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Some Second Thoughts on Wagner’s Law

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and
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Some Second Thoughts on Wagner’s Law

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Abstract

We examine whether the Samuelsonian definition of public goods can be reconciled with "Wagner’s Law", that is, public expenditures outpacing economic growth. While both predominantly focus on the demand-side, they differ with respect to their socio-political foundations. Taking the latter into account, and acknowledging that empirical studies are not generally supportive of individual income elasticities systematically differing between public and private goods, we find that Wagner’s notion of the role of public-sector issues is even at odds with his own dictum. Implicit in Samuelson, by contrast, is the prediction that public spending decreases relative to GNP when income grows, provided that the income distribution remains constant. If this is not the case it can be shown that a growing inequality increases government's share et vice versa which can lead to counteractive forces on the GNP ratio.

JEL-Classification: D11, H11, H41, H50

Keywords: Wagner’s Law, public expenditures, public goods, budget-to-GNP ratio

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1. Introduction

In his book on "Allgemeine und theoretische Volkswirtschaftslehre" (1876), Adolph Wagner first acquainted the German readership with his "law of increasing state activity", saying that (as an empirical regularity) government expenditures tend to grow faster than the economy. This notion of the state sector outgrowing the economy has ever since been referred to as "Wagner's Law". Peacock and Scott (2000) even write of a “curious attraction” of Wagner’s law, since numerous empirical studies and international comparisons examine whether budget to GDP ratios do indeed reflect Wagner's famous law (see Peacock/Scott 2000 for a critique of the most prominent studies). However, its theoretical and politico-economic foundations have less frequently been subjected to investigation. Nor are there studies available which address the question whether Wagner's notion of government’s expanding relative to GNP is compatible with Paul Samuelson's (1954; 1955) economic theory of public goods and the role of government in a modern economic perspective. The paper tries to fill this gap by going back to the roots.

Wagner – with some reservations – as well as Samuelson focuses on the demand rather than the supply of public goods. Therefore, we exclusively consider demand-side issues

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1 Actually, Wagner mentioned the relative growth of government for the first time in a very obscure Austrian source of 1863, and restated it more precisely in several publications thereafter, including his 1893 book which we prefer to quote from here. The 1911 publication where the Peacock/Scott (2000) article is based on is a shorter and partly asymmetric reformulation of the relevant parts of his 1893 book.

2 Empirical studies on Wagner's law, earlier most often of the cross-sectional determinant type, later as time-series-analyses had their boom in the 1950s and 1960s. Brown/Jackson (1982: 120) count almost 1,000 studies of this sort. Later on, the interest shifted towards the impact of the expansion of government on economic growth (e.g. Scully 1989; 1995; Barro 1991; Tanzi/Schuknecht 2000) or on the unemployment rate (Abrams 1999). Generally, Peacock and Scott (2000) criticize the empirical work absolutely correctly that all studies reviewed misspecify the "budget" - in Wagner’s (1893:905) thinking - by the omission of public utilities, one of the major growing parts of the economy due to take-overs from the private sector and a major force leading to the relative expansion of the state activity.

3 It would be better to say that Wagner is predominantly interpreted that way. Indeed, there are some hints in this direction when he says that "the expansion of state activity is connected to the need for higher, better and more perfect goods and services" (1893:904) or when he implicitly recurs to the household optimum delivering an optimal budget of private and public goods (1893:894). But it is necessary to remark that Wagner’s thinking is evolutionary in its core when he says that "this development (the relative expansion of the state, D/Z) explains itself and is justified because of the idea of the developed state" (1893:896) or "the circumstances of living on higher levels of culture" (1893:902). For Wagner, the decisive momentum behind that evolutionary process is the "change and progress in the production technique" (1893:902), and cultural and technical forces combined are seen as "mighty evolutionary phenomena against which the preference and will of the individual is a factor of
while supposing that there are no supply-side influences on the sectoral composition (private vs. public goods) of the economy. There are certainly many alternative explanations out there in the literature which compete with the demand-hypothesis when it comes to the relative growth of government. One popular example is Baumol's disease, that is, the proposition that the relative price of government services is generally higher and increasing; another explanation refers to population growth as it may affect public expenditures, and may imply a rise in budget to GNP ratios if the participation rate decreases and/or public expenditures on schooling and educational matters increase, as may other shifts in the age structure of the economy; the political process of preference aggregation may give rise to log-rolling with inefficient outcomes (like an oversized public sector), as may fiscal illusion, group interests and lobbying, and the incentives inherent in bureaucracy (see Blankart (1993), Miles/Myles/Preston (2003) or Borcherding/Lee (2004) for surveys).

