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Marx's Theory of Ground-Rent: A Suggested Reformulation

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Marx's Theory of Ground-Rent: A Suggested Reformulation

Deepankar Basu*

November 29, 2020

Abstract

This paper develops a simple theoretical model to analyze Marx's theory of ground rent. Using the model, I demonstrate two important results. First, if we take capital as exogenous, then total ground-rent can be decomposed into the three components: differential rent of the first variety (DRI), differential rent of the second variety (DRII), and absolute rent (AR). Second, if we endogenize capital outlays using profit-maximizing behaviour of capitalist farmers, then absolute rent becomes zero. Thus, under reasonable behavioural assumptions about landlords and capitalist farmers, there will be no absolute rent in a capitalist economy.
JEL Codes: B51.
Key words: ground rent, differential rent, absolute rent.

1 Introduction

The theory of ground-rent is an important but severely under-theorized part of Marxist political economy. Part of the reason for the lack of theoretical development in this area probably arises from the difficulty scholars face in clearly and precisely defining the meaning of key terms involved in the

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discussion: ground rent, and the three components into which Marx decomposed it, i.e. differential rent of the first variety (DRI), differential rent of the second variety ($DRII$) and absolute rent (AR). In this paper, I present a theoretical model to define ground-rent, to decompose it into DRI, $DRII$ and AR, and to analyze in detail the meaning and source of AR.

Marx understood ground-rent, in a capitalist economy, as a part of the surplus value that is appropriated by private owners of non-reproducible resources like land (Marx, 1993). Use of the non-reproducible resource can confer benefits on capitalist producers and generate what Marx called 'surplus profit', i.e. profit over and above the prevailing uniform (average) rate of profit. Therefore, bargaining between private owners of non-reproducible resources and capitalist producers who wish to use those resources leads to the latter appropriating the full amount of the surplus profit as ground-rent. Therefore, for Marx, ground-rent is the economic form that the monopoly of private ownership of non-reproducible resources takes in a capitalist economy, and its magnitude is the surplus profit. While Marx mainly discussed land and agricultural production, he was clear that his analysis applied to all non-reproducible resources in capitalist economies where a private monopoly of ownership obtains.

> Whenever natural forces can be monopolized and give the industrialist who make use of them a surplus profit, whether a waterfall, a rich mine, fishing grounds or a well situated building site, the person indicated as the owner of these natural objects, by virtue of his title to a portion of the earth, seizes this surplus profit from the functioning capital in the form of rent. (Marx, 1993, Chapter 46, pp. 908)

Using numerical examples, Marx had argued that the total ground-rent on any plot of land can be decomposed into three parts: DRI, which captures relative quality advantages of the plot; $DRII$, which captures the effect of the quantity of capital employed, and therefore diminishing marginal returns to the application of capital, on any plot of land; and AR, which, according to Marx, captures the effect of low organic composition of capital in agriculture or the class power of landlords. Using the model presented in this paper, we will be able to decompose total ground-rent along the lines proposed by Marx. The decomposition analysis demonstrates the following interesting results: if we take the capital outlays as exogenously given, then ground-rent can indeed be decomposed into three positive components, $DRI, DRII$

and AR; but, if we endogenize the magnitude of capital outlay using profit-maximizing behaviour of capitalist farmers, then absolute rent will be zero.

Perhaps previous discussions of Marx's rent theory have been mired in controversy and disagreements because scholars have not precisely defined the meaning of basic terms or moved beyond the use of simple numerical examples (Mandel, 1962; Ball, 1977; Fine, 1979; Haila, 1990; Ramirez, 2009; Fine and Filho, 2010; Ward and Aalbers, 2016). While there is broad agreement about the definition of ground-rent, there is much controversy about the decomposition of ground-rent into its three components: DRI, $DRII$ and AR. Two sources of disagreements are especially worth highlighting.

First, scholars have very different views about the existence, meaning and source of absolute rent (Fine, 1979; Ramirez, 2009; Fine and Filho, 2010). While some scholars view absolute rent as arising from the class power (or monopoly power) of landlords (Ramirez, 2009), others contend that the cause of absolute rent is the relatively low organic composition of capital in agriculture (Fine, 1979, 2019). Second, while there is agreement on the meaning of DRI and $DRII$, there is no theoretical framework available to implement the intuitions about these two components of differential rent. Part of the difficulty in carrying out this decomposition arises because DRI and $DRII$ use different references. While DRI is defined with respect to the worst-quality plot of land, $DRII$ is defined with respect to the worst-unit of capital. Most scholars, following Marx, have addressed DRI and $DRII$ one at a time, but have not been able to integrate them together into a single theoretical framework (Fine, 1979; Fine and Filho, 2010).

The first contribution of this paper is to offer a general theoretical framework in which key insights of Marx regarding ground-rent and its decomposition into DRI, $DRII$ and AR can be analyzed. In particular, I show that the total ground-rent on any plot of land can be decomposed into DRI, $DRII$ and AR. Thus, I present an integrated framework to decompose total ground-rent into its three components.

The second contribution of this paper is to throw some light on the meaning and existence of absolute rent. My analysis shows that absolute rent can be positive only when there is a gap between the marginal product of capital outlay and the opportunity cost of employing capital. If capitalist producers choose to maximize profits, then their choice of capital outlay will equate the marginal product of capital outlay and the opportunity cost of employing capital. Hence, in general, absolute rent will be zero. Positive AR will arise only when some gap can be opened up between the marginal product

of capital outlay and the opportunity cost of employing capital. The analysis in this paper shows that neither a lower organic composition of capital in agriculture compared to the rest of the economy nor the class power of landlords can generate such a gap. My analysis therefore suggests that, under reasonable behavioural assumptions, there will be no absolute rent in a capitalist economy.

In developing the analysis presented in this paper, I build on and extend two recent attempts to clarify Marx's theory of ground-rent, including its decomposition, using simple mathematical models: Basu (2018) and Das (2018). While Basu (2018) developed a simple mathematical framework to think about ground-rent and its decomposition, it had serious conceptual problems. The main problems were that it identified absolute rent with the worst-quality plot without any reference to magnitudes of capital outlay, which is conceptually problematic, and it could not distinguish consistently between the two components of differential rent. Das (2018) made a real advance and offered a way to deal with both problems in Basu (2018). The main theoretical drawbacks of the analysis in Das (2018) were, first, the use of discrete changes of capital outlay, which follows Marx's analysis but is difficult to justify, and second, taking the amount of capital outlay as exogenously given. In this paper, I extend the analysis in Das (2018) in three ways. First, I do away with the need to conceptualize changes in capital outlay in discrete units; second, I endogenise capital outlays across plots of land using profit-maximising behavior of capitalist farmers; and third, I explicitly model the determination of the price of the agricultural commodity.

The rest of the paper is organized as follows. In section 2, I discuss the set-up for the analysis and define the technological conditions of production. In section 3, I define ground-rent and derive explicit expressions for its three components when capital outlays are exogenously given: $DRI, DRII, AR$. In section 4, I endogenize capital outlays using profit maximising behaviour of capitalists and show that, in such a situation, absolute rent will be zero. In section 5, I close the model by determining the price of corn by the interaction of demand and supply of the agricultural product. The final section concludes the discussion.

