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## Welfare effects of the allocation of talent to financial trading: What does the Grossman-Stiglitz model say?

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**Welfare Effects of the Allocation of Talent to  
Financial Trading: What Does the Grossman-  
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# Welfare Effects of the Allocation of Talent to Financial Trading: What Does the Grossman-Stiglitz Model Say?<sup>1</sup>

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## Abstract

This paper investigates the implications of the Grossman-Stiglitz (1980) model on the informational efficiency of financial markets for the optimality of the allocation of talent to financial trading versus entrepreneurship. Informed traders make the financial market more informationally efficient, entrepreneurs create output and jobs. The model indicates that financial trading attracts too much, rather than too little, talent.

JEL classification: G14, J24

Key words: market efficiency, asymmetric information, allocation of talent, occupational choice

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## 1. Introduction

Is the allocation of talent to financial trading excessive or deficient? To investigate this question we incorporate occupational choice between financial trading and entrepreneurship and a labor market into the seminal Grossman-Stiglitz (1980) model on the informational efficiency of financial markets. Informed traders make the financial market more informationally efficient, entrepreneurs create output and jobs. In the model financial trading attracts too much, rather than too little, talent. For one thing, measures which raise entrepreneurial activity at the margin starting at the free markets equilibrium increase social welfare, in particular if there are labor market frictions and equilibrium unemployment. Second, social welfare is higher when agents are precluded from becoming informed traders altogether under fairly weak conditions. This is because informational efficiency is not generally conducive to social welfare: the fact that the asset price is more closely tied to stochastic fundamentals governing firms' profitability leads to a clustering of risks at entrepreneurs, thereby reducing the incentives to engage in entrepreneurship. Thus, in a context where the sole benefit of trading is to increase informational efficiency à la Grossman and Stiglitz (1980) the allocation of talent to trading tends to be excessive rather than deficient.<sup>3</sup> Other arguments than informational efficiency (mentioned in the literature review below) have to be invoked in order to argue for implicit or explicit support for financial trading in the competition for the best and brightest.

Finance has attracted an increasing amount of talent over the past decades. Goldin and Katz (2008) observe that the proportion of male Harvard graduates from selected classes who work in the finance sector 15 years after graduation rose from 5 percent for early-1970s cohorts to 15 percent for early-1990s cohorts. According to the *Harvard Magazine*, the figure peaked at more than 20 percent in 2007, before labor demand collapsed with the onset of the subprime crisis.<sup>4</sup> Philippon and Reshef (2012) report increasing average education of workers in the financial industry compared to the real sector since the 1980s. Despite rising relative employment, relative pay rose in "other finance" (i.e., mainly asset management and trading; cf. Greenwood and Scharfstein, 2013) compared to traditional banking and insurance. Insofar as wages reflect capabilities (and not compensating differentials), this reflects an increasing flow of talent to trading related activities. (Contrary evidence is presented by Böhm et al., 2018, for Sweden and by Lindley and McIntosh, 2017, for the U.K., who show that test scores and performance measures for finance workers have not increased since the 1990s.) Competition for talent does not stop when students have decided to specialize in science or engineering. Shortly before the financial crisis, serial entrepreneur and writer Vivek Wadhwa observed in his testimony to the the U.S. House of Representatives that "[T]hirty to forty percent of Duke Masters of Engineering Management students were accepting jobs outside of the engineering profession. They chose to become investment bankers or management consultants rather than engineers".<sup>5</sup> Similarly, *The Economist*

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<sup>3</sup>Contrary to Bolton et al.'s (2016, p. 711) conjecture that "the standard framework of trading in financial markets first developed by Grossman and Stiglitz (1980) ... seems to suggest that the financial sector could be too small."

<sup>4</sup>Elizabeth Gudrais, "Flocking to Finance", *Harvard Magazine*, May-June 2008, <http://harvardmagazine.com/2008/05/flocking-to-finance.html>.

<sup>5</sup>Quoted from Philippon (2010, p. 159).

reports that “[M]ost of the world’s top hedge funds prefer seasoned traders, engineers and mathematicians, people with insight and programming skills, to MBAs”.<sup>6</sup> Célérier and Vallée (2019) remark in their empirical study of French graduate engineers that a sizable portion of the post-2000 graduates worked in the City of London or on Wall Street. Oyer (2008, p. 2622) finds “mixed evidence that initial jobs on Wall Street lead Stanford MBAs to start fewer businesses”. He adds that there is path dependence in occupational choice: workers drawn into the financial sector by random events tend to stay there. As Cecchetti and Kharroubi (2012, p. 1) put it: “Finance literally bids rocket scientists away from the satellite industry.”<sup>7</sup>

While fierce competition for talent between finance and the real sector is undisputed, opinions diverge on whether this is a good thing. Esther Dufo replied to concerns that regulations would constrain the financial sector in the aftermath of the recent financial crisis: “Is there a risk of discouraging the most talented to work hard and innovate in finance? Probably. But it would almost certainly be a good thing.”<sup>8</sup> At *The Economist’s* 2013 Buttonwood Gathering, Robert Shiller (“When you study finance you are studying how to make things happen”) and Wadhwa (“Google – not Goldman Sachs – deserves our best minds”)<sup>9</sup> exchanged opinions. Long before the recent financial crisis Tobin (1984) bewailed that “we are throwing more and more of our resources, including the cream of our youth, into financial activities remote from the production of goods and services, into activities that generate high private rewards disproportionate to their social productivity.” Similarly, Baumol (1990, p. 915) holds that “arbitrageurs” at least occasionally engage in what he calls “unproductive entrepreneurship” as opposed to “productive entrepreneurship”, which is essential for long-term development. Murphy et al. (1991, p. 506) state in their classic paper on the allocation of talent: “Trading probably raises efficiency since it brings security prices closer to their fundamental values . . . But the main gains from trading come from the transfer of wealth to the smart traders . . . Even though efficiency improves, transfers are the main source of returns in trading.” Empirical studies of the impact of finance on real economic activity usually focus on economic growth as the dependent variable. Early studies, such as King and Levine (1993), found a positive impact of the financial sector on economic growth. Subsequent research points to an inverse U-shaped relation between finance and growth. For instance, in Cecchetti and Kharroubi (2012) the marginal effect of finance on growth turns negative when private credit exceeds 100 percent of GDP or financial sector employment exceeds 3.9 percent of total employment (see also Rousseau and Wachtel, 2011, Gruendler and Weitzel, 2013, Law and Singh, 2014, Arcand et al., 2015, Cornède et al., 2015, and Ductor and Grechyna, 2015). Consistent with the view that trading is more likely to be unproductive than intermediation, Beck and Degryse (2014) find that in a broad cross section of countries the size of the financial sector (measured by its value added share in GDP) is insignificant if intermediation is controlled for. Kneer (2013) and Boustanifar et al. (2017) relate this to brain drain from skill-intensive

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<sup>6</sup>Philip Delves Broughton, “Think twice”, *The Economist*, January 2011, <http://www.economist.com/whichmba/think-twice>.

<sup>7</sup>The positive impact of increased trading on the informational efficiency of asset prices is confirmed by Bai et al. (2016).

<sup>8</sup>*Vox*, October 8, 2008, <http://www.voxeu.org/article/too-many-bankers>.

<sup>9</sup>Washington Post, November 1, 2013, <http://www.washingtonpost.com/blogs/innovations/wp/2013/11/01/google-not-goldman-sachs-deserves-our-best-minds/>.

manufacturing industries and from foreign countries, respectively.

We investigate the welfare effects of financial trading by incorporating occupational choice (OC) and a labor market with or without frictions into the seminal Grossman-Stiglitz (1980, henceforth: “GS”) noisy rational expectations equilibrium (REE) model. As the “standard framework of trading in financial markets” (Bolton et al., 2016, p. 711) and one of the “top 20 articles published in the *American Economic Review* during its first 100 years” (Arrow et al., 2011), this is a natural starting point for the analysis of the allocation of talent to trading.<sup>10</sup> We conduct a second-best welfare analysis which takes agents’ investment and labor market decisions as given. Our measure of social welfare (SW) is the sum of *all* agents’ expected utilities, suitably transformed so that pure redistribution does not affect SW, assuming that the noise traders present in the GS model have the same utility function as the other agents. Equivalently, SW is the sum of all agents’ certainty equivalents. For the case of small noise trader shocks we show analytically that at equilibrium a marginal increase in the amount of resources devoted to entrepreneurship has a positive or zero first-order effect on SW, depending on whether there is equilibrium unemployment or not. Numerical analysis shows that for reasonably large noise trader shocks the marginal impact of entrepreneurship on SW remains positive in the vast majority of model specifications with unemployment and turns positive more often than not in the absence of labor market imperfections. That is, while the equilibrium allocation is constrained optimal with little noise and without labor market frictions, *the marginal benefits of job creation outweigh the marginal cost in terms of lower informational efficiency of financial markets in the presence of unemployment*. We also compare equilibrium with OC to the equilibrium that occurs when agents are precluded from becoming dealers, so that there is no informed trading at all. We analytically derive a set of weak sufficient conditions under which SW is higher in the absence of informed trading for small values of the variance of noise traders’ asset demand: first, neither noise traders nor rational agents short the asset on average, and, second, individuals are small relative to corporations, in that noise traders’ per capita demand for assets is less than the asset supply generated by a single entrepreneur. The reason why *agents are better-off with no informed trading at all* is that informational efficiency makes the asset price more responsive to stochastic fundamentals, which leads to an inefficient clustering of risks at entrepreneurs. Numerical analysis shows that equilibrium SW remains higher without OC for the vast majority of model parameterizations with reasonably large noise trader shocks. Our overall conclusion is that it is hard to argue that professional trading is socially beneficial on net if the only benefit it brings about is increased informational efficiency in the asset market as in GS.

Our model contributes to a growing literature which studies the efficiency of the allocation of resources to financial trading versus other activities. The GS model is regularly cited in the contributions but, despite its central role in financial economics, has not yet been used as the setting of the analysis. In Bolton et al. (2016) a class of agents have the option to become “dealers” and thus acquire the ability to assess the quality of newly issued stocks and buy the most profitable ones over-the-counter from originators. In the simplest version of their model (Bolton et al., 2016, Section II) such “cream skimming” is pure rent seeking, so if agents

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<sup>10</sup>The authoritative textbook treatment of the GS model is in Vives (2008, Subsection 4.2.2.)

specialize in dealing, this is socially wasteful. In an earlier version of the paper, Bolton et al. (2012) derive similar results in a variant of the model with OC between becoming a dealer or an originator. In Glode and Lowery (2016) financial firms hire “experts” either as “bankers”, who identify investment opportunities, or as “traders”, who identify valuable investments in other firms (hit by a negative liquidity shock). As in Bolton et al.’s (2016) simplest model, trading has no social benefit, so any employment it attracts is excessive (see also Glode et al., 2012).<sup>11</sup> Boot and Ratnovski (2016) study a bank-level model where engagement in low-level trading activity can create synergies, but strong commitment to financial trading activities is detrimental to traditional banking. In Biais et al. (2015) firms invest in fast trading capabilities. Since fast traders buy more when asset payoffs are high, and vice versa, a positive bid-ask spread is needed for market makers to break even. The bid-ask spread causes a welfare loss, because it prevents beneficial trades. There is over-investment in fast trading capabilities, as firms do not internalize their influence on the bid-ask spread. Bond and García (2018) analyze indexing in a model in which active trading entails a fixed cost compared to indexing. As in our model, better information due to more active trading reduces the scope for risk sharing and equilibrium welfare. In Angeletos et al. (2018), traders infer information about the profitability of new firms from entrepreneurs’ startup activity. This amplifies fluctuations caused by shocks to entrepreneurs’ private assessment of profitability and reduces welfare.

Other models concerned with the allocation of resources to financial trading assign a socially beneficial role to trading. For instance, Bolton et al. (2016, Section III) consider a version of their model with moral hazard in firms, in which cream skimming by dealers provides incentives for originators to supply high-quality assets.<sup>12</sup> Another branch of the literature (surveyed by Bond et al., 2012) emphasizes two further positive effects of trading in secondary financial markets on resource allocation via firm decisions. First, financial markets reveal information to producers they would otherwise not have (the “learning channel”). Second, asset prices can help improve efficiency when used as a determinant of managerial compensation (the “incentives channel”). Like our model, these models highlight potential tradeoffs between informational efficiency of asset prices and economic efficiency (see also Dow and Gorton, 1997, and Goldstein and Yang, 2014). In order for professional trading to be beneficial, the learning and incentive effects present in these models have to outweigh the rent seeking inefficiencies identified in the existing literature plus the opportunity cost of entrepreneurial activity and the negative impact of informational efficiency on entrepreneurship, which are the object of our paper. The paper is organized as follows. Section 2 introduces the model. Section 3 derives the price function and agents’ expected utilities. Sections 4 and 5 characterize equilibrium without a labor market, respectively. The welfare analysis is in Sections 6 and 7. Section 8 concludes. Proofs are delegated to an Appendix.

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<sup>11</sup>Both models also provide explanations for the finance wage premium (see, e.g., Oyer, 2008, Philippon and Reshef, 2012, Axelson and Bond, 2015, Boustanifar et al., 2017, and Lindley and McIntosh, 2017). Our model is silent on this issue, as agents are indifferent between entrepreneurship and trading at an equilibrium with a positive mass of agents active in each occupation.

<sup>12</sup>This relates to models with OC between finance and production in which agents who specialize in finance act as financial intermediaries (see, e.g., Phillipon, 2010, Cahuc and Challe, 2012, and Shakhnov, 2017).

## 2. Model

The model we consider is the GS model augmented to include OC between entrepreneurship and finance and a labor market.

### Preferences

Consider a CARA-Gaussian economy with three dates, “early”, “intermediate”, and “late”. There are three types of agents: a continuum of “high potential agents (hipos)” indexed by the interval  $[0, L]$  ( $L > 0$ ), who choose between becoming a “dealer” or an “entrepreneur”, a continuum of passive investors indexed by the interval  $[0, M]$  ( $M > 0$ ), and noise traders. There is a single homogeneous consumption good. Prices are quoted in terms of this consumption good. Hipos are endowed with  $e_L (\geq 0)$  units of the good early, passive investors with  $e_M$  units. Hipos and passive investors are characterized by the CARA utility function  $U(\pi) = -\exp(-\rho\pi)$ , where  $\pi$  is late consumption and  $\rho (> 0)$  is the coefficient of absolute risk aversion. All agents have access to a storage technology that transforms endowments one-for-one into late consumption.

### Occupational choice

Hipos who become entrepreneurs run firms, create real wealth, and employ workers. Hipos who decide to become GS-type dealers collect information about macroeconomic fundamentals and contribute to informational efficiency by trading in the financial market. There is no physical cost of becoming an entrepreneur or a dealer. Hipos choose the occupation which maximizes their expected utility at the early date. They also have the option not to become an entrepreneur or a dealer, in which case they act like the passive investors (since there is no physical cost of becoming a dealer, this can only be beneficial if holding private information is worthless). To assess the welfare effects of professional trading, we also consider the variant of the model without OC, in which hipos do not have the opportunity to become dealers, so that there is no informed trading.

### Firms

Each entrepreneur sets up a continuum of firms indexed by the interval  $[0, 1/a]$  ( $a > 0$ ). Denote the mass of hipos who decide to become entrepreneurs as  $L_E$ . Then a firm is an element of  $[0, L_E] \times [0, 1/a]$ . The mass of firms is  $L_E/a$ , and for each entrepreneur, the subset of firms he owns has measure zero, so entrepreneurs have no market power in the goods market. We start with the version of the model without a labor market, in which firm output is exogenous: each firm produces  $\theta$  units of output late.  $\theta$  is a macroeconomic shock, which is uniform across firms. It is the sum of two independent jointly normal random variables:  $\theta = s + \varepsilon$ , where  $s \sim N(\bar{s}, \sigma_s^2)$  and  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ . Later we introduce a labor market, in which output  $\theta$  depends on the input of unskilled labor, supplied by passive investors. While the passive investors do not play an important role in the model variants without a labor market, they are the beneficiaries of job creation by entrepreneurs.

### Financial market

The financial market is modeled as in GS (see also Grossman, 1976, and Hellwig, 1980). At the intermediate date shares in the firms are traded in a competitive stock market. Noise traders inelastically demand  $\nu \sim N(\bar{\nu}, \sigma_\nu^2)$  units of the risky asset. Dealers observe  $s$  and face residual uncertainty  $\varepsilon$  about firms’ payoff.



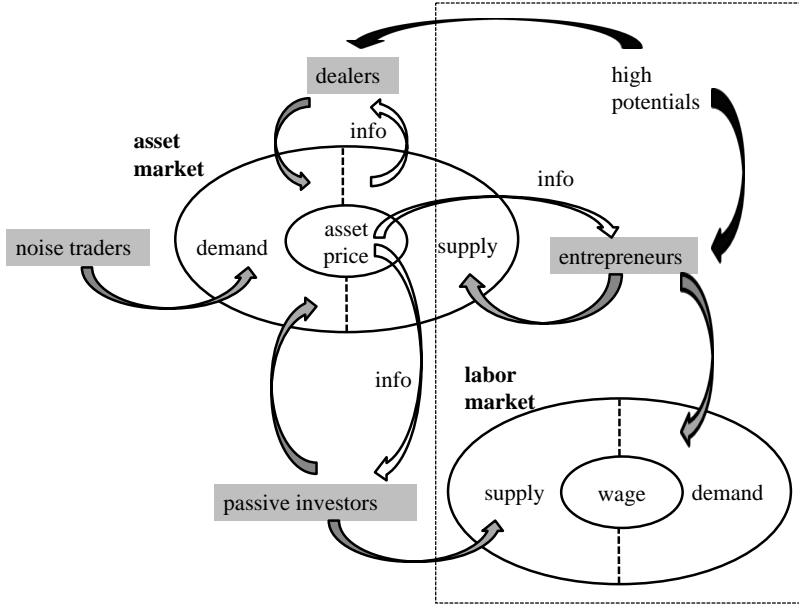


Figure 1: Structure of the model

Entrepreneurs and passive investors observe neither  $s$  nor  $\varepsilon$ , so dealers' private information about  $s$  contributes to informational efficiency in the stock market via its impact on the asset price (since there are no firm-specific shocks, this information structure does not entail that dealers have information about individual firms that entrepreneurs do not have. It also implies that there are no benefits of going private in terms of hiding information (see Ferreira et al., 2014). Entrepreneurs and passive investors do not observe the other agents' trades. So they cannot tell exactly if a high stock market value of the firms is due to high demand by noise traders or by dealers having obtained favorable private information about profitability.

### Model structure and variants

Figure 1 depicts the structure of the model (with OC and for the case in which hipos do not decide to act as passive investors). Agents interact in the asset and labor markets. The black, grey, and white arrows indicate hipos' OC decision, agents' market supplies and demands, and information flows, respectively. The part of the model outside the dashed box is the GS model. The novel parts are inside the dashed box: hipos' OC decision, endogenous supply in the asset market, and the labor market (introduced below).

The limitations of the model are similar as in Bolton et al. (2016). There is no moral hazard due to implicit or explicit state guarantees. There is no leverage, dealers trade only on their own account. The only input required to set up a firm is entrepreneurial labor, so there is no financial intermediation. Entrepreneurs set up and run firms, no distinction is made between engineering and management tasks.

In order to derive our main result, we have to consider  $2 \cdot 2 \cdot 3$  model variants: with or without OC; with no

labor market, full employment, or unemployment; and with or without noise. The first two distinctions are necessary in order to show that the welfare effects of OC crucially depend on the presence of labor market imperfections. The analysis of the case of no noise guides the choice of the parameter restrictions in the numerical analysis.

### 3. Price function and expected utilities

We start with the model without a labor market. We call this model  $M_{\zeta}^j$ . The superscript indicates whether there is OC ( $j = 1$ ) or not ( $j = 0$ ), and the subscript indicates whether the variance of noise trader demand  $\sigma_{\nu}^2$  is zero ( $\zeta = 0$ ) or positive ( $\zeta = \sigma$ ). This section defines equilibrium and derives the price function that relates the asset price to macroeconomic shocks and agents' expected utilities.

#### Equilibrium

The supply of stocks is equal to the mass of firms  $L_E/a$ . Let  $P$  denote the stock market value of each firm and  $I_E$ ,  $I_D$ , and  $I_M$  entrepreneurs', dealers', and passive investors stock holdings, respectively. Hipos make their OC and investment decisions so as to maximize expected utility conditional on available information. Consumption is  $\pi_E = e_L + P/a + (\theta - P)I_E$  for entrepreneurs and  $\pi_D = e_L + (\theta - P)I_D$  for dealers. Passive investors' consumption is  $\pi_M = e_M + (\theta - P)I_M$ . Since investment is independent of initial wealth, hipos who become neither entrepreneurs nor dealers invest the same amount  $I_M$  and their final wealth differs only by the constant  $e_L - e_M$ . While dealers know  $s$  when they make their investment decision, entrepreneurs and passive investors can only use the price level  $P$  they observe to infer information about  $s$ . Throughout the paper, we focus on equilibria at which the mass of entrepreneurs  $L_E$  is positive, since otherwise asset supply is zero. Moreover, in the model variants with OC we focus on equilibria at which hipos become either entrepreneurs or dealers (and not passive investors). This entails no loss of generality.

$(L_E, I_E, I_D, I_M, P)$  is an *equilibrium* (an REE) of  $M_{\zeta}^1$  ( $\zeta = 0, \sigma$ ) if  $I_E$  maximizes  $E[U(\pi_E)|P]$ ,  $I_D$  maximizes  $E[U(\pi_D)|s, P]$ ,  $I_M$  maximizes  $E[U(\pi_M)|P]$ , the market for the risky asset clears (i.e.,  $L_E/a = L_E I_E + (L - L_E)I_D + M I_M + \nu$ ), and OC is optimal (i.e.,  $E[U(\pi_E)] = E[U(\pi_D)]$  and  $0 < L_E \leq L$  or  $E[U(\pi_E)] \geq E[U(\pi_D)]$  and  $L_E = L$ ). An equilibrium of  $M_{\zeta}^0$  is defined analogously, except that  $I_D$  and the condition that it is chosen optimally drop out of the definition and the asset market clearing condition becomes  $L_E/a = L_E I_E + (L - L_E + M)I_M + \nu$ .

#### Price function

Utility maximization and asset market clearing yield the price function:

$$P = \frac{w + \frac{L_E + M}{\rho \text{var}(\theta|w)} E(\theta|w) - \frac{L_E}{a}}{\frac{L - L_E}{\rho \sigma_{\varepsilon}^2} + \frac{L_E + M}{\rho \text{var}(\theta|w)}}, \quad (1)$$

where

$$w \equiv \frac{L - L_E}{\rho \sigma_{\varepsilon}^2} s + \nu \quad (2)$$

(here and in what follows details of the derivations are delegated to the Appendix). A high value of ( $w$  and  $P$ ) can be due to good fundamentals (high  $s$ ) or high noise trader demand (high  $\nu$ ). This gives rise to the GS

signal extraction problem.

### Expected utilities

Let

$$z \equiv \frac{\mathbb{E}(\theta|w) - P}{[2 \text{var}(\theta|w)]^{\frac{1}{2}}}. \quad (3)$$

$z$  measures expected payoff relative to risk for financial investments conditional on  $w$  ( $z\sqrt{2}$  is the Sharpe ratio). An entrepreneur's unconditional expected utility is:

$$\begin{aligned} -\log\{-\mathbb{E}[U(\pi_E)]\} &= \rho e_L + \overbrace{\frac{\rho}{a} \left[ \mathbb{E}(P) - \frac{\rho}{2a} \text{var}(P) \right]}{=\text{GE}} \\ &+ \underbrace{\frac{[\mathbb{E}(z) - \frac{\rho}{a} \text{cov}(P, z)]^2}{1 + 2 \text{var}(z)} + \frac{1}{2} \log[1 + 2 \text{var}(z)]}_{=\text{GT}_E}. \end{aligned} \quad (4)$$

Notice that  $-\log\{-\mathbb{E}[U(\pi_E)]\}/\rho$  is equal to the certainty equivalent (CE) of an entrepreneurs' risky income  $\pi_E$ . For the sake of convenience, we often call this transformed value "expected utility" in what follows (and analogously for the other agents). If hipos merely stored and consumed their endowment, their expected utility would be given by  $\rho e_L$ . If they become entrepreneurs and sell the  $1/a$  firms they set up and carry out no further financial transactions, they get extra expected utility  $-\log\{-\mathbb{E}[U(e_L + P/a)]\} - \rho e_L = (\rho/a)[\mathbb{E}(P) - \rho/(2a) \text{var}(P)] \equiv \text{GE}$ . These "gains from entrepreneurship" are determined by the first two moments of the random asset price  $P$ . Define the additional terms in (4) as the "gains from trading" for entrepreneurs  $\text{GT}_E$ .  $\text{GT}_E$  reflects the marginal impact of an entrepreneur's trade in the stock market on his expected utility, after having sold his firm.  $\text{GT}_E$  depends on the first two moments of  $z$  and on the covariance of  $z$  and  $P$ . This covariance matters because changes in  $w$  (linearly) affect both the price  $P$  at which entrepreneurs sell their firms and the expected payoff-risk ratio  $z$ . (This effect is not present in GS, where agents are not engaged in entrepreneurial activity, and makes the application of a lemma in Demange and Laroque, 1995, necessary in the derivation of (4).)

