

Jung, Benjamin; Kohler, Wilhelm

**Working Paper**

## Input-output linkages and monopolistic competition: Input distortion and optimal policies

Hohenheim Discussion Papers in Business, Economics and Social Sciences, No. 06-2021

**Provided in Cooperation with:**

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*Suggested Citation:* Jung, Benjamin; Kohler, Wilhelm (2021) : Input-output linkages and monopolistic competition: Input distortion and optimal policies, Hohenheim Discussion Papers in Business, Economics and Social Sciences, No. 06-2021, Universität Hohenheim, Fakultät Wirtschafts- und Sozialwissenschaften, Stuttgart, <https://nbn-resolving.de/urn:nbn:de:bsz:100-opus-19375>

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Hohenheim Discussion Papers in Business, Economics and Social Sciences

# INPUT-OUTPUT LINKAGES AND MONOPOLISTIC COMPETITION: INPUT DISTORTION AND OPTIMAL POLICIES

**Benjamin Jung**

University of Hohenheim

**Wilhelm Kohler**

University of Tübingen

Research Area INEPA

06-2021

Discussion Paper 06-2021

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ISSN 2364-2084

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# Input-Output Linkages and Monopolistic Competition: Input Distortion and Optimal Policies\*

Benjamin Jung<sup>†</sup> and Wilhelm Kohler<sup>‡</sup>

August 12, 2021

## Abstract

In this paper, we provide a detailed analysis of a mechanism that distorts production towards too much use of primary factors like labor and too little use of intermediate inputs. The distortion results from two ingredients that are cornerstones of modern quantitative trade theory: monopolistic competition and input-output linkages. The distortion as such is unrelated to trade, but has important consequences for trade policy, including a positive first-order welfare effect from an import subsidy. For a crystal-clear view on the distortion, we first look at it in a single-sector, closed economy where the monopolistic competition equilibrium would be efficient without the presence of input-output linkages. We compare the social-planner-solution with the decentralized market equilibrium, and we identify first-best policies to correct the distortion. To analyze the trade policy implications we then extend our analysis to a setting with trade between two symmetric countries. We identify first-best cooperative policies, featuring nondiscriminatory subsidies of intermediate input use, as well as non-cooperative trade policies where countries use tariffs to weigh terms of trade effects against benefits from correcting the input distortion.

*JEL-Classification:* F12, F13, D57, D61, H21

*Keywords:* input-output linkages, monopolistic competition, international trade, allocational inefficiency, optimal policy

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\*We are grateful to Carsten Eckel, Hartmut Egger, Peter Egger, Gabriel Felbermayr, and Jens Südekum as well as participants of the Göttinger Workshop “Internationale Wirtschaftsbeziehungen” for helpful comments and discussions. Special thanks to Lorenzo Caliendo for a very revealing in-depth discussion about the paper. Benjamin Jung gratefully acknowledges support by the Faculty of Economics and Social Sciences at the University of Hohenheim within its research focus “Inequality and Economic Policy Analysis” (INEPA). Wilhelm Kohler is grateful for financial support received from Deutsche Forschungsgemeinschaft (DFG) under grant no. KO 1393/2-1.

<sup>†</sup>University of Hohenheim, 70593 Stuttgart, Germany; CESifo; IAW; jung.benjamin@uni-hohenheim.de.

<sup>‡</sup>University of Tübingen; IAW; CESifo, GEP; wilhelm.kohler@uni-tuebingen.de

# 1 Introduction

The presence of input-output linkages in an environment with monopolistic competition may constitute a case for a negative optimal import tariff. This surprising result can be seen from the numerical analysis in Caliendo et al. (2015) and is shown formally in a limiting case for a two-sector setting in the revised version of that paper which has appeared as Caliendo et al. (2017).<sup>1</sup> We argue that the mechanism responsible for the result is an input distortion that arises also in a single-sector setting: markup pricing implies that intermediate material inputs come at a price above their true opportunity cost. If technology allows for substitution between primary and intermediate inputs, then production of varieties will involve too much direct labor use, relative to material inputs (or roundabout use of labor). In such an environment, an import subsidy generates a positive first-order welfare effect by reducing the cost of imported intermediates, thus causing substitution towards heavier use of intermediate inputs, with the distinct possibility of a negative optimal tariff.

In this paper, we provide an anatomy of this input distortion by means of a detailed comparison between the social optimum and the decentralized market equilibrium, and we characterize first best government policies to deal with the distortion in closed economy settings as well as in a model featuring trade in intermediate inputs.

Since the input distortion is not directly related to trade, we start with a single sector, closed economy model where the distortion appears in its purest possible form. We distinguish between the case where the input distortion arises only in the variable cost, because the fixed cost uses only labor, and the case where the distortion arises in the fixed cost activity as well. In the former case, a decentralized equilibrium features efficient firm-level employment, but each firm uses too little of the material input. In the latter case, the use of material input is efficient, but firms use too much labor. In either case the equilibrium features a suboptimally low degree of roundaboutness in the use of labor. With the input dis-

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<sup>1</sup>Indeed, we were inspired to write this paper by a diagram in Caliendo et al. (2015) depicting welfare as a function of the import tariff and featuring a negative gradient at the level of a zero tariff, which suggests a negative optimum tariff.

tortion restricted to the variable cost, firm entry is efficient. With the distortion present in the fixed cost activity as well, firm entry is too low, compared to the social optimum. This holds true regardless of whether we have firm heterogeneity.<sup>2</sup>

A key feature of input output linkages is that any sector's output may be used for final consumption as well as for intermediate input use, including intermediate input use in the sector's own production. Therefore, the government has two instruments to deal with the input distortion. It may subsidize the intermediate input use in *production* of varieties (upstream, or backward subsidization), which seems like the natural way to address the distortion. Alternatively (or in addition), it may subsidize the *use* of varieties in production of final goods, which may in turn be used for final consumption or as an intermediate input (downward, or forward subsidization). We demonstrate that both instruments are equally suitable as a first best correction of the input distortion, even if the government cannot discriminate in downstream subsidization between intermediate input use and final consumption use of the final good. This latter assumption seems plausible since any such discrimination would likely prompt arbitrage activities. We also demonstrate that a wage tax cannot deal with the distortion, and we note in passing that the input distortion also arises in wage setting environments (fair wages or efficiency wages).

In the second part of the paper, we analyze the input distortion in a setting with trade, and we explore its implications for trade policy. We develop a symmetric two-economy model where each country produces its own version of an aggregate intermediate input (composed of its own varieties), which is then used, alongside the imported intermediate input, to assemble a non-tradable final good used for final consumption as well as for intermediate input use in production of each country's varieties. In this setting, the government may downward-subsidize the use of its own intermediate or the imported intermediate (negative import tariff), or it may upstream-subsidize the intermediate input use of the final good. Again, we compare the social optimum with the decentralized equilibrium and then identify coopera-

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<sup>2</sup>This paper offers a brief discussion of firm heterogeneity as an extension to the baseline model. A more detailed treatment of the input distortion under firm heterogeneity is found in an earlier version of the paper (Jung and Kohler, 2016).

tive (first-best) policies for the entire world. These involve a nondiscriminatory downward-subsidization in each country of both, the domestic and the imported aggregate intermediate (negative import tariff) in final goods assembly, or an ad-valorem equivalent upstream subsidization of each country's intermediate input use of the final good. In a policy regime that bans domestic subsidies, the government is left with downstream subsidization of the imported intermediate. Such a subsidy introduces a policy distortion of its own in that it favors the domestic over the foreign intermediate, without any underlying distortion (as the input distortion is present in both), but we find that the optimal second best-policy is a negative import tariff.

Finally, we address non-cooperative policies in either of the two countries, assuming a policy regime that bans domestic subsidies, leaving only pure trade policies to maximize national welfare. The novel element in this non-cooperative policy calculus is the familiar terms of trade effect which, in and of itself, calls for a positive import tariff. This, however, aggravates the input distortion which must be weighed against the benefit from a terms of trade improvement. We demonstrate that each country has an incentive to deviate from the cooperative policy equilibrium by lowering its import subsidy. Taking free trade as a reference point, we find that the terms of trade incentive dominates if the "Armington elasticity" is small enough and the strength of the input-output linkage (measured as the output elasticity with respect to the material input) is small enough, and vice versa. The optimal tariff is lower in the presence of the input-output linkage than without.

The importance of these results derives from the widespread use of monopolistic competition models in economics. In trade theory, this was sparked off by Krugman (1979,1980) and Ethier (1982) and reinforced by Melitz (2003).<sup>3</sup> The relevance of input-output linkages in such models was first noticed by Krugman and Venables (1995) and Venables (1996) who emphasize the potential of such linkages to cause (multiple) agglomeration equilibria. In large scale computational trade models, input-output linkages have always loomed large and

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<sup>3</sup>Blanchard and Kiyotaki (1987) have popularized the use of monopolistic competition models in new Keynesian macroeconomics. A third area of widespread use of such models is endogenous growth theory, sparked off by Romer (1990).

many such models also feature monopolistic competition; see, e.g., Keuschnigg and Kohler (1996), Balistreri et al. (2011) and Costinot and Rodríguez-Clare (2014).<sup>4</sup> However, in these strands of literature the presence of the above input distortion has gone unnoticed.<sup>5</sup>

We discuss the consequences of the input distortion for optimal policies. Costinot et al. (2020) also compare decentralized equilibrium to social optimum in order to characterize optimal unilateral policies in models with monopolistic competition. However, they neither consider input-output linkages nor cooperative policies. Lashkaripour and Lugovsky (2021) analyze optimal unilateral policy in a generalized multi-industry Krugman (1980) model. In an extension, they discuss the implications of input-output linkages for optimal policies, but they do not pay attention to the underlying distortions. In Caliendo et al. (2021), the authors provide an in-depth characterization of optimal trade policy in a two-sector small open economy with roundabout production and monopolistic competition. Our setting features two large countries and a broader set of policy instruments, which allows us to characterize cooperative (first-best) policies.<sup>6</sup>

As mentioned above, the input distortion as such is not directly related to trade. To facilitate a deeper understanding, we therefore start out by analyzing the input distortion in a closed economy setting. In particular, we compare our results to what we know from existing literature about the efficiency properties of a monopolistic competition equilibrium. A cornerstone of that knowledge is that in a *single sector* world with a *constant elasticity of substitution* (CES) in demand, the decentralized laissez faire equilibrium is efficient. The reason is that with standard CES preferences, “consumer surplus” and “profit destruction” distortions exactly cancel out, as noted by Baldwin (2005). Moreover, provided that all firms charge the same markup consumers’ spending decisions across firms are not distorted. Importantly,

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<sup>4</sup>The general importance of input-output linkages for trade policy effects was highlighted in Caliendo and Parro (2015), although that paper does not feature monopolistic competition.

<sup>5</sup>To be clear, this does not, per se, invalidate the results that such models generate regarding the positive and normative consequences of specific trade liberalization scenarios. But awareness of the above type of production inefficiency and the corresponding mechanisms should contribute to a full understanding of the welfare results obtained.

<sup>6</sup>There is a literature emphasizing the isomorphism between monopolistic competition models and Ricardian models with external economies of scale (Kucheryavyy et al., 2016). Bartelme et al. (2019) discuss optimal policies in a Ricardian model with external economies of scale.



in Melitz-type models with firm heterogeneity the efficiency-result also applies to the equilibrium mass of firm entry into the market and to the selection of firms taking up production (Dhingra and Morrow, 2019). However, efficiency is lost if the degree of external economies from product variety differs from the degree implicit in standard CES models, or if the elasticity of substitution in demand is variable; see Benassy (1996,1998) and Dhingra and Morrow (2019). Production efficiency is also lost in multi-sector models with CES preferences, as indicated above. Generally, industries with above average markups of prices over marginal cost will produce in less than optimal amounts; see Epifani and Gancia (2011) for a framework with constant markups and Behrens et al. (2018) for a model that features variable markups. The purpose of using a single sector, closed economy setup in the first part of our paper is to place the input distortion in a modeling environment where the monopolistic competition equilibrium is fully efficient, provided there are no input-output linkages. This allows for a crystal-clear view on the mechanism through which the input-output linkage causes a production distortion.

It is important to be clear about what we mean by an input-output linkage. In general terms, what we mean is the production of commodities by means of commodities, that is, goods serving as inputs in *their own* production, alongside primary inputs such as labor. The element of *circularity* is crucial here, and it separates input-output linkages from pure fragmentation of production, or multi-stage production. With pure fragmentation, the technology of production is *recursive*, meaning that along the value added chain no good is ever used, directly or indirectly, as an input in its own production. In this case, market power of intermediate input suppliers may lead to multiple markups in prices of intermediates and, thus, to a final goods price which is above its marginal cost, but this does not constitute a production distortion if markups are uniform, or – equivalently – if there is only one sector in the economy. In contrast, as we shall see below, input-output linkages generate an input distortion also in a single sector economy.

In a similar vein, pure multi-stage production does not involve roundaboutness in the use of primary inputs as input-output linkages do – even with a single stage of production. With input substitution, input-output linkages raise the issue of the optimal degree of roundabout-

ness in the use of primary inputs, and monopolistic competition lead to a less than optimal degree of roundaboutness, as emphasized above.<sup>7</sup> Thus, the input distortion is a potentially important but hitherto unnoticed case of resource misallocation; see Jones (2013).

The paper is structured as follows. Section two introduces the baseline model of a single sector, closed economy featuring the key ingredients of the input distortion, viz. an input-output linkage in a monopolistic competition environment. We first derive the social optimum to be compared in detail with a decentralized market equilibrium, with and without first-best policy intervention. In Section three, we extend our model to a trading environment involving two symmetric countries, each producing an aggregate intermediate input, composed of its own varieties, which is traded across countries. Each country assembles its own as well as the imported aggregate intermediate to a final good used for consumption as well as a material input into production of its own varieties. For this setting, we analyse first-best cooperative policies as well as non-cooperative trade policies where tariffs are used as second-best instruments. Section four concludes.

## 2 Input distortion in a closed-economy

We use a highly stylized model adopting features familiar from the literature in all aspects except input-output linkages. To bring the mechanisms of our input distortion into sharp focus, we begin with the simplest possible case of a closed economy producing a single aggregate good in quantity  $Q$  which may be used for final consumption  $C$  or as a material input

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<sup>7</sup>There is a long tradition in the literature, dating back to von Böhm-Bawerk, of discussing roundaboutness in the context of capital theory, where roundaboutness involves the lapse of time, which becomes key if individuals have time preference. See Hennings, K.H. (1987), “Capital as a Factor of Production,” in: Palgrave Macmillan (eds), *The New Palgrave Dictionary of Economics*, Palgrave Macmillan, London; and Hennings, K.H. (1987), Roundabout Methods of Production, in: Palgrave Macmillan (eds), *The New Palgrave Dictionary of Economics*, Palgrave Macmillan, London. This paper, however, is intended as a contribution, not to this literature, but – first and foremost – to modern trade literature where input-output linkages have played an important role which is more or less orthogonal to its role in growth theory. Awareness of the issue of optimal roundaboutness in the trade literature has emerged only recently; see Antràs and Chor (2021) and Caliendo et al. (2021).

in amount  $M$ . We write welfare as

$$U(C) = C = Q - M, \quad (1)$$

where  $U$  is a quasi-concave utility function. The material input  $M$  is used in production of differentiated varieties which are, in turn, used in assembly of the aggregate good  $Q$ . We assume a constant elasticity of substitution  $\sigma$  between different varieties in production of  $Q$ :

$$Q = N^{\nu+1}q, \quad (2)$$

where  $N$  denotes the number of symmetric firms, each producing its variety in amount  $q$ . This formulation implies love of variety (number of firms), measured by the parameter  $\nu$ . Writing  $\sigma = 1/(1 - \rho)$ , with  $\rho \in (0, 1)$ , we have  $\sigma > 1$  and  $\nu = 1/\rho > 1$ . Firm-level production of varieties is governed by

$$q = \left(\frac{\ell}{\gamma}\right)^\gamma \left(\frac{m}{1-\gamma}\right)^{1-\gamma}, \quad (3)$$

where  $\ell$  measures the representative firm's labor input and  $m$  denotes the level of its material input. We have  $M = Nm$ . The technological parameter  $\gamma \in (0, 1]$  measures the strength of the input-output linkage; without any such link we have  $\gamma = 1$ . In addition to these variable inputs, each firm must incur a fixed cost  $f$ . In our baseline case we assume  $f$  to be given in terms of labor, but we shall subsequently also consider the case where  $f$  is given in terms of the aggregate good. We assume a labor force of given size  $L$ , whence the resource constraint of the economy is

$$L \geq N(\ell + f). \quad (4)$$

In the decentralized market equilibrium analyzed below, we shall assume that firms act in a market environment of monopolistic competition. But we first explore the social optimum to establish a due reference point for our input distortion.

## 2.1 Social optimum

Society faces two trade-offs. The first relates to the roundaboutness of labor use. Increasing  $m/\ell$  economizes on labor in production of differentiated goods intermediates, but it also reduces the amount of output available for consumption. The second is the trade-off between variety and efficiency: Increasing the number of differentiated varieties increases output even for a constant intermediate input use, but at the same time it reduces output available for consumption on account of the fixed cost. Assuming strict equality in the resource constraint, the social optimum lies in the solution of the following problem:

$$\max_{\ell, m, N} N^{\nu+1} q - Nq \quad \text{s.t.} \quad L = N(\ell + f), \quad (5)$$

where  $q$  is given by Eq. (3). Using  $*$  to indicate optimal values, the first-order conditions for  $\ell^*$  and  $m^*$  are

$$(N^*)^{\nu+1} \gamma q^* / \ell^* = \lambda^* N^* \Rightarrow \ell^* = \gamma \frac{Q^*}{\lambda^* N^*} \quad (6)$$

$$\text{and} \quad (N^*)^{\nu+1} (1 - \gamma) q^* / m^* = N^* \Rightarrow m^* = (1 - \gamma) \frac{Q^*}{N^*} \quad (7)$$

In (6),  $\lambda^*$  denotes the shadow value of labor (Lagrangian multiplier). Note that the conditions on the two types of inputs are similar, but that optimal material input level  $m^*$  does not depend on the shadow value of labor. The reason is that the opportunity cost of intermediate input use in production is foregone consumption *of the same good*. Note also that equation (7) implies  $M^* = N^* m^* = (1 - \gamma) Q^*$ , which ensures viability ( $Q^* > M^*$ ).<sup>8</sup>

The first-order condition on  $N^*$  is  $(\nu + 1) (N^*)^\nu q^* - m^* = \lambda^* (\ell^* + f)$ , which may be written

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<sup>8</sup>In input-output analysis, this viability condition is known as the *Hawkins-Simon condition*. Writing the vector of final consumption quantities as  $\mathbf{c}$  and assuming a Leontief-technology with input coefficients collected in a Matrix  $\mathbf{A}$ , the Hawkins-Simon condition imposes a restriction on  $\mathbf{A}$  guaranteeing that there exists a vector  $\mathbf{x}$ , such that  $(\mathbf{I} - \mathbf{A})\mathbf{x} \geq 0$ . In our context,  $Q$  is the equivalent of  $\mathbf{I}\mathbf{x}$ , aggregate (gross) output, while  $M$  is the equivalent of  $\mathbf{A}\mathbf{x}$ , the inputs needed to generate  $\mathbf{x}$ . Obviously, in terms of the present notation, the Hawkins-Simon condition is equivalent to  $Q - M > 0$ . In input-output analysis, the coefficients  $\mathbf{A}$  are given exogenously. In our single sector case, the intermediate input intensity of production is chosen endogenously, and the equivalent of the Hawkins-Simon condition holds from the first-order condition on intermediate input use.

as

$$(\nu + 1)Q^* - M^* = \lambda^*L. \quad (8)$$

This condition simply states that the value deriving from additional *net* output obtained from a marginal increase of  $N$  must be equal to the marginal cost of  $N$ , which is equal to  $\lambda^*L/N$ . Note that this cost includes variable and fixed labor input into production. Finally, the derivative with respect to the Lagrange multiplier  $\lambda$  yields the resource constraint in (5).

Using these first-order conditions, we may now solve for the optimal levels of  $\ell^*$ ,  $m^*$ ,  $N^*$ , and  $\lambda^*$ . The optimal labor intensity of variety production is given by

$$\frac{\ell^*}{m^*} = \frac{\gamma}{1 - \gamma} \lambda^*. \quad (9)$$

Given  $M^* = (1 - \gamma)Q^*$ , the condition (8) may be rewritten as

$$(\nu + \gamma)Q^* = \lambda^*N^*(\ell^* + f). \quad (10)$$

Using the first equation in (6) to replace  $Q^*$ , we obtain the following variable labor use per firm:

$$\ell^* = \frac{\gamma}{\nu}f. \quad (11)$$

Intuitively, the variable labor use relative to labor required for the fixed cost is increasing  $\gamma$  and falling in  $\nu$ , the degree of economies from product variety.

