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## Time-frequency forecast of the equity premium

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# Time-frequency forecast of the equity premium\*

Gonçalo Faria<sup>†</sup>

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## Abstract

Any time series can be decomposed into cyclical components fluctuating at different frequencies. Accordingly, in this paper we propose a method to forecast the stock market's equity premium which exploits the frequency relationship between the equity premium and several predictor variables. We evaluate a large set of models and find that, by selecting the relevant frequencies for equity premium forecasting, this method significantly improves in both statistical and economic sense upon standard time series forecasting methods. This improvement is robust regardless of the predictor used, the out-of-sample period considered, and the frequency of the data used.

*Keywords:* time-frequency forecast, equity premium, multiresolution analysis

*JEL classification:* C58, G11, G17

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# 1 Introduction

Goyal and Welch (2008) and subsequent research show that the equity premium out-of-sample (OOS) predictability of several economic and financial variables is not robust and usually concentrated in specific periods of time (*i.e.* recessions). Notwithstanding, in recent years there has been increasing evidence that the equity premium is, at least to some extent, predictable when using more advanced econometric techniques. In particular, in the context of a single-variable predictive regression setup, recent methodological contributions that improved the OOS forecastability of the equity premium include regressions with time-varying coefficients (Dangl and Halling, 2012), with economic constraints (Pettenuzzo et al., 2014), with learning and time-varying volatility (Johannes et al., 2014), and quantile regression models with single predictor (Meligkotsidou et al., 2014). This paper contributes to this literature by proposing a forecasting method that combines a frequency-domain decomposition technique with traditional time series econometric methods.

In particular, our forecasting method exploits the frequency relationship between the equity premium and fourteen standard equity premium predictor variables. In the spirit of the trend-cycle decomposition of a time series (as proposed by *e.g.* Watson, 1986), we decompose both the equity premium and its predictors into time-frequency series components, each of them capturing the oscillations of the original variable within a specific frequency band. We do so by means of wavelet filtering methods, which allow to decompose a time series in a rather granular way, so that we can make use of the information embedded and aggregated in the time series of the variables in a more efficient way. Then, considering one predictor variable at a time, we forecast separately each of the time-frequency series component of the equity premium using the corresponding component of the predictor. Finally, to produce the forecast of the equity premium based on that predictor, we evaluate a large set of models that combine the forecasts from the different frequency components of the equity premium.

The main results can be summarized as follows. First, for *all* predictor variables considered, by using the proper time-frequency series components, the OOS forecasting performance is better than that using traditional time series forecasting methods. Second, when compared to the standard benchmark in the literature (*i.e.* the historical mean of returns), five predictor variables (the earnings-price ratio, the dividend-payout ratio, the inflation rate, the long-term government bond return and the term spread) deliver positive and statistically significant OOS R-squares ( $R_{OS}^2$ ), that is, they outperform the reference model in the literature. This result thus unveils that some variables considered to be poor equity premium predictors are ultimately good predictors once the frequencies that have the greatest predictive power are retained and the noisy frequencies are excluded. Third, this method informs us about the relative importance of the frequency components of each predictor variable. In particular, we find that the low frequency components are important whereas shorter business cycle frequencies are less so. Fourth, when examining the economic significance of our model predictive performance through an asset allocation analysis, we find that a mean-variance investor who allocates her wealth between equities and risk-free bills enjoys significant utility gains (both with respect to the historical mean and with the time series analysis) when making the forecasts using the proper time-frequency components of each predictor variable. Fifth, and differently from the time series analysis, using the proposed forecasting model some predictor variables outperform the historical mean benchmark also during periods of normal and good economic growth (where the regimes are based on sorted values of real GDP growth).

The rest of the paper is organized as follows. In section 2 we review related literature to provide context for our contribution. Section 3 presents the data and the methodology. Section 4 presents the OOS results and section 5 the results of the robustness exercises. Section 6 concludes.

## 2 Related literature

In the econometric literature, the recognition of frequency-specific modeling dates back at least to the work of Grether and Nerlove (1970) and to band-spectrum regression of Engle (1974). In this paper we use a relatively recent tool in economic and finance – wavelets filtering methods – to forecast the equity premium.<sup>1</sup> Differently from traditional frequency domain tools such as the Fourier analysis, wavelets are defined over a finite window in the time domain, with the size of that window being adjusted automatically according to the frequency of interest. This means that the high-frequency features of the time series can be captured by using a short window, whereas by looking at the same signal with a larger window, the low-frequency features are revealed. Hence, wavelets allow to extract both time-varying and frequency-varying features simultaneously just by changing the size of the window. They are thus better suited to handle variables (like *e.g.* financial variables) that exhibit jumps, structural breaks, and time-varying volatility.

The core of our wavelet decomposition is known as wavelet multiresolution analysis. It allows to decompose any variable (regardless of its time series properties) into a trend, a cycle, and a noise component in a way which is similar to the traditional time series trend-cycle decomposition approach (Watson, 1986), or other filtering methods like the Hodrick and Prescott (1997) or the Baxter and King (1999) bandpass filter. In particular, using the wavelet multiresolution decomposition allows to extract and forecast separately each frequency component of the time series. As wavelet methods allow for a rather granular decomposition of a time series, they could in principle help improving the forecast accuracy of the series as a whole.

Wavelet-based forecasting methods have indeed been successfully used to forecast (OOS) economic and financial variables. As regards economic variables, Rua (2011, 2017) proposes a

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<sup>1</sup> Crowley (2007) and Aguiar-Conraria and Soares (2014) provide reviews of economic and finance applications of wavelets tools.

wavelet-based multiscale principal component analysis to forecast GDP growth and inflation, while Kilponen and Verona (2016) forecast aggregate investment using the Tobin’s Q theory of investment. As regards financial variables, Mitra and Mitra (2006) and Caraianni (2017) forecast exchange rates, while Zhang et al. (2017) and Faria and Verona (2018, 2020) focus on stock return predictability. In particular, Faria and Verona (2018) run a time-frequency forecast of stock market returns in the context of Ferreira and Santa-Clara (2011) sum-of-the-part method. In this paper we generalize the idea of Faria and Verona (2018) and run the time-frequency forecast of the equity premium within a more general OOS predictive setting (single-variable predictive regression setup) instead of the sum-of-the-part method.

This paper is naturally related to the literature on the OOS forecasting of the equity premium, which was stimulated by Goyal and Welch (2008) findings that several equity premium predicting variables perform poorly OOS.<sup>2</sup> In particular, after running the OOS equity premium forecast on a frequency-by-frequency basis, we consider a large set of models that combine the forecasts from the different frequency components of the equity premium, and show that statistically and economically significant OOS gains can be obtained by removing some frequencies (of each individual predictor variable) from the forecasting exercise and only using the relevant frequencies.

Finally, this work links with recent literature analyzing the spectral properties of financial asset returns (Dew-Becker and Giglio, 2016 and Chaudhuri and Lo, 2016) and of equity returns predictability (Faria and Verona, 2018 and Bandi et al., 2019). In particular, we find that, for all predictors under analysis, their lowest frequency components are always selected as relevant frequencies for equity premium forecasting purposes (both with monthly

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<sup>2</sup> Besides the methodological contributions using single-variable predictive regression setup cited in section 1, methodological contributions that make use of several predictors to forecast the equity premium include dynamic factor models (Ludvigson and Ng, 2007, Kelly and Pruitt, 2013 and Neely et al., 2014), forecasts combination from different predictors (Rapach et al., 2010 and Pettenuzzo and Ravazzolo, 2016), regime-switching vector autoregression models (Henkel et al., 2011), the sum-of-the-parts method (Ferreira and Santa-Clara, 2011 and Faria and Verona, 2018), and Bayesian regime-switching combination or quantile combination approach (Zhu and Zhu, 2013 and Lima and Meng, 2017, respectively).



and quarterly series). Furthermore, in some cases (like *e.g.* the term spread), the lowest frequency component is the only relevant frequency. This finding adds to recent empirical evidence that the level and price of aggregate risk in equity markets are strongly linked to low-frequency economic fluctuations (see *e.g.* Dew-Becker and Giglio, 2016, Bianchi et al., 2017 and Gallegati and delli Gatti, 2018), that there are low-frequency, decades-long shifts in asset values relative to measures of macroeconomic fundamentals in the US (*e.g.* Bianchi et al., 2017), and also that accounting for time-varying macroeconomic trend components is crucial for understanding and forecasting long-term interest rates and bond returns (Bauer and Rudebusch, 2020).

