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## Article

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RESEARCH

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# On the volatility of daily stock returns of Total Nigeria Plc: evidence from GARCH models, value-at-risk and backtesting

Ngozi G. Emenogu<sup>1</sup>, Monday Osagie Adenomom<sup>2\*</sup>  and Nwaze Obini Nweze<sup>2</sup>

\* Correspondence: [adenomonmo@nsuk.edu.ng](mailto:adenomonmo@nsuk.edu.ng)

<sup>2</sup>Department of Statistics, Nasarawa State University, Keffi, Nasarawa State, Nigeria

Full list of author information is available at the end of the article

## Abstract

This study investigates the volatility in daily stock returns for Total Nigeria Plc using nine variants of GARCH models: sGARCH, girGARCH, eGARCH, iGARCH, aGARCH, TGARCH, NGARCH, NAGARCH, and AVGARCH along with value at risk estimation and backtesting. We use daily data for Total Nigeria Plc returns for the period January 2, 2001 to May 8, 2017, and conclude that eGARCH and sGARCH perform better for normal innovations while NGARCH performs better for student t innovations. This investigation of the volatility, VaR, and backtesting of the daily stock price of Total Nigeria Plc is important as most previous studies covering the Nigerian stock market have not paid much attention to the application of backtesting as a primary approach. We found from the results of the estimations that the persistence of the GARCH models are stable except for few cases for which iGARCH and eGARCH were unstable. Additionally, for student t innovation, the sGARCH and girGARCH models failed to converge; the mean reverting number of days for returns differed from model to model. From the analysis of VaR and its backtesting, this study recommends shareholders and investors continue their business with Total Nigeria Plc because possible losses may be overcome in the future by improvements in stock prices. Furthermore, risk was reflected by significant up and down movement in the stock price at a 99% confidence level, suggesting that high risk brings a high return.

**Keywords:** Volatility, Returns, Stocks, Total petroleum, Akaike information criterion (AIC), GARCH, Value-at-risk (VaR), Backtesting

## Introduction

Volatility is a statistical measure of the dispersion of returns for a given security or market index. It can be measured using the standard deviation or variance between returns from the same security or market index. It is often the case that higher levels of volatility, lead to higher risks associated with a particular security, a leading reason for why crude oil prices and corresponding stock prices fluctuate heavily and became more volatile during the World War II to early 1970s (Ulusoy and Ozdurak 2018). From an economic perspective, world resources are scarce, particularly in developing countries as Nigeria (Maxwell and Reuvey 2000; International Peace Institute (IPI) 2009). According to Milder et al. (2011), resource scarcity is increasingly perceived as one of the greatest security risks of the 21st, a characteristic of developing countries

that has encouraged investors to engage investment uncertainty, with more risk averse behaviour. Kou et al. (2014) proposed Multiple Criteria Decision Making (MCDM) for financial risk analysis. The authors found that MCDM is effective in clustering algorithm evaluation and can lead to good 2-way clustering solution on the selected financial risk data set. While Zhang et al. (2019) proposed soft consensus cost models for Group Decision Making (GDM) processes to aid decision makers in investment purposes in the financial market because it consider cost from both perspectives of a moderator and individual experts. Ahmadi et al. (2018) identified three characteristics associated with making investment decisions to be the cost of an investment, the uncertainty over likely future profits and the seeking of additional information in order to reduce this uncertainty. These are provided by stockbrokers, accountants, financial analysts, financial econometricians and financial time series analysts by sometime based on experiences of the financial market over the years which may be subjective and not scientific. The most dependable are the econometricians and time series analysts because they use long tested GARCH family models due to the volatility and value at risk presented by fluctuating stock prices.

For financial risk, Kou et al. (2019) suggest the use of machine learning to detect outbreak and contagion of systemic risk in financial markets and industry. Ahmed et al. (2018) recommend investors to invest in emerging markets like Nigeria. However, Chao et al. (2019) note that cross-border capital frequent flow (as in the case of trade-based money laundering), which is used as arbitrage in financial markets, can be harmful to the emerging financial markets and developing economies.

Nigeria is oil dependent (Adenomou 2016) and a major player in the oil market (Ahmadi et al. 2018) Investors who may be attracted to the Nigeria market should be guided by the literature that draws attention to volatile oil prices (Adeniyi 2011; Abdulkareem and Abdulkareem 2016). Despite volatility of oil prices, investors in Nigeria continue to invest in the stock of oil companies such as Total Nigeria Plc.

Total Nigeria Plc is a marketing and services subsidiary of Total S. A., a multinational energy company operating in more than 130 countries. In the Nigerian oil sector, Total has been a leading player for over 50 years. On September 11, 2001 the company had a successful merger with ELF Oil Nigeria Limited, after which the share capital of the company stood at NGN169,761,000 composed of 50,000 ordinary (<http://www.total.com.ng/pro/about-us.html>).

The share price for Total Nigeria Plc was high on November 16, 2018 at NGN 199.10 (<http://www.bloomberg.com/quote/TOTAL:NL>), therefore the risk and returns would also be expected to be high. However, the stock price has been fluctuating, reaching NGN 203.00 in January 2019 and trading down to NGN 100.00 on August 30, 2019 (<https://afx.kwayisi.org>, 30-08-2019). However, the trading down in the price of the stock of Total Nigeria Plc is affected by global crude oil price, which is currently unstable and in decline due to the trade war between China and United States ([Oilprice.com](http://oilprice.com), August 30, 2019). Investors in this company and sector need accurate and up to date information to reduce their risk and to enhance their investment decisions.

This study investigates the volatility, VaR and backtesting of the daily stock price of Total Nigeria Plc. This is important as most previous studies covering Nigerian stock market analysis have not paid much attention to the application of backtesting as a primary approach.

### Literature review

The relationship between oil price volatility and economic growth in Nigeria has attracted substantial attention from researchers. Asaola and Ilo (2012) investigated the relationship between the Nigerian stock market and world crude oil price. The study showed that the Nigerian stock market and oil price are tied together in the long run, as anticipated, given the dominance of the oil sector on the Nigerian economy. Similarly, Ogiri et al. (2013) studied oil price and stock market performance in Nigeria using VECM and VAR models. Their results revealed that oil price changes are important factors in explaining stock price movement in Nigeria. Also, Akinlo (2014) examined the relationship between oil prices and the stock market in Nigeria using the Vector Error Correction Model approach. The study revealed that oil prices, the exchange rate and stock market development are cointegrated, while the price of oil has a temporary positive impact on stock market growth in Nigeria. Alley et al. (2014) further investigated the effect of oil price shocks on the Nigerian economy using annual data from 1981 to 2012 and employed the general methods of moment (GMM) in the analysis of the data. They found that the oil price shocks insignificantly retard economic growth, while the price of oil itself significantly improved economic growth. The work of Akinlo and Apanisile (2015) examined the impact of the volatility of oil prices on economic growth in 20 Sub-Saharan African countries from 1986 to 2012. The panel-pooled OLS was employed, and the results revealed that the volatility of oil has a positive and significant effect on the economic growth of the oil-exporting countries, while, for non-oil-exporting countries, the volatility of oil prices has a positive and insignificant impact on economic growth.

Abdulkareem and Abdulkareem (2016) analyzed macroeconomic variables and oil price volatility in Nigeria using the GARCH model and its variants, employing daily, monthly and quarterly data sets. The study concluded that oil price is a major source of macroeconomic volatility in Nigeria, while Odupitan (2017) concluded that, government revenues also declined and the non-oil sector contracted because of a 2014 crash in global crude oil prices. This resulted in negative effects on the Nigerian economy, including job loss, stagnated savings, and increased external debt. Odupitan suggested that, for the country to overcome all these challenges, diversification of the economy should be considered and implemented by the Nigerian government. Jarrett et al. (2017) used the ARDL model to investigate the impact of local financial development and openness measures to mitigate oil volatility from data set of 194 countries between 1980 and 2014. They concluded that financial measures do in fact mitigate the effects of oil price volatility, and the introduction of these measures can reduce or completely eliminate the negative effects of oil price volatility on growth. Okere and Ndubuisi (2017) also investigated the relationship between crude oil price and stock market development and economic growth in Nigeria between 1981 and 2014 using the ARDL model. The study concluded that the dominant role of oil price is one of the engines driving economic growth in Nigeria.

