

Bems, Rudolfs; Kikkawa, Ayumu Ken

Working Paper

Measuring trade in value added with firm-level data

NBB Working Paper, No. 378

Provided in Cooperation with:

National Bank of Belgium, Brussels

Suggested Citation: Bems, Rudolfs; Kikkawa, Ayumu Ken (2019) : Measuring trade in value added with firm-level data, NBB Working Paper, No. 378, National Bank of Belgium, Brussels

This Version is available at:

<https://hdl.handle.net/10419/238165>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Measuring trade in value added with Firm-Level Data



Working Paper Research

by Rudolfs Bems and Ayumu Ken Kikkawa

November 2019 No 378

Editor

Pierre Wunsch, Governor of the National Bank of Belgium

Statement of purpose:

The purpose of these working papers is to promote the circulation of research results (Research Series) and analytical studies (Documents Series) made within the National Bank of Belgium or presented by external economists in seminars, conferences and conventions organised by the Bank. The aim is therefore to provide a platform for discussion. The opinions expressed are strictly those of the authors and do not necessarily reflect the views of the National Bank of Belgium.

The Working Papers are available on the website of the Bank: <http://www.nbb.be>

© National Bank of Belgium, Brussels

All rights reserved.

Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.

ISSN: 1375-680X (print)

ISSN: 1784-2476 (online)

Abstract

Global Value Chains have proliferated economic policy debates. Yet a key concept – trade in value added – is likely mismeasured because of sectoral aggregation bias stemming from reliance on input-output tables. This paper uses comprehensive firm-level data on both domestic and international transactions to study this bias. We find that sectoral aggregation leads to overstated trade in value added and, correspondingly, understated import content of gross exports. The economic magnitude of the estimated bias varies from moderate to large – at 2-5 p.p. of gross exports for Belgium and 17 p.p. for China. We study how the interplay between within-sector heterogeneities in firm import and export intensities and firm size determine the magnitude of the sectoral aggregation bias.

JEL classification: E01, F14, L14

Keywords: Global Value Chains, Input-Output tables, Aggregation Bias

Authors:

Rudolfs Bems, International Monetary Fund

– e-mail: RBems@imf.org

Ayumu Ken Kikkawa, Sauder School of Business, University of British Columbia

– e-mail: ayumukikkawa@gmail.com

First version: May 2019. Part of the analyses was done when Kikkawa was at the National Bank of Belgium as a researcher. We thank Cedric Duprez, Rob Johnson, and Heiwai Tang for helpful comments on this paper, JaeBin Ahn for sharing firm-level data sources, Matilde Bombardini for providing the codes to clean the Chinese datasets, Masao Fukui and Parisa Kamali for sharing statistics based on firm-level datasets. We would also like to thank the NBB for access to its datasets and for its assistance. Min Kyeong Baik and Yuting Chen provided excellent research assistance. The views expressed in this study are the sole responsibility of the authors and should not be attributed to the International Monetary Fund, its Executive Board, or its management.

The views expressed in this paper are those of the authors and do not necessarily reflect the views of the National Bank of Belgium or any other institution with which one of the authors are affiliated. All remaining errors are our own.

TABLE OF CONTENTS

1. Introduction	1
2. Framework for measuring trade in value added	4
2.1. Firm-Level vertical specialization measure	4
2.2. Aggregation bias	5
3. Data	8
3.1 Data sources	8
3.2 Construction of variables	9
3.3 Macro representativeness	10
4. Results	13
4.1 Aggregation bias	13
4.2 Direct and indirect contributions	14
5. Application to other countries	18
6. Conclusions	20
References	22
Appendix	24
Additional empirical results	26
Derivations	28
National Bank of Belgium - Working Papers series	33

1 Introduction

The last two decades have seen a rise in global value chains (GVCs). The topic has proliferated economic research both in international trade and macroeconomics. An important component of this research agenda has been the distinction between conventional gross exports and exports of value added, or the domestic value-added content of exports.¹ It is well documented that GVCs drive a wedge between these two trade concepts: the decline in value-added exports relative to gross exports, has been the flip-side of the rise in GVC participation.² The distinction between the two trade measures matters because it feeds into a wide range of economic questions, from the size of the U.S.-China bilateral trade balance to the magnitude of cross-border exposures to Brexit.

However, research in this area is constrained by the quality of data that underpin it. At the level of the macroeconomy, everything we know about a country's participation in GVCs and the trade in value added has been derived from sectoral input-output (I-O) tables. Reliance on I-O tables comes with stark assumptions that can significantly bias the widely-used GVC-related statistics.³

This paper aims to shed light on the bias that is generated when GVCs and value-added trade are measured using I-O table data. We focus on one key issue: I-O tables aggregate out well-documented firm-level heterogeneities in cross-border trade. The empirical regularity of our primary concern is that more export-oriented firms tend to be more reliant on imports (see for example Bernard, Jensen, Redding, and Schott, 2012; Amiti, Itskhoki, and Konings, 2014; Blaum, 2019). To see how aggregating out such heterogeneity can bias IO-based measures of trade in value added, consider a simple economy with two firms in Table 1.

Table 1: Economy with two firms

	Imports	Sales	Exports
Firm 1	50	100	50
Firm 2	10	100	10

An I-O table aggregates both firms and measures the import share of sector's exports as sector's (imported inputs)/(sales) = $(50 + 10) / (100 + 100) = 0.3$. Thus, import content of gross exports is 30% and the remaining 70% are exports of own value-added, i.e., value-added share of gross exports is 0.7. For many countries this share has decreased in recent decades, as they have become increasingly integrated in GVCs.

When the same calculation is implemented at the firm-level, imported input share of sales is $50/100 = 0.5$ for Firm 1 and $10/100 = 0.1$ for Firm 2. Weighting by firms' exports we arrive at the sector's import content in exports of $(50/60) * 0.5 + (10/60) * 0.1 = 0.43$, which is a larger number

¹See for example OECD's TiVA database, which reports statistics on value-added exports.

²See, for example, Hummels, Ishii, and Yi (2001), Johnson and Noguera (2012), Johnson (2014), Koopman, Wang, and Wei (2014), Timmer, Erumban, Los, Stehrer, and de Vries (2014), and Pahl and Timmer (2019).

³See Johnson (2018) for a discussion of this topic.

than the one obtained from the aggregated data. In this case, aggregation leads to understated import content of exports and overstated value-added trade flows. This simple example illustrates a more general concern about the neglect of firm-level heterogeneities: measurement based on sectoral I-O tables may bias downwards the extent of a country's GVC involvement.

Our paper studies this bias at the level of the macroeconomy. Using an accounting framework, we show that the bias can be expressed in terms of three firm-level characteristics—import intensity, export intensity, and size—and can be decomposed into two components. First, as depicted in the two-firm example in Table 1, the bias has a “direct” component that stems from firm-level heterogeneities in their import and export intensities. Second, there is also an “indirect” component where the bias stems from firm-level heterogeneities in their intensities of purchases from other domestic firms. For example, if export intensive firms purchase more from import intensive firms relative to other firms, then sectoral aggregation leads to understated import content of exports.

We apply the framework to Belgium. To do so, we use comprehensive data on firm-level cross-border trade and domestic firm-to-firm sales to measure trade in value added without relying on sectoral aggregation that underpins the construction of conventional I-O tables. We first establish that our dataset is representative of the macroeconomy. We then compute value-added trade measures within our dataset, contrasting the case where aggregation has been imposed versus the case where the variables of interest are constructed directly from firm-level data. Biases are identified by comparing the two cases.

We find that sectoral I-O tables for Belgium overstate the share of value-added exports in gross exports by 2-5 percentage points. To put this result in a context, note that this bias amounts to 20-50% of the 10 percentage point decline in Belgium's value added export share generated by the rise of GVCs over the last two decades. We also find that the “direct” component accounts for the majority of the aggregate bias throughout our sample period. Furthermore, a simple covariance statistic between firm-level import and export intensities can be used to proxy the overall bias. Armed with these findings, we extend our exercise to other countries and find a substantially larger bias of 10-17 percentage points for countries that are more deeply integrated in GVCs, such as China. The paper also finds large variation in the size of sector-by-sector biases, with heterogeneity in firm size contributing significantly to the variation.

It is important to emphasize at the outset that this is a “measurement” paper with “model-free” accounting of cross-border and within-country flows and decomposition exercises that shed light on aggregation biases implicit in measures that are derived using sectorally aggregated data. We see this as an essential—and overdue—step towards accurate measurement of trade in value-added and GVC-related statistics more broadly.

Our paper builds on a literature that studies the aggregation bias by incorporating within-sector heterogeneities into I-O tables. Several papers have estimated the aggregation bias stemming from the presence of a processing trade sector, which represents an acute case of firm-level heterogeneity

in openness to trade. Koopman, Wang, and Wei (2012) differentiate between import intensities of processing/non-processing economic activities and show that in the case of China measures based on the standard sectoral I-O table significantly understate import content of gross exports. De La Cruz, Koopman, Wang, and Wei (2011) report similar results for Mexico. The downward biases estimated by these papers are large, in the range of 10-25 percentage points of gross exports. More recently, statistical authorities in several countries have pursued similar analysis with a systematic treatment of heterogeneities for all sectors.⁴ Kee and Tang (2016) go a step further: authors use firm-level data on sales and cross-border trade combined with I-O table data on sectoral linkages to study the trend and drivers of domestic content of exports in China. They show that oversampling of large firms in China’s I-O table in the presence of firm-level heterogeneities biases the domestic content of exports downwards.

The contribution of our paper relative to this literature is twofold. First, our dataset allows us to bring the measurement of domestic/import content of exports (and the implementation of the Hummels, Ishii, and Yi (2001) VS measure in particular) fully into the realm of firm-level data, thus eliminating the sectoral aggregation bias entirely. In contrast, previous literature has at least partially relied on sectoral I-O tables. Second, our framework provides a joint treatment of heterogeneities in firm size, import intensities, and export intensities in generating the aggregation bias—three firm-level characteristics that can fully characterize the bias. Previous literature, instead, has focused on a specific dimension of heterogeneity (e.g., importer status, engagement in processing trade, firm size).

Our study complements a wider literature on measurement issues with GVC-related statistics that are based on sectoral I-O tables. Feenstra and Jensen (2012) assess the imported input “proportionality” assumption that underlies I-O table construction and argue that this assumption can significantly mismeasure the extent of offshoring. de Gortari (2018) shows that heterogeneities in input use by output destination can be a source of aggregation biases in conventional GVC-related statistics.

The structure of the rest of the paper is as follows: Section 2 lays out the accounting framework for decomposing gross exports into exports of domestic and imported contents. The section also defines and decomposes the aggregation bias and discusses how firm-level heterogeneities can create the aggregation bias. Section 3 describes the Belgian data. Section 4 presents the paper’s findings for Belgium. Section 5 extends our empirical results to other countries. Finally, Section 6 concludes and suggests avenues for future research on the topic.

⁴Hambÿe, Hertveldt, and Michel (2018) for Belgium split I-O table’s sectors based on export intensities and find a small (1 percentage point of gross exports) impact for heterogeneity on the import content of exports. Wu and Sabuhoro (2018) examine heterogeneities in firm size, export status, and ownership and find that standard I-O tables generate a 5 percentage point downward bias in the import content of exports for Canada. Yamano and Webb (2018) lay out a broader ongoing agenda for supply-use and input-output tables to better account for firm-level heterogeneities.

2 Framework for Measuring Trade in Value Added

Because we only have the detailed firm-level data for one country, the paper restricts attention to a measure of value-added exports that can be derived from national I-O tables. We focus on the import content of gross exports (Hummels, Ishii, and Yi, 2001), commonly referred to as the measure of “vertical specialization (VS)”. This measure separates imports and exports generated by vertical specialization from gross exports, with the difference proxying domestic value-added content of exports. More recent value-added trade measures utilize information from global I-O tables.⁵ However, in practice, results based on the VS and more full-fledged measures, such as the value-added to gross exports (VAX), are very similar. For example, Johnson and Noguera (2012) reports that for Belgium, VAX in 2004 is 0.475 and 1-VS is 0.478.⁶ For other countries the difference is equally small.⁷

2.1 Firm-Level Vertical Specialization Measure

We first define the VS measure allowing for heterogeneities at the firm-level and then study how aggregation of these firms to the sector-level may introduce biases. Consider the following notations, with $n, m, l \in N$ denoting firms, $s, k \in S$ denoting sectors, and $i, j \in \{H, R\}$ denoting countries where H stands for Home country and R stands for the rest of the world. $\mathbf{x}_{i,i}^{N \times N}$ is a matrix of firm-to-firm sales flows within country i , and $\mathbf{y}_i^{N \times 1}$ is a vector of firm-level gross outputs in country i . Cross-border flows are denoted by $\mathbf{x}_{H,R}^{N \times 1}$ or $\mathbf{x}_{R,H}^{N \times 1}$, where the former represents Home firms’ exports and the latter represents Home firms’ imports. Denote by tilde the input flows expressed as a share of destination’s gross output: $\tilde{\mathbf{x}}_{i,j}^{N \times N} = \mathbf{x}_{i,j}^{N \times N} \oslash \iota \mathbf{y}_j^{N \times 1}$, where $\iota \equiv \mathbf{1}^{N \times 1}$.