### 3 Data and methodology

We focus on the OOS predictability of monthly equity premium, measured by the difference between the log (total) return of the S&P500 index and the log return on a one-month Treasury bill. As it has been emphasized in the literature (*e.g.* Goyal and Welch, 2008 and Huang et al., 2015), the OOS exercise is more relevant to evaluate effective return predictability in real time while avoiding the in-sample over-fitting issue, eventual small-sample size distortions and the look-ahead bias concern. Moreover, we only focus on the one-month forecasting period as it has been documented that return predictability with a short horizon is usually magnified at longer horizons (see *e.g.* Cochrane, 2001).

We use monthly data from January 1973 to December 2018 for fourteen predictors from Goyal and Welch (2008) updated database. Specifically, we use the log dividend-price ratio (DP), the log dividend yield (DY), the log earnings-price ratio (EP), the log dividend-payout ratio (DE), the excess stock return volatility (RVOL), the book-to-market ratio (BM), the net equity expansion (NTIS), the Treasury bill rate (TBL), the long-term bond yield (LTY), the long-term bond return (LTR), the term spread (TMS), the default yield spread (DFY),

the default return spread (DFR) and the lagged inflation rate (INFL). In appendix 1 these predictors are briefly explained and their time series are plotted. Table 1 reports summary statistics for the equity premium and its predictors. The average monthly equity premium is 0.42%, which, together with a monthly standard deviation of 4.40%, corresponds to an average monthly Sharpe ratio of 0.10 in the sample period.

Our methodology to forecast the equity premium is based on the wavelet multiresolution analysis, which is described in sub-section 3.1. The OOS procedure is then explained in sub-section 3.2.

### 3.1 Wavelet multiresolution analysis

The wavelet multiresolution analysis (MRA) allows the decomposition of a time series into its constituent multiresolution (frequency) components.<sup>3</sup> Given a time series  $y_t$ , its wavelet multiresolution representation can be written as

$$y_t = y_t^{S_J} + \sum_{j=1}^J y_t^{D_j} , \quad (1)$$

where  $y_t^{S_J}$  is the wavelet smooth component and  $y_t^{D_j}$ ,  $j = 1, 2, \dots, J$ , are the  $J$  wavelet detail components. Equation (1) shows that the original variable  $y_t$ , exclusively defined in the time domain, can be decomposed in different components, each of them also defined in the time domain and representing the fluctuation of the original time series in a specific frequency band. In particular, for small  $j$ , the  $j$  wavelet detail components represent the higher frequency characteristics of the time series (*i.e.* its short-term dynamics). As  $j$  increases, the  $j$  wavelet detail components represent lower frequencies movements of the series. Finally,

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<sup>3</sup> In this section we limit the description to the basic concepts which are directly useful to understand our empirical analysis. A more detailed analysis of wavelets methods can be found in Percival and Walden (2000) and in appendix 2.

the wavelet smooth component captures the lowest frequency dynamics (*i.e.* its long-term behavior).

In this paper, we use the maximal overlap discrete wavelet transform (MODWT) MRA with the Haar wavelet filter and reflecting boundary conditions.<sup>4</sup> Given the sufficiently long data series, we set  $J$  to 6 so that the MRA decomposition delivers seven time-frequency series: six wavelet detail components ( $y_t^{D_1}$  to  $y_t^{D_6}$ ) and the wavelet smooth component ( $y_t^{S_6}$ ).<sup>5</sup> As we use monthly data, the first detail component  $y_t^{D_1}$  captures oscillations between 2 and 4 months, while detail components  $y_t^{D_2}$ ,  $y_t^{D_3}$ ,  $y_t^{D_4}$ ,  $y_t^{D_5}$  and  $y_t^{D_6}$  capture oscillations with a period of 4-8, 8-16, 16-32, 32-64 and 64-128 months, respectively. Finally, the smooth component  $y_t^{S_6}$ , which in what follows we re-denote  $y_t^{D_7}$ , captures oscillations with a period longer than 128 months (10.6 years).<sup>6</sup>

To illustrate the rich set of different dynamics aggregated (and therefore hidden) in the original time series, figure 1 plots the time series of the (log) equity premium (top left panel) and of its seven time-frequency series components (remaining panels). As expected, the lower the frequency, the smoother the resulting filtered time series.

Furthermore, wavelets allow to analyze the variability of a time series on a frequency-by-frequency basis. In particular, by running the so-called energy decomposition analysis, it is possible to compute the variance decomposition by frequency and, hence, to detect which frequency bands contribute relatively more to the overall volatility of the original time series.

Table 2 reports the results of the energy decomposition analysis for the variables under

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<sup>4</sup> Examples of papers using the MODWT MRA decomposition include Galagedera and Maharaj (2008), Bekiros and Marcellino (2013), Xue et al. (2013), Barunik and Vacha (2015), Berger (2016), and Faria and Verona (2020). While the Haar filter is simple and widely used (see *e.g.* Manchaldore et al., 2010, Malagon et al., 2015, Bandi et al., 2019 and Lubik et al., 2019), the results in this paper are qualitatively the same using other wavelet filters (like *e.g.* Daubechies).

<sup>5</sup> As regards the choice of  $J$ , the number of observations dictates the maximum number of frequency bands that can be used. In particular, if  $t_0$  is the number of observations in the in-sample period, then  $J$  has to satisfy the constraint  $J \leq \log_2 t_0$ .

<sup>6</sup> In the MODWT, each wavelet filter at frequency  $j$  approximates an ideal high-pass filter with passband  $f \in [1/2^{j+1}, 1/2^j]$ , while the smooth component is associated with frequencies  $f \in [0, 1/2^{j+1}]$ . The level  $j$  wavelet components are thus associated to fluctuations with periodicity  $[2^j, 2^{j+1}]$  (months, in our case).

analysis. For the variables with low persistence, most of the volatility (more than 70%) is concentrated at higher frequencies ( $D_1$  and  $D_2$ ), whereas for the more persistent variables the lowest frequencies components ( $D_5$  and above) account for the majority of the total variability of the series.

## 3.2 Out-of-sample forecasts

The one-step ahead OOS forecasts are generated using a sequence of expanding windows. We use an initial in-sample period (1973:01 to 1989:12) to make the first one-step ahead OOS forecast. The in-sample period is then increased by one observation and a new one-step ahead OOS forecast is produced. This is the procedure until the end of the sample. The full OOS period therefore spans from 1990:01 to 2018:12.

### 3.2.1 Single-variable predictive regression model: time series

Let  $r$  be the equity premium. For each individual predictor  $x_i$ ,  $i = 1, \dots, 14$ , the predictive regression model is

$$r_{t+1} = \alpha + \beta x_{i,t} + \varepsilon_{t+1} \quad , \quad (2)$$

and the one-step ahead OOS forecast of the equity premium,  $\hat{r}_{t+1}$ , is given by:

$$\hat{r}_{t+1} = \hat{\alpha}_t + \hat{\beta}_t x_{i,t} \quad , \quad (3)$$

where  $\hat{\alpha}_t$  and  $\hat{\beta}_t$  are the OLS estimates of  $\alpha$  and  $\beta$  in equation (2), respectively, using data from the beginning of the sample until month  $t$ . We denote this forecast as the TS (time series) forecast.

### 3.2.2 Wavelet-based forecasting model

To forecast with wavelets, we fit a model like (2) to each time-frequency component of the MODWT MRA decomposition of  $r$  and  $x_i$ . The overall forecast for  $r$  can then be obtained by aggregating the forecasts of its time-frequency components.<sup>7</sup> Importantly, as the MODWT MRA at a given point in time uses information of neighboring data points (both past and future), we recompute the time-frequency series components at each iteration of the OOS forecasting process. This ensures that our method does not suffer from look-ahead bias as the forecasts are made with current and past information only.

Let us explain in more detail the steps involved. First, we apply the MODWT MRA decomposition to the variable to be forecasted ( $r$ ) as well as to all predictors ( $x_i$ ). Second, for each predictor  $x_i$ , we estimate a model like (2) for each frequency level  $j = 1, \dots, 7$ . That is, we estimate – separately – each time-frequency component of the equity premium using the time-frequency component of the predictor at the same level  $j$ :<sup>8</sup>

$$r_{t+1}^{x_i, D_j} = \alpha_{t,j}^{x_i} + \beta_{t,j}^{x_i} x_{i,t}^{D_j} + \varepsilon_{t+1} \quad . \quad (4)$$

Third, we use the estimation results to produce the one-step ahead forecast of the corresponding time-frequency component of  $r$ :

$$\hat{r}_{t+1}^{x_i, D_j} = \hat{\alpha}_{t,j}^{x_i} + \hat{\beta}_{t,j}^{x_i} x_{i,t}^{D_j} \quad ,$$

where  $\hat{\alpha}_{t,j}^{x_i}$  and  $\hat{\beta}_{t,j}^{x_i}$  are the OLS estimates of  $\alpha_{t,j}^{x_i}$  and  $\beta_{t,j}^{x_i}$  in equation (4), respectively, using

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<sup>7</sup> The closest approaches in the literature are those suggested by Rua (2011) and Faria and Verona (2018). This is also the spirit of the scale predictability in Bandi et al. (2019), who explore a model where returns and predictors are linear aggregates of components operating over different frequencies, and where predictability is frequency-specific.

