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History-Based Price Discrimination with Imperfect Information Accuracy and Asymmetric Market Shares

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History-Based Price Discrimination with Imperfect Information Accuracy and Asymmetric Market Shares

Abstract

The paper considers a duopoly model in which firms inherited asymmetric market shares and history-based price discrimination is viable. However, firms can identify only a share of their own consumers depending to the degree of information accuracy. We derive the pricing strategies and we analyze the relationship between information accuracy and asymmetric market shares, showing under which circumstances there exists an equilibrium in pure strategies. We show that history-based price discrimination makes the dominant firm's profits always lower than those of the rival, with an ambiguous effect of the information accuracy on industry profits. Moreover, we prove that the level of information accuracy has a decreasing effect on social welfare, while it affects consumer surplus non-monotonically, according to the size of asymmetry in the inherited market shares.

JEL-Codes: D800, D430, L100.

Keywords: history-based price discrimination, information accuracy, asymmetric market shares.

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1. Introduction

The rapid growth of new technologies in the last decades has enabled firms to collect a large amount of information about their consumers and to tightly target pricing, advertising and products according to their preferences. The widespread use of tracking tools such as online cookies and web beacons, but also fidelity cards and questionnaires allow firms to profile consumers accurately (Choe *et al.*, 2018) and to offer them personalized prices. History-based price discrimination is a pricing strategy through which firms charge different prices according to the consumers' purchase histories. It has become widely used in many different sectors such as, for example, telecommunication, insurance, banking, e-commerce, media and entertainment and energy markets. This has raised a number of concerns in scholars and practitioners regarding the effect on firms' profits, consumer surplus and the competitive environment.¹ In particular, economists have wondered about the potential anti-competitive impact of this strategy: whether it contributes to the exclusion of smaller rivals (see e.g., Gerigh *et al.*, 2011 and Shaffer and Zhang, 2000) or the exploitation of the acquired information to the detriment of consumers (see e.g., Gerigh *et al.*, 2012).

Firms typically engage in a tough competition via history-based price discrimination in order to attract those consumers they have no trace in the past purchase behaviors. This kind of strategy can be enforced through personalized discounts, gift vouchers, free deliveries, post-sales services and included warranties (see Acquisti and Varian, 2005, Caillaud and De Nijs, 2014, and Garella *et al.*, 2021).² It is clear that this pricing strategy needs the possibility to collect some relevant information and recognize past consumers. However, this is not always feasible, at least not perfectly. The current regulations on data protection (see for instance the General Data Protection Regulation) imposed some restrictions on gathering information

¹In a recent case, the Italian Antitrust Authority (AGCM) sanctioned the energy company Enel, which is the former monopolist in the distribution and sale of energy in Italy. The AGCM fined Enel for having offered personalized discounts to those customers served through Servizio Elettrico Nazionale (the Enel retailer subsidiary providing the enhanced protection service), in order to induce them to switch to Enel Energia (the Enel retailer subsidiary), given the upcoming liberalization of energy market in Italy. See the case A511 – Enel /Condotte anticoncorrenziali nel mercato della vendita di energia elettrica (2019).

²For example, the CNBC (2020) has reported the price war to attract new customers in the streaming services, with Netflix offering slimmed-down subscription plans, while rivals, like Disney+ and Apple TV+, are offering discounts on long-term plans, locking in customers willing to pay a discounted price upfront. Similarly, the digital streaming platform DAZN offered a free trial month to new subscribers. Instead, The Guardian (2021) has recently documented three curious offers in banking: “First Direct is offering £100 cash to those customers switching to its 1st Account, while HSBC is offering £125 to anyone switching to its Advance or Premier current accounts. Virgin, meanwhile, is giving 12 bottles of wine and a £50 donation to charity”. This shows again the large variety of handouts to attract new customers. See Pazgal and Soberman (2008) for other examples.

about past purchases, search history and personal data in the digital markets.³ Now, firms need the (more or less conscious) approval of consumers to gather information about their online behavior. However, even in the absence of such a regulation, firms are not always able to recognize their past consumers perfectly. It is exemplificative the case of traditional shops which allow consumers to pay by cash. Although the identification of past purchase behaviors is relatively easy when consumers buy products on online platforms, it may be difficult to track them when they make their purchases in physical stores. Clearly, it depends on the degree of cash traceability and the spread of credit and debit cards. Instead, other forms of marketing techniques like surveys, questionnaires and loyalty cards are imperfect by definition and they cannot reach the entire audience of consumers (Esteves, 2014). Finally, when firms collect consumers' data indirectly, buying them from third parties, there are several reasons why the collected information may be imperfect. For instance, it may be too costly to gather detailed data at the disaggregated level⁴ or, alternatively, third parties may have no incentive to share such an information - e.g., if retailers want to keep a certain buyer power toward their own manufacturer (e.g., Li, 2002, He *et al.*, 2008, and Guo and Iyer, 2010). All these imperfections impede firms to know all the past purchase inherited histories.

History-based price discrimination have captured the attention of many economists (e.g., Shaffer and Zhang, 2000, Gehrig *et al.*, 2011 and 2012), who studied how this pricing practice affects industry profits and welfare in competitive contexts characterized by perfect information about past purchase behaviors. To the best of our knowledge, there are only few papers dealing with imperfect information and history-based price discrimination. In particular, noisy signals about the true preferences of consumers were firstly introduced by Chen *et al.* (2001) and Esteves (2014), who consider the case in which firms receive some private information about their customers: the more precise the information, the higher the possibility to price discriminate perfectly. Instead, Liu and Serfes (2004) study a model where consumers are classified in subgroups, which represent the imperfectness of the available information: the lower the number of these subgroups, the lower the degree of information accuracy. Finally, Colombo (2016) considers the case in which firms can collect information about only a share

³See the Regulation (EU) 2016/679 of the European Parliament and of the Council of 27 April 2016 on the protection of natural persons with regard to the processing of personal data and on the free movement of such data, and repealing Directive 95/46/EC (General Data Protection Regulation).

⁴Throughout the paper, we assume that gathering information is costless and the degree of information accuracy is exogenously given.

of their past consumers. Clearly, the larger the share of identified customers, the greater the degree of information accuracy.

Moreover, most of these works build on the assumption that firms are symmetric. This implies that the incentives to compete more or less aggressively are identical across firms. Therefore, in such a competitive framework, it turns out that rivals share the market equally. Yet, the symmetric case is a very special event and asymmetric duopoly represents better the reality. As argued by Carroni (2016),⁵ price discrimination is often used in markets characterized by some degrees of asymmetry. It may derive from a former situation of public monopoly which has been opened to competition through a liberalization of the market. Alternatively, firms may enjoy a competitive advantage thanks to the brand reputation built in sided markets.

In this paper, we follow the approach of Shaffer and Zhang (2000) and Gehrig *et al.* (2011 and 2012), by considering, however, a duopoly in which firms inherited asymmetric market shares. In this framework, we meet the challenge launched by Shaffer and Zhang (2000) by introducing some randomness in the accuracy of firms' information — i.e., firms know only with some probability whether a consumer belongs to their own turf.⁶ As in Colombo (2016), we assume that firms might not identify all their own consumers but only a defined quota of them. Such a quota of the inherited market represents the degree of information accuracy about the consumers' past purchase behavior. This allows us to provide a very general model that incorporates the perfect history-based price discrimination and uniform pricing as two limit cases, but it also shows a more realistic marketplace in which firms collect some imperfect information. Doing so, this paper contributes to the current debate on how more or less stringent laws on data protection can affect firms' profits and consumer surplus. Specifically, we explore the link between a greater information accuracy and firms' pricing strategies, highlighting if and when consumers are induced to switch seller, and the extent to which this relationship is affected by firms' asymmetries.

Our main results are as follows. We show under which circumstances there exists an

⁵Note that Carroni (2016) study firms' asymmetries concerning the quality of the products on sale while, in our model, we show how the asymmetry in the inherited market shares influence the firms' pricing strategies and the consequent equilibrium outcome.

⁶In their conclusions, Shaffer and Zhang (2000) admit several limitations to their paper. One of these includes the firms' information about consumers' past purchase behavior. They partition consumers into two groups, where α and β denote those consumers belonging to the turf of firms A and B respectively, which are perfectly known by the firms. The authors suggest the following extension: "*for any given consumer, it might be assumed that firms know only with some probability (less than one) whether he/she belongs to group α or group β* " (p. 419).

equilibrium in pure strategies according to the degree of information accuracy and asymmetry in the inherited market shares. Specifically, we find that, given a certain degree of information accuracy, a pure strategy equilibrium exists if and only if firms are almost symmetric or sufficiently asymmetric. Otherwise, there are no equilibria in pure strategies.⁷ Moreover, we show that the regions of parameters in which a pure strategy equilibrium exists are increasing in the degree of information accuracy. This highlights the relevance of detecting the available information and the difference in the inherited market shares in order to characterize the firms' behaviors and the consequent consumers' allocation. In particular, we show that poaching occurs on both sides of the market only if firms are almost symmetric – which is denoted as the weak dominance case –, while a large asymmetry in the inherited market shares leads only those consumers belonging to the turf of the dominant firm⁸ to switch – which is denoted as the strong dominance case. This result depends on the fact that products are horizontally differentiated and when the rival of the dominant firm inherits a small market share, the dominant firm should offer a very large price discount in order to induce rival's consumers to switch, which is suboptimal with respect to a uniform price charged to all perspective customers. Clearly, the pricing strategy depends on the degree of information accuracy: when the information accuracy is low, competition is focused on those consumers that firms are not able to recognize, but when it is high, offering price discounts to induce switching may be an optimal strategy for one firm if it is compensated by large profits from its own past consumers.

Hence, the size of asymmetry between firms may alter the nature of competition with a poaching or non-poaching behavior. We find that, depending on such a degree of asymmetry, a greater information accuracy has ambiguous and non-monotonic effects on firms' profits. Increasing the level of information accuracy has a decreasing (increasing) effect on the small firm's profits in case of weak (strong) dominance. By contrast, the dominant firm's profits depend crucially on the size of asymmetry. Indeed, in the weak dominance case, they are decreasing when firms are almost symmetric, they are U-shaped for an intermediate size of asymmetry and they are increasing if firms are sufficiently asymmetric. Instead, in the strong

⁷This also justifies the emergence of out-of-equilibrium behaviors. When a mixed strategy equilibrium exists firms may continuously adopt their pricing and marketing strategy according to the rivals' behavior. For example, HSBC has dramatically changed its pricing strategy toward new customers in the last six months, maybe in response to its rivals (The Guardian, 2021).

⁸We follow the terminology of Gerigh *et al.* (2012) in defining the dominant firm as the firm inheriting a market share larger than 50%, although it does not correspond perfectly to the legal definition of a dominant position. See, for example, the Guidance on the Commission's enforcement priorities in applying Article 82 of the EC Treaty to abusive exclusionary conduct by dominant undertakings.

dominance case, increasing the information accuracy has a decreasing effect on the dominant firm's profits. Although the effect of the information accuracy on firms' profits vary according to the degree of asymmetry, the dominant firm's profits are always the lower. This is because the small firm, inheriting a small market share, knows its own consumers and their willingness to pay better than the dominant firm, such that it can enforce a poaching strategy more profitably than a dominant firm. This result suggests that, when a history-based price discrimination is viable and there are no informational frictions in the market, a small firm is always able to compete fiercely with a dominant firm and it does not need any defending policy intervention. Moreover, we show that the small firm benefits from history-based price discrimination because it is more profitable than a uniform pricing strategy in case of strong dominance. Otherwise, history-based price discrimination is always detrimental for the industry profits with respect to uniform pricing – i.e., when the degree of information accuracy is nil.

