

Mestre, Roman

Article

A wavelet approach of investing behaviors and their effects on risk exposures

Financial Innovation

Provided in Cooperation with:

Springer Nature

Suggested Citation: Mestre, Roman (2021) : A wavelet approach of investing behaviors and their effects on risk exposures, Financial Innovation, ISSN 2199-4730, Springer, Heidelberg, Vol. 7, Iss. 1, pp. 1-37,
<https://doi.org/10.1186/s40854-021-00239-z>

This Version is available at:

<https://hdl.handle.net/10419/237255>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



<https://creativecommons.org/licenses/by/4.0/>

RESEARCH

Open Access



A wavelet approach of investing behaviors and their effects on risk exposures

Roman Mestre*

*Correspondence:
roman.mestre@live.fr
MRE Université de
Montpellier, UFR d'économie
Avenue Raymond Dugrand–
Site de Richter C.S. 79606,
34960 Montpellier Cedex 2,
France

Abstract

Exposure to market risk is a core objective of the Capital Asset Pricing Model (CAPM) with a focus on systematic risk. However, traditional OLS Beta model estimations (Ordinary Least Squares) are plagued with several statistical issues. Moreover, the CAPM considers only one source of risk and supposes that investors only engage in similar behaviors. In order to analyze short and long exposures to different sources of risk, we developed a Time–Frequency Multi-Betas Model with ARMA-EGARCH errors (Auto Regressive Moving Average Exponential AutoRegressive Conditional Heteroskedasticity). Our model considers gold, oil, and Fama–French factors as supplementary sources of risk and wavelets decompositions. We used 30 French stocks listed on the CAC40 (Cotations Assistées Continues 40) within a daily period from 2005 to 2015. The conjugation of the wavelet decompositions and the parameters estimates constitutes decision-making support for managers by multiplying the interpretive possibilities. In the short-run, (“Noise Trader” and “High-Frequency Trader”) only a few equities are insensitive to Oil and Gold fluctuations, and the estimated Market Betas parameters are scant different compared to the Model without wavelets. Oppositely, in the long-run, (fundamentalists investors), Oil and Gold affect all stocks but their impact varies according to the Beta (sensitivity to the market). We also observed significant differences between parameters estimated with and without wavelets.

Keywords: Risk exposures, CAPM, Multi-betas model, Time–frequency analysis, MODWT, Oil, Gold

JEL Classification: C32, C58, C65, G11, G40

Introduction

Markowitz (1952) Modern Portfolio Theory led the development of the Capital Asset Pricing Model (CAPM) which was created by Sharpe (1964), Lintner (1965) and Mossin (1966). Its mathematical equation, the Securities Market Line (SML), is similar to a simple regression model between the asset risk premium and the risks of the Market. This is also known as the Market Model. According to the CAPM hypothesis, the market is the only source of risk and the agents have homogeneous investing behavior. Systematic risk is measured through estimations of the traditional Market Model. Many authors such as Black et al. (1972) and then Fama and MacBeth (1973) highlighted several statistical anomalies in the model, more particularly, the non-robustness of the methods used

resulting from the autocorrelation-heteroscedasticity in the residuals of the estimations, and the potential absence of exogenous variables in the model.

Following the first tests of CAPM, Fabozzi and Francis (1978) indicated that the Beta parameter is unstable over time, resulting in confirmation by Bos and Newbold (1984). This characteristic of Beta implies that time-varying parameter estimation is required. Many methods have been tested such as rolling-window, recursive regression, or the Kalman-Bucy filter (Groenewold and Fraser 1997, 1999; Brooks et al. 1998; Faff et al. 2000; Yeo 2001). The multivariate GARCH (Generalised AutoRegressive Conditional Heteroskedasticity) approach is also developed to estimate time variance, according to Bollerslev et al. (1988). Advanced multivariate GARCH models such as the DCC-GARCH (Dynamic Conditional Correlation Generalised AutoRegressive Conditional Heteroskedasticity) or BEKK-GARCH (Baba, Engle, Kraft and Kroner Generalised AutoRegressive Conditional Heteroskedasticity) are also employed as attested by the studies of Choudhry and Wu (2008), Tsay et al. (2014), and Engle (2016).

Another way to improve the CAPM is for it to remain as an addition to explanatory variables in the Market Line equation. This kind of approach appears in Multi-Factors or Multi-Betas Models, originally initiated and theoretically constructed by Merton (1973) and Ross (1976) (with the Arbitrage Pricing Theory or APT). The number and the choice of selected variables vary according to the authors and their analysis. Bantz (1981) and Basu (1983) highlight the importance of considering the effects of accounting variables (specific to each stock) on equity returns, in accordance with their capitalization or size (in annual or quarterly frequencies). In an extension of these works, Fama and French (1992,1996) established three factors that CAPM generally referred to as the Fama–French Model, considering the variables of Price-to-Book and the Company Size (or more precisely, the relative performance of small companies versus big, and of companies with high Price-to-Book levels versus low). Otherwise, Chen et al. (1986) incorporated macro-economic variables as output or interest rates in the Market-Line equation.

Commodity prices also become an important source of risk. Since the oil shocks of the 1980s, several studies specifically highlight the links between financial markets and oil prices. Huang et al. (1996), Jones et al. (2004), Basher and Sadorsky (2006), and Boyer and Filion (2007) show the effects oil price variations have on stock returns. According to these authors, this variable positively affects oil and energy companies within oil producing countries. Lee and Zeng (2011), using Quantile Regressions, yielded similar results for G7 countries. These works establish that there is a relationship between oil and stock markets but there are few analyses about investing results for individual equities. Oil prices provide information about energy demand and it is therefore used as a macroeconomic indicator of economic health. Consequently, oil price risk can be measured by the Multi-Betas Model that provides information about stock's sensitivity/exposure to this type of risk.

Gold is also an interesting factor because it is generally considered a “safe haven” from counter-cyclical variations to the market as indicated by Baur and Lucey (2010) and Baur and McDermott (2010, 2016). There are many studies about gold-market relationships confirming this fact. Sumner et al. (2010) showed that the market affects gold prices during periods of crisis but the links are weaker in times of expansion. Miyazaki et al. (2012) confirm gold's interest in portfolio management as a counter-cyclical asset with a low

correlation in short-term markets. Mirsha et al. (2010) highlighted a bi-causal relationship between gold prices and the Indian stocks market. More recently, Arfaoui and Ben Rejeb (2017) using U.S data and Hussain Shahzad et al. (2017) referring to a panel of European countries (Greece, Ireland, Portugal, Spain, and Italy) confirm that gold prices influence the financial market overall.

These works mainly focus on the analyses of gold-markets relationships, but few studies directly introduce gold into the CAPM to measure equity exposure. Chua et al. (1990) include gold in the CAPM as a dependant variable. They then consider gold as an asset similar to equity and show that it has a weak Beta. These authors don't address the reverse relation and stock sensitivities to gold price fluctuations. However, Tufano (1998) analyzes the CAPM with gold as an explanatory variable in north-American mining stocks. He concludes that these stocks have greater sensitivity to gold prices compared to market variations because the Beta results related to gold are higher. This author also highlights the effect data frequencies have on the Beta value. Johnson and Lamdin (2015) and He et al. (2018) find similar results with more recent UK-US data (2005–2015). The combination of these different works leads us to introduce oil and gold prices as additional factors to the Market and the Fama–French Factors.

The hypothesis of Asset Pricing Models is discussed with a focus on the behavioral hypothesis of agents engaging in the same horizon investments and making homogeneous decisions towards portfolio allocation. Agents are supposed to have homogeneous investing behaviors. A large part of the literature tries to overcome this issue by considering a Behavioral Asset Pricing Model considering cognitive bias, heterogeneity of beliefs, investor attention, among others., as a factor in the model (Tuyon and Ahmad 2018; Wen et al. 2019; Gaffeo 2019; Heyman et al. 2019; Nanayakkara et al. 2019; Nasiri et al. 2019).

The objective of this paper is to analyze and compare the effects of short- and long-run exposures to these different sources of risk and discuss the behavioral hypothesis of agents' homogeneity underlining the CAPM and its extensions using a wavelet approach.

The main interest of this method is that wavelets methodology conserves both time and frequency information of financial time series. This approach is particularly useful for distinguishing short- and long-run co-movements and links between financial variables (Aguiar-Conraria and Soares 2014; Kahraman and Unal 2019).

In practice, investors have heterogeneous behaviors that result in different investing frequencies. In our case, frequency information is translated as the investment horizons of investors. We then use wavelet decompositions that allow for the distinction between short- and long-run sensibilities. We can compare the positions of High-Frequency Traders (HFT) having a short-run vision with those of mutual funds invested in the long-run. These two agents don't valorize the same market information but they still use the same models and methods for adapting their appetite for creating their own time series. The wavelets related to the time–frequency analysis represent a response to this type of problem. The discreet decompositions or Maximal Overlap Discrete Wavelets Transform MODWT (see Mallat and Meyer works) appear like the easiest and most suitable tool in this case. Gençay et al. (2005) reveal this with U.S data and then Mestre and Terraza (2018) with French data, which show that wavelets can indicate the

heterogeneous behavioral hypothesis leading to a Beta differentiation according to various investment horizons in the CAPM framework.

In this paper, we extend the Mestre and Terraza (2018) approach in a multivariate case to appreciate the stock sensitivities to various risks according to investment horizons. We estimate a Time–Frequency Multi-Betas Model considering oil and gold prices and the Fama–French factor with AR-EGARCH errors (AutoRegressive Exponential AutoRegressive Conditional Heteroskedasticity) in order to overcome the previous CAPM's limits. We use 30 French CAC40 Cotations Assistées Continues 40) equities for which quotations are perennial over a daily period from 2005 to 2015.

In the first part of this study, we estimated parameters of standard Multi-Betas-EGARCH (without wavelets). In the second part, we decompose the wavelets by the variables and we build time–frequency models considering heterogeneous investing behaviors. In the third part, we realize a portfolio application to highlight the usefulness of our model. We discuss the results and the financial perspectives for portfolio managers in the conclusion.

Standard estimation of multi-betas model

Theoretically, the Multi-Beta Model of Merton (1973) or the APT (Arbitrage Pricing Theory) of Ross (1976) are extensions of the CAPM in a multivariate regression framework where more risk factors are considered. The Fama–French Model is considered to be a reference as it includes two additional factors: the difference between the return of portfolios composed by big and small capitalizations called SMB (for Small-Minus-Big) and the difference between the return of portfolios composed by high book-to-market (B/M) Ratios and low B/M ratios called HML (for High-Minus-Low). We also add the oil and gold prices in the CAPM equation because of their specific characteristics: Gold as a “safe haven” asset and oil as a particular variable of global economic vitality.

However, the presence of autocorrelational and heteroscedasticity effects in the CAPM has been observed by many authors such as Diebold et al. (1988) and Giaccoto and Ali (1982). One of the consequences of this observation is that the Beta parameters are inefficient. The (G)ARCH family processes of Engle (1982) and Bollerslev (1986) are currently used to estimate a Beta parameter more efficient than the OLS estimator (Bera et al. 1988; Schwert and Seguin 1990; Corhay and Rad 1996). The consideration of heteroscedasticity seems, however, to affect only periods of the high volatility of the model, as shown by Morelli (2003) by comparing two versions of CAPM (with and without GARCH) for U.K stocks. More recently, Bendod et al. (2017) compared the CAPM and the GARCH-CAPM for the oil sector stocks of Arab and Gulf countries and concluded that the EGARCH model is better adapted to estimate the Beta. Mestre and Terraza (2020), for French stocks, come to similar conclusions. They also specify that the beta differences (between CAPM and EGARCH-CAPM) are not very significant when Betas are less than one whereas a correction is necessary for larger betas (greater than one). For all of these studies, there is a clear improvement of the market line residuals characteristics.

We take into account the statistical limits observed in the model residuals and also agents' heterogeneity by estimating a Multi-Betas Model with AR-EGARCH errors. We estimate 30 French stocks listed on the CAC40 (used as the Market reference) within a

daily period between 2005 and 2015. We used the same database as Mestre and Terraza's (2018) study of time–frequency CAPM and also the WTI oil price/barrel listed on the New York Mercantile Exchange, the gold price per ounce listed on the London Bullion Market, and the SMB and HML of the Fama–French Model to build a Time–Frequency based Multi-Betas Model.

The characteristics of the series in log-form and the results of the Unit-Root Tests (see Table 7a, b) reject the stationary hypothesis. As indicated in the CAPM, the Risk Premium is computed by subtracting the risk-free rate (OAT 10 years rate) from returns (the stationary variables by the first difference filter). Table 7c summarizes the characteristics of the risk premia series. These variables are stationary and zero-mean (see Table 7b).

The Multi-Betas Model, in its standard version (without wavelets), is written as follows:

$$r_{i,t} = \beta_{m,i}r_{m,t} + \beta_{o,i}r_{o,t} + \beta_{g,i}r_{g,t} + \beta_{SMB,i}SMB_{i,t} + \beta_{HML,i}HML_{i,t} + \varepsilon_{i,t} \quad (1)$$

where $r_{i,t}$ is the risk premium of asset i , $r_{m,t}$ the Market Premium, $r_{o,t}$ and $r_{g,t}$ are Oil and Gold Premia, $SMB_{i,t}$ and $HML_{i,t}$ are the Fama–French factors.

Under the OLS Hypothesis, $\varepsilon_{i,t}$ is an *i.i.d* $(0, \sigma_\varepsilon)$ process so in this case beta parameters are consistent estimators. The above studies reject this hypothesis concerning $\varepsilon_{i,t}$. As a substitute, we use the AR(1)-EGARCH(1,1) from Nelson's (1991) study to characterize it (see Mestre and Terraza, 2020). The parameters of Eq. (1) and those of this process are simultaneously estimated by the Maximum Likelihood methods associated with a non-linear optimization algorithm (see Ye 1997; Ghalanos and Theussl 2011).

The Table 1 summarizes the Model estimations for the 30 equities ranked according to the decreasing value of the β_m .

All of the Beta parameters are significant and thanks to the determination coefficients (R^2), we note that the three variables explain 30–70% of assets total risks. This ranking reveals a relatively significant relationship ($R^2=0.33$) between the 30 Betas β_m and their corresponding determination coefficients. Equities with strong (high) β_m have a globally high R^2 but this relationship is disrupted by the presence of outliers linked to a few stocks, as exemplified by Alcatel and PSA.

Residuals of the Multi-Betas Model are non-autocorrelated and homoscedastic, but the normality hypothesis is not respected. However, we consider this model as statistically acceptable.

Significance tests of Betas are used to appreciate if the Multi-Betas Model is selected for all stocks. In this context, if $\beta_o = \beta_g = \beta_{SMB} = \beta_{HML} = 0$ the CAPM is selected for the stock. The overall results lead to the following comments.