Though those approaches to the allocation of resources between the private and the public sector are interesting in and of themselves, in this paper we will ignore those reasons which may also contribute towards the growth of government. Instead, we will focus exclusively on the basic argument put forward in favor of Wagner's law, which originates from the demand for public goods, but which has to be interpreted differently in the worlds of Wagner or Samuelson.

However, even with reference to forces emanating from the demand-side, there are two competing explanations outstanding in the literature. Yet, none of them is really convincing: time and again, Wagner's law has been traced back to (exogenous) shifts in demand in favor of public goods due to individual income elasticities being larger than unity. However, those attempts to reconcile empirics with theory are fragile, for two reasons. First, rather than arriving at an individual income elasticity which is larger than one, the majority of empirical
studies conclude that individual income elasticities with respect to public goods are hovering around unity (see, for instance, Borcherding/Deacon (1972); Bergstrom/Goodman (1973); Pommerehne (1978), or, the overview of that older literature by Blankart (2003)). The studies analyzed by Blankart, however, are mainly cross section studies or studies based on very short time series. Auteri/Constantini (2004) show in their recent survey of the literature that typically cross section studies of the income elasticity of public goods lead to values around unity whereas time series analyses often reveal values above or below unity. Summarizing their findings it can be said that according to the object of inquiry (the specific public good), the locality of inquiry (region/land) and the method of inquiry (cross section or time series studies) elasticity values differ a lot - above and below unity. So, since there are no income elasticity estimations of the overall budget, and the existing estimates for single public goods are so diverse, we stick to Blankart’s statement of an overall elasticity around unity (which is also Peltzman's (1980) result), hence there seems to be no convincing empirical reason to assume that the overall elasticity values of public goods give rise to government expenditures systematically outpacing GNP.

Second, due to their ad-hoc character, "explanations" that hinge on preferences changing exogenously are generally vulnerable to criticism. Following Becker/Stigler (1977: 76) in that "... tastes neither change capriciously nor differ importantly between people", we thus dismiss the issue of preferences changing exogenously (in favor of public goods) as a driving force of an increase in public-goods related government expenditures relative to GNP. Rather, in this paper, we will track down the consequences for budget-to-GNP ratios by focusing on the (differing) socio-political assumptions implicitly underlying the perspectives of Wagner.

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4 Another interesting survey concentrating on time series analyses is presented by Borcherding/Ferris/Garzoni (2004).
5 Their own time series analysis of 18 selected OECD countries for different elements of social security expenditures between 1981 and 1999 show that dependent on the techniques of estimation income elasticities above or below unity can be calculated.
6 In very much the same manner as "explanations" that take recourse to changes in preferences, Becker/Stigler (1977: 89) consider arguments relying on differences in preferences "... a convenient crutch to lean on when the analysis has bogged down" and "... ad hoc arguments that disguise analytical failures".
and Samuelson on the aggregate demand for publicly provided goods. Hence, instead of preferences changing exogenously, in our analysis, the evolution of budget-to-GNP ratios in economic growth are exclusively attributed to changes in prices and incomes.

As we will show in this paper, Wagner's own approach implies that the relative size of government is in fact independent of the size of the economy while Samuelson's approach entails budget-to-GNP ratios that vary with GNP, even with preferences remaining stable and uniform. However, rather than increasing, the relative importance of the public sector declines with growing income when we adopt a Samuelsonian perspective, provided that the income distribution remains stable. Furthermore, in the Samuelsonian perspective, budget-to-GNP ratios increase when the income distribution becomes more skewed (with GNP unchanged), and even more so if some individuals lose their jobs or earnings capacity with incomes of the other individuals remaining the same. Although decreasing with preferences and income distributions being constant, there is a range of GNPs for which budget-to-GNP ratios under Samuelson are larger than those based on Wagner, whereas for all other values the opposite holds true. The latter result, though, is in line with socio-economic research and behavioral science in the tradition of Maslow (1987).