Finally, we show the effect of increasing the degree of information accuracy on consumer surplus and social welfare. Interestingly, social welfare – i.e., the sum of consumer surplus and industry profits – is always decreasing in the precision of the available information. This shows that the possibility to price discriminate is always detrimental from a social welfare point of view. By contrast, the impact of a greater degree of information accuracy on consumer surplus depends on the size of asymmetry between firms. It is harmful for consumers only in the weak dominance case, but firms are moderately asymmetric. Otherwise, the effect is beneficial and a Public Authority aiming at protecting the consumer's economic interests – beyond the issues related to data protection and privacy laws – should foster the collection of information about past consumers.

The analysis is concluded with two extensions of the baseline model which confirm our qualitative results. We characterize the pure strategy equilibria of the game when the dominant firm moves first, being the leader in the market, and the small firm follows. We show that, in a sequential game, there exists always an equilibrium in pure strategies for any degree of asymmetry and information accuracy. Yet, the weak and strong dominance cases are the unique equilibria of the game and they emerge for a degree of asymmetry sufficiently small or large respectively. Then, we investigate the equilibria of the game if we remove the hypothesis of symmetry in the precision of information accuracy. In particular we examine the two extreme situations where the smaller firm is either endowed with perfect information accuracy or it has no information at all. We show that the qualitative insights of the model are robust to this

generalization and that the main characteristics of the equilibria are preserved. In the weak dominance case equilibrium requires inherited market shares to be sufficiently symmetric and poaching occurs on both sides. Conversely, the strong dominance equilibrium, requires the inherited market shares to be highly asymmetric and poaching occurs only on one side.

2. Baseline Model

Consider a duopoly in which firms produce differentiated products at a constant marginal cost, which is normalized to zero without loss of generality. Firm A is located at the left endpoint of a Hotelling line of length 1, while firm B is located at the right endpoint of that unit interval. We assume there is a unit mass of consumers, who are distributed uniformly along this segment. We denote by $x \in [0, 1]$ the location of each consumer in the product space, which represents the preferred variety of a given good and the relative preference for the products on sale: the more the consumer is close to 0 (1), the higher the willingness to pay the product sold by firm A (B). Consumers are willing to purchase at most one of the products on sale, whose outside option is equal to zero. Thus, the utility function of a consumer i located in x , if she buys a product from firm j ($j = A, B$) is:

$$u_i^j(x) = v - p_j - t|x - l_j|,$$

where v is the intrinsic value of the product, which is assumed to be *large enough* so that each consumer always buys one product in equilibrium, p_j is the price charged by firm j , t represents the standard transportation cost — i.e., the disutility of consuming a variety of product which is different from the preferred one — and l_j is the location of each firm along the Hotelling line — i.e., $l_A = 0$ for firm A and $l_B = 1$ for firm B.

Following Shaffer and Zhang (2000) and Gehrig *et al.* (2011 and 2012), each consumer has a purchase history, which is exogenously given, that establishes an inherited market for each firm.⁹ The inherited market, or turf, of firm A, T_A , is represented by those consumers located at $x \leq k$ with $k \in [\frac{1}{2}, 1]$ — i.e., $T_A = \{x : x \leq k\}$. By contrast, the inherited market

⁹Although most of the standard literature on behavior-based price discrimination deals with a two-period model — e.g., Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000) and Esteves (2010) —, we consider the framework of Shaffer and Zhang (2000) and Gehrig *et al.* (2011 and 2012) that abstracts from any prior competition between firms. This is not a mere simplification but, as argued by Shaffer and Zhang (2000), it better represents those markets in which firms infer customers' preferences either from where they live or through their self-revelation via, for example, customer surveys. Alternatively, our model can be interpreted as a model where consumers face differentiated switching costs within a homogeneous market framework. Once consumers have purchased a product, some switching costs emerge and make the products differentiated. We study this case when switching costs have a non-symmetric distribution, which makes the firms' market shares asymmetric.

of firm B, T_B , coincides with those consumers located at $x \geq k$ — i.e., $T_B = \{x : x \geq k\}$. This implies that firm A has a larger market share with respect to firm B.¹⁰ For the sake of simplicity, we refer to firm A as the dominant firm.

As in Colombo (2016), we assume that firms have a common information technology that allows them to identify a share of their own turf. Such an information technology, which is available for the competing firms at zero costs, has a level of accuracy which is denoted by $\alpha \in [0, 1]$. The parameter α represents the quota of inherited market that firms are able to identify correctly at each point of the Hotelling line (or the probability that a consumer belonging to the turf of firm j is correctly identified from that firm). Therefore, if $\alpha = 1$, the information accuracy is perfect and it allows firms to recognize all the consumers who bought from them in the past. Conversely, if $\alpha = 0$, firms cannot distinguish their own consumers from the rival's consumers. For any $\alpha \in (0, 1)$, information is imperfect and firms have partial knowledge about their own turf: they can identify a quota α of their own consumers, while they do not know the past purchase history of the remaining fraction $1 - \alpha$. In the following, we refer to the group of "identified consumers" as the quota α of consumers, at each $x \in T_j$, which is correctly identified by firm j , and to the group of "unidentified consumers" as those consumers belonging to the turf of firm j , but who have not been identified by firm j , or belonging to the rival's turf — i.e., a quota $1 - \alpha$ at each $x \in T_j$ and all $x \in T_{-j}$.¹¹

Based on the size of the inherited market shares and the ability to recognize their own turf, firms can charge different prices for the identified and the unidentified consumers. Specifically, each firm j sets a bundle of prices $\varphi_j = \{p_j, \tilde{p}_j\}$, where p_j and \tilde{p}_j are the prices charged to the identified and unidentified consumers respectively. This pricing scheme should allow firms to extract more surplus from those consumers belonging to their own turf and to attract both the unidentified and rival's consumers. We assume that consumers only care about prices and they do not bear any further cost for switching. After observing all the prices offered, consumers decide which firm to patronize and make their purchase decisions. In the following of the analysis, we identify the region of parameters in which there exists an equilibrium in pure strategies, we verify whether poaching occurs and how the asymmetry in the inherited market shares affects the equilibrium outcomes.

¹⁰The game provides opposite and symmetric results if we consider $k \in [0, \frac{1}{2}]$.

¹¹Note that T_{-j} represents the turf of a firm different from firm j .

3. Equilibrium Analysis

The ability to recognize a share of consumers allows firms to price discriminate and offer a personalized price to their own turf. This pricing scheme can lead to two alternative consumers allocation according to the inherited market shares. Specifically, if firms are almost symmetric — i.e., k is sufficiently close to $\frac{1}{2}$ — poaching occurs on both sides of the Hotelling line. Intuitively, both firms can offer a price discount, which outweighs the relative preference for the products on sale, in order to attract consumers belonging to the rival's turf. We refer to this case, which is illustrated in Figure 1, as *weak dominance*.

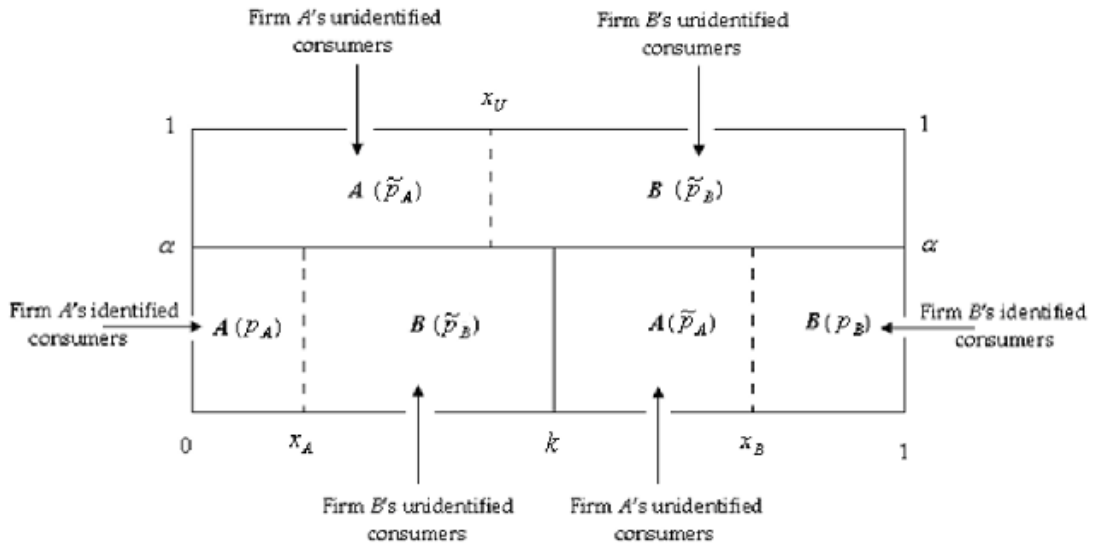


Figure 1: Weak dominance case

Instead, if firms are sufficiently asymmetric — i.e., k is sufficiently close to 1 — poaching occurs only on the left hand side of the market. The reason is that firm B's consumers are located so far from firm A that they are not willing to switch, because they have a relative high preference for firm B with respect to firm A. We refer to this case, which is illustrated in Figure 2, as *strong dominance*.

In what follows, we analyze the two cases separately and we verify under which circumstances these consumers allocations hold in equilibrium.

3.1 Weak dominance

Suppose that the inherited market shares are almost symmetric between firms and each firm j offers a personalized price p_j to the identified consumers belonging to the own turf, while it

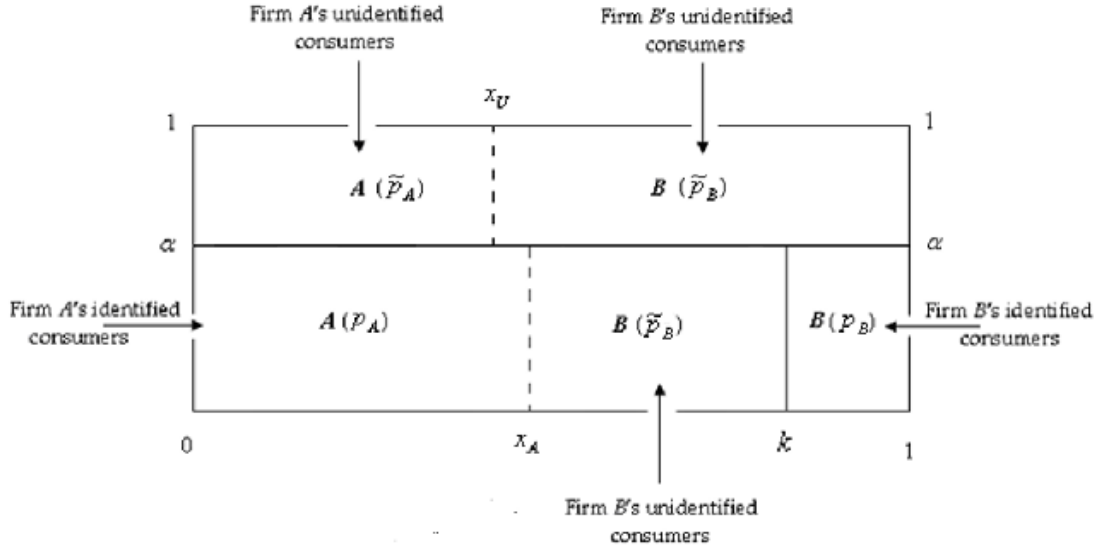


Figure 2: Strong dominance case

charges a price \tilde{p}_j to the rest of consumers.