2 Conditions of Production

2.1 The Set-Up

We conceive of the economy as being composed of an industrial sector and an agricultural sector. Industry is composed of I sectors, each producing a single product with a given technique of production. The agricultural sector produces a homogenous output, called corn, using land, labour, and inputs from industry. We work with the implicit assumption that corn is a non-basic product, i.e., it is not used as a direct means of production in any production process. In such a scenario, the prices of industrial products form a decomposable system of equation - which determines the prices of the industrial products and the average rate of profit (Kurz, 1978, pp. 27). Hence, for the analysis of rent, in this paper, we take the prices of industrial products, the wage rate and the average rate of profit, as given.

In agriculture, there are three classes: capitalists, workers and landlords. Capitalist organize production of corn by leasing in land from landlords and purchasing the labour-power of workers. The lease contract between capitalists and landlords specifies a fixed period of time - one production cycle - for which the latter hands over the right to use the relevant plot of land to the capitalist in return for a monetary payment known as ground-rent. The labour contract between capitalists and workers specifies a fixed period of time - one production cycle - for which the latter gives up the use of her labour-power to the capitalist for a monetary payment known as the wage. After production is completed, the capitalist sells the corn on the open market to recoup the wage and rent payments she made earlier and, in addition, make a profit income.

> The presuppositions for the capitalist mode of production [in agriculture] are thus as follows: the actual cultivators are wage-labourers, employed by a capitalist, the farmer, who pursues agriculture simply as a particular field of exploitation of capital, as an investment of his capital in a particular sphere of production. At certain specified dates, e.g. annually, this capitalist-farmer pays the landowner, the proprietor of the land he exploits, a contractually fixed sum of money ... for the permission to employ his capital in this particular field of production. This sum of money is known as ground-rent, irrespective of whether it is paid for

agricultural land, building land, mines, fisheries, forests, etc. It is paid for the entire period for which the landowner has contractually rented the land to the farmer. Marx (1993, pp. 755-756).

2.2 Technology of Production

Suppose the total available land used in agricultural production is divided into N plots and is indexed by $i = 1, 2, \ldots, N$. Let a subset of these plots, indexed by $i = 1, 2, \ldots, n$, be in use for agricultural production. An important point worth highlighting upfront is that n is a function of the price of corn, p, i.e. $n = n(p)$. The reason for this is easy to explain. As the price of corn changes, it has an impact on which plot is able to generate 'surplus profit' and hence provide ground-rent. Since plots of land will not be leased out by landlords unless they receive positive amount of ground-rent, the number of plots that will be used for agricultural cultivation will itself depend on the price of the agricultural commodity. Thus, we should note that the plots of land that are in use should be indexed by $i = 1, 2, \ldots, n(p)$, where p is the price of the agricultural commodity.

On the i-th plot of land that is in use, let total capital outlay by the capitalist producer of corn be denoted by $k_i = c_i + v_i$, where c_i and v_i are constant and variable capital respectively. Here c_i refers to the sum of money used by the capitalist to purchase non-labour inputs into production, and v_i refers to the sum of money used to purchase labour-power (for one production cycle).

Let $f_i(.)$ denote the 'production function' on the i-th plot of land, i.e. $f_i(.)$ captures the relationship between total capital outlays and the quantity of output on the i-th plot (which is why the function is indexed by i). We assume that the collection of 'production functions' have standard concavity properties.

Assumption 1. *Output on each plot of land is zero if capital outlay is zero, i.e.* $f_i(0) = 0$, *for all* $i = 1, 2, \ldots, n(p)$. *The quantity of output increases with the magnitude of capital outlay, i.e.* $f_i'(k) > 0$ *for* $k \geq 0$*; the marginal increase in output is always nonnegative and finite, i.e.* $0 \leq f_i'(k) < \infty$ *for* $k \geq 0$*; and, the marginal increase in output diminishes with the magnitude of capital outlay, i.e.* $f_i''(k) < 0$ *for* $k \geq 0$.

Assumption 1 is inspired by Pasinetti (1977, Chapter 1, Section 3), and its justification is straightforward. Land cannot be 'produced' by labour.

Thus, the amount of land in each plot (or parcel) is fixed, i.e. land is a non-reproducible resource. The above assumption says that as more capital is invested on this fixed plot of land, the output increases but only at a declining rate - as captured in Figure 1.[1]

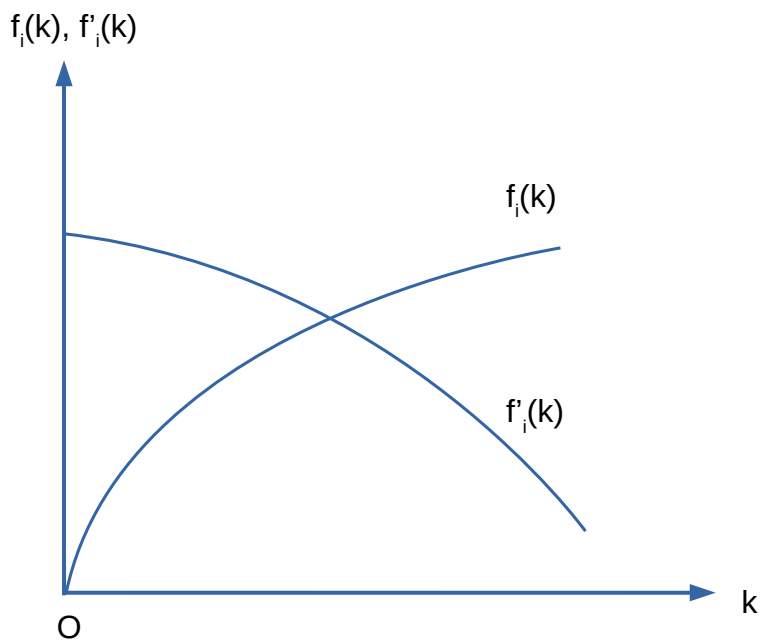

Figure 1: *This figure depicts the production function on th i-th plot of land that is in use for agricultural production. The curve $f_i(k)$ denotes the output, and $f_i'(k)$ denotes the marginal product, both as a function of the amount of capital outlay, k.*

2.3 Worst-Quality Plot

The plots of land are of unequal 'quality'; some plots are more fertile, or have better location, than others. But we cannot order plots of land by

[1]For a similar assumption see Figure 1.4 in Pasinetti (1977, Chapter 1, Section 3).

productivity without taking account of the amount of capital outlay in each plot, as has been stressed by Sraffian scholars (Kurz, 1978).[2] Because the area of each plot of land is fixed and the production function is concave by assumption 1, different amounts of capital outlay across plots can give rise to different ordering of land productivity (measured by the marginal product of capital outlay). There can be ordering reversals when capital outlays vary across plots. Hence, we cannot define the 'quality' of plots of land independently of capital outlay. This is highlighted in Figure 2, where we can see that the marginal product curves for the i-th and j-th plots cross over. For low levels of capital outlay, the j-th plot has higher marginal product; at higher levels of capital outlays, the position is reversed and the i-th plot has a higher marginal product. Our analysis in this paper is general enough to allow for such productivity reversals. But more importantly, we cannot order plots of land according to quality without taking account of capital outlays.

In this paper, we will adopt the convention of ordering plots of land by the marginal product of the very 'first' unit of capital outlay. The intuition for this is that the marginal product of capital at the very start of capital outlay on any plot of land comes closest to capturing the notion of the 'intrinsic' productivity of that plot of land. Considering a continuum of changes in capital outlay, it gives us the amount of increase in output (of corn) when capital outlay is increased from zero to a small positive amount. It is also able to capture any productivity that derives from past capital outlays on fixed capital, like irrigation, etc. that is now part of the plot of land.