The paper is organized as follows: In Section 2 we will construct the supply side of a model economy in which the sectoral composition is solely determined by demand; Section 3 illuminates how demand affects budget-to-GNP ratios with uniform preferences (and with a particular focus on implicit assumptions concerning preference aggregation). Section 4 is devoted to a comparison of the concepts of Wagner and Samuelson and their implications for the demand for publicly provided goods while Section 5 examines the impact of income distribution on budget-to-GNP ratios. Section 6 concludes.
2. Supply

Consider a model economy populated by two individuals who are going to decide on the budget-to-GNP ratio. Ignoring the difficulties posed by small numbers, the assumption concerning the population serves to facilitate the analysis, as the major differences between the approach by Wagner and by Samuelson already arise with the minimum number of two people in society. Each individual supplies inelastically some units of (homogenous) labor to the market, with total labor supply in the model economy $L$.

For the moment (that is Sections 2 to 4), we will assume that all members of society supply the same amount and quality of labor so that they also receive the same income. Since this assumption obviously need not hold true with respect to real world matters, we will come back to this when we discuss distributional issues and the implications of the (redistributive) welfare state in particular (see Section 5 of this paper).

Leaving distributional issues aside, the economy basically consists of two sectors, a private and a public sector which supply private ($x_p$) and (genuine) public goods ($x_s$) respectively. With respect to the latter, we will follow the Samuelsonian notion of goods provided by the government as being characterized by non-rivalry and non-excludability in consumption. Though Wagner was less advanced with respect to the economic characteristics of public goods, it was exactly for these (genuine) tasks of the state that he expected government expenditures to outgrow GNP as societies become richer.

Public and private goods are denoted by subscripts $s$ and $p$ respectively. In order to facilitate the analysis, we assume that both goods are produced with (homogenous) labor according to a simple linear production technology

$$x_j = AL_j$$  \hspace{1cm} (1)
with \( j = p, s \), and with factors fully employed so that the sectoral employment levels add up to total labor supply \( \bar{L} = L_s + L_p \). The (exogenous) technology parameter \( A \) serves to capture the impact of technical progress (and thus growth) on the economy and (in the subsequent analysis) on government growth relative to GNP. In order to concentrate on the role of demand for budget-to-GNP ratios, as did Wagner at least partly, we will assume that both of the goods are supplied efficiently. Hence, we will abstract from issues such as lack of competition in one or the other sector and the problems related to raising government revenue in the form of distortive taxation or to the revelation of preferences, all of which may affect supply. Rather, we will assume that Lindahl-pricing were possible with respect to public goods so that they have shadow prices which are in turn determined by their true costs of production, even if there is no market for those goods. Hence, we assume that they are produced according to least-cost technology.

With profit functions in each sector \( \pi_j = p_j x_j - w_j L_j \) and given wages, the first order conditions for a profit maximum are in any case \( A p_j = w_j \); if labor mobility is costless across sectors, both of the sectors pay the same wage, so that the first order conditions reduce to \( A p_j = w \). Moreover, since \( w_j = w \), it must be the case that \( p_j = p \) (provided that the technology parameter is the same in both of the sectors), so that the price of the private good in terms of the public good is unity. The corresponding national income and production accounts then imply that labor income is \( w \bar{L} = p_p x_p + p_s x_s = (w/A)x_s + (w/A)x_p \), or equivalently, after dividing by \( w \) and multiplying by \( A, R = A \bar{L} = x_s + x_p \).

For sake of completeness, we assume in addition that there is an extra good \( x_0 \) (the role of which will become clear in the next paragraph) which is produced by use of capital \( K \) (with \( K \) employed at the ongoing rental rate \( r \)) according to the production function \( x_0 = aK \). Taking the extra good into account, income in our model economy is thus \( GNP = r \bar{K} + w \bar{L} \).
However, since we are mainly interested in the size of the public sector relative to the private sector, we will for most of the time ignore the differences between \( A\bar{L} \) and \( GNP \) and will use both of the terms as if they were the same. Hence, \( R \equiv A\bar{L} \) also symbolizes the resource constraint of the economy. Since the production possibilities frontier with reference to public and private goods is linear, the allocation of resources between the two sectors is determined by demand only, as is sectoral employment, and therefore, the budget-to-GNP ratio.