Let us start by analyzing the group of those consumers recognized either from firm A or from firm B — i.e., a share α of the entire market. A consumer belonging to the turf of firm A is indifferent between buying again from firm A and switching to firm B if $v - p_A - tx = v - \tilde{p}_B - t(1 - x)$, which implies $x_A = \frac{\tilde{p}_B - p_A + t}{2t}$. Similarly, a consumer belonging to the turf of firm B is indifferent between buying again from firm B and switching to firm A if $v - \tilde{p}_A - tx = v - p_B - t(1 - x)$, which implies $x_B = \frac{p_B - \tilde{p}_A + t}{2t}$. Therefore, the segments $[0, x_A]$ and $[x_B, 1]$ represent those consumers who continue buying from the same firm. By contrast, the segments $[x_A, k]$ and $[k, x_B]$ represent those consumers who prefers to switch and buy from firm B and firm A respectively.

Finally, consider the group of consumers who are not recognized by either firm A or firm B — i.e., a share $1 - \alpha$ of the entire market. A consumer is indifferent between buying from one of the two firms if $v - \tilde{p}_A - tx = v - \tilde{p}_B - t(1 - x)$, which implies $x_U = \frac{\tilde{p}_B - \tilde{p}_A + t}{2t}$. Therefore, the segments $[0, x_U]$ and $[x_U, 1]$ represent the unidentified consumers who prefer buying from firm A and firm B respectively.

Based on the pricing scheme and the consumers allocation described above, the firms' profit functions are

$$\pi_A = p_A \alpha x_A + \tilde{p}_A [\alpha (x_B - k) + (1 - \alpha) x_U], \quad (1)$$

$$\pi_B = p_B \alpha (1 - x_B) + \tilde{p}_B [\alpha (k - x_A) + (1 - \alpha) (1 - x_U)]. \quad (2)$$

It is worth noting that, if $\alpha = 0$, firms can not price discriminate and charge a uniform price to all consumers. Conversely, if $\alpha = 1$, the model boils down to the case of perfect history-based price discrimination studied by Shaffer and Zhang (2000) and Gehrig *et al.* (2012). Instead, if $\alpha \in (0, 1)$, information technology is imperfect and firms can charge a personalized price only to a limited share of their own turf. Although poaching occurs for any $\alpha > 0$, when the information technology is imperfect, there exists a group of "unidentified consumers" for whom each firm j competes by charging \tilde{p}_j .

Differentiating the profit functions with respect to firms' prices and solving the system of best-replies yield the following equilibrium prices

$$p_A^w = \frac{t [12 - 4\alpha (2 - k) - \alpha^2 (1 - 2k)]}{3(4 - \alpha^2)}, \quad (3)$$

$$\tilde{p}_A^w = \frac{t [12 - 8\alpha (1 + k) + \alpha^2 (5 - 4k)]}{3(4 - \alpha^2)}, \quad (4)$$

$$p_B^w = \frac{t [12 - 4\alpha (1 + k) + \alpha^2 (1 - 2k)]}{3(4 - \alpha^2)}, \quad (5)$$

$$\tilde{p}_B^w = \frac{t [12 - 8\alpha (2 - k) + \alpha^2 (1 + 4k)]}{3(4 - \alpha^2)}. \quad (6)$$

where the superscript "w" stands for *weak dominance*. It can be easily shown that, given the pricing scheme described above, the market structure corresponds with Figure 1 — i.e., conditions $0 < x_A < k < x_B < 1$ and $0 < x_U < 1$ hold — only if $k < k_1(\alpha) \equiv \frac{12+4\alpha-7\alpha^2}{24-4\alpha-8\alpha^2}$, with $k_1(\alpha) \in [\frac{1}{2}, 1]$. The reason is quite intuitive. Firm A can profitably poach some rival's consumers only if firm B's turf is not too far from firm A in the Hotelling line. Indeed, the price discount that firm A would offer to the unidentified consumers is not large enough to attract some of those consumers identified by firm B. Hence, given the prices (3 – 6), the condition $k < x_B$ does not hold anymore for $k > k_1(\alpha)$.

Accordingly, the equilibrium profits, for $k < k_1$, are

$$\pi_A^w = \frac{t [144 - 48\alpha (1 + k) - 8\alpha^2 (1 - 16k - 8k^2) - 8\alpha^3 (5 + 4k - 10k^2) + \alpha^4 (41 - 80k + 32k^2) + \alpha^5 (1 - 2k)^2]}{18(4 - \alpha^2)^2} \quad (7)$$

$$\pi_B^w = \frac{t [144 - 48\alpha (5 - 4k) - 8\alpha^2 (23 - 32k + 8k^2) + 8\alpha^3 (1 - 16k + 10k^2) - \alpha^4 (7 - 16k - 32k^2) + \alpha^5 (1 - 2k)^2]}{18(4 - \alpha^2)^2} \quad (8)$$

Before concluding that these are equilibrium profits, we need to check for deviations by both firms. In particular, given the prices charged by the rival, does a firm post a couple of prices that induces a different consumers allocation? Clearly, there are two alternative deviation strategies: (i) setting a personalized price to the identified consumers in order to avoid rival's poaching; (ii) setting a price to the unidentified consumers such that the deviating firm does not attract rival's consumers but it extracts more surplus from those consumer who are not

recognized by any firm. While the first option is never optimal both for firm A and firm B,¹² the second deviation strategy may be optimal for firm A.¹³ Specifically, given p_B^w and \tilde{p}_B^w , firm A may have incentive to charge different prices to induce a market structure as illustrated in Figure 2, the one we called *strong dominance*. In such market structure, x^B must not be larger than k and firm A's profit is given by

$$\pi_A^d = p_A^d \alpha x_A(\tilde{p}_B^w) + \tilde{p}_A^d (1 - \alpha) x_U(\tilde{p}_B^w).$$

Maximizing with respect to p_A^d and \tilde{p}_A^d yields the following deviation prices

$$p_A^d = \tilde{p}_A^d = \frac{t [12 - 4\alpha(2 - k) - \alpha^2(1 - 2k)]}{3(4 - \alpha^2)},$$

which induce always the market structure illustrated in Figure 2. Intuitively, firm A may profitably deviate by not offering any price discount to attract rival's consumers, but it prefers to charge a uniform price to all consumers, obtaining the following profits

$$\pi_A^d = \frac{t [12 - 4\alpha(2 - k) - \alpha^2(1 - 2k)]^2}{18(4 - \alpha^2)^2}. \quad (9)$$

Hence, firm A has no incentive to deviate if and only if (7) does not fall short of (9) — i.e., an equilibrium in pure strategies exists as long as

$$k \leq k_2(\alpha) \equiv \frac{36(1 - \alpha) + 17\alpha^2 + \alpha^3 - 3\sqrt{R}}{2\alpha(2 + \alpha)(3 + \alpha)}$$

with $R \equiv 144 - 366\alpha + 280\alpha^2 - 104\alpha^3 + 17\alpha^4 - \alpha^5$. Note that $k_2(\alpha) < k_1(\alpha)$. This result leads to the following proposition.

Proposition 1 *There exists a threshold $k_2(\alpha) \in (\frac{1}{2}, k_1(\alpha))$ such that the weak dominance case has a unique equilibrium in pure strategies for $k \leq k_2(\alpha)$ with $k_2(\alpha)$ increasing in α .*

Proposition 1 shows that poaching occurs on both sides of the market only if firms are almost symmetric — i.e., k is sufficiently close to $\frac{1}{2}$. Otherwise, the dominant firm has incentive to deviate and extract more surplus from those consumers who are not recognized by any firm. Clearly, the region of parameters in which this equilibrium exists is increasing in the degree of information accuracy, other things being equal. Indeed, an increase in the degree of

¹²Details are available upon request.

¹³Being firm A the dominant firm, rival's consumers are more distant for firm A than for firm B. Therefore, the incentives that really matter are those of the dominant firm. If it has no incentive to deviate, the same holds for firm B.

information accuracy has a double effect. On the one hand, it lowers the share of unrecognized consumers and, as a consequence, the expected revenues from this group. On the other hand, it enlarges the share of rival's identified consumers and, therefore, the potential gains from poaching them. This makes deviation less appealing for firm A which has more incentive to attract rival's consumers as the information accuracy increases.

3.2 Strong dominance

Suppose now that firms inherited asymmetric market shares and let firm A be the dominant firm, as illustrated in Figure 2. In particular, we are interested in a situation where k is sufficiently close to 1 to make firm A unwilling to compete for firm B's consumers because their preferences are too close to the product offered by firm B. Similarly to the weak dominance case, we assume that firms can identify only a fraction α of their inherited consumers. The consumer indifferent between being loyal to firm A and switching to firm B is still given by $x_A = \frac{\tilde{p}_B - p_A + t}{2t}$. Also the condition that identifies x_B , $x_B = \frac{p_B - \tilde{p}_A + t}{2t}$, is unchanged. However, this market configuration relies on the assumption that the set of consumers loyal to firm B is given by $[k, 1]$, so that x_B must not be larger than k . Finally, among the $(1 - \alpha)$ consumers not recognized by either firm the indifferent one is still defined by $x_U = \frac{\tilde{p}_B - \tilde{p}_A + t}{2t}$.

The profit functions are given by:

$$\pi_A^s = p_A \alpha x_A + \tilde{p}_A (1 - \alpha) x_U \quad (10)$$

$$\pi_B^s = p_B (1 - k) + \tilde{p}_B \alpha (k - x_A) + \tilde{p}_B (1 - \alpha) (k - x_U) \quad (11)$$

where the superscript "s" indicates strong dominance. The profit function of firm A highlights that it sells to two groups of consumers only, the loyal customers located on the segment $[0, x_A]$ and the customers unrecognized by either firm located on the segment $[0, x_U]$. To find the optimal prices, we use the standard first order conditions for p_A , \tilde{p}_A and for \tilde{p}_B , while p_B , the price set to loyal consumers by firm B, is the highest price, given \tilde{p}_A , that firm B can charge to satisfy the constraint $x_B = k$, i.e., $p_B = \tilde{p}_A + t(2k - 1)$. Then, the optimal prices are:

$$p_A^s = \tilde{p}_A^s = t \frac{[3 - 2\alpha(1 - k)]}{3} \quad (12)$$

$$\tilde{p}_B^s = t \frac{[3 - 4\alpha(1 - k)]}{3} \quad (13)$$

$$p_B^s = 2t \frac{[k(3 + \alpha) - \alpha]}{3} \quad (14)$$

Note that the asymmetric market shares lead to asymmetric price strategies: firm B price discriminates charging a higher price to the loyal consumers, while firm A sets a uniform price. The reason is that, in this market configuration, firm B's turf is far away from firm A and poaching is not profitable for firm A. Conversely, firm B can exploit the closeness to firm A's consumers to price aggressively in order to induce them to switch.

