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The Neoclassical Theory of Aggregate Investment and its Criticisms

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The neoclassical theory of aggregate investment and its criticisms*

Daniele Girardi[†]

Abstract

This paper surveys the neoclassical theory of aggregate investment and its criticisms. We identify four main strands in neoclassical investment theory: (i) the traditional Wicksellian model; (ii) the Fisherian ‘array-of-opportunities’ approach; (iii) the Jorgensonian model; (iv) the now prevailing adjustment cost models. We summarize each approach, discuss the main conceptual issues, and highlight similarities and differences between them. We also provide a systematic summary and discussion of the main criticisms that have been leveled at each of these models and highlight some unresolved theoretical issues.

1 Introduction

What are we talking about when we talk about ‘neoclassical investment theory’? What are the main criticisms that have been leveled at this theory? What theoretical issues does it encounter?

To provide a systematic answer to the first question, this paper identifies four main strands and formulations of the neoclassical theory of aggregate investment:

1. the traditional Wicksellian model;
2. the Fisherian ‘array-of-opportunities’ approach;
3. the Jorgensonian model;
4. adjustment cost models.

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I summarize and discuss each of these models, highlighting the main underlying ideas and uncovering similarities and differences between them. I also identify the main theoretical issues and criticisms associated with each of these models. I consider both ‘mainstream’ critiques that have been addressed by subsequent developments within the approach, and more radical criticisms that tend to lead to alternative theories.

This study is motivated by the idea that a systematic perspective on the neoclassical theory of investment – a theory which implicitly or explicitly underlies many economic models, much heuristic thinking, and several policy discussions – can be helpful in clarifying important contemporary issues and debates. Indeed, investment theory bears implications for topics of first-order importance like short-run fluctuations, growth, the effects of fiscal and monetary policy, and the elasticity of substitution between capital and labor.

An important insight that emerges from this review is that adjustment cost models, which are now dominant and routinely embedded in macroeconomic general equilibrium models, have not simply ‘superseded’ the other three approaches. Rather, they have enriched them with an explicit theory of the short-run frictions that govern the process through which a firm’s capital stock tends to some desired level. But adjustment cost models still rely on previous neoclassical models in order to determine what moves the desired capital stock level in the first place. This confirms the usefulness of adopting a broad historical view of neoclassical investment theory.

Among other issues, I devote specific attention to the way in which each of these theories derives a negative relation between investment and the cost of capital – a fundamental cornerstone of neoclassical macroeconomics, without which there would exist no ‘natural’ rate of interest capable to ensure that in the long-run investment adapts to full-capacity savings.

A widespread view holds that “The notion that business spending on fixed capital falls when interest rates rise is a theoretically unambiguous relationship that lies at the heart of the monetary transmission mechanism” (Gilchrist and Zakrajsek, 2007, p.1). This paper leads to more nuanced conclusions: there are significant theoretical complications and controversies concerning the derivation of this relation. Of course, the interest rate might still affect investment through alternative mechanisms not considered here, but the mechanisms provided by neoclassical investment theory present more ambiguities than standard textbooks would suggest.

This survey is original in adopting a historical perspective on neoclassical investment theory,

starting from the model of investment adopted by Wicksell and other early marginalist economists, and in providing a systematic summary and discussion of the main criticisms that have been leveled at these models. This long-run perspective allows to see recent developments – including today’s workhorse model with adjustment costs – in historical context. This historical perspective is missing in most recent discussions of this literature (eg, Caballero 1999), which have focused more on providing a detailed description of the state of the art than in providing a comprehensive historical review.

The scope of analysis The field of investment theory is so vast that some clear delimitation of the scope of analysis is necessary. I set the boundary on two main dimensions.

First, this survey is about *neoclassical* theories of investment. I define those as models with complete and costless contracting that derive a negative relation between investment and the real interest rate.

Consistent with this delimitation of the scope of analysis, in my survey of criticisms raised by the theory, I will focus mostly on critiques that do not appeal to firms’ liquidity constraints, fundamental uncertainty or bounded rationality. To be sure, these factors matter– in fact, they are extremely important for a realistic understanding of investment dynamics. However, the object of this survey is the neoclassical model of investment under full information and certainty, therefore consistency requires that I focus mostly on criticisms that apply to this context. If I were to discuss critiques based on those issues, I would need to discuss models that have attempted to introduce them. But those are outside the scope of this paper.

Second, this survey does not directly discuss models of uncertainty and irreversibility, which have inspired important strands of recent literature.¹ There is, of course, no denying that the literature on uncertainty and irreversibility is important for the study of investment choices. Still, the neoclassical model of investment under certainty and reversibility remains an important benchmark for theory, worth surveying and investigating. Indeed, in order to provide a complete account of investment dynamics, empirical and theoretical analyses of investment under uncertainty need to specify what the relevant sources of uncertainty are – that is, what are the factors which determine the ‘certainty

¹Some main references on the theory of investment under uncertainty and irreversibility are, for example, Caballero (1991); Pindyck (1993); Dixit and Pindyck (1994); Bloom (2009); Gilchrist et al. (2014). Surveys have been provided for example by Dixit (1992); Pindyck (1991); Carruth et al. (2000).

equivalent' levels of demand for capital and investment in a benchmark theoretical model.²

2 Old school: the traditional 'Wicksellian' model

The traditional neoclassical model of aggregate investment is based on marginalist principles of factor substitution, and implies that aggregate investment is a negative function of the interest rate. This relation is derived in two steps. First, factor prices determine the optimal capital stock level, or 'demand for capital'. Second, investment is determined as a positive function of changes in the optimal capital stock. This approach is rooted in a marginalist general-equilibrium model of the economy, associated with authors such as Wicksell, Böhm Bawerk or Clark.³ It remains to this day a basic framework that, implicitly or explicitly, informs much thinking and policy-making.

The first step consists in deriving a demand function for capital, that is, a relation that gives the optimal aggregate capital stock K^* as a function of the real interest rate r :

$$K_t^* = f(r_t) \text{ with } f' < 0 \tag{1}$$

Equation 1 is derived from basic marginalist and profit-maximization principles and involves substitution between factors in production: a higher interest rate reduces demand for capital by decreasing the cost-minimizing capital-labor ratio of production processes.⁴

In the second step, aggregate investment I is derived as a function of demand for capital K^* . In the simplest setting with no fixed capital (i.e., all capital is circulating) and annual production cycles, yearly investment I_t is just equal to demand for capital K_t^* . With fixed capital, investment is a positive function of changes in demand for capital: investment rises above (falls below) the level needed to compensate depreciation when demand for capital increases (decreases). Formally,⁵

²In their survey of theory and evidence on investment under uncertainty and irreversibility, Carruth et al. (2000) write that "since the irreversible investment literature deals with the timing of investment rather than the level of investment activity, a complete model needs to incorporate an independent explanation of the 'certainty-equivalent' determinants of investment for a particular firm, industry or economy."

³This has also been called the 'long-period' formulation of general equilibrium theory, as opposed to the Walrasian (disaggregate and short-period) version. For the differences between these two approaches, see for example Garegnani (1976); Petri (1978); Kurz and Salvadori (1997).

⁴With multiple consumption goods, this pattern is reinforced by consumer optimization on the demand side: the relative prices of the more capital-intensive goods increase, so demand for them declines and their share in GDP gradually decreases, again reducing demand for capital.

⁵The derivation of eq.2 from eq.1 also assumes no 'perverse' effect of the interest rate on the rate of depreciation.

$$I_t = g(K_t^*(r_t), K_t) = f(r_t) \quad \text{with } g'(K^*) > 0 \text{ and } f' < 0, \quad (2)$$

Under putty-putty technology, the rate of investment per unit of time is actually not well defined in this model. In the absence of some form of adjustment costs, after a change in the interest rate the capital stock would jump instantaneously to its new optimal level, leaving the rate of investment undefined (Haavelmo, 1960; Söderström, 1976). Under putty-clay technology, however, the rate of investment per unit of time can be determined based on the rate of depreciation. Specifically, it would be equal to the optimal capital-labor ratio (that we can denote as $a = a(r)$) times the quantity of labor released by the scrapping of old plants (that we can denote as \hat{L}): $I_t = a(r_t)\hat{L}_t$ (Petri, 2015).

Appendix A provides a simple formal derivation of the Wicksellian model of demand for capital. Figure 1 illustrates the basic mechanism: given the level of employment, the optimal capital-labor ratio (as determined by the real interest rate) determines the desired capital stock.

As Figure 1 illustrates (and Appendix A shows formally), this traditional marginalist model of demand for capital takes total labor employment as fixed – determined exogenously by the conditions of equilibrium in the labor market. This is necessary to allow the optimal K/L ratio to univocally determine the optimal capital stock (that is, the long-period demand for capital). Without the fixed-employment assumption, the interest rate would determine the optimal K/L ratio but demand for capital would remain undetermined.

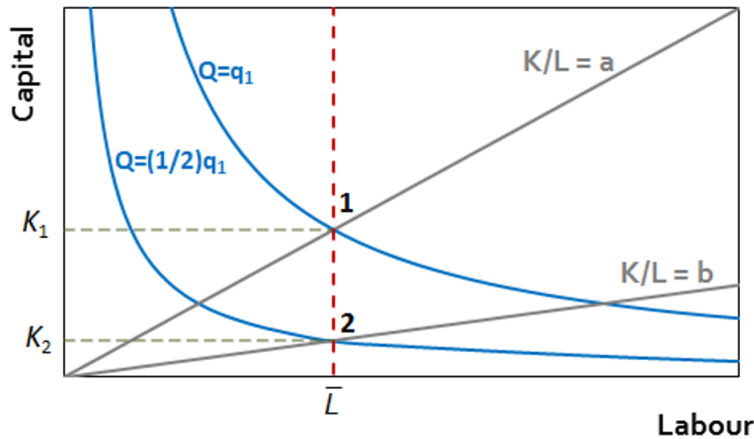


Figure 1: Determination of demand for capital in the traditional neoclassical model

2.1 Criticisms of the traditional neoclassical model

Two main issues have been the subject of criticism against this benchmark marginalist model. The first is the so-called Cambridge capital criticism (Sraffa, 1960; Samuelson, 1966; Garegnani, 1990; Harcourt, 1972; Petri, 2004; Lazzarini, 2011; Fratini, 2019; Baqaee and Farhi, 2019). It takes issues with the conception of capital as a single homogeneous production factor, and undermines the monotonic negative relation between capital intensity and the cost of capital. The second strand of criticism has a Keynesian flavor and takes issues with the absence of aggregate demand considerations in this theory of aggregate investment. It argues that, once endogenous demand-driven changes in employment are allowed, demand for capital is no longer uniquely determined by the interest rate, and the demand-side factors that determine aggregate employment must enter equations 1 and 2 (eg, Keynes 1936; Duesenberry 1958; Eisner 1978; Kalecki 1971; Steindl 1976).