Using this idea, we will order plots i and j according to intrinsic quality as follows: we will say that plot i is of *higher quality* than plot j if $f_i'(0) > f_j'(0)$. Since there are a finite number of plots, we can always arrange them in diminishing order of quality, as we have defined it here (with strict inequalities). Once we do so, we can renumber the plots and call plot 1 the most fertile, plot 2 the next most fertile, and so on. We state this as assumption about the ordering of the intrinsic quality of plots of land as

Assumption 2. $f_1'(0) > f_2'(0) > \cdots > f_{n-1}'(0) > f_n'(0)$, *where* $n = n(p)$ *and* p *is the price of the agricultural commodity.*

Our analysis of ground-rent will proceed in three steps. In the first step, we will derive expressions for ground-rent and its three components, DRI,

[2] One of the errors in Ricardo's analysis of rent was the assumption that plots of land could be ordered according to fertility without taking account of capital investment. (Kurz, 1978).

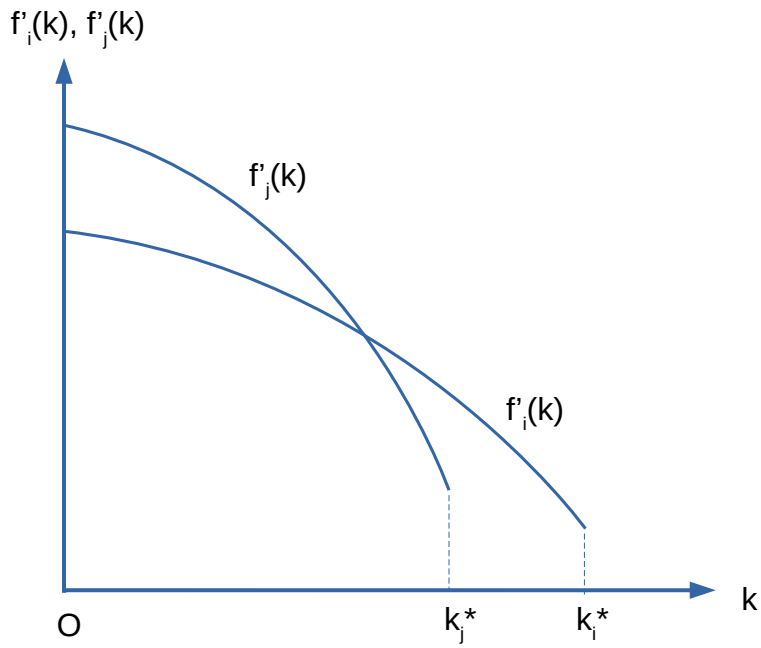

Figure 2: *This figure depicts the marginal product of capital outlay on the i-th and j-th plots of land. Note how there is a reversal of marginal productivity ranking of the two plots when we move from low to high capital outlays.*

$DRII$ and AR, taking the capital outlays on each plot of land, k_i, the price of corn, p, and the economy-wide rate of profit, r, as given. In the second step, we will endogenize the capital outlay, k_i, by positing profit-maximising behaviour of capitalist farmers. In the third, and final, step, we will close the model by endogenizing the price of corn, p, by allowing the interaction of demand and supply of corn to clear the market for corn.

3 Ground-Rent With Exogenous Capital Outlays

Taking the capital outlays on each plot of land, k_i, and the price of corn, p, and the economy-wide rate of profit, r, as exogenously given, we would like to determine the magnitude of ground-rent on each plot of land and decompose it into DRI, $DRII$ and AR.

3.1 Total Ground-Rent

Since the price of corn is given by p, and the economy-wide rate of profit is denoted by r, we can define what Marx calls the 'surplus profit' on the i-th plot of land as $pf_i(k_i) - (1+r)k_i$, where $pf_i(k_i)$ is the revenue earned and $(1+r)k_i$ is the counterfactual revenue that would have been earned if the total capital outlay, k_i, had earned the economy-wide average rate of profit. The key insight of Marx' analysis of rent is that private ownership of land by the class of landlords - which he refers to as the monopoly of landed property - allows them to appropriate the surplus profit as ground-rent (Marx, 1993, Chapter 37). Marx was not able to consistently implement this idea. In this paper, we begin with the definition that *ground-rent is the total surplus profit appropriated by the class of landlords*. Subsequently, we will decompose it into its components.

Implementing the definition of ground-rent in our model, we see that its magnitude on the i-th plot of land, *measured in units of corn*, is given by

$$GR_i = f_i(k_i) - \frac{(1+r)k_i}{p} = \int_0^{k_i} f_i'(k)dk - \frac{(1+r)k_i}{p}. \tag{1}$$

Since $f_i(k_i)$ is the total output on the i-th plot with capital outlay, k_i, and $(1+r)(k_i/p)$ is revenue earned, in real terms, on the same capital employed elsewhere in the economy (where the rate of profit is r), the surplus profit is given by the difference between the two. Using the fact that $f_i(k_i) = \int_0^{k_i} f_i'(k)dk$, which follows from assumption 1, and especially that $f_i(0) = 0$, we then get the expression in (1).

We can use Figure 3 to build some intuition about ground-rent. In this figure, we have plotted the marginal product of capital outlay, $f_i'(k)$, on the vertical axis against the amount of capital outlay, k, on the horizontal axis. The total amount of capital outlay on this plot of land is given by k_i. Hence,

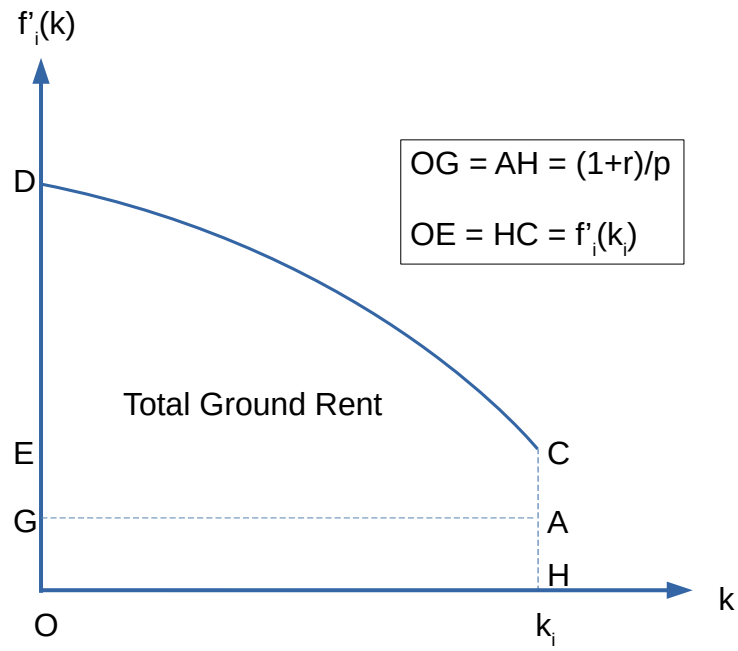

Figure 3: *Ground rent in agriculture on the i-th plot of land measured in units of corn. The horizontal axis measures total capital outlay; the vertical axis measures the marginal output as a function of capital outlay, $f_i'(k)$. The price of corn is p and the economy-wide rate of profit is r. Total ground rent on the i-th plot of land is given by the area $DCAG$.*

total output of corn is given by the area $DCHO$, which is the first term in (1). The area $GAHO$ represents the amount of corn that would be needed to ensure the economywide rate of profit, r, on the total capital outlay, k_i. This is the second term in (1). Hence, the total ground-rent on the i-th plot of land is represented by the area $DCAG$.