Solving the resource constraint for  $N^*$  and replacing for  $\ell^*$ , we obtain

$$N^* = \frac{1}{\gamma/\nu + 1} \frac{L}{f}. \quad (12)$$

As is common in the literature, we ignore the integer constraint on  $N$ . Assuming  $N^* > 1$  implies the parameter restriction  $L/f > \gamma/\nu + 1 = \gamma(\sigma - 1) + 1$ . Compared to the standard model without input-output linkages ( $\gamma = 1$ ), for any degree of scale economies  $\nu$ , the input-output linkage increases the efficient number of firms. Intuitively, since production of any variety now draws on intermediate inputs as well as labor, a given “endowment ratio”  $L/f$

allows for a larger number of production facilities (varieties).

Using the first-order condition on  $m^*$ , replacing  $Q^* = N^{*\nu+1}q^*$ , using the production function for  $q^*$ , and inserting the solution for  $N^*$  from above, we arrive at

$$m^* = \frac{(1-\gamma)f}{\nu} \left( \frac{1}{\gamma/\nu+1} \right)^{\frac{\nu}{\gamma}} \left( \frac{L}{f} \right)^{\frac{\nu}{\gamma}}. \quad (13)$$

If the number of firms is exogenous and equal to  $\bar{N}$ , we have

$$\bar{m}^* = \frac{(1-\gamma)f}{\nu} \bar{N}^{\frac{\nu}{\gamma}}.$$

The term  $(1-\gamma)f/\nu$  in an intuitive way mirrors (11) for optimal variable labor use. In addition, comparing the second term in (13) to (12), we recognize that the optimal intermediate input use per firm is also increasing in the number of firms. This essentially reflects the resource constraint of the economy. A higher number of firms implies a higher use of labor for fixed cost.

Taking the above solutions, we may write

$$\frac{m^*}{\ell^*} = \frac{1-\gamma}{\gamma} (N^*)^{\nu/\gamma}. \quad (14)$$

For the case of an exogenous firm-number  $\bar{N}$ , this ratio changes accordingly. Equating this to the right-hand side of (9) finally allows us to determine the shadow value of labor as

$$\lambda^* = \left( \frac{1}{\gamma/\nu+1} \right)^{\nu/\gamma} \left( \frac{L}{f} \right)^{\nu/\gamma}. \quad (15)$$

If the number of firms is exogenous, we have

$$\lambda^* = \bar{N}^{\frac{\nu}{\gamma}}.$$

Remember that this is expressed in terms of consumption. Intuitively, it increases with the ratio of  $L/f$ , due to aggregate economies of scale from the number of varieties of inputs in final good assembly. The parameter restriction  $L/f > \gamma/\nu + 1$  now implies that  $\lambda^* > 1$ , which

also holds for an exogenous number of firms  $\bar{N}$ .

It is instructive to investigate the degree of roundaboutness in the social optimum. We do this for the case where  $N$  is endogenous. We use  $A_\ell^* := \ell^* N^*/Q^*$  to denote the direct labor input coefficient in the variable cost part of production and accordingly for the direct material input coefficient,  $A_m^* := m^* N^*/Q^* = 1 - \gamma$ . Adding indirect material input use, the total material input coefficient may be written as  $A_m^* (1 - A_m^*)^{-1} = (1 - \gamma)/\gamma$ . The labor embodied in total material input is  $A_\ell^* (1 - \gamma)/\gamma$ , hence the ratio  $(1 - \gamma)/\gamma$  gives the ratio between direct and indirect labor use, looking only at variable inputs. From (11), the fixed labor input per unit of the aggregate good is  $f N^*/Q^* = (\nu/\gamma) \ell^* N^*/Q^* = (\nu/\gamma) A_\ell^*$ , which is all direct labor use. A useful measure of roundaboutness is the ratio of indirect labor use,  $A_\ell^* (1 - \gamma)/\gamma$ , relative to total labor use  $A_\ell^* (1 + \nu/\gamma + (1 - \gamma)/\gamma)$ . Denoting this measure by  $R$ , we have

$$R^* := \frac{1 - \gamma}{1 + \nu}, \quad (16)$$

which lies between zero and one.<sup>9</sup> Remember that  $\nu > 1$  measures the degree of economies from product variety. A high value of  $\nu$  means that it is economical to have many firms. But with many firms more of the overall labor endowment must be devoted to the fixed input, which does not involve any roundaboutness but only direct labor use. Hence  $R^*$  decreases in  $\nu$ .

Finally, the level of welfare (consumption) in the social optimum may be derived as follows. First, we have  $C^* = \gamma Q^* = \gamma (N^*)^{\nu+1} q^*$ . Since  $m^* = \ell^* \lambda^* (1 - \gamma)/\gamma$ , we may write  $q^* = (\ell^*/\gamma)(\lambda^*)^{1-\gamma}$ , whence  $C^* = \gamma Q^* = \ell^* (N^*)^{\nu+1} (\lambda^*)^{1-\gamma}$ . Using  $\lambda^* = (N^*)^{\nu/\gamma}$  from above and substituting  $\ell^* = \gamma f/\nu$ , we obtain

$$C^* = \frac{\gamma f}{\nu} \left( \frac{1}{\gamma/\nu + 1} \frac{L}{f} \right)^{(\gamma+\nu)/\gamma} = \frac{\gamma}{\nu} (\gamma/\nu + 1)^{-(\gamma+\nu)/\gamma} \left( \frac{L}{f} \right)^{\frac{\nu}{\gamma}} L. \quad (17)$$

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<sup>9</sup>Alternatively, we might measure roundaboutness as the ratio between indirect and direct labor use, inclusive of the fixed labor input:  $[(1 - \gamma)/\gamma]/(1 + \nu/\gamma) = (1 - \gamma)/(1 + \nu)$ , which lies between zero and infinity.

Equivalently, welfare may be written as

$$\frac{C^*}{L} = \frac{\gamma}{\gamma + \nu} \lambda^* \quad (18)$$

Consumption per capita is lower than the marginal shadow value of labor, as expected in a situation with (exogenously) increasing returns to scale ( $\nu > 1$ ). This is reinforced by the input output linkage ( $\gamma < 1$ ). For later reference, we also note that total factor productivity (TFP) in the social optimum is

$$\text{TFP} = \frac{Q^*}{L} = \frac{1}{\gamma + \nu} \lambda^* \quad (19)$$

which follows from  $C^* = \gamma Q^*$ .

The following proposition summarizes the role the input-output linkages play in the social optimum.

**Proposition 1** *(a) For a given ratio  $L/f$ , consumption per capita is a constant. Any increase in  $L/f$  leads to an over-proportional increase in per capita consumption ( $\nu/\gamma > 1$ ). (b) An increase in  $L/f$  also raises the shadow value of labor; in elasticity terms, this relationship is reinforced by the input-output linkage. (c) An increase in  $L/f$  lowers the labor intensity  $\ell^*/m^*$ , but the optimal degree of roundaboutness in production, measured by the ratio of indirect to total use of labor, is given parametrically by  $\frac{1-\gamma}{1+\nu}$ . (d) The level of consumption per capita is below the marginal shadow value of labor, and this discrepancy is reinforced by the input-output linkage.*

**Proof.** Part (a) follows from (17). Part (b) follows from (15). Part (c) follows from (14) and (15) as well as 16). And (d) follows writing (17) as  $C^* = \frac{\gamma}{\gamma + \nu} \lambda^* L$ . ■

In the present setup,  $\gamma$  is a primitive of the technology and not subject to policy influence. It is nevertheless instructive to explore how a change in technology, say a strengthening of the input-output linkage, represented by a reduction in  $\gamma$ , affects maximum consumption per capita. Equation (17) seems to suggest that the relationship between the strength of input-output linkages and the maximum level of consumption per capita,  $C^*/L$ , is ambiguous. A reduction in  $\gamma$  has two opposing effects. First, it lowers the share of aggregate output available



for consumption, and secondly, it raises aggregate output. The first effect follows directly from  $C^* = (1 - M^*/Q^*)Q^* = \gamma Q^*$ . The second effect follows from

$$Q^* = \frac{f}{\nu} (N^*)^{\nu/\gamma+1}, \quad (20)$$

which in turn follows from equations (11) through (13) above. It turns out that under the above mentioned parameter restriction the first effect always dominates:

**Proposition 2** *In the social optimum, a reduction in  $\gamma$  (which implies a higher roundaboutness of labor use) leads to an increase in the level of consumption per capita.*

**Proof.** Taking logs in (17) and differentiating with respect to  $\gamma$ , we obtain

$$\frac{\partial \ln C^*}{\partial \gamma} = \frac{\nu}{\gamma^2} \left[ \ln \left( \frac{\gamma}{\nu} + 1 \right) - \ln \left( \frac{L}{f} \right) \right].$$

Given the parameter restriction discussed in connection with  $N^*$  subsequent to equation (12), it follows that  $\frac{\partial C^*}{\partial \gamma} = C^* \frac{\partial \ln C^*}{\partial \gamma} < 0$ . ■

Why should a higher intermediate input intensity of production (lower  $\gamma$ ) lead to a higher level of aggregate output  $Q^*$ , as evidenced by equations (20) and (12)? Intuitively, a lower  $\gamma$  saves on direct labor use in production of varieties, thus freeing up labor for fixed input use. Due to economies from enhanced variety, the indirect labor requirement for production of the additional intermediate inputs (in line with a lower  $\gamma$ ) is lower than the incipient reduction in the direct use of labor. If the initial number of firms were equal to one, then the percentage increase in aggregate output would be exactly equal to the percentage reduction in direct labor use. But if  $N^* > 1$ , then the percentage increase in aggregate output is less than the percentage savings on direct labor use per initial variety produced. By implication, a higher degree of roundaboutness in production is a source of welfare increase.

## 2.2 Decentralized equilibrium with policy intervention

We now characterize a decentralized market equilibrium, in which producers of varieties behave under monopolistic competition, while assembly of the final good is governed by perfect competition. Writing  $\tilde{p}$  for the “demand-price” for a differentiated variety faced by final good producers, the minimum unit-cost function dual to (2) is  $N^{-\nu}\tilde{p}$ . Using  $\tilde{P}$  to denote the price of the final good, zero profits in final goods production implies

$$\tilde{P} = N^{-\nu}\tilde{p}. \quad (21)$$

Unit-demand for a variety follows as

$$q = \left( \frac{\tilde{p}}{\tilde{P}} \right)^{-\sigma}. \quad (22)$$

Given what we said in the introduction about the input distortion, it is useful to introduce a policy instrument that might correct this distortion. In this setup, there are two types of instruments that lend themselves for dealing with the input distortion. The first is an ad-valorem subsidy for material input use in variety production, such that the input price that variety producers face is  $(1 + s)\tilde{P}$ . In the following we use  $\theta := 1 + s$ . A subsidy implies  $\theta < 1$ , if  $\theta > 1$  this means material input use is taxed. The second policy instrument is a subsidy of differentiated varieties used in assembly of the final good. This introduces a wedge between the price  $p$  set by producers of varieties and the “demand-price”  $\tilde{p}$  faced by producers of the final good, such that  $p = \tilde{p}/(1 + t)$ . In the following, we write  $\tau := 1/(1 + t)$ , where a subsidy implies  $\tau > 1$ . If  $\tau < 1$  this means the use of varieties is taxed. We emphasize that  $\tau$  applies to the entire production of the composite good, including production for final consumption where there is no distortion. Hence, at first sight  $\tau$  seems an unlikely candidate for a first best correction of the distortion. However, we shall see that in the present setup the two types policy instruments,  $\theta$  and  $\tau$ , are isomorphic.

Cost-minimizing production of a variety gives rise to minimum unit-cost  $x := w^\gamma (\theta\tilde{P})^{1-\gamma}$ ,

and given demand as in equation (22), Bertrand pricing of variety producers yields

$$p = \mu x \quad \text{with} \quad \mu := \frac{\sigma}{\sigma - 1} > 1. \quad (23)$$

Conditional input demands in production of varieties are

$$\ell = \gamma \frac{x}{w} q \quad \text{and} \quad m = (1 - \gamma) \frac{x}{\theta \bar{P}} q. \quad (24)$$

Free entry into the market for varieties implies zero profits,  $(\mu - 1) x q = f w$ , which leads to the following equilibrium output per firm

$$q = \frac{1}{\mu - 1} \frac{w}{x} f. \quad (25)$$

The variable labor use per firm then immediately follows as

$$\ell = \frac{\gamma}{\mu - 1} f. \quad (26)$$

Comparing this with equation (11), and noting that  $\mu - 1 = \nu$ , we recognize that the decentralized equilibrium features a first-best level of direct labor use per firm. Notice the different contexts in which the two terms  $\mu - 1$  and  $\nu$  are placed. When describing the first best in the previous section, we have used  $\nu = 1/(\sigma - 1)$  as a measure of economies from the number of input varieties available for assembly of the aggregate good. Here,  $\mu - 1 = 1/(\sigma - 1)$  additionally captures the degree of market power enjoyed by variety producers. The equilibrium number of firms follows from the full employment condition,  $N(\ell + f) = L$ , which implies

$$N = \left( \frac{\gamma}{\mu - 1} + 1 \right)^{-1} \frac{L}{f}. \quad (27)$$

Comparing with (12), we find that the decentralized equilibrium also features a first-best number of firms.

The equilibrium material input use, however, is distorted. It emerges as

$$m = \frac{1 - \gamma}{\gamma} \frac{w}{\theta \tilde{P}} \ell. \quad (28)$$

Inserting the markup pricing equation (23) into the aggregate price equation (21), we obtain

$$\tilde{P} = \theta^{\frac{1-\gamma}{\gamma}} N^{-\nu/\gamma} \left( \frac{\mu}{\tau} \right)^{1/\gamma} w, \quad (29)$$

which implies

$$\frac{m}{\ell} = \left( \frac{\tau}{\theta \mu} \right)^{1/\gamma} \frac{1 - \gamma}{\gamma} N^{\nu/\gamma}. \quad (30)$$

Applying the logic of roundaboutness in production developed above, we use (24) to write  $A_m = Nm/Q$  as

$$A_m = (1 - \gamma) \left( \frac{w}{\theta \tilde{P}} \right)^\gamma / N^\nu = (1 - \gamma) \frac{\theta \mu}{\tau},$$

where the second equality uses (29) to replace  $\left( \frac{w}{\theta \tilde{P}} \right)^\gamma$ . The ratio of indirect to direct labor use in the variable input part of the technology is equal to

$$\frac{A_m}{1 - A_m} = \frac{1 - \gamma}{\gamma + \frac{\theta \mu}{\tau} - 1}. \quad (31)$$

If  $\theta \mu / \tau > 1$ , as in the laissez-faire case ( $\theta = 1$  and  $\tau = 1$ ) or with  $\theta = \tau$ , the decentralized equilibrium features a lower than socially optimal degree of roundaboutness in production. The total labor input per firm, inclusive of the fixed part, is equal to  $\ell(1 + \nu/\gamma)$ , as in the social optimum. The ratio of indirect to total labor use (inclusive of the fixed labor input), taking on the value  $R^*$  as given in (16), may generally be written as

$$R = \left( \frac{1 + \nu/\gamma}{A_m(1 - A_m)^{-1}} + 1 \right)^{-1} \quad (32)$$

Obviously, if  $A_m(1 - A_m)^{-1}$  is lower than in the social optimum, then  $R < R^*$  as well.

Finally, we may use equation (29) to derive the relative price of labor as

$$\frac{w}{\theta \tilde{P}} = \left( \frac{\tau}{\theta \mu} \right)^{\frac{1}{\gamma}} N^{\frac{v}{\gamma}}, \quad (33)$$

which we may compare to the shadow value of labor in the social planner's solution. From equations (12) and (15) we realize that  $w / (\tilde{P}\theta) = \left( \frac{\tau}{\theta \mu} \right)^{\frac{1}{\gamma}} \lambda^*$ . If  $\theta \mu / \tau > 1$ , as in the laissez-faire case ( $\theta = 1$  and  $\tau = 1$ ) or with  $\theta = \tau$ , then the relative price of labor lies below the shadow value of labor.

We may summarize our results on the decentralized equilibrium as follows:

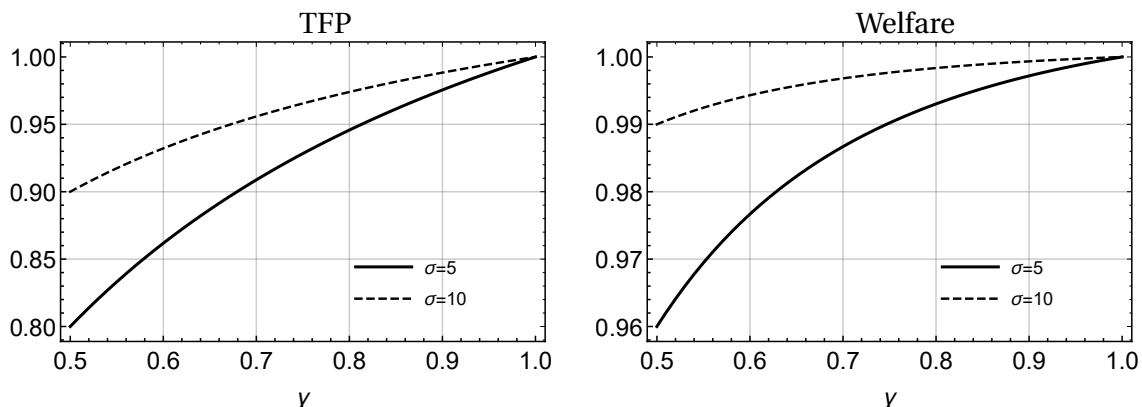
**Proposition 3** (a) *A decentralized laissez-faire equilibrium is characterized by a socially optimal level of employment in each firm as well as by an optimal number of firms in the market.* (b) *Compared to the social optimum, the material input use is lower than in the social optimum, and so is the degree of roundaboutness in production, causing an aggregate output loss as well as a consumption (welfare) loss.* (c) *In a subsidy/tax-ridden equilibrium the socially optimal level of consumption per capita is reached if the policy-wedges are such that  $\tau / \theta = \mu > 1$ .*

**Proof.** (a) The first-best nature of  $\ell$  as well as  $N$  is evidenced by equations (26) and (27) above. (b) Comparing (30) with equation (13), and setting  $\tau = \theta = 1$  (laissez faire), we find that  $m/m^* = \mu^{-1/\gamma} = [\sigma/(\sigma - 1)]^{-1/\gamma} < 1$ . As regards roundaboutness in production, see our remarks on (32) above. Similarly, inserting the above equations for  $\ell$ ,  $N$  and  $m$  (setting  $\tau = \theta = 1$ ) into the equation for  $Q$ , and comparing with  $Q^*$  in (20) above, gives  $Q/Q^* = \mu^{-(1-\gamma)/\gamma} = [\sigma/(\sigma - 1)]^{(1-\gamma)/\gamma} < 1$ . Finally, real consumption is  $C = Q - M = (1 - M/Q)Q$ , where  $M = mN$ . Since  $N = N^*$ , and given the ratios  $m/m^*$  and  $Q/Q^*$  from above, we have  $M/Q = \left( \frac{\tau}{\theta \mu} \right) (M^*/Q^*)$ . Remembering that  $M^*/Q^* = (1 - \gamma)$  and  $C^* = \gamma Q^*$ , we have

$$C = \frac{1}{\gamma} \left[ 1 - \frac{\tau}{\theta \mu} (1 - \gamma) \right] \left( \frac{\tau}{\theta \mu} \right)^{(1-\gamma)/\gamma} C^*. \quad (34)$$

For  $\tau = \theta = 1$ , we have  $0 < C/C^* < 1$ . Note that for a given size of the labor force  $C$  also serves as a welfare measure. (c) For a subsidy/tax-ridden equilibrium,  $\tau \neq 1$  and  $\theta \neq 1$ , it

Figure 1: Loss in total factor productivity and welfare induced by the input distortion



*Legend:* The curves depict the loss in total factor productivity (TFP) and consumption caused by the input distortion as a function of the labor cost share  $\gamma$  for different values of the elasticity of substitution.

is immediately obvious from (34) that  $\tau/\theta = \mu$  is a sufficient condition for  $C = C^*$ . A more thorough examination requires maximization of (34) with respect to  $\tau/\theta$ . It can be shown that the condition  $\tau/\theta = \mu$  is also a necessary condition. ■

What is the magnitude of the welfare loss caused by the input distortion, absent any policy? The loss is measured by the ratio  $C/C^*$  in (34). For  $\theta = \tau = 1$ , we have  $C/C^* = [(1 - 1/\mu + \gamma/\mu)/\gamma] \mu^{-(1-\gamma)/\gamma} < 0$ . We are also interested in the loss in total factor productivity which is simply measured by  $Q/Q^* = \mu^{-(1-\gamma)/\gamma}$ . Figure 1 depicts these losses for values of  $\gamma \in [0.5, 1]$ , and for values of  $\sigma \in \{5, 10\}$ .<sup>10</sup> The losses are substantial, ranging, up to 4 percent for welfare and up to 20 percent for TFP.