Ahmadi et al. (2018), studied the relationship between investment and uncertainty in the United States oil and gas industry using the SVAR-GARCH model. Their results

revealed that oil market uncertainty lowers investment only when it is caused by global consumption demand shocks, while market uncertainty is found to have a negative effect on investment with a one-year lag. Lastly, Okoye et al. (2018) empirically examined the interrelationship between the construction sector, oil prices and gross domestic product (GDP) in Nigeria, finding short-run linear relationships among these macro-economic variables. They argued that neither the construction sector nor oil prices directly influence the aggregate economy.

However, in Nigeria, investors often look at the performance of a company's stock in order to make investment decisions. Hence, the importance of this study is to critically investigate the performance of Total Nigeria Plc stock returns.

We also used value at risk (VaR) as an empirical basis for the study. VaR is a statistical measure of the riskiness of financial entities or portfolio of assets (Corkalo 2011). It is defined as the maximum a given amount of currency or price of stock is expected to be lost over a given time horizon, at a pre-defined confidence level (Best 1998; Bali and Cakici 2004). VaR has been described as a standard measure of market risk and is embraced by banks, trading firms, mutual funds and others, including non-financial firms, like Total Nigeria Plc (Tripathi and Aggarwal 2008). Okpara (2015) performed a risk analysis of the Nigerian stock market using the VaR approach. Based on Akaike information criterion (AIC), the study suggested that the EGARCH model with student t innovation distribution could furnish a more accurate estimate of VaR, and applying the likelihood ratio tests of proportional failure rates to VaR derived from the EGARCH model, Okpara (2015) concluded that investors and portfolio managers in the Nigerian stock market have long trading position.

Notwithstanding the assertion that VaR computation is a better measure for estimating portfolio risk than risk management (Tripathi and Aggarwal 2008), Eyisi and Oleka (2014) did not apply VaR in their study of risk management and portfolio analysis in the capital market in Nigeria but based their risk measurement on the size of the difference between the actual returns ( $R$ ) and the expected returns  $\Sigma(R)$ . In addition, Bali and Cakici (2004) stated that stock size, liquidity and VaR can explain the cross-sectional variation in expected returns than beta and total volatility and concluded that the relationship between average returns and VaR is robust for different investment horizons and loss probability levels, while VaR has additional explanatory power for stock returns.

Corkalo (2011) compared the main approaches of calculating VaR and implemented variance-covariance, historical simulation and bootstrapping approaches to stock portfolios and presented the results using a histogram. They recommended investors or risk managers to look at the composition of their portfolio and then choose an appropriate method to calculate VaR. A similar study was conducted by Van den Goorbergh and Vlaar (1999).

Literature covering the application of VaR on stocks in Nigeria, is limited, however. Global Innovation Exchange (GIE) (2019) carried out a comparative measurement of risk level using the VaR model for the Nigerian Stock Exchange (NSE) and Johannesburg South African Stock Exchange (JSE). Within the time frame of 2008–2014 daily returns, the NSE recorded its highest VaR in 2009 and JSE recorded its highest VaR in 2008. The results were consistent with expectations of normal market behavior. Oduwole (2015) explored the performance of

Nigerian mutual funds in the period from 2011 to 2014 using minimum conditional value at risk (MCVaR), which is similar to the VaR approach. The study revealed that the MCVaR approach outperformed the mutual funds and the Nigerian Stock Exchange index in the period from December 2012 to November 2014. Other market risk such as equity risk premiums estimation in Nigeria can be found in the work of Nwude (2013) and The Security and Exchange Commission (2019).

This study therefore seeks to contribute to the body of literature on the application of VaR and backtesting approach on daily oil stock returns in Nigeria with special interest in Total Nigeria Plc.

### Model specification

Financial theory states that an asset with high expected risk would, on average, pay a higher return (Xekalaki and Degiannakis 2010). This relationship between investors' expected return and risk was measured by Engle et al. (1987) using auto-regressive conditional heteroscedasticity (ARCH) and is given as

$$y_t = x_t' \beta + \phi(\sigma_t^2) + \varepsilon_t \varepsilon_t | I_{t-1} \sim f(0, \sigma_t^2), \sigma_t^2 = g(\sigma_{t-1}, \sigma_{t-2}, \dots, \varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \nu_{t-1}, \nu_{t-2}, \dots), \} \tag{1}$$

where  $x_t$  is a  $k \times 1$  vector of endogenous and exogenous explanatory variables included in the set  $I_{t-1}$ , and  $\phi(\sigma_t^2)$  represents the risk premium, which means the increase in the rate of return due to an increase in the variables of the returns.

Financial econometrics and financial time series analysis provides better understanding of how prices behave and how knowledge of price behavior can reduce risk or enhance better decision-making (Aas and Dimakos 2004). This is done using time series models for forecasting, option pricing and risk management. The remainder of this section focuses on some GARCH models and their extensions.

### Autoregressive conditional Heteroskedasticity (ARCH) family model

Every ARCH or GARCH family model requires two distinct specifications, namely: the mean and the variance equations (Atoi 2014). The mean equation for a conditional heteroskedasticity in a return series,  $y_t$ , is given as

$$y_t = E_{t-1}(y_t) + \varepsilon_t \tag{2}$$

where  $\varepsilon_t = \phi_t \sigma_t$

The mean equation in eq. (2) also applies to other ARCH family models.  $E_{t-1}(\cdot)$  is the expected value conditional on information available at time  $t - 1$ , while  $\varepsilon_t$  is the error generated from the mean equation at time  $t$ , and  $\phi_t$  is the sequence of independent and identically distributed random variables with zero mean and unit variance. The variance equation for an ARCH(p) model is given as

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \dots + \alpha_p a_{t-p}^2 \tag{3}$$

It can be seen in the equation that large values of the innovation of asset returns exert a bigger impact on the conditional variance because they are

squared, which means that a large shock tends to follow another large shock, similar to how clusters of the volatility behave. So, the ARCH(p) model becomes:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \dots + \alpha_p a_{t-p}^2 \tag{4}$$

where  $\varepsilon_t \sim N(0, 1)$  iid,  $\omega > 0$ , and  $\alpha_i \geq 0$  for  $i > 0$ . In practice,  $\varepsilon_t$  is assumed to follow the standard normal or a standard student- $t$  distribution or a generalized error distribution (Tsay 2005).

**Asymmetric power ARCH**

According to Rossi (2004), the asymmetric power ARCH model proposed by Ding et al. (1993) given below forms the basis for deriving the GARCH family models. Given that

$$\begin{aligned} r &= \mu + a_t, \\ \varepsilon_t &= \sigma_t \varepsilon_t, \\ \varepsilon_t &\sim N(0, 1) \\ \sigma_t^\delta &= \omega + \sum_{i=1}^p \alpha_i (|a_{t-i}| - \gamma_i a_{t-i})^\delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^\delta \end{aligned} \tag{5}$$

where:

$$\begin{aligned} \omega &> 0, \delta \geq 0. \\ \alpha_i &\geq 0 \quad i = 1, 2, \dots, p \\ -1 &< \gamma_i < 1 \quad i = 1, 2, \dots, p \\ \beta_j &> 0 \quad j = 1, 2, \dots, q \end{aligned}$$

This model imposes a Box-Cox transformation of the conditional standard deviation process and the asymmetric absolute residuals. The leverage effect is the asymmetric response of volatility to positive and negative “shocks.”

**Standard GARCH(p,q) model**

The mathematical model for the sGARCH(p,q) model is obtained from eq. (5) by letting  $\delta = 2$  and  $\gamma_i = 0, i = 1, 2, \dots, p$  to be:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i a_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \tag{6}$$

where  $a_t = r_t - \mu_t$  ( $r_t$  is the continuous compounding log return series), and  $\varepsilon_t \sim N(0, 1)$  iid, the parameter  $\alpha_i$  is the ARCH parameter and  $\beta_j$  is the GARCH parameter, and  $\omega > 0, \alpha_i \geq 0, \beta_j \geq 0$ , and  $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_j) < 1$ , (Rossi 2004; Tsay 2005; Jiang 2012).

The restriction on ARCH and GARCH parameters ( $\alpha_i, \beta_j$ ) suggests that the volatility ( $a_i$ ) is finite and that the conditional standard deviation ( $\sigma_i$ ) increases. It can be observed that if  $q = 0$ , then the model GARCH parameter ( $\beta_j$ ) becomes extinct and what is left is an ARCH(p) model.