<sup>8</sup> In principle it is possible to fit different forecasting models for each frequency components. For instance, we could use non-linear models when forecasting the high frequency components of the equity premium, or include more lags of the predictor when forecasting the lowest frequency components of the equity premium. We leave this for future research.

data from the beginning of the sample until month  $t$ .

The final step consists in evaluating the performance of a large set of models that combine the forecasts from the different frequency components of the equity premium, and to select the best one. We do so by searching, for each individual predictor, the combination of its time-frequency series components that maximizes the Campbell and Thompson (2008)  $R_{OS}^2$  statistic (as explained in sub-section 3.2.3). Taking the dividend-payout ratio (DE) as an example, the equity premium wavelet-based forecasting econometric model is given by:

$$\hat{r}_{t+1}^{DE} = \sum_{j=1}^{J+1} \delta_j \hat{r}_{t+1}^{DE, D_j} = \sum_{j=1}^{J+1} \delta_j \left[ \hat{\alpha}_{t,j}^{DE} + \hat{\beta}_{t,j}^{DE} DE_t^{D_j} \right] , \quad (5)$$

where  $DE_t^{D_j}$ ,  $j = 1, \dots, 7$ , are the time-frequency series components of DE. For each predictor, the weights  $\delta_j$  of each frequency component are chosen in order to maximize the predictor's statistical performance. In (5) we consider five possible values for each weight: 0, 0.25, 0.5, 0.75 and 1. A weight of 0 excludes a particular frequency from the forecast, that is, the information carried by that frequency to the forecast exercise is completely removed. We consider a limited number of possible values for the weights  $\delta_j$  mainly due to computational reasons. However, although the results are likely to improve by using a finer grid, the main message of this exercise would most likely be the same.

Moreover, in (5) we use a fixed weighting scheme for two main reasons. First, to effectively deal with the bias-variance trade-off it is preferable to fix the relative importance of various frequency components. In fact, estimating a model with time-varying weights would help reducing the bias in the forecasting exercise but would increase its forecast variance.<sup>9</sup> Second, several factors like *e.g.* market sentiment, monetary policies and uncertainty could motivate the use of time-varying schemes in order to assess the importance of each frequency at each point in time. However, in this paper we are interested in analyzing which frequencies of

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<sup>9</sup> We thank Christiane Baumeister for this insight.

each predictor are – on average – relevant to forecast the equity premium.

This model, which we will refer to as the WAV model, thus informs about the relevant frequencies of each predictor variable for the equity premium forecasting purposes.

### 3.2.3 Forecast evaluation

The forecasting performances of the time series (TS) and wavelet based (WAV) models are evaluated using the Campbell and Thompson (2008)  $R_{OS}^2$  statistic. As standard in the literature, the benchmark model is the average equity premium up to time  $t$  ( $\bar{r}_t$ ). The  $R_{OS}^2$  statistic measures the proportional reduction in the mean squared forecast error for the predictive model ( $MSFE_{PRED}$ ) relative to the historical mean ( $MSFE_{HM}$ ) and is given by

$$R_{OS}^2 = 100 \left( 1 - \frac{MSFE_{PRED}}{MSFE_{HM}} \right) = 100 \left[ 1 - \frac{\sum_{t=t_0}^{T-1} (r_{t+1} - \hat{r}_{t+1})^2}{\sum_{t=t_0}^{T-1} (r_{t+1} - \bar{r}_t)^2} \right],$$

where  $\hat{r}_{t+1}$  is the equity premium forecast for  $t+1$  from the TS or the WAV models considered and  $r_{t+1}$  is the realized equity premium from  $t$  to  $t+1$ . A positive (negative)  $R_{OS}^2$  indicates that the predictive model outperforms (underperforms) the historical mean (HM) in terms of MSFE.

The statistical significance of the results is evaluated using the Clark and West (2007) statistic. This statistic tests the null hypothesis that the MSFE of the HM model is less than or equal to the MSFE of the TS or WAV model against the alternative hypothesis that the MSFE of the HM model is greater than the MSFE of the TS or WAV model ( $H_0 : R_{OS}^2 \leq 0$  against  $H_A : R_{OS}^2 > 0$ ).

To assess the forecasting performance of the wavelet-based forecasting method with respect to the time series forecast, we also compute the  $R_{OS}^2$  (and its statistical significance) of the WAV model versus the TS model.

### 3.3 Asset allocation

We analyze the economic value of the different predictive models from an asset allocation perspective, considering a mean-variance investor who allocates her wealth between equities and risk-free bills. At the end of month  $t$ , the investor optimally allocates

$$w_t = \frac{1 \hat{R}_{t+1}}{\gamma \hat{\sigma}_{t+1}^2} \quad (6)$$

of the portfolio to equity for period  $t+1$ . In (6),  $\gamma$  is the investor's relative risk aversion coefficient,  $\hat{R}_{t+1}$  is the time  $t$  (TS or WAV) model forecast of equity premium for  $t+1$ , and  $\hat{\sigma}_{t+1}^2$  is the forecast of the variance of the equity premium. As in Rapach et al. (2016), we assume a relative risk aversion coefficient of three, use a ten-year moving window of past equity premium to estimate the variance forecast and constrain the weights  $w_t$  to lie between -0.5 and 1.5. These constraints limit the possibilities of short selling and leveraging the portfolio.

The realized portfolio return at time  $t+1$ ,  $RP_{t+1}$ , is given by  $RP_{t+1} = w_t R_{t+1} + RF_{t+1}$ , where  $RF_{t+1}$  denotes the risk-free return from time  $t$  to  $t+1$  (*i.e.* the market rate, which is known at time  $t$ ). The average utility (or certainty equivalent return, CER) of an investor that uses the portfolio rule (6) is given by  $CER = \overline{RP} - 0.5\gamma\sigma_{RP}^2$ , where  $\overline{RP}$  and  $\sigma_{RP}^2$  are the sample mean and variance of the portfolio return, respectively. We report the annualized utility gain, which is computed as the difference between the CER for an investor that uses the TS or WAV model to forecast equity premium and the CER for an investor who uses the HM benchmark for forecasting. The difference is multiplied by 12, which allows to interpret it as the annual portfolio management fee that an investor would accept to pay to have access to the alternative forecasting model versus the historical average forecast.



## 4 Out-of-sample forecasting performance

### 4.1 Statistical performance

The second and fourth columns of table 3 report the  $R_{O_S}^2$  statistics for each predictor using different model specifications versus the HM, for the entire OOS period (1990:01-2018:12).

The standard time series analysis (second column) confirms Goyal and Welch (2008) results, *i.e.* that traditional predictors perform badly OOS. As regards the WAV model, some predictors still underperform the HM benchmark ( $R_{O_S}^2 < 0$ ). However, there are five variables for which the  $R_{O_S}^2$ s are positive and statistically significant. This means that some of the equity premium predictors with reported poor performance in the literature have nevertheless predictability power, as long as their frequencies are properly chosen and used.

The weights of the frequency components ( $\delta_1 - \delta_7$ ) are listed in columns six to twelve of table 3. Regardless of the predictor used, the lowest frequency component is always included ( $\delta_7 > 0$ ). This finding is in line with recent empirical evidence that shows that the level and price of aggregate risk in equity markets are strongly linked to low-frequency economic fluctuations (*e.g.* Dew-Becker and Giglio, 2016) and also that there are low-frequency, decades-long shifts in asset values relative to measures of macroeconomic fundamentals in the US (*e.g.* Bianchi et al., 2017). For some predictors, it is also beneficial to include some high frequency fluctuations ( $\delta_1$ ), whereas shorter business cycle frequencies (especially  $\delta_3$  and  $\delta_4$ ) are usually less important. Finally, considering the entire spectrum of predictors/frequencies, more than 50% of the frequencies have zero weight. This means that a lot of information needs to be removed from the predictive regressions in order to improve the forecastability of the equity premium.

Column thirteen of table 3 reports the  $R_{O_S}^2$ s of the WAV model with respect to the TS model. Consistent with previously presented  $R_{O_S}^2$ s of both the TS and WAV models with respect to

the HM, results clearly indicate that the OOS forecasting performance of the WAV model is always better than that of the time series analysis.

