increasing production function with constant returns to scale

$$Y_i(S_i) = A_i(S_i) F(S_i, (X_{ij})_{j \in S_i}, L_i) \tag{2}$$

where $A_i(S_i) \in \mathbb{R}_{>0}$ is a Hicks-Neutral productivity shifter that depends on S_i , $(X_{ij})_{j \in S_i}$ is a vector of demands for primary goods and L_i is industry i 's labor demand.

Each intermediate industry i cannot use arbitrary combinations of primary inputs. Instead, firms in industry i are bound by a constraint

$$G(Y_i, (X_{ij})_{j \in S_i}, (\theta_j)_{j \in S_i}, S_i) \leq \tau_i Y_i \tag{3}$$

where G is a vector-valued function that has constant returns to scale in $(Y_i, (X_{ij})_{j \in S_i})$ and is quasi-convex as a function of $(Y_i, (X_{ij})_{j \in S_i})$. For some sets S_i , the set $\{(Y_i, (X_{ij})_{j \in S_i}) : G(Y_i, (X_{ij})_{j \in S_i}, (\theta_j)_{j \in S_i}, S_i) \leq \tau_i Y_i\}$ may be empty, so that production is not feasible using the set of inputs S_i . I denote by $\mathcal{F}(\theta, \tau_i)$ the collection of all feasible input subsets $S \subset \mathcal{J}$ for which there exists at least one vector $(Y_i, (X_{ij})_{j \in S_i})$ such that constraint (3) holds.

Example 2 *Constraint (3) can encode both continuous and discrete constraints. For example, it may encode a size constraint of the form*

$$\sum_{j \in S_i} \frac{X_{ij}}{Y_i} \theta_j \leq \tau_i. \quad (4)$$

It may also encode a constraint that does not depend on the quantities X_{ij} demanded by industry i , but only on the chosen input set, such as a Knapsack constraint

$$\sum_{j \in S_i} \gamma_j \theta_j \leq \tau_i. \quad (5)$$

Because the function G is vector-valued, both constraints may hold simultaneously. I emphasize that the discrete Knapsack constraint is not that different from the continuous size constraint. For example, the production of a car requires fuel, exhaust, cooling, lubrication and electrical systems, as well as an engine, wheels, tires, and a chassis. These components of a car have been increasing in quality, but have remained relatively constant in size. In this very stylized example, one can think of the production function for a car as taking the form $Y_i = A_i(S_i)F_i(S_i, \{X_{ij}\}_{j \in S_i}, L_i)$ subject to a combinatorial constraint $\sum_{j \in S_i} \theta_j \leq \theta_i$, where Y_i represents the quality-adjusted output of the car (e.g. adjusted for durability, mileage, safety, speed and comfort), X_{ij} represent the relative quality of the car's individual systems, θ_j represent the respective sizes of these systems, and τ_i represents the overall size of the car.

For the special case where F_i is Leontief, the discrete and continuous size constraints coincide. Throughout the paper, I do not impose any assumptions on the functional form of F_i besides continuity, monotonicity and quasi-concavity, and we may take F_i as Leontief whenever a discrete constraint is used.

The Final Industry The final industry's production function is given by

$$\log Y_f = \frac{\gamma}{\gamma - 1} \left(\int_0^\infty \log \left(\int_0^1 X_{f\tau\iota}^{\frac{\gamma-1}{\gamma}} d\iota \right) \nu(\tau) d\tau \right)$$

where $X_{f\tau\iota}$ is the final industry's demand for industry (τ, ι) 's good, $\gamma \in (1, \infty)$ is the elasticity of substitution for goods within the same industry, and the elasticity of substitution across industries is 1.

The final industry aggregates the idiosyncratic shocks of intermediate industries. This gives us—through the law of large numbers—a deterministic expression for how changes in primary product features affect both aggregate output as well as the probability of a primary industry being used in production. To perform this aggregation I make the following assumption about

the distribution of $A_i(S_i)$.

Assumption 1 *For each intermediate industry $i \in \mathcal{I}$, the vector of possible productivities $A_i = (A_i(S))_{S \in \mathcal{F}(\theta, \tau_i)} \in \mathbb{R}^{|\mathcal{F}(\theta, \tau_i)|}$ are independently drawn from distributions $\Psi(S)$ which:*

1. *may depend on S ;*
2. *do not depend on ι ;*
3. *are absolutely continuous with respect to Lebesgue measure; and*
4. *satisfy $\text{Var}[A(S)^\gamma] < \infty$ for all $A(S)$ drawn from $\Psi(S)$.*

Assumption 1 implies, among other things, that the prices $(P_i)_{i \in \mathcal{I}}$ are independently distributed, so that that price aggregates of the form $P_\tau = (\int_0^1 P_{\tau \iota}^{1-\gamma} d\iota)^{\frac{1}{1-\gamma}}$ are well defined. I will use the requirement that Ψ is absolutely continuous to show that most equilibrium quantities are unique.⁶

2.2 Equilibrium

In this subsection, I give a definition of equilibrium. To make some of the notation more compact, I introduce notational conventions that are carried throughout the paper. In addition, to address possible issues with the non-convexity of production technology, I define industries' marginal cost functions as in Baqaee and Farhi (2020) and Acemoglu and Azar (2020). To ensure that aggregates are well defined, I also introduce a tie-breaking rule for intermediate industries which have multiple potential cost-minimizing input sets, and show that this tie-breaking rule is used only by a subset of industries with measure zero. The subsection concludes with the definition of equilibrium.

Notation I will often work with equations that apply to all industries (primary, intermediate, and final) simultaneously. I will use the indices $g, h \in \mathcal{J} \cup \mathcal{I} \cup \{f\}$. to denote generic industries which may belong to any sector.

For any industry g , I denote its revenue by $\hat{Y}_g = P_g Y_g$. For any pair of industries g, h , I denote the revenue that industry h obtains from this transaction by $\hat{X}_{gh} = P_h X_{gh}$. I denote the amount of good h used in equilibrium to produce one unit of good g by $\alpha_{gh} = \frac{X_{gh}}{Y_g}$ and, industry h 's share of industry g 's revenue by $\hat{\alpha}_{gh} = \frac{P_h X_{gh}}{P_g Y_g}$.

⁶More precisely, for each intermediate industry $i \in \mathcal{I}$, there may exist a set of realizations of A_i such that the cost-minimizing input set S_i is not unique. This set of realizations will have Lebesgue measure zero. Since I assume that the distribution Ψ of A_i is absolutely continuous with respect to Lebesgue measure, the probability that any given industry will have multiple cost-minimizing sets is also zero. Thus, equilibrium quantities that are aggregated across all intermediate industries will always be unique.

I denote the aggregate demand of intermediate industries for a primary good j by $X_{int,j} = \int_{\mathcal{I}} X_{ij} di$, and the aggregate revenue received by primary industry j from its sales to intermediate industries by $\hat{X}_{int,j} = P_j X_{int,j}$.

Finally, given a set Ω , I use 2^Ω to refer to the collection of all subsets of Ω , and $\Delta(\Omega)$ to refer to the set of all probability distributions over Ω .

Marginal Cost Functions Because intermediate firms can choose their set of inputs, their overall production technologies may be non-convex. Thus, in some situations, it will be more appropriate to state firms' optimization objectives in terms of cost-minimization instead of profit maximization.⁷

Definition 1 (Marginal Cost Functions) Let $P = ((P_j)_{j=1}^J, (P_i)_{i \in \mathcal{I}}, P_f)$ denote a vector of prices. The marginal cost functions of primary industries, intermediate industries, and the final industry are defined as follows:

1. For a primary industry j , $K_j(P) = \min_{L_j, (X_{jk})_{k=1}^J} \sum_{k=1}^J P_k X_{jk} + L_j$, subject to $F_j((X_{jk})_{k=1}^J, L_j) = 1$.
2. For an intermediate industry i with threshold τ , $K_i(P, \tau) = \min_{S_i \in \mathcal{F}(\theta, \tau)} K(S_i, P, A_i, \tau)$ where $K(S_i, P, A_i, \tau) = \min_{(X_{ij})_{j \in S_i}, L_i} \sum_{j \in S_i} P_j X_{ij} + L_i$, subject to $A_i(S_i) F(S_i, (X_{ij})_{j \in S_i}, L_i) = 1$ and $G(1, (X_{ij})_{j \in S_i}, (\Theta_j)_{j \in S_i}, S_i) \leq \tau$.
3. For the final industry, $K_f(P) = \min_{(X_{fi})_{i \in \mathcal{I}}} \int_{\mathcal{I}} P_i X_{fi} di$, subject to $e^{\frac{\gamma}{\gamma-1} (\int_0^\infty \log(\int_0^1 X_{f\tau i}^{\frac{\gamma}{\gamma-1}} d\iota) \nu(\tau) d\tau)} = 1$.

Breaking Ties Between Cost-Minimizing Sets A technical issue that will arise is that, when intermediate industries minimize costs, there may be two different optimal input sets S_i^* and S_i^{**} , leading to two different allocations. In order to avoid this ambiguity, I define a rule to be used by all intermediate industries use to choose an input set in the case of ties. The tie-breaking rule is possibly randomized and always selects a set that minimizes costs.

Definition 2 (Tie Breaking Rule) When multiple input sets minimize costs, intermediate industries choose their input set according to a (possibly random) function $TieBreak : 2^{\mathcal{F}(\theta, \tau)} \rightarrow \Delta(\mathcal{F}(\theta, \tau))$ which satisfies

$$TieBreak(\arg \min_{S \in \mathcal{F}(\theta, \tau)} K(S, P, A(S), \tau)) \in \arg \min_{S \in \mathcal{F}(\theta, \tau)} K(S, P, A(S), \tau).$$

⁷More concretely, an intermediate firm i with a constant returns to scale production function will obtain zero profits by choosing any feasible set of inputs S_i and choosing $L_i, (X_{ij})_{j \in S_i}$ to maximize profits. This does not mean that all feasible sets S_i are equally likely to be used in equilibrium. Instead, firms which choose S_i to minimize marginal costs will be the only ones that will receive positive demand in equilibrium, since they can charge the lowest prices.

This tie-breaking rule is a technical construction to ensure that aggregates such as $X_{int,j} = \int_{\mathcal{I}} X_{ij} di$ are well defined. As the following Lemma shows, it is used only by a set of industries that has measure zero.

Lemma 1 *Let $\mathcal{A} = \{A \in \mathbb{R}^{|\mathcal{F}(\theta,\tau)|} : |\arg \min_{S \in \mathcal{F}} K(S, P, A(S), \tau)| \geq 2\}$ be the set of productivity parameters for which there are at least two input sets which minimize the intermediate cost function. Then \mathcal{A} has Lebesgue measure zero.*

I prove Lemma 1 in Appendix A. Since the distribution of productivity parameters is absolutely continuous with respect to Lebesgue measure, the probability that any given intermediate industry i has to use the tie breaking rule is zero.

Definition of Equilibrium

Definition 3 *An equilibrium is defined by a tuple (P, C, Y, X, S) such that*

1. *Prices equal marginal costs. For any industry $g \in \mathcal{J} \cup \mathcal{I} \cup \{f\}$,*

$$P_g = K_g(P). \quad (6)$$

2. *Intermediate firms choose input sets to minimize marginal costs*

$$S_i = \text{TieBreak}(\arg \min_{S' \in \mathcal{F}(\theta, \tau_i)} K(S', P, A_i(S'), \tau_i)). \quad (7)$$

3. *All firms choose their demands X, L to minimize marginal costs, subject to a given set of inputs.*

4. *Households choose consumption C to maximize utility subject to their budget constraint.*

5. *Markets clear, so that*

$$Y_j = F_j((X_{jk})_{k=1}^J, L_j) = \sum_{k=1}^J X_{kj} + \int_{\mathcal{I}} X_{ij} di \quad \forall j \in \mathcal{J} \quad (8)$$

$$Y_i = A_i(S_i) F(S_i, (X_{ij})_{j \in S_i}, L_i) = X_{fi} \quad \forall i \in \mathcal{I} \quad (9)$$

$$\log Y_f = \frac{\gamma}{\gamma - 1} \left(\int_0^\infty \log \left(\int_0^1 X_{f\tau i}^{\frac{\gamma-1}{\gamma}} d\tau \right) \nu(\tau) d\tau \right) = \log C \quad (10)$$

$$\sum_{j=1}^J L_j + \int_{\mathcal{I}} L_i di = 1. \quad (11)$$

3 Existence, Uniqueness and Characterization of Equilibrium

In this section, I show a mild necessary and sufficient condition for an equilibrium to exist, and for prices and aggregate equilibrium quantities to be unique. I also show that, under a standard assumption on the productivity term $A_i(S_i)$, there exists a tractable formula for aggregate output. In Section 4, this formula will allow us to derive useful comparative statics showing how output changes when the physical properties of primary products change. In Section 5, it will also allow us to expand the static model into a growth model, where growth is driven by improvements in the physical properties of primary inputs.

3.1 Existence

In this subsection, I show that an equilibrium can be obtained under a mild necessary and sufficient condition.

Theorem 1 *An equilibrium exists if and only if there exists a positive vector of primary industry prices $P_1, \dots, P_J > 0$ such that $P_j = K_j(P_0, P_1, \dots, P_J)$ for every primary industry $j \in \mathcal{J}$.*

I discuss the intuition behind the proof of Theorem 1 here and give a proof in Appendix B. It is immediate that if such a vector of prices does not exist, then an equilibrium cannot exist, so the condition is necessary. The intuition behind sufficiency is that the prices of intermediate and final industries are completely determined by primary prices and the parameters of the model. The cost-minimizing input sets S_i are determined by prices and the tie breaking rule, and equilibrium quantities are unique given S_i , since all production functions are strictly quasi-concave. I highlight that uniqueness is not immediate, since intermediate firms could have multiple cost-minimizing input sets. Nevertheless, as discussed below, one can show that all aggregate quantities are unique.

3.2 Uniqueness

In this subsection, I give mild conditions under which there is a unique equilibrium, and discuss what happens when these conditions do not apply. I begin by showing that, when the cost functions $K_j(P)$ of primary industries are strictly concave, the equilibrium price vector P^* is always unique (if it exists).

Proposition 1 *If the primary industry cost functions $(K_j(P))_{j=1}^J$ are strictly concave as a function of prices, then the equilibrium price vector P^* is unique if it exists.*

Proposition 1 holds, for example, when the primary industry production functions are CES with non-zero elasticity of substitution. For the special case where production functions are Leontief, the cost function $K_j(P)$ is linear in prices, and not strictly concave. Nevertheless, when the Leontief coefficients add up to less than 1, there is a unique primary price vector (and therefore a unique price vector P^* for all industries).

Proposition 2 *Let $F_j(A_j, L_j, X) = \min\{A_j L_j, \min_{k \in \mathcal{J}}\{\frac{X_{jk}}{\alpha_{jk}}\}\}$. If the matrix $(I - \alpha)$ is invertible and all entries in $(I - \alpha)^{-1}$ are non-negative, then there exists a unique equilibrium price vector P^* .*

Even when prices are unique, equilibrium quantities may not be. This is because there may exist, for a given intermediate industry $i \in \mathcal{I}$, two different input sets S_i^*, S_i^{**} , both of which minimize costs, and ties may be broken randomly between these sets. If this happens, intermediate industry i would have two different demand vectors X_i^*, X_i^{**} corresponding to the two different input sets.

While intermediate industry demands for primary goods may not always be unique, all other relevant quantities (including all industry outputs Y and profits) are unique. The intuition behind this is two-fold: (i) Lemma 1 tells us the probability that any given industry i has multiple input sets is 0, and (ii) these events of probability 0 do not affect the aggregate result. I now make this intuition more formal.

Theorem 2 *Assume that a unique price vector P^* satisfying equation (6) exists. Then,*

1. *The equilibrium output vector Y^* , final input demands $(X_{fi}^*)_{i \in \mathcal{I}}$ and primary input demands $((X_{jk}^*)_{k=1}^J, L_j^*)_{j=1}^J$ are unique.*
2. *If the tie breaking rule is deterministic, the equilibrium input sets S_i^* and intermediate demands $(X_{ij}^*)_{j=1}^J$ are also unique.*
3. *If the tie breaking rule is random, the the input set S_i^* and intermediate demands $(X_{ij}^*)_{j=1}^J$ are unique with probability 1, where the probability is taken over the random choice of $A_i \in \mathbb{R}^{|\mathcal{F}|}$ and the randomness of the tie breaking rule.*

3.3 A Tractable Formula for Aggregate Output as a Function of the Number of Feasible Production Sets

We want to understand how output depends on the physical properties θ of primary inputs. I show here that such an analysis is tractable under the assumption that intermediate industries' productivity terms $A_i(S_i) = \phi_i(S_i) \cdot \prod_{j \in S_i} A_j$ are the product of deterministic

input-specific productivities A_j , and of a random variable $\phi_i(S_i)$ which is drawn identically and independently from a Frechet distribution for every $i \in \mathcal{I}$ and every $S_i \in \mathcal{F}$. This assumption is used by Acemoglu and Azar (2020) on their model of endogenous production networks, and is inspired by the use of Frechet productivity draws in Kortum (1997) and Eaton and Kortum (1998) to microfound gravity equations in international trade models.

Assumption 1' *For all intermediate industries $i \in \mathcal{I}$ and all sets $S_i \in \mathcal{F}$, the productivity term $A_i(S_i) = \phi_i(S_i) \cdot \prod_{j \in S_i} A_j$ where $A_j > 0$ is a deterministic term corresponding to input j , and $\phi_i(S_i)$ is a random variable drawn independently from a Frechet distribution with CDF $\Psi(x) = e^{-x^{-\kappa}}$. The shape parameter κ satisfies $\kappa > 2(\gamma - 1)$.*

To give a tractable formula for aggregate output, I need to define what the cost of intermediate products would be if all productivity terms are deterministic (that is, if $A_i(S_i) = \prod_{j \in S_i} A_j$, and $\phi_i(S_i) = 1$).

Definition 4 (Deterministic Cost Function) *Given an input set $S \in \mathcal{F}$ and an equilibrium vector P of primary prices, the deterministic cost function for intermediate goods with threshold τ is given by*

$$\bar{K}(S, P, \tau) = \min_{(X_{ij})_{j \in S}, L_i} \sum_{j \in S} P_j X_{ij} + L_i$$

$$\text{Subject to : } \left(\prod_{j \in S} A_j \right) \cdot F(S, X_i, L_i) = 1 \text{ and } G(1, (X_{ij})_{j \in S_i}, (\Theta_j)_{j \in S_i}, S_i) \leq \tau$$

I can now characterize the equilibrium output as a function of deterministic cost functions and the feasible collection $|\mathcal{F}(\theta, \tau)|$.

Theorem 3 (Static Output Characterization) *Suppose that Assumption 1' holds. Let $\Gamma : \mathbb{R}_{>0} \rightarrow \mathbb{R}$ be the Gamma function $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$. Then the output of the final industry is given by*

$$\log Y_f = \frac{1}{\kappa} \int_0^\infty \log \left(\sum_{S \in \mathcal{F}(\theta, \tau)} \bar{K}(S, P, \tau)^{-\kappa} \right) \nu(\tau) d\tau - \log \left(\Gamma \left(1 - \frac{\gamma - 1}{\kappa} \right)^{\frac{1}{1-\gamma}} \right) \quad (12)$$

I give a proof of Theorem 3 in Appendix B. The intuition for the result follows from the fact that the production function for the final industry is a nested CES function, where the inner nest has elasticity γ and the outer nest is Cobb-Douglas. For each threshold τ , the inner nest's output will be $Y_\tau = (\sum_{S \in \mathcal{F}(\theta, \tau)} \bar{K}(S', P, \tau)^{-\kappa})^{-\frac{1}{\kappa}}$. Informally, this follows from the fact that each firm with threshold τ faces a standard discrete choice problem where they have to choose among $|\mathcal{F}(\theta, \tau)|$ alternatives, and each alternative has a cost which is Frechet distributed with shape parameter κ . The outer nest's output is given by $\int_0^\infty \log Y_\tau \nu(\tau) d\tau$,

minus a constant term $\log(\Gamma(1 - \frac{\gamma-1}{\kappa})^{\frac{1}{1-\gamma}})$ that arises from the Cobb-Douglas structure of production.