To expatiate on the properties of GARCH models, the following representation is necessary,

Let  $\eta_t = a_t^2 - \sigma_t^2$  so that  $\sigma_t^2 = a_t^2 - \eta_t$ . By substituting  $\sigma_{t-i}^2 = a_{t-i}^2 - \eta_{t-i}$ , ( $i = 0, \dots, q$ ) into Eq. (4), the GARCH model can be rewritten as

$$a_t = \alpha_0 + \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_j) a_{t-i}^2 + \eta_t - \sum_{j=1}^q \beta_j \eta_{t-j} \tag{7}$$

It can be seen that  $\{\eta_t\}$  is a martingale difference series (i.e.,  $E(\eta_t) = 0$  and  $cov(\eta_t, \eta_{t-j}) = 0$ , for  $j \geq 1$ ). However,  $\{\eta_t\}$  in general is not an iid sequence.

A GARCH model can be regarded as an application of the ARMA idea to the squared series  $a_t^2$ . Using the unconditional mean of an ARMA model results in

$$E(a_t^2) = \frac{\alpha_0}{1 - \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_j)}$$

provided that the denominator of the prior fraction is positive, (Tsay 2005). When  $p = 1$  and  $q = 1$ , we obtain the GARCH(1,1) model given by

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{8}$$

**GJR-GARCH(p,q) model**

The GJosten-Jagannathan-Runkle GARCH (GJR-GARCH) model, which is a model that attempts to address volatility clustering in an innovation process, is obtained by letting  $\delta = 2$ .

When  $\delta = 2$  and  $0 \leq \gamma_i < 1$ ,

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}|^2 + \gamma_i^2 \varepsilon_{t-i}^2 - 2\gamma_i |\varepsilon_{t-i}| \varepsilon_{t-i}) + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \tag{9}$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^p \alpha_i^2 (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \\ \omega + \sum_{i=1}^p \alpha_i^2 (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \end{cases}$$

i.e.:

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{i=1}^p \alpha_i \{ (1 + \gamma_i)^2 - (1 - \gamma_i)^2 \} S_i^- \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p 4\alpha_i \gamma_i S_i^- \varepsilon_{t-i}^2$$

where:  $S_i^- = \begin{cases} 1 & \text{if } \varepsilon_{t-i} < 0 \\ 0 & \text{if } \varepsilon_{t-i} \geq 0 \end{cases}$ .

Now, we define

$$\alpha_i^* = \alpha_i (1 - \gamma_i)^2 \text{ and } \gamma_i^* = 4\alpha_i \gamma_i$$

then



$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \gamma_i^* S_i^- \varepsilon_{t-i}^2 \tag{10}$$

which is the GJR GARCH model (Rossi 2004).

However, when  $-1 \leq \gamma_i < 0$ , then recall Eq. (9)

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}|^2 + \gamma_i^2 \varepsilon_{t-i}^2 - 2\gamma_i |\varepsilon_{t-i}| \varepsilon_{t-i}) + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^p \alpha_i^2 (1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \\ \omega + \sum_{i=1}^p \alpha_i^2 (1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \end{cases}$$

$$\begin{aligned} \sigma_t^2 &= \omega + \sum_{i=1}^p \alpha_i (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \alpha_i \{ (1 + \gamma_i)^2 - (1 - \gamma_i)^2 \} S_i^+ \varepsilon_{t-i}^2 \\ &= \omega + \sum_{i=1}^p \alpha_i (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \alpha_i \{ 1 + \gamma_i^2 - 2\gamma_i - 1 - \gamma_i^2 - 2\gamma_i \} S_i^+ \varepsilon_{t-i}^2 \end{aligned}$$

where:  $S_i^+ = \begin{cases} 1 & \text{if } \varepsilon_{t-i} > 0 \\ 0 & \text{if } \varepsilon_{t-i} \leq 0 \end{cases}$ .

Also define

$$\alpha_i^* = \alpha_i (1 + \gamma_i)^2 \text{ and } \gamma_i^* = -4\alpha_i \gamma_i$$

then

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i^* \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \gamma_i^* S_i^+ \varepsilon_{t-i}^2 \tag{11}$$

which allows positive shocks to have a stronger effect on volatility than negative shocks (Rossi 2004). However, when  $p = q = 1$ , the GJR GARCH(1,1) model will be written as

$$\sigma_t^2 = \omega + \alpha \varepsilon_t^2 + \gamma S_i \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{12}$$

### IGARCH(1,1) model

Integrated GARCH (IGARCH) models are unit-root GARCH models. The IGARCH(1, 1) model is specified in Tsay (2005) as

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t; \\ \sigma_t^2 &= \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1-\beta_1) a_{t-1}^2 \end{aligned} \tag{13}$$

where

$\varepsilon_t \sim N(0, 1)$  iid, and  $0 < \beta_1 < 1$ . Ali (2013) used  $\alpha_i$  to denote  $1 - \beta_i$ . The model is also an exponential smoothing model for the  $\{a_t^2\}$  series. To see this, we rewrite the model as

$$\sigma_t^2 = (1-\beta_1)a_{t-1}^2 + \beta_1\sigma_{t-1}^2 = (1-\beta_1)a_{t-1}^2 + \beta_1[(1-\beta_1)a_{t-2}^2 + \beta_1\sigma_{t-1}^2] = (1-\beta_1)a_{t-1}^2 + (1-\beta_1)\beta_1a_{t-2}^2 + \beta_1^2\sigma_{t-2}^2 \tag{14}$$

By repeated substitution, we obtain

$$\sigma_t^2 = (1-\beta_1)(a_{t-1}^2 + \beta_1a_{t-1}^2 + \beta_1^2a_{t-3}^2 + \dots) \tag{15}$$

which is a well-known exponential smoothing formation in which  $\beta_1$  is the discounting factor (Tsay 2005).

**TGARCH(p,q) model**

The threshold GARCH model is another model used to handle leverage effects, and a TGARCH(p,q) model is given by the following:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p (\alpha_1 + \gamma_i N_{t-i}) a_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \tag{16}$$

where  $N_{t-i}$  is an indicator for negative  $a_{t-i}$ ; that is,

$$N_{t-i} = \begin{cases} 1 & \text{if } a_{t-i} < 0, \\ 0 & \text{if } a_{t-i} \geq 0, \end{cases}$$

and  $\alpha_i$ ,  $\gamma_i$  and  $\beta_j$  are nonnegative parameters satisfying conditions similar to those of GARCH models, (Tsay 2005). When  $p = 1$ ,  $q = 1$ , the TGARCH(1,1) model becomes:

$$\sigma_t^2 = \omega + (\alpha + \gamma N_{t-1}) a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{17}$$

**NGARCH(p,q) model**

The nonlinear GARCH model has been presented variously in the literature by the following scholars: Hsieh and Ritchken (2005), Lanne and Saikkonen (2005), Malecka (2014) and Kononovicius and Ruseckas (2015). The following model can be shown to represent all representations:

$$h_t = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i \varepsilon_{t-i} + \sum_{j=1}^p \beta_j h_{t-j} \tag{18}$$

where  $h_t$  is the conditional variance, and  $\omega$ ,  $\beta$  and  $\alpha$  satisfy  $\omega > 0$ ,  $\beta \geq 0$  and  $\alpha \geq 0$ .

This can also be written as

$$\sigma_t = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i \varepsilon_{t-i} + \sum_{j=1}^p \beta_j \sigma_{t-j} \tag{19}$$

**The EGARCH model**

The exponential GARCH (EGARCH) model was proposed by Nelson (1991) to overcome some weaknesses of the GARCH model in handling financial time series, as pointed out by Enocksson and Skoog (2012). In particular, to allow for asymmetric effects between positive and negative asset returns, he considered the weighted innovation:

$$g(\varepsilon_t) = \theta \varepsilon_t + \gamma [|\varepsilon_t| - E(|\varepsilon_t|)] \tag{20}$$

where  $\theta$  and  $\gamma$  are real constants. Both  $\varepsilon_t$  and  $|\varepsilon_t| - E(|\varepsilon_t|)$  are zero-mean iid sequences

with continuous distributions. Therefore,  $E[g(\varepsilon_t)] = 0$ . The asymmetry of  $g(\varepsilon_t)$  can easily be seen by rewriting it as

$$g(\varepsilon_t) = \begin{cases} (\theta + \gamma)\varepsilon_t - \gamma E(|\varepsilon_t|) & \text{if } \varepsilon_t \geq 0, \\ (\theta - \gamma)\varepsilon_t - \gamma E(|\varepsilon_t|) & \text{if } \varepsilon_t < 0. \end{cases} \tag{21}$$