In writing the expression for total ground-rent in (1) and in constructing the corresponding visual representation in figure 3, we have implicitly assumed that $f_i'(k_i) > (1+r)/p$ for $i = 1, 2, \ldots, n$. This assumption means that on each plot of land that is in use, the marginal product of capital out-

lay, $f_i'(k_i)$, is greater than the opportunity cost of investing capital elsewhere in the economy, $(1+r)/p$, where r is the uniform rate of profit and p is the price of the agricultural commodity. This is a crucial assumption for the analysis of absolute rent and we will come back to it several times in the rest of the paper.

3.2 Marginal-Capital Plot

We would now like to decompose the magnitude of total rent given in (1) into two components: differential rent and absolute rent. To do so, we need to identify the 'marginal' unit of capital outlay, i.e. the unit of capital outlay that is least productive, and then use this benchmark to define differential rent. Note that each plot of land has diminishing marginal productivity of capital outlay because $f_i''(.) < 0$. Hence, to identify the marginal capital, we just need to find the minimum of the marginal output on each plot of land at their capital outlay levels. Recall that the amount of capital outlay on the i-th plot of land is given by k_i. Hence, we can define the output associated with the marginal unit of capital as

$$y^m \equiv y^m(p) = \min_{i \in \{1,2,\ldots,n(p)\}} f_i'(k_i), \tag{2}$$

where y^m is the marginal output associated with the least productive unit of capital outlay used among all plots of land. Note that y^m is a function of p, the price of the agricultural commodity, because it is the minimum of $f_i'(k_i)$ where i ranges over $\{1, 2, \ldots, n(p)\}$. Since $n = n(p)$, y^m is also a function of p. To make this explicit, we have specified $y^m = y^m(p)$.

Let us identify this plot of land with the index m, where $1 \leq m \leq n(p)$, and call it the *marginal-capital plot of land*. Thus, the m-th plot of land has the lowest marginal product of capital outlay for the last unit of capital, i.e. $y^m = f_m'(k_m)$. Note that, in general, $m \neq n(p)$, i.e. the worst plot of land in terms of quality that has been identified in assumption 2 is not, in general, the plot of land with the 'marginal' unit of capital outlay.

3.3 Differential and Absolute Rent

The first step of the decomposition of total rent is given as,

$$GR_i = DR_i + AR_i, \tag{3}$$

where the first component in (3) is *total differential rent*,

$$DR_i(p) = \int_0^{k_i} [f_i'(k) - y^m(p)]\, dk, \tag{4}$$

and the second component in (3) is *absolute rent*,

$$AR_i(p) = k_i \left[y^m(p) - \frac{(1+r)}{p} \right]. \tag{5}$$

An important point to keep in mind is that both DR_i and AR_i are functions of p, the price of the agricultural commodity, and r, the economy-wide rate of profit. This is easy to see from (3) and (5).

Figure 4 shows the decomposition of rent into differential and absolute rent. Total ground rent, represented by area $DCAG$, is the sum of differential rent, represented by the area $DCBF$, and absolute rent, represented by the area $FBAG$. We call the first component, DR_i, as differential rent because its magnitude depends on *productivity differences* across plots of land arising both from its 'intrinsic' productivity and from differences in magnitudes of capital outlay. The second component, AR_i, is known as absolute rent because it does not depend on differences across plots of land.

Since there is large disagreement in the extant literature about absolute rent, let us spend some time studying it. From the expression in (5), we see that absolute rent on the i-th plot, AR_i, arises from the *gap* between the minimum marginal product, y^m, and the opportunity cost of capital outlay, $(1+r)/p$. As long as this gap exists, there will be positive absolute rent; if this gap is wiped out, there will be no absolute rent. This is precisely where the implicit assumption that $f_i'(k_i) > (1+r)/p$ becomes important. Since $f_i'(k_i) > (1+r)/p$ for $i = 1, 2, \ldots, n$, this implies that the minimum of $f_i'(k_i)$, i.e. y^m, is also larger than $(1+r)/p$. Hence, $y^m > (1+r)/p$. This ensures that there is positive absolute rent on the i-th plot of land.

Marx had conceived of absolute rent as the total rent earned on a new plot of land that is brought under cultivation to satisfy rising demand for corn (Marx, 1993, Chapter 45). Implicit in Marx's analysis is the idea that the new land brought under cultivation is also the least fertile plot of land, a fact that has been emphasized by Fine (1979). In Marx's analysis, the source of absolute rent is the relatively lower organic composition of capital in agriculture compared to the rest of the economy. Fine (1979, pp. 263) has an algebraic expression to capture this idea. Following Marx's claim, Fine

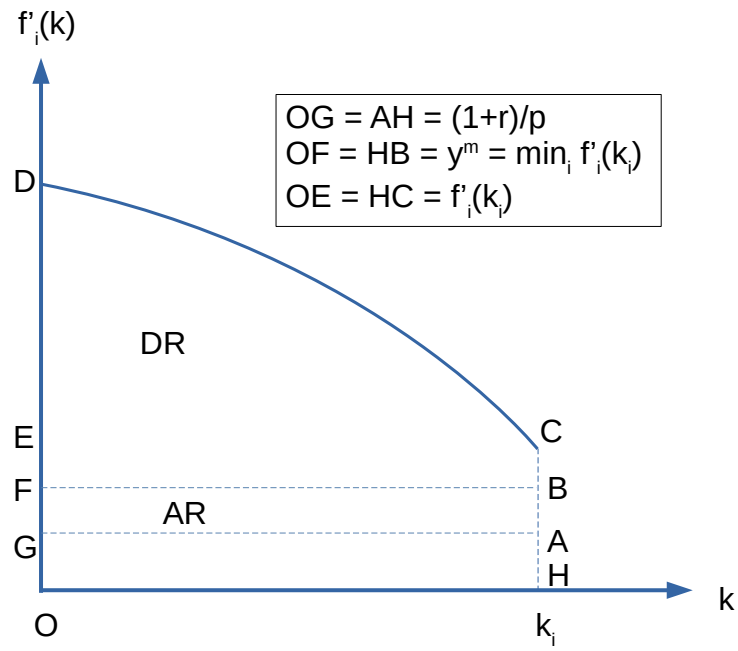

Figure 4: *Ground rent in agriculture on the i-th plot of land measured in units of corn. The horizontal axis measures total capital outlay; the vertical axis measures the marginal output as a function of capital outlay,* $f_i'(k)$*. The price of corn is* p *and the economy-wide rate of profit is* r*. Total ground rent on the i-th plot of land is given by the area* $DCAG$*, differential rent by* $DCBF$ *and absolute rent by* $FBAG$*.*

(1979) shows that if the organic composition of capital rises in the rest of the economy even as the organic composition of capital in agriculture remains unchanged, there should be an increase in absolute rent, as Marx (1993) claims. In our model, this effect is easy to capture. The relative rise in the organic composition of capital in the rest of the economy *ceteris paribus* leads to a fall in the uniform rate of profit, r. From the expression in (5), we can see that this will lead to a rise in absolute rent if y^m and p does not change.