A passive investor's unconditional expected utility is obtained analogously:

$$-\log\{-\mathbb{E}[U(\pi_M)]\} = \rho e_M + \underbrace{\frac{[\mathbb{E}(z)]^2}{1 + 2 \text{var}(z)} + \frac{1}{2} \log[1 + 2 \text{var}(z)]}_{=\text{GT}_M}. \quad (5)$$

As passive investors do not own shares in firms early, this is (4) with  $P = 0$ . The final two terms in the sum on the right-hand side,  $\text{GT}_M$  say, give the passive investor's gains from trading. As  $\text{cov}(P, z)$  is negative,  $\text{GT}_E > \text{GT}_M$  whenever  $\mathbb{E}(z) > 0$  (which holds true whenever rational agents do not short the asset in the aggregate at equilibrium). Under this condition, even though entrepreneurs trade on the same information as passive investors, they derive greater benefits from their trades, since fluctuations in  $z$  provide a hedge against the entrepreneurial risk they carry. The expected utility of hipos who act like passive investors is  $-\log\{-\mathbb{E}[U(\pi_M)]\} + \rho(e_L - e_M)$ .

A dealer's expected utility is

$$\begin{aligned}
-\log\{-E[U(\pi_D)]\} &= \rho e_L + \overbrace{\frac{1}{2} \log \left[ \frac{\text{var}(\theta|w)}{\sigma_\varepsilon^2} \right]}{=\text{GI}} \\
&\quad + \frac{[E(z)]^2}{1+2\text{var}(z)} + \frac{1}{2} \log [1+2\text{var}(z)].
\end{aligned} \tag{6}$$

The sum on the right-hand side can be rewritten as  $\rho e_L + \text{GI} + \text{GT}_M$ , where  $\text{GI} \equiv (1/2) \log[\text{var}(\theta|w)/\sigma_\varepsilon^2]$  represents the “gains from being informed”, i.e., from knowing  $s$  rather than having to infer information about it from the asset price.  $\text{GI}$  is positive or zero, depending on whether  $P$  reveals  $s$  or not, respectively.

#### 4. Equilibrium without a labor market

This section analyzes equilibrium in the model without a labor market. We start with the case of no noise trader shocks (i.e., model  $M_0^1$ ) and then introduce noise.

##### Equilibrium with no noise

Consider the model with OC  $M_0^1$ . Suppose to begin with that the mass of dealers is positive (i.e.,  $L_E < L$ ). The condition for optimal OC is that entrepreneurs are as well-off as dealers:  $\text{GE} + \text{GT}_E - \text{GT}_M = \text{GI}$ . Since the asset price is fully revealing in the case of no noise, there are no gains from being informed for dealers:  $\text{GI} = 0$ . Moreover,  $z$  is non-random, so  $\text{cov}(P, z) = 0$  and  $\text{GT}_E = \text{GT}_M$ . It follows that  $\text{GE} = 0$ . That is, hipos earn no rents in equilibrium compared to passive investors. This condition can be written as:

$$\Delta_0(L_E) \equiv \frac{\rho}{a} \left[ \bar{s} - \frac{\rho\sigma_\varepsilon^2}{L+M} \left( \frac{L_E}{a} - \bar{v} \right) - \frac{\rho\sigma_s^2}{2a} \right] = 0. \tag{7}$$

The left-hand side of (7) maps the half-open interval  $[0, L)$  to the reals. (The mass of entrepreneurs  $L_E$  must not take on the value  $L$ , because there would be no-one left to reveal information about  $s$  then.) So we have:

**PROPOSITION 4.1.** *If there is  $L_E^1$  ( $0 < L_E^1 < L$ ) such that  $\Delta_0(L_E^1) = 0$ , then there is an equilibrium of  $M_0^1$  with  $L_E = L_E^1$ .*

This is illustrated in the left panel of Figure 2. The downward-sloping line  $\Delta_0(L_E)$  gives the expected utility of an entrepreneur over and above a passive investor's,  $\text{GE}$ . The expected utility differential for a dealer compared to a passive investor  $\text{GI}$  is zero. So equilibrium occurs at the point of intersection of  $\Delta_0(L_E)$  and the horizontal axis (see the filled circle). Evidently, from (7), the equilibrium has the expected comparative statics properties: for instance,  $L_E$  increases when  $\bar{s}$  or  $L$  rises. Here and in what follows remarks on the formal properties of the equilibrium (existence, uniqueness, comparative statics) are delegated to the Appendix.

Next, suppose there are no dealers. Since no market participant observes  $s$ , the asset price is uninformative and non-random then:  $P = (a/\rho)\Delta_0^0(L_E)$ , where

$$\Delta_0^0(L_E) = \frac{\rho}{a} \left[ \bar{s} - \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{L+M} \left( \frac{L_E}{a} - \bar{v} \right) \right]. \tag{8}$$

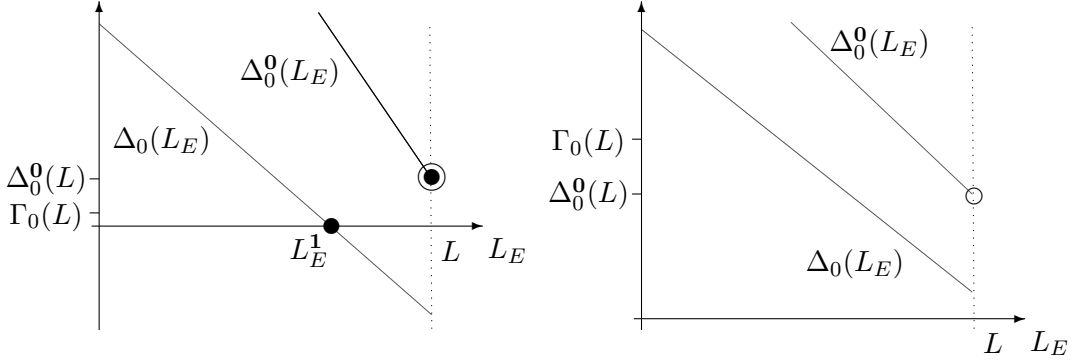


Figure 2: Equilibrium with no noise

$\Delta_0^0(L_E)$  gives the gains from entrepreneurship GE. It maps  $[0, L]$  to the reals. Being a dealer is preferred to being a passive investor whenever the mass of dealers is zero, as valuable information about the asset price can be obtained at no cost. So equilibrium without dealers implies that all  $L$  hipos become entrepreneurs, the asset supply is  $L/a$ , and the asset price is  $P = (a/\rho)\Delta_0^0(L)$ . Optimal OC implies that a single agent must not have an incentive to become a dealer, given that the others are entrepreneurs. This is the case if  $\Delta_0^0(L) \geq \Gamma_0(L)$ , where

$$\Gamma_0(L) = \frac{1}{2} \log \left( \frac{\sigma_s^2 + \sigma_\varepsilon^2}{\sigma_\varepsilon^2} \right).$$

**PROPOSITION 4.2.** *If  $\Delta_0^0(L) \geq \Gamma_0(L)$ , then there is an equilibrium of  $M_0^1$  with  $L_E = L$ .*

Equilibrium is not necessarily unique. If the conditions of both Proposition 4.1 and Proposition 4.2 are satisfied, the two types of equilibria, with or without dealers, coexist (as illustrated in the left panel of Figure 2). It can also happen that an equilibrium fails to exist. This is illustrated in the right panel of Figure 2.  $\Delta_0(L_E) > 0$  for all  $L_E < L$  implies that an equilibrium with  $L_E < L$  entrepreneurs and a positive mass of dealers does not exist (if the price is fully informative, it does not pay to be a dealer).  $\Gamma_0(L) > \Delta_0^0(L)$  implies that an equilibrium without dealers does not exist either (if the price is uninformative, a single agent has an incentive to incur a low cost of becoming informed). This is the classic GS non-existence result.

Finally, suppose hipos do not have the opportunity to become dealers (model  $M_0^0$ ). Hipos who do not become entrepreneurs act as passive investors then. As the gains from entrepreneurship GE are given by  $\Delta_0^0(L_E)$ , we have:

**PROPOSITION 4.3.** *(i) If there is  $L_E^0$  ( $0 < L_E^0 < L$ ) such that  $\Delta_0^0(L_E^0) = 0$ , then there is an equilibrium of  $M_0^0$  with  $L_E = L_E^0$ . (ii) If  $\Delta_0^0(L) \geq 0$ , then there is an equilibrium of  $M_0^0$  with  $L_E = L$ .*

Equilibria of these types are illustrated by the open circles in Figure 2. Suppose there are equilibria with  $L_E^1 < L$  in  $M_0^1$  and with  $L_E^0 < L$  in  $M_0^0$ , respectively. One might expect that  $L_E^0 > L_E^1$ , since hipos do not have the opportunity to become dealers in the absence of OC. While this is not generally true, a simple

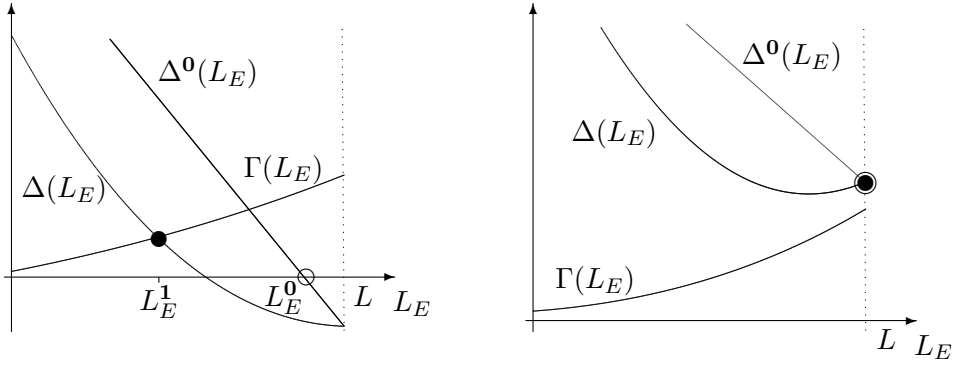


Figure 3: Equilibrium with noise

sufficient condition is

$$L - a\bar{\nu} < \frac{L + M}{2}. \quad (9)$$

Inequality (9) is in turn valid if “talent is scarce” in that there are fewer hipos than passive investors ( $L \leq M$ ) and noise traders do not short the asset ( $\bar{\nu} > 0$ ).

### Equilibrium with noise

Next, consider the model with a positive variance of noise traders’ demand for assets  $\sigma_\nu^2$ , i.e.,  $M_\sigma^i$ . If hipos face the OC decision (i.e., in  $M_\sigma^1$ ) and a positive fraction of them become entrepreneurs, this must be no less attractive than becoming a dealer. The benefits of being an entrepreneur as compared to a passive trader  $GE + GT_E - GT_M$  (cf. (4)) can be expressed as a function of  $L_E$  alone:

$$\Delta(L_E) \equiv \frac{\frac{\rho}{a} \text{cov}(P, z) \left[ \frac{\rho}{a} \text{cov}(P, z) - 2E(z) \right]}{1 + 2 \text{var}(z)} + \frac{\rho}{a} \left[ E(P) - \frac{\rho}{2a} \text{var}(P) \right]. \quad (10)$$

Closed-form solutions for the moments of  $P$  and  $z$  on the right-hand side as functions of  $L_E$  are in the Appendix. Likewise, the gains from being informed  $GI$  (cf. (6)) can be expressed as a function of  $L_E$  alone:

$$\Gamma(L_E) \equiv \frac{1}{2} \log \left[ \frac{\text{var}(\theta|w)}{\sigma_\varepsilon^2} \right]. \quad (11)$$

An expression that relates  $\text{var}(\theta|w)$  to  $L_E$  alone is in the Appendix. Both  $\Delta(L_E)$  and  $\Gamma(L_E)$  are continuous real-valued functions on  $[0, L]$ .

**PROPOSITION 4.4.** (i) If there is  $L_E^1$  ( $0 < L_E^1 < L$ ) such that  $\Delta(L_E^1) = \Gamma(L_E^1)$ , then there is an equilibrium of  $M_\sigma^1$  with  $L_E = L_E^1$ . (ii) If  $\Delta(L) \geq \Gamma(L)$ , then there is an equilibrium of  $M_\sigma^1$  with  $L_E = L$ .

Other than in the absence of noise, hipos earn positive rents in equilibrium. Either the rents are identical in both occupations (case (i)), or all hipos become entrepreneurs because this yields higher expected utility than trading (case (ii)). Contrary to the case of no noise, these equilibria are consistent with a positive cost

of becoming an entrepreneur or a dealer (viz., no greater than  $\Delta(L_E^1)$  or  $\Delta(L)$ , respectively). The two types of equilibria are illustrated in Figure 3. The left and right panels refer to cases (i) and (ii), respectively. The coordinates of the filled circles indicate the equilibrium mass of entrepreneurs  $L_E$  and the equilibrium difference in the expected utilities of entrepreneurs and passive investors  $\text{GE} + \text{GT}_E - \text{GT}_M$  ( $= \text{GI}$  in case (i)).

Without OC (in  $M_\sigma^0$ ), the expected utility differential for entrepreneurs compared to passive investors  $\text{GE} + \text{GT}_E - \text{GT}_M$  is also given by (10). However, since no-one gathers information about  $s$ , the price is uninformative, so the moments of  $P$  and  $z$  in (10) differ from their counterparts in  $M_\sigma^1$ . As in  $M_\sigma^1$ , the moments can be expressed as continuous real-valued functions of  $L_E$  alone. Denote the composite function obtained by substituting these moments into the right-hand side of (10) as  $\Delta^0(L_E)$  (with domain  $[0, L]$ ).

**PROPOSITION 4.5.** *(i) If  $\Delta^0(L_E^0) = 0$  for some  $L_E^0$  ( $0 < L_E^0 < L$ ), then there is an equilibrium of  $M_\sigma^0$  with  $L_E = L_E^0$ . (ii) If  $\Delta^0(L) \geq 0$ , then there is an equilibrium of  $M_\sigma^0$  with  $L_E = L$ .*

Such equilibria are illustrated by the open circles in Figure 3.

An important property of equilibria of  $M_\zeta^1$  and  $M_\sigma^0$  (proved in the Appendix) is that they are continuous functions of the variance of noise trader demand  $\sigma_\nu^2$  at  $\sigma_\nu^2 = 0$ . That is, given an equilibrium of  $M_0^j$  with mass of entrepreneurs  $L_E^j$ , there is an equilibrium of  $M_\sigma^j$  with a mass of entrepreneurs arbitrarily close to  $L_E^j$  for  $\sigma_\nu^2$  sufficiently small ( $j = 0, 1$ )

## 5. Equilibrium with a labor market

This section incorporates a labor market into the model (cf. Figure 1). We consider both a full employment version of the model and a specification with real wage rigidity and equilibrium unemployment. Workers benefit from entrepreneurship in both models: a higher mass of entrepreneurs raises the real wage rate at a full employment equilibrium and lowers unemployment in the presence of labor market frictions.

### Model

We maintain all assumptions made in Section 2 unless stated otherwise. Passive investors are now endowed with one unit of unskilled labor per capita and also called “workers”. Hipos do not require unskilled labor to set up firms or to gather information. Irrespective of their OC decision (i.e., either as an entrepreneur or as a dealer or acting like a passive investor), they do not supply unskilled labor.

As before, an entrepreneur sets up firms indexed  $[0, 1/a]$  early. Firm output and profit are  $Y = \tilde{\theta} + F(m)$  and  $\theta \equiv Y - Wm$ , respectively, where  $m$  is firm-level employment and  $W$  is the wage rate. The production function  $F$  is twice continuously differentiable, strictly increasing, and strictly concave, with  $\lim_{m \rightarrow 0} F'(m) = \infty$  and  $\lim_{m \rightarrow \infty} F'(m) = 0$ , so that profit maximization yields an interior solution.  $\tilde{\theta}$  is the sum of two independent jointly normal random variables  $\tilde{s} \sim N(\hat{s}, \sigma_s^2)$  and  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ .

Wages and employment are determined early, so  $W$  and  $m$  are non-random, and an employed worker’s initial wealth is  $e_M + W$ . The disutility of working is equivalent to  $D$  ( $\geq 0$ ) safe units of consumption. So the aggregate supply of labor is  $M$  for  $W \geq D$  and zero otherwise. An entrepreneur’s expected utility is an

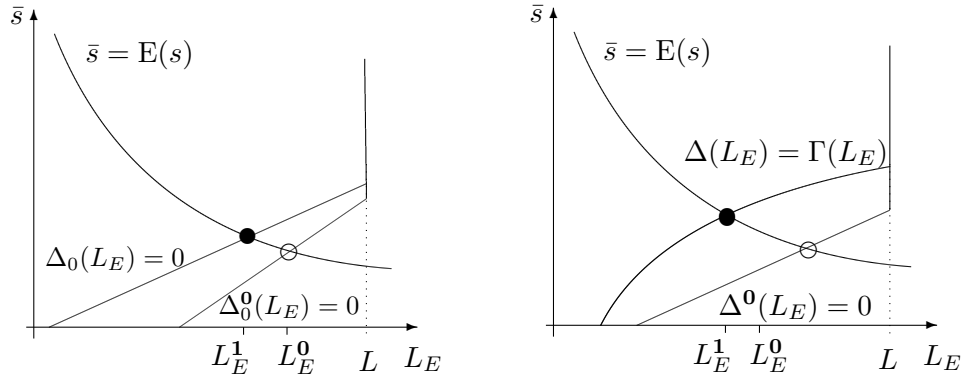


Figure 4: Full employment equilibrium

increasing function of  $F(m) - Wm$ , so he chooses employment  $m$  so as to maximize this objective function (see the Appendix).<sup>13</sup>

Dealers observe  $\bar{s}$  at the intermediate date, while entrepreneurs and workers have to infer information about  $\bar{s}$  from the equilibrium stock price.

### Full employment

Denote the full employment version of the model as  $F_\zeta^j$ , with  $j$  equal to  $\mathbf{1}$  or  $\mathbf{0}$  depending on whether there is OC or not, respectively, and  $\zeta$  equal to  $0$  or  $\sigma$  depending on whether  $\sigma_v^2$  is zero or positive. For simplicity, let  $D = 0$  here, so that workers supply labor for any positive wage rate. Let  $\hat{M} \equiv M/(L_E/a)$  denote the number of workers per firm.  $(L_E, I_E, I_D, I_M, P, m, W)$  is an *equilibrium* of  $F_\zeta^j$  if, in addition to the conditions stated in Section 3, employment  $m$  maximizes  $F(m) - Wm$  and the labor market clears (i.e.,  $m = \hat{M}$ ).

Denote the wage rate given full employment of unskilled labor as  $\hat{W} = F'(\hat{M})$ . Define

$$s \equiv F\left(\frac{aM}{L_E}\right) - F'\left(\frac{aM}{L_E}\right) \frac{aM}{L_E} + \tilde{s}, \quad \bar{s} \equiv E(s). \quad (12)$$

$s + \varepsilon$  is firm profit given full employment.  $s$  is normal with mean  $E(s) = \bar{s}$  and variance  $\sigma_s^2$ . Given the definitions of  $s$  and  $\bar{s}$  in (12), the equilibrium analysis of  $M_\zeta^j$  in Section 4 goes through without modification, and we have:

**PROPOSITION 5.1.** *Let  $s$  and  $\bar{s}$  be given by (12). If  $(L_E, I_E, I_D, I_M, P)$  is an equilibrium of  $M_\zeta^j$ , then  $(L_E, I_E, I_D, I_M, P, \hat{M}, \hat{W})$  is an equilibrium of  $F_\zeta^j$  ( $j = \mathbf{0}, \mathbf{1}$ ,  $\zeta = 0, \sigma$ ).*

(It is understood that  $I_D$  drops out for  $j = \mathbf{0}$ .) Figure 4 illustrates the determination of the equilibrium values of  $L_E$  and  $\bar{s}$ . The left panel applies to the model without noise trader shocks  $F_\zeta^j$ . The upward-sloping

<sup>13</sup>This would not be true if the impact of  $\tilde{\theta}$  on firm profit  $\theta$  were non-additive, so that setting up a firm creates an asset with different return characteristics.



lines depict the relations between  $\bar{s}$  and  $L_E$  at an equilibrium of  $\mathbf{M}_0^{\mathbf{j}}$  with  $L_E < L$ . The left line applies to the model with OC (i.e.,  $\mathbf{j} = \mathbf{1}$ ). It depicts the  $(L_E, \bar{s})$  combinations which satisfy  $\Delta_0(L_E) = 0$  (see (7)). The right line applies to the case of no OC (i.e.,  $\mathbf{j} = \mathbf{0}$ ). It is determined by  $\Delta_0^{\mathbf{0}}(L_E) = 0$  (see (8)). The positive slopes of the lines reflect the fact that a higher value of  $\bar{s}$  makes entrepreneurship more attractive. For sufficiently large values of  $\bar{s}$ , equilibria of  $\mathbf{M}_\zeta^{\mathbf{j}}$  are characterized by  $L_E = L$ . This is illustrated by the vertical line segment. The downward-sloping curve depicts the relation between  $\bar{s}$  and  $L_E$  implied by (12). The negative slope reflects the fact that an increase in  $L_E$  decreases firm-level employment  $\hat{M}$  and, therefore,  $\bar{s}$ :  $d\bar{s}/dL_E = F''(\hat{M})\hat{M}^2/L_E < 0$ . The filled circle and the open circle represent equilibria of  $\mathbf{F}_0^{\mathbf{1}}$  and  $\mathbf{F}_0^{\mathbf{0}}$ , respectively. The right panel of 4 applies to the model with stochastic noise trader demand  $\mathbf{F}_\sigma^{\mathbf{j}}$ . The two upward sloping curves represent  $(L_E, \bar{s})$  combinations such that  $\Delta(L_E) = \Gamma(L_E)$  and  $\Delta^{\mathbf{0}}(L_E) = 0$ , respectively. Intersections with the downward sloping curve that represents (12) correspond to equilibria of  $\mathbf{F}_\sigma^{\mathbf{1}}$  and  $\mathbf{F}_\sigma^{\mathbf{0}}$ , respectively.

Increased entrepreneurial activity causes higher wages for workers:  $d\hat{W}/dL_E = -F''(\hat{M})\hat{M}/L_E > 0$ . Therefore, any parameter change that raises  $L_E$  (and leaves  $F$  and the other parameters in the definition of  $\hat{M}$  unaffected, such as an increase in  $E(\bar{s})$ ) raises workers' wages.

### Unemployment

Next, consider the model with unemployment. The model is denoted  $\mathbf{U}_\zeta^{\mathbf{j}}$ , where, as before,  $\mathbf{j}$  is equal to  $\mathbf{1}$  or  $\mathbf{0}$  depending on whether there is OC or not, respectively, and  $\zeta$  is equal to 0 or  $\sigma$  depending on whether  $\sigma_v^2$  is zero or positive. There is equilibrium unemployment due to union wage setting (cf. McDonald and Solow, 1981). Workers are organized in decentralized firm-level unions. They are spread evenly across firms, so there are  $\hat{M}$  workers per firm. Unions monopolistically set the wage rate. Firms have the “right to manage” and choose the profit maximizing level of employment. The production function  $F$  is Cobb-Douglas:  $F(m) = Am^{1-b}$ , where  $A > 0$  and  $0 < b < 1$ . If there is unemployment, the probability of being employed is  $m/\hat{M}$  for each worker. Unions maximize workers' expected utility, taking their asset demands as given. In the Appendix we show that the gains from trading are separable from the gains of having a job, so that unions maximize the following expression which gives the expected gain from having a job:

$$-\log \left( 1 - \frac{m}{\hat{M}} \{1 - \exp[-\rho(W - D)]\} \right). \quad (13)$$

$(L_E, I_E, I_D, I_M, P, m, W)$  is an *equilibrium* of  $\mathbf{U}_\zeta^{\mathbf{j}}$  if in addition to the conditions stated in Section 3, employment  $m$  maximizes  $F(m) - Wm$ ,  $W$  maximizes (13) given firms' optimal choice of  $m$ , and there is unemployment (i.e.,  $m < \hat{M}$ ).