3. Demand
Since in any case, Wagner as well as Samuelson, the driving force of budget-to-GNP ratios originates from the demand rather than the supply side, we will examine the former more closely, while supposing that production takes place on the production possibilities frontier, that is with all resources fully and efficiently employed. Suppose, then, the \( i \)th individual of our small society (with \( i = 1, 2 \)) has the following additively separable utility function in \( x_0 \) on the one hand (which shall also serve as numéraire) and \( x_p \) and \( x_s \) on the other hand

\[
U_i(x_0, \{x_p, x_s\}) = U_i(x_0) + U_i(x_p, x_s)
\]  

(2)

The introduction of the extra good \( x_0 \) ensures that the marginal utility of income is fixed, and, with \( p_0 = 1 \), is unity. However, in the following analysis, and as suggested in Section 2, we will largely ignore the extra good. Ignoring the extra good is legitimate in this framework, for two reasons: (i) we are exclusively interested in the allocation of resources between sectors \( p \) and \( s \) and the size of the government sector relative to the private sector. Yet, in our set up, the latter is unaffected by what happens to the extra good. (ii) Wagner's as well as Samuelson's considerations are in fact partial equilibrium approaches to the allocation of
resources with respect to the two alternative uses, private and public. The partial equilibrium approach implicitly assumes that the marginal utility of income is fixed.

Since what matters are relative numbers, we will also disregard in what follows the part of income which is spent on the extra good. More specifically, we will assume that individuals maximize the following quadratic sub-utility function in goods $x_p$ and $x_s$

$$U_i(x_p, x_s) = \alpha_{ip} x_p + \alpha_{ix} x_s - \frac{1}{2} \left( \beta_{ip} x_p^2 + \beta_{ix} x_s^2 \right)$$

subject to the budget constraint $\sum_{j=1}^{2} p_j x_{ij} \leq wL_i$, and (recalling that $p_j$ equals $w/A$) income in terms of public goods $\sum_{j=1}^{2} x_{ij} \leq E_i \equiv AL_i$, or, ignoring the extra good, GNP with $\sum_{i=1}^{2} \sum_{j=1}^{2} x_{ij} \leq R = \sum_{i=1}^{2} E_i$. The corresponding inverse and direct demand functions for private and public goods of the $i$–th member of society are thus

$$p_{ij} = \alpha_{ij} - \beta_{ij} x_j$$

with $j = p, s$.

4. Wagner vs. Samuelson

Starting with a society in which income is evenly distributed so as to exclude distributional issues allows us to focus on the pure economic-growth effect on demand and the resulting allocation of resources between the private and the public sector. In any case, we are in the realm of social rather than private choice. Yet, Wagner himself focused on a single member of society, from which he extrapolated the socio-economic outcome on the aggregate level. Implicitly, he thus assumed that one person decides in lieu of all others on the allocation of
resources. Mathematically, there is no problem with this sort of preference aggregation. We simply define a statistical individual which completely reflects the prevailing preference mix of a society. Since according to (3) sub-utility functions are of the same quadratic type, individual demand functions only differ with respect to their slope and/or the intercepts with either the $x$- or the $y$-axis. Hence, we can easily construct a representative individual whose demand functions reflect sort of the "average" preference in society.

Adopting a Wagnerian perspective, the equilibrium allocation is achieved if the consumption of the private and the public good yield the same marginal utility. The corresponding budget-to-GNP ratio is obtained by equating the right hand sides of (4) for $j = p, s$. With $R$ either spent on private or on public goods, we can rewrite the resulting equation by substituting $(R - x_p)$ for $x_p$. Solving for $x_s$ and dividing by GNP, that is $R$ (or, equivalently, $A\bar{L}$), then yields the budget to GNP ratio according to Wagner

$$
\left(\frac{x_s}{R}\right)_W = \frac{\sum_{i=1}^{2}(\alpha_{is} - \alpha_{ip}) + \left[\sum_{i=1}^{2}\beta_{ip}\right]R}{\left[\sum_{i=1}^{2}(\beta_{is} + \beta_{ip})\right]R}
$$

(5)

