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A unified framework for CBDC design: remuneration, collateral haircuts and quantity constraints

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A unified framework for CBDC design:
remuneration, collateral haircuts
and quantity constraints

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Abstract

We study the macroeconomic effects of central bank digital currency (CBDC) in a dynamic general equilibrium model. Timing and information frictions create a need for inside (bank deposits) and outside money (CBDC) to finance production. To steer the quantity of CBDC, the central bank can set the lending and deposit rates for CBDC as well as collateral and quantity requirements. Less restrictive provision of CBDC reduces bank deposits. A positive interest spread on CBDC or stricter collateral or quantity constraints reduce welfare but can contain bank disintermediation, especially if the elasticity of substitution between bank deposits and CBDC is small.

Keywords: Central bank digital currency, monetary policy, search and matching.

JEL Classification: E58, E41, E42, E51, E52.

Non-technical summary

The digital payments landscape has evolved rapidly over the past years. The emergence of crypto-assets and big tech companies' reflections on issuing private currencies have prompted concerns about safety and data protection related to private currencies. As a response to these developments, central banks have started their own work programmes to assess the prospects of issuing central bank digital currency (CBDC) for retail transactions. The issuance of a CBDC, however, entails important implications for monetary policy implementation, monetary policy transmission and financial stability, which depend on the specific design features of CBDC, such as remuneration, holding limits or the choice and the pricing of assets held against CBDC.

To study these questions, we construct a general equilibrium model with search and matching frictions, which require entrepreneurs to borrow inside money (bank deposits) and outside money (CBDC). These two types of money are needed to pay for two different kinds of inputs that are used in production. The central bank chooses the interest rate it charges on loans to entrepreneurs and the interest rate it pays on workers' deposits of CBDC. It can also set a limit on the size of each loan and apply a haircut to future revenue posted as collateral for a loan. The model reflects a setup where CBDC exists in equilibrium, as postulated by the production function, while it allows us to analyse the impact of the different CBDC design parameters on credit allocation and welfare within a unified framework.

The equilibrium allocation under the first-best solution is characterised by CBDC being unconstrained by neither collateral requirements nor a quantitative cap and by its lending spread being zero. With these CBDC policy parameter settings the central bank can eliminate welfare losses arising from the matching friction in the investment market. Restrictions on the supply of CBDC give rise to inefficiently low investment and, consequently, production. This result follows from production requiring an input that is produced by workers remunerated in CBDC and costly to substitute. Consequently, welfare gains from CBDC depend on the degree of substitutability of the production inputs paid for in inside or outside money. For increasing degrees of substitution between both forms of money, welfare gains from CBDC decline, approaching zero if both types of money are perfect substitutes.

Furthermore, we analyse how CBDC design choices affect bank lending, which we interpret as evidence for the potential of CBDC to disintermediate the banking sector. If the degree of substitution between inside money and outside money is relatively high, a higher interest

rate spread on CBDC and stricter collateral requirements for CBDC increase bank lending. If substitution is low, bank lending falls together with — although less than — CBDC demand as the latter is now more difficult to replace by inside money. Setting a fixed cap for CBDC loans unambiguously increases the demand for bank loans. Overall, the central bank therefore can contain bank disintermediation by adjusting its policy parameters which, however, may lead to losses in output and welfare.

We leave the impact of CBDC on monetary policy transmission over the business cycle or on financial stability to future research. In our setting, prices are flexible and money is neutral. In addition, assets are safe and liquid. Our modelling framework is therefore informative of the impact of CBDC on the steady state, i.e. structural changes in the financial system.

1 Introduction

The digital payments landscape has evolved rapidly over the past years. The emergence of crypto-assets and big tech companies' reflections about issuing private currencies have prompted concerns about safety and data protection related to private currencies. A widespread usage of private currencies could also entail risks to monetary sovereignty and raise financial stability concerns. As a response to these developments, central banks have started their own work programmes to assess the prospects of issuing central bank digital currency (CBDC) for retail transactions. A recent survey by the Bank for International Settlements (BIS) (Boar and Wehrli, 2021) found that central banks representing a fifth of the world's population are likely to issue a general purpose CBDC in the next three years, demonstrating a further significant advancement in these projects compared to two earlier surveys (Barontini and Holden, 2019; Boar et al., 2020).

Central banks' motivations for issuing CBDC emanate from its potential to provide a secure, efficient and universally accessible means of payment for everybody (European Central Bank, 2020; Bank for International Settlements, 2020). Beyond this primary motivation, however, the existence of CBDC entails important implications for monetary policy implementation, monetary policy transmission and financial stability.¹ These implications depend on the specific design features of CBDC, such as remuneration, holding limits or the choice and the pricing of assets held against CBDC. These parameters may be set by the central bank in such a way that undesired consequences for monetary transmission and financial stability are mitigated, see e.g. Bindseil (2020); Bindseil and Panetta (2020). Little is known, however, about the effectiveness of these parameters in steering the demand for CBDC and the resulting macroeconomic effects; in particular the implications that variations of them may have on equilibrium allocations and welfare.

To study these questions, we construct a general equilibrium model in the spirit of Lagos and Wright (2005) with search and matching frictions, which require entrepreneurs to borrow inside and outside money. These two types of money – inside money in the form of bank deposits and outside money in the form of CBDC – are needed to pay for two different kinds of inputs that are needed for production. The setup can be motivated by preferences for specific forms of payment on part of the workers who produce these inputs. Some workers, for instance, may hold bank accounts that allow them to access a range of bank services and they therefore prefer

¹ The literature on monetary policy and financial stability effects of CBDC is evolving quickly and includes Pollock (2018); Carstens (2019); Mancini-Griffoli et al. (2018); Agur et al. (forthcoming).

deposits although these may offer less protection of private payment data. Other workers, in contrast, may prefer the security of CBDC, which is a liability of the central bank although it may not give access to services that banks typically provide for their customers. Alternatively, this modelling setup can be motivated by spatial separation, where some workers produce and sell their inputs (goods or services) in locations where access to bank deposits is limited whereas others operate in locations in which they dislike accepting CBDC.² In our model, we do not consider cash. The model reflects a setup where CBDC exists in equilibrium, as postulated by the production function, while the macroeconomic effects of different central bank policies with respect to the parameters governing CBDC supply can be studied.

The model comprises three markets that operate sequentially: a settlement, a loan and an investment market. In the settlement market, which constitutes a centralised market, banks, firms, and workers produce and consume a generic good and settle borrowing contracts. Entrepreneurs live for one period only and are born during the loan market with an investment opportunity and a small endowment. Each period, entrepreneurs receive an idiosyncratic shock that determines the optimal size of their production. To realise their projects they need capital goods that are produced by workers. As entrepreneurs cannot commit to repay debt, workers in the investment market require payment in the form of a generally accepted medium of exchange; a friction that gives rise to the demand for money. Therefore, in the loan market, the entrepreneurs borrow inside and outside money from commercial banks and from the central bank. But, because of the limited commitment friction, they are required to pledge collateral.³

The capital goods produced in the (decentralised) investment market are nonstorable and, therefore, cannot be carried from one market to the next. There are two types of workers who produce capital: A worker of type C only accepts CBDC as payment, whereas type D workers only accept commercial bank deposits. An entrepreneur needs to borrow from both a commercial bank and the central bank to be able to buy the inputs needed for production. The central bank chooses the interest rate it charges on loans to entrepreneurs and the interest rate it pays on workers' deposits of CBDC. It can also set a limit on the size of each loan and apply a haircut to future revenue posted as collateral for a loan.

Our modelling approach allows for analysing the impact of the different CBDC design

² As in Duffie et al. (2005) the demand for different monetary assets could also be motivated by the different monetary assets generating different flows of disutility or by originating from spatial separation as in Diamond (1982) where production and trade occurs on distinct island economies.

³ In the baseline version of the model we assume that the central bank and commercial banks impose the same collateral requirements. Section 5.2 investigates the case of only the central bank requiring collateral.

parameters on credit allocation and welfare within a unified framework. Interest rates and haircuts already exist in today's central banks' operational frameworks and are used to influence the demand for central bank reserves (see Sylvestre and Coutinho (2020) for the Eurosystem). The main difference of the existence of CBDC to today's framework would be that also non-banks had to pledge collateral and could earn interest (although potentially at a negative rate) on their holdings of central bank liabilities. A quantity constraint on CBDC loans does not exist today but is one of the safeguards that have been proposed to limit potential adverse effects of CBDC on monetary policy transmission and financial stability (European Central Bank, 2020; Bank for International Settlements, 2020).⁴

The equilibrium allocation under the first-best solution is characterised by CBDC being unconstrained by collateral requirements or a quantitative cap and by the outside money lending spread being zero.⁵ With these CBDC policy parameter settings the central bank can eliminate welfare losses arising from the matching friction in the investment market. If the central bank imposes an interest rate spread between the CBDC lending and deposit rates or collateral or quantity constraints, some investment projects that otherwise would be profitable at market interest rates are not financed, giving rise to inefficiently low investment and consequently production. This result follows from production requiring an input that is produced by workers requiring CBDC and which is costly to substitute. Consequently, welfare gains from CBDC depend on the degree of substitutability of production inputs paid for in inside or outside money. For increasing degrees of substitution between both forms of money, welfare gains from CBDC decline, approaching zero if both types of money are perfect substitutes.

Besides the effects on the capital allocation and welfare, we are interested in how CBDC design choices affect bank deposits, which we interpret as evidence of the potential of CBDC to disintermediate bank lending. With this notion we want to capture the so-called "structural" disintermediation resulting from the existence of CBDC (see Bindseil (2020)). If the degree of substitution between commercial bank credit and CBDC is relatively high, a higher interest rate spread and stricter collateral requirements for CBDC increase bank lending. If the degree of substitution is low, bank lending falls together with — although less than — CBDC demand as

⁴ Our model considers a quantity constraint as a measure to prevent a large demand for CBDC at the expense of bank deposits. Quantity constraints are also proposed to address concerns related to money laundering or terrorist financing. These issues, however, are more closely related to the question of privacy for CBDC transactions, which our model does not capture.

⁵ The spread between the lending and deposit rate for inside money turns out to be irrelevant for the capital allocation and welfare as interest paid on inside money is internalised in a bargaining setup between the firm and the bank.

the latter is now more difficult to replace by inside money. Setting a fixed cap for CBDC loans unambiguously increases the demand for bank loans. Overall, the central bank therefore can contain bank disintermediation by adjusting its policy parameters which, however, may lead to losses in output and welfare.

Within the rapidly expanding literature on CBDC, our approach falls into a class of new-monetarist models assigning an essential role to money on account of search and matching frictions, as in Lagos and Wright (2005). Accordingly, our approach shares some important commonalities with the model by Keister and Sanches (2019) featuring perfectly competitive banks and credit frictions. CBDC is possibly interest bearing, might crowd out bank deposits, and leads to an increase in welfare by reducing credit frictions. By comparison, in addition to alleviating matching frictions, Chiu et al. (2019) and Andolfatto (2018) derive further welfare improvements from CBDC through reductions in the market power of an imperfectly competitive banking sector. A common feature with these papers is that in our model investment projects succeed with certainty and all assets are safe. This assumption contrasts with Williamson (2020), where bank deposits mitigate the risk of theft, and Hu and Rocheteau (2015) and Lagos and Zhang (2020), where deposits obscure the risk of the entire asset endowment disappearing with a certain probability.

Yet, there are some important differences. While in Keister and Sanches (2019) some investment projects may be socially wasteful, a credit friction that can be alleviated by introducing CBDC can lead to inefficiently low level of deposits and exchange in the decentralised market. In our case, all investment projects are socially valuable, with banks and entrepreneurs negotiating on lending volumes and rates. Furthermore, in Keister and Sanches (2019) the central bank fixes the price level by buying or selling goods against cash or CBDC, whereas we assume that entrepreneurs borrow from the central bank and commercial banks, allowing us to focus on safeguards such as quantity or collateral constraints in addition to variations in the interest rate on CBDC.

Within the class of models with search and matching frictions, Davoodalhosseini (2019) also analyses welfare implications from CBDC, but the model does not feature credit, and welfare changes are rather derived from the central bank being able to accommodate different household preferences with respect to holding cash or CBDC.

An alternative modelling approach to analysing welfare implications from CBDC rests on including money in the utility function originating from Sidrauski (1967). Niepelt (2020) finds

that CBDC is neutral with respect to the allocation of capital in the economy. As in Brunnermeier and Niepelt (2019), a shift of deposits into CBDC in the event of a crisis triggers the central bank's lender of last resort role, resulting in a swap of banks' funding sources from private sector deposits to central bank credit. With CBDC resulting in a mere change in the composition of bank funding, resource allocation may not be affected and bank funding may even be more stable. A similar neutrality result due to an explicit lender of last resort role of the central bank is modelled in Gross and Schiller (2020) who incorporate CBDC in a New Keynesian DSGE model featuring money in the utility function as well.

Our approach also differs from studies based on bank-run models in the tradition of Diamond and Dybvig (1983), showing under which conditions CBDC can either pose a threat to financial stability or reinforce it through more stable bank funding conditions. In Fernandez-Villaverde et al. (2020) the central bank is competing with commercial banks for deposits. As depositors internalise the possibility of a banking panic, CBDC will crowd out bank deposits entirely. The same result is found in Böser and Gersbach (2020), although CBDC, due to the threat of bank runs, enhances monitoring incentives and enforces higher market discipline in the short run.