The Cambridge capital criticism This critique concerns the monotonic relation between the optimal K/L ratio and the interest rate, on which the traditional neoclassical model relies to derive the aggregate investment function (equation 1). It holds that this relation cannot be derived in an economy with heterogeneous capital goods— a key result from the so-called Cambridge capital controversy (Sraffa 1960; Samuelson 1966; Garegnani 1990; Petri 2004; Lazzarini 2011; Fratini 2019; Harcourt 1972; a recent constructive study of the issue is Baqaee and Farhi 2019). In what follows I summarize the argument, while Appendix B provides a more detailed exposition.

In an economy with heterogeneous capital goods, the aggregate capital stock must be measured in value, given that a common physical measure is not available. But then – whatever the numeraire chosen and even in the absence of any change in production methods employed and quantities produced – the quantity of capital varies with the vector of relative prices, which in turn is a function of factor remunerations.

In the absence of a measure of the ‘capital-intensity’ of production techniques that is independent of factor prices, it is not possible to derive a decreasing demand curve for capital based on factor substitution. It has been demonstrated that the same technique can actually be cost-minimizing at two different levels of the rental cost of capital but not in between them (a phenomenon known in the literature as ‘reswitching of techniques’) and that a decrease in the interest rate may well induce profit-maximizing firms to decrease their capital intensity, however the latter is defined, if

the direct effect of the interest rate reduction is more than compensated by changes in the relative prices of the heterogeneous capital goods ('reverse capital deepening'). I provide a more detailed summary and illustration of these results in Appendix B.

According to the Cambridge capital criticism, these findings imply that the very conception of capital on which the traditional neoclassical approach is based – capital as a single homogeneous production factor, a 'fluid' measured in value and adaptable to different forms – turns out to be untenable. And even when adopting this incorrect definition of capital – the criticism goes – there is no general negative relation between the long-period K/L ratio and the interest rate.

Keynesian criticisms Broadly speaking, the Keynesian criticism of the traditional neoclassical investment model is that it focuses only on the effect on factor prices, neglecting the influence of quantity variables and thus leaving out important aggregate demand considerations.

If the employment level is not fixed by structural factors but sensitive to aggregate demand fluctuations, then the fixed-employment assumption should be substituted with the condition that aggregate output and labor employment are determined by effective demand. For a given optimal capital/labor ratio, the desired capital stock and investment would thus be positive functions of the (demand-determined) output level, as in so-called 'accelerator' models (eg, Clark 1917; Harrod 1939; Samuelson 1939; Duesenberry 1958; Eisner 1978). This is illustrated in Figure 2, where for a given cost-minimizing capital-labor ratio, demand for capital depends positively on the level of output.

A version of the Keynesian criticism – associated for example with the work of Steven Fazzari – focuses on liquidity constraints. It holds that under imperfect credit markets, credit is rationed, therefore not all desired investment is financed. This provides another channel for aggregate demand shocks to affect aggregate investment. Higher capacity utilization and sales would directly relax liquidity constraints by providing internal funds to finance investment. Moreover, they can also improve access to external financing, if cash flows and realized profits are among the variables that lenders use to screen potential borrowers (see, eg, Fazzari and Athey 1987; Fazzari et al. 1988; Fazzari 1993).

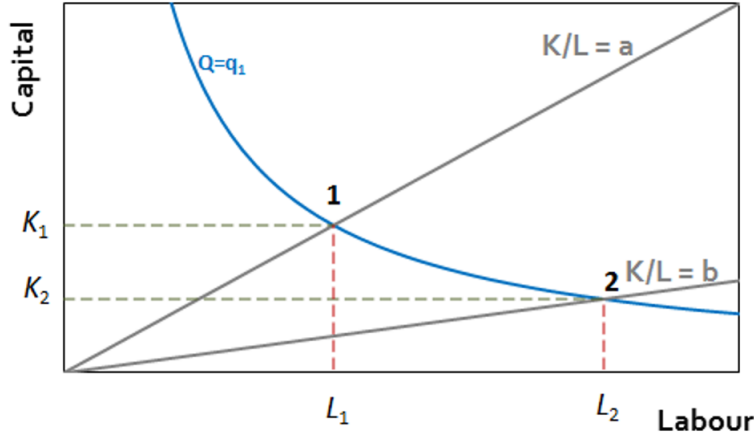


Figure 2: Neoclassical accelerator: Demand for capital when employment depends on demand

3 The ‘array of opportunities’ approach: a disequilibrium model

The ‘array of opportunities’ approach provides a different derivation of the neoclassical interest-elastic investment function of equation 2, that does not rely on capital-labor substitution.⁶

The idea is very simple. At any given point in time, each firm is presented with several possible investment projects, which can be ranked on the basis of their expected rate of return. Profit-maximization implies that the projects offering a rate of return higher than the ongoing interest rate – and thus providing positive profits – will be undertaken.⁷

Consider what happens if the interest rate decreases. For each firm, investment projects that were unfeasible will become profitable. Aggregate investment, defined as the sum of all investments realized in the economy, will thus increase. Figure 3 provides a graphical example: there are 12 potential investment projects, each defined by its expected rate of return (on the vertical axis) and its size in monetary value (on the horizontal axis). As the interest rate decreases from r_0 to r_1 , the amount of investment undertaken increases from I_0 to I_1 .

Drawing on Marglin (1970), the ‘array of opportunities’ approach can be formalized as follows. A density function $J(\pi)$ gives the density of investment opportunities yielding a expected discounted rate of return π . For example, for a small interval (π_1, π_2) , $J(\pi)(\pi_2 - \pi_1)$ gives the amount of

⁶The term ‘array of opportunities’ comes from Witte (1963, 445) and is used also in Ackley (1978, 623) and Petri (2004, 262), among others. Some version of this theory can be found in many contributions, starting from Fisher (1930). A version of this approach is employed, for example, in the popular Romer (2019) macroeconomics textbook (p.421). As we will see, there is also a parallel between this model and the convex adjustment cost model of investment.

⁷In the presence of risk, the rate of return should be higher than the risk-adjusted interest rate.

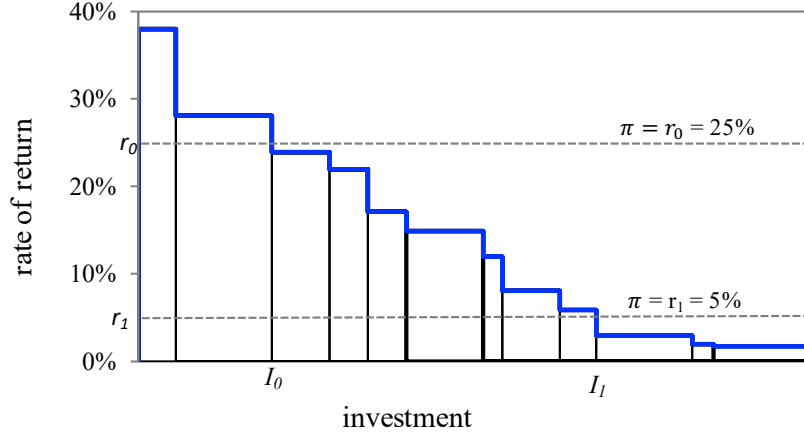


Figure 3: Investment determination in the ‘array of opportunities’ approach

investment that yields a rate of return between π_1 and π_2 . A representative firm undertakes all the investment which yields $\pi \geq r$, where r is the real interest rate. Therefore investment is $I = f(r) = \int_{\pi=r}^{\infty} J(\pi)d\pi$, and $f'(r) < 0$.

3.1 Theoretical issues with the ‘array of opportunities’ approach

The ‘array-of-opportunities’ approach encounters one major theoretical difficulty, highlighted for example by Alchian (1955, 942), Junankar (1972, 23) and Ackley (1978, 623-624). This approach assumes that prices are unaffected by interest rate changes— an assumption that is hard to square with any general equilibrium model of the economy. Indeed, competition implies that interest rate changes are bound to cause variations in prices, modifying the expected yields of all projects and their ranking. To grasp this point, note that (i) the rate of return of an investment project depends on the price at which the output will be sold and (ii) the interest rate enters in the determination of prices, being one of the costs of production. A change in the interest rate will change both the ratio between the prices of different products (i.e., relative prices) and the ratio between price and wage in each sector (i.e., rates of return).

This criticism therefore holds that a change in the interest rate is bound to modify the returns of all projects and their ranking. It is thus incorrect to take the array of opportunities as fixed while the interest rate varies, as done in Figure 3. Even worse for this approach, in equilibrium competition imposes a uniform profit rate on all investment projects (after allowing for risk differentials), so

there is no ‘array of opportunities’ to start with (Ackley, 1978, 623). The equilibrium rate of return, moreover, moves in step with the interest rate: because of competition, an autonomous decrease in the interest rate will result in a reduction in prices relative to wages, thus decreasing also the normal rate of return (Pivetti, 1991).