Generally speaking, the ratio decreases in $R$ for $\sum_{i=1}^{2}(\alpha_{is} - \alpha_{ip}) > \sum_{i=1}^{2}\alpha_{ip}$ and increases otherwise. Yet, if we adhere to the Becker-Stigler assumption of uniform preferences, this property vanishes. If the intercept of each direct demand function with the $y$-axis is the same for all individuals, the budget-to-GNP ratio in Wagnerian perspective reduces to

$$
\left(\frac{x_s}{R}\right)_W \bigg|_{s = \alpha} = \frac{\left[\sum_{i=1}^{2}\beta_{ip}\right]}{\left[\sum_{i=1}^{2}(\beta_{is} + \beta_{ip})\right]}
$$

(6)

---

7 Wagner's idea of the political system was clearly underdeveloped, better to say: he had none. He occasionally writes about the parliament exerting some control power in being less inclined to an expansion of state activity than the government but that was it. Basically, he took the position of a welfare economist, assuming a social planner (the government) working in the "public interest", and, characteristically, he speaks of an "authoritative measurement of needs" (1893:894), where "authoritative" – according to Webster's Collegiate Dictionary – means "preceeding from authority" and has an unmasking synonym: "dictatorial". Taking together this dictatorial touch with the pursuit of the public interest we arrive at the regime of the representative individual – being the average individual or the median voter where the latter would clearly be very remote from Wagner's thinking.
Notably, in this case, the size of the public sector is independent of the size of the economy. Yet, empirical studies do not lend support even to this (generalized) version. Rather, empirical studies found no systematic differences between both groups of goods with respect to individual income elasticities. If we consequently assume that the inverse demand functions for private and for public goods are identical, that is \( \alpha_{ij} = \alpha \) and \( \beta_{ij} = \beta \), the budget-to-GNP ratio attains exactly fifty percent – independent of GNP

\[
\left( \frac{x_p}{R} \right)_{\alpha_{ij} = \alpha; \beta_{ij} = \beta} = 0.5 \tag{7}
\]

Hence, if both individuals share the same preferences with respect to private and public goods which in turn give rise to the same direct demand functions (or at least with the \( \alpha ' \)s the same), the budget-to-GNP ratio remains constant, despite of income growth.

This result is not only at odds with Wagner's own dictum, according to which the public sector takes an ever larger bite of GNP as societies become richer, but also with the modern theory of public goods à la Samuelson. Rather than the representative individual deciding on social matters in an authoritarian (but authoritative) manner, following Samuelson, the society as a whole, that is, each and every member of society, democratically decides on everybody's consumption of public goods.\(^8\) Note the difference: in case of Wagner the budget-to-GNP ratio was obtained by assuming that the macro-result could be inferred from (the fiction of) a representative individual which decides in place of all members of society – notably, while incorporating possible differences in individual preferences. Yet, implicitly, this amounts to the notion that one individual is able to decide for the community as a whole (like a "benevolent dictator"). However, Samuelson's approach is fundamentally different:

\(^8\) To be more precise: The Samuelsonian view primarily has to do with efficiency in a world of public goods. But adding up the individual willingness to pay has a political meaning too: Instead of the representative (average) individual, the whole society is part of the process finding the optimal amount of public goods. We are aware of some difficulties of definition but call this procedure – relative to Wagner’s authoritative-authoritarian style – "democratic" because nobody with a positive willingness to pay is left out and everyone can "vote" according to his/her monetary preferences.
applying his criteria according to which public goods are characterized by non-rivalry and non-excludability in consumption implies that we have to add the marginal willingness to pay of all individuals rather than to add up the quantities demanded individually in order to obtain the value the society places on public goods in terms of opportunities forgone.

Applying Samuelson's theory of public goods we thus arrive at the budget-to-GNP ratio

\[
\left( \frac{x_s}{R} \right)_s = \frac{\left( \alpha_{1s} - \alpha_{1p} \right) \beta_{2p} + \left( \alpha_{2s} - \alpha_{2p} \right) \beta_{1p} + \sum_{i=1}^{2} \alpha_{ip} \beta_{ip} + \Pi_{i=1}^{2} \beta_{ip} R}{\left[ \sum_{i=1}^{2} \beta_{is} \right] \left[ \sum_{i=1}^{2} \beta_{ip} \right] + \Pi_{i=1}^{2} \beta_{ip} R} \tag{8}
\]