We observe that the CAPM is retained for only three equities (Vivendi, Carrefour, and Schneider). Thus, the addition of other variables is not relevant because we accept $\beta_o = \beta_g = \beta_{SMB} = \beta_{HML} = 0$. In this case, the CAPM results remain valid for these stocks. For the other 27 equities, there is at least one significant additional variable. For 43.33% of the stocks (13 equities) we notice a significant β_o while β_g is significant for 53.33% of stocks (16 equities). Finally, these two additional variables are significant for 30% of the sample (Vivendi, Total, Technip, GDF, LVMH, Alcatel, AXA, BNP, and Crédit Agricole). Concerning the Fama–French factors, β_{SMB} is significant for 16 stocks

Table 1 Multi-betas-AR-EGARCH model estimates

MB-EGARCH	β_m	TSTAT	β_o	TSTAT	β_g	TSTAT	β_{SMB}	TSTAT	β_{HML}	TSTAT	R2	JB	LB	ARCH
Essilor	0.5465	34.3755	-0.0138	-1.5048	0.0374	1.8193	-0.0727	-2.1948	-0.0825	-2.6446	0.3156	14,886.8106	2.2929	3.4083
Sodexo	0.5764	34.0822	-0.0265	-3.0530	0.0126	1.4286	-0.0516	-1.4035	-0.0664	-1.8927	0.3523	9486.8058	0.5854	0.7400
Ricard	0.6592	41.3370	-0.0171	-1.6340	0.0366	2.2164	-0.0937	-2.2163	-0.2243	-5.0019	0.3594	8457.2479	5.1244	0.4965
Publicis	0.6888	41.5041	0.0012	0.1216	0.0246	3.9324	0.0989	3.5915	0.0577	1.6953	0.4273	1956.4191	5.6180	0.6882
Danone	0.7101	51.3373	-0.0053	-0.6735	0.0147	0.7169	-0.0294	-0.8576	-0.1754	-6.2064	0.4071	4941.3222	3.6895	2.0217
Orange	0.7371	47.5652	-0.0145	-1.8401	-0.0410	-2.2798	-0.0524	-1.7019	-0.0855	-3.1268	0.4348	4729.6820	2.3237	0.4009
L'Oréal	0.7862	56.3575	-0.0242	-3.3899	0.0071	0.5172	-0.0988	-2.9723	-0.1663	-5.9745	0.4892	4428.9182	1.9431	2.4978
Vivendi	0.7922	57.3108	-0.0080	-2.2791	-0.0103	-2.9845	-0.0033	-1.5092	-0.0262	-6.1989	0.5234	7736.5512	1.6092	0.1586
Veolia	0.8520	40.4459	-0.0297	-2.2036	-0.0253	-0.9384	-0.0639	-2.7895	0.0385	0.8809	0.3888	152,573.1525	0.7125	11.8468
Air Liquide	0.8541	63.7216	-0.0034	-0.7886	0.0386	3.0939	0.0792	2.9755	-0.0770	-2.8033	0.6509	7907.8450	2.7418	0.3281
Total	0.8663	78.3249	0.0770	11.1593	0.0772	6.7149	-0.0776	-3.9230	-0.0260	-2.1333	0.6953	2313.8447	2.9854	1.1112
Carrefour	0.8769	45.7226	-0.0159	-1.8985	-0.0015	-0.3979	-0.0228	-0.5863	-0.0023	-0.0567	0.4837	3807.5330	1.3692	1.1464
Technip	0.9178	36.4579	0.1388	11.1147	0.1029	4.5024	0.1105	1.9307	0.1286	2.0498	0.4139	7678.6395	5.1697	4.9311
Airbus	0.9448	41.6451	0.0045	0.5873	0.0213	3.7259	-0.0400	-1.0343	0.0575	2.5552	0.3551	104,366.3980	3.7567	0.0794
Accor	0.9446	40.8456	-0.0177	-1.4160	0.0354	1.2486	0.1471	3.4207	0.0625	1.2867	0.4788	5669.3152	1.6773	1.1744
GDF	0.9966	127.1810	-0.0182	-3.3635	-0.0416	-3.7110	-0.0919	-1.4589	-0.0728	-1.4691	0.4977	150,612.7368	5.7682	0.1908
Bouygues	0.9951	40.3635	-0.0194	-1.7900	0.0384	2.4603	-0.1034	-3.1617	0.1739	3.9969	0.5050	18,377.1038	0.5454	0.4945
Gemini	1.0042	44.3010	-0.0274	-2.1425	0.0181	0.7373	0.1185	2.4446	-0.0315	-0.6087	0.4855	2924.4063	2.6322	0.3840
Michelin	1.0136	49.0846	-0.0229	-1.8517	0.0370	1.4420	0.1794	3.9525	0.0932	1.8507	0.4899	3228.7508	4.0973	1.2892
LVMH	1.0319	75.9458	-0.0128	-1.9659	0.0185	2.2401	0.0567	2.2029	-0.0307	-1.4042	0.6250	10,589.6791	1.6496	0.7156
Vinci	1.0595	74.3620	-0.0183	-1.8476	0.0148	1.0902	0.0601	1.2580	0.0306	0.9531	0.6695	5088.6765	1.3454	1.0304
Alcatel	1.1255	41.6015	-0.0220	-0.6312	-0.0530	-0.6781	0.1500	1.9781	0.1130	0.7856	0.3928	1578.7975	3.8509	1.1019
PSA	1.1318	32.1611	-0.0236	-2.4479	-0.0831	-2.5733	0.2242	4.0979	-0.0907	-1.3327	0.3258	13,927.3025	0.9150	1.3106
Schneider	1.1898	67.4567	-0.0103	-1.0074	0.0194	1.0206	0.0659	1.6223	0.0416	1.0323	0.6763	1128.1090	9.8194	2.4801

Table 1 (continued)

MB-EGARCH	β_m	TSTAT	β_o	TSTAT	β_g	TSTAT	β_{SMB}	TSTAT	β_{HML}	TSTAT	R ²	JB	LB	ARCH
St-Gobain	1.2373	69.2948	− 0.0026	− 0.3686	0.0056	0.5795	0.1707	5.7325	0.0551	1.3654	0.6693	16,152.7003	2.6272	0.2183
Renault	1.2602	51.7172	− 0.0200	− 1.4395	0.0157	0.6035	0.1284	2.3379	0.0978	1.4316	0.5498	2219.4141	1.1933	1.9139
AXA	1.2893	67.0776	− 0.0444	− 4.4087	− 0.0577	− 2.9230	− 0.0733	− 1.7649	0.2083	4.5534	0.6726	45,909.1043	5.0398	0.0089
BNP	1.2925	64.2483	− 0.0405	− 3.9577	− 0.0827	− 5.1730	− 0.0317	− 0.7545	0.2717	5.9755	0.6185	40,068.3805	2.4614	0.5333
SG	1.3068	27.7305	− 0.0087	− 0.7971	− 0.0955	− 3.7482	− 0.0274	− 1.1462	0.3006	5.9691	0.5674	10,487.5738	5.0467	4.5046
CA	1.3262	55.4297	− 0.0332	− 3.5000	− 0.0812	− 3.3929	0.0003	0.0055	0.4087	6.2493	0.5679	7916.5054	1.9223	1.9958

At 5% significance level, Column LB (Ljung–Box test); $\chi^2(5) = 11.1$; Column ARCH (ARCH-LM test); $\chi^2(5) = 11.1$; Column J-B (Jarque–Bera Line); $\chi^2(2) = 5.99$. We use Weighted Tests of Ljung–Box and ARCH-M of Fisher and Gallagher (2012). Moreover, in this model the non-collinearity between exogenous variables are tested (see Table 8). The names of certain actions are abbreviated, see the list of abbreviations for more details on their full name

(53.33% of the sample) and β_{HML} for 15 (50%). They are both significant for five equities (Essilor, Ricard, L'Oréal, Air Liquide, and Total, Bouygues).

We built an adjustment of OLS Betas in order to quickly appreciate a more consistent β_m without reestimating the model with AR(1)-EGARCH¹ processes. This adjustment remains valid in the case of the Multi-Betas Model, because we observe no significant differences between the β_m (and residuals) of Multi-Betas Model in Table 2 and the β_m of a CAPM with AR(1)-EGARCH(1,1) errors (see Tables 9, 10). The addition of these additional variables has a limited impact on the vast majority of equities because the β_m are not affected. However, the Multi-Betas Model results have an interest to portfolio managers for analyzing and interpreting the β_o and β_g (their sign and their value) illustrating the sensitivities to oil and gold fluctuations.

For a large part of equities having a significant β_o , we notice that estimators are almost negative, however, sensitivities to oil movements are relatively low varying between -0.001 and -0.045% . Considering the classification of stocks by β_m , as in Table 1, we observe that oil affects the different equities profiles in the same way. Technip and Total are a notable exception to this case because their β_o have higher positive values compared to the other stocks. Thereby, an oil price rise by 1% entails a stock price rise by 0.08% for Total and by 0.14% for Technip. We conclude that stocks of the oil and gas sectors are the most sensitive to oil price fluctuations, which is relevant to their activities.

We note significant and negative β_g for seven stocks (Alcatel, SG, BNP, CA, AXA, Orange, Vivendi, and GDF) and positive for seven others equities (Publicis, Essilor, Ricard, Air Liquide, Bouygues, Total, and Technip). The Financial sector equities, classified as “risky” due to their β_m greater than one, are negatively affected by the gold, justifying in part its “safe haven” characteristic. We can extend this result to stocks with strong β_m (Financial Sector + Alcatel) having a negative and relatively higher sensitivity to gold fluctuations. On the contrary, stocks with $\beta_m < 1$ have positive and lower β_g . Once again, Total and Technip are exceptions because they are more sensitive to gold than other stocks.

By comparing these three sensitivity estimators, the market represents a major source of risk because the β_m are greater than β_g and β_o in absolute values. We also note that Gold sensitivity is higher (in absolute values) than oil sensitivity, particularly in financial sectors and for stocks with $\beta_m > 1$.

Time–frequency multi-betas model estimates

In the financial markets, the assumption of homogeneity of agents' behavior is difficult to maintain. The investment frequency of a global and a mutual funds portfolio, for example, depends on their buying or selling intentions based on various calculation/financial models. These models don't differentiate the agents and considers only an aggregation of behaviors (i.e. an “average behavior”) from the financial time series used. The use of wavelet time–frequency decompositions of these time series is justified in the Multi-Betas Model framework because the high-frequencies are related to HFT and the low-frequencies to fundamentalist investors. Wavelets represent a relevant solution to

¹ See Mestre and Terraza (2020).

Table 2 Percentages of significantly different Betas between Standard Multi-Betas Model and Time–Frequency Multi-Betas Model (TFMB)

	D1 (%)	D2 (%)	D3 (%)	D4 (%)	D5 (%)	D6 (%)
β_m	13.33	13.33	33.33	56.67	66.67	76.67
β_o	16.67	23.33	40.00	46.67	70.00	83.33
β_g	6.67	6.67	23.33	46.67	66.67	73.33
β_{SMB}	23.33	13.33	26.67	40.00	40.00	66.67
β_{HML}	13.33	13.33	33.33	50.00	70.00	63.33

We test if the difference between the two estimators are significant with Student Test. We count the number of significant differences and we expressed them as a percentage of the total number of actions (i.e. 30)

analyzing the behavior of agents that use this kind of financial model. In the rest of the paper, we name Standard Multi-Beta Models, the results of which are summarized in Table 1, to distinguish from its time–frequency versions estimated in this section.

The first paragraph is a brief reminder of wavelets methodology applied to our model before comparing results obtained in the previous part for the Standard Multi-Betas Model with its time–frequency version in the second paragraph. In a third paragraph, we analyse the frequency sensitivities of stock prices to exogenous additional variables.

Wavelets methodology reminder: the maximal overlap discrete wavelets transform (or MODWT):

A Wavelets-mother $\Psi(t)$ with zero-mean and normalised is written as follows²:

$$\int_{-\infty}^{+\infty} \psi(t) dt = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} |\psi(t)|^2 dt = 1 \quad (2)$$

These properties ensure the Variance/Energy preservation during the decomposition of a series and also guarantee the respect of admissibility condition (Grossman and Morlet 1984).

This wavelet-mother is shifted by the τ parameter and dilated by scale parameters to create “wavelets-daughters” regrouping in the wavelets family used as filtering basis:

$$\Psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \Psi\left(\frac{t - \tau}{s}\right) \quad (3)$$

The decomposition of time function $x(t)$ creates/lead to the wavelets coefficients $W(s, \tau)$ as follows:

$$W(s, \tau) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi^*\left(\frac{t - \tau}{s}\right) dt = \langle x(t), \psi_{\tau,s}(t) \rangle \quad (4)$$

ψ^* is the complex conjugate of ψ

τ and s parameters indicate the time and frequency localization of the coefficient. Thanks to the wavelets, we can represent the temporal localization of the frequency components, hence the name of the time–frequency analysis. These previous equations are a theoretical presentation of wavelet decompositions based on continuous wavelets. A time discreet version is used to decompose time series x_t but the principle remains

² We use the notation of Mallat (2001).

similar because frequencies are still continuous. The practical use of this kind of decomposition implies important computational time and effort, consequently, a frequency discretization is realized for a fast-decomposing time series, as is the MODWT. In this framework, wavelets are defined by a succession repeated J times of high-pass and low-pass filter combinations (Mallat Algorithm 1989, 2009). J is the decomposition order representing the optimal number of repetitions necessary to reconstruct a time series x_t of length N such as $J = \frac{\ln(N)}{\ln(2)}$.

Despite this simplified process, the MODWT is still variance/energy preserving. It ensures the perfect reconstruction of the decomposed series, without losses, by adding the high and low-frequencies components:

$$x_t = S_{J,t} + \sum_{j=1}^{j=J} D_{j,t} \quad (5)$$

$S_{J,t}$ is a basic approximation of the series and $D_{j,t}$ are the details, called also frequency bands, of scale j regrouping the frequencies in the interval $\left[\frac{1}{2^{j+1}}; \frac{1}{2^j}\right]$.

In Finance, the frequencies interpretation is simplified by translating them in periods that have the same time unit as the original data (for example, days, weeks, etc.). In this case, frequencies represent the different time investment horizons (short-medium-long run). The Table 11 records the frequency bands and the corresponding time horizons in days.

Considering the series length, through wavelets decomposition, we have 11 frequency bands and one approximation. The high-frequencies bands (D1–D2) are related to short-run investments whereas the low-frequencies illustrate long-run horizons. In order to simplify the analysis, we focus on the first six frequency bands: D1 bands are related to 2–4 days investment horizon (high-frequencies) whereas D6 band represents a 3–6 month investment (low-frequencies).

In the Multi-Betas Model framework, we decompose the dependent and the three independent variables by the MODWT. In the Multi-Betas Model, each frequency band of the stock are associated with the corresponding bands of the market, oil, gold, and Fama–French factors. By construction, the frequency bands means are equal to zero.

For an asset i , the Time–Frequency Multi-Betas Model is written as follows:

$$\begin{aligned} D_{j,t}^{asset} &= \beta_j^m D_{j,t}^{Market} + \beta_j^o D_{j,t}^{Oil} + \beta_j^g D_{j,t}^{Gold} + \beta_j^{SMB} D_{j,t}^{SMB} + \beta_j^{HML} D_{j,t}^{HML} + \varepsilon_{j,t} \\ \forall j &= 1, \dots, 6 \text{ the frequency band} \\ \varepsilon_{j,t} &\sim AR(1) - EGARCH(1, 1) \end{aligned} \quad (6)$$

Betas parameters are estimated in the time–frequency space and represent the asset sensitivities to the five factors considering agents' investment frequencies and considering stock risk profiling. The different time–frequency regression models are estimated previously by conserving the hypothesis of AR(1)-EGARCH(1,1) errors.

