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Bisin, Alberto; Rubin, Jared; Seror, Avner; Verdier, Thierry

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Culture, Institutions & the Long Divergence

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Alberto Bisin, Jared Rubin, Avner Seror, Thierry Verdier

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Culture, Institutions & the Long Divergence

Abstract

Recent theories of the Long Divergence between Middle Eastern and Western European economies focus on Middle Eastern (over-)reliance on religious legitimacy, use of slave soldiers, and persistence of restrictive proscriptions of religious (Islamic) law. These theories take as exogenous the cultural values that complement the prevailing institutions. As a result, they miss the role of cultural values in either supporting the persistence of or inducing change in the economic and institutional environment. In this paper, we address these issues by modeling the joint evolution of institutions and culture. In doing so, we place the various hypotheses of economic divergence into one, unifying framework. We highlight the role that cultural transmission plays in reinforcing institutional evolution toward either theocratic or secular states. We extend the model to shed light on political decentralization and technological change in the two regions.

JEL Codes: O100, P160, P480, N340, N350, Z120, O330.

Keywords: long divergence, cultural transmission, institutions, legitimacy, religion.

Alberto Bisin
Department of Economics
New York University
19 West Fourth Street
USA - New York, NY, 10012
alberto.bisin@nyu.edu

Jared Rubin
Chapman University
1 University Drive
USA - Orange, CA, 92866
jrubin@chapman.edu

Avner Seror
Aix-Marseille School of Economics
5-9 Boulevard Maurice Bourdet / France
avner.seror@univ-amu.fr

Thierry Verdier
Paris School of Economics
48 Boulevard Jourdan
France - 75014 Paris
verdier@pse.ens.fr

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1 Introduction

Around the year 1000 C.E., the Muslim Middle East was far ahead of Christian Western Europe in terms of socio-economic development. By the dawn of the industrial period (circa 1750), however, the Middle East severely lagged behind along several dimensions, including technology, innovation, literacy, wages, and financial development (Bosker, Buringh and Van Zanden 2013; Kuran 2011; Mokyr 1990; Özmucur and Pamuk 2002). In the course of the medieval and early modern periods, economic institutions in the Middle East failed to keep pace with those of the West. This is what Timur Kuran calls the *Long Divergence* (Kuran 2011).

Several attempts to explain the Long Divergence have recently been put forward. Kuran (2011) highlights the inability of the Muslim world to create or adopt those fundamental commercial and financial institutions which were responsible for significant socio-economic growth in the West, such as banking, the corporation (and corporate law), and institutions supporting impersonal exchange. Kuran (2011) identifies the root cause of Middle East stagnation in the religious legal system (Islamic law or Sharia) in governing all human activities, including economic activities. He argues that certain aspects of Islamic law, such as its inheritance system and partnership law, placed impediments that were difficult for economic actors to overcome, especially as the world changed and opportunities for long-distance exchange flourished. As Islamic law remained the law of the land in most of the Muslim world well into the 19th century, these religious proscriptions retarded the region's economic growth. Rubin (2017) stresses the role of the political power conceded to Muslim religious authorities due to their ability to provide legitimacy to rulers.[1] He argues that this power was used to block important socio-economic advancements, a leading example being the printing press. In Europe, on the other hand, where the Catholic Church had a much weaker legitimating role, novel ideas and reforms spread more quickly (thanks also to the printing press), the economic elite developed laws and policies that portend economic

[1] Legitimacy is defined as the degree to which individual citizens believe they have a moral obligation to obey the ruler. The study of political legitimacy has a long history in the social sciences. Perhaps most famously, Weber (1947) defined political legitimacy as either charismatic, traditional, or legal-rational. Our definition follows more closely in the footsteps of the definition of political legitimacy employed by Lipset (1959, p. 86): "the capacity of a political system to engender and maintain the belief that existing political institutions are the most appropriate or proper ones for the society." For similar definitions of political legitimacy, see Levi, Sacks and Tyler (2009), Greif and Tadelis (2010), Rubin (2017), and Greif and Rubin (2020).

success, and long-run economic growth resulted.[2] Blaydes and Chaney (2013) concentrate on the different role of leadership in the Muslim world and Western Europe. In particular, they argue that the relative weakness of Western European rulers, who had to rely on feudal institutions for tax collection and military recruitment, led to a balance of power more favorable to local (feudal) elites, which turned out to promote economic growth in the long run. Muslim sultans, on the other hand, relied much more on centralized power, derived in large part due to their access to slave soldiers, to satisfy both fiscal and military needs. This limited the political power of economic elites and instead furthered the socio-economic power of religious elite.