Remember that  $\tau > 1$  ( $\tau < 1$ ) means subsidizing (taxing) production of differentiated goods, while  $\theta < 1$  ( $\theta > 1$ ) means subsidizing (taxing) the intermediate input use of the aggregate good. Hence, the optimal policy requires that the combined effect of the two types of policy amounts to net-subsidization at a rate equal to  $\mu$ , which is the markup rate. This is as expected, since it is precisely this markup which is behind the input distortion. Moreover, it is unsurprising that subsidizing *intermediate input use* should be a first-best instrument to

<sup>10</sup>In the appendix, we show that the share of intermediate inputs in gross output is between 40 and 60 percent. In an undistorted world, this share would reflect  $1 - \gamma$ .

correct the input distortion. What is remarkable, however, is that the two types of policies considered here are equally suitable as a first-best correction of this distortion. Specifically, subsidizing *production* of differentiated goods, is an equally suitable first-best policy. After all, this policy effectively subsidizes production of the aggregate good irrespective of whether such production takes place for intermediate input use, or for final consumption, whereas the distortion is present only in intermediate input use.

Two things are important to understand this result. First, given perfect competition in aggregate goods production, subsidization of varieties used in assembly fully feeds into a lower price of this good. And secondly, while this lower price applies to both production and consumption (which is not distorted to start with), it does not distort consumption since by assumption all labor income is spent on this good. Moreover, we have implicitly assumed that the revenue needed to finance the subsidy is raised in a lump-sum (i.e., non-distortionary) way. One might wonder about the subsidy bill in the above analysis. The reason why this bill never showed up simply has to do with the fact that we did not approach consumption from the income side of the household sector. Instead, we have identified real consumption directly as what is left from aggregate output after taking account of intermediate input use.

What is the trade-off behind the optimal policy  $\tau/\theta = \mu$ ? Consider a rise in  $\tau/\theta$ . This has two opposing effects on aggregate welfare  $C$  in (34). First, due to  $M/Q = \left(\frac{\tau}{\theta\mu}\right) (M^*/Q^*)$  it increases the *share* of aggregate output  $Q$  devoted to intermediate input use, thus lowering the share available for consumption. But secondly, it also increases the *level* of aggregate output, due to  $Q = [\tau/(\theta\mu)]^{(1-\gamma)/\gamma} Q^*$ . Thus, we have two opposing forces emanating from the policy instrument  $\tau/\theta$ , and  $\tau/\theta = \mu$  ensures an optimal trade-off between these two forces.

The industrial economists's immediate response to upstream markup pricing is vertical integration. It should be obvious that this is not a viable solution to our input distortion. There simply is no single material input supplier that the variety producer could possibly identify for vertical integration. Material inputs are bought on perfectly competitive markets. Moreover, the input-output linkage implies that it is the variety producer's own markup pricing which is at the heart of the problem.

### 2.3 Wage tax and wage setting

In the decentralized equilibrium, the material input intensity of variety production is governed by the price of the aggregate good relative to the wage rate; see equation (28). It is tempting to argue that a wage tax might be an equally suitable policy instrument to correct the input distortion. In a similar vein, one might argue that a wage setting environment leading to a wage that lies above the opportunity cost constitutes an offsetting distortion, mitigating the need for policy intervention or potentially even calling for a tax, rather than a subsidy on material input. It turns out that in our single-sector setting both arguments, while plausible in partial equilibrium, are wrong in general equilibrium.

Suppose that the government introduces an ad-valorem wage tax  $\phi$  generating a wedge between  $w$ , the wage earned by workers, and  $\tilde{w}$ , the price for labor paid by variety producers:

$$\tilde{w} = (1 + \phi) w. \quad (35)$$

With homogeneous labor, it seems reasonable to assume that that wage tax is uniformly imposed on variable and fixed labor input. A uniform wage tax does not distort the allocation between the variable and fixed type of labor use and, therefore, also does not distort the number of firms. The minimum unit cost in production of a variety is  $x = \tilde{w}^\gamma (\theta \tilde{P})^{1-\gamma}$ . In *partial equilibrium*, i.e., for a *given* price of the aggregate good  $\tilde{P}$ , the wage tax clearly is an incentive to increase the material input intensity of production. Moreover, it is an incentive to increase output per firm according to  $q = (\mu - 1)^{-1} \tilde{w} f / x$ .

*General equilibrium* requires that we close the price-loop implied by the input-output linkage. A one percent increase in  $1 + \phi$  increases the minimum unit cost  $x$  as well as prices  $p$  and  $\tilde{p}$  by  $\gamma$  percent. This follows from technology and markup pricing for varieties. Zero profits in assembly of the aggregate good then imply that the price of the aggregate good similarly increases by  $\gamma$  percent; see equation (21). Formally, the general equilibrium price adjustment of  $\tilde{P}$  is governed by

$$\tilde{P} = \theta^{\frac{1-\gamma}{\gamma}} N^{-\nu/\gamma} \left(\frac{\mu}{\tau}\right)^{1/\gamma} \tilde{w}. \quad (36)$$



Comparing this with (29), we see that the relative price of the two inputs for variety production is invariant to the wage tax. So are the ratios  $x/\tilde{w}$  and  $x/\tilde{P}$  and, hence, input demands as well as outputs and the number of firms; see equations (24), (25) and (27). Put simply, in this model a wage tax doesn't entail any distortion.

We may summarize this result as follows:

**Proposition 4** *(a) In our single-sector economy, a wage tax does not constitute a distortion and is, therefore, not suited to address the input distortion generated by monopolistic competition in the presence of an input-output linkage. (b) In an economy with monopolistic competition in the presence of an input-output linkage, an input distortion arises regardless of the underlying wage setting mechanism.*

**Proof.** Part (a) follows from the text above. Part (b) follows from considering alternative wage setting mechanisms (e.g., fair wages, efficiency wages, search and matching, or trade union wages) as deviations from the reference case of a perfect labor market. These deviations are isomorphic to the wage tax  $\phi$  above. ■

The intuition for this proposition is that with the input-output linkage and the price loop a wage tax doesn't affect any decision margin. We must, however, emphasize two caveats. The first is that we have assumed a completely inelastic labor supply. With elastic labor supply, a decision margin (e.g., consumption-leisure) arises which is affected by a wage tax as well as by wage setting mechanisms. The second is that our model has but one sector. In contrast to the input distortion, the entry distortion that arises in multi-sector settings interacts with wage setting mechanisms. One might also wonder about tax revenue, or the rents generated by wage setting. As with the subsidy instruments considered above, the reason why we need not consider tax revenue here is that in our approach we do not treat consumption from the income side of the household sector. Instead, we identify real consumption directly as what is left from aggregate output after taking account of intermediate input use.

## 2.4 Extensions

### 2.4.1 Fixed input in terms of the final good

In the above model, input-output linkages are restricted to the variable input part of production activities. From an empirical perspective, in many cases technology is such that intermediate inputs also loom large in the fixed input activities. We now develop a version of the model where the fixed input into production requires  $f$  quantities of the composite good rather than  $f$  units of labor. Except for this modification, the model remains as in the baseline case above.

**Social optimum.** The social planner maximizes  $C = Q - M$  by choosing  $\ell$ ,  $m$ , and  $N$ , as above, but the resource constraint now reads as  $L \leq N\ell$ , whereas the total material input (in terms of the composite good) in production of varieties is  $M = N(m + f)$ . The corresponding Lagrangian reads as

$$\mathcal{L} = Q - M - \lambda [N\ell - L]. \quad (37)$$

As in the baseline, the conditions on  $\ell^*$  and  $m^*$  emerge as

$$\ell^* = \gamma \frac{Q^*}{\lambda^* N^*} \text{ and } m^* = (1 - \gamma) \frac{Q^*}{N^*}. \quad (38)$$

The condition on  $N^*$  reads as

$$(\nu + 1) Q^* - M^* = \lambda^* L, \quad (39)$$

where  $M^*$  now contains total use of the composite good in variable and fixed input into production. The resource constraint requires  $\ell^* = L/N^*$ .

Employing the production function in the condition on  $m^*$ , using relative input demand, and observing the resource constraint, we may see that the Lagrange parameter emerges as in the baseline case above:

$$\lambda^* = (N^*)^{\frac{\nu}{\gamma}}. \quad (40)$$

Using the condition on  $m^*$  in the condition on  $N^*$ , we obtain

$$(\nu + \gamma) Q^* = fN^* + \lambda^*L. \quad (41)$$

Using the condition on  $\ell^*$  and using the resource constraint to get rid of  $Q^*$  and  $N^*$  in the above expression, we obtain a second relationship between  $\lambda^*$  and  $N^*$ :

$$\lambda^* = \frac{\gamma f}{\nu L} N^*. \quad (42)$$

Combining the two expressions, we can solve for  $N^*$  as

$$N^* = \left( \frac{\nu L}{\gamma f} \right)^{\frac{\gamma}{\gamma - \nu}}. \quad (43)$$

Optimal material input and labor input emerge as

$$m^* = \frac{1 - \gamma}{\gamma} \lambda^* \ell^* = \frac{1 - \gamma}{\nu} f \quad \text{and} \quad \ell^* = (\lambda^*)^{-1} \frac{\gamma f}{\nu}. \quad (44)$$

Finally, we check the parameter restriction implied by viability,  $Q^* \geq N^* (m^* + f)$ . Using the condition on  $m^*$  to substitute out  $Q^*$ , we obtain

$$\frac{N^* m^*}{(1 - \gamma)} \geq N^* m^* \left( 1 + \frac{f}{m^*} \right). \quad (45)$$

Employing the solution for  $m^*$ , we may rewrite this as  $\gamma \geq \nu$ .

**Decentralized equilibrium with policy intervention.** While conditional demands of the final good producer and the price of the composite good are the same as in the baseline case, the zero profit condition now implies

$$q = \frac{\theta \tilde{P}}{x} \frac{f}{\mu - 1}. \quad (46)$$

Using this expression to substitute out firm size from conditional demands for material input and labor input, respectively, we obtain

$$m = \frac{1 - \gamma}{\mu - 1} f \quad \text{and} \quad \ell = \left( \frac{w}{\theta \tilde{P}} \right)^{-1} \frac{\gamma f}{\mu - 1}, \quad (47)$$

where

$$\frac{w}{\theta \tilde{P}} = \left( \frac{\tau}{\mu \theta} \right)^{\frac{1}{\gamma}} N^{\frac{\nu}{\gamma}}. \quad (48)$$

**Allocational efficiency and optimal policy.** In this version of the model, material input is efficient, while in the absence of policy intervention ( $\tau = \theta = 1$ ), labor input is inefficiently large and – by implication – the number of firms is inefficiently low. As in the baseline, the optimal policy to offset this input distortion is

$$\frac{\tau}{\theta} = \mu > 1. \quad (49)$$

We now explore this implications for roundaboutness of production. The direct labor input coefficient reads as  $A_\ell = \ell N/Q$ . The direct material input coefficients for production fixed input are, respectively,

$$A_{mp} = \frac{mN}{Q} \quad \text{and} \quad A_{mf} = \frac{fN}{Q}. \quad (50)$$

The combined material input,  $A_{mp} + A_{mf}$ , requires further material input use in the amount of  $A_{mp}(A_{mp} + A_{mf})$ . Reiterating, we obtain a total material input use per unit of final output equal to  $(A_{mp} + A_{mf})(1 - A_{mp})^{-1}$ . Note that  $A_{mp} + A_{mf}$  requires no further (indirect) material input use on account of the fixed cost. The indirect labor use (embodied in material inputs) is  $A_\ell(A_{mp} + A_{mf})(1 - A_{mp})^{-1}$ . For easier writing, we now measure roundaboutness by the indirect labor use relative to the direct labor use. This measure is

$$R_1 = (A_{mp} + A_{mf})(1 - A_{mp})^{-1}. \quad (51)$$

In the social optimum, we have  $A_{mp}^* = 1 - \gamma$  and  $A_{mf}^* = \frac{f}{m^*} A_{mp}^* = \nu$ , which results in

$$R_1^* = \frac{1 - \gamma + \nu}{\gamma}. \quad (52)$$

In the decentralized equilibrium without policy intervention, we have  $A_{mp} = A_{mp}^*/\mu$  and thus  $A_{mf}^* = A_{mf}^*/\mu$ . Summing up, we obtain

$$R_1 = \frac{1 - \gamma + \nu}{(\mu - 1) + \gamma} < R_1^* \quad (53)$$

In the decentralized equilibrium the degree of roundaboutness is too small, as in the baseline case.

We can also quantify the implications for total factor productivity and consumption. With respect to  $\text{TFP} = Q/L$ , we have

$$\text{TFP} = \left(\frac{N}{N^*}\right)^{\nu+1} \left(\frac{\ell}{\ell^*}\right)^\gamma \text{TFP}^*, \quad (54)$$

where  $(N/N^*)^{\nu+1} < 1$  and  $(\ell/\ell^*)^\gamma > 1$  reflect the effects of, respectively, an inefficiently small number of firms and an inefficiently large labor input into production. On net, we have

$$\text{TFP} = \mu^{-\frac{1-\gamma+\nu}{\gamma-\nu}} \text{TFP}^*. \quad (55)$$

Compared to the baseline above, the TFP discrepancy is larger.<sup>11</sup>

Turning to real (per capita) consumption, we have

$$C = \mu^{-\frac{1-\gamma+\nu}{\gamma-\nu}} \frac{1 - \frac{1-\gamma}{\mu}}{\gamma} C^*. \quad (56)$$

The distortion in the *share* of the composite output used as material input is the same as in the baseline. Hence, the larger discrepancy in TFP compared to the baseline directly translated into a larger discrepancy in real consumption.

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<sup>11</sup>This follows from noting that  $\frac{1-\gamma+\nu}{\gamma-\nu} > \frac{1-\gamma}{\gamma}$ .

We may summarize the result of this extension in the following proposition:

**Proposition 5** *If the input-output linkage affects the fixed cost on the same footing as it does the variable inputs, then the decentralized market equilibrium has the following properties, compared to the social optimum: (a) The equilibrium level of material inputs is undistorted, relative to the social optimum, but the labor input is inefficiently large. As in the baseline case, the degree of roundaboutness is suboptimally low. (b) The equilibrium number of firms is no longer undistorted, but is suboptimally low. (c) The welfare loss caused by the input distortion is now larger than in the baseline case; so is the loss in terms of total factor productivity.*

**Proof.** All parts follow from the text above. ■

Part (a) of the proposition is intuitive. The distortion works in the same direction regarding the input intensity as in the baseline case, but this time due to an inefficiently large amount of labor, rather than too little material input use. Part (b) implies a new channel through which the input distortion causes inefficiency, viz. the number of firms. Intuitively, this leads to a larger welfare loss, relative to the benchmark case. Figure 2 shows that the loss is now very large indeed, with a maximum value of almost 40 percent in the case where  $\sigma = 5$ . Note that a lower  $\sigma$  implies a high variety effect, meaning that the loss in the number of firms is now felt more strongly in welfare terms.

#### 2.4.2 Heterogeneous firms

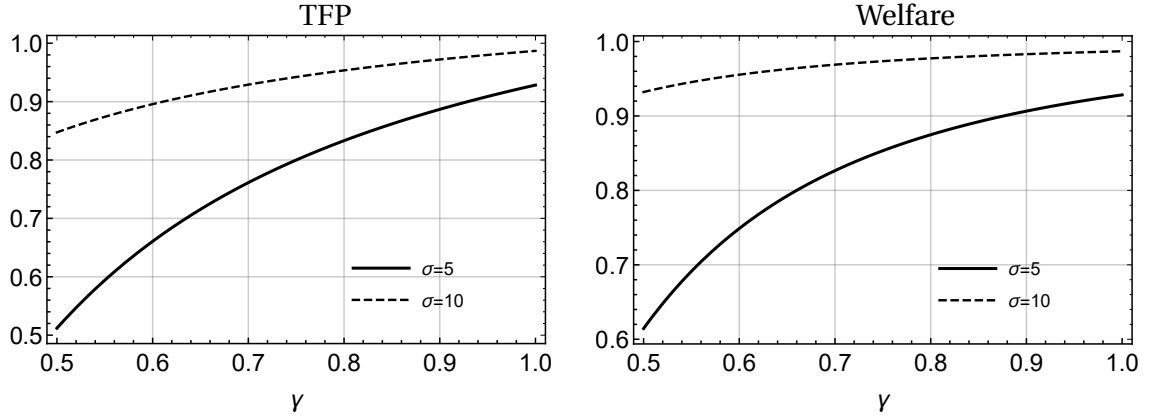
We now allow differentiated good producers to differ in terms of their productivity levels  $\varphi$  such that

$$q(\varphi) = \varphi \left( \frac{l(\varphi)}{\gamma} \right)^\gamma \left( \frac{m(\varphi)}{1-\gamma} \right)^{1-\gamma}. \quad (57)$$

Let productivities be distributed according to the cumulative density function  $G(\varphi)$ . With heterogeneous firms, real consumption is given by

$$C = Q - M = \left[ N \int_{\varphi_c}^{\infty} q(\varphi)^{\frac{\sigma-1}{\sigma}} dG(\varphi) \right]^{\frac{\sigma}{\sigma-1}} - N \int_{\varphi_c}^{\infty} m(\varphi) g(\varphi) dG(\varphi) \quad (58)$$

Figure 2: Loss in TFP and welfare with a fixed input in terms of the composite good



*Legend:* The curves depict the loss in total factor productivity (TFP) and consumption caused by the input distortion as a function of the labor cost share  $\gamma$  for different values of the elasticity of substitution when the fixed input requires the final good.

where  $N$  now denotes the mass of entrants and  $\varphi_c$  denotes the cutoff productivity level. The following observations stand out from equation (58):

1. For a given mass of entrants  $N$  and a given productivity cutoff  $\varphi_c$ , the first term is increasing in  $l(\varphi)$ . It is disciplined, however, by the resource constraint of the primary factor (labor), i.e., (i) for a given cutoff  $\varphi_c$ , a higher  $l(\varphi)$  implies a lower  $N$ , and (ii) for a given  $N$ , a higher  $l(\varphi)$  implies a higher cutoff  $\varphi_c$ .
2. Material input  $m(\varphi)$  can be changed *independently* of labor input  $l(\varphi)$ . The corresponding “resource constraint” is  $M \leq Q$ , but  $m(\varphi)$  is not directly bound by the resource constraint of the primary factor (labor).
3. For given  $N$  and  $\varphi_c$ , the first term rises in  $m(\varphi)$  (higher output), while the second falls in  $m(\varphi)$  (higher material input).

These observations imply that the planner’s real consumption maximization problem can be decomposed into two stages. In the first stage, aggregate output  $Q$  is maximized for given  $\{m(\varphi)\}$ . In the second stage, the planner finds  $\{m(\varphi)\}$  that maximizes real consumption  $C$  for given  $N$ ,  $\varphi_c$ , and  $\{l(\varphi)\}$ . The market, however, maximizes aggregate revenue for given

$\{m(\varphi)\}$  in the first stage. In the second stage, material input is chosen to minimize firms' cost, which is guided by the relative price of labor

$$\frac{w}{\theta \tilde{P}} = \left( \frac{\tau}{\theta \mu} \right)^{\frac{1}{\gamma}} N^{\frac{\nu}{\gamma}} \left( \int_{\varphi_c} \varphi^{\sigma-1} dG(\varphi) \right)^{\frac{\nu}{\gamma}}. \quad (59)$$

Consider the *first stage*. Dhingra and Morrow (2019) show that *for given*  $\{m(\varphi)\}$ , the market and optimal allocations  $N$ ,  $\varphi_c$ , and  $\{l(\varphi)\}$  can be expressed as solutions to

$$\begin{aligned} \max R &= N \int_{\varphi_c} u'(q(\varphi)) q(\varphi) dG(\varphi) \text{ s.t. resource constraint (market) and} \\ \max Q &= N \int_{\varphi_c} u(q(\varphi)) dG(\varphi) \text{ s.t. resource constraint (optimum).} \end{aligned}$$

With CES, we have  $u(q(\varphi)) = q(\varphi)^{\frac{\sigma-1}{\sigma}}$  and  $u'(q(\varphi)) q = \rho q(\varphi)^{\frac{\sigma-1}{\sigma}}$ . Revenue maximization is perfectly aligned with welfare maximization such that the mass of entrants  $N$ , the entry cutoff  $\varphi_c$ , and variable labor input per firm  $\{l(\varphi)\}$  are efficient.

Turn now to the *second stage*. As the mass of entrants  $N$  and the cutoff  $\varphi_c$  are efficient, markup pricing is the only source of distortion of the price of the aggregate good. Hence, firms use too little material input, as in the case of homogeneous firms. Again, a subsidy on the purchase of intermediate inputs that exactly offsets the mark-up yields the efficient allocation of material input.

