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Working Paper

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Working Papers, No. 2018-16

Provided in Cooperation with:
Bank of Mexico, Mexico City

Suggested Citation: García-Verdú, Santiago; Ramos-Francia, Manuel; Sánchez-Martínez, Manuel (2018) : TIIE-28 swaps as risk-adjusted forecasts of monetary policy in Mexico, Working Papers, No. 2018-16, Banco de México, Ciudad de México

This Version is available at:
https://hdl.handle.net/10419/240657

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Working Papers

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August 2018

Documento de Investigación
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TIIE-28 Swaps as Risk-Adjusted Forecasts of Monetary Policy in Mexico*

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Abstract: Information extracted from financial derivatives on interest rates is commonly used to forecast movements in such rates. Yet, such an extraction generally assumes that agents are risk-neutral. Thus, it might be useful to account for their risk-aversion when doing forecasts. This can be done adding a risk-correction. In this context, we use TIIE-28 swaps to forecast changes in monetary policy in Mexico, using financial variables to account for the risk-correction. We assess whether models with a risk-correction outperform the TIIE-28 swaps rates. Their *in-sample* explained variability improves when using a risk-correction. Our main model's *out-of-sample* forecasts are similar for short horizons (3-month), and statistically better for longer horizons (9 to 24-month), compared to the direct use of TIIE-28 swaps interest rates.
Keywords: TIIE-28, Swaps, Interest Rates, Expected Monetary Policy
JEL Classification: E-52, G-12

Resumen: La información extraída de derivados financieros sobre tasas de interés es comúnmente utilizada para pronosticar movimientos en las tasas de interés. Sin embargo, dicha extracción generalmente supone que los agentes son neutrales al riesgo. Así, podría ser útil considerar la aversión al riesgo de los agentes al hacer pronósticos, lo que se puede hacer agregando una corrección por riesgo. En este contexto, utilizamos los swaps de TIIE-28 para pronosticar cambios en la política monetaria en México, utilizando un conjunto de variables financieras para considerar la corrección por riesgo. Analizamos si los modelos con corrección por riesgo son mejores que las tasas swaps de TIIE-28. La variabilidad explicada *dentro de la muestra* mejora cuando se usa una corrección por riesgo. Los pronósticos *fuera de muestra* de nuestro modelo principal son similares para horizontes cortos (3-meses) y estadísticamente mejores para horizontes más largos (9 a 24 meses), en comparación con el uso directo de las tasas de interés de swaps de TIIE-28.
Palabras Clave: TIIE-28, Swaps, Tasa de Interés, Política Monetaria Esperada

*The opinions in this paper are those of the authors and do not necessarily reflect those of Banco de México. We are grateful for suggestions and comments from Raúl Alvarez del Castillo, Raúl Ibarra Ramírez, Fabrizio López-Gallo, Monika Piazzesi, José-Gonzalo Rangel and Martín Tobal. We thank the Financial Stability Division at Banco de México for providing us with most of the data.

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1. Introduction

Having an estimate of the expected monetary policy is of great importance to financial institutions, the government, and market participants at large. In addition, in monetary models, expectations play a central role. Thus, having a dependable measure of the expected monetary policy is relevant, for instance, as it aids agents in taking their investment decisions and policy makers in gauging how their policy announcements could have affected such expectations.

One can think of three ways of obtaining a forecast: i) model-based; ii) survey-based; and iii) market-based. Model-based forecasts are well known and quite popular. One typically estimates an econometric model based on financial data, and with it, produces a forecast. Survey-based forecasts use a combination of forecasts obtained through surveys, without paying particular attention to how surveyees individually obtain their forecasts. Finally, market-based forecasts rely on market data and, typically, on a model to extract the forecast. We focus on this type of forecasts.

In particular, a possible method to obtain forecasts extracts information from financial derivatives that have interest rates as their underlying. This is fitting as derivatives carry market information. As mentioned, another possibility is to obtain such forecasts from surveys. An important difference is that while agents trading financial assets have a pecuniary stake, surveys' participants have a reputational one. For that reason, we compare the performance of our model's forecasts against a survey.

As an example of such a procedure, we have that market participants use futures as predictors for the associated underlying interest rates. Nonetheless, under the assumption of risk-adverse agents, asset pricing theory implies that there needs to be a risk-correction to such a forecast. Thus, one of our main objectives in this paper is to obtain risk-corrections for TIIE-28 and government funding interest rates' forecasts for Mexico.

To that end, we follow Piazzesi and Swanson (2008) who risk-adjust the federal funds rate futures to forecast changes in U.S. monetary policy. Naturally, we account for the characteristics of the Mexican financial markets. For instance, one consideration is that there

are no futures that have as an underlying the reference interest rate. Nonetheless, there is an important market for TIIE-28 interest rate swaps. TIIE-28 is the acronym (in Spanish) for equilibrium interbank rate and has, as indicated in its name, an associated 28-day horizon. It is the representative interest rate of Mexican interbank operations and benchmark rate for banking loans. Apart from its own relevance, it is highly correlated with the government-funding interest rate. Another consideration is that although there is a market for TIIE-28 futures, its volume dropped in the past years. Thus, we focus on the TIIE-28 swaps market.

As for our main findings, we have the following comments. The *in-sample* explained variability improves when using a risk-correction. Centrally, we document that our main model's *out-of-sample* forecasts are similar for short horizons (3-month), and statistically significantly better for long horizons (9 to 24-month), relative to the direct use of TIIE-28 swaps rates. These findings are in line with the theory in that the risk-correction becomes more relevant for forecasts with a medium horizon. In addition, several of the risk-correction factors that we consider are associated with the business cycle, indicating that the bias relates to it.