3.4 Differential Rent I and II

The second step of the decomposition is to break up total differential rent in (4) into DRI and $DRII$. To do so, we will use the least productive plot, indexed by n according to assumption 2, as the benchmark to define DRI_i. To implement this decomposition let us define a level of capital outlay on the i-th plot of land, k_i^*, such that the marginal product of capital outlay on the i-th plot of land at this level of capital outlay is exactly equal to $f'_{n(p)}(0)$, i.e.

$$f'_i(k_i^*) = f'_{n(p)}(0). \tag{6}$$

Assumption 1 and 2 guarantees that a positive value of $k_i^*(p)$ always exists (as long as all plots have positive capital outlays), which is a function of p, the price of the agricultural commodity, because n is a function of p, i.e. $n = n(p)$. With this definition of $k_i^*(p)$, we can define *differential rent of the first variety* as,

$$\begin{aligned} DRI_i\,(p) &= \int_0^{k_i^*(p)} f'_i(k)dk - k_i^*\,(p)\, f'_{n(p)}(0) \\ &= \int_0^{k_i^*(p)} \left[f'_i(k) - f'_{n(p)}(0)\right] dk. \end{aligned} \tag{7}$$

Note that assumption 2 ensures that $DRI_i\,(p)$ is positive. If we weaken assumption 2 and use weak inequalities, then we will be ensured a nonnegative $DRI_i\,(p)$. Thus, we will always have $DRI_i\,(p) \geq 0$.

What is the intuition for the definition of $DRI_i\,(p)$? Using the worst-quality plot of land as the benchmark, we are identifying all units of capital outlay that have higher marginal product, i.e. productivity, than the 'intrinsic' quality of the worst-quality plot. When we add up all the additional output produced by these units of capital, we get $DRI_i\,(p)$, as defined in (7). Thus, this definition captures Marx's intuition that differential rent of the first variety arises due to differences in the quality (or fertility) of plots of land - the extensive margin of David Ricardo. Using Figure 5, this method would translate into identifying differential rent of first variety, $DRI_i\,(p)$, as the area DIJ. This area depends of p, the price of the agricultural commodity, because $k_i^* = k_i^*(p)$.

We can now define the second component of differential rent as the difference of total differential rent and differential rent of the first variety, i.e.

$$DRII_i\,(p) = \int_0^{k_i} \left[f'_i(k) - y^m\,(p)\right] dk - DRI_i\,(p)\,. \tag{8}$$

Using Figure 5, differential rent of second variety would be identified with the area $IJCBF$, which is the difference in total differential rent, $DCBF$, and differential rent of the first variety, DIJ. Since $y^m(p) \leq f_i'(k_i) < f_{n(p)}'(0)$, where the last (strict) inequality is true if $k_i > 0$ for all plots, we can see that the area $IJBF$ will always be nonnegative. Hence, $DRII_i(p) \geq 0$.

What is the intuition for $DRII_i(p)$? We have seen earlier that total differential rent $DR_i(p)$, defined in (4), arises from a combination of productivity differences that come from differences in quality of plots of land and differences in the magnitude of capital outlay. We have also seen that $DRI_i(p)$, defined in (7), arises from differences in quality of a plot with respect to the benchmark worst-quality plot of land. When we remove $DRI_i(p)$ from $DR_i(p)$, we are left with the component of differential rent that arises due to differences in magnitude of capital outlay. Hence, $DRII_i(p)$, as defined in (8), captures Marx's intuition that differential rent of the second variety arises from differences in the magnitude of capital outlay - the intensive margin of David Ricardo.

We can now bring together the above discussion to see that Marx's ideas about ground-rent, including his claim about its decomposition into three parts, can be rigorously established. Defining total ground-rent as the transformation of surplus profit, we have shown that it can be decomposed on any plot of land into differential rent of first variety, differential rent of second variety, and absolute rent, i.e.

$$GR_i(p) = DRI_i(p) + DRII_i(p) + AR_i(p),$$

where $GR_i(p)$ is defined in (1), $DRI_i(p)$ is defined in (7), $DRII_i(p)$ is defined in (8), and $AR_i(p)$ is defined in (5). It is important to note that not only the total ground-rent, but each component of ground-rent as well depends on the price of the agricultural commodity, p.

3.5 Two Special Plots of Land

The analysis of rent presented above uses two special plots of land as reference plots, the m-th plot (the worst capital plot), and the n-th plot of land (the worst quality plot of land, according to the convention captured in Assumption 2). What can we say about the components of rent on these two reference plots of land?

On the worst capital plot, $y^m = f_m'(k_m)$, i.e. y^m is the marginal product on the worst capital plot. The decomposition of total rent on the worst capital

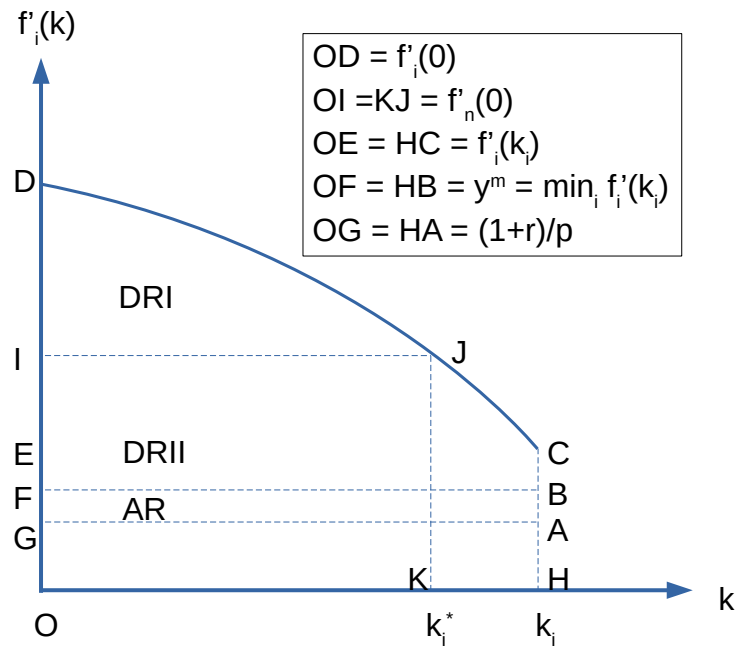

Figure 5: *Ground rent in agriculture on the i-th plot of land measured in units of corn. The horizontal axis measures total capital outlay; the vertical axis measures the marginal output as a function of capital outlay, $f_i'(k)$. The price of corn is p and the economy-wide rate of profit is r. Total ground rent on the i-th plot of land is given by the area $DCAG$, differential rent by $DCBF$ and absolute rent by $FBAG$. DRI_i is given by DJI, and $DRII_i$ by $IJCBF$.*

plot is depicted in Figure 6. There is no qualitative difference between the m-th plot and any other plot of land. Much like on any other plot, total rent on the m-th plot is also the sum of DRI, $DRII$ and AR. A more interesting case is presented by the worst quality plot, i.e. the plot indexed with $n = n(p)$.