Note that in the special case where $\overline{K}(S, P, \tau) = 1$,⁸ one can obtain a simplified formula

$$\log Y_f = \frac{1}{\kappa} \int_0^\infty \log |\mathcal{F}(\theta, \tau)| \nu(\tau) d\tau - \log(\Gamma(1 - \frac{\gamma-1}{\kappa})^{\frac{1}{1-\gamma}}). \quad (13)$$

This simplification makes more explicit the connection between output and the number of feasible combinations $|\mathcal{F}(\theta, \tau)|$ and will be important in Sections 4 and 5 for deriving tractable comparative statics and growth rates.

4 Comparative Statics

In this section, I analyze how changes in primary goods' physical properties affect aggregate productivity. In particular, I give formulas for the first-order and second-order partial derivatives $\frac{\partial \log Y_f}{\partial \theta_j}$ and $\frac{\partial^2 \log Y_f}{\partial \theta_j \partial \theta_k}$ of log-output with respect to physical properties. I then apply these formulas to show how changes in products' physical properties can propagate through the supply chain, and have spillover effects on log-output.

4.1 Direct Comparative Statics

Throughout this section, Assumption 1' holds. Furthermore, I assume that the feasibility constraint $G(Y_i, (X_{ij})_{j \in S_i}, (\theta_j)_{j \in S_i}, S_i) \leq \tau_i Y_i$ depends only on S_i and $(\theta_j)_{j \in S_i}$, so that the constraint becomes $G((\theta_j)_{j \in S_i}, S_i) \leq \tau_i$. For simplicity of analysis, I make the additional assumption that $\overline{K}(S, P, \tau) = 1$ for all sets $S \in \mathcal{F}$. In subsection 4.3, I show comparative statics results when $\overline{K}(S, P, \tau)$ can be arbitrary.

With these assumptions, log-output is given by equation (13). Its derivative with respect to θ_j is given by

$$\frac{\partial \log Y_f}{\partial \theta_j} = \frac{1}{\kappa} \int_0^\infty \frac{\frac{\partial}{\partial \theta_j} |\mathcal{F}(\theta, \tau)|}{|\mathcal{F}(\theta, \tau)|} \nu(\tau) d\tau, \quad (14)$$

where $\frac{\partial}{\partial \theta_j} |\mathcal{F}(\theta, \tau)|$ is a distributional derivative. The following Lemma gives a simple formula for this derivative in terms of Dirac delta functions.

⁸This may happen, for example, if all primary prices P_j are equal to 1, and the intermediate production function is Cobb-Douglas so that $\overline{K}_i = \prod_{j \in S_i} P_j^{-\alpha_j}$.

Lemma 2 *Let $\delta(\cdot)$ be the Dirac delta function. Then,*

$$\frac{\partial}{\partial \theta_j} |\mathcal{F}(\theta, \tau)| = - \sum_{S \subset \mathcal{J}} \delta(\tau - G(\theta, S)) \frac{\partial G}{\partial \theta_j} \quad (15)$$

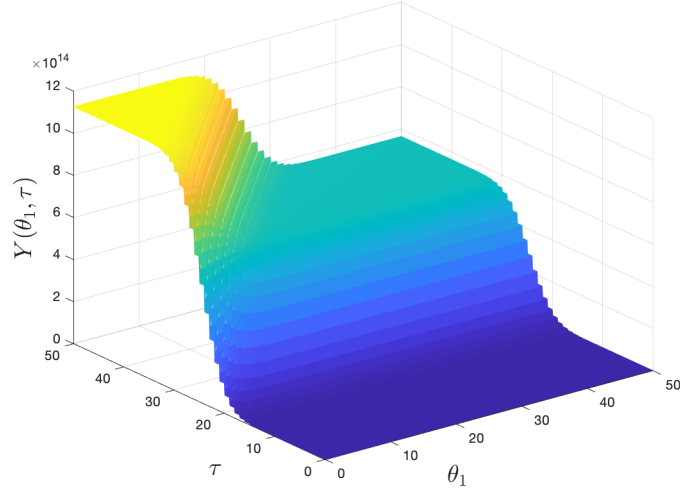
I give a proof of Lemma 3 in Appendix C, and briefly discuss the intuition here. Recall that $|\mathcal{F}(\theta, \tau)|$ is the number of sets that satisfy the combinatorial constraint $G(\theta, S) \leq \tau$. If $G(\theta, S)$ is differentiable and increasing in θ , then, for any fixed τ , $|\mathcal{F}(\theta, \tau)|$ is a decreasing step function which takes integer values and ranges between 0 and 2^J . Its derivative is 0 at almost every point, except for those values of θ which satisfy $G(\theta, S) = \tau$ for some set S . If such a set S contains j , then increasing θ_j by an infinitesimally small amount would make S infeasible, and would decrease $|\mathcal{F}(\theta, \tau)|$. The size of this decrease is exactly the number of sets S which contain j , and for which $G(\theta, S) = \tau$.

A Note on Computation, and an Example The sum in equation (15) is over 2^J sets, and may be daunting to compute. However, in many situations of interest, we can either compute approximately or simulate the values of $|\mathcal{F}(\theta, \tau)|$. Appendix E gives both exact and approximation algorithms for computing \mathcal{F} when the combinatorial constraints are multi-dimensional Knapsack constraints.

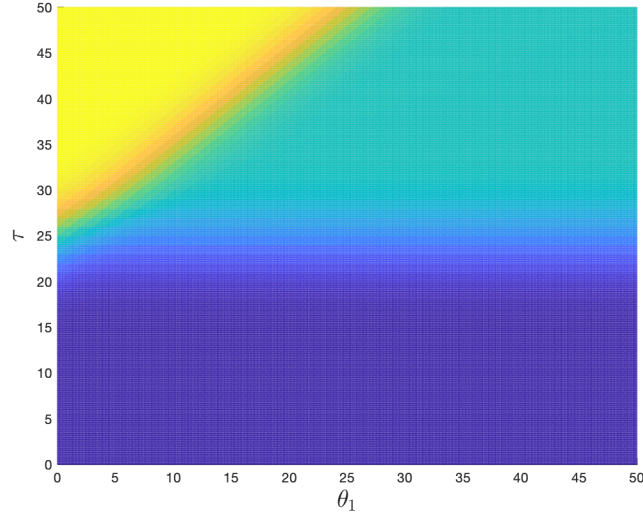
The following numerical example shows how $|\mathcal{F}(\theta, \tau)|$ changes when there is a large number of primary goods which do not change size, and one primary good which shrinks. This is reflective, for example, of an economy where electronic goods miniaturize, but other goods remain at relatively constant sizes.

Example 3 *Consider an economy where there are 51 primary goods. The first good has a variable size θ_1 and the 50 remaining goods have a constant size equal to 1. The collection of feasible sets is given by $\mathcal{F}(\theta, \tau) = \{S : \sum_{j \in S} \theta_j \leq \tau\}$. The density $\nu(\tau)$ is uniform over $[0, 50]$.*

Figure 2 gives an illustration of the discontinuous nature of $|\mathcal{F}(\theta, \tau)|$ for this economy. We can see from the ridges in the plot that $|\mathcal{F}(\theta, \tau)|$ is a step function, that it is very low for low values of τ , and that decreasing the size of θ_1 starts having a significant effect on $|\mathcal{F}(\theta, \tau)|$ only when $\tau \geq 25$ and more than half of the goods can be combined. Subfigure (b) shows a heatmap for $|\mathcal{F}(\theta, \tau)|$, and clearly illustrates that phase transitions occur in the value of $|\mathcal{F}(\theta, \tau)|$ as τ increases above 25, and when θ decreases below $\tau + 25$. These phase transitions appear exactly when θ is small enough or τ is large enough that more than half of the goods can be combined.



(a) $|\mathcal{F}(\theta, \tau)|$



(b) Level curves for $|\mathcal{F}(\theta, \tau)|$

Figure 2: This figure shows how $|\mathcal{F}(\theta, \tau)|$ changes as a function of θ and τ in the economy given in Example 3. In this example, there are 51 goods. The first good has a variable size θ_1 , and the 50 other goods have a constant size equal to 1. Subfigure (a) shows a 3D plot of this function. We can see from the ridges in the plot that $|\mathcal{F}(\theta, \tau)|$ is a step function, that it is very low for low values of τ , and that decreasing the size of θ_1 starts having an effect on $|\mathcal{F}(\theta, \tau)|$ only when $\tau \geq 25$ and more than half of the goods can be combined. Subfigure (b) shows a heatmap for $|\mathcal{F}(\theta, \tau)|$, and clearly illustrates that phase transitions occur as τ increases above 25, and when θ decreases below $\tau + 25$.

First-Order Comparative Statics Applying Lemma 3 to equation (14), I obtain the following Proposition.

Proposition 3

$$\frac{\partial \log Y_f}{\partial \theta_j} = -\frac{1}{\kappa} \sum_{S \subset \mathcal{J}} \frac{\nu(G(\theta, S))}{|\mathcal{F}(\theta, G(\theta, S))|} \frac{\partial G}{\partial \theta_j}. \quad (16)$$

Each term in the expression $\sum_{S \subset \mathcal{J}} \frac{\nu(G(\theta, S))}{|\mathcal{F}(\theta, G(\theta, S))|} \frac{\partial G}{\partial \theta_j}$ in Proposition 3 has a natural interpretation. Let τ be fixed, and suppose that θ_j increases. The number of feasible sets $|\mathcal{F}(\theta, \tau) = \{S : G(\theta, S) \leq \tau\}|$ will decrease only for those values of τ for which there exists at least one set S such that $\tau = G(\theta, S)$. The density of such sets is $\nu(\tau) = \nu(G(\theta, s))$. Now, \log output does not depend on $|\mathcal{F}(\theta, G(\theta, S))|$ directly, but rather through $\log |\mathcal{F}(\theta, G(\theta, S))|$. Using the chain rule, the rate at which the expression $\log |\mathcal{F}(\theta, G(\theta, S))|$ decreases will be $\frac{\partial G}{\partial \theta_j} \frac{1}{|\mathcal{F}(\theta, G(\theta, S))|}$. The sum aggregates these output changes over all possible affected sets S .

Second-Order Comparative Statics An important feature of the model is that, as the physical properties of primary inputs change, they can relax intermediate industries' combinatorial constraints, and increase demand for other primary goods as well. We can show this effect quantitatively by taking second-order derivatives of output with respect to θ .

Proposition 4 *If $G(\theta, S)$ is given by a linear function $G(\theta, S) = \sum_{j \in S} \gamma_j \theta_j$, then the matrix of second partial derivatives of log output with respect to θ is almost-everywhere given by*

$$\frac{\partial^2 \log Y_f}{\partial \theta_j \partial \theta_k} = -\frac{\gamma_j \gamma_k}{\kappa} \sum_{S \subset \mathcal{J}} \frac{\nu'(G(\theta, S))}{|\mathcal{F}(\theta, G(\theta, S))|}. \quad (17)$$

I give a proof of Proposition 4 in the Appendix. It is important to note that Equation (17) holds for all values of θ , except for a set of measure zero. The values of θ for which Equation (17) does not hold are those for which the number of feasible combinations $|\mathcal{F}(\theta, G(\theta, S))|$ may change with a small change in θ . More formally, for $|\mathcal{F}(\theta, G(\theta, S))|$ to change with θ , there must exist a set $S' \neq S$ such that $G(\theta, S) = G(\theta, S')$. Since G is linear in θ , the set of θ for which such condition holds has measure zero.

The second partial derivatives are always positive if ν is a decreasing function and $\gamma_j, \gamma_k > 0$. The economic interpretation of this fact is that—if the majority of intermediate industries have small thresholds, then a decrease in the size θ_k of good k will amplify the positive effect of a decrease in the size θ_j of good j . As θ_k decreases, any decrease in θ_j will make intermediate industries feasible at a faster rate, and therefore increase productivity at a faster rate, compared with the scenario where θ_k remained constant. This captures the

notion that complementarity exists between the physical properties of different goods. If the physical properties of good improve, then that increases the value of improving the physical properties of other goods.

4.2 Spillovers of Physical Properties

A fundamental fact in the production networks literature is that increases in productivity in one industry will lead, through spillover effects, to increases in productivity to downstream industries. In this paper, increases in productivity are microfounded through changes in the physical properties of primary inputs. In practice, miniaturization of one product such as transistors, can lead to physical changes, such as greater portability and energy efficiency, in other products.

In this subsection, I give a quantitative formula for how changes in the physical properties of one primary good propagate through the economy and have an effect on aggregate output. As in Section 2, the vector θ of physical properties is a differentiable function $\theta(\zeta)$ of some fundamental properties ζ_1, \dots, ζ_K .⁹ The following result is an immediate corollary from Proposition 3 and the chain rule.

Corollary 1

$$\frac{\partial \log Y_f}{\partial \zeta_k} = -\frac{1}{\kappa} \sum_{j=1}^J \sum_{S \subset \mathcal{J}} \frac{\nu(G(\theta, S))}{|\mathcal{F}(\theta, G(\theta, S))|} \frac{\partial G}{\partial \theta_j} \frac{\partial \theta_j}{\partial \zeta_k} \quad (18)$$

This result is very general, and applies to many areas outside miniaturization. To give a concrete example, room temperature superconductors that can be produced reliably would have impacts across the energy, transportation, defense, industrial and medical sectors (Johns et. al. 1990). The conductivity, temperature, and size of superconducting inputs could be modeled as property vectors $\theta(\zeta)$ which may depend on the properties ζ of the materials used to create these inputs. The impact of superconductors across industries could be modeled through a constraint function $G(\theta, S)$, which would specify size, temperature and conductivity constraints of different industries.

⁹Here I emphasize that the number of fundamental properties K may be different from the number of primary industries J , or the dimension p of the property vector θ .

4.3 Comparative Statics with Heterogeneous Prices

I conclude this section by showing how to compute comparative statics when $\bar{K}(S, P, \tau)$ can be arbitrary. In this case, log-output is given by Equation (20), and equation (14) becomes

$$\frac{\partial \log Y_f}{\partial \theta_j} = -\frac{1}{\kappa} \int_0^\infty \frac{\frac{\partial}{\partial \theta_j} \sum_{S \in \mathcal{F}(\theta, \tau)} \bar{K}(S, P, \tau)^{-\kappa}}{\sum_{S \in \mathcal{F}(\theta, \tau)} \bar{K}(S, P, \tau)^{-\kappa}} \nu(\tau) d\tau.$$

The following analogue of Lemma 3 holds:

Lemma 3 *Let $\delta(\cdot)$ be Dirac's Delta function. Then,*

$$\frac{\partial}{\partial \theta_j} \sum_{S \in \mathcal{F}(\theta, \tau)} \bar{K}(S, P, \tau)^{-\kappa} = \sum_{S \subset \mathcal{J}} \delta(\tau - G(\theta, S)) \bar{K}(S, P, \tau)^{-\kappa} \frac{\partial G}{\partial \theta_j} \quad (19)$$

Applying Lemma 3, we obtain

$$\frac{\partial \log Y_f}{\partial \theta_j} = -\frac{1}{\kappa} \sum_{S \subset \mathcal{J}} \frac{\frac{\partial G}{\partial \theta_j} \bar{K}(S, P, G(\theta, S))^{-\kappa} \nu(G(\theta, S))}{\sum_{S' \in \mathcal{F}(\theta, G(\theta, S'))} \bar{K}(S', P, G(\theta, S'))^{-\kappa}}.$$

This result is completely analogous to Proposition 3, and yields analogous results for spillovers with physical properties. One important thing to note is that now there is an additional channel for second-order effects. When θ_k increases, it not only affects $\nu(G(\theta, S))$, but it also affects $\bar{K}(S, P, G(\theta, S))$. Effectively, an increase in θ_k tightens the combinatorial constraint for all intermediate industries using product k , and increases the marginal cost of producing any intermediate product that uses both goods j and k in production.

5 A Growth Model

In this section, I show how growth arises from the miniaturization of primary inputs. I extend the static model of Section 2 to a dynamic analogue that captures the intuition that, as transistors become smaller, other electronic components also become smaller and new combinations of inputs arise. These components can themselves be combined into machines, and the exponential growth of combinations leads to sustained GDP growth.

The model has some interesting dynamics. Intermediate industries—such as electricity, computer hardware, internet services and nanotechnology—arrive over time as primary components miniaturize. Each intermediate industry has a threshold τ , and arrives only when there are primary components with sizes smaller than τ which can be used in production. Thus, when an intermediate industry first arrives, its productivity is very low since there are

not many components that can be combined to create machines in that sector. As inputs keep shrinking, the new sector's productivity increases.

Primary Industries There are a countable number of primary industries, indexed by $j \in \mathbb{N}$. Time is continuous and indexed by $t \in \mathbb{R}_{\geq 0}$. Industry 1 uses labor as input, and its production function is $Y_1(t) = L_1(t)$. Industry j uses industry $j - 1$'s good as input, and its production function is $Y_j(t) = X_{j,j-1}(t)$.¹⁰ The size of good 1 is given by $\theta_1(t) = \zeta(t)$, where $\zeta(t)$ is a parameter whose dynamics I describe below. Since the size of good j is the sum of sizes of its inputs, I have $\theta_j(t) = \theta_{j-1}(t) = \dots = \zeta(t)$.

To ensure that the set of primary industries at any given point is finite, I introduce a new constraint on primary good production, not present in the static model. This is a size constraint $\theta_j(t) \leq \tau_j$, where τ_j is a threshold. The constraint captures the idea that some primary goods cannot be feasibly produced unless their size is below a given threshold. Since in this model all primary goods have the same size $\zeta(t)$, primary goods arrive one at a time, in order of descending τ_j . I assume that $\tau_1 = 1$ and that the conditional random variable $\log \tau_j - \log \tau_{j+1}$ is exponentially distributed with mean $\frac{1}{\lambda} > 0$. I also assume that $\zeta(t) = e^{-Zt}$, where $Z > 0$ is some exogenous level of research. With these dynamics, I can show the following Lemma.

Lemma 4 *New primary industries arrive according to a Poisson process with arrival rate $Z\lambda$.*

Intermediate and Final Industries There is a continuum of intermediate industries with a feasibility set $\mathcal{F}(\theta(t), \tau) = \{S : \sum_{j \in S} \theta_j(t) \leq \tau, S \neq \emptyset\}$. As in Section 4, I assume that the intermediate deterministic cost function satisfies $\bar{K}(S, P, \tau) = 1$.¹¹ The productivity term for industry i if it chooses set S is given by $A_i(S) = \phi_i(S)$, which is drawn from a Frechet distribution as in Assumption 1'. The output of the final industry is given by the aggregate
$$\log Y_f = \frac{\int_0^\infty \log((\int_0^1 X_{f\tau t}^{\frac{\gamma-1}{\gamma}} dt)^{\frac{\gamma}{\gamma-1}}) \nu(\tau) d\tau}{\int_0^\infty \nu(\tau) d\tau}.$$

Density of Intermediate Industries In contrast with the model in Section 2, the mass of intermediate industries $\int_0^\infty \nu(\tau) d\tau$ can be infinite. At any time t , the only intermediate

¹⁰Here I am deviating from the assumption that labor is essential for every primary industry. I used that assumption only to prove Theorem 1. Since in this simple model, the existence of equilibrium is trivial to verify, it is acceptable to deviate from the assumption.

¹¹Since all primary good prices are equal to 1, one way to satisfy this assumption would be to have Cobb-Douglas production functions with cost functions $\bar{K}(S, P, \tau) = \prod_{j \in S} P_j^{-\alpha_j}$. Another way to satisfy this assumption would be to have Leontief production function with cost functions $\bar{K}(S, P, \tau) = \frac{1}{|S|} \sum_{j \in S} P_j$. The asymptotic characterization of the growth rate would still hold if we assume that all intermediate cost functions are in a bounded interval $\bar{K}(S, P, \tau) \in [K_{lower}, K_{upper}]$ that does not depend on t .

industries that will be active in the economy are those for which $\zeta(t) \leq \tau$. Any industry with $\tau < \zeta(t)$ will not be able to use any inputs and will not be able to produce its good. Intermediate industry thresholds are distributed in the interval $(0, 1]$, and there's a mass $\nu(\tau) = \frac{1}{\tau} d\tau$ of industries with threshold τ . At any given time t , the active intermediate industries are in the interval $[\zeta(t), 1]$, and the total mass of intermediate industries is $\int_{\zeta(t)}^1 \frac{d\tau}{\tau} = \log(1) - \log(e^{-Zt}) = Zt$.

Dynamic Output Characterization Even though there is no longer a unit mass of intermediate industries, the following variation of Theorem 3 holds.