### 3 Country borders and trade costs

We assume two countries, home ( $h$ ) and foreign ( $f$ ), producing large numbers  $N_h$  and  $N_f$ , respectively, of differentiated goods, which symmetrically enter production of a composite (or aggregate) good according to a CES production function with an (Armington) elasticity of



substitution  $\sigma = 1/(1 - \rho) > 1$ :

$$Y_i = (N_i q_i^{\rho_i})^{1/\rho_i} = N_i^{\nu_i+1} q_i, \quad i \in \{h, f\} \quad (60)$$

$$Y_i = Y_{ii} + \delta Y_{ij}, \quad i, j \in \{h, f\}, i \neq j, \quad (61)$$

$$Q_i = (Y_{ii}^\rho + Y_{ji}^\rho)^{1/\rho}, \quad i, j \in \{h, f\}, i \neq j. \quad (62)$$

In these equations,  $Q_i$  is the quantity of the final good produced in country  $i$ , while  $Y_i$  is the quantity of a country-specific aggregate good, assembled from  $N_i$  varieties produced in country  $i$ , and used in amount  $Y_{ii}$  for country  $i$ 's own final good assembly. The remainder is available for the final good assembly in country  $j$ , albeit subject to an iceberg-type trade cost  $\delta > 1$ . The parameter  $\rho_i \in (0, 1)$  measures the degree of substitution between different varieties produced in country  $i$ , related to the elasticity of substitution  $\sigma_i > 1$  according to  $\rho_i = (\sigma_i - 1)/\sigma_i$ . It is also inversely related to the strength of economies from enhanced variety, measured by  $\nu_i = 1/(\sigma_i - 1) > 1$ . The final good is produced according to a CES technology using the two country-specific aggregate goods as inputs (pure assembly) with an elasticity of substitution  $\sigma > 1$  (Armington elasticity), related to the parameter  $\rho$  according to  $\rho = (\sigma - 1)/\sigma$  and assumed the same for both countries. Importantly,  $Q_i$  may be used for consumption in country  $i$  or as a material input in production of country  $i$ 's varieties  $q_i$ .

We now explore how two symmetric countries set their policies if they are separated by borders. Symmetry allows us to drop the country indices  $i$  and  $j$ . To simplify further, we also set  $\rho$  appearing in Eq. (62) equal to  $\rho_i$  and  $\rho_j$  in Eq. (60), so that the elasticities of substitution between varieties in production of the two country-specific aggregate intermediates and the elasticity of substitution in assembly of intermediates to the final good are all the same. We discuss optimal cooperative and non-cooperative policies. In doing so, we focus on the baseline-case where input-output linkages are restricted to variable input use.

### 3.1 Cooperative policies

The presence of borders gives rise to two additional margins. First, the aggregate output levels, equal to  $Y$  for each country, have to be distributed among the two countries for production of the nontrable final goods. We write  $Y$  and  $Y_m$  for conditional domestic and import demand, respectively. Second, the government may now subsidize the use of the domestically produced and the imported country-specific aggregates differently by  $\tau_d$  and  $\tau_m$ . Due to the symmetry assumption, we suppress the country indices. We have to bear in mind, however, that  $L$  and  $N$  now refer symmetrically to endowments and the number of firms at the country level, reflecting immobility of labor across countries.

#### 3.1.1 Full set of policies

We assume that the governments have all policy instruments  $\tau_d$ ,  $\tau_m$ , and  $\theta$  at their disposal. As in Section 3, we proceed by first identifying the social optimum for a cooperative world and then comparing this with a decentralized equilibrium with policy interventions.

**Social optimum.** The socially optimal allocation follows from solving

$$\max_{\ell, m, N, Y_m} 2 \left[ ((Y - \delta Y_m)^\rho + Y_m^\rho)^{\frac{1}{\rho}} - Nm \right] \quad (63)$$

subject to the production functions in (Eq. (60)) and the resource constraint for each country. The number 2 appears in this expression since we deal with two symmetric countries. Compared to the baseline, there is a new margin: the optimal choice of the imported quantity  $Y_m$ . In the statement of the maximization problem, we already impose the goods market clearing condition. The corresponding Lagrangian is

$$\mathcal{L} = ((Y - \delta Y_m)^\rho + Y_m^\rho)^{\frac{1}{\rho}} - Nm - \lambda [N(\ell + f) - L]. \quad (64)$$

The first-order conditions for  $\ell^*$  and  $m^*$  emerge as

$$\ell^* = \gamma \frac{\Gamma^* Y^*}{\lambda^* N^*} \text{ and } m^* = (1 - \gamma) \frac{\Gamma^* Y^*}{N^*}, \text{ where } \Gamma^* \equiv \left( \frac{Q^*}{Y_d^*} \right)^{1-\rho}. \quad (65)$$

The first-order condition for imports  $Y_m^*$  is

$$-\rho (Y^* - \delta Y_m^*)^{\rho-1} \delta + \rho (Y_m^*)^{\rho-1} = 0 \Rightarrow Y_m^* = \frac{\delta^{\frac{1}{\rho-1}}}{1 + \delta^{1+\frac{1}{\rho-1}}} Y^*. \quad (66)$$

Invoking goods market clearing,  $Y = Y_d + \delta Y_m$ , relative import demand for intermediate inputs in the assembly industry merges as

$$\frac{Y_m^*}{Y_d^*} = \delta^{-\sigma}. \quad (67)$$

Intuitively, trade costs drive a wedge between import and domestic demand. Using equation (67) and goods market clearing, we obtain<sup>12</sup>

$$\Gamma^* Y^* = \left( \frac{Q^*}{Y_d^*} \right)^{1-\rho} (Y_d^* + \delta Y_m^*) = Q^*. \quad (68)$$

Hence, the conditions on  $\ell^*$  and  $m^*$  in equation (65) collapse to their counterparts in the closed economy; see (6) and (7).

The first-order condition on  $N^*$  is  $\Gamma^*(\nu + 1)N^*q^* = \lambda^*(\ell^* + f)$ . Multiplying with  $N^*$  and observing the resource constraint, this may be written as

$$\Gamma^* (\nu + 1) Y^* - N^* m^* = \lambda^* L. \quad (69)$$

Using the conditions on  $m^*$  and  $\ell^*$  to substitute out  $m^*$  and  $\lambda^*$ , respectively, from equation

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<sup>12</sup>The term  $(Q/Y_d)^{1-\delta}$  is the marginal productivity of the domestic input. By virtue of (66), in the social optimum the marginal productivity of the imported input is  $\delta$  times  $(Q/Y_d)^{1-\delta}$ , whence the equation above is an instance of Euler's theorem.

(69) and again invoking the resource constraint, we can solve for variable labor input as

$$\ell^* = \frac{\gamma}{\nu} f. \quad (70)$$

Thus, in the social optimum the variable labor input into production of a differentiated variety is not affected by the presence of borders. Employing the resource constraint, the optimal number of firms in each country emerges as

$$N^* = \frac{1}{\frac{\gamma}{\nu} + 1} \frac{L}{f}. \quad (71)$$

This expression is structurally equivalent to the one obtained in the closed economy above, the difference being, of course, that  $L$  now refers to a single country's labor endowment, while in the closed economy equilibrium  $L$  represents world labor endowments.

Using again the first-order condition on  $m^*$  to substitute out  $m^*$  from equation (69) and employing equation (70) and the resource constraint, the Lagrange parameter now reads as

$$\lambda^* = \Gamma^{\frac{1}{\gamma}} (N^*)^{\frac{\nu}{\gamma}}, \quad (72)$$

where, in contrast to the baseline,  $N^*$  refers to the number of firms in one of the countries. The term  $\Gamma^{\frac{1}{\gamma}}$  appears because a relaxation of the resource constraint not only affects consumption through the domestic, but also through the imported country-specific intermediate input. Employing equation (67), we obtain

$$\Gamma = \left( \frac{Q^*}{Y_d^*} \right)^{1-\rho} = \left( \frac{((Y_d^*)^\rho + (Y_m^*)^\rho)^{\frac{1}{\rho}}}{Y_d^*} \right)^{1-\rho} = \left( 1 + \left( \frac{Y_m^*}{Y_d^*} \right)^\rho \right)^{\frac{1-\rho}{\rho}} = (1 + \delta^{1-\sigma})^\nu. \quad (73)$$

In the absence of trade costs ( $\delta = 1$ ),  $\Gamma$  collapses to  $2^\nu$ , and the Lagrange parameter reads as  $\lambda^* = (2N^*)^{\frac{\nu}{\gamma}}$ , which is in line with the closed economy equilibrium above. In general, however, the Lagrange multiplier is decreasing in trade costs.

Rearranging terms in equation (73), and employing equation (67), optimal aggregate out-

put emerges as

$$Q^* = (1 + \delta^{1-\sigma})^{\frac{1}{\rho}} Y_d^* = (1 + \delta^{1-\sigma})^\nu Y^*, \quad (74)$$

where  $Y^* = (N^*)^{\nu+1} q^*$ . In the absence of trade costs ( $\delta = 1$ ), aggregate output is  $Q^* = 2^\nu Y^* > 2Y^*$ . The inequality is a consequence of external economies of scale. With scale economies, restricting labor mobility results in lower aggregate output, even in the absence trade costs. Employing the production functions and using the conditions on  $\ell^*$  and  $m^*$ , we obtain

$$Y^* = \frac{f}{\nu} \Gamma^{\frac{1-\gamma}{\gamma}} (N^*)^{\frac{\nu}{\gamma}+1} \quad \text{and} \quad Q^* = \frac{f}{\nu} \Gamma^{\frac{1}{\gamma}} (N^*)^{\frac{\nu}{\gamma}+1}. \quad (75)$$

It immediately follows from the condition on  $m^*$  and equation (68) that the share of output used as input into production of differentiated varieties is equal to  $M^*/Q^* = 1 - \gamma$ , as in the reference case of the closed economy. Hence, consumption is  $C^* = \gamma Q^*$ .

**Decentralized equilibrium with policy intervention.** In a decentralized equilibrium, conditional demands for inputs at the level of the differentiated good producers are the same as the basic setting; see equation (24). Moreover, their pricing behavior and the zero profit condition are the same, giving rise to the following solution in decentralized equilibrium; see equations (26) to (28):

$$\ell = \frac{\gamma}{\mu - 1} f, \quad N = \left( \frac{\gamma}{\mu - 1} + 1 \right)^{-1} \frac{L}{f}, \quad \text{and} \quad m = \frac{1 - \gamma}{\gamma} \frac{w}{\theta \tilde{P}} \ell. \quad (76)$$

We stick to the above notation, using a  $\tilde{p}$  for the “demand price” for varieties and  $\tilde{P}$  to the price of the aggregate good received by producers.

The new margin is that final good producers have to combine the two country-specific aggregate intermediates. Cost minimizing behavior implies

$$\frac{Y_m}{Y_d} = \left( \frac{\tau_d}{\tau_m} \delta \right)^{-\sigma}, \quad (77)$$

where, on top of trade costs, the relative policies drive a wedge between the cost of the two

types of inputs. The price of the final good is given by

$$\tilde{P} = (N\tilde{p}_d^{1-\sigma} + N\tilde{p}_m^{1-\sigma})^{\frac{1}{1-\sigma}} = N^{-\nu} \frac{\mu w^\gamma (\theta \tilde{P})^{1-\gamma}}{\tau_d} \left( 1 + \left( \frac{\tau_d}{\tau_m} \delta \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \quad (78)$$

Solving the loop in equation (78) for  $\tilde{P}$ , the relative price of labor emerges as

$$\frac{w}{\theta \tilde{P}} = \left( \frac{\tau_d}{\theta \mu} \right)^{\frac{1}{\gamma}} \left[ \left( 1 + \left( \frac{\tau_d}{\tau_m} \delta \right)^{1-\sigma} \right) N \right]^{\frac{\nu}{\gamma}}. \quad (79)$$

The first term is exactly the same as in the closed economy equilibrium. The second term highlights the effects of the trade costs and relative policies. In the absence of trade costs ( $\delta = 1$ ) and policy differentials ( $\tau_d = \tau_m$ ), it collapses to  $(2N)^{\frac{\nu}{\gamma}}$ . For a given policy differential  $\tau_d/\tau_m$ , the relative price of labor is decreasing in trade costs.

Employing the production function and invoking goods market clearing, aggregate output emerges as

$$Q = \left( 1 + \left( \frac{\tau_d}{\tau_m} \delta \right)^{1-\sigma} \right)^{\frac{1}{\rho}} Y_d = \frac{\left( 1 + \left( \frac{\tau_d}{\tau_m} \delta \right)^{1-\sigma} \right)^{\frac{1}{\rho}}}{1 + \delta^{1-\sigma} \left( \frac{\tau_d}{\tau_m} \right)^{-\sigma}} Y, \quad (80)$$

where we may rewrite

$$\left( 1 + \left( \frac{\tau_d}{\tau_m} \delta \right)^{1-\sigma} \right)^{\frac{1}{\rho}} = \left( 1 + \left( \frac{\tau_d}{\tau_m} \delta \right)^{1-\sigma} \right) \left( 1 + \left( \frac{\tau_d}{\tau_m} \delta \right)^{1-\sigma} \right)^{\nu}$$

in order to facilitate easy comparison to  $Q^*$  in (74) above. In the absence of policy differentials ( $\tau_d = \tau_m$ ), the expression collapses to  $Q = (1 + \delta^{1-\sigma})^\nu Y$ , which resembles equation (74).

Employing the production functions and using conditional input demands  $\ell^*$  and  $m^*$ , we obtain

$$Q = \frac{f}{\nu} \left( \frac{\tau_d}{\theta \mu} \right)^{\frac{1-\gamma}{\gamma}} \frac{1 + \left( \frac{\tau_d}{\tau_m} \delta \right)^{1-\sigma}}{1 + \delta^{1-\sigma} \left( \frac{\tau_d}{\tau_m} \right)^{-\sigma}} \left( 1 + \left( \frac{\tau_d}{\tau_m} \delta \right)^{1-\sigma} \right)^{\frac{\nu}{\gamma}} N^{\frac{\nu}{\gamma} + 1}. \quad (81)$$

In the absence of policy differentials, aggregate output reads as  $Q = \left( \frac{\tau_d}{\theta \mu} \right)^{\frac{1-\gamma}{\gamma}} Q^*$ , as in the reference case. Again employing the production functions and using conditional input de-

mands for  $\ell$  and  $m$ , the share of output used as input into production emerges as

$$\frac{M}{Q} = (1 - \gamma) \frac{\tau_d}{\theta\mu} \frac{1 + \delta^{1-\sigma} \left(\frac{\tau_d}{\tau_m}\right)^{-\sigma}}{1 + \left(\frac{\tau_d}{\tau_m} \delta\right)^{1-\sigma}}. \quad (82)$$

In the absence of policy differentials, the share collapses to  $(1 - \gamma) \frac{\tau_d}{\theta\mu}$ , again as in the baseline. Consumption is given by  $C = Q(1 - M/Q)$ .

Comparing the decentralized equilibrium to the social optimum, two observations stand out. First, the allocation of the country-specific aggregate to markets  $Y_m/Y_d$  is optimal if the government does not condition the subsidy on the country of origin:  $\tau_d = \tau_m$ . Second, conditional on  $\tau_d = \tau_m$ , the decentralized equilibrium replicates social optimum if the government subsidizes either the use of the country-specific aggregates in the production of the final good or the use of the final good in the production of differentiated varieties to offset the markup. The following proposition straightforwardly generalizes Proposition 3 to the case of borders.

**Proposition 6 (a)** *In a cooperative setting with borders and symmetric countries, a decentralized laissez-faire equilibrium is characterized by a socially optimal level of employment in each firm as well as by an optimal number of firms in each market. (b) Compared to the social optimum, the material input use is lower than in the social optimum, causing an aggregate output loss as well as a consumption (welfare) loss. (c) In a subsidy/tax-ridden equilibrium the socially optimal level of consumption per capita is reached if the policy-wedges are such that  $\frac{\tau_d}{\theta} = \mu$  and  $\tau_d = \tau_m$ .*

**Proof. (a)** The first-best nature of  $\ell$  and  $N$  follow from comparing (76) to (70) and (71). Follows from noting that  $\frac{m}{m^*} = \left(\frac{\tau_d}{\theta\mu}\right)^{\frac{1}{\gamma}} \left(\frac{1 + \left(\frac{\tau_d}{\tau_m} \delta\right)^{1-\sigma}}{1 + \delta^{1-\sigma}}\right)^{\nu/\gamma}$ . **(c)** A sufficient condition for  $C = C^*$  is  $\frac{\tau_d}{\theta} = \mu$  and  $\tau_d = \tau_m$ . ■

An important corollary to part (c) of the proposition is that trade costs do not affect optimal policies.

### 3.1.2 Restriction to trade policy

We continue to assume that the two symmetric countries set their policies cooperatively, but now we suppose that they are restricted to the use of trade policy measures, while domestic subsidies are not available.<sup>13</sup> With cooperation, the terms-of-trade externality is internalized. While in the standard two-country, single-sector CES setting, free trade is optimal, in our setting with an input-output linkages, cooperative trade policy might be used as a second-best instrument to address the input distortion. The intuition is the following. An import subsidy lowers the price of the imported country-specific aggregate, and this reduction is partly passed on to the price of composite good, which, in turn, alleviates the input distortion. However, the import subsidy causes a distortion in and of itself, raising the relative price of the imported aggregate intermediate, relative to the domestic intermediate. We now determine the optimal level of this second-best policy instrument.

To formalize the argument, we consider a setting in which governments take the behavior of all types of firms and  $\tau_d = \theta = 1$  as given and cooperatively choose  $\tau_m$  to maximize consumption  $C = Q - M$ . For the sake of clear argument, we abstract from trade costs, setting  $\delta = 1$ . Equations (75) and (81) imply that in this restricted setting aggregate output emerges as

$$Q = \mu^{-\frac{1-\gamma}{\gamma}} \frac{(1 + \tau_m^{\sigma-1})^{\frac{\sigma}{\gamma} + 1}}{1 + \tau_m^\sigma} Q^*. \quad (83)$$

The effect of an import subsidy on aggregate output is ambiguous. Totally differentiating the above expression with respect to  $\tau_m$ , we obtain

$$\frac{\partial \ln Q}{\partial \ln \tau_m} = \tau_m^{\sigma-1} \left[ \frac{\frac{1}{\gamma} + \sigma - 1}{1 + \tau_m^{\sigma-1}} - \frac{\sigma \tau_m}{1 + \tau_m^\sigma} \right]. \quad (84)$$

Evaluated at  $\tau_m = 1$ , we have

$$\left. \frac{\partial \ln Q}{\partial \ln \tau_m} \right|_{\tau_m=1} = \frac{1-\gamma}{2\gamma} > 0. \quad (85)$$

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<sup>13</sup>The Agreement on Subsidies and Countervailing Measures disciplines the use of production subsidies.



Hence, a *small* import subsidy ( $\tau_m > 1$ ) raises aggregate output.

Employing the production functions and using the expressions for labor and material input, the share of output used as material input emerges as

$$\frac{M}{Q} = \frac{1 - \gamma}{\mu} \frac{1 + \tau_m^\sigma}{1 + \tau_m^{\sigma-1}}. \quad (86)$$

It is easy to check that the share of output used as material input into production is increasing in the import subsidy. By implication, the import subsidy (larger  $\tau_m$ ) lowers the share of output available for consumption. To demonstrate that the net effect a *small* import subsidy on consumption is positive, we totally differentiate the share of output used for final consumption:

$$\frac{\partial \ln \left(1 - \frac{M}{Q}\right)}{\partial \ln \tau_m} = - \frac{1 - \gamma}{\mu} \frac{\sigma (\tau_m - 1) + 1 + \tau_m^\sigma}{(1 + \tau_m^{\sigma-1})^2} \frac{\tau_m^{\sigma-1}}{1 - \frac{1 - \gamma}{\mu} \frac{1 + \tau_m^\sigma}{1 + \tau_m^{\sigma-1}}} < 0. \quad (87)$$

Evaluating at  $\tau_m = 1$  and using equation (85), we obtain

$$\left. \frac{\partial \ln C}{\partial \ln \tau_m} \right|_{\tau_m=1} = \frac{1 - \gamma}{\gamma} \frac{\mu - 1}{2(\mu - 1 + \gamma)} > 0. \quad (88)$$

Thus, a *small* import subsidy imposed by each country is welfare enhancing. Intuitively, the effect of a small import subsidy on consumption becomes negligible when either  $\gamma \rightarrow 1$  or  $\mu \rightarrow 1$  ( $\rho \rightarrow 1$ ). Recall that in the standard of cooperative policy formation without the input-output linkage, free trade is optimal.

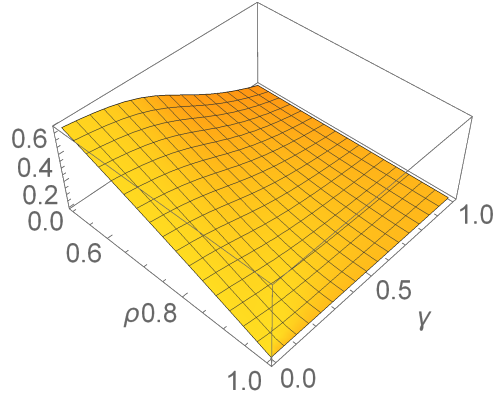
We may summarize our result as follows:

**Proposition 7** *In a world of cooperative policy formation, a small import subsidy imposed by each country serves as a second-best policy to address the input distortion generated by monopolistic competition in the presence of an input-output linkage.*

**Proof.** See equation (88). ■

Figure 3 illustrates the welfare maximizing cooperative import subsidies  $|t| = 1 - 1/\tau_m$

Figure 3: Optimal cooperative import subsidy in the absence of domestic policies



The graph shows the optimal cooperative import subsidy  $|t| = 1 - 1/\tau_m$  in the absence of domestic policies ( $\tau_d = \theta = 1$ ).  $\rho = (\sigma - 1)/\sigma \in (0.5, 1)$  is a transformation of the elasticity of substitution.  $\gamma \in (0, 1)$  is labor cost share in production.

in  $(\rho, \gamma)$ -space.<sup>14</sup> The smaller the labor cost share  $\gamma$  and/or the elasticity of substitution (or:  $\rho$ ), the larger the optimal cooperative import subsidy. For  $\sigma = 5$  ( $\rho = 0.8$ ) and  $\gamma = 0.5$ , the optimal cooperative import subsidy amounts to 7.4 percent.