Hence, in the most general case, that is with \( \alpha_{1j} \neq \alpha_{2j} \) and \( \beta_{1j} \neq \beta_{2j} \) for all \( j = p, s \), budget-to-GNP ratios differ depending on the perspective adopted, old or modern, Wagner or Samuelson. Nevertheless, both results have some properties in common: in the generalized version and employing Samuelson's theory of public goods, the impact of growth on the relative size of the public sector is again ambiguous: the budget-to-GNP ratio decreases if

\[
\left( \sum_{i=1}^{2} \alpha_{is} \right) \left( \sum_{i=1}^{2} \beta_{ip} \right) > \alpha_{1p} \beta_{2p} + \alpha_{2p} \beta_{1p},
\]

and increases otherwise. However, if the intercepts of the inverse demand curves with respect to the \( y \)-axis are the same for both, individuals and groups of goods,9 that is, if \( \alpha_{ij} = \alpha \), the expression reduces to

\[
\left( \frac{x_s}{R} \right)_{s, \alpha_{ij} = \alpha} = \frac{\alpha \left( \sum_{i=1}^{2} \beta_{ip} \right) + \Pi_{i=1}^{2} \beta_{ip} R}{\left[ \sum_{i=1}^{2} \beta_{is} \right] \left[ \sum_{i=1}^{2} \beta_{ip} \right] + \Pi_{i=1}^{2} \beta_{ip} R} \tag{9}
\]

Unlike in the case of Wagner, the equilibrium ratio remains a function of GNP, which even extends to the case of preferences and thus inverse demand curves being identical in every respect.

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9 Rather than sharing the \( y \)-intercept, the inverse demand functions could also be sharing the same \( x \)-intercept. In this case, the budget-to-GNP ratio decreases if \( \alpha_{1s} \beta_{12p} / (\beta_{1p} + \beta_{1s}) < \alpha_{1s} \beta_{2p} / (\beta_{1s} + \beta_{1p}) \) and increases otherwise provided that Wagner applies. If we adopt a Samuelsonian perspective instead, it decreases if \( 2 \alpha_{1s} \beta_{12p} (\beta_{1p} + \beta_{1s}) < \alpha_{1s} \beta_{1p} / (\beta_{1s} + \beta_{1p}) \) and increases otherwise. However, naturally, with the demand curves identical, both concepts (that is the similarity with respect to either the \( x \)- or the \( y \)-intercept) collapse into one.
\[
\left( \frac{x_i}{R} \right)_{\alpha \beta \beta} = \frac{1}{5} + \frac{2\alpha}{5\beta R}
\]

(10)

Hence, if Samuelson applies, the budget-to-GNP decreases in \( R \): rather than the budget outpacing the economy in terms of growth the reverse holds true.\(^{10}\)

If income is evenly distributed, we thus can summarize our main results on the Samuelsonian equilibrium budget to GNP ratio in the following manner:

1. for GNP large, the hyperbola (10) converges asymptotically to \( 1/(n^2 + 1) \). Hence, for GNP large enough, the budget-to-GNP ratio only depends (inversely) on the number of individuals;

2. economically meaningful results require \( R > \alpha/2\beta \). For \( R \) smaller than \( \alpha/2\beta \), there is no private sector. Rather, demand for public goods is so strong that these are the only type of goods which are produced and consumed. This applies independent of their marginal costs of production, provided that consumption is associated with net welfare gains.

3. however, for GNPs larger than \( \alpha/2\beta \), the expenditure share spent on public goods decreases in \( R \), provided that preferences and thus the marginal value placed on public and private goods remain stable.