As in the standard right-to-manage model without asset demands, the conditions for optimum union wage setting and profit maximization jointly determine the wage rate,  $\tilde{W}$  say, and employment per firm  $\tilde{M}$ , independently of the other variables which make up an equilibrium  $(L_E, I_E, I_D, I_M, P, m, W)$  (see the Appendix). In particular, the real wage rate is rigid in that it does not depend on the mass of firms  $L_E/a$ . So while an increase in the mass of entrepreneurs does not affect employment at the firm level (the intensive margin), it

increases aggregate employment by raising the mass of firms (the extensive margin).<sup>14</sup>

The model has a block-recursive structure, which makes equilibrium even easier to characterize than in the full employment model. Analogously as in  $F_{\zeta}^{\mathbf{j}}$  (cf. (12)), define  $s \equiv F(\tilde{M}) - \tilde{W}\tilde{M} + \tilde{s}$ . Since  $\tilde{M}$  and  $\tilde{W}$  are constants,  $s$  is normal with mean

$$\bar{s} \equiv F(\tilde{M}) - \tilde{W}\tilde{M} + \hat{s} \quad (14)$$

and variance  $\sigma_s^2$  and independent of  $L_E$ .

**PROPOSITION 5.2.** *Let  $\bar{s}$  be given by (14). If  $(L_E, I_E, I_D, I_M, P)$  is an equilibrium of  $M_{\zeta}^{\mathbf{j}}$  and  $\tilde{M} < \hat{M}$ , then  $(L_E, I_E, I_D, I_M, P, \tilde{M}, \tilde{W})$  is an equilibrium of  $U_{\zeta}^{\mathbf{j}}$  ( $\mathbf{j} = \mathbf{0}, \mathbf{1}, \zeta = 0, \sigma$ ).*

More entrepreneurship means more jobs: since employment at the firm level  $\tilde{M}$  is constant, aggregate employment  $\tilde{M}L_E/a$  goes up when  $L_E$  rises. For instance, consider an increase in  $E(\hat{s})$  and, hence,  $\bar{s}$ . Since an increase in  $\bar{s}$  raises  $L_E$  at an equilibrium of  $M_{\zeta}^{\mathbf{j}}$ , the same is true at an equilibrium of  $U_{\zeta}^{\mathbf{j}}$ . In terms of Figure 4, the  $\bar{s} = E(s)$  curve is horizontal, and the upward-sloping curves shift to the right.

## 6. Welfare

This section presents our main results on the welfare effects of financial trading.

### Social welfare

Let  $N$  denote the mass of noise traders and  $e_N$  their endowment per capita. Each noise trader invests  $\nu/N$ , so his final wealth is  $\pi_N = e_N + (\theta - P)\bar{\nu}/N$ . The presence of noise traders, who do not optimize, raises the question of how to treat them in the calculation of SW. We (i) assess noise traders' well-being using the same CARA utility function  $U(\pi)$  that also characterizes rational agents and (ii) define SW as the unweighted sum of *all* agents', including noise traders', *transformed* expected utilities  $-\log\{-E[U(\pi_i)]\}$ .<sup>15</sup> Assumption (i) amounts to assuming (in model  $M_{\zeta}^{\mathbf{j}}$ ) that a fraction  $N/(M + N)$  of the  $M + N$  non-hipos experience a shock at the intermediate date that keeps them from carrying out their utility maximization problem and randomize their asset demands. According to assumption (ii), since the transformed expected utilities are equal to  $\rho$  times the respective CEs, SW is  $\rho$  times the unweighted sum of CEs and SW maximization is equivalent to maximization of the sum of the CEs. This is a standard welfare criterion in risky environments (see, e.g., Chambers and Echenique, 2012). It implies that SW rises one-for-one with endowments, which rules out that redistribution of safe income affects SW.

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<sup>14</sup>We have solved the model with alternative sources of wage rigidity: union wage setting aimed at maximizing the wage bill (cf. Dunlop, 1944) and efficiency wages due to moral hazard (cf. Shapiro and Stiglitz, 1984) or fairness considerations (cf. Solow, 1979). The subsequent results carry over one-to-one to these models (see the Appendix).

<sup>15</sup>A similar approach is used by Allen (1984) and Albagli et al. (2017). Allen (1984, p. 4) attributes a risk neutral utility function to the noise traders in his welfare analysis of the GS model. Albagli et al. (2017, p. 7) use total expected dividends (accruing to rational agents and noise traders) as the measure of welfare in their analysis of corporate risk taking in a model with risk neutral rational agents. An alternative is to only consider rational agents' welfare, which leads to similar conclusions in our model but ignores the impact of resource flows to noise traders on their well-being. Attempts to avoid the problem altogether by setting up fully rational models in which noise trader demand is derived from stochastic liquidity needs or portfolio churning by asset managers are surveyed by Dow and Gorton (2008). A recent contribution is in Han et al. (2016).

We conduct a second-best welfare analysis, which analyzes SW given agents' investment and labor market decisions. We investigate the impact of marginal changes in entrepreneurial activity on SW at an equilibrium with OC as well as the question of whether SW is higher with or without OC. We show below that any given level of  $L_E$  can be achieved with an appropriate set of taxes. The current section treats the case of small noise trader shocks analytically, the next section investigates large noise trader shocks numerically. Our overall conclusion is that the allocation of talent to trading tends to be excessive from a welfare point of view. This conclusion holds true in the absence of labor market imperfections and is strongly reinforced by the inclusion of labor market frictions.

### Constrained efficiency of equilibrium without noise trader shocks

Ignore noise trader shocks to begin with. Let the mass of entrepreneurs  $L_E$  ( $< L$ ) be given, the other  $L - L_E$  ( $> 0$ ) hipos act as dealers. All agents make optimizing investment decisions. Then SW in the model without a labor market  $M_0^1$  is

$$S^1(L_E, \bar{s}) = \rho e + L_E \Delta_0(L_E) + \frac{\rho^2 \sigma_\varepsilon^2}{2(L+M)} \left[ \left( \frac{L_E}{a} \right)^2 - \left( 1 + \frac{L+M}{N} \right) \bar{v}^2 \right], \quad (15)$$

where  $e = Le_L + Me_M + Ne_N$  denotes aggregate endowments. SW without OC, i.e., in model  $M_0^0$ , is

$$S^0(L_E, \bar{s}) = \rho e + L_E \Delta_0^0(L_E) + \frac{\rho^2 (\sigma_s^2 + \sigma_\varepsilon^2)}{2(L+M)} \left[ \left( \frac{L_E}{a} \right)^2 - \left( 1 + \frac{L+M}{N} \right) \bar{v}^2 \right]. \quad (16)$$

In the full employment economy  $F_0^j$ , the expressions for SW,  $S^1$  and  $S^0$ , contain the additional term  $\rho \hat{W} M$ , which represents workers' extra expected utility drawn from their safe labor income. In the economy with unemployment  $U_0^j$ , SW contains  $M$  times the expression in (13) as an additional term representing workers' aggregate gains from employment.

**PROPOSITION 6.1.** (i) Suppose a solution  $L_E^1 < L$  to  $\Delta_0(L_E^1) = 0$  exists, a solution  $(L_E^1, \bar{s})$  with  $L_E^1 < L$  to  $\Delta_0(L_E^1) = 0$  and (12) exists, and a solution  $L_E^1 < L$  to  $\Delta_0(L_E^1) = 0$  with  $\bar{s}$  given by (14) exists. Then  $L_E^1$  maximizes  $S^1$  on  $[0, L]$  in  $M_0^1$  and  $F_0^1$ , and  $L_E^1$  falls short of the value that maximizes  $S^1$  in  $U_0^1$ . (ii) Suppose a solution  $L_E^0 < L$  to  $\Delta_0^0(L_E^0) = 0$  exists, a solution  $(L_E^0, \bar{s})$  with  $\bar{s}$  given by (12) and  $L_E^0 < L$  exists, and a solution  $L_E^0 < L$  with  $\bar{s}$  given by (14) exists. Then  $L_E^0$  maximizes  $S^0$  on  $[0, L]$  in  $M_0^0$  and  $F_0^0$ , and  $L_E^0$  falls short of the value that maximizes  $S^0$  in  $U_0^0$ .

This is our first main result on the welfare implications of OC. It is illustrated in the left panel of Figure 5. Equilibrium without noise trader shocks and without labor market frictions is constrained efficient in the following sense: (i) given that the hipos who do not become entrepreneurs observe macro fundamentals and reveal them with their asset trades, the equilibrium mass of entrepreneurs is at its second-best optimum level in  $M_0^1$  and  $F_0^1$ . (ii) The analogous result holds under the premise that the non-entrepreneurs act as passive investors, so that the asset price is uninformative. This is not surprising, since, conditional on the distribution of information, there are no market imperfections. The mere fact that entrepreneurs create jobs in  $F_0^j$  does not imply that there is too little entrepreneurship. Viewed the other way round, the assertion of the proposition lends support to the welfare criterion chosen (with noise traders' expected utilities included): conditional on

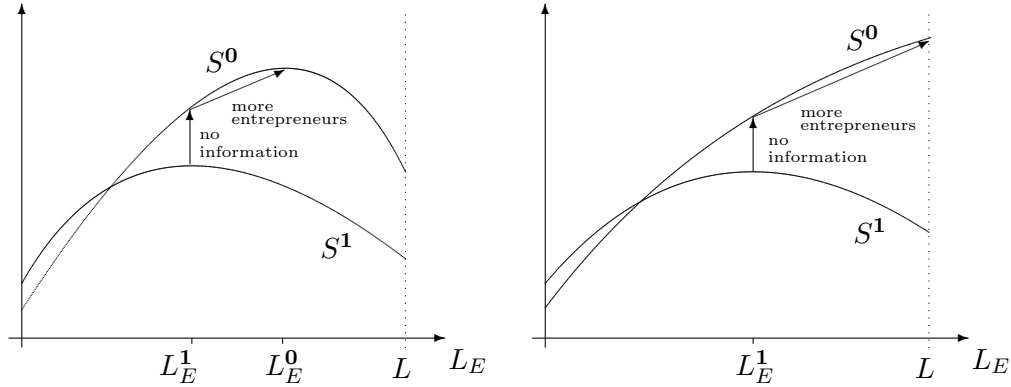


Figure 5: Social welfare

the distribution of information, it entails welfare maximization at the equilibria of the frictionless economies. Equilibrium is also constrained-efficient if all hipos become entrepreneurs in the absence of OC (see the right panel of Figure 5).<sup>16</sup>

Equilibrium is not constrained efficient, given the distribution of information, in the presence of labor market rigidities. An increase in the mass of entrepreneurs, starting at equilibrium, raises SW in  $U_0^j$ . This reflects the fact that job creation by entrepreneurs is inefficiently low in the presence of labor market frictions. As a result, measures which induce a shift of resources from professional trading to production enhance SW.<sup>17</sup> In the Appendix we show that social welfare is a continuous function of  $\sigma_\nu^2$  at  $\sigma_\nu^2 = 0$  (which is a non-trivial task, as the calculation of noise trader expected utility for  $\sigma_\nu^2 > 0$  requires the derivation of a variant of the Demange-Laroque, 1995, lemma). Together with the fact that equilibrium is continuous in  $\sigma_\nu^2$  at  $\sigma_\nu^2 = 0$ , it follows that the assertion of Proposition 6.1 also holds true for small noise trader shocks.

### Informational efficiency and real efficiency

According to Proposition 6.1, the equilibrium mass of entrepreneurs is at its constrained-efficient level in the absence of noise and labor market frictions conditional on whether prices are informative or not. We now proceed to compare the welfare levels at the two constrained optima. The welfare effect of switching from  $L_E^1$  entrepreneurs and  $L - L_E^1$  dealers to  $L_E^0$  entrepreneurs and no dealers can be decomposed into two effects (see Figure 5): the difference in  $S^0$  and  $S^1$  at  $L_E^1$  is the welfare effect of having no information about  $s$ , and the difference in  $S^0$  evaluated at  $L_E^0$  and at  $L_E^1$  is the impact of additional entrepreneurial activity.

The welfare effect of having no information about macro fundamentals  $s$  tends to be positive in our model

<sup>16</sup>Another implicit condition for second-best optimality of equilibrium in  $M_0^1$  and  $F_0^1$  is that the allocation of roles to agents is given. SW would be higher if the  $N$  noise traders acted as passive investors (see the Appendix).

<sup>17</sup>These are third-best measures, however, as entrepreneurial activity is at its constrained optimum level once labor market frictions are removed, and SW is lower in  $U_0^j$  than in  $F_0^j$  for each  $L_E$  (see the Appendix).

(for related results in different setups, see Allen, 1984, Gorton, 1997, Goldstein and Yang, 2014, Bond and García, 2018, and Angeletos et al., 2018). The reason is that price informativeness leads to a concentration of risk at entrepreneurs. This is easy to see in the case without noise trader shocks. At an equilibrium with no short selling by noise traders (i.e., with  $\bar{\nu} > 0$ ), each entrepreneur supplies a positive net amount of assets equal to  $(1/a - I_E) = (1/a)[1 - (L_E - a\bar{\nu})/(L + M)]$ . While the selling price is safe without dealers, it fluctuates with the observable part of the macro fundamentals  $s$  when there is a positive mass of dealers. That is, the revelation of macro information makes it harder for entrepreneurs to get rid of the risks inherent in their production activity.

Entrepreneurs' expected utility is in fact lower at an equilibrium of  $M_0^1$  than at an equilibrium of  $M_0^0$  if

$$\bar{\nu} < \frac{L_E^1}{a} < \frac{L_E^0}{a}. \quad (17)$$

The first inequality requires that rational agents do not short the asset at equilibrium with OC (their aggregate equilibrium assets holdings are  $L_E^1/a - \bar{\nu}$ ). The second inequality states that the mass of hipos who become entrepreneurs rises when they are precluded from becoming dealers (e.g., because (9) holds). Since the other rational agents obtain the same level of expected utility as entrepreneurs at equilibrium, they are also worse-off with OC. Thus, given (17), no OC is Pareto-preferred by the set of rational agents. The following result compares equilibrium SW, including noise traders' expected utilities, with and without OC in the absence of noise trader shocks:

**PROPOSITION 6.2.** *Let  $L_E^1 < L$ . Then the difference  $S^1 - S^0$  in the equilibrium values of SW in economies  $M_0^1$  and  $M_0^0$  is negative if*

$$0 < \bar{\nu} < \frac{L_E^1}{a}, \quad \frac{\bar{\nu}}{N} < \frac{1}{a}. \quad (18)$$

*The same holds true in economies  $F_0^1$  versus  $F_0^0$  and, if  $L_E^1 < L_E^0$ , in economies  $U_0^1$  versus  $U_0^0$ .*

This is our second main result on the welfare implications of OC. It is illustrated by Figure 5. The proposition states a set of very simple sufficient conditions which ensure that equilibrium SW is lower with than without OC in the absence of noise trader shocks. The second inequality in (18) also appears in (17). Jointly with the first one it says that neither noise traders nor rational agents short the asset. The third inequality states that individuals are small relative to corporations, in that noise traders' per capita demand for assets is less than the asset supply generated by a single entrepreneur. This inequality ensures that noise traders benefit, or at least are not too strongly negatively affected, by the absence of OC. To see why, note that with OC both the mean and the variance of noise traders' final wealth  $\pi_N = e_N + (\theta - P)\bar{\nu}/N$  are lower than without OC (as the asset price  $P$  is more closely tied to fundamentals  $\theta$ ). The impact of lower expected wealth on expected utility is linear in  $\bar{\nu}/N$ . It dominates the quadratic effect of lower risk for  $\bar{\nu}/N$  small enough. In models  $M_0^j$  and  $U_0^j$  with  $L_E^0 < L$ ,  $\bar{\nu}/N < 1/a$  ensures that noise trader demand is small enough in this sense, so that having no OC is not only superior in terms of SW, but Pareto-preferred. In model  $F_0^j$ , a stronger assumption is required. If one includes the second inequality in (17), then the SW differential  $S^1 - S^0$  is also negative in the model with unemployment.

Due to continuity of equilibrium and SW in  $\sigma_\nu^2$  at  $\sigma_\nu^2 = 0$ , it follows that the assertion of Proposition 6.2

also holds true for small noise trader shocks. As  $\bar{\nu}$  is the *mean* of noise traders' asset demand  $\nu$  ( $\sim N(\bar{\nu}, \sigma_\nu^2)$ ), (18) then says that neither noise traders nor rational traders short the asset *on average*, and noise traders' *mean* per capita demand is less than the supply generated by a single entrepreneur.

Price informativeness has traditionally been considered conducive to real efficiency: “the ideal is a market in which prices provide accurate signals for resource allocation” (Fama, 1970, p. 383). This traditional view has been challenged by Hirshleifer (1971), who emphasizes that the revelation of information precludes ex ante efficient mutual insurance, and others (e.g., Allen, 1984, Hu and Qin, 2013, and Bond and García, 2018). Our extension of the GS model adds two further caveats to the traditional view. First, there is an opportunity cost of informational efficiency in terms of foregone real economic activity. And, second, increased informational efficiency may discourage entrepreneurial activity by making its returns more risky.

## 7. Large noise trader shocks

The preceding section has established two results on the benefits of more entrepreneurship for the case of small noise trader shocks. First, at equilibrium a marginal increase in the mass of entrepreneurs has a positive effect on SW when there is unemployment due to labor market frictions (Proposition 6.1). Second, given weak sufficient conditions (see (18)), SW is higher if hipos who do not become entrepreneurs act as passive investors and not as dealers (Proposition 6.2). This section investigates the case of large noise trader shocks numerically. We show that the two results carry over to the vast majority of model specifications with reasonably large volatility of noise trader demand and that the impact of a marginal increase in entrepreneurship also tends to be positive in the absence of labor market imperfections.

### Example and strategy

The closed-form solutions for the moments in (10) and (11) in the Appendix allow the computation of the equilibrium mass of entrepreneurs  $L_E^j$  via Propositions 4.4, 4.5, 5.1, and 5.2. Closed-form solutions for SW as functions of  $L_E$  (generalizing (15) and (16)) are used to compute SW.

Figure 6 illustrates an example of model  $M_\sigma^j$ . The model parameters listed in the first column of Table 1 are set as multiples of the terms in the third column. The values in the second column are 0.05 or 0.2 for  $\sigma_\nu$  and 0.2, 1, 1, 100, 0.5, 0.75, 0.1, 1, 10, in that order, for the other variables. Initial endowments are zero. The solid curves in Figure 6 depict the levels of SW  $S^1$  and  $S^0$  with and without noise, respectively. The dashed curves give the corresponding welfare levels without noise, investigated analytically in Section 6. SW without OC  $S^0$  is hump-shaped. By contrast, while  $S^1$  is also hump-shaped in the absence of noise, it converges to the level without OC as  $L_E \rightarrow L$  when  $\sigma_\nu > 0$ . This is because, in the presence of noise, the informational efficiency goes to zero as the mass of dealers goes to zero. Consider the case with  $\sigma_\nu$  equal to 5 percent of  $L/a$  (see the left panel of Figure 6). SW is very close to the noiseless case (for even smaller values of  $\sigma_\nu$ , the solid and dashed curves become indistinguishable) except  $S^1$  at the right end.  $L_E^1$  and  $L_E^0$  fall short of the constrained welfare-maximizing levels by 6.36 and 0.24 percent, respectively. The resulting deviations from maximum SW are less than 0.25 percent. So an increase in  $L_E$  starting at equilibrium has a weak positive impact SW in both cases. Equilibrium SW without OC is eleven times as high as equilibrium SW with OC (47.170 compared to 4.271). This confirms the conclusions of Propositions 6.1 and 6.2 for the

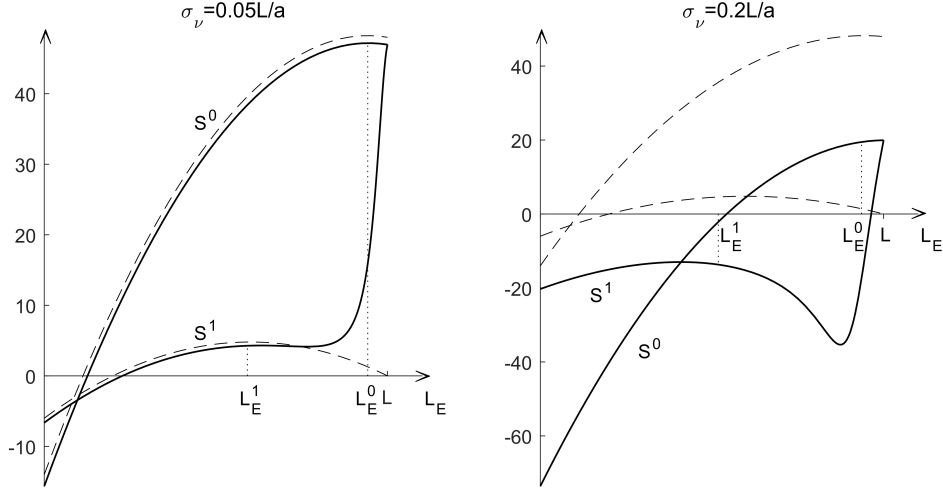


Figure 6: Example with  $\sigma_\nu = 0.05L/a$  (left panel) or  $\sigma_\nu = 0.2L/a$  (right panel)

noiseless case. In the example with  $\sigma_\nu = 0.2L/a$  (see the right panel of Figure 6), while  $L_E^0$  is 6.43 percent lower than its constrained optimal value,  $L_E^1$  is 26.94 percent higher. So a marginal increase in  $L_E$  has a positive effect on SW starting at the equilibrium without OC but a negative effect on SW starting at the equilibrium with OC (observing negative marginal effects becomes more common as  $\sigma_\nu^2$  gets large, but still remains highly unlikely for  $\sigma_\nu$  up to 20% of  $L/a$ , see Table 2). While the differences between equilibrium and optimum SW are relatively small ( $-13.680$  versus  $-12.987$  with OC,  $19.473$  versus  $19.954$  without OC), equilibrium SW is much larger without than with OC. As is also clear from Figure 6, while the presence of information collected by dealers reduces SW at equilibrium (i.e., at  $L_E^1$ ), information is socially beneficial at lower values of  $L_E$  in both cases.

We simulate the model for a large set of parameter values using Matlab. The strategy is as follows. We specify parameters such that for zero and small noise trader shocks, equilibrium exists and is unique and condition (18) holds, so that the analytical results on under-investment in entrepreneurship in Section 6 apply. We then consider increasing values of the volatility of noise trader demand  $\sigma_\nu$  and check whether the under-investment results remain valid. In doing so, we restrict attention to the subset of the original parameter combinations for which equilibrium exists and is unique and (18) holds (i.e., no-one shorts the asset on average and corporations are large relative to individual noise traders).

### Basic model

Consider first the model without a labor market  $M_\sigma^j$ . Agents' initial endowments  $e_i$  ( $i = L, M, N$ ) play no role in the determination of equilibrium and SW, so they are set equal to zero. Table 1 summarizes the values chosen for the other parameters. The parameters in the first column are specified as multiples of the magnitudes in the third column.

The maximum feasible supply of assets, which would materialize if all hipos became entrepreneurs, is  $L/a$ . For

Table 1: Parameter values in the simulation of  $M_0^1$

parameter	values	multiple of
$\sigma_\nu$	0.001, 0.01, 0.05, 0.1, 0.2, 0.5	$\frac{L}{a}$
$\bar{\nu}$	0.001, 0.01, 0.05, 0.1, 0.2, 0.5	$\frac{L}{a}$
$M$	1, 2, 3, 5, 10, 100	$L$
$N$	1, 2, 3, 5, 10, 100	$0.25(L + M)$
$L$	100	1
$\bar{s} - \frac{\rho\sigma_\varepsilon^2}{2a}$	0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99	$\frac{\rho\sigma_\varepsilon^2}{L+M} \left( \frac{L}{a} - \bar{\nu} \right)$
$\sigma_\varepsilon^2$	0.1, 0.25, 0.5, 0.75, 1	$\sigma_s^2$
$\sigma_s^2$	0.0001, 0.0005, 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, 1	$16 \left( \frac{a}{\rho} \right)^2$
$\rho$	1	1
$a$	10	1

the standard deviation of noise traders' asset demand  $\sigma_\nu$ , we consider different proportions of this maximum asset supply. While 10 percent or 20 percent appears to be a reasonable upper bound, we also consider  $\sigma_\nu = 0.5L/a$  to allow for very large noise trader shocks. Similarly, the average noise trader demand for assets  $\bar{\nu}$  rises from 0.1 percent up to 50 percent of  $L/a$  (so the first inequality in (18) is satisfied).