By substituting the prices given by (12), (13) and (14) in the expression for x_A , x_B and x_U it can be easily shown that the above prices are consistent with the assumed market configuration since $1 > k > x_A = \frac{1}{2} - \alpha(\frac{1-k}{3}) > 0$ and $k = x^B$.

Firms' profits are:

$$\pi_A^s = t \frac{(3 - 2\alpha(1 - k))^2}{18} \quad (15)$$

$$\pi_B^s = t \frac{(9 - 12\alpha(2 - 5k + 3k^2) + 4\alpha^2(1 - k)^2)}{18} \quad (16)$$

To verify that (12), (13) and (14) are the equilibrium prices, let us check if firm A has any profitable deviation. In particular, we need to check whether, given p_B and \tilde{p}_B , firm A has an incentive to deviate to induce a market structure with $x_B > k$, i.e. the weak dominance market structure where firm A profit is given by

$$\pi_A = p_A \alpha x_A + \tilde{p}_A (1 - \alpha) x_U + \alpha \tilde{p}_A (x_B - k)$$

Maximizing this profit, given the prices p_B^s and \tilde{p}_B^s determined above, we find:

$$p_A^{sd} = p_A$$

$$\tilde{p}_A^{sd} = t \frac{(2 - \alpha)(3 - 2\alpha(1 - k))}{6}$$

Observe that now poaching occurs and firm A decreases the price to unrecognized consumers ($\tilde{p}_A^d < \tilde{p}_A$) to attract some of firm B customers. We need to verify whether these prices support the weak dominance market configuration where: $1 > x_B > k > x_A$. The values of x_B and x_A when firm A deviates are given by: $x_B = k + \frac{1}{4}\alpha - \frac{1}{6}\alpha^2(1 - k)$ and $x_A = \frac{1}{3}k\alpha - \frac{1}{3}\alpha + \frac{1}{2}$ respectively. It is easy to verify that the relevant constraint is $1 > x_B$ that requires

$$k < k_3(\alpha) \equiv \frac{12 - 3\alpha + 2\alpha^2}{12 + 2\alpha^2}.$$

Hence, the deviation is feasible only when $k < k_3(\alpha)$. Consider now firms profits. Firm A profit when deviation occurs is larger than the alleged "equilibrium" profit:

$$\pi_A^d - \pi_A = \frac{1}{72}t\alpha^2(2k\alpha - 2\alpha + 3)^2 > 0$$

Hence, we can conclude that the deviation, when feasible, is also profitable.

Let us verify now whether, given the prices in (12), firm B has an incentive to deviate. Indeed, if firm B maximizes $\pi_B = \alpha p_B(1 - x_B) + (1 - \alpha)\tilde{p}_B(1 - x_U) + \alpha\tilde{p}_B(k - x_A)$ we obtain:

$$p_B^d = t \frac{(3 - \alpha(1 - k))}{3} \tag{17}$$

$$\tilde{p}_B^d = \tilde{p}_B \tag{18}$$

Note that the conditions $1 > x_B > k > x_A$ are never satisfied at prices (17) and (18) when $k \geq k_3(\alpha)$. Observe also that $k_3(\alpha) > k_2(\alpha)$. Then, we can state the following proposition.

Proposition 2 *There exists a threshold $k_3(\alpha) \in (k_2(\alpha), 1)$ such that the strong dominance case has a unique equilibrium in pure strategies for $k \geq k_3(\alpha)$ with $k_3(\alpha)$ decreasing in α .*

Proposition 2 shows that the strong dominance configuration exhibits an equilibrium only when the inherited market shares are such that firm B's turf is small (k close to 1). In this case, the customers of firm B have strong preference for this brand and it would be very costly for firm A to induce them to switch. When instead the inherited market share of firm B increases (k decreases) the preferences of its consumers are less geared to its brand, and poaching by firm A becomes profitable inducing it to deviate. The negative relationship between $k_3(\alpha)$ and α indicates that the higher is the firms' ability to identify their consumers the more likely is the equilibrium. This follows from the fact that a higher information accuracy, by increasing the profit from the loyal consumers and decreasing the profit from the unrecognized consumers, makes firm A less interested in winning firm B's customers.

3.3 Equilibrium outcomes

This equilibrium analysis allows us to identify the region of parameters in which there exists an equilibrium in pure strategies. Suppose that firms are almost symmetric — i.e., $k \leq k_2(\alpha)$. There exists a unique equilibrium in which firm A is in a position of *weak dominance* and both

firms profitably charge a discounted price to unidentified consumers in order to attract them. By contrast, if firms are highly asymmetric — i.e., $k \geq k_3(\alpha)$ with $k_3(\alpha) > k_2(\alpha)$ — there exists a unique equilibrium in which firm A is in a position of *strong dominance* but only firm B adopts a poaching strategy. It is worth noting that both regions of parameters, which are never overlapping, are increasing in the degree of information accuracy. This result suggests that increasing the ability to recognize consumers enlarges the parameter space in which a pure strategy equilibrium exists. Indeed, for any $k \in (k_2(\alpha), k_3(\alpha))$, it has been proved that firm A has incentive to deviate from both the equilibrium paths identified in the previous sections. This result is stated in the following proposition.

Proposition 3 For any $\alpha \in (0, 1)$:

- If k is sufficiently small ($k \leq k_2(\alpha)$), there exists a unique equilibrium featuring the weak dominance case;
- If k is intermediate ($k \in (k_2(\alpha), k_3(\alpha))$), there are no equilibria in pure strategies;
- If k is sufficiently large ($k \geq k_3(\alpha)$), there exists a unique equilibrium featuring the strong dominance case.

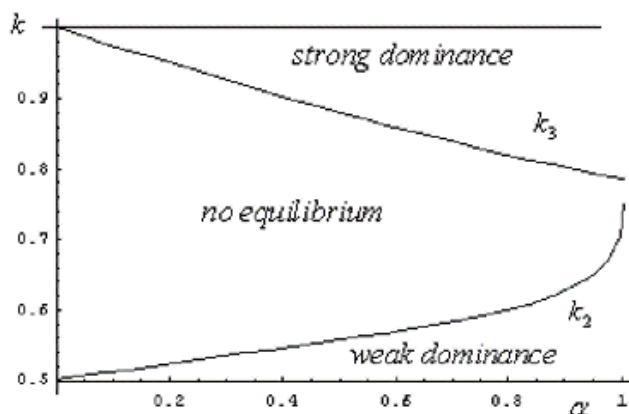


Figure 3: Equilibrium outcome

Figure 3 graphically summarizes all the equilibrium outcomes described in the proposition above. It is evident from the figure that the degree of information accuracy plays a crucial role in identifying the optimal pricing strategies for the competing firms. Consider the weak dominance case. Given a defined size of asymmetry, the dominant firm discriminates and exerts a downward pressure on prices charged to unidentified consumers, which allows to poach

some rival's consumers, only if the information accuracy is sufficiently high. Basically, a high degree of information accuracy allows firm B to charge a high price to a large share of its own turf, making firm A's poaching strategy profitable. Otherwise, firm A would deviate from the equilibrium path, without poaching rival's consumers but charging a uniform price in order to extract more surplus from those consumers who are not identified by any firm.

Consider, instead, the strong dominance case. Given a defined size of asymmetry, the dominant firm does not discriminate but it charges a uniform price to identified and unidentified consumers only if the information accuracy is sufficiently high. Intuitively, being able to recognize a large quota of its own turf leads the dominant firm not to lower the price in order to attract rival's consumers. Conversely, a low degree of information accuracy would lower the gains from its own turf and it would lead the dominant firm to deviate from the equilibrium path by poaching rival's consumers. This highlights a novel tension between market power and information accuracy. Specifically, increasing the degree of information accuracy may or may not lead a dominant firm to poach rival's consumers according to the size of the inherited market shares: when firms are almost symmetric (highly asymmetric), poaching is more (less) likely to occur as the information accuracy increases.

Finally, there exists an intermediate region of parameters in which there are no equilibria in pure strategies. Hence, none of the consumers allocations analyzed above is sustainable in equilibrium because, given firm B's prices, firm A has always incentive to deviate. As a result, only mixed strategy equilibria can exist in this intermediate parameter space. Being prices defined in a continuum set, it is not possible to identify, with some probability, both the prices charged by the firms and the consumers allocation. However, comparing the firm A's equilibrium prices in weak and strong dominance, we can notice that $p_A^s = \tilde{p}_A^s > p_A^w > \tilde{p}_A^w$, which allows us to reasonably infer the price charged by firm A to the unidentified consumers. Specifically, we can reasonably expect that \tilde{p}_A^s is likely to be the maximum price that firm A may charge in order to extract more surplus, while the minimum price is likely to correspond with the marginal cost, which is normalized to zero, in order to induce switching as much as possible. These two extreme cases can represent the least and most competitive scenario respectively. Clearly, these prices cannot represent the equilibrium prices because, given firm A's deviating prices, firm B would be induced to deviate as well.

The existence of a mixed strategy equilibrium could partly explain why firms producing differentiated products, with asymmetric inherited market shares, may adopt different pricing

strategies featuring (or not) personalized offers to loyal consumers and why they change them across time. For instance, in the software industry, Microsoft, Symantec and Adobe offered price discounts to their existing customers, but Adobe, in contrast to its competitors, has also offered lower prices to its rival's consumers in order to attract them (Shaffer and Zhang, 2000). Other examples are in the telecommunication sector, in which firms like Vodafone, Tim and WindTre are used to make special offers to rivals' consumers, but they sometimes charge discounted prices to their loyal customers. Similarly, traditional retailers, like supermarkets, offer personalized discounts to their loyal customers through, for instance, fidelity cards (Choe *et al.*, 2018), but they also send price discounts for online shopping to new prospective consumers.

3.4 Comparative statics

In this section, we analyze the effects of history-based price discrimination on firms' profits and we provide some comparative statics related to the equilibrium outcomes characterized above. Let us start by considering the differences in profits between firm A and firm B. This simple comparison yields the following result.

Proposition 4 *Firm A's profits are lower than firm B's profits both in the weak and strong dominance case.*

Proposition 4 states that, when firms price discriminate among consumers, the dominant firm earns profits that are always lower than its rival, regardless of the size of the market shares. This result, which may seem counter intuitive, can be explained as follows. Inheriting a large market share and the possibility to identify its own turf make the dominant firm able to charge a personalized price to a large share of consumers. However, being the inherited market large, the personalized price does not match each consumer's willingness to pay perfectly. By contrast, inheriting a small market share allows the rival to know better its own turf and each consumer's willingness to pay. This makes poaching rival's consumers more difficult for the dominant firm which is also subject to the competitive pressure of the small firm. Indeed, the latter, with an undercutting logic, can profitably attract a large share of the dominant firm's consumers. Hence, the possibility to identify a quota of the inherited market and to price discriminate benefits the small firm that obtains profits larger than the dominant firm. This is in sharp contrast with the results of Gehrig *et al.* (2012) according to which a dominant firm may obtain larger profits than its rival. Such a difference crucially depends on the size

of the switching costs, which are exogenous and common across consumers:¹⁴ they must be strictly positive in the strong dominance case or sufficiently large in the weak dominance case. Otherwise, the small firm obtains the larger profits. Therefore, if there are no switching costs (or they are negligible), the history-based price discrimination is detrimental for the dominant firms' profits.