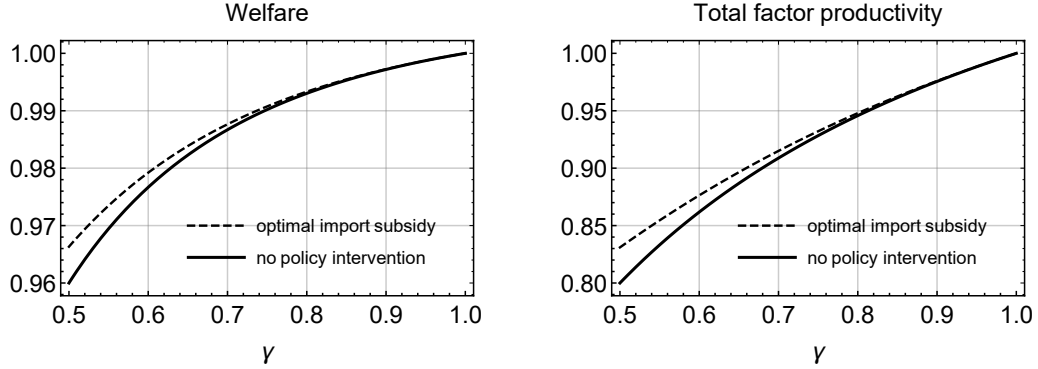
We can also quantify the welfare consequences of prohibiting the use of domestic policies. In the presence of an import tariff, the consumption (welfare) discrepancy emerges as

$$\frac{C}{C^*} = \mu^{-\frac{1-\gamma}{\gamma}} \frac{(1 + \tau_m^{\sigma-1})^{\frac{\nu}{\gamma} + 1}}{1 + \tau_m^\sigma} \frac{1 - \frac{1-\gamma}{\mu} \frac{1 + \tau_m^\sigma}{1 + \tau_m^{\sigma-1}}}{\gamma}. \quad (89)$$

Figure 4 illustrates the TFP and consumption discrepancies as a function of  $\gamma$  for  $\sigma = 5$ , evaluated at the optimal cooperative import subsidy. Compared to the situation without any policy intervention, the use of an optimal cooperative import subsidy has only small effects on the discrepancies. For  $\gamma = 0.5$ , optimally subsidizing imports shrinks the consumption and TFP loss induced by the input distortion, respectively, from 4 to 3.4 percent and from 20 to 17 percent.

<sup>14</sup>The optimal cooperative import subsidy is determined by the first-order condition of the welfare maximization problem:  $\frac{\partial C}{\partial \tau_m} = 0$ . In order to compute the policy-induced change in consumption, we employ equations (84) and (87).

Figure 4: Discrepancies with and without optimal cooperative import subsidies



*Legend:* The curves depict the loss in total factor productivity (TFP) and welfare caused by the input distortion as a function of the labor cost share  $\gamma$  for different values of the elasticity of substitution. The *solid line* refers to a situation without policy intervention, the *dashed line* to a situation in which the optimal cooperative import subsidy is employed.

### 3.2 Non-cooperative policies

By non-cooperative policies we mean a national government pursuing maximum welfare using trade policy, and assuming that the other government keeps its policy unchanged. It proves convenient to reintroduce country indices first introduced at the beginning of this Section. In our analysis, we abstract from subsidies on the use of the final good (setting  $\theta_h = \theta_f = 1$ ) and focus on policy wedges in the prices of differentiated varieties. We first characterize a decentralized equilibrium with asymmetric policies and then analyze the home country's incentive to deviate (i) from the cooperative subsidy of varieties and (ii) from free trade. The new mechanism arising in this context is the familiar terms of trade effect. In pursuit of a beneficial terms of trade effect, the home government would impose a tariff on imported varieties. However, in raising the price of material inputs, this aggravates the input distortion. Hence an optimal non-cooperative policy weighs the input distortion against the terms of trade distortion.

The price of a variety produced in country  $i$  and used in country  $j$  inclusive of trade costs and the policy wedge emerges as

$$\tilde{p}_{ij} = \delta_{ij} (1 + t_{ij}) p_i, \quad (90)$$

where  $t_{ij} > 0$  implies a tax and  $t_{ij} < 0$  implies a subsidy.<sup>15</sup> Proposition 6 implies that in this kind of setting, the optimal cooperative policy is setting  $1 + t_{ij} = \mu^{-1} < 1$  for all countries  $i \in \{h, f\}$  and  $j \in \{h, f\}$ , which is a uniform subsidy on domestically produced and imported varieties. Without loss of generality, we take country  $h$ 's perspective, define  $\bar{t} := t_{hh} = t_{ff} = t_{hf}$ , and assume that country  $h$  chooses its trade policy  $t_{fh}$  non-cooperatively.<sup>16</sup>

### 3.2.1 Decentralized equilibrium with asymmetric policy intervention

Extending equation (21) and (78), the price of the final good emerges as

$$\tilde{P}_i = \left( \sum_{j \in \{h, f\}} N_j [\delta_{ji} (1 + t_{ji}) p_j]^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \quad (91)$$

where domestic trade costs are normalized to unity,  $\delta_{ii} = 1$ . Extending equation (22), unit-demand for varieties follow as

$$q_{ji} = \left( \frac{\delta_{ji} (1 + t_{ji}) p_j}{\tilde{P}_i} \right)^{-\sigma}. \quad (92)$$

It is useful to focus on the cost share of country  $j$ 's varieties in country  $i$ 's assembly of the final good

$$\lambda_{ji} := \frac{N_j [\delta_{ji} (1 + t_{ji}) p_j]^{1-\sigma}}{\tilde{P}_i^{1-\sigma}}. \quad (93)$$

As will become clear below, the term  $\lambda_{ii}$ ,  $i \in \{h, f\}$  takes the role of the domestic expenditure share in Arkolakis et al. (2012). Following this notation will facilitate an easy interpretation of our results below.

Balanced trade in differentiated varieties requires  $N_f \delta_{fh} q_{fh} p_f = N_h \delta_{hf} q_{hf} p_h$ , where exports and imports are inclusive of real trade costs  $\delta$ , but exclusive of policy wedges  $t$ . Using optimal demands (92) and employing equation (93), the balanced-trade-condition emerges

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<sup>15</sup>For the sake of simplicity, we cast the analysis in terms of  $t$ , rather than  $\tau := 1/(1+t)$  as above.

<sup>16</sup>We could also explore deviation in terms of the domestic policy  $t_{ii}$ ,  $i \in \{h, f\}$ . In the light of footnote 13 and in order to facilitate comparison to the optimal tariff literature, we focus on trade policy.

as

$$\frac{\lambda_{fh}}{1+t_{fh}} \tilde{P}_h Q_h = \frac{\lambda_{hf}}{1+t_{hf}} \tilde{P}_f Q_f, \quad (94)$$

where  $\tilde{P}_i Q_i$  is the value of output county  $i$ 's final good. The final good is used by consumers and by differentiated good producers. Consumers spend their labor income  $w_i L_i$  net of the lump-sum transfer  $T_i$  on the final good, while differentiated goods producers spend a share  $(1-\gamma)/\mu$  of their revenues  $Z_i := N_i p_i q_i$  on material inputs. Remember that in our baseline case there are no input-output linkages in the fixed cost part of the technology. Zero profits imply that they pay out the rest of their revenues to the workers whence

$$Z_i = \kappa w_i L_i, \quad \text{where} \quad \kappa := \left(1 - \frac{1-\gamma}{\mu}\right)^{-1}. \quad (95)$$

The net transfers  $T_i$  is given by

$$T_i := \sum_{j \in \{h,f\}} t_{ji} N_j \delta_{ji} q_{ji} p_i = \tilde{P}_i Q_i \sum_{j \in \{h,f\}} \frac{t_{ji} \lambda_{ji}}{1+t_{ji}}, \quad (96)$$

where the last expression follows from employing equation (93). Note that for a subsidy  $T_i < 0$ . The value of output is equal in value to consumption demand ( $w_i L_i + T_i$ ) and material input demand:

$$\tilde{P}_i Q_i = w_i L_i + T_i + \frac{1-\gamma}{\mu} Z_i = \frac{\kappa w_i L_i}{1 - \sum_{j \in \{h,f\}} \frac{t_{ji} \lambda_{ji}}{1+t_{ji}}}. \quad (97)$$

Using this expression to substitute out the value of aggregate output and using symmetry in labor endowments, balanced trade can be rewritten as

$$\omega := \frac{w_h}{w_f} = \frac{1+t_{fh}}{1+t_{hf}} \frac{1-\lambda_{ff}}{1-\lambda_{hh}} \frac{1 - \sum_{j \in \{h,f\}} \frac{t_{jh} \lambda_{jh}}{1+t_{jh}}}{1 - \sum_{j \in \{h,f\}} \frac{t_{jf} \lambda_{jf}}{1+t_{jf}}}, \quad (98)$$

where we have used  $\lambda_{ji} = 1 - \lambda_{ii}$ .

Using the markup pricing rule and the minimum unit cost function, equation (93) implies the following relationship between the relative price of labor (the real wage) and the share of

expenditure (for consumption as well as intermediate input use) falling on domestic varieties:

$$\frac{w_i}{\bar{P}_i} = (\bar{\chi}_i \lambda_{ii})^{-\frac{1}{\gamma(\sigma-1)}}, \quad \text{where} \quad \bar{\chi}_i := N_i^{-1} [\mu (1 + t_{ii})]^{\sigma-1}. \quad (99)$$

We know from above that the number of firms in any country is independent of policy wedges  $t_{ij}$  and we rule out changes in domestic policies  $t_{ii}$ , whence we can treat  $\bar{\chi}_i$  as a constant in the subsequent analysis. Moreover, symmetry in country characteristics and domestic policy allows us to write  $\bar{\chi} := \bar{\chi}_h = \bar{\chi}_f$ .

Writing  $x_h/x_f = w_h/w_f [(w_h/P_h)/(w_f/P_f)]^{1-\gamma}$  and employing equation (99), relative marginal costs emerge as

$$x := \frac{x_h}{x_f} = \omega \left( \frac{\lambda_h}{\lambda_f} \right)^{\frac{1-\gamma}{\gamma(\sigma-1)}}. \quad (100)$$

In the absence of input-output linkages, this expression collapses to the relative wage  $\omega$ . In their presence, relative marginal costs additionally depend on the prices of the final good, which, in turn, depend on prices of domestic and imported differentiated varieties, whence the presence of  $\lambda_h := \lambda_{hh}$  and  $\lambda_f := \lambda_{ff}$  in the above expression.

Using equation (91) to substitute out the price of final good from equation (93), letting  $\bar{t} := t_{hh} = t_{ff} = t_{hf}$  and using  $t$  to denote country  $h$ 's trade policy, the equilibrium values of  $\lambda_h$ ,  $\lambda_f$ , and  $\omega$  are determined by the following system of equations:

$$\begin{aligned} \lambda_h &= \left( 1 + \left( \frac{1+t}{1+\bar{t}} \delta \right)^{1-\sigma} x^{\sigma-1} \right)^{-1} \\ \lambda_f &= \left( 1 + (\delta x)^{1-\sigma} \right)^{-1} \\ \omega &= \frac{1-\lambda_f}{1-\lambda_h} \left( 1 + \frac{t-\bar{t}}{1+\bar{t}} \lambda_h \right), \end{aligned} \quad (101)$$

where relative marginal costs  $x$  are a function of the relative wage and the domestic shares; see equation (100).

We are now ready to explore any one country's incentive to deviate from a cooperative

policy. For the sake of concreteness we look at country  $h$ .<sup>17</sup> Recall that households's aggregate income is the wage  $w_h L_h$  income net of the lump-sum transfer  $T_h$ . We define the policy multiplier as:

$$\zeta_h := \frac{w_h L_h + T_h}{w_h L_h}. \quad (102)$$

Then, real per capita income emerges as

$$W_h := \frac{w_h L_h + T_h}{\bar{P}_h L_h} = \zeta_h \times (\bar{\chi}_h \lambda_h)^{-\frac{1}{\gamma(\sigma-1)}}, \quad \text{where } \zeta_h = 1 + \kappa \frac{\bar{t} \lambda_h + t(1 - \lambda_h + \bar{t})}{1 + \bar{t}(1 - \lambda_h) + t \lambda_h} \quad (103)$$

where country  $h$ 's income multiplier follows from using equation (96). This expression is a straightforward generalization of the welfare formula presented by Arkolakis et al (2012). In the absence of policy intervention ( $t = \bar{t} = 0$ ), the income multiplier collapses to unity. Real income is then determined by the (endogenously determined) domestic cost share, the trade elasticity ( $\sigma - 1$ ), and with the labor cost share  $\gamma$ . For a given domestic cost share  $\lambda_h$ , a given trade elasticity ( $\sigma - 1$ ), and a given  $\bar{\chi}_h$ , real income is larger in the presence of input-output linkages ( $\gamma < 1$ ) than without such linkages. With policy intervention, welfare is multiplied by the policy multiplier, as noted by Felbermayr et al. (2015). Note that this multiplier is less than one if the policy is a subsidy. In the following, we explore the welfare consequences of a deviation of  $t$  implications of a.

The non-cooperative policy analysis now requires that we partially differentiate equation (103) with respect to  $t$ , taking into account the effect on  $\lambda_h$  as determined by the system (101), holding  $\bar{t}$  constant. The change in country  $i$ 's welfare is given by

$$\frac{dW_h}{W_h} = \frac{d\zeta_h}{\zeta_h} - \frac{1}{\gamma(\sigma-1)} \frac{d\lambda_h}{\lambda_h}, \quad (104)$$

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<sup>17</sup>Analogous expressions also emerge for country  $f$ . Without loss of generality, we explore country  $h$ 's incentive to deviate from cooperative policies in the subsequent analysis, whence the focus on country  $h$ .

where

$$\begin{aligned} \frac{\partial \zeta_h}{\partial t} &= \kappa \frac{(1 + \bar{t} + (t - \bar{t}) \lambda_h) \left[ (\bar{t} - t) \frac{\partial \lambda_h}{\partial t} + (1 - \lambda_h + \bar{t}) \right]}{(1 + \bar{t}(1 - \lambda_h) + t \lambda_h)^2} \\ &\quad - \kappa \frac{(t(1 + \bar{t}) + (\bar{t} - t) \lambda_h) \left[ \lambda_h + (t - \bar{t}) \frac{\partial \lambda_h}{\partial t} \right]}{(1 + \bar{t}(1 - \lambda_h) + t \lambda_h)^2} \end{aligned} \quad (105)$$

$$\frac{d\lambda_h}{\lambda_h} = -(1 - \lambda_h) \left[ \frac{1 - \gamma}{\gamma} \left( \frac{d\lambda_h}{\lambda_h} - \frac{d\lambda_f}{\lambda_f} \right) + (\sigma - 1) \frac{d\omega}{\omega} - \frac{\sigma - 1}{1 + t} dt \right], \quad (106)$$

$$\frac{d\lambda_f}{\lambda_f} = -(1 - \lambda_f) \left[ \frac{1 - \gamma}{\gamma} \left( \frac{d\lambda_f}{\lambda_f} - \frac{d\lambda_h}{\lambda_h} \right) - (\sigma - 1) \frac{d\omega}{\omega} \right], \text{ and} \quad (107)$$

$$\frac{d\omega}{\omega} = \frac{1 + t}{1 + \bar{t}(1 - \lambda_h) + t \lambda_h} \frac{\lambda_h}{1 - \lambda_h} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda_f}{1 - \lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{\lambda_h}{1 + \bar{t}(1 - \lambda_h) + t \lambda_h} dt. \quad (108)$$

Intuitively, the expressions for the changes in the domestic cost shares are similar for the two countries. They differ as a change in  $t$  directly affects country  $h$ 's domestic cost share, while country  $f$ 's domestic cost is only affected through general equilibrium adjustments in the country  $h$ 's domestic cost share and the relative wage. Given import (and thus domestic) cost shares, a change in  $t$  directly affects the relative wage. In order to back out the general equilibrium adjustment in the relative wage, we have to take into account the changes in the cost shares.

### 3.2.2 Deviation from equilibrium with unrestricted cooperative policy formation

We now explore whether country  $h$  has an incentive to deviate from the cooperative policy equilibrium. As countries are symmetric in the initial equilibrium, we can suppress country indices and write  $\lambda := \lambda_h = \lambda_f$ . Evaluated at the cooperative policy equilibrium  $\bar{t} = -1/\sigma$ , the policy-induced changes in the income multiplier and the domestic cost share in country  $h$  are given by

$$\begin{aligned} \left. \frac{d\zeta_h/\zeta_h}{dt} \right|_{t=-1/\sigma} &= \frac{\kappa(1 - \lambda)}{\zeta_h} > 0 \quad \text{and} \\ \left. \frac{d\lambda_h/\lambda_h}{dt} \right|_{t=-1/\sigma} &= \frac{(\sigma - 1)(1 - \lambda) \left( \frac{1 - \gamma}{\gamma} (1 - \lambda) + (\sigma - 1)\lambda + 1 - \lambda \right)}{2 \frac{1 - \gamma}{\gamma} (1 - \lambda) + 2(\sigma - 1)\lambda + 1} \frac{1}{1 + \bar{t}} > 0, \end{aligned}$$



where the first line follows from equation (105) and the second from solving the system of equations (106) to (108), which requires somewhat tedious but straightforward calculus. With  $t = \bar{t}$ , the income multiplier reduces to  $\zeta_h = 1 + \kappa\bar{t}$ . Collecting terms, evaluated at the cooperative policy equilibrium, the policy-induced change in country  $h$ 's welfare emerges as

$$\left. \frac{dW_h/W_h}{dt} \right|_{t=-1/\sigma} = \frac{\mu(1-\lambda)}{\gamma} \frac{\frac{1-\gamma}{\gamma}(1-\lambda) + (\sigma-1)\lambda + \lambda}{2\frac{1-\gamma}{\gamma}(1-\lambda) + 2(\sigma-1)\lambda + 1} > 0, \quad (109)$$

where the inequality follows from  $1 - \lambda < 1$ . Thus, lowering the import subsidy (an increase in  $t$ ) raises country  $h$ 's welfare. Given the underlying symmetry, this generalizes to the following proposition:

**Proposition 8** *Each country has an incentive to unilaterally deviate from a first-best cooperative policy by lowering its subsidy on imported varieties.*

**Proof.** Follows directly from equation (109). ■

The result reflects the standard terms-of-trade externality. Deviation from the first-best cooperative policy does not exhibit a first-order welfare effect from the input distortion, because that is perfectly corrected at the outset. It can be shown that the welfare change induced by an increase in  $t$  starting from  $t = \bar{t} = -1/\sigma$  is decreasing in  $\lambda$  (in trade costs  $\delta$ ) and in  $\gamma$ . The incentive to deviate from the first-best cooperative policy is larger when trade costs are low. This finding is in line with result that without input-output linkages, the optimal non-cooperative tariff is decreasing in real trade costs. In the presence of prohibitively high trade costs ( $\lambda \rightarrow 1$ ), the terms-of-trade effect does not materialize. The incentive to deviate is also larger when when input-output linkages are stronger. As pointed out above, this is not the result of the input *distortion*, because the distortion is corrected at the outset, but of the sheer presence of input-output linkages.

### 3.2.3 Deviation from free trade

We now explore the implications of the input distortion for the optimal conduct of trade policy. As standard in the optimal trade policy literature, we use free trade as the benchmark,

although free trade is not an optimal policy, given the input distortion. More specifically, we explore the non-cooperative trade policy incentive in a laissez faire situation. Formally, this means we set  $\bar{t} = 0$ . We consider the effects of marginal changes in  $t$  starting from a symmetric equilibrium with  $t = \bar{t} = 0$  and characterize the optimal trade policy in turn.