Without money playing an essential role, DSGE models do not lend themselves easily to analysing welfare effects from CBDC. A prominent example is Barrdear and Kumhof (2016) who model CBDC as an additional short-term central bank liability and issued against longer-term government bonds. Like quantitative easing, such government bond purchases lower the free-float of bonds, thereby sovereign yields, and raise GDP. In this vein, economic benefits arise from a large central bank asset portfolio rather than the change in the composition of the central bank's liabilities.⁶ Likewise, in Bitter (2020) macro-financial effects from assets backing CBDC are found to alleviate bank run risk and thereby support financial stability. Ferrari et al. (2020) show that in an open economy DSGE model the introduction of CBDC amplifies international spillovers of shocks.

⁶ Accordingly, Kumhof and Noone (2018) suggest to stave off fears of bank disintermediation risk by making CBDC distinct from reserves, bank deposits, and cash. Specifically, they stipulate that CBDC should be issued against eligible securities and be nonconvertible into cash or reserves, thus removing the key feature of what constitutes money. It thereby is conceived to become a separate policy instrument.



Figure 1
Market sequencing

2 The model environment

2.1 Market sequencing

There are three types of agents in the model: workers, entrepreneurs, and bankers. Workers and bankers are infinitely lived and have a measure of 1. Entrepreneurs live for one period only and have a measure of 1 as well. Time is discrete and the discount factor across periods is $0 < \beta < 1$.

In each period three markets open sequentially (see Figure 1): a *settlement market*, where credit contracts are settled and a generic good is produced and consumed; a *loan market*, where borrowing takes place; and an *investment market*, where the production of capital goods takes place. All goods are perfectly divisible and nonstorable, which means that they cannot be carried from one market to the next. There are two perfectly divisible financial assets: deposits (inside money) and central bank digital currency (CBDC; virtual outside money).

In the *settlement market*, a generic good can be produced and consumed by all agents. They all have the same constant returns to scale production technology with one unit of the good being produced with one unit of labour generating one unit of disutility. Thus, producing h units of generic goods implies disutility $-h$. Furthermore, for all agents the utility of consuming x units of the generic goods yields utility x . As in Lagos and Wright (2005), these assumptions yield a degenerate distribution of portfolios at the beginning of the loan market.⁷

In the *loan market*, entrepreneurs are born with an investment opportunity. To implement their projects they need capital goods that are produced by the workers in the investment market. Since entrepreneurs cannot commit to repay debt, the workers require payment in the form of a generally accepted medium of exchange. Thus, this limited commitment friction generates a demand for money. The entrepreneurs borrow money from the central bank and the commercial banks. Due to the limited commitment friction, lenders require collateral. There are two types of collateral: future output y and an endowment \tilde{e} that entrepreneurs receive when they are born

⁷ All agents have the same production technology for the generic good and the same utility function in the settlement market. However, in equilibrium, workers and bankers will consume only and entrepreneurs will sell a fraction of their output for money in this market.

in the loan market. Since the entrepreneur can divert a fraction $1 - \bar{\eta}$ of the output y , only a fraction $\bar{\eta}$ of future output is available as collateral. The endowment \tilde{e} is idiosyncratic and yields discounted utility $e = \beta\tilde{e}$ to the entrepreneur when consumed in the settlement market. It has no consumption utility to any other agent in the model.

In the loan market entrepreneurs are matched with bankers pairwise at random. Since both types of agents have measure 1 we assume that every banker and every entrepreneur has a match. A banker-entrepreneur pair bargains about the size of the loan and the interest rates. The terms of the loan are determined according to a surplus splitting rule.⁸

In the *investment market*, capital goods k are produced by workers and bought by the entrepreneurs, with an investment of k yielding

$$y = \beta^{-1}\varepsilon^{1-\alpha}f(k) \tag{1}$$

units of goods in the following settlement market. Capital $k = (k_C, k_D)$ consists of two different types, k_C and k_D , that have to be paid for by the entrepreneur either with CBDC, denoted by the subscript C , or with deposits, D . The function $f(k)$ satisfies

$$f(k) = (ak_C^\rho + (1-a)k_D^\rho)^{\alpha/\rho} \tag{2}$$

with a being the input share of k_C and k_D in the production function, $0 < \alpha < 1$ the degree of homogeneity and $\rho \leq 1$ the substitution parameter.⁹ The term ε is an idiosyncratic investment shock that becomes known at the beginning of the loan market when entrepreneurs are born. The investment shock has a continuous log-normal distribution $G(\varepsilon)$ with support $(0, \infty)$, is i.i.d. across banks and serially uncorrelated. The investment shock generates a distribution of investment projects that allows us to study how CBDC design affects firms or investment projects of different size in the economy.

The capital goods are produced and sold by two types of workers at competitive prices. Some workers can produce k_C and live in a location where they do not have access to banking services. We call this location island C . Others can produce k_D and live on island D . The precise measure

⁸ We consider the bargaining setup to capture important features of interest rate determination. A perfectly competitive loan market, in which all firms face the same risk-adjusted interest rate, does not exist in practice. Interest rates are rather negotiated between borrowers and lenders with the agreed rate depending on firm characteristics and the state of the economy.

⁹ We assume that the project succeeds with certainty. However, this would be easy to relax.

of workers on these islands is irrelevant since we will specify preferences and technologies such that the workers are indifferent with regard to how much to produce.

Holding money i provides the flow disutility φ_{ij} to workers living on island j .¹⁰ We assume that workers that can produce k_C dislike inside money but are neutral towards CBDC:

$$\varphi_{CC} = 0 \text{ and } \varphi_{DC} \ll 0.$$

In contrast, workers that can produce k_D dislike CBDC and are neutral towards inside money:

$$\varphi_{CD} \ll 0 \text{ and } \varphi_{DD} = 0.$$

These flow utilities generate a demand for both assets even when the rates of return of the two monies are not equal.¹¹ Nevertheless, in our model the flow utilities are irrelevant for investment decisions when k_C and k_D are perfect substitutes, i.e. $\rho \rightarrow 1$. In this case, the two monies can coexist only if they have the same rate of return because entrepreneurs will exclusively use the type of money that is less costly for them.

In this paper, we assume that φ_{DC} and φ_{CD} are sufficiently large negative numbers such that on island j workers only accept money $j = C, D$.¹² Note further that we assume that each worker on an island produces the same amount and each bank has the same customer base on island D . This implies that banks have a symmetric inflow of inside money at the end of the investment market.

2.2 Money supply

The central bank maintains two standing facilities. At the borrowing facility entrepreneurs and bankers can borrow CBDC against collateral. The central bank sets the collateral haircut η so that $\eta \leq \bar{\eta}$ and the lending rate i_{ℓ_C} . The central bank can also impose a quantity constraint on CBDC loans in form of a maximum loan amount, \bar{k} . In the baseline version of the model we assume that the commercial banks set the same haircut as the central bank, whereas in section 5.2 we investigate the case when the commercial banks do not request collateral. At the deposit

¹⁰ Similar assumptions are made in the finance search and matching literature pioneered by Duffie et al. (2005).

¹¹ This also allows us to circumvent the famous Kareken and Wallace (1981) indeterminacy result.

¹² An interesting extension of the model would be to assume that φ_{DC} and φ_{CD} are small, but strictly negative numbers. In this case, the rate of return of the two monies can be different without driving out one of the monies as long as the difference is not too large.

facility all agents can deposit CBDC and earn the deposit rate i_C . The deposit facility is a form of ‘central bank accounts for all’ and we assume that all entrepreneurs and workers have such an account.¹³ In equilibrium, the workers hold the CBDC overnight and, therefore, earn i_C on their CBDC holdings.

The stock of CBDC is completely endogenous in the model. When an entrepreneur receives a loan from the central bank, CBDC is created. When the loan is redeemed, CBDC is destroyed.¹⁴ Let M_C denote the aggregate quantity of CBDC at the beginning of the settlement market. Note that

$$M_C = \int_0^\infty \ell_C(\varepsilon) dG(\varepsilon),$$

where $\ell_C(\varepsilon)$ is the CBDC loan taken out by an entrepreneur that drew the idiosyncratic investment shock ε . The real budget constraint of the central bank satisfies

$$T = M_C \phi_C(i_{tC} - i_C).$$

We assume that the central bank hands out any profit or loss in form of lump sum transfers to the agents in the model. Without loss of generality, we assume that the transfers go to the entrepreneurs. Thus, since the measure of entrepreneurs is 1, each entrepreneur receives T .

2.3 First-best allocation

In the Appendix we derive the first-best investment quantities that would be attained in the absence of the limited commitment friction. Optimality requires that the marginal return from an investment project equals the marginal cost of capital. That is,

$$\varepsilon^{1-\alpha} f_j(k_j) = 1, \quad j = C, D. \quad (3)$$

The optimal ratio satisfies

$$\omega^* \equiv \frac{k_C^*}{k_D^*} = \left(\frac{a}{1-a} \right)^{\frac{1}{1-\rho}}. \quad (4)$$

Closed-form solutions for k_C^* , k_D^* , and output y^* are stated in the Appendix. Note that $\beta y^* - k_C^* - k_D^* \geq 0$ for all ε . The implication is that from a societal point of view all projects

¹³ Entrepreneurs and workers on island D also have inside money accounts at the commercial banks. They have access to the respective accounts if they are on the respective island and in the settlement market.

¹⁴ For the same reasons, the aggregate stock of inside money is endogenous as well.

should be implemented. We will compare the market allocation with this first-best solution. Furthermore, we will discuss how CBDC design can affect the market solution and bring it closer to the first best.

3 Decision problems

In this section, we study the decision problems of all agents in the three markets. We start with the investment market, where workers decide how much capital to produce and sell. Then, we move to the settlement market, where output is consumed, loans are settled, and banks decide how much money they bring to the loan market. Finally, we describe the loan market, where entrepreneurs make decisions on investment and loans, negotiating the latter with banks.

3.1 Investment market

Let $p = (p_C, p_D)$ denote the prices of the two capital goods. On island $j = \{C, D\}$, workers produce capital goods k_j with linear disutility $c(k_j) = k_j$ and then sell the goods at the competitive price p_j to the entrepreneurs for money j . They hold the money overnight in their accounts, earning interest on it. In the settlement market, they spend the money on the generic good obtaining linear utility $U(x) = x$. It is straightforward to show that workers are indifferent as to how much they sell in the investment market if

$$p_j \beta \phi_j^+ (1 + i_j) = 1, \quad (5)$$

where ϕ_j^+ is the value of money j in next period's settlement market and i_j is the interest rate earned for holding money j overnight. Since workers are indifferent with regard to how much they produce, we assume that each worker on an island produces the same amount, which implies that each bank has the same customer base. Therefore, banks have a symmetric inflow of inside money at the end of the investment market.

3.2 Settlement market

3.2.1 Banks

Let $B_S(m, \ell)$ denote a bank's expected value of entering the settlement market with money holdings $m = (m_C, m_D)$ and loans $\ell = (\ell_C, \ell_D)$. The quantity m_D is a liability to the bank

(deposits) and m_C is a liability of the central bank (central bank deposits). In contrast, ℓ_D is an asset for the bank (the loan extended to the entrepreneur) and ℓ_C is a liability for the bank (borrowing from the central bank).

The marginal values of holding the assets at the beginning of the settlement market are

$$B_S^D = -\phi_D (1 + i_D) \text{ and } B_S^C = \phi_C (1 + i_C), \quad (6)$$

$$B_S^{\ell_C} = -\phi_C (1 + i_{\ell_C}) \text{ and } B_S^{\ell_D} = \phi_D (1 + i_{\ell_D}), \quad (7)$$

where B_S^i is the partial derivative of B_S with respect to m_D and m_C , respectively, and $B_S^{\ell_j}$ is the partial derivative of B_S with respect to ℓ_D and ℓ_C , respectively. Consider, for example, the derivative B_S^D . The banker pays the interest rate i_D and thus the marginal debt is $(1 + i_D)$. The value of deposits in terms of the generic good is ϕ_D . That is, $(1 + i_D)$ deposits buy $\phi_D (1 + i_D)$ generic goods. Given the banker's linear consumption preferences, the disutility of redeeming a marginal unit of deposits is $-\phi_D (1 + i_D)$. The interpretation of the other derivatives is similar.

Note that bankers bring no money into the loan market. The reason is that they can produce inside money on the spot by extending loans to entrepreneurs. Furthermore, they hold no CBDC because the entrepreneurs can borrow CBDC directly from the central bank.

3.2.2 Workers

Workers produce capital goods in the investment market and sell them for deposits or CBDC, for which they earn interest payments in the beginning of the next settlement market. Because of the linearity of their preferences for consumption and leisure (disutility of working), they spend all their money holdings for units of the generic good and move into the following loan and investment markets without any money holdings. For workers the marginal values of bringing an additional unit of CBDC or inside money into the settlement market are

$$W_S^C = \phi_C (1 + i_C) \text{ and } W_S^D = \phi_D (1 + i_D). \quad (8)$$

3.2.3 Entrepreneurs

$E_S(y, m, \ell)$ denotes an entrepreneur's expected value of entering the settlement market with goods $y = \beta^{-1} \varepsilon^{1-\alpha} f(k)$, money holdings $m = (m_C, m_D)$, and loans $\ell = (\ell_C, \ell_D)$. For them, ℓ_C

and ℓ_D are liabilities and m_C and m_D are assets. The marginal values of holding these assets at the beginning of the settlement market are

$$E_S^y = 1, \quad (9)$$

$$E_S^C = \phi_C (1 + i_C) \text{ and } E_S^D = \phi_D (1 + i_D), \quad (10)$$

$$E_S^{\ell_C} = -\phi_C (1 + i_{\ell_C}) \text{ and } E_S^{\ell_D} = -\phi_D (1 + i_{\ell_D}). \quad (11)$$

The marginal values of the assets have the same interpretation as above. The marginal value of bringing goods into the settlement market is 1 because the marginal disutility of producing generic goods in the settlement market is 1.