This result has a strong policy implication for Antitrust and Regulatory Authorities, which should ensure a fair competition between firms. Our model shows that if there are no barriers to entry or informational frictions, history-based price discrimination allows even small firms to compete fairly and profitably with a dominant firm. This does not exclude that a small firm can gain lower profits than the dominant firm but, if this is the case, it is because of some inefficiencies. Therefore, a public policy aiming to defend small firms or to foster the entrance of new competitors should properly be justified by the presence of some market failures.¹⁵

Now, let us consider the impact of the degree of information accuracy on firms' profits. The equilibrium analysis has shown that firms adopt different strategies according to the size of k , i.e., the degree of asymmetry between firms. In case of weak dominance, both firms poach rival's consumers, while in the strong dominance case, the dominant firm charges a uniform price and it does not attract rival's consumers. These alternative strategies lead to remarkable differences in profits and, consequently, in the impact of the degree of information accuracy. Hence, in answering the question "what is the impact of greater information accuracy on the equilibrium profits?", we need to disentangle the weak and the strong dominance cases. The results of this comparative statics are summarized in the following proposition.

Proposition 5 *Suppose that the degree of information accuracy increases.*

In the weak dominance case:

- *Firm A's profits are decreasing if k is sufficiently small ($k \leq 0.56$), U-shaped if k is intermediate ($k \in (0.56, 0.61)$) and increasing if k is sufficiently large ($k \geq 0.61$);*
- *Firm B's profits are decreasing.*

¹⁴This differs from Shaffer and Zhang (2000), where consumers are not identical and they do not have the same switching costs.

¹⁵This conclusion follows the considerations of Motta (2004) according to which "competition policy should be about *defending competition* and not about *defending competitors*, which are less efficient" (p. 52).

In the strong dominance case:

- *Firm A's profits are decreasing;*
- *Firm B's profits are increasing.*

Proposition 5 allows us to identify the effects of greater information accuracy when firms engage in history-based price discrimination. Specifically, in the weak dominance case, firms' profits are both decreasing in α when firms inherit very similar market shares. This is because more precise information makes competition fiercer, yielding lower equilibrium profits (Thisse and Vives, 1988, Fudenberg and Tirole, 2000). However, if firms are moderately asymmetric in their market shares, the dominant firm's profits are increasing in the degree of information accuracy, except for an intermediate region of k in which they are decreasing when the degree of information accuracy is sufficiently low. The intuition is the following. In the weak dominance case, the dominant firm offers a price discount to the group of "unidentified consumers" and an increase in the degree of information accuracy allows to extract more surplus from its own turf (an increase in α implies charging a higher price to a larger share of "identified consumers"). Clearly, this positive effect on profits dominates a more intense competition on the group of "identified consumers" only if the inherited market share is sufficiently large or, at least, the degree of information accuracy is sufficiently high.

By contrast, the strong dominance case features a monotonic (and opposite) impact of α on firms' profits. Indeed, although all the equilibrium prices are decreasing in α , increasing the degree of information accuracy benefits the small firm which can extract a larger surplus from its "identified consumers", knowing that the dominant firm has no incentive to poach some of its loyal customers. This positive effect on the gains deriving from the "identified consumers" more than compensates the downward pressure on prices charged to the rest of consumers. Instead, by charging a uniform price, the dominant firm does not take an advantage from an increase in the precision of identifying its own turf, which has the unique effect of making the competition tough. Hence, an increase in the degree of information accuracy enlarges the difference in profits for the benefit of the small firm.

Now, by using Proposition 3 and 5, we can briefly discuss the effect of price discrimination on firms' profits with respect to a uniform pricing regime. Indeed, by setting $\alpha = 0$, our model boils down to a standard Hotelling game in which firms cannot recognize their own turf and charge a uniform price. While in the weak dominance case both firms' profits are

negatively affected by the possibility to price discriminate,¹⁶ in the strong dominance case only the dominant firm suffers such a negative effect. Indeed, as shown in Proposition 5, firm B's profits are increasing in α in the strong dominance case. These results are in contrast with those of Gehrig *et al.* (2011), in which only one firm (the incumbent) could recognize its own turf and the unique differences in profits between a price discrimination and a uniform regime derive from the emergence of switching costs. This can be attributed to the fact that, in their model, the incumbent has previously served the entire market — which corresponds to the limit case $k = 1$ —, while we consider a more general setting in which both firms can acquire information and the dominant firm inherits a share of consumers.

Finally, we conclude this section by comparing our results with those of Shaffer and Zhang (2000) and Gehrig *et al.* (2012). Let us start from the former who show, however, some differences in the baseline model. Specifically, they assume that firms' market shares are asymmetric, as in our setup, but each consumer has a defined degree of loyalty toward one of the two firms and, in particular, toward the one he/she belongs to. By contrast, in our model, only the distribution along the Hotelling line leads the relative preference toward one of the products on sale, regardless the past purchase behavior. In other words, in Shaffer and Zhang (2000), the consumers who have bought from firm j in the past are *assumed* to be more loyal to that firm than to the rival, whereas we do not make such an assumption. This proves why our results (with $\alpha = 1$) converge to their findings only if market shares are symmetric. Nevertheless, there are some common qualitative insights with our analysis. When firms inherit asymmetric market shares and price discrimination is viable, firms tend to lower their prices to induce rivals' customers to switch, although history-based price discrimination decreases both firms' profits if consumers show the same degree of loyalty. Moreover, in spite of the differences mentioned above, this marketing practice appears always more profitable for the small firm rather than the dominant firm. This is because price discrimination allows to compete more aggressively for rival's turf and the larger the own turf, the greater the negative effect on the own profit.

Now, let us discuss the relationship between our results and those of Gehrig *et al.* (2012). In the weak dominance case, considering an imperfect information technology, we generalize their model when there are no switching costs. Indeed, the equilibrium prices and profits

¹⁶Note that when firm A's profits are increasing in α , the weak dominance structure exists only for sufficiently large values of α . Hence, it is not possible to make such a comparison (with $\alpha = 0$) in those regions of parameters.

identified in our model — i.e., with imperfect information — do tend to those characterized by Gehrig *et al.* (2012) — i.e., with perfect information — if we simplify our setup by setting $\alpha = 1$. Yet, this is not true in the strong dominance case. Our model highlights a point of discontinuity in $\alpha = 1$. This is not surprising because the presence of even very small degrees of uncertainty in recognizing the inherited market shares change the firms' pricing strategies dramatically. This difference can be explained as follows. With perfect information, the dominant firm charges a poaching price \tilde{p}_A equal to the marginal cost — i.e., zero in our setup — because there are no consumers that both firms are unable to recognize. Therefore, charging a price larger than the marginal cost does not increase the dominant firm's profit, then there is no incentive to deviate. Instead, the small firm charges a personalized price to its own turf given $\tilde{p}_A = 0$. This is the unique equilibrium when $\alpha = 1$. However, it is sufficient to introduce some degrees of uncertainty, such that α is strictly lower than 1, that the dominant firm's pricing strategy changes. The existence of a group of "unidentified consumers" changes its profit function and explains why the dominant firm has incentive to charge prices which are always larger than its marginal cost. Moreover, while the equilibrium pricing strategy for the dominant firm does not imply price discrimination if $\alpha < 1$, as shown in the equilibrium analysis, the equilibrium prices feature a history-based price discrimination if $\alpha = 1$ (with $\tilde{p}_A = 0$). Therefore, there is no reason why \tilde{p}_A must tend to zero for α that goes to 1, because of such a discontinuity in the profit function and in the optimal pricing strategy. Nevertheless, in the strong dominance case, the dominant firm does not poach rival's consumers and its profits actually tend to those identified by Gehrig *et al.* (2012). Instead, as shown in Figure 4, this does not occur for the equilibrium profits of the small firm.

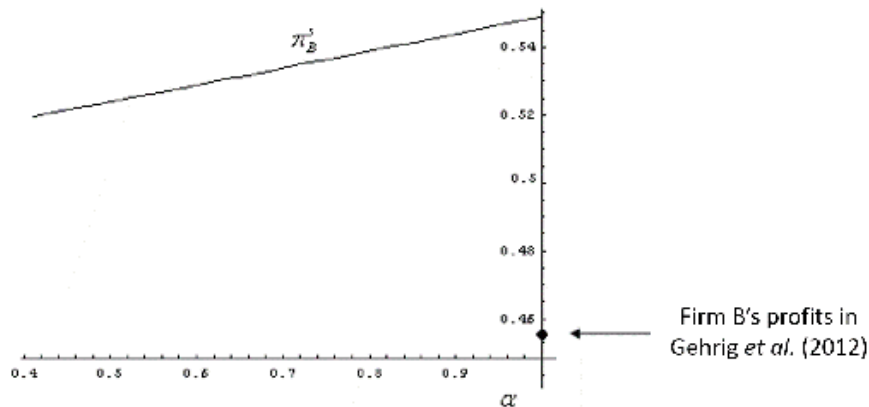


Figure 4: The impact of uncertainty on firm B's profit

Setting $\alpha = 1$ represents a very special case in which the dominant firm exerts a very competitive pressure on prices without decreasing its profit. This leads the small firm to lower the price charged to its own turf, with a negative effect on its profit. Introducing uncertainty by setting $\alpha < 1$ softens competition and allows the small firm to earn larger profits always. For this reason the small firm's profits in our model can not tend to those in Gehrig *et al.* (2012) as α goes to 1. This puts more emphasis on our model which seems to be more than a simply generalization of Gehrig *et al.* (2012), but it helps in characterizing the equilibrium outcomes in a more realistic competitive environment.

4. Welfare Analysis

In this section, we discuss the impact of a marginal increase in the degree of information accuracy on consumer surplus and social welfare. The final goal is to characterize the optimal regulatory regimes from these two points of view, where the latter is simply the sum of industry profits and consumer surplus. To identify the optimal enforcement, we need to explicit the consumer and the social welfare. Following the scheme of analysis adopted so far, we disentangle the weak and strong dominance cases.