Table 12 summarizes all results³ of Time–Frequency Multi-Betas AR-EGARCH Model estimates.

³ We use the “rugarch” R—package developed by Ghalanos and Theussl (2011).

Time–frequency multi-betas model estimates

β_m coefficients are highly significant for all equities and frequencies. The coefficients of determination computed on the high-frequencies (D1–D2) are relatively closed to the overall Model of Table 1. But they become more important for medium–low-frequencies (D4–D5) and they are almost equal to 100% on the D6 band. The order of magnitude of the D5–D6 wavelets coefficients are small, therefore, a range of residuals estimates are also small, and then values of R^2 are high on low-frequencies. However, for all frequency band regressions, we notice a deterioration of residual characteristics. Particularly, the AR-EGARCH process no longer properly captures the heteroscedasticity. By increasing the order of the process, we reduce the autocorrelation and heteroscedasticity without significantly modifying the values of the Betas parameters. Despite these reservations, the Time–Frequency Multi-Betas Model has sufficient statistical properties to analyze its economic results.

Estimates of these parameters play an important role in investor strategies who question the choice of the model according to the significant parameters. Globally, we remark that stocks with a strong $\beta_m (> 1)$ have negative β_o and β_g for all frequency bands more particularly in the long-run. The stocks with low $\beta_m (< 1)$ have positive and relatively high β_g whereas β_o still mainly remain negative. Portfolio managers can thus appreciate the different sources of risk affecting their portfolios when making choices.

Table 2 summarizes the differences between parameters of standard and time–frequency models and represents an additional help to interpret results.

The Betas estimated without wavelets (Standard Multi-Betas) are globally similar to short-run Betas (D1–D2) for the majority of equities whereas the differences are more significant in the long-run. For example, in the long-run (D6), we note that differences between β_m of the Standard Model and TFMB are significant for 76.67% of equities.

Wavelets provide a differentiated beta estimate according to the investment frequencies, which are useful for identifying and analyzing the effects of investment horizons on systematic risk measures/indicators and on sensitivities to different factors. The intensity of β_o and β_g is greater in the long-run than in the short-run. For all equities, the selected variables more strongly affect assets for long-run investments. Therefore, we confirm the results of Gençay et al. (2005) as well as Mestre and Terraza (2018) concerning the interest of wavelets in market models for long-run investments. Both studies indicated that CAPM's Beta is frequency-varying for long-run investment horizons and the standard estimation of Beta does not hold. Therefore, equities risk profile changes. The Time–Frequency Multi-Betas Model is therefore of strategic interest for long-term investments by estimating its low-frequency exposures to risks in order to adjust their allocation if the initial characteristic is lost (see Part III).

Stocks sensitivities to oil and gold movements

By testing the significance of frequency parameters $\beta_o, \beta_g, \beta_{SMB}$ and β_{HML} for all stocks, we establish the following statements:

- If $\beta_o = \beta_g = \beta_{SMB} = \beta_{HML} = 0$, the addition of the four variables is not appropriate for this asset. In this case, the time–frequency CAPM⁴ is selected.

⁴ The time–frequency CAPM is already estimated in the study Mestre and Terraza (2018).

- If $\beta_o \neq 0, \beta_g \neq 0, \beta_{SMB} \neq 0$ and $\beta_{HML} \neq 0$ the five additional variables are relevant and the Full Multi-Betas Model (FULL MB) is retained.
- If $\beta_o \neq 0, \beta_g \neq 0, \beta_{SMB} = 0$ and $\beta_{HML} = 0$, only two additional variables are relevant and the Multi-Betas Model with Oil and Gold (MB OIL GOLD) is retained.
- If $\beta_o = 0, \beta_g = 0, \beta_{SMB} \neq 0$ and $\beta_{HML} \neq 0$, only Fama–French factors are relevant and the Fama–French Model (FF) is retained.
- If at least one of the five Beta is significant, we always retain the Multi-Betas Model under a mixed version.

We use this framework to summarize results in the Tables 3 and 13. We count for each frequency band the number of shares for which the CAPM or Multi-Betas is retained.

By reading Tables 3 and 12 in Appendix, it is possible to establish the following comments:

- There is no stocks having the CAPM on all frequency bands, while lot of equities retained a version of Multi-Betas Model whatever the investment horizon.
- In the short-run (D1), the CAPM is valid for only five stocks (16.66% of the sample) and results are similar to the previous estimate of the Standard Model. This percentage decreases as the time horizon increase, so more stocks retain the Full Multi-Betas Model in the long-run. After D3 bands, there are no equities having non-significant additional variables. Stocks are therefore more impacted/affected by oil and/or gold in the long-term than in the short-term as the Fama–French Factors affected a large part of stocks.

The Time Frequency Multi-Betas Model is therefore of statistical interest for a majority of equities whatever the investment horizons. This model is complementary to the CAPM results as it introduces the decompositions of risk sources. It can be decision-making support for investors by synthesizing the results of the stock sensitivities.

Table 4 synthesizes, for each frequency band, the percentage of $\beta_o, \beta_g, \beta_{SMB}, \beta_{HML}$ significantly greater, lower, or equal to zero and also their means. To improve the analysis, we also indicate the mean of the corresponding β_m .

The number of Multi-Betas Model having a positive β_g is relatively stable on all frequency bands (around 50%) whereas it increases for the model with positive β_o . Furthermore, we note that their means increase from D1 (short-run) to D6 (long-run). Stocks with positive β_g and/or positive β_o have an average β_m less or equal to one. The oil-sector stocks (Total, Technip) are strongly and positively sensitive to oil and gold for all frequency bands.

The number of stocks negatively sensitive to oil is higher in the short-term (D1–D2) than the long-term (D6), whereas it is stable for gold. The value of β_g and β_o increases as the investment horizon increases (in average). We note similar results for stocks with $\beta_o < 0$ as the mean of β_m increases accross frequencies, it is less than one until the D3-Bands but is greater beyond. Financial Equities (SG, BNP, CA, and AXA) are not sensitive to oil short-run variations. However, these stocks have strongly negative β_o when investment horizon increases (D5–D6). Financial stocks are also strongly negatively affected by gold prices both in the short- and long-run, however, the effect is greater. The equities are negatively sensitive to gold, which has an average β_m greater than one

Table 3 Analysis of Time–Frequency Multi-Betas Model results. Number of stocks for which the CAPM or the Multi-Betas is valid (in values and percentage)

	D1	D2	D3	D4	D5	D6
<i>In values</i>						
CAPM	5	2	3	0	0	0
FULL MB	2	3	5	13	14	25
MB OIL GOLD	3	2	0	0	1	0
FF	1	4	2	0	0	0
Mixed	19	19	20	17	15	5
<i>In %</i>						
CAPM	16.67	6.67	10.00	0.00	0.00	0.00
FULL MB	6.67	10.00	16.67	43.33	46.67	83.33
MB OIL GOLD	10.00	6.67	0.00	0.00	3.33	0.00
FF	3.33	13.33	6.67	0.00	0.00	0.00
Mixed	63.33	63.33	66.67	56.67	50.00	16.67

except on D1. However, we notice that the intensity of negative β_g is greater (on average) than the positive β_g mean. Gold negatively affects the stocks with an important systematic risk in the financial sector and Alcatel. During an expansion period, a rise in the market leads to a stronger increase in the stock's price and a decrease in the gold prices confirms the upward dynamic on stocks. Oppositely, during periods of crisis, the decreasing trend of the market pushes down the stock prices. In this context, investors close their positions and buy gold. Therefore, the demand for gold becomes increasingly important as its price naturally increases, confirming the investors' choices, while consequently, stock prices decrease. The “safe haven” characteristic of gold is partially justified even if half the stocks have positive β_g .

We note similar conclusions for β_{SMB} , β_{HML} . The percentage of stocks with positive beta is quietly stable across frequencies (between 55–65%) even if a break appears for β_{HML} while the value of the parameters increases as investment horizons increase with the average β_m value. Same observations are made for negative β_{SMB} . Stocks with high β_m tend to have a positive value of β_{SMB} , β_{HML} . The performances of small companies and high B/M ratio firms then have a stronger impact on equities, especially in the long-run. This result is coherent with the Fama–French Model.

To highlight the usefulness of time–frequency parameters for portfolios managers, we present in the third part, a portfolios approach.

Portfolios application

As the betas are differentiated across frequency bands and investment horizons, a portfolio, initially constructed with proper specificity, could get lost in the long-run as exposure to risk factors are different. Consequently, we build three portfolios respectively that are negatively sensitive to oil price variations (Pf1), insensitive to oil fluctuation (Pf2) and positively sensitive to oil (Pf3). In a similar way, we also create three portfolios that indicate negatively sensitive (Pf4), insensitive (Pf5), and positively sensitive (Pf6) to gold price variations. An insensitive to oil and gold variation portfolios (Pf7) is also created. To finish, we elaborate the last portfolio (Pf8) as a tracker of the Market ($\beta_m = 1$). All stocks included in a portfolio are equally weighted.

Table 4 Synthesis of *Betas* signs and means

	D1	D2	D3	D4	D5	D6
% of $\beta_o > 0$	28.57	20.00	31.25	25.00	18.52	44.83
Mean of β_o	0.05	0.07	0.08	0.08	0.08	0.08
Mean of β_m	0.91	0.93	1.03	0.88	0.89	0.89
% of $\beta_g > 0$	50.00	54.55	46.67	52.38	56.00	56.67
Mean of β_g	0.05	0.04	0.06	0.07	0.08	0.11
Mean of β_m	0.85	0.82	0.88	1.01	1.01	1.02
% of $\beta_{SMB} > 0$	54.55	78.57	65.00	45.83	58.33	64.29
Mean of β_{SMB}	0.12	0.15	0.17	0.27	0.26	0.29
Mean of β_m	0.90	1.04	1.06	1.12	1.06	1.03
% of $\beta_{HML} > 0$	53.33	60.00	52.17	46.15	57.69	28.57
Mean of β_{HML}	0.15	0.18	0.28	0.40	0.20	0.40
Mean of β_m	1.16	1.12	1.15	1.17	1.06	1.14
% of $\beta_o < 0$	71.43	80.00	68.75	75.00	81.48	55.17
Mean of β_o	-0.03	-0.03	-0.04	-0.05	-0.06	-0.11
Mean of β_m	0.88	1.04	1.08	1.06	1.05	1.11
% of $\beta_g < 0$	50.00	45.45	53.33	47.62	44.00	43.33
Mean of β_g	-0.08	-0.11	-0.07	-0.08	-0.10	-0.12
Mean of β_m	1.17	1.30	1.17	1.14	1.14	1.02
% of $\beta_{SMB} < 0$	45.45	21.43	35.00	54.17	41.67	35.71
Mean of β_{SMB}	-0.096	-0.11	-0.14	-0.12	-0.15	-0.27
Mean of β_m	0.83	0.72	0.94	0.94	0.99	0.92
% of $\beta_{HML} < 0$	46.67	40.00	47.83	53.85	42.31	71.43
Mean of β_{HML}	-0.12	-0.17	-0.18	-0.19	-0.34	-0.21
Mean of β_m	0.81	0.78	0.81	0.86	0.94	0.97

Table 5 presents the results of the Standard Multi-Beta estimation for the eight portfolios.

We note that all portfolios respect their specificity as Pf1 has a negative significant β_o while it is non-significant for Pf2, and significantly positive for Pf3. Note also that the gold sensitivities have some similar signs to oil sensitivities for these three portfolios. Concerning the Portfolio 4, 5, and 6, all gold sensitivities respect the expected signs. For the last portfolio, Pf8, it β_m is equal to 1 and it is insensitive to oil and gold prices fluctuations.

Then, we re-estimate the Model in the Time–Frequency space to check if these characteristics are still valid according to investment horizons (see Table 14). Table 6 provides the parameters estimated on each band and indicates if they are significantly different from the parameters of the Standard Model estimations in Table 5. One star indicates a significant difference at a 10% risk level while two stars is for a 5% risk level.

For the Pf8, we note that the β_m keeps its property in the short-run (D1–D2) while it is greater than one for D3 to D5 frequency bands and lesser than one on D6. These results indicate that the initial tracker profile of Pf8 is only valid for short-run investments and so an adjustment of portfolio allocation is required if a long-run tracker profile is expected. In addition, this portfolio has sensitives to oil and gold price variations in the long-run as this exposure is non-observed in the Standard Model.

For Pf1 (initially negatively sensitive to oil), it conserves this characteristic for all frequencies, but this initial exposure is significantly greater in the long-run. A similar result is noted for Pf3 (initially positively sensitive to gold) as the sensitivities are significantly greater in the long-run than in the short-run. The Pf2 keeps its insensitive property until D4 bands but not in the long-run as it becomes negatively sensitive to oil (and positively to gold). We also observe similar conclusions concerning the sensitivities to Gold for these three portfolios.

For Pf4, the initial profile with negative β_g is conserved no matter the investment horizons, but in the long-run, its exposure to gold is significantly greater than estimated by the Standard Model. Similar observations are noted for the Pf6, however for some horizons, the sensitivity is greater than estimated in Table 5. Results for a gold insensitive portfolio (Pf5) highlight significant difference between parameters on D2, D4 and D6 frequency bands as β_g is positive.

Concerning the Pf7, initially non sensitive to oil and gold, the insensitive characteristic is lost as the β_o is significantly negative starting D3 bands even if β_g is still equal to 0. To conserve this property for long-run investments, a new allocation is required.

For all portfolios, we note a differentiation of parameters across frequency bands confirming previous observations.

The results obtained by wavelet estimators of risks sensibilities are useful for investors to support their decision-making on portfolio allocation and could be completed by multiple criteria decisions making (MCDM) methods and clusters algorithms based on different risks measured in the assessment of financial risks or in predictions of variables (Kou et al. 2014, 2021).

Conclusion

The Time–Frequency Multi-Betas Model effectively complements the different instruments used by stock investors to build their portfolios. In the first hand, it can substitute the CAPM by considering the residual anomalies by using ARMA-EGARCH processes to model the errors of the regression. On the other hand, it improves the CAPM by adding exogeneous variables and it considers the heterogeneity of agents' behaviors by the wavelet decompositions. Despite some statistical shortcomings, particularly those concerning the characteristics of its frequency residuals, this model brings a significant gain of information to model the risk premiums.

In the short-run, the β_m parameter of the Time–Frequency Multi-Betas Model, measuring the sensitivity to market fluctuation, is not significantly different to the Standard Multi-Betas Model and the CAPM. For a short-run investor, the use of the CAPM can be sufficient to make investment choices based on the β_m . However, he can consider the Multi-Betas Model and the sensitivities to gold and oil in order to modulate its choices.