To evaluate the consistency over time of the OOS performance of the forecasting model, we report the dynamics of the difference between the cumulative square forecasting error for the HM forecasting model and the cumulative square forecasting error when the TS or the WAV model for each predictor is used. Results, plotted in figure 2, should be read as follows. When the line increases/decreases, the predictive regression of the WAV model (in blue) or of the TS model (in black) outperforms/underperforms that of the HM. A forecasting model that consistently outperforms the HM will thus always have a positively sloped curve.

In the time series analysis (black lines), all predictors underperform the HM, so their corresponding lines are almost always below zero. Looking at the WAV models (blue lines), it is possible to broadly classify the predictors into four different groups as regards the consistency of their OOS performance over time. The first group includes predictors (DP, DY, NTIS, TBL and DFY) with an OOS performance close to that of the HM most of the time (*i.e.* the lines are relatively stable around zero). A second group includes predictors (RVOL and LTY) with an erratic forecasting performance, as the slopes of their plotted graphs swing between positive and negative values. A third group includes predictors (EP, DE, BM, DFR and INFL) which post a strong OOS outperformance versus the HM only during the last NBER-dated recession. Finally, two predictors (LTR and TMS) post a consistent positive outperformance throughout the entire OOS period (except for the first 5 years), with their corresponding lines featuring smooth upward-sloping trends.

## 4.2 Economic performance

In the previous sub-section we have shown that the proposed wavelet-based forecasting method delivers statistically significant gains. We now analyze the performance of this

method from an asset allocation perspective. Results are reported in the third, fifth and last columns of table 3.

Eleven out of fourteen predictors deliver positive CER gains (compared to the HM) with the WAV model, with the highest utility gains (570 basis points) being obtained when using the term spread (TMS). More importantly, the CER gains using the WAV forecasting method are usually larger than those in the TS analysis (last column).

Figure 3 provides a dynamic perspective of the portfolio and cumulative wealth for an investor that uses the HM model, the WAV model for the dividend-payout ratio (DE) and for the term spread (TMS), which obtain the highest  $R_{OS}^2$  and CER gains in the OOS sample period under analysis, respectively.

Panel A presents the dynamic equity weights (constrained to lie between -0.5 and 1.5) for those three alternative portfolios. Two results stand out. First, the equity exposure of the HM portfolio (black line) changes more smoothly than in the alternative portfolios under analysis. Second, changes in the equity allocation in a portfolio based on the WAV TMS (blue line) are smoother than those on the WAV DE (red line). This can be explained by the fact that the WAV TMS only considers the lowest frequency (*i.e.* the long run) of the TMS, while the WAV DE considers both higher and lower frequencies of the DE.

Panel B shows the log cumulative wealth for an investor that invests 1\$ in January 1990 and reinvests all proceeds. Both strategies based on the WAV models clearly outperform the strategy based on the HM, with that outperformance being particularly strong during recession periods. This is essentially due to the improved market timing of both WAV model based strategies versus the HM based strategy, as illustrated in Panel A.

## 5 Robustness tests

We run two tests to evaluate the robustness of the wavelet-based equity premium forecast methodology. We first analyze the forecasting performance in different sample periods (sub-section 5.1), and then run the forecasting exercise using quarterly data (sub-section 5.2).

### 5.1 Different sample periods

#### 5.1.1 Great moderation and great financial crisis

We divide the OOS period into two sub-periods: from January 1990 to December 2006, which broadly corresponds to the so-called great moderation period, and from January 2007 onward, which corresponds to the great financial crisis and aftermath.

Table 4 reports the  $R_{OS}^2$  and the CER gains (compared to the HM) for all predictors. Regardless of the forecasting method used (TS or WAV), the OOS predictability in the first period is usually weaker than in the second period.<sup>10</sup> In any case, in both sample periods there are significant OOS forecasting improvements for almost all predictors using the WAV forecasting model. In the first period, five variables yield positive and statistically significant  $R_{OS}^2$ s using the WAV model, while in the time series analysis no predictor outperforms the HM benchmark in a statistically significant way. A similar pattern is visible in the second period. Interestingly, with the WAV model two predictors (dividend-payout ratio and term spread) outperform the HM benchmark in both sub-sample periods. Very similar conclusions arise from the utility gains analysis. The maximum CER gains obtained are 499 and 786 basis points in the first and second sub-sample periods (term spread and inflation with WAV model, respectively), which are significantly higher than the gains achieved in the time

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<sup>10</sup> Regarding the WAV model, for each predictor and for each sub-sample period, we use the same weights for the frequencies as the ones in the full OOS period (reported in table 3). This is a conservative approach, as we would expect to improve the performance of the WAV models by choosing the optimal weights of different frequencies for each predictor and for each sub-sample period.

series analysis (115 and 129 basis points for Treasury bill rate and default return spread, respectively).

### 5.1.2 Bad, normal, and good growth periods

A typical finding in the equity premium forecasting literature is that there is no (or very weak) predictability during expansions or good times (see *e.g.* Henkel et al., 2011 and Neely et al., 2014) using standard single variable predictive regressions.<sup>11</sup> Accordingly, and following Rapach et al. (2010), we evaluate the forecasts during periods of bad, normal, and good economic growth. Those regimes are defined as the bottom, middle, and top third of sorted growth rates of industrial production in the US, respectively.<sup>12</sup> We report the  $R_{OS}^2$  and the CER gains (compared to the HM) for each regime in table 5.

Looking at the  $R_{OS}^2$  during bad growth periods, no predictor variable is statistically significant in the time series analysis, whereas five predictors are statistically significant and obtain expressive CER gains when using the WAV models. In particular, the maximum  $R_{OS}^2$  and CER gains are 7.07% and 1080 basis points, respectively, both achieved using the dividend-payout ratio.

The same qualitative conclusions can be extended to the normal growth period, even if for this regime only two predictors are statistically significant using the WAV model. Although the maximum  $R_{OS}^2$  and CER gains are usually lower than during bad periods, the levels are still quite high: the maximum  $R_{OS}^2$  and CER gains are 2.11% and 470 basis points using the term spread.

As regards the good period regime, two predictors (earnings-price ratio and term spread)

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<sup>11</sup> Dangl and Halling (2012) and Huang et al. (2017) find positive and statistically significant levels of OOS predictability during expansions using time-varying coefficients regression and state-dependent predictive regression models, respectively.

<sup>12</sup> The data for the industrial production in the US was downloaded from Federal Reserve Economic Data at <http://research.stlouisfed.org/fred2/>.

are statistically significant when using the WAV model, with  $R_{OOS}^2$  of 2.72% and 1.01%, respectively. From an utility perspective, results are also strong, as their annualized CER gains are 583 and 507 basis points, respectively.

Overall, for some predictors the wavelet-based forecasting method allows to improve the OOS forecast performance also when splitting the OOS period in bad, normal, and good growth periods.

## 5.2 Quarterly data

At last, we test the robustness of the wavelet-based equity premium forecasting method using quarterly data. As before, the OOS forecasts are made using a sequence of expanding windows. To have a sufficiently large initial sample period, we use data from 1952:Q1. The initial in-sample period is 1952:Q1 to 1989:Q4, and the full OOS period spans from 1990:Q1 to 2018:Q4. As in the analysis using monthly data, we set  $J$  to 6 but we perform the MODWT MRA using the Daubechies filter with length 8 and reflecting boundary conditions. We adopt this filter, instead of the Haar filter used with monthly data, as it is more suited for (and more commonly used with) quarterly data (see *e.g.* Gallegati and Ramsey, 2013).

We consider sixteen predictors: the same fourteen predictors used in the monthly data analysis plus the (lagged) investment to capital ratio (IK) and the consumption-wealth ratio (CAY).<sup>13</sup> Table 6 reports the  $R_{OOS}^2$  for each predictor for both the time series analysis and the WAV model specification. The main conclusion is that the wavelet-based equity premium forecasting method is robust towards the use of quarterly data. In particular, results in the last column of the table show that there are indeed significant OOS forecasting improvements for almost all predictors using the WAV forecasting model when compared to the TS analy-

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<sup>13</sup> The quarterly time series of the IK and the CAY are available from the Goyal and Welch (2008) updated database. These variables, which are briefly explained in appendix 1, have been used as equity premium predictors when using quarterly data (see *e.g.* Rapach et al., 2010, Lettau and Ludvigson, 2001).

sis. Furthermore, four variables (earnings-price ratio, dividend-payout ratio, long-term yield and investment rate) yield positive and statistically significant  $R_{OOS}^2$ s using the WAV model, while in the time series analysis only investment rate outperforms the HM benchmark in a statistically significant way.