Theorem 4 (Dynamic Output Characterization) *Suppose that Assumption 1' holds. The output of the final industry is given by the formula*

$$\log Y_f(t) = \frac{1}{\kappa} \frac{\int_{e^{-Zt}}^1 \log |\mathcal{F}(\theta, \tau)|^{\frac{1}{\tau}} d\tau}{Zt} - \log(\Gamma(1 - \frac{\gamma - 1}{\kappa})^{\frac{1}{1-\gamma}}) \quad (20)$$

Theorem 4 tells us that log output is a weighted average of the logarithms of the sizes of feasible input set collections $|\mathcal{F}(\theta, \tau)|$, with the weight for the collection $\mathcal{F}(\theta, \tau)$ given by $\frac{1}{\tau}$. The proof of the Theorem is analogous to that of Theorem 3 and takes into account that households may choose to save some output for future periods. Even though consumption is not equal to output, we still have a tractable formula for output that only depends on production technology and not on household preferences.

5.1 Solving for the Long-Run Growth Rate

In this subsection, I characterize the long-run growth rate of output, defined as $g^* = \lim_{t \rightarrow \infty} \frac{\log Y_f(t)}{t}$.¹² This long-run growth rate will depend both on the rate λ at which new primary industries arrive, as well as the rate Z at which existing inputs miniaturize.

Theorem 5 *The long-run growth rate is given by*

$$g^* = \lim_{t \rightarrow \infty} \frac{\log Y_f}{t} = \frac{\lambda z \log 2}{\kappa}. \quad (21)$$

I give the proof of Theorem 5 in the Appendix, and give some interpretation here. Growth is driven by the joint arrival of new materials and new applications. Without new materials,

¹²Traditionally, the long-run growth rate of output is defined as $\lim_{t \rightarrow \infty} \frac{\partial \log Y_f(t)}{\partial t}$. When this limit exists, L'Hopital's rule tells us that $g^* = \lim_{t \rightarrow \infty} \frac{\log Y_f(t)}{t}$ is equal to the traditional definition. However—even though the derivative $\frac{\partial \log Y_f(t)}{\partial t}$ always exists in our model—the limit of this derivative may not exist due to small oscillations as the number of newly discovered input combinations fluctuates with time. In contrast, the limit $\lim_{t \rightarrow \infty} \frac{\log Y_f(t)}{t}$ will always exist.

no new combinations are possible, and there is no growth. Without applications, the new materials are not useful, and there also isn't any growth. It is the combined development of materials and applications at the same time that leads to sustained growth.

In this model, new primary industries have log-thresholds τ_j distributed along $(-\infty, 0)$, with the average distance between log-thresholds being $\frac{1}{\lambda}$. If $\lambda \rightarrow 0$, then the average distance between thresholds goes to infinity. This means that, even if the existing industries keep miniaturizing, new primary industries will not be unlocked. Without the arrival of new primary industries, intermediate industries cannot attempt new combinations, and growth goes to zero. On the other hand, if $\lambda > 1$, then there will be a large number of primary industries that are “discovered” over time. However, these newly discovered primary industries cannot be fully used because the rate of miniaturization Z is smaller than the rate λZ at which new primary industries arrive. Thus, these newly discovered primary goods are “too large” to be combined in arbitrary ways. In this case, the growth rate in this case is dictated entirely by the rate Z at which primary industries miniaturize, which also governs the rate at which new intermediate industries arrive into the market.

6 Empirical Analysis

I test the predictions of the model in the context of computer and electronic miniaturization during the late 20th and early 21st centuries. The intuition behind the empirical analysis is that industries with larger products, such as the aerospace and agricultural machinery industries, are early beneficiaries of electronic miniaturization. In contrast, industries with smaller products, such as precision instruments or medical devices, cannot incorporate the newest electronic devices until those devices become small enough.

Miniaturization can affect a single industry in multiple time periods. For example, in agriculture, automated control and navigation systems were adopted in the 1980s and 90s once powerful CPUs and GPS systems were small enough to fit inside agricultural vehicles. As more electronics, such as sensors and cameras became miniaturized, they were also incorporated into agricultural machinery.

The effect of electronic and computer expenditure on TFP growth is fundamentally endogenous, since industries with higher productivity growth are more likely to spend more on computers and electronics. To address this issue, I use an instrumental variables strategy motivated by the model. I use the physical features of industry i 's products, namely their volume, weight, and density as instruments for industry i 's expenditures on computers and

electronics. More precisely, I estimate a two-stage least squares regression

$$\Delta \log Comp_{i,t} = \alpha_S \log Size_i + \alpha_D \log Density_i + \nu \log Rev_{i,t-1} + \psi_{I \times t} + \tau_{i,t} \quad (22)$$

$$\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t} \quad (23)$$

where equation (22) is the first-stage regression and equation (23) is the second-stage. In equation (23), $\Delta \log A_{i,t} = \log A_{i,t} - \log A_{i,t-1}$ is the change in log TFP for industry i at time t , $\Delta \log Comp_{i,t} = \log Comp_{i,t} - \log Comp_{i,t-1}$ is the change in industry i 's log-expenditures on computers and electronics, $\log Rev_{i,t-1}$ is revenue at the beginning of the period, $\delta_{I \times t}$ is a sector-by-time fixed effect,¹³ and $\Delta \log A_{i,t-1}$ captures autoregressiveness in changes in log TFP. In equation (22), $\log Size_i$ is the median log size of industry i 's products, $\log Weight_i$ is the median log weight, and $\log Density_i = \log Weight_i - \log Size_i$.

The intuition behind this approach, which I test in the regression results, is that industries with larger products will have larger Knapsack thresholds, and will be able to invest more in computers and electronics as they shrink. Empirically, this would be reflected by having $\alpha_{Size} > 0$. The main regression results also show that $\alpha_{Density} > 0$, potentially implying that producers of more dense products are more likely to invest in intermediate inputs, including computers and electronics. The results still hold—albeit with lower significance—if I omit density as an instrument and use only a product's median weight or size.

Implicit in this instrumental variables approach is the assumption that the physical characteristics of representative products in industry i have not changed. This is a reasonable assumption for most manufacturing industries excluding computers and electronics. The sizes of airplanes, trucks, and industrial machines have not significantly changed. Even in industries such as medical devices, where some products like pacemakers have miniaturized, most other products have remained at relatively the same size and weight.

Finally, there may be remaining endogeneity concerns about using an industry's revenue at the beginning of the period as a control. I include this control because it is possible that an industry's size (in terms of its revenue) will affect both its growth in the subsequent period and its investment in computers and electronics. As a robustness check, I report regressions both with and without revenue as a control variable. Under the assumption that a good's physical features are exogenous, the coefficient β from regressions without the control should still be unbiased. The results show that the coefficient β does not change significantly whether

¹³In the OLS regression, I could also add industry fixed effects. However, the instrumental variables will be industry specific and have no variation over time. Because of this, the IV regressions do not allow for industry fixed effects. To be able to compare the OLS and IV results, I use only sector-by-time fixed effects.

Since equation (23) is a first-differences regression, adding industry-level fixed effects would account for industry-specific changes in log-productivity. Industry-specific differences in levels of log-productivity are already taken into account by taking first-differences.

revenue is included as a control or not.

6.1 Data

I implement my empirical analysis with a new dataset on product weights and sizes and combine it with existing datasets on input-output tables and industry productivity. The product data comes from IHS Markit’s PIERS database of Schedule B information for imports into the United States. This dataset contains shipment-level information on products, including 6-digit harmonized-system (HS) codes, price paid, volume (in Twenty-Foot Equivalent Units, or TEUs),¹⁴ weight (in pounds), units and quantities. The data on computers and electronics usage comes from Acemoglu, Autor, and Patterson’s (2017) harmonized input-output tables for the United States, which are derived from the BEA benchmark input-output tables. The harmonized tables cover industries at the SIC 1987 level; they span the years 1972-2007 with 5-year gaps. Finally, the data on productivity and revenue comes from the NBER-CES manufacturing dataset, which covers 459 manufacturing industries (at the 1987 SIC level) from 1958 to 2011. I define the computer and electronics sectors as those industries whose SIC codes start with 357 or 367.

I use a crosswalk matching HS and SIC codes to merge the datasets and compute median weight-per-unit and volume-per-unit for every manufacturing SIC code. Observations where the units are not boxes, packages or containers are dropped. The remaining observations represent 87.1 % of the data.

Even though the dataset contains all manufacturing industries, some industries such as food, tobacco, apparel, and glass manufacturing are less likely to have embedded electronics inside them, and therefore unlikely to have directly benefited from electronic miniaturization. Because of this, I focus on *heavy manufacturing* industries, with 2-digit SIC codes 34 (Fabricated Metal Products), 35 (Industrial Machinery and Equipment), 36 (Electronic and Other Electric Equipment), 37 (Transportation Equipment), 38 (Instruments and Related Products) and 39 (Miscellaneous Manufacturing). I further exclude the computers and electronics sectors (that is, those with SIC codes starting with 357 or 367), because I cannot reasonably assume that their size has remained constant during the sample period. While most of the tables use heavy manufacturing industries, Table 5 shows the regression results using all manufacturing industries except computers and electronics, and shows significant results.

Table 1 shows the top 5 and bottom 5 industries separately ranked by four metrics. Panel A shows industries ranked by volume, measured in TEUs. We can see that the largest

¹⁴A TEU is the volume of a standard 20-foot cargo container, which is approximately 1172 cubic feet or 33 cubic meters.

Panel A: Manufacturing Industries Ranked by Volume (TEUs)			
Top 5 Industries		Bottom 5 Industries	
Truck and bus bodies	12.000	Other transportation equipment, nspf, and parts, nspf	0.001
Truck trailers	1.000	Dental equipment, supplies, and parts, nspf	0.001
Travel trailers and campers	0.670	Small arms ammunition, nspf	0.001
Machine tools, metal-forming, and parts, nspf	0.220	Electric lamps	0.002
Conveyors and conveying equipment, and parts, nspf	0.208	Telephone and telegraph apparatus, and parts, nspf	0.002
Panel B: Manufacturing Industries Ranked by Weight (Lbs)			
Top 5 Industries		Bottom 5 Industries	
Truck and bus bodies	31,378.010	Guided missiles and space vehicles, and parts, nspf	9.645
Truck trailers	7,607.040	Telephone and telegraph apparatus, and parts, nspf	10.907
Travel trailers and campers	3,179.060	Dolls and stuffed toy animals	11.352
Machine tools, metal-forming, and parts, nspf	2,601.450	Electric lamps	12.138
Rolling mill machinery, and parts, nspf	2,416.508	Dental equipment, supplies, and parts, nspf	13.553
Panel C: Manufacturing Industries Ranked by Density (Lbs/TEUs)			
Top 5 Industries		Bottom 5 Industries	
Other transportation equipment, nspf, and parts, nspf	36,788.140	Guided missiles and space vehicles, and parts, nspf	857.333
Fabricated plate work	28,377.130	Truck and bus bodies	2,614.834
Structural metal parts, nspf	28,208.570	Aircraft	3,306.925
Fabricated structural metal products, nspf	25,487.860	Aircraft equipment, nspf	3,748.971
Small arms ammunition, nspf	23,185.870	Travel trailers and campers	4,744.866
Panel D: Manufacturing Industries Ranked by Price (\$/Lbs)			
Top 5 Industries		Bottom 5 Industries	
Guided missiles and space vehicles, and parts, nspf	259.448	Structural metal parts, nspf	0.617
Aircraft	117.643	Architectural and ornamental metal work, nspf	0.797
Missile and rocket engines	90.623	Fabricated structural metal products, nspf	1.008
Aircraft equipment, nspf	68.309	Truck trailers	1.102
X-ray apparatus and tubes and related irradiation apparatus	38.773	Bolts, nuts, screws, rivets, and washers	1.367

Table 1: Top and Bottom Industries by Physical Characteristics. This table shows how different SIC industries are ranked according to different physical features of their median product. Panel A ranks industries by volume. Panel B ranks industries by weight. Panel C ranks industries by density. Panel D ranks industries by Price-Per-Pound. The data is obtained from Schedule B reports of imports into the United States, which provide shipment level details on product quantities, weights, volumes, units and prices.

products are trucks, buses, conveyors, and machine tools.¹⁵ The smallest products are parts of other transportation equipment, parts of dental equipment, parts of telephones, ammunition and lamps. Panel B shows the top and bottom 5 industries ranked by weight. Again, the heaviest products are trucks and buses, followed now by metal-forming machine tools and rolling mill machinery. The lightest products are missile and aerospace parts, phone parts, dental equipment parts, toys, and lamps. Panel C shows products ranked by density, with fabricated metal products and parts of transportation equipment and ammunition being the most dense, and missiles, space vehicles, trucks and aircraft being the least dense. Panel D shows the industries ranked by price per pound. Advanced industries such as the aerospace and x-ray apparatus industries are the most expensive per pound. The cheapest products per pound are structural metal products, ornamental metal work, and bolts, nuts, and screws.

¹⁵Except for truck and bus bodies, all products are smaller in volume than one 20-foot container. While it may seem that the volume of truck and bus bodies (12 TEUs) is too large, the average semi truck is around 8,262 cubic feet (72 feet long, 8.5 feet wide, 13.5 feet tall) or 7.05 TEUs which is in the ballpark of the data. The weight of an average semi truck (without any cargo) is around 35,000 pounds, which is also in the ballpark of the data, as Panel B shows. For these reasons, I do not consider the size and weight of truck and bus bodies to be an error in the data.

6.2 Regression Results for the 1982-2007 Period

I focus my main analysis on the 25 years from 1982 through 2007, a period during which computers and other electronic devices were introduced into virtually every household, office, and factory. In 1981, the year immediately preceding this period, IBM introduced the PC. Throughout the subsequent 25 years, new innovations such as laptop computers, Internet browsers, the Windows operating system, smartphones, tablets and many others arrived to the market.

Description of Table 2 Table 2 shows OLS and IV results illustrating the effect of computer purchases on industry TFP for the 1982-2007 period. Each row in the table contains 6 columns, depending on which controls are used. All columns control for time and 2-digit SIC sector fixed effects, to account for the fact that different periods and different sectors may have different rates TFP growth. Column (2) additionally controls for $SIC_2 \times Time$ fixed effects. These fixed effects account for the fact that different sectors may have had different growth rates at different periods in history. Column (3) adds lagged changes in log TFP as a control, to account for autoregression in changes in TFP. Columns (4)-(6) repeat each of these control combinations, with the addition of $\log Rev_{i,t-1}$ as a control variable. This accounts for the fact that changes in industry productivity between time $t - 1$ and time t may be explained by that industry's output at the beginning of the period, and not just by investment in computers and electronics. All errors are clustered at the industry level to account for heteroskedasticity. Correspondingly, the F-statistics reported for the instrumental variables results are the Effective F-statistics proposed by Montiel Olea and Pflueger (2013) and advocated by Andrews, Stock and Sun (2019).

Table 2 contains six panels. Panel A shows the result of running an OLS regression, using the controls specified in each column. Panel B shows first stage IV results, and Panel C shows second-stage IV results. In Panels A, B and C, the $(i, t)^{th}$ observation is weighted by its value added $ValueAdded_{i,t-1}$ at the beginning of the period. This weighting by value-added accounts for the fact industries are aggregates of multiple firms, and firms themselves are aggregates of smaller production units. At a disaggregate level, industries with higher value added represent more observations than industries with lower value added. Panels D, E and F are analogous, with unweighted observations.

The results in Table 2 are significant and the coefficients for the first and second stages do not vary much across specifications. For the weighted regressions, the IV estimates are around 0.06, and the F-statistics are all above 10. For the unweighted regressions, the IV estimates are around 0.07, and the F statistics are mostly above 9.¹⁶

¹⁶The only exception is column (2), where the F-statistic is 8.961

The table also reports counterfactual effects from expenditure on computers and electronics, and from electronic miniaturization. I give more details on how these counterfactual effects are computed in Subsection 6.7. Across all specifications, the effect of electronic miniaturization on aggregate TFP growth is 0.03% per year. This accounts for 3.4% of the annualized TFP growth for the period, which is 0.88% per year, and for 37.5% of the productivity growth of heavy manufacturing industries. After controlling for lagged TFP and lagged revenue, computer and electronics investment account for 0.07% – 0.08% of annualized TFP growth. This accounts for almost all the productivity improvements in heavy manufacturing.

6.3 Robustness Results and Additional Regressions

In the main regression analysis, I focused on the years 1982-2007 when the IT revolution was in full force. Table 3 shows the regression results for the full 1972-2007 sample. One can see that, for value-weighted regressions, the IV coefficients are around 0.055 and the F-statistics are above 9. The counterfactual effect from electronic miniaturization on TFP is 0.01%, which accounts for 1.7% of all TFP growth in this 35-year period, and 20% of all TFP growth in the heavy manufacturing sector.

6.4 Using all Manufacturing Industries: A Validation of The Identification Strategy

So far I have restricted the industry observations to be in the heavy manufacturing sectors whose 2-digit SIC code is in the 34-39 range. This includes industrial machinery, precision instruments and more, but excludes industries such as apparel and food manufacturing. I should not expect electronic miniaturization to lead to computers being directly adopted in these light manufacturing industries.¹⁷ The results in Table 4—which runs the analysis for all manufacturing industries¹⁸ in the 1982-2007 time period—confirms this intuition. In particular, Panels B and E show that an industry’s product size has no effect on that industry’s investment in electronics and computers, and that the corresponding F-statistics (shown in panels C and F) are very weak. By looking at the estimated effect of computer investment on manufacturing productivity, we can see that the coefficients have an upward bias. The predicted annualized growth in productivity is at least 0.62%, whereas the observed annualized productivity growth in manufacturing is 0.15%. Even with this upward bias, the predicted effect of electronic miniaturization on aggregate TFP is at most 0.06%, which is not

¹⁷This is especially true for the sample that ends in 2007, but even recent developments in wearable technology during the 2010s such as smart watches are likely not significant enough to show up in such aggregate statistics.