We divide the rest of the paper into the following sections. The second one presents the theory supporting our assertions and the linear regression model. The third section describes the data we use and their characteristics. The next one has a brief literature review. The fifth section presents the main estimations, along with their interpretation. The final section offers some concluding remarks.

2. The Model

Consider the standard asset pricing equation:

$$P_t = \mathbb{E}_t[M_{t+1}P_{t+1}] \qquad (1)$$

where P_t is the price of a given financial asset at time t, M_{t+1} is the stochastic discount factor (SDF), and $\mathbb{E}_t$ is the expectation conditional on the information available at time t. Thus, its

price today is the expected value of its price tomorrow times the stochastic discount factor. As is well known, the existence of the stochastic discount factor follows from assuming the absence of arbitrage opportunities (e.g., Duffie, 2001).

While we will be using swaps on an interest rate to obtain our forecasts, it is clearer to explain the key argument considering a futures contract on an interest rate. Thus, a long (short) position in a futures contract gives the holder the obligation to buy (sell) an underlying asset at a specific time in the future at an initially mutually agreed price or interest rate. For those futures that have a commodity as an underlying asset, the delivery location and its specific characteristics have to be previously agreed.

In the case of an interest rate future, the holder of the long position commits to buy a notional multiplied by the difference between the futures rate (F_t) and the realized interest rate at maturity (i_{t+1}). Assuming a notional of a peso, we have that the future for such a position is worth $F_t - i_{t+1}$ in period $t+1$. Conversely, the short position is worth $i_{t+1} - F_t$ in period $t+1$. Thus, for a one period interest rate future contract, equation (1) specializes to

$$0 = \mathbb{E}_t[M_{t+1}(F_t - i_{t+1})]. \qquad (2)$$

The contract is settled in such a way that no resources are exchanged in period t, thus, $P_t = 0$. At time $t+1$, the flow $(F_t - i_{t+1})$ would then be exchanged. Of course, i_{t+1} is not known until the next period.

It is common to observe in the financial press (e.g., Infosel, 2015), as well as some banks' publications (e.g., Citibanamex, 2015) that F_t is used as an approximate forecast of i_{t+1}. Similarly, market participants use futures to forecast the expected movements of other financial indices and commodities. In such cases, they typically assume that $F_t = \mathbb{E}_t(i_{t+1})$.

To see why this is not generally the case, one can rewrite (2) as follows:[1]

$$F_t = \mathbb{E}_t[i_{t+1}] + \mathbb{cov}_t(M_{t+1}, i_{t+1})(\mathbb{E}_t[M_{t+1}])^{-1} \qquad (3)$$

[1] We have used the equality: $\mathbb{E}_t[M_{t+1}i_{t+1}] = \mathbb{E}_t[M_{t+1}]\mathbb{E}_t[i_{t+1}] + \mathbb{cov}_t(M_{t+1}, i_{t+1})$.

The term involving the covariance biases F_t as a predictor of the interest rate i_{t+1}. There are, however, some specific cases when the covariance is equal to zero.

Consider then the following two cases. First, if agents were risk-neutral, we have that M_{t+1} would be constant and, thus, the covariance term zero. It would then follow that $F_t = \mathbb{E}_t[i_{t+1}]$. Second, in the case of a short-term horizon, the stochastic discount factor plays a minor role. To see this, assume that the stochastic discount factor is given by $M_{t+1} = \beta u'(C_{t+1})/u'(C_t)$, where β is the subjective discount factor, u' is the marginal utility, and C_t consumption in period t. For short horizons, we can reasonably assume that consumption will not vary much; i.e., $C_t \approx C_{t+1}$. Accordingly, M_{t+1} is close to being constant, and the covariance term would be close to zero. Moreover, the referred SDF in terms of consumption growth provides a clearer intuition on the relation of the risk premium to the business cycle and monetary policy. However, it is worth underscoring that our results do not rely on assuming a specific SDF.

The central idea in this paper is to account for the term $\mathbb{cov}_t(M_{t+1}, i_{t+1})(E_t[M_{t+1}])^{-1}$ when making forecasts for the government-funding rate. There are then two criteria for choosing a variable or a set of variables to approximate this term. From the economic point of view, it should measure macroeconomic risk. In effect, the identification of the sources of macroeconomic risk is a central goal in finance and macroeconomics (Cochrane, 2005). From the econometric point of view, it should improve the forecasts' performance. We have that the first criteria imposes discipline on the model, since there has to be economic content to each variable. The second criteria assesses its empirical relevance.

Let us then extend the idea for an interest rate swap contract. In an interest rate swap with notional N, the parts agree to exchange a floating interest rate flow for a fixed interest rate one over k periods. In other words, the long (short) part of the contract pays (receives) the notional times the floating interest rate and receives (pays) the notional times the agreed swap rate for k periods. As in the case of a future contract, consider the particular version of equation (1), which prices, in a simplified way, a two-period swap:

$$0 = \mathbb{E}_t\left[M_{t+1}\left(s_{t,2} - i_t\right) + M_{t+2}\left(s_{t,2} - i_{t+1}\right)\right] \qquad (4)$$

Similarly, we assume that the notional is one peso and no resources are exchanged in period 0; i.e., $P_t = 0$. The swap rate $s_{t,2}$ is the same for each period. Of course, both parties agree to it in period t. In addition, the floating interest rates are i_t and i_{t+1} respectively, for each period.