The Standard Multi-Betas Model (without wavelets) is retained for a majority of the stocks in its full or one mixed version. The stock sensitivities to oil and gold are lower than the sensitivity to the market, but we can appreciate potential positive effects on some sectors such as the Petroleum/Gaz-stocks, and Financial sectors. For example, oil negatively affects the majority of stocks, however, its impact is stronger for high β_m equities than low β_m equities. Gold negatively and more strongly affects equities with a high systematic risk (such as the Financial sector) but the effect is reversed for equities with a

Table 5 Standard Multi Beta Model for Portfolios

Portfolios	β_m	TSTAT to 1	β_o	TSTAT	β_g	TSTAT	β_{SMB}	TSTAT	β_{HML}	TSTAT	R ²	JB	LB	ARCH
Pf1	1.0156	2.0886	-0.0237	-5.6760	-0.0383	-5.3654	0.0372	7.6100	0.0504	2.9568	0.8841	2212.3582	1.3103	0.3452
Pf2	0.9847	-2.1364	-0.0048	-1.2709	-0.0060	-0.7328	0.0683	3.8880	0.0516	2.7766	0.8715	2916.2014	11.1718	0.8293
Pf3	0.8930	-6.3892	0.1076	11.6205	0.0877	5.3983	0.0138	0.6030	0.0629	2.7109	0.6351	3444.3284	4.8302	6.7425
Pf4	1.1143	12.5686	-0.0192	-3.6791	-0.0740	-7.7902	0.0050	0.2477	0.1178	5.5040	0.8421	3413.1451	1.4233	0.8095
Pf5	0.9591	-5.6721	-0.0172	-5.0278	0.0037	0.5122	0.0607	5.3748	0.0063	0.6233	0.8835	1812.4577	0.9471	0.8533
Pf6	0.8705	-15.7448	0.0242	5.6821	0.0384	4.8830	0.0455	2.5922	0.0263	1.5475	0.8569	2711.4237	2.2448	2.8390
Pf7	1.0206	2.3786	-0.0083	-1.6909	-0.0061	-0.6627	0.0533	2.5999	-0.0040	-0.1837	0.8566	1837.1235	10.7708	3.1274
Pf8	0.9907	-0.7784	-0.0117	-1.7242	0.0088	0.7233	0.0631	2.0728	0.0265	1.0479	0.7856	1804.8054	1.1307	0.0919

Table 6 Time–Frequency sensitivities of Portfolios compared to Standard Multi Beta Model

	β_m	β_o	β_g	β_{SMB}	β_{HML}
Pf1D1	1.0128	−0.0147	−0.0271	0.0137**	0.0111
Pf1D2	1.0293	−0.0272	−0.0463	0.0710**	0.0850
Pf1D3	1.0420**	−0.0264	−0.0429	0.0971**	0.1141**
Pf1D4	1.0647**	−0.0514**	−0.0187**	0.0092**	0.1505**
Pf1D5	1.0397**	−0.0313	−0.0945**	0.1586**	0.1510**
Pf1D6	1.0733**	−0.0766**	−0.1362**	0.2349**	0.1661**
Pf2D1	0.9885	−0.0016	−0.0065	0.0343	0.0483
Pf2D2	0.9934	−0.0141	0.0003	0.0589	0.0820
Pf2D3	0.9944	−0.0141*	0.0021	0.1075*	0.0089**
Pf2D4	0.9873	−0.0066	−0.0170	0.0982	0.0187
Pf2D5	1.0446*	−0.0349**	0.0191**	0.0430	−0.0143**
Pf2D6	1.0543*	−0.0253**	0.0110**	0.2653**	0.0509
Pf3D1	0.9007	0.0841*	0.0814	−0.0160	0.0718
Pf3D2	0.8970	0.1280*	0.0864	−0.0293	0.0391
Pf3D3	0.8950	0.1728**	0.0418**	0.0514	0.0833
Pf3D4	0.9037	0.2112**	0.0900	0.1715**	−0.1186**
Pf3D5	0.9480**	0.1289*	0.1681**	−0.0005	−0.1963**
Pf3D6	0.8555*	0.2942**	0.0317**	−0.3159**	0.1596**
Pf4D1	1.1218	−0.0145	−0.0594	0.0049	0.0812
Pf4D2	1.1128	−0.0205	−0.0717	0.0492	0.1634
Pf4D3	1.1200	−0.0116	−0.0821	0.0491	0.2383**
Pf4D4	1.1252	−0.0386**	−0.0900	−0.0489**	0.2568**
Pf4D5	1.1236	−0.0547**	−0.0614	0.0899**	0.1965**
Pf4D6	1.0841*	−0.0737**	−0.1897*	0.0319	0.1994**
Pf5D1	0.9590	−0.0170	0.0028	0.0355	−0.0127
Pf5D2	0.9819**	−0.0299*	−0.0042	0.0534	−0.0151
Pf5D3	1.0034**	−0.0195	−0.0112	0.0162	−0.0138
Pf5D4	0.9909**	−0.0232	0.0186*	0.0467	−0.0022
Pf5D5	1.0405**	−0.0478**	0.0735**	0.1658	−0.0968
Pf5D6	0.9506	−0.0004**	0.0366**	0.2996	−0.1889
Pf6D1	0.8737	0.0187	0.0281	−0.0305	0.0286
Pf6D2	0.8878	0.0268	0.0598**	0.0167	0.0454
Pf6D3	0.8766	0.0432**	0.0446	0.0704**	−0.0348
Pf6D4	0.8704	0.0406**	0.0554*	0.0990	−0.0644
Pf6D5	0.9172**	0.0238	0.0427	0.0207**	−0.0078**
Pf6D6	0.9040**	0.0826**	0.0914**	−0.0105**	−0.0724**
Pf7D1	1.0379	−0.0093	−0.0036	0.0221**	0.0335
Pf7D2	1.0327	−0.0157	−0.0019	0.0557	0.0352
Pf7D3	1.0351	−0.0322**	0.0011	−0.0112**	−0.0142
Pf7D4	1.0493**	−0.0262**	−0.0096	−0.0128**	−0.0828
Pf7D5	0.9905**	−0.0324**	−0.0170	0.1034	0.1163**
Pf7D6	0.8777**	−0.0322**	−0.0040	−0.0196**	−0.0251**
Pf8D1	0.9905	−0.0113	0.0165	0.0122	0.0521
Pf8D2	0.9980	−0.0110	0.0071	0.0558	0.0773
Pf8D3	1.0520**	−0.0175	−0.0202*	0.0096	−0.0050**
Pf8D4	1.0544**	−0.0545**	−0.0002	0.0437	0.0799**
Pf8D5	1.0579**	−0.0492**	0.0389**	0.1166	−0.0528
Pf8D6	0.9568**	−0.0144	−0.0880**	0.1087	0.3633**

β_m lower than one. The Time–frequency Multi-Betas Model multiplies the possibilities of analysis by crossing the betas and the sectors with the investment horizons. We confirm the differentiations of risk according to investment horizons observed by Gençay et al. (2005) and Mestre and Terraza (2018). We also find similar results with the Mestre and Terraza analysis, as the Standard Multi-Beta and Time–Frequency Model provide slightly similar Beta coefficients in the short-run with CAPM estimates, but the more the investment horizon increases the more the differences between Models coefficients are significant.

The Time–Frequency Multi-Betas Model is more useful for fundamentalist investors (in the long-run) as there are significant differences with Standard Model estimations. At low-frequencies (D6), the CAPM is not retained whereas for the others, oil and gold variables and Fama–French factors have significant effects on equities. Their effects increase as the time horizons increase. The application to portfolios highlights the potential effect of variation of risk exposure across frequencies on the property and characteristic of portfolios in the long-run, and some initial features do not hold.

Wavelets represent a powerful tool to differentiate the stock sensitivities to various factors according to the agents investment horizons. The combination of the time–frequency estimates of the Multi-Betas Model improves the investment choice possibilities and risk analysis.

Appendix

See Tables 7, 8, 9, 10, 11, 12, 13 and 14.

Table 7 Equities characteristics

Stocks prices	Means		SD	Skewness	Kurtosis
(a) Means and standard deviations of Ln(Prices)					
Accor	3.15		0.34	0.15	2.76
Airbus	3.15		0.52	0.28	2.18
Alcatel	1.20		0.74	0.19	2.05
Air Liquide	4.05		0.40	−0.15	2.16
AXA	2.48		0.31	0.08	2.71
BNP	3.72		0.25	−0.98	3.68
Bouygues	3.13		0.26	0.10	2.17
CA	2.29		0.48	−0.46	2.87
CARREFOUR	3.23		0.28	−0.74	3.31
DANONE	3.70		0.23	−0.39	2.78
ESSILOR	3.84		0.47	0.27	2.00
GDF	2.76		0.19	−0.11	2.75
Gemini	3.51		0.37	0.78	3.22
St Gobain	3.48		0.26	−0.08	3.22
LVMH	4.42		0.41	−0.10	1.79
Michelin	3.90		0.35	0.04	2.28
L'Oréal	4.36		0.36	0.50	2.30
Orange	2.29		0.19	−0.10	3.09
PSA	2.72		0.56	−0.47	2.36
Publicis	3.46		0.43	0.29	1.98
Renault	3.81		0.47	−0.62	2.87
Ricard	4.12		0.31	−0.03	2.21
Schneider	3.51		0.43	−0.16	1.88
SG	3.70		0.50	0.05	2.51
Sodexo	3.77		0.41	−0.06	2.28
Technip	3.85		0.36	−0.65	3.21
Total	3.43		0.18	0.29	2.88
Veolia	2.77		0.49	0.03	2.24
Vinci	3.45		0.30	−0.03	2.84
Vivendi	2.57		0.18	0.68	3.10
Gold	6.64		0.40	−0.44	2.00
Oil	4.05		0.25	−0.58	2.81
CAC	8.33		0.19	−0.02	2.31
Ln (prices)	M3	M2	M1	Premia	M3
(b) Phillips-Perron stationarity test on Ln(Prices) and on risk premia					
CAC	−1.89	−1.85	0.23	CAC	−56.11
Oil	−2.57	−2.13	−0.07	Oil	−57.32
Gold	−1.22	−2.23	1.77	Gold	−56.18
Accor	−3.1	−2.06	−1.11	Accor	−52.93
Airbus	−1.53	−0.52	0.93	Airbus	−53.39
Alcatel	−1.15	−1.74	−1.47	Alcatel	−51.26
Air Liquide	−3.27	−1.51	2.08	Air Liquide	−60.49
AXA	−2.41	−2.05	0.72	AXA	−51.22
BNP	−2.48	−2.48	0.24	BNP	−53.9
Bouygues	−1.88	−1.88	0.44	Bouygues	−55.34
CA	−1.5	−1.49	−0.51	CA	−51.42
CARREFOUR	−1.87	−1.67	−0.03	CARREFOUR	−54.48
DANONE	−3.06	−2.19	1.34	DANONE	−56.78

Table 7 (continued)

Ln (prices)	M3	M2	M1	Premia	M3
ESSILOR	− 2.32	− 0.45	2.38	ESSILOR	− 57.77
GDF	− 3.09	− 3.02	0.33	GDF	− 54.64
Gemini	− 1.58	− 0.81	1.4	Gemini	− 53.5
St Gobain	− 2.36	− 2.36	0.03	St Gobain	− 54.64
LVMH	− 1.96	− 0.65	1.73	LVMH	− 59.99
Michelin	− 2.54	− 1.41	1.1	Michelin	− 55.7
L'Oréal	− 2.46	− 1.57	0.69	L'Oréal	− 52.28
Orange	− 1.42	− 1.38	0.32	Orange	− 54.42
PSA	− 1.36	− 1.46	− 0.59	PSA	− 49.27
Publicis	− 1.97	− 0.7	1.26	Publicis	− 54.03
Renault	− 1.27	− 1.25	0.3	Renault	− 49.21
Ricard	− 2.64	− 1.48	1.43	Ricard	− 55.28
Schneider	− 2.53	− 1.91	1.19	Schneider	− 57.69
SG	− 1.72	− 1.49	− 0.33	SG	− 48.76
Sodexo	− 3.19	− 1.6	2.04	Sodexo	− 54.67
Technip	− 1.89	− 2.04	0.24	Technip	− 53.73
Total	− 3.31	− 2.77	0.78	Total	− 55.66
Veolia	− 1.1	− 1.14	0.04	Veolia	− 510,257
Vinci	− 2.59	− 1.99	1.48	Vinci	− 56.9
Vivendi	− 2.17	− 1.86	0.56	Vivendi	− 56.31
Critical (%)					Values
1					− 3.96
5					− 3.41
10					− 3.13
Premia	Means	Nullity tests	SD	Skewness	Kurtosis
<i>(c) Means and standard deviations of risk premia</i>					
CAC	− 0.0000494	0.19	0.0143	0.02	9.55
Oil	0.0000309	0.07	0.0232	− 0.01	8.44
Gold	0.000392	1.78	0.0118	− 0.47	8.27
Accor	0.000355	0.93	0.0205	0.17	7
Airbus	0.000316	0.74	0.0228	− 0.95	16.65
Alcatel	− 0.000506	0.88	0.0307	− 0.26	9.46
Air Liquide	0.000368	1.34	0.0147	0.04	7.34
AXA	0.000351	0.72	0.026	0.45	12.15
BNP	0.0000518	0.11	0.0254	0.27	11.53
Bouygues	0.000105	0.26	0.0212	0.31	10.41
CA	− 0.000212	0.41	0.0275	0.21	9.03
DANONE	0.000245	0.9	0.0145	− 0.05	7.18
CARREFOUR	− 0.0000784	0.23	0.0185	− 0.06	6.34
ESSILOR	0.000466	1.84	0.0136	0.36	9.08
GDF	0.0000471	0.13	0.019	1.12	23.11
Gemini	0.000425	1.07	0.0212	0.02	6.65
St Gobain	− 0.0000298	0.07	0.0234	0.04	9.52
L'Oréal	0.00031	1.12	0.0148	0.23	8.75
LVMH	0.000298	0.89	0.018	0.09	8.53
Orange	− 0.0000008	0	0.0158	0.28	6.68
Michelin	0.000228	0.56	0.0219	− 0.1	6.59
PSA	− 0.000269	0.54	0.0267	− 0.02	5.42

Table 7 (continued)

Premia	Means	Nullity tests	SD	Skewness	Kurtosis
Publicis	0.000271	0.92	0.0157	0.01	6.39
Renault	0.000123	0.25	0.0261	−0.16	7.44
Ricard	0.000293	0.95	0.0165	−0.33	12.43
Schneider	0.000351	0.88	0.0214	0.09	7.92
SG	−0.000179	0.34	0.0281	−0.07	9.04
Sodexo	0.00047	1.65	0.0152	−0.11	8.88
Technip	0.0000608	0.14	0.024	−0.34	8.17
Total	0.000137	0.45	0.0162	0.17	10.01
Veolia	−0.0000174	0.04	0.0209	−0.79	17.07
Vinci	0.000379	1.05	0.0194	0.27	10.59
Vivendi	0.0000598	0.21	0.0155	0.07	7.5

For tests on premia, the statistics for model 1 and 2 are similar to model 3 values. M1 referring to the model with Trend and Constant, M2 the model with Constant and M3 the model with no trend and no constant