An EGARCH(m,s) model, according to Tsay (2005), Dhamija and Bhalla (2010), Jiang (2012), Ali (2013) and Grek (2014), can be written as

$$a_t = \sigma_t \varepsilon_t, \\ \ln(\sigma_t^2) = \omega + \sum_{i=1}^s \alpha_i \frac{|a_{t-i}| + \theta_i a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^m \beta_j \ln(\sigma_{t-j}^2) \tag{22}$$

which specifically results in EGARCH(1,1) being written as

$$a_t = \sigma_t \varepsilon_t \\ \ln(\sigma_t^2) = \omega + \alpha(|a_{t-1}| - E(|a_{t-1}|)) + \theta a_{t-1} + \beta \ln(\sigma_{t-1}^2) \tag{23}$$

where  $|a_{t-1}| - E(|a_{t-1}|)$  are iid and mean zero. When the EGARCH model has a Gaussian distribution of error term, then  $E(|\varepsilon_t|) = \sqrt{2/\pi}$ , which gives:

$$\ln(\sigma_t^2) = \omega + \alpha\left[|a_{t-1}| - \sqrt{2/\pi}\right] + \theta a_{t-1} + \beta \ln(\sigma_{t-1}^2) \tag{24}$$

**The AVGARCH model**

An asymmetric GARCH (AGARCH), according to Ali (2013), is simply

$$a_t = \sigma_t \varepsilon_t; \\ \sigma^2 = \omega + \sum_{i=1}^p \alpha_i |\varepsilon_{t-i} - b|^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \tag{25}$$

while the absolute value GARCH (AVGARCH) model is specified as

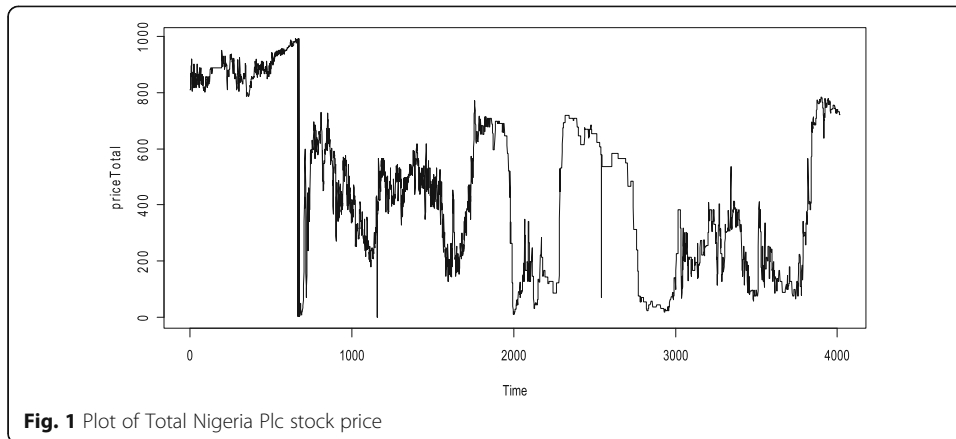
$$a_t = \sigma_t \varepsilon_t; \\ \sigma^2 = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i} + b| - c(\varepsilon_{t-i} + b))^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \tag{26}$$

**The N(a)GARCH or NAGARCH model**

The nonlinear (Asymmetric) GARCH (NAGARCH or N(A)GARCH) model plays key role in option pricing with stochastic volatility because, as we shall see later on, NAGARCH allows one to derive closed-form expressions for European option prices in spite of rich volatility dynamics. Because a NAGARCH may be written as

$$\sigma_{t+1}^2 = \omega + \alpha \sigma_t^2 (z_t - \delta)^2 + \beta \sigma_t^2 \tag{27}$$

If  $z_t \sim IIDN(0, 1)$ ,  $z_t$  is independent of  $\sigma_t^2$  as  $\sigma_t^2$  is only a function of an infinite number of past-squared returns, it is possible to easily derive the long run, unconditional variance under NGARCH and the assumption of stationarity:



$$\begin{aligned}
 E[\sigma_{t+1}^2] &= \bar{\sigma}^2 = \omega + \alpha E[\sigma_t^2(z_t - \delta)^2] + \beta E[\sigma_t^2]E[\sigma_{t+1}^2] = \bar{\sigma}^2 = \omega + \alpha E[\sigma_t^2(z_t - \delta)^2] + \beta E[\sigma_t^2] \\
 &= \omega + \alpha E[\sigma_t^2]E(z_t^2 + \delta^2 - 2\delta z_t) + \beta E[\sigma_t^2] = \omega + \alpha \bar{\sigma}^2(1 + \delta^2) + \beta \bar{\sigma}^2
 \end{aligned}
 \tag{28}$$

where  $\bar{\sigma}^2 = E[\sigma_t^2]$  and  $E[\sigma_t^2] = E[\sigma_{t+1}^2]$  because of stationarity. Therefore,

$$\bar{\sigma}^2 [1 - \alpha(1 + \delta^2) + \beta] = \omega \Rightarrow \bar{\sigma}^2 = \frac{\omega}{1 - \alpha(1 + \delta^2) + \beta}
 \tag{29}$$

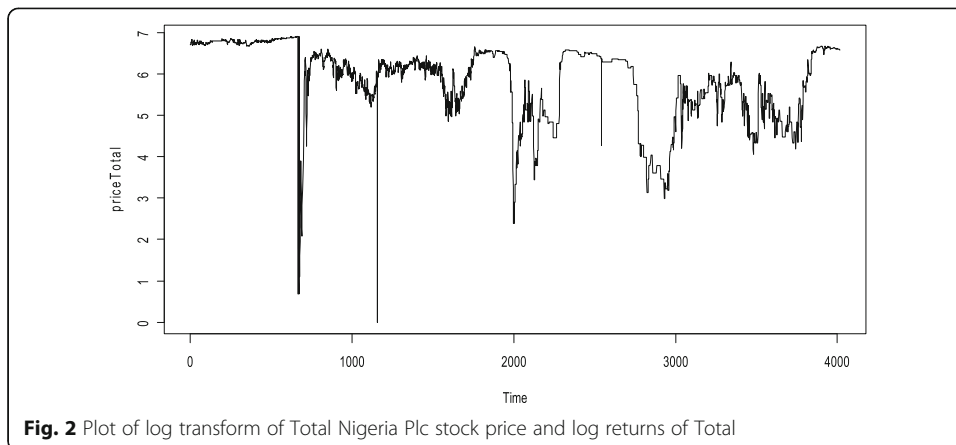
which exists and is positive if, and only if,  $\alpha(1 + \delta^2) + \beta < 1$ . This has two implications

- (i) the persistence index of a NAGARCH(1,1) is  $\alpha(1 + \delta^2) + \beta$  and not simply  $\alpha + \beta$ ; n and
- (ii) an NAGARCH(1,1) model is stationary if, and only if,  $\alpha(1 + \delta^2) + \beta < 1$ .

Further details about these implications can be found in Nelson (1991), Hall and Yao (2003), Enders (2004), Christoffersen et al. (2008) and Engle and Rangel (2008).

**Table 1** Summary statistics, daily stock price and returns of Total Nigeria Plc

Statistics	Actual Daily Stock Price	Log of Daily Stock Price	Log of returns of Daily Stock price
Min	1	0	-2.3194
Max	991	6.8987	2.3557
Median	456.5	6.1236	0
Mean	460.261	5.8258	0.0004
Estimated sd	280.7877	0.9404	0.0771
Estimated skewness	0.1352	-1.3744	0.8950
Estimated kurtosis	1.8106	5.3150	814.9014
Jarque-Bera Normality Test	X-squared: 248.9706 p Value: < 2.2e-16	X-squared: 2156.7212 p Value: < 2.2e-16	X-squared: 110001718.9071 p Value: < 2.2e-16
Number of Observations	4016	4016	4015



**Materials and methods**

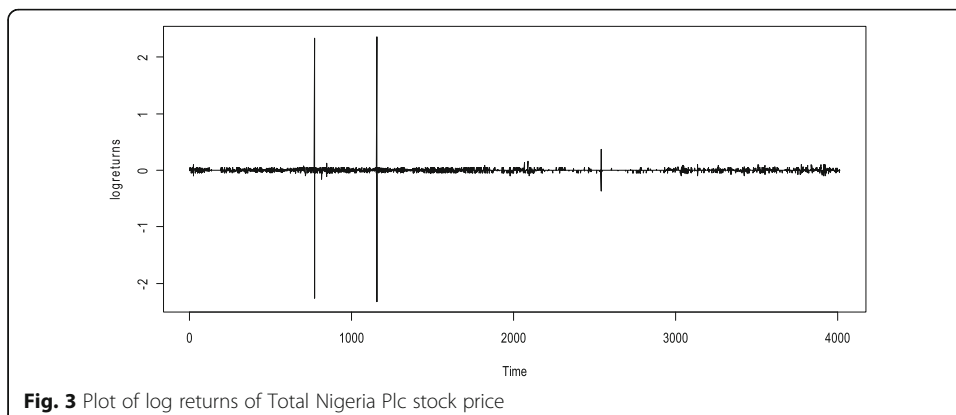
The data used in this study were collected from [www.cashcraft.com](http://www.cashcraft.com) under stock trend and analysis. Daily stock prices for Total Nigeria Plc from January 2, 2001, to May 8, 2017 (a total of 4016 observations), were collected from the website. The returns were calculated using the following formula:

$$R_t = \ln P_t - \ln P_{t-1} \tag{30}$$

where  $R_t$  is return at time  $t$ ;  $\ln$  is the natural logarithm;  $P_t$  is the current daily stock price at time  $t$ , and  $P_{t-1}$  is the previous daily stock price at time  $t - 1$ . After the time lag is accounted for, the total number of observations becomes 4015.