We have that each floating interest rate is determined one period before its associated payment is due, a common convention when pricing a swap (Wilmott, 2006). We can then rewrite (4) as follows:

$$s_{t,2}\mathbb{E}_t[M_{t+1}(1) + M_{t+2}(1)] = \mathbb{E}_t[M_{t+1}(i_t) + M_{t+2}(i_{t+1})]$$

We have expressed the last equation setting the fixed leg of the swap on the left hand side, and the floating leg on the right hand side. Of course, if correctly priced, their expected SDF-adjusted values should be equal. In addition, we have added and subtracted a one (twice) in the right hand side, implying that:

$$s_{t,2}(\mathbb{E}_t[M_{t+1}] + \mathbb{E}_t[M_{t+2}]) = \mathbb{E}_t[M_{t+1}(1 + i_t - 1) + M_{t+2}(1 + i_{t+1} - 1)]$$

We note that $\mathbb{E}_t[M_{t+1}(1 + i_t)] = 1$ in t, and $\mathbb{E}_{t+1}[M_{t+2}(1 + i_{t+1})] = 1$ in $t + 1$. Using the law of iterated expectations, we obtain the following formula to price this swap:

$$s_{t,2} = \frac{\left(2 - (1+i_{t,1})^{-1} + (1+2i_{t,2})^{-1}\right)}{\left((1+i_{t,1})^{-1} + (1+2i_{t,2})^{-1}\right)}.$$

We can also express the pricing equation for the swap as follows (as a convention, we let $i_t = s_{t,1}$).

$$\mathbb{E}_t[i_{t+1}] = \left(s_{t,2} - s_{t,1}\right)(\mathbb{E}_t[M_{t+1}])(\mathbb{E}_t[M_{t+2}])^{-1} + s_{t,2} - \mathbb{COV}_t(M_{t+2}, i_{t+1})(\mathbb{E}_t[M_{t+2}])^{-1}.$$

Thus, we have that $\mathbb{E}_t[i_{t+1}] \approx \left(2s_{t,2} - s_{t,1}\right)$. Market participants commonly consider a change in slope of the swap curve as signaling a future adjustment in the associated interest rates. This last expression is telling in this respect. In addition, one can see such an approximation as a 'forward rate' obtained from the swaps rate curve. In effect, if we define $f_t^{1\to 2}$ as the rate that satisfies the equation $\left(1 + s_{t,1}\right)(1 + f_t^{1\to 2}) = \left(1 + 2s_{t,2}\right)$, we then have that $f_t^{1\to 2} \approx 2s_{t,2} - s_{t,1}$, an expression that coincides with the approximation for $\mathbb{E}_t(i_{t+1})$ above.

As mentioned, the term $\mathbb{cov}_t(M_{t+2}, i_{t+1})(\mathbb{E}_t[M_{t+2}])^{-1}$ is a risk-premium. To see this, note that as a general result we have that:

$$\mathbb{E}_t\left[i_{t+1} - r_f\right] = -\mathbb{cov}_t(M_{t+2}, i_{t+1})(\mathbb{E}_t[M_{t+2}])^{-1} \qquad (5)$$

where r_f is the risk-free rate. We know that interest rate premiums vary along the business cycle (e.g., see Cochrane, 2005). One would then expect that the bias changes with the business cycle and should be time varying. In addition, as mentioned, the swap has as an underlying interest rate the TIIE-28, while our main interest is on forecasting the government funding, a one-day rate. There is at least one additional consideration in this regard. The TIIE-28 and the funding rate differ in their associated maturity by a few days. Although their discrepancy is small, we implicitly account for this, as we explain in more detail below.

In this context, to motivate our problem, we consider the apparent relations among the following three time series. First, the TIIE-28 (in black) is the realized rate along with two of its possible forecasts, which we explain in detail in the following sections. On the one hand, the swap interest rate (in orange) and, on other hand, the swap interest with a risk-correction (in blue). Both forecasts have an associated horizon of 6 months (Figure 1).

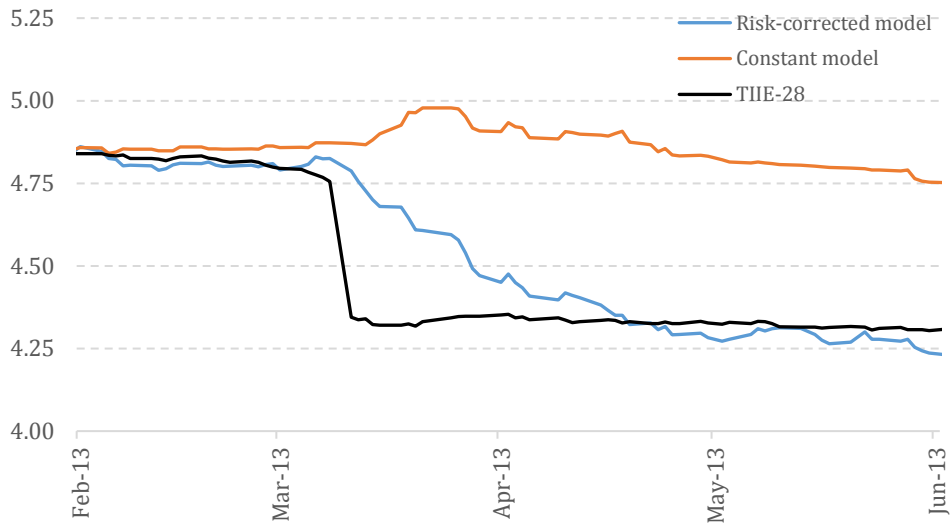

Figure 1. Realized Interest Rates and Forecasts:
TIIE-28 Interest Rates and Two of its Possible Forecasts.
Notes: Realized TIIE-28 (in black) and two forecasts: the swap rates (in orange) and the swap rates with the multi-variable risk-correction (in blue) in percent. Each of the forecasts has an associated 6-month horizon. We note how the risk-correction anticipates the change in the policy rate on March 8th, 2013. It takes around four months for the swap rate to account for the eventual change in the monetary policy stance.
Sources: Banco de México and own estimations with data from Valor de Mercado, Bloomberg, and Banco de México.