Several common themes underlie all these explanations. Fundamentally, economic growth in Western Europe and the Middle East is seen as the outcome of the development of institutional and technological progress brought about or hindered by the interactions between rulers and elites—clerical and secular elites in particular. These explanations have taken us far in explaining the reversal of economic fortunes between Western Europe and the Middle East. Yet, each of these explanations raises a new set of puzzles. Consider first Kuran's explanation of how the persistence of religious proscriptions and the use of religious law can slow economic growth. Muslim rulers must have understood this. Why, then, did they continue to use Islamic institutions (like courts) that promote inefficiencies? The answers to these questions are not obvious from Kuran's framework. Or, consider explanations focusing on the role religious legitimacy plays in dampening economic growth (Kuru 2019; Platteau 2017; Rubin 2017). Religious legitimacy is only effective if people are religious and thus care about what religious authorities dictate. Why then, given that religions carry with them economic proscriptions, do people remain religious, and thereby yield religious legitimacy effective? Is religiosity and religious identity a cause or a consequence of institutional arrangements? These questions are left unanswered in theories

[2] Platteau (2017) and Auriol and Platteau (2017) present a related theory of political divergence between the Christian and Islamic worlds, stressing the role of the centralization of Christian religious institutions and the decentralization of Islamic religious institutions. They argue that this entails that Islamic rulers have to co-opt the marginal cleric, leaving out the more radical clerics from the political decision-making process. Chaney (2016) makes a related argument, noting that the relative fall of Islamic science coincided with the rise of the madrasa system, which was a reflection of the increasing power of the religious establishment in politics. Kuru (2019) likewise argues for the importance of the religion-state alliance in determining long-run economic and political outcomes in the Middle East. He dates the rise of this alliance to the 11th century, when the madrasa system came to prominence. Relatedly, Iyigun (2015) argues that because Islam and Christianity are both monotheisms in which there is "one true god", existential (and political) conflict was more likely to arise between states following the two religions than between states with polytheistic gods.

focusing on legitimacy. Finally, consider the role that slave soldiers played in constraining Middle Eastern rulers and, on the other hand, the greater constraint placed by the feudal elite on European rulers (Blaydes and Chaney 2013). The logic of this argument is that greater unchecked power of Muslim rulers meant they had less incentive to delegate power to local elites, who could more effectively provide revenue but wanted rights in return. One question that arises from this logic is that if Muslim rulers were sufficiently powerful, why should they have been afraid to delegate tax collection to (non-religious) elites? A ruler with a (near) monopoly on violence should not fear such elites, and he could increase tax revenue by delegating power to them.

In this paper we account for these puzzles, while generating new insights, with a stylized model of the joint dynamics of culture and institutions. In doing so, we elucidate the historical mechanisms which might have contributed to the divergent growth paths of Western Europe and the Middle East since the late medieval period. The fundamental abstract dynamics of culture and institutions in the model are the result of three basic elements, as in Bisin and Verdier (2017). First, institutions represent the relative political power of different groups in civil society to affect policy decisions, and institutional change is a mechanism to internalize externalities and other distortions characterizing the equilibrium. Second, the cultural profile of values and preferences in society evolves according to socio-economic incentives. Third, interdependence between institutions and culture is the fundamental factor determining their joint dynamics and their effects on economic performance.

The logic of our model is as follows. Legitimacy is an indirect choice of the ruler. She might want to delegate part of her political power to clerics, obtaining in turn religious legitimacy through the services provided by clerics to the religious component of civil society. These services in turn shape and control civil society's moral beliefs. The ruler, therefore, by delegating to the clerics and obtaining legitimacy, internalizes an externality which facilitates her ability to govern civil society. Since this externality operates through the religious component of civil society, its extent is related to the relative size of this component, and hence to the religious composition of society. The ruler's incentive to delegate to religious authorities also depends on the degree to which religious proscriptions dampen economic activity. Although greater proscriptions decrease the size of the param-

eter space in which a theocratic equilibrium arises, they limit economic development in that equilibrium.[3]

The socio-economic dynamics of society depend on the complementarity of culture and institutions. Institutional change delegating power to the clerics reinforces the incentives of religious individuals to transmit their cultural values, increasing their relative share in the population. A higher fraction of religious individuals in the population in turn augments the political incentives for the ruler to delegate power to clerics to increase legitimacy. The resulting joint dynamics of culture and institutions in this society display two types of stationary states: a *theocratic regime* where clerics have strong political power and the share of religious individuals in the population is high (in spite of religious proscriptions on economic activity); and a *secular regime* where clerics have little political power and the share of religious individuals is low.

Projecting this general abstract framework into the most prominent accounts of the socio-economic dynamics of Western Europe and the Middle East in the period from approximately 1000–1800 C.E., we model the socio-economic policy interactions of rulers, clerics, and civil society in a religious environment. Our base model accounts for two features central in the literature on the long divergence (and the economics of religion literature more generally): i) rulers derive *legitimacy* from the religious elite (Auriol and Platteau 2017; Coşgel, Miceli and Rubin 2012; Coşgel and Miceli 2009; Kuru 2019; Lewis 1974, 2002; Platteau 2017; Rubin 2011, 2017); and ii) religious authorities impose *proscriptions* that impinge on economic development (Berman 2000; Carvalho 2013; Iannaccone 1992; Kuran 2001, 2005, 2011; Seror 2018). In this context, we study whether different initial conditions and parameters of the model could generate distinct dynamic growth paths, converging to distinct stationary states, interpreted to represent Western Europe and the Middle East over the period 1000–1800 C.E. We also study whether the pattern of institutional formation and the spread of culture (religious beliefs) implied by the model is consistent with stylized facts which identify the histories of the regions in this period.

