

Loumeau, Gabriel

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Locating Public Facilities: Theory and Micro Evidence from Paris

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Poschingerstr. 5, 81679 Munich, Germany

Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de

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Locating Public Facilities: Theory and Micro Evidence from Paris

Abstract

This paper proposes a novel approach to evaluate location decisions for public facilities. The approach addresses, not only the standard distance-minimizing problem, but also the endogenous location decisions of individuals. To do so, I develop a quantifiable general equilibrium model with endogenous (residential and commercial) densities, housing prices, commutes to work and public facilities, as well as public facility characteristics. The latter includes a facility's location, quality, district and capacity. I apply the framework to secondary schools in Paris' greater region at a 1km² geographical scale. The analysis reveals that the observed location decisions made between 2001 and 2015 underestimate the endogenous reaction of individuals. A more decentralized strategy is predicted to increase welfare growth by 10 percentage points on average, mostly via shorter commutes and lower housing prices.

JEL Codes: R530, H110, R120.

Keywords: location, facility, general equilibrium, commuting, interrupted search.

Gabriel Loumeau
Department of Management, Technology, and Economics
ETH Zurich
Leonhardstrasse 21
Switzerland – 8092 Zurich
loumeau@ethz.ch

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1 Introduction

How to choose the location of public facilities? Naturally, minimizing the distance between users and facilities is crucial for the facilities’ accessibility and efficiency. In the case of health care services, [Buchmueller, Jacobson, and Wold \(2006\)](#) and [Nicholl, West, Goodacre, and Turner \(2007\)](#) show that proximity to a hospital strongly influences the chances of recovery after an accident. Looking at educational services, shorter travel distances imply shorter daily commutes and higher attendance rates ([Card, 1993](#); [Frenette, 2006](#)).¹ However, travel distance to a public facility is not the only factor to consider. Individuals react to the location of facilities in deciding their own residential location. Typical evidence of such an endogenous reaction to facility location is the capitalization of public goods provision into housing prices.² Hence, to answer the question of where to locate public facilities appropriately, one needs to analyze not only the pure distance-minimizing problem, but also how individuals react to facility locations.

Despite widespread policy interest in this question, the literature has not yet investigated the problem of locating public facilities in a comprehensive manner. To fill this gap, I propose a quantifiable spatial general equilibrium model of public service location to evaluate location decisions.³ In the model, the central planner decides on a location strategy, i.e., *location, quality, district and capacity* for all public facilities, while observing the location’s characteristics and anticipating how individuals and firms will react. I then apply this framework to Paris’ greater region (“Ile-de-France”) – at a 1km² geographical scale – to evaluate 54 new secondary school location decisions made between 2001 and 2015. To derive a suitable set of comparison strategies for the observed one, I propose and apply a novel interrupted search algorithm. As key feature, this algorithm accepts local welfare optima to constitute a robust comparison set. Overall, I find that the observed location decisions appear to ignore the endogenous reactions of individuals and firms. This leads to significant inefficiencies in terms of density, housing prices and commuting times.

In this paper, I start by developing a framework combining state-of-the-art approaches from urban and public economics. While the spatial characteristics are modeled following a

¹Longer distances to public services also induce several adverse indirect effects (see, among others, [Alvarez-Pedrerol, Rivas, López-Vicente, Suades-González, Donaire-Gonzalez, Cirach, de Castro, Esnaola, Basagana, Dadvand, Nieuwenhuijsen, and Sunyer, 2017](#) and [Liu, Ma, Liu, Han, Chuang, and Chuang, 2015](#)).

²Early literature includes [Tiebout \(1956\)](#), [Oates \(1969\)](#), and [Yinger \(1982\)](#), among others. See [Reback \(2005\)](#), [Cellini, Ferreira, and Rothstein \(2010\)](#), and [Fack and Grenet \(2010\)](#) for more recent estimates of the magnitude of such capitalization effects.

³In the remainder of the paper, I will refer to a “facility” as a particular establishment (e.g., a specific school) and to a “service” as the full set of facilities of a particular type (e.g., all schools).

framework à la [Ahlfeldt, Redding, Sturm, and Wolf \(2015\)](#), it combines this approach with a model of endogenous public good provision along several dimensions, i.e., location, quality, capacity and districts. Overall, the model innovates on several fronts. First, the framework distinguishes individuals depending on their usage, or not, of the public service. Second, individuals decide on a triplet of locations (i.e. residence, workplace and public facility) in order to maximize their utility. Floor prices and wages are then endogenously determined. Third, the model accounts for differences in the quality of public facilities, which affects the willingness of individuals to locate nearby. Finally, a location strategy for public facilities is an outcome of the model, rather than an exogenous location fundamental.

The evaluation of any observed location strategy – as for many other public policies – requires defining a suitable set of strategies which can then be used as comparison point. Defining this set using only the welfare maximizing strategy is likely to be too restrictive. This follows from the fact that the derivations of both, the observed and the welfare maximizing strategies suffer from inherent limitations. For instance, planner’s computational capacity may be limited, data may suffer from random mis-measurement, or general equilibrium frameworks may rely on stronger or weaker functional form assumptions. To circumvent these limitations and inspired by recent work in computer science to evaluate strategic gerrymandering for the US Supreme Court ([Cho, 2019](#); [Liu and Cho, 2020](#)), I propose an interrupted search algorithm. This approach evaluates a large number of randomly-chosen different location strategies and retains a few locally optimal ones.⁴ As such it attempts to represent the choice set faced by the actual planner when deciding on the observed location strategy. As the set of local optima may contain very different strategies, this algorithm is more robust to the limitations listed above than a standard welfare maximizing approach. Yet, as it retains only few good local optima, it is restrictive enough to notice if the observed strategy fails to improve regional welfare.

I then apply this framework to the case of secondary schools in Paris’ greater region (“Ile-de-France”). The choice of the application is directed by the availability of precise data on public facilities over time, residential densities, as well as on bilateral residential-workplace and residential-facility commutes. The size and the quality of the data used in the application stands out. The analysis is conducted using micro-spatial data, in which each cell has a size of 1km². As an example of public services, the use of French “Collèges”, i.e., lower secondary schools, offers several crucial advantages. First, there are no fundamental differences across

⁴Precisely, 165 such good local optima are derived after evaluating 27,500 different locations strategies.

public lower secondary schools (i.e., the same national degree is offered, teachers' salaries do not vary by location, teachers must pass the same qualifying exams, and state spending per pupil is equal across facilities). Second, as students take the same national exam at the end of the cycle, the school success rate can be used as a measure of quality, which is naturally comparable across facilities. Third, due to the centralized financing of the secondary school system, there are (almost) no spatial differences in the amount that individuals pay for schooling. Fourth, the location of private secondary schools has little effect on the location strategy of public secondary schools. This is due to, both, the location strategy adopted by private secondary schools, and their generally non-secular nature. Lastly, I can precisely identify the pupils in the secondary schools, since education is compulsory in France until the age of 16.⁵

In the calibration exercise, I pay particular attention to the estimation of two key effects: (i) commuting costs for public service users and non-users separately; (ii) the peer effect affecting facility quality. To obtain commuting elasticities to work and to school, I estimate micro-founded gravity equations for both types of individuals.⁶ To retrieve the peer effect affecting facility quality, I adopt an instrumental variable approach using the dominant wind direction in Paris' region as instrument for local residential wages. Historically, dominant wind direction (from West to East) have led to the unequal spatial distribution of air pollutants; and consequently, to a clear West-East wage pattern (Heblich, Trew, and Zylberberg, 2016).

After successfully testing for the validity of the model, three key results appear. First, the endogenous reaction of individuals in their location decisions is underestimated – if not ignored – by the observed location strategy between 2001 and 2015. The observed strategy is well rationalized by a simple distance minimizing approach taking residential density as exogenous. The welfare maximizing strategies, however, encourage a more decentralized approach by building significantly more secondary schools outside of Paris metropolitan area. Second, accounting for the endogenous reactions of individuals is predicted to increase welfare growth by 10 percentage points on average. Finally, the analysis of the differences across strategies further reveals significant inefficiencies in residential densities, commutes and housing prices due to the observed location decisions made between 2001 and 2015. For instance, within 30km of Paris city center, the observed strategy inefficiently raises housing prices up to 10 €/m² and commutes to facilities by up to 30%.

⁵In France, the average pupil attends lower secondary education between the ages of 11 and 15.

⁶This implies relying – at the municipal level – on 2,342,271 trilateral links for users, and 132,997 bilateral links for non-users.

Looking at these findings in the context of the urban-rural divide provides further policy implications. In many countries, among which the United States, the United Kingdom, Germany, and France,⁷ representatives of rural areas point to the increasing centralization of the public services as the source of rural decline. The central government often counters by reversing the causality, i.e., centralization reforms do not cause rural decline, it is rural decline that renders centralization reforms necessary. The findings in this paper tend to agree with the direction of causality supported by representatives of rural areas.

The paper proceeds as follows. In Section 2, I briefly discuss the related literature. I describe the model and its main assumptions in Section 3 and the interrupted search algorithm in Section 4. In Section 5, I introduce the data. I calibrate the model to fit the economic geography of Paris greater region and assesses its validity in Section 6. I evaluate the observed location strategy between 2001 and 2015 in Section 7. Section 8 discusses the limitations of using optimality as a policy metric, and briefly outlines an approach robust to these limitations. Finally, Section 9 concludes.

2 Contribution to the literature

This paper relates to the literature in various ways. In its general intention, this paper is part of a recent strand of the literature that has approached economic geography questions from an optimal design perspective. Examples of such work are [Fajgelbaum and Schaal \(2020\)](#), who study the optimal design of a transport network, [Albouy, Behrens, Robert-Nicoud, and Seegert \(2019\)](#), who investigate optimal city size, and [Allen, Arkolakis, and Li \(2015\)](#), who look at the optimal structure of a city. In its approach, this paper builds upon recent developments in urban economics (see, among others, [Ahlfeldt et al., 2015](#)), which applies quantitative spatial general equilibrium models in rich geographic contexts.

The specific question of the endogenous location of discrete public facilities has received relatively little attention in economic research.⁸ [Fujita \(1986\)](#) and [Thisse and Wildasin \(1992\)](#) were the first to analyze the endogenous location problem in the context of an urban area. In these models, the location decisions of firms and households are a function of the location of

⁷Link to media coverage in the US ([NY times, 2018](#)), the UK ([The Guardian, 2019](#)), Germany ([Süddeutsche Zeitung, 2019](#)) and France ([Le Monde, 2016](#)).

⁸The case of transport infrastructure constitutes an exception. As a key determinant of both trade and commuting costs, the relationship between transport infrastructure and the spatial distribution of economic activity has received significantly more attention (see, among others, [Baum-Snow, 2007](#); [Redding, Sturm, and Wolf, 2011](#); [Fajgelbaum and Schaal, 2020](#); [Donaldson, 2018](#)).

the public facility.⁹ However, compared to this paper, these analyses remain theoretical. In fact, most of the work on the issue at hand comes from operations research and the question has been to solve various forms of the classical Fermat-Weber location problem.¹⁰ This body of literature often takes the location of individuals (or costumers) as well as land prices as given; hence, ignoring the endogeneity of individuals' location decisions. [Thisse and Zoller \(1983\)](#) provide a comprehensive review of the literature on public facility location at the intersection of economics and operations research.

More generally, abstracting from the question of the location of public facilities at discrete points, a large literature has analyzed how local amenities and public goods determine individuals' residential locations (see, among others, [Schuler, 1974](#); [Turnbull, 1989](#); [Bénabou, 1993, 1996](#); [Brueckner, Thisse, and Zenou, 1999](#)). Recently, [Gaigné, Koster, Moizeau, and Thisse \(2017\)](#) focus on within-city sorting of income-heterogeneous individuals with exogenously given amenities. Naturally, this paper also relates to the literature on the capitalization effects of public good provision. The capitalization of a better tax-benefit linkage into housing prices has been described in early literature ([Tiebout, 1956](#); [Oates, 1969](#); [Hamilton, 1976](#); [Yinger, 1982](#)). More recently, [Reback \(2005\)](#), [Cellini et al. \(2010\)](#) and [Fack and Grenet \(2010\)](#) have focused on estimating the magnitude of the capitalization effect into property prices. [Epple, Gordon, and Sieg \(2010\)](#) propose a multi-jurisdiction public finance model with location amenities.

With secondary schools as the subject of the application, this paper also relates to the literature studying education systems in a general equilibrium context. [Epple and Romano \(1998\)](#) study the provision of education with competition between public and private schools. [Epple, Romano, and Sieg \(2006\)](#) and [Epple, Romano, Sarpça, and Sieg \(2017\)](#) focus on higher education in the U.S. context. This literature has generally focused more on the interaction between tax-financed public provision and tuition-financed private provision and how these affect the quality and level of provision, rather than on the question of the location of facilities.

Finally, this paper also contributes to the general discussion about urban structure formation by focusing on a relatively understudied type of agglomeration force, namely the provision of public services.¹¹ In addition to the more standard agglomeration forces studied in the

⁹More recently, [Berliant, Peng, and Wang \(2006\)](#) and [Bellettini and Kempf \(2013\)](#) have also modeled the location problem by accounting for the endogeneity of individual locations.

¹⁰For a review of the facility location problem literature in Operations Research, see [ReVelle and Eiselt \(2005\)](#) and [Farahani and Hekmatfar \(2009\)](#).