An ε -entrepreneur who has invested $k = (k_C, k_D)$ holds y units of the production good at the beginning of the settlement market. To repay the loans, the entrepreneur sells some of them for CBDC and some for inside money to repay the loans and consumes the rest. Upon repaying the loans the entrepreneur receives back the collateral \tilde{e} , consumes it yielding utility e , and leaves the economy.

3.3 Loan market

In the loan market, entrepreneurs borrow CBDC and inside money in order to pay for the capital goods in the investment market. Since the entrepreneurs cannot commit to repay loans, the central bank and the commercial bank require collateral.

For an ε -entrepreneur who borrows ℓ the indirect utility function in the loan market, $E_L(\varepsilon)$, satisfies

$$E_L(\varepsilon) = \beta E_S(y, m, \ell),$$

where $m = \ell - pk$ is the amount of money left after spending pk units on capital. Since the entrepreneur has linear utility in the settlement market and, in equilibrium, pays back the loan, receiving back his collateral, it follows:

$$\beta E_S(y, m, \ell) = e + \varepsilon^{1-\alpha} f(k) - \beta \phi_C \ell_C (1 + i_{\ell_C}) - \beta \phi_D \ell_D (1 + i_{\ell_D}).$$

This expression shows that if borrowing rates exceed deposit rates, the entrepreneur will always spend all of his money on capital goods; that is, if $i_{\ell_C} \geq i_C$ and $i_{\ell_D} \geq i_D$, then $m_C = \ell_C - p_C k_C = 0$

and $m_D = \ell_D - p_D k_D = 0$.¹⁵ For the rest of this paper we will assume that $i_{\ell_j} \geq i_j$, implying $\ell_j = p_j k_j$ for $j = \{C, D\}$.

For a banker matched with an ε -entrepreneur, the indirect utility function $B_L(\varepsilon)$ satisfies

$$B_L(\varepsilon) = \beta B_S(m_D, \ell_D).$$

This expression takes into account that bankers enter the loan market with no money and no liabilities. If they extend a loan ℓ_D to the entrepreneurs, they create inside money $m_D = \ell_D$. Note again that m_D is a liability for the bank. Using the linearity of preferences in the settlement market, we can write this expression as follows:

$$\beta B_S(m_D, \ell_D) = \beta \phi_D^+ \ell_D (1 + i_{\ell_D}) - \beta \phi_D^+ m_D (1 + i_D) + EB_L(\varepsilon).$$

This expression includes a continuation value $EB_L(\varepsilon)$ because the bankers are infinitely lived and will be matched in the future with entrepreneurs drawn from the distribution $G(\varepsilon)$, which is reflected in the expectation operator.

3.3.1 Bargaining

In the loan market, a banker-entrepreneur pair negotiates the loan quantities ℓ_C and ℓ_D and the interest rate on the inside money loan i_{ℓ_D} . After the negotiation the entrepreneur receives ℓ_D from the bank and borrows the required amount ℓ_C directly from the central bank at the given policy rate i_{ℓ_C} .

We assume that bargaining follows a bargaining splitting rule. Specifically, bankers maximise their payoff subject to the constraint that the entrepreneurs receive at least a fraction $1 - \theta$ of the total surplus $TS = S_B + S_E$, which is the sum of the individual surpluses. In what follows, we derive these surpluses and the bargaining solution.

The banker's surplus from the loan is

$$S_B = \beta B_S(m_D, \ell_D) - \beta B_S(0, 0),$$

where $\beta B_S(0, 0)$ is the banker's utility if there is an exogenous breakdown of the negotiation.

¹⁵ If $i_{\ell_C} = i_C$ and $i_{\ell_D} = i_D$, entrepreneurs are indifferent on how much to borrow. Without loss of generality, we assume that in this case they only borrow what they will spend on capital goods.

Exploiting the linearity of payoffs in the settlement market and the workers' first-order condition in the investment market, (5), the banker's surplus satisfies

$$S_B = k_D \frac{(i_{\ell_D} - i_D)}{1 + i_D}. \quad (12)$$

The entrepreneur takes out a CBDC loan at the central bank and an inside money loan from the banker. The entrepreneur's surplus is

$$S_E = \beta E_S(y, m, \ell) - \beta E_S(0, 0, 0),$$

where $\beta E_S(0, 0, 0) = e$ is the entrepreneur's utility if there is an exogenous breakdown of the negotiation.¹⁶ The entrepreneur's surplus satisfies

$$S_E = \varepsilon^{1-\alpha} f(k_C, k_D) - \beta \phi_C \ell_C (1 + i_{\ell_C}) - \beta \phi_D \ell_D (1 + i_{\ell_D}).$$

Using the workers' first-order conditions yields

$$S_E = \varepsilon^{1-\alpha} f(k_C, k_D) - \iota_C k_C - \iota_D k_D,$$

where $\iota_C \equiv \frac{1+i_{\ell_C}}{1+i_C}$ and $\iota_D \equiv \frac{1+i_{\ell_D}}{1+i_D}$. The total surplus satisfies

$$TS = S_E + S_B = \varepsilon^{1-\alpha} f(k_C, k_D) - \iota_C k_C - k_D. \quad (13)$$

Notice that the total surplus does not directly depend on the loan rate i_{ℓ_D} . As we show further below, the role of the inside money loan rate is to split the total surplus between the banker and the entrepreneur.

The bankers maximise their surplus S_B subject to the condition that the entrepreneurs receive the fraction $1 - \theta$ of the total surplus. That is,

$$S_E \geq (1 - \theta) TS. \quad (14)$$

¹⁶ We assume here that in case of an exogenous breakdown of the negotiation, the entrepreneurs produce nothing and just consume their endowment in the settlement market.

Furthermore, the banker takes into account the limited commitment problem of the entrepreneur via the following collateral constraint:

$$\iota_C k_C + k_D + \theta TS \leq e + \eta \varepsilon^{1-\alpha} f(k_C, k_D). \quad (15)$$

In the Appendix, we derive (15) from basic principles. The left-hand side is the sum of the loans to be repaid, including the interest on the loans since $\iota_D k_D = k_D + \theta TS$. The right-hand side of (15) reflects that the entrepreneur can only pledge the fraction η of future output plus the discounted utility of the idiosyncratic collateral \tilde{e} . Note that (15) is measured in terms of utility.

The banker also takes into account the quantity constraint on CBDC, which is one of the design instruments available to the central bank. The constraint is $\ell_C \leq \overline{M}_C$, where \overline{M}_C is a nominal cap, or in real terms,

$$k_C \leq \bar{k} \equiv \overline{M}_C / p_C. \quad (16)$$

We assume that the central bank indexes \overline{M}_C to inflation so that \bar{k} is constant.

With these constraints, the banker's optimization problem solves

$$(k_C, k_D) = \arg \max_{k_C, k_D} S_B \text{ s.t. (14) - (16)}. \quad (17)$$

4 Market equilibrium

We now derive the equilibrium quantities for capital, k_C and k_D , and the corresponding interest rates and prices. In this context, a key variable is the size of the investment shock, ε , that determines whether and when the collateral and quantity constraints start to bind. Therefore, we first investigate the different regions in which these constraints are binding before we turn to the effects of the different policy parameters on capital allocation, bank lending rates, and welfare.

4.1 Collateral constraint and quantitative cap

Definition 1. The quantities k_C and k_D that solve (17) are an equilibrium for this economy.

Once we have solved for k_C and k_D , we can derive all other real and nominal quantities from the solution of (17). Note first that in any solution, condition (14) holds with equality since it

can not be optimal for the banker to provide more surplus to the entrepreneur than necessary. Thus, we can use (14) to replace S_B and obtain

$$(k_C, k_D) = \arg \max_{k_C, k_D} \theta TS \text{ s.t. (15) and (16).}$$

Let λ be the multiplier for constraint (15) and let κ be the multiplier for constraint (16). Then, the first-order conditions for k_C and k_D satisfy

$$\varepsilon^{1-\alpha} f_C = \frac{[\theta + \lambda(1-\theta)] \iota_C + \kappa}{\theta + \lambda(\eta - \theta)} \text{ and } \varepsilon^{1-\alpha} f_D = \frac{\theta + \lambda(1-\theta)}{\theta + \lambda(\eta - \theta)}. \quad (18)$$

The two multipliers characterise the equilibrium input quantities. Depending on whether the constraints are binding or not, we can distinguish four regions, as shown in Table 1.

Regions	Multipliers	Quantities: k_C and k_D solve
1	if $\lambda = 0$ and $\kappa = 0$	$\varepsilon^{1-\alpha} f_C = \iota_C$ and $\varepsilon^{1-\alpha} f_D = 1$
2	if $\lambda = 0$ and $\kappa > 0$	eq. (16) and $\varepsilon^{1-\alpha} f_D = 1$
3	if $\lambda > 0$ and $\kappa = 0$	$f_C = f_D \iota_C$ and eq. (15)
4	if $\lambda > 0$ and $\kappa > 0$	eq. (16) and eq. (15)

Table 1
Multipliers and equilibrium conditions for quantities k_C and k_D

If none of the constraints binds, we are in region 1 with $\lambda = 0$ and $\kappa = 0$ and, hence, k_C and k_D solve $\varepsilon^{1-\alpha} f_C = \iota_C$ and $\varepsilon^{1-\alpha} f_D = 1$. In region 2, the quantity constraint is binding, in region 3, the collateral constraint is binding, and in region 4, both constraints bind.

Whether the multipliers are binding or not depends on the relative size of the model parameters. Moreover, the size of the investment shock plays an important role, as entrepreneurs that draw a small shock will less likely face constraints whereas for larger shocks the collateral or the quantity constraint may become binding. In the following, we investigate the conditions for the order in which the collateral and the quantity constraints start to bind. Let

$$\psi \equiv (1 - \theta) \alpha - (\eta - \theta),$$

and assume for the moment that the central bank does not impose a quantity constraint; i.e. $\bar{k} \rightarrow \infty$.

Lemma 1. If $\psi < 0$, the collateral constraint never binds while for $\psi > 0$ there exists a critical value ε^{13} such that the collateral constraint is not binding if $\varepsilon \leq \varepsilon^{13}$ and it is binding if $\varepsilon > \varepsilon^{13}$ with ε^{13} denoting the cut-off value for the regions in Table 1 at which region 1 is exited and region 3 is entered.

The closed-form solution for ε^{13} can be found in the Appendix.

If $\eta > (1 - \theta)\alpha + \theta$ the collateral constraint never binds, meaning that the entrepreneurs are not constrained by their inability to commit because the limited commitment problem is small in the sense that only a small fraction $1 - \eta$ of future output can be diverted. Note that when $\alpha > \eta$, i.e. when the degree of homogeneity exceeds the share of output that cannot be diverted, $\psi > 0$ for any θ . For the rest of the paper we assume that $\psi > 0$ so that the collateral constraint is binding for some investment shocks.

We now turn to the conditions under which the quantity constraint is binding. Let

$$\Phi \equiv \frac{e\alpha}{\psi(\iota_C + 1/\omega)}, \quad \text{where } \omega \equiv \frac{k_C}{k_D}.$$

Proposition 1. *If*

$$\bar{k} > \Phi, \tag{19}$$

there exist critical values $\varepsilon^{34} > \varepsilon^{13} > 0$, defined in the proof with ε^{34} denoting the cut-off value at which region 3 is exited and region 4 is entered, such that for $\varepsilon \leq \varepsilon^{13}$, $\lambda = \kappa = 0$, for $\varepsilon^{13} < \varepsilon \leq \varepsilon^{34}$, $\lambda > \kappa = 0$, and, finally, for $\varepsilon > \varepsilon^{34}$, $\lambda, \kappa > 0$.

Proposition 1 describes a situation where we move from region 1 to region 3 and then to region 4 as we increase ε from $\varepsilon = 0$. The condition $\bar{k} > \Phi$ ensures that the collateral constraint binds prior to the quantity constraint. For $\bar{k} < \Phi$ we move from region 1 to region 2 and then to region 4. For the rest of the paper we assume $\bar{k} > \Phi$ as this is the more interesting case with regard to the evolution of k_C in regions 1, 3, and 4.¹⁷

To provide some intuition for these analytical results, we present a numerical example to illustrate how the different constraints start to bind as ε increases. We use the parameter values shown in Table 3, which imply that with an increasing ε we move from region 1 to region 3 and, finally, to region 4.

¹⁷ In the alternative case the binding quantity constraint would keep k_C constant over regions 2 and 4, leading only to a change in the slope for the path of k_D .