It appears that the risk-corrected forecast would had correctly anticipated the change in the TIIE-28 interest rate on March 8, 2013. Moreover, it took approximately three more months for the swap rate to account for the forthcoming variation in the monetary policy stance. Evidently, this is only one particular case in which the risk-correction seems to have accounted for the bias. In the rest of the paper, we explain the specific data we use, the rationale supporting the risk-correction, how we build it and, centrally, provide statistical evidence on the forecasts' improvement.

3. Data

We use daily time series from different sources. The TIIE-28 and Government Bond rates are from *Valor de Mercado*.[2] The Government Funding Rate and the TIIE-28 interest rates are from the Banco de México.

Our key empirical challenge is to measure the risk-correction. In this context, equation (5) is particularly useful. It suggests the interest bonds rates differences are natural candidates. Of course, such differences are associated with the business cycles (see, e.g., Reyna et al. 2008, for the Mexican case). On its part, the volatility of a financial variable is also a plausible candidate. In particular, we use the volatility of the Mexican stock exchange index (S&P/BMV IPC, for its acronym in Spanish). We also use the Vimex index, published by Bloomberg. Vimex is the implied volatility of the IPC index of the Mexican Stock Exchange, akin to the VIX index estimated by the CBOE, the implicit volatility of options on the S&P 500 index. In addition, we use the volatility of the exchange rate, with data from Banco de México.

The time series go from January 2, 2008 to August 16, 2017. The forecasting horizons we focus on are three, six, and nine months, and one and two years. We note that a one-year swap is composed by 13 periods of 28 days. Of course, a variables' frequency restricts if we can consider it as a risk-correction. For instance, several macroeconomic variables have a monthly or quarterly frequency. [3, 4] Relatedly, while being only a few cases, for those data points that are not available, we take the previous available one. In other words, we assume they have stayed constant.

[2] One of the two Mexican price vendors.
[3] Still, time disaggregating a variable would be an option.
[4] In the case of Mexico the correlation between some business cycle indicators such as the output gap and interest rates have changed in magnitude. In particular, after the Global Financial Crisis, the output gap has remained close to zero, while the short-term interest rates have changed considerable. This brings into question the potential relevance of macroeconomic indicators such as the output gap as a risk-correction. We do not actually face such a problem because, as we explain in the main text, we only used financial variables. Nonetheless, it is worth emphasizing.

Finally, somewhat recently MexDer along with the CBOE proposed changing some of the pricing procedures and conventions for the TIIE-28 swaps. These involved using the exchange rate futures and foreign interest rates based on no-arbitrage arguments. We, nonetheless, decided to base our pricing on the domestic interest rates.[5] Still, we do not think that such a difference is driving our results.

4. An Abridged Literature Review

Although there is an extensive literature on interest rate predictability using U.S. financial data, the literature on the same topic but for emerging market economies (EMEs) is less extensive. As EMEs financial markets keep on developing, we think there will be more interest in learning about them and about the extent to which they share traits with U.S. financial markets.

The seminal papers studying interest rates predictability in the U.S. are Fama and Bliss (1987), and Campbell and Shiller (1991). On this topic, a more recent paper is Piazzesi and Cochrane (2005). They explore bond risk premiums extending the tests implemented in the initial two references. Closer to our paper, we have Piazzesi and Swanson (2004). They propose a risk-correction for the federal funds futures to forecast U.S. monetary policy. We refer the interested reader to Piazzesi and Swanson (2004), and the sources cited therein, for a more detailed review of the related U.S. literature.

In the case of papers using data from the Mexican bonds markets, we have the following ones. Sod (1995) explores the expectations hypothesis for Mexican bond interest rates, finding evidence against it. Of course, at the time, there were a limited number of maturities and a short history. More recently, for instance, Ramos-Francia, Espada, and Torres (2008) provide evidence against the expectations hypothesis using Mexican bond data. They estimate regressions parallel to those in Fama and Bliss (1987), and Campbell and Shiller (1991).

[5] See Valmer and CBOE (2014).

On their part, Castellanos and Camero (2002) examine the predictability content of interest rates. They find that interest rates have informational content to forecast interest rates and economic activity. Reyna et al. (2008) assess the relationship between the yield curve and economic activity. In particular, they document that the slope of the yield curve predicts changes in the Mexican GDP growth rate. This last result is in line with Castellanos and Camero (2002).

The predictability of interest rates is part of the phenomenon we study in this paper. In effect, one can see it as deviations from what models would predict assuming risk-neutrality, as the risk compensation changes through the business cycle (Cochrane, 2005). In addition, the idea to obtain an empirical risk-correction is not unique to interest rates. For instance, Hamilton and Wu (2013) have estimated a risk-correction for oil futures under a similar logic.

5. Linear Regression Models

Empirically, we consider the following linear regression model:

$$i_{t+1} - f_t = \alpha + \beta x_t + e_{t+1} \qquad (6)$$

where i_t is the interest rate, f_t is a forecasting rate, x_t is a variable approximating the risk premium, α and β are coefficients, and e_t is the error term, in period t.