¹¹See [Fujita, Krugman, and Venables \(2001\)](#) for an overview of the theoretical literature on urban structures and [Overman, Redding, and Venables \(2001\)](#) for an overview of the empirical literature on the same topic.

literature on New Economic Geography following [Krugman \(1991\)](#), this paper emphasizes the importance of public services in explaining the formation of core-periphery structures. In a way, the reasoning presented in this paper follows the line of classical economic geography theories, such as the Central Place Theory developed by [Christaller \(1933\)](#), which focuses on the provision of services to surrounding areas in order to explain the existence of cities.

3 Theoretical framework

Consider an area that occupies a closed and bounded subset S of a two-dimensional Euclidean space. A location is a point in S . There are two sets of locations: $I \in S$ city blocks and $K \in S$ public facility locations. Each block i has an effective supply of floor space L_i which can be used for residential or commercial purposes. The fraction of floor space used commercially or residentially are denoted θ_i and $1 - \theta_i$, respectively. I further consider two types of individuals: users and non-users of the public service. Users of the public service, denoted by \cdot^F , value both the quality and the proximity of a given facility. Conversely, non-users, denoted by \cdot^E , are indifferent to the quality and distance to public facilities. The area is populated by a total of $H = H^F + H^E$ individuals. Each individual is endowed with one unit of labor each, which they supply inelastically.

3.1 Individual utility and location choices

Users of the public service

The utility of individual o of type F living in place i , working in place j , and using a facility in site k is linear in an aggregate consumption index C_{ijko}^F , such that: $U_{ijko}^F = C_{ijko}^F$. This consumption index depends on consumption of the single final good (c_{ijko}), consumption of residential floor space (l_{ijko}), the utility from residential amenities in i (B_i), the disutility from commuting from residential place i to public facility location k ($d_{ik} \geq 1$), the disutility from commuting from residential location i to workplace j ($d_{ij} \geq 1$), the quality g_k of public facility k , and an idiosyncratic shock that is specific to the individual and varies with the individual's location choice (z_{ijko}). The disutility from commuting from place i to place k is modeled as an iceberg cost $d_{ik} = e^{\kappa \rho_{ik}} \in [1, \infty)$, which increases with the Euclidean distance ρ_{ik} between places i and k . Similarly, the disutility of commuting from residential location i to workplace j is defined as $d_{ij} = e^{\eta \rho_{ij}} \in [1, \infty)$, where ρ_{ij} is the distance between i and j . The aggregate

consumption index is assumed to take the following Cobb-Douglas form:

$$C_{ijk o}^F = z_{ijk o} \frac{B_i}{d_{ik} d_{ij}} \left(\frac{c_{ijk o}^F}{\beta} \right)^\beta \left(\frac{l_{ijk o}^F}{1 - \beta} \right)^{1-\beta} g_k, \quad 0 < \beta < 1. \quad (1)$$

The idiosyncratic shock ($z_{ijk o}$) describes the heterogeneity in utility that individuals derive from living in i , working in j , and using the public service in k . For each individual, this idiosyncratic component is drawn from an independent Fréchet distribution following [McFadden \(1974\)](#) and [Eaton and Kortum \(2002\)](#):

$$F(z_{ijk o}) = e^{-z_{ijk o}^{-\epsilon}}, \quad (2)$$

where $\epsilon > 1$ is the shape parameter that controls the dispersion of idiosyncratic utility.

After observing their realization of the idiosyncratic utility, individuals choose their place of residence, their workplace, and their public facility to maximize their utility. Assuming the price of the final good to be the numeraire, I have $c_{ijk o}^F = \beta w_j$, and $l_{ijk o}^F = (1 - \beta) \frac{w_j}{Q_i}$. w_j is the income received by individuals working in j and Q_i is the housing price in location i . Utility maximization implies that a fraction $(1 - \beta)$ of worker's wages is spent on residential land. To incorporate the benefits of rents to landowners, I suppose that a share $(0 < \iota < 1)$ of the rents is redistributed lump sum to all local residents. The remaining share $(1 - \iota)$ is accrued by absentee landlords and therefore not spent within the city. Hence, workers' wages $w_j = \bar{w}_j + \frac{\iota \sum_{i=1}^S (1 - \theta_i) L_i Q_i + \theta_i L_i q_i}{H}$, where \bar{w}_j is the income from work and L_i is the local floor space. Substituting into (1), I obtain the following indirect utility function:

$$U_{ijk o}^F = \frac{B_i z_{ijk o} w_j Q_i^{\beta-1} g_k}{d_{ik} d_{ij}}. \quad (3)$$

Non-users of the public service

Similarly, the aggregate consumption index of individual o of type E living in place i and working in place j is:

$$C_{ij o}^E = z_{ij o} \frac{B_i}{d_{ij}} \left(\frac{c_{ij o}^E}{\beta} \right)^\beta \left(\frac{l_{ij o}^E}{1 - \beta} \right)^{1-\beta}. \quad (4)$$

Following the same reasoning as for public service users, I obtain the following indirect utility function for non-users:

$$U_{ij o}^E = \frac{B_i z_{ij o} w_j Q_i^{\beta-1}}{d_{ij}}. \quad (5)$$

Individuals' location choices

Define the attractiveness – also sometimes labeled mean utility – of an ijk -triplet for public service users as $v_{ijk}^F = \frac{B_i w_j g_k}{d_{ik} d_{ij} Q_i^{1-\beta}}$. Thanks to the monotonic relationship between the indirect utility (3) and the idiosyncratic component of utility (z_{ijk}), I derive the probability that public service users choose a particular combination of residence, workplace and public service location:¹²

$$\pi_{ijk}^F = Pr[u_{ijk}^F \geq \max\{u_{ijk}^F\}; \forall i, j, k] = \frac{(v_{ijk}^F)^\epsilon}{\sum_{r=1}^I \sum_{s=1}^I \sum_{t=1}^K (v_{rst}^F)^\epsilon}. \quad (6)$$

Similarly, define the attractiveness of an ij -pair for non-users as $v_{ij}^E = \frac{B_i w_j}{d_{ij} Q_i^{1-\beta}}$. The probability that non-users of the public service choose a particular combination of residence, workplace and public service location is then:

$$\pi_{ij}^E = Pr[u_{ij}^E \geq \max\{u_{ij}^E\}; \forall i, j] = \frac{(v_{ij}^E)^\epsilon}{\sum_{r=1}^I \sum_{s=1}^I (v_{rs}^E)^\epsilon}. \quad (7)$$

3.2 Production

I assume a single final good is traded without costs within the metropolitan area. Production occurs under perfect competition and constant returns to scale. In their employment decisions, firms are indifferent between service users and non-users. For simplicity, I assume the following Cobb-Douglas production function:

$$y_j = A_j \left(H_{Mj} \right)^\alpha \left(L_{Mj} \right)^{1-\alpha}, \quad (8)$$

where y_j is the output of the final good in workplace j , A_j is the final goods productivity, H_{Mj} is the total workplace employment, and L_{Mj} is the land used.

Firms choose their place of production, their level of employment, and their level of floor space consumption. From the first order conditions for profit maximization, the number of workers working in j is:

$$H_{Mj} = \left(\frac{\alpha A_j}{\bar{w}_j} \right)^{\frac{1}{1-\alpha}} L_{Mj}. \quad (9)$$

Therefore, employment in place j is increasing in productivity (A_j), decreasing in wages (\bar{w}_j), and increasing in commercial land use (L_{Mj}). The equilibrium commercial floor price is

¹²For full derivation, see Section C of the Appendix. In what follows, I will refer to residential locations using indices i and r , to workplace locations using indices j and s , and to facility locations by k and t .

determined by the zero profit condition along with profit maximization:

$$q_j = (1 - \alpha) \left(\frac{\alpha}{\bar{w}_j} \right)^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}}. \quad (10)$$

3.3 Districts and public facility quality

This paper attempts to provide a general structure to analyze public facility locations. Hence, public facilities are characterized by a *location*, a *capacity*, a *quality* and a *district*. Whereas capacity has indirectly already been defined by the sum over residential and commercial locations of π_{ijk}^F in (6), this section models the quality and districts of public facilities. Finally, their location is the key optimization variable which is discussed in Section 3.5.

Ignoring districts when discussing public location strategies leaves out a crucial determinant of public facilities (see, among others, [Cellini et al., 2010](#)). In this paper, I model districts as a probabilistic function of inverse distance. Beyond its simplicity appeal, this choice appears to be a suitable representation of the world for at least two reasons. First, as in many real world case such as the one in the application, users are likely to use the nearest facility but districts are rarely an absolute assignment. Second, it creates convex areas around public facilities, which is what is mostly observed. Formally, consider that the probability to attend a public facility is inversely related to the distance between residence location i and facility location k . Formally, I define the assignment variable a_{ik} as:

$$a_{ik} = \left(\frac{1/\rho_{ik}}{\sum_{k=1}^K 1/\rho_{ik}} \right)^\xi. \quad (11)$$

ξ in (11) is a device for smoothing the underlying distance minimizing objective, see [Iyigun and Ben-Israel \(2010\)](#) and the seminal article by [Teboulle \(2007\)](#).

I model differences in public facility quality as arising from differences in peer-effects. The assignment weighted residential income represents the possible benefits from local peer-effects. This modeling choice is, for instance, in line with the observation that pupils in schools located in richer neighborhoods perform better at national exams. Beyond possible differences in school financing or teachers' education, this is due to the double fact that care about education of the kids rises with the income of the parents, and that kids of richer parents inherit a cultural background which allows them to perform better at national exams (see, among others, [Fack and Grenet, 2010](#)). Formally, the quality of a public facility in k , g_k , is then function the sum

over all residential locations of the local residential wages, w_i , weighted by the assignment, a_{ik} , as follows:

$$g_k = \left(\sum_{i=1}^I w_i a_{ik} \right)^\lambda \bar{g}_k. \quad (12)$$

\bar{g}_k is a local random unobserved shifter with $\mathbb{E}[\bar{g}_k] = 0$. It captures, for instance, a particularly competent (or not) staff.

3.4 Land market

The fraction of floor space used for commercial purposes is $\theta_i \in (0, 1)$. Given positive residential and productive fundamentals across all locations, no location is fully specialized at the equilibrium. Therefore, in equilibrium, $q_i = Q_i$ in all locations. In the model, land prices are determined by the individuals' and firms' location choices.

As is often done in the urban literature, I consider that floor space L is provided by a competitive construction sector. I follow [Epple et al. \(2010\)](#) and [Ahlfeldt et al. \(2015\)](#) and assume that the production function takes the Cobb-Douglas form: $L_i = M_i^\mu N_i^{1-\mu}$, where M is capital and N is land. The corresponding dual cost function for floor space is then $Q_i = \mu^{-\mu} (1 - \mu)^{-(1-\mu)} P^\mu R_i^{1-\mu}$, where Q_i is the price for floor space, P is the common price for capital, and R_i is the price for land. Since the price for capital is the same across all locations, the relationships between the quantities and prices for geographical land is $L_i = \phi_i N_i^{1-\mu}$ and for floor space is $Q_i = \chi R_i^{1-\mu}$. $\phi_i = M_i^\mu$ determines the density of development and χ is a constant. The clearing condition for total demand for space can then be expressed as: $\phi_i N_i^{1-\mu} = (1 - \theta_i) L_i + \theta_i L_i$.

The residential land market clearing condition can then be written as:

$$(1 - \beta) \frac{\mathbb{E}[w_j | i]}{Q_i} H_{Ri} = (1 - \theta_i) L_i. \quad (13)$$

The commercial land market clearing imposes:

$$\left(\frac{(1 - \alpha) A_j}{q_j} \right)^{1/\alpha} H_{Mj} = \theta_j L_i. \quad (14)$$

3.5 Central planner

The central planner decides on the location and capacity of public facilities, under consideration of location characteristics and anticipating how individuals and firms will react. To model the fact that facilities are often built on publicly owned land, I assume that the central planner does not consume land (see case study in Section B). Note also that, in order to properly investigate the location side of the problem, I deliberately abstract from the financing side by considering the number of facilities as exogenous. This is done in order to avoid entering lengthy discussions on the appropriate level of public spending, as well as discussions on the appropriate capacity of a public facility.¹³

For brevity, I refer to any planner's combined choice as a *location strategy*. The planner's objective is to maximize the aggregate welfare, of *both* users and non users, associated with providing the public service. Formally, the planner decides on a location strategy by choosing the set of geographic coordinates $\mathbf{k} = [\tilde{k}_1, \dots, \tilde{k}_K]$ of the facilities.¹⁴ Hence, the planner's maximization problem is:¹⁵

$$\begin{aligned} \max_{\tilde{k}_1, \dots, \tilde{k}_K} \mathbb{E}[u] = & \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right) \left[\left(\sum_{r=1}^I \sum_{s=1}^I \sum_{t=1}^K (d_{rt} d_{rs} Q_r^{1-\beta})^{-\epsilon} (B_r w_s g_t)^\epsilon \right)^{\frac{1}{\epsilon}} \right. \\ & \left. + \left(\sum_{r=1}^I \sum_{s=1}^I (d_{rs} Q_r^{1-\beta})^{-\epsilon} (B_r w_s)^\epsilon \right)^{\frac{1}{\epsilon}} \right]. \end{aligned} \quad (15)$$

$\mathbb{E}[u]$ represents the expected utility level in the area. The first term in the main bracket represent the expected utility of public service users, whereas the second the expected utility of non-users. Γ is the Gamma function.

3.6 Equilibrium conditional on a location strategy

Conditional on a location strategy, the general equilibrium of the model can be referenced by the following seven vectors $\{\boldsymbol{\pi}^F, \boldsymbol{\pi}^E, \mathbf{Q}, \mathbf{q}, \mathbf{g}, \mathbf{w}, \boldsymbol{\theta}\}$.

¹³See [Afsa \(2014\)](#) for an empirical analysis of the various effects of a facility's capacity using the French secondary education facilities.