Table 8 Multicollinearity analysis

	CAC	Oil	Gold	SMB	HML
<i>(a) Matrix of Correlation</i>					
CAC	1	0.28	−0.065	0.05	0.23
Oil	0.28	1	0.15	0.02	0.12
Gold	−0.065	0.15	1	0.04	−0.026
SMB	0.05	0.02	0.04	1	−0.096
HML	0.23	0.12	−0.026	−0.096	1
ViF					
CAC		Oil	Gold	SMB	HML
<i>(b) Variance inflation factors (ViF)</i>					
1.15		1.12	1.04	1.017	1.075

Table 9 CAPM-EGARCH Estimates

CAPM-EGARCH	β_m	T-stat	R ²	LB	ARCH	JB
$\beta_m < 1$						
Essilor	0.525	33.527	0.31	2.02	3.59	16,301
Sodexo	0.559	38.830	0.35	0.41	0.72	9359
Ricard	0.627	35.094	0.355	5.79	0.386	9427
Danone	0.697	49.456	0.41	4.11	2.26	4732
Publicis	0.7	43.134	0.43	6	0.76	2019
Orange	0.722	51.499	0.43	2.27	0.46	4566
L'Oréal	0.755	55.404	0.486	1.34	2.04	4675
Vivendi	0.788	57.146	0.52	1.51	0.16	7736
Veolia	0.838	41.000	0.39	0.24	1.35	152,130
Air Liquide	0.851	65.115	0.65	2.9	0.21	7518
Carrefour	0.866	49.282	0.48	1.32	1.14	3813
Total	0.887	73.458	0.67	2.47	0.68	2273
GDF	0.942	62.613	0.49	11.6	0.33	148,170
Airbus	0.945	42.384	0.35	3.94	0.17	105,930
Accor	0.954	41.580	0.48	1.4	1.07	5617
$\beta_m = 1$						
Technip	0.991	35.798	0.39	3.22	1.22	6331
Bouygues	0.994	51.512	0.5	0.24	0.84	17,744
Gemini	0.996	46.108	0.484	2.27	0.35	2782
Michelin	1.017	44.993	0.49	4.46	2.41	3560
$\beta_m > 1$						
LVMH	1.028	72.783	0.62	1.79	0.31	10,806
Vinci	1.062	72.543	0.67	1.12	0.875	5150
Alcatel	1.131	37.546	0.32	0.846	1.56	14,264
PSA	1.135	46.521	0.39	3.63	0.07	1539
Schneider	1.192	70.704	0.68	9.93	2.533	1063
St-Gobain	1.25	64.465	0.67	2.04	0.23	15,825
Renault	1.268	54.896	0.55	1.16	2.05	2262
AXA	1.288	70.891	0.67	4.38	0.013	43,151
BNP	1.289	77.316	0.61	2.94	0.96	40,217
SG	1.305	62.943	0.56	5.07	4.53	10,525
CA	1.351	37.302	0.56	2.62	1.58	7842

At 5% risk level, Column LB (Ljung–Box test): $\chi^2(5) = 11.1$; Column ARCH (ARCH-LM test): $\chi^2(5) = 11.1$ Column J–B (Jarque–Bera Line): $\chi^2(2) = 5.99$. We use Weighted Tests of Ljung–Box and ARCH-M of Fisher and Gallagher (2012)

Table 10 Comparison of β_m between CAPM-EGARCH and Multi-Betas-EGARCH

Stocks	Beta (CAPM-EGARCH)	Beta (MB-EGARCH)	
$\beta_m < 1$			
Essilor	0.53	0.55	NS
Sodexo	0.56	0.58	NS
Danone	0.7	0.71	NS
Ricard	0.63	0.66	NS
Publicis	0.7	0.69	NS
L'Oréal	0.75	0.79	NS
Orange	0.72	0.74	NS
Vivendi	0.79	0.79	NS
Air Liquide	0.85	0.85	NS
Carrefour	0.87	0.88	NS
Veolia	0.83	0.85	NS
Total	0.89	0.87	NS
GDF	0.95	0.99	S
Airbus	0.94	0.94	NS
Accor	0.95	0.94	NS
$\beta_m = 1$			
LVMH	1.02	1.03	NS
Gemini	0.99	1.00	NS
Technip	0.99	0.92	NS
Bouy	0.99	1.00	NS
Michelin	1.03	1.01	NS
Vinci	1.06	1.06	NS
PSA	1.13	1.13	NS
Alcatel	1.13	1.13	NS
Schneider	1.19	1.19	NS
$\beta_m > 1$			
St-Gobain	1.25	1.24	NS
Renault	1.27	1.26	NS
BNP	1.29	1.3	NS
CA	1.35	1.33	NS
SG	1.31	1.31	NS
AXA	1.29	1.29	NS

NS = Non-significant differences according to tests of comparison of parameters

Table 11 Frequency Bands corresponding days

Level J	Time Horizons in days	
	Inferior border	Superior border
D1	2	4
D2	4	8
D3	8	16
D4	16	32
D5	32	64
D6	64	128
D7	128	256
D8	256	512
D9	512	1024
D10	1024	2048
D11	2048	4096
S11	4096	—

Table 12 Time-frequency multi-beta-AR-EGARCH estimates

Stocks	Bands	β_m	TSAT	β_o	TSAT	β_g	TSAT	β_{SMB}	TSAT	β_{HML}	TSAT	R ²	JB	LB	ARCH
$\beta_m = \text{MarketBeta}; \beta_o = \text{OilBeta}; \beta_g = \text{GoldBeta}; \beta_{SMB} = \text{SMBBeta}; \beta_{HML} = \text{HMLBeta}$															
ACCOR	D1	0.920	0.018	-0.013	0.010	0.004	0.019	-0.011	0.000	0.044	0.000	0.640	2176.073	778.535	5.493
	D2	0.971	0.019	-0.027	0.009	0.000	0.020	0.202	0.000	0.026	0.000	0.564	1474.454	1318.361	12.234
	D3	0.993	0.016	-0.016	0.009	0.010	0.019	0.236	0.000	0.044	0.000	0.830	322.185	1798.456	278.172
	D4	0.982	0.018	-0.020	0.006	-0.042	0.014	0.222	0.000	0.168	0.000	0.944	379.816	1941.241	1.051
	D5	1.050	0.012	-0.093	0.007	0.083	0.008	0.034	0.000	0.103	0.000	0.985	66.916	3537.109	7.639
	D6	1.063	0.008	0.085	0.004	0.223	0.008	0.739	0.000	-0.072	0.000	0.997	1028.739	4967.133	1.614
AIRBUS	D1	0.919	0.017	0.007	0.004	0.005	0.005	0.037	0.000	0.068	0.000	0.593	15636.202	742.288	3.223
	D2	0.988	0.019	0.022	0.016	0.024	0.025	-0.050	0.001	-0.006	0.000	0.423	9826.253	1343.410	25.286
	D3	1.027	0.029	0.041	0.012	0.072	0.026	0.126	0.001	-0.166	0.000	0.764	3868.726	1938.664	372.142
	D4	0.938	0.014	0.069	0.008	-0.014	0.014	0.302	0.000	-0.305	0.000	0.931	1937.247	1870.148	1.403
	D5	1.131	0.013	-0.008	0.003	0.038	0.010	0.033	0.000	-0.006	0.000	0.983	95.790	3429.199	17.998
	D6	1.102	0.005	-0.038	0.003	-0.149	0.006	0.242	0.000	-0.417	0.000	0.996	537.513	5012.739	1.186
ALCATEL	D1	1.125	0.026	-0.006	0.014	-0.075	0.026	0.166	0.001	-0.250	0.001	0.551	4031.673	721.389	10.167
	D2	1.142	0.027	-0.022	0.012	-0.029	0.025	0.382	0.001	-0.214	0.001	0.408	2200.818	1245.312	16.828
	D3	1.143	0.035	-0.039	0.017	-0.080	0.032	0.643	0.001	0.308	0.001	0.771	900.446	1879.142	242.899
	D4	1.143	0.053	-0.031	0.010	-0.154	0.021	0.338	0.001	0.062	0.000	0.928	243.412	1956.377	3.894
	D5	1.566	0.015	0.000	0.001	-0.216	0.014	0.501	0.000	0.156	0.000	0.984	74.009	3794.851	14.986
	D6	1.154	0.009	0.133	0.006	-0.559	0.012	0.376	0.000	-0.017	0.000	0.996	116.509	4750.106	13.906
Air liquide	D1	0.881	0.011	-0.005	0.003	0.045	0.011	0.079	0.000	-0.061	0.000	0.787	3795.818	810.253	2.477
	D2	0.841	0.012	0.003	0.006	0.020	0.011	0.101	0.000	-0.118	0.000	0.687	1224.184	1318.454	20.637
	D3	0.890	0.010	-0.010	0.008	0.056	0.012	0.004	0.000	-0.192	0.000	0.861	827.119	1812.126	353.381
	D4	0.900	0.010	-0.011	0.005	0.018	0.005	-0.059	0.000	-0.189	0.000	0.957	5809.939	1902.848	8.859
	D5	0.851	0.008	0.001	0.001	-0.023	0.008	-0.115	0.000	0.070	0.000	0.989	1344.158	3379.998	14.369
	D6	0.768	0.006	0.054	0.003	0.011	0.003	-0.065	0.000	-0.241	0.000	0.996	113.036	5368.044	44.493
AXA	D1	1.291	0.015	-0.015	0.008	-0.073	0.015	-0.088	0.000	0.106	0.000	0.776	20,133.506	737.104	7.078

Table 12 (continued)

Stocks	Bands	β_m	TSAT	β_o	TSAT	β_g	TSAT	β_{SMB}	TSAT	β_{HML}	TSAT	R ²	JB	LB	ARCH
BNP	D2	1.363	0.017	-0.037	0.005	-0.122	0.015	-0.024	0.000	0.149	0.000	0.726	14,570.375	1234.680	19,001
	D3	1.350	0.018	-0.052	0.008	-0.071	0.013	0.139	0.000	0.533	0.000	0.870	5845.542	1927.288	322.951
	D4	1.364	0.012	-0.070	0.003	-0.099	0.027	-0.033	0.001	0.522	0.000	0.969	2214.427	1971.828	13.605
	D5	1.286	0.016	-0.065	0.006	-0.113	0.010	-0.098	0.000	0.298	0.000	0.988	7572.606	3523.380	7.081
	D6	1.370	0.007	-0.339	0.003	-0.237	0.006	0.341	0.000	0.754	0.000	0.998	523.648	4534.706	10.913
	D1	1.273	0.015	-0.011	0.008	-0.085	0.016	0.016	0.000	0.194	0.000	0.770	10,491.978	742.459	14.941
Bouygues	D2	1.299	0.015	-0.013	0.009	-0.118	0.015	0.036	0.000	0.325	0.000	0.661	11,679.448	1308.148	21.161
	D3	1.357	0.016	-0.072	0.007	-0.032	0.007	-0.132	0.000	0.487	0.001	0.859	14,623.156	1912.274	241.976
	D4	1.289	0.018	-0.098	0.017	-0.104	0.022	-0.092	0.000	0.696	0.000	0.963	11,820.936	1831.152	1.065
	D5	1.136	0.008	-0.058	0.004	-0.045	0.007	0.048	0.000	0.593	0.000	0.983	7788.796	3151.869	3.174
	D6	1.335	0.005	-0.170	0.002	0.039	0.004	-0.009	0.000	-0.131	0.000	0.996	35,778.314	4451.578	0.006
	D1	0.953	0.017	0.019	0.006	0.023	0.022	0.011	0.000	0.162	0.000	0.690	6126.830	774.433	3.172
CA	D2	0.928	0.017	0.021	0.010	0.036	0.020	0.034	0.001	0.226	0.000	0.567	3827.927	1262.173	16.608
	D3	0.991	0.019	-0.009	0.010	0.070	0.013	-0.074	0.000	0.209	0.000	0.814	23,142.967	1818.726	373.859
	D4	0.985	0.013	-0.055	0.006	0.053	0.018	0.011	0.000	0.362	0.000	0.944	2829.643	1914.276	3.426
	D5	0.925	0.013	-0.050	0.006	0.031	0.012	-0.164	0.000	0.502	0.000	0.983	91.635	4106.282	15.544
	D6	0.871	0.005	-0.242	0.005	0.065	0.004	0.176	0.000	0.487	0.000	0.997	25.896	5029.820	24.496
	D1	1.280	0.023	0.004	0.011	-0.104	0.019	-0.048	0.000	0.116	0.000	0.700	3159.649	775.354	8.470
Carrefour	D2	1.266	0.021	-0.001	0.012	-0.171	0.020	0.068	0.000	0.430	0.000	0.624	7633.193	1345.555	20.711
	D3	1.349	0.021	0.011	0.004	-0.189	0.033	-0.019	0.000	0.664	0.000	0.854	7645.835	1849.888	275.519
	D4	1.395	0.012	-0.096	0.005	-0.029	0.009	0.179	0.000	0.974	0.000	0.962	596.908	1790.770	5.988
	D5	1.314	0.024	-0.068	0.014	-0.135	0.011	-0.245	0.000	0.237	0.000	0.983	2066.322	3126.477	1.998
	D6	1.359	0.007	-0.125	0.003	-0.015	0.007	0.026	0.000	0.393	0.000	0.997	7361.710	4621.910	0.034
	D1	0.897	0.012	-0.003	0.009	0.015	0.012	-0.005	0.001	0.049	0.000	0.674	1113.131	768.269	8.085
	D2	0.896	0.018	-0.009	0.009	-0.009	0.017	0.002	0.000	0.011	0.000	0.557	1210.173	1229.252	16.780

 $\beta_m = \text{MarketBeta}; \beta_o = \text{OilBeta}; \beta_g = \text{GoldBeta}; \beta_{SMB} = \text{SMBBeta}; \beta_{HML} = \text{HMLBeta}$

Table 12 (continued)