¹⁸Once again, I exclude computer and electronics industries from the sample.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: OLS Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.025*** (0.005)	0.019*** (0.005)	0.020*** (0.005)	0.025*** (0.005)	0.020*** (0.005)	0.021*** (0.005)
TFP Change From Investment in Electronics and Computers	0.04%	0.03%	0.02%	0.05%	0.04%	0.02%
Panel B: First Stage Results (Weighted by Value Added)						
$\log Size_i$	0.074*** (0.024)	0.074*** (0.024)	0.074*** (0.024)	0.075*** (0.024)	0.075*** (0.024)	0.075*** (0.024)
$\log Density_i$	0.244*** (0.051)	0.244*** (0.052)	0.241*** (0.052)	0.250*** (0.054)	0.251*** (0.055)	0.248*** (0.054)
Panel C: IV Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.060** (0.029)	0.060** (0.029)	0.065** (0.032)	0.056** (0.028)	0.056** (0.028)	0.062** (0.031)
F-test	10.91	10.61	10.67	11.17	10.91	10.98
TFP Change From Investment in Electronics and Computers	0.13%	0.13%	0.08%	0.12%	0.12%	0.08%
TFP Change From Electronic Miniaturization	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%
Panel D: OLS Results (Unweighted)						
$\Delta \log Comp_{it}$	0.024*** (0.005)	0.019*** (0.006)	0.019*** (0.006)	0.025*** (0.005)	0.020*** (0.006)	0.020*** (0.006)
TFP Change From Investment in Electronics and Computers	0.05%	0.04%	0.02%	0.05%	0.04%	0.02%
Panel E: First Stage Results (Unweighted)						
$\log Size_i$	0.065*** (0.022)	0.065*** (0.022)	0.065*** (0.022)	0.066*** (0.022)	0.066*** (0.022)	0.066*** (0.022)
$\log Density_i$	0.207*** (0.050)	0.207*** (0.050)	0.205*** (0.050)	0.214*** (0.052)	0.216*** (0.053)	0.214*** (0.052)
Panel F: IV Results (Unweighted)						
$\Delta \log Comp_{it}$	0.070** (0.032)	0.070** (0.032)	0.073** (0.034)	0.065** (0.031)	0.065** (0.030)	0.068** (0.033)
F-test	9.216	8.961	9.055	9.621	9.400	9.529
TFP Change From Investment in Electronics and Computers	0.11%	0.11%	0.07%	0.10%	0.10%	0.07%
TFP Change From Electronic Miniaturization	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%
Observations	735	735	735	735	735	735
Benchmark TFP Change	0.08%	0.08%	0.08%	0.08%	0.08%	0.08%
$SIC_2 \times Time$ Trend	No	Yes	Yes	No	Yes	Yes
Control for lagged change in TFP	No	No	Yes	No	No	Yes
Control for lagged revenue	No	No	No	Yes	Yes	Yes

Table 2: Regression Results for the 1982-2007 Sample. This table presents OLS and IV estimates of the effect of computer and electronics investment on TFP for the years 1982-2007, using a dataset of five-year stacked differences for heavy manufacturing industries (SIC Codes 3401-3999), excluding computers and electronics (SIC codes 3571-3579, 3671-3679). Panels A, B and C show results when observations are weighted by the value added of industry i at the beginning of the period. Panel A shows the estimate $\hat{\beta}$ in the OLS regression $\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma \Delta \log Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t}$, where $\log A_{i,t}$ is industry i 's TFP at time t , $Comp_{i,t}$ represents investment in computers and electronics, $Rev_{i,t-1}$ represents lagged revenue, and $\delta_{I \times t}$ is a 2-digit sector X time fixed effect. Panel B shows the first-stage estimates obtained from a 2SLS regression with first-stage $\Delta \log Comp_{i,t} = \alpha_{Size} \log Size_i + \alpha_{Density} \log Density_i + \nu \Delta \log Capex_{i,t} + \psi_{I \times t} + \tau_{i,t}$, where $Size_i$, $Density_i$ are the median size and density of industry i 's products. Panel C shows the corresponding second-stage results. Panels D, E and F show analogous results where observations are not weighted. Column (1) includes Sector and Time fixed effects, Column (2) adds Sector X Time fixed effects, and Column (3) adds lagged TFP as a control. Columns (4)-(6) are analogous to Columns (1)-(3), while adding lagged revenue as a control. Counterfactual effects of electronics investment on TFP are computed using equation (24). Counterfactual effects of electronic miniaturization on TFP are computed using equation (25). All standard errors are clustered at the industry level.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: OLS Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.026*** (0.005)	0.019*** (0.005)	0.021*** (0.005)	0.026*** (0.005)	0.020*** (0.005)	0.021*** (0.005)
TFP Change From Investment in Electronics and Computers	0.05%	0.03%	0.02%	0.05%	0.04%	0.02%
Panel B: First Stage Results (Weighted by Value Added)						
$\log Size_i$	0.054*** (0.019)	0.054*** (0.019)	0.060*** (0.020)	0.054*** (0.019)	0.055*** (0.019)	0.061*** (0.020)
$\log Density_i$	0.187*** (0.039)	0.186*** (0.040)	0.195*** (0.044)	0.191*** (0.041)	0.191*** (0.042)	0.200*** (0.046)
Panel C: IV Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.056* (0.033)	0.055* (0.033)	0.059* (0.035)	0.050 (0.033)	0.050 (0.033)	0.054 (0.034)
F-test	10.01	9.625	9.642	10.34	10.04	9.934
TFP Change From Investment in Electronics and Computers	0.13%	0.12%	0.05%	0.11%	0.11%	0.04%
TFP Change From Electronic Miniaturization	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
Panel D: OLS Results (Unweighted)						
$\Delta \log Comp_{it}$	0.025*** (0.005)	0.019*** (0.005)	0.021*** (0.005)	0.026*** (0.005)	0.020*** (0.005)	0.022*** (0.005)
TFP Change From Investment in Electronics and Computers	0.05%	0.04%	0.02%	0.05%	0.04%	0.02%
Panel E: First Stage Results (Unweighted)						
$\log Size_i$	0.027** (0.013)	0.027** (0.013)	0.041** (0.016)	0.027** (0.012)	0.028** (0.012)	0.041*** (0.016)
$\log Density_i$	0.107*** (0.028)	0.105*** (0.028)	0.123*** (0.037)	0.111*** (0.027)	0.111*** (0.028)	0.129*** (0.039)
Panel F: IV Results (Unweighted)						
$\Delta \log Comp_{it}$	0.068 (0.056)	0.066 (0.056)	0.055 (0.049)	0.058 (0.055)	0.057 (0.055)	0.049 (0.048)
F-test	6.209	5.874	6.450	7.033	6.874	6.810
TFP Change From Investment in Electronics and Computers	0.10%	0.10%	0.05%	0.09%	0.09%	0.04%
TFP Change From Electronic Miniaturization	0.02%	0.02%	0.02%	0.02%	0.02%	0.01%
Observations	1,011	1,011	882	1,011	1,011	882
Benchmark TFP Change	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%
$SIC_2 \times Time$ Trend	No	Yes	Yes	No	Yes	Yes
Control for lagged change in TFP	No	No	Yes	No	No	Yes
Control for lagged revenue	No	No	No	Yes	Yes	Yes

Table 3: Regression Results for the 1972-2007 Sample. This table presents OLS and IV estimates of the effect of computer and electronics investment on TFP for the years 1972-2007, using a dataset of five-year stacked differences for heavy manufacturing industries (SIC Codes 3401-3999), excluding computers and electronics (SIC codes 3571-3579, 3671-3679). Panel A shows the estimate $\hat{\beta}$ in the OLS regression $\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma \Delta \log Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t}$, where $\log A_{i,t}$ is industry i 's TFP at time t , $Comp_{i,t}$ represents investment in computers and electronics, $Rev_{i,t-1}$ represents lagged revenue, and $\delta_{I \times t}$ is a 2-digit sector X time fixed effect. Panel B shows the first-stage estimates obtained from a 2SLS regression with first-stage $\Delta \log Comp_{i,t} = \alpha_{Size} \log Size_i + \alpha_{Density} \log Density_i + \nu \Delta \log Capex_{i,t} + \psi_{I \times t} + \tau_{i,t}$, where $Size_i$, $Density_i$ are the median size and density of industry i 's products. Panel C shows the corresponding second-stage results. Column (1) includes Sector and Time fixed effects, Column (2) adds Sector X Time fixed effects, and Column (3) adds lagged TFP as a control. Columns (4)-(6) are analogous to Columns (1)-(3), while adding lagged revenue as a control. Counterfactual effects of electronics investment on TFP are computed using equation (24). Counterfactual effects of electronic miniaturization on TFP are computed using equation (25). All standard errors are clustered at the industry level.

much higher than the predicted effect when the sample included only heavy manufacturing industries.

6.5 Regression Results without using Density as an Instrument

Using the median product density as an instrument in Table 2 gives the regressions some power, but may not be seen as immediately justified by the model. In this section I show that using only median product weight as an instrument yields significant results, with similar counterfactual effects of miniaturization on TFP growth as the main regression.

Table 5 shows the results using median product weight as the sole instrumental variable. The first-stage coefficients are significant and positive, and the F-statistics are strong. The counterfactual effects of electronic miniaturization on productivity growth are around 0.02%, which are slightly attenuated to the counterfactual effects obtained in Table 2. One drawback of using only the median product weight as the instrumental variable is that the second-stage results are not significant.

For completeness, Table 6 shows the same results using median product volume as the sole instrumental variable. The results in this case are still positive, but not as strong, with very weak F-statistics and attenuated effects of electronic miniaturization on TFP growth. A potential explanation for why these results are weaker is that the measurements of weight—which come from customs data—are much more precise than the measurements of volume. In the raw data, the weight-per-unit of a shipment is computed by dividing the shipment’s weight by the number of units reported. The volume of the shipment is computed by measuring the TEUs of containers that the shipment came in, and dividing by the number of units reported. Since weight is precisely measured on a scale whereas volume is estimated from TEUs, the latter measurement is naturally more noisy.

6.6 Rolling Regression Analysis

Finally, I analyze how the estimates change through time. To do this, I compute rolling regression results covering the period of 35 calendar years from 1972 through 2007. I divide the overall sample period into five partially overlapping 15-year periods. The second through fifth windows each begin five years after the first year of the preceding period, as follows: 1972-1987, 1977-1992, 1982-1997, 1987-2002, and 1992-2007. The results are reported in Appendix F, Tables 7 through 11. For the first two of these five windows—which end before 1992—the results are not significant. However, for the last three windows, covering the 1982-2007 period, the results become stronger over time. The most robust results, with very strong F-statistics, appear in the last period (1992-2007), which saw the explosion in the use of personal

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: OLS Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.019*** (0.004)	0.016*** (0.004)	0.016*** (0.004)	0.020*** (0.004)	0.019*** (0.004)	0.018*** (0.004)
TFP Change From Investment in Electronics and Computers	0.09%	0.07%	0.05%	0.09%	0.09%	0.06%
Panel B: First Stage Results (Weighted by Value Added)						
$\log Size_i$	0.016 (0.016)	0.016 (0.016)	0.020 (0.016)	0.020 (0.015)	0.020 (0.015)	0.020 (0.015)
$\log Density_i$	0.092** (0.040)	0.092** (0.041)	0.109*** (0.039)	0.101*** (0.038)	0.098** (0.039)	0.102*** (0.039)
Panel C: IV Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.159** (0.066)	0.160** (0.066)	0.181*** (0.070)	0.173** (0.069)	0.174** (0.069)	0.195*** (0.073)
F-test	3.979	3.909	4.244	3.849	3.746	4.074
TFP Change From Investment in Electronics and Computers	0.89%	0.89%	0.73%	0.99%	1.00%	0.81%
TFP Change From Electronic Miniaturization	0.04%	0.04%	0.05%	0.06%	0.05%	0.06%
Panel D: OLS Results (Unweighted)						
$\Delta \log Comp_{it}$	0.019*** (0.004)	0.016*** (0.004)	0.016*** (0.004)	0.020*** (0.004)	0.019*** (0.004)	0.018*** (0.004)
TFP Change From Investment in Electronics and Computers	0.09%	0.07%	0.05%	0.09%	0.09%	0.06%
Panel E: First Stage Results (Unweighted)						
$\log Size_i$	0.012 (0.015)	0.012 (0.015)	0.015 (0.015)	0.015 (0.014)	0.015 (0.014)	0.015 (0.014)
$\log Density_i$	0.085** (0.039)	0.085** (0.040)	0.100*** (0.038)	0.091** (0.036)	0.089** (0.038)	0.092** (0.037)
Panel F: IV Results (Unweighted)						
$\Delta \log Comp_{it}$	0.194** (0.078)	0.194** (0.077)	0.211*** (0.081)	0.215** (0.085)	0.216** (0.085)	0.232*** (0.088)
F-test	3.324	3.295	3.571	3.125	3.067	3.335
TFP Change From Investment in Electronics and Computers	0.73%	0.74%	0.62%	0.79%	0.80%	0.67%
TFP Change From Electronic Miniaturization	0.04%	0.04%	0.06%	0.06%	0.06%	0.06%
Observations	1,848	1,848	1,848	1,848	1,848	1,848
Benchmark TFP Change	0.15%	0.15%	0.15%	0.15%	0.15%	0.15%
$SIC_2 \times Time$ Trend	No	Yes	Yes	No	Yes	Yes
Control for lagged change in TFP	No	No	Yes	No	No	Yes
Control for lagged revenue	No	No	No	Yes	Yes	Yes

Table 4: Regression Results for the 1982-2007 Sample and All Manufacturing Industries. This table presents OLS and IV estimates of the effect of computer and electronics investment on TFP for the years 1982-2007, using a dataset of five-year stacked differences for all manufacturing industries (SIC Codes 2000-3999), excluding computers and electronics (SIC codes 3571-3579, 3671-3679). Panel A shows the estimate $\hat{\beta}$ in the OLS regression $\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma \Delta \log Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t}$, where $\log A_{i,t}$ is industry i 's TFP at time t , $Comp_{i,t}$ represents investment in computers and electronics, $Rev_{i,t-1}$ represents lagged revenue, and $\delta_{I \times t}$ is a 2-digit sector X time fixed effect. Panel B shows the first-stage estimates obtained from a 2SLS regression with first-stage $\Delta \log Comp_{i,t} = \alpha_{Size} \log Size_i + \alpha_{Density} \log Density_i + \nu \Delta \log Capex_{i,t} + \psi_{I \times t} + \tau_{i,t}$, where $Size_i$, $Density_i$ are the median size and density of industry i 's products. Panel C shows the corresponding second-stage results. Column (1) includes Sector and Time fixed effects, Column (2) adds Sector X Time fixed effects, and Column (3) adds lagged TFP as a control. Columns (4)-(6) are analogous to Columns (1)-(3), while adding lagged revenue as a control. Counterfactual effects of electronics investment on TFP are computed using equation (24). Counterfactual effects of electronic miniaturization on TFP are computed using equation (25). All standard errors are clustered at the industry level.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: OLS Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.025*** (0.005)	0.019*** (0.005)	0.020*** (0.005)	0.025*** (0.005)	0.020*** (0.005)	0.021*** (0.005)
TFP Change From Investment in Electronics and Computers	0.04%	0.03%	0.02%	0.05%	0.04%	0.02%
Panel B: First Stage Results (Weighted by Value Added)						
$\log Weight_i$	0.080*** (0.024)	0.080*** (0.024)	0.080*** (0.024)	0.081*** (0.024)	0.081*** (0.024)	0.081*** (0.024)
Panel C: IV Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.041 (0.035)	0.041 (0.035)	0.040 (0.038)	0.037 (0.035)	0.037 (0.035)	0.037 (0.038)
F-test	10.98	10.67	10.85	11.41	11.20	11.33
TFP Change From Investment in Electronics and Computers	0.08%	0.08%	0.05%	0.07%	0.08%	0.05%
TFP Change From Electronic Miniaturization	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%
Panel D: OLS Results (Unweighted)						
$\Delta \log Comp_{it}$	0.024*** (0.005)	0.019*** (0.006)	0.019*** (0.006)	0.025*** (0.005)	0.020*** (0.006)	0.020*** (0.006)
TFP Change From Investment in Electronics and Computers	0.05%	0.04%	0.02%	0.05%	0.04%	0.02%
Panel E: First Stage Results (Unweighted)						
$\log Weight_i$	0.070*** (0.022)	0.070*** (0.023)	0.070*** (0.022)	0.071*** (0.022)	0.072*** (0.022)	0.072*** (0.022)
Panel F: IV Results (Unweighted)						
$\Delta \log Comp_{it}$	0.045 (0.038)	0.045 (0.038)	0.043 (0.041)	0.041 (0.037)	0.041 (0.037)	0.039 (0.040)
F-test	9.849	9.577	9.741	10.46	10.28	10.43
TFP Change From Investment in Electronics and Computers	0.07%	0.07%	0.04%	0.07%	0.07%	0.04%
TFP Change From Electronic Miniaturization	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%
Observations	735	735	735	735	735	735
Benchmark TFP Change	0.08%	0.08%	0.08%	0.08%	0.08%	0.08%
$SIC_2 \times Time$ Trend	No	Yes	Yes	No	Yes	Yes
Control for lagged change in TFP	No	No	Yes	No	No	Yes
Control for lagged revenue	No	No	No	Yes	Yes	Yes

Table 5: Regression Results for the 1982-2007 Sample using Weight as the sole instrumental variable. This table presents OLS and IV estimates of the effect of computer and electronics investment on TFP for the years 1982-2007, using a dataset of five-year stacked differences for heavy manufacturing industries (SIC Codes 3401-3999), excluding computers and electronics (SIC codes 3571-3579, 3671-3679). Panel A shows the estimate $\hat{\beta}$ in the OLS regression $\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma \Delta \log Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t}$, where $\log A_{i,t}$ is industry i 's TFP at time t , $Comp_{i,t}$ represents investment in computers and electronics, $Rev_{i,t-1}$ represents lagged revenue, and $\delta_{I \times t}$ is a 2-digit sector X time fixed effect. Panel B shows the first-stage estimates obtained from a 2SLS regression with first-stage $\Delta \log Comp_{i,t} = \alpha_{Weight} \log Weight_i + \nu \Delta \log Capex_{i,t} + \psi_{I \times t} + \tau_{i,t}$, where $Weight_i$ is the median weight of industry i 's product. Panel C shows the corresponding second-stage results. Column (1) includes Sector and Time fixed effects, Column (2) adds Sector X Time fixed effects, and Column (3) adds lagged TFP as a control. Columns (4)-(6) are analogous to Columns (1)-(3), while adding lagged revenue as a control. Counterfactual effects of electronics investment on TFP are computed using equation (24). Counterfactual effects of electronic miniaturization on TFP are computed using equation (25). All standard errors are clustered at the industry level.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: OLS Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.025*** (0.005)	0.019*** (0.005)	0.020*** (0.005)	0.025*** (0.005)	0.020*** (0.005)	0.021*** (0.005)
TFP Change From Investment in Electronics and Computers	0.04%	0.03%	0.02%	0.05%	0.04%	0.02%
Panel B: First Stage Results (Weighted by Value Added)						
$\log Size_i$	0.060** (0.026)	0.060** (0.026)	0.060** (0.026)	0.060** (0.026)	0.060** (0.026)	0.061** (0.026)
Panel C: IV Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.023 (0.044)	0.023 (0.044)	0.018 (0.048)	0.021 (0.045)	0.021 (0.045)	0.016 (0.048)
F-test	5.346	5.198	5.363	5.469	5.347	5.499
TFP Change From Investment in Electronics and Computers	0.04%	0.04%	0.02%	0.04%	0.04%	0.02%
TFP Change From Electronic Miniaturization	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%
Panel D: OLS Results (Unweighted)						
$\Delta \log Comp_{it}$	0.024*** (0.005)	0.019*** (0.006)	0.019*** (0.006)	0.025*** (0.005)	0.020*** (0.006)	0.020*** (0.006)
TFP Change From Investment in Electronics and Computers	0.05%	0.04%	0.02%	0.05%	0.04%	0.02%
Panel E: First Stage Results (Unweighted)						
$\log Size_i$	0.053** (0.024)	0.053** (0.024)	0.053** (0.024)	0.054** (0.024)	0.054** (0.024)	0.054** (0.024)
Panel F: IV Results (Unweighted)						
$\Delta \log Comp_{it}$	0.023 (0.046)	0.023 (0.046)	0.016 (0.050)	0.020 (0.047)	0.020 (0.047)	0.014 (0.050)
F-test	4.977	4.840	4.993	5.133	5.016	5.162
TFP Change From Investment in Electronics and Computers	0.04%	0.04%	0.02%	0.04%	0.04%	0.02%
TFP Change From Electronic Miniaturization	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
Observations	735	735	735	735	735	735
Benchmark TFP Change	0.08%	0.08%	0.08%	0.08%	0.08%	0.08%
$SIC_2 \times Time$ Trend	No	Yes	Yes	No	Yes	Yes
Control for lagged change in TFP	No	No	Yes	No	No	Yes
Control for lagged revenue	No	No	No	Yes	Yes	Yes

Table 6: Regression Results for the 1982-2007 Sample using Size as the sole instrumental variable. This table presents OLS and IV estimates of the effect of computer and electronics investment on TFP for the years 1982-2007, using a dataset of five-year stacked differences for heavy manufacturing industries (SIC Codes 3401-3999), excluding computers and electronics (SIC codes 3571-3579, 3671-3679). Panel A shows the estimate $\hat{\beta}$ in the OLS regression $\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma \Delta \log Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t}$, where $\log A_{i,t}$ is industry i 's TFP at time t , $Comp_{i,t}$ represents investment in computers and electronics, $Rev_{i,t-1}$ represents lagged revenue, and $\delta_{I \times t}$ is a 2-digit sector X time fixed effect. Panel B shows the first-stage estimates obtained from a 2SLS regression with first-stage $\Delta \log Comp_{i,t} = \alpha_{Size} \log Size_i + \nu \Delta \log Capex_{i,t} + \psi_{I \times t} + \tau_{i,t}$, where $Size_i$ is the median size of industry i 's product. Panel C shows the corresponding second-stage results. Column (1) includes Sector and Time fixed effects, Column (2) adds Sector X Time fixed effects, and Column (3) adds lagged TFP as a control. Columns (4)-(6) are analogous to Columns (1)-(3), while adding lagged revenue as a control. Counterfactual effects of electronics investment on TFP are computed using equation (24). Counterfactual effects of electronic miniaturization on TFP are computed using equation (25). All standard errors are clustered at the industry level.

computers both at home and at the workplace. For each of the last three 15-year windows, the estimated effect of electronic miniaturization on TFP growth is around $0.02 - 0.03\%$.

6.7 Computing The Effect of Electronic Miniaturization on Aggregate Productivity

In the above analysis, I used the regression results to estimate how much of the increase in aggregate TFP can be explained by industries using more computers and electronics, and furthermore, how much of this effect is due to electronic miniaturization. In this subsection, I show how these computations are performed.