¹⁴Note that, while k indexes a facility, \tilde{k}_k refers to its coordinates.

¹⁵For full derivation of the expected utility in the area, see Section D.

Proposition 1 *Assuming strictly positive, finite, and exogenous characteristics, there exist unique general equilibrium vectors $\{ \pi^F, \pi^E, Q, q, g, w, \theta \}$.*

PROOF: See the proof of Proposition 1 in Section E.

Q.E.D.

4 Interrupted Searches

To assess the performance of a central planner, one is faced with the difficulty of defining a suitable set of successful policies, which may then serve as comparison point for the observed one. Including in this set the globally optimal policy *only* is too restrictive for at least two general reasons.

First, it appears reasonable to allow a small margin of error to the central planner, especially in problems with many different local optima. To illustrate this point in the context of this paper, suppose that it is globally welfare maximizing (i.e., optimal) to locate the next facility in the North-East of the considered area. However, further suppose that locating a facility in the South-East is only slightly worse, whereas choosing any location in the West is much worse. In such context, it appears reasonable and prudent to acknowledge that a central planner choosing the South-East is actually performing well at its task.

Second, there are inherent limitations to predicting accurately the true welfare effects of any given policy. Such limitations typically include random data mis-measurement, or the reliance on structural assumptions in any applied modeling work. Even if these limitations remain small (i.e., if data depicts well the observed world despite marginal mis-measurement, or if the framework does capture the key mechanisms at play), one can never be certain that the derived optimal policy is indeed better than a slightly sub-optimal one.

Overall, these remarks imply that the evaluation strategy is required to be generous enough to acknowledge slightly sub-optimal strategies, but not more. With this in mind, I propose an *interrupted search algorithm* to derive a suitable comparison set to evaluate the observed location strategy. This algorithm is inspired by recent work at the frontier between Law and Computer Science to evaluate strategic gerrymandering for the US Supreme Court (Cho, 2019; Liu and Cho, 2020).¹⁶ Based on a global-search numerical method that builds upon

¹⁶Cho (2019) proposed to draw a large number of electoral partition as an impartial planner may (i.e., dividing

standard simulated annealing technique, the algorithm searches for a large number of local optima starting from randomly chosen location strategies.¹⁷ The optimization is, *on purpose*, stopped early to avoid convergence to the global optima. The goal is then to compare the large set of derived local optima to the observed strategy. Provided that the framework captures the key mechanisms at play, the observed location strategy is likely to be similar to one or more local minima if the central planner did attempt to maximize welfare.

Formally, the algorithm proceeds as follows.

Interrupted search algorithm

1. Define stopping criteria for the inner and outer loop, s_i and s_o , respectively. Define $a < 1$ as the convergence criteria that controls the speed of convergence of the simulated annealing algorithm in the inner loop.
2. Set an iteration count for the inner loop, $iter_o := 1$.
3. Randomly select initial locations \mathbf{k}^l .
4. Set an iteration count for the inner loop, $iter_i := 1$.
 - (a) Compute the welfare, $\mathbb{E}[u]_{\mathbf{k}^l}$, associated to \mathbf{k}^l . Set the initial “temperature” of the system T to some arbitrary threshold.
 - (b) Obtain a new candidate strategy \mathbf{k}^{l+1} by perturbing \mathbf{k}^l . Compute the associated welfare, $\mathbb{E}[u]_{\mathbf{k}^{l+1}}$.
 - (c) Accept the new candidate strategy if the corresponding $\mathbb{E}[u]_{\mathbf{k}^{l+1}}$ is such that $\mathbb{E}[u]_{\mathbf{k}^{l+1}} > \mathbb{E}[u]_{\mathbf{k}^l}$ or, with probability $\frac{1}{1 + \exp(\frac{\mathbb{E}[u]_{\mathbf{k}^{l+1}} - \mathbb{E}[u]_{\mathbf{k}^l}}{T})}$, accept a new strategy \mathbf{k}^{l+1} for which $\mathbb{E}[u]_{\mathbf{k}^{l+1}} < \mathbb{E}[u]_{\mathbf{k}^l}$.
 - (d) Stop when $iter_i = s_i$. Otherwise, let $iter_i := iter_i + 1$ and $T := aT$ and return to Step (b).

an area without strategically targeting groups of people). Then, the observed partition can be plotted relative to the wide array of possible impartial partitions. If the observed partition differs significantly, it is likely to suffer from strategic gerrymandering.

¹⁷A simulated annealing optimizer is chosen over other Matlab alternative global search algorithms (i.e., genetic algorithm or pattern search) as it proved more efficient (i.e., better solutions derived faster) in small sized versions of the problem at hand.

5. Stop when $iter_o = s_o$. Otherwise, let $iter_o := iter_o + 1$ and return to Step 2.

Step 4 of the algorithm corresponds to a standard simulated annealing global optimization technique. I run three configurations of the algorithm: $\{s_i = 100, s_o = 100\}$, $\{s_i = 200, s_o = 50\}$, and $\{s_i = 500, s_o = 15\}$. Hence, a total of 27,500 different location strategies are evaluated.¹⁸

5 Data description

This section describes the data used in the application, motivates the use of French secondary schools in the application over other public services, and provides a short overview of the French secondary education system.

Data sources

The core data employed in the application combines three main datasets: geo-localized individual data, geo-localized public service data, and commuting data (from 2010). First, information about the location, capacity and success rate of the French secondary schools is provided by the French Ministry of Education. School success rate at the national exam closing the cycle of lower secondary education (i.e., the “diplôme national du brevet”) is used to measure its quality. Second, geo-localized individual data is given by the 2010-gridded population data, which registers the number of persons residing in $1\text{km} \times 1\text{km}$ squares covering the whole country. This data also includes the population by age group in each grid. Third, data about the average commuting distance to school is provided by the National Institute for Statistics and Economic Studies (INSEE). From any given residential location, the probability of commutes to school, π_{ik} , and to workplace, π_{ij} , are independently recorded at the municipal level. To retrieve the probability to commute from residential location i to workplace j and facility k , π_{ijk} , I assume that conditional on a residential location, the choice of the public facility and the workplace are independent. This allows the derivation of π_{ijk} as follows:

¹⁸In practice and without loss of generality, it is efficient to use the observed housing prices as starting values when solving for the endogenous housing prices. Given that starting point, the evaluation of a location strategy takes on average 25 seconds on ETH Zurich’s Remote Desktop (CPU: 18 Cores, RAM: 256GB). Hence, this leads to a total running time of 7days 23hours and 48minutes. Initial “temperature” T is kept as Matlab’s default value (i.e., 100), and a is randomly drawn on the interval (0,1).

$$\pi_{ijk} = \pi_{ijk|i} \times \pi_i = \pi_{ij|i} \times \pi_{ik|i} \times \pi_i = \frac{\pi_{ij} \times \pi_{ik}}{\pi_i}. \quad (16)$$

Additionally, I obtain wage data and other demographic information at the municipal level from the INSEE, such as residential wages. The website “Meilleursagents.com” provides municipal housing price data. These data are based on the notary database “BIEN” and their own transaction records. I assume that housing prices correspond to an hypothetical annual mortgage payment if one was to borrow the total purchasing price. The generic annual mortgage payment M is calculated using the formula $M = P \frac{\varsigma(1+\varsigma)^n}{(1+\varsigma)^n - 1}$, where P is the generic total purchasing price, ς is the yearly interest rate, and n is the total tenure of the mortgage. I assume $\varsigma = 2.2\%$ and $n = 20$ years. Finally, land use data is provided by the regional urban planning institute.

I create the units of analysis by designing a grid of $1\text{km} \times 1\text{km}$ cells over the Paris region, i.e., “Ile-de-France”. Overall, 7,700 cells of this grid contain at least one inhabitant. As all data described above are geo-localized, it is then straightforward to match the data onto the chosen grid. Lastly, I compute Euclidean distances between each cell and existing public facilities.

French secondary school system

To empirically analyze the location of public services, it is useful to focus on each public service separately, apart from all others. Considering the location of all public services jointly, without drawing distinctions between them might lead to bad spatial coverage for each particular service. In the application, I focus on the location of “Collèges”, i.e., French lower secondary education facilities. In France, secondary education is organized in two stages: the lower secondary education level called “Collège” for pupils aged between 11 and 15, and the higher secondary education level called “Lycée” for pupils aged between 15 and 18.¹⁹

Compared to other public services and other national settings, studying the location of French secondary schools offers several unique advantages:

1. *No difference in nature across facilities:* All schools offer the same national degree to pupils, teachers earn the same salary, teachers must pass the same qualifying exams, and average spending per pupil by the State is equal across facilities. Note that, even though there are no difference in nature (or in other words no fundamental differences), there

¹⁹For simplicity, I will refer to the French “Collège” as secondary school from this point forward.

are however differences in quality, mostly due to peer-effects, as modeled in Section 3.

2. *Straightforward measure of quality:* Students take the same national degree at the end of the cycle. Hence, the school success rate can be used as a measure of quality, which is naturally comparable across facilities.
3. *No spatial differences in financing:* Due to the centralized financing of the secondary school system, there are (almost) no spatial differences in the amount households pay for the schooling of one student. Hence, individuals do not locate strategically to pay more or less for education.
4. *Clear identification of users:* As education is compulsory in France until the age of 16, I am able to easily and precisely identify the users of the public service. According to UNESCO statistics, the enrollment rate in lower secondary education in France was 99,79% in 2010. I, therefore, consider that all individuals between the ages 11 and 15 are users of secondary schools.²⁰
5. *Commuting data to school is available:* The full matrix containing the universe of commutes from residential locations to schools is available in 2010. Hence, the commuting elasticities can be precisely estimated in a manner that is consistent with the model.

However, when looking at public education, one important consideration to keep in mind is that the private sector offers a competing service. In France, 17% of all pupils attend a private establishment. For simplicity and clarity of the approach, the model assumes that no private education is offered and that only pupils going to a public establishment need to be offered a seat. This assumption should not harm the generality of the framework for two reasons. First, the strategy of private schools in France is to locate very close to public schools and to draw away and enroll the good students from the respective public schools. Hence, the key moment targeted by the framework – the spatial coverage – should not be significantly impacted by the existence of private schools. Second, 97% of all private schools are religious schools while public schools obey strict secular rules. Hence, the switch from public to private (or vice-versa) might not be as straightforward as in other countries.

²⁰Note that I also include the parents in the number of users, as their residential location decision is naturally affected by the secondary school attended by their kids. Following French demographics, I do so by assuming 1.97 kids for two parents.

Secondary schools in Paris' region between 2001 and 2015

I focus on public secondary school openings between 2001 and 2015 in the Paris' greater region.²¹ Figure A1 illustrates the location of existing secondary schools as a function of distance to Paris city center in 2000. Table A1 summarizes the number of openings and closures by year. There are between 0 and 8 school openings and between 0 and 2 closures per year. As there are *only* two school closures during the period studied for non-renovation/asbestos removal, I focus on the more relevant case of school openings. The overall stock increased from 841 in 2000 to 895 in 2015 (i.e., 7%).

6 Calibration

In this section, I estimate the parameters of the model, calibrate it for local characteristics and assesses its performance using an over-identification check.

6.1 Set of locations

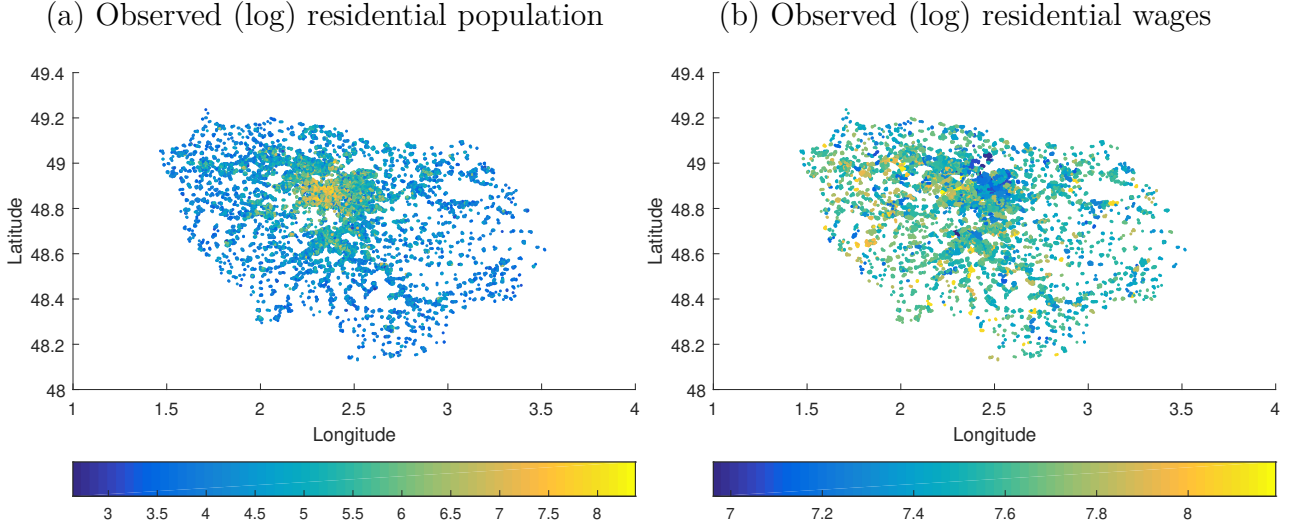
The theoretical framework and the solution algorithm proposed in Sections 3 and 4 permit the analysis of a large-scale grid of finely defined locations. I include in the analysis all 1km×1km cells in Paris region with at least one inhabitant. This leads to a total of 7,700 cells. Figure 1 displays the location of all these cells, and highlights (log) residential population (Figure 1a) and (log) residential wages (Figure 1b) in color from low levels (blue) to high levels (yellow). Population density is high around Paris city center and decreases as one moves away. Wages are high at Paris center and in the West of the French capital. This pattern in wages reflects a well-known observations that wind directions have led polluting industrial activities to locate in the East of Paris. The resulting wage patterns still persist today.