\(^{10}\) Supposing instead that preferences are described by the otherwise due to its specific properties popular Cobb-Douglas function, i.e. \( U = x_1^\alpha x_2^{1-\alpha} \), yields the empirically implausible result that budget-to-GNP ratios are generally independent of the size of the economy: since the Cobb-Douglas function is characterized by the fraction of incomes spent on one or the other group of goods being constant, the Wagner-result is straightforward, and in the above mentioned case \( (x_i/R)_\alpha = \alpha \). However, taking the economic properties of public goods into consideration, we also obtain the result of a constant budget-to-GNP ratio. With the marginal rate of substitution described by the parameter \( \alpha \) we obtain \( (x_i/R)_\alpha = 2^{\alpha-\alpha}(t-\alpha)+2^{\alpha-\alpha} \), which is strictly increasing in \( \alpha \) (though at a decreasing rate), and generally larger than \( \alpha \) as \( 2^{\alpha-\alpha} > 1 \). The latter result though is also hard to reconcile with the empirical fact that people tend to buy more privately instead of publicly provided goods as they become richer, even if those goods are associated with (positive) externalities. The shift of demand from public to private schools is just one case in point. We therefore reject the hypotheses of preferences along Cobb-Douglas lines and stick to the alternative of a utility function which gives rise to linear demand functions or which are obtained by linear approximation.
4. nevertheless, there is a GNP for which budget-to-GNP ratios along the lines of Wagner and Samuelson are the same, that is
\[
\left(\frac{x_s}{R}\right)_{\left(\alpha_i, \beta_i, \gamma_i, \delta_i\right)} = \left(\frac{x_s}{R}\right)_{\left(\alpha_i, \beta_i, \gamma_i, \delta_i\right)},
\]\nif
\[
R\bigg|_{\gamma_s} = \frac{4\alpha}{3\beta}.
\]

Hence, in a modern perspective, the budget-to-GNP ratio can be smaller or larger than in case of Wagner. However, in any case, we should observe that the budget-to-GNP ratio declines as GNP increases.

Figure 1 displays the equilibrium budget-to-GNP ratio according to Wagner and Samuelson with preferences across individuals and thus direct demand for both groups of goods identical (the latter are the dotted lines sloping downwards): the intersection of the solid lines (that is the aggregation of the individual perspectives) in both of the panels show budget-to-GNP ratios if the representative individual à la Wagner decides on the allocation of resources. As can be seen, the budget-to-GNP ratio is always 50 percent, independent of GNP. Hence, the expenditure shares are the same in both of the panels, for GNP small (LHS) and large (RHS). Rather than increasing in GNP, Wagner's own approach to the allocation of resources between the private and the public sector results in government expenditures growing in proportion to GNP.
Figure 1. Wagner versus Samuelson as a function of the size of the economy

These results not only differ from "Wagner's Law"; they are also hard to reconcile with Samuelson's theory of public goods. Following Samuelson, the collective marginal willingness to pay for public goods is the sum of the willingness to pay of all the individuals (the dashed curve in both of the diagrams), hence the budget-to-GNP ratio may be either smaller or larger than in case of Wagner (or exactly the same). On the LHS, that is for GNP comparatively small, the budget-to-GNP ratio is larger in case of Samuelson than in case of Wagner while on the RHS, that is a relatively rich society, the opposite holds true. Or to put it differently: if Samuelson applies, poor countries are characterized by a comparatively large government sector while for rich countries the opposite holds true. And, it must be the case that there is exactly one size of GNP for which countries switch sides. This latter observation may give rise to some further (though somewhat tentative and speculative) thoughts: the Samuelsonian result may give rise to virtuous and vicious cycles (or poverty traps for that matter): if the country is poor and if a large government sector is detrimental to growth, the
budget-to-GNP ratio may continue to increase which further hampers growth, thus choking down the process of take off. However, once countries have managed to attain or even surpass a critical size in terms of GNP, the process described above may dominate and may feed into a self-sustaining process of economic growth.

5. Distribution(al) matters

Thus far, we have assumed that both of our individuals contribute equally to GNP and thus have the same income at their disposal. Admittedly, this is a very strong assumption as societies are seldom characterized by an absolutely even income distribution. Therefore, we will relax the assumption concerning the income distribution in the following paragraph. And, indeed, distributional matters will prove crucial for the results obtained. However, modifying our assumption by allowing for other income distributions requires an assumption about the existence or non-existence of a welfare state. For instance, in the US and in Europe there is a different attitude with respect to income inequality as is reflected in the relative size of the redistributive state (see Alesina et al 2004). Interestingly, these attitudes also yield different results with respect to the provision of public goods (relative to the private sector).