The effect of computer use on an individual industry's TFP is relatively straightforward to estimate. One can use the coefficient $\hat{\beta}$ obtained from regression (23) to compute $\Delta \log \widehat{TFP}_{i,t} \approx \hat{\beta} \Delta \log Comp_{i,t}$.

To aggregate this effect across industries, I rely on Hulten's Theorem (1978). In an economy with N industries, each of which has a Hicks-Neutral production technology, the change in aggregate TFP obtained from industry-specific TFP shocks $\Delta \log TFP_i$ is given by

$$\Delta \log TFP_t = \sum_{i=1}^N D_{i,t} \cdot \Delta \log TFP_{i,t}$$

where $D_{i,t} = \frac{P_{i,t}Y_{i,t}}{GDP_t}$ is industry i 's sales share as a fraction of GDP. The share $D_{i,t}$ is commonly referred to as industry i 's Domar weight.

Because there are multiple time periods t , each of which spans five years, the annualized average effect of industrial computer use on aggregate TFP is given by

$$\Delta \log \widehat{TFP} = \frac{1}{5T} \sum_{t=1}^T \Delta \log \widehat{TFP}_t. \quad (24)$$

Computing the Effect of Electronic Miniaturization on Productivity using Instrumental Variables

To estimate the effect of electronic miniaturization on aggregate productivity, I use the first-stage regression to compute how much of industry i 's computer use $\Delta \log Comp_i$ can be attributed to miniaturization. The logic behind the first stage regression (22) is that industries with larger products, such as the aerospace industry, will have less binding size constraints (higher τ_i), and will be able to incorporate larger and more powerful computers into their products, making them more productive.

Because the data on product size and density is static, I do not observe changes in

electronic sizes throughout the sample. Instead, I use Moore’s Law, which states that the size of transistors has been shrinking by a factor of 2 every 18 months. At an annualized level, this means that transistors have been shrinking by a factor of $2^{1/1.5} = 1.5874$ every year. As discussed in the introduction, this shrinking of transistors has been accompanied by a corresponding shrinking of computers and electronic devices.

To apply Moore’s Law in computing counterfactuals, recall that in the structural model, firms will choose a set of inputs S_i which maximizes their profits subject to the size constraint (5). If we decompose $S_i = S_{i,Comp} \cup S_{i,-Comp}$ as the union of computer and electronics industries ($S_{i,Comp}$)—which shrink by a factor of 1.5874 every year—and all other industries ($S_{i,-Comp}$)—which do not substantially change in size—then we can write the size constraint for industry i at time t as

$$\sum_{j \in S_{i,Comp}} \frac{\theta_j}{1.5874^t} + \sum_{j \in S_{i,-Comp}} \theta_j \leq \tau_i.$$

The key step to compute the counterfactual is to notice that the above size constraint is equivalent to

$$\sum_{j \in S_{i,Comp}} \theta_j + \sum_{j \in S_{i,-Comp}} \theta_j \cdot 1.5874^t \leq \tau_i \cdot 1.5874^t.$$

In the model, the production technology would change in *exactly the same way* if all inputs *except* for computers and electronics were to grow in size and weight by a factor of 1.5874 every year. The intuition is that, if offices, airplanes, and cars all grew at the same rate, while computers stayed the same size, then firms would be able to embed more computers into machines, and obtain more productivity-enhancing combinations.

The first-stage regression (22) then asks: for each industry i and time period t , how much higher would the left-hand side variable $\Delta \log Comp_{i,t}$ be if $\Delta \log Size_i$ increased by 1.5874?¹⁹ The answer is

$$\widehat{\Delta Comp_{i,t}} = \widehat{\alpha_{Size}} \cdot 1.5874.$$

This is the effect of shrinking computers (or equivalently, growing industry i ’s product) on industry i ’s computer and electronic investment.

The estimated change in computer expenditures $\widehat{\Delta Comp_{i,t}}$ is then used as an input in the computation of aggregate TFP, weighing by the appropriate Domar weights. Putting all equations together, I obtain that the effect of electronic miniaturization on aggregate TFP

¹⁹In practice, since each time period t is 5 years, the increase is $5 \cdot 1.5874$.

is given by

$$\Delta \widehat{\widehat{\log TFP}}_t = \frac{1}{T} \sum_{i=1}^T \sum_{i=1}^N D_{i,t} \widehat{\beta_{size}} \cdot 1.5874. \quad (25)$$

7 Conclusion

Over the past few decades, transistors have been getting exponentially smaller and cheaper over time, leading to significant changes across industries and countries. The effect of cheaper transistors on GDP has been widely studied. This paper is the first attempt to build a model where the miniaturization of electronic components leads to increases in aggregate productivity.

To study the effect of smaller electronics on GDP, I have introduced a new model that incorporates physical constraints into the profit-maximization problem of the firm. Even though the constraints are discrete, one can still derive tractable formulas for aggregate output, compute comparative statics on the effect of size spillovers on output, and estimate growth rates.

My model is flexible enough to allow for arbitrary combinatorial constraints instead of the baseline size constraint. This opens the door to more general models that capture physical constraints in production. In future work, I hope to explore these more general constraints.

The model also has testable implications for which industries are more likely to adopt new electronic innovations: industries with larger products such as aviation and heavy machinery will have more slack in their physical constraints, and will be able to incorporate newly arrived products more easily than industries with smaller products such as medical and precision instruments.

I have used these implications to apply the model to the data. Using a new dataset on the sizes and weights of goods imported into the United States, I have shown that industries with larger products are more likely to use electronic inputs. This increase in computer and electronic expenditures accounts for about 37.5% of the annualized increase in TFP for heavy manufacturing sectors, and about 3.5% of the annualized increase in TFP for the whole American economy between 1982 and 2007.

I have left unexplored the competitive aspects of semiconductor innovation, and the races between firms to develop ever-shrinking transistors. In future work, I hope to develop the model further to include markups, an oligopolistic market structure and an endogenous growth model where firms invest in research and compete to develop smaller varieties within an industry. Combining this model with micro-level data on semiconductor production would yield new insights on how overcoming physical constraints in production leads to economic

growth.

References

- [1] Acemoglu, D., Autor, D. and Patterson C. (2017). Bottlenecks: Sectoral Imbalances and the US Productivity Slowdown. *Manuscript*.
- [2] Acemoglu D., and Azar ,P. (2020). Endogenous Production Networks. *Econometrica*, 88(1), 33-82.
- [3] Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5), 1977-2016
- [4] Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2017). Microeconomic origins of macroeconomic tail risks. *The American Economic Review*, 107(1), 54-108.
- [5] Acemoglu, D., and Tahbaz-Salehi, A. (2020). Firms, failures, and fluctuations: the macroeconomics of supply chain disruptions *National Bureau of Economic Research Working Paper*. (No. w27565).
- [6] Aghion, P., and Howitt, P. (1992). A model of Growth through Creative Destruction. *Econometrica*, 60(2), 323-351.
- [7] Andrews, I., Stock, J.H. and Sun, L. (2019). Weak instruments in instrumental variables regression: Theory and practice. *Annual Review of Economics*, 11, pp.727-753.
- [8] Auerswald, P., Kauffman, S., Lobo, J., & Shell, K. (2000). The production recipes approach to modeling technological innovation: An application to learning by doing. *Journal of Economic Dynamics and Control*, 24(3), 389-450.
- [9] Baqaee, D. R. 2018: Cascading Failures in Production Networks, *Econometrica*, 86 (5), 1819–1838.
- [10] Baqaee, D. R., and Farhi, E. (2019a). The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten’s Theorem . *Econometrica*, 87 (4), 1155–1203.
- [11] Baqaee, D.R., and Farhi, E. (2019b). Productivity and Misallocation in General Equilibrium. *The Quarterly Journal of Economics*, 135 (1), 105–163.
- [12] Bartelme, D., and Gorodnichenko Y. (2015) Linkages and Economic Development. *National Bureau of Economic Research Working Paper No. 21251*.

- [13] Bigio, S., and La'O, J. (2016). Financial frictions in production networks. *National Bureau of Economic Research Working Paper No. 22212*.
- [14] Boehm, J. and Oberfield, E. (2018). Misallocation in the Market for Inputs: Enforcement and the Organization of Production, NBER Working Paper 24937, August 2018.
- [15] Caliendo, L., Parro, F., and Tsyvinski, A. (2017). Distortions and the Structure of the World Economy. *National Bureau of Economic Research Working Paper No. w23332*.
- [16] Eaton, J., and Kortum, S. (2001). Technology, trade, and growth: A unified framework. *European economic review*, 45(4), 742-755.
- [17] Fadinger, H., Ghiglino C. and Teteryatnikova M. (2016) Income differences and input-output structure. *CEPR Discussion Paper No. 11547*.
- [18] Gabaix, X. (2011). The granular origins of aggregate fluctuations. *Econometrica*, 79(3), 733-772.
- [19] Ghiglino, C. (2012). Random walk to innovation: Why productivity follows a power law. *Journal of Economic Theory*, 147(2), 713-737.
- [20] Grossman, G. M., and Helpman, E. (1991). Quality ladders in the theory of growth. *The Review of Economic Studies*, 58(1), 43-61.
- [21] Grossman, G. M., and Helpman, E. (1992). Innovation and growth in the global economy. *MIT press*.
- [22] Hulten, C. R. (1978). Growth accounting with intermediate inputs. *The Review of Economic Studies*, 511-518.
- [23] Jones, C. I. (1995). R & D-based models of economic growth. *Journal of political Economy*, 103(4), 759-784.
- [24] Jones, C. I. (2005). The shape of production functions and the direction of technical change. *The Quarterly Journal of Economics* 120(2), 517-549.
- [25] Jones, C. I. (2011). Intermediate goods and weak links in the theory of economic development. *American Economic Journal: Macroeconomics* 3(2), 1-28.
- [26] Jones, C. I. (2021). Recipes and Economic Growth: A Combinatorial March Down an Exponential Tail. *National Bureau of Economic Research Working Paper (No. w28340)*.

- [27] Jorgenson, D.W. (2005). Accounting for Growth in the Information Age. *Handbook of Economic Growth* 1 (A), 743-815.
- [28] Jorgenson, D.W., Stiroh, K.J. (1999). Information technology and growth. *American Economic Review* 89 (2), 109–115.
- [29] Jorgenson, D.W., Stiroh, K.J. (2000). U.S. economic growth at the industry level. *American Economic Review* 90 (2), 161–167.
- [30] Kennan, J. (2001). Uniqueness of Positive Fixed Points for Increasing Concave Functions on \mathbb{R}^n : An Elementary Result. *Review of Economic Dynamics*, 4(4), 893–899.
- [31] Liu, E. (2019). Industrial policies and economic development. *Quarterly Journal of Economics* 139(4).
- [32] Liu, E. and Tsyvinski, A. (2020). Dynamical Structure and Spectral Properties of Input-Output Networks *National Bureau of Economic Research Working Paper (No. w28178)*.
- [33] Long, J. B., and Plosser, C. I. (1983). Real business cycles. *Journal of political Economy*, 91(1), 39-69. Chicago
- [34] Olea, J.L.M. and Pflueger, C. (2013). A robust test for weak instruments. *Journal of Business & Economic Statistics*, 31(3), pp.358-369.
- [35] Oberfeld, E. (2017). A theory of input-output architecture. *Technical report*.
- [36] Romer, P. M. (1990). Endogenous technological change. *Journal of political Economy*, 98(5, Part 2), S71-S102.
- [37] Taschereau-Dumouchel, M. (2020). Cascades and fluctuations in an economy with an endogenous production network. *Manuscript*.
- [38] Weitzman, M. L. (1998). Recombinant growth. *The Quarterly Journal of Economics*, 113(2), 331-360.

A Proofs from Section 2

Proof of Lemma 1. Let $\mathcal{A}_i = \{A_i \in \mathbb{R}^{|\mathcal{F}|} : |\arg \min_S K(S, P^*, A_i)| \geq 2\}$ be the set of productivity parameters which admit more than one cost-minimizing set. Since the distribution Ψ of A_i is absolutely continuous with respect to Lebesgue measure, it suffices to show that \mathcal{A}_i has Lebesgue measure zero. Let S_i^*, S_i^{**} be two different possible input sets, and define $\mathcal{A}_i(S^*, S^{**}) = \{A_i : K(S^*, P^*, A_i(S^*)) = K(S^{**}, P^*, A_i(S^{**}))\}$. The following set inclusion holds

$$\mathcal{A}_i \subset \cup_{S^*, S^{**}} \mathcal{A}_i(S^*, S^{**}).$$

Since the right-hand side of this set inclusion is a countable union of sets, it suffices to show that each $\mathcal{A}_i(S^*, S^{**})$ has measure zero for each pair S^*, S^{**} . For any such pair, define the function

$$\Delta_i(A_i, S^*, S^{**}) = K(S_i^*, P^*, A_i(S_i^*)) - K(S_i^{**}, P^*, A_i(S_i^{**}))$$

and note that $\mathcal{A}_i(S^*, S^{**})$ is exactly the set of productivity parameters A_i for which $\Delta_i(A_i, S^*, S^{**}) = 0$. Since K is strictly increasing in A , we have $A_i \in \mathcal{A}_i(S^*, S^{**})$ if and only if $A_i(S_i^*) = A_i(S_i^{**})$. But this implies that $\mathcal{A}_i(S^*, S^{**})$ is an $|\mathcal{F}| - 1$ dimensional subset of a $|\mathcal{F}|$ dimensional space, so it must have measure zero. ■

B Proofs from Section 3

Proof of Proposition 1. The primary price vector, if it exists, satisfies the system of equations

$$P_j = \sum_{k=1}^K \alpha_{jk} P_k + \frac{1}{A_j}.$$

Letting $B = (\frac{1}{A_1}, \dots, \frac{1}{A_J})'$, write this in matrix form as

$$(I - \alpha)P = B.$$

Since $(I - \alpha)^{-1}$ exists and all of its coefficients are non-negative, the unique price vector is given by $P = (I - \alpha)^{-1}B$. ■

Proof of Theorem 1. First, I show that the condition is necessary. Equation (6) implies that if an equilibrium exists, then $P_j = K_j(P)$ for every primary industry.

Sufficiency To prove sufficiency, I proceed in three steps:

1. I show that intermediate and final industry prices can be defined so that equation (6) is always satisfied.
2. I construct equilibrium quantities for the special case where $Y_f = 1$, so that only one unit of the final good is produced in equilibrium. In this special case, I denote equilibrium intermediate demands, labor demands and output by the lowercase vector variables x, ℓ, y .
3. I solve for the equilibrium level of final output Y_f , and use the fact that all production functions have constant returns to scale to solve for the equilibrium quantities $(X, Y, L) = Y_f \cdot (x, y, \ell)$.

Step 1: Solving for Prices and Intermediate Input Sets To prove the opposite direction, note that for any intermediate industry $i \in \mathcal{I}$, the cost function $K(S, P, A_i(S), \tau_i)$ depends only on the price vector $P_{pri} = (P_0, P_1, \dots, P_J)$ of primary industries, and not on any intermediate or final prices. Define $P_i = \min_{S' \in \mathcal{F}(\theta, \tau_i)} K_i(S', P_{pri}, A_i(S'), \tau_i)$ and $P_f = K_f((P_i)_{i \in \mathcal{I}})$, so that equation (6) is satisfied for all industries. The equilibrium input set S_i for firm i is given by $S_i = TieBreak(\arg \min_{S' \in \mathcal{F}(\theta, \tau_i)} K(S', P, A_i(S')), \tau_i)$, so that equation (7) is satisfied for all intermediate industries i .

For convenience of notation, it will be helpful to define, for every intermediate industry $i = (\tau, \iota)$, the normalized price $\bar{P}_{\tau\iota} = \frac{P_{\tau\iota}}{A_{\tau\iota}(S_{\tau\iota})}$. Since intermediate production functions are Hicks-Neutral, $\bar{P}_{\tau\iota}$ does not depend on $A_{\tau\iota}$.

Step 2a: Final Industry Demands when $Y_f = 1$ I now solve for the special case where only one unit of the final good is produced. Define $x_{f\tau\iota} = \arg \min_{x'} \int_0^\infty \int_0^1 P_{\tau\iota} x'_{f\tau\iota} \nu(\tau) d\tau d\iota$ subject to the constraint $(\int_0^\infty (\int_0^1 (x'_{f\tau\iota})^{\frac{\gamma-1}{\gamma}} d\iota)^{\frac{\gamma}{\gamma-1}} \frac{\beta-1}{\beta} \nu(\tau) d\tau)^{\frac{\beta}{\beta-1}} = 1$. Because this is a nested-CES unit cost minimization problem, the first order conditions for this optimization problem yield

$$x_{f\tau\iota} = \left(\frac{P_{\tau\iota}}{P_\tau}\right)^{-\gamma} \left(\frac{P_\tau}{P_f}\right)^{-\beta} = \left(\frac{\bar{P}_{\tau\iota}}{P_\tau}\right)^{-\gamma} \left(\frac{P_\tau}{P_f}\right)^{-\beta} A_{\tau\iota}(S_{\tau\iota})^\gamma, \quad (26)$$

where $P_\tau = (\int_0^1 P_{\tau\iota}^{1-\gamma} d\iota)^{\frac{1}{1-\gamma}}$ is a price index over all industries with threshold τ . Note that P_τ is deterministic, since $P_\tau = (\int_0^1 P_{\tau\iota}^{1-\gamma} d\iota)^{\frac{1}{1-\gamma}} = \mathbb{E}[A_{\tau\iota}(S_{\tau\iota})^{\gamma-1}]^{\frac{1}{\gamma-1}} (\int_0^1 \bar{P}_{\tau\iota}^{1-\gamma} d\iota)^{\frac{1}{1-\gamma}}$, where the expectation is taken over all industries (τ, ι) with the same threshold τ . Define normalized demands that do not depend on the random productivity terms $A_{\tau\iota}(S_{\tau\iota})$ as $\bar{x}_{f\tau\iota} = \frac{x_{f\tau\iota}}{A_{\tau\iota}(S_{\tau\iota})^\gamma} = \left(\frac{\bar{P}_{\tau\iota}}{P_\tau}\right)^{-\gamma} \left(\frac{P_\tau}{P_f}\right)^{-\beta}$.

Step 2b: Intermediate Industry Demands when $Y_f = 1$ Since markets clear, and intermediate industry outputs are used solely by the final industry, we have $y_i = x_{fi}$. Define normalized output as $\bar{y}_i = \bar{x}_{fi}$.

Given industry i 's equilibrium input set S_i and productivity A_i , intermediate input and labor demands are given by

$$(x_i, \ell_i) = \arg \min_{(x'_{ij})_{j \in S_i}, \ell'_i} \sum_{j \in S} P_j x'_{ij} + \ell'_i \text{ subject to: } A_i(S_i) F(S_i, x', \ell'_i) = y_i \text{ and } G(y_i, x, \theta, S_i) \leq \tau_i y_i. \quad (27)$$

So far, the intermediate demand x_i is well defined. However, aggregate quantities $\int_{\mathcal{I}} x_i di$ may diverge to infinity. I show that, under Assumption 1, this does not happen. Since the intermediate production function is Hicks-Neutral, one can write $(x_i(A_i), \ell_i(A_i)) = A_i(S_i)^\gamma (\bar{x}_i(S_i), \bar{\ell}_i(S_i))$, where $\bar{x}, \bar{\ell}$ are independent of A_i and defined as

$$(\bar{x}(S_i), \bar{\ell}(S_i)) = \arg \min_{(x_{ij})_{j \in S_i}, \ell_i} \sum_{j \in S_i} P_j x_{ij} + \ell_i \text{ subject to: } F(S_i, x, \ell) = \bar{y}_i \text{ and } G(\bar{y}_i, x, \theta, S_i) \leq \tau_i \bar{y}_i.$$

The aggregate demand of intermediate industries can be written as $\int_{\mathcal{I}} x_i(A_i) di = \int_{\mathcal{I}} A_i(S_i)^\gamma \bar{x}_i(S_i) di \leq \mathbb{E}[A_i(S_i)^\gamma] \max_{S \subset \mathcal{J}} \bar{x}(S)$. Analogously, the aggregate demand for labor can be written as $\int_{\mathcal{I}} \ell_i(A_i) di = \int_{\mathcal{I}} A_i(S_i)^\gamma \bar{\ell}_i(S_i) di \leq \mathbb{E}[A_i(S_i)^\gamma] \max_{S \subset \mathcal{J}} \bar{\ell}(S)$. From Assumption 1, the expectation of $A_i(S_i)^\gamma$ is finite.²⁰ Therefore, the aggregate demand of intermediate industries is finite.