The mass of passive investors  $M$  is a multiple of the mass of hipos  $L$ , so that, from (9), talent is scarce (i.e.,  $L_E^0 > L_E^1$ ) in the absence of noise. The mass of noise traders  $N$  is at least 25 percent of the mass of rational agents  $L + M$ , so that  $N/a \geq 0.5L/a$ . Jointly with the fact that  $\bar{\nu} \leq 0.5L/a$ , this implies the validity of the third inequality in (18).

The moments which appear in rational agents' expected utility functions  $E[U(\pi_i)]$  and in (10) and (11) are homogeneous of degree zero in  $\sigma_\nu$ ,  $\bar{\nu}$ ,  $L$ ,  $M$ ,  $N$ , and  $L_E$  jointly (see the formulas in the Appendix). Since the former five variables vary proportionately with  $L$  in the simulations, so does the equilibrium mass of entrepreneurs  $L_E$ . Like rational agents' expected utilities,  $E[U(\pi_N)]$  is also unaffected by proportionate changes in  $\sigma_\nu$ ,  $\bar{\nu}$ ,  $L$ ,  $M$ ,  $N$ , and  $L_E$ . So whether there is under-investment in entrepreneurship or not is independent of the choice of  $L$ , and we confine attention to a single value, 100 say.

The mean of the macro fundamentals  $\bar{s}$  is such that  $\bar{s} - \rho\sigma_s^2/(2a)$  is in between zero and  $\rho\sigma_\varepsilon^2(L/a - \bar{\nu})/(L + M)$ . From (7), this ensures that the equilibrium mass of entrepreneurs  $L_E^1$  is in between  $a\bar{\nu}$  ( $> 0$ ) and  $L$  in  $M_0^1$ , so that the second inequality in (18) and, hence, all conditions of Proposition 6.2 are satisfied.

The variance of the unobservable productivity parameter  $\sigma_\varepsilon^2$  is between 10 and 100 percent of the variance of the observable macro productivity parameter  $\sigma_s^2$ . It is well known that large gamble sizes potentially lead to very low CEs with CARA utility (see, e.g., Babcock et al., 1993, p. 19). In our model, when  $\sigma_s^2$  and, hence,  $\sigma_\varepsilon^2$  grow large, the variance of entrepreneurs' payoff  $\pi_E$  becomes so high that the CE of  $\pi_E$  becomes very small. To rule out excessive risk aversion on behalf of entrepreneurs, we restrict the admissible values for  $\sigma_s^2$  appropriately. The CE of  $\pi_E$  is below the 95 percent confidence interval if  $\text{var}(\pi_E) < 16/\rho^2$ . In  $M_0^1$ , the variance of  $\text{var}(\pi_E)$  is in  $[\sigma_s^2/a^2, 1.25\sigma_s^2/a^2]$  (see the Appendix). So the CE is in the confidence interval at least in the case of minimum payoff variance if  $\sigma_s^2 < 16(a/\rho)^2$ . The admissible values of  $\sigma_s^2$  are specified as proportions of this threshold. For all proportions below unity, we have  $1.25\sigma_s^2/a^2 < 16/\rho^2$ , so that the CE



of  $\pi_E$  is inside the 95 percent confidence interval in the noiseless case.

Given the specifications of  $\bar{s} - \rho\sigma_s^2/(2a)$ ,  $\sigma_\varepsilon^2$ , and  $\sigma_s^2$  as multiples of the respective expressions in the third column of Table 1 and the homogeneity properties of the model, the equilibrium levels of  $L_E^j$  and the resulting welfare levels  $S^j$  are independent of  $\rho$  and of  $a$ . So we can fix these two parameters arbitrarily. We set the coefficient of relative risk aversion  $\rho$  equal to unity, which is common in numerical analysis (cf. Biais et al., 2010), and  $a = 10$ .

The parameters in Table 1 yield a total of 87,480 combinations for each given value of  $\sigma_\nu$ . By construction, an equilibrium exists and the conditions of Proposition 6.2 are satisfied. To rule out multiplicity in the noiseless case, we impose the condition ( $U_0$  for “uniqueness”) that  $\Delta_0^0(L) < \Gamma_0(L)$  (cf. Proposition 4.2), which leaves us with 68,112 combinations. (Almost all cases ruled out entail high values of  $\sigma_s^2$ . Multiplicity does not occur at all for the first five values of  $\sigma_s^2$  in Table 1 but for more than 80 percent of the parameters with  $\sigma_s^2 = 16(a/\rho)^2$ .) From the analysis in Section 6, we know that the impact of a marginal change in the mass of entrepreneurs on SW is zero (Proposition 6.1) and SW is higher without OC (Proposition 6.2) without noise for each parameter combination.

We add two regularity conditions, which apply to the noisy version of the model. First, to make sure that a unique equilibrium with  $L_E^1/a > \bar{\nu}$  exists for each parameterization and rational agents do not go short on average, we focus on cases with  $\Delta(a\bar{\nu}) > \Gamma(a\bar{\nu})$  and  $\Delta(L) < \Gamma(L)$  and rule out cases with multiple solutions to  $\Delta(L_E) = \Gamma(L_E)$  (see Proposition 4.4). This condition is called  $EU_\sigma$  (for “existence and uniqueness”). Second, while noise trader expected utility  $E[U(\pi_N)]$  is well defined in the case without noise, it does not generally exist when  $\sigma_\nu^2 > 0$  (as the square root of a potentially negative term, which depends on  $L_E$ , appears in the expression for  $E[U(\pi_N)]$  in the Appendix). We confine attention to parameters for which noise trader expected utility is well defined at the equilibria with and without OC (condition  $BNU_\sigma$ , for “bounded noise trader utility”). Applying these two regularity conditions reduces the number of admissible parameter combinations further to between 61,290 for  $\sigma_\nu = 0.05L/a$  and 17,529 for maximum noise trader demand volatility (where condition  $BNU_\sigma$ , applied after condition  $EU_\sigma$ , is responsible for less than 1,707 additional cases lost).

Table 2 summarizes the results of the Matlab simulation of model  $M_\sigma^j$ . For given values of  $\sigma_\nu$  relative to  $L/a$ , columns 2–4 report the following figures: the number of parameter combinations which satisfy conditions  $U_0$ – $BNU_\sigma$ , the proportion of cases in which the marginal impact of an increase in  $L_E$  on SW  $S^1$  at an equilibrium of  $M_\sigma^1$  is positive, and the proportion of cases such that equilibrium SW is higher without than with OC. The overall conclusions are that noise turns the marginal impact of  $L_E$  on SW at equilibrium positive and the assertion of Proposition 6.2 carries over to model specifications with sizable noise trader shocks.

**RESULT 7.1:** *Consider the parameters in Table 1, with a standard deviation of noise trader demand  $\sigma_\nu$  up to 50 percent of the maximum feasible asset supply  $L/a$ , and let conditions  $U_0$ ,  $EU_\sigma$ , and  $BNU_\sigma$  hold. (i) The marginal impact of an increase in  $L_E$  at equilibrium is most often positive in  $M_\sigma^1$  for  $\sigma_\nu$  up to 20 percent of  $L/a$ . (ii) Equilibrium SW is most often higher in  $M_\sigma^0$  than in  $M_\sigma^1$ .*

SW is higher without than with OC not only in the majority of cases but also by a large amount. For  $\sigma_\nu$  up

Table 2: Matlab simulation of  $M_\sigma^j$ 

$\frac{\sigma_\nu}{L/a}$	# cases	$\frac{dS^1(L_E^1)}{dL_E} > 0$	$S^0(L_E^0) > S^1(L_E^1)$
0.001	68,112	99.63%	100.00%
0.01	67,926	99.63%	100.00%
0.05	61,290	98.89%	100.00%
0.1	52,287	97.72%	99.95%
0.2	39,180	89.04%	99.54%
0.5	17,529	17.75%	95.29%

to  $0.2L/a$ , the ratio of the two SW levels is greater than ten on average. This huge difference is due to the fact that equilibrium SW without OC is very close to its maximum, which is far greater than equilibrium SW with OC.

### Labor market

Next, consider the model with full employment  $F_\sigma^j$ . The model parameters which also appear in  $M_\sigma^j$  take on the values in Table 1. Further parameters are summarized in the upper part of Table 3.

Table 3: Parameter values in the simulations of $F_\sigma^j$ and $U_\sigma^j$			
parameter	values	multiple of	
$F_\sigma^j$	$\bar{s}' - \frac{\rho\sigma_s^2}{2a}$	0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99	$\frac{\rho\sigma_s^2}{L+M} \left(\frac{L}{a} - \bar{\nu}\right) + \bar{s}' - \bar{s}''$
	$b$	0.10, 0.25, 0.40, 0.55	1
	$A$	0.25, 0.5, 0.75, 1	$\frac{\rho\sigma_s^2}{2ab} \left(\frac{\bar{\nu}}{M}\right)^{1-b}$
$U_\sigma^j$	$\bar{s} - \frac{\rho\sigma_s^2}{2a}$	0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99	$\frac{\rho\sigma_s^2}{L+M} \left(\frac{L}{a} - \bar{\nu}\right)$
	$A$	0.25, 0.5, 0.75, 1	$\frac{\rho\sigma_s^2}{2ab} \left(\frac{L}{aM}\right)^{1-b}$
	$D$	$\tilde{W} - \frac{1}{\rho} \ln \left(1 + \rho b \tilde{W}\right)$	1

Consider  $F_0^j$ . Let  $\bar{s}' (= bA(M/\bar{\nu})^{1-b} + \hat{s})$  and  $\bar{s}'' (= bA(aM/L)^{1-b} + \hat{s})$  denote the values of  $\bar{s}$  given by (12) for  $L_E = a\bar{\nu}$  and  $L_E = L$ , respectively. To make sure that an equilibrium at which no-one goes short exists for  $\sigma_\nu = 0$ ,  $\Delta_0(L_E)$  has to be positive for  $L_E = a\bar{\nu}$  and  $\bar{s} = \bar{s}'$  and negative for  $L_E = L$  and  $\bar{s} = \bar{s}''$  (see Proposition 5.1). The choice of  $\hat{s}$  and, hence,  $\bar{s}'$  and  $\bar{s}''$  as in the first row of the  $F_\sigma^j$  part of Table 3 makes sure that this is the case:  $\Delta_0(a\bar{\nu}) > 0 > \Delta_0(L)$  (from (7)).

The production function is  $F(m) = Am^{1-b}$ . To have a labor elasticity of about 0.75, we consider  $b$  around 0.25.  $A$  is set such that the two terms which add up to expected firm profit  $\bar{s} = Abm^{1-b} + \hat{s}$  are of comparable size. From (7) and (12), the two terms are equally large at an equilibrium with  $\sigma_\nu^2 = 0$  and  $L_E^1/a = \bar{\nu}$  for  $A$  equal to 0.5 times the term in the final column in Table 3. Accordingly we consider values of  $A$  scattered around this value. (In the simulations, the two terms account for 37 percent and 63 percent, respectively, of  $\bar{s}$  on average for  $\sigma_\nu = 0.05L/a$ , for instance.)

The total number of parameter combinations is 1,399,680 for each given  $\sigma_\nu$ . As before, we require  $\Gamma_0(L) > \Delta_0^0(L)$  to rule out multiplicity in the noiseless case (condition  $U_0$ ), which leaves us with 1,159,134 combinations. In order to have a unique equilibrium and no short sales on average in the noisy case too, we

focus on parameters such that  $\Delta(L_E)$  with  $\bar{s}$  given by (12) is larger than  $\Gamma(L_E)$  for  $L_E = a\bar{\nu}$  and vice versa for  $L_E = L$  and rule out cases with multiple intersections of  $\Delta(L_E)$  with  $\bar{s}$  given by (12) and  $\Gamma(L_E)$  (condition  $EU_\sigma$ ). As in the basic model, we omit parameter combinations which yield unbounded noise trader expected utility at equilibrium (condition  $BNU_\sigma$ ). Applying  $EU_\sigma$  and  $BNU_\sigma$  leaves us with no less than 971,974 cases for  $\sigma_\nu$  up to  $0.2L/a$  and 661,087 cases for  $\sigma_\nu = 0.5L/a$ .

Columns 2 and 3 of Table 4 report the results of the simulations for under- versus over-investment in entrepreneurship. The results are similar to the basic model without a labor market: the marginal impact of an increase in  $L_E$  at equilibrium is most often positive except for the maximum admissible value for  $\sigma_\nu$ , and equilibrium SW is almost always higher without than with OC.

Table 4: Matlab simulation of  $F_\sigma^j$  and  $U_\sigma^j$

$\frac{\sigma_\nu}{L/a}$	$F_\sigma^j$		$U_\sigma^j$	
	$\frac{dS^1(L_E^1)}{dL_E} > 0$	$S^0(L_E^0) > S^1(L_E^1)$	$\frac{dS^1(L_E^1)}{dL_E} > 0$	$S^0(L_E^0) > S^1(L_E^1)$
0.001	99.62%	100.00%	100.00%	100.00%
0.01	99.62%	100.00%	100.00%	100.00%
0.05	91.85%	99.99%	99.99%	100.00%
0.1	84.92%	99.78%	99.93%	99.97%
0.2	74.26%	98.80%	99.63%	99.67%
0.5	23.00%	93.51%	97.00%	97.89%

Finally, consider model  $U_\sigma^j$ . The parameters are as in the model with full employment unless stated otherwise in the bottom part of Table 3.  $\bar{s}$  is given by (14) and independent of  $L_E$ .  $\hat{s}$  is chosen such that, similarly as in model  $F_\sigma^j$ ,  $\bar{s}$  is as in the table, so that  $L_E^1$  is in between  $a\bar{\nu}$  and  $L$  for  $\sigma_\nu = 0$ . Similarly as before,  $A$  is chosen around the value (0.5 times the expression in the final column) for which  $Ab(aM/L)^{1-b}$  and the expected macro shock  $\hat{s}$  contribute equal amounts to expected firm profit  $\bar{s}$  for  $\sigma_\nu^2 = 0$  and  $L_E^1/a = \bar{\nu}$ .

Workers' disutility of effort  $D$  is chosen such that labor demand equals labor supply exactly if all hipos become entrepreneurs. This condition,  $JC_\sigma$  say (for "job creation"), ensures that the job creation effect of entrepreneurship is operative at equilibrium: employment rises when  $L_E$  rises. The condition is implemented as follows. Profit maximization and  $JC_\sigma$  jointly imply  $A(1-b)(aM/L)^{-b} = \tilde{W}$ . For each single case (i.e., for given values of  $A$ ,  $b$ ,  $a$ ,  $M$ , and  $L$ ), we set the disutility of work  $D$  such that the wage rate which maximizes workers' expected utility (which also depends on  $\rho$  and  $b$ ) coincides with this value of  $\tilde{W}$ . We maintain conditions  $U_0$ ,  $EU_\sigma$ , and  $BNU_\sigma$ .

Out of a total of 1,399,680 parameter combinations for each value of  $\sigma_\nu$ , 1,089,792 satisfy condition  $U_0$ . Applying conditions  $EU_\sigma$  and  $BNU_\sigma$  reduces the number of cases further, to no less than 836,500 for  $\sigma_\nu$  up to  $0.1L/a$ , to 626,880 for  $\sigma_\nu = 0.2L/a$ , and to 280,464 for  $\sigma_\nu = 0.5L/a$ . As is clear from the analytical treatment of the noiseless case, the marginal impact of entrepreneurship is positive for an even bigger proportion of cases than in the frictionless case. To sum up:

**RESULT 7.2:** *Consider the parameters in Tables 1 and 3, with a standard deviation of noise trader demand  $\sigma_\nu$  up to 50 percent of the maximum feasible asset supply  $L/a$ , and let conditions  $U_0$ ,  $EU_\sigma$ ,  $BNU_\sigma$ , and, in*

$U_\sigma^1$ ,  $JC_\sigma$  hold. (i) The marginal impact of an increase in  $L_E$  at equilibrium is most often positive in  $F_\sigma^1$  and almost universally positive in  $U_\sigma^1$  for  $\sigma_\nu$  up to 20 percent of  $L/a$ . (ii) Equilibrium SW is most often higher in  $F_\sigma^0$  than in  $F_\sigma^1$  and in  $U_\sigma^0$  than in  $U_\sigma^1$ .

In sum, the numerical analysis of the case of large noise trader shocks confirms the results of the analytical investigation of the case of small noise trader shocks: there tends to be too much, rather than too little, financial trading in the augmented GS model.

### Implementation

The implementation of a constrained-efficient allocation is simple in theory. A second-best allocation with no professional trading and employment in entrepreneurship  $L'_E$  can be achieved by a (prohibitive) tax  $t_D > \Gamma(L)/\rho$  on dealers and a tax  $t_E = \Delta^0(L'_E)/\rho$  on entrepreneurs. Generally, an equilibrium with  $L'_E$  entrepreneurs and  $L - L'_E$  dealers is achieved with a tax differential  $t_D - t_E = [\Gamma(L'_E) - \Delta(L'_E)]/\rho$  (details in the Appendix).

### 8. Conclusion

If the sole benefit of trading is to increase informational efficiency à la GS, it is unlikely that finance, as opposed to entrepreneurship, attracts too little talent. The increased price informativeness brought about by professional trading is not necessarily beneficial on net. And even if it is, the benefits tend to be outweighed by the opportunity cost in terms of foregone output and lower wages or lower employment in manufacturing. This lends some support to popular concerns (several of them cited in the Introduction) that financial trading attracts too much, rather than too little, talent. The main policy conclusion is that, insofar as policy has an impact on the allocation of talent, it should make jobs in entrepreneurial activities rather than in finance more attractive, or at least not subsidize, explicitly or implicitly, financial institutions specialized in professional trading. Like any other model, the GS model provides only a partial view of financial trading and economic welfare. A comprehensive assessment also has to take into account the incentive effects of professional trading on issuers of assets (see Bolton et al., 2016, Section III), the impact of information contained in asset prices on firm decisions, and the opportunity to link pay to performance (see Bond et al., 2012). The GS model itself, however, does not lend additional support to studies which attribute a beneficial role to trading but points to a downside that other effects would have to outweigh.

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## Appendix

### Demange-Laroque lemma:

The derivations below make use of Lemma 1 in Demange and Laroque (1995, p. 252), which says that for jointly normal random variables  $x$  and  $y$ ,

$$\mathbb{E} [\exp (x - y^2)] = \frac{\exp \left\{ \mathbb{E}(x) + \frac{1}{2} \text{var}(x) - \frac{[\mathbb{E}(y) + \text{cov}(x, y)]^2}{1 + 2 \text{var}(y)} \right\}}{[1 + 2 \text{var}(y)]^{\frac{1}{2}}}. \quad (\text{A.1})$$

### Price function (1):

The optimal investment levels are

$$I_E = I_M = \frac{\mathbb{E}(\theta|P) - P}{\rho \text{var}(\theta|P)}, \quad I_D = \frac{s - P}{\rho \sigma_\varepsilon^2}. \quad (\text{A.2})$$

To see this, use (A.1) with  $y$  identically equal to zero to get

$$\mathbb{E}[U(\pi_E)|P] = -\exp \left( -\rho \left\{ e_L + \frac{P}{a} + [\mathbb{E}(\theta|P) - P] I_E \right\} + \frac{\rho^2}{2} \text{var}(\theta|P) I_E^2 \right). \quad (\text{A.3})$$

Maximizing with respect to  $I_E$  yields the first equation in (A.2). Given that passive investors trade on the same information as entrepreneurs,  $I_M = I_E$  follows from the fact that optimum investment does not depend on endowments. Similarly, using  $\mathbb{E}(\theta|s) = s$  and  $\text{var}(\theta|s) = \sigma_\varepsilon^2$ ,

$$\mathbb{E}[U(\pi_D)|s, P] = -\exp \left\{ -\rho [e_L + (s - P) I_D] + \frac{\rho^2}{2} \sigma_\varepsilon^2 I_D^2 \right\}, \quad (\text{A.4})$$

and maximization with respect to  $I_D$  yields the expression for  $I_D$  in (A.2).

Substituting the optimum investments in (A.2) into the market clearing condition for the risky asset yields the price function

$$P = \frac{\frac{L - L_E}{\rho \sigma_\varepsilon^2} s + \frac{L_E + M}{\rho \text{var}(\theta|P)} \mathbb{E}(\theta|P) - \left( \frac{L_E}{a} - \nu \right)}{\frac{L - L_E}{\rho \sigma_\varepsilon^2} + \frac{L_E + M}{\rho \text{var}(\theta|P)}},$$

which, using the definition of  $w$  in (2), can be rewritten as (1).

From the updating rule for the mean of a normal random variable,

$$\mathbb{E}(\theta|w) = \bar{s} + \frac{\text{cov}(\theta, w)}{\text{var}(w)} [w - \mathbb{E}(w)]. \quad (\text{A.5})$$

From  $\text{var}(\theta|w) = \text{var}(s|w) + \sigma_\varepsilon^2$  and the updating rule  $\text{var}(s|w) = \sigma_s^2 - [\text{cov}(s, w)]^2 / \text{var}(w)$ , it follows that

$$\text{var}(\theta|w) = \sigma_s^2 - \frac{[\text{cov}(s, w)]^2}{\text{var}(w)} + \sigma_\varepsilon^2 \quad (\text{A.6})$$

is non-random. The conditional payoff variance  $\text{var}(\theta|w)$  in (A.6) is a measure of the informational efficiency of the asset market: a lower value of  $\text{var}(\theta|w)$  means that the asset price helps make a more accurate

prediction of macroeconomic fundamentals. The fact that  $\text{var}(\theta|w)$  is lower than the unconditional variance  $\sigma_s^2 + \sigma_\varepsilon^2$  whenever  $L_E < L$  means that the information contained in  $P$  (or  $w$ ) is valuable to agents who do not know  $s$ .

**Expected utilities (4)–(6):**

Substituting for  $I_E$  from (A.2) into (A.3) yields

$$\mathbb{E}[U(\pi_E)|P] = -\exp\left\{-\rho e_L - \rho \frac{P}{a} - \frac{[\mathbb{E}(\theta|P) - P]^2}{\text{var}(\theta|P)} + \frac{1}{2} \frac{[\mathbb{E}(\theta|P) - P]^2}{\text{var}(\theta|P)}\right\}.$$

Collecting terms and using the definition of  $z$  yields

$$\mathbb{E}[U(\pi_E)|P] = -\exp(-\rho e_L) \exp\left(-\rho \frac{P}{a} - z^2\right).$$

From (1), (2), (A.5), and (A.6),  $z$  is a linear function of  $w$ . It can be shown that the linear dependence is negative, so that  $\text{cov}(P, z) = -[\text{var}(P) \text{var}(z)]^{1/2}$ . Using the law of iterated expectations, we obtain

$$\mathbb{E}[U(\pi_E)] = -\exp(-\rho e_L) \mathbb{E}\left[\exp\left(-\rho \frac{P}{a} - z^2\right)\right]. \quad (\text{A.7})$$

Since  $P$  and  $z$  are normal, we can apply the Demange-Laroque lemma (A.1) to get

$$\mathbb{E}\left[\exp\left(-\rho \frac{P}{a} - z^2\right)\right] = \frac{\exp\left\{\mathbb{E}\left(-\rho \frac{P}{a}\right) + \frac{1}{2} \text{var}\left(-\rho \frac{P}{a}\right) - \frac{[\mathbb{E}(z) + \text{cov}\left(-\rho \frac{P}{a}, z\right)]^2}{1 + 2 \text{var}(z)}\right\}}{[1 + 2 \text{var}(z)]^{\frac{1}{2}}}.$$

Substituting this into (A.7) and rearranging terms gives

$$\mathbb{E}[U(\pi_E)] = -\exp(-\rho e_L) \frac{\exp\left\{-\frac{\rho}{a} \mathbb{E}(P) + \frac{1}{2} \left(\frac{\rho}{a}\right)^2 \text{var}(P) - \frac{[\mathbb{E}(z) - \frac{\rho}{a} \text{cov}(P, z)]^2}{1 + 2 \text{var}(z)}\right\}}{[1 + 2 \text{var}(z)]^{\frac{1}{2}}},$$

which can be rewritten as (4).

A passive investor's expected utility (5) is obtained analogously. The terms containing  $P/a$  drop out.