Consider first a situation in which there is no redistributive state. If, rather than being equally distributed, all income accrues to just one individual while the other gets none (with GNP the same as previously though), only the first individual can develop a demand for private goods. However, in the political process, both are active and both decide democratically on the consumption of public goods which means that there is a fictitious

11 Amazingly, Wagner as a member of a group of scientists disparagingly called "Kathedersozialisten" (academic socialists) was obviously not aware of the momentum that social policy and redistribution could exert on his law one time – his view is very static when he speaks of the "right combination of the private, public and caritative systems" (1893:900) and rather defensive when he advocates a transfer of firms from the private to the public sector if private firms have no better performance than public ones but have unfavourable characteristics from a social policy view (1893:903).
equal distribution of GNP among the two individuals. The resulting equilibrium budget-to-GNP ratio in Samuelsonian perspective is thus

\[ \frac{x_s}{R} = \frac{1}{3} + \frac{\alpha}{3\beta R} \]  

(11)

which is larger than in case the same income is equally distributed (10) for \( R > \alpha/2\beta \), and since \( R = \alpha/2\beta \) represents the lower bound for government's share to be meaningful (that is, \( \leq 1 \)) in both cases, (11) is always larger than (10). Moreover, the absolute and relative decrease in the budget-to-GNP ratio is smaller than if the income is equally distributed when incomes or GNP grow by the same amount. Hence, if the income distribution changes (with GNP unchanged), so does the budget-to-GNP ratio. That is, for any given GNP, a rise in society’s homogeneity decreases budget-to-GNP ratios. Likewise, (in a dynamic perspective,) the more homogeneous its distribution, the faster does growing income reduce the government's share of GNP.

The extreme case of a homogeneous distribution is the equal distribution which was a decisive part of our model developed in Sections 2-4, and can be interpreted also as tax revenues being redistributed in a lump sum fashion so that both individuals again have the same resources at their disposal.

Notably, the fact that budget-to-GNP ratios may increase if inequality increases is not due to inefficiencies related to government such as lack of competition and distortions related to taxes and transfers as we assumed that public goods are perfectly supplied to the market and that redistribution takes place in a lump sum fashion; the result above is solely the consequence of the characteristics of private and public goods. In any case, with GNP growing, budget-to-GNP ratios are getting smaller et vice versa, but it may be possible that a temporary polarization of the income distribution during the growth process may spoil the result of the budget-to-GNP ratio decreasing in GNP which can be seen as a typical part of a
dualistic development process as in Russia today. When GNP is high enough and the peak of the polarization process is surpassed, those counteractive forces will vanish, and both forces – income growth and decreasing inequality – will work together in only one direction: lowering government's share of GNP.

6. Conclusions

The paper shows that "Wagner's Law" is hard to reconcile with the modern theory of public goods as developed by Paul Samuelson. The two views deliver dissenting results, in particular if, (i) preferences are uniform (as Becker/Stigler suggest) and (ii) individual income elasticities for public goods hover around unity (as a large part of the empirical studies claim and as we state as a plausible hypothesis for the overall budget). Rather than increasing, the Samuelsonian perspective yields a budget-to-GNP ratio which is decreasing in GNP. Hence, public-goods related expenditures will tend to grow more slowly than the economy. Yet, "Wagner's Law" is even incompatible with his own statement as the relative size can be expected to remain constant if the Wagnerian approach is formalized with (i) and (ii). The differences between Wagner and Samuelson can be largely attributed to differences in the philosophy of the state, namely authoritative-authoritarian vs. democratic.

However, distributional issues matter. While budget-to-GNP ratios in the Samuelsonian perspective continue to decline as economies become richer, changes in the income distribution may be associated with changes in the budget-to-GNP, too. Hence, there certainly is a GNP at which the distribution is neutral with respect to the relative size of the public sector. Yet, the budget-to-GNP ratio unambiguously increases if the primary income distribution becomes more uneven due to some individuals losing their earnings capacity with the others retaining theirs – independent of whether there is a redistributive welfare state or not, provided that the first operates in lump-sum manner. With a growing GNP and
decreasing inequality as (has been observed) in the continental states of Europe for the last 30 years, both forces work in the same direction, and hence, the relative growth of government cannot be traced back to the basic allocative functions of government but must be rooted elsewhere.

References


Bisher erschienen:

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• Dewenter, Ralf & Justus Haucap, Mobile Termination with Asymmetric Networks, No. 23 (October 2003), erschienen unter dem Titel “The Effects of Regulating Mobile Termination Rates for Asymmetric Networks” erschienen in: European Journal of Law and Economics 20, 2005, 185-197.


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