Once we estimate the model, we calculate the forecast as $(\hat{\alpha} + \hat{\beta} x_t) + f_t$. It is worth pointing out that it is in terms of information available at time t. To see this, consider the expectation conditional on the information at time t of (6). This leads to the following expression:

$$\mathbb{E}_t(i_{t+1}) - f_t = \alpha + \beta x_t.$$

More generally, we have that:

$$i_{t+n} - f_{t,n} = \alpha_n + \boldsymbol{\beta}_{\boldsymbol{n}}' \boldsymbol{x}_{\boldsymbol{t,n}} + e_{t+n},$$

where n is the forecasting horizon $\boldsymbol{x_{t,n}}$ is a vector of variables accounting for the risk-correction. Accordingly, we obtain estimates for α_n and $\boldsymbol{\beta'_n}$, (respectively, a scalar and a vector). Thus, in such a case, we calculate the forecast as $\hat{\alpha}_n + \widehat{\boldsymbol{\beta}}'_{\boldsymbol{n}}\boldsymbol{x_{t,n}} + f_{t,n}$.

In this context, one could extract the relevant informational content of a time series by using a transformation of such series. For instance, one can use the moving average of some time series. What is crucial for these transformations is that they, of course, cannot involve information ahead of period t. Thus, for example, $z_t = g(x_t, x_{t-1}, \dots)$ is a valid transformation, since it follows how the information unfolds through time.

As a first general step, we document that there is indeed a bias when swaps interest rates are used as predictors and a risk-correction is omitted, particularly so as the horizon increases. To that end, we consider the following regression:

$$i_{t+n} - f_{t,n} = \alpha_n + e_{t+n};$$

and individually tests whether $\alpha_n = 0$, where we will use the swap rates for $f_{t,n}$. We present our estimates in Table 1.

Several remarks are in order. First, it is clear that there is a statistically significant bias when using the proposed predictor, regardless of the horizon. Second, the bias increases with the horizon, in all cases. This suggests that the correction will play a more relevant role when the forecasting horizon increases. This is as anticipated based on the theory. Third, as a rule of thumb, practitioners seem to correct by 30 basis points the TIIE-28 forecast to obtain the bank-funding rate forecast. In effect, the results in the table suggests that such difference is on average 30 basis points for long-term forecasts. It, nonetheless, varies with the horizon.

Constant (α)		
	Forecast	
n	s	b(s)
3m	-4.32	-8.20
	(-5.95)	(-13.82)
6m	-10.00	-19.61
	(-7.13)	(-15.51)
9m	-18.18	-35.01
	(-8.81)	(-17.71)
1y	-32.07	-58.82
	(-11.54)	(-21.93)
2y	-72.71	-134.71
	(-18.66)	(-38.01)

TIIE-28

Constant (α)		
	Forecast	
n	s	b(s)
3m	-37.99	-41.88
	(-50.49)	(-65.92)
6m	-43.04	-52.65
	(-30.73)	(-41.79)
9m	-50.69	-67.52
	(-24.60)	(-34.41)
1y	-63.24	-89.98
	(-22.55)	(-33.27)
2y	-102.97	-164.97
	(-26.36)	(-46.35)

Government Funding Rate

Table 1. Estimates of α_n.
Estimates of α_n are based on the following linear model:

$$i_{t+n} - f_{t,n} = \alpha_n + e_{t+n}$$

Notes: The forecast rates are denoted by s and $b(s)$. The b indicates that the associated interest rate has been obtained by bootstrapping. Full-sample estimations. T-stats between parentheses. Statistically significant coefficients at the 90%, 95% and 99% confidence levels have a t-stat with an absolute value greater than 1.645, 1.96 and 2.576, respectively.
Sample: January 2, 2008–August 16, 2017.
Sources: Own estimations with data from Valmer and Banco de México.

In addition, we point out that the regressions have been implemented with the swap fixed rate directly, which has been denoted by s, as well as the bootstrapped swap fixed rate, denoted by $b(s)$. A possible interpretation of these two interest rates is that the former is akin to an YTM rate, and the latter is similar to a zero-coupon interest rate.

Second, once we have provided evidence on the bias, we are in a position to estimate a more general model:

$$i_{t+n} - f_{t,n} = \alpha_n + \boldsymbol{\beta_n'} \boldsymbol{x_{t,n}} + e_{t+n}$$

In this context, standard errors might need a correction. This could be the case since observations overlap and, thus, their error terms might be auto-correlated. In addition, they might also be subject to heteroscedasticity. Thus, we use the HAC standard error estimates (Newey and West, 1987).

Our interest is in correcting the bias, and in improving the forecasting prowess. We estimate rolling-window regressions, and perform the forecasts. Hence, as a first step, we assess the *in-sample* fit and, as a second one, we assess the forecasts. To this end, we use 220-day windows, which approximate a year.[6] To calculate the following statistics, we estimate their goodness of fit, roll the window one day, and estimate the linear regression model again. Although we have estimated the model for several predictors, we focus on a specific subset.