Stocks	Bands	β_m	TSAT	β_o	TSAT	β_g	TSAT	β_{SMB}	TSAT	β_{HML}	TSAT	R2	JB	LB	ARCH
Danone	D3	0.859	0.019	-0.019	0.010	-0.027	0.021	0.059	0.001	-0.114	0.001	0.787	591.771	1917.111	307.732
	D4	0.957	0.015	0.033	0.006	-0.067	0.013	-0.083	0.000	-0.250	0.000	0.944	1007.144	2020.457	3640
	D5	0.936	0.011	-0.107	0.005	0.028	0.012	0.032	0.000	0.101	0.000	0.984	108.545	3562.459	8079
	D6	1.022	0.010	-0.020	0.004	0.042	0.005	-0.347	0.000	-0.483	0.000	0.995	3.797	5250.012	31.341
	D1	0.739	0.013	0.000	0.007	-0.005	0.015	-0.079	0.000	-0.132	0.000	0.637	3280.263	828.343	24.427
	D2	0.717	0.013	0.011	0.008	0.001	0.014	-0.054	0.000	-0.218	0.000	0.495	2992.399	1281.406	21.271
Essilor	D3	0.714	0.012	-0.034	0.008	0.020	0.015	-0.005	0.000	-0.324	0.000	0.743	4227.122	1822.328	288.702
	D4	0.586	0.014	-0.004	0.010	0.000	0.043	-0.028	0.000	-0.173	0.001	0.919	161.991	2070.157	15.987
	D5	0.765	0.009	0.027	0.003	0.059	0.008	-0.072	0.000	-0.428	0.000	0.977	889.174	3518.043	8608
	D6	0.714	0.004	0.075	0.004	-0.032	0.006	-0.252	0.000	-0.132	0.000	0.997	4.467	4834.703	20.771
	D1	0.546	0.046	-0.013	0.029	-0.026	0.087	-0.049	0.001	0.011	0.002	0.576	2886.330	919.175	1.893
	D2	0.562	0.014	-0.018	0.009	0.038	0.017	-0.038	0.000	-0.100	0.000	0.382	1843.298	1208.356	16.063
GDF	D3	0.555	0.013	-0.023	0.005	0.075	0.011	-0.015	0.000	-0.167	0.000	0.708	18,297.375	1819.191	369.214
	D4	0.601	0.010	-0.028	0.004	-0.011	0.008	-0.058	0.000	-0.100	0.000	0.931	2918.413	2065.624	4.481
	D5	0.773	0.008	-0.012	0.005	-0.004	0.005	0.322	0.000	-0.745	0.000	0.983	1345.026	3605.996	1.575
	D6	0.496	0.005	-0.100	0.003	0.222	0.005	0.222	0.000	-0.341	0.000	0.994	395.314	5259.630	5.813
	D1	0.911	0.014	-0.032	0.008	-0.004	0.004	0.000	0.000	-0.053	0.000	0.651	70,131.470	795.346	4.891
	D2	0.919	0.015	-0.031	0.009	-0.020	0.013	-0.050	0.000	-0.031	0.000	0.551	24,741.872	1249.029	16.222
Gemini	D3	1.015	0.014	-0.028	0.006	-0.010	0.004	-0.183	0.000	-0.120	0.000	0.793	2677.702	1821.894	309.631
	D4	1.072	0.014	-0.029	0.008	0.052	0.019	-0.217	0.000	-0.197	0.000	0.945	2620.328	2000.362	3.325
	D5	0.985	0.011	-0.030	0.007	-0.092	0.007	-0.185	0.000	-0.203	0.000	0.983	831.342	3364.322	7.272
	D6	0.918	0.006	-0.087	0.003	-0.166	0.004	-0.215	0.000	-0.238	0.000	0.996	45.244	4874.324	14.415
	D1	1.002	0.024	-0.014	0.012	0.015	0.021	0.047	0.000	0.034	0.000	0.653	744.791	796.809	7.717
	D2	1.022	0.017	-0.036	0.010	-0.015	0.019	0.106	0.000	-0.048	0.000	0.560	789.774	1277.122	23.776
	D3	1.036	0.017	-0.001	0.003	-0.052	0.015	0.103	0.000	-0.161	0.000	0.819	140.007	1821.964	323.112

Table 12 (continued)

Stocks	Bands	β_m	TSAT	β_o	TSAT	β_g	TSAT	β_{SMB}	TSAT	β_{HML}	TSAT	R2	JB	LB	ARCH
Saint Gobain	D4	1.050	0.142	-0.076	0.169	0.057	0.004	0.036	0.002	-0.141	0.000	0.944	222.010	1973.764	4.285
	D5	1.140	0.015	-0.091	0.007	0.047	0.006	0.256	0.000	-0.291	0.000	0.985	462.736	3320.640	1.2338
	D6	1.139	0.007	-0.093	0.004	-0.064	0.006	1.306	0.000	-0.161	0.000	0.998	2.611	4901.625	33.979
	D1	1.241	0.021	-0.011	0.009	-0.002	0.008	0.062	0.000	0.116	0.000	0.786	4177.040	779.518	6.229
	D2	1.237	0.014	-0.034	0.007	-0.006	0.006	0.098	0.000	0.145	0.000	0.728	3331.047	1192.037	17.597
	D3	1.278	0.019	-0.043	0.012	-0.013	0.017	0.134	0.000	0.153	0.000	0.869	12,965.452	1907.500	338.638
L'Oréal	D4	1.232	0.017	-0.004	0.007	-0.054	0.013	0.332	0.000	0.049	0.000	0.966	22,088.674	1786.112	2.831
	D5	1.149	0.010	0.021	0.004	-0.165	0.008	0.575	0.000	0.353	0.000	0.991	3974.724	3434.694	15.783
	D6	1.301	0.010	-0.045	0.003	-0.005	0.001	0.372	0.000	0.048	0.000	0.998	3083.831	4976.617	2.261
	D1	0.812	0.013	-0.023	0.004	0.009	0.006	0.023	0.000	-0.070	0.000	0.681	2681.395	781.366	4.920
	D2	0.810	0.012	-0.037	0.006	0.002	0.013	0.019	0.000	-0.103	0.000	0.545	900.646	1275.442	11.514
	D3	0.761	0.016	-0.022	0.016	0.005	0.019	-0.119	0.001	-0.238	0.000	0.788	315.272	1860.329	356.612
LVMH	D4	0.683	0.012	0.027	0.006	-0.015	0.012	-0.085	0.000	-0.226	0.000	0.935	163.162	1978.358	6.407
	D5	0.793	0.010	-0.062	0.005	0.022	0.011	-0.096	0.000	-0.317	0.000	0.981	152.812	3707.103	9.366
	D6	0.762	0.004	-0.014	0.002	-0.004	0.002	-0.370	0.000	-0.190	0.000	0.997	167.057	4635.446	19.827
	D1	0.989	0.012	-0.019	0.006	0.014	0.006	0.018	0.000	0.006	0.000	0.752	3829.072	798.574	5.536
	D2	1.052	0.014	-0.032	0.007	0.030	0.008	0.058	0.000	-0.036	0.000	0.683	1655.684	1272.906	11.404
	D3	1.005	0.017	0.009	0.003	-0.003	0.017	0.083	0.000	0.068	0.000	0.875	439.422	1810.031	244.999
Michelin	D4	0.999	0.013	0.002	0.008	0.071	0.012	0.055	0.000	-0.254	0.000	0.956	870.275	1993.383	8.246
	D5	1.087	0.007	-0.042	0.004	-0.036	0.006	0.056	0.000	-0.455	0.000	0.988	2102.990	3550.206	17.234
	D6	1.011	0.004	0.072	0.003	0.162	0.006	0.230	0.000	-0.151	0.000	0.997	3.065	5085.204	29.959
	D1	0.963	0.018	-0.039	0.010	0.047	0.020	0.196	0.000	0.040	0.000	0.642	1488.474	871.319	11.123
	D2	1.048	0.018	-0.030	0.012	-0.012	0.017	0.216	0.000	0.055	0.000	0.564	1292.620	1248.943	21.046
	D3	1.118	0.017	-0.016	0.010	-0.023	0.019	-0.058	0.000	-0.011	0.000	0.831	695.225	1896.961	266.888
	D4	1.118	0.012	-0.068	0.005	0.026	0.008	0.086	0.000	0.197	0.000	0.940	2519.702	1869.416	3.893

 $\beta_m = \text{MarketBeta}; \beta_o = \text{OilBeta}; \beta_g = \text{GoldBeta}; \beta_{SMB} = \text{SMBBeta}; \beta_{HML} = \text{HMLBeta}$

Table 12 (continued)

Stocks	Bands	β_m	TSAT	β_o	TSAT	β_g	TSAT	β_{SMB}	TSAT	β_{HML}	TSAT	R2	JB	LB	ARCH
Orange	D5	0.969	0.016	-0.053	0.006	0.164	0.010	0.407	0.000	-0.005	0.000	0.984	29.378	3434.106	13.710
	D6	0.804	0.010	0.013	0.005	0.225	0.008	0.309	0.000	0.816	0.000	0.997	1173.151	4545.535	0.251
	D1	0.800	0.014	-0.019	0.008	-0.045	0.014	0.091	0.000	-0.013	0.000	0.656	989.843	694.556	3.689
	D2	0.773	0.013	-0.021	0.008	-0.018	0.016	0.040	0.000	-0.053	0.000	0.500	522.283	1300.408	17.854
	D3	0.772	0.016	-0.015	0.008	-0.034	0.015	-0.041	0.000	-0.204	0.000	0.759	644.112	2005.976	290.776
	D4	0.742	0.017	-0.047	0.004	-0.028	0.005	-0.163	0.000	-0.144	0.000	0.927	311.981	2001.328	1.242
PSA	D5	0.783	0.010	0.012	0.006	0.038	0.008	-0.023	0.000	-0.246	0.000	0.979	140.265	3475.948	4.634
	D6	0.766	0.005	0.056	0.003	-0.030	0.004	-0.845	0.000	-0.272	0.000	0.995	6957.442	4598.645	0.047
	D1	1.106	0.023	-0.034	0.013	-0.078	0.024	0.053	0.001	0.004	0.001	0.577	699.244	815.923	11.687
	D2	1.187	0.021	-0.057	0.011	-0.012	0.010	0.135	0.000	0.249	0.001	0.491	1009.296	1229.218	12.826
	D3	1.241	0.023	-0.034	0.015	-0.002	0.025	0.182	0.001	0.121	0.000	0.798	895.692	1859.303	267.642
	D4	1.159	0.017	-0.030	0.005	-0.026	0.015	0.132	0.000	0.228	0.000	0.936	352.923	2076.192	0.803
Publicis	D5	1.363	0.016	-0.059	0.006	0.143	0.010	0.300	0.000	-0.157	0.000	0.983	289.911	3706.570	7.769
	D6	1.264	0.007	0.003	0.010	-0.094	0.008	0.178	0.000	-0.202	0.000	0.995	1318.906	4744.839	0.354
	D1	0.697	0.014	-0.020	0.006	0.034	0.014	0.041	0.000	-0.013	0.000	0.595	1230.444	824.103	6.375
	D2	0.697	0.015	-0.012	0.006	0.027	0.014	0.138	0.000	0.114	0.000	0.494	470.473	1288.363	9.815
	D3	0.719	0.015	0.015	0.010	0.000	0.017	0.107	0.000	0.080	0.000	0.809	104.162	1789.086	385.927
	D4	0.765	0.010	-0.007	0.002	0.017	0.010	0.413	0.000	0.211	0.000	0.946	976.689	1890.475	3.629
Renault	D5	0.755	0.015	-0.038	0.005	-0.022	0.022	0.069	0.000	0.069	0.000	0.983	7885.513	3451.310	0.444
	D6	0.874	0.007	0.034	0.004	0.041	0.005	0.185	0.000	-0.299	0.000	0.996	423.897	4963.553	3.554
	D1	1.237	0.021	-0.048	0.009	-0.032	0.025	-0.002	0.001	0.007	0.000	0.681	894.989	819.858	20.352
	D2	1.285	0.073	-0.047	0.015	-0.015	0.029	0.105	0.000	0.122	0.001	0.610	1497.253	1339.134	19.217
	D3	1.316	0.021	0.008	0.007	0.042	0.020	0.055	0.000	0.073	0.000	0.856	548.021	1802.699	346.795
	D4	1.283	0.017	-0.028	0.011	-0.022	0.043	0.472	0.000	0.548	0.000	0.957	389.128	1994.560	9.071
Ricard	D5	1.424	0.016	-0.155	0.010	0.106	0.010	0.145	0.000	-0.049	0.000	0.986	138.556	3714.436	9.046
	D6	1.792	0.033	-0.039	0.003	0.128	0.008	0.143	0.000	0.259	0.001	0.998	1187.605	4993.387	6.400
	D1	0.649	0.017	-0.018	0.007	0.036	0.017	-0.138	0.000	-0.172	0.000	0.587	2602.471	777.385	6.198

Table 12 (continued)

Stocks	Bands	β_m	TSAT	β_o	TSAT	β_g	TSAT	β_{SMB}	TSAT	β_{HML}	TSAT	R2	JB	LB	ARCH
Schneider	D2	0.661	0.014	-0.008	0.010	0.016	0.017	-0.165	0.000	-0.342	0.000	0.460	1736.476	1271.797	23.078
	D3	0.644	0.014	-0.010	0.007	0.093	0.012	-0.067	0.000	-0.274	0.000	0.766	8004.109	1796.284	352.040
	D4	0.731	0.011	0.010	0.004	-0.005	0.007	-0.027	0.000	-0.414	0.000	0.925	3030.111	1834.277	1.774
	D5	0.700	0.011	0.015	0.004	0.005	0.005	-0.113	0.000	-0.270	0.000	0.981	963.250	3528.947	16.162
	D6	0.930	0.005	-0.011	0.003	0.061	0.003	-0.391	0.000	-0.182	0.000	0.997	2556.860	4546.120	2.090
	$\beta_m = MarketBeta; \beta_o = OilBeta; \beta_g = GoldBeta; \beta_{SMB} = SMBBeta; \beta_{HML} = HMLBeta$														
Société Générale	D1	1.246	0.016	-0.010	0.007	-0.013	0.011	0.034	0.000	0.064	0.000	0.804	319.223	812.444	6.158
	D2	1.193	0.015	-0.009	0.004	-0.015	0.006	0.031	0.000	0.150	0.000	0.717	351.236	1296.270	14.686
	D3	1.163	0.016	-0.025	0.009	0.010	0.016	0.167	0.000	0.180	0.000	0.863	485.350	1921.742	392.911
	D4	1.248	0.025	-0.004	0.030	0.068	0.016	0.253	0.000	-0.153	0.000	0.966	88.112	2005.775	10.428
	D5	1.220	0.009	-0.042	0.005	0.043	0.009	0.031	0.000	0.096	0.000	0.989	29.977	3645.952	5.169
	D6	1.108	0.014	0.022	0.003	0.138	0.020	0.092	0.000	-0.256	0.000	0.997	5.184	5362.512	19.295
Sodexo	D1	1.296	0.019	-0.002	0.006	-0.075	0.012	-0.025	0.000	0.186	0.001	0.694	12,141.047	782.822	5.849
	D2	1.373	0.019	-0.045	0.018	-0.116	0.014	0.048	0.001	0.344	0.002	0.593	15,146.487	1242.610	13.284
	D3	1.312	0.019	-0.043	0.006	-0.122	0.014	-0.068	0.000	0.332	0.000	0.838	8283.915	1969.654	274.576
	D4	1.540	0.014	-0.073	0.004	-0.182	0.007	-0.336	0.000	0.629	0.000	0.957	4189.733	1934.230	2.015
	D5	1.353	0.010	-0.128	0.006	-0.093	0.005	-0.322	0.000	0.265	0.000	0.986	1486.861	3069.794	2.130
	D6	1.563	0.012	-0.153	0.004	-0.109	0.006	-0.002	0.000	-0.059	0.000	0.998	4376.942	4798.076	0.051
Technip	D1	0.604	0.014	-0.020	0.008	-0.006	0.015	-0.091	0.000	-0.117	0.000	0.567	2170.828	779.146	7.379
	D2	0.635	0.013	-0.008	0.009	0.014	0.014	-0.100	0.000	-0.171	0.000	0.463	1521.985	1229.036	12.471
	D3	0.514	0.014	0.001	0.008	-0.003	0.016	0.074	0.000	-0.046	0.000	0.756	608.417	1835.932	298.508
	D4	0.628	0.009	-0.021	0.004	0.124	0.011	-0.047	0.000	-0.091	0.000	0.930	264.991	2125.131	3.394
	D5	0.595	0.011	-0.054	0.005	0.122	0.010	0.457	0.000	0.095	0.000	0.981	161.283	3376.248	10.878
	D6	0.659	0.005	0.043	0.004	0.219	0.008	-0.144	0.000	-0.076	0.000	0.995	22.967	4854.986	14.704
Technip	D1	0.882	0.022	0.101	0.011	0.119	0.027	0.088	0.000	0.288	0.000	0.616	2349.743	824.456	4.035
	D2	0.908	0.021	0.142	0.012	0.081	0.019	0.088	0.000	0.105	0.000	0.494	1023.357	1279.892	19.541
	D3	0.937	0.023	0.198	0.015	0.027	0.027	0.200	0.001	0.237	0.001	0.797	2011.793	1898.289	398.722