With these notations, the VS measure for Home can be expressed as

$$VS_H = \left[\tilde{\mathbf{x}}_{R,H}^{N \times 1'} \left[\mathbf{I}^{N \times N} - \tilde{\mathbf{x}}_{H,H}^{N \times N} \right]^{-1} \mathbf{x}_{H,R}^{N \times 1} \right] / \left[\iota' \mathbf{x}_{H,R}^{N \times 1} \right]. \quad (1)$$

The VS measure is expressed relative to gross exports, so that the value always falls between 0 and 1. We alternatively also consider the measure in terms of nominal flows, in which we consider the numerator of the above,

$$\begin{aligned} XVS_H &= \iota' \mathbf{x}_{H,R}^{N \times 1} VS_H \\ &= \tilde{\mathbf{x}}_{R,H}^{N \times 1'} \left[\mathbf{I}^{N \times N} - \tilde{\mathbf{x}}_{H,H}^{N \times N} \right]^{-1} \mathbf{x}_{H,R}^{N \times 1}. \end{aligned} \quad (2)$$

⁵The main difference is that these measures account for third-country effects, which by construction are absent in a Belgium versus the rest of the world setting of a national I-O table.

⁶Note that 1-VS is the proxy for value-added exports. To simplify the expressions, this paper focuses instead on the import content of exports, i.e. VS, rather than the domestic value added contents in exports, i.e., 1-VS.

⁷See Table 3 June 2009 working paper version of Johnson and Noguera (2012).

To gain intuition about this measure, we decompose the nominal VS measure in the following way:

$$\begin{aligned}
XVS_H &= \widetilde{\mathbf{x}}_{R,H}' \left[\mathbf{I}^{N \times N} + \widetilde{\mathbf{x}}_{H,H}^{N \times N} + \widetilde{\mathbf{x}}_{H,H}^{N \times N} \widetilde{\mathbf{x}}_{H,H}^{N \times N} + \dots \right] \mathbf{x}_{H,R}^{N \times 1} \\
&= \underbrace{\widetilde{\mathbf{x}}_{R,H}' \mathbf{x}_{H,R}^{N \times 1}}_{XVS_H^{dir}} + \underbrace{\widetilde{\mathbf{x}}_{R,H}' \widetilde{\mathbf{x}}_{H,H}^{N \times N} \mathbf{x}_{H,R}^{N \times 1} + \widetilde{\mathbf{x}}_{R,H}' \widetilde{\mathbf{x}}_{H,H}^{N \times N} \widetilde{\mathbf{x}}_{H,H}^{N \times N} \mathbf{x}_{H,R}^{N \times 1} + \dots}_{XVS_H^{indir}}. \tag{3}
\end{aligned}$$

In a special case when $\left[\mathbf{I}^{N \times N} - \widetilde{\mathbf{x}}_{H,H}^{N \times N} \right]^{-1} = \mathbf{I}$, no domestic inputs are used in production. Hummels, Ishii, and Yi (2001) denote this case as the “direct” case. This assumption simplifies the formula considerably. The resulting measure, XVS_H^{dir} , computes gross exports that represent the *direct* import content of exports within each firm. For the more general case when $\left[\mathbf{I}^{N \times N} - \widetilde{\mathbf{x}}_{H,H}^{N \times N} \right]^{-1} \neq \mathbf{I}$, the additional term XVS_H^{indir} takes into account the impact of domestic input linkages on the import content of exports as imports can be embedded in domestic inputs and exported *indirectly* across domestic firms.

The nominal VS measure XVS_H defined above and its direct and indirect components, XVS_H^{dir} and XVS_H^{indir} , are scalars when they are defined at the country-level. We will also work with the analogous measures defined at the sector-level, where $XVS_H = \sum_s XVS_s$, and study the aggregation bias in these sector-level measures. In particular, the direct component of the nominal VS measure for sector s , XVS_s^{dir} , is defined as

$$XVS_s^{dir} = \sum_{n \in N_s} \frac{x_{R,n}}{y_n} x_{n,R}, \tag{4}$$

where N_s denotes the set of firms in sector s . Analogously, the indirect component of the nominal VS measure for sector s is defined as

$$\begin{aligned}
XVS_s^{indir} &= \underbrace{\sum_{n \in N_s} \sum_m \frac{x_{R,m}}{y_m} \frac{x_{m,n}}{y_n} x_{n,R}}_{XVS_s^{indir1}} + \sum_{n \in N_s} \sum_m \sum_l \frac{x_{R,l}}{y_l} \frac{x_{l,n}}{y_m} \frac{x_{m,n}}{y_n} x_{n,R} + \dots, \tag{5}
\end{aligned}$$

where we denote its first term coming from firms’ first-link suppliers by XVS_s^{indir1} .

2.2 Aggregation Bias

Next we turn to the aggregation bias. Define the “direct” bias as the difference between the measures of XVS_s^{dir} and its analogous measure but with sectoral aggregation, $XVS_s^{bias,dir} = XVS_s^{dir} - XVS_{IO,s}^{dir}$. The computation of the term $XVS_{IO,s}^{dir}$ requires aggregating firm-level variables (sales, exports, and

imports), in line with the conventional practice that utilizes I-O tables:

$$XVS_{IO,s}^{dir} = \frac{\sum_{n \in N_s} x_{R,n}}{\sum_{n \in N_s} y_n} \sum_{n \in N_s} x_{n,R}. \quad (6)$$

Firm-level studies have identified three key dimensions of firm-level heterogeneity: (i) size, (ii) export intensity, and (iii) intensity of imported inputs. The direct bias, $XVS_s^{bias,dir}$, can be further decomposed in terms of firm-level deviations from the “representative” sectoral firm along these three dimensions as follows:

$$\begin{aligned} XVS_s^{bias,dir} &= \sum_{n \in N_s} \Delta \alpha_n^M \Delta \alpha_n^X y_n, \\ \Delta \alpha_n^M &= \alpha_n^M - \bar{\alpha}_s^M \\ \Delta \alpha_n^X &= \alpha_n^X - \bar{\alpha}_s^X \end{aligned} \quad (7)$$

where $\alpha_n^M \equiv x_{R,n}/y_n$ denotes firm’s import intensity and $\alpha_n^X \equiv x_{n,R}/y_n$ denotes export intensity.⁸ For import and export intensities, we define sectoral weighted means as $\bar{\alpha}_s^M = \sum_n x_{R,n} / \sum_n y_n$ and $\bar{\alpha}_s^X = \sum_n x_{n,R} / \sum_n y_n$, and denote firm-level deviations from means as $\Delta \alpha_n^M$ and $\Delta \alpha_n^X$. Equation (7) shows that if all firms in a sector had import intensity of $\bar{\alpha}_s^M$ and export intensity of $\bar{\alpha}_s^X$, then we obtain $XVS_s^{dir} = XVS_{IO,s}^{dir}$ and the sectoral aggregation would not generate any biases.

Equation (7) also implies that the direct bias is the numerator of the weighted covariance between the intensities α_n^M and α_n^X , with weights being firms’ sales.⁹

$$XVS_s^{bias,dir} = \sum_{n \in N_s} y_n \frac{\sum_{n \in N_s} \Delta \alpha_n^M \Delta \alpha_n^X y_n}{\sum_{n \in N_s} y_n} = N_s \bar{y}_s * cov^{w(y_n)}(\alpha_n^M, \alpha_n^X),$$

where we denote the unweighted mean of firms’ sales in sector s by \bar{y}_s . If firms that are import intensive also tend to be export intensive, then the term $XVS_s^{bias,dir}$ becomes positive and the I-O table based VS measure would be downward biased.

Moreover, the bias can be impacted by any systematic variation between trade intensities and firm size. To see this, we decompose the direct bias term in equation (7), into unweighted covariance

⁸See Appendix C.1 for derivation.

⁹See Price (1972).

terms that are easier to interpret. We obtain

$$\begin{aligned}
XVS_s^{bias,dir} &= \bar{y}_s \sum_{n \in N_s} (\alpha_n^M - \bar{\alpha}_s^M) (\alpha_n^X - \bar{\alpha}_s^X) + (\bar{\alpha}_s^X - \tilde{\alpha}_s^X) \sum_{n \in N_s} (\alpha_n^M - \bar{\alpha}_s^M) (y_n - \bar{y}_s) \\
&\quad + (\bar{\alpha}_s^M - \tilde{\alpha}_s^M) \sum_{n \in N_s} (\alpha_n^X - \bar{\alpha}_s^X) (y_n - \bar{y}_s) + N_s (\bar{\alpha}_s^M - \tilde{\alpha}_s^M) (\bar{\alpha}_s^X - \tilde{\alpha}_s^X) \bar{y}_s \\
&\quad + \sum_{n \in N_s} (\alpha_n^M - \bar{\alpha}_s^M) (\alpha_n^X - \bar{\alpha}_s^X) (y_n - \bar{y}_s) \\
&= N_s \bar{y}_s cov(\alpha_n^M, \alpha_n^X) + N_s (\bar{\alpha}_s^X - \tilde{\alpha}_s^X) cov(\alpha_n^M, y_n) + N_s (\bar{\alpha}_s^M - \tilde{\alpha}_s^M) cov(\alpha_n^X, y_n) \\
&\quad + N_s \bar{y}_s (\bar{\alpha}_s^M - \tilde{\alpha}_s^M) (\bar{\alpha}_s^X - \tilde{\alpha}_s^X) + \sum_{n \in N_s} (\alpha_n^M - \bar{\alpha}_s^M) (\alpha_n^X - \bar{\alpha}_s^X) (y_n - \bar{y}_s), \tag{8}
\end{aligned}$$

where the first term contains the unweighted covariance between import and export intensities, the second term relates to the unweighted covariance between import intensity and firms' size, and the third term relates to the unweighted covariance between export intensity and firms' size. The fourth term is a constant, and the last term is a residual coming from the triple covariance term.

Equation (8) demonstrates how the three dimensions of firm heterogeneity—firm size and import and export intensities—interact with each other to create the direct bias. The first term—unweighted covariance of import and export intensities—accounts for the contribution to the bias from systematic firm-level differences in import/export exposures, but switches off interactions with the third heterogeneity—variation in firm size. The remaining four terms of equation (8) account for the impact of variation in firm size on the bias. These four additional terms contain two key adjustments. First, simple import/export covariance needs to be adjusted for interactions with firm size. This is done via the $cov(\alpha_n^M, y_n)$, $cov(\alpha_n^X, y_n)$ and the triple covariance terms. Second, in the presence of firm size heterogeneities, the simple covariance between export and import intensities is based on a distorted average import and export intensities, as captured by the $\bar{\alpha}_s^X - \tilde{\alpha}_s^X$ and $\bar{\alpha}_s^M - \tilde{\alpha}_s^M$ terms. For example, if firm-level trade intensities are positively correlated with firm size, then an unweighted sectoral average for export and import intensities will understate actual sectoral trade intensities (i.e., $\bar{\alpha}_s^X < \tilde{\alpha}_s^X$ and $\bar{\alpha}_s^M < \tilde{\alpha}_s^M$). This distortion will impact $XVS_s^{bias,dir}$. In the examined example, understated sectoral trade intensities will lead to a downward bias in the import content of exports and a corresponding upward bias for trade in value added. While it is difficult to disentangle these two mutually interacting adjustments to the unweighted covariance statistic, $cov(\alpha_n^M, \alpha_n^X)$, one can show that they tend to have opposing impacts on $XVS_s^{bias,dir}$. For example, if the firm size heterogeneity magnifies the bias resulting from unweighted export/import covariance, then the correction for sectoral trade exposure reduces (or even reverses) the bias.