Step 2c: Demand and Output of Primary Industries For any primary industry j , define $x_{int,j} = \int_{\mathcal{I}} x_{ij} di$ to be the aggregate intermediate demand for industry j 's good. Let y_j be the output of industry j when one unit of the final good is produced, and let x_{kj} be primary industry k 's demand for industry j 's good when exactly y_k units of good k are produced. The market-clearing condition for industry j yields

$$y_j = \sum_{k=1}^J x_{kj} + P_j x_{int,j}.$$

²⁰ Assumption 1 is slightly stronger and requires the variance of $A_i(S_i)^\gamma$ to be finite. Since the terms $A_i(S_i)$ are independently but not identically distributed, I need to assume finite variances to apply the law of large numbers. Throughout most of the paper, I assume that $A_i(S) = \phi_i(S) \cdot \prod_{j \in S} A_j$, where the A_j are deterministic and $\phi_i(S)$ are iid random variables. Under this functional form, I only need to assume that $(A_i(S))^\gamma$ has finite expectation to apply the law of large numbers.

Note that x_{kj} is a function of y_k , so it suffices to solve for the vector of primary outputs (y_0, y_1, \dots, y_J) .²¹

To solve for the vector of primary revenues, multiply the market clearing condition by P_j to obtain an analogous expression in terms of revenues.

$$P_j y_j = \sum_{k=1}^J P_j x_{kj} + P_j x_{int,j}. \quad (28)$$

Multiply and divide the term $P_j x_{kj}$ by $P_k y_k$, and rewrite equation (28) as a linear system in terms of the variables $\hat{y}_j =_{def} P_j y_j$ and $\hat{x}_{int,j} = P_j x_{int,j}$.

$$\hat{y}_j = \sum_{k=1}^J \frac{P_j x_{kj}}{P_k y_k} \hat{y}_k + \hat{x}_{int,j} \quad (29)$$

Define industry j 's share of industry k 's revenue as $\hat{\alpha}_{jk} = \frac{P_j x_{kj}}{P_k y_k}$.²² Write equation (29) in matrix form as

$$(I - \hat{\alpha}') \hat{y} = \hat{x}_{int}. \quad (30)$$

The system of equations (30) has a solution only if $(I - \hat{\alpha}')$ is invertible. Note that $\sum_{k=1}^J \hat{\alpha}_{jk} = \sum_{k=1}^J \frac{P_k x_{jk}}{P_j y_j} < 1$. The inequality follows from the fact that labor is essential in the production of good j , so that some of the revenue obtained by good j is always paid to labor. Since $\sum_{k=1}^J \hat{\alpha}_{jk} < 1$, the spectral radius of the matrix $\hat{\alpha}'$ is less than 1, and the matrix $(I - \hat{\alpha}')$ is invertible. Denoting its inverse by $\hat{\mathcal{L}}'$, primary industry revenues are given by

$$\hat{y} = \hat{\mathcal{L}}' \hat{x}_{int}. \quad (31)$$

From the revenue vector, one can solve for $y_j = \frac{\hat{y}_j}{P_j}$, and from y_j , one can solve for industry j 's demand vector x_j, ℓ_j .

Step 3: Solving for Y_f So far, I have solved for the price vector P^* , for the input sets $(S_i^*)_{i \in \mathcal{I}}$, and for the quantities $q^* = ((y_i^*)_{i \in \mathcal{I}}, (y_j^*)_{j=1}^J, (x_{fi}^*)_{i \in \mathcal{I}}, (x_{ij}^*)_{i \in \mathcal{I}, j \in \mathcal{J}}, (x_{jk}^*)_{j,k \in \mathcal{J}}, (\ell_j^*)_{j \in \mathcal{J}})$ that would be observed if $Y_f^* = 1$. Because of constant returns to scale in the production function, the equilibrium quantities Q^* are given by $Q^* = Y_f^* q^*$. Furthermore, the equilibrium

²¹Given knowledge of y_k , one can solve for $(x_{k1}, \dots, x_{kJ}, \ell_k)$ as the unique solution to the cost minimization problem $\min_{x'_k, \ell'_k} \sum_{j=1}^J P_j x'_{kj} + \ell'_k$ subject to $F_j(A_j, \ell'_k, x'_k) = y_k$.

²²Note that, because of constant returns to scale, $\hat{\alpha}_{jk}$ only depends on the ratio $\frac{x_{kj}}{y_k}$ and not on the particular value of y_k . For example, one can compute $\hat{\alpha}_{jk} = \frac{P_j x_{kj}^*}{P_k}$ where x_{kj}^* is the amount of good j necessary to produce one unit of good k .

input sets S_i^* do not change if production is scaled by a factor of Y_f^* .

Combining the market clearing condition for the final good and the budget constraint for the household, expenditure on final goods is given by $P_f^* Y_f^* = P_f^* C_f^* = 1$. One can write $Y_f^* = \frac{1}{P_f^*}$, and obtain all the equilibrium quantities $Q^* = Y_f^* q^*$. ■

Proof of Proposition 1. Recall, from the proof of Theorem 1, that if there exists a vector of primary prices (P_0, P_1, \dots, P_J) which satisfies the equations $P_j = K_j(P_0, P_1, \dots, P_J)$, then there exists a unique equilibrium price vector P^* such that $P_j^* = P_j$. Thus, it suffices to show that if $K_j(\cdot)$ is strictly concave, then there exists at most one vector of primary prices that satisfies equation (6).

I use the following Lemma from Kennan (1999):

Lemma 5 (Kennan) *If $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is an increasing, strictly concave function such that*

1. $f_i(0, \dots, 0) \geq 0$,
2. *there exists $a = (a_1, \dots, a_n)$ such that $a_i > 0$ and $f_i(a) > a_i$ for all $i \in \{1, \dots, n\}$, and*
3. *there exists $b = (b_1, \dots, b_n)$ such that $b_i > a_i$ and $f_i(b) < b_i$ for all $i \in \{1, \dots, n\}$,*

then there exists a unique positive vector x such that $f(x) = x$.

In the statement of Lemma 5, define $f : \mathbb{R}^J \rightarrow \mathbb{R}^J$ such that $f_j(P) = K_j(P)$. Because equation (6) holds, f has a fixed point. I use Lemma 5 to prove that this fixed point is unique. I proceed to show that all the conditions of the Lemma hold. First, note that, by assumption f is strictly concave.

I now show that $f(0) > 0$. Recall that $K_j(P) = \min_{X,L} \sum_{k=1}^{P_k} X_k + L$ subject to $F_j(X, L) = 1$, and that labor is an essential factor of production, so that the optimal L must be greater than 0. This means that $K_j(0) > 0$, since—even if prices were zero—the wage would still be equal to 1 (since labor is the numeraire), and a non-zero quantity of labor will be demanded. Thus, the condition $f_j(0, \dots, 0)$ in Lemma 5 is satisfied.

I now show that there exists $a > 0$ such that $f(a) > a$. This follows from the continuity of f . Let $\{a(n)\}_{n=1}^\infty$ be a sequence of positive vectors $a(n)$ such that $\lim_{n \rightarrow \infty} a(n) = 0$. Since f is continuous, $\lim_{n \rightarrow \infty} f(a(n)) = f(0) > 0 = \lim_{n \rightarrow \infty} a(n)$. Thus, for n large enough, $f(a(n)) > a(n)$.

Finally, I show that there exists $b > a$ such that $f(b) < b$. Let P be a fixed point of f (which exists by assumption). Then $f(P) = P$. Since f is strictly concave, $f(2P) < \frac{1}{2}f(P) + \frac{1}{2}f(P) = f(P)$. Thus, by setting $b = 2P$, the last condition of Lemma 5 is satisfied.

I conclude that f has a unique fixed point. Therefore, there exists only one vector P_1^*, \dots, P_J^* of primary prices such that equation (6) is satisfied, and a unique equilibrium

price vector P^* . ■

Proof of Theorem 2.

Deterministic Tie Breaking Rule I begin with the case where the tie breaking rule is deterministic. Recall from the proof of existence of equilibrium that lowercase variables y^*, x^*, ℓ^* denote equilibrium quantities for the special case where only one unit of the final good is produced. From the proof of existence of equilibrium, we have $y_i^* = y_{\tau\iota}^* = x_{f\tau\iota}^* = (\frac{P_{\tau\iota}}{P_f})^{-\gamma} (\frac{P_f}{P_f})^{-\beta}$, so that final industry demands and intermediate outputs are uniquely determined.

Furthermore, since the intermediate production function is strictly quasi-concave, for every possible input set S_i to intermediate industry i there exists a unique vector $x_i^*(S_i) = (x_{ij}^*)_{j \in S_i}$ which minimizes costs when the set S_i is used and y_i^* units of output are produced. Since the tie-breaking rule is deterministic, there exists a unique set $S_i^* = \text{TieBreak}(\arg \min_{S_i} \sum_{j \in S_i} P_j^* x_{ij}^*(S_i))$ that firms in industry i will choose in equilibrium. Therefore, intermediate input sets and input demands are uniquely determined.

Given intermediate inputs x_{ij}^* , I defined $x_{int,j}^* = \int_{\mathcal{I}} x_{ij}^* di$, and showed that there exists a non-negative matrix $\hat{\mathcal{L}} \in \mathbb{R}^{J \times J}$ such that $P_j^* y_j^* = \sum_{k=1}^J \hat{\mathcal{L}}_{kj} P_k^* x_{int,k}^*$. Thus, primary industry outputs are also uniquely determined. Since primary production functions are strictly quasi-concave, there exist unique input demand vectors $(x_j^*, \ell_j^*) = ((x_{jk}^*)_{k=1}^J, \ell_j^*)$ such that firms in industry J choose (x_j^*, ℓ_j^*) when producing y_j^* units of output.

Since Y_f^* can be computed as a function of the unique price vector P^* , Y_f^* is unique. Therefore, the allocations $(Y^*, X^*, L^*) = Y_f^* \cdot (y^*, x^*, \ell^*)$. Since Y_f^*, y^*, x^* and ℓ^* are unique, the constructed equilibrium allocations are unique.

Randomized Tie Breaking Rule The uniqueness argument with a randomized tie breaking rule is identical for final industry demands and industry output $y_i^* = x_{fi}^*$, since both of these depend only on the price vector. Because the tie breaking rule is now randomized, there may be multiple equilibrium sets S_i^* . Since all intermediate industries use the same tie-breaking rule and have independent productivity draws, the random variables $S_i^*(\tau), (x_{ij}^*(S_i^*(\tau)))_{j=1}^J$ will be independently distributed for all intermediate industries $i \in \mathcal{I}$ with the same threshold τ .²³ Let $i = (\tau, \iota)$ and $x_{ij} = x_{\tau\iota j}$. From the law of large numbers, one can write $\int_0^1 x_{\tau\iota j}^*(S_{\tau\iota j}^*) d\iota = \mathbb{E}[x_{\tau\iota j}^*(S_{\tau\iota j}^*)]$, where the expectation is taken over all $\iota \in [0, 1]$.²⁴

The aggregate $x_{int,j}^* = \int_{\mathcal{I}} x_{ij}^*(S_i^*) di$ is given by $\int_0^\infty \mathbb{E}[x_{\tau\iota j}^*(S_{\tau\iota j}^*)] \nu(\tau) d\tau$ and is uniquely determined in equilibrium. Since $x_{int,j}^*$ is uniquely determined, primary industry outputs

²³Here I have emphasized the dependence of the random variable S^* on τ .

²⁴I showed in the proof of Theorem 1 that this expectation is finite.

$(y_j^*)_{j=1}^J$ and demands (x_j^*, ℓ_j^*) are also uniquely determined. Since Y_f^* only depends on y^*, P^* , it is also uniquely determined.

Finally, note that Lemma 1 implies that, for any given industry $i \in \mathcal{I}$, the input sets S_i^* and intermediate demands $X_{ij}^*(S_i^*)$ are unique with probability 1. ■

B.1 Proof of Theorem 3

I begin by recalling some properties of the Frechet distribution. I use these properties to solve for equilibrium quantities and prices.

Properties of the Frechet Distribution In this subsection, I recall the definition of a Frechet distribution's shape and scale parameters and show how these parameters change when under multiplication by a constant, exponentiation, and the maximum operator.

Definition 5 *Given a scale parameter $s > 0$ and a shape parameter $\kappa > 0$, a Frechet distribution with scale s and shape κ is given by the CDF $\Psi(x) = e^{-(\frac{x}{s})^{-\kappa}}$.*

Lemma 6 *Let $\gamma, c > 0$, and let X be a random variable drawn from a Frechet distribution scale parameter s and shape parameter κ . Then cX^γ is a random variable drawn from a Frechet distribution with scale parameter cs^γ and shape parameter $\frac{\kappa}{\gamma}$.*

Proof.

$$\Phi(x) = \text{Prob}[cX^\gamma \leq x] = \text{Prob}[X \leq (\frac{x}{c})^{\frac{1}{\gamma}}] = \Psi((\frac{x}{c})^{\frac{1}{\gamma}}) = e^{-(\frac{(\frac{x}{c})^{\frac{1}{\gamma}}}{s})^{-\kappa}} = e^{-(\frac{x}{cs^\gamma})^{-\frac{\kappa}{\gamma}}}$$

■

Lemma 7 *Let X_1, \dots, X_n be Frechet random variables drawn from independent distributions with shape parameter κ and scale parameters s_1, \dots, s_n , respectively. Then $\max(X_1, \dots, X_n)$ is drawn from a Frechet distribution with scale parameter $(\sum_{i=1}^n s_i^\kappa)^{\frac{1}{\kappa}}$ and shape parameter κ .*

Proof. Let $\Psi_i(x) = e^{-(\frac{x}{s_i})^{-\kappa}}$ be the CDF of X_i . Then

$$\text{Prob}[\max(X_1, \dots, X_n) \leq x] = \prod_{i=1}^n \Psi_i(x) = \prod_{i=1}^n e^{-(\frac{x}{s_i})^{-\kappa}} = e^{-\left(x(\sum_{i=1}^n s_i^\kappa)^{-\frac{1}{\kappa}}\right)^{-\kappa}}$$

■

Lemma 8 *If U is a standard exponential distribution with CDF $\Phi(x) = 1 - e^{-x}$ and $X = sU^{-\frac{1}{\kappa}}$, then X is a Frechet distribution with scale parameter s and shape parameter κ .*

Proof.

$$Prob[X \leq x] = Prob[sU^{-\frac{1}{\kappa}} \leq x] = Prob[U \geq (\frac{x}{s})^{-\kappa}] = e^{-(\frac{x}{s})^{-\kappa}}$$

where the sign change in $Prob[sU^{-\frac{1}{\kappa}} \leq x] = Prob[U \geq (\frac{x}{s})^{-\kappa}]$ is justified because $f(x) = (\frac{x}{s})^{-\kappa}$ is a decreasing function. ■

Lemma 9 *If X is a Frechet distribution with scale parameters s_X and shape parameter κ , and Y is a Frechet distribution with scale parameter s_Y and shape parameter κ which is independent of X , then*

$$\mathbb{E}[X|X > Y] \cdot Prob[X > Y] = \Gamma(1 - \frac{1}{\kappa}) s_X^\kappa (s_X^\kappa + s_Y^\kappa)^{\frac{1}{\kappa}-1}.^{25}$$

Proof.

Using Lemma 8, there exist independent standard exponential random variables U, V such that $X = s_X U^{-\frac{1}{\kappa}}$ and $V = s_Y V^{-\frac{1}{\kappa}}$. Furthermore, since the function $f(x) = x^{-\kappa}$ is decreasing, the event $X > Y$ happens if and only if $s_X^{-\kappa} U < s_Y^{-\kappa} V$. Equivalently, $X > Y$ happens if and only if $U(\frac{s_Y}{s_X})^\kappa < V$. I use this fact to first compute $Prob[X > Y]$ and then to compute $\mathbb{E}[X|X > Y]$.

I first compute $Prob[X > Y]$, as follows

$$Prob[X > Y] = \int_0^\infty Prob[V > (\frac{s_Y}{s_X})^\kappa u] e^{-u} du = \int_0^\infty e^{-(1+(\frac{s_Y}{s_X})^\kappa)u} du$$

Using the change of variables $t = (1 + (\frac{s_Y}{s_X})^\kappa)u$, $dt = (1 + (\frac{s_Y}{s_X})^\kappa)du$, I conclude that

$$Prob[X > Y] = (1 + (\frac{s_Y}{s_X})^\kappa)^{-1} \int_0^\infty e^{-t} dt = (1 + (\frac{s_Y}{s_X})^\kappa)^{-1}.$$

Now I compute $\mathbb{E}[X|X > Y] \cdot Prob[X > Y]$. I can write this as

$$\mathbb{E}[X|X > Y] \cdot Prob[X > Y] = \mathbb{E}[s_X U^{-\kappa} | U(\frac{s_Y}{s_X})^\kappa < V] Prob[U(\frac{s_Y}{s_X})^\kappa < V].$$

I can write this expression as an integral

$$\int_0^\infty \int_{(\frac{s_Y}{s_X})^\kappa u}^\infty s_X u^{-\kappa} e^{-u} e^{-v} dv du = s_X \int_0^\infty u^{-\frac{1}{\kappa}} e^{-u} \int_{(\frac{s_Y}{s_X})^\kappa u}^\infty e^{-v} dv du = s_X \int_0^\infty u^{-\frac{1}{\kappa}} e^{-(1+(\frac{s_Y}{s_X})^\kappa)u} du$$

²⁵The proof of this Lemma is adapted from <https://math.stackexchange.com/questions/2330661/conditional-expectation-e-leftxx-right-if-x-and-y-are-independent>

Using the change of variables $t = (1 + (\frac{s_Y}{s_X})^\kappa)u$, $dt = (1 + (\frac{s_Y}{s_X})^\kappa)du$, I conclude that

$$\mathbb{E}[X|X > Y] \cdot \text{Prob}[X > Y] = s_X(1 + (\frac{s_Y}{s_X})^\kappa)^{-1}(1 + (\frac{s_Y}{s_X})^\kappa)^{\frac{1}{\kappa}} \int_0^\infty t^{-\frac{1}{\kappa}} e^{-t} dt = s_X(1 + (\frac{s_Y}{s_X})^\kappa)^{\frac{1}{\kappa}-1} \Gamma(1 - \frac{1}{\kappa})$$

Finally, I can multiply and divide this expression by $(s_X^\kappa)^{\frac{1}{\kappa}-1} = s_X^{1-\kappa}$ to obtain

$$\mathbb{E}[X|X > Y] \cdot \text{Prob}[X > Y] = s_X^\kappa (s_X^\kappa + s_Y^\kappa)^{\frac{1}{\kappa}-1} \Gamma(1 - \frac{1}{\kappa}).$$

■

Lemma 10 *Let $\log Y = \int_0^\infty \log Y_\tau \nu(\tau) d\tau$ be a Cobb-Douglas production function, where the input with index τ has a price P_τ . The marginal cost of producing one unit of output is given by*

$$\log P = \frac{\int_0^\infty \log P_\tau \nu(\tau) d\tau}{\int_0^\infty \nu(\tau) d\tau} + \log \int_0^\infty \nu(\tau) d\tau.$$

Proof. Write the unit cost minimization problem as $\min_{Y_\tau} \int_\tau Y_\tau P_\tau \nu(\tau)$ subject to $\log Y = \int_0^\infty \log Y_\tau \nu(\tau) d\tau = 0$. The first order condition is

$$P_\tau = \frac{\lambda}{Y_\tau}.$$

Plugging this into the constraint, one can solve for λ as follows,

$$\int_0^\infty (\log \lambda - \log P_\tau) \nu(\tau) d\tau = 0 \implies \log \lambda = \frac{\int_0^\infty \log P_\tau \nu(\tau) d\tau}{\int_0^\infty \nu(\tau) d\tau}.$$

Finally, compute $P = \int_0^\infty Y_\tau P_\tau \nu(\tau) d\tau = \lambda \int_0^\infty \nu(\tau) d\tau$. Taking logarithms, one can write

$$\log P = \log \lambda + \log \int_0^\infty \nu(\tau) d\tau = \frac{\int_0^\infty \log P_\tau \nu(\tau) d\tau}{\int_0^\infty \nu(\tau) d\tau} + \log \int_0^\infty \nu(\tau) d\tau.$$

■

Corollary 2 *Let the conditions of Lemma 10 hold. When $\int_0^\infty \nu(\tau) d\tau = 1$, then*

$$\log P = \int_0^\infty \log P_\tau \nu(\tau) d\tau.$$

Proof of Theorem 3.