Possible defenses of the ‘array of opportunities’ approach can be based on lack of competition, or on interpreting it as a disequilibrium phenomenon. However, while market power is present in some degree in all industries, the assumption of lack of *any* mechanism that makes prices react to changes in production costs appears very strong. Of course the economy can be in disequilibrium in the short run. But as long as some type of investment emerges, which is expected to yield a rate of return higher than some equilibrium profit rate, some competition between firms to exploit it, even if limited, should push down the rate of return (Ackley, 1978, 623). In this sense the ‘array of opportunities’ can perhaps be seen as one of the mechanisms that contribute to ensure some (not necessarily complete) tendency toward uniformity of the risk-adjusted rate of profit over the supply prices of the various capital goods – but it seems difficult to see it as the main mechanism that explains the rate of aggregate investment.

4 The Jorgensonian approach: one model, two different theories

The Jorgensonian investment model, first presented in Jorgenson (1963), derives a neoclassical investment function from the dynamic optimization problem of a firm maximizing the present value of future profits. Jorgenson’s model has gained recognition as a influential benchmark model of investment.⁸ ⁹ The declared aim was “*to present a theory of investment behavior based on the neoclassical theory of optimal accumulation of capital*” (Jorgenson, 1963, 248). This might have contributed to some confusion in the literature: the Jorgensonian model is often presented as a formalization of the traditional neoclassical investment function described in Section 2 above. It is,

⁸This status is explicitly recognized, for example, in the stated motivation for the inclusion of Jorgenson’s 1963 article in the list of the Top 20 most influential papers published in the first 100 years of the *American Economic Review* (Arrow et al., 2011, 4-5). Indeed in the literature Jorgenson’s 1963 model is almost universally presented as the baseline standard neoclassical investment model. Another exposition of the same model is provided in an often quoted empirical work by Hall and Jorgenson (1967).

⁹It may be worth noting, to clear a possible source of confusion, that in a different paper, also published in 1967, Jorgenson proposed a different model, with different and more restrictive assumptions (Jorgenson, 1967). It is however fair to say that Jorgenson’s 1967 model (strongly criticized by Tobin (1967, pp.156-158) in a comment appeared in the same volume) has not received widespread acceptance, as testified by the fact that it is not discussed at all in any major recent survey or textbook, nor in most empirical and theoretical studies on investment.

in fact, a distinct and different model, as I hope to make clear in what follows.

The setup of the Jorgensonian model is a dynamic version of the standard neoclassical theory of the firm. The firm maximizes the present value of future net revenues, equal to $PV_0 = \int_0^\infty e^{-rt} [pQ - wL - p^k(r + \delta)K] dt$, where Q is output, p is the price of output, p^K the price of the capital good, w the wage and δ the rate of depreciation. Under a neoclassical production function ($Q = F(K, L)$) with some positive elasticity of substitution between labor and capital, this yields the marginal productivity conditions

$$\frac{\partial Q}{\partial L} = \frac{w}{p} \quad \text{and} \quad \frac{\partial Q}{\partial K} = \frac{c}{p} \quad (3)$$

where the composite term $c = p^K(r + \delta)$ is labeled the ‘user cost of capital’, and takes into account the interest rate r , the relative price of capital goods p^K and the rate of depreciation δ . Indeed, one of the contributions of the Jorgenson model is the introduction of an explicit definition of the cost of capital that takes into account the relative price of investment goods and the rate of depreciation, besides the interest rate.¹⁰

The optimality conditions in equation 3 are akin to those in the traditional Wicksellian model, except for the fact that they are derived at the level of a single representative firm rather than at the aggregate level.¹¹ Although the Jorgensonian model is formally dynamic, the absence of adjustment costs and irreversibilities rules out intertemporal trade-offs, and the firm just needs to satisfy its (static) marginal productivity conditions at each point in time.

Unlike the traditional Wicksellian model, however, the Jorgensonian model – given its microeconomic level of analysis – cannot be closed with a full (or fixed) employment assumption. A single firm operating under perfect competition is by definition able to hire any desired amount of labor at the prevailing wage rate. This represents the crucial departure of the Jorgensonian model from the traditional Wicksellian one.

¹⁰Here for simplicity we are neglecting taxes, that Jorgenson (1963) includes. Another difference between the exposition here and the original formulation in Jorgenson (1963) is that the latter assumes that the firm owns its capital stock and buys capital goods, rather than renting capital, and writes the revenue function as $R = pQ - wL - p^K I$. The formulation presented here is more general from this point of view, because cK can be seen either as the cost of renting capital or as the opportunity cost of using own capital. In any case, in a frictionless model, ownership of capital does not alter the optimality conditions. Capital gains (\dot{q}) are neglected for simplicity (here and also in Jorgenson’s original analysis).

¹¹Specifically, the two conditions of equation 3 correspond to equations 14 and 15 (in Appendix A) in the traditional Wicksellian model, although in writing equations 14 and 15 I have assumed capital homogeneous to output – thus neglecting the price of capital relative to output – and no depreciation.

A way to close the Jorgensonian model is to impose a decreasing returns to scale assumption. Under decreasing returns to scale, the optimality conditions in equation 3 determine a well-defined firm-level demand for capital function, of the same form of equation 1, but adopting the Jorgensonian (broader) definition of the cost of capital:¹²

$$K_t^* = f(c_t) \text{ with } f' < 0 \quad (4)$$

Under constant returns to scale, however, the optimality conditions in equation 3 define only the optimal K/L ratio, while the optimal capital stock remains indeterminate. Indeed, with constant returns to scale, the profit function of a price-taking firm is linear in the capital stock, so the optimal quantity of capital is not well defined.¹³

An alternative closure of the model, that allows to determine K^* also under constant returns to scale, is to assume that the firm exercises market power and faces a downward-sloping demand curve $Q = Q(p)$. This leads to a well defined demand for capital. The optimal capital stock then depends both on the cost of capital and aggregate demand. Consider for simplicity a case in which $Q = \bar{Q}$ if $p \leq \bar{p}$, and $\bar{Q} = 0$ if $p > \bar{p}$, and assume a constant elasticity of substitution (CES) production function. Then the optimality conditions in equation 3 imply the demand for capital function

$$K_t^* = f(c_t, \bar{Q}_t) = \alpha(\bar{p}_t \bar{Q}_t) c_t^{-\sigma} \quad (5)$$

Equation 5 is known as the ‘neoclassical accelerator’ because it combines neoclassical factor substitution mechanisms with an influence of aggregate demand. This closure of the Jorgensonian model leads to the same determination of demand for capital discussed in the context of Keynesian critiques of the traditional neoclassical model, and illustrated in Figure 2, with the only difference that here the analysis is carried out at the level of a single representative firm rather than the whole economy. This closure, indeed, leads the model outside of the neoclassical paradigm in an important sense, because it abandons the perfect competition framework, assuming instead market power.

The distinction between these two ways to derive the demand for capital function from the

¹²Of course, in a one-good economy without depreciation, equations 1 and 4 would have exactly the same form.

¹³This is of course just a manifestation of a well-known fact in neoclassical production theory: under a linearly homogeneous production function, the optimal size of a price-taking firm is either zero, infinite or undefined, depending on whether the unit production cost is higher, lower or equal to the unit price of output.

Jorgensonian optimality conditions is hard to overstate. The two closures actually define two alternative theories of the optimal capital stock. In the first, aggregate demand exerts no autonomous influence: given returns to scale, K^* depends only on the relative cost of capital. By contrast in the second approach, with demand-constrained production, an influence of aggregate demand (the accelerator effect) is admitted.¹⁴

Theoretical work has often followed the decreasing-returns-to-scale version of the Jorgensonian theory of demand for capital summarized by equation 4 (eg, Lucas 1967, Caballero and Engel 1999, Bachmann et al. 2013, Winberry 2021.). Econometric work, instead, has traditionally been dominated by the ‘neoclassical accelerator’ model in equation 5 (eg, Hall and Jorgenson 1971; Chirinko et al. 1999; Bloom et al. 2007; Kang et al. 2014). This is most likely due to pragmatic considerations: output or sales have been found to have strong explanatory power for investment in virtually all samples and periods (Chirinko, 1993). In this sense, it has been natural for empirical work to employ the version of the theory that allows inclusion of this term.

On the other hand, empirical estimations of demand for capital functions consistent with equation 5 have traditionally taken output or sales as exogenous, neglecting that in general output and desired capital are simultaneously determined. Of course, the same problem applies to the assumption, widespread until recently, that the cost of capital can be taken as exogenous. While these endogeneity problems have always been clear theoretically, empirical work has started to take them seriously only recently, in line with the broader movement towards a ‘credibility revolution’ in empirical economics (Angrist and Pischke, 2010). For example, Cloyne et al. (2018) exploit high frequency surprises in interest rate futures contracts for identification.

Given a definition of demand for capital K^* , the Jorgensonian formulation derives an investment function by assuming delivery lags. Net investment is therefore a distributed lag of new orders, which are made by firms in each period to fill the gap between the initial and the optimal capital stock. Given the assumption of ‘radioactive’ depreciation, replacement investment is proportional to the capital stock. Indicating net investment with I^N and replacement investment with I^R , we thus have

¹⁴Due to some ambiguities in the discussion in Jorgenson (1963) and in subsequent work, it is not easy to assess which of these two approaches is the one actually followed by Jorgenson’s original contribution. What matters for this survey is that these two approaches represent the two most influential theories of demand for capital in the literature.

$$I_t = I_t^N + I_t^R = \sum_{j=0}^J \beta_j \Delta K_{t-j}^* + \delta K_t \quad (6)$$

with β_s and J depending on the speed of delivery.¹⁵ Given the assumption that in each period firms order the amount of capital goods that would fill the gap between actual and optimal capital stock, the theory also implies $\sum_{j=0}^J \beta_j = 1$.

This represents a further difference between the Jorgensonian model and the traditional neoclassical approach described in Section 2. The latter could justify a finite rate of investment per unit of time based on the gradual adjustment due to putty-clay capital. In the Jorgensonian approach, instead, putty-putty capital is assumed, and delivery lags are introduced to avoid discontinuous jumps in the capital stock in response to shocks to the determinants of demand for capital.