**Results and discussions**

The analyses of this study were carried in R environment using rugarch package by Ghalanos (2018) and the PerformanceAnalytics package by Peterson et al. (2018). The section begins with the descriptive statistics of the daily stock price of Total Nigeria Plc. Figures 1, 3 present the daily stock price of Total, its log transform and log returns of Total Nigeria Plc. Figure 3 shows some level of stability except in few cases. Table 1 shows the descriptive statistics of the daily stock price of Total, its log transform and log returns of Total Nigeria Plc: they all exhibited



**Table 2** GARCH models and their performance on the log returns of daily stock price of Total Nigeria Plc

Model	Information criteria	Normal innovation	Student t innovation
sGARCH (1,1)	Akaike	-4.7049	N.A
	Bayes	-4.7002	
	Shibata	-4.7049	
	Hannan-Quinn	-4.7032	
gjrGARCH(1,1)	Akaike	-4.7103	N.A
	Bayes	-4.7040	
	Shibata	-4.7103	
	Hannan-Quinn	-4.7081	
eGARCH (1,1)	<b>Akaike</b>	<b>-4.7221</b>	-5.6080
	<b>Bayes</b>	<b>-4.7158</b>	-5.6002
	<b>Shibata</b>	<b>-4.7221</b>	-5.6080
	<b>Hannan-Quinn</b>	<b>-4.7199</b>	-5.6052
iGARCH (1,1)	Akaike	-4.6949	-6.1100
	Bayes	-4.6918	-6.1053
	Shibata	-4.6949	-6.1100
	Hannan-Quinn	-4.6938	-6.1084
apARCH (1,1)	Akaike	-4.7111	-9.3760
	Bayes	-4.7033	-9.3666
	Shibata	-4.7111	-9.3760
	Hannan-Quinn	-4.7083	-9.3727
TGARCH (1,1)	Akaike	-2.0986	-7.6480
	Bayes	-2.0923	-7.6402
	Shibata	-2.0986	-7.6480
	Hannan-Quinn	-2.0964	-7.6452
NGARCH (1,1)	Akaike	-4.7057	<b>-22.057</b>
	Bayes	-4.6994	<b>-22.049</b>
	Shibata	-4.7057	<b>-22.057</b>
	Hannan-Quinn	-4.7034	<b>-22.054</b>
NAGARCH (1,1)	Akaike	-4.7068	-6.0847
	Bayes	-4.7006	-6.0768
	Shibata	-4.7068	-6.0847
	Hannan-Quinn	-4.7046	-6.0819
AVGARCH (1,1)	Akaike	-4.7068	-7.3255
	Bayes	-4.6990	-7.3160
	Shibata	-4.7068	-7.3255
	Hannan-Quinn	-4.7040	-7.3221

the characteristics of financial time series, and the variables were not normally distributed at a 5% level of significance (i.e., evidence of volatility) (Abdulkareem and Abdulkareem 2016).

Figure 1 shows the plot of the up and down movement in the daily stock price of Total Nigeria Plc. This gives evidence of volatility in the price of the stock which is expected because oil prices are highly volatile (Adeniyi 2011). Sudden

jumps in the price of stock are evident in Fig. 1, one may suspect structural break, but this is outside the scope of the present study.

Figure 2 shows the plot of the log transform of the daily stock price of Total Nigeria Plc. The series show ups and downs movement in the stock price, giving some evidence of volatility in the price of the stock. This is expected because oil prices are highly volatile (Adeniyi 2011). The essence of log transformation is to reduce or stabilize variability in the series.

Figure 3 shows the plot of the log returns of the daily stock price of Total Nigeria Plc. The plot of the returns shows some high and low spikes. In the case one may suspect outliers in the series, the data needed to be cleansed for possible outliers.

The log returns of the daily stock price of Total Nigeria Plc were modeled with nine different GARCH models (sGARCH, gjrGARCH, eGARCH, iGARCH, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH), and are given in Table 2. We used GARCH(1,1) because many studies have found its usefulness and performance when compared to higher order GARCH models (Bollerslev 1986; Gonzalez-Rivera et al. 2004; Panait and Slavescu 2012). Using the Akaike information criterion (AIC), the eGARCH outperformed the other models for normal innovation, while, for student t innovation, the NGARCH model outperformed the other models. The performance of NGARCH was found to be in line with the work of Emenogu and Adenomom (2018). Table 3 shows the persistence and half-life volatility of the models. The persistence values of the models reveal the stability of the model, except for iGARCH, which has a value of 1. This means that the volatility of the Total Nigeria Plc daily stock price and returns can be modeled and forecasted. For normal innovation, the half-life of the models sGARCH, gjrGARCH, eGARCH, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH for mean reverting takes about 7 days, 8 days, 4 days, 7 days, 90 days, 6 days, 7 days and 5 days, respectively. This should boost the confidence of the stockholders of Total Nigeria Plc and indicate that any drop in the price of the stock can be regained in the future. For student t innovation, sGARCH and gjrGARCH values for persistence and half-life volatility were not available, while iGARCH persistence value was equal to 1. The eGARCH, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH models were stable with a half-life of about 1031 days, 19 days, 2 days, 38 days, 11 days and 3 days, respectively.

**Table 3** Persistence and half-life volatility of log returns of daily stock price of Total Nigeria Plc

	Normal innovation		Student t innovation	
	Persistence	Half-life volatility	Persistence	Half-life volatility
sGARCH (1,1)	0.8985	6.4765	NA	NA
<b>gjrGARCH(1,1)</b>	0.9068	7.0831	NA	NA
<b>eGARCH(1,1)</b>	0.8242	3.5859	0.9993	1030.535
<b>iGARCH(1,1)</b>	1	Infinity	1	Infinity
<b>aPARCH(1,1)</b>	0.8960	6.3101	0.9638	18.7753
<b>TGARCH(1,1)</b>	0.9923	89.4049	0.6300	1.5000
<b>NGARCH(1,1)</b>	0.8853	5.6903	0.9816	37.2784
<b>NAGARCH(1,1)</b>	0.9009	6.6394	0.9364	10.5541
<b>AVGARCH(1,1)</b>	0.8546	4.4127	0.7293	2.1961

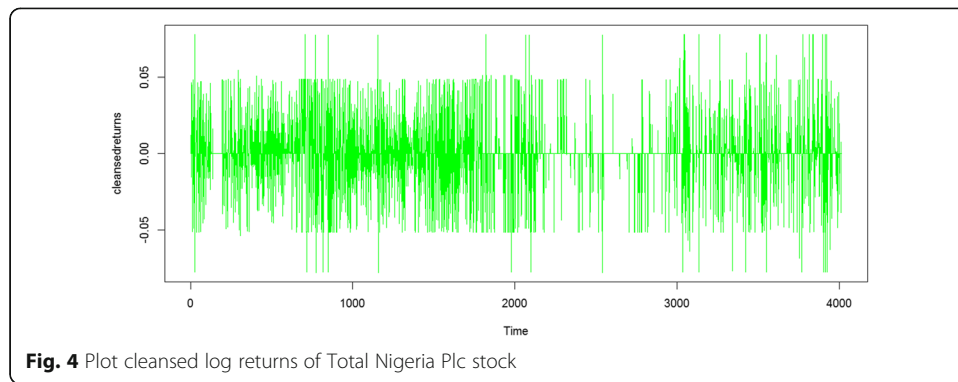


Figure 4 presents the cleansed log returns of Total Nigeria Plc stock price. This is done to reduce the effects of outliers (Peterson et al. 2018). Figure 4 shows the plot of the cleansed log transform of the returns of the daily stock price of Total Nigeria Plc. The series was cleansed using the methods Boudt and Goltner using the PerformanceAnalytics package by Peterson et al. (2018). These methods provide multiple accesses for cleaning outliers from return data there creating more robust and stable series. The cleaning for possible outliers is necessary to avoid the problems outliers can pose to estimation.