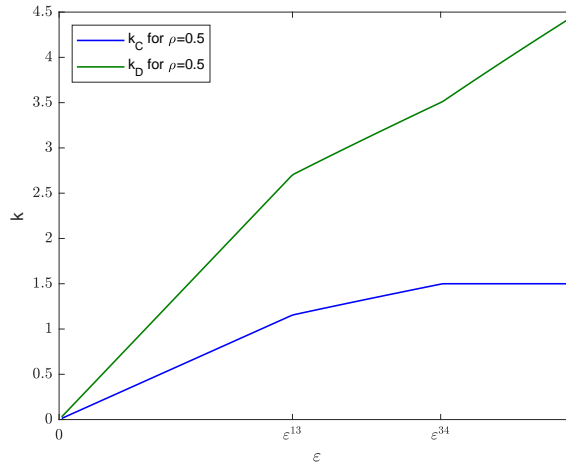


Figure 2
Optimal capital investment

Figure 2 plots the optimal quantities k_C and k_D as a function of the idiosyncratic investment shock, ε . Because of the value of a that determines the input shares in the production function and the positive spread ι_C on CBDC, entrepreneurs in this example demand more inside money than CBDC. As ε becomes larger, the optimal quantities of both k_C and k_D increase. From the threshold ε^{13} on, the collateral constraint starts to bind but not yet the quantity constraint. At this point, k_C and k_D still increase in ε but with a smaller slope since only a part of the higher output that results from a larger ε can be pledged as collateral. The amount of invested k_C attains its maximum at ε^{34} when the quantity constraint starts to bind. Independent of a further increase in ε , k_C is now capped and equal to \bar{k} . At the same time, k_D rises with a steeper slope because all additional investment resulting from a higher investment shock is now paid for with inside money.

4.2 Nominal equilibrium quantities

So far we have derived the real quantities k_C , k_D , y and the threshold values for the transition between the different regimes. It remains to determine the prices p_C , p_D , ϕ_C , and ϕ_D and the interest rates i_D and i_{ℓ_D} . Note that in any equilibrium all prices must grow at the same rate γ :

$$\gamma \equiv \frac{p_C^+}{p_C} = \frac{\phi_C}{\phi_C^+} = \frac{p_D^+}{p_D} = \frac{\phi_D}{\phi_D^+}. \quad (20)$$

We assume that the central bank controls γ .¹⁸ In what follows we show how the prices and the interest rates are determined.

First, from the workers' arbitrage condition for inside money, (5), we have

$$p_D \beta \phi_D^+ (1 + i_D) = 1.$$

Using (20), this equation determines the effective price of k_D :

$$p_D \phi_D (1 + i_D) = \frac{\gamma}{\beta}. \quad (21)$$

The product $p_D (1 + i_D)$ is the effective price of k_D from a worker's point of view. It is the amount of inside money that is available in the settlement market to a worker who has produced and sold one unit of k_D in the investment market to the entrepreneur against a payment of p_D , and the quantity $p_D i_D$ is obtained in the settlement market from the banker who issued the unit of money. The term $p_D \phi_D (1 + i_D)$ is the real quantity of inside money obtained from producing one unit of capital k_D . This arbitrage equation also shows that the effective price of k_D does not change in i_D for a given value of inside money ϕ_D . An increase in i_D lowers only p_D as inside money becomes more valuable to workers so that they are willing to work more for one unit, keeping the product $p_D (1 + i_D)$ constant.

Second, from the workers' arbitrage condition in the CBDC market, (5), we have

$$p_C \beta \phi_C^+ (1 + i_C) = 1.$$

This equation determines the effective price of k_C :

$$p_C \phi_C (1 + i_C) = \frac{\gamma}{\beta}. \quad (22)$$

The product $p_C (1 + i_C)$ is the effective price of k_C , following the same intuition as for the effective price of k_D . There is one important difference, however, because the interest rate i_C is a policy variable and, hence, exogenous. Note that an increase in i_C will lower the price of k_C , p_C , for a given value of CBDC, ϕ_C . The intuition is the same as for an increase in i_D .

¹⁸ As money is super-neutral in our model, the central bank can announce a specific inflation rate γ and agents will set their prices accordingly as they would not benefit by deviating from the announced rate.

Third, inside money is a promise to pay central bank money, i.e. CBDC in our model, on demand. We assume that commercial banks keep their promises and, hence,

$$\phi_D = \phi_C.$$

Fourth, this allows us to combine (21) and (22) as follows:

$$p_C (1 + i_C) = p_D (1 + i_D). \quad (23)$$

Fifth, we can normalize one price since the initial price level is indeterminate. That is, we set

$$p_C(t = 0) = 1.$$

This choice determines the path for the product $p_D (1 + i_D)$, although the individual components p_D and i_D are not yet determined.

Sixth, we can rewrite (12) as follows:

$$\frac{(i_{\ell_D} - i_D)}{1 + i_D} = \theta TS/k_D. \quad (24)$$

The right-hand side of this equation depends only on k_C and k_D which are determined by equilibrium conditions and therefore pin down $\frac{(i_{\ell_D} - i_D)}{1 + i_D} = \iota_D - 1$. Note that ι_D depends on ε since $\theta TS/k_D$ depends on ε .

Finally, the price path for p_D and the two interest rates i_{ℓ_D} and i_D still need to be determined, but there are only two equations (23) and (24). Thus, although all real quantities are uniquely determined, the model has a nominal indeterminacy. Rather than adding a theory of how the deposit rate is determined to remove this nominal indeterminacy, we investigate the model under two alternative assumptions:

Assumption 1: We assume that the inside money deposit rate $i_D = 0$, which is in line with the level of deposit rates in developed countries for the last couple of years (see, for example, Berentsen et al. (2020)).

Assumption 2: We assume that the inside money deposit rate equals the CBDC deposit rate, $i_D = i_C$, which could be motivated by the existence of arbitrageurs that can profitably exploit any deviation from $i_D = i_C$.

All real quantities and welfare are independent from whether we use Assumption 1 or 2 and only the results for the bank lending rate, i_{ℓ_D} , are affected. In the following, we therefore assume that $i_D = 0$ so that the spread ι_D can be interpreted as the gross bank lending rate. We discuss in section 6.4 how the bank lending rate i_{ℓ_D} changes if we instead assume that i_D follows i_C .

5 Optimal CBDC design: remuneration and haircuts

In this section we first compare the market solution of the model to the first-best allocation and derive the optimal CBDC design. We then investigate the effects of different rates of remuneration and collateral haircuts analytically. We also consider whether the introduction of CBDC drives out inside money and, hence, leads to bank disintermediation. In section 6, we present numerical simulations of the model and use them to further discuss the tools for an optimal CBDC design.

Corollary 1. For any $\varepsilon \leq \varepsilon^{34}$, the ratio k_C/k_D satisfies

$$\omega \equiv \frac{k_C}{k_D} = \left(\frac{a}{\iota_C(1-a)} \right)^{\frac{1}{1-\rho}} \quad (25)$$

and the policy rate ι_C has a direct effect on capital investments.

The first-best allocation as described in (3) requires that the ratio of k_C to k_D is constant and satisfies ω^* . From (25), the central bank can achieve this ratio for any $\varepsilon \leq \varepsilon^{34}$ by setting

$$\iota_C = 1.^{19}$$

Note that the first-best allocation is still not attained because for entrepreneurs with an $\varepsilon > \varepsilon^{13}$ the collateral constraint binds in the market solution. The underlying friction cannot be solved by the central bank although a lowering of collateral haircuts would allow entrepreneurs to borrow more and would move the market allocation closer towards the first-best allocation.²⁰

¹⁹ This is the minimum value for ι_C . If the central bank were to set the spread to negative values, the entrepreneur would want to borrow money and deposit it with the central bank instead of investing. We rule this case out by assumption.

²⁰ Note that we assume in the baseline version of the model that the central bank and commercial banks impose the same collateral requirements. Section 5.2 investigates the case where commercial banks do not require collateral.

5.1 Remuneration, bank intermediation and bank lending rates

We now analyse how a change in the policy rate affects bank intermediation and bank lending rates. In the Appendix, we show that $\frac{dk_C}{d\iota_C} < 0$ when the quantity constraint does not bind, implying that with an increase in the CBDC lending rate relative to the deposit rate the central bank can depress the demand for CBDC. We know from (25) that an increase in ι_C decreases the ratio k_C/k_D , which can be interpreted as a higher policy rate on CBDC reducing bank disintermediation but it needs to be examined whether k_D increases in absolute terms. In the Appendix, we derive an explicit solution for k_D when the quantity constraint does not bind and show that

$$\frac{dk_D}{d\iota_C} = -\frac{(\alpha - \rho) k_D a \omega^\rho}{(1 - \alpha) [a \omega^\rho + 1 - a] (1 - \rho) \iota_C}.$$

Thus, if $\rho \leq \alpha$, $\frac{dk_D}{d\iota_C} \leq 0$. Hence, if the two inputs are sufficiently close substitutes in the sense that $\rho > \alpha$, an increase in ι_C increases k_D . In this context, we can think of a high ρ as CBDC features that make its functionality close to those of bank deposits. In contrast, if $\rho < \alpha$, an increase in ι_C decreases k_D .

Turning to the effect of ι_C on bank lending rates, ι_D , we use the explicit solution for ι_D derived from the banker's surplus, as outlined in more detail in the Appendix, and obtain

$$\frac{d\iota_D}{d\iota_C} = \theta \frac{1 - \alpha}{\alpha} \frac{\rho}{\rho - 1} \omega < 0.$$

Table 2 summarises the effects of changes in the policy rate on bank intermediation for different degrees of substitutability.

	$\frac{dk_D}{d\iota_C}$	$\frac{dk_C}{d\iota_C}$	$\frac{d\iota_D}{d\iota_C}$
$\rho > \alpha$	+	-	-
$\rho < \alpha$	-	-	-

Table 2
Effects of the policy rate on bank intermediation

Section 6 shows simulations of aggregate capital and the bank lending rate as a function of ι_C for $\rho \leq \alpha$.

5.2 Collateral haircuts and bank intermediation

The impact of a change in the collateral haircut on bank intermediation can be seen from the first-order conditions (18). If the collateral constraint is binding, an increase in η (the share of output that cannot be diverted) relaxes the constraint and decreases the marginal products f_C and f_D , which results in an increase in k_D . The evolution of capital and bank lending rates as a function of the collateral haircut is simulated in section 6. Note that we are assuming that η applies to both central bank and commercial bank loans.

We can also analyse how results change when haircuts applied by the central bank differ from those of commercial banks. If only the central bank requires collateral, the decision problem changes to²¹

$$\begin{aligned} (k_C^N, k_D^N) &= \arg \max_{k_C, k_D} \theta [\varepsilon^{1-\alpha} f(k_C, k_D) - \iota_C k_C - k_D] \\ \text{s.t. } \iota_C k_C &\leq e + \eta \varepsilon^{1-\alpha} f(k_C, k_D). \end{aligned} \quad (26)$$

The first-order conditions for k_C and k_D would then satisfy

$$\varepsilon^{1-\alpha} f_C = \frac{\iota_C (\theta + \lambda)}{\theta + \lambda \eta} \text{ and } \varepsilon^{1-\alpha} f_D = \frac{\theta}{\theta + \lambda \eta}. \quad (27)$$

If the collateral constraint does not bind, k_C and k_D solve

$$\varepsilon^{1-\alpha} f_C = \iota_C \text{ and } \varepsilon^{1-\alpha} f_D = 1.$$

By contrast, if the collateral constraint is binding, k_C , k_D and λ solve (26) and (27).

Note that the ratio k_C/k_D then satisfies

$$\frac{k_C}{k_D} = \left(\frac{a}{(1-a)} \frac{\theta}{\iota_C (\theta + \lambda)} \right)^{\frac{1}{1-\rho}}.$$

When assuming that only the central bank requires collateral, the ratio thus depends on the policy rate ι_C . Then we can define the implicit interest rate charged by the central bank as

$$\iota_C^\lambda = \frac{\iota_C (\theta + \lambda)}{\theta},$$

²¹ For simplicity, we assume here that the quantity constraint does not bind.

which includes the costs arising from the interest rate ι_C and the collateral constraint.²²

6 Simulations

To illustrate the mechanics of the model in more detail and to elaborate further on the results outlined above, we simulate how the key quantities depend on the size of the investment shock, ε , and show the effects of the collateral and the quantity constraints and changes in the policy rate. Since these effects depend on the degree of substitutability, ρ , we generally show simulations for two different values of ρ . We illustrate how the central bank can steer the demand for CBDC using different policy instruments, thereby also affecting welfare. After presenting simulations of the evolution of capital and bank lending rates as a function of the investment shock, ε , we aggregate these variables across the entire distribution of ε and show how the use of different policy tools affects the evolution of aggregate capital, bank lending rates and welfare. Finally, in section 6.4, we discuss how the results change if we assume that $i_D = i_C$ instead of setting $i_D = 0$ as for the simulations in sections 6.1 to 6.3.

Parameter	Value
Mean and variance of log-normal investment shock distribution	μ_ε 1.00 σ_ε^2 2.00
Discount factor	β 0.96
Degree of homogeneity	α 0.65
Share of banker's surplus	θ 0.05
Input share for CBDC	a 0.40
Endowment	e 1.00
Substitution parameter	ρ 0.50
Spread on CBDC	ι_C 1.02
Deposit interest rate	i_D 0.00
Output share that cannot be diverted	η 0.50
Quantity constraint for CBDC	\bar{k} 1.50

Table 3
Parameter values for simulations

For the following simulations we use the parameter values shown in Table 3 (unless stated otherwise) with a log-normal distribution for the investment shock. The discount factor $\beta = 0.96$ reflects an interpretation of a period as a year, which is the time an investment project needs

²² Alternatively, it could be assumed that the central bank applies larger collateral haircuts than the private sector.

to mature. Because of the nominal indeterminacy, the model solution does not depend on the specific level of deposit or lending rates but only on the spread between them, i.e. ι_C and ι_D . To anchor the inside money spread, we assume for the following results that deposits are not remunerated, i.e. $i_D = 0$, such that ι_D equals the loan rate on inside money. We set $\iota_C = 1.02$, which corresponds to an interest rate spread of 2 percentage points at the central bank, matching the ECB's corridor width before 2009. An increase in ι_C can thus either be interpreted as a decrease in the CBDC deposit rate, i_C or an increase in the CBDC lending rate, i_{ℓ_C} . For the collateral constraint, we choose $\eta = 0.5$.²³ We set the input share for CBDC in the production function to $a = 0.4$.²⁴ The other parameter values are chosen such that the collateral and the quantity constraints start to bind in a range where there is still a non-negligible density of the investment shock. Furthermore, with these parameters, $\psi > 0$ and $\bar{k} > \Phi$, which means that we move from region 1 to region 3 and then to region 4 as ε increases (see section 4.1 for a detailed discussion).