The Jorgensonian derivation of the investment function from a demand for capital function implicitly assumes static (or myopic) expectations. The firm orders the quantity of capital goods that would make it reach the *current* optimal capital stock, even if it knows that they will be delivered with some lags, as if it always expected the optimal capital stock to remain the same in the future. In other words, all changes to the optimal capital stock are always assumed to be permanent. Alternatively, one can interpret the theory as assuming that delivery lags are unforeseen: the firm optimizes as if delivery was instantaneous, and then is continually surprised by delivery lags (Nerlove 1972, Junankar 1972, p.371, Söderström 1976). Of course both these interpretations are hard to square with rationality and perfect foresight, that the theory otherwise assumes

4.1 Criticisms of the Jorgensonian model

We can identify four broad strands of criticism that have been leveled at the Jorgensonian model of investment: (i) the ‘elasticity of substitution’ debate; (ii) the ‘speed of adjustment’ problem; (iii) the ‘Cambridge capital’ criticism; (iv) aggregation problems. As I will explain, the first three criticisms concern both the decreasing-returns-to-scale and the ‘neoclassical accelerator’ versions of the theory, while the fourth applies only to the former.

¹⁵Note that, for simplicity, delivery lags do not apply to replacement investments, but only to the share of investment that enlarges the capital stock.

The elasticity of substitution debate The ‘elasticity of substitution’ debate centers around the assumption of unitary elasticity of substitution between labor and capital ($\sigma = 1$). The original and most common formulation of the Jorgensonian model imposes this assumption through adoption of a Cobb-Douglas production function.

This criticism is famously associated with the pioneering empirical work of Robert Eisner. While early empirical work by Jorgenson and others had imposed the $\sigma = 1$ assumption, effectively constraining the elasticity of investment with respect to output and to the cost of capital to be the same, Eisner and coauthors showed that empirical tests of this constraint were unfavorable. The evidence found by Eisner, instead, pointed to a elasticity of substitution much closer to zero than to one. Debate between Eisner and Jorgenson on this topic was famously lively (Hall and Jorgenson, 1967; Eisner and Nadiri, 1968; Jorgenson and Hall, 1969; Eisner, 1970).¹⁶

The debate on whether the elasticity of substitution between labor and business fixed capital is high or low is still alive and very relevant today, and a consensus on whether σ is closer to one or to zero has not been reached (see, eg, Chirinko et al. 1999, 2011; Antràs 2004; Schaller 2006; Schaller and Voia 2017; Knoblach et al. 2020). The issue bears important implications for the expected consequences of economic policy on investment. Its theoretical significance is substantial as well: a low elasticity of substitution weakens the main mechanism through which the interest rate affects investment in neoclassical theory.¹⁷

The speed of adjustment problem The ‘speed of adjustment’ problem concerns the derivation of the investment function from the capital demand function. Jorgenson assumes that adjusting the capital stock is costless (besides the cost of acquiring capital goods). He takes into account delivery lags, but assumes that firms take every change in the capital stock as permanent, so they don’t influence the optimization problem. As a result, the firm makes its choices as if adjustment was instantaneous and costless. It has been argued, however, that to explain short-run investment dynamics one needs to model not only the optimal capital stock, but also the choice of the speed at which gaps between actual and optimal capital stock are filled. In order to do this, the adjustment process must be modeled explicitly (Söderström, 1976, 371). This point of view had been forcefully

¹⁶The econometric specifications employed by both Jorgenson and Eisner would not be considered conclusive today, of course, because of the strong assumptions that they required about the exogeneity of the regressors of interest.

¹⁷This survey focuses on investment, but the value of σ is relevant also for the theoretical and empirical study of other important topics, including growth and income distribution.

advocated by Haavelmo (1960), who argued in an often cited passage that

“The demand for investment cannot simply be derived from the demand for capital. Demand for a finite addition to the stock of capital can lead to any rate of investment, from almost zero to infinity, depending on the additional hypothesis we introduce regarding the speed of reaction of capital-users. I think the sooner this naive, and unfounded, theory of the demand-for-investment schedule is abandoned, the sooner we shall have a chance of making some real progress in constructing more powerful theories to deal with the capricious short-run variations in the rate of private investment.” (p.216)

This critique is at the basis of the most influential development of investment theory in the last decades – adjustment costs models – which can be seen as an attempt to enrich the neoclassical framework by explicitly modeling adjustment costs.

Cambridge capital criticism The Cambridge capital criticism – discussed in Section 2 in the context of the traditional Wicksellian model – applies to, and has been leveraged against, the Jorgensonian model as well (eg, Petri 2004, pp.256-291). Indeed the Jorgensonian approach, like the Wicksellian one, determines the optimal capital stock on the basis of a monotonic negative relation between the K/L ratio and the relative cost of capital. However, as we have already illustrated (Section 2.1 and Appendix B), the Cambridge capital criticism holds that the heterogeneity of capital goods creates analytical difficulties for this relation. In relation to the elasticity of substitution debate mentioned above, this criticism can be seen as arguing, on theoretical grounds, in favor of a $\sigma = 0$ assumption in equation 5– that is, in favor of a theory of demand for capital where quantities and not prices play the main role.

Aggregation problems The Jorgensonian model makes a representative firm assumption. The aggregate investment function is simply assumed to be a scaled-up version of that of the single firm. Aggregation problems have not been among the most prominent topics in debates on the neoclassical Jorgensonian model (exceptions are Lund 1971 and Ackley 1978).¹⁸ However, they are worth discussing, because they pose serious difficulties when the model is closed by assuming that the representative firm faces decreasing returns to scale.

¹⁸As we will see (Section 5), aggregation problems have instead been more extensively discussed in reference to adjustment-costs models of investment.

Indeed, the neoclassical Jorgensonian investment model with decreasing returns to scale suffers from a fallacy of composition. Decreasing returns to scale can determine the optimal level of output and capital for each firm, but not for the whole economy, in which the number of firms can vary. Naturally, decreasing returns to scale at the firm-level do not imply decreasing returns to scale for the whole economy.

This aggregation problem implies that the Jorgensonian theory of demand for capital under decreasing returns to scale is best interpreted as a theory of the optimal size of the competitive firm, rather than a theory of aggregate investment. The main implication of this theory – that investment can be determined only on the basis of prices and technology – might apply to firm-level investment but not to aggregate investment, even if all assumptions of the firm-level model hold. For example imagine that aggregate demand is expanding but it is not convenient for the existing price-taking firms to increase their size, because of strongly decreasing returns to scale. Other firms will be created - each one with the optimal size and K/L ratio - until the entire demand is met at cost-covering prices. So decreasing returns at the firm level will have no influence on aggregate output and on the aggregate capital stock, they will just determine the optimal size of the firm, and thus the optimal number of firms for each given level of aggregate demand.¹⁹

This aggregation problem, however, does not concern the ‘neoclassical accelerator’ version of the theory (equation 5). The latter can legitimately be seen as a theory of aggregate investment without committing a fallacy of composition. In that case the representative firm can be interpreted as a scale copy of the whole business sector (or of an industry), because the closure of the model relies on a decreasing demand function that holds also at the aggregate level. In other words, one can take the marginal productivity conditions as applying to the whole economy, due to both direct and indirect substitution mechanisms (as in the traditional Wicksellian approach), and at the same

¹⁹The fallacy of composition incurred by treatments of investment that rely on decreasing returns to scale is stated very effectively by Ackley (1978): “[An] incorrect derivation [of the neoclassical investment function] rests on factors internal to a firm, which have little relevance for an economy in which the number of firms is free to vary. We assume that a firm’s long-run (as well as its short-run) production is subject to diminishing returns – either because some factor of production, perhaps the contribution of the ultimate decision maker, is fixed in amount or because of inevitable diseconomies of organization, communication, or management that arise as the scale of a firm increases. This means that any firm’s calculation of profitability must recognize that, beyond some point, additional investment implies less than proportionate increases in output and/or more than proportionate increases in employment. Thus, the [interest rate level] that will equate [the marginal profitability of capital] and [the interest rate] must decline in order for [investment] to increase. True enough. But this explains only the size of the firm, not the amount of aggregate investment. The number of firms is not pre-ordained.[T]he preceding comments seem applicable to the “neoclassical” investment analyses of Jorgenson and others, who derive the specifications of an investment function from a micro-economic analysis of a profit maximizing-firm, without apparent reference to problems of aggregation ” (p.624)

time let aggregate output be a function of effective demand.

5 Adjustment cost models

Adjustment cost models aim to determine the rate at which the capital stock catches up with its desired level. Costs of altering the capital stock, additional to the costs of purchasing capital goods, are seen as the reason why adjustment is not instantaneous but gradual, making the investment rate well defined. Adjustment costs can include installation costs, reorganization of the plant, retraining of workers, delivery lags, and so on. These costs of adjustment are generally assumed to be a function of the rate of investment and (possibly) of the capital stock level.

In general terms, the approach can be summarized as follows. Given some definition of the desired capital stock K^* , the investment rate is a non-negative function of discrepancies between current and optimal capital stock: $I = f(K^* - K)$. Some assumption about the nature of adjustment costs determines the functional form of $f(\cdot)$, and therefore the optimal path of investment. Convex adjustment costs – which increase at an increasing rate in I_t – tend to imply a smooth and gradual path of adjustment. Non-convex adjustment costs can produce ‘lumpy’ investment: large, infrequent adjustment of the capital stock.

Adjustment cost models originate from the seminal contribution of Eisner and Strotz (1963). The canonical model features convex adjustment costs. A key theoretical result is that under convex adjustment costs Tobin’s q – the ratio of the market value of the marginal unit of installed capital to its replacement cost – can be a sufficient statistic for the firm’s investment decision (Lucas and Prescott, 1971; Yoshikawa, 1980; Abel, 1983). Hayashi (1982) clarified the relation between marginal and average q – a result of substantial importance because average q is, at least in principle, readily observable from market valuations.

Subsequent work has relaxed the assumption of convex adjustment costs, in order to explore alternative and more general shapes of the adjustment-cost function (Abel and Eberly, 1994; Caballero, 1999; Caballero and Engel, 1999; Cooper and Haltiwanger, 2006; Fiori, 2012). Models with non-convexities – for example, a fixed cost of adjustment – aim to better match observed patterns of plant-level investment, which tends to be intermitted and lumpy, and therefore inconsistent with the canonical convex adjustment cost models (Doms and Dunne, 1998; Cooper and Haltiwanger,

2006).