On the worst quality plot, which is depicted in Figure 7, the graph of the marginal product starts at $f_n'(0)$. From (6), we see that for the n-th plot, $k_n^* = 0$. This implies, using the expression in (7), that $DRI_n = 0$. By the

concavity of the production function, we have $f_n'(.) < 0$. Hence $DRII_n > 0$. We also know that $y^m > (1+r)/p$, so that $AR_n > 0$. Thus, on the worst quality plot, total rent is composed of $DRII$ and AR; there is no DRI. This is an important conclusion and worth commenting on. Marx thought that total rent on the worst quality plot of land could be identified with absolute rent, i.e. there would be no differential rent on the worst quality plot (Marx, 1993, Chapter 45). Our analysis shows that that is not correct. On the worst quality plot of land, total rent is composed of both AR and $DRII$. Since $DRII$ arises from the concavity of the production function, i.e. the diminishing marginal product of capital outlay, Marx's conclusion seems to derive from his ignoring this latter factor when analyzing rent on the worst quality plot of land.

There is a deeper problem in the analysis presented so far. We have completely ignored the decision making process of the capitalist farmers. In more concrete terms, by assuming a given amount of capital outlay on each plot of land, we have ignored the process and implications of the behaviour of capitalist farmers. This is a serious shortcoming because, once we allow a reasonable behaviour of capitalist farmers, there will be a serious implication for the analysis of rent. To that we now turn.

4 Ground-Rent With Endogenous Capital Outlays

Thus far, the analysis of ground-rent has treated the amount of capital outlays on each plot of land as exogenously given. We would now like to endogenize capital outlay by positing a simple behavioural rule. On each plot of land, capitalist producers choose the amount of capital outlay that will maximize the surplus profit they can earn vis-a-vis what they can earn if they were to employ their capital elsewhere in the economy.

4.1 There is No Absolute Rent

Since the economy-wide rate of profit is exogenously given to be r, capitalist producers can always earn the total revenue of $(1+r)k_i$ by employing their capital, k_i, elsewhere in the economy. If they choose to employ the capital in agricultural production on the i-th plot of land, then they can expect to earn total revenue of $pf_i(k_i)$, if the price of the agricultural commodity is p. If

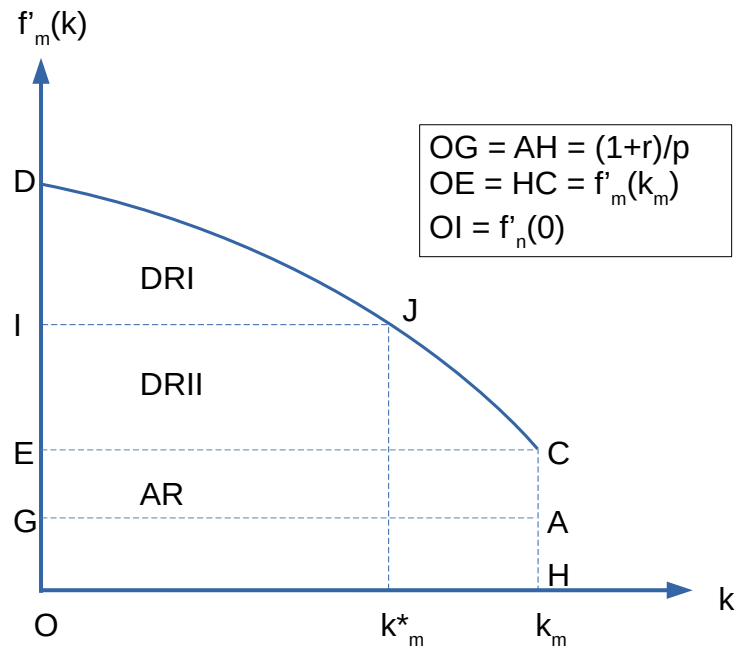

Figure 6: *Ground rent in agriculture on the m-th plot (i.e. worst capital plot) of land measured in units of corn.*

total rent is a lump-sum monetary payment given by GR_i, then the amount of revenue they can expect to earn by employing their capital in agriculture is $pf_i(k_i) - GR_i$. Hence, the extra revenue a capitalist can earn by investing her capital in agriculture is given by $pf_i(k_i) - GR_i - (1+r)k_i$. We posit that capitalist producers choose the level of capital outlay, k_i, on the i-th plot to maximize this surplus, i.e. her choice problem on the i-th plot of land can be represented as follows:

$$\max_{k_i} pf_i(k_i) - GR_i - (1+r)k_i.$$

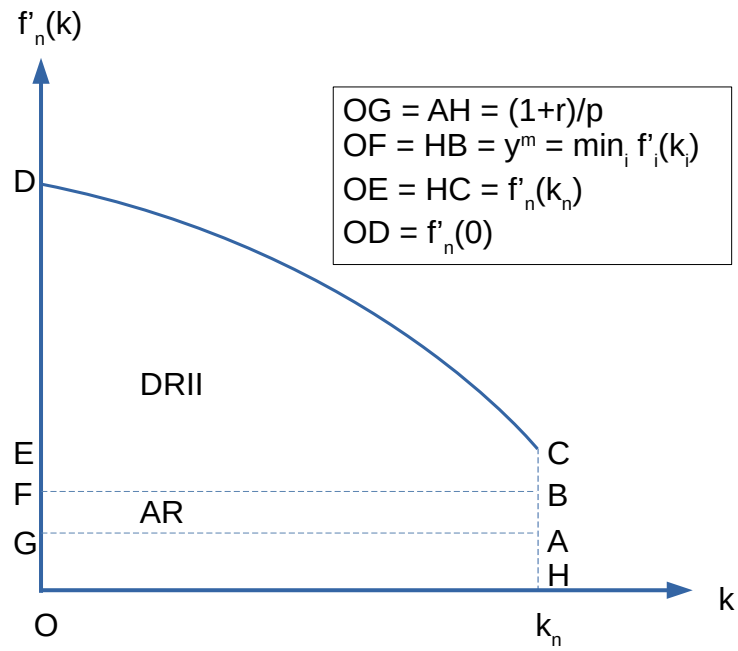

Figure 7: *Ground rent in agriculture on the n-th plot (i.e. worst quality plot) of land measured in units of corn. Note that DRI is zero.*

The first order condition of this maximisation problem gives us the optimal choice of capital outlay as k_i^*, where,

$$f_i'(k_i^*) = \frac{1+r}{p}. \tag{9}$$

This condition means that on the i-th plot of land, the optimal amount of capital outlay that will be chosen by profit-maximising capitalist farmers will be such that the marginal product of capital outlay will be equalized to $(1+r)/p$. Since this latter magnitude is exogenously given, because r and p are exogenously given to the capitalist farmer, this implies that the marginal product of capital outlay on each plot of land will be equalized. This is intuitively clear. If there were differences in the marginal product of capital

outlay, inter-plot movement of capital would allow capitalist producers to increase surplus profit. The equilibrium configuration of the distribution of capital outlays will rule this out. Hence, the marginal product of capital outlay has to be equalized across plots of land.