Intuitively, the policy multiplier simplifies to  $\zeta_h = 1$  at  $t = 0$ . Evaluated at the symmetric free trade equilibrium, the policy-induced changes in the income multiplier and the domestic cost share are:

$$\left. \frac{d\zeta_h/\zeta_h}{dt} \right|_{t=0} = \kappa(1 - \lambda) > 0 \text{ and} \quad (110)$$

$$\left. \frac{d\lambda_h/\lambda_h}{dt} \right|_{t=0} = (\sigma - 1)(1 - \lambda) \frac{\frac{1-\gamma}{\gamma}(1 - \lambda) + (\sigma - 1)\lambda + 1 - \lambda}{2\frac{1-\gamma}{\gamma}(1 - \lambda) + 2(\sigma - 1)\lambda + 1} > 0, \quad (111)$$

where the first line again follows from equation (105) and the second line follows from solving the system of equations (106) to (108). The policy-induced welfare change emerges as

$$\left. \frac{dW_h/W_h}{dt} \right|_{t=0} = (1 - \lambda) \left[ \kappa - \frac{1}{\gamma} \frac{\frac{1-\gamma}{\gamma}(1 - \lambda) + (\sigma - 1)\lambda + 1 - \lambda}{2\frac{1-\gamma}{\gamma}(1 - \lambda) + 2(\sigma - 1)\lambda + 1} \right]. \quad (112)$$

At a symmetric free trade equilibrium with zero trade costs ( $\delta = 1$ ), the domestic cost share is  $\lambda = 0.5$ , and the above expression reduces to

$$\left. \frac{dW_h/W_h}{dt} \right|_{t=0} = (1 - \lambda) \left( \kappa - \frac{1}{2\gamma} \right) = \frac{(1 - \lambda)(2\mu - 1)}{2\gamma[\mu - (1 - \gamma)]} \left( \gamma - \frac{\mu - 1}{2\mu - 1} \right). \quad (113)$$

In the presence of input-output linkages and the absence of trade costs, the introduction of an import tariff is welfare enhancing, if and only if the labor cost share is sufficiently large:  $\gamma > \tilde{\gamma} := \frac{\mu - 1}{2\mu - 1} = \frac{1 - \rho}{2 - \rho} = \frac{1}{\sigma + 1}$ . When the labor cost share is smaller than this threshold, the optimal policy is an import subsidy. With  $\gamma = \tilde{\gamma}$ , there is no incentive to deviate from the symmetric free trade equilibrium. Note that in the limiting case  $\sigma \rightarrow 1$ ,  $\tilde{\gamma} \rightarrow 0.5$ . Thus, in the empirically relevant cases with  $\gamma > 0.5$ , an import subsidy will never be optimal. Nevertheless, we can conclude that in the presence of an input distortion, the optimal import tariff will be lower than without. We will return to the characterization of the optimal trade policy below.

Evaluated at a symmetric free trade equilibrium, an increase in  $t$  improves county  $h$ 's the terms of trade:<sup>18</sup>

$$\frac{dx/x}{dt} = \frac{\frac{\lambda}{1-\lambda} + \frac{1-\gamma}{\gamma(\sigma-1)}}{\frac{1-\gamma}{\gamma}(1-\lambda) + 1 + (\sigma-1)\lambda} \frac{d\lambda_h/\lambda_h}{dt} + \frac{\lambda}{\frac{1-\gamma}{\gamma}(1-\lambda) + 1 + (\sigma-1)\lambda} > 0, \quad (114)$$

where the inequality follows from noting that  $d\lambda_h/dt > 0$ ; see equation (111). The terms-of-trade improvement constitutes a first-order welfare effect. Two effects potentially run counter to this terms-of-trade improvement. First, as in the standard model, a deviation of  $t$  from  $\bar{t}$  distorts relative imports according to

$$\frac{N_f q_{fh} p_{fh}}{N_h q_{hh} p_{hh}} = \left( \frac{1+t}{1+\bar{t}} \delta \right)^{1-\sigma}. \quad (115)$$

This consumption distortion vanishes if  $t = \bar{t}$ , in which case a marginal change in  $t$  does not cause a first-order welfare loss. Second, trade policy affects the input distortion. Relative material input is determined by the relative price of labor, which is equivalent to the real wage:

$$\frac{m_h}{\ell_h} = \frac{1-\gamma}{\gamma} \frac{w_h}{\bar{P}_h} \propto \lambda_h^{-\frac{1}{\gamma(\sigma-1)}}. \quad (116)$$

An increase in  $t$  raises  $\lambda_h$ , thus lowering relative material input and causing a first-order welfare loss. Clearly, this channel only materializes if  $\gamma < 1$ . Intuitively, our result implies that, absent trade costs, the input distortion dominates the standard terms-of-trade effect, if the labor cost share is sufficiently small, and vice versa.

Things change if there are real trade costs,  $\delta > 1$ . To explore this change, we start out observing that for  $\delta = 1$  we have  $\lambda = 0.5$ . Then, the ratio in equation (112) is equal to 0.5. Taking the derivative of this ratio with respect to  $\lambda$  and evaluating at  $\lambda = 0.5$ , we obtain

$$\left. \frac{\partial \frac{\frac{1-\gamma}{\gamma}(1-\lambda) + (\sigma-1)\lambda + 1 - \lambda}{2 \frac{1-\gamma}{\gamma}(1-\lambda) + 2(\sigma-1)\lambda + 1}}{\partial \lambda} \right|_{\lambda=1/2} = -\frac{1}{\frac{1-\gamma}{\gamma} + \sigma} < 0. \quad (117)$$

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<sup>18</sup>The result also holds in the presence of trade costs.

This implies that in the knife-edge case where costless trade renders free trade optimal, the introduction of trade costs creates an incentive to impose a tariff. The tariff incentive is decreasing in the elasticity of substitution  $\sigma$  and increasing in the labor cost share  $\gamma$ . The intuition is that in the presence of trade costs, the response in the domestic cost share  $\lambda_h$  to a marginal change in  $t$  is smaller than in their absence. The welfare cost of a tariff deriving from the consumption distortion thus weighs less heavily in the welfare calculus. This same logic further implies that the parameter restriction on  $\gamma$  for the optimal policy to call for an import subsidy becomes more binding. Put simply, with trade costs a small deviation  $\gamma < 1/(\sigma + 1)$  still calls for an import tariff.

We may summarize our results as follows:

**Proposition 9** *(a) Given a laissez faire equilibrium, each country faces (i) an incentive to impose a small tariff when  $\gamma > 1/(\sigma + 1)$ , and (ii) an incentive to introduce a small import subsidy when  $\gamma < 1/(\sigma + 1)$ . (iii) With  $\gamma = 1/(\sigma + 1)$ , no country faces an incentive to deviate from free trade. (b) In a neighborhood of  $\gamma$ -values around the knife-edge case of a zero deviation incentive from laissez faire without trade cost, the presence of trade cost generates a tariff incentive, but this discrepancy is falling in the elasticity of substitution  $\sigma$  and increasing in the labor cost share  $\gamma$ .*

**Proof.** Parts (a) and (b) follow from, respectively, equations (113) and (117). ■

Finally, we characterize the optimal non-cooperative trade policy  $t^*$ . The first-order condition of the non-cooperative welfare maximization problem is  $dW_h/dt = 0$ . Evaluating the changes in the income multiplier, the domestic cost shares, and the relative wage in equa-

tions (105) and (108) at  $\bar{t} = 0$ , this first-order condition emerges as

$$\begin{aligned}
0 &= \frac{1 - \lambda_h \frac{1-\gamma}{\mu}}{1 - \frac{1-\gamma}{\mu} + t \left(1 - \lambda_h \frac{1-\gamma}{\mu}\right)} - \frac{\lambda_h}{1 + t\lambda_h} \\
&- \left( \frac{t\lambda_h \frac{1-\gamma}{\mu}}{1 - \frac{1-\gamma}{\mu} + t \left(1 - \lambda_h \frac{1-\gamma}{\mu}\right)} + \frac{t\lambda_h}{1 + t\lambda_h} + \frac{1}{\gamma(\sigma - 1)} \right) \\
&\times \frac{(\sigma - 1)(1 - \lambda_h)}{1 + t} \frac{\frac{1-\gamma}{\gamma} (1 - \lambda_f) + (\sigma - 1)\lambda_f + \frac{1-\lambda_h}{1+t\lambda_h}}{\frac{1-\gamma}{\gamma} (2 - \lambda_h - \lambda_f) + 1 + (\sigma - 1) \left(\lambda_f + \frac{1+t}{1+t\lambda_h} \lambda_h\right)}, \quad (118)
\end{aligned}$$

where each country's domestic cost share depends on country  $h$ 's trade policy. The system of equilibrium conditions in equation (101) together with the first order condition in equation (118) characterize country  $h$ 's optimal trade policy. Since this system defies analytic solution, we use numerical methods to describe interesting features of the solution.

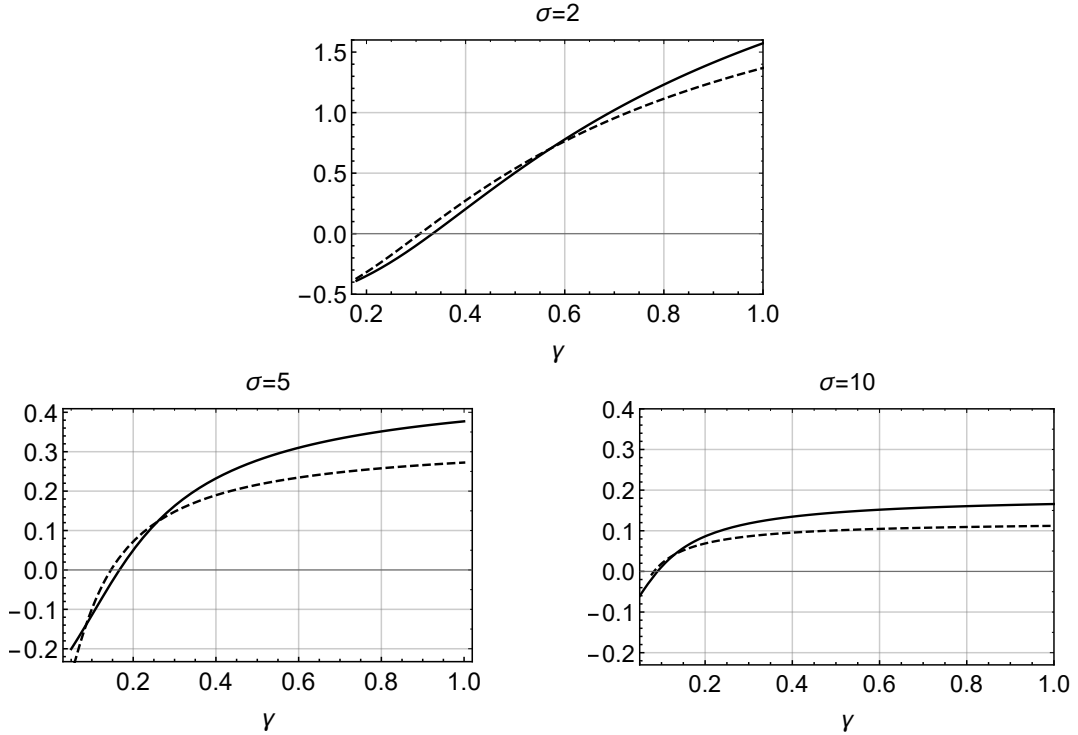
Figure 5 displays the optimal trade policy as function of  $\gamma$  for different levels of the elasticity of substitution (different panels) and trade costs. In line with Anderson and van Wincoop (2003), we set  $\sigma = 5$  and  $\sigma = 10$  (top panels). We also set  $\sigma = 2$  (lower panel) as Feenstra et al. (2018) document Armington elasticities lower than 2. With respect to real trade costs, we consider their absence ( $\delta = 1$ , solid line) and alternatively set  $\delta = 1.6$  (dashed line), in line with Bernard et al. (2007).

The critical labor cost shares amount to  $\tilde{\gamma} = 1/(\sigma + 1)$ , in line with part (a) of proposition 9. An increase in trade costs increases the range for which an import tariff is optimal, which generalizes part (b) of proposition 9 to discrete changes in trade costs.

In the absence of input-output linkages ( $\gamma = 1$ ), the optimal non-cooperative tariff is *decreasing* in real trade costs. This result is well-known from Gros (1987) and Felbermayr et al. (2013). Two observations stand out. First, for sufficiently weak input-output linkages, the optimal tariff also decreases in the strength of input-output linkages (measures as a reduction in the exogenous parameter  $\gamma$ ). The decline, however, is smaller when trade costs are larger.

Importantly, with input-output linkages, however, the optimal trade policy  $t$  is *increasing* in real trade costs over a parameter range  $(\underline{\gamma}, \bar{\gamma})$ . The existence of  $\bar{\gamma}$  is a consequence of the

Figure 5: Optimal non-cooperative trade policy



*Legend:* We use the system of equilibrium conditions in equation (101) and the first-order condition to the non-cooperative welfare maximization problem in equation (118) to numerically solve for the optimal trade policy  $t^*$  in the absence of domestic policies and a foreign country that pursues free trade. We set  $\sigma = 2$  (top diagram),  $\sigma = 5$  (bottom left diagram), and  $\sigma = 10$  (bottom right diagram), and real trade costs  $\delta = 1$  (solid line) and  $\delta = 1.6$  (dashed line).

observations that at  $\tilde{\gamma}$  where laissez-faire is optimal, the introduction of trade cost renders an import tariff optimal, while for  $\gamma = 1$ , the optimal tariff is lower with trade costs than in their absence. For small labor cost shares, the model is not well behaved (see also Caliendo et al., 2017), so it is not always possible to observe  $\underline{\gamma}$ ; see for example the diagram with  $\sigma = 10$ . Starting from  $\gamma = \tilde{\gamma}$ , a marginal increase in  $\gamma$  dampens the input distortion, calling for an increase in the optimal tariff. This increase should be larger, the lower the trade costs (which implies larger trade volumes). While for  $\sigma = 5$ , the critical labor cost share at which the comparative statics result with respect to trade costs reverses, is  $\bar{\gamma} \approx 0.4$ , for  $\sigma = 2$  the critical value is  $\bar{\gamma} \approx 0.58$ .

We may summarize the result as follows:

**Proposition 10** (a) *In a non-cooperative world with an input-output linkage, the optimal trade policy is characterized by the system of equations in (101) and (118). (b) The optimal tariff is lower than in a world without an input linkage. It turns into an import subsidy for a sufficiently small labor cost share. (c) For labor cost shares in an intermediate range, the optimal tariff is increasing and the optimal subsidy is decreasing in real trade costs.*

**Proof.** Part (a) follows from the text. For part (b), remember that the standard optimal tariff analysis weighs the terms of trade externality against the welfare-cost of the tariff generated by distorting the relative demand in favor of the imported intermediate. With the input distortion generated by the input-output linkage, the welfare cost of the tariff is aggravated by increasing the price of the aggregate good, which now has a first-order welfare cost on account of raising the price of the material input in variety production. Incorporating this effect when deriving the optimal tariff a setting clearly results in a lower optimal tariff. For part (c), recall that at  $\gamma = 1/(\sigma + 1)$  where laissez-faire is optimal, the introduction of trade costs renders an import tariff optimal, while at  $\gamma = 1$ , the optimal tariff is decreasing in real trade costs. This proves the existence of a critical  $\bar{\gamma} \in (\tilde{\gamma}, 1)$ . ■

As Caliendo et al. (2017), we find that the traditional positive optimal-tariff argument can be reversed, but for a different reason. They focus on a domestic *entry* distortion which is already present in the absence of input-output linkages, but magnified by their presence and by the presence of real trade costs. We carve out our argument in the absence of an entry distortion and real trade costs and purely focus on the *input* distortion generated an input-output linkage. We also demonstrate that in the presence of an input-output linkage, the optimal tariff (import subsidy) might increase (decrease) in real trade costs, a comparative static result that does not show up in Caliendo et al. (2017).

## 4 Summary and conclusions

Modern trade literature emphasizes that product differentiation is an important source of consumer welfare, while firms gain from the availability of differentiated intermediate inputs.

But product differentiation comes at the cost of market power and prices above marginal cost, which in turn is a source of welfare loss. There is a voluminous literature addressing the various distortions that the twin feature of market power and product differentiation may entail. In general, the distortions arising from monopolistic competition are well understood. Most of the literature focuses on environments of monopolistic competition. Oftentimes, it also assumes an input-output linkage, meaning that production of differentiated goods uses material inputs alongside primary inputs. In this paper we demonstrate that the combination of monopolistic competition and input-output linkages gives rise to a distortion that has so far mostly gone unnoticed. The reason is that markup pricing for goods means prices of material inputs lie above their opportunity cost, which distorts the mix of material and primary inputs. This causes a welfare loss over and above the potential loss deriving from distortions highlighted by the existing literature.

We develop a stylized model that zooms in on this input distortion by assuming away all other potential distortions deriving from monopolistic competition. In particular, we assume a single sector and we model monopolistic competition based on the CES-version of love of variety. Following existing literature, we model the input output linkage by means of a Cobb-Douglas production function for differentiated varieties, using material inputs as well as labor. This nests the simpler case without input-output linkage and allows us to explore what the presence and strength of input-output linkages means for welfare and total factor productivity. We first provide a full description of the social optimum for a closed economy featuring imperfect competition in the presence of such an input-output linkage. In doing so, we also analyze the optimal degree of roundaboutness in the use of labor through using output as an intermediate input rather than direct consumption. The social optimum of a closed economy presents a benchmark for our analysis of the production inefficiency caused by the input-output linkage in a decentralized market equilibrium. It also establishes a reference case against which we discuss the implications of trade in intermediate goods.

It is well known from the literature that the comparative statics of a monopolistic competition equilibrium much depends on whether or not the fixed cost relies on the same input bundle as the variable cost. In our baseline case we assume that the fixed cost arises in the



form of labor (the primary input), whereas the variable cost arises from both labor and material inputs. In an extension, we allow for the input-output linkage to be present also in the fixed cost. In the baseline case, markup pricing leaves the labor input as well as the number of firms undistorted, relative to the social optimum, whereas the material input is used in a less than optimal level which is also responsible for a suboptimally low degree of roundaboutness. Importantly, these deviations from the social optimum generate a sizable welfare loss. For plausible values of the key parameters, the welfare loss is in the vicinity of two to four percent, relative to the first best. The loss in total factor productivity is even larger, between 10 and 20 percent. In the alternative case where the input-output linkage extends to the fixed cost, we again find a distorted input mix, but this time it comes from a higher than optimal labor use, relative to material inputs which are now first-best. Interestingly, in this case we also find that there is a further channel through which the input distortion works out in the decentralized equilibrium, which is a lower than optimal number of firms. This, in turn, is responsible for a magnified welfare loss, which for plausible parameter values lies between 10 and 20 percent. In a further extension, we allow for Melitz-type heterogeneity among firms. It turns out that this does not alter our main results in any way.

What are suitable policies to address the input distortion? Intuitively, the first-best policy is to subsidize material input use in production of differentiated varieties. In principle, subsidizing production of differentiated varieties would also seem a suitable instrument to address the distortion, but one expects this to be a second-best policy since it does not directly target the distortion which lies with the input mix. We describe the decentralized market equilibrium simultaneously allowing for both of these policy instruments, and it turns out that they are perfect substitutes for each other. The reason is that in the integrated world equilibrium the output subsidy does not, in and of itself, involve any distortion. One would expect that a wage tax is similarly able to address the input distortion. However, we demonstrate that this is partial equilibrium intuition and that in general equilibrium the input-output linkage implies the wage tax is fully passed on to the price of material input. Therefore, it is unable to influence the input mix chosen by producers. Indeed, it turns out that in our stylized model a wage tax simply doesn't constitute a distortion and cannot, therefore, serve in offsetting the

input distortion. The same applies for deviations from the benchmark model in the form of wage setting environments, instead of a perfectly competitive labor market. An important caveat here is that these conclusions would be altered in a model allowing for many sectors and/or endogenous labor supply.

The most interesting implications of the input distortion arise in a trading environment where countries have completely segmented labor markets and where trade is subject to trade barriers. We look at the simple case of two symmetric countries that may trade in intermediate inputs. We readdress the question of optimal cooperative policies in two different settings. The first is a setting where domestic subsidies and trade policies are available. In this setting, it is optimal to subsidize domestic and imported varieties in the same way in order to avoid a distortion of relative import demand. The second is one where only trade policy instruments are at the disposal of the policy makers. Absent any input-output linkage, in a cooperative setting there would be no case for any trade policy intervention. With input-output linkages, absent the above mentioned first-best policies, there is a case for a second-best use of trade policy, which in this case is an import subsidy. This finding has important implications for optimal non-cooperative trade policy. The input distortion runs counter to the standard terms-of-trade considerations, thereby calling for an import tariff that is smaller in the standard setting, or for an import subsidy, even in the absence of multiple sectors.

Summing up, the general thrust of our paper is that the presence of input-output linkages in an environment of monopolistic competition establishes an economic rationale for subsidizing producers relying on material inputs. Ideally the subsidy would target the input distortion directly, but under plausible assumptions the distortion may be also addressed by means of a production subsidy. In a framework that allows for trade policy interventions, the thrust of our analysis is that the input distortion counteracts the terms-of-trade argument for an import tariff and may even call for an import subsidy.

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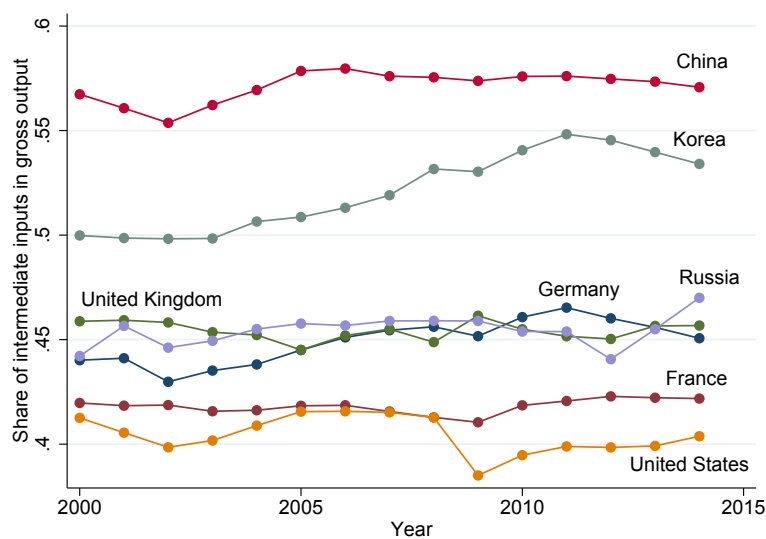
## A Empirical importance of input-output linkages

We use national input-output tables for Germany, France, the UK, the US, China, and South Korea from the World input-output Database (WIOD) over the time period to compute share of intermediate inputs in gross output. Let  $i$  denote a sector. For each year, country-level shares are computed as the value-added share weighted average of sectoral intermediate input shares:

$$\sum_i \frac{\text{value added}_i}{\sum_k \text{value added}_k} \frac{\text{gross output}_i - \text{value added}_i}{\text{gross output}_i}$$

Figure 6 shows that the shares are substantial. At the country level, they range between around 40 percent for the US, and a bit less than 60 percent for China. Moreover, there is not much variation over time, although the period includes the year 2008, which may explain the drop for the US at that time.