6.2 Estimation of commuting parameters

From the commuting shares of public service users in (6), the model predicts a semi-log gravity equation for commuting flows between the place of residence i , the workplace j , and the public facility location k :

²¹The choice of the period is driven by data availability.

Figure 1: Set of locations



$$\ln \pi_{ijk}^F = -\kappa\epsilon\rho_{ik} - \zeta^F\epsilon\tau_{ij} + \omega_i + \xi_j + v_k + \epsilon_{ijk}, \quad (17)$$

where ω_i are residence-fixed effects capturing residence characteristics, ζ_j refers to workplace characteristics, and v_k are public-facility-fixed effects. The parameters products $\kappa\epsilon$ and $\zeta^F\epsilon$ are the semi-elasticities of commuting flows with respect to the Euclidean distance, where κ is the commuting cost parameter for journeys to the public facilities, η^F is the commuting cost parameter for journeys to the workplace for service users, and ϵ is the heterogeneity parameter from the Fréchet-distributed shock on individuals' utility.

Similarly, from the commuting shares of non-users in (7), the model predicts the following semi-log gravity equation for commuting flows:

$$\ln \pi_{ij}^E = -\zeta^E\epsilon\tau_{ij} + \omega_i + \xi_j + \epsilon_{ij}. \quad (18)$$

The semi-elasticity of commuting flows with respect to the distance to the workplace for non-users is represented by $\zeta^E\epsilon$, where η^E is the commuting cost parameter for journeys to the workplace for service non-users.

To empirically estimate the semi-elasticities of commuting flows for public service users and non-users, I use municipal data from the 1,276 municipalities and the 20 Parisian districts. In total, 2,342,246 tri-dimensional links for users are observed. Furthermore, I estimate the semi-elasticity of distance to the workplace for non-users of the public service using bilateral

commuting data between the same geographical units. In total, 133,376 bilateral links are observed. As modeling the complexity of travel mode choice, which is a function of travel distance and transport networks, is beyond the scope of that paper, I measure distances (ρ_{ik} and τ_{ij}) using the travel time along the great circle distance in minutes per kilometers. An effective average travel speed of 30km/h is assumed.²² As bilateral distances are measured at the cell level, I compute the great circle distance between two municipalities by taking the average distance between all cells in a municipality to all cells in another.

Table 1 displays the estimation of (17) and (18) using these data. It is organized in two column pairs, where the first pair focuses on commuting for public service users, and the second on non-users. Within each pair, the first column displays a linear model without fixed effects whereas the second includes them. In Column II for users, I obtain a semi-elasticity of commuting to work of -0.025 and a semi-elasticity of commuting to school of -0.029, both significant at the 1-percent level. The semi-elasticity of commute to school is significantly larger than the semi-elasticity of commuting to work even though the magnitude difference is small. This reflects the fact that parents generally prefer that their kids not commute long distances to their school, as pupils (aged 11 to 15) might commute on their own. Columns (IV)-(V) mirror Columns (I)-(III) for non-users. In Column IV, for non-users, I obtain a semi-elasticity of commuting to work of -0.02. This implies that non-users (i.e., workers without kids in secondary school age) are more likely to commute larger distances. The estimated coefficient are of the same order of magnitude than those estimated in Ahlfeldt et al. (2015), but a bit smaller. This is not too surprising as the area considered in the paper includes rural areas where commuters are likely to commute longer distances.

6.3 Endogenous school quality

Determining the causal effect of wages on school performance is not trivial. A reverse causality problem naturally arises. To tackle this issue, I exploit the fact that, as shown in Figure 1, residential wages are higher in the West of Paris and lower in the East. Dominant winds in France flow West to East. Historically, this has lead to the unequal spatial distribution of air pollutants as detailed by Heblich et al. (2016). Consequently, to avoid pollution, richer households have been more likely to locate on the West of the region. As such longitude

²²The chosen average speed is a middle ground between an average effective driving speed of 50km/h on congested highways in Paris' metropolitan area and an average effective driving speed of 15km/h in agglomerations. *Source:* French Ministry of Transport.

Table 1: Gravity estimations of commutes

	I	II	III	IV
	Users (π_{ijk}^F)		Non-users (π_{ij}^E)	
Travel time to work (min)	-0.014***	-0.025***	-0.011***	-0.020***
$(-\eta\epsilon)$	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Travel time to school (min)	-0.014***	-0.029***	-	-
$(-\kappa\epsilon)$	(0.0001)	(0.0001)	-	-
Fixed effects	No	Yes	No	Yes
Obs.	2,342,273	2,342,246	133,376	132,997
Adj. R ²	0.15	0.57	0.15	0.57

Notes: Estimates based on bilateral commuting probability in 2010.

predicts well wages, and it is safe to assume that average grades at the national exam are independent of the wind direction in Paris region, except through the wage effect.

Table 2 displays the outcome of regressing the average grade at the national exam on residential wages in 2010 following (12). Column (I) displays the results of an OLS regression, whereas Column (II) reports the estimated value for λ using a IV approach with $\left(\sum_{i=1}^I x_i a_{ik}\right)$ as instrument, where x_i is the longitude of location i , for the expected income background of the pupils, $\left(\sum_{i=1}^I w_i a_{ik}\right)$. First stage results are displayed in Column (III). The instrument is revealed highly relevant (F-statistic: 1.7×10^4). As expected, the average grade at the national exam increases with residential wages as I obtain a value $\lambda = 0.039$.

6.4 Calibration of location characteristics

Using the structure of the model, I show that there is a one-to-one mapping between the observed and the unobserved location characteristics. I can then derive the unobserved location characteristics, namely, residential amenities, locational productivity, and density of development.

Table 2: Endogenous school quality

	I	II	III
	OLS	2SLS	IV-First stage
(log) Residential wages	0.042*** (0.009)	0.039*** (0.009)	-
(log) Longitude	-	-	0.951*** (0.010)
Observations	895	895	895
Weak inst. test (F-statistic)	-	-	1.7×10^4

Notes: (log) Residential wages refer to $\log(\sum_{i=1}^I w_i a_{ik})$. The instrumentation strategy is highly statistically significant and the Cragg-Donald F-statistic is largely above the critical values derived by [Stock and Yogo \(2005\)](#), i.e., 16.38.

Proposition 2 *Given known values for the parameters and the observed data, there exist unique vectors of residential amenities $\{\mathbf{B}\}$, locational productivity $\{\mathbf{A}\}$, and density of development $\{\phi\}$ that close the model for a given location strategy.*

PROOF: See the proofs of Proposition 2 in Section F.

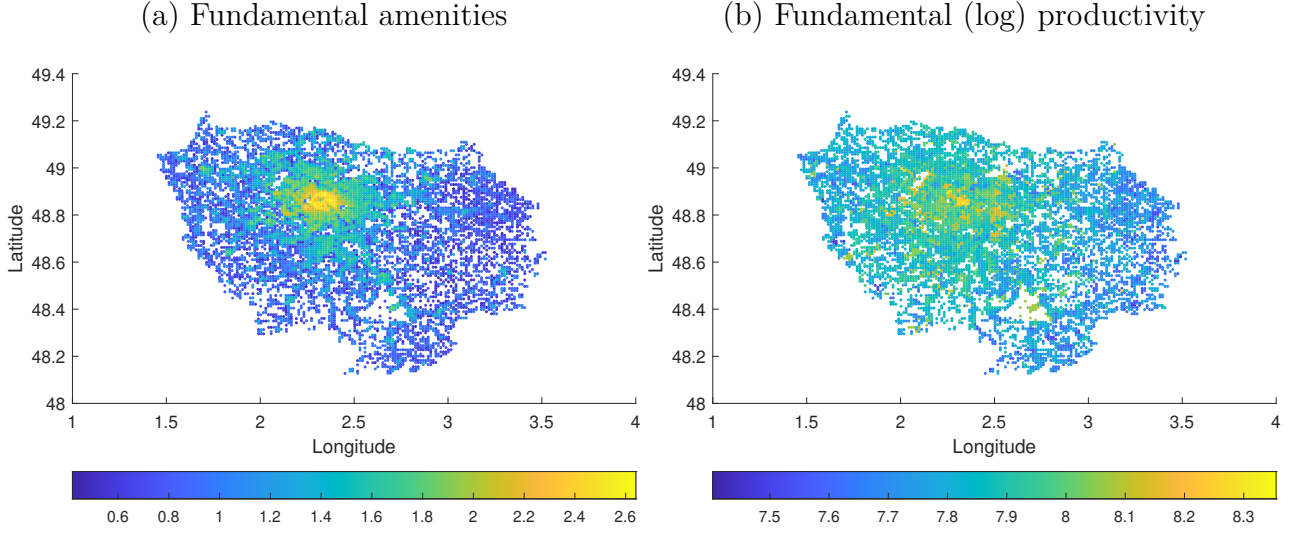
Q.E.D.

Figure 2 maps the residential amenities (Panel a) and locational productivity (Panel b) in Paris region. Amenities appear particularly high as one gets closer to Paris city center. These amenities explain why I observe “over-proportional” densities close to the city center. Typical examples for such amenities are cultural attractions (i.e., museums, theaters, cinemas, ...). Productivity values are also higher closer to Paris city center, but the concentration pattern is less pronounced.

6.5 Calibration of remaining parameters

I set the values of the remaining parameters in the model using standard sources in the literature. The consumption share in utility is set to $\beta = 0.75$ following [Davis and Ortalo-Magne \(2011\)](#). I choose the Fréchet shape parameter $\epsilon = 6.83$ following [Ahlfeldt et al. \(2015\)](#). Using this value of the shape parameter ϵ , I can retrieve the commuting to facility cost parameter $\kappa = \frac{0.029}{6.83} = 0.04$, the commuting to work cost parameter for user $\zeta^F = \frac{0.025}{6.83} = 0.04$, and the

Figure 2: Fundamental amenities and productivity



Notes: Map based on 7,700 inhabited cells of 1km². Fundamental amenities are normalized such that the

$$\text{geometric mean amenity is equal to 1, i.e., } \left[\prod_{i=1}^I B_i \right]^{1/I} = 1.$$

commuting to work cost parameter for non-user $\zeta^E = \frac{0.02}{6.83} = 0.03$. The share of labor in firm costs is set to 0.8 based on [Valentinyi and Herrendorf \(2008\)](#). The share of capital in floor space production is set to $\mu = 0.75$ following [Ahlfeldt et al. \(2015\)](#). Based on the Notary database BIEN, I set the share of local housing income to local residents to $\iota = 0.9$. Finally, following [Iyigun and Ben-Israel \(2010\)](#), I set $\xi = 2$ to smooth the distance minimizing objective. Table [A2](#) presents the values and sources of these parameters.

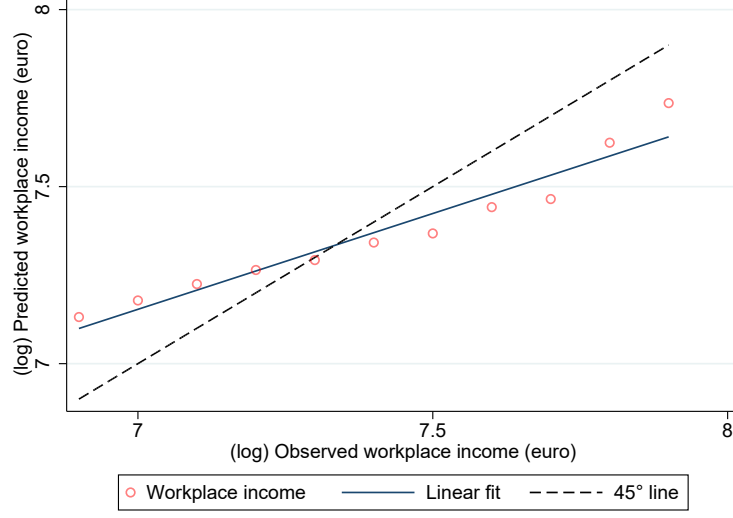
6.6 Model validation

Validating calibrated quantitative models is generally difficult, because all key model parameters are chosen to minimize any gaps between data and model counterparts. Yet, not all endogenous variables are targeted in the calibration exercise. For such moment, it is then possible to test whether the model's predictions align well with the data. In this paper, predicted workplace wages are endogenously determined, but their observed counterpart are not targeted in the calibration.²³ Figure 3 plots predicted workplace wages against the observed ones. Small red circles represent predicted averages for each bin of observed outcome. The full blue line is a linear fit of these data, while the dashed black line represents the 45° line. Overall, the linear fit is generally close to the 45° line, indicating that the framework performs well at representing

²³Data and the observed location strategy as of 2015 is used.

the real world.

Figure 3: Over-identification check: Model vs. data



Notes: Based on 7,700 inhabited cells of 1km². Average taken over bins of 0.1 (log) €.

7 Evaluating location strategies

Using the framework described above, I now aim to evaluate the location decisions of secondary schools in the Paris region in the period 2001-2015. This implies taking location choices prior to 2001 as given. As the location of new public facilities is the key moment of the optimization, I start the presentation of the results by comparing the location of facilities under three strategies: (i) observed one, (ii) welfare maximizing, and (iii) distance minimizing strategy with exogenous residential density (Section 7.1). Then, I measure the welfare difference between the optimal and the observed location strategies (Section 7.2). Finally, at the 1km² cell level, I identify key drivers of this difference (Section 7.3).