Although there seems to be consensus on the fact that non-convex models do a better job in matching firm-level investment patterns, the issue of what adjustment cost function is more useful for modeling aggregate investment in macroeconomic models remains open. If firm-level lumpiness washes out in the aggregate, the canonical convex adjustment cost model might be a better approximation to aggregate investment dynamics, which are empirically smooth rather than lumpy (Wang and Wen, 2012). Thomas (2002), Veracierto (2002) and Khan and Thomas (2003, 2008) provide neoclassical general equilibrium models in which non-convexity at the firm-level does not matter for aggregate investment dynamics, while Bachmann et al. (2013) reach the opposite conclusion.

5.1 The canonical convex adjustment cost model

In the canonical model of investment with convex adjustment costs, the firm faces the following optimization problem

$$\begin{aligned}
\max \quad & V(0) = \int_0^{\infty} e^{-rt} R(t) dt \\
\text{with} \quad & R(t) \equiv Y(K(t), L(t)) - G(I(t), K(t)) - w(t)L(t) - p^K(t)I(t); \\
& G_I > 0; \quad G_{II} > 0; \quad G_K \leq 0; \quad G(0, K) = 0; \\
\text{subject to} \quad & \dot{K}(t) = I(t) - \delta K(t); \quad \delta > 0
\end{aligned} \tag{7}$$

where R are net revenues; Y output; K capital; L labour; w the real wage; p^K the relative price of capital; G adjustment costs; δ the depreciation rate and r the interest rate (assumed constant here for simplicity). Output price is taken as the numeraire and adjustment costs are expressed in terms of units of output.

Without adjustment costs, the model would be practically equivalent to the Jorgensonian formulation, thus yielding conditions analogous to equation 3. With adjustment costs, instead, the first order conditions for an optimum imply

$$\mu(t) = p^K(t) + G_I(t); \quad \mu(t) = \frac{Y_K(t) - G_K(t) + \dot{\mu}(t)}{r + \delta} \tag{8}$$

where μ is the marginal value of installed capital. μ can be interpreted as the present value of the

increase in net revenues that would be provided by an additional unit of capital, after other inputs have been optimally adjusted.²⁰

The economic interpretation of equation 8 is that the profit-maximizing investment rate equates the marginal profitability of capital with its marginal cost. The marginal cost is comprehensive not only of the price of buying an additional unit of capital, but also of the increase in adjustment costs that its installation would generate. The marginal profitability of capital, in turn, depends on its marginal productivity, the depreciation rate, the effect of a greater capital stock on adjustment costs, the rate at which future profits are discounted, and the capital gains it will provide.

Given the assumed convex shape of the adjustment-costs function $G(\cdot)$, and assuming a large but fixed number of identical firms, equation 8 implicitly defines the following aggregate investment function

$$I = F(\mu, p^K, K) \quad \text{with} \quad F_\mu > 0; \quad F_K \geq 0; \quad F_{p^K} < 0 \quad (9)$$

This is the general result of models with convex and symmetric adjustment costs:²¹ investment is increasing in the marginal profitability of capital and in the capital stock and decreasing in the relative price of capital.²²

From the perspective of this survey, two points are especially worth noting. First, unlike Jorgensonian models, convex adjustment cost models yield definite results also under constant returns to scale and price-taking. The optimal capital stock under constant returns to scale and price-taking is still unbounded in this model (positively or negatively infinite), but the investment rate is nevertheless finite: the presence of convex adjustment costs limits the optimal rate of expansion or contraction.

In this sense, in terms of the discussion in Section 4, we can interpret the assumption of convex adjustment costs as a third alternative closure of the Jorgensonian neoclassical investment model.

²⁰It is important to keep in mind that the shadow price of capital is different from the marginal product of capital Y_K , because the latter is defined by taking the employment of other factors as fixed.

²¹By symmetric here it is meant that, keeping everything else fixed, the adjustment cost of increasing the capital stock by some amount is equal to the adjustment cost of decreasing it by the same quantity.

²²Imposing additional assumptions regarding the revenue function, the adjustment cost function and the efficiency of information-gathering in financial markets, the model can be shown to imply the same relation proposed by the Keynesian theory of Tobin and Brainard (1977) between stock market valuations and investment. In particular, indicating the stock market valuation of the firm as V and defining Tobin's q as $q = V/p^K K$, it can be shown that investment is a (positive) function of Tobin's q only (Hayashi, 1982). It is however more useful, for the purposes of this survey, to stick to this simple but more general formulation.

This closure can be seen as somehow similar in nature to the assumption of decreasing returns to scale, in the sense that it relies on factors internal to the firm, that impose some cost of expansion. The difference is that, while the Jorgensonian model with decreasing returns to scale allows to determine the desired capital stock level but not (without additional ad-hoc assumptions) a well defined rate of investment, convex adjustment costs allow to determine an optimal rate of investment without a well defined desired capital stock.

A second point worth noting is that this model implies a negative elasticity of investment to the interest rate – a defining feature of neoclassical theory – even in the absence of marginalist factor substitution mechanisms. As apparent from equation 8, a lower interest rate increases the present value of an additional unit of capital μ , resulting in a faster speed of adjustment and therefore a higher investment rate. Here the interest rate affects negatively the marginal benefit of increasing the rate of expansion through its role in discounting future profits, even independently of any role in the choice of technique. Indeed, the negative effect of the interest rate on investment would obtain even under a linear production technique of the $Y = aK$ type, with constant marginal product of capital.

5.2 Non-convex adjustment costs

To rationalize the ‘lumpy’ investment dynamics usually observed in manufacturing plants, with infrequent sizable waves of investment, recent literature has introduced non-convexities.

To illustrate descriptively the general features of non-convex adjustment cost models, we can consider a parsimonious model with fixed costs of adjustment– the most common model of lumpy investment in the recent literature (eg, Caballero and Engel 1999; Bachmann et al. 2013; Winberry 2021; Baley and Blanco forthcoming).²³

Firms incur a stochastic fixed cost ω every time they alter their capital stock. The size of this fixed adjustment cost varies randomly across firms and in time. Define the ‘frictionless’ or target optimal capital stock K^* as the capital stock level that would maximize profits in the absence of adjustment costs.

Of course the problem of deriving the frictionless optimal capital stock is exactly equivalent to that of deriving demand for capital in Jorgenson’s neoclassical model. It thus incurs in the same

²³Specifically, this simple exposition of the model follows Caballero (1999).

difficulty: the firm's 'frictionless' optimal capital stock is indeterminate under constant returns to scale and price-taking. To avoid indeterminacy, decreasing returns to scale are typically assumed. The necessity to determine a well-defined target (frictionless) optimal capital stock prior to deriving the rate of investment through adjustment costs is not specific to this simple version of the model, but a general feature of adjustment cost models with fixed costs.

The probability that a firm adjusts, given its imbalance and the adjustment cost it faces, is given by the adjustment hazard function

$$\Lambda = \Lambda(z, \omega) \quad \text{with} \quad \Lambda_z > 0; \quad \Lambda_\omega < 0 \quad (10)$$

where z is the deviation from the frictionless optimal capital stock (the *imbalance*), defined as $z_{it} = \ln(K_{it}) - \ln(K_{it}^*)$.

In this simple case in which adjustment costs are only fixed,²⁴ if the firm adjusts it does so fully, reaching instantaneously its optimal capital stock (an 'all-or-nothing' investment policy). The expected investment rate of a firm is thus equal to its imbalance times the probability of adjustment

$$E(I_{it}/K_{it}|z) = -z\Lambda(z) \quad (11)$$

In order to infer aggregate investment dynamics from this microeconomic model, it is assumed that the number of firms is large but fixed. $\Lambda(z)$ is then interpreted as the share of firms with imbalance equal to z that will adjust, given the distribution function of ω . Aggregate investment is the integral of $\Lambda(z)$ over the cross-sectional distribution of z .

$$I_t/K_t = - \int_{-\infty}^{\infty} z\Lambda(z)f(z, t)dz \quad (12)$$

where $f(z, t)$ is the distribution of z across firms at time t . Intuitively, aggregate investment in the fixed set of firms is the sum of the expected adjustment of each firm.

A consequence of equation 12 is that not only the average imbalance ($K - K^*$) in the economy matters for aggregate investment, but also its distribution across firms. For a given average imbalance in the economy, higher variance and positive skewness in its distribution across firms are likely

²⁴See Caballero and Engel (1999) for the version of this model with both fixed and variable adjustment costs.

to lead to higher aggregate investment.

5.3 Can adjustment costs explain aggregate investment? Theoretical issues and criticisms.

The main questions, from the perspective of this survey, are (i) what novel insights adjustment cost models have provided to investment theory; (ii) whether they address the theoretical issues with the Wicksellian and Jorgensonian models; and (iii) whether they present additional theoretical difficulties.

Adjustment costs and the determinants of investment From the perspective of investment theory, adjustment cost models contribute a theory of the optimal adjustment path of a firm's capital stock towards its desired level. They thus provide appealing and tractable models of short-run investment patterns at the firm level. With the introduction of non-convexities, they can rationalize the lumpy pattern of plant-level investment that we observe empirically. It is of course hard to assess whether non-convex adjustment costs are really the cause of lumpy investment, given that adjustment costs are multiform and unobservable. But these models do certainly provide parsimonious models of firm-level investment rates.

On the other hand, adjustment cost models generally do not add much to the study of the deeper factors which determine long-run trends in demand for capital, because they do not provide novel insights on the determinants of desired capital stock levels. By construction, they are better apt to study short-run fluctuations in firm-level investment than long-run aggregate investment trends.

Investment and the interest rate in the convex adjustment cost model The canonical convex adjustment cost model, nevertheless, provides a way to derive a negative elasticity of investment to the interest rate without relying on problematic capital-labor substitution mechanisms. In this model, the interest rate affects investment through the discount factor applied to the future stream of profits. As shown by equations 8 and 9, a decrease in the interest rate, *keeping all other prices fixed*, raises the optimal investment rate by shifting upwards the shadow price of capital (μ) curve. This is an important contrast with the Wicksellian and Jorgensonian theories, in which the interest rate affects investment through the choice of technique. In this sense, the conclusions of

convex adjustment cost models do not seem to depend on the assumption of a single homogeneous capital good, unlike traditional neoclassical models.