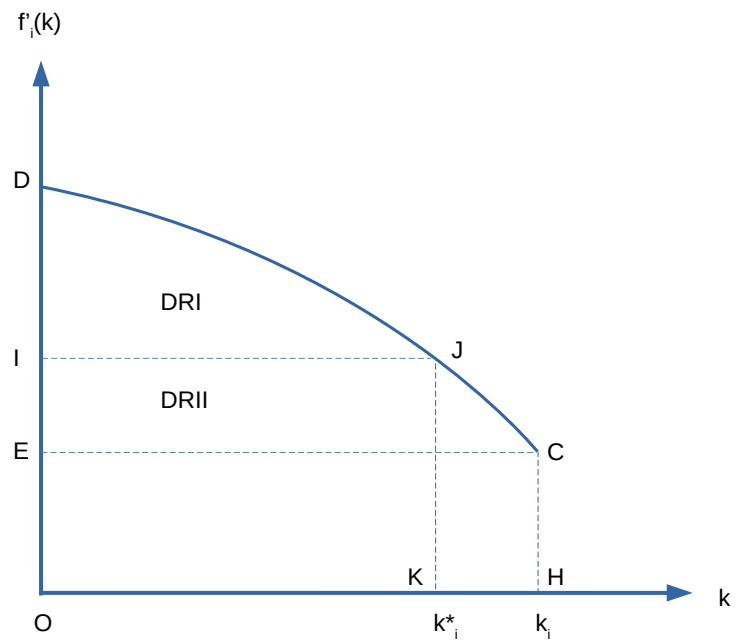

Figure 8: *Decomposition of ground rent in agriculture on the i-th plot of land, measured in units of corn, when rent is specified as a lump-sum monetary payment. In this case, there is no absolute rent. Total ground-rent is the sum of differential rent of first variety (DRI) and differential rent of second variety (DRII).*

The condition represented in (9) has an important implication for the decomposition of ground-rent. Turning to our previous definition of the marginal-capital plot of land in (2), we can see that once we endogenize capital outlays using the principle of surplus maximization, all plots become marginal-capital plots of land. Since $f_i'(k_i)$ is equal for each plot, hence,

$y^m = f_i'(k_i)$ for each $i \in \{1, 2, \ldots, n(p)\}$. Using (9), we see further that $y^m = f_i'(k_i) = (1+r)/p$. From the expression in (5), we see that absolute rent, AR_i, is zero. In terms of Figure 4, this means that $OF = OE = OG$, so that $AR_i = 0$.

One strand of Marxist literature argues, in line with Marx's argument in Chapter 45 of Volume III of *Capital*, that the source of absolute rent is the low organic composition of capital in agriculture relative to the rest of the economy (Fine, 1979; Fine and Filho, 2010; Fine, 2019). The analysis in this section raises doubts on such claims because the conclusion that $AR_i = 0$ does not depend on the price level, p, or the uniform rate of profit, r; it is true for any combination of p and r. If the organic composition of capital were to rise in the rest of the economy relative to agriculture, as used in the argument in Fine (1979), then the uniform rate of profit, r, would fall to r', say. Capitalist farmers would choose the level of capital outlay to ensure that $f_i'(k_i)$ is equal to $(1+r')/p$. Hence, absolute rent would still be zero. The relatively lower organic composition of capital in agriculture does not generate any absolute rent.

To emphasize the conclusions of this section, I have visually represented the configuration of ground-rent and its decomposition, when capital outlay is endogenous, in Figure 8. The total amount of ground-rent on plot i is represented by the area DCE. This is, as before, the surplus profit. $DRI_i(p)$ is represented, just like in Figure 3, by the area DIJ. This is the part of ground-rent that can be attributed to quality differentials across plots of land. $DRII_i(p)$ is now represented by the area $IJCE$, and there is no AR. Thus, we now have a two-part decomposition of ground-rent on the i-th plot as

$$GR_i(p) = DRI_i(p) + DRII_i(p),$$

where

$$DRI_i(p) = \int_0^{k_i^*(p)} \left[f_i'(k) - f_{n(p)}'(0) \right] dk, \tag{10}$$

and

$$DRII_i(p) = \int_0^{k_i} \left[f_i'(k) - \frac{1+r}{p} \right] dk - DRI_i(p). \tag{11}$$

4.2 Can Absolute Rent be Positive?

Let us now return to the condition that we had implicitly assumed when analysing rent with exogenous capital outlays: $f_i'(k_i) > (1+r)/p$. This

assumption was crucial for generating positive amount of absolute rent on the i-th plot of land (see equation 5).[3] The importance of this assumption can be visually seen from Figure 4 and 5. This assumption implies that there is a 'gap' between the marginal product of capital outlay, $f_i'(k_i)$, and the opportunity cost of capital outlay, $(1+r)/p$. Thus, only when there is a gap between $f_i'(k_i)$ and $(1+r)/p$, can the i-th plot generate positive AR. If there are mechanisms available in a capitalist economy that can put a wedge between the marginal product of capital outlay and the opportunity cost of capital outlay, then it can generate positive absolute rent.

The two main contenders for explaining absolute rent are the relatively low organic composition of capital and the class power of landlords. In the previous section, we have seen that a relatively low organic composition of capital cannot generate any wedge between the marginal product of capital outlay and the opportunity cost of capital outlay. Hence, a relatively low organic composition of capital cannot be the source of absolute rent. Now, we would like to investigate the efficacy of the other explanation: class power of landlords.

To see how the class power of landlords might generate absolute rent, let us consider one mechanism that might generate a wedge between the marginal product of capital outlay and the opportunity cost of capital outlay: taxation of capital outlay. For instance, if the state were to tax capital outlays at the rate of $t > 0$, then the capitalist farmers choice problem would become

$$\max_{k_i} pf_i(k_i) - GR_i - tk_i - (1+r)k_i,$$

so that the optimal choice of capital outlay would satisfy the condition that

$$f_i'(k_i^*) = \frac{1+r+t}{p}. \tag{12}$$

Since $t > 0$, this would ensure that $f_i'(k_i^*) > (1+r)/p$. From (5), we can see that the absolute rent would just be the tax revenue collected by the state: $AR_i = tk_i^*/p$. If the class of landlords were strong enough to ensure that the state redistributes the tax revenue, tk_i^*/p, to them, then we could consider it to be absolute rent. Thus, there seems to be some justification for the claim that class power of landlords can generate absolute rent. But this is only apparently correct.

[3] We had assumed that $f_i'(k_i) > (1+r)/p$ for $i = 1, 2, \ldots, n$. This ensured that y^m, which is the minimum $f_i'(k_i)$ across the n plots, was also larger than $(1+r)/p$.

To see the problem with the class power argument, let $k^*_{i,T}$ and $k^*_{i,NT}$ denote the optimal capital outlays on the i-th plot of land with and without taxation of capital outlay, respectively. The first order condition in (9) determines $k^*_{i,NT}$ as, $f'_i(k^*_{i,NT}) = (1+r)/p$; and the first order condition in (12) determines $k^*_{i,T}$ as $f'_i(k^*_{i,T}) = (1+r+t)/p$. The concavity of $f_i(.)$ and the fact that $t > 0$ implies that $k^*_{i,NT} > k^*_{i,T}$. Thus, when there is taxation of capital outlays, it will lead to a loss of rent for the class of landlords, which is represented by the area JCF in Figure 9. Therefore, it seems that if the class of landlords were strong enough to impact public policy, they would in fact push for removing the taxation of capital outlays, rather than ensuring the redistribution of the tax revenue towards themselves (and call it absolute rent). And if there is no taxation of capital outlays, then absolute rent would be zero.