## 6 Concluding remarks

Goyal and Welch (2008) and subsequent research have documented the poor out-of-sample (OOS) equity premium forecasting performance of an extensive list of economic and financial variables. In this paper we propose a wavelet-based method to forecast the equity premium. The series are decomposed into their time-frequency components, forecasted separately, and then aggregated to obtain the forecast of the equity premium. Regardless of the predictor variable used, the OOS period, and the frequency of the data considered, this method significantly improves upon the OOS forecast done using traditional time series tools. The proposed wavelet-based method allows for a more granular analysis, leading to its strong and robust empirical performance. In particular, the crucial step to improve the forecasting performance of the predictors is to retain the frequencies that have the greatest predictive power and to exclude the noisy frequencies.

The proposed wavelet-based forecasting method could, in principle, be helpful to improve the forecast of other financial variables (*e.g.* equity market volatility) and returns in other markets such as fixed income, currency and commodities. Moreover, given the role that the equity premium forecast has in asset allocation decisions, the proposed method may bring relevant insights about the frequency-domain implications in the optimal dynamic asset allocation decisions. At last, this forecasting method can also be useful for policymakers in their attempt of anticipating possible “over-heated” equity markets that could, ultimately, pose a threat to macroeconomic and financial stability.

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	mean	median	min	max	std. dev.	AR(1)
Equity premium (%)	0.42	0.84	-24.8	14.9	4.40	0.04
DP	-3.64	-3.65	-4.52	-2.75	0.43	0.99
DY	-3.64	-3.64	-4.53	-2.75	0.43	1.00
EP	-2.84	-2.88	-4.84	-1.90	0.48	0.99
DE	-0.80	-0.85	-1.24	1.38	0.33	0.99
RVOL (ann.)	0.14	0.14	0.05	0.32	0.05	0.96
BM	0.47	0.35	0.12	1.21	0.28	1.00
NTIS	0.01	0.01	-0.06	0.05	0.02	0.98
TBL (% , ann.)	4.68	4.95	0.01	16.3	3.50	0.99
LTY (% , ann.)	6.77	6.76	1.75	14.8	2.91	1.00
LTR (%)	0.69	0.72	-11.2	15.2	3.10	0.05
TMS (% , ann.)	2.09	2.24	-3.65	4.55	1.46	0.95
DFY (% , ann.)	1.09	0.95	0.55	3.38	0.46	0.96
DFR (%)	0.01	0.05	-9.75	7.37	1.49	-0.04
INFL (%)	0.32	0.30	-1.92	1.81	0.38	0.61

Table 1: Summary statistics

This table reports summary statistics for the (log) equity premium and for the set of predictive variables. The sample period is from 1973:01 to 2018:12. Equity premium, LTR, DFR, and INFL (TBL, LTY, TMS, and DFY) are measured in percent (annual percent).

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>
Equity premium	48	26	12	7	4	2	2
DP	0	0	1	2	3	5	89
DY	0	0	1	2	3	5	89
EP	1	1	3	7	11	11	66
DE	1	2	8	19	29	26	15
RVOL	2	3	6	14	24	27	24
BM	0	0	1	2	2	4	91
NTIS	1	2	4	10	18	17	48
TBL	0	1	2	3	6	10	79
LTY	0	0	1	1	3	5	90
LTR	47	28	13	6	3	1	1
TMS	3	4	7	11	20	31	25
DFY	2	4	7	13	18	18	38
DFR	52	26	11	6	3	1	0
INFL	19	19	15	7	5	5	29

Table 2: Energy decomposition (%)

This table reports the variance decomposition by frequency for the time series under analysis. The sample period is from 1973:01 to 2018:12. Percentages may not add up to 100 because of rounding.

	TS vs HM		WAV vs HM									WAV vs TS	
	$R_{OS}^2$	CER gains	$R_{OS}^2$	CER gains	$\delta_1$	$\delta_2$	$\delta_3$	$\delta_4$	$\delta_5$	$\delta_6$	$\delta_7$	$R_{OS}^2$	CER gains
DP	-1.90	-2.79	-0.38	0.00	0	0	0	0	0	0	0.5	1.49***	2.79
DY	-2.03	-2.60	-0.38	0.06	0	0	0	0	0	0.5	0.5	1.61***	2.67
EP	-1.05	-0.34	2.47**	3.97	0	0.5	1	0.25	0	0	0.75	3.49***	4.31
DE	-2.06	-0.93	2.69***	3.87	1	0	0.25	0.5	0	0	1	4.65***	4.81
RVOL	-0.78	-2.16	-0.12	1.21	0	0	0	0	0.5	0	0.75	0.66**	3.37
BM	-0.52	-0.68	0.16	1.10	0	0	0	0	1	1	0.5	0.68**	1.79
NTIS	-2.95	-2.17	-0.15	0.52	0.75	1	0	0	0	0	0.5	2.72***	2.69
TBL	-0.35	0.81	-0.23	0.64	0	0	0	0	0	1	0.75	0.12	-0.17
LTY	-0.29	0.11	-0.14	0.54	0	0	0.25	1	0.25	1	0.5	0.15	0.43
LTR	-0.34	-0.47	0.91**	2.56	0	0.25	0	0.5	0	0	1	1.25**	3.03
TMS	-0.72	0.23	1.83***	5.70	0	0	0	0	0	0	1	2.53***	5.48
DFY	-2.84	-4.37	-0.62	-0.44	0	0	0	0	0	0	0.75	2.16***	3.93
DFR	-1.83	1.10	0.43	1.11	1	0	0	0	0.75	1	0.25	2.22*	0.02
INFL	-0.66	-0.61	1.64**	3.76	0.75	1	0	0.25	1	1	0.75	2.28**	4.37

Table 3: Out-of-sample R-squares ( $R_{OS}^2$ ) and annualized CER gains

This table reports the out-of-sample R-squares (in percentage) for the equity premium forecasts at monthly (non-overlapping) frequencies from the model as given by equation (3) for each of the original predictors (TS, second column) and from the WAV model (equation 5, fourth column) for each predictor, where the frequency components used and corresponding weights ( $\delta_j$ ,  $j = 1, 2, \dots, 7$ ) are listed in columns six to twelve. The out-of-sample R-squares ( $R_{OS}^2$ ) measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean (HM). The 1-month ahead out-of-sample forecast of equity premium is generated using a sequence of expanding windows. In columns three and five are reported the annualized certainty equivalent return (CER) gain (in percent) for an investor who allocates her wealth between equities and risk free bills according to the rule (6), using stock return forecasts from models in equations (3) and (5) instead of the forecasts based on the HM. The last two columns report the  $R_{OS}^2$ s and CER gains of the WAV model compared to the TS model. The sample period is from 1973:01 to 2018:12. The full out-of-sample forecasting period is from 1990:01 to 2018:12, monthly frequency. Asterisks denote significance of the out-of-sample *MSFE*-adjusted statistic of Clark and West (2007). \*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% levels, respectively.

	1990:01 - 2006:12				2007:01 - 2018:12			
	TS vs HM		WAV vs HM		TS vs HM		WAV vs HM	
	$R_{OS}^2$	CER gains	$R_{OS}^2$	CER gains	$R_{OS}^2$	CER gains	$R_{OS}^2$	CER gains
DP	-3.29	-4.49	-0.37	-0.04	-0.14	-0.39	-0.39	0.07
DY	-3.61	-4.36	-0.27	0.33	-0.04	-0.12	-0.53	-0.29
EP	-1.05	-1.02	0.94*	2.81	-1.06	0.63	4.39	5.60
DE	-1.82	-1.60	0.88*	2.18	-2.36	0.01	4.97*	6.26
RVOL	-1.73	-3.02	1.60**	4.12	0.41	-0.94	-2.28	-2.86
BM	-0.86	-1.14	0.06	0.70	-0.09	-0.05	0.29	1.67
NTIS	-2.84	-1.59	0.13	1.56	-3.09	-2.99	-0.50	-0.93
TBL	-0.49	1.15	-0.38	0.06	-0.18	0.32	-0.04	1.46
LTY	-0.41	-0.33	-0.49	-0.31	-0.15	0.73	0.29	1.74
LTR	-0.83	-0.98	0.83*	2.14	0.28	0.23	1.01	3.15
TMS	-1.11	1.05	1.52***	4.99	-0.22	-0.95	2.22***	6.70
DFY	-3.37	-4.43	-0.42	-0.20	-2.17	-4.29	-0.88	-0.78
DFR	-2.64	0.96	0.20	0.95	-0.80	1.29	0.72	1.35
INFL	-0.37	0.56	-0.51	0.86	-1.04	-2.25	4.33**	7.86