6.1 Simulations for different realisations of the investment shock

Figure 3 shows the demand for capital and the bank lending rate as a function of the investment shock for two different policy rates. To make the differences clearly visible we compare central bank lending rates of 2% (solid line) and 20% (dashed line). In region 1 where neither the collateral nor the quantity constraint bind, k_C and k_D increase linearly with an increasing ε . Due to the parameter value chosen for the input share a , the demand for bank loans is higher than for CBDC loans. From the threshold ε^{13} on the pledgeable collateral rises with a fraction η of output, which is less than the efficient amount of investment would require. Consequently, both k_C and k_D continue to increase with the size of the project but with a lower slope than before. At ε^{34} the quantity constraint \bar{k} starts to bind, preventing a further increase in k_C . By contrast, k_D increases with an even steeper slope since all additional investment has to be financed by bank loans.

A higher policy rate ι_C mainly dampens demand for k_C with limited impact on k_D . With the lower demand for k_C , both the collateral and the quantity constraint start to bind at higher

²³ This is in the range of haircuts the Euro system applies to credit claims with a rating of BBB, see Table 4 in Bindseil et al. (2017).

²⁴ This choice determines the location of the curves and thus the readability of the Figures whereas it does not affect the dynamics.

values of ε .²⁵ The slight downward shift in k_D in Figure 3 can be explained by the distortion caused by a higher ι_C that, as both types of capital are imperfect substitutes, also dampens the demand for k_D .

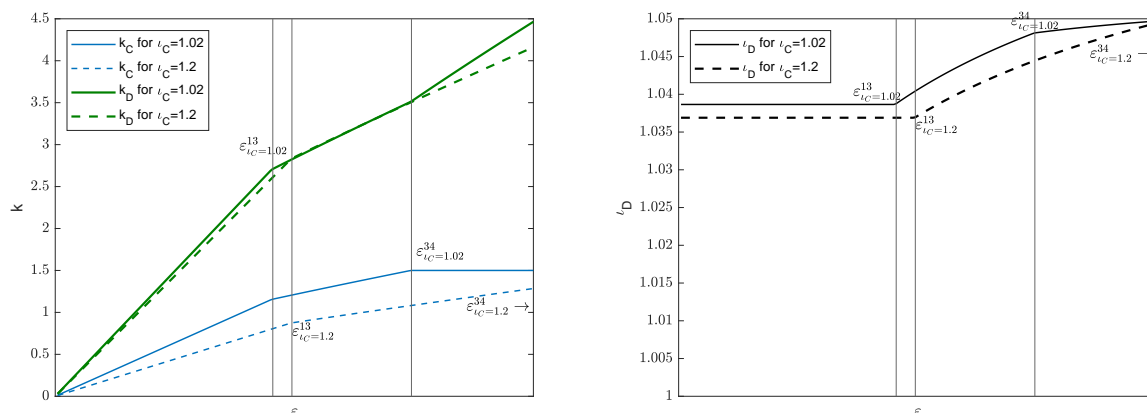


Figure 3
Simulated k_C , k_D , and ι_D for different ι_C

The right panel in Figure 3 shows the corresponding bank lending rates, which are determined by the banker's share in the total surplus over k_D , see (24). The key parameter determining the level of the bank lending rate is the banker's share in the total surplus, θ , which we set to $\theta = 0.05$. A decline in the total surplus also lowers ι_D , i.e. the share that the banker obtains as a remuneration for the k_D loan. Note that ι_D exceeds the interest rate spread on CBDC for $\iota_C = 1.02$. In the unconstrained region the lending rate is independent of the size of the investment project. With a binding collateral constraint, entrepreneurs invest less than the optimal amount of capital for the size of the investment shock they have drawn. Therefore, the marginal gain from an additional unit of invested capital increases with ε , leading to an increase in ι_D , as the total surplus rises more quickly than k_D . Once the quantity constraint starts to bind as well, all additional loans are made in inside money, k_D . Relative to the increase in the total surplus, more inside money is now invested, which leads to a decrease in the slope of ι_D in this region. The dashed line in Figure 3 shows that with a higher policy rate the lending rate on inside money decreases. At the first sight, this may seem counter-intuitive but the reason is that the higher cost entrepreneurs incur for their CBDC loans reduces the total surplus and thus translates into a lower profit for the bankers for the same size of an investment project.

²⁵ With the parameter values chosen for our simulations, in this case the quantity constraint does not bind over the range of ε that we show in Figure 3.

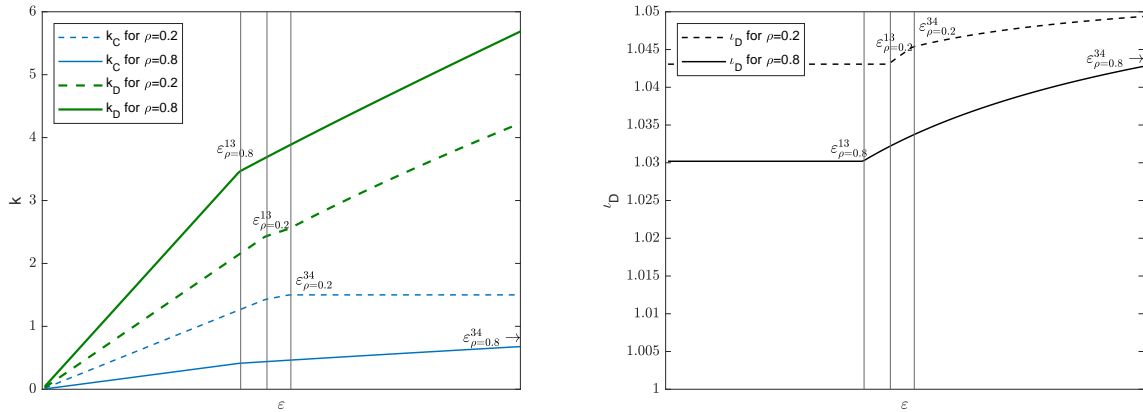


Figure 4
Simulated k_C, k_D , and ι_D of ε for different degrees of substitutability

We next explore how k_C , k_D , and ι_D depend on the degree of substitution. While Figure 3 shows simulations for $\rho = 0.5$, Figure 4 plots k_C , k_D , and ι_D as a function of the investment shock ε for $\rho = 0.2$ and $\rho = 0.8$. A low ρ can be interpreted as CBDC having more cash-like features that make it less suitable as a substitute for bank deposits whereas a high ρ could be seen as CBDC offering similar functionalities as bank deposits. Consequently, a high ρ boosts the demand for k_D at the expense of k_C (solid lines), whereas a low ρ leads to a more similar demand for both types of capital.

The right panel in Figure 4 shows the impact of the substitution parameter, ρ , on the bank lending rate. As before, the lending rate starts to rise when the collateral constraint is binding. With a high degree of substitutability (solid line), the demand for k_D is higher relative to k_C which reduces the ratio of the total surplus to k_D and, hence, also ι_D .

6.2 Simulation of aggregated variables

To investigate the effects of the three policy tools – namely the policy rates ι_C , the collateral haircut, η , and the quantity constraint \bar{k} – on the capital allocation, bank lending rates and welfare, we aggregate all variables over the entire distribution of ε .

Invested capital, aggregated over the entire distribution, is defined as:

$$k_C^{aggr} = \int_0^{\infty} k_C(\varepsilon) dG(\varepsilon) \text{ and } k_D^{aggr} = \int_0^{\infty} k_D(\varepsilon) dG(\varepsilon) .$$

Accordingly, the average spread between deposit and lending rates for inside money is defined as:

$$\iota_D^{mean} = \int_0^\infty \iota_D(\varepsilon) dG(\varepsilon) = \int_0^\infty 1 + \frac{S_B(\varepsilon)}{k_D(\varepsilon)} dG(\varepsilon) = 1 + \theta \int_0^\infty \frac{TS(\varepsilon)}{k_D(\varepsilon)} dG(\varepsilon) .$$

Finally, welfare is calculated as the difference between output that can be consumed in the next period and invested capital, integrated over all investment shocks:

$$Welfare = \int_0^\infty \beta y(\varepsilon) - k(\varepsilon) dG(\varepsilon) = \int_0^\infty \varepsilon^{1-\alpha} f(k_C(\varepsilon), k_D(\varepsilon)) - k_C(\varepsilon) - k_D(\varepsilon) dG(\varepsilon) .$$

6.2.1 Different policy rates

Of particular interest is whether the demand for CBDC and the impact on bank intermediation can be steered by varying the interest rates on CBDC. The left panel of Figure 5 shows aggregate capital investment in inside money and CBDC as a function of the policy rate spread, ι_C . As the latter increases and, therefore, CBDC loans become more expensive, the demand for k_C declines. The entrepreneurs' capital demand for inside money increases with ι_C if ρ is relatively high (solid line), as the lower CBDC investment can more easily be substituted by inside money investment. In this case, the remuneration of CBDC is an effective instrument to affect bank intermediation which we interpret as the ratio between k_D and k_C . If ρ is small, however, both k_D and k_C decrease with an increasing ι_C , which means that the degree of disintermediation does not react strongly to changes in the remuneration of CBDC.²⁶ The right panel of Figure 5 shows the corresponding bank lending rates. For both values of ρ they slightly decrease in ι_C , since the bank lending rates reflect the banker's share of the total surplus, which becomes smaller when CBDC loans are more expensive. When ρ is smaller (dashed line), less k_D and more k_C are used than with a higher ρ , corresponding to a higher ι_D .

²⁶ Note that disintermediation occurs in relative terms, meaning that there is no abrupt disintermediation of k_D -loans at any specific interest rate i_C , as opposed to Keister and Sanches (2019).

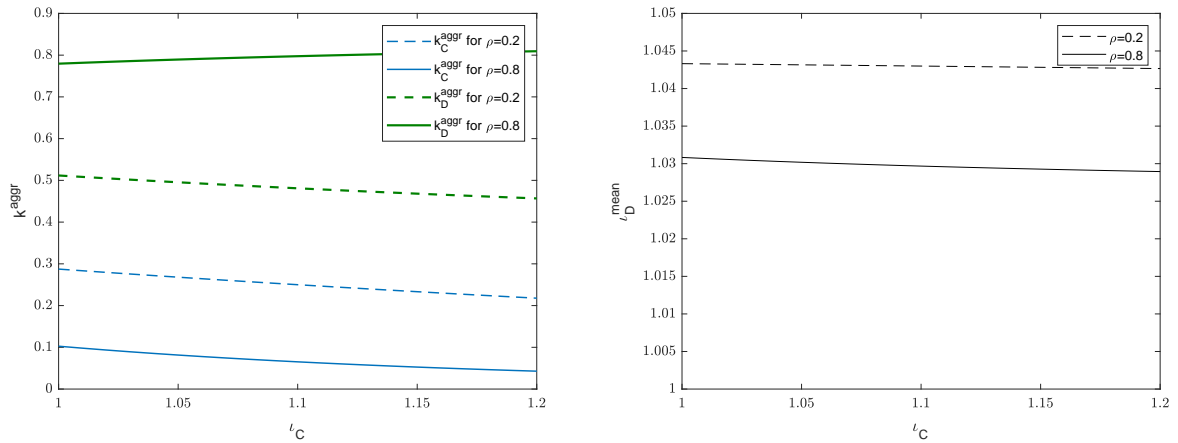


Figure 5
 Simulated aggregated capital, k_C^{aggr} and k_D^{aggr} , and average bank lending rate, ι_D , as a function of ι_C for different degrees of substitutability

Figure 6 shows the evolution of welfare as a function of the policy rate, ι_C . To maximize welfare, a social planner would choose $\iota_C = 1$, as this value eliminates the distortion introduced by the central bank's lending spread. An increase in the interest rate spread between the central bank's lending and deposit rate reduces welfare, the more so if entrepreneurs cannot easily switch from CBDC to inside money, hence, when ρ is small. With a higher degree of substitutability, welfare is less affected by high policy rates as entrepreneurs can more easily substitute away from k_C . At intermediate policy rates, however, welfare falls less quickly with low substitutability. This is due to production taking place at a combination of k_C and k_D which implies a ratio that is closer to the optimal one and thus more efficient.