In the weak dominance case, the consumer surplus is:

$$\begin{aligned}
 CS^w = & v - \alpha \left[\underbrace{\int_0^{x_A^w} (p_A^w + tx) dx + \int_{x_A^w}^k (\tilde{p}_B^w + t(1-x)) dx + \int_k^{x_B^w} (\tilde{p}_A^w + tx) dx + \int_{x_B^w}^1 (p_B^w + t(1-x)) dx}_{\text{Identified Consumers}} \right] \\
 & - (1 - \alpha) \left[\underbrace{\int_0^{x_U^w} (\tilde{p}_A^w + tx) dx + \int_{x_U^w}^1 (\tilde{p}_B^w + t(1-x)) dx}_{\text{Unidentified Consumers}} \right]
 \end{aligned}$$

which can be rewritten as follows

$$CS^w = v - \frac{t \left[720 - 144\alpha(5 - 4k + 4k^2) + 8\alpha^2(43 - 64k + 64k^2) + 8\alpha^3(11 - 80k + 80k^2) - \alpha^4(35 - 32k + 32k^2) - \alpha^5(1 - 76k + 76k^2) \right]}{36(4 - \alpha^2)^2}. \quad (19)$$

Note that an increase in the degree of information accuracy has a non-monotone effect on consumer surplus. Specifically, the consumer surplus is increasing in α if firms are almost symmetric ($k \leq 0.55$); it is inverse U-shaped if firms are moderately asymmetric ($k \in [0.55, 0.62]$); it is decreasing in α if firms are sufficiently asymmetric ($k \geq 0.62$). Therefore, the consumer surplus starts decreasing when the competitive advantage of the dominant firm is sufficiently large but poaching occurs on both sides of the market.

By considering also the industry profits, we can compute the social welfare

$$\begin{aligned}
SW^w &= \pi_A^w + \pi_B^w + CS^w \\
&= v - \frac{t \left[\begin{aligned} &144 - 144\alpha(1-2k)^2 - \alpha^2(8 + 256k - 256k^2) + \\ &+ 8\alpha^3(19 - 40k + 40k^2) - \alpha^4(103 - 160k + 160k^2) - \alpha^5(5 - 92k + 92k^2) \end{aligned} \right]}{36(4 - \alpha^2)^2} \quad (20)
\end{aligned}$$

which is decreasing in α . This implies that, although consumer surplus is increasing in α when firms are not too asymmetric, the negative effect on the industry profits more than compensates the potential benefit for consumers.

Now, let us consider the strong dominance case. The consumer surplus is

$$\begin{aligned}
CS^s &= v - \alpha \left[\underbrace{\int_0^{x_A^s} (p_A^s + tx) dx + \int_{x_A^s}^k (\tilde{p}_B^s + t(1-x)) dx + \int_k^1 (p_B^w + t(1-x)) dx}_{\text{Identified Consumers}} \right] \\
&\quad - (1-\alpha) \left[\underbrace{\int_0^{x_U^s} (\tilde{p}_A^s + tx) dx + \int_{x_U^s}^1 (\tilde{p}_B^s + t(1-x)) dx}_{\text{Unidentified Consumers}} \right]
\end{aligned}$$

which can be rewritten as follows

$$CS^s = v - \frac{t \left[45 - 72\alpha(1-k)^2 + 20\alpha^2(1-k)^2 \right]}{36}. \quad (21)$$

It is worth noting that, in the strong dominance case, the consumer surplus is increasing in α . This is because, in that market structure, an increase in the degree of information accuracy makes the competition tough with a decreasing effect on the equilibrium prices.

Instead, the impact of α on the social welfare confirms the same result found in the weak dominance case. In fact, the social welfare is

$$\begin{aligned}
SW^s &= \pi_A^s + \pi_B^s + CS^s \\
&= v - \frac{t \left[9 + 4\alpha^2(1-k)^2 \right]}{36} \quad (22)
\end{aligned}$$

which is decreasing in α . As in the weak dominance case, the negative effect of α on industry profits outweighs the positive impact on consumer surplus in the region of parameters characterizing a strong dominance case.

In a nutshell, greater information accuracy always reduces the social welfare. However, a Public Authority aiming at protecting consumers should be aware that the impact of information accuracy on consumer surplus may differ according to the size of asymmetry between firms and the consequent market structure. In particular, an increase in α is detrimental for

consumers only in the weak dominance case when firms are sufficiently asymmetric, otherwise it is beneficial.

5. Extensions

This section provides two main extensions of the baseline model. First, we show how the presence of a Stackelberg competition between competing firms allows to identify pure strategy equilibria for any combination of the parameters. Second, we point out that the qualitative insights of the baseline model do not change if we remove the hypothesis of symmetry between firms in the precision of identifying their own consumers. This proves that our results are robust to several extensions and it allows us to provide some general policy implications for Regulatory and Antitrust Authorities, which are applicable to different competitive environments.

5.1 Stackelberg Competition

In Section 3.3, we have shown that a pure strategy equilibrium does not exist for certain parameters. In this section, we tackle this issue by switching to Stackelberg competition between the two firms. Namely, we consider the case where the dominant firm, firm A, moves first, whereas the smaller firm, firm B, moves second. Indeed, as shown above, the non-existence of a pure strategy equilibrium in case of simultaneous moves is due to the incentive of the dominant firm to deviate in order to induce a different market structure given the rival's prices. Clearly, when the dominant firm sets the prices by anticipating the rival's best-reply (Stackelberg leadership) this incentive disappears.

We consider first the case of weak dominance. In this case, firm B maximizes (2), yielding the following best-reply functions

$$p_B^w(\tilde{p}_A) = \frac{t + \tilde{p}_A}{2}, \quad (23)$$

$$\tilde{p}_B^w(p_A, \tilde{p}_A) = \frac{\tilde{p}_A(1 - \alpha) + p_A\alpha + t[1 - 2\alpha(1 - k)]}{2}. \quad (24)$$

Plugging (23) and (24) into (1) and maximizing with respect to p_A and \tilde{p}_A , we get the following first-stage equilibrium prices of the dominant firm

$$p_A^w = \frac{t[6 - 4\alpha - (1 - 2k)\alpha^2]}{2(2 - \alpha^2)}, \quad (25)$$

$$\tilde{p}_A^w = \frac{t[6 - 4\alpha(1 - k) + \alpha^2]}{2(2 - \alpha^2)}. \quad (26)$$

By using the equilibrium prices just defined, we can easily observe that the conditions $1 > x_B > k > x_A > 0$ and $x_U > 0$ are satisfied if and only if $k \leq k_4(\alpha) \equiv \frac{6+4\alpha-7\alpha^2}{16-4\alpha-8\alpha^2}$.¹⁷ As for the case of simultaneous moves, the only relevant constraint is $x_B > k$. Therefore, when $k > k_4$, the incumbent does not poach the rival's consumers. The explanation is akin to the one illustrated in the baseline model.

The equilibrium profits derive immediately from the substitution of (25) and (26) into (23), (24) and the profit functions (1) and (2).¹⁸ Now, we check for deviations. Clearly, being firm A the first mover in the game, it has no incentive to deviate. Instead, firm B might deviate in the second period in order to induce a different market structure given the prices set by the dominant firm in the first period — i.e., p_A^w and \tilde{p}_A^w . Suppose that firm B deviates to induce the strong dominance market structure. In other words, given p_A^w and \tilde{p}_A^w , firm B maximizes (11) with respect to \tilde{p}_B^w , while p_B^w is such that $x_B = k$. This yields the following deviation prices

$$p_B^d = \frac{t [2 - 4\alpha + 3\alpha^2 + 4k(2 - \alpha - \alpha^2)]}{2(2 - \alpha^2)}, \quad (27)$$

$$\tilde{p}_B^d = \frac{t [10 - 4\alpha(3 - k) + \alpha^2(1 - 4k) + 2\alpha^3(1 - k)]}{4(2 - \alpha^2)}. \quad (28)$$

It can be observed that deviation is always feasible — i.e., the conditions $1 > x_B = k > x_A > 0$ and $x_U > 0$ are always satisfied — but it is never profitable for firm B. Indeed, given the deviation prices, we can immediately derive the deviation profits and we can compare them with the candidate equilibrium profits. It can be easily shown that the deviation profits are lower than the candidate equilibrium profits for any k and α in the region of parameters under analysis.

Next, we consider the case of strong dominance. Now, firm B maximizes (11), yielding the following best-reply functions

$$p_B^s(\tilde{p}_A) = \tilde{p}_A - t(1 - 2k), \quad (29)$$

$$\tilde{p}_B^s(p_A, \tilde{p}_A) = \frac{\tilde{p}_A(1 - \alpha) + p_A\alpha + t[1 - 2\alpha(1 - k)]}{2}. \quad (30)$$

It is worth noting that, while the best-reply price in the own turf varies between the cases of weak and strong dominance, the best-reply poaching price is always the same. Moving in the first stage, the dominant firm maximizes (10) by anticipating $p_B^s(\tilde{p}_A)$ and $\tilde{p}_B^s(p_A, \tilde{p}_A)$, yielding

¹⁷Note that this requires that $\alpha \geq 1 - \frac{\sqrt{3}}{3}$. Indeed, $k_4 < \frac{1}{2}$ if $\alpha < 1 - \frac{\sqrt{3}}{3}$.

¹⁸The complete expressions of the equilibrium profits are quite long and, therefore, they are omitted. They are available upon request.

the following first-stage equilibrium prices

$$p_A^s = \tilde{p}_A^s = \frac{t[3 - 2\alpha(1 - k)]}{2}. \quad (31)$$

Firm A does not price discriminate when it does not poach consumers from the rival (see Section 3.2). First, it can be noted that the conditions $1 > x_B = k > x_A > 0$ and $x_U > 0$ are always satisfied. By using the equilibrium prices, the equilibrium profits are easily derived.¹⁹ Finally, we check for the deviation of firm B, that might want to induce the weak dominance market structure when firm A plays (31) in the first stage. In this case, the deviation prices of firm B are

$$p_B^d = \frac{t[5 - 2\alpha(1 - k)]}{4}, \quad (32)$$

$$\tilde{p}_B^d = \frac{t[5 - 6\alpha(1 - k)]}{4}. \quad (33)$$

However, it can be observed that the weak dominance market structure, which requires $1 > x_B > k > x_A > 0$ and $x_U > 0$, is never sustained at the deviation prices. Therefore, deviation is never feasible. This allows us to state the following result.

Proposition 6 *When firms compete à la Stackelberg and the dominant firm moves first, there exists a unique equilibrium in pure strategies that features weak dominance if and only if $k \leq k_4(\alpha)$ and strong dominance otherwise.*

Figure 5 represents graphically the equilibrium outcomes described in Proposition 6 when firms compete à la Stackelberg. It can be easily observed that there always exists a pure strategy equilibrium for any value of k and α in the region of parameters under analysis. Moreover, a simple comparison of the equilibrium profits shows that the small firm's profits are larger than the dominant firm's profits both in the weak and the strong dominance case. This confirms the results in case of simultaneous moves without loss of generality.

Finally, we conclude this section by comparing the relationship between the degree of information accuracy and firms' profits in a sequential game with that identified in the baseline model. Interestingly, the effect of α on the dominant firm's profits features the same paths described in a simultaneous move game both in the weak²⁰ and the strong dominance case.

¹⁹The complete expression is omitted.

²⁰In the weak dominance case, there are only mild differences in the thresholds of k above and under which the effect of α on firm A's profit changes. Specifically, for $k \leq 0.54$, firm A's profits are decreasing; they are U-shaped for $k \in [0.54, 0.63]$; they are increasing otherwise.