Table 4: Out-of-sample R-squares ( $R_{OS}^2$ ) and annualized CER gains

This table reports the out-of-sample R-squares (in percentage) for equity premium forecasts at monthly (non-overlapping) frequencies from the model as given by equation (3) for each of the original predictors (TS) and from the WAV model in equation (5) for each predictor, where the frequency components used and corresponding weights ( $\delta_j$ ,  $j = 1, 2, \dots, 7$ ) are listed in columns six to twelve of Table 3. The out-of-sample R-squares ( $R_{OS}^2$ ) measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean (HM). The 1-month ahead out-of-sample forecast of equity premium is generated using a sequence of expanding windows. It is also reported the annualized certainty equivalent return (CER) gain (in percent) for an investor who allocates her wealth between equities and risk free bills according to the rule (6), using stock return forecasts from above mentioned models in equations (3) and (5) instead of forecasts based on the HM. The sample period is from 1973:01 to 2018:12. Two out-of-sample forecasting periods are considered: from 1990:01 to 2006:12 and from 2007:01 to 2018:12, monthly frequency. Asterisks denote significance of the out-of-sample *MSFE*-adjusted statistic of Clark and West (2007). \*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% levels, respectively.



	Bad				Normal				Good			
	TS vs HM		WAV vs HM		TS vs HM		WAV vs HM		TS vs HM		WAV vs HM	
	$R_{OS}^2$	CER gains	$R_{OS}^2$	CER gains	$R_{OS}^2$	CER gains	$R_{OS}^2$	CER gains	$R_{OS}^2$	CER gains	$R_{OS}^2$	CER gains
DP	-1.26	-2.34	-0.61	-0.79	-2.02	-2.75	0.13	0.57	-2.52	-3.27	-0.52	0.24
DY	-1.38	-2.07	-0.60	-0.55	-2.13	-2.68	0.46	1.11	-2.70	-3.04	-0.82	-0.36
EP	-1.83	-0.75	6.72*	9.11	0.28	0.52	-3.79	-2.95	-1.26	-0.76	2.72**	5.83
DE	-2.67	0.63	7.07***	10.8	-2.48	-2.87	-2.55	-3.29	-1.04	-0.55	1.97	4.21
RVOL	-0.82	-3.21	-0.87	-0.06	-0.65	-2.09	1.94**	4.45	-0.85	-1.20	-0.94	-0.74
BM	-0.69	-1.54	0.23	1.58	0.04	-0.25	0.24	0.63	-0.79	-0.27	0.01	1.09
NTIS	-4.62	-4.47	1.27	3.17	-1.15	-0.42	-2.07	-3.04	-2.53	-1.61	-0.21	1.48
TBL	0.18	0.96	-0.06	1.20	-0.19	0.97	-0.09	0.54	-1.08	0.47	-0.53	0.19
LTY	-0.13	-0.11	-0.21	0.29	0.05	0.22	0.44	1.33	-0.76	0.21	-0.54	0.01
LTR	0.20	-0.33	1.73**	5.02	-1.20	-0.64	0.24	0.49	-0.26	-0.46	0.51	2.18
TMS	0.33	0.13	2.35***	7.33	-1.79	0.45	2.11***	4.70	-1.04	0.09	1.01*	5.07
DFY	-2.81	-4.64	-0.85	-1.21	-3.76	-5.18	0.02	0.30	-2.13	-3.27	-0.88	-0.41
DFR	-11.40	-2.24	-1.90	1.18	1.11	1.59	1.50	1.63	6.62	3.95	2.20	0.55
INFL	-0.80	-0.85	2.90**	5.70	0.34	0.42	-0.02	-0.07	-1.32	-1.42	1.55	5.65

Table 5: Out-of-sample R-squares ( $R_{OS}^2$ ) and annualized CER gains

This table reports the out-of-sample R-squares (in percentage) for equity premium forecasts at monthly (non-overlapping) frequencies from the model as given by equation (3) for each of the original predictors (TS) and from the WAV model in equation (5) for each predictor, where the frequency components used and corresponding weights ( $\delta_j$ ,  $j = 1, 2, \dots, 7$ ) are listed in columns six to twelve of Table 3. The out-of-sample R-squares ( $R_{OS}^2$ ) measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean (HM). The 1-month ahead out-of-sample forecast of equity premium is generated using a sequence of expanding windows. It is also reported the annualized certainty equivalent return (CER) gain (in percent) for an investor who allocates her wealth between equities and risk free bills according to the rule (6), using stock return forecasts from above mentioned models in equations (3) and (5) instead of forecasts based on the HM. The sample period is from 1973:01 to 2018:12. Three out-of-sample forecasting periods are considered, each with 108 monthly observations: bad growth, normal growth and good growth. Those regimes are defined as the bottom, middle and top third of sorted growth rates of industrial production in the US, respectively. Asterisks denote significance of the out-of-sample *MSFE*-adjusted statistic of Clark and West (2007). \*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% levels, respectively.

	TS vs HM	WAV vs HM								WAV vs TS
	$R_{OS}^2$	$R_{OS}^2$	$\delta_1$	$\delta_2$	$\delta_3$	$\delta_4$	$\delta_5$	$\delta_6$	$\delta_7$	$R_{OS}^2$
DP	-4.63	-1.47	0	0	0	0	1	0	0.5	3.01**
DY	-6.42	-1.29	0	0	0	0	1	0.25	0.5	4.82**
EP	-2.38	6.48**	0.75	0	0	0	0.25	0	0.5	8.65**
DE	-4.24	2.27*	0	0.5	0	0	0	0	1	6.25***
RVOL	-4.67	-2.67	0	0	0	0	0	0	0.75	1.91
BM	-1.06	0.23	0	0	1	1	0	0	0.5	1.27
NTIS	-5.22	-1.42	0	0.5	0	0	0	0	0.75	3.61**
TBL	-1.16	-0.59	1	0.25	0	0	0	0	0.75	0.56
LTY	0.06	1.96*	1	1	0	1	0.75	0	0.75	1.90*
LTR	-2.37	0.79	1	0	0.25	0	1	0	1	3.08*
TMS	-3.18	0.00	0	0	0	0	0	0.25	0.75	3.08*
DFY	-1.87	-1.41	1	0	0	0	0	0	0.5	0.45
DFR	-2.36	-0.55	0	0	0.25	0.5	0.25	0	0.5	1.77*
INFL	1.73	1.65	0	0	0.25	1	1	1	0.75	-0.08
IK	1.89*	3.20*	0	0.75	0.5	0	0	0	1	1.34
CAY	-1.97	-0.05	0	0	0	0	1	0	0.5	1.88*

Table 6: Out-of-sample R-squares ( $R_{OS}^2$ ) using quarterly data

This table reports the out-of-sample R-squares (in percentage) for the equity premium forecasts at quarterly (non-overlapping) frequencies from the model as given by equation (3) for each of the original predictors (TS, second column) and from the WAV model (equation 5, third column) for each predictor where the frequency components used and corresponding weights ( $\delta_j$ ,  $j = 1, 2, \dots, 7$ ) are listed in columns four to ten. The out-of-sample R-squares ( $R_{OS}^2$ ) measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean (HM). The 1-quarter ahead out-of-sample forecast of equity premium is generated using a sequence of expanding windows. The last column reports the  $R_{OS}^2$  of the WAV model compared to the TS model. The sample period is from 1952:Q1 to 2018:Q4. The full out-of-sample forecasting period is from 1990:Q1 to 2018:Q4, quarterly frequency. Asterisks denote significance of the out-of-sample *MSFE*-adjusted statistic of Clark and West (2007). \*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% levels, respectively.