7.1 Observed, distance minimizing and welfare maximizing strategies

I start by looking at the location of the facilities opened under the observed, distance minimizing and welfare maximizing strategies as a function of the distance to Paris city center. Whereas the welfare maximizing strategy is obtained using the framework (Sections 3 and 4), the distance minimizing strategy is derived by minimizing the residential-population-weighted

distance between residential locations and public facilities as follows:

$$\min_{\tilde{k}_1, \dots, \tilde{k}_K} \sum_i \sum_k H_{Ri} d_{ik}. \quad (19)$$

In (19), the residential mass H_{Ri} is taken as given. To solve this minimization problem, I employ the interrupted search algorithm presented in Section 4 with $s_o = 15$ and $s_i = 1,000$.²⁴ This leads to the evaluation of 15,000 different strategies to derive 15 locally optimal location strategies.

Figure 4 reports the kernel density function of each strategy as a function of distance to Paris' city center. In each case, a kernel smoothing function is estimated with a bandwidth size of 7. The observed strategy and the distance minimizing strategies are represented in full blue and dashed back lines, respectively. For the welfare minimizing strategy, I display the distribution using the different stopping criterion for the inner loop. As these thresholds are arbitrary, it is useful to test whether results depend significantly on their value.

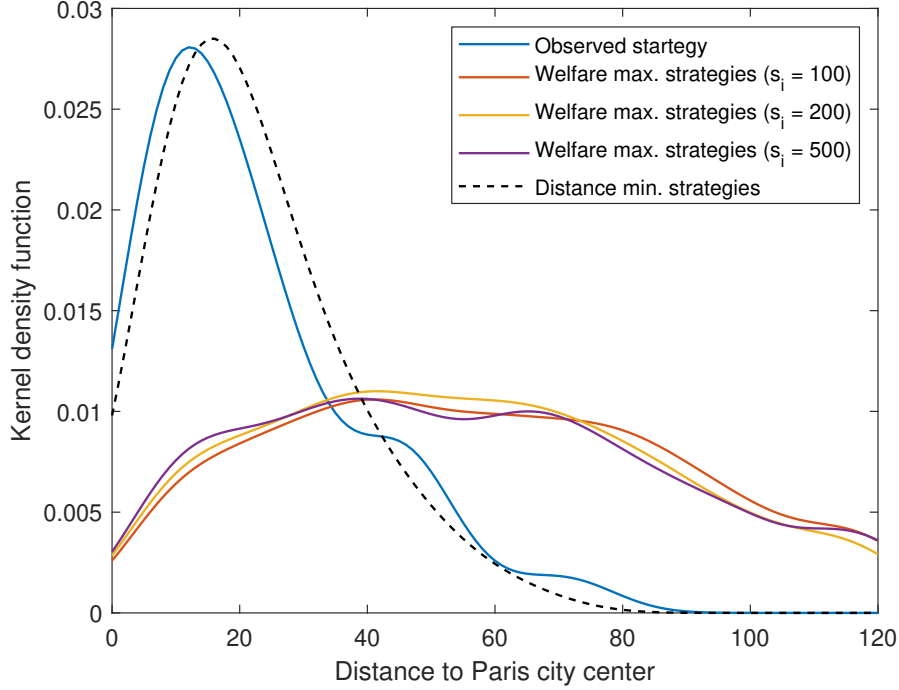
Overall, two main findings stand out. First, the distance minimizing strategy appears to predict very well the observed strategy. Under both strategies, a location peak is observed at about 20km of Paris' city center. Beyond that point significantly less facilities are newly located. At more than 80km, no new facilities are located. Hence, this result offers strong evidence that actual Parisian planners take population density as given, and do not account for the endogenous locational responses of individuals. Second, independently of the stopping criterion of the inner loop, all welfare maximizing strategies, i.e., solving (15), lead to a very similar density function. Accounting for the endogenous locational responses of individuals leads to a much flatter distribution. Even though a peak is still observed around 40km to 60km of the city center, welfare maximization still encourages locating a non-trivial share of facilities far from the city center.

Facilities, land use and land price

The framework introduced in Section 3 supposes that the costs for locating a facility are the same across all locations. Given the large disparities in housing prices between Paris center and the rural areas considered, this assumption may appear surprising. The key motivation behind this assumption is that – as discussed in the Montreuil case study in Section B – public

²⁴As the evaluation of a single strategy does not require solving the framework, using a relatively larger s_i and a smaller s_o is more efficient.

Figure 4: Observed, distance minimizing and welfare maximizing locations



Notes: Locations of the facility under the observed and the welfare maximizing location strategies are displayed as a function of distance to Paris city center. For smoothing, the selected bandwidth size is 7. s_i refers to the stopping criterion of the inner loop of the interrupted search algorithm presented in Section 4. Overall, the welfare maximizing distributions are based on 10,000, 10,000 and 7,500 different locally optimal strategies, respectively. The distance minimizing strategy is based on 15,000 different locally optimal strategies.

facilities are commonly built on publicly owned land. Hence, assuming that the central planner faces the same land prices as individuals or firms would create a false congestion force.

Moreover, the welfare maximizing strategies displayed in Figure 4 already recommend more decentralized location choices *without considering land prices*. Hence, supposing that the central planner would need to pay more closer to the center would likely reinforce this result.

Upon scrutiny, the constraint of building on public land may actually raises another difficulty. Additionally to equal costs, the framework also assumes that facilities can be built anywhere in space. If the results recommended to build more in denser areas, this assumption would naturally be a source of concern. However, once again, the results actually hint in the other direction, i.e., new facilities should be built relatively more in less dense areas where competition for land is lower.

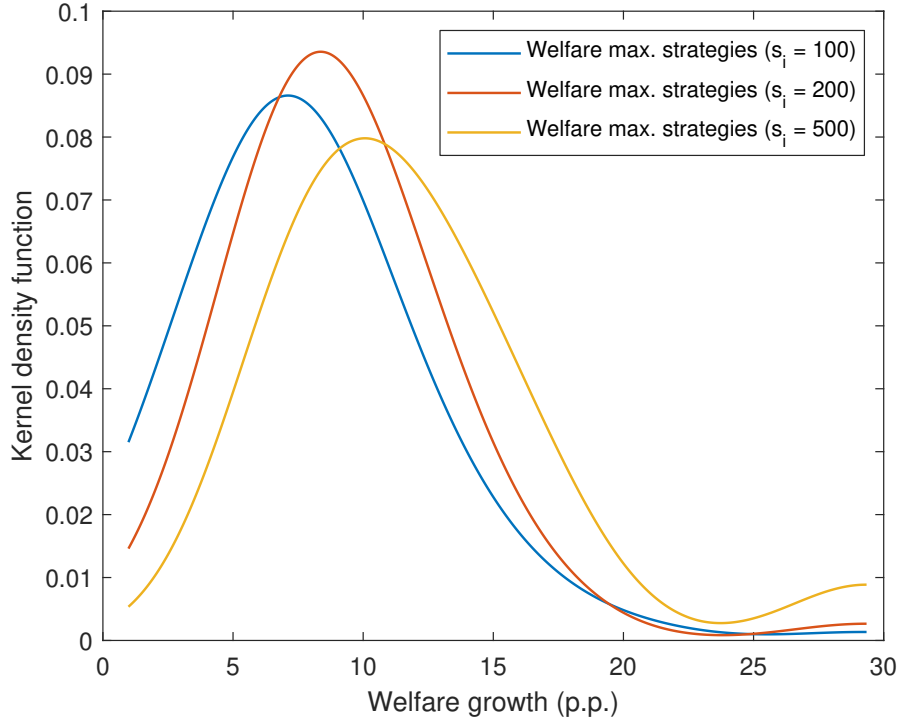
Overall, the two assumptions on the possible set of locations for public facilities (i.e., *equal costs* and *anywhere in space*) are indeed approximations of the real world – as in any modeling framework. Yet, they are unlikely to confound the results presented.

7.2 Welfare difference across strategies

I quantitatively compare the welfare maximizing and observed location strategies on the period 2001-2015. To interpret the size of the welfare gains after adopting a welfare maximizing strategy, I normalize the difference between the optimal and observed welfare levels in 2015 by the observed welfare difference between 2000 and 2015. Precisely, the welfare growth is computed as: $100 \times \left(\frac{\mathbb{E}[u]_{2015}^{opt} - \mathbb{E}[u]_{2015}^{obs}}{\mathbb{E}[u]_{2015}^{obs} - \mathbb{E}[u]_{2000}^{obs}} \right)$. $\mathbb{E}[u]_{2015}^{opt}$ is the welfare level under the optimal strategies in year 2015. $\mathbb{E}[u]_{*}^{obs}$ is the welfare level under the observed strategy in year 2000 and 2015, alternatively.

Figure 5 reports kernel density functions, with the horizontal axis referring to the welfare growth in percentage points. Overall, all locally optimal welfare maximizing strategies perform better than the observed strategy – and so, independently of the stopping criterion for the inner loop, s_i . Naturally, when allowing more iterations, i.e., $s_i = 500$ versus $s_i = 200$ versus $s_i = 100$, the average derived welfare level is higher. Relative to the observed strategy, the overall welfare gains are significant – with mean of 10 p.p., a minimum of 1 p.p. and a maximum of 30 p.p.

Figure 5: Observed and welfare maximizing strategies



Notes: s_i refers to the stopping criterion of the inner loop of the interrupted search algorithm presented in Section 4. For smoothing, the selected bandwidth size is 3.

7.3 Understanding the difference across strategies at the cell-level

Using the structure of the model, I attempt to break down the gains from adopting the welfare maximizing strategy for key model variables across all grid cells. Consider location i belonging to $r \in R$ rings of width 5km around Paris' city center. I identify local effects using the following spatial event-study approach:

$$y_{it}^r = \sum_{r=1}^R \beta^r D_i^r + u_{it}^r, \quad (20)$$

y_{it}^r is the outcome of interest. D_i^r is indicator equal to 1 $\forall i \in r$, and 0 otherwise. u_{it}^r is the error term.

I define the outcomes of interest as the difference between the *observed* and *optimal* strategies. As optimal strategies, I consider the 165 local optima derived using the interrupted search algorithm. I consider residential density ($\overline{H_{Ri}}$), equilibrium floor prices ($\overline{Q_i}$), commutes to public facility ($\overline{d_{ik}}$), and commutes to work ($\overline{d_{ij}}$) at the $1\text{km} \times 1\text{km}$ cell level for each optima. This leads to a final dataset of $165 \times 7,700 = 1,270,500$ observations.

$$\begin{aligned} \overline{H_{Ri}} &= H_{Ri}^{\text{Obs}} - H_{Ri}^{\text{Opt}}, \quad \overline{Q_i} = Q_i^{\text{Obs}} - Q_i^{\text{Opt}}, \\ \overline{d_{ik}} &= \left(\sum_k H_{ik} \rho_{ik} \right)^{\text{Obs}} - \left(\sum_k H_{ik} \rho_{ik} \right)^{\text{Opt}}, \\ \overline{d_{ij}} &= \left(\sum_j H_{ij} \tau_{ij} \right)^{\text{Obs}} - \left(\sum_j H_{ij} \tau_{ij} \right)^{\text{Opt}}. \end{aligned} \quad (21)$$

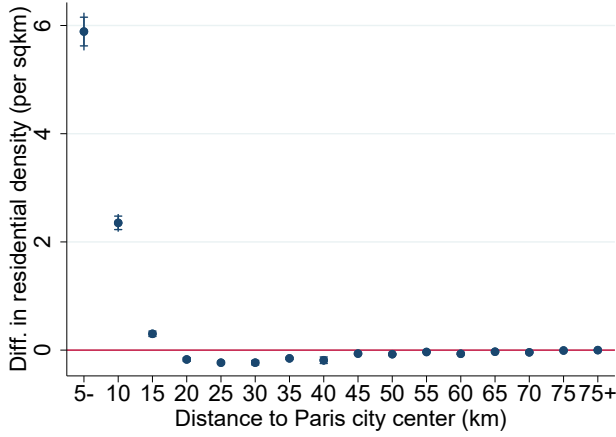
A positive value of $\overline{H_{Ri}}$ implies that the cell is a more attractive residential location under the observed than under the welfare maximizing strategy. Conversely, a negative value implies that the cell is more attractive under the welfare maximizing than the observed strategy. A positive value of $\overline{d_{ik}}$, $\overline{d_{ij}}$, or $\overline{Q_i}$ implies that for a given location the sum of commutes to public facilities, to the workplace or the cell equilibrium floor price per m^2 is larger under the observed than under the optimal strategy.

Figure 6 displays the results of estimating (20) on each outcome. Cells at more than 75km of Paris' city center constitute the omitted category. Panel (a) looks at the difference in overall residential density. The observed strategy appears to increase residential density by up to 6 inhabitants per km^2 within 15km of Paris' center. Symmetrically, cells between 20km and 45km of Paris hosts significantly more inhabitants under the welfare maximizing strategies.