Substituting for  $I_D$  from (A.2) into (A.4) yields

$$\mathbb{E}[U(\pi_D)|s, P] = -\exp\left\{-\rho e_L - \left[\frac{s - P}{(2\sigma_\varepsilon^2)^{\frac{1}{2}}}\right]^2\right\}. \quad (\text{A.8})$$

Let  $y \equiv (s - P)/(2\sigma_\varepsilon^2)^{1/2}$ . Notice that  $\mathbb{E}(s|P) = \mathbb{E}(\theta|P)$  and  $\text{var}(s|P) = \text{var}(\theta|P) - \sigma_\varepsilon^2$ , so that  $\mathbb{E}(y|P) = [\mathbb{E}(\theta|P) - P]/(2\sigma_\varepsilon^2)^{1/2}$  and  $\text{var}(y|P) = [\text{var}(\theta|P) - \sigma_\varepsilon^2]/(2\sigma_\varepsilon^2)$ . Applying the law of iterated expectations to

(A.8) and using (A.1), we obtain

$$E[U(\pi_D)|P] = -\exp(-\rho e_L) \frac{\exp\left\{-\frac{[\mathbb{E}(\theta|P)-P]^2}{2\sigma_\varepsilon^2}\right\}}{\left[\frac{\text{var}(\theta|P)}{\sigma_\varepsilon^2}\right]^{\frac{1}{2}}}.$$

Rearranging terms and using the definition of  $z$  yields

$$E[U(\pi_D)|P] = -\exp(-\rho e_L) \left[\frac{\sigma_\varepsilon^2}{\text{var}(\theta|w)}\right]^{\frac{1}{2}} \exp(-z^2).$$

Taking expectations, again making use of the law of iterated expectations and (A.1), yields

$$E[U(\pi_D)] = -\exp(-\rho e_L) \left[\frac{\sigma_\varepsilon^2}{\text{var}(\theta|w)}\right]^{\frac{1}{2}} \frac{\exp\left\{-\frac{[\mathbb{E}(z)]^2}{1+2\text{var}(z)}\right\}}{[1+2\text{var}(z)]^{\frac{1}{2}}},$$

which can be rewritten as (6).

#### Equilibrium of $\mathbf{M}_0^1$ with $\mathbf{L}_E < \mathbf{L}$ :

Since noise trader demand is non-random,  $w$  defined in (2) reveals  $s$  to entrepreneurs and passive investors. From (2) and (A.5),  $E(\theta|w) = s$ . From (A.6),  $\text{var}(\theta|w) = \sigma_\varepsilon^2$ . So  $I_E$  equals  $I_D$ , as given by (A.2). The price function (1) simplifies to

$$P = s - \frac{\rho\sigma_\varepsilon^2}{L+M} \left(\frac{L_E}{a} - \bar{v}\right). \quad (\text{A.9})$$

From (3), since  $E(\theta|w) - P = s - P$  is non-random,  $z$  is non-random, even though both  $s$  and  $P$  are risky. In fact, from (3) and (A.9),

$$z = \left(\frac{\sigma_\varepsilon^2}{2}\right)^{\frac{1}{2}} \frac{\rho}{L+M} \left(\frac{L_E}{a} - \bar{v}\right). \quad (\text{A.10})$$

Proposition 4.1 follows from the definition of  $\Delta_0(L_E)$  in (7). The following remarks characterize the formal properties of the equilibrium of  $\mathbf{M}_0^1$  (existence, uniqueness, comparative statics).

*Remark A.4.1.1.* Having determined the equilibrium value of  $L_E$ , the other variables in the definition of equilibrium of  $\mathbf{M}_0^1$  are obtained as follows:  $I_E = I_D$ ,  $I_M = I_D$ ,  $I_D$  is given by (A.2), and  $P$  is given by (A.9). The analogous remark applies to subsequent propositions.

*Remark A.4.1.2.* There is an equilibrium with  $0 < L_E^1 < L$  if  $\Delta_0(0) > 0$  and  $\Delta_0(L_E) < 0$  for  $L_E$  large enough. As  $\Delta_0(L_E)$  is monotonically decreasing, this type of equilibrium is unique. (However, an equilibrium without dealers may coexist, as will be seen below.) Clearly, the condition of the proposition is also necessary: if it is not satisfied, then an equilibrium of  $\mathbf{M}_0^1$  with  $L_E < L$  does not exist.

*Remark A.4.1.3.* The fact that, other than in GS, a fully revealing REE possibly exists in the absence of noise is due to the fact that the only cost of becoming a dealer is the opportunity cost of not becoming an entrepreneur, which is zero at equilibrium. This type of equilibrium would vanish if there was a physical cost of becoming a dealer.

*Remark A.4.1.4.* The allocation is indeterminate in that dealers would be equally well-off as passive investors. However, since investments and the asset price are the same if only a subset of non-zero measure of the  $L-L_E$

non-entrepreneurs become dealers,  $L_E$  and equilibrium  $(L_E, I_E, I_D, I_M, P)$  are uniquely determined. In this sense the focus on equilibria at which no hipo becomes a passive investor is without loss of generality.

*Remark A.4.1.5.* An equilibrium with  $L_E^1$  entrepreneurs has the expected comparative statics properties: whatever raises  $\text{GE}$  ( $= \Delta_0(L_E)$ ) raises the equilibrium mass of entrepreneurs.

*Remark A.4.1.6.* That neither dealers nor entrepreneurs earn any rents at equilibrium (as  $\text{GI} = \text{GE} = 0$ ) is due to the fact that the asset price is fully revealing. Positive and sizable rents at equilibrium will occur in the model with stochastic noise trader demand (see Remarks A.4.4.5 and A.7.1.3 below).

### Equilibrium of $\mathbf{M}_0^1$ with $\mathbf{L}_E = \mathbf{L}$ :

Since  $w$  is non-stochastic,  $\text{E}(\theta|w) = \bar{s}$  and  $\text{var}(\theta|w) = \sigma_s^2 + \sigma_\varepsilon^2$ . Agents' asset demand is

$$I_E = I_M = \frac{\bar{s} - P}{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}. \quad (\text{A.11})$$

Setting asset demand equal to supply  $L_E/a$  yields  $P = (a/\rho)\Delta_0^0(L_E)$  with  $\Delta_0^0(L_E)$  given by (8). From (3),  $z$  is also non-random:

$$z = \frac{\bar{s} - P}{[2(\sigma_s^2 + \sigma_\varepsilon^2)]^{\frac{1}{2}}}. \quad (\text{A.12})$$

An entrepreneur's expected utility (4) is

$$-\log\{-\text{E}[U(\pi_E)]\} = \rho e_L + \frac{\rho}{a}P + z^2. \quad (\text{A.13})$$

The second and third terms on the right-hand side represent the gains from entrepreneurship  $\text{GE}$  and the gains from trading  $\text{GT}_E$ , respectively. A passive investor's expected utility is  $-\log\{-\text{E}[U(\pi_M)]\} = \rho e_M + z^2$ . The gains from trading  $\text{GT}_M$  are the same as for entrepreneurs.

Consider a single hipo who contemplates whether it pays to become a dealer when no-one else does. His expected utility conditional on  $s$  is given by (A.8). Taking expectations, using (A.1) with  $x$  identically equal to zero,  $I_D = (s - P)/(\rho\sigma_\varepsilon^2)$ , the fact that  $P$  is safe, and (A.12) yields his unconditional expected utility:

$$-\log\{-\text{E}[U(\pi_D)]\} = \rho e_L + \frac{1}{2} \log\left(\frac{\sigma_s^2 + \sigma_\varepsilon^2}{\sigma_\varepsilon^2}\right) + z^2.$$

Using the definition of  $\Gamma_0(L)$  in the main text, it does not pay to become a dealer if  $\Delta_0^0(L)$  is no less than  $\Gamma_0(L)$ .

*Remark A.4.2.1.* The condition of the proposition is also necessary: if it is not satisfied, then an equilibrium of  $\mathbf{M}_0^1$  with  $L_E = L$  does not exist.

*Remark A.4.2.2.* The assertion of Proposition 4.2 is similar as in GS: an equilibrium without dealers exists if the gains from trading  $\text{GI}$  ( $= \Gamma_0(L)$ ) are sufficiently small (see the upper right filled circle in the left panel of Figure 2). In GS the gains from trading are compared to the physical cost of information gathering. Here they are compared to the benefits of being an entrepreneur  $\text{GE}$  ( $= \Delta_0^0(L)$ ).

*Remark A.4.2.3.* Since the gains from entrepreneurship are positive, this type of equilibrium would survive the introduction of a sufficiently small positive physical cost (no greater than  $\Delta_0^0(L)/\rho$ ) of becoming an entrepreneur (cf. Remark A.4.1.3).

*Remark A.4.2.4.* There is at most one equilibrium with  $L_E < L$  (cf. Remark A.4.1.2), and there is at most one equilibrium with  $L_E = L$ . But, as mentioned in the main text, the two types of equilibria can coexist.

*Remark A.4.2.5.* As explained in the main text, it can also happen that an equilibrium fails to exist (the GS non-existence result).

### Equilibrium of $\mathbf{M}_0^0$ :

*Remark A.4.3.1.*  $\Delta_0^0(0) > 0$  ensures existence of equilibrium. As  $\Delta_0^0(L_E)$  is monotonically decreasing, equilibrium is unique. The equilibrium mass of entrepreneurs  $L_E^0$  has the expected comparative statics properties. There are no positive rents for entrepreneurs at an equilibrium with  $L_E^0 < L$ .

*Remark A.4.3.2.* Suppose there are equilibria with  $L_E^1$  and  $L_E^0$  (both less than  $L$ ) entrepreneurs with and without OC, respectively. Then  $L_E^0 > L_E^1$  if (9) holds.

*Proof:*  $L_E^0 < L$  implies

$$\bar{s} - \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{L + M} \left( \frac{L}{a} - \bar{\nu} \right) < 0.$$

Together with condition (9), it follows that

$$\bar{s} - \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{2a} < 0. \quad (\text{A.14})$$

From (7) and (8),

$$\Delta_0^0(L_E^1) = -\frac{\rho \sigma_s^2}{a \sigma_\varepsilon^2} \left[ \bar{s} - \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{2a} \right].$$

Suppose  $L_E^0 \leq L_E^1$ . Since  $\Delta_0^0(L_E)$  is a decreasing function, this implies  $\Delta_0^0(L_E^1) \leq 0$ . This contradicts (A.14), so  $L_E^0 > L_E^1$ . ///

### Equilibrium of $\mathbf{M}_\sigma^1$ :

Equations (1), (2), (A.5), and (A.6) determine the moments and the covariance of  $P$  and  $z$  as continuous functions of  $L_E$  alone. To see this, let

$$\alpha \equiv \frac{L - L_E}{\rho \sigma_\varepsilon^2}, \quad \beta \equiv \frac{L_E + M}{\rho \text{var}(\theta|w)}, \quad \gamma \equiv \frac{1}{\alpha^2 \sigma_s^2 + \sigma_\nu^2}. \quad (\text{A.15})$$

Then,

$$\text{var}(\theta|w) = \gamma \sigma_s^2 \sigma_\nu^2 + \sigma_\varepsilon^2 \quad (\text{A.16})$$

$$\text{E}(P) = \bar{s} - \frac{\frac{L_E}{a} - \bar{\nu}}{\alpha + \beta} \quad (\text{A.17})$$

$$\text{var}(P) = \frac{1}{\gamma} \left( \frac{1 + \alpha\beta\gamma\sigma_s^2}{\alpha + \beta} \right)^2 \quad (\text{A.18})$$

$$\text{E}(z) = \frac{\frac{L_E}{a} - \bar{\nu}}{(\alpha + \beta) [2(\gamma\sigma_s^2\sigma_\nu^2 + \sigma_\varepsilon^2)]^{\frac{1}{2}}} \quad (\text{A.19})$$

$$\text{var}(z) = \frac{\gamma (\sigma_\nu^2)^2}{(\alpha + \beta)^2 2(\gamma\sigma_s^2\sigma_\nu^2 + \sigma_\varepsilon^2)} \quad (\text{A.20})$$

$$\text{cov}(P, z) = -\frac{(1 + \alpha\beta\gamma\sigma_s^2)\sigma_\nu^2}{(\alpha + \beta)^2 [2(\gamma\sigma_s^2\sigma_\nu^2 + \sigma_\varepsilon^2)]^{\frac{1}{2}}}. \quad (\text{A.21})$$

Note that  $\bar{s}$  affects only  $E(P)$ . For future reference, also notice that the right-hand sides of (A.16)–(A.21) are homogeneous of degree zero in  $\sigma_\nu$ ,  $\bar{\nu}$ ,  $L$ ,  $M$ ,  $N$ , and  $L_E$  jointly.

By definition,  $w = \alpha s + \nu$ , so  $\text{var}(w) = \alpha^2 \sigma_s^2 + \sigma_\nu^2$  and  $\text{cov}(s, w) = \alpha \sigma_s^2$ . Substituting this into (A.6) yields

$$\text{var}(\theta|w) = \sigma_\varepsilon^2 \left( 1 - \frac{\alpha^2 \sigma_s^2}{\alpha^2 \sigma_s^2 + \sigma_\nu^2} \right) + \sigma_\varepsilon^2.$$

Equation (A.16) follows from the definition of  $\gamma$  in (A.15). From the definition of  $\alpha$  in (A.15), it follows that  $\text{var}(\theta|w)$  decreases when  $L_E$  increases. That is, an increase in the mass of dealers increases informational efficiency.  $\text{var}(\theta|w)$  converges to  $\sigma_\varepsilon^2$  as  $\sigma_\nu^2$  goes to zero.

According to the updating rule for the mean of a normal random variable,  $E(\theta|w) = E(\theta) + [\text{cov}(\theta, w)/\text{var}(w)][w - E(w)]$ . Using  $E(\theta) = \bar{s}$ ,  $\text{var}(w) = \alpha^2 \sigma_s^2 + \sigma_\nu^2$ ,  $\text{cov}(\theta, w) = \alpha \sigma_s^2$ , and the definitions of  $w$ ,  $\alpha$ , and  $\gamma$ ,

$$E(\theta|w) = \bar{s} + \alpha \gamma \sigma_s^2 [\alpha(s - \bar{s}) + \nu - \bar{\nu}]. \quad (\text{A.22})$$

This can be used to rewrite (1) as

$$P = \frac{\alpha s + \nu + \beta \left\{ \bar{s} + \alpha \gamma \sigma_s^2 [\alpha(s - \bar{s}) + \nu - \bar{\nu}] \right\} - \frac{L_E}{a}}{\alpha + \beta}$$

or, rearranging terms,

$$P = \bar{s} + \frac{(1 + \alpha \beta \gamma \sigma_s^2) [\alpha(s - \bar{s}) + \nu - \bar{\nu}] - \left( \frac{L_E}{a} - \bar{\nu} \right)}{\alpha + \beta}. \quad (\text{A.23})$$

Equation (A.17) follows upon taking expectations.

The variance of  $P$  is

$$\text{var}(P) = \frac{(1 + \alpha \beta \gamma \sigma_s^2)^2 (\alpha^2 \sigma_s^2 + \sigma_\nu^2)}{(\alpha + \beta)^2}.$$

Using the definition of  $\gamma$ , we obtain (A.18).

Substituting  $E(\theta|w)$  from (A.22) and  $P$  from (A.23) into the definition of  $z$  yields

$$z = \frac{\bar{s} + \alpha \gamma \sigma_s^2 [\alpha(s - \bar{s}) + \nu - \bar{\nu}] - \bar{s} - \frac{(1 + \alpha \beta \gamma \sigma_s^2) [\alpha(s - \bar{s}) + \nu - \bar{\nu}] - \left( \frac{L_E}{a} - \bar{\nu} \right)}{\alpha + \beta}}{[2(\gamma \sigma_s^2 \sigma_\nu^2 + \sigma_\varepsilon^2)]^{\frac{1}{2}}}.$$

Simplifying terms, using  $1 - \alpha^2 \gamma \sigma_s^2 = \gamma \sigma_\nu^2$ , we get

$$z = \frac{-\gamma \sigma_\nu^2 [\alpha(s - \bar{s}) + \nu - \bar{\nu}] + \frac{L_E}{a} - \bar{\nu}}{(\alpha + \beta) [2(\gamma \sigma_s^2 \sigma_\nu^2 + \sigma_\varepsilon^2)]^{\frac{1}{2}}}. \quad (\text{A.24})$$

Taking expectations yields (A.19). Clearly,  $E(z) \geq 0$  if  $L_E/a \geq \bar{\nu}$ .

The variance of  $z$  is

$$\text{var}(z) = \frac{\gamma^2 (\sigma_\nu^2)^2 (\alpha^2 \sigma_s^2 + \sigma_\nu^2)}{(\alpha + \beta)^2 2 (\gamma \sigma_s^2 \sigma_\nu^2 + \sigma_\varepsilon^2)}.$$

Equation (A.20) follows from the definition of  $\gamma$ .

From (A.23) and (A.24),

$$\text{cov}(P, z) = \frac{1 + \alpha\beta\gamma\sigma_s^2}{\alpha + \beta} \frac{-\gamma\sigma_\nu^2}{(\alpha + \beta) [2(\gamma\sigma_s^2\sigma_\nu^2 + \sigma_\varepsilon^2)]^{\frac{1}{2}}} (\alpha^2\sigma_s^2 + \sigma_\nu^2).$$

Equation (A.21) follows from the definition of  $\gamma$ . Using (A.18) and (A.20), (A.21) can be rewritten as  $\text{cov}(P, z) = -[\text{var}(P) \text{var}(z)]^{1/2}$ , which proves that  $P$  and  $z$  are perfectly negatively correlated.

Using the definitions of  $\Delta(L_E)$  in (10) and  $\Gamma(L_E)$  in (11), we obtain Proposition 4.4.

*Remark A.4.4.1.*  $\Delta(0) > \Gamma(0)$  is sufficient to ensure existence of equilibrium. Together with continuity of  $\Delta(L_E)$  and  $\Gamma(L_E)$ , this condition implies that either there is  $L_E^1 < L$  such that  $\Delta(L_E^1) = \Gamma(L_E^1)$  or else  $\Delta(L) \geq \Gamma(L)$ . This is in line with the result that the GS non-existence result (cf. Remark A.4.2.5) vanishes in the presence of noise.

*Remark A.4.4.2.* There exist parameterizations of the model such that there are multiple intersections of the functions  $\Delta(L_E)$  and  $\Gamma(L_E)$ , so multiplicity of equilibria cannot be ruled out (cf. Remark A.4.2.4 above and Remark A.4.6.2 below).

*Remark A.4.4.3.* An equilibrium  $(L_E^1, I_E, I_D, I_M, P)$  with a positive mass of dealers would also be an equilibrium in the presence of a physical cost of not being a passive investor no greater than  $\Delta(L_E^1)/\rho$ .

*Remark A.4.4.4.* If  $\Delta(L_E)$  intersects  $\Gamma(L_E)$  from above, then the corresponding equilibrium of  $M_\sigma^1$  has the expected comparative statics properties. For instance, a decrease in firms' expected profitability  $\bar{s}$  reduces entrepreneurial activity, raises the number of dealers, and fosters informational efficiency (i.e., reduces  $\text{var}(\theta|w)$ ).

*Remark A.4.4.5.* Hipos earn positive rents  $\text{GE} + \text{GT}_E - \text{GT}_M (= \Delta(L_E) > 0)$  compared to passive investors at equilibrium either as entrepreneurs or, if  $L_E < L$ , as dealers. The numerical analysis below shows that these rents can be sizable (see Remark A.7.1.3).

### Equilibrium of $M_\sigma^0$ :

In  $M_\sigma^0$ , since no-one gathers information about  $s$ , the price is uninformative:  $E(\theta|P) = \bar{s}$  and  $\text{var}(\theta|P) = \sigma_s^2 + \sigma_\varepsilon^2$ . Entrepreneurs' optimal investment level is given by (A.11) and the price function is given by (8) with  $\nu$  instead of  $\bar{\nu}$ .

The first and second moments of  $P$  and  $z$  without OC can be expressed as functions of  $L_E$  alone:

$$E(P) = \bar{s} - \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{L + M} \left( \frac{L_E}{a} - \bar{\nu} \right) \quad (\text{A.25})$$

$$\text{var}(P) = \left( \frac{\rho}{L + M} \right)^2 (\sigma_s^2 + \sigma_\varepsilon^2)^2 \sigma_\nu^2 \quad (\text{A.26})$$

$$E(z) = \frac{\rho}{L + M} \left( \frac{\sigma_s^2 + \sigma_\varepsilon^2}{2} \right)^{\frac{1}{2}} \left( \frac{L_E}{a} - \bar{\nu} \right) \quad (\text{A.27})$$

$$\text{var}(z) = \left( \frac{\rho}{L + M} \right)^2 \frac{(\sigma_s^2 + \sigma_\varepsilon^2)}{2} \sigma_\nu^2 \quad (\text{A.28})$$

$$\text{cov}(P, z) = - \left( \frac{\rho}{L + M} \right)^2 \frac{(\sigma_s^2 + \sigma_\varepsilon^2)^{\frac{3}{2}}}{2^{\frac{1}{2}}} \sigma_\nu^2. \quad (\text{A.29})$$

Clearly, the right-hand sides of (A.25)–(A.29) are homogeneous of degree zero in  $\sigma_\nu$ ,  $\bar{\nu}$ ,  $L$ ,  $M$ ,  $N$ , and  $L_E$ .

Equations (A.25) and (A.26) follow immediately from (8).

Inserting  $E(\theta|w) = \bar{s}$  and  $\text{var}(\theta|w) = \sigma_s^2 + \sigma_\varepsilon^2$  into the definition of  $z$  in (3) yields

$$z = \frac{\rho}{L+M} \left( \frac{\sigma_s^2 + \sigma_\varepsilon^2}{2} \right)^{\frac{1}{2}} \left( \frac{L_E}{a} - \nu \right). \quad (\text{A.30})$$

Equations (A.27) and (A.28) follow immediately.

Equations (8) and (A.30) yield (A.29).

Equations (A.25)-(A.29) hold true for all  $L_E \leq L$ . This is because in the absence of OC there is no jump in the informational efficiency of prices at  $L_E = L$ .

It is easily checked that the moments in (A.25)-(A.29) coincide with their counterparts (A.17)-(A.21) for  $L_E = L$ , so that  $\Delta^0(L) = \Delta(L)$ .

Substituting the moments of  $P$  and  $z$  from (A.25)-(A.29) into the right-hand side of (10) gives  $\Delta^0(L_E)$ . Differentiating with respect to  $L_E$  yields  $\Delta^0(L_E)$  is a linear, decreasing function:

$$(\Delta^0)'(L_E) = \left( \frac{\rho}{a} \right)^2 \frac{(\sigma_s^2 + \sigma_\varepsilon^2)}{L+M} \left[ \frac{\left( \frac{\rho}{L+M} \right)^2 (\sigma_s^2 + \sigma_\varepsilon^2) \sigma_\nu^2}{1 + \left( \frac{\rho}{L+M} \right)^2 (\sigma_s^2 + \sigma_\varepsilon^2) \sigma_\nu^2} - 1 \right] < 0. \quad (\text{A.31})$$

Notice that, since  $E(P)$  is a linear function of  $\bar{s}$  and the other moments are independent of  $\bar{s}$ , the equilibrium value of  $L_E$  is a linear function of  $\bar{s}$ .

*Remark A.4.5.1.*  $\Delta^0(L_E)$  is a linear, decreasing function. So  $\Delta^0(0) > 0$  is sufficient for existence of equilibrium, and equilibrium is unique.  $L_E^0$  has the expected comparative statics properties.

*Remark A.4.5.2.*  $\Delta^0(L) = \Delta(L)$  (see Figure 3): whether the non-entrepreneurs are passive investors or dealers does not make a difference for their expected utility compared to a passive investor's as their mass goes to zero.