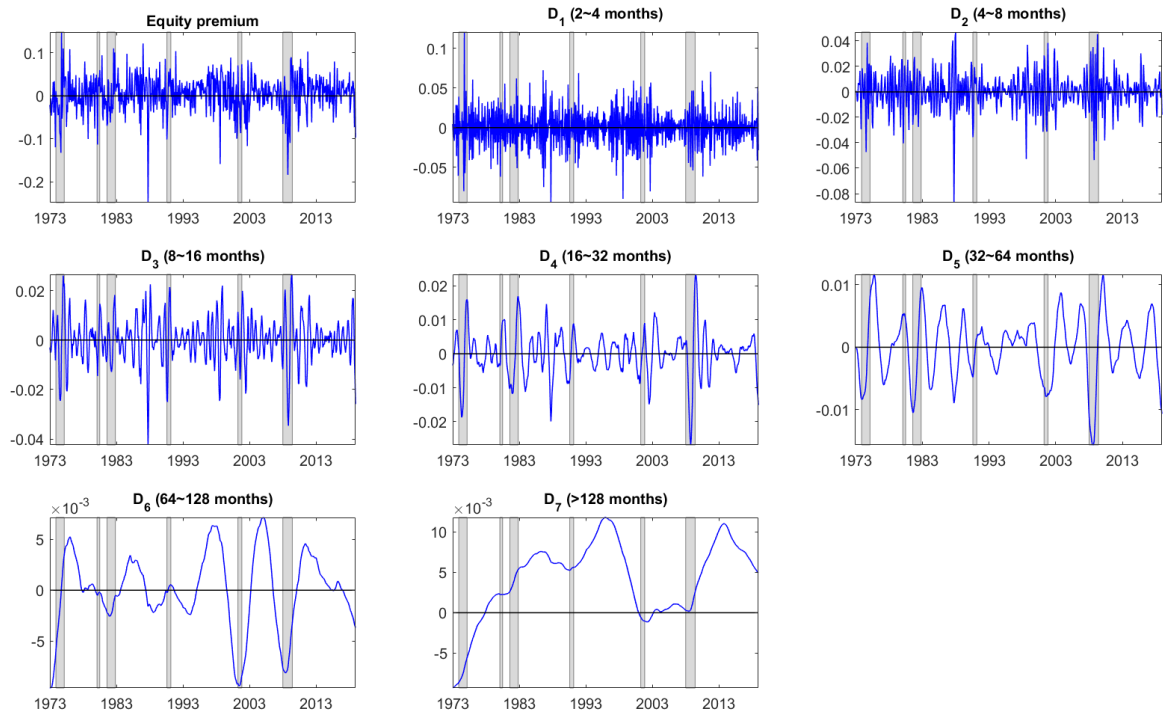


Figure 1: Equity premium, time series and wavelet decomposition

The time series of the (log) equity premium as proxied by the log S&P 500 index total return minus the log return on a one-month Treasury bill is presented in the top left panel. In the remaining panels are plotted the seven frequency components into which the equity premium time series is decomposed. It is applied a  $J = 6$  level wavelet decomposition which leads to six wavelet details ( $D_1, D_2, \dots, D_6$ ), representing the higher-frequency characteristics of the series, and a wavelet smooth ( $D_7$ ), that captures the low-frequency dynamics of the series. Sample period from 1973:01 to 2018:12, monthly frequency.

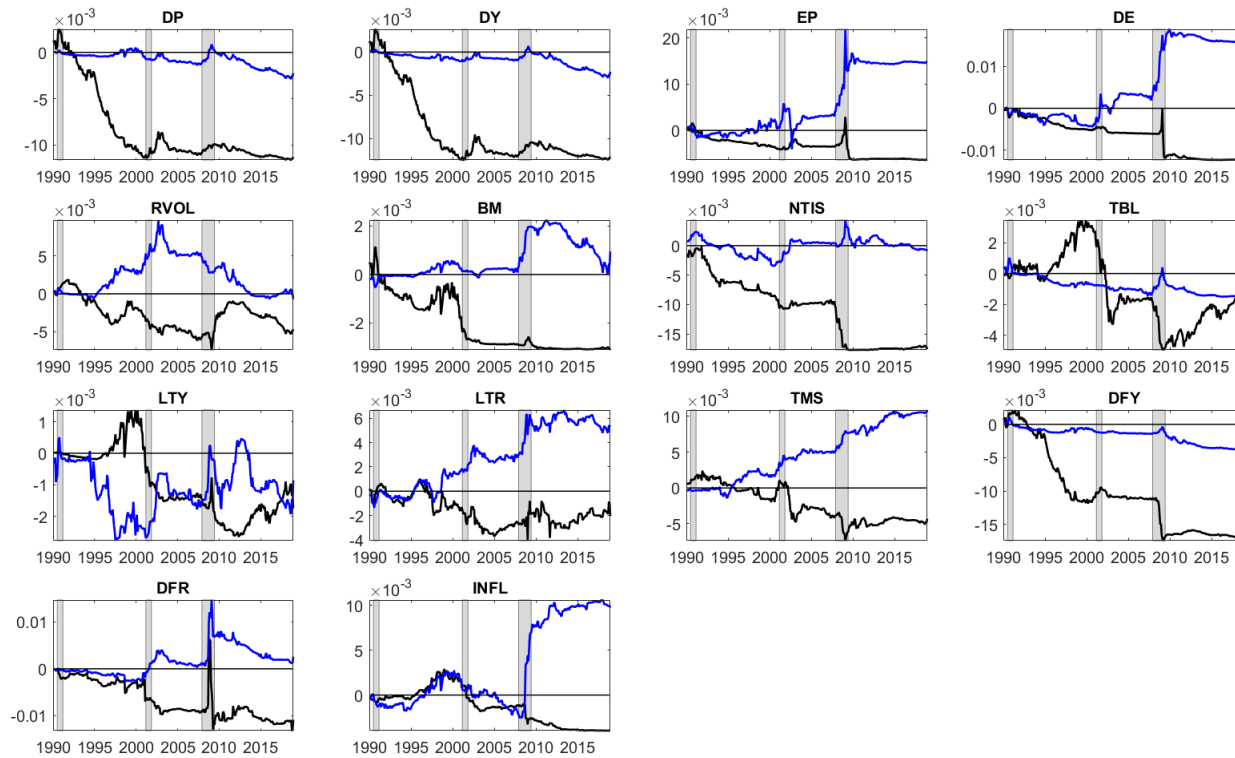
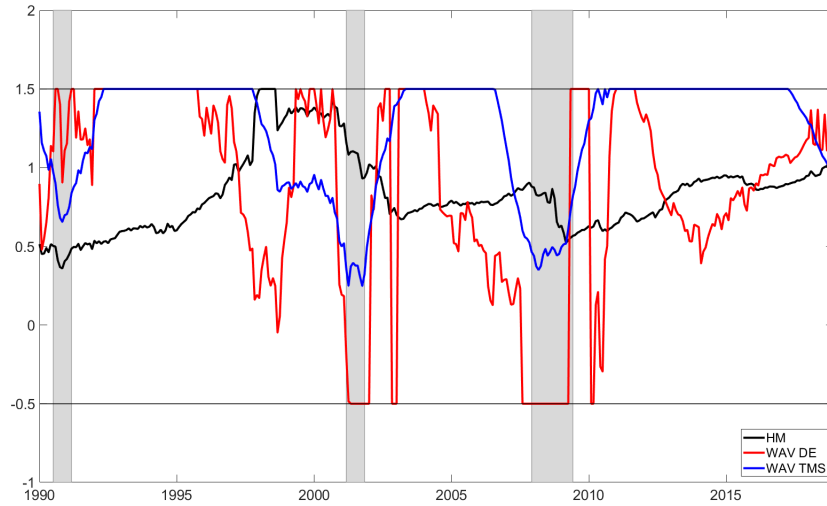


Figure 2: Difference between cumulative square forecasting error for the HM forecasting model and the cumulative square forecasting error for the individual predictive regression forecasting model

This figure reports the dynamics of the difference between the cumulative square forecasting error for the HM forecasting model and the cumulative square forecasting error for the predictive regression forecasting based on the WAV model (5) for each predictor with frequency components reported in table 3 (blue line), and when each predictor is considered in its original monthly time series (TS, black line). The sample period is from 1973:01 to 2018:12. The full out-of-sample forecasting period is from 1990:01 to 2018:12, monthly frequency.