We also characterize the bias in measuring the term XVS_s^{indir1} , the indirect component of the nominal VS measure coming from firms' first-link suppliers. The bias in measuring XVS_s^{indir1} becomes

$XVS_s^{bias,indir1} = XV S_s^{indir1} - XV S_{IO,s}^{indir1}$, where

$$XVS_{IO,s}^{indir1} = \sum_{k \in S} \frac{\sum_{m \in N_k} x_{R,m}}{\sum_{m \in N_k} y_m} \frac{\sum_{n \in N_s} \sum_{m \in N_k} x_{m,n}}{\sum_{n \in N_s} y_n} \sum_{n \in N_s} x_{n,R}. \quad (9)$$

Similarly to the direct bias, the term XVS_s^{indir1} can be decomposed into terms of firm-level deviations from the “representative” I-O firm, along the import and export intensities, and intensities of purchases from other firms. Consider firm n in sector s , and firm m in sector k . Denoting firm n ’s share of purchases from firm m by $\alpha_{m,n} = \frac{x_{m,n}}{y_n}$, and sector s firms’ weighted average share of purchases from a firm in sector k by $\tilde{\alpha}_{k,s} = \frac{1}{N_k} \frac{\sum_{n \in N_s} \sum_{m \in N_k} x_{m,n}}{\sum_{n \in N_s} y_n}$, and the firm-pair deviation from these weighted averages as $\Delta\alpha_{m,n} = \alpha_{m,n} - \tilde{\alpha}_{k,s}$, we obtain the following.¹⁰

$$\begin{aligned} XV S_s^{bias,indir1} = & \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \Delta\alpha_m^M \tilde{\alpha}_{k,s} \tilde{\alpha}_s^X y_n + \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \tilde{\alpha}_k^M \Delta\alpha_{m,n} \Delta\alpha_n^X y_n \\ & + \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \Delta\alpha_m^M \Delta\alpha_{m,n} \tilde{\alpha}_s^X y_n + \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \Delta\alpha_m^M \Delta\alpha_{m,n} \Delta\alpha_n^X y_n. \end{aligned} \quad (10)$$

The first term in equation (10) comes from firm-level heterogeneity in import intensities, α_m^M . If large firms tend to be more import intensive within sectors, then the sum of the firm-level deviations in import intensities relative to the sector’s weighted average, $\sum_m (\alpha_m^M - \tilde{\alpha}_k^M)$, becomes negative. The second term comes from the correlation between export intensities and the intensities of purchases from other firms. If firms that are more export intensive in a sector supply more intensively from other firms, then this term becomes positive. Similarly, the third term comes from the correlation between import intensities and the intensities of purchases from other firms. If firms supply more intensively from firms that are more import intensive, then this term becomes positive. Finally, the last term comes from the interaction between the three intensities. If export intensive firms in a sector supply more intensively from import intensive firms in the supplying sector, then this term becomes positive.

3 Data

3.1 Data Sources

We use information from the National Bank of Belgium (NBB) Business-to-Business (B2B) transactions database. This database records VAT-ID to VAT-ID yearly transactions among all Belgian enterprises in the private non-financial sector in the form of a panel from years 2002 to 2014. As long as the value exceeds 250 Euro, all enterprises in Belgium are required to report their yearly sales to

¹⁰See Appendix C.2 for derivation.

each individual VAT-ID. See Dhyne, Magerman, and Rubinova (2015) for the detailed construction of this dataset.

We combine this information with data from the annual account filings and the international trade dataset. The annual account filings contain the primary sector (NACE Rev. 2, 4-digit), total sales, value added, location (ZIP code) for each Belgian VAT-ID, and other standard balance sheet items. The international trade dataset combines information from the Belgian customs records and the intra-EU trade declarations, and records the values of imported and exported goods at the VAT-ID-country-product (CN 8-digit)-year level.¹¹

We use the VAT-ID as unit of observation. Papers such as Kikkawa, Magerman, and Dhyne (2018) and Tintelnot, Kikkawa, Mogstad, and Dhyne (2018) aggregate VAT-IDs up to the firm-level using ownership filings from the annual accounts and the foreign ownership filings in the Balance of Payments survey, as they focus on pricing decisions or linkage forming decisions across firms. In this paper we focus on constructing VS measures using the firm-to-firm transactions data, and since the sectors of Belgian enterprises' primary economic activities are recorded for each VAT-ID, we keep the VAT-IDs as the unit of observation. That said, in this paper we use the terms enterprises, VAT-IDs, and firms interchangeably.

3.2 Construction of Variables

Here we explain how we construct the VS measure. From equation (2), we need measures of domestic firm-to-firm sales, $\mathbf{x}_{H,H}^{N \times N}$, firm-level measures on domestically absorbed imports, $\mathbf{x}_{R,H}^{N \times 1}$, exports, $\mathbf{x}_{H,R}^{N \times 1}$, and sales, $\mathbf{y}_H^{N \times 1}$.

Domestic firm-to-firm sales, $\mathbf{x}_{H,H}^{N \times N}$, are directly read-off from the firm-to-firm transactions data.¹² When we construct VS measures at sectoral levels, we use information on the primary sector of economic activity of each VAT-ID, and aggregate the domestic transactions to sector-to-sector level. For value added, we use the values reported in the annual accounts in our baseline specification. Each enterprise's total output, $\mathbf{y}_H^{N \times 1}$, is computed as the sum of value added, inputs purchased from other firms, and imports. Using this measure of total output, we also construct firms' outputs that are for capital formation or for final consumption. We identify these as the residual values of firms' output after subtracting their sales to other firms and their exports.

It is worth pointing out that the total output of firms calculated by using the reported value added does not necessarily match the total output reported in the annual accounts. As an alternative measure, we also construct value added measures that are consistent with the output reported in the annual accounts. Value added, in this case, is computed as the residual of firms total output reported in the annual accounts, after subtracting the sum of purchases from other firms and imports. Sales

¹¹See Appendix A.1 for more details of the international trade dataset.

¹²We use all the firm-to-firm transactions recorded in the data. Since we do not directly observe the types of goods that are transacted, we cannot exclude deliveries of capital or final goods in a consistent manner.

for capital formation/final consumption in this case become the value of total output reported in the annual accounts less of their sales to other firms and their exports.¹³

Following the 2008 System of National Accounts (SNA), domestically absorbed goods imports, $\mathbf{x}_{R,H}^{N \times 1}$, exclude re-exports.¹⁴ We identify cross-border trade as re-exports if a Belgian VAT-ID imports and exports the same good in a given year.¹⁵ After excluding re-exports, we reclassify domestically absorbed imports from CN 8-digit level to NACE 4-digit level. We also classify these imports to either imported intermediate goods or to capital/consumption goods, using the Broad Economic Categories (BEC) classification.¹⁶ When we construct VS measures at the sectoral level, domestically absorbed intermediate imports are aggregated up using the imported goods' NACE classification and the importers' NACE classification. Domestically absorbed capital/consumption goods are aggregated up using the imported goods' NACE classification.

Exports of goods, $\mathbf{x}_{H,R}^{N \times 1}$, also exclude re-exports.¹⁷ Analogously to imported goods, we also reclassify CN 8-digit codes to NACE 4-digit codes. When we construct VS measures at the sectoral level, these exports are aggregated using the exported goods' NACE classification and the exporters' NACE classification.

We consider VS measures constructed using VAT-IDs that report positive labor costs. This sample selection removes VAT-IDs that are inactive or VAT-IDs that are mostly self-employed. This reduces the value added covered in the sample by around 8 percent in 2010.

3.3 Macro Representativeness

This section examines macro representativeness of our data. Representativeness is essential, as we aim to study bias in the measurement of *aggregate* value-added trade. We start by comparing with national accounts the nominal levels of key macro aggregates that impact the VS measure, followed by their sectoral composition. Separately, we compare VS measures from a sectorally aggregated version of our firm-level dataset and national I-O tables. WIOD's 2016 vintage, aggregated to Belgium and the-rest-of-the-world, is used as the national accounts' counterpart in all comparisons.

¹³This alternative measure of sales to final demand require the total output reported the annual accounts, and is only well defined for large firms that are required to report the value in the annual accounts. For smaller firms that do not report the total output, we treat the sales to final demand as zero. See footnote 9 of Dhyne et al. (2015) for the reporting thresholds.

¹⁴Re-exports are a specific subcategory of gross exports and imports, which 2008 SNA defines as import/export activities, where the nature of the good does not change and there is change in ownership from a non-resident to resident, thus requiring some treatment by the national accounts and balance of payments statistics. According to 2008 SNA, re-exports should be excluded from gross exports and imports.

¹⁵Re-exports are identified at the VAT-ID-CN8-product level, and defined as the minimum of the value of exports and the value of imports.

¹⁶See Appendix A.2 for details.

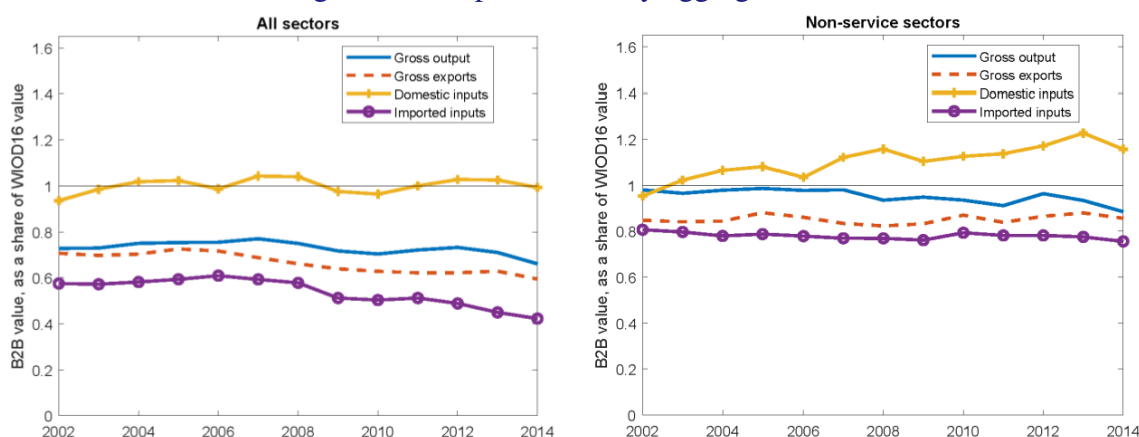
¹⁷For consistency, identified re-exports are also excluded from firms' sales.

Aggregate variables

Figure 1 compares the four key variables that enter the VS calculation in equation (2): gross output, gross exports as well as intermediate domestic and imported non-service inputs. We report the comparison separately for non-service sectors and the aggregate economy.¹⁸

Results reveal that for non-service sectors we can broadly match the nominal levels of macro variables.¹⁹ Any deviations are within a 20% range. When services are included, the fit deteriorates. This is to be expected, as our data does not cover cross-border trade in services and likely underreports aggregate economic activity in the service sector. We find that representativeness is broadly stable over time.²⁰

Figure 1: Comparison of key aggregate series



Sectoral shares

A representative aggregate series can potentially hide sectoral disparities. To address this concern, Figure 2 compares sectoral composition for non-service sectors of economic activity in the WIOD and our firm-level data for year 2010, using the most detailed sectoral breakdown in WIOD.²¹ Despite significant deviations for some sectors, overall, we find that sectoral shares for the four variables of interest are highly correlated. Correlations for other sample years are equally strong, with the mean correlation of 0.88.²²

¹⁸Re-exports are not separately identified in WIOD, which lumps them together with gross trade (see discussion on page 21 in Timmer et al., 2012 and Appendix B in Timmer et al., 2016). Hence, to compare our firm-level data with WIOD, we add identified re-exports back to the firm-level imports, exports and sales.

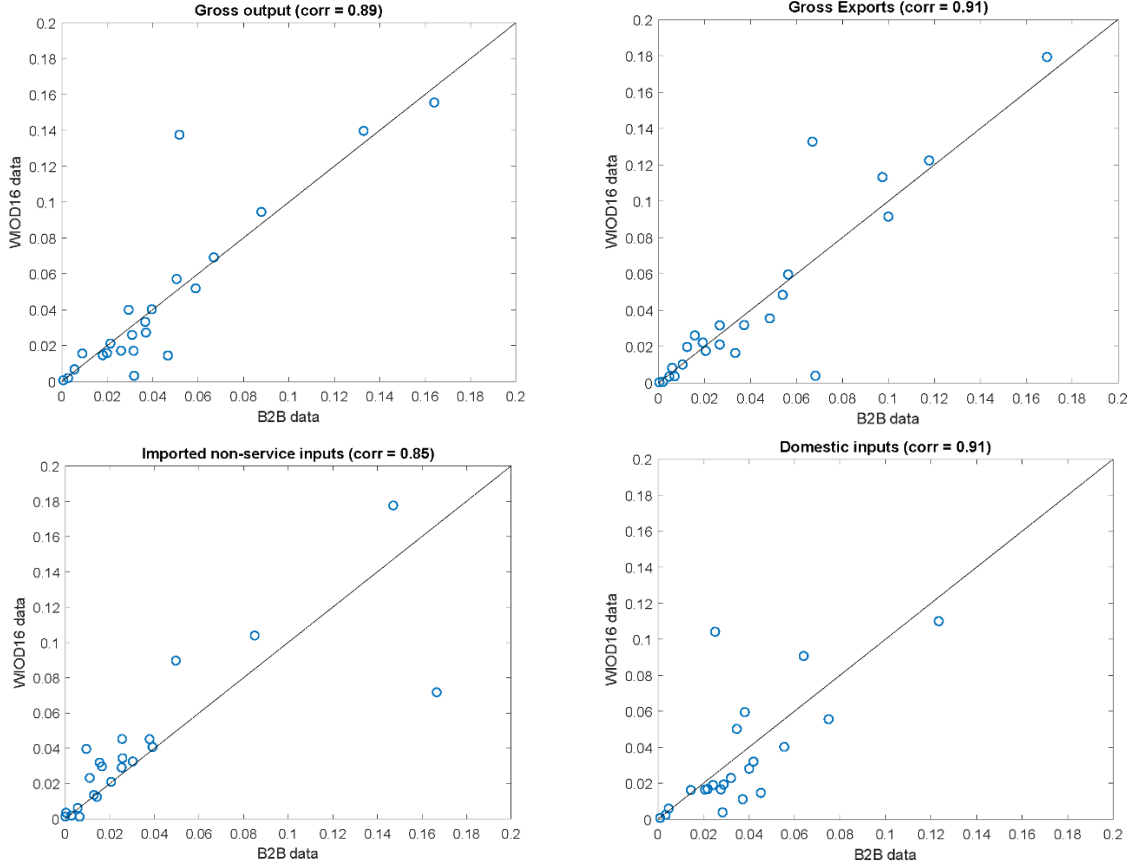
¹⁹We obtain similar results when using alternative measures of value added explained on page 9. See Appendix B.1.