Table 12 (continued)

Stocks	Bands	β_m	TSAT	β_o	TSAT	β_g	TSAT	β_{SMB}	TSAT	β_{HML}	TSAT	R2	JB	LB	ARCH
Total	D4	1.028	0.021	0.207	0.012	0.180	0.033	0.291	0.001	0.182	0.000	0.945	57.401	2066.978	7.243
	D5	0.923	0.016	0.211	0.006	0.235	0.013	0.244	0.000	-0.315	0.000	0.982	224.973	3447.854	2.608
	D6	0.738	0.012	0.301	0.005	0.091	0.011	0.071	0.000	0.230	0.000	0.996	2545.784	4874.279	1.545
	D1	0.882	0.012	0.067	0.006	0.065	0.010	-0.085	0.000	-0.065	0.000	0.805	2135.402	823.451	13.276
	D2	0.855	0.012	0.092	0.015	0.064	0.009	-0.063	0.000	-0.064	0.000	0.732	1058.616	1278.823	15.340
	D3	0.813	0.013	0.133	0.007	0.036	0.022	-0.160	0.000	-0.066	0.000	0.881	372.020	1879.790	345.446
Veolia	D4	0.930	0.012	0.130	0.005	0.037	0.012	-0.056	0.000	-0.050	0.000	0.964	2794.304	1924.299	1.507
	D5	0.932	0.007	0.119	0.005	0.045	0.005	-0.070	0.000	0.020	0.000	0.991	102.071	3425.487	10.295
	D6	0.921	0.005	0.169	0.003	0.161	0.004	-0.062	0.000	0.198	0.000	0.997	40.234.841	4299.488	0.008
	D1	0.799	0.021	-0.003	0.009	-0.015	0.019	0.080	0.000	-0.031	0.000	0.602	18.492.765	766.970	4.304
	D2	0.865	0.016	-0.031	0.010	0.024	0.010	-0.001	0.000	-0.008	0.000	0.448	8070.179	1230.665	16.844
	D3	1.022	0.015	-0.038	0.021	-0.055	0.066	-0.211	0.001	0.029	0.001	0.787	9409.466	1781.057	273.058
Vinci	D4	1.042	0.013	-0.057	0.008	0.007	0.017	-0.181	0.000	0.033	0.000	0.942	2734.859	2025.850	3.759
	D5	1.024	0.012	-0.039	0.005	-0.020	0.012	0.141	0.000	-0.331	0.000	0.983	1394.997	3359.564	10.078
	D6	1.087	0.006	-0.162	0.004	0.063	0.004	0.030	0.000	0.005	0.000	0.997	434.629	4556.958	3.699
	D1	1.053	0.013	0.001	0.008	0.007	0.014	0.009	0.000	0.063	0.000	0.795	2220.474	806.155	9.823
	D2	1.031	0.012	0.016	0.008	0.019	0.015	0.039	0.000	0.082	0.000	0.725	3343.302	1225.840	10.051
	D3	1.116	0.015	0.003	0.008	0.015	0.014	-0.014	0.000	-0.043	0.000	0.871	2595.684	1896.660	455.472
Vivendi	D4	1.101	0.010	-0.041	0.005	0.079	0.013	-0.128	0.000	0.147	0.000	0.957	441.046	2031.859	0.706
	D5	0.998	0.008	-0.036	0.005	-0.039	0.012	0.112	0.000	0.001	0.000	0.986	146.291	3637.523	11.878
	D6	0.926	0.006	0.027	0.002	-0.046	0.003	0.019	0.000	-0.009	0.000	0.997	178.049	5206.353	21.990
	D1	0.828	0.014	-0.010	0.007	0.015	0.014	0.123	0.000	-0.027	0.000	0.705	1573.615	797.581	7.533
	D2	0.804	0.013	-0.023	0.005	0.003	0.010	0.021	0.000	-0.051	0.000	0.583	733.598	1295.779	21.415
	D3	0.771	0.013	-0.033	0.005	0.030	0.009	0.022	0.000	-0.040	0.000	0.804	700.668	1985.771	405.793
	D4	0.779	0.014	-0.012	0.007	-0.051	0.015	0.036	0.000	-0.057	0.000	0.946	214.969	1922.169	7.903
	D5	0.840	0.007	-0.026	0.002	-0.098	0.004	-0.002	0.000	0.034	0.000	0.983	248.090	3435.890	7.938
	D6	0.765	0.004	-0.127	0.002	0.015	0.005	0.160	0.000	-0.233	0.000	0.996	36.408	5437.728	5.365

Table 13 Synthesis of time–frequency model for each stocks

	D1	D2	D3	D4	D5	D6
Accor	CAPM	MIXED	MIXED	FULL	MIXED	FULL
Airbus	MIXED	CAPM	FULL	MIXED	MB	FULL
Alcatel	MIXED	FF	FULL	MIXED	MIXED	FULL
Air Liquide	MIXED	FF	MIXED	FULL	MIXED	FULL
AXA	MIXED	MIXED	FULL	MIXED	FULL	FULL
BNP	MIXED	MIXED	FULL	FULL	FULL	FULL
Bouygues	MIXED	MIXED	MIXED	MIXED	FULL	FULL
CA	MIXED	MIXED	MIXED	FULL	FULL	MIXED
Carrefour	CAPM	CAPM	CAPM	FULL	MIXED	FULL
Danone	FF	MIXED	MIXED	MIXED	FULL	FULL
Essilor	CAPM	MIXED	MIXED	MIXED	MIXED	FULL
GDF	MIXED	MIXED	FULL	FULL	FULL	FULL
Gemini	CAPM	MIXED	MIXED	MIXED	FULL	FULL
St-Gobain	MIXED	MIXED	MIXED	MIXED	FULL	FULL
L'Oréal	MIXED	MIXED	FF	MIXED	MIXED	FULL
LVMH	MB	MIXED	MIXED	MIXED	FULL	FULL
Michelin	MIXED	MIXED	CAPM	FULL	MIXED	FULL
Orange	MIXED	MIXED	MIXED	FULL	MIXED	FULL
PSA	MB	MIXED	MIXED	MIXED	FULL	MIXED
Publicis	MB	FULL	FF	MIXED	MIXED	FULL
Renault	MIXED	MIXED	MIXED	MIXED	MIXED	FULL
Ricard	FULL	FF	MIXED	MIXED	MIXED	FULL
Schneider	CAPM	MIXED	MIXED	MIXED	MIXED	FULL
SG	MIXED	MB	MIXED	FULL	FULL	MIXED
Sodexo	MIXED	FF	MIXED	FULL	FULL	FULL
Technip	MIXED	FULL	MIXED	FULL	FULL	FULL
Total	FULL	FULL	MIXED	FULL	FULL	FULL
Veolia	MIXED	MB	MIXED	MIXED	MIXED	MIXED
Vinci	MIXED	MIXED	CAPM	FULL	MIXED	MIXED
Vivendi	MIXED	MIXED	MIXED	MIXED	MIXED	FULL

Table 14 Time-frequency multi-beta-AR-EGARCH estimates

	Bm	TSTAT	Bo	TSTAT	Bg	TSTAT	Bsmb	TSTAT	Bhml	TSTAT	R2	JB	LB	ARCH
$\beta_m = \text{MarketBeta}; \beta_o = \text{OilBeta}; \beta_g = \text{GoldBeta}; \beta_{SMB} = \text{SMBBeta}; \beta_{HML} = \text{HMLBeta}$														
Pf1D1	1.0128	1.8260	-0.0147	-4.0513	-0.0271	-4.1888	0.0137	1.3197	0.0111	0.9067	0.9198	5054.1431	770.8507	10.6967
Pf1D2	1.0293	4.6444	-0.0272	-10.7529	-0.0463	-7.2919	0.0710	5.6888	0.0850	5.9507	0.9049	1318.0062	1376.3509	11.7707
Pf1D3	1.0420	5.5350	-0.0264	-6.7614	-0.0429	-5.6053	0.0971	7.6941	0.1141	6.1202	0.9627	632.0573	1847.2088	323.9638
Pf1D4	1.0647	9.4863	-0.0514	-17.7020	-0.0187	-5.2777	0.0092	1.7401	0.1505	14.9950	0.9890	108.3324	1989.3269	5.3578
Pf1D5	1.0397	8.0482	-0.0313	-13.5745	-0.0945	-22.9679	0.1586	21.5679	0.1510	16.6384	0.9967	1025.6481	3557.1430	15.1779
Pf1D6	1.0733	22.3863	-0.0766	-38.7805	-0.1362	-52.3561	0.2349	39.7099	0.1661	34.5741	0.9994	388.6896	4842.1533	2.8870
Pf2D1	0.9885	-1.7824	-0.0016	-0.4237	-0.0065	-0.8588	0.0343	2.2726	0.0483	2.8409	0.9152	1982.3226	766.0051	7.0794
Pf2D2	0.9934	-1.0095	-0.0141	-3.2003	0.0003	0.0343	0.0589	3.7068	0.0820	4.1472	0.8843	1268.8381	1361.8681	24.5851
Pf2D3	0.9944	-0.9029	-0.0141	-4.2682	0.0021	0.4989	0.1075	10.2505	0.0089	0.9911	0.9524	526.5796	2018.7980	468.0411
Pf2D4	0.9873	-2.2911	-0.0066	-3.0118	-0.0170	-8.0105	0.0982	11.3004	0.0187	2.3086	0.9880	273.3564	1924.1235	6.0320
Pf2D5	1.0446	10.7182	-0.0349	-16.8265	0.0191	6.0452	0.0430	4.5824	-0.0143	-1.3943	0.9966	18.2804	3385.2359	7.2749
Pf2D6	1.0543	32.7148	-0.0253	-17.4259	0.0110	5.4796	0.2653	47.2160	0.0509	11.1970	0.9994	105.4132	4948.7422	26.5995
pf3D1	0.9007	-6.8954	0.0841	10.3003	0.0814	5.9320	-0.0160	-0.5192	0.0718	2.3146	0.7624	1267.2178	831.7440	9.0860
pf3D2	0.8970	-7.9242	0.1280	16.2711	0.0864	6.5131	-0.0293	-1.4324	0.0391	1.3449	0.6929	628.2041	1304.3842	26.3066
pf3D3	0.8950	-7.1751	0.1728	20.6186	0.0418	2.8580	0.0514	1.4915	0.0833	2.7162	0.8740	702.2852	1925.7050	386.8339
pf3D4	0.9037	-5.9604	0.2112	30.6337	0.0900	6.6366	0.1715	5.3644	-0.1186	-4.9075	0.9639	478.0363	2058.4045	11.0169
pf3D5	0.9480	-7.6244	0.1289	14.2375	0.1681	34.6976	-0.0005	-0.0286	-0.1963	-11.5581	0.9884	250.9343	3574.9310	2.9612
pf3D6	0.8555	-11.2671	0.2942	27.2611	0.0317	4.9733	-0.3159	-15.0591	0.1596	22.7156	0.9975	12,005.5624	4593.1584	0.0498
pf4D1	1.1218	14.2701	-0.0145	-5.1229	-0.0594	-7.9133	0.0049	0.3654	0.0812	6.0728	0.8945	3912.0928	759.5884	3.6846
pf4D2	1.1128	12.9494	-0.0205	-4.1248	-0.0717	-8.5750	0.0492	2.7191	0.1634	9.1622	0.8675	2389.9123	1372.2471	10.6827
pf4D3	1.1200	12.0426	-0.0116	-2.3942	-0.0821	-7.1891	0.0491	2.6198	0.2383	11.6878	0.9472	2718.4934	1877.9496	302.0437
pf4D4	1.1252	20.6822	-0.0386	-17.6717	-0.0900	-17.8351	-0.0489	-4.6542	0.2568	20.0316	0.9824	417.7878	1927.0607	4.9735
pf4D5	1.1236	24.9265	-0.0547	-32.5790	-0.0614	-19.0736	0.0899	18.6292	0.1965	24.3080	0.9948	1046.7269	3292.5984	12.6606
pf4D6	1.0841	5.3668	-0.0737	-40.5908	-0.1897	-21.1726	0.0319	8.0065	0.1994	37.2976	0.9989	1774.3733	4833.1435	0.0679
pf5D1	0.9590	-5.9736	-0.0170	-5.4061	0.0028	0.4165	0.0355	2.5711	-0.0127	-0.7911	0.9234	1928.9845	792.0324	7.3002

Table 14 (continued)