However, this derivation is based on keeping present and future wages and prices fixed while the interest rates varies— a dubious exercise in a model that features a competitive price-taking firm.²⁵ This is exactly the same theoretical difficulty encountered by the ‘array of opportunities’ approach (Section 3). The interest rate on employed capital is a component of average costs. Assuming that the relation between output and input prices remains unchanged in the aftermath of an interest rate reduction, is tantamount to assuming that extra-profits can be earned over an infinite time-horizon. Clearly, the rate of return on capital would be expected to react to interest rate changes, in the presence of competitive forces.

Aggregation problems An important theoretical issue with adjustment cost models is the aggregation problem: under free entry, adjustment cost models are generally unable to determine aggregate investment. This is of course a major difficulty with this approach, insofar as it is used to explain aggregate dynamics.

This problem is not new: as we have seen, Ackley (1978) warned against explanations of aggregate investment that rely “on factors internal to a firm, which have little relevance for an economy in which the number of firms if free to vary” (p.624). Similarly, in an early survey, Söderström (1976) wrote that in models with adjustment costs “market equilibrium (...) may be indeterminate under free entry”. Recent discussions of this aggregation problem are found in Pindyck (1993, p.274) and Petri (2004, p.280). Both Pindyck (1993, p.274) and Petri (2004, p.280) note that in convex adjustment cost models, when a gap opens up between current and desired investment and existing firms will not cover it entirely because of adjustment costs, an indefinitely large number of very small new firms would enter. These newly-created very small firms would pay lower average costs due to convexity of the adjustment cost function.

This issue is typically side-stepped in this literature by assuming a fixed set of firms in the economy. Of course, however, it is difficult to justify ruling out entry in models of aggregate investment. But with free entry, factors such as internal adjustment costs or decreasing returns to scale can determine the optimal expansion rate of the single firm, but not necessarily the rate of

²⁵This problem is discussed in Petri (2004, pp.279-280).

aggregate investment. This problem is of course similar to that discussed in relation to Jorgensonian models with decreasing returns to scale.

Indeterminacy in non-convex adjustment cost models As we have seen, models of lumpy investment typically need to specify a ‘frictionless’ optimal capital stock– the capital stock level that would maximize firm value in absence of adjustment costs. This introduces an indeterminacy problem – the same faced by Jorgensonian model – because under constant returns to scale and price-taking, this firm-level optimal capital stock is indeterminate. But with indeterminacy in the optimal capital stock, the model cannot define the imbalance z , equal to the distance between the current and the frictionless capital stock, a crucial and necessary ingredient for deriving the investment function. As already discussed, this problem can be addressed in models featuring imperfect competition and allowing for an ‘accelerator’ effect, in which demand determines the optimal output level.

6 Conclusions

We have reviewed the neoclassical determination of demand for capital and investment, and its criticisms. We have distinguished between theories of the optimal capital stock (as conceived by the traditional ‘Wicksellian’ theory and later by Jorgensonian models) and theories of the short-run frictions which govern the adjustment process.

A key takeaway that emerges from this comparative analysis is that the development of adjustment cost models does not make the debate on the traditional ‘early marginalist’ determination of demand for capital less relevant or outdated. Underlying the modeling of short-run frictions, there is still a logically prior determination of the factors that determine deeper trends in desired (‘frictionless’) capital stocks. At the core of the most recent models with non-convexities, there are still Wicksellian mechanisms of capital-labor substitution, and a conception of long-run investment as mainly a choice of technique problem.

The q model with convex adjustment costs is a partial exception, in that its conclusions do not necessarily rely on capital-labor substitution, and because it can derive the rate of investment without needing a prior well-defined determination of the ‘frictionless optimal capital stock’. However,

the assumption of convex adjustment costs is restrictive, and incompatible with empirically lumpy firm-level investment. Moreover, the implication of the convex adjustment cost model with price-taking, that adjustment costs are the only factor preventing boundless capital expansion, does not seem realistic. More recent models with non-convex frictions are able to rationalize the infrequent large investment bursts that we observe at the firm level, but lose the property of not requiring a well defined ‘frictionless’ optimal capital stock to start with.

From this perspective, there is a striking contrast between continuing innovation and progress in the modeling of short-run frictions, and a theory of the fundamental forces determining underlying long-run investment trends that arguably did not change very much relative to late 19th Century neoclassical authors. Of course, this might just mean that Wicksell was right on the broad determinants of investment. However, the criticisms we have surveyed highlight some unresolved theoretical issues, suggesting that the debate should not be considered settled.

First, the neoclassical mechanisms producing a negative relation between investment and the cost of capital are less theoretically robust than textbooks suggest. They encounter difficulties related to heterogeneity of capital goods, aggregation problems, and the interdependence of prices and distributive variables. This also implies that recent re-examinations of the ‘microfoundations of aggregate production functions’ (Baqae and Farhi, 2019) can bear significant implications for contemporary investment theory.

These theoretical ambiguities might help explain, for example, why a sharp cut in dividend taxation in the US in 2013 failed to exert any effect on corporate investment (Yagan, 2015). In fact, it is commonplace to consider the influence of the cost of capital on investment as almost obvious theoretically but, because of challenging identification problems, not easy to detect empirically.²⁶ The review presented here suggests that the relation might be dubious theoretically as well as empirically.

Another conclusion that emerges from this survey is that, even within a neoclassical framework, it is hard to escape the conclusion that aggregate demand is a main determinant of business investment. Unless one assumes Say’s law to start with – as the Wicksellian (and pre-Keynesian) model

²⁶The passage by Gilchrist and Zakrajsek (2007) quoted in the introduction, in which the theoretical relation between interest rates and investment is considered unambiguous, continues as follows: “Nevertheless, the presence of a robust negative relationship between investment expenditures and real interest rates—or the user cost of capital more generally—has been surprisingly difficult to document in actual data (p.1).” (Gilchrist and Zakrajsek 2007 do find a substantial negative effect of the cost of capital on investment in their firm-level study.)

does – investment appears bound to depend on output. Decreasing returns to scale, often imposed to derive an investment function that depends only on the interest rate, cannot actually do the trick: if existing firms did not increase investment in the face of positive demand shocks because of decreasing returns, increases in capacity would come from entry of new firms.

Coupled with strong empirical evidence that investment depends on demand (eg, Girardi and Pariboni 2020), this suggests that the influence of aggregate demand on investment can be a promising explanation for hysteresis, defined as the idea that aggregate demand effects can persist beyond the short-run (Yellen et al. 2016, pp.1-2, Fatás and Summers 2018, Girardi et al. 2020, Fazzari et al. 2020).

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Appendix A Formal derivation of the benchmark ‘Wicksellian’ investment function

This Appendix presents a simple formalization of the benchmark marginalist (‘Wicksellian’) determination of investment.

Take a one good economy (steel is produced by means of labour and steel)²⁷ with no taxes. Specifically, we assume a Cobb-Douglas production function, which implies perfect ex-ante substitutability between capital and labour.

Perfect substitutability applies only to new plants in the short-run but to the whole economy in the long-run (even if technology is putty-clay, in the long-period²⁸ all productive plants will eventually come to adopt the optimal K/L ratio, provided that the latter remains constant for a sufficient lapse of time).

We also assume, for simplicity, a rigid labor supply; the market-clearing condition thus reduces to an equality between labor employment and the given supply of labor.

Profit maximization requires that the value marginal product of each factor is equal to its price.

²⁷Here steel is both the capital good and the consumption good: one unit of steel can either be consumed or employed as a means of production. Wages are obviously also paid in steel.

²⁸Note that here the ‘long-period’ does not correspond to that horizon of time in which one can expect prices to converge to production costs, but rather to the longer time horizon in which the equilibrium K/L ratio is adopted by all production units. Some authors refer to this kind of long-period equilibrium as a fully-adjusted position (Vianello, 1985, 70).

In a long-period position we thus have

$$Q = AK^\alpha L^\beta \quad (13)$$

$$w = \frac{\delta Q}{\delta L} \quad (14)$$

$$r = \frac{\delta Q}{\delta K} \quad (15)$$

$$L = \bar{L} \quad (16)$$

Q is output (the quantity of steel produced); K the capital stock (steel used as capital); L labour (worker-hours); w the wage; r the interest rate; p the (normalized) long-period price of steel.

Equations 13 to 15 determine the optimal K/L ratio:

$$K/L = (\alpha/\beta)(r/w)^{-1} \quad (17)$$

Eq.16 (the full labor employment condition) closes the model, allowing the interest rate to univocally determine demand for capital:

$$K^* = \left(\frac{r\bar{L}^{-\beta}}{\alpha A}\right)^{\frac{1}{\alpha-1}} \quad \text{and} \quad \frac{\delta K^*}{\delta r} = \gamma r^{\left(\frac{1}{\alpha-1}-1\right)} \quad (18)$$

where γ is a constant.²⁹ Decreasing marginal productivity of capital ($\alpha < 1$) is a sufficient condition for the effect of the interest rate on the optimal capital stock to be negative ($\gamma < 0$), as long as r is non-negative.

Figure 1 in the main text illustrates an example in the K-L space with $\alpha = \beta = 0.5$. Imagine that initially the interest rate and wage rate are such that the optimal K/L ratio is equal to a . Since L is exogenously fixed, the intersection between the vertical dotted line indicating the quantity of labour and the grey slope corresponding to the optimal K/L ratio determines a definite long-period position (point 1) with a determinate optimal capital stock (K_1) and level of output (the isoquant q_1). Imagine that the interest rate increases to such a level that the optimal K/L ratio decreases to b . As old plants are gradually scrapped and replaced with new plants with a lower K/L ratio, the

²⁹More precisely, $\gamma = \frac{(\alpha A)^{\frac{1}{\alpha-1}} \bar{L}^{-\frac{\beta}{\alpha-1}}}{\alpha-1}$

capital-intensity of the economy will gradually decrease. Eventually, all plants would adopt the new optimal K/L ratio and the economy would reach the new long-period position (point 2). During the slow transition, a cumulated flow of net (dis)investment equal to $K_2 - K_1$ will take place. Also aggregate output decreases.