²⁰Intermediate (domestic and imported) input comparison for subsectors can in principle be based on the source or the destination of the input. In Figure 1 results for imported inputs are reported by source (i.e., imports of non-service inputs only), while for domestic inputs results are reported by destination (i.e., domestic inputs into non-service sectors only). The fit deteriorates for other subsets in the input matrix. This is to be expected, because of the more limited coverage of the service sector inputs and activities, and the lack of data on imports of services.

²¹Firm-level data is aggregated accordingly by mapping NACE 2-digit sectors into WIOD's sectoral definitions.

²²Again, we obtain similar results when using alternative measures of value added, with results reported in Appendix B.1.

Figure 2: Comparison of sectoral shares in non-service sectors, 2010



VS measure based on sectoral aggregates

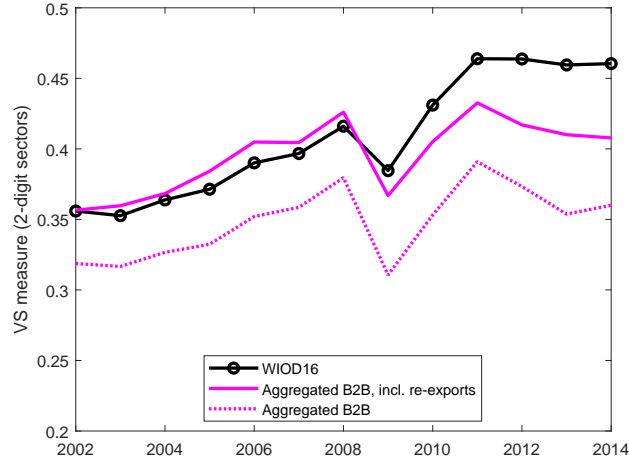
Figure 3 replicates the VS measure using our firm-level data by first aggregating data to the WIOD's 56 sectors and then calculating the VS measure for 2002-2014. The resulting comparison simultaneously relies on all input variables at the sectoral level and can be interpreted as a summary statistic for the macro representativeness of the firm-level dataset. As was the case with macro series in Figure 1, comparison of the VS measure with WIOD requires that re-exports be added back to the firm-level imports, exports and sales.

Results show that, when sectorally aggregated, firm-level data capture the gradual rise in the WIOD-based VS measure during 2002-2008, as well as the relative stagnation after 2010, although some discrepancies emerge between the two series in the latter part of the sample. For completeness, the figure also reports the VA measure based on our baseline data that excludes re-exports, as required by 2008 SNA. As expected, re-exports shift the VS measure upwards level-wise, as by definition import content of re-exports is close to 100 percent (i.e., VS measure is close to 1).

It is worth stressing that the objective of the comparison in Figure 3 is to establish representativeness. The identification of the aggregation bias in the measurement of the VS is based exclusively on the firm-level dataset. Hence, deviations in the VS measure between the I-O table data and the firm-level dataset in Figure 3 do not impact the identified aggregation bias.

Overall, the firm-level dataset is representative of the macroeconomy and provides a suitable laboratory for studying the impact of the aggregation bias on the measurement of trade in value added.

Figure 3: Comparison of the VS measure (based on 2-digit sector data)



4 Results

This section applies the Belgian data to assess the size of the bias in the VS measure resulting from sectoral aggregation of firm-level data. The quantification is implemented using the value-added trade measurement framework of Section 2.

4.1 Aggregation Bias

Macro-level results for the role of the aggregation bias are reported in Figure 4, which compares the VS measure calculated directly from firm-level data (equation (1)) with the same measure computed by first aggregating firm-level data into sectors and then applying the VS formula.²³ As expected, we find that aggregating out firm-level heterogeneities leads to an *underestimate* of the import content of gross exports. In the figure this result is captured by the persistently lower VS measure when computed using 2-digit sectoral data. A key implication for our paper is that value-added exports (i.e., $1 - VS$) are *overestimated*.²⁴

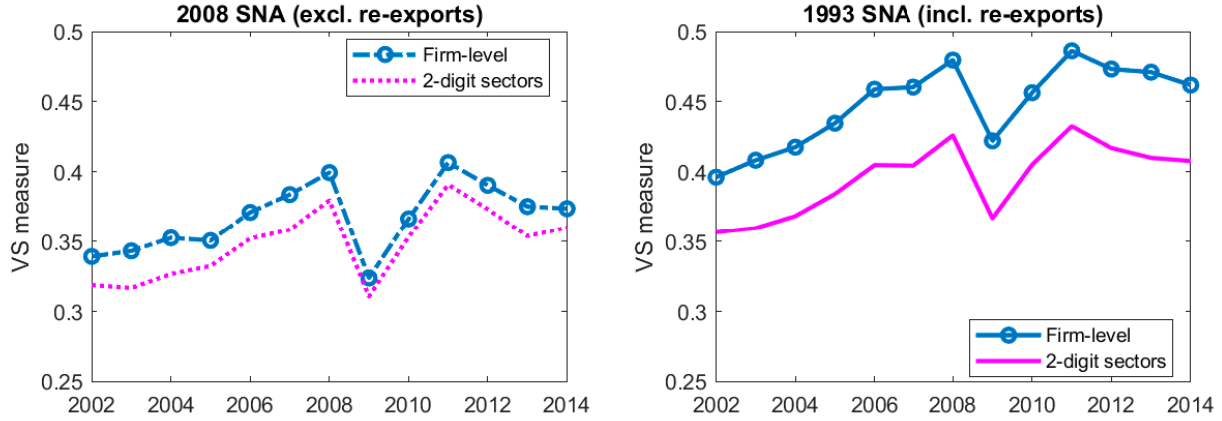
In terms of magnitudes, estimates based on firm-level data suggest that on average over the sample period for each 1 euro of gross exports, 37 cents represent import content (see dashed line in left-side panel in Figure 4) and the remaining 63 cents are value-added exports. Aggregating out firm level heterogeneities (dotted line), as done by I-O tables, on average, reduces the import content of

²³In line with conventional global I-O tables, NACE Rev.2 two-digit sectoral definitions (divisions) were used in the aggregation. To facilitate comparability with WIOD, the NACE two-digit sectors were further aggregated to match the WIOD's 56 sectors.

²⁴The convention used in this paper labels this bias as *positive*, based on the definitions of the bias in Section 2.

gross exports by 2 cents and, correspondingly, overstates the value-added exports by 2 cents (i.e., an increase to 65 cents). The size of the bias is stable over time, varying in the range of 1.3-2.6 percentage points of gross exports during the 2002-2014 period.

Figure 4: Aggregation bias in the VS measure



Two pertinent findings warrant a discussion. First, the size of the bias increases when re-exports are added back to the identified firm-level imports, exports, and sales. While this modification deviates from the 2008 SNA, the inclusion of re-exports is more consistent with historical (1993 SNA-based) national accounts and I-O table compilation practices and therefore provides a more relevant estimate of the aggregation bias for I-O table based statistics on trade in value added. With re-exports included, the average bias increases from 2 to 5 cents for each euro of gross exports (see right-side panel in Figure 4).²⁵

Second, an obvious course of action to reduce the bias would be to increase sectoral detail. Available data allows us to examine the extent to which the bias in Figure 4 can be reduced via this avenue. We find that zooming in from 56 2-digit sectors used by WIOD to 250 3-digit NACE sectors leaves the aggregation bias essentially unchanged, reduced it by mere 5%. A breakdown of sectoral data into 600 4-digit NACE sectors reduces the aggregation bias in Belgian data by around 50%. Thus, an increase in sectoral detail to beyond 250 sectors (e.g., 4-digit NACE breakdown) would be required to start reducing the aggregation bias through increased sectoral detail.²⁶

4.2 Direct and Indirect Contributions

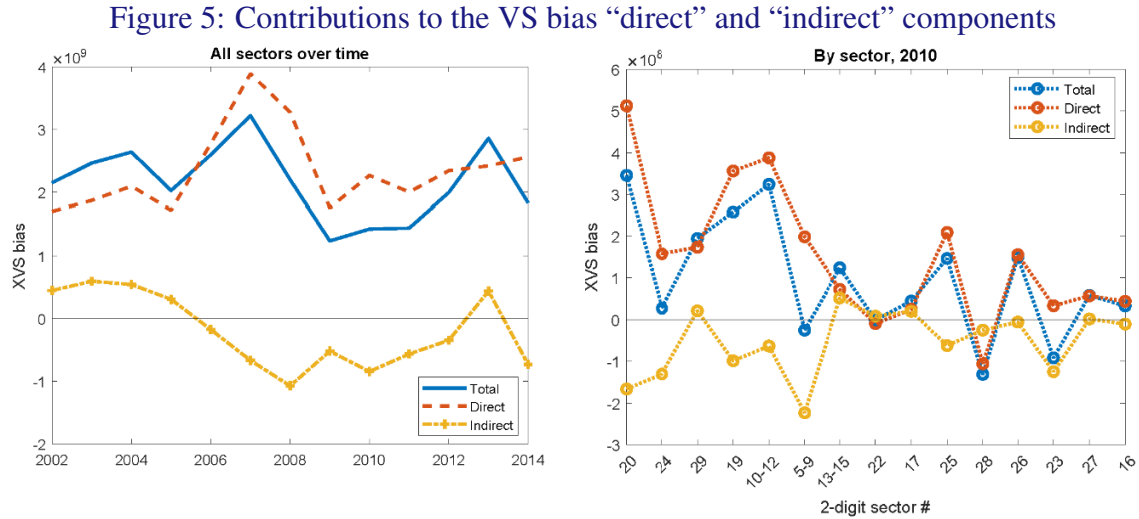
To shed more light on the bias, Figure 5 decomposes it into the direct and indirect components (see equations (7) and (10)), using the baseline data that excludes re-exports.²⁷ The bias plotted in the figure is identical to the one in Figure 4, but expressed in nominal terms, XVS_s . The left panel

²⁵Utilizing a proxy identified in the next section, Appendix C.3 uses the accounting framework of Section 2 to argue that the inclusion of re-exports systematically increases the aggregation bias.

²⁶The findings are similar if re-exports are included in firm-level data.

²⁷Results are broadly similar when re-exports are included.

reports the evolution of the direct and indirect components of the bias over time, $\sum_s XVS_{s,t}^{bias,dir}$ and $\sum_s XVS_{s,t}^{bias,indir}$, focusing on the manufacturing part of the economy, which generates 84% of Belgian import content of gross exports.²⁸ The right panel shows the decomposition by 2-digit manufacturing sectors in a representative year, $XVS_{s,2010}^{bias,dir}$ and $XVS_{s,2010}^{bias,indir}$, with sectors ranked, left-to-right, by their contribution to the overall import content of gross exports in Belgium.



We find that the direct component of the bias closely mimics the overall bias. It accounts for 109% of the aggregate bias over the sample period, while the contribution of the indirect component, at -9%, is not systematically different from zero. The positive bias is present in the majority of sectors. Both in sectors that are the main generators of import content of gross exports in Belgium (for example, NACE Rev. 2 division 20: manufacture of chemicals), as well as in classic GVC sectors (i.e., NACE Rev. 2 division 29: manufacture motor vehicles), the dominant contribution stems from the direct component of the bias. For the median 2-digit manufacturing sector in 2010 the contribution of the direct component is 94%. The indirect component is not accounting for any sizable systematic bias across sectors, with a 6% contribution for the median manufacturing sector.