### A. Equity weights



### B. Log cumulative wealth

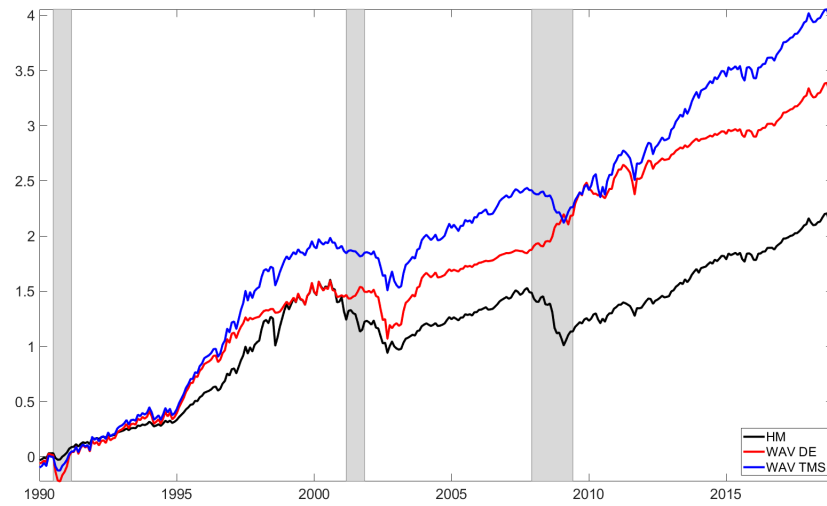


Figure 3: Equity weights and log cumulative wealth

Panel A plots the dynamics of the equity weight for a mean-variance investor who allocates monthly her wealth between equities and risk free bills according to the rule (6), using stock return forecasts based on the HM benchmark (black line), on the forecast with the WAV model (5) for the TMS (WAV TMS, blue line) and the DE (WAV DE). The equity weight is constrained to lie between -0.5 and 1.5. Panel B delineates the corresponding log cumulative wealth for the investor, assuming that she begins with 1\$ and reinvests all proceeds. Grey bars denote NBER-dated recessions. Sample period from 1990:01 to 2018:12, monthly frequency.

## Appendix 1. Definition of equity premium predictors

- Log dividend-price ratio (DP): difference between the log of dividends (12-month moving sums of dividends paid on S&P 500) and the log of prices (S&P 500 index).
- Log dividend yield (DY): difference between the log of dividends (12-month moving sums of dividends paid on S&P 500) and the log of lagged prices (S&P 500 index).
- Log earnings-price ratio (EP): difference between the log of earnings (12-month moving sums of earnings on S&P 500) and the log of prices (S&P 500 index price).
- Log dividend-payout ratio (DE): difference between the log of dividends (12-month moving sums of dividends paid on S&P 500) and the log of earnings (12-month moving sums of earnings on S&P 500).
- Excess stock return volatility (RVOL): calculated using a 12-month moving standard deviation estimator.
- Book-to-market ratio (BM): ratio of book value to market value for the Dow Jones Industrial Average.
- Net equity expansion (NTIS): ratio of 12-month moving sums of net equity issues by NYSE-listed stocks to the total end-of-year NYSE market capitalization.
- Treasury bill rate (TBL): three-month Treasury bill rate.
- Long-term yield (LTY): long-term government bond yield.
- Long-term return (LTR): long-term government bond return.

- Term spread (TMS): difference between the long-term government bond yield and the T-bill.
- Default yield spread (DFY): difference between Moody's BAA- and AAA-rated corporate bond yields.
- Default return spread (DFR): difference between long-term corporate bond and long-term government bond returns.
- Inflation rate (INFL): calculated from the Consumer Price Index (CPI) for all urban consumers.

For quarterly data, we also use:

- Investment to capital ratio (IK): ratio of aggregate (private nonresidential fixed) investment to aggregate capital for the whole economy.
- Consumption-wealth ratio (CAY): log consumer spending minus log asset wealth (total household net worth) and minus log labor income, all measured on an aggregate basis.

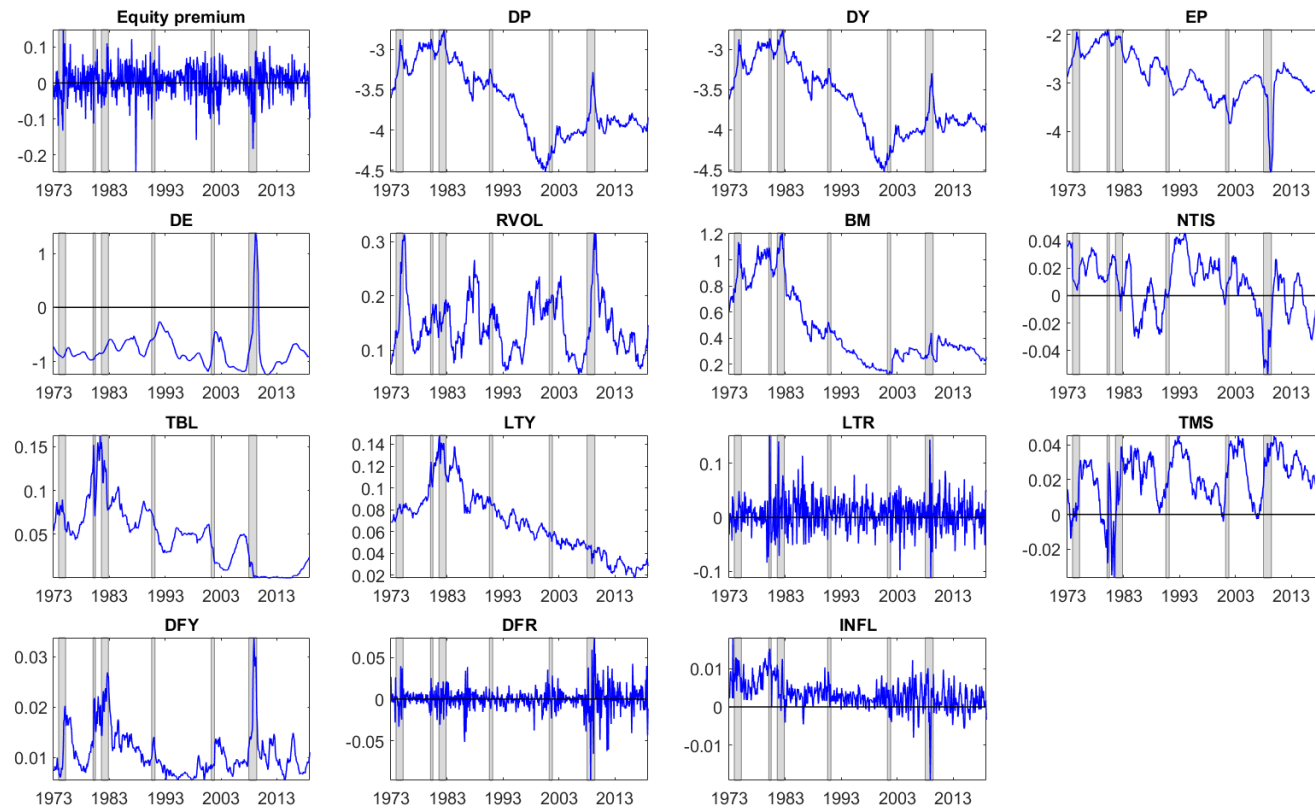


Figure 4: Monthly time series of the equity premium and the predictors



## Appendix 2

The discrete wavelet transform (DWT) multiresolution analysis (MRA) allows the decomposition of a time series into its constituent multiresolution (frequency) components. There are two types of wavelets: father wavelets ( $\phi$ ), which capture the smooth and low frequency part of the series, and mother wavelets ( $\psi$ ), which capture the high frequency components of the series, where  $\int \phi(t) dt = 1$  and  $\int \psi(t) dt = 0$ .

Given a time series  $y_t$  with a certain number of observations  $N$ , its wavelet multiresolution representation is given by

$$y_t = \sum_k s_{J,k} \phi_{J,k}(t) + \sum_k d_{J,k} \psi_{J,k}(t) + \sum_k d_{J-1,k} \psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k} \psi_{1,k}(t) \quad , \quad (7)$$

where  $J$  represents the number of multiresolution levels (or frequencies),  $k$  defines the length of the filter,  $\phi_{J,k}(t)$  and  $\psi_{j,k}(t)$  are the wavelet functions and  $s_{J,k}$ ,  $d_{J,k}$ ,  $d_{J-1,k}$ ,  $\dots$ ,  $d_{1,k}$  are the wavelet coefficients.

The wavelet functions are generated from the father and mother wavelets through scaling and translation as follows

$$\begin{aligned} \phi_{J,k}(t) &= 2^{-J/2} \phi(2^{-J}t - k) \\ \psi_{j,k}(t) &= 2^{-j/2} \psi(2^{-j}t - k) \quad , \end{aligned}$$

while the wavelet coefficients are given by

$$\begin{aligned} s_{J,k} &= \int y_t \phi_{J,k}(t) dt \\ d_{j,k} &= \int y_t \psi_{j,k}(t) dt , \end{aligned}$$

where  $j = 1, 2, \dots, J$ .

Due to the practical limitations of DWT in empirical applications, we perform wavelet decomposition analysis here by applying the maximal overlap discrete wavelet transform (MODWT). The MODWT is not restricted to a particular sample size, is translation-invariant so that it is not sensitive to the choice of the starting point of the examined time series, and does not introduce phase shifts in the wavelet coefficients (so peaks or troughs in the original time series are correctly aligned with similar events in the MODWT MRA). This last property is especially relevant in the forecasting exercise.

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