Appendix B The Cambridge capital controversy and the relation between distribution and the choice of technique

In this appendix we provide a simple exposition of the theoretical problems of early marginalist investment theory – in particular the impossibility of deriving a monotonic functional relation between the optimal K/L ratio and the relative price of capital when there is more than one capital good.³⁰ This critique of the marginalist determination of demand for capital was brought up during the so-called Cambridge-capital controversy, mainly by economists working at Cambridge, UK.

In this exposition we take quantities produced as given, assume that no scarce resources enter production, no joint production is undertaken and the non-substitution theorem applies.³¹ Hence in the long-period prices converge, due to competitive pressure, to minimum average costs. Long-period prices can thus be determined as functions of the rate of interest, according to the system of equations

$$\mathbf{p} = \mathbf{p}(\mathbf{d} + r\mathbf{I})\mathbf{A}(r) + w\mathbf{l}(r) \tag{19}$$

where \mathbf{p} is the (row) vector of product prices; \mathbf{A} is the matrix of technical coefficients of non-labor inputs, r the rate of interest (here identical with the rate of profit), \mathbf{I} the identity matrix, \mathbf{d} a diagonal matrix which has the vector of input depreciation rates as its diagonal (depreciation is assumed to be of the ‘radioactive’ type), \mathbf{l} the vector of technical coefficients of labor input and w the wage rate.

³⁰Obviously, I claim no originality at all for this appendix: the results summarized here are well-known.

³¹The restrictiveness of these assumptions – and of the ones that will be introduced later – is not a problem here, because our aim is to show an internal inconsistency in the neoclassical determination of the optimal K/L ratio. If this can be shown in the simplest setting, it will apply also to more nuanced models.

B.1 Dependence of the quantity of capital on distribution

The quantity of capital in the economy is an aggregate measure of the several existing capital goods. As these different capital goods are physically heterogeneous, this aggregate measure cannot be expressed in physical quantities. It must be expressed in value.³² But then – as shown by the seminal contribution of Sraffa (1960) – independently of whatever numeraire is chosen, the value of the aggregate capital stock depends on distributive variables.

A simple representation of this result is found in the Sraffian literature (for example Petri (2004, 208-210)). Consider the simplest case in which technical coefficients are taken as given (only one technique of production is available for each production process) and all capital is circulating ($d = 1$), so the price equations in eq.19 become

$$\mathbf{p} = [1 + r]\mathbf{p}\mathbf{A} + w\mathbf{l} \quad (20)$$

The economy-wide net product is taken as a numeraire ($py = 1$, with y representing the vector of quantities produced and p the vector of prices). Furthermore, measure labor input in such a way that total labour employment L is $L = 1$ (total labor employment is exogenously determined, because we are taking technical coefficients and quantities produced as given).

Consider the system of price equations (eq. 20). The vector of prices p and the wage rate w can be determined on the basis of the interest rate r and – due to the Perron-Frobenius theorem for non-negative matrices – a wage curve $w(r)$ exists, is downward-sloping and crosses both axes³³ in the positive orthant of the w - r space³⁴. We thus have a decreasing wage curve, as in Figure B.1.³⁵ Given our assumptions and choice of numeraire, the wage rate equals both the total wage bill and the wage share. The two intercepts of the wage curve represent the extreme cases in which all income goes to labor (intercept with vertical axis: $r = 0 \Rightarrow w = 1$) or to capital (intercept with horizontal axis: $w = 0 \Rightarrow r = 1$).

We thus have the income identity $p(r)y = 1 = rk + w(r)$ and we can write k as a function of

³²See Petri (2004, 89-91) for a thorough explanation of why capital, as conceived by the early neoclassical approach, can only be measured in value.

³³We also assume that there exists at least one good, entering some production process as an input, that also enters directly or indirectly in its own production process. Without this assumption, the interest rate would tend to infinite as the real wage tends to zero, so the $w(r)$ curve would not intersect the horizontal axis.

³⁴See Kurz and Salvadori (1997) for the mathematical demonstration.

³⁵We depict a concave wage curve, but there is no guarantee that this is the case: it may be well convex or have inflections. This is irrelevant for our exposition.

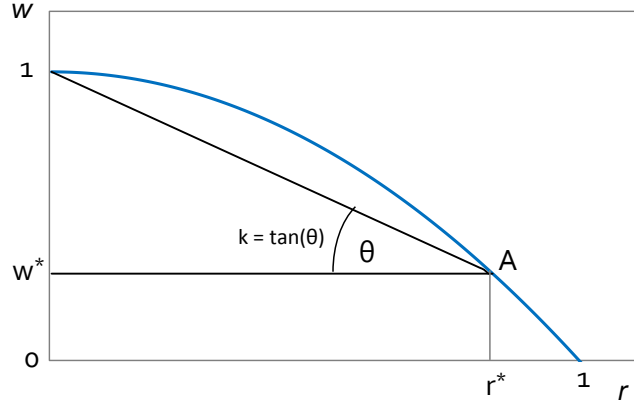


Figure B.1: Relation between income distribution and the value of capital

the interest rate as follows

$$k = k(r) = \frac{1 - w(r)}{r} \quad (21)$$

Meaning that the value of capital is equal to the ratio between the sine and the cosine – and thus the tangent – of angle θ in Figure B.1. It is thus clear that the value of capital is a function of the interest rate.

B.2 Inversion in relative price movements

As already mentioned, given the price system in eq.20, the vector of relative prices and the wage rate can be determined as functions of the interest rate. An important result demonstrated by Sraffa (1960) is that the price of a commodity in terms of another commodity need not be a *monotonic function* of the interest rate. To accomplish this, he first demonstrated that in a system like eq.19 the price of a commodity can be determined by ‘reduction to dated quantity of wages’. This means that the price of a commodity can be calculated as an infinite series: the remuneration of the quantity of labor directly employed in the production of the given commodity, plus the (discounted) remuneration of the direct labor necessary to produce the inputs, plus the (twice discounted) remuneration of the direct labor necessary to produce the inputs to these inputs, and so on, in an infinite regress.

Formally, this can be proved by applying the Perron-Frobenius theorem, which implies that the following equality holds

$$[I - (1 + r)A]^{-1} = I + (1 + r)A + (1 + r)^2 A^2 + (1 + r)^3 A^3 + \dots \quad (22)$$

We can rewrite the price equations in eq.20 as $p = w\ell[I - (1 + r)A]^{-1}$ and then apply eq.22 to obtain

$$p = w\ell[I + (1 + r)A + (1 + r)^2 A^2 + (1 + r)^3 A^3 + \dots] \quad (23)$$

Let us now introduce the notation $L_{i(t)}$ to indicate the dated quantities of labor that must be applied at each time period $T - t$, for the production process of a good i to happen at time T . $L_{i(0)}$ is thus the quantity of labor directly applied in the production process that generates good i at time T ; $L_{i(1)}$ is the quantity of labor directly applied at time $T - 1$ in the production processes that will produce the intermediate goods that are necessary to produce i ; $L_{i(2)}$ is the quantity of labor directly applied at time $T - 2$ to produce the means of production that at period $T - 1$ are employed to produce the inputs necessary to produce i , and so on. And of course we will have $L_{i(0)} = \ell$; $L_{i(1)} = \ell A$; $L_{i(2)} = \ell A^2$; $L_{i(3)} = \ell A^3$, ... and in general $L_{i(t)} = \ell A^t$. Applying this notation to eq.23, we can express the price of a commodity as an infinite series of discounted dated wages

$$p_i = wL_{i(0)} + wL_{i(1)}(1 + r) + wL_{i(2)}(1 + r)^2 + wL_{i(3)}(1 + r)^3 + \dots \quad (24)$$

Sraffa (1960, 37) provides the example of two commodities – ‘a’ and ‘b’ – which differ in three of their ‘dated labour’ terms, all the others being equal. In other words, the series of dated wage payments determining the prices of a and b are identical, except for three terms. Commodity ‘a’ implies a greater number of work hours by 20 units at time $T - 8$, while commodity ‘b’ necessitates 19 more work hours at the time of production T and 1 more unit applied 25 years earlier. (Sraffa notes that this can resemble the classical example of ‘wine aged in the cellar’ and of ‘old oak made into a chest’.) The difference between the prices of these two commodities is thus equal to

$$p_a - p_b = 20w(1 + r)^8 - [19w + w(1 + r)^{25}] \quad (25)$$

Sraffa takes the ‘Standard Commodity’ as the numeraire.³⁶ This implies that the wage-curve is

³⁶Given a certain economy (defined by its price equations), the Standard Commodity is a composite commodity,

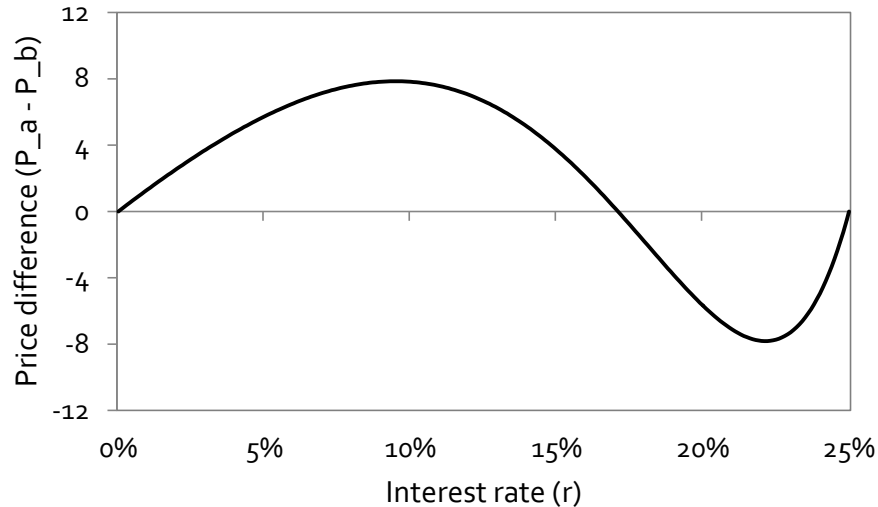


Figure B.2: Difference between the prices of commodities a and b as a function of the interest rate
Source: Sraffa (1960)

linear and equal to $w = 1 - \frac{r}{R}$, where R is the maximum rate of profit. (But note that this choice of numeraire – and so the particular shape of the wage curve – are not necessary for proving the point, which would hold under any numeraire.) By assuming a maximum rate of profit of 25%, we can appreciate the dynamics of $p_a - p_b$ as a function of the interest rate (Figure B.2).