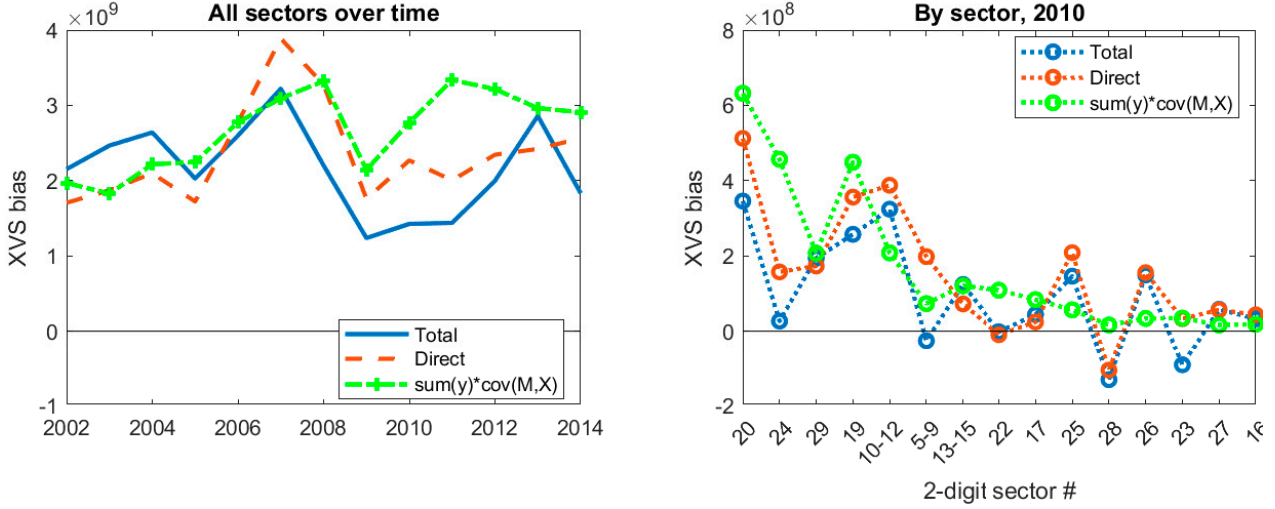
Characterizing the Direct Bias

These findings suggest that a further examination of the direct component can help our understanding of the overall bias. As shown in equation (7), the direct component of the overall bias is a sales-weighted covariance of firm-level export and import intensities. This covariance can be usefully decomposed further into (i) a simple *unweighted* covariance of export and import intensities and (ii) a set of residual terms that capture the impact of the interplay between firms' import/export intensities and firms' size on the bias (see equation (8)). Figure 6 conveys the extent to which the simple covariance term can approximate the direct bias term. As a benchmark, the figure also reports

²⁸Based on WIOD.

the size of the total bias.

Figure 6: Weighted and unweighted covariance terms



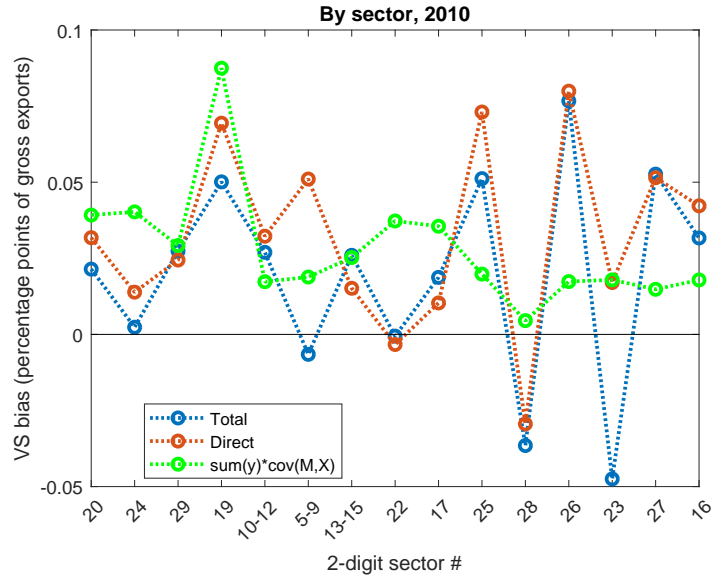
We find that the simple firm-level covariance of import and export intensities can closely capture the *aggregate* direct bias, as well as the total bias (left-side figure). However, it does less well at capturing the variation of the bias *across sectors* (right-side figure). The differences between the direct bias term and the simple covariance term across sectors (i.e., gaps between red and green lines in right-side figure) are driven by two countering forces captured by the residual terms in equation (8), and stemming from the positive covariance between trade intensities and firm size. On the one hand, when firm size co-varies with trade intensities, the bias increases. Not only exporters import more, but they are also larger in size. Sectoral aggregation averages out these three co-varying heterogeneities—import intensity, export intensity, and firm size. In the decomposition this effect is captured by the positive covariance terms between trade intensities and firm size.

On the other hand, the *simple* unweighted export-import covariance term understates actual import and export intensities of a sector. The residual terms in equation (8) correct for this problem by using sales-weighted trade intensities instead, which are larger when firm size co-varies positively with trade intensities ($\bar{\alpha}_s^M < \tilde{\alpha}_s^M$ and $\bar{\alpha}_s^X < \tilde{\alpha}_s^X$). Higher sectoral trade intensities, in turn, increase import content as a share of gross exports (i.e., the VS measure), reducing the bias, or even reversing it for some sectors.

Additional insights into sectoral results can be obtained by recasting the bias in a more familiar form as percentage points of sectoral gross exports, thus emphasizing sectors with high VS measures rather than high nominal values of import content of gross exports (see Figure 7). We find that the small aggregate bias (at 1.3 percentage points of gross exports in 2010) hides sizable sectoral variation. For example, 2-digit sectors that are commonly associated with supply chains (sectors 29, 26, 27) show elevated biases in the 0.03-0.08 range. At the same time, several sectors have negative biases. Consistent with results in Figure 6, the simple covariance term (i.e., green line) captures only

a fraction of the sectoral variation in the VS measure.

Figure 7: Direct bias by sector

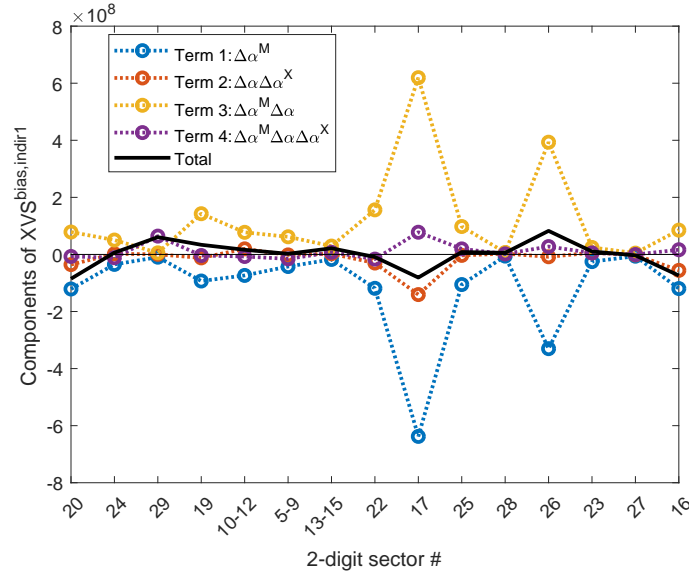


Characterizing the Indirect Bias

The indirect bias term accounts for a small fraction of the total bias. To characterize how the biases are generated, Section 2 focused on the first term of the indirect bias, and in Figure 8 we apply equation (10) to the data.²⁹ The four terms largely cancel each other out. Notably, the first term is always negative, as large firms tend to be import intensive (Tintelnot, Kikkawa, Mogstad, and Dhyne, 2018). We also find that across sectors, firms source more intensively from firms that are more import intensive as the third term is consistently positive. The second and the fourth terms tend to be smaller in magnitudes and exhibit both positive and negative signs, indicating that firms that are more export intensive are not always ones sourcing more intensively from other firms.

²⁹In Appendix B.2 we plot the fraction that the first indirect terms explain out of the total bias.

Figure 8: Components of the first indirect term of the bias



Notes: The four dashed lines correspond to the four terms on the RHS of equation (10), and the solid black line corresponds to the sum of the four terms.

In sum, our two key findings of this section are that (i) the direct bias accounts for the bulk of the overall aggregation bias and (ii) the bias can vary significantly across sectors. A simple covariance of export and import intensities can closely trace the aggregate bias, but captures only a fraction of the variation in the bias across sectors. Nevertheless, we find the simple covariance proxy useful, as it allows one to decompose the bias into economically meaningful contributing factors.

5 Application to Other Countries

A key limitation of our findings so far is that they are limited to one country—Belgium. Even if Belgium is viewed as a representative small open economy, it would be important to know how the estimated bias differs with the extent and nature of a country’s participation in global supply chains.

Unfortunately, the full-fledged approach implemented for Belgium is very data intensive and, to the best of our knowledge, comparable data are not publicly available for other countries of interest. To extend the paper’s methodology to other countries, this section leans on the finding that the direct component dominates the aggregation bias for Belgium. We assume that this is the case also for other countries and estimate the direct bias for a larger set of countries for which we could find firm-level data on sales, imports, exports, and the main sector of economic activity.

Following the approach for Belgium, we first examine data of each country for macro representativeness in terms of macro variables and the VS measure. The size of the direct component of the aggregation bias is then computed by applying equation (7). As we could not identify re-exports,

results refer to the case with re-exports included. Furthermore, to ensure cross-country comparability of findings, we focus on the manufacturing sector, as the firm-level data in some countries were limited to manufacturing establishments.³⁰

A summary of findings for countries with available data is reported in Table 2. We start by noting that results for Belgium are in line with findings of the previous section—limiting the focus to the manufacturing sector leaves the size of the direct bias, $VS_H^{bias,dir}$, broadly unchanged at 4 p.p. of gross exports. Beyond Belgium, results reveal large variation in the size of the direct aggregation bias across countries. The very sizable aggregation bias for China is comparable to results reported in Koopman, Wang, and Wei (2012), where the authors, using a different methodology, estimate the direct aggregation bias for China to be in the range of 16-29 p.p.³¹ We also find a sizable aggregation bias for Vietnam, at 10 p.p., which is another country with a significant supply chain participation. On the other end of the spectrum, based on a limited time frame of 2010-11, our estimates suggest close-to-zero bias in the case of Chile, which exports mostly commodities.³²

Table 2: The Size of Direct Aggregation Bias in Select Countries

	Time frame (1)	Direct VS measure (2)	Bias (in p.p.) (3)
Belgium	2002-14	0.36	4
Chile	2010-11	0.10	0
China	2000-07	0.39	17
India	2014-15	0.18	3
Indonesia	1992-96	0.17	3
Korea	2006-17	0.23	2
Latvia	2006-12	0.30	4
Vietnam	2011-13	0.45	10

Notes: For each country column 1 reports the time frame used for calculating the results; column 2 reports the average values of the direct VS measure, VS^{dir} ; column 3 reports the average direct VS bias, $VS^{bias,dir}$, where a positive value implies that sectoral aggregation biases the direct VS measure downwards (see Section 2.2 for definition). All calculations based on firm-level data.

Overall, these findings suggest that in countries with elevated supply chain participation the aggregation bias can significantly distort the import content of exports, trade in value added and other GVC-related statistics.

³⁰See Appendix A.3 for the descriptions of data sources.

³¹See Table 2 in Koopman, Wang, and Wei (2012).

³²Additional results on the size of sector-by-sector bias and the performance of a simple covariance statistic for import-export intensities as a proxy for the bias are available from the authors upon request. Details of the representativeness tests are also available.

6 Conclusions

This paper studies the aggregation bias in widely-used GVC measures. It would be preferable for measures such as trade in value added to be constructed using firm-level data. In practice, however, sectorally aggregated data, such as I-O tables, are used. The problem with this practice is that sectorally aggregated data can bias the GVC measures.

We start by defining and deriving the aggregation bias conceptually. Focusing on the VS measure (Hummels, Ishii, and Yi, 2001), we show that the bias has direct (capturing firms' "direct" import and export engagements) and indirect (capturing firms' "indirect" exposure to cross-border trade via other domestic firms) components. The direct component can be further decomposed into a simple covariance term for export and import intensities and a residual that accounts for heterogeneities in firm size and the interaction between trade intensities and firm size. A decomposition of the indirect component is also provided.

The paper then applies these concepts to Belgium by comparing the VS measure derived from firm-level data with the same measures derived from data that has been sectorally aggregated. We find that in the case of Belgium the bias in the VS measure, computed from sectorally aggregated data, is at 2-5 percentage points of gross exports. Furthermore, the bias is mostly stemming from its direct component, while the contribution of the indirect component is on average close to zero. A simple covariance statistic for import and export intensities, capturing positive correlation between imports and exports across firms within sectors, is found to be a good proxy for the aggregate bias. However, this proxy does not account for the variation in the bias for sector-specific VS measures. We show that to account for sector-by-sector biases, the interplay between heterogeneities in trade intensities and firm size needs to be taken into account, even though these additional firm size considerations broadly cancel out for the aggregate VS bias.