Figure 6: Share of intermediate inputs in gross output



Source: World Input Database (WIOD) - National Input-Output Tables

## B Detailed derivations

In this appendix, we show detailed derivations of the equations presented in section 4.2.

**Domestic cost share and real wage.** The price of a variety produced in country  $i$  and used in country  $j$  inclusive of trade costs and the policy wedge emerges as  $\tilde{p}_{ij} = \delta_{ij} (1 + t_{ij}) p_i$ . The price of the final good is  $\tilde{P}_i = \left( \sum_{j \in \{h, f\}} N_j [\delta_{ji} (1 + t_{ji}) p_j]^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ . Unit demand in country  $i$  for a variety from  $j$  reads as  $q_{ji} = \left( \frac{\delta_{ji} (1 + t_{ji}) p_j}{\tilde{P}_i} \right)^{-\sigma}$ . The share of country  $j$ 's varieties in the costs of country  $i$ 's assembly of the final is

$$\lambda_{ji} := \frac{N_j q_{ji} \tilde{p}_{ji}}{\sum_k N_k q_{ki} \tilde{p}_{ki}} = \frac{N_j (\delta_{ji} (1 + t_{ji}) p_j)^{1-\sigma}}{\sum_k N_k (\delta_{ki} (1 + t_{ki}) p_k)^{1-\sigma}} = \left( \frac{N_j^{\frac{1}{1-\sigma}} \delta_{ji} (1 + t_{ji}) p_j}{\tilde{P}_i} \right)^{1-\sigma},$$

where profit maximization implies that  $p_j = \mu x_j$  and cost-minimizing production that  $x_j = w_j^\gamma \left( \tilde{P}_j \right)^{1-\gamma}$ . The domestic cost share (with  $\delta_{ii} = 1$ ) emerges as

$$\begin{aligned} \lambda_{ii} &= \left( \frac{N_i^{\frac{1}{1-\sigma}} \delta_{ii} (1 + t_{ii}) p_i}{\tilde{P}_i} \right)^{1-\sigma} = N_i (1 + t_{ii})^{1-\sigma} \left( \frac{\mu x_i}{\tilde{P}_i} \right)^{1-\sigma} \\ &= N_i (1 + t_{ii})^{1-\sigma} \left( \frac{\mu w_i^\gamma \left( \tilde{P}_i \right)^{1-\gamma}}{\tilde{P}_i} \right)^{1-\sigma} = N_i (1 + t_{ii})^{1-\sigma} \left( \mu \left( \frac{w_i}{\tilde{P}_i} \right)^\gamma \right)^{1-\sigma}, \end{aligned}$$

which implies that the real wage (or: the relative price of labor), is given by

$$\frac{w_i}{\tilde{P}_i} = (\bar{\chi}_i \lambda_{ii})^{-\frac{1}{\gamma(\sigma-1)}}, \text{ where } \bar{\chi}_i := N_i^{-1} [\mu (1 + t_{ii})]^\sigma.$$

**Relative marginal costs.** Relative marginal costs read as

$$x := \frac{x_h}{x_f} = \frac{w_h^\gamma \tilde{P}_h^{1-\gamma}}{w_f^\gamma \tilde{P}_f^{1-\gamma}} = \frac{w_h}{w_f} \left( \frac{\tilde{P}_h/w_h}{\tilde{P}_f/w_f} \right)^{1-\gamma} = \omega \left( \frac{\bar{\chi}_h \lambda_h}{\bar{\chi}_f \lambda_f} \right)^{\frac{1-\gamma}{\gamma(\sigma-1)}},$$

where  $\omega := w_h/w_f$  denotes the relative wage. As policies do not affect the number of domestic firms and countries are supposed to be symmetric in all dimensions but trade policy,  $\bar{\chi}_h/\bar{\chi}_f =$



1, and the expression simplifies to the one in the main text.

**Value of aggregate domestic demand.** We now establish a link between wage income  $w_i L_i$  and the value of aggregate demand. Only differentiated good producers employ workers. The total revenue of differentiated good producers amounts to  $Z_i := N_i p_i q_i$ . It is well known that under monopolistic competition with zero profits, firms spend a constant share  $1/\sigma$  of their revenue on the fixed input and the remaining share  $1 - 1/\sigma = 1/\mu$  on the variable input into the production, which requires labor and material input. The Cobb-Douglas technology assumption implies that the cost share of material input amounts to  $1 - \gamma$ . Then, total payments to workers emerges as

$$Z_i - \frac{1-\gamma}{\mu} Z_i = w_i L_i \Leftrightarrow Z_i = \kappa w_i L_i, \text{ where } \kappa := \left(1 - \frac{1-\gamma}{\mu}\right)^{-1}.$$

The net income generated by taxing and/or subsidizing the use of domestic and imported varieties reads as

$$T_i = \sum_{j \in \{h, f\}} t_{ji} N_j \delta_{ji} q_{ji} p_i = \tilde{P}_i Q_i \sum_{j \in \{h, f\}} \frac{t_{ji} \lambda_{ji}}{1 + t_{ji}},$$

where we have used optimal demand and employed the definition of the cost shares. Net income is redistributed to households in a lump-sum fashion.

The final good is used by households and by differentiated good producers. Households spend all their labor income  $w_i L_i$  net of the lump-sum transfer  $T_i$  on the final good. As argued above, differentiated good producers spend a share  $(1 - \gamma) / \mu$  of their revenues on the final good. Then, the value of aggregate demand is given

$$\begin{aligned} \tilde{P}_i Q_i &= w_i L_i + T_i + \frac{1-\gamma}{\mu} Z_i = w_i L_i + \tilde{P}_i Q_i \sum_{j \in \{h, f\}} \frac{t_{ji} \lambda_{ji}}{1 + t_{ji}} + \frac{1-\gamma}{\mu} \kappa w_i L_i \Leftrightarrow \\ \tilde{P}_i Q_i &= \frac{\kappa w_i L_i}{1 - \sum_{j \in \{h, f\}} \frac{t_{ji} \lambda_{ji}}{1 + t_{ji}}}. \end{aligned}$$

**Balanced trade.** Balanced trade requires that  $N_f \delta_{fh} q_{fh} p_f = N_h \delta_{hf} q_{hf} p_h$ , where the value of trade flows is inclusive of trade costs and exclusive of the trade policy wedge. Using optimal

demand and the pricing rule, we obtain

$$\begin{aligned} N_f \delta_{fh} \left( \frac{\delta_{fh} (1 + t_{fh}) p_f}{\tilde{P}_h} \right)^{-\sigma} p_f Q_h &= N_h \delta_{hf} \left( \frac{\delta_{hf} (1 + t_{hf}) p_h}{\tilde{P}_f} \right)^{-\sigma} p_h Q_f \Leftrightarrow \\ N_f \left( \frac{\delta_{fh} (1 + t_{fh}) p_f}{\tilde{P}_h} \right)^{1-\sigma} \frac{\tilde{P}_h Q_h}{1 + t_{fh}} &= N_h \left( \frac{\delta_{hf} (1 + t_{hf}) p_h}{\tilde{P}_f} \right)^{1-\sigma} \frac{\tilde{P}_f Q_f}{1 + t_{hf}}. \end{aligned}$$

Employing the definition of the cost share, balanced trade emerges as

$$\frac{\lambda_{fh}}{1 + t_{fh}} \tilde{P}_h Q_h = \frac{\lambda_{hf}}{1 + t_{hf}} \tilde{P}_f Q_f.$$

Using the expressions for the values of aggregate demand, balanced trade can be rewritten as

$$\frac{\lambda_{fh}}{1 + t_{fh}} \frac{w_h L_h}{1 - \sum_{j \in \{h, f\}} \frac{t_{jh} \lambda_{jh}}{1 + t_{jh}}} = \frac{\lambda_{hf}}{1 + t_{hf}} \frac{w_f L_f}{1 - \sum_{j \in \{h, f\}} \frac{t_{jf} \lambda_{jf}}{1 + t_{jf}}}.$$

**Policy multiplier.** We define the policy multiplier as

$$\zeta_i := \frac{w_i L_i + T_i}{w_i L_i} = \frac{w_i L_i + \tilde{P}_i Q_i \sum_{j \in \{h, f\}} \frac{t_{ji} \lambda_{ji}}{1 + t_{ji}}}{w_i L_i} = \frac{w_i L_i + \frac{\kappa \sum_{j \in \{h, f\}} \frac{t_{ji} \lambda_{ji}}{1 + t_{ji}} w_i L_i}{1 - \sum_{j \in \{h, f\}} \frac{t_{ji} \lambda_{ji}}{1 + t_{ji}}}}{w_i L_i} = 1 + \kappa \frac{\sum_{j \in \{h, f\}} \frac{t_{ji} \lambda_{ji}}{1 + t_{ji}}}{1 - \sum_{j \in \{h, f\}} \frac{t_{ji} \lambda_{ji}}{1 + t_{ji}}}.$$

**System of equilibrium conditions and welfare.** We define  $\bar{t} := t_{hh} = t_{ff} = t_{hf}$  and  $t := t_{fh}$ .

Then, country  $h$ 's domestic cost share emerges as

$$\lambda_h = \frac{N_h [(1 + \bar{t}) x_h]^{1-\sigma}}{N_h [(1 + \bar{t}) x_h]^{1-\sigma} + N_f [\delta (1 + t) x_f]^{1-\sigma}} = \frac{1}{1 + \left( \delta \frac{1+t}{1+\bar{t}} x^{-1} \right)^{1-\sigma}},$$

where the markup  $\mu$  and the relative number of firms  $N_h/N_f$  disappeared due to the symmetry assumption. By analogy, we have

$$\lambda_f = \frac{1}{1 + (\delta x)^{1-\sigma}},$$

where the relative policy wedges disappeared to the uniformity assumption.

Using again the definitions of  $\bar{t}$  and  $t$ , balanced trade and noting that cost share add to unity ( $\lambda_{ii} + \lambda_{ji} = 1$ ), balanced trade can be rewritten as

$$\begin{aligned} \frac{1 - \lambda_h}{1 + t} \frac{w_h}{1 - \left[ \frac{\bar{t}}{1 + \bar{t}} \lambda_h + \frac{t}{1 + t} (1 - \lambda_h) \right]} &= \frac{1 - \lambda_f}{1 + \bar{t}} \frac{w_f}{1 - \left[ \frac{\bar{t}}{1 + \bar{t}} \lambda_f + \frac{\bar{t}}{1 + \bar{t}} (1 - \lambda_f) \right]} \Leftrightarrow \\ \frac{1 - \lambda_h}{1 + t} \frac{w_h}{1 - \left[ \frac{\bar{t}}{1 + \bar{t}} \lambda_h + \frac{t(1 - \lambda_h)}{1 + t} \right]} &= \frac{1 - \lambda_f}{1 + \bar{t}} \frac{w_f}{1 - \frac{\bar{t}}{1 + \bar{t}}} \Leftrightarrow \\ \frac{(1 - \lambda_h) w_h}{1 + t - \bar{t} \frac{1 + t}{1 + \bar{t}} \lambda_h - t(1 - \lambda_h)} &= \frac{(1 - \lambda_f) w_f}{1 + \bar{t} - \bar{t}}, \quad \text{which implies} \end{aligned}$$

$$\omega = \frac{1 - \lambda_f}{1 - \lambda_h} \left( 1 - \bar{t} \frac{1 + t}{1 + \bar{t}} \lambda_h + t \lambda_h \right) = \frac{1 - \lambda_f}{1 - \lambda_h} \left( 1 + \frac{t + \bar{t}t - \bar{t} - \bar{t}t}{1 + \bar{t}} \lambda_h \right) = \frac{1 - \lambda_f}{1 - \lambda_h} \left( 1 + \frac{t - \bar{t}}{1 + \bar{t}} \lambda_h \right).$$

With our policy configuration, the income multiplier  $\zeta_h$  is determined by

$$\begin{aligned} \frac{\zeta_h - 1}{\kappa} &= \frac{\frac{\bar{t}}{1 + \bar{t}} \lambda_h + \frac{t}{1 + t} (1 - \lambda_h)}{1 - \frac{\bar{t}}{1 + \bar{t}} \lambda_h - \frac{t}{1 + t} (1 - \lambda_h)} = \frac{\bar{t}(1 + t) \lambda_h + t(1 + \bar{t})(1 - \lambda_h)}{(1 + t)(1 + \bar{t}) - \bar{t}(1 + t) \lambda_h - t(1 + \bar{t})(1 - \lambda_h)} \\ &= \frac{\bar{t}(1 + t) \lambda_h + t(1 + \bar{t})(1 - \lambda_h)}{(1 + t)(1 + \bar{t}) - \bar{t}(1 + t) \lambda_h - t(1 + \bar{t}) \lambda_h} = \frac{\bar{t}(1 + t) \lambda_h + t(1 + \bar{t})(1 - \lambda_h)}{1 + \bar{t} + [t + \bar{t}t - \bar{t} - \bar{t}t] \lambda_h} \\ &= \frac{\bar{t}(1 + t) \lambda_h + t(1 + \bar{t}) \lambda_h}{1 + \bar{t} + (t - \bar{t}) \lambda_h} = \frac{[\bar{t} + \bar{t}t - t - \bar{t}t] \lambda_h + t(1 + \bar{t})}{1 + \bar{t} + (t - \bar{t}) \lambda_h} \\ &= \frac{(\bar{t} - t) \lambda_h + t(1 + \bar{t})}{1 + \bar{t} + (t - \bar{t}) \lambda_h} = \frac{\bar{t} \lambda_h + t(1 - \lambda_h + \bar{t})}{1 + \bar{t}(1 - \lambda_h) + t \lambda_h} \end{aligned}$$

Then, real income emerges as

$$W_h = \zeta_h \times (\bar{x}_h \lambda_h)^{-\frac{1}{\gamma(\sigma-1)}}, \quad \text{where } \zeta_h = 1 + \kappa \frac{\bar{t} \lambda_h + t(1 - \lambda_h + \bar{t})}{1 + \bar{t}(1 - \lambda_h) + t \lambda_h}.$$

**Equations in differentiated form.** In differentiated form, the domestic cost shares read as

$$\begin{aligned} \frac{d\lambda_h}{\lambda_h} &= -(1 - \lambda_h) \left[ \frac{1 - \gamma}{\gamma} \left( \frac{d\lambda_h}{\lambda_h} - \frac{d\lambda_f}{\lambda_f} \right) + (\sigma - 1) \frac{d\omega}{\omega} - \frac{\sigma - 1}{1 + t} dt \right], \\ \frac{d\lambda_f}{\lambda_f} &= -(1 - \lambda_f) \left[ \frac{1 - \gamma}{\gamma} \left( \frac{d\lambda_f}{\lambda_f} - \frac{d\lambda_h}{\lambda_h} \right) - (\sigma - 1) \frac{d\omega}{\omega} \right]. \end{aligned}$$

The balanced condition emerges as

$$\begin{aligned}
\frac{d\omega}{\omega} &= \frac{\lambda_h}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{\frac{t-\bar{t}}{1+\bar{t}}\lambda_h}{1+\frac{t-\bar{t}}{1+\bar{t}}\lambda_h} \frac{d\lambda_h}{\lambda_h} + \frac{\frac{1}{1+\bar{t}}\lambda_h}{1+\frac{t-\bar{t}}{1+\bar{t}}\lambda_h} dt \\
&= \left(1 + \frac{\frac{t-\bar{t}}{1+\bar{t}}(1-\lambda_h)}{1+\frac{t-\bar{t}}{1+\bar{t}}\lambda_h}\right) \frac{\lambda_h}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{\frac{1}{1+\bar{t}}\lambda_h}{1+\frac{t-\bar{t}}{1+\bar{t}}\lambda_h} dt \\
&= \frac{1+\frac{t-\bar{t}}{1+\bar{t}}\lambda_h + \frac{t-\bar{t}}{1+\bar{t}}(1-\lambda_h)}{1+\frac{t-\bar{t}}{1+\bar{t}}\lambda_h} \frac{\lambda_h}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{\frac{1}{1+\bar{t}}\lambda_h}{1+\frac{t-\bar{t}}{1+\bar{t}}\lambda_h} dt \\
&= \frac{1+\frac{t-\bar{t}}{1+\bar{t}}}{1+\frac{t-\bar{t}}{1+\bar{t}}\lambda_h} \frac{\lambda_h}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{\frac{1}{1+\bar{t}}\lambda_h}{1+\frac{t-\bar{t}}{1+\bar{t}}\lambda_h} dt \\
&= \frac{1+\bar{t}+t-\bar{t}}{1+\bar{t}+(t-\bar{t})\lambda_h} \frac{\lambda_h}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{\lambda_h}{1+\bar{t}+(t-\bar{t})\lambda_h} dt \\
&= \frac{1+t}{1+\bar{t}(1-\lambda_h)+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{\lambda_h}{1+\bar{t}(1-\lambda_h)+t\lambda_h} dt.
\end{aligned}$$

The income multiplier reads as

$$\begin{aligned}
\frac{d\zeta_h}{dt} &= \kappa \frac{(1+\bar{t}(1-\lambda_h)+t\lambda_h) \left[ \bar{t} \frac{d\lambda_h}{dt} + (1-\lambda_h+\bar{t}) - t \frac{d\lambda_h}{dt} \right] - (\bar{t}\lambda_h + t(1-\lambda_h+\bar{t})) \left[ \lambda_h + (t-\bar{t}) \frac{d\lambda_h}{dt} \right]}{(1+\bar{t}(1-\lambda_h)+t\lambda_h)^2} \\
&= \kappa \frac{(1+\bar{t}+(t-\bar{t})\lambda_h) \left[ (\bar{t}-t) \frac{d\lambda_h}{dt} + (1-\lambda_h+\bar{t}) \right] - ((\bar{t}-t)\lambda_h + t(1+\bar{t})) \left[ \lambda_h + (t-\bar{t}) \frac{d\lambda_h}{dt} \right]}{(1+\bar{t}(1-\lambda_h)+t\lambda_h)^2}.
\end{aligned}$$

**Deviations from symmetric equilibrium with  $t = \bar{t}$ .** Evaluated at a symmetric initial equilibrium with  $t = \bar{t}$ , we have

$$\begin{aligned}
\frac{d\lambda_h}{\lambda_h} &= -(1-\lambda) \left[ \frac{1-\gamma}{\gamma} \left( \frac{d\lambda_h}{\lambda_h} - \frac{d\lambda_f}{\lambda_f} \right) + (\sigma-1) \frac{d\omega}{\omega} - \frac{\sigma-1}{1+\bar{t}} dt \right], \\
\frac{d\lambda_f}{\lambda_f} &= -(1-\lambda) \left[ \frac{1-\gamma}{\gamma} \left( \frac{d\lambda_f}{\lambda_f} - \frac{d\lambda_h}{\lambda_h} \right) - (\sigma-1) \frac{d\omega}{\omega} \right], \\
\frac{d\omega}{\omega} &= \frac{\lambda}{1-\lambda} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda}{1-\lambda} \frac{d\lambda_f}{\lambda_f} + \frac{\lambda}{1+\bar{t}} dt, \text{ and} \\
\frac{d\zeta_h}{dt} &= \kappa(1-\lambda),
\end{aligned}$$

where  $\lambda := \lambda_h = \lambda_f$  denotes the domestic cost share in the symmetric initial equilibrium.