Risk-correction (x): Gvt. 22		
	Forecast	
n	s	b(s)
3m	0.15	0.21
6m	0.31	0.36
9m	0.33	0.38
1y	0.36	0.38
2y	0.33	0.40

TIIE-28

Risk-correction (x): Gvt. 22		
	Forecast	
n	s	b(s)
3m	0.15	0.19
6m	0.29	0.35
9m	0.30	0.37
1y	0.33	0.37
2y	0.34	0.40

Government Funding Rate

Table 2. In-sample Adjusted R2 Single-Factor Regression.
The following regression is estimated. We report the R^2 for different n.

$$i_{t+n} - f_{t,n} = \alpha_n + \beta x_t + e_{t+n}$$

Notes: The forecast rates are denoted by s and $b(s)$. The b indicates that the associated rate has been obtained by bootstrapping. Rolling-Window Regressions. Window size 220-day. Gvt. 22 stands for the difference in nominal interest rates of bonds with a maturity of 24 and 22 months. Sample: January 2, 2008-August 16, 2017.
Sources: Own estimations with data from Valmer and Banco de México.

To set the stage, Table 2 provides the *in-sample* relative variability explained by the single-factor linear model; i.e., the adjusted R^2s. The factor we use is the difference between the 24-month year government nominal bond rate and that of the 22-month. In short, an interest rate spread. We note that, the R^2's generally increase with the horizon. This suggests that as the horizon increases, the risk correction gains relevance, as hinted by the theory. Still, the larger the horizon, the more factors could have a role in the determination of the risk-correction. As explained, this is a business cycle phenomenon. Thus, the forecast could deteriorate if the forecasting horizon is longer than that of the business cycle.

[6] We have implemented similar estimations using 6-month windows. The associated results (not reported) are comparable. We conjecture there could be an optimal size for the estimation window in that it minimizes forecasting errors, an issue we do not explore.

As a natural exercise, we consider a set of regressors to improve our forecasts. Our choice of these factors depend in their economic content to account for the risk-correction, as described above. Within a general set, we choose those variables that improve upon the *in-sample* statistically properties. In this context, we choose the following variables: government spreads for 3, 6, and 22 months, the z-scores of the volatility of IPC, and the FX volatility. We estimate the volatility based on the moving average standard deviation with a window of 220 days.

Table 3 provides the *in-sample* coefficient of determination of the multi-factor linear model. In effect, there is a notable increase in the R^2, both, with respect to the single-factor model and as the forecasting horizon increases. Their values for the 24-month horizon are high, indicative of the risk-correction relevance when formulating a forecast.

An important issue relates to the estimates of $\boldsymbol{\beta}_{\boldsymbol{n}}'$ (which are not reported). Their statistical significance tends to increase with the horizon as well.

	Risk-correction (x): Multi-factor Forecast	
n	**s**	**b(s)**
3m	0.51	0.54
6m	0.70	0.74
9m	0.73	0.76
1y	0.80	0.79
2y	0.80	0.79

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	Risk-correction (x): Multi-factor Forecast	
n	**s**	**b(s)**
3m	0.47	0.49
6m	0.66	0.71
9m	0.70	0.75
1y	0.76	0.77
2y	0.80	0.79

Government Funding Rate

Table 3. In-sample Adjusted R^2 Multi-Factors
The following regression is estimated. We report the R^2 for different n.

$$i_{t+n} - f_{t,n} = \alpha_n + \boldsymbol{\beta}_{\boldsymbol{n}}'\boldsymbol{x}_{\boldsymbol{t,n}} + e_{t+n}$$

Notes: The forecast rates are denoted by s and $b(s)$. The b indicates that the associated rate has been obtained by bootstrapping. Rolling-Window Regressions. Window size 220-day. Volatilities are estimated with the moving average standard deviation with a window of 220 days. Regressors**:** Government Spread 3, Government Spread 6, Government Spread 22, Z-score Vimex, Z-score Volatility IPC, and Z-score FX Volatility.
Sample: January 2, 2008–August 16, 2017.
Sources: Own estimations with data from Valmer, Bloomberg, Yahoo Finance and Banco de México.

Next, we compare their forecasting performance based on two parameters. First, as an initial assessment, we compare the absolute value of the forecasting errors for various horizons (Table 4). To this end, we similarly estimate a rolling-window regression, make the forecast and record the forecasting error. Then, we estimate a new rolling-window regression having shifted the sample by three days, make a forecast and record its error, iteratively.

Thus, we present the mean errors of three models: i) the constant model; ii) a single-factor model; and, iii) a multi-factor model (Tables 4, 5 and 6, respectively). On these results, we have the following comments. First, as foreseen by the R^2s, we note that in all three cases the absolute errors increase as the forecasting horizon increases. This is intuitive since, in general, the longer the horizon the harder is to obtain a small error.

Second, crucially, we have that the absolute errors decrease when accounting for the risk-correction in the forecasts. In other words, while the gain in the risk-correction is marginal for the short-term forecasts, it becomes more relevant for longer-term forecast.

	Constant (α)			Constant (α)	
	Forecast			Forecast	
n	s	b(s)	n	s	b(s)
3m	21.13	18.64	**3m**	20.52	18.40
6m	34.62	34.74	**6m**	34.29	34.25
9m	39.19	40.65	**9m**	39.21	40.41
1y	46.40	47.66	**1y**	45.50	46.50
2y	57.16	56.26	**2y**	56.89	56.27
	TIIE-28			**Government Funding Rate**	

Table 4. Mean Absolute Errors Constant (*Out-of-Sample*)
(Basis Points)
The errors are based on the following model.

$$i_{t+n} - f_{t,n} = \alpha_n + e_{t+n}$$

Notes: The forecast rates are denoted by s and $b(s)$. The b indicates that the associated rate has been obtained by bootstrapping. These are based on Rolling-Window Regressions. Window size 220-day**.**
Sample: January 2, 2008–August 16, 2017.
Sources: Own estimations with data from Valmer and Banco de México.