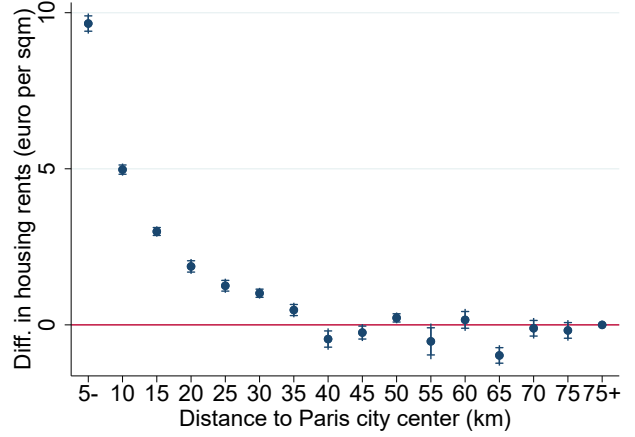
Panel (b) focuses on the equilibrium floor price (expressed in $\text{€}/\text{m}^2$) at the cell level. Floor

Figure 6: Cell-level spatial event study

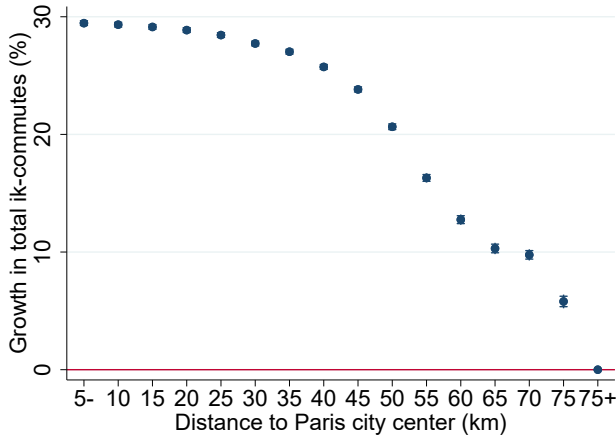
(a) Difference in overall residential density



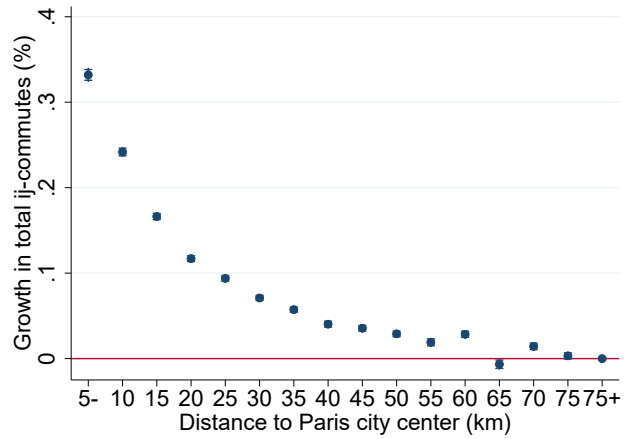
(b) Difference in equilibrium floor price



(c) Growth in total residence to school commutes



(c) Growth in total residence to workplace commutes



Notes: Locations at more than 75km of Paris city center are omitted. 5- refers to locations at less than 5km of the center. 95% and 99% confidence intervals are displayed.

prices are inefficiently increased by up to 10€/m² within 30km of Paris' city center. In a context of high floor prices in Paris, this result is particularly relevant for central planners and policy makers. Smaller peripheral centers at 55km and 65km experience an increase in housing prices following the welfare maximizing strategies.

Finally, Panels (c) and (d) look at the sum of total commutes to the public facility and to the workplace, respectively. Overall, the observed strategy appears to increase the commuting in central locations as opposed to further away. The total number of kilometers traveled to public facilities is made 30% larger within 40km of Paris city center. This result is the direct implication of an inefficiently large density induced by a over-centralized location strategy. A

qualitatively similar effect on commuting distance to workplaces is observed, albeit of much smaller magnitude, i.e., up to 0.4%.

8 Discussing optimality

Solving for optimality in a quantifiable general equilibrium approach offers several advantages. First, it allows to compare the performance of alternative policies along several dimensions of interests, i.e., welfare, density, commutes, housings prices, etc. Second, it allows to jointly account for key mechanisms at play *in a consistent manner*, which may not be the case when attempting to combine different reduced form results. Yet, any optimality results rely heavily on a set of structural assumptions, e.g., functional forms for individual utility and production (see Section 3). If these assumptions significantly misrepresent the real world, then the derived optimality will also fail as metric to evaluate different policies.

Here let me first note that, in the application presented above, the chosen structural assumptions appear to represent well the real world as revealed by the over-identification check in Section 6.6. That being said, I briefly outline an approach which, even though based on the structural framework, is robust to misspecifying the optimal policies. Overall, this approach offers the key advantage of not relying on optimality, but instead, on a smaller set of structural assumptions, namely, the *exogeneity assumption of economic fundamentals* (i.e., local productivity, A_j ; local amenities, B_i ; land development, ϕ_i ; and bilateral distances, ρ_{ik} and τ_{ij}). In what follows, I present the approach and apply it to estimate the effect of public facility proximity on residential density.

Consider the following linear model of local residential density change:

$$\Delta H_{Ri} = \gamma \Delta d_i + \mathbf{X}'\beta + \varepsilon_i. \quad (22)$$

Generically, ΔH_{Ri} refers to residential density change (which may be expressed in, e.g., growth or simple difference). Δd_i refers to the change in proximity to public facilities. \mathbf{X} refers to a matrix of local control variables and ε_i to the error term. As shown in Figure 4, the OLS estimation of (22) is likely to suffer from reverse causality as past population density is a strong determinant of the location of new public facilities.

To circumvent this source of bias, the endogenous public facility location framework presented above can be exploited to generate instruments in a Instrumental Variable (IV) ap-

proach. Importantly, instruments are only required *to be relevant and not optimal*, on top of satisfying the exclusion restriction. Then, even if the framework only offers an inaccurate representation of the world, the model outcomes may still provide relevant instruments. Moreover, in the present context, the exclusion restriction will then be valid if the economic fundamentals are exogenous. Hence, *under the exogeneity assumption* of these economic fundamentals (even if improperly measured), any local welfare optimal strategy is likely to be a valid instrument to predict the observed location strategy.

The use of a solution to a model-based optimization problem as instrument is not novel in economics. It has been used in other contexts, e.g., [Faber \(2014\)](#); [Alder and Kondo \(2018\)](#); [Egger, Loumeau, and Loumeau \(2020\)](#) derive model-driven optimal transport networks to tackle the non-random connection placement problem.

Application to Paris secondary schools

To apply the proposed mixed approach strategy to the case of Paris secondary schools, I exploit the two waves of the French gridded population data in 2010 and 2015 (i.e., *données carroyées* in French).²⁵ These data record the residential count within 1km² in 2010 and 2015. The 2010 vintage was used in the calibration exercise above. Hence, I can track the 1km² cell level residential population difference and growth between 2010 and 2015, ΔH_{Ri} . Furthermore, using the same 1km² cells, I can measure different moments of proximity to public facilities. Precisely, I measure the distance to the nearest facility, the average distance within a 2km radius, within a 5km radius, and within a 50km radius. I compute these moments both in 2000 and 2015, and take the difference in distance (in km) between these two years, Δd_i . The difference in the time period considered is motivated by the fact that one may expect residential density to react with some lag.

The results are displayed in Table 3. Panels (a) and (b) report the second stage estimation results, whereas Panel (c) reports the first stage. In Panel (a), the simple residential count difference between 2015 and 2010 is used as outcome, whereas Panel (b) reports growth. Each of the four columns pertains to a distance metric (i.e., distance to the nearest facility, the average distance within a 2km radius, within a 5km radius, and within a 50km radius). Across specifications, the instrumentation strategy is statistically significant and the Cragg-Donald F-statistic is – except in Column III – above the critical values derived by [Stock and Yogo](#)

²⁵Source: [INSEE](#), 2010 and 2015.

(2005), i.e., 16.38. Interestingly, and in line with the results in Figure 4, the observed and welfare location strategies lead to very different local changes in proximity to public facilities. A highly significant negative coefficient is observed across specifications (Panel c).

Table 3: Population change and public facility locations

	I	II	III	IV
Distance metric	Minimum distance	Avg distance within 2km	Avg distance within 5km	Avg distance within 50km
(a) Population difference of 1km ² cells (2015 vs 2010)				
Observed change in distance (km)	-208.463*** (37.719)	-215.235*** (39.147)	-265.079*** (50.588)	-87.492*** (17.058)
(b) Population growth of 1km ² cells (2015 vs 2010)				
Observed change in distance (km)	-49.051*** (16.249)	-51.928*** (16.835)	-68.371*** (21.893)	-11.065* (5.851)
(c) IV-First stage				
Optimal change in distance (km)	-0.019*** (0.002)	-0.018*** (0.002)	-0.015*** (0.002)	-0.198*** (0.010)
Obs.	9213	9213	9213	9213
Weak instr. test (F-statistic)	19	17	11	1,145

Notes: Change in distance to public facilities for distance metric is measured between 2015 and 2000. The instrumentation strategy is statistically significant and the Cragg-Donald F-statistic is – except in Column III – above the critical values derived by [Stock and Yogo \(2005\)](#), i.e., 16.38.

Overall, across distance metrics, an increase in the observed distance to public facilities leads to a significant decrease in population (both in levels and growth). As key results, an increase by 1km of the distance to the nearest facility decreases the population count in the average 1km² cell by 208 inhabitants, which is equivalent to a staggering 49% decrease. However, given that schools have mostly opened in the region between 2000 and 2015 (Table A1), a more accurate interpretation of the results is that individuals react strongly to the opening of a new school, and are willing to relocate close-by. This is in line with the results illustrated in Figure 4. Hence, receiving a new school constitute an important factor for local growth, which channel would be very beneficial for smaller urban centers in more peripheral areas.

9 Conclusion

In this paper, I develop and apply a spatial quantifiable general equilibrium model to study the endogenous public facility location problem. The framework features a central planner deciding on a location strategy – which implies the location of facilities, their quality, capacity and districts – while anticipating how individuals and firms will react. To derive a suitable set of comparison strategies which may serve as a benchmark to evaluate the observed strategy, I propose and apply a novel interrupted search algorithm. This algorithm accepts local welfare optima after evaluating a large number of randomly chosen strategies. Relative to using solely the welfare maximizing strategy as comparison point, this approach is more resilient to different limitations, such as planner’s computational limitations, random data mis-measurement, functional form assumptions, etc.

The framework is applied to the case of secondary schools in the Paris’ greater region at a fine geographical scale. After calibrating the framework to fit the economic geography of the region, I evaluate the new school location decisions made between 2001 and 2015. Three key results arise from this application. First, the observed location strategy appears to ignore the endogenous location decisions of individuals and firms, as it is almost perfectly explained by a distance minimization exercise with exogenous residential density. Second, large welfare gains – amounting to 10 percentage point higher welfare growth on average – can be enjoyed by adopting a more decentralized strategy. Finally, the observed strategy leads to large inefficiencies in terms of residential density, housing prices and commutes.

The proposed framework is sufficiently flexible such that it could also be used to analyze the location of other types of public services. Natural candidates for such further analyses are, among others, education facilities other than secondary schools, green areas, health care facilities, police and fire stations. As such the framework proposed may constitute a stepping stone for further research aimed at better understanding the spatial dependencies between public services and individuals’ location choices. The flexibility of the approach also make it particularly suited to help planning authorities decide where to locate a wide range of public services in the future.

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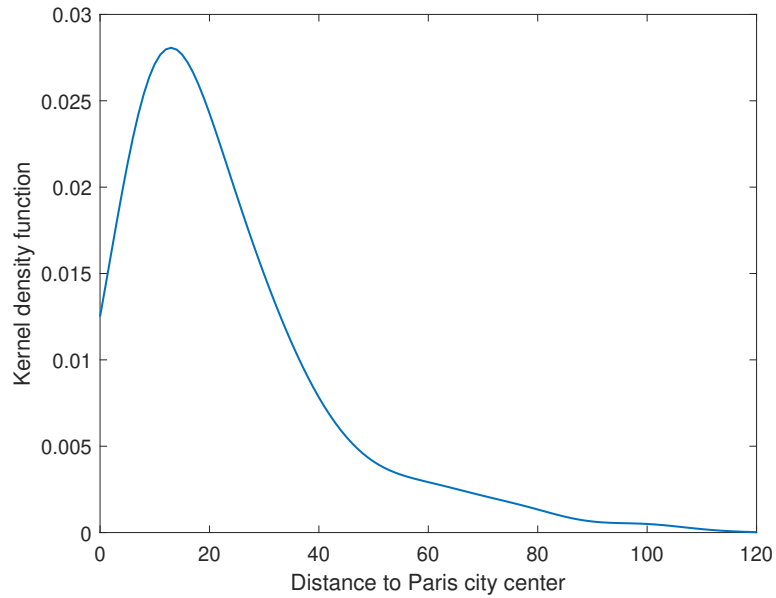
A Supporting material

Table A1: Openings and closures of facilities in Paris region (2001-2015)

Year	Stock	Open	Close	Year	Stock	Open	Close	Year	Stock	Open	Close
2001	843	2	0	2006	871	7	2	2011	884	1	2*
2002	851	8	0	2007	875	4	0	2012	885	1	0
2003	857	6	0	2008	881	7	1*	2013	885	0	0
2004	859	2	0	2009	882	1	0	2014	893	8	0
2005	866	7	0	2010	885	3	0	2015	895	2	0

*: Facility closures for renovation or asbestos removal.

Figure A1: Secondary school locations



Notes: Stock of secondary schools in 2000 in Paris' greater region ("Ile-de-France"). For smoothing, the selected bandwidth size is 7. Distance to Paris city center is taken in km.