Using the balanced trade condition in country  $f$ 's domestic cost share, we obtain

$$\begin{aligned} -\frac{1}{1-\lambda} \frac{d\lambda_f}{\lambda_f} &= \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda} \right) \left( \frac{d\lambda_f}{\lambda_f} - \frac{d\lambda_h}{\lambda_h} \right) - \frac{(\sigma-1)\lambda}{1+\bar{t}} dt \Leftrightarrow \\ -\left( \frac{1}{1-\lambda} + \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda} \right) \frac{d\lambda_f}{\lambda_f} &= -\frac{1-\gamma}{\gamma} \frac{d\lambda_h}{\lambda_h} - \frac{(\sigma-1)\lambda}{1-\lambda} \frac{d\lambda_h}{\lambda_h} - \frac{(\sigma-1)\lambda}{1+\bar{t}} dt \Leftrightarrow \\ \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda}{1-\lambda} \right) \frac{d\lambda_f}{\lambda_f} &= \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda} \right) \frac{d\lambda_h}{\lambda_h} + \frac{(\sigma-1)\lambda}{1+\bar{t}} dt. \end{aligned}$$

Similarly, using the balanced trade condition in country  $h$ 's domestic cost share, we obtain

$$\begin{aligned} -\frac{1}{1-\lambda} \frac{d\lambda_h}{\lambda_h} &= \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda} \right) \left( \frac{d\lambda_h}{\lambda_h} - \frac{d\lambda_f}{\lambda_f} \right) + \frac{(\sigma-1)\lambda}{1+\bar{t}} dt - \frac{\sigma-1}{1+\bar{t}} dt \\ \left( \frac{1}{1-\lambda} + \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda} \right) \frac{d\lambda_h}{\lambda_h} &= \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda} \right) \frac{d\lambda_f}{\lambda_f} + \frac{(\sigma-1)(1-\lambda)}{1+\bar{t}} dt. \end{aligned}$$

Combining two previous expressions, we find

$$\begin{aligned} \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda}{1-\lambda} \right) \frac{d\lambda_h}{\lambda_h} &= \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda} \right) \frac{\left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda} \right) \frac{d\lambda_h}{\lambda_h} + \frac{(\sigma-1)\lambda}{1+\bar{t}} dt}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda}{1-\lambda}} \\ &\quad + \frac{(\sigma-1)(1-\lambda)}{1+\bar{t}} dt \\ &= \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda} \right) \frac{\frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda}}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda}{1-\lambda}} \frac{d\lambda_h}{\lambda_h} \\ &\quad + \left( \frac{\frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda}{1-\lambda}}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda}{1-\lambda}} \lambda + 1 - \lambda \right) \frac{(\sigma-1) dt}{1+\bar{t}}. \end{aligned}$$

Collincting terms, the change in country  $h$ 's domestic cost share induced by change in  $t$  emerges as

$$\frac{d\lambda_h/\lambda_h}{dt} = \frac{(\sigma-1)(1-\lambda) \left( \frac{1-\gamma}{\gamma} (1-\lambda) + (\sigma-1)\lambda + 1 - \lambda \right)}{2 \frac{1-\gamma}{\gamma} (1-\lambda) + 2(\sigma-1)\lambda + 1} \frac{1}{1+\bar{t}} > 0.$$

At a symmetric equilibrium with  $t = \bar{t}$ , the domestic multiplier reads as

$$\zeta_h = 1 + \kappa \frac{\bar{t}\lambda + \bar{t}(1 - \lambda + \bar{t})}{1 + \bar{t}(1 - \lambda) + \bar{t}\lambda} = 1 + \kappa \bar{t} \frac{1 + \bar{t}}{1 + \bar{t}} = 1 + \kappa \bar{t}.$$

Then, at a symmetric equilibrium with  $t = \bar{t}$ , the change in welfare induced by a change in  $t$  is given by

$$\begin{aligned} \frac{dW_h/W_h}{dt} &= \frac{\kappa(1 - \lambda)}{1 + \kappa \bar{t}} - \frac{1(1 - \lambda) \left( \frac{1-\gamma}{\gamma} (1 - \lambda) + (\sigma - 1)\lambda + 1 - \lambda \right)}{\gamma \left( 2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2(\sigma - 1)\lambda + 1 \right)} \frac{1}{1 + \bar{t}} \\ &= \frac{1 - \lambda}{\gamma(1 + \bar{t})} \left( \frac{\gamma(\kappa + \kappa \bar{t})}{1 + \kappa \bar{t}} - \frac{\frac{1-\gamma}{\gamma} (1 - \lambda) + (\sigma - 1)\lambda + 1 - \lambda}{2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2(\sigma - 1)\lambda + 1} \right). \end{aligned}$$

Evaluated at an initial symmetric with unrestricted cooperative policy formation,  $\bar{t} = -1/\sigma$ , we obtain

$$\frac{\gamma/\mu}{1 - \lambda} \frac{dW_h/W_h}{dt} = \frac{\gamma \kappa \frac{\sigma-1}{\sigma}}{1 - \frac{\kappa}{\sigma}} - \frac{\frac{1-\gamma}{\gamma} (1 - \lambda) + (\sigma - 1)\lambda + 1 - \lambda}{2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2(\sigma - 1)\lambda + 1},$$

where

$$\frac{\gamma \kappa \frac{\sigma-1}{\sigma}}{1 - \frac{\kappa}{\sigma}} = \frac{\gamma \frac{\mu}{\mu - (1-\gamma)} \frac{1}{\mu}}{1 - \frac{\kappa}{\sigma}} = \frac{\gamma \frac{1}{\mu - (1-\gamma)}}{1 - \frac{1}{\sigma} \frac{\mu}{\mu - (1-\gamma)}} = \frac{\gamma}{\mu - 1 + \gamma - \frac{\mu}{\sigma}} = \frac{\gamma}{\mu \left( 1 - \frac{1}{\sigma} \right) - 1 + \gamma} = \frac{\gamma}{1 - 1 + \gamma} = 1.$$

Thus,

$$\begin{aligned} \frac{\gamma/\mu}{1 - \lambda} \frac{dW_h/W_h}{dt} &= 1 - \frac{\frac{1-\gamma}{\gamma} (1 - \lambda) + (\sigma - 1)\lambda + 1 - \lambda}{2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2(\sigma - 1)\lambda + 1} \\ &= \frac{2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2(\sigma - 1)\lambda + 1 - \frac{1-\gamma}{\gamma} (1 - \lambda) - (\sigma - 1)\lambda - (1 - \lambda)}{2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2(\sigma - 1)\lambda + 1} \\ &= \frac{\frac{1-\gamma}{\gamma} (1 - \lambda) + (\sigma - 1)\lambda + 1 - (1 - \lambda)}{2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2(\sigma - 1)\lambda + 1} > 0. \end{aligned}$$

Evaluated at an initial symmetric free trade equilibrium,  $\bar{t} = 0$ , we have  $\zeta_h = 1$ . The

change in welfare is given by

$$\frac{dW/W}{dt} = (1 - \lambda) \left( \kappa - \frac{1}{\gamma} \underbrace{\frac{\frac{1-\gamma}{\gamma} (1 - \lambda) + (\sigma - 1) \lambda + 1 - \lambda}{2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2 (\sigma - 1) \lambda + 1}}_{:=\Psi} \right),$$

where in the absence of real cost cost,  $\lambda = 0.5$ , and  $\frac{1}{1-\lambda} \frac{dW/W}{dt} = \kappa - \frac{1}{2\gamma}$ . Taking the derivative of the ratio  $\Psi$  with respect to  $\lambda$ , we obtain

$$\begin{aligned} \frac{\partial \Psi}{\partial \lambda} &= \frac{\left(-\frac{1-\gamma}{\gamma} + \sigma - 1 - 1\right) \left(2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2 (\sigma - 1) \lambda + 1\right)}{\left(2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2 (\sigma - 1) \lambda + 1\right)^2} \\ &\quad - \frac{2 \left(\frac{1-\gamma}{\gamma} (1 - \lambda) + (\sigma - 1) \lambda + 1 - \lambda\right) \left(-\frac{1-\gamma}{\gamma} + \sigma - 1\right)}{\left(2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2 (\sigma - 1) \lambda + 1\right)^2} \\ &= \frac{\left(-\frac{1-\gamma}{\gamma} + \sigma - 1 - 1\right) \left(2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2 (\sigma - 1) \lambda + 1\right)}{\left(2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2 (\sigma - 1) \lambda + 1\right)^2} \\ &\quad - \frac{\left(2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2 (\sigma - 1) \lambda + 2 (1 - \lambda)\right) \left(-\frac{1-\gamma}{\gamma} + (\sigma - 1)\right)}{\left(2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2 (\sigma - 1) \lambda + 1\right)^2} \\ &= \frac{\left(-\frac{1-\gamma}{\gamma} + \sigma - 1\right) (1 - 2(1 - \lambda)) - 2 \left(\frac{1-\gamma}{\gamma} (1 - \lambda) + (\sigma - 1) \lambda + \frac{1}{2}\right)}{\left(2 \frac{1-\gamma}{\gamma} (1 - \lambda) + 2 (\sigma - 1) \lambda + 1\right)^2}, \end{aligned}$$

Evaluated at  $\lambda = 1/2$ , the term is

$$\left. \frac{\partial \Psi}{\partial \lambda} \right|_{\lambda=1/2} < 0,$$

which implies that in the presence of trade costs (which cause the increase in  $\lambda$ ), for  $\gamma = 1/(\sigma + 1)$  an import tariff is welfare improving.

**Optimal non-cooperative trade policy.** Consider again the system of differentiated equilibrium conditions and the differentiated income multiplier. We now set  $\bar{t} = 0$  right from the

start, but allow for a flexible  $t$ . Then, balanced trade condition emerges as

$$\frac{d\omega}{\omega} = \frac{1+t}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{\lambda_h}{1+t\lambda_h} dt.$$

Using this balanced trade condition in country  $f$ 's domestic cost share, we obtain

$$\begin{aligned} -\frac{1}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} &= \frac{1-\gamma}{\gamma} \left( \frac{d\lambda_f}{\lambda_f} - \frac{d\lambda_h}{\lambda_h} \right) - (\sigma-1) \left( \frac{1+t}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{\lambda_h}{1+t\lambda_h} dt \right) \\ &= \frac{1-\gamma}{\gamma} \frac{d\lambda_f}{\lambda_f} - \frac{1-\gamma}{\gamma} \frac{d\lambda_h}{\lambda_h} - \frac{(\sigma-1)(1+t)}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} - \frac{(\sigma-1)\lambda_h}{1+t\lambda_h} dt \\ &= \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f} \right) \frac{d\lambda_f}{\lambda_f} - \left( \frac{(\sigma-1)(1+t)}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} + \frac{1-\gamma}{\gamma} \right) \frac{d\lambda_h}{\lambda_h} - \frac{(\sigma-1)\lambda_h}{1+t\lambda_h} dt. \end{aligned}$$

Hence,

$$\begin{aligned} \left( \frac{1}{1-\lambda_f} + \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f} \right) \frac{d\lambda_f}{\lambda_f} &= \left( \frac{(\sigma-1)(1+t)}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} + \frac{1-\gamma}{\gamma} \right) \frac{d\lambda_h}{\lambda_h} + \frac{(\sigma-1)\lambda_h}{1+t\lambda_h} dt \Leftrightarrow \\ \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} \right) \frac{d\lambda_f}{\lambda_f} &= \left( \frac{(\sigma-1)(1+t)}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} + \frac{1-\gamma}{\gamma} \right) \frac{d\lambda_h}{\lambda_h} + \frac{(\sigma-1)\lambda_h}{1+t\lambda_h} dt. \end{aligned}$$

Similarly, using this balanced trade condition in country  $h$ 's domestic cost share, we obtain

$$-\frac{1}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} = \frac{1-\gamma}{\gamma} \left( \frac{d\lambda_h}{\lambda_h} - \frac{d\lambda_f}{\lambda_f} \right) + (\sigma-1) \left( \frac{1+t}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} \frac{d\lambda_h}{\lambda_h} - \frac{\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{\lambda_h}{1+t\lambda_h} dt \right) - \frac{\sigma-1}{1+t} dt.$$

Collecting terms, we have

$$\begin{aligned} \Upsilon_h \frac{d\lambda_h}{\lambda_h} &= \frac{1-\gamma}{\gamma} \frac{d\lambda_f}{\lambda_f} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} - \frac{(\sigma-1)\lambda_h}{1+t\lambda_h} dt + \frac{\sigma-1}{1+t} dt \\ &= \frac{1-\gamma}{\gamma} \frac{d\lambda_f}{\lambda_f} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} - \left( \frac{\lambda_h}{1+t\lambda_h} - \frac{1}{1+t} \right) (\sigma-1) dt \\ &= \frac{1-\gamma}{\gamma} \frac{d\lambda_f}{\lambda_f} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \left( \frac{1}{1+t} - \frac{\lambda_h}{1+t\lambda_h} \right) (\sigma-1) dt \\ &= \frac{1-\gamma}{\gamma} \frac{d\lambda_f}{\lambda_f} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{1+t\lambda_h - \lambda_h(1+t)}{(1+t)(1+t\lambda_h)} (\sigma-1) dt \\ &= \frac{1-\gamma}{\gamma} \frac{d\lambda_f}{\lambda_f} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f} \frac{d\lambda_f}{\lambda_f} + \frac{(1-\lambda_h)(\sigma-1)}{(1+t)(1+t\lambda_h)} dt \\ &= \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f} \right) \frac{d\lambda_f}{\lambda_f} + \frac{(1-\lambda_h)(\sigma-1)}{(1+t)(1+t\lambda_h)} dt, \end{aligned}$$



where

$$\begin{aligned}
\Upsilon_h &: = \frac{1}{1-\lambda_h} + \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)(1+t)}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} \\
&= \frac{1}{1-\lambda_h} + \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)(1+t)}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} \\
&= \frac{1}{1-\lambda_h} + \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)(1+t)}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h} \\
&= \frac{1-\gamma}{\gamma} + \left(1 + \frac{(\sigma-1)(1+t)\lambda_h}{1+t\lambda_h}\right) \frac{1}{1-\lambda_h} \\
&= \frac{1-\gamma}{\gamma} + \left(\frac{1+t\lambda_h + (\sigma-1)(1+t)\lambda_h}{1+t\lambda_h}\right) \frac{1}{1-\lambda_h} \\
&= \frac{1-\gamma}{\gamma} + \frac{1+t\lambda_h + (\sigma-1)(1+t)\lambda_h}{(1+t\lambda_h)(1-\lambda_h)}.
\end{aligned}$$

Combining these expressions, we obtain

$$\begin{aligned}
&\left( \frac{1-\gamma}{\gamma} + \frac{1+t\lambda_h + (\sigma-1)(1+t)\lambda_h}{(1+t\lambda_h)(1-\lambda_h)} - \frac{\left(\frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f}\right) \left(\frac{1-\gamma}{\gamma} + \frac{(\sigma-1)(1+t)\lambda_h}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h}\right)}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f}} \right) \frac{d\lambda_h}{\lambda_h} \\
&= \frac{\frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f}}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f}} \frac{(\sigma-1)\lambda_h}{1+t\lambda_h} dt + \frac{(1-\lambda_h)(\sigma-1)}{(1+t)(1+t\lambda_h)} dt.
\end{aligned}$$

Collecting terms on the left hand side of this expression, we obtain

$$\begin{aligned}
& \left( \frac{1-\gamma}{\gamma} + \frac{1+t\lambda_h + (\sigma-1)(1+t)\lambda_h}{(1+t\lambda_h)(1-\lambda_h)} - \frac{\left(\frac{1-\gamma}{\gamma} + \frac{(\sigma-1)(1+t)\lambda_h}{1+t\lambda_h} \frac{\lambda_h}{1-\lambda_h}\right) \left(\frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f}\right)}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f}} \right) \frac{d\lambda_h}{\lambda_h} \\
&= \left( \left( 1 - \frac{\frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f}}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f}} \right) \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)(1+t)\lambda_h}{(1+t\lambda_h)(1-\lambda_h)} \right) + \frac{1}{1-\lambda_h} \right) \frac{d\lambda_h}{\lambda_h} \\
&= \left( \left( \frac{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} - \frac{1-\gamma}{\gamma} - \frac{(\sigma-1)\lambda_f}{1-\lambda_f}}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f}} \right) \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)(1+t)\lambda_h}{(1+t\lambda_h)(1-\lambda_h)} \right) + \frac{1}{1-\lambda_h} \right) \frac{d\lambda_h}{\lambda_h} \\
&= \left( \frac{\frac{1}{1-\lambda_f}}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f}} \left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)(1+t)\lambda_h}{(1+t\lambda_h)(1-\lambda_h)} \right) + \frac{1}{1-\lambda_h} \right) \frac{d\lambda_h}{\lambda_h} \\
&= \frac{\frac{1-\gamma}{\gamma} \frac{1}{1-\lambda_f} + \frac{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f}}{1-\lambda_h} + \frac{(\sigma-1)(1+t)\lambda_h}{(1+t\lambda_h)(1-\lambda_h)} \frac{1}{1-\lambda_f}}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f}} \frac{d\lambda_h}{\lambda_h} \\
&= \frac{\frac{1-\gamma}{\gamma} (1-\lambda_h + 1-\lambda_f) + 1 + (\sigma-1)\lambda_f + \frac{(\sigma-1)(1+t)\lambda_h}{1+t\lambda_h}}{(1-\lambda_f)(1-\lambda_h) \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} \right)} \frac{d\lambda_h}{\lambda_h} \\
&= \frac{\frac{1-\gamma}{\gamma} (2-\lambda_h-\lambda_f) + 1 + (\sigma-1) \left( \lambda_f + \frac{(1+t)\lambda_h}{1+t\lambda_h} \right)}{(1-\lambda_f)(1-\lambda_h) \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} \right)} \frac{d\lambda_h}{\lambda_h}.
\end{aligned}$$

The right hand side of the above expression can be rewritten as

$$\begin{aligned}
& \frac{\sigma - 1}{1 + t\lambda_h} \left( \frac{\frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f}}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f}} \lambda_h + \frac{1 - \lambda_h}{1 + t} \right) dt \\
= & \frac{\sigma - 1}{1 + t\lambda_h} \frac{\left( \frac{1-\gamma}{\gamma} + \frac{(\sigma-1)\lambda_f}{1-\lambda_f} \right) \lambda_h + \frac{1-\lambda_h}{1+t} \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} \right)}{\frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f}} dt \\
= & \frac{\sigma - 1}{1 + t\lambda_h} \frac{\left( \left( \frac{1-\gamma}{\gamma} (1 - \lambda_f) + (\sigma - 1) \lambda_f \right) \lambda_h + \frac{1-\lambda_h}{1+t} \left( \frac{1-\gamma}{\gamma} (1 - \lambda_f) + 1 + (\sigma - 1) \lambda_f \right) \right)}{(1 - \lambda_f) \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} \right)} dt \\
= & \frac{\sigma - 1}{1 + t\lambda_h} \frac{\frac{1-\gamma}{\gamma} (1 - \lambda_f) \lambda_h + (\sigma - 1) \lambda_f \lambda_h + \frac{1-\lambda_h}{1+t} \frac{1-\gamma}{\gamma} (1 - \lambda_f) + \frac{1-\lambda_h}{1+t} + \frac{1-\lambda_h}{1+t} (\sigma - 1) \lambda_f}{(1 - \lambda_f) \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} \right)} dt \\
= & \frac{\sigma - 1}{1 + t\lambda_h} \frac{\frac{1-\gamma}{\gamma} (1 - \lambda_f) \left( \lambda_h + \frac{1-\lambda_h}{1+t} \right) + (\sigma - 1) \lambda_f \left( \lambda_h + \frac{1-\lambda_h}{1+t} \right) + \frac{1-\lambda_h}{1+t}}{(1 - \lambda_f) \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} \right)} dt \\
= & \frac{\sigma - 1}{1 + t\lambda_h} \frac{\frac{1-\gamma}{\gamma} (1 - \lambda_f) \frac{\lambda_h + t\lambda_h + 1 - \lambda_h}{1+t} + (\sigma - 1) \lambda_f \frac{\lambda_h + t\lambda_h + 1 - \lambda_h}{1+t} + \frac{1-\lambda_h}{1+t}}{(1 - \lambda_f) \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} \right)} dt \\
= & \frac{\sigma - 1}{1 + t\lambda_h} \frac{\frac{1-\gamma}{\gamma} (1 - \lambda_f) \frac{t\lambda_h + 1}{1+t} + (\sigma - 1) \lambda_f \frac{t\lambda_h + 1}{1+t} + \frac{1-\lambda_h}{1+t}}{(1 - \lambda_f) \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} \right)} dt \\
= & \frac{\sigma - 1}{1 + t} \frac{\frac{1-\gamma}{\gamma} (1 - \lambda_f) + (\sigma - 1) \lambda_f + \frac{1-\lambda_h}{1+t\lambda_h}}{(1 - \lambda_f) \left( \frac{1-\gamma}{\gamma} + \frac{1+(\sigma-1)\lambda_f}{1-\lambda_f} \right)} dt.
\end{aligned}$$

Hence,

$$\frac{d\lambda_h/\lambda_h}{dt} = \frac{(\sigma - 1)(1 - \lambda_h)}{1 + t} \frac{\frac{1-\gamma}{\gamma} (1 - \lambda_f) + (\sigma - 1) \lambda_f + \frac{1-\lambda_h}{1+t\lambda_h}}{\frac{1-\gamma}{\gamma} (2 - \lambda_h - \lambda_f) + 1 + (\sigma - 1) \left( \lambda_f + \frac{1+t}{1+t\lambda_h} \lambda_h \right)} > 0.$$

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University of Hohenheim  
Dean's Office of the Faculty of Business, Economics and Social Sciences  
Palace Hohenheim 1 B  
70593 Stuttgart | Germany  
Fon +49 (0)711 459 22488  
Fax +49 (0)711 459 22785  
[wiso@uni-hohenheim.de](mailto:wiso@uni-hohenheim.de)  
[wiso.uni-hohenheim.de](http://wiso.uni-hohenheim.de)