### Small noise trader shocks

Consider the limiting case  $\sigma_\nu^2 \rightarrow 0$  of  $\mathbf{M}_\sigma^{\mathbf{j}}$ . The following result states that, with or without OC, the equilibrium mass of entrepreneurs is a continuous function of  $\sigma_\nu^2$  at  $\sigma_\nu^2 = 0$ :

**PROPOSITION A.4.6.** *Given an equilibrium of  $\mathbf{M}_0^{\mathbf{j}}$  with mass of entrepreneurs  $L_E^{\mathbf{j}}$ , there is an equilibrium of  $\mathbf{M}_\sigma^{\mathbf{j}}$  with mass of entrepreneurs  $L_E^{\mathbf{j} \prime}$  arbitrarily close to  $L_E^{\mathbf{j}}$  for  $\sigma_\nu^2$  sufficiently small ( $\mathbf{j} = \mathbf{0}, \mathbf{1}$ ).*

The proof relies on the fact that the functions which determine an equilibrium of  $\mathbf{M}_\sigma^{\mathbf{j}}$  (i.e.,  $\Delta(L_E)$  and  $\Gamma(L_E)$  for  $\mathbf{j} = \mathbf{1}$  and  $\Delta^0(L_E)$  for  $\mathbf{j} = \mathbf{0}$ ) are close to the functions that determine an equilibrium of  $\mathbf{M}_0^{\mathbf{j}}$  (i.e.,  $\Delta_0(L_E)$  and the horizontal line at height zero for  $\mathbf{j} = \mathbf{1}$  and  $\Delta_0^0(L_E)$  for  $\mathbf{j} = \mathbf{0}$ ) for  $\sigma_\nu^2$  small (see Figure A.1).

*Proof:* For  $L_E < L$ , inserting the limits of the functions defined in (A.15)-(A.21) as  $\sigma_\nu^2 \rightarrow 0$  into (10) yields  $\Delta(L_E) \rightarrow \Delta_0(L_E)$  pointwise,  $\Gamma(L_E) \rightarrow 0$  pointwise, and  $\Delta^0(L_E) \rightarrow \Delta_0^0(L_E)$  pointwise.

For  $L_E = L$ , we get (A.25)-(A.29) evaluated at  $L_E = L$ . Inserting these expressions into (10) and taking the limit as  $\sigma_\nu^2 \rightarrow 0$  yields  $\Delta^0(L) \rightarrow \Delta_0^0(L)$ , and substitution into (11) and taking the limit  $\sigma_\nu^2 \rightarrow 0$  yields  $\Gamma_0(L)$ . These convergence properties are proved below.



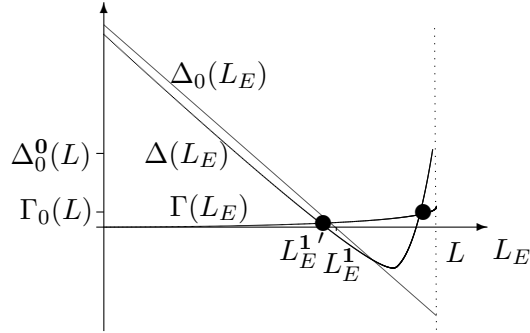


Figure A.1: Equilibrium with small noise trader shocks

Consider first the case with OC. An equilibrium of  $M_0^1$  with  $L_E^1 < L$  entrepreneurs is determined by the requirement that  $\Delta_0(L_E^1) = 0$  (see Proposition 4.1). Since  $\Delta(L_E)$  and  $\Gamma(L_E)$  converge pointwise to  $\Delta_0(L_E)$  and zero, respectively, for all  $L_E < L$ , there is  $L_E^{1'}$  arbitrarily close to  $L_E^1$  such that  $\Delta(L_E^{1'}) = \Gamma(L_E^{1'})$  for  $\sigma_\nu^2$  small enough (see the left panel of Figure A.1). From part (i) of Proposition 4.4, there is an equilibrium of  $M_\sigma^1$  with  $L_E^{1'}$  entrepreneurs.

An equilibrium with mass of entrepreneurs  $L$  exists if  $\Delta_0^0(L) > \Gamma_0(L)$  (see Proposition 4.2). Since  $\Delta(L) \rightarrow \Delta_0^0(L)$  and  $\Gamma(L) \rightarrow \Gamma_0(L)$  for  $\sigma_\nu^2 \rightarrow 0$ , if  $\Delta_0^0(L) > \Gamma_0(L)$ , then  $\Delta(L) > \Gamma(L)$  for  $\sigma_\nu^2$  small enough, and, from part (ii) of Proposition 4.4, there is an equilibrium of  $M_\sigma^1$  at which all hipos become entrepreneurs.

Next, consider the case of no OC. An equilibrium of  $M_0^0$  with  $L_E^0 < L$  entrepreneurs and  $L - L_E^0$  hipos acting as passive investors exists if there is  $L_E^0$  such that  $\Delta_0^0(L_E^0) = 0$  (see part (i) of Proposition 4.3). Since  $\Delta^0(L_E)$  converges pointwise to  $\Delta_0^0(L_E)$  as  $\sigma_\nu^2 \rightarrow 0$ , for  $\sigma_\nu^2$  small enough, there is  $L_E^{0'}$  arbitrarily close to  $L_E^0$  such that  $\Delta^0(L_E^{0'}) = 0$  (see the right panel of Figure A.1) and, from part (i) of Proposition 4.5, an equilibrium of  $M_\sigma^0$ .

An equilibrium with no OC at which all hipos become entrepreneurs exists if  $\Delta_0^0(L) > 0$  (see part (ii) of Proposition 4.3). Since  $\Delta(L) \rightarrow \Delta_0^0(L)$  as  $\sigma_\nu^2 \rightarrow 0$ , it follows from part (ii) of Proposition 4.4 that there is an equilibrium of  $M_\sigma^0$  with  $L$  entrepreneurs for  $\sigma_\nu^2$  small enough.

To prove the pointwise convergence properties, note that for  $L_E < L$ , the limits of the functions defined in

(A.15)–(A.21) as  $\sigma_\nu^2 \rightarrow 0$  are:

$$\begin{aligned}
\alpha\gamma\sigma_s^2 &\rightarrow \frac{1}{\alpha} \\
\text{var}(\theta|w) &\rightarrow \sigma_\varepsilon^2 \\
\beta &\rightarrow \frac{L_E + M}{\rho\sigma_\varepsilon^2} \\
\alpha + \beta &\rightarrow \frac{L + M}{\rho\sigma_\varepsilon^2} \\
\text{E}(P) &\rightarrow \bar{s} - \frac{\rho\sigma_\varepsilon^2}{L + M} \left( \frac{L_E}{a} - \bar{\nu} \right) \\
\text{var}(P) &\rightarrow \sigma_s^2 \\
\text{E}(z) &\rightarrow \frac{\rho}{L + M} \left( \frac{\sigma_\varepsilon^2}{2} \right)^{\frac{1}{2}} \left( \frac{L_E}{a} - \bar{\nu} \right) \\
\text{var}(z) &\rightarrow 0 \\
\text{cov}(P, z) &\rightarrow 0.
\end{aligned} \tag{A.32}$$

Inserting these expressions into (10) yields  $\Delta_0(L_E)$ . This proves that  $\Delta(L_E) \rightarrow \Delta_0(L_E)$  pointwise. From (A.32), it follows that (11) goes to zero, i.e.,  $\Gamma(L_E) \rightarrow 0$  pointwise.

Similarly, Substituting the limits of (A.25)–(A.29) as  $\sigma_\nu^2 \rightarrow 0$  into (11) proves  $\Delta^0(L_E) \rightarrow \Delta_0^0(L_E)$  pointwise. For  $L_E = L$ , we get (A.25)–(A.29) evaluated at  $L_E = L$ . Inserting these expressions into (10) and taking the limit as  $\sigma_\nu^2 \rightarrow 0$  yields  $\Delta_0^0(L)$ . Since  $\Delta^0(L) = \Delta(L)$ , this also proves  $\Delta^0(L) \rightarrow \Delta_0^0(L)$ . Substitution into (11) and taking the limit  $\sigma_\nu^2 \rightarrow 0$  yields  $\Gamma_0(L)$ . ///

*Remark A.4.6.1.* There is a qualitative difference between the models with and without noise for  $L_E$  values in the vicinity of  $L$ . For  $\sigma_\nu^2$  small,  $\Gamma(L_E)$  is close to zero for  $L_E < L$  and  $\Gamma(L)$  is close to  $\Gamma_0(L)$ . That is, the graph of  $\Gamma(L_E)$  is almost kinked at  $L_E$  close to  $L$ .

*Remark A.4.6.2.* This sheds light on the multiplicity result in Remark A.4.4.2. If there is  $L_E^1 (< L)$  such that  $\Delta_0(L_E^1) = 0$  and  $\Delta_0^0(L) > \Gamma_0(L)$ , then two equilibria with  $L_E^1$  and  $L$  entrepreneurs, respectively, coexist in  $\mathbf{M}_0^1$  (see Remark A.4.2.4). These conditions also imply that for  $\sigma_\nu^2$  small enough,  $\Delta(L_E)$  and  $\Gamma(L_E)$  intersect twice, viz., close to  $L_E^1$  and close to  $L$ , so that there are two equilibria of  $\mathbf{M}_\sigma^1$  with  $0 < L_E < L$  (see Figure A.1).

*Remark A.4.6.3.* Since  $L_E^1$  and  $L_E^0$  are continuous functions of  $\sigma_\nu^2$ , condition (9) in Remark A.4.3.2 ensures that  $L_E^0 > L_E^1$  for  $\sigma_\nu^2$  positive but sufficiently small.

### Proof that entrepreneurs maximize profit:

Suppose all firms employ  $m$  workers and make profit  $\theta = \tilde{\theta} + F(m) - Wm$ . Consider a single firm which deviates with employment  $m' \neq m$ , thereby creating a new asset. Given the fact that the productivity shock is additive, an arbitrage argument is sufficient in order to prove that this is not beneficial to the firm. The deviating firm makes profit  $\theta' = \theta + \delta$ , where

$$\delta \equiv F(m') - F(m) - W(m' - m).$$

Since the firm's profit differs from the other firms' profit by the non-random amount  $\delta$ , buying a fraction  $\lambda$  of the firm's shares at cost  $\lambda P'$  generates the same cash flow as buying a fraction  $\lambda$  of one of the other firms at cost  $\lambda P$  and storing  $\lambda \delta$ . Hence, arbitrage-freeness implies  $P' = P + \delta$ .

The final wealth of an entrepreneur who employs  $m'$  workers in each of his firms is  $\pi'_E = e_L + P'/a + (\theta - P)I_E = \pi_E + \delta/a$ . Since the price differential  $\delta$  is non-random, we have

$$\mathbb{E}[U(\pi'_E)] = \exp\left(-\rho \frac{\delta}{a}\right) \mathbb{E}[U(\pi_E)].$$

So the entrepreneurs' objective is to maximize  $\delta$  or, equivalently,  $F(m') - Wm'$ .

### Remarks to Proposition 5.1:

*Remark A.5.1.1.* With  $\bar{s}$  given by (12) the conditions for existence of equilibrium in  $\mathbb{F}_\zeta^j$  are given by the respective remarks to Propositions 4.1, 4.2, 4.3, 4.4, and 4.5. Since equilibrium in  $\mathbb{M}_\sigma^1$  is not generally unique (see Remarks A.4.2.4 and A.4.4.2), the same holds true for  $\mathbb{F}_\sigma^1$ . Uniqueness of equilibrium in  $\mathbb{M}_\sigma^0$  (see Remarks A.4.3.1 and A.4.5.1) implies uniqueness in  $\mathbb{F}_\sigma^0$ .

*Remark A.5.1.2.* Equilibria of  $\mathbb{F}_0^j$  with  $L_E < L$  have the expected comparative statics properties. Parameter changes which raise GE in (7) or (8), respectively, shift the upward-sloping lines to the right. Parameter changes which raise expected firm profit in (12) shift the downward-sloping curve upward. In either case the equilibrium mass of entrepreneurs  $L_E$  goes up. For  $\sigma_\nu^2 > 0$ , the equilibrium mass of entrepreneurs in  $\mathbb{F}_\sigma^1$  has the expected comparative statics properties if  $\Delta(L_E)$  intersects  $\Gamma(L_E)$  from above.  $L_E^0$  in  $\mathbb{F}_\sigma^0$  has the expected comparative statics properties. Because of diminishing marginal productivity, wages rise when the number of firms increases:  $d\hat{W}/dL_E = -F''(\hat{M})\hat{M}/L_E > 0$ . Therefore, any parameter change that raises  $L_E$  (and leaves  $F$  and  $\hat{M}$  unaffected) increases workers' wages.

*Remark A.5.1.3.* If condition (9) is satisfied, then the  $\Delta_0(L_E) = 0$  line in the left panel of Figure 4 is located to the left of the  $\Delta_0^0(L_E) = 0$  line and the equilibrium mass of entrepreneurs is smaller in  $\mathbb{F}_0^1$  than in  $\mathbb{F}_0^0$ , i.e.,  $L_E^1 < L_E^0$ .

*Proof:* Let  $\bar{s}^1$  and  $\bar{s}^0$  denote the equilibrium levels of  $\bar{s}$  in  $\mathbb{F}_0^1$  and  $\mathbb{F}_0^0$ , respectively.  $L_E^0 < L$  and (9) jointly imply that (A.14) holds for  $\bar{s} = \bar{s}^0$ :

$$\bar{s}^0 - \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{2a} < 0. \quad (\text{A.33})$$

From (7) and (8),

$$\Delta_0^0(L_E^1) = -\frac{\rho \sigma_s^2}{a \sigma_\varepsilon^2} \left[ -\frac{\sigma_\varepsilon^2}{\sigma_s^2} (\bar{s}^0 - \bar{s}^1) + \bar{s}^1 - \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{2a} \right].$$

Suppose  $L_E^0 \leq L_E^1$ . This implies  $\bar{s}^0 \geq \bar{s}^1$  and  $\Delta_0^0(L_E^1) \leq 0$  (since  $\bar{s}$  and  $\Delta_0^0(L_E)$  are decreasing functions of  $L_E$ ). This contradicts (A.33), so  $L_E^0 > L_E^1$ . ///

*Remark A.5.1.4.* If  $(L_E, I_E, I_D, I_M, P, \hat{M}, \hat{W})$  is an equilibrium of  $\mathbb{F}_0^j$ , then there is an equilibrium of  $\mathbb{F}_\sigma^j$  with a mass of entrepreneurs close to  $L_E$  for  $\sigma_\nu^2$  positive but sufficiently small. This follows from Proposition A.6.3 together with the fact that  $\sigma_\nu^2$  does not show up in (12).

### Proof that the real wage is rigid in $\mathbb{U}_\zeta^j$ :

Whenever firms pay a uniform wage  $W$  and employment per firm  $m$  falls short of the mass of workers per

firm  $\hat{M}$ , a worker's expected utility is

$$E[U(\pi_M)] = \exp(-\rho e_M) \left( \frac{m}{\hat{M}} \{1 - \exp[-\rho(W - D)]\} - 1 \right) E \{ \exp[-\rho(\theta - P)I_M] \}. \quad (\text{A.34})$$

The same argument as in the full employment case proves that firms choose the profit maximizing level of employment  $m = (F')^{-1}(W)$  if unions set a uniform wage  $W$ . If a union deviates with a wage rate  $W' \neq W$ , firm profit becomes  $\theta' = \theta + \delta$ , where  $\delta = F(m') - F(m) - W'm' + Wm$ , and arbitrage implies that the firm value is  $P' = P + \delta$ . By the same argument as above, firms choose  $m' = (F')^{-1}(W')$ . Hence, unions anticipate that firms react to the wage they set by choosing employment on the labor demand curve.

Equation (A.34) is unions' objective function. The three factors on the right-hand side are non-random. So, irrespective of workers' subsequent investment decision, unions' objective is to maximize the second factor or, equivalently, the expression in (13).

The firm's labor demand curve is  $m = [A(1 - b)/W]^{1/b}$ . Maximization of (13) subject to this constraint is equivalent to maximization of  $bW^{-1/b}\{1 - \exp[-\rho(W - D)]\}$ . Setting the derivative equal to zero yields

$$W^{-\frac{1}{b}-1} \exp[-\rho(W - D)] \{1 + \rho bW - \exp[\rho(W - D)]\} = 0.$$

There is a unique positive  $\tilde{W}$  ( $> D$ ) such that the condition holds for  $W = \tilde{W}$ , and the derivative changes from positive to negative at  $\tilde{W}$ , so that  $\tilde{W}$  maximizes expected utility. Employment is  $\tilde{M} = [A(1 - b)/\tilde{W}]^{1/b}$ . There is unemployment if  $\tilde{M} < \hat{M}$ .

### Remarks to Proposition 5.2:

*Remark A.5.2.1.* With  $\bar{s}$  given by (14), the existence and uniqueness properties of  $M_\zeta^j$  carry over to  $U_\zeta^j$ .

*Remark A.5.2.2.* More entrepreneurship means more jobs: parameter changes that increase  $L_E$  at an equilibrium of  $M_\zeta^j$  increase  $L_E$  at an equilibrium of  $U_\zeta^j$ . This follows from the fact that  $\bar{s}$  does not change and the increase in the mass of entrepreneurs raises aggregate employment  $\tilde{M}L_E/a$ . In terms of Figure 4, the  $\bar{s} = E(s)$  curve is horizontal, and the upward-sloping curves shift to the right.

*Remark A.5.2.3.* Changes in labor market parameters which reduce the equilibrium wage rate (and leave the production function unaffected) lead to increases in employment at the intensive margin (as  $\tilde{M}$  rises) and at the extensive margin (whenever  $\bar{s}$  raises  $L_E$  in  $M_\zeta^j$ ). This is due to the fact that expected firm profitability rises:  $d\bar{s}/d\tilde{W} = [F'(\tilde{M}) - \tilde{W}]d\tilde{M}/d\tilde{W} - \tilde{M}$  or, using the condition for profit maximization,  $d\bar{s} = -\tilde{M}d\tilde{W} > 0$ . In terms of Figure 4, the horizontal  $\bar{s} = E(s)$  curve shifts upwards.

*Remark A.5.2.4.* From the fact that  $\bar{s}$  is independent of  $L_E$ , it follows that condition (9) in Remark A.4.3.2 is sufficient for  $L_E^1 < L_E^0$ .

*Remark A.5.2.5.* From Proposition A.6.3 and the fact that  $\bar{s}$  is independent of  $L_E$ , it follows that if  $(L_E, I_E, I_D, I_M, P, \tilde{M}, \tilde{W})$  is an equilibrium of  $U_0^j$ , then there is an equilibrium of  $U_\sigma^j$  with a mass of entrepreneurs close to  $L_E$  for  $\sigma_\nu^2$  positive but sufficiently small.

### Alternative sources of wage rigidity:

Denote model  $U_\zeta^j$  in the running text as  ${}_1U_\zeta^j$ . Consider the following models  ${}_2U_\zeta^j$ – ${}_4U_\zeta^j$ .

${}_2U_\zeta^j$ : Employees can “work” or “shirk” at their workplace (cf. Shapiro and Stiglitz, 1984). A worker who works gets the wage rate  $W$ . Effort is not perfectly observable: a shirker is caught shirking with probability

$q$  ( $0 < q < 1$ ). So he gets the wage rate  $W$  with probability  $1 - q$  and no payment otherwise. If all workers work, firm output is  $Y = F(m)$ . If everyone shirks, output is zero. So firms have to pay workers such that they choose not to shirk.  $(L_E, I_E, I_D, I_M, P, m, W)$  is an *equilibrium* if, in addition to the conditions stated in Section 3, employment  $m$  maximizes  $F(m) - Wm$ ,  $W$  is such that workers' expected utility is as high if they work as if they shirk, and there is unemployment (i.e.,  $m < \hat{M}$ ).

♣ $\mathcal{U}_\zeta^j$ : Unions are organized as in  $\mathcal{U}_1(\sigma_\nu^2, j)$ . Firms have the right to manage. Rather than maximizing a utility function, firm-level unions set the wage rate  $W$  such that the wage bill  $Wm$  is maximal (cf. Dunlop, 1944).  $F$  is CES with low substitutability:  $F(m) = A[b + (1 - b)m^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$ , where  $A > 0$ ,  $0 < b < 1$ , and the elasticity of substitution  $\eta$  obeys  $0 < \eta < 1$ .  $(L_E, I_E, I_D, I_M, P, m, W)$  is an *equilibrium* if, in addition to the conditions stated in Section 3, employment  $m$  maximizes  $F(m) - Wm$ ,  $W$  maximizes  $Wm$  given firms' optimal choice of  $m$ , and  $m < \hat{M}$ .

♠ $\mathcal{U}_\zeta^j$ : Firm output is  $Y = F[E(W)m]$ , where  $E(W)$  is the effort provided by workers given the wage they receive (cf. Solow, 1979). Workers' provision of effort is determined by how fair they conceive the wage  $W$  they are paid. It is assumed that there is a unique "efficiency wage"  $\tilde{W}$  that maximizes  $E(W)/W$ . Effort is normalized such that  $E(\tilde{W}) = 1$ .  $(L_E, I_E, I_D, I_M, P, m, W)$  is an *equilibrium* if, in addition to the conditions stated in Section 3, employment  $m$  and the wage rate  $W$  jointly maximize  $F(m) - Wm$ , and  $m < \hat{M}$ .

These models display the same kind of wage rigidity as the right-to-manage model in the main text.

♣ $\mathcal{U}_\zeta^j$ : Workers' asset demand  $I_M$  is independent of their employment status. If an employed worker doesn't shirk, his expected utility is

$$E[U(\pi_M)] = \exp(-\rho e_M) \exp[-\rho(W - D)] E\{-\exp[-\rho(\theta - P)I_M]\}.$$

If he shirks,

$$\begin{aligned} E[U(\pi_M)] &= (1 - q) \exp(-\rho e_M) \exp(-\rho W) E\{-\exp[-\rho(\theta - P)I_M]\} \\ &\quad + q \exp(-\rho e_M) E\{-\exp[-\rho(\theta - P)I_M]\} \end{aligned}$$

Equalizing these expected utilities yields the efficiency wage, necessary to prevent shirking:

$$\tilde{W} = \frac{1}{\rho} \log \left[ \frac{\exp(\rho D) - (1 - q)}{q} \right]. \quad (\text{A.35})$$

Employment is  $\tilde{M} = (F')^{-1}(\tilde{W})$ . There is unemployment for  $M$  sufficiently large.

♣ $\mathcal{U}_\zeta^j$ : Equating the marginal product of labor to the wage rate yields the following expression for each firm's wage bill:

$$Wm = A(1 - b) \left[ b + (1 - b)m^{\frac{\eta-1}{\eta}} \right]^{\frac{1}{\eta-1}} m^{\frac{\eta-1}{\eta}}.$$

Setting the derivative equal to zero gives:

$$\frac{1 - \eta}{\eta} W \left[ \frac{1 - b}{1 - \eta} \frac{m^{\frac{\eta-1}{\eta}}}{b + (1 - b)m^{\frac{\eta-1}{\eta}}} - 1 \right] = 0.$$

Optimum employment is given by the value  $m = \tilde{M}$  at which the derivative changes from positive to negative:

$$\tilde{M} = \left( \frac{\eta}{1-\eta} \frac{1-b}{b} \right)^{\frac{\eta}{1-\eta}}.$$

The corresponding wage rate is

$$\tilde{W} = A \left[ \frac{1-\eta}{(1-b)^\eta} \right]^{\frac{1}{1-\eta}}.$$

$\mathcal{U}_\zeta^j$ : Firm profit can be expressed as

$$F \left[ \frac{E(W)}{W} Wm \right] - Wm.$$

Maximization with respect to  $m$  and  $W$  is equivalent to maximizing  $E(W)/W$  by an appropriate choice of  $W$  and then maximizing profit for given  $E(W)/W$  by an appropriate choice of the wage bill  $Wm$ . The former step gives  $W = \tilde{W}$ , the latter  $\tilde{M} = (F')^{-1}(\tilde{W})$ .

With  $k\mathcal{U}_\zeta^j$  ( $k = 2, 3, 4$ ) instead of  $\mathcal{U}_\zeta^j$ , the assertions of Proposition 5.2 and the subsequent remarks carry over one-to-one to these models.

### Social welfare for $\sigma_\nu^2 = 0$ :

For  $\sigma_\nu^2 = 0$ , from (A.9) and (8), respectively,

$$\theta - P = \frac{\rho\sigma_\varepsilon^2}{L+M} \left( \frac{L_E^1}{a} - \bar{\nu} \right) + \varepsilon \quad (\text{A.36})$$

at an equilibrium of  $\mathcal{M}_0^1$  and

$$\theta - P = s - \bar{s} + \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{L+M} \left( \frac{L_E^0}{a} - \bar{\nu} \right) + \varepsilon \quad (\text{A.37})$$

at an equilibrium of  $\mathcal{M}_0^0$  with  $L_E^0 < L$  entrepreneurs. With OC  $\theta - P$  ( $= s - P + \varepsilon$ ) does not depend on  $s$  since  $P$  varies one-for-one with  $s$  (see (A.9)). That reduces the ex ante volatility of the return on investing in the risky asset.