Table A2: Calibration overview

PARAMETERS COMMON TO ALL LOCATIONS		
1. Preferences		
$\beta = 0.75$	Consumption share in utility	Davis and Ortalo-Magne (2011)
$\epsilon = 6.83$	Shape parameter	Ahlfeldt et al. (2015)
2. Production		
$\alpha = 0.8$	Share of labor in firm costs	Valentinyi and Herrendorf (2008)
3. Commuting		
$\kappa = 0.004$	Elasticity of commuting to public facilities	Own estimation
$\zeta^F = 0.004$	Elasticity of commuting to workplace for service users	Own estimation
$\zeta^E = 0.003$	Elasticity of commuting to workplace for service non-users	Own estimation
4. Public sector characteristics		
$\lambda = 0.02$	Peer-effect parameter	Own estimation
$\xi = 2$	Smoothness of assignment parameter	Iyigun and Ben-Israel (2010)
5. Housing Market characteristics		
$\mu = 0.75$	Share of capital in floor space production	Ahlfeldt et al. (2015)
$\iota = 0.9$	Share of local housing income to local residents	Notary database BIEN
LOCATION-SPECIFIC CHARACTERISTICS		
B_i	Residential amenities	Own estimation
A_j	Locational productivity	Own estimation
ϕ_i	Intensity of development	Own estimation

B Case study: Locating a secondary school in Ile-de-France

In this section, I provide anecdotal evidence of how a facility location is chosen. Understanding the actual decision process leading to a particular location choice helps the development of a realistic and relevant framework. Precisely, I detail the process that led to the opening of a new secondary school in the municipality of Montreuil in September 2018.²⁶

Who took the decision? The decision *to open* a new school in Montreuil was taken by the Département of Seine-Saint-Denis (NUTS3 region).²⁷ The decision *on the location* of the newly-built secondary school in Montreuil was taken jointly by a large number of administrative bodies. On top of the municipality and the Département (NUTS3 region), the central government, the Région (NUTS2 region), the cooperation of municipalities in which Montreuil is a member, the public transport authority, the department of national and regional parks, as well as several local associations were all involved in deciding where to locate the new school.

Where was it located? The school was ultimately built on unoccupied land bordering a municipal park owned by the Département.²⁸ On the map, the green point represents the location of the new secondary school. The blue points represent the already existing public secondary schools, whereas the red ones represent the private secondary school locations. The first striking observation when investigating the chosen location is that five existing public schools and one private schools are within a radius of 1km from the new location. The second observation is that the new school location is next to two development zones (Zone d’Aménagement Concerté (ZAC), in French) created in 2011. These zones are areas in which new housing space was built; and hence, where the population density increased.

How was the location justified? The project proposal argues that, without a new school, the local demand for school seats would get close to full capacity in 2022.²⁹ It was argued

²⁶Montreuil is a municipality on the eastern border of Paris intra-muros; hence, within the area studied in the application.

²⁷The “Plan Ambition Collège 2015-2020” was passed on November 27th, 2014. [Link to project website](#) (in French).

²⁸Figure A2 displays the Local Urban Plan (Plan Local d’Urbanisme (PLU), in French) which was active when the new school location was selected. Author’s own illustration based on the project proposal and the PLU from September 13, 2012.

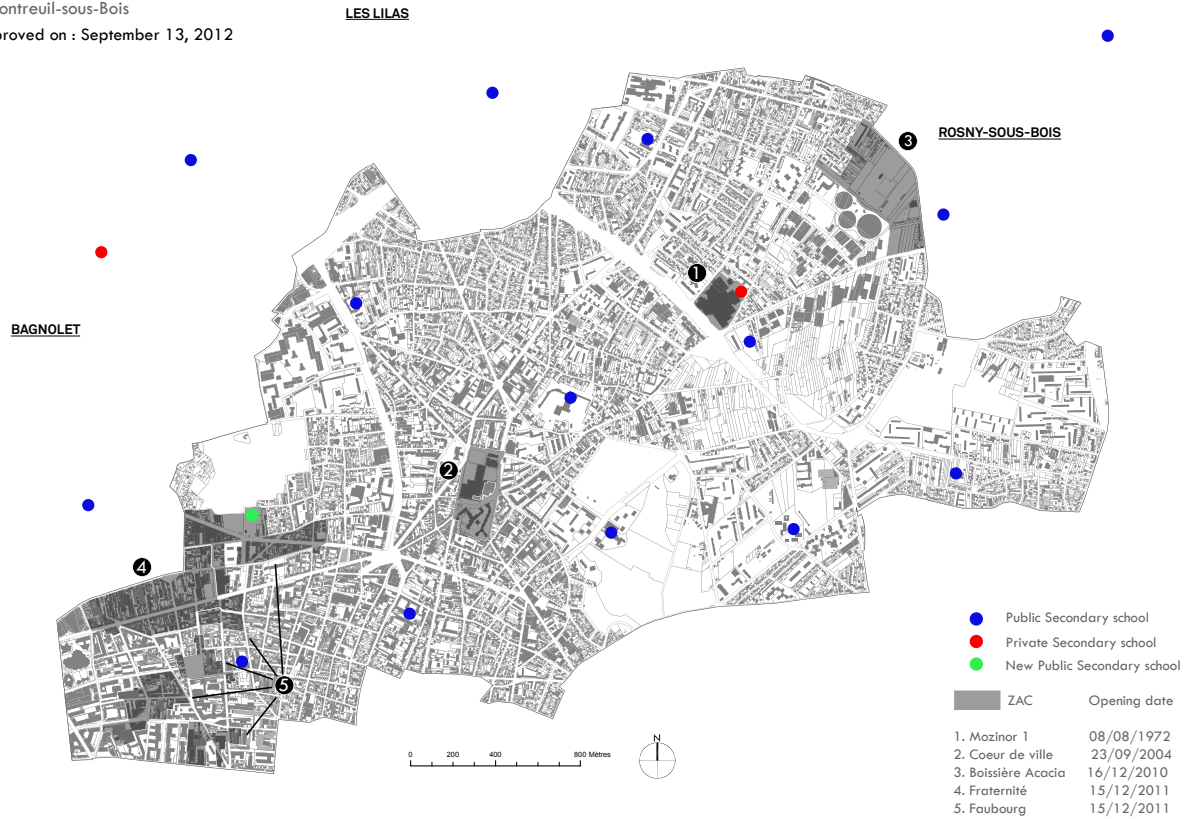
²⁹Figure A3 display the evolution of offer and demand of school seats with and without a new school. [Link to project proposal](#) (in French).

Figure A2: Montreuil's local urban plan (2012)

PLU

Plan: Montreuil-sous-Bois

PLU approved on : September 13, 2012



that building a new school would lead to a total capacity that is significantly higher than the number of students (by about 23% in 2018 and by about 18% in 2022). Note, however, that full capacity would not have been reached by 2022, even without a new school.

Why is the justification not enough? In looking at the project proposal, two pitfalls become apparent: (i) lack of accounting for effects on economic variables, such as housing prices and commutes and (ii) lack of comparison to other locations in order to justify the choice of this particular location in that municipality. Such pitfalls are not specific to the French context. The same shortcomings can be observed in the otherwise very informative “Oregon School Siting Handbook” (Kunec, 2005) or in more standard research in urban planning (Moussa, Mostafa, and Elwafa, 2017). In both practical and more academic literature in urban planning such effects are often ignored. Vincent (2006) constitute a notable exception by arguing in favor of a higher integration of school location decisions into general city planning in the U.S. context.

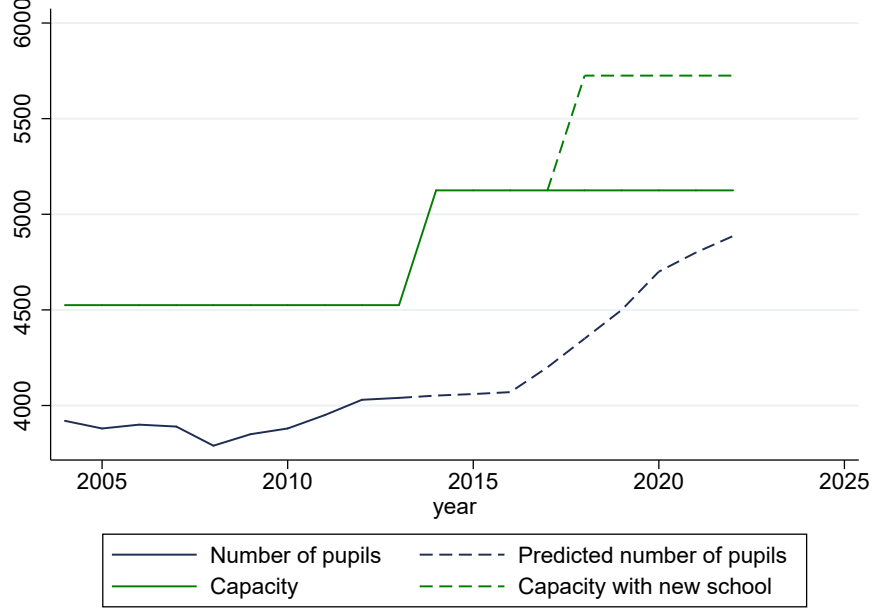


Figure A3: Capacity and demand for secondary schools in Montreuil

C Theory Appendix: Deriving individual location choices

In the Appendix, I detail the analytical derivation of individual location choices *for public service users*. The derivation for non-users follows exactly the same line, to the exception that all k or t indexed variables are taken out. This follows directly from the fact that non-users are indifferent to quality and proximity of the public service.

For the sake of completeness, I will sometimes repeat what has already been presented in the main part of the paper. As the relationship between the aggregate consumption index (1) and the idiosyncratic component of utility is monotonic, the distribution of the utility of an individual living in i , working in j and using public services in k is also Fréchet-distributed:

$$G_{ijk}^F = Pr[U \leq u] = F\left(\frac{ud_{ik}d_{ij}Q_i^{1-\beta}}{w_j B_i g_k}\right), \quad (23)$$

$$G_{ijk}^F = e^{-\Phi_{ijk}^F u^{-\epsilon}}, \quad \Phi_{ijk}^F = (d_{ik}d_{ij}Q_i^{1-\beta})^{-\epsilon} (B_i w_j g_k)^\epsilon.$$

I first derive the probability that individuals choose a particular combination of residence, workplace and public service location.

$$\begin{aligned}
\pi_{ijk}^F &= Pr[u_{ijk}^F \geq \max\{u_{rst}^F\}; \forall r, s, t] \\
&= \int_0^\infty \prod_r \prod_s \prod_t \epsilon \Phi_{ijk}^F u^{-(\epsilon+1)} e^{-\Phi_{rst}^F u^{-\epsilon}} du \\
&= \int_0^\infty \epsilon \Phi_{ijk}^F u^{-(\epsilon+1)} e^{-\Phi^F u^{-\epsilon}} du
\end{aligned} \tag{24}$$

Noting that

$$\frac{d}{du} \left[-\frac{1}{\Phi^F} e^{-\Phi u^{-\epsilon}} \right] = \epsilon u^{-(\epsilon+1)} e^{-\Phi u^{-\epsilon}}, \tag{25}$$

I obtain the probability that a individual resides in i , works in j and use public service in k :

$$\pi_{ijk}^F = \frac{(d_{ik}d_{ij}Q_i^{1-\beta})^{-\epsilon}(B_iw_jg_k)^\epsilon}{\sum_{r=1}^I \sum_{s=1}^I \sum_{t=1}^K (d_{rt}d_{rs}Q_r^{1-\beta})^{-\epsilon}(B_rw_sg_t)^\epsilon} \equiv \frac{\Phi_{ijk}^F}{\Phi^F}. \tag{26}$$

Equation (26) shows that individuals will sort across all combinations of residence, work-places and public service locations depending on their idiosyncratic preferences and the characteristics of these locations.

D Theory appendix: Expected utility

Here, I detail the derivation of the expected utility in the metropolitan area. The derivation follows the derivation in Section C.

Among all possible combinations of place of residence, work and public service, individuals choose the combination that offers the highest utility. Since the maximum of a sequence of Fréchet-distributed random variables is also Fréchet-distributed, the distribution of utilities across all combinations is:

$$1 - G^F(u) = 1 - \prod_r \prod_s \prod_t e^{-\Phi_{rst}^F u^{-\epsilon}}, \tag{27}$$

where the left-hand side is the probability that an individual has a utility lower than u and the right-hand side is one minus the probability that an individual has a utility level lower than u for all possible pairs of blocks of residence, workplace and public service. This leads to:

$$G^F(u) = e^{-\Phi^F u^{-\epsilon}}, \quad \Phi^F = \sum_r \sum_s \sum_t \Phi_{rst}^F. \tag{28}$$

Given that utility is Fréchet-distributed, I can derive the expected utility of being in the metropolitan area:

$$\mathbb{E}[u]^F = \int_0^\infty \epsilon \Phi^F u^{-\epsilon} e^{-\Phi^F u^{-\epsilon}} du. \quad (29)$$

Setting the following variable changes,

$$y^F = \Phi^F u^{-\epsilon}, \quad dy^F = -\epsilon \Phi^F u^{-(\epsilon+1)} du, \quad (30)$$

I can then write the expected utility of moving to the city as:

$$\mathbb{E}[u] = \int_0^\infty (\Phi^F)^{1/\epsilon} (y^F)^{-1/\epsilon} e^{-y^F} dy. \quad (31)$$

This is equivalent to:

$$\mathbb{E}[u] = \gamma (\Phi^F)^{1/\epsilon}, \quad \gamma = \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right), \quad (32)$$

where $\Gamma(\cdot)$ is the Gamma function, \mathbb{E} is the expectation operator, and the expectation is taken over by the distribution of idiosyncratic utility.

$$\mathbb{E}[u] = \gamma \Phi^{1/\epsilon} = \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right) \left[\sum_{r=1}^S \sum_{s=1}^I \sum_{t=1}^K (d_{rt} d_{rs} Q_r^{1-\beta})^{-\epsilon} (B_r w_s g_t)^\epsilon \right]^{(1/\epsilon)} \quad (33)$$

E Theory Appendix: Existence and uniqueness of equilibrium given a location strategy

The present existence and uniqueness proof follows closely the equilibrium proof in [Ahlfeldt et al. \(2015\)](#). Three differences are nonetheless present. First, endogenous public facility quality is introduced in individuals' utility and it is a function of surrounding wages. Second, commuting happens along two dimensions (i.e., to work and to the facility). Third, heterogeneous agents are present (i.e., users and non-users of the public service).