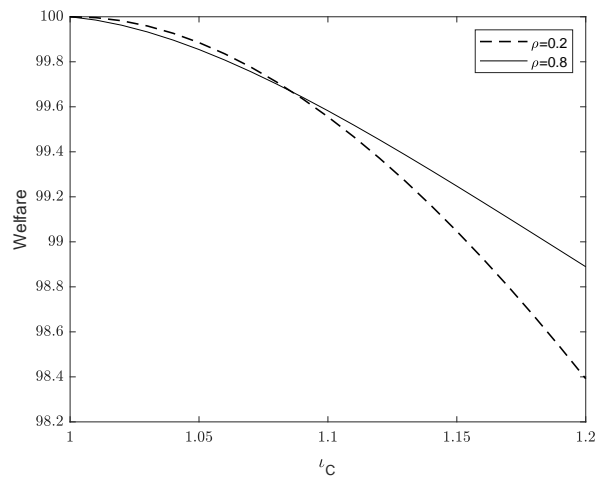


Figure 6
 Simulated welfare of ι_C for different degrees of substitutability

6.2.2 Different collateral constraints

Besides the interest rate spread on CBDC, another tool that affects capital investment, bank lending rates and welfare is the collateral haircut. As illustrated in the left panel of Figure 7, capital increases with a higher share of pledgeable output, η . If ρ is low and, hence, the two forms of capital are not easily substitutable, both k_C and k_D increase less with a higher η because of the quantity constraint on CBDC. When ρ is relatively high, however, k_D continues to increase also when η becomes high, since CBDC can be substituted by inside money relatively easily.

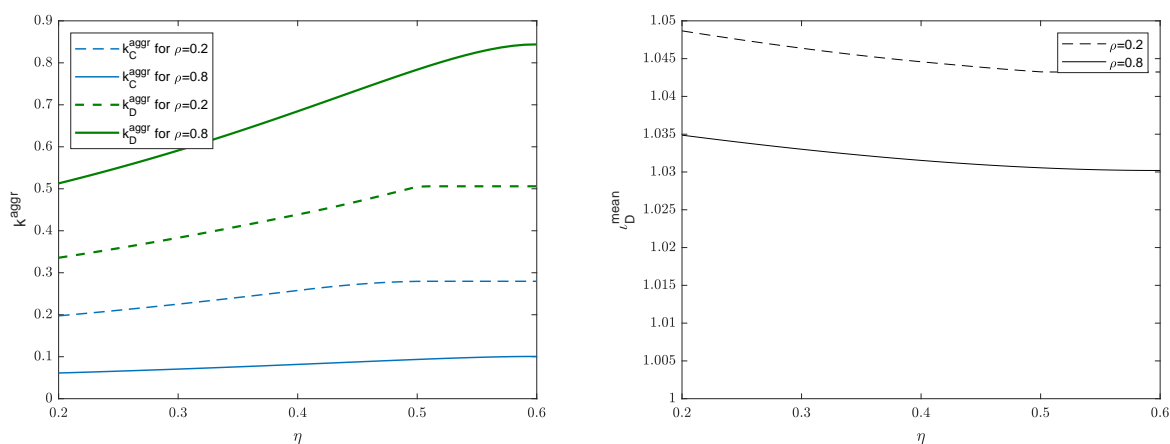


Figure 7
Simulated aggregated capital, k_C^{aggr} and k_D^{aggr} , and average bank lending rate ι_D as a function of η for different degrees of substitutability

The average bank lending rate decreases as η increases, as shown in the right panel of Figure 7, since the total surplus increases less than invested inside money. With a higher η , fewer entrepreneurs are constrained in their capital investments and, therefore, more entrepreneurs are closer to their optimal capital allocation (as long as the quantity constraint does not bind), leading to a lower marginal surplus of an additional unit of capital. As inside money is less substitutable by CBDC when ρ is smaller, less k_D is invested for a given total surplus, and therefore, the bank lending rate is higher than with a high degree of substitutability.

Figure 8 shows that welfare increases with a higher share of pledgeable output, η . As η increases, the amount of invested capital increases so that at some point the quantity constraint on CBDC starts to bind. Therefore, as η increases further, welfare increases with a smaller slope, which is even smaller if ρ is rather small and CBDC can be substituted by inside money less easily. For any level of η , welfare is higher when the degree of substitution between inside money

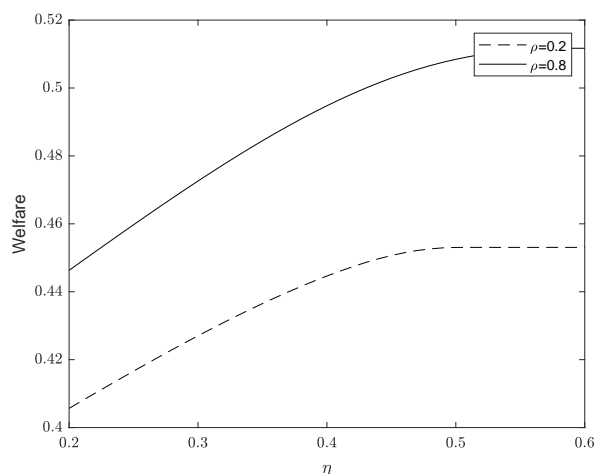


Figure 8
Simulated welfare of η for different degrees of substitutability

and CBDC is higher, since then entrepreneurs are more flexible in their capital allocation and can choose a combination that is closer to the optimum.

6.2.3 Different quantity constraints

Finally, we study different quantity constraints on CBDC, \bar{k} , as another policy tool of the central bank to steer the amount of invested CBDC. Figure 9 (left panel) shows capital investment as a function of the cap on CBDC. As the latter increases the invested amount of CBDC increases. Inside money investments depend on the degree of substitutability, though. At low rates of substitution, the demand for inside money can even increase more quickly than the demand for CBDC as the quantity constraint is loosened. If the degree of substitutability is high, however, demand for inside money falls, reflecting disintermediation of banks given that policy rates are kept constant. From a certain level of \bar{k} onward, bank intermediation seems to reach a steady state, which is reached faster, when ρ is high and more inside money is used.

The right panel of Figure 9 shows that for a low ρ the increase of k_D corresponds to a decrease in the bank lending rate as the quantity constraint becomes more slack. When ρ is high, these patterns are reversed. As the capital investments reach a constant level when \bar{k} is high enough, the bank lending rates do so as well.

Figure 10 shows that raising the upper bound on CBDC leads to an increase in welfare, which is not surprising as the model by construction assigns an important role to CBDC. While the quantity constraint matters less for welfare if substitutability between k_D and k_C is high, welfare

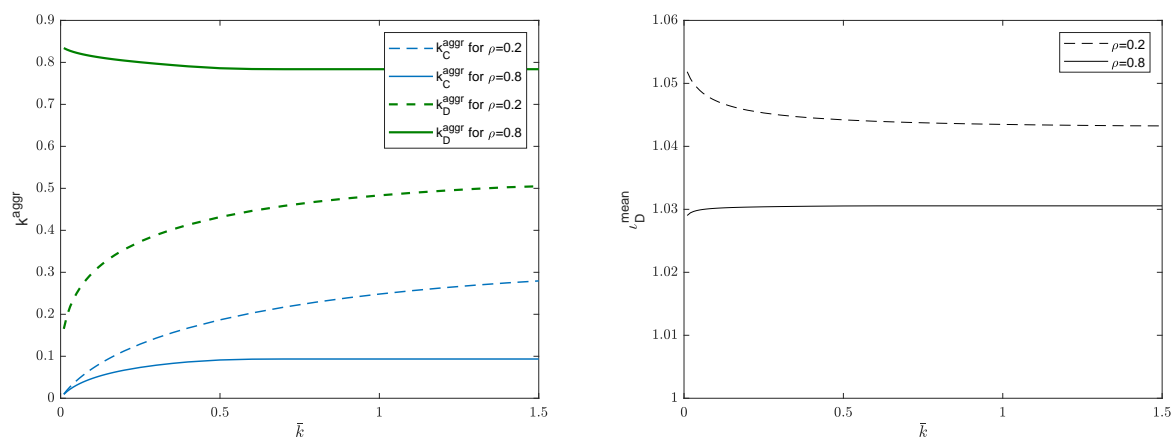


Figure 9
 Simulated aggregated capital, k_C^{aggr} and k_D^{aggr} , and average bank lending rate r_D as a function of \bar{k} for different degrees of substitutability

gains are more significant if inside and outside money are not easily substitutable, hence, if ρ is low.

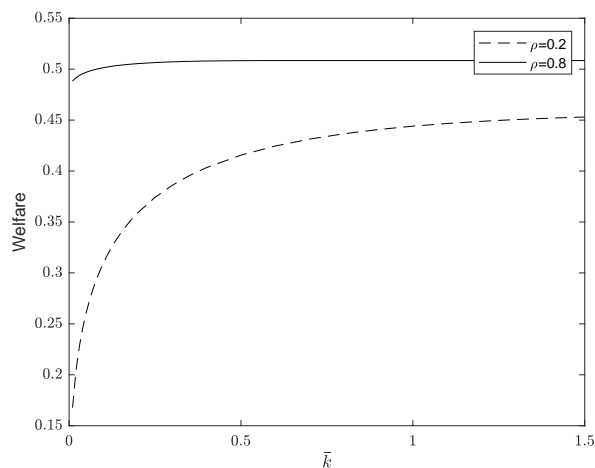


Figure 10
 Simulated welfare of \bar{k} for different degrees of substitutability

6.3 Welfare gains of CBDC and the degree of substitutability

Finally, we investigate how an increased prevalence of CBDC affects welfare. As it has become clear above, the answer depends crucially on the assumption for the degree of substitutability between CBDC and inside money. Therefore, we compare for a range of $\rho \in (0, 1)$ welfare obtained in the case of a high quantity constraint (\bar{k}^{high}) with welfare obtained in the case of a

low quantity constraint (\bar{k}^{low}). The welfare gain of expanding CBDC supply as a function of ρ is then calculated according to the following equation:²⁷

$$\text{Welfare gain}(\rho) = \text{Welfare}(\bar{k}^{high}, \rho) - \text{Welfare}(\bar{k}^{low}, \rho) .$$

Figure 11 shows that the welfare gain of expanding CBDC supply is high for a low degree of substitutability and approaches 0 as ρ becomes close to 1. When CBDC and inside money are not close substitutes (low ρ), welfare is much lower in a case where almost no CBDC can be used for investment than it is in a case where the quantity constraint only binds for large investment projects. Therefore, for low values of ρ , expanding CBDC supply leads to considerable welfare gains. In contrast, when these two forms of money can be substituted rather easily (high ρ), the increased supply of CBDC has almost no effect on welfare. The elasticity of substitution between CBDC and bank deposits will be affected by the design features of CBDC, such as its ease of use in specific situations. Our results suggest that the welfare gain from CBDC is rather small if CBDC covers similar use cases as bank deposits. If CBDC provides benefits to users in situations where bank deposits are less useful or not universally accepted, welfare gains from expanding CBDC supply are larger.

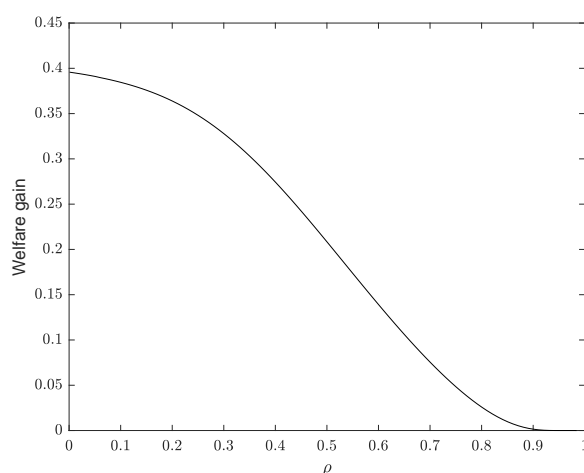


Figure 11
Simulated welfare gain of expanding CBDC supply as a function of ρ

²⁷ We set $\bar{k}^{high} = 1.5$ as in our baseline simulations and $\bar{k}^{low} = 0.001$.

6.4 Discussion of the assumptions on i_D

In this section, we explore how the assumptions on the bank deposit rate, i_D , affect the results for the bank loan rate, i_{ℓ_D} . Until now, we have assumed that $i_D = 0$. We will compare this case with the case $i_D = i_C$. For both assumptions, we will analyse how a change in the policy rates, expressed in ι_C , affects the inside money loan rate.

Figure 12 shows the bank loan rate, i_{ℓ_D} , as a function of ι_C for assumption 1 (left panel) and assumption 2 (right panel). Recall that $\iota_C = \frac{1+i_{\ell_C}}{1+i_C}$ is the spread between the CBDC loan rate, i_{ℓ_C} , and the CBDC deposit rate, i_C . For assumption 1, it does not matter whether a change in ι_C stems from a change in i_C or i_{ℓ_C} , since $i_D = 0$ by assumption. With $i_D = 0$, $i_{\ell_D} = \iota_D - 1$, and we can interpret ι_D as the (gross) bank loan rate. Therefore, the left panel in Figure 12 is basically the same as the right panel in Figure 5.

For assumption 2, $i_D = i_C$ (shown in the right panel of Figure 12), we need to distinguish, whether an increase in ι_C arises from an increase in i_{ℓ_C} or a reduction in i_C , since i_D follows the latter. Let us assume that $i_{\ell_C} = 0.2$ is constant. That means that a gradual increase of ι_C from 1 to 1.2, as depicted in Figure 12, is equivalent to a gradual decrease in i_C and, hence, generates a gradual decrease of i_D from 0.2 to 0. Since the spread ι_D only decreases very little as ι_C changes (as it can be seen in Figure 5), i_{ℓ_D} decreases as ι_C increases. Differently speaking, as the central bank decreases the CBDC deposit rate and the bank deposit rate follows this decrease, the bank loan rate decreases as well, and even slightly more.