The descriptive statistics of the cleansed returns of Total Nigeria Plc in Table 4 also exhibits the characteristics of financial time series. In Table 4, the effect of cleansing series for possible outliers is evident as the standard deviation of the series reduced from 0.0771 in Tables 1 to 0.0224 in Table 4.

The cleansed log returns of the daily stock price of Total Nigeria Plc were modeled with nine different GARCH models (sGARCH, gjrGARCH, eGARCH, iGARCH, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH) in Table 5. We used GARCH(1,1) because many studies have found its usefulness and performance (Bollerslev 1986; Gonzalez-Rivera et al. 2004; Panait and Slavescu 2012). Using AIC, the sGARCH outperformed other models for normal innovation while for student t innovation; the NGARCH model outperformed other models. The performance of NGARCH is in line with the work of Emenogu and Adenomon (2018). Table 6 shows the persistence and half-life volatility of the models. The persistence values of the models reveal the stability of the model, except for iGARCH, which has a value of 1. This means that the

**Table 4** Descriptive statistics of the cleansed returns of daily stock price of Total Nigeria Plc

Statistics	Cleansed Returns of Total Nigeria Plc
Min	-0.0778
Max	0.0780
Median	0
Mean	0.0003
Estimated sd	0.0224
Estimated skewness	-0.0023
Estimated kurtosis	
Jarque-Bera Normality Test	X-squared: 632.2164 Asymptotic p Value: < 2.2e-16
Number of Observations	4015



**Table 5** GARCH models and their performance on the cleansed log returns of daily stock price of Total Nigeria Plc

Model	Information criteria	Normal innovation	Student t innovation
sGARCH (1,1)	Akaike	<b>-4.9438</b>	NA
	Bayes	<b>-4.9391</b>	
	Shibata	<b>-4.9438</b>	
	Hannan-Quinn	<b>-4.9421</b>	
gjrGARCH(1,1)	Akaike	-4.9434	NA
	Bayes	-4.9371	
	Shibata	-4.9434	
	Hannan-Quinn	-4.9411	
eGARCH(1,1)	Akaike	-4.9401	-5.8066
	Bayes	-4.9338	-5.7988
	Shibata	-4.9401	-5.8066
	Hannan-Quinn	-4.9379	-5.8039
iGARCH(1,1)	Akaike	-4.9363	-6.3708
	Bayes	-4.9331	-6.3661
	Shibata	-4.9363	-6.3708
	Hannan-Quinn	-4.9352	-6.3691
apARCH(1,1)	Akaike	-4.9429	-12.693
	Bayes	-4.9350	-12.684
	Shibata	-4.9429	-12.693
	Hannan-Quinn	-4.9401	-12.690
TGARCH(1,1)	Akaike	-2.8546	-7.5955
	Bayes	-2.8483	-7.5876
	Shibata	-2.8546	-7.5955
	Hannan-Quinn	-2.8523	-7.5927
NGARCH (1,1)	Akaike	-4.9433	<b>-21.080</b>
	Bayes	-4.9370	<b>-21.072</b>
	Shibata	-4.9433	<b>-21.080</b>
	Hannan-Quinn	-4.9411	<b>-21.077</b>
NAGARCH (1,1)	Akaike	-4.9433	-6.3209
	Bayes	-4.9371	-6.3131
	Shibata	-4.9433	-6.3209
	Hannan-Quinn	-4.9411	-6.3181
AVGARCH(1,1)	Akaike	-4.9363	-8.0452
	Bayes	-4.9284	-8.0358
	Shibata	-4.9363	-8.0452
	Hannan-Quinn	-4.9335	-8.0419

volatility of the cleansed Total Nigeria Plc daily stock price can be modeled and forecasted. For normal innovation, the half-life of the models sGARCH, gjrGARCH, eGARCH, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH for mean reverting takes about 33 days, 33 days, 9 days, 33 days, 172 days, 33 days, 33 days and 18 days, respectively. This should boost the confidence of the stockholders of Total Nigeria Plc that any drop in stock price can

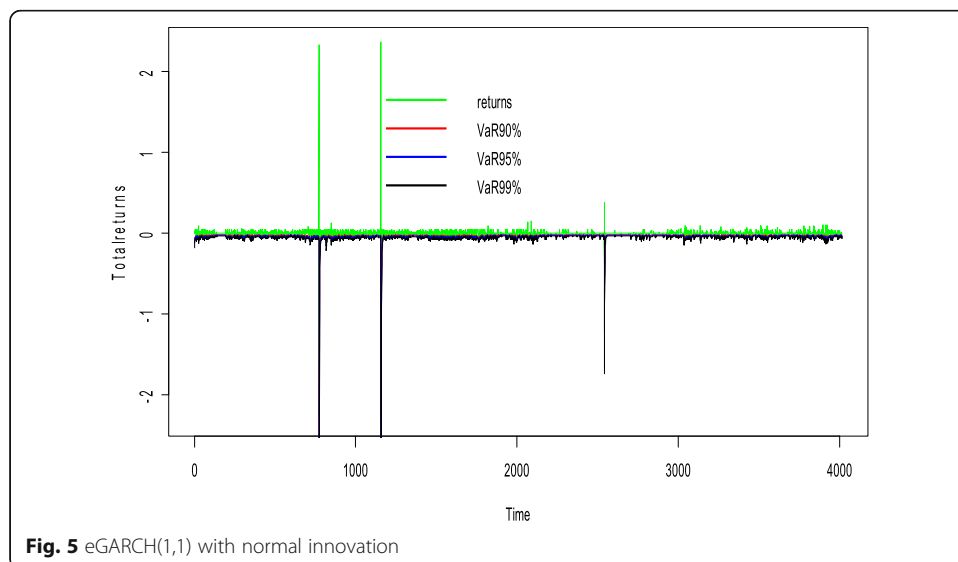
**Table 6** Persistence and half-life volatility of the cleansed log returns of daily stock price of Total Nigeria Plc

	Normal innovation		Student t innovation	
	Persistence	Half-life volatility	Persistence	Half-life volatility
sGARCH (1,1)	0.9791	32.8884	NA	NA
gjrGARCH(1,1)	0.9791	32.7524	NA	NA
eGARCH(1,1)	0.9187	8.1793	1	Infinity
iGARCH(1,1)	1	Infinity	1	Infinity
apARCH(1,1)	0.9791	32.8002	0.9667	20.4769
TGARCH(1,1)	0.9960	171.5258	0.6387	1.5460
NGARCH(1,1)	0.9791	32.8943	0.9851	46.0923
NAGARCH(1,1)	0.9790	32.6689	0.8924	6.0899
AVGARCH(1,1)	0.9612	17.5026	0.5455	1.1437

be regained in the future. For student t innovation, sGARCH and gjrGARCH values for persistence and half-life volatility are not available, while eGARCH and iGARCH persistence values are equal to 1. However, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH models are stable with a half-life of about 21 days, 2 days, 47 days, 7 days and 2 days, respectively. The performance of NGARCH model for student t innovation for cleansed daily stock returns is similar to that of log returns of Total Nigeria Plc.

**Value-at-risk (VaR) analysis of Total Nigeria Plc daily stock returns**

VaR analysis is important for two reasons: first, it provides a common consistent measure of risk for stock returns, and second, it takes into account the correlation between risk factors (Nieppola 2009). In this study, the in-sample VaR was calculated using normal distribution. The choice of in-sample is for the purpose of large data set, while for out-of-sample, VaR calculation will be done in the next two to 3 years to be able to obtain a data set with a time series length of approximately 1000.