Several takeaways from this paper are relevant for future work on this topic. First, we show that the indirect component of the aggregate VS bias does not contribute sizably to the overall bias in case of Belgium, including in classic supply chain sectors. We see this as an encouraging news for the measurement of supply chains directly from firm-level data, as it suggests that firm-level imports, exports and sales are sufficient to derive a good proxy for the VS measure. In the penultimate section of the paper we pursue this approach and compute the "direct" biases for multiple countries and find that there are large heterogeneities in the magnitudes of the bias across countries. This proposed approach offers several advantages over the current practice based on I-O tables. Yearly firm-level microdata are becoming increasingly available in many countries, and our approach allows one to compute more up-to-date estimates of the VS measure relative to time lags that comes with I-O tables. The approach would also avoid sectoral aggregation biases, and might be preferable to the alternative of incorporating firm-level heterogeneities to the I-O tables. An extension of our analysis to other countries is needed to assess the broader applicability of this proposal.

Another takeaway from the paper is that aggregation biases might be particularly concerning for sectoral GVC measures. We find that for Belgium, despite the small overall bias, biases in sectoral measures can be sizable. Thus, one needs to be particularly careful when interpreting empirical findings that are based on sectoral GVC measures, more so than on aggregate measures.

Lastly, we shed light on the role of re-exports in generating the aggregation bias. Results for Belgium reveal that the inclusion of re-exports significantly increases the bias. One implication of this finding is that, as countries gradually switch from the 1993 SNA to 2008 SNA standards, the aggregation bias decreases. This switching can be misinterpreted in IO-based GVC-related statistics as an increase in the import content of exports over time (i.e., a continued rise in global supply chains) if the change in the aggregation bias is not accounted for.

References

- AMITI, M., O. ITSCHKOKI, AND J. KONINGS (2014): “Importers, exporters, and exchange rate disconnect,” *American Economic Review*, 104, 1942–1978.
- BERNARD, A. B., J. B. JENSEN, S. J. REDDING, AND P. K. SCHOTT (2012): “The empirics of firm heterogeneity and international trade,” *Annual Review of Economics*, 4, 283–313.
- BLAUM, J. (2019): “Global Firms in Large Devaluations,” *mimeo*.
- DE GORTARI, A. (2018): “Disentangling Global Value Chains,” *mimeo*.
- DE LA CRUZ, J., R. B. KOOPMAN, Z. WANG, AND S.-J. WEI (2011): “Estimating Foreign Value-Added in Mexico’s Manufacturing Exports,” *USITC Office of Economics Working Paper*.
- DHYNE, E., G. MAGERMAN, AND S. RUBINOVA (2015): “The Belgian production network 2002-2012,” *National Bank of Belgium Working Paper Series*, 288.
- FEENSTRA, R. C. AND J. B. JENSEN (2012): “Evaluating estimates of materials offshoring from US manufacturing,” *Economics Letters*, 117, 170–173.
- HAMBÛE, C., B. HERTVELDT, AND B. MICHEL (2018): “Value Chain Integration of Export-Oriented and Domestic Market Manufacturing Firms,” *Federal Planning Bureau Working Paper*.
- HUMMELS, D., J. ISHII, AND K. M. YI (2001): “The nature and growth of vertical specialization in world trade,” *Journal of International Economics*, 54, 75–96.
- JOHNSON, R. C. (2014): “Five Facts about Value-Added Exports and Implications for Macroeconomics and Trade Research,” *Journal of Economic Perspectives*, 28, 119–142.
- (2018): “Measuring Global Value Chains,” *Annual Review of Economics*, 10, 207–236.
- JOHNSON, R. C. AND G. NOGUERA (2012): “Accounting for intermediates: Production sharing and trade in value added,” *Journal of International Economics*, 86, 224–236.
- KAMALI, P. (2019): “Indirect Exporter Dynamics and the Impact of Trade Liberalization,” *mimeo*.
- KEE, H. L. AND H. TANG (2016): “Domestic Value Added in Exports: Theory and Firm Evidence from China,” *American Economic Review*, 106, 1402–1436.
- KIKKAWA, A. K., G. MAGERMAN, AND E. DHYNE (2018): “Imperfect Competition in Firm-to-Firm Trade,” *mimeo*.
- KOOPMAN, R., Z. WANG, AND S. J. WEI (2012): “Estimating domestic content in exports when processing trade is pervasive,” *Journal of Development Economics*, 99, 178–189.

- (2014): “Tracing value-added and double counting in gross exports,” *American Economic Review*, 104, 459–494.
- MA, Y., H. TANG, AND Y. ZHANG (2014): “Factor Intensity, product switching, and productivity: Evidence from Chinese exporters,” *Journal of International Economics*, 92, 349–362.
- PAHL, S. AND M. P. TIMMER (2019): “Patterns of Vertical Specialisation in Trade: Long-run Evidence for 91 Countries,” *Review of World Economics*, 155, 459–486.
- PRICE, G. R. (1972): “Extension of covariance selection mathematics,” *Annals of Human Genetics*, 35, 485–490.
- TIMMER, M. P., A. ERUMBAN, R. GOUMA, B. LOS, U. TEMURSHOEV, G. J. DE VRIES, I. ARTO, A. GENTY, F. NEUWAHL, J. M. RUEDA-CANTUCHE, A. VILLANUEVA, J. FRANCOIS, O. PINDYUK, J. POSCHL, R. STEHRER, AND G. STREICHER (2012): “The World Input-Output Database (WIOD): Contents, Sources and Methods,” *mimeo*.
- TIMMER, M. P., A. A. ERUMBAN, B. LOS, R. STEHRER, AND G. J. DE VRIES (2014): “Slicing Up Global Value Chains,” *Journal of Economic Perspectives*, 28, 99–118.
- TIMMER, M. P., B. LOS, R. STEHRER, AND G. J. DE VRIES (2016): “An Anatomy of the Global Trade Slowdown based on the WIOD 2016 Release,” *GGDC Research Memorandum*, 162.
- TINTELNOT, F., A. K. KIKKAWA, M. MOGSTAD, AND E. DHYNE (2018): “Trade and Domestic Production Networks,” *mimeo*.
- WU, W. AND J. B. SABUHO (2018): “Extended Supply-Use Tables and Trade in Value Added Measures for Canada: Preliminary Estimates, Issues and Challenges,” *mimeo*.
- YAMANO, N. AND C. WEBB (2018): “Future Development of the Inter-Country Input-Output (ICIO) Database for Global Value Chain (GVC) and Environmental Analyses,” *Journal of Industrial Ecology*, 22, 487–488.
- YU, M. AND W. TIAN (2012): “China’s Processing Trade: A Firm-level Analysis,” in *Rebalancing and Sustaining Growth in China*, ed. by H. McKay and L. Song, ANU Press.

A Data appendix

A.1 International trade dataset

The international trade dataset contains information from the Belgian customs records which cover extra-EU trade, and the intra-EU trade declarations. The customs records cover all extra-EU imports and exports at the VAT-ID level with values higher than 1,000 Euro or with weights bigger than 1,000kg. As for the intra-EU trade, for years prior to 2006, the dataset covers all imports and exports by Belgian enterprises whose combined imports from other EU countries that are more than 250,000 Euro a year. For years 2006 onward, the thresholds for exports and imports changed to 1,000,000 Euro and 400,000 Euro, respectively. The reporting threshold for intra-EU imports was changed to 700,000 Euro per year in 2010.

A.2 Classifying imported goods

The raw international trade dataset comes at the CN 8-digit level. We map these codes to NACE Rev. 2, 4-digit level, and also to either intermediate goods or capital/consumption goods.

To map CN 8-digit classifications to NACE 4-digit classifications, we first convert the CN codes to Harmonized System (HS) 6-digit codes. The CN codes are constructed so that their first 6 digits are identical to the contemporary HS codes. We then convert these HS codes to Classification of Products by Activity (CPA) 2008 codes, and then final these CPA codes to NACE Rev. 2 codes using the fact that CPA 2008 codes are identical to NACE Rev.2 codes up to the first 4 digits.

To map CN 8-digit classifications to intermediate or capital/consumption goods, we use the Broad Economic Categories (BEC) classification. First we convert the contemporary CN 8-digit codes to HS 2002 6-digit codes, and reclassify them to BEC Rev. 4 codes. The BEC Rev. 4 codes provide us with a broad classification of whether the good is used as intermediate goods, or for capital formation, or for final consumption. Using this information, we classify each CN 8-digit code to goods for intermediate goods, or to goods for capital/consumption goods.

A.3 Data sources of Section 5

Here we briefly describe the data sources for the analyses done in Section 5. We obtain firm-level data on sales, imports, exports, and their main sector at the 2-digit level. We focus our attention to manufacturing firms, since some datasets only provide representative sample for the manufacturing sectors. Results on the representativeness tests for these countries are available from the authors upon request.

Chile

Our data source for Chile is the plant-level panel data from the Encuesta Nacional Industrial Anual (ENIA). The ENIA contains survey data on all manufacturing establishments of more than 10 employees, including data on exports and imports. We use the ISIC Revision 3 for sectoral classification.

China

For China we merge two datasets. One is the manufacturing survey data from the National Bureau of Statistics (NBS) that covers all state-owned firms and non-state-owned firms with sales above 5 million Chinese yuan. This data contains information on firms' sales, export values, and their main sector (Classification of National Economy Industries). However, the data does not record firms' imports. Therefore we merge the NBS survey with the Chinese customs data, using information on firms' names as identifiers.

As is well-known (see for example, Yu and Tian, 2012 and Ma et al., 2014), the merged data contains some discrepancies, which we address through two additional adjustments. First, we drop firms that report positive exports in the NBS survey but are not matched with the customs data. Second, for firms that are matched in the two datasets but show different total export values in the NBS survey and the customs data, we take the export values in the NBS survey, and recover their import values as the export values in the NBS survey multiplied by the ratio of imports over exports in the customs data.

India

We use the Annual Survey of Industries (ASI) for India. The ASI survey covers all manufacturing establishments with over 100 workers, and smaller establishments are typically surveyed every three to five years. We adjust for this sample selection issue by using sampling weights for smaller establishments. We use India's National Industrial Classification (NIC) codes for establishments' sectoral classification.

Indonesia

The data for Indonesia comes from the Manufacturing Survey of Large and Medium-sized firms (Survei Industri). The survey covers all large and medium manufacturing firms with twenty or more employees. We use the Indonesia Standard Commodity Classification (Klasifikasi Komoditi Indonesia, KBKI) for firms' sectoral classification.

Korea

The Korean data is from the Survey of Business Activities (SBA) from Statistics Korea. The SBA covers all firms with 50 or more employees and 300 million won or greater capital, each year. The SBA follows the Korean Standard Industry Classification for industry classification.

Latvia

Results are based on merged data covering firms' financial statements and customs activities, obtained from the National Statistical Office. Resulting dataset includes a universe of manufacturing firms.

Vietnam

Firm-level data is from the Vietnamese Enterprise Survey, conducted by the General Statistics Office of the ministry of Planning and Investment of Vietnam. The survey includes data on cross-border trade flows. All firms that are at least 50% state-owned as well as all foreign-owned firms are included. For other private enterprises, all firms with at least 20 employees are included, while a random sample is chosen for firms with less than 20 employees. For further details see Kamali (2019).

B Additional empirical results

B.1 Alternative value added measures

In our baseline, our measure of value added is what firms report in their annual accounts. As an alternative, we also consider an alternative measure of value added that is consistent with the output reported in the annual accounts. Value added in this case, is computed as the residual of firms total output reported in the annual accounts, after subtracting the sum of purchases from other firms and imports. Using these measures, we plot in Figures 9 and 10 the analogous figures from Figures 2 and 2 in the main text.