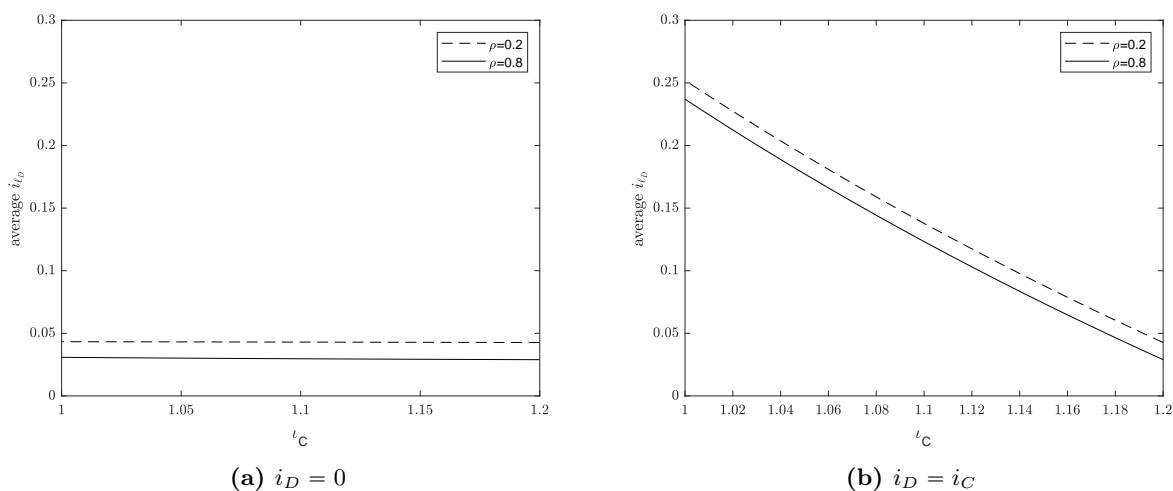


Figure 12
Average bank lending rate i_{ℓ_D} of ι_C for different degrees of substitutability

Note that when $i_C = 0$ and, hence, $\iota_C = 1.2$, both graphs in Figure 12 display the same values for the bank lending rate, i_{ℓ_D} , since in this case, $i_D = 0$ under both assumptions. We have only shown the bank lending rate in this section, since, as explained above, the real quantities and welfare are the same under the two assumptions. For these variables, only the value of ι_C – that is, only the spread between the CBDC loan rate and the CBDC deposit rate – matters.

7 Conclusions

We build a general equilibrium search model in which the central bank and commercial banks compete in supplying money to finance investment projects paid for in CBDC or inside money. As entrepreneurs cannot commit to repay debt, workers require payment in the form of a generally accepted medium of exchange, a friction that gives rise to the demand for money. Entrepreneurs borrow inside and outside money from commercial banks and from the central bank. However, because of the limited commitment friction, they are required to pledge collateral.

We define CBDC as a public or central bank liability that is interest bearing or paying and may be quantitatively constrained by the central bank. The central bank fixes interest rates on CBDC and possible quantity or collateral constraints exogenously, whereas banks and firms negotiate on bank lending rates and volumes.

We show that capital allocation and the welfare gains depend on the degree by which collateral or quantity constraints are binding, as well as on the spread between the CBDC deposit and lending rate. All three parameters constitute distortions lowering output. Conversely, relaxing CBDC caps and collateral constraints is strictly welfare improving. CBDC improves the overall allocation of resources and thereby increases output and welfare (as long as production inputs are not perfect substitutes), as it always reduces frictions in credit provision. At the same time, the provision of CBDC can reduce commercial bank credit and thereby disintermediate banks to some extent. Increasing the interest rate on CBDC is effective in containing bank disintermediation, in particular if inside and outside money are close substitutes.

We leave the impact of CBDC on monetary policy transmission over the business cycle or on financial stability to future research. In our setting prices are flexible and money is neutral over the business cycle. In addition, assets are safe and liquid. Our modelling framework is therefore informative of the impact of CBDC on the steady state, i.e. structural changes in the financial system.

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Appendix: Derivations and Proofs

Derivation of the first-best quantities. In what follows we calculate the first-best investment quantities that would be attained in the absence of the limited commitment friction. Note that the disutility of producing capital goods occurs in period t while the utility of consuming the generic good output occurs in period $t + 1$. Hence, the optimization problem satisfies

$$(k_C^*, k_D^*) = \arg \max_{k_C, k_D} \beta y - k_C - k_D = \varepsilon^{1-\alpha} f - k_C - k_D.$$

The first-order conditions for k_D and k_C satisfy

$$\varepsilon^{1-\alpha} f_C = 1 \text{ and } \varepsilon^{1-\alpha} f_D = 1. \quad (28)$$

Note that the ratio k_C/k_D satisfies²⁸

$$\omega^* \equiv \frac{k_C^*}{k_D^*} = \left(\frac{a}{1-a} \right)^{\frac{1}{1-\rho}}. \quad (29)$$

Then, from (29) we obtain $k_C^* = k_D^* \omega^*$. Use this expression to derive f as a function of k_D^* :

$$f = (k_D^*)^\alpha \Omega^*, \text{ where } \Omega^* \equiv [a(\omega^*)^\rho + 1 - a]^{\alpha/\rho}$$

Then, use $\varepsilon^{1-\alpha} f_D = 1$ to solve for k_D^* :

$$k_D^* = \varepsilon^{\frac{1-\alpha}{1-\rho}} \alpha^{\frac{1}{1-\rho}} f^{\frac{\alpha-\rho}{\alpha(1-\rho)}} (1-a)^{\frac{1}{1-\rho}}$$

Substitute f to get a closed-form solution for k_D^* :

$$k_D^* = \varepsilon \alpha^{\frac{1}{1-\alpha}} (\Omega^*)^{\frac{\alpha-\rho}{\alpha(1-\alpha)}} (1-a)^{\frac{1}{1-\alpha}}$$

Use k_D^* to get a closed-form solution for f :

$$f = \varepsilon^\alpha \alpha^{\frac{\alpha}{1-\alpha}} (\Omega^*)^{\frac{1-\rho}{1-\alpha}} (1-a)^{\frac{\alpha}{1-\alpha}}$$

²⁸ To see this, note that $\frac{f_C}{f_D} = \frac{\alpha f^{\frac{\alpha-\rho}{\alpha}} a k_C^{\rho-1}}{\alpha f^{\frac{\alpha-\rho}{\alpha}} (1-a) k_D^{\rho-1}} = 1$.

The total surplus $TS = \beta y^* - k_C^* - k_D^*$ satisfies:

$$TS^* = \beta y^* (1 - \alpha)$$

Thus, $\beta y^* - k_C^* - k_D^* \geq 0$ for all ε . To see this, consider (29) and write it as follows:

$$(1 - a) \omega^* = a (\omega^*)^\rho$$

We can use this expression to obtain:

$$\begin{aligned} \Omega^* &= [(1 - a) \omega^* + 1 - a]^{\alpha/\rho} \\ &= (1 - a)^{\alpha/\rho} [1 + \omega^*]^{\alpha/\rho}, \end{aligned}$$

implying that

$$\begin{aligned} k_D^* + k_C^* &= k_D^* (1 + \omega^*) \\ &= k_D^* (\Omega^*)^{\rho/\alpha} (1 - a)^{-1} \\ &= \varepsilon \alpha^{\frac{1}{1-\alpha}} (1 - a)^{\frac{\alpha}{1-\alpha}} (\Omega^*)^{\frac{1-\rho}{1-\alpha}} \\ &= \alpha \beta y. \end{aligned}$$

In summary, the optimal quantities and the total surplus satisfy:

$$\begin{aligned} \beta y^* &= \varepsilon \alpha^{\frac{\alpha}{1-\alpha}} (1 - a)^{\frac{\alpha}{1-\alpha}} (\Omega^*)^{\frac{1-\rho}{1-\alpha}} \\ k_D^* &= \varepsilon \alpha^{\frac{1}{1-\alpha}} (1 - a)^{\frac{1}{1-\alpha}} (\Omega^*)^{\frac{\alpha-\rho}{\alpha(1-\alpha)}} \\ k_C^* &= k_D \omega^*. \\ TS^* &= \beta y^* (1 - \alpha) \end{aligned}$$

■

Derivation of the collateral constraint. The collateral constraint satisfies

$$(1 + i_{\ell_C}) \ell_C + (1 + i_{\ell_D}) \ell_D \leq e + \eta \varepsilon^{1-\alpha} f(k_C, k_D). \quad (30)$$

If the entrepreneur borrows ℓ_C at the standing facility, then he has to pay back $(1 + i_{\ell_C})\ell_C$. In real terms that quantity satisfies $(1 + i_{\ell_C})\ell_C\phi_C^+$. Finally, to convert it into utility we have to discount it and obtain

$$\beta\ell_C\phi_C^+(1 + i_{\ell_C}).$$

Using the workers' first-order condition, we can write this quantity as $\iota_C k_C$.

The banker's surplus is $S_B = \theta TS = \beta\phi_D^+\ell_D(1 + i_{\ell_D}) - \beta\phi_D^+\ell_D(1 + i_D)$. To guarantee this surplus, the banker requires that the entrepreneur pays back $(1 + i_{\ell_D})\ell_D$. In real terms that quantity satisfies $(1 + i_{\ell_D})\ell_D\phi_D^+$. Finally, to convert it into utility we have to discount it and obtain

$$\beta\ell_C\phi_D^+(1 + i_{\ell_D}).$$

Using the workers' first-order condition, we can write this quantity as $\iota_D k_D$.

Consider, next, the right-hand side of the collateral constraint. The entrepreneur is endowed with an idiosyncratic endowment which yields utility e/β when consumed in the settlement market. Since the collateral constraint is applied in the investment market, we need to discount the utility and so the discounted consumption utility of the idiosyncratic endowment is e .

The entrepreneur can also pledge the fraction η of the output $y = \beta^{-1}\varepsilon^{1-\alpha}f(k_C, k_D)$. The quantity y is also the consumption utility generated from consuming y . We need again to discount the consumption utility. Hence, the discounted utility of the pledgable output is

$$\eta\beta y = \eta\varepsilon^{1-\alpha}f(k_C, k_D).$$

Optimality requires that (14) holds with equality. That is, $(1 - \theta)TS = S_E$, or equivalently $S_B = \theta TS$. Furthermore, from (12), we have

$$S_B = k_D \frac{(i_{\ell_D} - i_D)}{1 + i_D} = k_D \frac{(1 + i_{\ell_D} - (1 + i_D))}{1 + i_D} = \iota_D k_D - k_D.$$

Since $S_B = \theta TS$, we have

$$\iota_D k_D = \theta TS + k_D.$$

Use this expression to rewrite the collateral constraint (30) as follows:

$$\iota_C k_C + k_D + \theta TS \leq e + \eta\varepsilon^{1-\alpha}f(k_C, k_D).$$

■

Proof of Lemma 1. In what follows, we impose parameter restrictions such that for $\varepsilon^{1-\alpha} f_C = \iota_C$ and $\varepsilon^{1-\alpha} f_D = 1$, we have

$$\iota_C k_C + k_D + \theta TS > \eta \varepsilon^{1-\alpha} f(k_C, k_D). \quad (31)$$

This restriction on parameters implies that for $e = 0$, all entrepreneurs' collateral constraints are binding. It then follows that for $e > 0$ entrepreneurs with a small ε are unconstrained and entrepreneurs with large ε are constrained.

To derive the critical value ε^{13} we need to derive k_C and k_D under the assumption that constraints are not binding. Then, from (25) we get $k_C = k_D \omega$. As before, we can use this expression to derive $f(k_C, k_D)$ as a function of k_D :

$$f(k_C, k_D) = k_D^\alpha \Omega, \text{ where } \Omega \equiv [a\omega^\rho + 1 - a]^{\alpha/\rho} \text{ with } \omega = \frac{a}{\iota_C(1-a)}^{\frac{1}{1-\rho}}.$$

Following the same steps as before, we obtain

$$k_D = \varepsilon \alpha^{\frac{1}{1-\alpha}} \Omega^{\frac{\alpha-\rho}{\alpha(1-\alpha)}} (1-a)^{\frac{1}{1-\alpha}}.$$

Using k_D , we obtain an explicit solution for f and βy :

$$\begin{aligned} f &= k_D^\alpha \Omega = \varepsilon^\alpha \alpha^{\frac{\alpha}{1-\alpha}} \Omega^{\frac{1-\rho}{1-\alpha}} (1-a)^{\frac{\alpha}{1-\alpha}} \\ \beta y &= \varepsilon^{1-\alpha} f = \varepsilon \alpha^{\frac{\alpha}{1-\alpha}} \Omega^{\frac{1-\rho}{1-\alpha}} (1-a)^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

Also for the other variables, we can follow the same approach as in the derivations for the first-best solution. The only difference is that now $\omega = \left(\frac{a}{\iota_C(1-a)}\right)^{\frac{1}{1-\rho}}$ with ι_C possibly larger

than 1. In summary, the equilibrium quantities in region 1), in which none of the constraint is binding, yet, satisfy

$$\beta y = \varepsilon \alpha^{\frac{\alpha}{1-\alpha}} \Omega^{\frac{1-\rho}{1-\alpha}} (1-a)^{\frac{\alpha}{1-\alpha}}, \quad (32)$$

$$k_D = \varepsilon \alpha^{\frac{1}{1-\alpha}} \Omega^{\frac{\alpha-\rho}{\alpha(1-\alpha)}} (1-a)^{\frac{1}{1-\alpha}}, \quad (33)$$

$$k_C = k_D \omega = \varepsilon \alpha^{\frac{1}{1-\alpha}} \Omega^{\frac{\alpha-\rho}{\alpha(1-\alpha)}} (1-a)^{\frac{1}{1-\alpha}} \omega, \quad (34)$$

$$TS = (1-\alpha) \beta y, \quad (35)$$

$$k_D + \iota_C k_C = \alpha \beta y. \quad (36)$$

We now use these quantities to derive an explicit condition such that $\iota_C k_C + k_D + \theta TS > \eta \varepsilon^{1-\alpha} f(k_C, k_D)$. Use (32)-(36) to get

$$\alpha + \theta(1-\alpha) > \eta.$$

Hence, the condition is

$$\psi \equiv (1-\theta)\alpha - (\eta - \theta) > 0.$$

Note that $\alpha > \eta$ is a sufficient condition for $\psi > 0$.