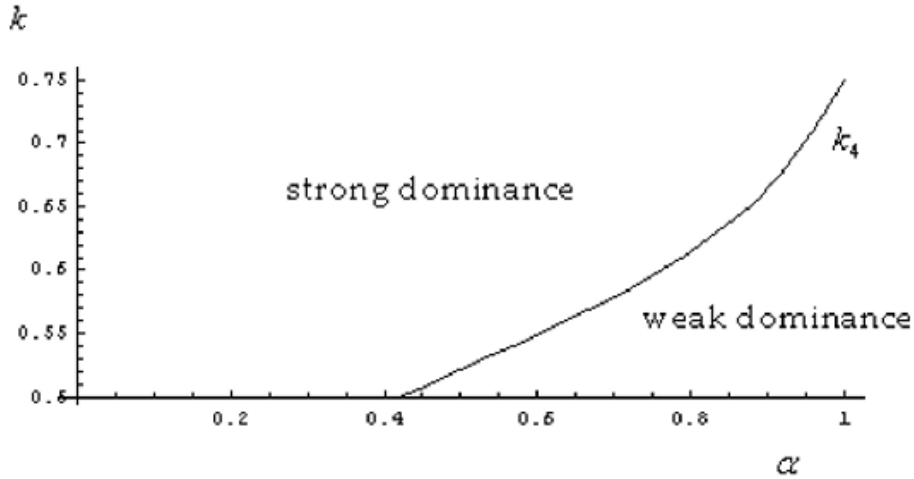


Figure 5: Equilibrium outcome with a Stackelberg leader

Instead, the effect of α on the smaller firm's profits is different than in the baseline model. Indeed, being the smaller firm a second mover in the Stackelberg game, it is subject to a competitive disadvantage which affects its own profits. In the weak dominance case, increasing the degree of information accuracy has a negative effect on firm B's profit if firms are almost symmetric — i.e., k is sufficiently close to $\frac{1}{2}$ — as in the baseline model. However, if firms are moderately asymmetric, firm B's profit is U-shaped in an intermediate region of k and increasing in α for k sufficiently large (but small enough such that the weak dominance case continues to be an equilibrium).²¹ In the strong dominance case, instead, firm B's profit is increasing in α , as in the baseline model, only if firms are sufficiently asymmetric — i.e., k is sufficiently close to 1. By contrast, when firms are almost symmetric, firm B's profit is decreasing in α , while it is U-shaped in an intermediate region of k .²² Yet, we need to be cautious in making this comparison because we are discussing the effect of an increase in the degree of information accuracy in different competitive environments, characterized by simultaneous or sequential moves, which influence, in turn, the creation of different market structures — i.e., weak or strong dominance cases.

²¹For the sake of completeness, in the weak dominance case, firm B's profit is decreasing for $k \leq 0.68$, U-shaped for $k \in [0.68, 0.71]$ and increasing for $k \in [0.71, 0.75]$.

²²Similar to the previous case, when the market structure features a strong dominance, firm B's profit is decreasing for $k \leq 0.65$, U-shaped for $k \in [0.65, 0.68]$ and increasing for $k \in [0.68, 1]$.

5.2 Asymmetric Information Accuracy

In this section we relax the assumption that the firms have the same information accuracy, $\alpha_B = \alpha_A = \alpha$, and we provide some insights on the effect on pricing strategies of a different degree of information about consumers. Specifically, we want to verify whether the pricing schedules derived in a symmetric setting are robust to the introduction of informational advantages. The firms may have different information accuracy even when operating in the same industry because often the information about own customers is the result of past decisions reflecting the particular history the firm. Different ability to identify own consumers may result, for example, from the decision to use different information technologies or from different distribution channels.

We focus on two limit cases where firm B has either a perfect knowledge or no knowledge at all of its consumers while we maintain the assumption that the dominant firm has less than perfect information.

Case 1. Firm B has an information advantage with respect to A

Let us assume that firm B, having a small market share, could easily recognize all its consumers. By contrast, firm A can not recognize all its consumers because of a large market share. Hence, we analyze here a setting where firm B is perfectly able to identify all its customers ($\alpha_B = 1$), while the dominant firm A is endowed with an imprecise level of accuracy ($\alpha_A < 1$). As before, we distinguish between the weak and strong dominance cases according to size of inherited market shares.

Let consider first the weak dominance case where firms have inherited similar market shares. Firms profits are:

$$\begin{aligned}\pi_A^w(\alpha_A < 1, \alpha_B = 1) &= p_A \alpha_A x_A + \tilde{p}_A (x_B - k) + \tilde{p}_A (1 - \alpha_A) x_U \\ \pi_B^w(\alpha_A < 1, \alpha_B = 1) &= p_B (1 - x_B) + \tilde{p}_B \alpha_A (k - x_A) + \tilde{p}_B (1 - \alpha_A) (k - x_U)\end{aligned}$$

Given $\alpha_B = 1$ firm B has a particularly large set of identified consumers to whom it may charge a high price. At the same time firm A can poach a large share of firm B consumers. The prices obtained by maximizing the above profit functions²³ show that both firms price discriminate and charge a discounted price to unidentified consumers. It is worth noting that all prices are lower than the corresponding prices determined in the symmetric weak dominance

²³They are available upon request.

case ($\alpha_B = \alpha_A = \alpha$). This suggests that a different degree of information accuracy intensifies competition. Indeed, there is a positive relationship between all prices and α_A that confirms this. It indicates that when the ability to identify the consumers of the two firms gets closer, firm A extracts a higher surplus from its loyal consumers and decreases the discount offered to firm B's consumers, softening competition. This in turn allows firm B to increase its own prices.

These prices sustain the weak dominance market configuration²⁴ when $k \leq \frac{28-19\alpha_A}{44-28\alpha_A}$.

Firms profits are equal to

$$\pi_A^w(\alpha_A < 1, \alpha_B = 1) = \frac{t(2k^2(32 + 41\alpha_A - 28\alpha_A^2) - 2k(128 - 106\alpha_A + 23\alpha_A^2) + 91\alpha_A^2 - 302\alpha_A + 256)}{9(5\alpha_A - 8)^2},$$

$$\pi_B^w(\alpha_A < 1, \alpha_B = 1) = \frac{t(2k^2(17\alpha_A^2 - 76\alpha_A + 104) + 2k(7\alpha_A^2 + 28\alpha_A - 80) + 61\alpha_A^2 - 224\alpha_A + 208)}{9(5\alpha_A - 8)^2}.$$

By checking for deviations, it can be verified that firm B has no profitable deviations, while firm A has an incentive to deviate for large values of k . Then, the equilibrium requires

$$k \leq k_5(\alpha_A) = \frac{161\alpha_A - 178 - 37\alpha_A^2}{2Z} - \frac{15}{2} \sqrt{\frac{Y}{Z}},$$

where $Z = (32\alpha_A^2 - 61\alpha_A - 7)$ and $Y = 2(64 - \alpha_A(176 - 177\alpha_A)) - 5\alpha_A^3(31 + 5\alpha_A)$.

Intuitively, firm A can profitably deviate only when poaching firm B's consumers is not profitable and this in turn requires that k is sufficiently large. When instead k is close to $1/2$, firm A prefers the weak dominance structure similarly to the case with symmetric information accuracy.

Let turn now to the strong dominance case where firm B customers are far away from firm A because k is large. This market configuration is characterized by $x_B = k$ and firms profits are:

$$\pi_A^s(\alpha_A < 1, \alpha_B = 1) = p_A \alpha_A x_A + \tilde{p}_A (1 - \alpha_A) x_U$$

$$\pi_B^s(\alpha_A < 1, \alpha_B = 1) = p_B (1 - k) + \tilde{p}_B \alpha_A (k - x_A) + \tilde{p}_B (1 - \alpha_A) (k - x_U)$$

The prices obtained by maximizing the above profit functions show that firm A charges a uniform price, while firm B price discriminates, similarly to the case with symmetric information accuracy. Observe that, given $\alpha_B = 1$, firm B has a particularly large set of loyal customers to

²⁴This means that the following conditions must be satisfied: $1 \geq x_B \geq k \geq x_A \geq 0$ and $1 \geq x_U \geq 0$

whom it can charge a high price without fearing the competition of firm A that is too distant from firm B's customers.

Firms profits are:

$$\begin{aligned}\pi_A^s(\alpha_A < 1, \alpha_B = 1) &= \frac{1}{18}t(2k+1)^2, \\ \pi_B^s(\alpha_A < 1, \alpha_B = 1) &= \frac{1}{18}t(-32k^2 + 52k - 11).\end{aligned}$$

Firm B benefits from its ability to identify all customers, and its profit is higher than those obtained in the strong dominance setting when both firms have the same (imperfect) information accuracy. Conversely, firm A is worse off. As expected, the dominant firm has lower profit than firm B and such a difference is larger than the one with $\alpha_A = \alpha_B = \alpha < 1$. This finding is not surprising and Proposition 5 has already highlighted the opposite impact of α on firm profit in the strong dominance configuration.

Before concluding that these are the equilibrium profits, we need to verify whether there are profitable deviations. Let first check firm A. As we may expect, given its low profit in this setting, firm A is better off offering a discounted price to unidentified consumers in order to induce the weak dominance configuration that is feasible when $k < k_6(\alpha_A) = \frac{23-12\alpha_A}{26-12\alpha_A}$. The deviation profit of firm A is always higher than the equilibrium profit in this range of values, so that the deviation, when feasible, is always profitable. Thus, the equilibrium requires $k \geq k_6(\alpha_A)$. Lastly, by looking at the incentive to deviate for firm B it can be verified that for small values of k also firm B has an incentive to decrease the price to loyal consumers in order to induce the weak dominance configuration. This reduction has a twofold effect: it reduces the profit obtained from these consumers but, at the same time, it increases the number of loyal consumers and therefore it may be profitable when the second effect dominates the first.

The following proposition summarizes the discussion so far.

Proposition 7 *In the asymmetric case with firm B enjoying perfect information about its customers, $\alpha_B = 1$, there exist two threshold values $k_5(\alpha_A)$ and $k_6(\alpha_A)$ such that the weak dominance case has a unique equilibrium in pure strategies for $k \leq k_5(\alpha_A)$ and the strong dominance case has a unique equilibrium in pure strategies for $k \geq k_6(\alpha_A)$.*

The Proposition shows that, similarly to the symmetric case, when firm B has an information advantage over the dominant firm A, an equilibrium in pure strategies requires a small value of k for the weak dominance case and a large one for the strong dominance configuration.

Note that $k_6(\alpha_A)$ is a decreasing function of α_A , so that the closer is the information accuracy of the two firms, the more likely the equilibrium. The intuition is that firm B benefits from its perfect information technology and this increases the difference in firms' profits making firm A deviation profitable for a large range of parameter values. A higher value of α_A reduces the asymmetry between the two firms, increases the profit of firm A and, as a result of this, mitigates the incentive to deviate.

Case 2. Firm A has an information advantage

Consider now the opposite case where firm B cannot distinguish among its consumers, $\alpha_B = 0$, while firm A has an imperfect information accuracy $0 < \alpha_A < 1$. This configuration can be justified by the presence of a dominant firm which has accumulated enough financial resources to invest in a more sophisticated informational tools, allowing it to recognize at least a part of its past consumers, and a small firm which does not own such a technology. Lacking any information about its consumers firm B can charge only a uniform price, \tilde{p}_B . As usual, we distinguish two market structures, the weak dominance with k close to $\frac{1}{2}$ and the strong dominance with k close to 1.