Figure 9: Comparison of key aggregate series (alternative value added measure)

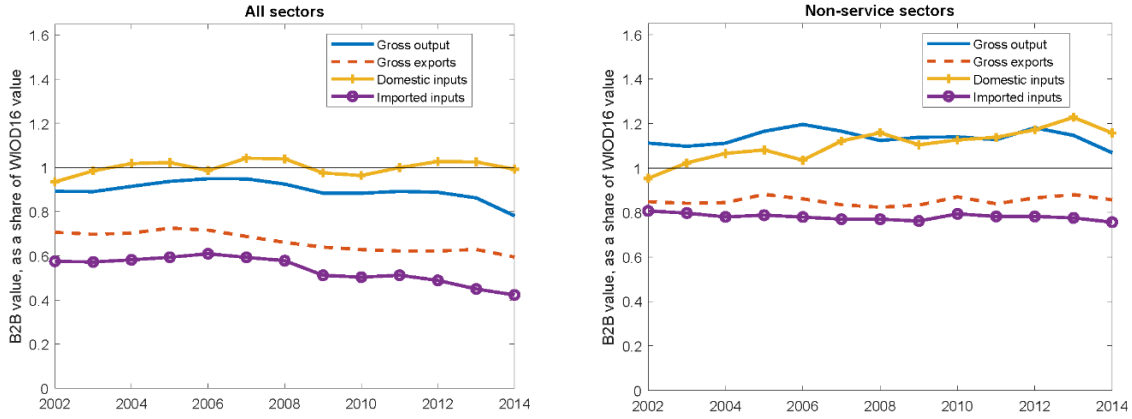
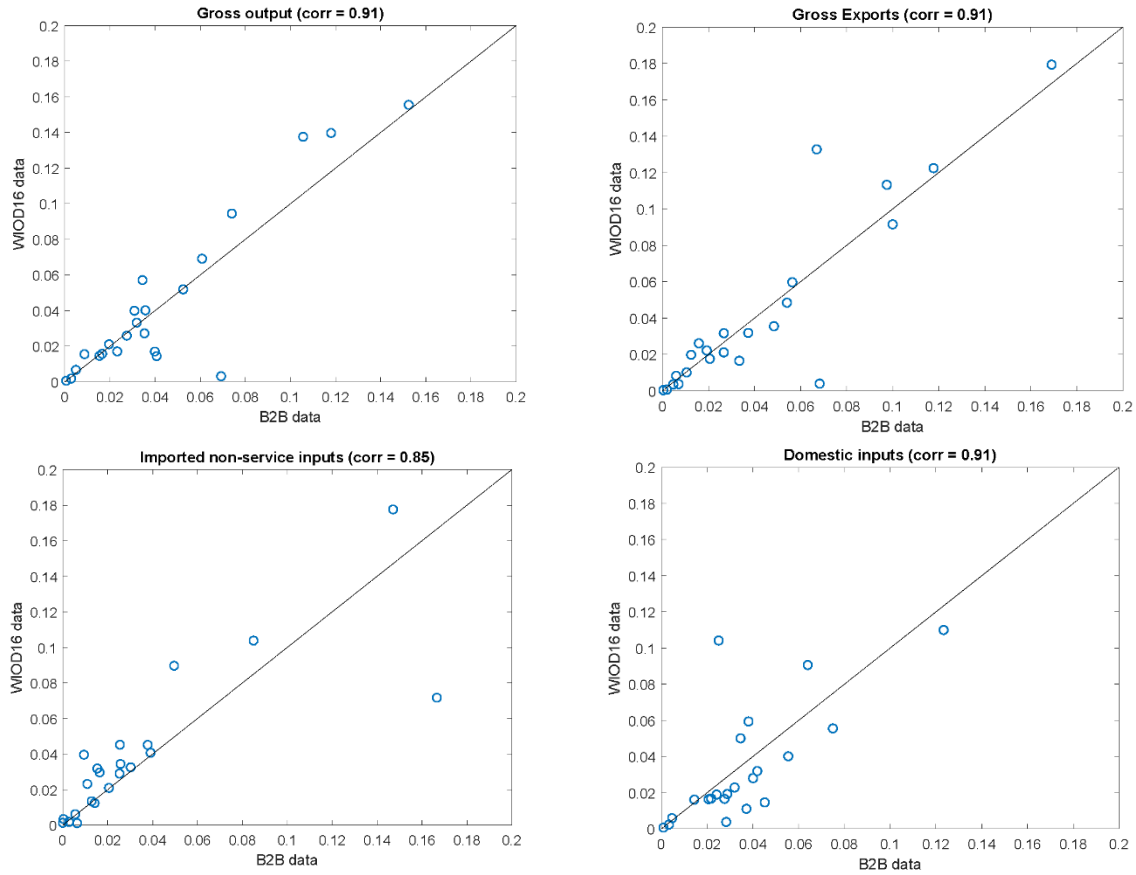


Figure 10: Comparison of sectoral shares in non-service sectors, 2010 (alternative value added measure)

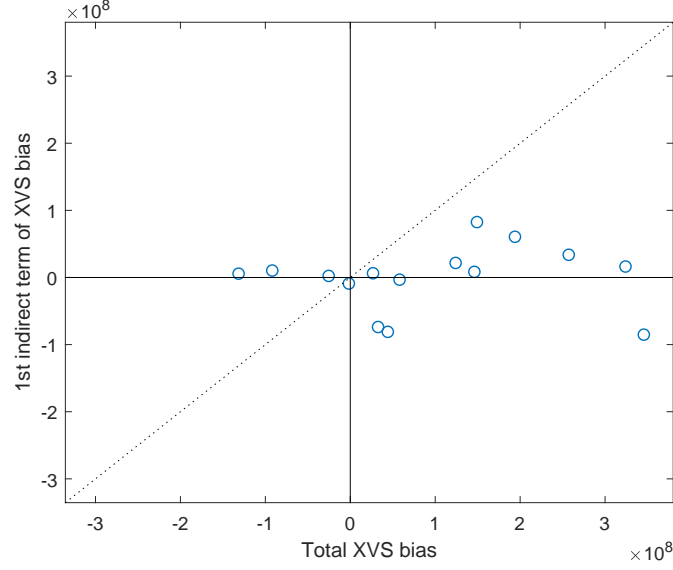


B.2 The share of the first term of the indirect bias

Figure 11 plots the first indirect term of the bias against the total bias for 15 manufacturing sectors with the highest shares in total import content of gross exports for Belgium. There is a positive relationship between the two sectoral biases, but the first indirect term accounts for only a fraction

of the total sectoral bias. This findings is consistent with the direct term being the main driver of the bias.

Figure 11: Comparison of total and first indirect biases for 2-digit manufacturing sectors



C Derivations

C.1 Characterizing the “direct” bias

Arranging equation (4) gives

$$XVS_s^{dir} = \sum_{n \in N_s} \alpha_n^M \alpha_n^X y_n,$$

where $\alpha_n^M = \frac{x_{R,n}}{y_n}$ and $\alpha_n^X = \frac{x_{n,R}}{y_n}$. We can rewrite these components of XVS_s^{dir} in terms of deviations from sectoral means:

$$\begin{aligned} XVS_s^{dir} &= \sum_{n \in N_s} (\tilde{\alpha}_s^M + \Delta \alpha_n^M) (\tilde{\alpha}_s^X + \Delta \alpha_n^X) y_n \\ &= \tilde{\alpha}_s^M \tilde{\alpha}_s^X \sum_{n \in N_s} y_n + \tilde{\alpha}_s^M \sum_{n \in N_s} \Delta \alpha_n^X y_n + \tilde{\alpha}_s^X \sum_{n \in N_s} \Delta \alpha_n^M y_n + \sum_{n \in N_s} \Delta \alpha_n^M \Delta \alpha_n^X y_n. \end{aligned}$$

If all firms in the sector are identical, then we obtain the measure with sectoral aggregation in equation (6):

$$\begin{aligned} XVS_{IO,s}^{dir} &= \tilde{\alpha}_s^M \tilde{\alpha}_s^X \sum_{n \in N_s} y_n \\ &= \tilde{\alpha}_s^M \tilde{\alpha}_s^X \sum_{n \in N_s} \bar{y}_s, \end{aligned}$$

where \bar{y}_s is the mean firm-level sales in sector s . Taking the difference, we obtain

$$XVS_s^{bias,dir} = \tilde{\alpha}_s^M \sum_{n \in N_s} \Delta \alpha_n^X y_n + \tilde{\alpha}_s^X \sum_{n \in N_s} \Delta \alpha_n^M y_n + \sum_{n \in N_s} \Delta \alpha_n^M \Delta \alpha_n^X y_n.$$

This expression can be simplified further. Using

$$\begin{aligned} \tilde{\alpha}_s^M \sum_n \Delta \alpha_n^X y_n &= \tilde{\alpha}_s^M \sum_{n \in N_s} \left(\frac{x_{n,R}}{y_n} - \frac{\sum_{n \in N_s} x_{n,R}}{\sum_{n \in N_s} y_n} \right) y_n \\ &= \tilde{\alpha}_s^M \left(\sum_{n \in N_s} x_{n,R} - \frac{\sum_{n \in N_s} x_{n,R}}{\sum_{n \in N_s} y_n} \sum_{n \in N_s} y_n \right) \\ &= 0, \end{aligned}$$

and similarly $\tilde{\alpha}_s^X \sum_{n \in N_s} \Delta \alpha_n^M y_n = 0$, we obtain

$$XVS_s^{bias,dir} = \sum_{n \in N_s} \Delta \alpha_n^M \Delta \alpha_n^X y_n.$$

We further decompose this term into unweighted covariances. We then obtain the following:

$$\begin{aligned} XVS_s^{bias,dir} &= \sum_{n \in N_s} (\alpha_n^M - \bar{\alpha}_s^M + \bar{\alpha}_s^M - \tilde{\alpha}_n^M) (\alpha_n^X - \bar{\alpha}_s^X + \bar{\alpha}_s^X - \tilde{\alpha}_n^X) (y_n - \bar{y}_s + \bar{y}_s) \\ &= \bar{y}_s \sum_{n \in N_s} (\alpha_n^M - \bar{\alpha}_s^M) (\alpha_n^X - \bar{\alpha}_s^X) \\ &\quad + (\bar{\alpha}_s^X - \tilde{\alpha}_s^X) \sum_{n \in N_s} (\alpha_n^M - \bar{\alpha}_s^M) (y_n - \bar{y}_s) + (\bar{\alpha}_s^M - \tilde{\alpha}_s^M) \sum_{n \in N_s} (\alpha_n^X - \bar{\alpha}_s^X) (y_n - \bar{y}_s) \\ &\quad + N_s (\bar{\alpha}_s^M - \tilde{\alpha}_s^M) (\bar{\alpha}_s^X - \tilde{\alpha}_s^X) \bar{y}_s \\ &\quad + \sum_{n \in N_s} (\alpha_n^M - \bar{\alpha}_s^M) (\alpha_n^X - \bar{\alpha}_s^X) (y_n - \bar{y}_s), \end{aligned}$$

where the second and the third term are identical.

C.2 Characterizing the “first indirect” bias

Arranging the term XVS_s^{indir1} in equation (5) and expressing the terms in terms of deviations from sectoral means gives

$$\begin{aligned} XVS_s^{indir1} &= \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \alpha_m^M \alpha_{m,n} \alpha_n^X y_n \\ &= \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} (\tilde{\alpha}_k^M + \Delta \alpha_m^M) (\tilde{\alpha}_{k,s} + \Delta \alpha_{m,n}) (\tilde{\alpha}_s^X + \Delta \alpha_n^X) y_n, \end{aligned}$$

where $\alpha_{m,n} = \frac{x_{m,n}}{y_n}$, and $\tilde{\alpha}_{k,s} = \frac{1}{N_k} \frac{\sum_{n \in N_s} \sum_{m \in N_k} x_{m,n}}{\sum_{n \in N_s} y_n}$. If all firms in each sector are identical, then we obtain the measure with sectoral aggregation in equation (9):

$$\begin{aligned} XVS_{IO,s}^{indir1} &= \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \tilde{\alpha}_k^M \tilde{\alpha}_{k,s} \tilde{\alpha}_s^X y_n \\ &= \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \tilde{\alpha}_k^M \tilde{\alpha}_{k,s} \tilde{\alpha}_s^X \bar{y}_s. \end{aligned}$$