	Bm	TSTAT	Bo	TSTAT	Bg	TSTAT	Bmb	TSTAT	Bhml	TSTAT	R2	JB	LB	ARCH
pf5D2	0.9819	-2.9366	-0.0299	-9.4215	-0.0042	-0.6262	0.0534	3.9681	-0.0151	-0.9699	0.9051	1114.3730	1316.5605	10.3169
pf5D3	1.0034	0.5089	-0.0195	-6.0161	-0.0112	-1.7126	0.0162	1.2892	-0.0138	-0.9361	0.9587	540.7671	1859.9520	344.4924
pf5D4	0.9909	-1.9855	-0.0232	-11.0116	0.0186	4.3092	0.0467	4.1252	-0.0022	-0.2312	0.9856	1053.9387	1902.7966	3.3969
pf5D5	1.0405	9.7558	-0.0478	-23.5468	0.0735	27.1366	0.1658	25.7960	-0.0968	-12.5002	0.9969	147.0312	3290.8140	8.1607
pf5D6	0.9506	-15.2984	-0.0004	-0.1757	0.0366	15.2212	0.2996	49.8728	-0.1889	-26.8280	0.9991	15,758.1129	4587.7523	0.0094
pf6D1	0.8737	-17.1138	0.0187	3.7732	0.0281	4.6470	-0.0305	-2.5606	0.0286	1.9040	0.9150	728.3765	786.3586	1.7048
pf6D2	0.8878	-15.6270	0.0268	8.0587	0.0598	11.5432	0.0167	1.8168	0.0454	4.3768	0.8871	478.9190	1332.8522	11.8096
pf6D3	0.8766	-17.6389	0.0432	16.1826	0.0446	6.7467	0.0704	4.8289	-0.0348	-3.5503	0.9438	3435.1631	1921.9218	339.6454
pf6D4	0.8704	-20.6654	0.0406	12.9371	0.0554	9.1593	0.0990	7.5633	-0.0644	-3.4686	0.9820	1720.4819	2118.0799	2.1100
pf6D5	0.9172	-13.7235	0.0238	8.1041	0.0427	9.4687	0.0207	1.8452	-0.0078	-1.1847	0.9949	1025.5750	3580.5899	9.3538
pf6D6	0.9040	-51.2927	0.0826	56.5789	0.0914	48.7542	-0.0105	-2.5330	-0.0724	-23.2545	0.9988	1021.3129	5128.3632	34.9478
Pf7D1	1.0379	4.8018	-0.0093	-2.2685	-0.0036	-0.6998	0.0221	1.2601	0.0335	2.2809	0.9125	1549.7345	746.1414	3.7055
Pf7D2	1.0327	4.8484	-0.0157	-3.9411	-0.0019	-0.6123	0.0557	4.8466	0.0352	4.7899	0.8854	747.5027	1305.3457	20.3845
Pf7D3	1.0351	3.9255	-0.0322	-12.6129	0.0011	0.5556	-0.0112	-0.7803	-0.0142	-0.9359	0.9428	271.1022	1956.2690	380.4895
Pf7D4	1.0493	7.2107	-0.0262	-9.8505	-0.0096	-2.4314	-0.0128	-1.4769	-0.0828	-9.2075	0.9858	335.6903	1860.5534	5.8824
Pf7D5	0.9905	-1.8724	-0.0324	-9.0003	-0.0170	-3.3819	0.1034	7.9933	0.1163	8.4322	0.9958	30.6748	3573.5427	4.0842
Pf7D6	0.8777	-43.8547	-0.0322	-14.3253	-0.0040	-1.8821	-0.0196	-9.3017	-0.0251	-5.8013	0.9988	1820.0950	4921.4491	4.5020
pf8D1	0.9905	-0.9161	-0.0113	-1.8357	0.0165	1.4676	0.0122	0.5680	0.0521	2.3787	0.8592	876.7466	825.9724	7.5587
pf8D2	0.9980	-0.1947	-0.0110	-2.1039	0.0071	1.2473	0.0558	2.0518	0.0773	2.8974	0.8202	562.8547	1298.3455	18.0321
pf8D3	1.0520	5.4799	-0.0175	-3.0904	-0.0202	-1.8986	0.0096	0.4702	-0.0050	-0.2193	0.9220	1379.8857	1842.5713	302.5731
pf8D4	1.0544	5.6999	-0.0545	-11.7887	-0.0002	-0.0090	0.0437	1.6324	0.0799	3.4195	0.9749	233.1310	1969.8363	4.0433
pf8D5	1.0579	9.5613	-0.0492	-13.5931	0.0389	6.1059	0.1166	8.0279	-0.0528	-4.8712	0.9926	150.7068	3524.0528	16.8286
pf8D6	0.9568	-3.4414	-0.0144	-7.1454	-0.0880	-16.4707	0.1087	4.4857	0.3633	42.1324	0.9987	33.3478	4913.5588	35.2440

Abbreviations

CAPM: Capital Assets Pricing Model; SML: Securities Market Line; APT: Arbitrage Pricing Theory; MB: Multi-Beta (referring to Multi-Beta Model); TFMB: Time-Frequency Multi-Betas Model; OLS: Ordinary Least Squares; TSTAT: Student Statistic; BLUE: Best Linear Unbiased Estimator; JB: Jarque-Bera; LB: Ljung-Box; ARCH: Autoregressive Conditional Heteroskedasticity; ARMA: Auto Regressive Moving Average; E/GARCH: Exponential/Generalised AutoRegressive Conditional Heteroskedasticity; DCC-GARCH: Dynamic Conditional Correlation Generalised AutoRegressive Conditional Heteroskedasticity; BEKK-GARCH: Baba Engle Kraft Kroner Generalised AutoRegressive Conditional Heteroskedasticity; CAC40: Cotations Assistées Continues 40 (main index of Bourse de Paris); OAT: Obligation Assimilable du Trésor (French Treasury Bonds Rate); HFT: High-Frequency Trader; MODWT: Maximal Overlap Discrete Wavelets Transform; GDF: Gaz de France; LVMH: Louis Vuitton Moët Hennessy; PSA: Peugeot Société Anonyme; BNP: Banque Nationale de Paris-Paribas; SG: Société Générale; CA: Crédit Agricole.

Acknowledgements

Not applicable.

Authors' contributions

As unique author, RM fully analyse, compute, write and approved the final manuscript. RM is the corresponding author. The author read and approved the final manuscript.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Availability of data and materials

The datasets used and analysed during the current study are available from the corresponding author on reasonable request. In others, data are free available on any data providers. For Fama-French Factors data are available on French website: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Declarations

Competing interests

The authors declare that they have no competing interests.

Received: 6 July 2020 Accepted: 25 March 2021

Published online: 02 April 2021

References

- Aguiar-Conraria L, Soares MJ (2014) The continuous Wavelet Transform: moving beyond uni and bivariate analysis. *J Econ Surv* 28(2):344–375
- Arfaoui M, Ben Rejeb A (2017) Oil, gold, US dollar and stock market interdependencies: a global analytical insight. *Eur J Manag Bus Econ* 26(3):278–293. <https://doi.org/10.1108/EJMBE-10-2017-016>
- Bantz R (1981) The relationship between return and market value of common stocks. *J Financ Econ* 9(1):3–18
- Basher SA, Sadorsky P (2006) Oil price risk and emerging stock markets. *Glob Financ J* 17(2):224–251
- Basu S (1983) The relationship between earnings yield, market value and return for NYSE common stocks: further evidence. *J Financ Econ* 12(1):129–156
- Baur DG, Lucey B (2010) Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *Financ Rev* 45(2):217–229
- Baur DG, McDermott TK (2010) Is gold a safe haven? International evidence. *J Bank Finance* 34:1886–1898
- Baur DG, McDermott TKJ (2016) Why is gold a safe haven? *J Behav Exp Financ* 10:63–71
- Bendod A, Chikhi M, Bennaceur F (2017) Testing the CAPM-GARCH MODELS in the GCC-wide equity sectors. *Asian J Econ Model* 5(4):413–430
- Bera A, Bubnys E, Park H (1988) Conditional heteroscedasticity in the market model and efficient estimates of betas. *Financ Rev* 23(2):201–214
- Black F, Jensen M, Scholes M (1972) The capital asset pricing model: some empirical test; studies in the theory of capital markets edited by M. Jensen. Praeger Publishers, New York
- Bollerslev T (1986) Generalized autoregressive conditional heteroskedasticity. *J Econ* 31:307–327
- Bollerslev T, Engle R, Wooldridge H (1988) A Capital asset pricing model with time-varying covariance. *J Polit Econ* 96(1):116–131
- Bos T, Newbold P (1984) An empirical investigation of the possibility of stochastic systematic risk in the market model. *J Bus* 57(1):37–47
- Boyer MM, Filion D (2007) Common and fundamental factors instock returns of Canadian oil and gas companies. *Energy Econ* 29(3):428–453
- Brooks RD, Faff RW, McKenzie MD (1998) Time-varying beta risk of Australian industry portfolios: a comparison of modelling techniques. *Aust J Manag* 23(1):1–22
- Chen NF, Roll R, Ross SA (1986) Economic forces and the stock market. *J Bus* 59(3):383–403
- Choudry T, Wu H (2008) Forecasting ability of GARCH vs Kalman filter method evidence from daily UK time-varying beta. *J Forecast* 27:670–689
- Chua JH, Sick G, Woodward RS (1990) Diversifying with gold stocks. *Financ Anal J* 46:76–79
- Corhay A, Rad A (1996) Conditional heteroscedasticity adjusted market model and an event study. *Q Rev Econ Finance* 36(4):529–538

- Diebold F, Jang I, Jevons L (1988) Conditional heteroskedasticity in the market, finance and economics discussion series, 42, Division of Research and Statistics, Federal Reserve Board, Washington
- Engle R (1982) Autoregressive conditional heteroskedasticity with estimates of the variance of UK inflation. *Econometrica* 50:987–1008
- Engle R (2016) Dynamic conditional beta. *J Financ Economet* 14(4):643–667
- Fabozzi F, Francis J-C (1978) Beta as a random coefficient. *J Financ Quant Anal* 13(1):101–116
- Faff RW, Hiller D, Hiller J (2000) Time varying beta risk: an analysis of alternative modelling techniques. *J Bus Financ Acc* 27(2):523–555
- Fama E (1996) Multifactor portfolio efficiency and multifactor asset pricing. *J Financ Quant Anal* 31(4):441–465
- Fama E, French K (1992) Common risk factors in the returns on stocks and bonds. *J Financ Econ* 33(1):3–56
- Fama E, French KR (1996) The CAPM is wanted, dead or alive. *J Finance* 51(5):1947–1958
- Fama E, MacBeth J (1973) Risk, return, and equilibrium: empirical tests. *J Polit Econ* 81(3):607–636
- Fisher TJ, Gallagher C (2012) New weighted portmanteau statistics for time series goodness of fit testing. *J Am Stat Assoc* 107(498):777–787
- Gaffeo E (2019) Leverage and evolving heterogeneous beliefs in a simple agents-based financial market. *Financ Res Lett* 29:272–279
- Gençay R, Selçuk F, Whitcher B (2005) Systematic risk and timescales. *Quant Finance* 3(2):108–116
- Ghalanos A, Theussl S (2011) Rsolnp: general non-linear optimization using augmented Lagrange multiplier method, 1.1.1 edition
- Giacotto C, Ali MM (1982) Optimal distribution free tests and further evidence of heteroskedasticity in the market model. *J Finance* 37:1247–1257
- Groenewold N, Fraser P (1997) Share prices and macroeconomics factors. *J Bus Financ Acc* 24(9):1367–1384
- Groenewold N, Fraser P (1999) Time varying estimates of CAPM betas. *Math Comput Simul* 48:531–539
- He Z, O'Connor F, Thijssen J (2018) Is gold a sometime safe haven or an always hedge for equity investor? A Markov switching CAPM approach for US and Uh stocks indices. *Int Rev Financ Anal* 60:30–37
- Heyman D, Lescrauwaet M, Stieperaere H (2019) *Finance Res Lett* 29:1–6
- Huang RD, Masulis RW, Stoll HR (1996) Energy shocks and financial markets. *J Futur Mark* 16(1):1–27
- Hussain Shahzad SJ, Raza N, Shahbaz M, Ali A (2017) Dependence of stock market with gold and bonds under bullish and bearish Market States. *Resour Policy* 52(C):308–319
- Johnson M, Lamdin D (2015) New evidence on whether gold mining stock are more like gold or like stocks. *Altern Invest Anal Rev* 5(2):31–38
- Jones DW, Leiby P, Paik I (2004) Oil price shocks and the macroeconomy: what has been learned since 1996. *Energy J* 25(2):1–32
- Kahraman E, Unal G (2019) Multiple wavelet coherency analysis and forecasting of metal prices. *Financ Innov* 5(22)
- Kou G, Peng Y, Wang G (2014) Evaluation of clustering algorithms for financial risk analysis using MCDM methods. *Inf Sci* 275:1–12
- Kou G, Xu Y, Peng Y, Shen F, Chen Y, Chang K, Kou S (2021) Bankruptcy prediction for SMEs using transactional data and two-stage multi-objective feature selection. *Decis Support Syst* 140
- Lee CC, Zeng JH (2011) The impact of oil price shocks on stock market activities: asymmetric effect with quantile regression. *Math Comput Simul* 81(7):1910–1920
- Lintner J (1965) The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Rev Econ Stat* 47(1):13–37
- Lintner J (1981) Some new perspectives on tests of CAPM and other capital asset pricing models and issues of market efficiency; edited by Harvard Institute of Economic Research, discussion paper
- Mallat S (1989) A theory for multiresolution signal decomposition: the wavelet representation. *IEEE Trans Pattern Anal Mach Intell* 11(7)
- Mallat S (2001) Une exploration des signaux en ondelettes. *Ecole polytechnique*
- Mallat S (2009) Wavelet tour of signal processing: the sparse way. Academic Press
- Markowitz H (1952) Portfolio selection. *J Finance* 7(1):77–91
- Merton RC (1973) Theory of rational option pricing. *Bell J Econ Manag Sci* 4(1):141–183
- Mestre R, Terraza M (2018) Time-frequency analysis of the CAPM—application to the CAC 40. *Manag Glob Transit Int Res J* 16–2:141–157
- Mestre R, Terraza M (2020) Adjusted beta based on an empirical comparison of OLS CAPM and the CAPM with EGARCH errors. *Int J Financ Econ*. <https://doi.org/10.1002/ijfe.1977>
- Meyer Y (1990) Ondelettes et algorithmes concurrents. *Actualités mathématiques Hermans éditions des sciences et des arts* xii: 217–381
- Mishra PK, Das JR, Mishra SK (2010) Gold price volatility and stock market returns in India. *Am J Sci Res* 9:49–55
- Miyazaki T, Toyoshima Y, Hamori S (2012) Exploring the dynamic interdependence between gold and other financial markets. *Econ Bull AccessEcon* 32(1):37–50
- Morelli D (2003) Capital asset pricing models on UK securities using ARCH. *Appl Financ Econ* 13(3):211–223
- Mossin J (1966) Equilibrium in a capital asset market. *Econometrica* 34:768–783
- Nanayakkara N, Nimal PD, Weerakoo YK (2019) Behavioural asset pricing a review. *Int J Econ Financ Issues* 9(4):101–108
- Nasiri M, Nourollahzadeh N, Sarraf F, Hamidian M (2019) Modeling assets pricing using behavioral patterns Fama-French approach. *Iran J Finance* 3(3):35–60
- Nelson D (1991) Conditional heteroskedasticity in asset return a new approach. *Econometrica* 59(2):347–370
- Ross S (1976) The arbitrage theory of capital pricing. *J Econ Theory* 13:341–360
- Schwert G, Seguin P (1990) Heteroscedasticity in stock returns. *J Finance* 45(4):1129–1155
- Sharpe W (1964) Capital asset prices: a theory of market equilibrium under risk. *J Finance* 19(3):425–442
- Sumner S, Johnson R, Soenen L (2010) Spillover effects between gold, stocks, and bonds. *J Cent Cathedra* 3(2):106–120
- Tsay H-J, Chen M-C, Yang C-Y (2014) A time-varying perspective on the CAPM and downside betas. *Int Rev Econ Financ* 29:440–454

- Tufano P (1998) The determinants of stock price exposure: Financial engineering and the gold mining industry. *J Finance* 53(3):1015–1052
- Tuyon J, Ahmad Z (2018) Behavioural asset pricing determinants in a factor and style investing framework. *Cap Mark Rev* 26(2):32–52
- Wen F, Xu L, Ouyang G, Kou G (2019) Retail investor attention and stock price crash risk: evidence from China. *Int Rev Financ Anal* 65:101376
- Ye Y (1997) Interior point algorithms: theory and analysis. Wiley
- Yeo J (2001) Modelling time-varying systematic risk in Australia. In: Ghessemi M, McAleer L, Oxley MS (eds) *Proceedings of the international congress on modelling & simulation*, vol 3, Canberra. ed. Western Australia: Modelling and Simulation Society of Australia and New Zealand Inc., pp 1565–1570

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Submit your manuscript to a SpringerOpen[®] journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

Submit your next manuscript at ► [springeropen.com](https://www.springeropen.com)