In the weak dominance case firm profits are:

$$\begin{aligned}\pi_A^w(\alpha_A < 1, \alpha_B = 0) &= p_A \alpha_A x_A + \tilde{p}_A (1 - \alpha_A) x_U + \tilde{p}_A \alpha_A (x_U - k) \\ \pi_B^w(\alpha_A < 1, \alpha_B = 0) &= \tilde{p}_B (1 - x_U) + \tilde{p}_B \alpha_A (k - x_A)\end{aligned}$$

The prices obtained by maximizing these functions indicate that firm A price discriminates and offers a discounted price to the unidentified consumers. Firms profit are equal to

$$\begin{aligned}\pi_A^w(\alpha_A < 1, \alpha_B = 0) &= \frac{1}{18} \frac{t}{\alpha_A + 1} (9(1 + k^2 \alpha_A^3) + \alpha_A^2 (1 + 4k(k - 1)) + 6\alpha_A (1 - 2k)), \\ \pi_B^w(\alpha_A < 1, \alpha_B = 0) &= \frac{1}{18} \frac{t}{\alpha_A + 1} (3 - \alpha_A + 2k\alpha_A)^2.\end{aligned}$$

In the range of k values that satisfy the requirement for this market configuration ²⁵, firm A may profitably drop the discount to unidentified consumers and charge a uniform price to induce the strong dominance configuration. It can be verified that firm A equilibrium profit is higher than the deviation profit only if $k \leq k_7(\alpha_A) \equiv \frac{9+9\alpha_A+2\alpha_A^2-3(3+4\alpha_A+\alpha_A^2)\sqrt{1-\alpha_A}}{\alpha_A(3+13\alpha_A+9\alpha_A^2)}$. Firm B instead has no profitable deviations in the relevant range of values.

²⁵This market structure requires $k < \frac{3+\alpha_A}{6+2\alpha_A-3\alpha_A^2}$.

Consider now the strong dominance case where firm A inherits a large share of the market so that k is close to 1 and in particular $k > x_U$. Firms profits are:

$$\begin{aligned}\pi_A^s(\alpha_A < 1, \alpha_B = 0) &= p_A \alpha_A x_A + \tilde{p}_A (1 - \alpha_A) k \\ \pi_B^s(\alpha_A < 1, \alpha_B = 0) &= \tilde{p}_B (1 - k) + \tilde{p}_B \alpha_A (k - x_A)\end{aligned}$$

and both firms set a uniform price equal to t , the same price that would result if no firm were able to identify its consumers. The resulting profits are $\pi_A = \pi_B = t/2$ as in the standard Hotelling model with uniform prices. However, if k is not too large, firm A can deviate and offer a discounted price $\tilde{p}_A^d = t(1 - k\alpha_A)$ to unidentified customers while still charging $p_A = t$ to the identified ones. Doing this, firm A can induce the weak dominance market structure where it can profitably poach some of firm B's consumers. Thus, to avoid firm A deviation, k must be sufficiently close to 1 and the outcome with uniform prices equal to t is indeed an equilibrium only if $k \geq k_8(\alpha_A) = \frac{1 - \sqrt{1 - \alpha_A}}{\alpha_A}$. Instead, firm B has no feasible deviation in this setting.

Observe that the firm A benefits from its information advantage. Contrary to the symmetric case, where the dominant firm A has lower profit than firm B both in the weak and in the strong dominance case, here it has higher profit in the weak dominance case if k is sufficiently close to 1/2 and profit equal to B in the strong dominance case. The results are summarized in the following proposition.

Proposition 8 *In an asymmetric setting where firm B cannot recognize its own turf ($\alpha_B = 0$) there exist two thresholds $k_7(\alpha_A)$ and $k_8(\alpha_A)$ such that the weak dominance case has a unique equilibrium in pure strategies for $k \leq k_7(\alpha_A)$, while the strong dominance case has an equilibrium in pure strategies for $k \geq k_8(\alpha_A)$.*

Finally, let compare our findings with those of Gehrig *et al.* (2011) who analyze a situation in which an entrant competes with an incumbent firm that has imperfect information about its consumers. Of course, the entrant has no previous customers and it charges a uniform price while the incumbent can price discriminate. The authors find that for a sufficiently high switching cost the incumbent can maintain its dominant position despite entry. If we simplify their model by setting the switching cost equal to zero, our model provides the same result in the strong dominance case where prices and profits are the same as with the uniform pricing scheme. However, our model does not rely on the assumption that the small firm is new entrant

(that is, $k = 1$). Our setting generalizes their results to the case where $k < 1$ and it allows us to analyze also the weak dominance case where firm A discriminates.

Concluding remarks

The analysis of the two cases where firms have asymmetric information accuracy confirms the robustness of the firms pricing strategies found in section 4. In the weak dominance case firm A, irrespective of any informational advantage, always discriminates and charges a discounted price to unidentified consumers in order to poach some rival's customers. Conversely, in the strong dominance case firm A always finds more profitable to charge a uniform price without poaching firm B's consumers.

The asymmetry in the information available to the firms allows us to analyze the impact of different degrees of information accuracy on firms profits. We have already pointed out that while in the symmetric case firm A has always a lower profit than firm B, now the relationship between their profits depends on which firm has a more accurate information. By comparing the firms' profit under the different information structures it is possible to rank them.

In the weak dominance case the "best situation" for firm A is always the symmetric one where firms have the same degree of information accuracy. The ranking between the other two cases depends on the values of k and the level of information accuracy of firm A. When firm A has a relatively low information accuracy, the ranking is:

$$\pi_A^w(\alpha_A = \alpha_B = \alpha) \geq \pi_A^w(\alpha_A < 1, \alpha_B = 0) \geq \pi_A^w(\alpha_A < 1, \alpha_B = 1),$$

while as firm A level of information accuracy grows larger it becomes:

$$\pi_A^w(\alpha_A = \alpha_B = \alpha) \geq \pi_A^w(\alpha_A < 1, \alpha_B = 1) \geq \pi_A^w(\alpha_A < 1, \alpha_B = 0).$$

Note that firm A profit is higher the more symmetric is the situation, so when α_A is large, firm A is better off when firm B has full information about its customers and viceversa when α_A is small, firm A is better off when firm B has no information about its customers.

The ranking for the rival is the opposite and the symmetric information case, that is the "best case" for firm A, coincides with the "worst case" for firm B, yielding the lowest level of profit. Moreover, the preference over the other cases depends on the level of information accuracy of firm A. When α_A is low we have:

$$\pi_B^w(\alpha_A < 1, \alpha_B = 0) \geq \pi_B^w(\alpha_A < 1, \alpha_B = 1) \geq \pi_B^w(\alpha_A = \alpha_B = \alpha),$$

while when α_A is high it becomes:

$$\pi_B^w(\alpha_A < 1, \alpha_B = 1) \geq \pi_B^w(\alpha_A < 1, \alpha_B = 0) \geq \pi_B^w(\alpha_A = \alpha_B = \alpha).$$

The ranking changes in the strong dominance case but firms' preferences are always opposite, as it emerges from the following ranking:

$$\begin{aligned} \pi_A^s(\alpha_A < 1, \alpha_B = 0) &\geq \pi_A^s(\alpha_A = \alpha_B = \alpha) \geq \pi_A^s(\alpha_A < 1, \alpha_B = 1), \\ \pi_B^s(\alpha_A < 1, \alpha_B = 1) &\geq \pi_B^s(\alpha_A = \alpha_B = \alpha) \geq \pi_B^s(\alpha_A < 1, \alpha_B = 0). \end{aligned}$$

Note that, in the strong dominance case each firm is better off when the information accuracy is different. However, as expected, they have opposite preferences on who should have the information advantage.

6. Conclusions

In this article, we contributed to the growing debate about the collection and use of consumers' personal data, often referred as *big data*, in order to tightly target personalized pricing offers. Although collecting, processing and exploiting personal data by firms is often seen as a matter of consumer protection and privacy laws, they have strong implications on the results of a competitive game. The OECD (2018) has identified several concerns about the possibility to price discriminate among consumers through data analytics and pricing algorithms based on the information previously collected. Price discrimination may lead to a consumer and welfare loss especially when it is enforced by a dominant firm. Hence, according to the OECD, an anti-competitive effect is more likely to occur in the presence of an asymmetric market. This has pushed us to investigate the impact of the degree of information accuracy about consumers' past purchase behavior on the equilibrium outcomes in a market characterized by asymmetric market shares.

Building on the literature dealing with history-based price discrimination (see Gehrig *et al.*, 2011 and 2012, and Shaffer and Zhang, 2000), we design an asymmetric duopoly and we introduce some uncertainty about consumers' past purchase histories. Following Colombo (2016), we consider a market in which information is incomplete in the sense that firms can recognize only a quota of their own consumers and they may charge a different price to their own turf. This allowed us to highlight a novel aspect of the relationship between information accuracy, asymmetric market shares and price discrimination.

We find that a dominant firm has incentive to poach rival's consumers only if the asymmetry in the inherited market shares is not too high. Otherwise, it charges a uniform price. By contrast, the small firm has always incentive to offer a discounted price to those consumers not belonging to its own turf. Yet, the existence of such pure strategy equilibria hinges crucially on the degree of information accuracy and the asymmetry between firms. We identify two non-overlapping regions of parameters, characterizing a pure strategy equilibrium, that allow us to disentangle two distinct cases (weak and strong dominance). The regions of parameters in which weak and strong dominance cases exist are increasing in the degree of information accuracy, highlighting the relevance of detecting the available information to characterize the equilibrium outcomes.

Interestingly, the effect of a greater information accuracy on firms' profits is ambiguous and non-monotonic. Specifically, we argue that the effect is negative (positive) on the small firm's profits in the weak (strong) dominance case. Conversely, the impact on the dominant firm's profits depends on the size of asymmetry. In the weak dominance case, dominant firm's profits are decreasing, U-shaped or increasing for a small, intermediate and large size of asymmetry respectively. Instead, in the strong dominance case, the dominant firm's profits are decreasing in the degree of information accuracy. Moreover, in accordance with the most prominent literature on history-based price discrimination, we find that uniform price tends to soften competition, while price discrimination makes the competition tougher. Said this, for any degree of information accuracy, the small firm's profits are always the larger, given a better knowledge of its own turf, being smaller than the dominant firm's one, and its consumers' willingness to pay. This result has a clear and straightforward policy implication: when there is no ban to price discrimination and information about consumers can be easily collected, there is no reason to defend a small firm in the absence of market failures (e.g., informational frictions or barriers to entry).

Finally, our analysis implies a discussion on the impact of increasing the level of information accuracy on consumer surplus and total welfare. We show that, while total welfare is always decreasing in the degree of information accuracy, the effect on consumer surplus varies depending on the size of asymmetry. Increasing the information accuracy is harmful for consumers only in the weak dominance case and if firms are moderately asymmetric. Otherwise, the effect on consumer surplus is positive. Hence, a Public Authority aiming at fostering the protection and safeguard of consumers should encourage the collection and use of consumers'

personal data with the final goal of charging personalized prices. Clearly, this policy address does not take into account all the other objective that a Public Authority could pursue — such as maximizing total welfare, protecting consumers' personal data, hindering discriminatory practices. However, our analysis sheds some doubts on the actual (policy) attitude to protect consumers' data through privacy laws and guidelines that do not consider the parallel effect on competition.

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