Taking the difference, we obtain

$$\begin{aligned} XVS_s^{bias,indir1} &= \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \Delta \alpha_m^M \tilde{\alpha}_{k,s} \tilde{\alpha}_s^X y_n + \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \tilde{\alpha}_k^M \Delta \alpha_{m,n} \Delta \alpha_n^X y_n \\ &\quad + \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \Delta \alpha_m^M \Delta \alpha_{m,n} \tilde{\alpha}_s^X y_n + \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \Delta \alpha_m^M \Delta \alpha_{m,n} \Delta \alpha_n^X y_n, \end{aligned}$$

from the fact that

$$\begin{aligned} \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \tilde{\alpha}_k^M \tilde{\alpha}_{k,s} \Delta \alpha_n^X y_n &= \sum_{n \in N_s} \Delta \alpha_n^X y_n \sum_{k \in S} N_k \tilde{\alpha}_k^M \tilde{\alpha}_{k,s} \\ &= \sum_{n \in N_s} \left(\frac{x_{n,R}}{y_n} y_n - \frac{\sum_{n \in N_s} x_{n,R}}{\sum_{n \in N_s} y_n} y_n \right) \sum_{k \in S} N_k \tilde{\alpha}_k^M \tilde{\alpha}_{k,s} \\ &= 0, \end{aligned}$$

$$\begin{aligned} \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \Delta \alpha_m^M \tilde{\alpha}_{k,s} \Delta \alpha_n^X y_n &= \sum_{n \in N_s} \Delta \alpha_n^X y_n \sum_{k \in S} \sum_{m \in N_k} \Delta \alpha_m^M \tilde{\alpha}_{k,s} \\ &= 0, \end{aligned}$$

and

$$\begin{aligned} \sum_{n \in N_s} \sum_{k \in S} \sum_{m \in N_k} \tilde{\alpha}_k^M \Delta \alpha_{m,n} \tilde{\alpha}_s^X y_n &= \tilde{\alpha}_s^X \sum_{n \in N_s} \sum_{k \in S} \tilde{\alpha}_k^M \sum_{m \in N_k} \left(\frac{x_{m,n}}{y_n} - \frac{1}{N_k} \frac{\sum_{n \in N_s} \sum_{m \in N_k} x_{m,n}}{\sum_{n \in N_s} y_n} \right) y_n \\ &= \tilde{\alpha}_s^X \sum_{k \in S} \tilde{\alpha}_k^M \left(\sum_{n \in N_s} \sum_{m \in N_k} x_{m,n} - \sum_{n \in N_s} y_n \sum_{m \in N_k} \frac{1}{N_k} \frac{\sum_{n \in N_s} \sum_{m \in N_k} x_{m,n}}{\sum_{n \in N_s} y_n} \right) \\ &= 0. \end{aligned}$$

C.3 Simple covariance when including re-exports

Here we focus on the simple covariance to study how the inclusion of re-exports impacts the aggregation bias. Let us consider a sector with $N + 1$ firms with initial import and export intensities α_n^M and α_n^X for $n \in \{1, \dots, N, N + 1\}$, with unweighted means of $\bar{\alpha}^M$ and $\bar{\alpha}^X$.

We explore how a sector's simple covariance of the two intensities changes when we include re-exports for the firm indexed with $N + 1$. That is, we change the import and export intensities of the firm indexed with $N + 1$ to $\hat{\alpha}_{N+1}^M$ and $\hat{\alpha}_{N+1}^X$, where $\hat{\alpha}_{N+1}^M > \alpha_{N+1}^M$ and $\hat{\alpha}_{N+1}^X > \alpha_{N+1}^X$. We denote the differences between the new and old intensities by $\hat{M} = \hat{\alpha}_{N+1}^M - \alpha_{N+1}^M$ and $\hat{X} = \hat{\alpha}_{N+1}^X - \alpha_{N+1}^X$, and the new unweighted means by $\hat{\alpha}^M$ and $\hat{\alpha}^X$.

The numerator of the simple covariance after the inclusion of re-exports can be written as

$$\sum_{n=1}^N (\alpha_n^M - \hat{\alpha}^M)(\alpha_n^X - \hat{\alpha}^X) + (\hat{\alpha}_{N+1}^M - \hat{\alpha}^M)(\hat{\alpha}_{N+1}^X - \hat{\alpha}^X). \quad (11)$$

Since the new unweighted means can be written as

$$\begin{aligned} \hat{\alpha}^M &= \frac{1}{N+1} \left(\sum_{n=1}^N \alpha_n^M + \alpha_{N+1}^M + \hat{M} \right) \\ &= \bar{\alpha}^M + \frac{1}{N+1} \hat{M} \\ \hat{\alpha}^X &= \bar{\alpha}^X + \frac{1}{N+1} \hat{X}, \end{aligned}$$

we can rearrange equation (11) as follows:

$$\begin{aligned} &\sum_{n=1}^N (\alpha_n^M - \hat{\alpha}^M)(\alpha_n^X - \hat{\alpha}^X) + (\hat{\alpha}_{N+1}^M - \hat{\alpha}^M)(\hat{\alpha}_{N+1}^X - \hat{\alpha}^X) \\ &= \sum_{n=1}^N \left(\alpha_n^M - \bar{\alpha}^M - \frac{1}{N+1} \hat{M} \right) \left(\alpha_n^X - \bar{\alpha}^X - \frac{1}{N+1} \hat{X} \right) + \left(\alpha_{N+1}^M - \bar{\alpha}^M + \frac{N}{N+1} \hat{M} \right) \left(\alpha_{N+1}^X - \bar{\alpha}^X + \frac{N}{N+1} \hat{X} \right) \\ &= \sum_{n=1}^N (\alpha_n^M - \bar{\alpha}^M)(\alpha_n^X - \bar{\alpha}^X) + (\alpha_{N+1}^M - \bar{\alpha}^M)(\alpha_{N+1}^X - \bar{\alpha}^X) \\ &\quad + \hat{M}(\alpha_{N+1}^X - \bar{\alpha}^X) + \hat{X}(\alpha_{N+1}^M - \bar{\alpha}^M) + \frac{N}{(N+1)^2} \hat{M} \hat{X} + \frac{N^2}{(N+1)^2} \hat{M} \hat{X}, \end{aligned}$$

where the first two terms in the last equation correspond to the numerator of the initial simple covariance.

If the initial import and export intensities of the firm indexed with $N + 1$ are larger than the sector's average, $\alpha_{N+1}^M > \bar{\alpha}^M$ and $\alpha_{N+1}^X > \bar{\alpha}^X$, then the simple covariance will increase after the inclusion of re-exports.

NATIONAL BANK OF BELGIUM - WORKING PAPERS SERIES

The Working Papers are available on the website of the Bank: <http://www.nbb.be>.

334. "Exchange rate movements, firm-level exports and heterogeneity", by A. Berthou and E. Dhyne, *Research series*, January 2018.
335. "Nonparametric identification of unobserved technological heterogeneity in production", by L. Cherchye, T. Demuynck, B. De Rock and M. Verschelde, *Research series*, February 2018.
336. "Compositional changes in aggregate productivity in an era of globalisation and financial crisis", by C. Fuss and A. Theodorakopoulos, *Research series*, February 2018.
337. "Decomposing firm-product appeal: How important is consumer taste?", by B. Y. Aw, Y. Lee and H. Vandenbussche, *Research series*, March 2018.
338. "Sensitivity of credit risk stress test results: Modelling issues with an application to Belgium", by P. Van Roy, S. Ferrari and C. Vespro, *Research series*, March 2018.
339. "Paul van Zeeland and the first decade of the US Federal Reserve System: The analysis from a European central banker who was a student of Kemmerer", by I. Maes and R. Gomez Betancourt, *Research series*, March 2018.
340. "One way to the top: How services boost the demand for goods", by A. Ariu, F. Mayneris and M. Parenti, *Research series*, March 2018.
341. "Alexandre Lamfalussy and the monetary policy debates among central bankers during the Great Inflation", by I. Maes and P. Clement, *Research series*, April 2018.
342. "The economic importance of the Belgian ports: Flemish maritime ports, Liège port complex and the port of Brussels – Report 2016", by F. Coppens, C. Mathys, J.-P. Merckx, P. Ringoot and M. Van Kerckhoven, *Document series*, April 2018.
343. "The unemployment impact of product and labour market regulation: Evidence from European countries", by C. Piton, *Research series*, June 2018.
344. "Trade and domestic production networks", by F. Tintelnot, A. Ken Kikkawa, M. Mogstad, E. Dhyne, *Research series*, September 2018.
345. "Review essay: Central banking through the centuries", by I. Maes, *Research series*, October 2018.
346. "IT and productivity: A firm level analysis", by E. Dhyne, J. Konings, J. Van den Bosch, S. Vanormelingen, *Research series*, October 2018.
347. "Identifying credit supply shocks with bank-firm data: methods and applications", by H. Degryse, O. De Jonghe, S. Jakovljević, Klaas Mulier, Glenn Schepens, *Research series*, October 2018.
348. "Can inflation expectations in business or consumer surveys improve inflation forecasts?", by R. Basselier, D. de Antonio Liedo, J. Jonckheere and G. Langenus, *Research series*, October 2018.
349. "Quantile-based inflation risk models", by E. Ghysels, L. Iania and J. Striaukas, *Research series*, October 2018.
350. "International food commodity prices and missing (dis)inflation in the euro area", by G. Peersman, *Research series*, October 2018.
351. "Pipeline pressures and sectoral inflation dynamics", by F. Smets, J. Tielens and J. Van Hove, *Research series*, October 2018.
352. "Price updating in production networks", by C. Duprez and G. Magerman, *Research series*, October 2018.
353. "Dominant currencies. How firms choose currency invoicing and why it matters", by M. Amiti, O. Itskhoki and J. Konings, *Research series*, October 2018.
354. "Endogenous forward guidance", by B. Chafwehé, R. Oikonomou, R. Priftis and L. Vogel, *Research series*, October 2018.
355. "Is euro area lowflation here to stay? Insights from a time-varying parameter model with survey data", by A. Stevens and J. Wauters, *Research series*, October 2018.
356. "A price index with variable mark-ups and changing variety", by T. Demuynck and M. Parenti, *Research series*, October 2018.
357. "Markup and price dynamics: Linking micro to macro", by J. De Loecker, C. Fuss and J. Van Biesebroeck, *Research series*, October 2018.
358. "Productivity, wages and profits: Does firms' position in the value chain matter?", by B. Mahy, F. Rycx, G. Vermeylen and M. Volral, *Research series*, October 2018.
359. "Upstreamness, social upgrading and gender: Equal benefits for all?", by N. Gagliardi, B. Mahy and F. Rycx, *Research series*, December 2018.
360. "A macro-financial analysis of the corporate bond market", by H. Dewachter, L. Iania, W. Lemke and M. Lyrio, *Research series*, December 2018.
361. "Some borrowers are more equal than others: Bank funding shocks and credit reallocation", by O. De Jonghe, H. Dewachter, K. Mulier, S. Ongena and G. Schepens, *Research series*, December 2018.

362. "The origins of firm heterogeneity: A production network approach", by A. B. Bernard, E. Dhyne, G. Magerman, K. Manova and A. Moxnes, *Research series*, January 2019.
363. "Imperfect competition in firm-to-firm trade", by A. Ken Kikkawa, G. Magerman and E. Dhyne, *Research series*, January 2019.
364. "Forward guidance with preferences over safe assets", by A. Rannenberg, *Research series*, January 2019.
365. "The distinct effects of information technologies and communication technologies on the age-skill composition of labour demand", by S. Blanas, *Research series*, January 2019.
366. "A survey of the long-term impact of Brexit on the UK and the EU27 economies", by P. Bisciari, *Document series*, January 2019.
367. "A macroeconomic model with heterogeneous and financially-constrained intermediaries", by Th. Lejeune and R. Wouters, *Research series*, February 2019.
368. "The economic importance of the Belgian ports: Flemish maritime ports, Liège port complex and the port of Brussels – Report 2017", by E. Gueli, P. Ringoot and M. Van Kerckhoven, *Document series*, March 2019.
369. "Does banks' systemic importance affect their capital structure and balance sheet adjustment processes?", by Y. Bakkar, O. De Jonghe and A. Tarazi, *Research series*, March 2019.
370. "A model for international spillovers to emerging markets", by R. Houssa, J. Mohimont and C. Otrok, *Research series*, April 2019.
371. "Estimation methods for computing a branch's total value added from incomplete annual accounting data", S. Vansteelandt, F. Coppens, D. Reynders, M. Vackier and L. Van Belle, *Research series*, April 2019.
372. "Do SVARs with sign restrictions not identify unconventional monetary policy shocks?", by J. Boeckx, M. Dossche, A. Galesi, B. Hofmann and G. Peersman, *Research series*, June 2019.
373. "Research and development activities in Belgium: A snapshot of past investment for the country's future", by S. Vennix, *Research series*, July 2019.
374. "State dependent fiscal multipliers with preferences over safe assets" by A. Rannenberg, *Research series*, July 2019.
375. "Inequality, the risk of secular stagnation and the increase in household debt", by A. Rannenberg, *Research series*, August 2019.
376. "Welfare effects of business cycles and monetary policies in a small open emerging economy", by J. Mohimont, *Research series*, October 2019.
377. "Learning about demand abroad from wholesalers: a B2B analysis", by W. Connell, E. Dhyne and H. Vandenbussche, *Research series*, November 2019.
378. "Measuring trade in value added with Firm-level Data, by R. Bems and A. K. Kikkawa, *Research series*, November 2019.

National Bank of Belgium
Limited liability company
RLP Brussels – Company's number: 0203.201.340
Registered office: boulevard de Berlaimont 14 – BE-1000 Brussels
www.nbb.be

Editor
Pierre Wunsch
Governor of the National Bank of Belgium

© Illustrations: National Bank of Belgium

Layout: Analysis and Research Group
Cover: NBB CM – Prepress & Image

Published in November 2019