Formula for P_f . I begin by giving a formula for the final price. Recall that the final industry's production function is given by $\log Y_f = \frac{\gamma}{\gamma-1} (\int_0^1 \log(\int_0^1 X_{f\tau\iota}^{\frac{\gamma-1}{\gamma}} d\iota) \nu(\tau) d\tau)$. This is a nested CES function where the inner nest has elasticity of substitution γ , and the outer nest has elasticity of substitution 1. Using Lemma 10 and the fact that $\int_0^\infty \nu(\tau) d\tau = 1$, write

$$\log P_f = \int_0^\infty \log P_\tau \nu(\tau) d\tau, \quad (32)$$

where $P_\tau = (\int_0^1 P_{\tau\iota}^{1-\gamma} d\iota)^{\frac{1}{1-\gamma}}$. Using the law of large numbers, write

$$P_\tau = \mathbb{E}[(\frac{\bar{K}(S_{\tau\iota}^*, P, \tau)}{\phi_{\tau\iota}(S_{\tau\iota}^*)})^{1-\gamma}]^{\frac{1}{1-\gamma}}, \quad (33)$$

where $S_{\tau\iota}^*$ is a random variable denoting the cost-minimizing set of intermediate firm (τ, ι) . Taking conditional expectations over $S_{\tau\iota}^*$, one can write this as

$$P_\tau = (\sum_{S \in \mathcal{F}(\theta, \tau)} Pr[S^* = S] \mathbb{E}[(\frac{\bar{K}(S, P, \tau)}{\phi_{\tau\iota}(S)})^{1-\gamma} | S_{\tau\iota}^* = S])^{\frac{1}{1-\gamma}}. \quad (34)$$

Note that the event $S_{\tau\iota}^* = S$ is equivalent to the events

$$\begin{aligned} \frac{\bar{K}(S, P, \tau)}{\phi_{\tau\iota}(S)} &\leq \min_{S' \neq S} \frac{\bar{K}(S', P, \tau)}{\phi_{\tau\iota}(S')} \\ \frac{\phi_{\tau\iota}(S)}{\bar{K}(S, P, \tau)} &\geq \max_{S' \neq S} \frac{\phi_{\tau\iota}(S')}{\bar{K}(S', P, \tau)} \\ (\frac{\phi_{\tau\iota}(S)}{\bar{K}(S, P, \tau)})^{\gamma-1} &\geq \max_{S' \neq S} (\frac{\phi_{\tau\iota}(S')}{\bar{K}(S', P, \tau)})^{\gamma-1} \end{aligned}$$

For any $S \in \mathcal{F}(\theta, \tau)$, Lemma 6 tells us that $(\frac{\phi_{\tau\iota}(S)}{\bar{K}(S, P, \tau)})^{\gamma-1}$ has a Frechet distribution with shape parameter $\frac{\kappa}{\gamma-1}$ and scale parameter $\frac{1}{\bar{K}(S, P, \tau)^{\gamma-1}}$. Lemma 7 tells us that the distribution of $\max_{S' \neq S} (\frac{\phi_{\tau\iota}(S')}{\bar{K}(S', P, \tau)})^{\gamma-1}$ is Frechet with shape parameter $\frac{\kappa}{\gamma-1}$ and scale parameter $(\sum_{S' \neq S} \bar{K}(S', P, \tau)^{-\kappa})^{\frac{\gamma-1}{\kappa}}$. Lemma 9 tells us that

$$Prob[S^* = S] \cdot \mathbb{E}[(\frac{\phi_{\tau\iota}(S)}{\bar{K}(S, P, \tau)})^{\gamma-1} | S^* = S] = \Gamma(1 - \frac{\gamma-1}{\kappa}) \bar{K}(S, P, \tau)^{-\kappa} (\sum_{S' \in \mathcal{F}} \bar{K}(S', P, \tau)^{-\kappa})^{\frac{\gamma-1}{\kappa}-1}. \quad (35)$$

Adding this up over all sets $S \in \mathcal{F}(\theta, \tau)$, one can write

$$\sum_{S \in \mathcal{F}(\theta, \tau)} \text{Prob}[S^* = S] \cdot \mathbb{E}\left[\left(\frac{\phi_{\tau}(S)}{\bar{K}(S, P, \tau)}\right)^{\gamma-1} | S^* = S\right] = \Gamma\left(1 - \frac{\gamma-1}{\kappa}\right) \left(\sum_{S' \in \mathcal{F}} \bar{K}(S', P, \tau)^{-\kappa}\right)^{\frac{\gamma-1}{\kappa}}.$$

Plugging this into equation (34) yields

$$P_\tau = \Gamma\left(1 - \frac{\gamma-1}{\kappa}\right)^{\frac{1}{1-\gamma}} \left(\sum_{S \in \mathcal{F}(\theta, \tau)} \bar{K}(S', P, \tau)^{-\kappa}\right)^{\frac{1}{-\kappa}}. \quad (36)$$

Finally, combining equations (32) and (36), one can write

$$\log P_f = \log\left(\Gamma\left(1 - \frac{\gamma-1}{\kappa}\right)^{\frac{1}{1-\gamma}}\right) - \frac{1}{\kappa} \int_0^\infty \log\left(\sum_{S \in \mathcal{F}(\theta, \tau)} \bar{K}(S', P, \tau)^{-\kappa}\right).$$

Since $P_f Y_f = 1$, we have $\log Y_f = -\log P_f$. ■

C Proofs from Section 4

Proof of Lemma 3. Take τ as fixed, and consider the function $f(\theta) = |\mathcal{F}(\theta, \tau)| = |S : G(\theta, S) \leq \tau| = \sum_{S \in \mathcal{J}} \mathbf{1}_{G(\theta, S) \leq \tau}$, where $\mathbf{1}_{G(\theta, S) \leq \tau}$ is an indicator function. The distributional derivative of this indicator function is $\frac{\partial}{\partial \theta_j} \mathbf{1}_{G(\theta, S) \leq \tau} = -\delta(\tau - G(\theta, S)) \frac{\partial G}{\partial \theta_j}$. Adding up over all sets $S \in \mathcal{J}$, we obtain $\frac{\partial}{\partial \theta_j} |\mathcal{F}(\theta, \tau)| = -\sum_{S \in \mathcal{J}} \delta(\tau - G(\theta, S)) \frac{\partial G}{\partial \theta_j}$. ■

Proof of Proposition 3. This proposition follows from applying Lemma 3 to equation (14). From Lemma 3, we have $\frac{\partial}{\partial \theta_j} |\mathcal{F}(\theta, \tau)| = -\sum_{S \in \mathcal{J}: j \in S} \delta(\tau - G(\theta, S)) \frac{\partial G}{\partial \theta_j}$. From equation (14), we have $\frac{\partial \log Y_f}{\partial \theta_j} = \frac{1}{\kappa} \int_0^\infty \frac{\frac{\partial}{\partial \theta_j} |\mathcal{F}(\theta, \tau)|}{|\mathcal{F}(\theta, \tau)|} \nu(\tau) d\tau$. Plugging in the partial derivative for $|\mathcal{F}(\theta, \tau)|$ and using the fact that $\int \delta(\tau - G(\theta, S)) f(\tau) d\tau = f(G(\theta, S))$ for any function $f(\cdot)$, we obtain

$$\frac{\partial \log Y_f}{\partial \theta_j} = -\frac{1}{\kappa} \sum_{S \in \mathcal{J}} \frac{\nu(G(\theta, S))}{|\mathcal{F}(\theta, G(\theta, S))|} \frac{\partial G}{\partial \theta_j}.$$

■

Proof of Proposition 4. From Proposition 3, the first-order partial derivative $\frac{\partial \log Y_f}{\partial \theta_j}$ is

given by $-\frac{1}{\kappa} \sum_{S \subset \mathcal{J}} \frac{\nu(G(\theta, S))}{|\mathcal{F}(\theta, G(\theta, S))|} \frac{\partial G}{\partial \theta_j}$. Since $G(\theta, S) = \sum_{j \in S} \gamma_j \theta_j$, one can write this as

$$\frac{\partial \log Y_f}{\partial \theta_j} = -\frac{1}{\kappa} \sum_{S \subset \mathcal{J}} \frac{\nu(G(\theta, S))}{|\mathcal{F}(\theta, G(\theta, S))|} \gamma_j.$$

We want to compute the second-order partial derivative $\frac{\partial^2 \log Y_f}{\partial \theta_j \partial \theta_k}$.

Write the denominator as $|\mathcal{F}(\theta, G(\theta, S))| = |S' : G(\theta, S') \leq G(\theta, S)| = |S' : \sum_{j \in S'} \gamma_j \theta_j \leq \sum_{j \in S} \gamma_j \theta_j|$. Now suppose that θ_k increases by an infinitesimal amount to $\theta'_k = \theta_k + d\theta_k$. The expression $|\mathcal{F}(\theta, G(\theta, S))|$ will change if and only if there exists some set S' such that increasing θ_k to θ'_k transforms S' from feasible to infeasible, or from infeasible to feasible.

Fix a pair of sets S, S' , and consider the following four cases: (1) $k \notin S \cup S'$, (2) $k \in S \cap S'$, (3) $k \in S \setminus S'$, (4) $k \in S' \setminus S$. In case (1), neither $G(\theta, S')$ nor $G(\theta, S)$ depend on θ_k . Thus, a change in θ_k will not affect the feasibility of the set S' . In case (2), both $G(\theta, S)$ and $G(\theta, S')$ increase by $\gamma_k d\theta_k$, so that the feasibility of set S' is also not affected. In case (3), the feasibility of S' is affected only if $\sum_{j \in S} \gamma_j \theta_j = \sum_{j \in S'} \gamma_j \theta_j$. Let $\mathcal{T}(S, S') = \{\theta : \sum_{j \in S} \gamma_j \theta_j - \sum_{j \in S'} \gamma_j \theta_j = 0\}$ denote the set of vectors θ for which this condition holds and note that $\mathcal{T}(S, S')$ has measure zero. Analogously, in case (4), the feasibility of S' is affected only if $\theta \in \mathcal{T}(S, S')$.

Since there is only a finite number of possible supplier sets $S, S' \subset \mathcal{J}$, the union

$$\mathcal{T} =_{\text{def}} \bigcup_{S, S' \subset \mathcal{J}} \mathcal{T}(S, S')$$

also has measure zero. We conclude that the set of values of θ for which any of the denominators $|\mathcal{F}(\theta, G(\theta, S))|$ changes when θ_k increases by an infinitesimal amount has measure zero. Thus, we can treat the denominators as constant almost everywhere.

We can now easily compute the second-order derivative almost everywhere as

$$\frac{\partial^2 \log Y_f}{\partial \theta_j \partial \theta_k} = -\frac{1}{\kappa} \sum_{S \subset \mathcal{J}} \frac{\frac{d\nu}{d\tau}|_{\tau=G(\theta, S)}}{|\mathcal{F}(\theta, G(\theta, S))|} \gamma_j \gamma_k.$$

■

Proof of Lemma 3. The proof is analogous to the proof of Lemma 3. Take τ as fixed, and consider the function $f(\theta) = \sum_{S \in \mathcal{F}(\theta, \tau)} \bar{K}(S, P, \tau)^{-\kappa} = \sum_{S \subset \mathcal{J}} \mathbf{1}_{G(\theta, S) \leq \tau} \bar{K}(S, P, \tau)^{-\kappa}$. The derivative of the term $\mathbf{1}_{G(\theta, S) \leq \tau} \bar{K}(S, P, \tau)^{-\kappa}$ with respect

to θ_j is $-\delta(\tau - G(\theta, S)) \frac{\partial G}{\partial \theta_j} \overline{K}(S, P, \tau)^{-\kappa}$. Adding up over all sets $S \subset \mathcal{J}$, we conclude that

$$\frac{\partial}{\partial \theta_j} \sum_{S \in \mathcal{F}(\theta, \tau)} \overline{K}(S, P, \tau)^{-\kappa} = \sum_{S \subset \mathcal{J}} \delta(\tau - G(\theta, S)) \overline{K}(S, P, \tau)^{-\kappa} \frac{\partial G}{\partial \theta_j}.$$

■

D Proofs from Section 5

Proof of Lemma 4. Let t_j be the arrival time of primary industry j . Since industry j has a threshold τ_j , we must have $\zeta(t_j) = \tau_j$. Since $\zeta(t_j) = e^{-Zt_j}$, we must have $t_j = -\frac{\log \tau_j}{Z}$ and $t_{j+1} - t_j = \frac{\log \tau_j - \log \tau_{j+1}}{Z}$. Since $\log \tau_j - \log \tau_{j+1} | \log \tau_j$ has an exponential distribution with mean $\frac{1}{\lambda}$, the conditional difference $t_{j+1} - t_j | t_j$ has an exponential distribution with mean $\frac{1}{\lambda Z}$. Thus, new industries arrive following a Poisson process with arrival rate $Z\lambda$. ■

Proof of Theorem 5. Write log-output as

$$\log Y_f(t) = \frac{1}{\kappa} \frac{\int_{e^{-Zt}}^1 \log |\mathcal{F}(\theta, \tau)| \frac{1}{\tau} d\tau}{\int_{e^{-Zt}}^1 \frac{1}{\tau} d\tau}.$$

The denominator in this expression can be written as $g(t) = \int_{e^{-Zt}}^1 \frac{1}{\tau} d\tau = Zt$.

To compute the numerator, define $\tau^*(t) = J(t)e^{-Zt}$. Since $\theta_j(t) = e^{-Zt}$ for all primary industries $j \in \{1, \dots, J\}$, all intermediate industries with threshold $\tau \geq \tau^*(t)$ will be able to feasibly combine any subset $S \subset \{1, \dots, J\}$ of primary inputs. For these industries, the number of feasible combinations $|\mathcal{F}(\theta, \tau)| = 2^{J(t)}$, and $\log |\mathcal{F}(\theta, \tau)| = J(t) \ln 2$.

Conversely, any intermediate industry with $\tau < \tau^*(t)$ will not be able to attempt arbitrary combinations of inputs. Industries with τ in the interval $[e^{-Zt}, 2e^{-Zt}]$ will be able to attempt only one combination, and therefore $\log |\mathcal{F}(\theta, \tau)| = 0$ for these industries. Industries with τ in the interval $[je^{-Zt}, (j+1)e^{-Zt}]$ will be able to attempt $\sum_{k=0}^{j-1} \binom{J}{k}$ combinations of inputs. Thus, one can write the numerator as

$$\begin{aligned} f(t) &= \sum_{j=1}^J \int_{je^{-Zt}}^{(j+1)e^{-Zt}} \log \sum_{k=0}^j \binom{J}{k} \frac{1}{\tau} d\tau + \int_{(J+1)e^{-Zt}}^1 \log 2 \cdot J \frac{1}{\tau} d\tau \\ &= \sum_{j=1}^J [\log(j+1) - \log(j)] \cdot \log \left(\sum_{k=0}^j \binom{J}{k} \right) + \log 2 \cdot J \cdot Zt - \log 2 \cdot J \cdot \log(J+1). \end{aligned} \quad (37)$$

To compute the limit $\lim_{t \rightarrow \infty} \frac{\log Y_f(t)}{t} = \frac{1}{\kappa} \frac{f(t)}{Zt^2}$, note that the following bounds apply to each term in $f(t)$:

1. $\sum_{j=1}^J [\log(j+1) - \log(j)] \cdot \log(\sum_{k=0}^j \binom{J}{k}) \leq \log(J+1) \cdot \log 2 \cdot J = O(J \log J),$
2. $\log 2 \cdot J \cdot Zt = O(Jt),$ and
3. $\log 2 \cdot J \cdot \log(J+1) = O(J \log J).$

Since $J(t)$ is a Poisson process with arrival rate λZ , it satisfies $\lim_{t \rightarrow \infty} \frac{J(t)}{t} = \lambda Z$ almost surely. This implies that $J = O(t)$ almost surely. Any term that is $O(J \log J) = O(t \log t)$ grows asymptotically slower than the denominator Zt^2 . The only term in the limit that does not vanish is the second term. Thus, we have

$$\lim_{t \rightarrow \infty} \frac{\log Y_f(t)}{t} = \lim_{t \rightarrow \infty} \frac{1}{\kappa} \frac{\log 2 \cdot J \cdot Zt}{Zt^2} = \lim_{t \rightarrow \infty} \frac{1}{\kappa} \frac{\log 2 \cdot J}{t} = \frac{1}{\kappa} \log 2 \cdot \lambda Z.$$

■

E Algorithms for Computing the Number of Feasible Combinations with Size Constraints

In this Appendix, I give a recursive formula to compute $|\mathcal{F}(\theta, \tau)| = |S : \sum_{j \in S} \theta_j \leq \tau|$, the number of feasible combinations under a size constraint. Without loss of generality, I assume that all θ_j and τ are rational numbers with p digits of precision. Multiplying all of them by 10^p , I can further assume that they are all integers in $\{0, 1, \dots, M\}$ for some large integer M .

For any $j \in \mathcal{J}$ and $m \in \{0, \dots, M\}$, define $\mathcal{C}(j, m) = |\{S \subset \{1, \dots, j\} : \sum_{j \in S} \theta_j \leq m\}|$ to be the number of feasible sets which only contain the first j industries and whose total size is less than or equal to m . For any $j \in \mathcal{J}$, let $N_0(j) = |\{j' : 1 \leq j' \leq j \text{ and } \theta_{j'} = 0\}|$ be the number of industries whose index is less than equal to j and which have size $\theta_{j'} = 0$. Note that $\mathcal{C}(J, M) = \mathcal{C}$ is the quantity that we want to compute, and that $\mathcal{C}(j, 0) = 2^{N_0(j)}$ for any $j \in \mathcal{J}$. Furthermore, note that $\mathcal{C}(1, m) = 2$ if $\theta_1 \leq m$ and $\mathcal{C}(1, m) = 1$ if $\theta_1 > m$. Using $\{\mathcal{C}(j, 0), \mathcal{C}(1, m)\}_{j \leq J, m \leq M}$ as the base cases, one can use the recursive formula

$$\mathcal{C}(j, m) = \mathcal{C}(j-1, m) + \mathcal{C}(j-1, m - \theta_j) \quad (38)$$

to build up the dynamic programming table all the way up to $\mathcal{C}(J, M) = \mathcal{C}$.

The recursive formula (38) is justified because there are two kinds of sets $S \subset \{1, \dots, j\}$ which satisfy the constraint $\sum_{j' \in S} \theta_{j'} \leq m$. The first kind is those sets which do not contain j . The number of such sets which do not contain j is $\mathcal{C}(j-1, m)$. The second type of set is those that contain j . One can write each of these sets in a unique way as $S = \{j\} \cup S'$ where

$S' \subset \{1, \dots, j-1\}$ and $\sum_{j' \in S} \theta_{j'} \leq m - \theta_j$. There are exactly $\mathcal{C}(j-1, m - \theta_j)$ of these sets, which justifies formula (38).

This dynamic programming algorithm runs in time $O(J \times M)$. This is tractable when M is small, but quickly becomes intractable when M is very large. Dyer (2003) gives a tractable approximation algorithm whose running time does not depend on M , and which computes \mathcal{C} with arbitrary precision.

Both the exact and approximate counting algorithms can be generalized to situations where the feasible sets S have to satisfy multiple knapsack constraints.²⁶ These running time of these algorithms increases exponentially with the number of constraints (see Dyer (2003) for more details on these generalizations.)

F Rolling Regression Results

In this Appendix, I report rolling regression results discussed in Section 6.6 for the following 15-year rolling periods: 1972-1987, 1977-1992, 1982-1997, 1987-2002, and 1992-2007 (Tables 7-11.)