Using  $\pi_N = e_N + (\theta - P)\bar{\nu}/N$  and (A.36), noise traders' expected utility in the case with OC can be written as

$$\mathbb{E}[U(\pi_N)] = -\exp(-\rho e_N) \mathbb{E} \left( \exp \left\{ \left[ \frac{\rho\sigma_\varepsilon^2}{L+M} \left( \frac{L_E^1}{a} - \bar{\nu} \right) + \varepsilon \right] \frac{\bar{\nu}}{N} \right\} \right).$$

As final wealth is normal, we can apply (A.1) to get

$$-\log\{-\mathbb{E}[U(\pi_N)]\} = \rho e_N + \rho \frac{\bar{\nu}}{N} \frac{\rho\sigma_\varepsilon^2}{L+M} \left( \frac{L_E^1}{a} - \bar{\nu} \right) - \frac{1}{2} \left( \rho \frac{\bar{\nu}}{N} \right)^2 \sigma_\varepsilon^2. \quad (\text{A.38})$$

Following the same steps, using (A.37) instead of (A.36), we get noise traders' expected utility in the absence of OC:

$$-\log\{-\mathbb{E}[U(\pi_N)]\} = \rho e_N + \rho \frac{\bar{\nu}}{N} \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{L+M} \left( \frac{L_E^0}{a} - \bar{\nu} \right) - \frac{1}{2} \left( \rho \frac{\bar{\nu}}{N} \right)^2 (\sigma_s^2 + \sigma_\varepsilon^2). \quad (\text{A.39})$$

With  $L$  instead of  $L_E^0$  the formulas also apply to the case in which all hipos become entrepreneurs with no

OC.

Since (A.9) and (8) are also valid in the economies with full employment or unemployment, the formulas are likewise valid in these models.

Consider SW, as defined in the main text. Consider model  $M_0^1$  with  $L_E$  ( $< L$ ) entrepreneurs and  $L - L_E$  ( $> 0$ ) dealers. Entrepreneurs' expected utility is  $-\log\{-E[U(\pi_E)]\} = \rho e_L + GE + GT_E$ . Substituting  $GE = \Delta_0(L_E)$  (see (7)),  $GT_E = z^2$ , and the expression for  $z$  in (A.10) yields

$$-\log\{-E[U(\pi_E)]\} = \rho e_L + \Delta_0(L_E) + \frac{\sigma_\varepsilon^2}{2} \left[ \frac{\rho}{L+M} \left( \frac{L_E}{a} - \bar{\nu} \right) \right]^2. \quad (\text{A.40})$$

Passive investors' expected utility is  $\rho e_M + z^2$ . Similarly, as  $GI = 0$ , dealers' expected utility is  $\rho e_L + z^2$ . Summing all agents' expected utilities yields (15).

Next, consider model  $M_0^0$ . Hipos who are not entrepreneurs act as passive investors. Following the same steps as above, entrepreneurs' expected utility can be written as

$$-\log\{-E[U(\pi_E)]\} = \rho e_L + \Delta_0^0(L_E) + \frac{\sigma_s^2 + \sigma_\varepsilon^2}{2} \left[ \frac{\rho}{L+M} \left( \frac{L_E}{a} - \bar{\nu} \right) \right]^2. \quad (\text{A.41})$$

So SW is given by (16).

From (A.40) with  $\Delta_0(L_E^1) = 0$  and (A.41) with  $\Delta_0^0(L_E^0) \geq 0$ , it is evident that entrepreneurs' expected utility is lower at an equilibrium with OC than at an equilibrium without OC if (17) holds.

**Proposition 6.1:**

*Proof:* Denote the functions defined in (7) and (8) as  $\Delta_0(L_E, \bar{s})$  and  $\Delta_0^0(L_E, \bar{s})$  (instead of  $\Delta_0(L_E)$  and  $\Delta_0^0(L_E)$ , respectively, as in the main text). Making the dependence on  $\bar{s}$  explicit is essential in the full employment economy, where it depends on  $L_E$  via (12).

Consider first  $M_0^1$ . (i) Differentiating (15) twice yields

$$\frac{\partial S^1(L_E, \bar{s})}{\partial L_E} = \Delta_0(L_E, \bar{s}) \quad (\text{A.42})$$

and  $\partial^2 S^1(L_E, \bar{s}) / \partial L_E^2 < 0$ . So if  $\Delta_0(L_E^1, \bar{s}) = 0$  for some  $L_E^1$  ( $< L$ ), then  $S^1$  is a hump-shaped function of  $L_E$  with its maximum at  $L_E^1$ . Otherwise  $S^1$  is monotonically increasing.

In  $F_0^1$ , workers' expected utility is  $-\log\{-E[U(\pi_M)]\} = \rho(e_M + \hat{W}) + z^2$ , and SW is

$$\hat{S}^1(L_E, \bar{s}) = S^1(L_E, \bar{s}) + \rho \hat{W} M,$$

where  $\bar{s}$  is given by (12) (in another slight abuse of notation, SW is denoted  $S^1$  in the main text and "includes the term  $\rho \hat{W} M$ "). Differentiating with respect to  $L_E$  yields

$$\frac{d\hat{S}^1(L_E, \bar{s})}{dL_E} = \frac{\partial \hat{S}^1(L_E, \bar{s})}{\partial L_E} + \frac{\partial \hat{S}^1(L_E, \bar{s})}{\partial \bar{s}} \frac{d\bar{s}}{dL_E} + \rho \frac{d\hat{W}}{dL_E} M.$$

Using  $\partial \hat{S}^1 / \partial L_E = \partial S^1 / \partial L_E$ ,  $\partial \hat{S}^1 / \partial \bar{s} = \rho L_E / a$ ,

$$\frac{d\bar{s}}{dL_E} = \left[ F'(\hat{M}) - \hat{W} \right] \frac{d\hat{M}}{dL_E} - \frac{d\hat{W}}{dL_E} \hat{M},$$

$F'(\hat{M}) = \hat{W}$ , and  $\hat{M} = M / (L_E / a)$ , it follows that

$$\frac{d\hat{S}^1(L_E, \bar{s})}{dL_E} = \frac{\partial S^1(L_E, \bar{s})}{\partial L_E}.$$

From (A.42) and the fact that  $\Delta_0(L_E^1, \bar{s}) = 0$  at equilibrium, it follows that  $d\hat{S}^1(L_E^1, \bar{s}) / dL_E = 0$ .

In  $U_0^1$ , a worker is employed with probability  $\tilde{M} / \hat{M}$ , in which case he gets extra payoff  $\tilde{W} - D$ . From (A.34) with  $m = \tilde{M}$  and  $W = \tilde{W}$ , SW is (with the same slight abuse of notation as above)

$$\tilde{S}^1(L_E, \bar{s}) = S^1(L_E, \bar{s}) - M \log \left( 1 - \frac{\tilde{M}}{\hat{M}} \left\{ 1 - \exp[-\rho(\tilde{W} - D)] \right\} \right), \quad (\text{A.43})$$

and  $\bar{s}$  is given by (14) and does not depend on  $L_E$ . The log term on the right-hand side is decreasing in  $\hat{M} = M / (L_E / a)$  (since the term in braces is negative). So  $\partial \tilde{S}^1(L_E, \bar{s}) / \partial L_E > \partial S^1(L_E, \bar{s}) / \partial L_E$  for all  $L_E$ , and  $\partial \tilde{S}^1(L_E^1, \bar{s}) / \partial L_E > \partial S^1(L_E^1, \bar{s}) / \partial L_E = 0$ .

(ii) Next, suppose the  $L - L_E$  non-entrepreneurs act as passive investors. SW  $S^0$  is given by (16). Taking the first two derivative yields

$$\frac{\partial S^0(L_E, \bar{s})}{\partial L_E} = \Delta_0^0(L_E, \bar{s})$$

and  $\partial^2 S^0(L_E, \bar{s}) / \partial L_E^2 < 0$ . That is, if there is  $L_E^0 < L$  such that  $\Delta_0^0(L_E^0, \bar{s}) = 0$ , then it maximizes  $S^0(L_E, \bar{s})$  on  $[0, L]$ . Otherwise  $S^0$  is monotonically increasing on the interval  $[0, L]$ .

In  $F_0^0$ , SW  $\hat{S}^0(L_E, \bar{s})$  encompasses the additional term  $\rho \hat{W} M$  representing the contribution of wage income to passive investors' expected utility. Using the same results as in the previous case, it follows that  $d\hat{S}^0(L_E, \bar{s}) / dL_E = \partial S^0(L_E, \bar{s}) / \partial L_E$ . In  $U_0^0$  SW encompasses the log term on the right-hand side of (A.43), which is increasing in  $L_E$ , so that  $d\tilde{S}^0(L_E^0, \bar{s}) / dL_E > 0$ . ///

*Remark A.6.1.1.* If there is no  $L_E^0 < L$  such that  $\Delta_0^0(L_E^0) = 0$ , then  $S^0$  increases monotonically with  $L_E$  in  $M_0^0$ ,  $F_0^0$  and  $U_0^0$  (see the right panel of Figure 5). From part (ii) of Proposition 4.3, the equilibrium mass of entrepreneurs is  $L$  in this case and maximizes  $S^0$ .

*Remark A.6.1.2.* From (A.38) and (A.39), noise trader expected utility increases with  $L_E$  for  $\bar{\nu} > 0$ . Hence, if one deletes noise traders' expected utilities from the SW function,  $S^j$  decreases with  $L_E$  at  $L_E^j$ , and there is too much entrepreneurship, conditional on information, at the margin in the absence of labor market frictions. This alleviates our overall conclusion that there is too little entrepreneurship in our model. However, the marginal impact of more entrepreneurship on SW is most often positive in the model with unemployment (see the numerical analysis).

*Remark A.6.1.3.* According to the proposition, SW attains a local maximum at the equilibrium mass of employment  $L_E^j$  in the absence of noise and labor market frictions. For  $\sigma_\nu^2$  small, there is a local maximum of  $S^j$  at a mass of entrepreneurs close to  $L_E^j$ . This follows from the fact that all agents' expected utilities and, hence, SW  $S^j$  are continuous in  $\sigma_\nu^2$  at  $\sigma_\nu^2 = 0$ . Equations (4)–(6) and the formulas for the moments of  $P$



and  $z$  and  $\text{var}(\theta|w)$  imply continuity of  $-\log\{-\mathbb{E}[U(\pi_i)]\}$  for  $i = E, D, M$ . Below, we derive a closed-form solution for noise trader expected utility  $-\log\{-\mathbb{E}[U(\pi_N)]\}$  (which requires the use of a modified version of the Demange-Laroque, 1995, lemma) and show that it is also continuous in  $\sigma_\nu^2$  at  $\sigma_\nu^2 = 0$ .

*Remark A.6.1.4.* From Proposition A.6.3 and Remark A.5.1.4, for  $\sigma_\nu^2$  small, there is an equilibrium of the economy without labor market frictions with a mass of entrepreneurs close to the noiseless equilibrium value. From Remark A.6.1.3, SW is close to its local maximum then.

*Remark A.6.1.5.* Recall that the model with OC behaves very differently with and without noise for  $L_E$  values in the vicinity of  $L$  (see Remark A.4.6.1). This also holds true for SW.  $S^1$  is continuous in  $\sigma_\nu^2$  at  $\sigma_\nu^2 = 0$  for  $L_E < L$  (see Remark A.6.1.3). However, price informativeness converges to zero and, hence,  $S^1(L_E, \bar{s})$  converges to  $S^0(L, \bar{s})$  as  $L_E \rightarrow L$  when  $\sigma_\nu^2$  is positive (whereas the price remains perfectly informative when  $\sigma_\nu^2 = 0$ ).

*Remark A.6.1.6.* Suppose that for  $\sigma_\nu^2 = 0$  two equilibria exist, one with  $L_E^1 < L$  and one with  $L_E^1 = L$  (cf. Remark A.4.2.4). Even though the former equilibrium yields SW at its *local* maximum, SW is higher at the latter equilibrium if  $S^0(L, \bar{s}) > S^1(L_E^1, \bar{s})$  (a set of simple sufficient conditions is given in Remark A.6.2.5). Under the same conditions, but with  $\sigma_\nu^2$  positive but small, SW for  $L_E \rightarrow L$  is higher than SW at its local maximum (see Figure 6 below; cf. Remark A.4.6.2).

### Economy without noise traders:

Let the  $N$  noise traders act as passive investors, so that the mass of the latter is  $M + N$  and there is no noise trading. From (7) and (15) with  $M + N$  instead of  $M$  and  $\bar{\nu} = 0$ , SW with OC for given  $L_E$  is

$$S^{1'} = \rho e + L_E \frac{\rho}{a} \left( \bar{s} - \frac{\rho \sigma_\varepsilon^2}{L + M + N} \frac{L_E}{a} - \frac{\rho \sigma_s^2}{2a} \right) + \frac{\rho^2 \sigma_\varepsilon^2}{2(L + M + N)} \left( \frac{L_E}{a} \right)^2.$$

Subtracting  $S^1$  yields

$$S^{1'} - S^1 = \frac{\rho \sigma_\varepsilon^2}{2} \frac{N}{(L + M)(L + M + N)} \left[ \frac{L_E}{a} - \left( 1 + \frac{L + M}{N} \right) \bar{\nu} \right]^2 > 0$$

for all  $L_E$ . Let  $L_E^{1'}$  denote the equilibrium mass of entrepreneurs in the economy without noise trading.  $L_E^{1'}$  maximizes  $S^{1'}$  (this follows from the fact that  $L_E^1$  maximizes  $S^1$ , considering the special case with  $\bar{\nu} = 0$ ). Together with the result that  $S^{1'}$  exceeds SW with noise trading  $S^1$  at  $L_E^1$ , it follows that equilibrium SW is higher without than with noise trading.

### Comparison of SW in the models with or without labor market frictions:

Compare model  $F_\sigma^1$  to  $U_\sigma^1$ . To do so, let  $D = 0$ , in the latter, since this is assumed in the former. The levels of SW differ for two reasons. First,  $L_E$  and, hence,  $\bar{s}$  differ (see (12) and (14)). Second, workers' extra utility is given by  $\rho \hat{W}$  in  $F_\sigma^1$  and by  $M$  times (13) in  $U_\sigma^1$ . As noise traders' utility,  $GI$ ,  $GT_E$ ,  $GT_M$ , and  $\text{var}(P)$  are independent of  $\bar{s}$ , making use of (A.17), we get the difference in SW for given  $L_E$ :

$$\hat{S}^1 - \tilde{S}^1 = \frac{L_E}{a} \left( \rho \left[ F(\hat{M}) - F(\tilde{M}) + \tilde{W} \tilde{M} \right] + \hat{M} \log \left\{ 1 - \frac{\tilde{M}}{\hat{M}} \left[ 1 - \exp(-\rho \tilde{W}) \right] \right\} \right).$$

As  $F(\hat{M}) - F(\tilde{M}) > 0$ , the SW difference is positive if the sum composed of the remaining terms on the

right-hand side is positive, which is equivalent to

$$\frac{\tilde{M}}{\hat{M}} \left[ 1 - \exp(-\rho\tilde{W}) \right] < 1 - \exp\left(-\rho\tilde{W}\frac{\tilde{M}}{\hat{M}}\right).$$

Both sides of this inequality take on the same value for  $\tilde{M}/\hat{M} = 0$  and for  $\tilde{M}/\hat{M} = 1$ . The validity of the inequality for all  $\tilde{M}/\hat{M}$  in between follows from the fact that the left-hand side is linear and the right-hand side is strictly concave.

The proof for  $F_\sigma^0$  compared to  $U_\sigma^0$  proceeds analogously, using (A.25) instead of (A.17). Comparing  $F_\zeta^0$  to  $U_\zeta^0$  gives the same results.

**Proposition 6.2:**

*Proof:* Consider the difference in SW  $S^1 - S^0$  for given  $L_E$ . From (7), (8), (15), and (16),

$$S^1(L_E) - S^0(L_E) = \frac{\rho^2 \sigma_s^2}{2(L+M)} \left[ \left(\frac{L_E}{a}\right)^2 - 2\frac{L_E}{a} \left(\frac{L+M}{2a} + \bar{\nu}\right) + \left(1 + \frac{L+M}{N}\right) \bar{\nu}^2 \right]$$

(where use is made of the fact that for given  $L_E$ ,  $\bar{s}$  is the same with and without OC; see (12) and (14)).  $S^1(L_E) - S^0(L_E)$  is a convex function with a minimum at  $L_E = (L+M)/2 + a\bar{\nu}$ . Evaluating the difference at  $a\bar{\nu}$  and at  $L$  yields

$$S^1(a\bar{\nu}) - S^0(a\bar{\nu}) = -\frac{\rho^2 \sigma_s^2}{2N} \bar{\nu} \left(\frac{N}{a} - \bar{\nu}\right)$$

and

$$S^1(L) - S^0(L) = -\frac{\rho^2 \sigma_s^2}{2(L+M)} \left[ \frac{M}{N} \left(\frac{L}{a} \frac{N}{a} - \bar{\nu}^2\right) + \bar{\nu} \left(\frac{L}{a} - \bar{\nu}\right) + \bar{\nu} \frac{L}{N} \left(\frac{N}{a} - \bar{\nu}\right) \right],$$

respectively. From (18), both differences are negative. This implies  $S^1(L_E) - S^0(L_E) < 0$  for  $a\bar{\nu} < L_E < L$ . In economies  $M_0^j$  and  $F_0^j$ , since  $L_E^0$  maximizes  $S^0(L_E)$ , we have

$$S^1(L_E^1) < S^0(L_E^1) < S^0(L_E^0), \tag{A.44}$$

which proves that *equilibrium* SW is lower with OC. If  $L_E^1 < L_E^0$ , then the inequalities in (A.44) also hold true in  $U_0^j$ . Here the second inequality follows from the fact that  $L_E^0$  falls short of the  $L_E$  value that maximizes  $S^0(L_E)$ , so that  $S^0(L_E)$  increases as  $L_E$  rises from  $L_E^1$  to  $L_E^0$ . ///

*Remark A.6.2.1.* To see that noise trader utility is higher with no OC for  $\bar{\nu}/N$  sufficiently small, subtract (A.38) from (A.39), evaluated at the respective equilibrium levels of  $L_E$ , to get

$$\rho \left\{ \frac{\rho}{L+M} \left[ \sigma_s^2 \left(\frac{L_E^0}{a} - \bar{\nu}\right) + \sigma_\varepsilon^2 \frac{L_E^0 - L_E^1}{a} \right] \frac{\bar{\nu}}{N} - \frac{\rho}{2} \sigma_s^2 \left(\frac{\bar{\nu}}{N}\right)^2 \right\}.$$

For  $\bar{\nu}/N$  small enough, the first term in braces dominates the second one and the difference in the utilities without and with OC is positive.

*Remark A.6.2.2.* Jointly (17) and (18) imply that no OC is Pareto-preferred to OC in  $M_0^j$  and  $U_0^j$ : (17) ensures that rational agents are better-off and the final inequality in (18) ensures that the same holds true for noise traders.

*Remark A.6.2.3.* An alternative set of sufficient conditions for  $S^1 - S^0 < 0$  (in  $\mathbf{M}_0^j$  as well as in the models with a labor market) is

$$0 < \left(1 + \frac{L + M}{N}\right)^{\frac{1}{2}} \bar{\nu} < \frac{L_E^0}{a}, \quad L_E^1 < L_E^0.$$

These conditions strengthen the requirement that the aggregate supply of assets is large relative to noise traders' aggregate demand but make the condition that a single noise trader's demand falls short of the supply generated by a single entrepreneur obsolete.

*Remark A.6.2.4.* In the economy with no noise traders and  $M + N$  passive investors equilibrium SW is generally higher without than with OC. Letting  $S^{0'}$  denote SW without noise traders and without OC,  $S^{1'} < S^{0'}$  at equilibrium follows immediately from replacing  $M$  with  $M + N$  and  $\bar{\nu} = 0$  in expression for  $S^1 - S^0$  in the proof of Proposition 6.2.

*Proof:* Consider  $\mathbf{M}_0^j$ . Let  $L_E^1 < L$ , so that  $\Delta_0(L_E^1) = 0$ . From (15), (16), and  $\Delta_0(L_E^0) \geq 0$ , it follows that  $S^1 - S^0$  is negative if

$$\sigma_\varepsilon^2 \left[ \left(\frac{L_E^1}{a}\right)^2 - \left(\frac{L_E^0}{a}\right)^2 \right] - \sigma_s^2 \left[ \left(\frac{L_E^0}{a}\right)^2 - \left(1 + \frac{L + M}{N}\right) \bar{\nu}^2 \right] < 0.$$

The assertion of the remark follows immediately.

In  $\mathbf{F}_0^j$  and  $\mathbf{U}_0^j$  workers' aggregate welfare includes the extra terms  $\rho \hat{W} M$  and  $M$  times (13), respectively. Since these terms are increasing in  $L_E$ , workers benefit from a greater mass of entrepreneurs. So the conditions in the remark are sufficient for higher SW without OC. ///

*Remark A.6.2.5.* The conditions in (18) plus  $L < M$  and  $\sigma_\varepsilon^2 < \sigma_s^2$  jointly imply that in  $\mathbf{M}_0^j$  we have  $(S^0(L, \bar{s}) =) S^1(L, \bar{s}) > S^1(L_E^1, \bar{s})$  (see Remark A.6.1.6).

*Proof:* Consider  $\mathbf{M}_0^j$ . Let the conditions in (18) be satisfied. The difference between  $S^0(L)$  and  $S^1(L_E^1)$  can be written as

$$\begin{aligned} S^0(L) - S^1(L_E^1) &= \rho \bar{s} \left(\frac{L}{a} - \bar{\nu}\right) - \frac{L + M}{2\sigma_\varepsilon^2} \left(\bar{s} - \frac{\rho\sigma_s^2}{2a}\right)^2 + \frac{\rho^2\sigma_s^2}{2} \bar{\nu} \left(\frac{1}{a} - \frac{\bar{\nu}}{N}\right) \\ &\quad - \frac{1}{2} \frac{\rho^2(\sigma_s^2 + \sigma_\varepsilon^2)}{L + M} \left(\frac{L}{a} - \bar{\nu}\right)^2. \end{aligned} \tag{A.45}$$

We proceed to show that this expression is non-negative under the additional conditions of Remark A.6.2.5. Consider the expression on the right-hand side as a function of  $\bar{s}$ . This function takes on a unique maximum for

$$\bar{s} = \frac{\rho\sigma_s^2}{2a} + \frac{\rho\sigma_\varepsilon^2}{L + M} \left(\frac{L}{a} - \bar{\nu}\right).$$

The assumption that there are a positive mass of dealers when there is OC (i.e.,  $L_E^1 < L$ ) implies that  $\bar{s}$  is less than this maximizing value. So  $S^0(L) - S^1(L_E^1)$  is an increasing function for the admissible values of  $\bar{s}$ . The condition that rational agents do not go short at equilibrium (i.e.  $L_E^1/a > \bar{\nu}$ ) puts a lower bound on the set of admissible values of  $\bar{s}$ , viz.,  $\bar{s} = \rho\sigma_s^2/(2a)$ . Evaluating (A.45) at this value gives

$$\frac{\rho^2\sigma_s^2}{2a} \left(\frac{L}{a} - \bar{\nu}\right) + \frac{\rho^2\sigma_s^2}{2} \bar{\nu} \left(\frac{1}{a} - \frac{\bar{\nu}}{N}\right) - \frac{1}{2} \frac{\rho^2(\sigma_s^2 + \sigma_\varepsilon^2)}{L + M} \left(\frac{L}{a} - \bar{\nu}\right)^2.$$

If  $L < M$  and  $\sigma_\varepsilon^2 < \sigma_s^2$ , then a sufficient condition for this expression to be greater than zero is

$$\frac{\rho^2 \sigma_s^2}{2a} \bar{\nu} \left[ \left( \frac{1}{a} - \frac{\bar{\nu}}{N} \right) + \frac{1}{L} \left( \frac{L}{a} - \bar{\nu} \right) \right] > 0.$$

This proves  $S^0(L) > S^1(L_E^1)$  under the assumptions made. ///

*Remark A.6.2.6.* From Proposition 6.1, the assertion of Proposition 6.2 is also valid for the *maximum* (rather than *equilibrium*) welfare levels in  $M_0^j$  and  $F_0^j$ . From the proof of Proposition 6.2, it can be seen that the same holds true for  $U_0^j$  (irrespective of whether  $L_E^1 < L_E^0$  or not).

*Remark A.6.2.7.* Propositions 6.1 and 6.2 jointly imply that in the absence of noise trader shocks and labor market imperfections the only market intervention needed to achieve the second-best welfare-maximizing allocation is to keep hipos from becoming dealers. In the presence of labor market frictions, increases in entrepreneurship further raises SW.