Risk-correction (x): Gvt. 22	Forecast	
n	s	b(s)
3m	18.54	18.03
6m	27.59	28.19
9m	28.66	29.49
1y	35.23	36.01
2y	45.29	43.65

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Risk-correction (x): Gvt. 22	Forecast	
n	s	b(s)
3m	17.55	17.29
6m	27.20	27.73
9m	29.07	29.60
1y	35.27	35.86
2y	44.53	42.71

Government Funding Rate

Table 5. Mean Absolute Errors Single-Factor (*Out-of-Sample*)
(Basis Points)
The errors are based on the following model

$$i_{t+n} - f_{t,n} = \alpha_n + \beta_n \, x_{t,n} + e_{t+n}$$

Notes: The forecast rates are denoted by s and $b(s)$. The b indicates that the associated rate has been obtained by bootstrapping. These are based on Rolling-Window Regressions. Window size 220-day. Gvt. 22 stands for the difference in nominal interest rates of Mexican bonds with a maturity of 24 and 22 months.
Sample: January 2, 2008– August 16, 2017.
Sources: Own estimations with data from Valmer and Banco de México.

Risk-correction (x): Multi-factor	Forecast	
n	s	b(s)
3m	15.91	16.49
6m	15.04	16.27
9m	13.46	14.77
1y	17.02	18.73
2y	19.41	20.30

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Risk-correction (x): Multi-factor	Forecast	
n	s	b(s)
3m	15.69	16.38
6m	15.60	16.69
9m	13.96	15.10
1y	16.96	18.58
2y	18.97	19.84

Government Funding Rate

Table 6. Mean Absolute Errors Multiple-Factor (*Out-of-Sample*)
(Basis Points) The errors are based on the following model

$$i_{t+n} - f_{t,n} = \alpha_n + \boldsymbol{\beta_n' x_{t,n}} + e_{t+n}$$

Notes: The forecast rates are denoted by s and $b(s)$. The b indicates that the associated rate has been obtained by bootstrapping. Rolling-Window Regressions. Window size 220-day. Volatilities are estimated with the moving average standard deviation with a window of 220 days. Regressors**:** Government Spread 3, Government Spread 6, Government Spread 22, Z-score Vimex, Z-score Volatility IPC, and Z-score FX Volatility.
Sample: January 2, 2008–August 16, 2017.
Sources: Own estimations with data from Valmer, Bloomberg, Yahoo Finance and Banco de México.

Third, the improvement in the reduction of the forecast is notable when considering a multi-factor risk-correction. In effect, in the constant model case, the error increases more than two-fold when moving from three to 24-month (Table 4), and in the case of the single-factor risk-correction (Table 5), while relative better, its improvement along the horizon is similar. However, when considering a multi-factor risk-correction (Table 6), we underscore the improvement as the horizon increases. In fact, the absolute error is less than twice an already low value.

An important issue is the performance of these forecasts relative to that of surveys. Such comparisons are not direct since the number of forecasts based on surveys have a smaller number of data points. To that end, we use Banco de México's Survey (i.e., *Encuesta sobre las expectativas de los especialistas en economía del sector privado*).[7] Table 7 presents the associated estimates. As the horizon increases, the errors increase in tandem. The performance of the survey is, in general, comparable to that of the constant model. In particular, the pace at which the errors increase is similar to those of the constant model; for instance, the 24-month error is more than two-fold the one associated with that of 12-month.

Evidently, while the average absolute errors are indicative of the forecasting performance, they are not a formal statistical test. Hence, we turn to the standard Diebold and Mariano (1995) Test, which formally compares the relative performance of two forecasting models.[8]

We next compare the single-factor model with the constant one. The test compares the error series of two possible forecasts. The errors are in terms of a function g, i.e., $g(e_1)$ and $g(e_2)$. The null hypothesis is that their difference is zero.[9] Under the null hypothesis, the Diebold-

[7] Albeit surveys report the bank-funding rate, our estimations are comparable because of the correlation between the two time series. Their difference is about three basis points and they have a correlation coefficient of 0.99.

[8] We could have implemented this comparison with the Giacomini-White (2006) test, which one can see as an extension of the Diebold and Mariano (1995) and West (1996) tests. It is important to mention, however, that the Giacomini-White (2006) test does not directly apply in our case since the model with the risk-correction nests the model with the constant.

[9] The function g has to be such that $g(0) = 0$, g is strictly increasing and $g > 0$. Of course, the absolute function, which we use, satisfies these criteria.

Mariano statistic has a standard normal distribution. More specifically, a statistically significant positive (negative) statistic indicates that the constant model (single-factor) forecast performs better.[10]

Banco de Mexico Survey	
3m	13.65
6m	26.30
9m	44.46
1y	62.66
2y	140.08

Bank-Funding Rate

Table 7. Mean Absolute Errors from Banco de México Surveys
(Basis Points)
Note: Banco de Mexico's surveys are over the bank-funding rate.
Sample period: August 2010–July 2017.
Source: Own calculations based on data from Banco de México and Valor de Mercado.

Tables 8 and 9 present the estimates of the Diebold-Mariano statistics. These compare the relative performance of the forecasts based on the single-factor and multi-factor models. On these statistics, we have the following remarks. The single-factor model performs, in general, better than the constant model, and statistically significantly better at the three and 24-month horizons.

When the horizon increases, the importance of implementing such corrections becomes more apparent. We have that a single-factor does not conclusively outperform the constant model (Table 8). Yet, the multi-factor risk-correction is consistently better than that of the constant model (Table 9). We have that the Diebold-Mariano tests are statistically significant better at the 5% critical level. Moreover, the p-values tend to decrease as the horizon increases up to a value less than 0.01. This is in line with our previous results.