Conditional on a location strategy, the equilibrium of the model is referenced by the vectors $\{\pi^F, \pi^E, \mathbf{Q}, \mathbf{q}, \mathbf{g}, \mathbf{w}, \boldsymbol{\theta}\}$. The following expressions define the elements of the equilibrium.

$$\pi_{ijk}^F = \frac{(v_{ijk}^F)^\epsilon}{\sum_{r=1}^I \sum_{s=1}^I \sum_{t=1}^K (v_{rst}^F)^\epsilon}. \quad (34)$$

$$\pi_{ij}^E = \frac{(v_{ij}^E)^\epsilon}{\sum_{r=1}^I \sum_{s=1}^I (v_{rs}^E)^\epsilon} \quad (35)$$

$$g_k = \left(\sum_{i=1}^I w_i a_{ik} \right)^\lambda \bar{g}_k \quad (36)$$

$$(1 - \theta_i) L_i = \frac{(1 - \beta) \mathbb{E}[w_j | i]}{Q_i} H_{Ri} \quad (37)$$

$$\theta_j L_j = \left(\frac{(1 - \alpha) A_j}{q_j} \right)^{1/\alpha} H_{Mj} \quad (38)$$

$$q_j = (1 - \alpha) \left(\frac{\alpha}{w_j} \right)^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}} \quad (39)$$

$$\theta_i \in [0, 1], \quad \forall i \quad (40)$$

Assuming strictly positive, finite, and exogenous characteristics ($H \in (0, \infty)$, $H^F \in (0, \infty)$, $H^E \in (0, \infty)$, $A_i \in (0, \infty)$, $B_i \in (0, \infty)$, $N_i \in (0, \infty)$, $\rho_{ik} \in (1, \infty) \times (1, \infty)$, $\rho_{ij} \in (1, \infty) \times (1, \infty)$), there exist unique general equilibrium vectors $\{\boldsymbol{\pi}^F, \boldsymbol{\pi}^E, \mathbf{Q}, \mathbf{q}, \mathbf{w}, \mathbf{g}, \boldsymbol{\theta}\}$.

Proof:

In equilibrium, as no location is fully specialized given positive amenity and productivity fundamentals, I have $q_i = Q_i$. Given the production's zero-profit condition, I can express wages as follows:

$$w_j = \alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} A_j^{\frac{1}{\alpha}} Q_j^{\frac{\alpha-1}{\alpha}}. \quad (41)$$

Using this expression for wages, I can express the public facility quality as a function of fundamentals and equilibrium housing prices. Expected wages in residential location i can be expressed as:

$$\mathbb{E}[w_j | i] = \alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} \sum_{s=1}^S \frac{(A_s^{\frac{1}{\alpha}} Q_s^{\frac{\alpha-1}{\alpha}} / d_{is})^\epsilon}{\sum_{r=1}^S (A_r^{\frac{1}{\alpha}} Q_r^{\frac{\alpha-1}{\alpha}} / d_{ir})^\epsilon} A_s^{\frac{1}{\alpha}} Q_s^{\frac{\alpha-1}{\alpha}} \quad (42)$$

Then, public facility quality can be expressed as:

$$g_k = \left(\sum_{i=1}^I \mathbb{E}[w_j|i] a_{ik} \right)^\lambda \bar{g}_k. \quad (43)$$

Assuming that the utility of the chosen units is such that $(\frac{\gamma}{\mathbb{E}[u]})^\epsilon = 1$, I can rewrite the land market clearing condition using (6), (7), (9), (42) and (43) as the following system:

$$\begin{aligned} D_i(Q) &= \frac{(1-\beta)\mathbb{E}[w_j|i]}{Q_i} \pi_{Ri} + \left(\frac{(1-\alpha)A_i}{q_i} \right)^{1/\alpha} \pi_{Mi} \\ &= \frac{(1-\beta)\mathbb{E}[w_j|i]}{Q_i^{\frac{1}{\alpha}}} \\ &\quad \times \left[H^F \sum_{s=1}^I \sum_{t=1}^K \frac{\left((\sum_{i=1}^I \mathbb{E}[w_j|i] a_{it})^\lambda \bar{g}_t B_i \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} A_s^{\frac{1}{\alpha}} \right)^\epsilon}{\left(d_{it} d_{is} Q_i^{1-\beta} Q_s^{\frac{1-\alpha}{\alpha}} \right)^\epsilon} \right. \\ &\quad \left. + H^E \sum_{s=1}^I \frac{(B_i \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} A_s^{\frac{1}{\alpha}})^\epsilon}{\left(d_{is} Q_i^{1-\beta} Q_s^{\frac{1-\alpha}{\alpha}} \right)^\epsilon} \right] \\ &\quad + \left(\frac{(1-\alpha)A_i}{q_i} \right)^{1/\alpha} \sum_{r=1}^I \sum_{t=1}^K \frac{\left((\sum_{i=1}^I \mathbb{E}[w_j|i] a_{it})^\lambda \bar{g}_t B_r \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}} Q_i^{\frac{\alpha-1}{\alpha}} \right)^\epsilon}{\left(d_{rt} d_{ri} Q_r^{1-\beta} \right)^{-\epsilon}} \\ &= \frac{\phi_i N_i^{1-\mu}}{H} \end{aligned} \quad (44)$$

This land market condition provides a system of I equations for I unknown residential floor space prices, Q_i , which have the following properties:

$$\begin{aligned} 1. \quad & \lim_{Q_i \rightarrow 0} D_i(Q) = \infty > L_i \\ 2. \quad & \lim_{Q_i \rightarrow \infty} D_i(Q) = 0 < L_i \\ 3. \quad & \frac{dD_i(Q)}{dQ_i} < 0 \\ 4. \quad & \frac{dD_i(Q)}{dQ_j} < 0 \\ 5. \quad & \left| \frac{dD_i(Q)}{dQ_i} \right| > \left| \frac{dD_i(Q)}{dQ_j} \right| \end{aligned} \quad (45)$$

Hence, there exists a unique vector \mathbf{Q} which clears the land market system. In equilibrium, \mathbf{q} follows. Given housing prices, I can then obtain the vector \mathbf{w} of wages using the zero-profit condition, as well as the public facility quality vector \mathbf{g} . I can then derive the vectors $\boldsymbol{\pi}^F$ and

π^E . θ follows.

Q.E.D.

F Calibration Appendix

Residential amenities

The share of individuals residing in i as been defined as follows:

$$\begin{aligned} H_{Ri} &= H^F \pi_{Ri}^F + H^E \pi_{Ri}^E \\ &= H^F \frac{\sum_{s=1}^I \sum_{t=1}^K (d_{is} d_{it} Q_i^{1-\beta})^{-\epsilon} (g_t B_i w_s)^\epsilon}{\sum_{r=1}^I \sum_{s=1}^I \sum_{t=1}^K (d_{rs} d_{rt} Q_r^{1-\beta})^{-\epsilon} (g_t B_r w_s)^\epsilon} + H^E \frac{\sum_{s=1}^I (d_{is} Q_i^{1-\beta})^{-\epsilon} (B_i w_s)^\epsilon}{\sum_{r=1}^I \sum_{s=1}^I (d_{rs} Q_r^{1-\beta})^{-\epsilon} (B_r w_s)^\epsilon}. \end{aligned} \quad (46)$$

I can rewrite this condition as a system of I equations for the I unknown residential amenities B_i as follows:

$$\begin{aligned} D_i(B) &= H_{Ri} - \left[H^F \frac{\sum_{s=1}^I \sum_{t=1}^K (d_{is} d_{it} Q_i^{1-\beta})^{-\epsilon} (g_t B_i w_s)^\epsilon}{\sum_{r=1}^I \sum_{s=1}^I \sum_{t=1}^K (d_{rs} d_{rt} Q_r^{1-\beta})^{-\epsilon} (g_t B_r w_s)^\epsilon} \right. \\ &\quad \left. + H^E \frac{\sum_{s=1}^I (d_{is} Q_i^{1-\beta})^{-\epsilon} (B_i w_s)^\epsilon}{\sum_{r=1}^I \sum_{s=1}^I (d_{rs} Q_r^{1-\beta})^{-\epsilon} (B_r w_s)^\epsilon} \right] \\ &= 0. \end{aligned} \quad (47)$$

Lemma 3: Given the parameters $\{\epsilon, \kappa, \zeta, \beta\}$, and observables $\{\mathbf{Q}, \boldsymbol{\rho}, \boldsymbol{\rho}, \mathbf{w}, \boldsymbol{\pi}_R\}$, the system in (47) exhibits the following properties:

Property 1: $D(B)$ is continuous.

Property 2: $D(B)$ is homogeneous of degree zero.

Property 3: $\sum_{i=1}^I D_i(B) = 0$.

Property 4: $D(B)$ exhibits gross substitution:

$$\frac{\partial D_i(B)}{\partial B_j} > 0, \quad \forall i, j, i \neq j \quad (48)$$

$$\frac{\partial D_i(B)}{\partial B_i} < 0, \quad \forall i \quad (49)$$

PROOF: Properties 1 and 2 of Lemma 3 follow directly from an inspection of (47). Property 3 is satisfied by noting:

$$\sum_{i=1}^I D_i(B) = H - H = 0 \quad (50)$$

Property 4 can be established by noting:

$$\begin{aligned} \frac{\partial D_i(B)}{\partial B_j} = & H^F \frac{\epsilon B_i^{2\epsilon-1} (\sum_{s=1}^I \sum_{t=1}^K (d_{is} d_{it} Q_i^{1-\beta})^{-\epsilon} (g_t w_s)^\epsilon)^2}{[\sum_{r=1}^I \sum_{s=1}^I \sum_{t=1}^K B_r^\epsilon (d_{rt} d_{rs} Q_r^{1-\beta})^{-\epsilon} (g_t w_s)^\epsilon]^2} \\ & + H^E \frac{\epsilon B_i^{2\epsilon-1} (\sum_{s=1}^I (d_{is} Q_i^{1-\beta})^{-\epsilon} (w_s)^\epsilon)^2}{[\sum_{r=1}^I \sum_{s=1}^I B_r^\epsilon (d_{rs} Q_r^{1-\beta})^{-\epsilon} (w_s)^\epsilon]^2} > 0. \end{aligned} \quad (51)$$

Using property 2, which implies $\nabla D_i(B)B = 0$, it follows that:

$$\frac{\partial D_i(B)}{\partial B_i} < 0, \quad \forall i. \quad (52)$$

Thus, gross substitution is established.

Q.E.D.

Lemma 4: Given the parameters $\{\epsilon, \kappa, \beta\}$ and observables $\{Q_i, g_k, \rho_{ik}, \mathbb{W}_{ij}, w_j, H_{Ri}^E, H_{Ri}^F\}$, there exists a unique vector B which solves (47).

PROOF: I proceed in two steps. First, I show that there exists at most one (normalized) vector B which solves (47). Second, I show a vector B that solves (47) exists.

Gross substitution requires that $D(B) = D(B')$ cannot occur if B and B' non-collinear vectors. By homogeneity of degree zero, I can assume that $B' \geq B$ and $B_i = B'_i$ for some i . Now suppose that I lower (or keep constant) B' in all locations except in i one at a time. By gross substitution, B_i will increase in at least one step. Hence, $D(B) > D(B')$ which is a contradiction.

By homogeneity of degree zero, the search for an equilibrium amenity vector can be restricted to the unit simplex $\Delta = \{\sum_{i=1}^I B_i = 1\}$. Define on Δ the function $D^+(\cdot)$ by $D_i^+(B) = \max\{D_i(B), 0\}$. $D^+(\cdot)$ is continuous. Denote $\alpha(B) = \sum_{i=1}^I [B_i + D_i^+(B)]$ with $\alpha(B) \geq 1, \forall B$. Then define the function $f(\cdot)$ from the closed convex set Δ into itself as:

$$f(B) = [1/\alpha(B)][B + D^+(B)]. \quad (53)$$

By Brouwer's Fixed Point Theorem, there exist a $B^* \in \Delta$ such that $B^* = f(B^*)$. Since $\sum_{i=1}^I D_i(B) = 0$, it follows that at the fixed point for amenity, $B^* = f(B^*)$ and $D_i(B) = 0$ for all i .

Q.E.D.

Homogeneity of degree zero implies that the equilibrium amenity vector is unique up to a normalization. I impose the normalization that the geometric mean amenity is equal to 1, i.e., $\left[\prod_{i=1}^I B_i \right]^{1/I} = 1$.

Locational productivity

Given equilibrium housing market prices and observed workplace wages, I obtain the vector fundamental productivity, A_j using:

$$q_j = (1 - \alpha) \left(\frac{\alpha}{w_j} \right)^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}} \quad (54)$$

Density of development

The density of development is derived using the land market clearing condition,

$$\phi_i N_i^{1-\mu} = \frac{(1 - \beta) \mathbb{E}[w_j|i]}{Q_i} H_{Ri} + \left(\frac{(1 - \alpha) A_i}{q_i} \right)^{1/\alpha} H_{Mi}, \quad (55)$$

accounting for the fact that total wages are defined as

$$w_j = \bar{w}_j + \frac{\iota \sum_{i=1}^S L_i Q_i}{H}. \quad (56)$$

In practice, I use a standard contraction mapping approach to retrieve ϕ_i .