*Remark A.6.2.8.* Together with Remark A.6.1.3, it follows that for  $\sigma_\nu^2$  positive but small enough equilibrium SW is higher without than with OC if the conditions of Proposition 6.2 are satisfied.

#### Conditions for lower noise trader expected utility with than without OC:

Subtract (A.39) from (A.38) to obtain the difference in noise traders' expected utility with versus without OC:

$$\rho \left\{ -\frac{\rho}{L+M} \left[ \sigma_s^2 \left( \frac{L_E^0}{a} - \bar{\nu} \right) + \sigma_\varepsilon^2 \frac{L_E^0 - L_E^1}{a} \right] \frac{\bar{\nu}}{N} + \frac{\rho}{2} \sigma_s^2 \left( \frac{\bar{\nu}}{N} \right)^2 \right\}.$$

Let  $L_E^0 < L$ . Substituting for  $L_E^1$  and  $L_E^0$  from (7) and (8), respectively, we find that in  $M_0^j$  and  $U_0^j$  this condition can be written as

$$\frac{\bar{\nu}}{N} < \frac{1}{a} - \frac{2}{\rho \sigma_s^2} (\bar{s}^1 - \bar{s}^0).$$

In model  $M_0^j$ ,  $\bar{s}$  is exogenous. In  $U_0^j$ ,  $\bar{s}$  is pinned down by (16). So in both cases  $\bar{s}^1 = \bar{s}^0$ , and the inequality is satisfied if the first and third inequalities in (18) are satisfied. In  $F_0^j$ , the condition is stronger, since  $\bar{s}^1 > \bar{s}^0$  when  $L_E^1 < L_E^0$  (see the proof of Remark A.5.1.3).

#### Modified Demange-Laroque lemma:

For the calculation of noise traders' expected utility for  $\sigma_\nu^2 > 0$  we need a variant of the Demange-Laroque (1995) lemma (A.1):

**PROPOSITION A.6.3.** *For normal random variables  $x$  and  $y$ ,*

$$E [\exp (x + y^2)] = \frac{\exp \left\{ E(x) + \frac{1}{2} \text{var}(x) + \frac{[E(y) + \text{cov}(x, y)]^2}{1 - 2 \text{var}(y)} \right\}}{[1 - 2 \text{var}(y)]^{\frac{1}{2}}} \quad (\text{A.46})$$

for  $\text{var}(y) < 1/2$ .  $E[\exp(x + y^2)]$  does not exist otherwise.

*Proof:* By direct calculation

$$E [\exp (x + y^2) | y] = \exp (y^2) \exp \left[ E(x|y) + \frac{1}{2} \text{var}(x|y) \right] \int_{-\infty}^{\infty} \frac{\exp \left( -\frac{\{x - [E(x|y) + \text{var}(x|y)]\}^2}{2 \text{var}(x|y)} \right)}{[2\pi \text{var}(x|y)]^{\frac{1}{2}}} dx.$$

The integral is unity, since the integrand is the density of  $N[\mathbb{E}(x|y) + \text{var}(x|y), \text{var}(x|y)]$ . Using the updating rules for normal random variables, it follows that

$$\begin{aligned} \mathbb{E}[\exp(x + y^2) | y] &= \exp\left[y^2 + \frac{\text{cov}(x, y)}{\text{var}(y)}y\right] \\ &\cdot \exp\left[\mathbb{E}(x) + \frac{1}{2}\text{var}(x) - \frac{\text{cov}(x, y)}{\text{var}(y)}\mathbb{E}(y) - \frac{1}{2}\frac{\text{cov}(x, y)^2}{\text{var}(y)}\right]. \end{aligned} \quad (\text{A.47})$$

The unconditional expectation of the first exponential on the right-hand side can be rewritten as

$$\begin{aligned} \mathbb{E}\left\{\exp\left[y^2 + \frac{\text{cov}(x, y)}{\text{var}(y)}y\right]\right\} &= \frac{\exp\left[\frac{2\mathbb{E}(y)\text{cov}(x, y) + \text{cov}(x, y)^2 + 2\mathbb{E}(y)^2\text{var}(y)}{2[1 - 2\text{var}(y)]\text{var}(y)}\right]}{[1 - 2\text{var}(y)]^{\frac{1}{2}}} \\ &\cdot \int_{-\infty}^{\infty} \frac{\exp\left\{-\frac{\left[y - \frac{\mathbb{E}(y) + \text{cov}(x, y)}{1 - 2\text{var}(y)}\right]^2}{\frac{\text{var}(y)}{1 - 2\text{var}(y)}}\right\}}{\left[2\pi \frac{\text{var}(y)}{1 - 2\text{var}(y)}\right]^{\frac{1}{2}}} dy. \end{aligned}$$

The integral is unity, since the integrand is the density of  $N\{\mathbb{E}(y) + \text{cov}(x, y)/[1 - 2\text{var}(y)], \text{var}(y)/[1 - 2\text{var}(y)]\}$ . So applying the law of iterated expectations to (A.47) yields (A.46). ///

#### Noise trader utility for $\sigma_\nu^2 > 0$ :

A noise trader's expected utility conditional on  $\nu$  is

$$\mathbb{E}[U(\pi_N) | \nu] = -\exp(-\rho e_N) \exp\left[-\rho(\theta - P)\frac{\nu}{N} \mid \nu\right].$$

Applying (A.1) or (A.46) with  $x = (\theta - P)\nu/N$  (which is normal) and  $y = 0$  yields

$$\mathbb{E}[U(\pi_N) | \nu] = -\exp(-\rho e_N) \exp\left[-\rho\frac{\nu}{N}\mathbb{E}(\theta - P | \nu) + \frac{1}{2}\left(\rho\frac{\nu}{N}\right)^2 \text{var}(\theta - P | \nu)\right]$$

or, using  $\mathbb{E}(\theta | \nu) = \bar{s}$  and the standard updating rules,

$$\begin{aligned} \mathbb{E}[U(\pi_N) | \nu] &= -\exp(-\rho e_N) \exp\left(\overbrace{-\rho\frac{\nu}{N}\left[\bar{s} - \mathbb{E}(P) + \frac{\text{cov}(P, \nu)}{\sigma_\nu^2}\bar{\nu}\right]}^{=\Phi}\right) \\ &+ \underbrace{\rho\left(\frac{\nu}{N}\right)^2\left\{\frac{\text{cov}(P, \nu)}{\sigma_\nu^2}N + \frac{\rho}{2}\left[\text{var}(\theta - P) - \frac{(\text{cov}(\theta - P, \nu))^2}{\sigma_\nu^2}\right]\right\}}_{=\Psi^2}. \end{aligned}$$

Let  $\Phi$  be defined as the first term in the sum in the second exponential and  $\Psi$  as the square root of the

second term, so that

$$\begin{aligned}
\mathbb{E}(\Phi) &= -\rho \frac{\bar{\nu}}{N} \left[ \bar{s} - \mathbb{E}(P) + \frac{\text{cov}(P, \nu)}{\sigma_\nu^2} \bar{\nu} \right] \\
\text{var}(\Phi) &= \left[ \frac{\mathbb{E}(\Phi)}{\bar{\nu}} \right]^2 \sigma_\nu^2 \\
\mathbb{E}(\Psi) &= \rho^{\frac{1}{2}} \frac{\bar{\nu}}{N} \left\{ \frac{\text{cov}(P, \nu)}{\sigma_\nu^2} N + \frac{\rho}{2} \left[ \text{var}(\theta - P) - \frac{(\text{cov}(\theta - P, \nu))^2}{\sigma_\nu^2} \right] \right\}^{\frac{1}{2}} \\
\text{var}(\Psi) &= \left[ \frac{\mathbb{E}(\Psi)}{\bar{\nu}} \right]^2 \sigma_\nu^2 \\
\text{cov}(\Phi, \Psi) &= \frac{\mathbb{E}(\Phi)\mathbb{E}(\Psi)}{\bar{\nu}^2} \sigma_\nu^2.
\end{aligned}$$

Since both  $\Phi$  and  $\Psi$  are normal, from the law of iterated expectations and (A.46),

$$\mathbb{E}[U(\pi_N)] = -\exp(-\rho e_N) \frac{\exp \left\{ \mathbb{E}(\Phi) + \frac{1}{2} \text{var}(\Phi) + \frac{[\mathbb{E}(\Psi) + \text{cov}(\Phi, \Psi)]^2}{1 - 2 \text{var}(\Psi)} \right\}}{[1 - 2 \text{var}(\Psi)]^{\frac{1}{2}}}. \quad (\text{A.48})$$

In the presence of dealers  $\mathbb{E}(P)$  is given by (A.17). From (A.23), the other moments in the definitions of  $\Phi$  and  $\Psi$  are

$$\begin{aligned}
\text{cov}(P, \nu) &= \frac{1 + \alpha\beta\gamma\sigma_s^2}{\alpha + \beta} \sigma_\nu^2 \\
\text{var}(\theta - P) &= \left( 1 - \alpha \frac{1 + \alpha\beta\gamma\sigma_s^2}{\alpha + \beta} \right)^2 \sigma_s^2 + \left( \frac{1 + \alpha\beta\gamma\sigma_s^2}{\alpha + \beta} \right)^2 \sigma_\nu^2 + \sigma_\varepsilon^2 \\
\text{cov}(\theta - P, \nu) &= -\text{cov}(P, \nu).
\end{aligned}$$

Given (A.15) and these formulas,  $\mathbb{E}[U(\pi_N)]$  can be expressed as a composite function of  $L_E$  alone.

In the absence of dealers  $\mathbb{E}(P)$  is given by (A.25). From (8) with  $\nu$  instead of  $\bar{\nu}$ , the other moments in the definitions of  $\Phi$  and  $\Psi$  are

$$\begin{aligned}
\text{cov}(P, \nu) &= \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{L + M} \sigma_\nu^2 \\
\text{var}(\theta - P) &= \sigma_s^2 + \sigma_\varepsilon^2 + \left[ \frac{\rho(\sigma_s^2 + \sigma_\varepsilon^2)}{L + M} \right]^2 \sigma_\nu^2 \\
\text{cov}(\theta - P, \nu) &= -\text{cov}(P, \nu).
\end{aligned}$$

Given these formulas,  $\mathbb{E}[U(\pi_N)]$  can be expressed as a composite function of  $L_E$  alone.

$\mathbb{E}[U(\pi_N)]$  goes to minus infinity as  $\text{var}(\Psi)$  rises towards  $1/2$ . For  $\text{var}(\Psi) \geq 1/2$ , noise trader expected utility is undefined.

It is easily checked that  $\mathbb{E}[U(\pi_N)]$  is homogeneous of degree zero in  $\sigma_\nu$ ,  $\bar{\nu}$ ,  $L$ ,  $M$ ,  $N$ , and  $L_E$ .

**Continuity of noise trader expected utility at  $\sigma_\nu^2 = 0$ :**

For  $L_E < L$ , taking the limits as  $\sigma_\nu^2 \rightarrow 0$  in the moments which appear in (A.48) yields (A.38):

$$\begin{aligned}
\text{cov}(P, \nu) &\rightarrow 0 \\
\text{var}(\theta - P) &\rightarrow \sigma_\varepsilon^2 \\
\text{cov}(\theta - P, \nu) &\rightarrow 0 \\
\text{E}(\Phi) &\rightarrow -\rho \frac{\bar{\nu}}{N} \left[ \frac{\rho \sigma_\varepsilon^2}{L + M} \left( \frac{L_E}{a} - \bar{\nu} \right) + \frac{\rho \sigma_\varepsilon^2}{L - L_E} \bar{\nu} \right] \\
\text{var}(\Phi) &\rightarrow 0 \\
\text{E}(\Psi) &\rightarrow \rho^{\frac{1}{2}} \frac{\bar{\nu}}{N} \left( \frac{\rho \sigma_\varepsilon^2}{L - L_E} N + \frac{\rho \sigma_\varepsilon^2}{2} \right)^{\frac{1}{2}} \\
\text{var}(\Psi) &\rightarrow 0 \\
\text{cov}(\Phi, \Psi) &\rightarrow 0.
\end{aligned}$$

Substitution of the analogous expressions for the case  $L_E = L$  into (A.48) yields (A.39):

$$\begin{aligned}
\text{cov}(P, \nu) &\rightarrow 0 \\
\text{var}(\theta - P) &\rightarrow \sigma_s^2 + \sigma_\varepsilon^2 \\
\text{cov}(\theta - P, \nu) &\rightarrow 0 \\
\text{E}(\Phi) &\rightarrow -\frac{\rho^2 \bar{\nu} (\sigma_s^2 + \sigma_\varepsilon^2) L}{N(L + M) a} \\
\text{var}(\Phi) &\rightarrow 0 \\
\text{E}(\Psi) &\rightarrow \rho^{\frac{1}{2}} \frac{\bar{\nu}}{N} \left[ \frac{\rho (\sigma_s^2 + \sigma_\varepsilon^2)}{L + M} N + \frac{\rho (\sigma_s^2 + \sigma_\varepsilon^2)}{2} \right]^{\frac{1}{2}} \\
\text{var}(\Psi) &\rightarrow 0 \\
\text{cov}(\Phi, \Psi) &\rightarrow 0.
\end{aligned}$$

**Specification of  $\sigma_s^2$  in the numerical analysis:**

The CE of  $\pi_E$  is  $\text{E}(\pi_E) - (\rho/2)\text{var}(\pi_E)$ , and the lower bound of the 95-percent confidence interval for  $\pi_E$  is  $\text{E}(\pi_E) - 2\text{var}(\pi_E)^{1/2}$ , so the CE is not below the lower bound if  $\text{var}(\pi_E) < 16/\rho^2$ .

Using (A.2) and (A.9), the variance of  $\pi_E = e_L + P/a + (\theta - P)I_E$  in  $\mathbf{M}_0^1$  is

$$\text{var}(\pi_E) = \frac{\sigma_s^2}{a^2} + \left( \frac{L_E - a\bar{\nu}}{L + M} \right)^2 \frac{\sigma_\varepsilon^2}{a^2}.$$

It is bounded below by  $\sigma_s^2/a^2$ . As  $L_E \leq L$  and  $M \geq L$ , the term in parentheses is no greater than 0.5. So, given  $\sigma_\varepsilon^2 \leq \sigma_s^2$ , the variance is bounded above by  $1.25\sigma_s^2/a^2$ .

**Remarks to Result 7.1:**

*Remark A.7.1.1.* SW is higher without than with OC not only in the majority of cases but also by a large amount. For  $\sigma_\nu$  up to  $0.2L/a$ , the ratio of the two SW levels is greater than ten on average. This huge

difference is due to the fact that equilibrium SW without OC is very close to its maximum, which is far greater than equilibrium SW with OC.

*Remark A.7.1.2.* The overall conclusion that the allocation of talent to finance tends to be excessive does not hinge on the use of the *transformed* expected utilities (i.e., CEs) in the calculation of SW. With SW defined as the sum of all agents' *untransformed* expected utilities, the figures in columns 3 and 4 of Table 2 are similar.

*Remark A.7.1.3.* Hipos earn sizable rents compared to passive investors in the model specifications with positive noise trader shocks (contrary to the noiseless case; cf. Remarks A.4.1.6 and A.4.3.1). To see this, consider the ratio of the CEs for entrepreneurs and passive investors ( $-\log\{-E[U(\pi_E)]\}/(-\log\{-E[U(\pi_M)]\})$ ). This ratio equals 8.96 on average for  $\sigma_\nu = 0.001L/a$  and rises strongly with increases in the volatility of noise trader demand (the differences are much less pronounced in terms of untransformed expected utility, however).

There are two further interesting outcomes of the simulations. First, equilibrium asset price volatility tends to be lower rather than higher in the presence of noise trader shocks. The direct positive impact of volatility of noise trader demand  $\nu$  on the asset price  $P$  is more than offset by the effect that noise makes  $P$  less sensitive to the macro fundamentals  $s$ . Second, the equilibrium mass of entrepreneurs is always higher in the absence of dealers.

### SW with and without OC in the numerical analysis:

Table A.1 explains by how much and why SW is higher without than with OC.  $\check{S}^0$  denotes maximum SW without OC.  $\Delta(x, y) = |x - y| / \max\{|x|, |y|\}$  denotes the absolute difference between  $x$  and  $y$  relative to the greater of the two.<sup>18</sup> Since the domain of  $\Delta$  is usually confined to  $x$  and  $y$  with the same sign (see Törnqvist et al., 1980, p. 3), we calculate  $\Delta(x, y)$  only for the subset of cases in which this condition is satisfied, thereby losing between 16.7% (for  $\sigma_\nu = 0.001L/a$ ) and 88.4% (for  $\sigma_\nu = 0.5L/a$ ) of the cases stated in the second column of Table 2 in the main text. For a given value of  $\sigma_\nu$  relative to  $L/a$ , column 2 of Table A.1 reports the average difference between the equilibrium SW levels without and with OC (standard deviations in percentage points in parentheses). When  $S^0(L_E^0)$  is positive and greater than  $S^1(L_E^1)$ ,  $\Delta(S^1(L_E^1), S^0(L_E^0)) > 0.9$  implies  $S^0(L_E^0) > 10S^1(L_E^1)$ . Hence, for  $\sigma_\nu$  up to  $0.2L/a$ , SW is more than ten times as large without than with OC on average. Columns 3 and 4 give the average difference between equilibrium SW with OC and maximum SW without OC and the average difference between equilibrium and optimum SW without OC, respectively. Here we additionally restrict attention to cases where noise trader expected utility is bounded not only at equilibrium (condition  $\text{BNU}_\sigma$ ) but for all  $L_E$  and where  $S^0(L_E^0)$  and  $\check{S}^0$  have the same sign (this costs between 0 and 152 cases for given  $\sigma_\nu$ ). The large difference between the equilibrium levels of SW without and with OC for  $\sigma_\nu \leq 0.2L/a$  is due to the fact that equilibrium SW without OC is very close to its maximum (column 4), which is far greater than equilibrium SW with OC (column 3).

### Implementation

To implement second-best allocations via subsidies and taxes, if there are too few entrepreneurs and too many dealers, what is required is a subsidy to the former and/or a tax on the latter. Note that a tax on dealers is not a tax on trading, as it is not levied on other agents' asset trades. While this distinction is

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<sup>18</sup>This measure ranges between 0 and 100 percent. Symmetry and boundedness make it preferable to a simple percentage difference within our setting.



Table A.1: Matlab simulation of  $M_\sigma^j$ 

$\frac{\sigma_\nu}{L/a}$	$\Delta(S^1(L_E^1), S^0(L_E^0))$	$\Delta(S^1(L_E^1), \check{S}^0)$	$\Delta(S^0(L_E^0), \check{S}^0)$
0.001	93.82% (09.50%)	93.92% (09.50%)	0.00% (0.00%)
0.01	93.29% (10.14%)	93.29% (10.14%)	0.00% (0.00%)
0.05	91.57% (12.32%)	91.57% (12.31%)	0.00% (0.06%)
0.1	90.34% (13.35%)	90.39% (13.31%)	0.05% (0.80%)
0.2	90.29% (14.09%)	90.51% (13.44%)	0.39% (4.40%)
0.5	50.06% (28.06%)	47.94% (28.51%)	1.01% (5.92%)

clear in theory, a tax on informed but not on uninformed trading would be hard to implement in practice. We think of it as a proxy for fiscal and regulatory measures aimed at constraining agents and institutions specialized in trading securities in secondary markets.

The implementation of the SW maximum is easy in the model without noise trader shocks. Suppose there are (possibly negative) lumpsum taxes  $t_i$  on type- $i$  agents ( $i = E, D, M, N$ ). (For simplicity, the tax on hipos who act as passive investors is zero.) Consider first models  $M_0^j$  and  $F_0^j$ . For the sake of brevity, assume that without taxation there are unique equilibria with and without OC and  $L_E^1 < L$ . With OC any  $t_D > \Gamma_0(L)/\rho$  ( $> 0$ ) is a prohibitive tax on dealers, given that all other tax rates are zero, since it exceeds the gains from being informed GI, irrespective of whether  $L_E < L$  (so that  $GI = 0$ ) or  $L_E = L$  (so that  $GI = \Gamma_0(L)$ ). So the model with  $t_D > \Gamma_0(L)$  behaves exactly like the model without OC. From Propositions 6.1 and 6.2, this is sufficient to implement the constrained-efficient allocation. As the tax base is zero, the budget is balanced. In model  $U_0^j$ , the second-best optimal mass of entrepreneurs,  $\hat{L}_E^0$  say, exceeds the free markets equilibrium value  $L_E^0$  (see Proposition 6.1). The equilibrium mass of entrepreneurs is determined by  $\Delta_0^0(L_E) - \rho t_E = 0$  (or it is equal to  $L$ , if  $\Delta_0^0(L) - \rho t_E > 0$ ), so the optimum value is achieved with  $t_E = \Delta_0^0(\hat{L}_E^0)/\rho$  ( $< 0$ ). That is, a tax on dealers and a subsidy to entrepreneurs are required in order to implement the second-best optimum with labor market frictions. The budget can be balanced by taxing workers and/or noise traders:  $t_M M + t_N N = -t_E \hat{L}_E^0$  ( $> 0$ ).

The following result states how to implement an allocation with  $L'_E$  entrepreneurs in the presence of noise:

**PROPOSITION A.7.3.** *Let the mass of dealers be zero and  $I_D$  arbitrary. Given  $L'_E$  ( $0 < L'_E \leq L$ ), let  $t_D > \Gamma(L)/\rho$ ,  $t_E = \Delta^0(L'_E)/\rho$ ,  $t_M \geq 0$ , and  $t_M M + t_N N = -t_E L'_E$ . Then: (i)  $L'_E, I_E$  given by (A.11),  $I_D, I_M = I_E$ , and  $P$  given by (8) with  $\nu$  instead of  $\bar{\nu}$  are an equilibrium of  $M_\sigma^1$ . (ii) With  $s$  and  $\bar{s}$  given by (12),  $(L'_E, I_D, I_E, I_M, P, \hat{M}, \hat{W})$  is an equilibrium of  $F_\sigma^1$ . (iii) With  $\bar{s}$  given by (14),  $(L'_E, I_D, I_E, I_M, P, \tilde{M}, \tilde{W})$  is an equilibrium of  $U_\sigma^1$ .*

The reasoning is the same as in the noiseless case. As  $\Gamma(L_E)$  is strictly increasing, the tax  $t_D$  ensures that no-one chooses to be a dealer and makes the models with OC and taxes behave like the ones without OC and taxes. The tax  $t_E$  (or subsidy  $-t_E$ ) implies that  $L'_E$  hipos decide to become entrepreneurs. And  $t_M$  and  $t_N$  balance the budget, where non-negativity of  $t_M$  ensures that hipos are not better-off as passive investors than as entrepreneurs.

*Remark A.7.3.1.* Suppose the maximum of  $S^0(L_E)$  exceeds the maximum of  $S^1(L_E)$ , for instance because the conditions of Proposition 6.2 are satisfied and  $\sigma_\nu^2$  is small enough. Setting  $L'_E$  equal to the value  $\hat{L}_E^0$

that maximizes  $S^0(L_E)$ , the taxes in the proposition implement the constrained-optimal solution. The tax on dealers  $t_D$  is positive. For  $\sigma_v^2$  small, the tax on entrepreneurs  $t_E$  is small in the absence of labor market frictions and negative in  $U_\sigma^1$  (viz., close to the noiseless case).

*Remark A.7.3.2.* The tax on dealers in the proposition is prohibitive. In practice, taxes and regulations are likely not to be aimed at shutting down professional trading altogether. Given non-prohibitive taxes, the equilibrium mass of entrepreneurs  $L_E$  is determined by  $\Delta(L_E) - \rho t_E = \Gamma(L_E) - \rho t_D$ . Given a target level of entrepreneurship  $L'_E$ , this equality gives the tax differential needed to implement  $L'_E$ :  $t_D - t_E = [\Gamma(L'_E) - \Delta(L'_E)]/\rho$ . Suppose, as usual, that  $\Delta(L_E)$  intersects  $\Gamma(L_E)$  from above and the intersection is unique (as in Figure 3). Then, in order to achieve  $L'_E$  higher than the equilibrium level without taxes, a preferential tax treatment of entrepreneurship compared to trading is required:  $t_D - t_E > 0$ . This can be achieved by taxing dealers and/or by subsidizing entrepreneurship.

### Additional references

Törnqvist, L., Vartia, P., and Vartia, Y. (1980), “How should relative changes be measured?”, *Keskusteluaiheita Discussion Paper, No. 68*, Elinkeinoelämän Tutkimuslaitos Research Institute of the Finnish Economy, Helsinki.