²⁶These arise naturally, for example, if there are both size and weight constraints.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: OLS Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.016* (0.009)	0.016 (0.010)	0.019* (0.011)	0.019** (0.009)	0.019** (0.010)	0.022** (0.011)
TFP Change From Investment in Electronics and Computers	0.03%	0.03%	0.00%	0.04%	0.04%	0.00%
Panel B: First Stage Results (Weighted by Value Added)						
$\log Size_i$	-0.039** (0.016)	-0.039** (0.016)	-0.034 (0.022)	-0.039** (0.016)	-0.039** (0.016)	-0.034 (0.022)
$\log Density_i$	-0.110** (0.050)	-0.110** (0.050)	-0.134** (0.067)	-0.103** (0.042)	-0.103** (0.042)	-0.127* (0.066)
Panel C: IV Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	-0.021 (0.093)	-0.023 (0.093)	-0.094 (0.117)	-0.011 (0.096)	-0.013 (0.096)	-0.085 (0.120)
F-test	4.634	4.500	2.773	5.168	5.007	2.526
TFP Change From Investment in Electronics and Computers	0.06%	0.05%	0.00%	0.07%	0.07%	0.00%
TFP Change From Electronic Miniaturization	-0.01%	-0.01%	0.01%	-0.01%	-0.01%	0.00%
Panel D: OLS Results (Unweighted)						
$\Delta \log Comp_{it}$	0.015* (0.008)	0.015* (0.009)	0.021* (0.011)	0.018** (0.008)	0.019** (0.009)	0.024** (0.011)
TFP Change From Investment in Electronics and Computers	0.03%	0.03%	0.00%	0.04%	0.04%	0.00%
Panel E: First Stage Results (Unweighted)						
$\log Size_i$	-0.047*** (0.014)	-0.046*** (0.015)	-0.041** (0.020)	-0.046*** (0.013)	-0.046*** (0.014)	-0.041** (0.020)
$\log Density_i$	-0.113** (0.046)	-0.113** (0.047)	-0.156** (0.063)	-0.107*** (0.036)	-0.107*** (0.037)	-0.150** (0.063)
Panel F: IV Results (Unweighted)						
$\Delta \log Comp_{it}$	0.026 (0.079)	0.025 (0.079)	-0.040 (0.092)	0.034 (0.080)	0.032 (0.080)	-0.033 (0.094)
F-test	6.745	6.543	4.383	8.746	8.497	4.116
TFP Change From Investment in Electronics and Computers	-0.04%	-0.05%	0.00%	-0.02%	-0.03%	0.00%
TFP Change From Electronic Miniaturization	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%
Observations	423	423	294	423	423	294
Benchmark TFP Change	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%
$SIC_2 \times Time$ Trend	No	Yes	Yes	No	Yes	Yes
Control for lagged change in TFP	No	No	Yes	No	No	Yes
Control for lagged revenue	No	No	No	Yes	Yes	Yes

Table 7: Regression Results for the 1972-1987 Sample. This table presents OLS and IV estimates of the effect of computer and electronics investment on TFP for the years 1972-1987, using a dataset of five-year stacked differences for heavy manufacturing industries (SIC Codes 3401-3999), excluding computers and electronics (SIC codes 3571-3579, 3671-3679). Panel A shows the estimate $\hat{\beta}$ in the OLS regression $\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma \Delta \log Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t}$, where $\log A_{i,t}$ is industry i 's TFP at time t , $Comp_{i,t}$ represents investment in computers and electronics, $Rev_{i,t-1}$ represents lagged revenue, and $\delta_{I \times t}$ is a 2-digit sector X time fixed effect. Panel B shows the first-stage estimates obtained from a 2SLS regression with first-stage $\Delta \log Comp_{i,t} = \alpha_{Size} \log Size_i + \alpha_{Density} \log Density_i + \nu \Delta \log Capex_{i,t} + \psi_{I \times t} + \tau_{i,t}$, where $Size_i$, $Density_i$ are the median size and density of industry i 's products. Panel C shows the corresponding second-stage results. Column (1) includes Sector and Time fixed effects, Column (2) adds Sector X Time fixed effects, and Column (3) adds lagged TFP as a control. Columns (4)-(6) are analogous to Columns (1)-(3), while adding lagged revenue as a control. Counterfactual effects of electronics investment on TFP are computed using equation (24). Counterfactual effects of electronic miniaturization on TFP are computed using equation (25). All standard errors are clustered at the industry level.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: OLS Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.025*** (0.008)	0.026*** (0.009)	0.026*** (0.009)	0.028*** (0.008)	0.029*** (0.009)	0.029*** (0.009)
TFP Change From Investment in Electronics and Computers	0.03%	0.03%	0.01%	0.04%	0.04%	0.01%
Panel B: First Stage Results (Weighted by Value Added)						
$\log Size_i$	-0.011 (0.018)	-0.011 (0.018)	-0.011 (0.018)	-0.010 (0.018)	-0.010 (0.018)	-0.010 (0.018)
$\log Density_i$	-0.124** (0.058)	-0.124** (0.059)	-0.123** (0.059)	-0.114** (0.052)	-0.113** (0.052)	-0.113** (0.052)
Panel C: IV Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	-0.140 (0.107)	-0.140 (0.107)	-0.138 (0.106)	-0.138 (0.118)	-0.139 (0.118)	-0.137 (0.117)
F-test	2.247	2.195	2.190	2.173	2.118	2.131
TFP Change From Investment in Electronics and Computers	-0.10%	-0.10%	-0.03%	-0.09%	-0.09%	-0.02%
TFP Change From Electronic Miniaturization	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
Panel D: OLS Results (Unweighted)						
$\Delta \log Comp_{it}$	0.025*** (0.008)	0.026*** (0.009)	0.026*** (0.009)	0.028*** (0.008)	0.029*** (0.009)	0.029*** (0.009)
TFP Change From Investment in Electronics and Computers	0.03%	0.03%	0.01%	0.04%	0.04%	0.01%
Panel E: First Stage Results (Unweighted)						
$\log Size_i$	-0.020 (0.017)	-0.020 (0.017)	-0.020 (0.017)	-0.020 (0.017)	-0.020 (0.017)	-0.020 (0.017)
$\log Density_i$	-0.140** (0.055)	-0.140** (0.056)	-0.141** (0.056)	-0.133*** (0.050)	-0.132*** (0.051)	-0.133*** (0.050)
Panel F: IV Results (Unweighted)						
$\Delta \log Comp_{it}$	-0.076 (0.088)	-0.076 (0.088)	-0.074 (0.084)	-0.066 (0.093)	-0.067 (0.093)	-0.065 (0.089)
F-test	3.565	3.482	3.523	3.557	3.470	3.535
TFP Change From Investment in Electronics and Computers	-0.19%	-0.19%	-0.05%	-0.18%	-0.18%	-0.05%
TFP Change From Electronic Miniaturization	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
Observations	441	441	441	441	441	441
Benchmark TFP Change	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%
$SIC_2 \times Time$ Trend	No	Yes	Yes	No	Yes	Yes
Control for lagged change in TFP	No	No	Yes	No	No	Yes
Control for lagged revenue	No	No	No	Yes	Yes	Yes

Table 8: Regression Results for the 1977-1992 Sample. This table presents OLS and IV estimates of the effect of computer and electronics investment on TFP for the years 1977-1992, using a dataset of five-year stacked differences for heavy manufacturing industries (SIC Codes 3401-3999), excluding computers and electronics (SIC codes 3571-3579, 3671-3679). Panel A shows the estimate $\hat{\beta}$ in the OLS regression $\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma \Delta \log Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t}$, where $\log A_{i,t}$ is industry i 's TFP at time t , $Comp_{i,t}$ represents investment in computers and electronics, $Rev_{i,t-1}$ represents lagged revenue, and $\delta_{I \times t}$ is a 2-digit sector X time fixed effect. Panel B shows the first-stage estimates obtained from a 2SLS regression with first-stage $\Delta \log Comp_{i,t} = \alpha_{Size} \log Size_i + \alpha_{Density} \log Density_i + \nu \Delta \log Cape_{i,t} + \psi_{I \times t} + \tau_{i,t}$, where $Size_i$, $Density_i$ are the median size and density of industry i 's products. Panel C shows the corresponding second-stage results. Column (1) includes Sector and Time fixed effects, Column (2) adds Sector X Time fixed effects, and Column (3) adds lagged TFP as a control. Columns (4)-(6) are analogous to Columns (1)-(3), while adding lagged revenue as a control. Counterfactual effects of electronics investment on TFP are computed using equation (24). Counterfactual effects of electronic miniaturization on TFP are computed using equation (25). All standard errors are clustered at the industry level.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: OLS Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.022*** (0.007)	0.014* (0.008)	0.014* (0.008)	0.022*** (0.007)	0.015* (0.008)	0.015* (0.008)
TFP Change From Investment in Electronics and Computers	0.07%	0.05%	0.03%	0.07%	0.05%	0.04%
Panel B: First Stage Results (Weighted by Value Added)						
$\log Size_i$	0.084** (0.033)	0.084** (0.033)	0.083** (0.033)	0.086*** (0.033)	0.086*** (0.033)	0.086** (0.033)
$\log Density_i$	0.243*** (0.088)	0.243*** (0.089)	0.244*** (0.087)	0.253*** (0.092)	0.257*** (0.094)	0.258*** (0.093)
Panel C: IV Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.035 (0.030)	0.035 (0.030)	0.035 (0.030)	0.032 (0.028)	0.032 (0.028)	0.033 (0.028)
F-test	5.769	5.635	5.781	5.890	5.778	5.935
TFP Change From Investment in Electronics and Computers	0.16%	0.16%	0.12%	0.15%	0.15%	0.11%
TFP Change From Electronic Miniaturization	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%
Panel D: OLS Results (Unweighted)						
$\Delta \log Comp_{it}$	0.022*** (0.007)	0.015* (0.008)	0.015* (0.008)	0.022*** (0.007)	0.015* (0.008)	0.015* (0.008)
TFP Change From Investment in Electronics and Computers	0.07%	0.05%	0.03%	0.07%	0.05%	0.03%
Panel E: First Stage Results (Unweighted)						
$\log Size_i$	0.069** (0.028)	0.069** (0.029)	0.069** (0.029)	0.071** (0.028)	0.072** (0.029)	0.071** (0.028)
$\log Density_i$	0.185** (0.075)	0.185** (0.076)	0.187** (0.075)	0.197** (0.079)	0.200** (0.081)	0.201** (0.079)
Panel F: IV Results (Unweighted)						
$\Delta \log Comp_{it}$	0.051 (0.036)	0.051 (0.036)	0.051 (0.036)	0.046 (0.034)	0.046 (0.033)	0.046 (0.034)
F-test	4.938	4.823	4.971	5.188	5.101	5.271
TFP Change From Investment in Electronics and Computers	0.11%	0.11%	0.08%	0.10%	0.10%	0.08%
TFP Change From Electronic Miniaturization	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%
Observations	441	441	441	441	441	441
Benchmark TFP Change	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
$SIC_2 \times Time$ Trend	No	Yes	Yes	No	Yes	Yes
Control for lagged change in TFP	No	No	Yes	No	No	Yes
Control for lagged revenue	No	No	No	Yes	Yes	Yes

Table 9: Regression Results for the 1982-1997 Sample. This table presents OLS and IV estimates of the effect of computer and electronics investment on TFP for the years 1982-1997, using a dataset of five-year stacked differences for heavy manufacturing industries (SIC Codes 3401-3999), excluding computers and electronics (SIC codes 3571-3579, 3671-3679). Panel A shows the estimate $\hat{\beta}$ in the OLS regression $\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma \Delta \log Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t}$, where $\log A_{i,t}$ is industry i 's TFP at time t , $Comp_{i,t}$ represents investment in computers and electronics, $Rev_{i,t-1}$ represents lagged revenue, and $\delta_{I \times t}$ is a 2-digit sector X time fixed effect. Panel B shows the first-stage estimates obtained from a 2SLS regression with first-stage $\Delta \log Comp_{i,t} = \alpha_{Size} \log Size_i + \alpha_{Density} \log Density_i + \nu \Delta \log Capex_{i,t} + \psi_{I \times t} + \tau_{i,t}$, where $Size_i, Density_i$ are the median size and density of industry i 's products. Panel C shows the corresponding second-stage results. Column (1) includes Sector and Time fixed effects, Column (2) adds Sector X Time fixed effects, and Column (3) adds lagged TFP as a control. Columns (4)-(6) are analogous to Columns (1)-(3), while adding lagged revenue as a control. Counterfactual effects of electronics investment on TFP are computed using equation (24). Counterfactual effects of electronic miniaturization on TFP are computed using equation (25). All standard errors are clustered at the industry level.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: OLS Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.023*** (0.008)	0.018** (0.009)	0.019** (0.008)	0.024*** (0.008)	0.018** (0.009)	0.019** (0.008)
TFP Change From Investment in Electronics and Computers	0.06%	0.04%	0.03%	0.06%	0.04%	0.03%
Panel B: First Stage Results (Weighted by Value Added)						
$\log Size_i$	0.076** (0.031)	0.076** (0.031)	0.075** (0.031)	0.077** (0.031)	0.077** (0.031)	0.077** (0.032)
$\log Density_i$	0.290*** (0.085)	0.290*** (0.086)	0.287*** (0.085)	0.295*** (0.087)	0.296*** (0.089)	0.293*** (0.088)
Panel C: IV Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.032 (0.035)	0.032 (0.035)	0.041 (0.039)	0.031 (0.035)	0.031 (0.035)	0.041 (0.038)
F-test	6.715	6.560	6.476	6.733	6.593	6.501
TFP Change From Investment in Electronics and Computers	0.07%	0.07%	0.07%	0.07%	0.07%	0.07%
TFP Change From Electronic Miniaturization	0.02%	0.02%	0.02%	0.01%	0.01%	0.02%
Panel D: OLS Results (Unweighted)						
$\Delta \log Comp_{it}$	0.025*** (0.008)	0.019** (0.008)	0.020*** (0.007)	0.025*** (0.008)	0.019** (0.009)	0.020*** (0.008)
TFP Change From Investment in Electronics and Computers	0.05%	0.04%	0.03%	0.05%	0.04%	0.03%
Panel E: First Stage Results (Unweighted)						
$\log Size_i$	0.073** (0.028)	0.073** (0.029)	0.072** (0.029)	0.074** (0.029)	0.074** (0.029)	0.074** (0.029)
$\log Density_i$	0.262*** (0.077)	0.262*** (0.078)	0.259*** (0.078)	0.267*** (0.079)	0.269*** (0.081)	0.266*** (0.081)
Panel F: IV Results (Unweighted)						
$\Delta \log Comp_{it}$	0.030 (0.038)	0.030 (0.038)	0.041 (0.041)	0.029 (0.037)	0.029 (0.037)	0.041 (0.040)
F-test	6.927	6.766	6.627	6.980	6.837	6.700
TFP Change From Investment in Electronics and Computers	0.07%	0.07%	0.07%	0.07%	0.07%	0.07%
TFP Change From Electronic Miniaturization	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%
Observations	441	441	441	441	441	441
Benchmark TFP Change	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%
$SIC_2 \times Time$ Trend	No	Yes	Yes	No	Yes	Yes
Control for lagged change in TFP	No	No	Yes	No	No	Yes
Control for lagged revenue	No	No	No	Yes	Yes	Yes

Table 10: Regression Results for the 1987-2002 Sample. This table presents OLS and IV estimates of the effect of computer and electronics investment on TFP for the years 1987-2002, using a dataset of five-year stacked differences for heavy manufacturing industries (SIC Codes 3401-3999), excluding computers and electronics (SIC codes 3571-3579, 3671-3679). Panel A shows the estimate $\hat{\beta}$ in the OLS regression $\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma \Delta \log Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t}$, where $\log A_{i,t}$ is industry i 's TFP at time t , $Comp_{i,t}$ represents investment in computers and electronics, $Rev_{i,t-1}$ represents lagged revenue, and $\delta_{I \times t}$ is a 2-digit sector X time fixed effect. Panel B shows the first-stage estimates obtained from a 2SLS regression with first-stage $\Delta \log Comp_{i,t} = \alpha_{Size} \log Size_i + \alpha_{Density} \log Density_i + \nu \Delta \log Capex_{i,t} + \psi_{I \times t} + \tau_{i,t}$, where $Size_i, Density_i$ are the median size and density of industry i 's products. Panel C shows the corresponding second-stage results. Column (1) includes Sector and Time fixed effects, Column (2) adds Sector X Time fixed effects, and Column (3) adds lagged TFP as a control. Columns (4)-(6) are analogous to Columns (1)-(3), while adding lagged revenue as a control. Counterfactual effects of electronics investment on TFP are computed using equation (24). Counterfactual effects of electronic miniaturization on TFP are computed using equation (25). All standard errors are clustered at the industry level.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: OLS Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.023*** (0.006)	0.018*** (0.005)	0.020*** (0.005)	0.023*** (0.006)	0.018*** (0.006)	0.020*** (0.005)
TFP Change From Investment in Electronics and Computers	0.04%	0.03%	0.03%	0.04%	0.03%	0.03%
Panel B: First Stage Results (Weighted by Value Added)						
$\log Size_i$	0.100*** (0.035)	0.100*** (0.035)	0.100*** (0.035)	0.101*** (0.035)	0.101*** (0.036)	0.101*** (0.035)
$\log Density_i$	0.379*** (0.080)	0.379*** (0.081)	0.374*** (0.080)	0.382*** (0.083)	0.383*** (0.084)	0.377*** (0.082)
Panel C: IV Results (Weighted by Value Added)						
$\Delta \log Comp_{it}$	0.044* (0.023)	0.044* (0.023)	0.051* (0.028)	0.042* (0.023)	0.042* (0.023)	0.050* (0.027)
F-test	10.44	10.20	10.24	10.33	10.12	10.10
TFP Change From Investment in Electronics and Computers	0.08%	0.08%	0.07%	0.07%	0.08%	0.07%
TFP Change From Electronic Miniaturization	0.03%	0.03%	0.02%	0.03%	0.03%	0.02%
Panel D: OLS Results (Unweighted)						
$\Delta \log Comp_{it}$	0.022*** (0.006)	0.018*** (0.006)	0.019*** (0.005)	0.022*** (0.006)	0.018*** (0.006)	0.019*** (0.005)
TFP Change From Investment in Electronics and Computers	0.04%	0.03%	0.03%	0.04%	0.03%	0.03%
Panel E: First Stage Results (Unweighted)						
$\log Size_i$	0.101*** (0.036)	0.101*** (0.037)	0.100*** (0.037)	0.101*** (0.037)	0.101*** (0.037)	0.101*** (0.037)
$\log Density_i$	0.391*** (0.087)	0.391*** (0.088)	0.386*** (0.086)	0.394*** (0.089)	0.395*** (0.091)	0.389*** (0.089)
Panel F: IV Results (Unweighted)						
$\Delta \log Comp_{it}$	0.042** (0.021)	0.042** (0.021)	0.048* (0.026)	0.040* (0.021)	0.040* (0.021)	0.047* (0.026)
F-test	9.705	9.480	9.563	9.599	9.396	9.432
TFP Change From Investment in Electronics and Computers	0.08%	0.08%	0.08%	0.08%	0.08%	0.08%
TFP Change From Electronic Miniaturization	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%
Observations	441	441	441	441	441	441
Benchmark TFP Change	0.08%	0.08%	0.08%	0.08%	0.08%	0.08%
$SIC_2 \times Time$ Trend	No	Yes	Yes	No	Yes	Yes
Control for lagged change in TFP	No	No	Yes	No	No	Yes
Control for lagged revenue	No	No	No	Yes	Yes	Yes

Table 11: Regression Results for the 1992-2007 Sample. This table presents OLS and IV estimates of the effect of computer and electronics investment on TFP for the years 1992-2007, using a dataset of five-year stacked differences for heavy manufacturing industries (SIC Codes 3401-3999), excluding computers and electronics (SIC codes 3571-3579, 3671-3679). Panel A shows the estimate $\hat{\beta}$ in the OLS regression $\Delta \log A_{i,t} = \beta \Delta \log Comp_{i,t} + \gamma \Delta \log Rev_{i,t-1} + \delta_{I \times t} + \eta \Delta \log A_{i,t-1} + \epsilon_{i,t}$, where $\log A_{i,t}$ is industry i 's TFP at time t , $Comp_{i,t}$ represents investment in computers and electronics, $Rev_{i,t-1}$ represents lagged revenue, and $\delta_{I \times t}$ is a 2-digit sector X time fixed effect. Panel B shows the first-stage estimates obtained from a 2SLS regression with first-stage $\Delta \log Comp_{i,t} = \alpha_{Size} \log Size_i + \alpha_{Density} \log Density_i + \nu \Delta \log Capex_{i,t} + \psi_{I \times t} + \tau_{i,t}$, where $Size_i$, $Density_i$ are the median size and density of industry i 's products. Panel C shows the corresponding second-stage results. Column (1) includes Sector and Time fixed effects, Column (2) adds Sector X Time fixed effects, and Column (3) adds lagged TFP as a control. Columns (4)-(6) are analogous to Columns (1)-(3), while adding lagged revenue as a control. Counterfactual effects of electronics investment on TFP are computed using equation (24). Counterfactual effects of electronic miniaturization on TFP are computed using equation (25). All standard errors are clustered at the industry level.