Finally, the critical value ε^{13} solves

$$\iota_C k_C + k_D + \theta TS = e + \eta \beta y.$$

where k_C , k_D , βy , and TS solve (32)-(35). Thus,

$$\alpha \beta y + \theta(1-\alpha) \beta y = e + \eta \beta y.$$

Solving for βy yields

$$\beta y = \frac{e}{(1-\theta)\alpha - (\eta - \theta)}.$$

Replacing βy yields

$$\varepsilon^{13} = \frac{e}{[(1-\theta)\alpha - (\eta - \theta)] \left[\alpha^{\frac{\alpha}{1-\alpha}} \Omega^{\frac{1-\rho}{1-\alpha}} (1-a)^{\frac{\alpha}{1-\alpha}} \right]}. \quad (37)$$

Note that ε^{13} is increasing in ι_C since Ω is decreasing in ι_C . Note further that output at ε^{13} satisfies

$$y^{13} = \frac{e}{(1-\theta)\alpha - (\eta - \theta)},$$

which is independent of ι_C . This implies that in the region where entrepreneurs are unconstrained, output decreases as ι_C increases. ■

Proof of Proposition 1. We first derive the critical values, ε^{13} , ε^{34} , ε^{12} , and ε^{24} , and then the condition under which the collateral constraint binds earlier than the quantity constraint.

Collateral constraints binds first. In this case, there are two critical values: ε^{13} (moving from region 1 to region 3) and ε^{34} (moving from region 3 to 4).

The critical value ε^{13} is given in (37). The critical value ε^{34} and the corresponding quantities k_C^{34} and k_D^{34} can be derived by solving the following equations simultaneously:

$$\begin{aligned} \frac{k_C^{34}}{k_D^{34}} &= \left(\frac{a}{\iota_C(1-a)} \right)^{\frac{1}{1-\rho}} \text{ with } k_C^{34} = \bar{k} \text{ and} \\ \iota_C k_C^{34} + k_D^{34} + \theta TS &= e + \eta (\varepsilon^{34})^{1-\alpha} f. \end{aligned}$$

Quantity constraints binds first. Assume now that, as we increase ε from $\varepsilon = 0$, we move from 1) to 2) and then to 4). There are two critical values: ε^{12} (moving from region 1 to region 2) and ε^{24} (moving from region 2 to 4).

From (34), the critical value ε^{12} solves $\bar{k} = \varepsilon^{12} \alpha^{\frac{1}{1-\alpha}} \Omega^{\frac{\alpha-\rho}{\alpha(1-\alpha)}} (1-a)^{\frac{1}{1-\alpha}} \omega$, leading to

$$\varepsilon^{12} = \frac{\bar{k}}{\alpha^{\frac{1}{1-\alpha}} \Omega^{\frac{\alpha-\rho}{\alpha(1-\alpha)}} (1-a)^{\frac{1}{1-\alpha}} \omega}.$$

The critical value ε^{24} and the corresponding quantities k_C^{24} and k_D^{24} solve

$$\begin{aligned} \varepsilon^{1-\alpha} f_D &= 1 \text{ with } k_C^{24} = \bar{k} \text{ and} \\ \iota_C k_C^{34} + k_D^{34} + \theta TS &= e + \eta (\varepsilon^{34})^{1-\alpha} f. \end{aligned}$$

Which constraint binds first? Whether the collateral constraint or the quantity constraint

binds first, depends on the critical values ε^{12} and ε^{13} . If $\varepsilon^{12} > \varepsilon^{13}$, the collateral constraint binds first. We have

$$\varepsilon^{13} = \frac{e}{[(1-\theta)\alpha - (\eta-\theta)]\alpha^{\frac{\alpha}{1-\alpha}}\Omega^{\frac{1-\rho}{1-\alpha}}(1-a)^{\frac{\alpha}{1-\alpha}}} \text{ and}$$

$$\varepsilon^{12} = \frac{\bar{k}}{\alpha^{\frac{1}{1-\alpha}}\Omega^{\frac{\alpha-\rho}{\alpha(1-\alpha)}}(1-a)^{\frac{1}{1-\alpha}}\omega}.$$

Hence, $\varepsilon^{12} > \varepsilon^{13}$ if

$$\begin{aligned} \frac{\bar{k}}{\alpha^{\frac{1}{1-\alpha}}\Omega^{\frac{\alpha-\rho}{\alpha(1-\alpha)}}(1-a)^{\frac{1}{1-\alpha}}\omega} &> \frac{e}{[(1-\theta)\alpha - (\eta-\theta)]\alpha^{\frac{\alpha}{1-\alpha}}\Omega^{\frac{1-\rho}{1-\alpha}}(1-a)^{\frac{\alpha}{1-\alpha}}} \\ \bar{k} &> \frac{e\omega\alpha(1-a)}{[(1-\theta)\alpha - (\eta-\theta)]\Omega^{\frac{\rho}{\alpha}}} \\ \bar{k} &> \frac{e(1-a)\omega\alpha}{[(1-\theta)\alpha - (\eta-\theta)][a\omega^\rho + 1 - a]} \\ \bar{k} &> \frac{e(1-a)\omega\alpha}{[(1-\theta)\alpha - (\eta-\theta)][\omega\iota_C(1-a) + 1 - a]} \\ \bar{k} &> \frac{e(1-a)\omega\alpha}{\psi(1-a)(\omega\iota_C + 1)} \\ \bar{k} &> \frac{e\alpha}{\psi(\iota_C + \frac{1}{\omega})} \equiv \Phi. \end{aligned}$$

■

Effects of CBDC interest rates. Here we derive the derivatives $\frac{dk_D}{d\iota_C}$ and $\frac{dk_C}{d\iota_C}$.

If unconstrained, we have

$$\begin{aligned} k_D &= \varepsilon\alpha^{\frac{1}{1-\alpha}}(1-a)^{\frac{1}{1-\alpha}}[a\omega^\rho + 1 - a]^{\frac{\alpha-\rho}{\rho(1-\alpha)}}. \\ &= \varepsilon\alpha^{\frac{1}{1-\alpha}}(1-a)^{\frac{1}{1-\alpha}}[(1-a)\iota_C\omega + 1 - a]^{\frac{\alpha-\rho}{\rho(1-\alpha)}} \\ &= \varepsilon\alpha^{\frac{1}{1-\alpha}}(1-a)^{\frac{\alpha}{\rho(1-\alpha)}}[\iota_C\omega + 1]^{\frac{\alpha-\rho}{\rho(1-\alpha)}}, \end{aligned}$$

since $\omega = \left(\frac{a}{\iota_C(1-a)}\right)^{\frac{1}{1-\rho}}$, implying that $\omega\iota_C(1-a) = a\omega^\rho$. Next, note that $\iota_C\omega = \left(\frac{a}{(\iota_C)^\rho(1-a)}\right)^{\frac{1}{1-\rho}}$ and, hence, $\frac{d\iota_C\omega}{d\iota_C} < 0$.

The closed form solution can be derived as follows:

$$\begin{aligned}
\frac{dk_D}{d\iota_C} &= \varepsilon \alpha^{\frac{1}{1-\alpha}} (1-a)^{\frac{1}{1-\alpha}} \frac{\alpha-\rho}{\rho(1-\alpha)} [a\omega^\rho + 1 - a]^{\frac{\alpha-\rho}{\rho(1-\alpha)}-1} a\rho\omega^{\rho-1} \\
&\quad \frac{1}{1-\rho} \left(\frac{a}{\iota_C(1-a)} \right)^{\frac{1}{1-\rho}-1} \frac{-(1-a)a}{(\iota_C(1-a))^2} \\
\frac{dk_D}{d\iota_C} &= \frac{\frac{\alpha-\rho}{\rho(1-\alpha)} k_D}{[a\omega^\rho + 1 - a]} a\rho\omega^{\rho-1} \frac{1}{1-\rho} \frac{\omega\iota_C(1-a)}{a} \frac{-(1-a)a}{(\iota_C(1-a))^2} \\
\frac{dk_D}{d\iota_C} &= -\frac{(\alpha-\rho)k_D a\omega^\rho}{(1-\alpha)[a\omega^\rho + 1 - a](1-\rho)\iota_C} \leq 0
\end{aligned}$$

Note that $\frac{d\iota_C\omega}{d\iota_C} < 0$ implies that $\frac{dk_D}{d\iota_C} > 0$ if $\rho > \alpha$.

The effect on k_C is always negative:

$$\begin{aligned}
\frac{dk_C}{d\iota_C} &= \frac{dk_D}{d\iota_C}\omega + k_D \frac{d\omega}{d\iota_C} = -\frac{(\alpha-\rho)k_D a\omega^\rho\omega}{(1-\alpha)[a\omega^\rho + 1 - a](1-\rho)\iota_C} - k_D \frac{1}{1-\rho} \left(\frac{a}{\iota_C(1-a)} \right)^{\frac{1}{1-\rho}} \frac{1}{\iota_C} \\
&= -\frac{(\alpha-\rho)k_D a\omega^\rho\omega}{(1-\alpha)[a\omega^\rho + 1 - a](1-\rho)\iota_C} - \frac{k_D\omega}{(1-\rho)\iota_C} \\
&= \frac{\omega k_D}{(1-\rho)\iota_C} \left[-\frac{(\alpha-\rho)a\omega^\rho}{(1-\alpha)[a\omega^\rho + 1 - a]} - 1 \right] \\
&= -\frac{\omega k_D}{(1-\rho)\iota_C} \left[\frac{(\alpha-\rho)a\omega^\rho + (1-\alpha)[a\omega^\rho + 1 - a]}{(1-\alpha)[a\omega^\rho + 1 - a]} \right] \\
&= -\frac{\omega k_D}{(1-\rho)\iota_C} \left[\frac{a\omega^\rho(1-\rho) + (1-\alpha)(1-a)}{(1-\alpha)[a\omega^\rho + 1 - a]} \right] < 0
\end{aligned}$$

Let us now analyze the effect of ι_C on ι_D . From the banker's surplus, we have:

$$\iota_D = \theta \frac{TS}{k_D} + 1 = \theta \frac{\varepsilon^{1-\alpha} f(k_C, k_D) - \iota_C k_C - k_D}{k_D} + 1 = \theta \left[\frac{\varepsilon^{1-\alpha} f(k_C, k_D) - \iota_C k_C}{k_D} - 1 \right] + 1,$$

which, using the explicit forms derived in the appendix, can be written as:

$$\begin{aligned}
\iota_D &= \theta \left[\varepsilon^{1-\alpha} k_D^{\alpha-1} \Omega - \iota_C \omega - 1 \right] + 1 = \theta \left[\varepsilon^{1-\alpha} \left(\varepsilon \alpha^{\frac{1}{1-\alpha}} \Omega^{\frac{\alpha-\rho}{\alpha(1-\alpha)}} (1-a)^{\frac{1}{1-\alpha}} \right)^{\alpha-1} \Omega - \iota_C \omega - 1 \right] + 1 \\
&= \theta \left[\alpha^{-1} (1-a)^{-1} \Omega^{\frac{\rho}{\alpha}} - \iota_C \omega - 1 \right] + 1 = \theta \left[\alpha^{-1} (1-a)^{-1} [a\omega^\rho + 1 - a] - \iota_C \omega - 1 \right] + 1
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{d\iota_D}{d\iota_C} &= \theta \frac{a}{\alpha(1-a)} \rho \omega^{\rho-1} \frac{1}{1-\rho} \left(\frac{a}{\iota_C(1-a)} \right)^{\frac{1}{1-\rho}-1} \frac{-a(1-a)}{(\iota_C(1-a))^2} \\
- &\quad \theta \left(\omega + \iota_C \frac{1}{1-\rho} \left(\frac{a}{\iota_C(1-a)} \right)^{\frac{1}{1-\rho}-1} \frac{-a(1-a)}{(\iota_C(1-a))^2} \right) \\
= &\quad \theta \left[\frac{a}{\alpha(1-a)} \omega^{\rho-1} \frac{\rho}{1-\rho} \omega \frac{-(1-a)}{\iota_C(1-a)} - \left(\omega + \iota_C \frac{1}{1-\rho} \omega \frac{-(1-a)}{\iota_C(1-a)} \right) \right] \\
= &\quad \theta \left[\frac{1}{\alpha} \frac{\rho}{\rho-1} \omega - \frac{\rho}{\rho-1} \omega \right] \\
= &\quad \theta \frac{1-\alpha}{\alpha} \frac{\rho}{\rho-1} \omega < 0 \text{ with a steeper slope for high } \rho \text{ and low } \iota_C.
\end{aligned}$$

■

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