We thus see that the price of a commodity relative to another can well be a non-monotonic function of the interest rate. Specifically, in this example, the price of a relative to b increases as the interest rate rises from 0% to 9%, then decreases as r passes from 9% to 22%, and then rises again when r increases from 22% to 25%.

From the point of view of neoclassical analysis, this means that for some values of the interest rate ($r < 9\%$ and $r > 22\%$) a is more ‘capital-intensive’ than b , but for others ($9\% < r < 22\%$) it is b that is more capital-intensive than a . So the relative capital intensity of the two commodities changes *even if their production methods are held constant*.

constructed in such a way that if the net product of an economy was constituted by the Standard Commodity, the set of non-wage inputs necessary for its production would present exactly the same composition. Sraffa takes as the unit of measure of the Standard Commodity the quantity of it whose production would necessitate the same quantity of labor (per period) employed by the economy under study (Sraffa, 1960, pp.18-25).

B.3 Reswitching of techniques

The result just presented undermines the mechanisms of factor substitution on which the neoclassical capital demand function is based. Indirect substitution predicts that the composition of demand would shift towards more capital intensive goods as the interest rate decreases. It thus needs a *purely technical* measure of capital intensity, independent of distribution. But the latter, as the example above demonstrates, does not in fact exist. If some arbitrary criterion is adopted to establish that commodity a or commodity b is more capital-intensive irrespective of distribution, we will have the paradoxical (from the point of view of neoclassical theory) result that, in some relevant ranges, increases in the cost of capital will reduce the price of the more capital intensive technique, thus *increasing* demand for capital.

By interpreting ‘a’ and ‘b’ as two alternative techniques for producing the same commodity, instead of two different commodities, it is possible to show that also the mechanism of direct substitution is based on shaky theoretical foundations. p_a and p_b are thus now interpreted as the unitary costs of production of a and b , and of course cost-minimizing firms will adopt the technique with lower p . Following Petri (2004, 214-218), the point can be shown very simply by modifying Sraffa’s example in such a way that the $p_a - p_b$ curve (eq.25 and Fig.B.2) is shifted downwards by some small amount. (This can be done by simply assuming uniformly smaller production coefficients for commodity a – see (Petri, 2004, 215)). The point of this modification is to yield an example in which the $p_a - p_b$ curve crosses twice the horizontal axis, as in Figure B.3, thus showing how is it possible for a technique to be cost-minimizing at relatively low and high values of the interest rate, but not at intermediate values. This phenomenon is known in the literature as *reswitching of techniques*. Besides the one that we just considered (taken from Petri (2004, 215)), other numerical examples of reswitching have been presented for example by Samuelson (1966) and Pasinetti (1966). . Notably, Han and Schefold (2006) provide empirical (as opposed to theoretical) examples of reswitching, using the OECD input-output tables dataset.

B.4 Reverse capital deepening

Reverse capital deepening occurs when a rise in the interest rate leads to an increase in the ratio

of aggregate capital (measured in value)³⁷ to total labor employment. It is straightforward to see that reswitching of techniques leads to reverse capital deepening. Consider the example with one commodity and two available techniques (a and b) presented in the previous section and depicted in Figure B.3. The corresponding wage curves associated with the two techniques are displayed in Figure B.4. For each given level of the interest rate, the cost-minimizing technique is the one associated with the most external wage curve, i.e., the one that lies on the outer envelope.³⁸ Again we see that technique a is more profitable at low and high levels of the interest rate, but not at ‘intermediate’ values. Recall from Sec. B.1 (in particular Fig.B.1) that the value of capital can be inferred from the prevailing wage curve.³⁹ We can thus appreciate the evolution of the value of capital per worker as a function of the interest rate in this example. This is depicted in Figure B.5. A discrete increase in the interest rate from (for example) 1% to 4% would have in this case the effect predicted by neoclassical theory: capital intensity would decrease; however a further interest rate increase, for example from 4% to 10% (of from 10% to 15%, or from 15% to 20%), would cause an *increase* in capital intensity.

But while reswitching of techniques is a sufficient condition for reverse capital deepening, it is not a necessary one. Reverse capital deepening can occur also in the absence of reswitching. A trivial (but possibly empirically relevant) case is the following. Imagine that the same technique is cost-minimizing for all relevant values of the interest rate. This is equivalent to (but more general than) the case in which only one production technique is available. Suppose, furthermore, that the wage curve generated by this production method is concave, like the wage curve associated with technique b in Figure B.4. The corresponding relation between the interest rate and the value of capital per worker will be monotonically *increasing*, like in the segment of Figure B.5 in which technique b remains dominant. This is due to the so-called *price Wicksell effects*: in the absence of any switch of technique, the aggregate value of capital is altered by the price changes caused by interest rate variations. Note that also in the presence of multiple switches of techniques that are all well-behaved from a neoclassical point of view (that is, interest rate increases are always

³⁷Of course the discussion in the preceding sections implies that the conception of capital as a single factor measurable in value independently of distribution is flawed. In this section we adopt this mistaken definition only to show that it brings to results that run counter to neoclassical capital theory.

³⁸This is a well-known result, which demonstration can be found for example in Kurz and Salvadori (1997).

³⁹Note that in this case, with different wage curves compared in the same diagram, we do not set the intercept of both wage curves at $w = y = 1$: the real wage associated with $r = 0$ (that is, y) is different for the two techniques. So the formula determining the value of capital is, more generally, $k = (y - w)/r$.

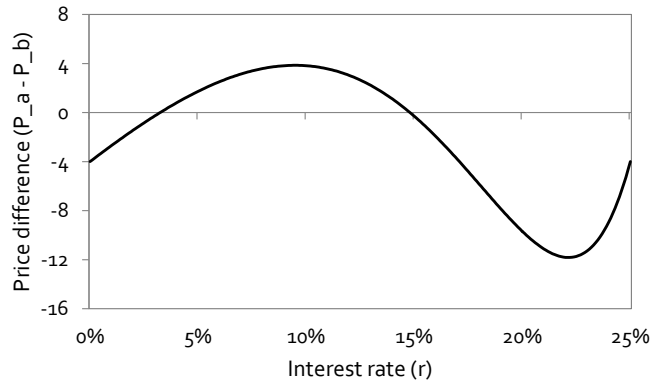


Figure B.3: Difference between the unit production cost of techniques a and b as a function of the interest rate

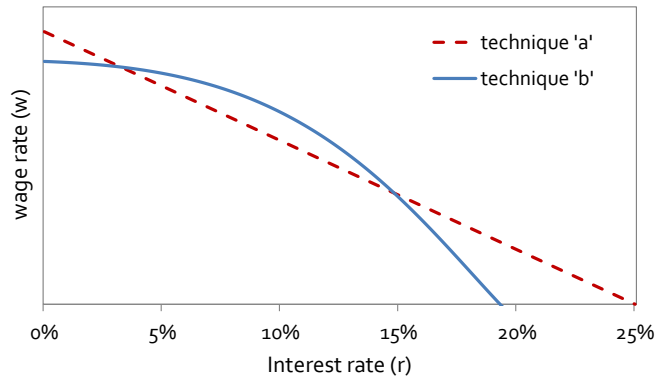


Figure B.4: Reswitching between techniques a and b in terms of wage curves

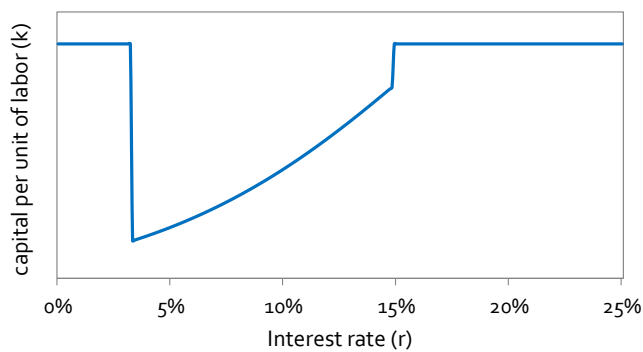


Figure B.5: Capital intensity as a function on the interest rate, resulting from reswitching between production techniques a and b

Source: Our own elaboration on Petri (2004, 215-217) (Sraffa example modified to yield reswitching)

associated with switches toward less capital intensive techniques), positive price Wicksell effects may still dominate them and produce a non-neoclassical demand curve for aggregate capital.

In addition to the cases just discussed, based on price Wicksell effects, reverse capital deepening in the absence of reswitching can be also be caused by technique switches that run counter to marginalist theory. This can happen when there are more than two available techniques. Then it is possible that two techniques cross more than once, but only one of the switches is on the outer envelope. In this case we would have reverse capital deepening whenever there is a switch between two techniques whose wage curves have already crossed at a lower interest rate. And whenever this first intersection between the two wage curves is situated below the outer envelope (while the second, at a higher interest rate, is on the outer envelope) reverse capital deepening happens in the absence of reswitching.

The bottom line is that capital, defined as the set of all commodities that are employed as means of production, cannot be conceived as a single homogeneous factor, that will be employed more intensively in production whenever its price decreases. Capital cannot be conceived as a single homogeneous factor because there is no way to measure its quantity in ‘technical’ units – that is, independently of distributive variables. And even if that erroneous conception of capital is adopted, there is no guarantee that the resulting demand curve for capital is monotonically decreasing in the interest rate.