**Fig. 5** eGARCH(1,1) with normal innovation

**Table 7** VaR violation of the actual Total Nigeria Plc returns

Model	VaR alpha	No. of Violation	Ratio	Percentage
eGARCH(1,1) with normal	1%	86	86/4015	2.1%
	5%	216	216/4015	5.4%
	10%	340	340/4015	8.5%
NGARCH(1,1) with std	1%	85	85/4015	2.1%
	5%	206	206/4015	5.1%
	10%	320	320/4015	8.0%

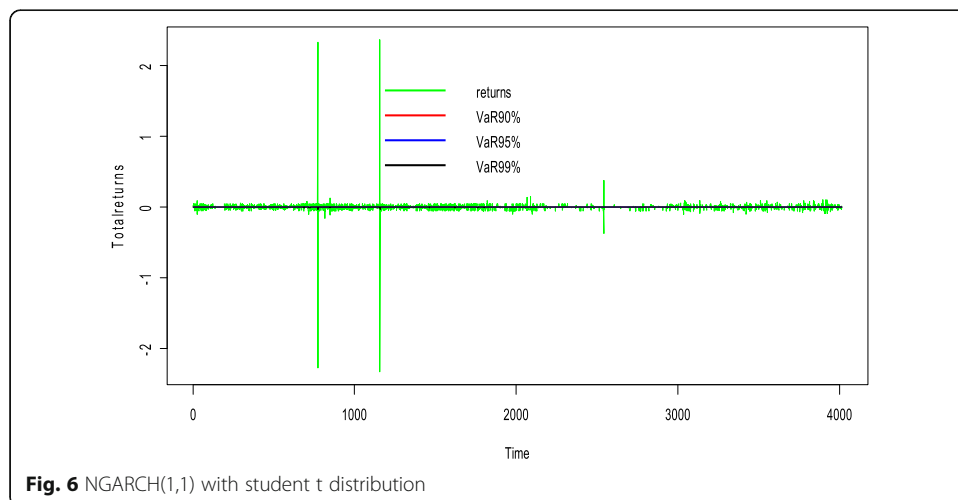
In Fig. 5, the VaR values are less than the returns for eGARCH model. In addition, the percentage of VaR violation (see Table 7) at 1% is 2.1%, providing a rejection of the model, while at 5% the model is slightly rejected, but at 10% the model is accepted, having a VaR violation of 8.5%.

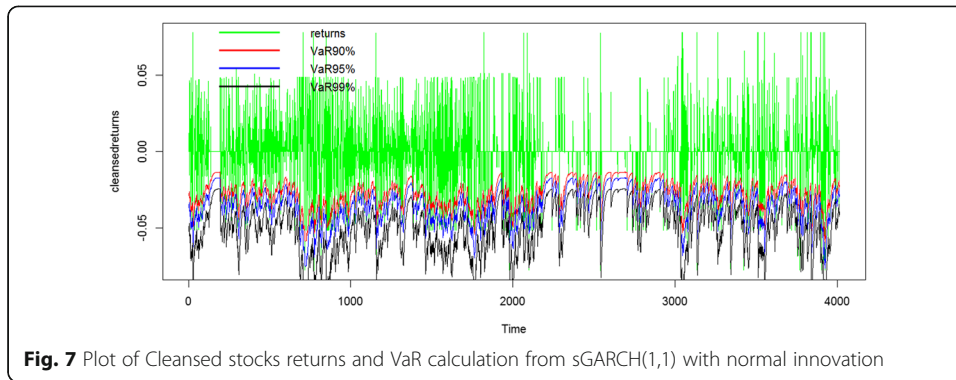
In Fig. 6, the VaR values are less than the returns for the NGARCH model. In addition, the percentage of VaR violation (see Table 7) at 1% is 2.1%, providing for a rejection of the model, while at 5% the model is slightly rejected, but at 10% the model is accepted, having a VaR violation of 8.0%.

The above results from VaR analysis for the log returns of Total Nigeria Plc means that VaR calculation for the eGARCH model with normal innovation and the NGARCH model with student t innovation is rejected at the 99% confidence level and slightly rejected at 95% confidence and accepted at 90% confidence. This means the risk in Total Nigeria Plc stock is high at the 99% confidence level, suggesting that high risk brings high return.

In Fig. 7, the VaR values are less than the returns for the sGARCH model. In addition, the percentage of VaR violation (see Table 8) at 1% is 2.2%, providing for a rejection of the model, while at 5% the model is slightly rejected and at 10% the model is accepted, having a VaR violation of 7.9%. This means that the risk in Total Nigeria Plc stock is high at a 99% confidence level, suggesting that high risk brings high return.

In Fig. 8, the VaR values are less than the returns for the NGARCH model. In addition, the percentage of VaR violation (see Table 8) revealed strange





results, as the model is rejected at all confidence levels, and the percentages of violation are very high.

**Backtesting VaR model**

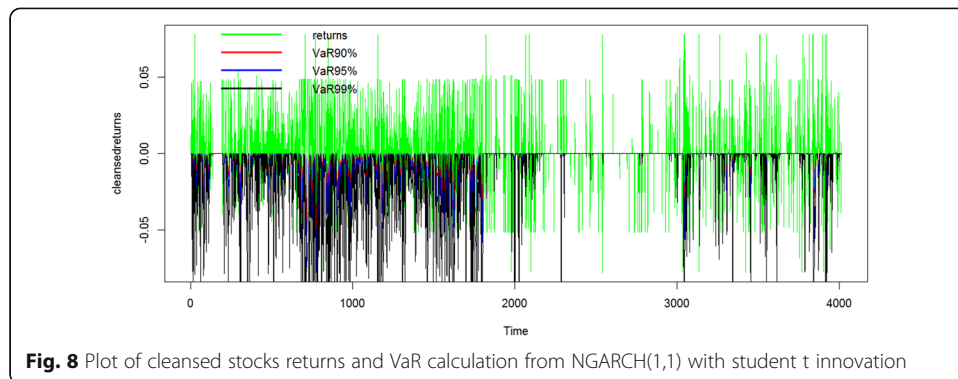
The evaluation of financial risk models or backtesting - is an important part of the internal model’s approach to market risk management, as put out by Basle Committee on Banking Supervision (Christoffersen and Pelletier 2004). Backtesting is a statistical procedure where actual profits and losses are systematically compared to corresponding VaR estimates (Nieppola 2009). This study adopted the backtesting techniques of Christoffersen and Pelletier (2004); the VARTest in rugarch package in R that implements both the unconditional (Kupiec) and conditional (Christoffersen) coverage tests for the correct number of exceedances (see details in Christoffersen 1998; Christoffersen et al. 2001).

First, we conducted duration-based tests of independence. Under the null hypothesis that the risk model is correctly specified, the no-hit duration should have no memory of  $1/p$  days. This test is suitable for a series with a length of at least 1000.

The duration-based tests of independence conducted (in Tables 9 and 10) reveal that the models are correctly specified since, in all cases, the null hypotheses were accepted. This means that the probability of an exception on any day did not depend on the outcome of the previous day. This will go a long way to boost the confidence of shareholders of Total Nigeria Plc stock in Nigeria. A more detailed backtesting technique is the conditional and unconditional coverage rate, presented in Tables 11 and 12.

**Table 8** VaR violation of cleansed Total Nigeria Plc returns

Model	VaR alpha	No. of Violation	Ratio	Percentage
sGARCH(1,1) with normal	1%	88	88/4015	2.2%
	5%	203	203/4015	5.1%
	10%	319	319/4015	7.9%
NGARCH(1,1) with std	1%	960	960/4015	23.9%
	5%	971	971/4015	24.2%
	10%	974	974/4015	24.3%



The VARTest in rugarch package in R implements both the unconditional (Kupiec) and conditional (Christoffersen) coverage tests for the correct number of exceedances for both Total Nigeria Plc stock returns and cleansed total returns (results presented in Tables 11 and 12). The results reject the models at 1% level of significance, which is similar to results we obtained in the percentages of violation rates presented in Tables 7 and 8. This shows that unconditional (Kupiec) and conditional (Christoffersen) coverage tests for the correct number of exceedances are reliable.