In the mid-19th century, when Marx turned his attention to the problem of absolute rent, feudal obligations were still enforced in many parts of Germany. One could perhaps think of these feudal obligations as taxes on capital, which would then explain why Marx theorised about absolute rent. It might also be argued that once landlords became capitalist revenue maximizers, they realized that the feudal dues were in fact reducing the total ground rent by creating a wedge between the marginal rate of profit in industry and agriculture. This might have been one of the important factors that led to the outlawing of feudal obligations.[4]

5 Price of Corn

So far we have taken the price of the agricultural commodity is given. To complete the analysis, we need to investigate how this price is determined. To do so we look at the market for corn. Let us posit a demand function for corn, $D(p;\gamma)$, where $\partial D/\partial p < 0$ and γ captures shift factors like population growth, urbanization, regulatory aspects of the corn market, etc. The total supply of corn can be expressed as

$$S(p) = \sum_{i=1}^{n(p)} f_i(k_i(p)). \tag{13}$$

[4] I would like to thank Duncan Foley for pointing this out.

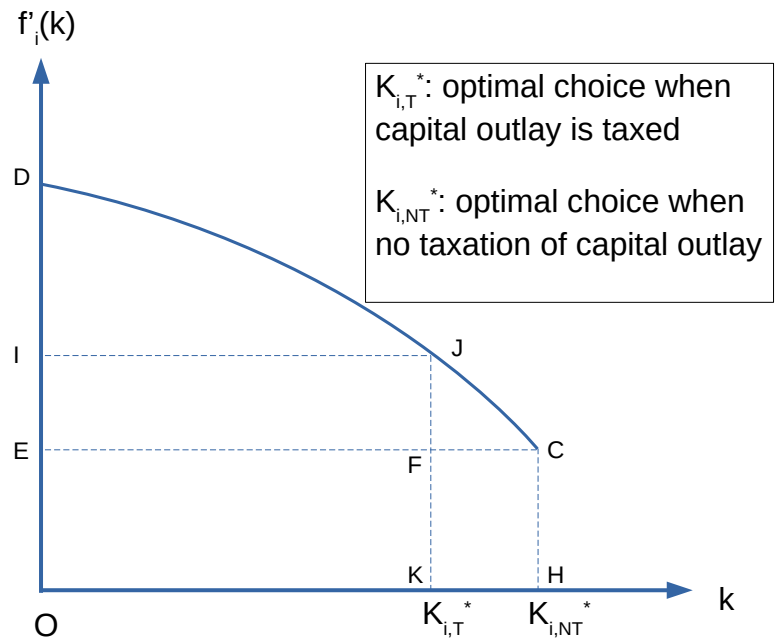

Figure 9: *A comparison of the total ground-rent on the i-th plot with and without taxation of capital outlay.*

Note that total supply of corn is an upward sloping function of price because of two reasons. First, the actual number of plots in use for agricultural production is a function of p, i.e. $n = n(p)$. We had made this assumption right at the outset, and it is now clear why this was important. The dependence of the number of plot of land in use is a function of the price of corn because as the price of corn rises, it makes worse plots of land profitable to bring under cultivation. Second, on any plot in use, we know that k_i is an (increasing) function of p, so that output is an (increasing) function of price, i.e. $f_i\left(k_i(p)\right)$. Hence, total supply, $S\left(p\right)$ is an (increasing) function of p. The equilibrium price of corn is the level of $p = p^*$ which bring supply and

demand into balance, i.e.

$$D\left(p^{*};\gamma\right)=S\left(p^{*}\right)\equiv\sum_{i=1}^{n(p^{*})}f_{i}\left(k_{i}(p^{*})\right). \tag{14}$$

To ensure that our model is properly closed, let us conduct a simple counting argument. On the one hand, when there is no taxation of capital outlays, our model is captured by (12) and (14). Thus, we have $n+1$ equations because (9) gives us n equations and (14) gives us one more. The model has $n+1$ endogenous variables, $k_1,\ldots,k_n,p^*$, and one exogenous variables, r. Hence, the equation system can be solved to arrive at the equilibrium magnitudes of all the endogenous variables. On the other hand, when the state taxes capital outlays, our model is captured by (12) and (14). Thus, we have $n+1$ equations and $n+2$ endogenous variables, $k_1,\ldots,k_n,t,p^*$. Once we choose a value of $t=t^*$, the model is closed and we can solve for all the endogenous variables.

We can now ready present the condition under which any plot of land will be in use in a capitalist economy with landed property, i.e. private ownership of land by the class of landlords. In such a context, any plot of land will be in use it can be leased in profitably by a capitalist farmer. And it can be leased in only when it pays a positive amount of ground-rent to the landlord in addition to ensuring the uniform rate of profit for capital investment. Let us consider the most extreme case when the demand for corn is so low that even the 'best' plot of land cannot be profitably used. In this situation, there will be no corn production and hence there will be zero ground-rent. This immediately identifies the minimum threshold for the price, if rent is to be positive, as

$$p^{**}=\frac{1+r}{f_1'(0)}, \tag{15}$$

where, it is to be recalled from Assumption 2 that plot 1 is the best quality plot. If population growth, urbanization, etc. leads to growth of demand for the agricultural product such that $p>p^{**}$, this will bring land under cultivation. As soon as *any* land is brought under cultivation, this will generate positive ground-rent. As demand rises further, progressively more (worse plots) land will come under cultivation and total rent will increase. Technical change will have a more ambiguous effect.

In our model, technical change can be captured by the upward shifts of the marginal product curves. Such upward shifts of the marginal product

curves would imply that the same plots of land can satisfy rising demand. Hence, more, and worse quality, parcels will not need to be brought under cultivation as the demand for the agricultural commodity rises. This would reduce the total amount of ground-rent that would otherwise have to be paid by capitalist farmers. On the other hand, upward shifts of the marginal product curves also imply larger surplus profit on all the existing plots in use. Hence, this would have a tendency to raise the total amount of ground-rent. Therefore, the overall impact of technical change on the total ground-rent earned by the class of landlords will depend on the relative strengths of these opposing effects.

6 Conclusion

In this paper I have offered a rigorous way to conceptualize Marx's ideas about ground-rent. In developing the analysis of this paper, I have extended the discussion in two recent, noteworthy attempts to clarify Marx's theory of ground-rent, Basu (2018) and Das (2018). Previous attempts to discuss Marx's theory of ground-rent has been marked by disagreements and confusions partly because of the difficulty in clearly defining the meaning of the terms involved in the discussion, especially the components of ground-rent (Fine, 1979; Ramirez, 2009; Fine and Filho, 2010). In this respect, Basu (2018) and Das (2018) advanced the literature by trying to clearly specify Marx's ideas, avoid unnecessary exegetical debate, and formalize the ideas in simple mathematical models.

In this paper, I have extended the analysis further by endogenising the capital outlay on each plot of land using a simple and intuitive profit-maximising principle to model the behaviour of capitalist farmers. This simple extension has far reaching conclusions. Our analysis shows that total ground-rent can be decomposed into differential rent of the first variety, differential rent of the second variety and absolute rent only when capital outlays are taken to be exogenously given. As soon as we endogenize capital outlays using the principle of profit maximisation, the decomposition of ground-rent changes. While we always have differential rent of the first and second varieties, the existence of absolute rent now hinges on their being a gap between the marginal product of capital outlay and the opportunity cost of investing capital outside agriculture. In general, profit maximization by capitalist farmers will ensure that this is not true, i.e. they will choose to invest capital precisely

in the magnitude that equates the marginal product of capital outlay and the opportunity cost of investing capital outside agriculture. Thus, absolute will be zero. I have argued in this paper that neither a relatively low organic composition of capital in agriculture nor the class power of landlords can, in general, ensure positive absolute rent.

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