[10] Specifically, if its absolute value is greater than 1.65 it is significant at the 10% confidence level, and if it is greater than 1.96, it is significant at the 5% confidence level.

Risk-correction (x): Gvt. 22

n	Forecast s	Forecast b(s)
3m	-1.71	-0.49
	(0.09)	(0.63)
6m	-1.24	-0.72
	(0.21)	(0.47)
9m	-1.20	-0.93
	(0.23)	(0.35)
1y	-1.67	-1.62
	(0.09)	(0.11)
2y	-2.52	-2.35
	(0.01)	(0.02)

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Risk-correction (x): Gvt. 22

n	Forecast s	Forecast b(s)
3m	-1.77	-0.46
	(0.08)	(0.65)
6m	-1.39	-0.89
	(0.17)	(0.37)
9m	-1.37	-1.11
	(0.17)	(0.27)
1y	-1.61	-1.63
	(0.11)	(0.10)
2y	-2.56	-2.43
	(0.01)	(0.02)

Government Funding Rate

Table 8. Average Diebold-Mariano Statistics for Single-Factor vs Constant Model
Notes: The forecast rates are denoted by s and $b(s)$. The b indicates that the associated rate has been obtained by bootstrapping. A negative sign in the Diebold-Mariano (DM) statistic indicates that the current model forecasts statistically significantly better than the model with a constant. A positive sign indicates the opposite result. Rolling-Window Regressions. Window size 220-day. Volatilities are estimated with the moving average standard deviation with a window of 220 days. P-values between parentheses.
Sample: January 2, 2008–August 16, 2017.
Sources: Own estimations with data from Valmer and Banco de México.

Risk-correction (x): Multi-factor

n	Forecast s	Forecast b(s)
3m	-1.27	-0.59
	(0.203)	(0.555)
6m	-2.63	-2.69
	(0.008)	(0.007)
9m	-2.94	-3.14
	(0.003)	(0.002)
1y	-3.01	-3.73
	(0.003)	(0.000)
2y	-4.58	-5.93
	(0.000)	(0.000)

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Risk-correction (x): Multi-factor

n	Forecast s	Forecast b(s)
3m	-1.35	-0.52
	(0.18)	(0.61)
6m	-2.80	-2.96
	(0.005)	(0.003)
9m	-2.92	-3.11
	(0.004)	(0.002)
1y	-2.97	-3.65
	(0.003)	(0.000)
2y	-4.65	-6.07
	(0.000)	(0.000)

Government Funding Rate

Table 9. Average Diebold-Mariano Statistics for Multiple-Factors vs Constant
Notes: The forecast rates are denoted by s and $b(s)$. The b indicates that it has been obtained by bootstrapping. A negative sign in the DM statistic indicates that the current model forecasts statistically significantly better than the model with just a constant. A positive sign indicates the opposite result. Rolling-Window Regressions. Window size 220-day. Volatilities are estimated with the moving average standard deviation with a window of 220 days. The p-values are between parentheses. **Regressors:** Government Spread 3, Government Spread 6, Government Spread 22, Z-score Vimex, Z-score Volatility IPC, and Z-score FX Volatility. **Sample:** January 2, 2008–August 16, 2017. **Sources:** Own estimations with data from Valmer, Bloomberg, Yahoo Finance and Banco de México.

6. Final Remarks

We have explored key asset pricing implications for the formulation of forecasts for interest rates. We have provided some evidence that a risk-correction can improve the forecasting performance in the case of short-term interest rates. Statistically and financially, the motivation for improving such a forecast is direct. Economically, it is relevant to point out that predictability seems to be associated with the business cycle.

The fact that the rationale of a risk-adjustment comes from various areas of economics can give place to some important issues in terms of assessing the model. For instance, in including a given risk factor, how should one weight its relevance? By its economic content or by the extent to which it improves the forecasts? While some of the coefficients associated with the *in-sample* estimation of the regression model might be marginally statistically significant, in some cases, they improve the forecasting performance relative to the no correction model.

Overall, our main goals was not to assess the performance between our models, this is, the risk-corrections. However, we have provided sufficient evidence to compare them and conclude that the multi-factor model appears to perform better. At this point, we want to bring home that using them is statistically and economically relevant. Their refinement can be a pertinent line of future research.

In this context, there are possible lines of future research, for example, in terms of the forecasts' performance. For instance, while we have maintained the original variables, say, using Factor Analysis to obtain variables could improve the forecasts' prowess by conveying the relevant information more efficiently. There is then an inherent interest for market participants, in particular the monetary authority, in obtaining better forecast for monetary policy. In effect, it is difficult to overemphasize the importance of expectations in the conduct of monetary policy. This is relevant for various reasons, particularly so, because agents take their decisions based on the expectations they have on various variables, not to mention the expected evolution of monetary policy.

We emphasize that one could implement this approach based on financial derivatives that have as their underlying the inflation rate. As a corollary, in an inflation swap, the fixed leg is generally a *biased* estimate of expected inflation. In the same vein, the financial press provides some probabilities for changes Federal Funds Rate based on its features. The estimation relies on some approximation assumptions as well.

As a more general approach, consider the complete density function, as oppose to only the mean or median. For example, Fleckenstein et al. (2013) have implemented this approach in the context of U.S. inflation. Yet, instead of adding a risk-correction, they obtained the complete *objective* density, as opposed to risk-neutral density. For the same reason, using the risk-neutral density as a forecasting density is akin for not adding a risk-correction.[11]

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