**Conclusion and recommendations**

This study investigates the volatility of the stock price of Total Nigeria Plc using nine variants of GARCH models, namely sGARCH, gjrGARCH, eGARCH, iGARCH, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH. We also investigated the VaR and backtesting of the models. This study therefore seeks to contribute to the body of the literature on the application of VaR and backtesting on oil stock in Nigeria, with a special interest in Total Nigeria Plc. This investigation of the volatility, VaR, and backtesting of the daily stock price of Total Nigeria Plc is important as most previous studies covering the Nigerian stock market have not paid much attention to the application of backtesting as a primary approach. We obtained and analyzed the daily stock prices for Total Nigeria Plc from secondary sources. The study used both normal and student t innovations with AIC to select the best model. For normal innovations, for log returns and cleansed log returns of Total Nigeria Plc, the eGARCH and sGARCH models performed the best, while the NGARCH model performed best for student t innovation for both log returns

**Table 9** Implements the VaR Duration Test of Christoffersen and Pelletier on Total Nigeria Plc Returns

$H_0$ : "Duration Between Exceedances have no memory (Weibull  $b = 1 =$  Exponential)"

Model	VaR alpha	B	uLL	rLL	LRp	Decision
eGARCH(1,1) with normal	1%	0.9113	-411.8762	-412.687	0.2029	Accept
	5%	1.0604	-843.6371	-844.3382	0.2363	Accept
	10%	1.0353	-1176.545	-1176.938	0.3758	Accept
NGARCH(1,1) With std	1%	0.8837	-407.4371	-408.826	0.0956	Accept
	5%	1.0022	-814.8295	-814.8304	0.9660	Accept
	10%	0.9996	-1126.9	-1126.9	0.9926	Accept

Note: b: the estimated Weibull parameter that, when restricted to the value of 1, results in exponential distribution; uLL: the unrestricted log-likelihood value; rLL: the restricted log-likelihood value; LRp: the likelihood-ratio test statistic

**Table 10** Implements the VaR Duration Test of Christoffersen and Pelletier on Cleansed Total Nigeria Plc returns

$H_0$ : "Duration Between Exceedances have no memory (Weibull  $b = 1 =$  Exponential)"

Model	VaR alpha	b	uLL	rLL	LRp	Decision
sGARCH(1,1) with normal	1%	0.9277	-419.8567	-420.374	0.3091	Accept
	5%	1.0387	-805.6184	-805.884	0.4661	Accept
	10%	1.0063	-1124.353	-1124.366	0.8754	Accept
NGARCH(1,1) With std	1%	0.9974	-2332.186	-2332.194	0.9030	Accept
	5%	0.9941	-2347.843	-2347.882	0.7806	Accept
	10%	0.9936	-2352.093	-2352.139	0.7620	Accept

Note: b: the estimated Weibull parameter, which when restricted to the value of 1, results in exponential distribution; uLL: the unrestricted log-likelihood value; rLL: the restricted log-likelihood value; LRp: the likelihood-ratio test statistic

and cleansed returns of Total Nigeria Plc. The persistence of the models was stable except in few cases where iGARCH and eGARCH were unstable. Additionally, for student t innovation, the sGARCH and gjrGARCH models failed to converge. The mean-reverting number of day for the returns of Total Nigeria Plc differed from model to model. The performance of NGARCH was in line with the work of Emenogu and Adenomon (2018). Evidence from the VaR analysis of the selected models revealed that the risk of VaR losses was high at a 99% confidence level, slightly high at a 95% confidence level and better at a 90% confidence level. Although duration-based tests of independence conducted revealed that the models were correctly specified, in all cases, the null hypotheses were accepted. This indicates that the probability of an exception on any day did not depend on the outcome of the previous day. Finally, both the unconditional (Kupiec) and conditional (Christoffersen) coverage tests for the correct number of exceedances for both Total Nigeria Plc stock returns and cleansed Total Nigeria Plc returns revealed arejection of the models at a 1% level of significance, which is similar to results obtained for the percentages of violation rates. This confirms that unconditional (Kupiec) and conditional (Christoffersen) coverage tests for the correct number of exceedances are reliable compared to the duration-based tests of independence (Nieppola 2009). This study recommends shareholders and investors to continue their business with Total Nigeria Plc because losses may be recouped in the future, based on a long-term view of the price of the stock. Furthermore, risk was found to be high at a 99% confidence level, suggesting that high risk brings high return. This is in line with financial theory, which states that an asset with high expected risk would, on average, pay higher return (Xekalaki and Degiannakis 2010).

**Future study**

We studied Total Nigeria Plc because of its potential in the Nigeria Stock Exchange. In the future, we will examine the stock price with GARCH-M models and other more advanced GARCH models, out-of-sample VaR and Backtesting. We also suggest the need to investigate Total Nigeria Plc stocks in relation to the interest rate, inflation rate, exchange rate and crude oil price in the global market during the global financial crisis of 2007 to 2008 using multivariate GARCH (MGARCH) models.

**Table 11** VaRTest for Total Nigeria Plc daily stock returns

Model	H <sub>0</sub> : Correct Exceedances				H <sub>0</sub> : Correct Exceedances & Independent			
	Alpha	expected Exceed	actual Exceed	Decision	cc.LRstat	cc.critical	cc.LRp	decision
eGARCH norm	1%	40	86	Reject	43.61369	9.21034	3.383828e-10	Reject
	5%	200	216	NA	NaN	5.991465	NaN	NA
	10%	401	340	NA	NaN	4.60517	NaN	NA
NGARCH with std	1%	40	85	Reject	41.99094	9.21034	7.616976e-10	Reject
	5%	200	206	NA	NaN	5.991465	NaN	NA
	10%	401	320	NA	NaN	4.60517	NaN	NA

Note: uc.LRstat: the unconditional coverage test likelihood-ratio statistic; uc.critical: the unconditional coverage test critical value; uc.LRp: the unconditional coverage test p-value; cc.LRstat: the conditional coverage test likelihood-ratio statistic; cc.critical: the conditional coverage test critical value; cc.LRp: the conditional coverage test p-value; NA not available

**Table 12** VaRTest for cleansed Total stock returns

Model	H <sub>0</sub> : Correct Exceedances*				H <sub>0</sub> : Correct Exceedances & Independent				decision		
	Alpha	expected.exceed	actual.Exceed	uc.LRstat	uc.critical	uc.LRp	Decision	cc.LRstat		cc.critical	cc.LRp
sGARCH norm	1%	40	88	42.98808	6.634897	5.507428e-11	Reject	46.9334	9.21034	6.434908e-11	Reject
	5%	200	203	NaN	3.841459	NaN	NA	NaN	5.991465	NaN	NA
	10%	401	319	NaN	2.705543	NaN	NA	NaN	4.60517	NaN	NA
NGARCH with std	1%	40	960	NaN	6.634897	NaN	NA	NaN	9.21034	NaN	NA
	5%	200	971	NaN	3.841459	NaN	NA	NaN	5.991465	NaN	NA
	10%	401	974	NaN	2.705543	NaN	NA	NaN	4.60517	NaN	NA

Note: uc.LRstat: the unconditional coverage test likelihood-ratio statistic; uc.critical: the unconditional coverage test critical value; uc.LRp: the unconditional coverage test p-value; cc.LRstat: the conditional coverage test likelihood-ratio statistic; cc.critical: the conditional coverage test critical value; cc.LRp: the conditional coverage test p-value; NA not available



### Abbreviations

apARCH: Asymmetric Power Autoregressive Conditional Heteroskedasticity; ARCH: Autoregressive Conditional Heteroskedasticity; AVGARCH: Absolute Value Generalized Autoregressive Conditional Heteroskedasticity; EGARCH: Exponential Generalized Autoregressive Conditional Heteroskedasticity; GARCH: Generalized Autoregressive Conditional Heteroskedasticity; GJR-GARCH: Glosten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroskedasticity; IGARCH: Integrated Generalized Autoregressive Conditional Heteroskedasticity; NAGARCH: Nonlinear Asymmetric Generalized Autoregressive Conditional Heteroskedasticity; NGARCH: Nonlinear Generalized Autoregressive Conditional Heteroskedasticity; sGARCH: Standard Generalized Autoregressive Conditional Heteroskedasticity; SVAR: Structural Vector Autoregression; TGARCH: Threshold Generalized Autoregressive Conditional Heteroskedasticity; VaR: Value-at-Risk; VAR: Vector Autoregression; VECM: Vector Error Correction Model

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### Authors' contributions

Ngozi G. Emenogu: Initiated the subject, contributed to the methodologies and review of literature, Write and proof read the drafts. Monday Osagie Adenomom: co-Initiated the subject, Analyzed the data in R and interpretation and discussion of results. Nwaze Obini Nweze: Editing of the manuscripts and contributed to the discussion of the results. The author(s) read and approved the final manuscript.

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### Competing interests

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### Author details

<sup>1</sup>Department of Statistics, Federal Polytechnic, Bida, Niger State, Nigeria. <sup>2</sup>Department of Statistics, Nasarawa State University, Keffi, Nasarawa State, Nigeria.

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