We extend the model to account for a third key feature of Middle Eastern and Western European political and economic development: constraints on executive power and the de-

[3]Religious proscriptions can have welfare-enhancing features. For instance, Iannaccone (1992) shows how seemingly inefficient sacrifice and stigma common among religious groups can serve as a mechanism to enhance commitment within the group. For a theoretical treatment of religious proscriptions, see Seror (2018). However, these welfare-enhancing features are inherently limited to smaller religious groups, since this entails that the non-committed do not join the religious "club." In this paper, we are concerned with mainstream religions that do not have this feature.

centralization of political power (Acemoglu and Robinson 2012, 2019; Acemoglu, Johnson and Robinson 2005*b*; Blaydes and Chaney 2013; Mann 1986; North and Weingast 1989; Tilly 1990). We show how a society's political centralization interacts with religious legitimacy and religious proscriptions to determine its long-run economic and political paths. While political decentralization can increase tax revenue by placing constraints on executive power (Besley and Persson 2009, 2010; Dincecco 2009; Johnson and Koyama 2017; North and Weingast 1989), it may come at the cost of undermining the efficacy of religious legitimacy. We further extend the model to consider the role of religion and religious legitimacy in innovation and technological change. Our model is consistent with recent theories which argue that culture (Davids 2013; Mokyr 1990, 2010, 2016; White 1972, 1978), and religious proscriptions in particular (Bénabou, Ticchi and Vindigni 2015, 2020; Coşgel, Miceli and Rubin 2012; Squicciarini 2020) can inhibit technological change.[4]

Our framework not only accounts for the long-run economic divergence between Western Europe and the Middle East, but it also accounts for the puzzles raised by each of the prevailing explanations. People remained religious in the face of religious proscriptions—a puzzle raised by the works of Kuran (2011) and Rubin (2017)—because there is feedback between religious institutions and cultural evolution. As institutions evolved so that religious legitimacy became more important, cultural values spread (optimally) throughout the population that were complementary to these institutions. Likewise, the model explains why Muslim rulers did not decentralize power to local elites, even though they had a near monopoly on violence and should not have feared these elites. Doing so would have reduced the power of religious authorities and could have triggered cultural evolution toward a more secular society. Given the significant legitimating power of Muslim religious authorities, this equilibrium was suboptimal from the perspective of Muslim rulers. In short, by combining the three prevailing theories in one unifying framework *and* by introducing the interaction between cultural and institutional evolution, our model clarifies the logic underlying the primary puzzles raised by the literature while providing many new insights.

The paper proceeds as follows. In Section 2 we lay out the basic model. Section 3 solves the model and presents key propositions. Section 4 presents extensions to the model, and Section 5 concludes.

[4]In is certainly not the case that religion and religious proscriptions always have a negative impact on economic development. See Barro and McCleary (2003) and McCleary and Barro (2019) for an overview of the literature and a theory of the positive associations between religion and economic development.

2 Ruler, clerics, and civil society

Consider a society populated by there are three types of agents: a ruler, clerics, and civil society. Civil society is composed of two types i of citizens: religious individuals ($i = Re$) in proportion q, and secular individuals ($i = S$) in proportion $1 - q$. Citizens work. Total production is $qe_{Re} + (1 - q)e_S$, where $e_i,\ i = Re, S$ is the per capita work effort generated by an individual of type i. The ruler lives off taxing civil society at a tax rate τ. The tax base which the ruler has access to is the total production of citizens: $E = qe_{Re} + (1 - q)e_S$. The ruler also builds and maintains religious infrastructures, m, for the clerics to provide religious services. The total religious services provided for the society are $\alpha_c\, m$, where α_c is the effort of the (representative) cleric. The building of religious infrastructures has cost $C(m)$ that the ruler pays for. On the other hand the clerics pay for the daily maintenance costs $F(m)$ of these infrastructures.[5]

The fundamental mechanism in the model, which induces a role for *legitimacy*, is the assumption that the provision of religious services facilitates governance and obedience for religious individuals. We capture this by assuming that religious individuals, when taxed by the ruler, subjectively perceive a tax rate τ^e_{Re} smaller than the actual τ chosen by the ruler and decreasing in the religious effort of the clerics, α_c:[6]

$$\tau^e_{Re} = \tau(1 - \theta\alpha_c). \tag{1}$$

The parameter θ represents the efficiency of the "legitimizing" technology of the clerics. Likewise, θ can be interpreted as the efficiency of religious legitimacy in encouraging compliance with authority (or, similarly, discouraging tax evasion) (Coşgel and Miceli 2009; Greif and Rubin 2020). For secular individuals, $\tau^e_S = \tau$.

On the other hand, religious services have an indirect cost, by imposing *proscriptions* (i.e., regulations and constraints) on individual behavior for both religious and secular individuals. We capture this effect by assuming that the cost of individual production

[5] These costs are assumed to be increasing in m and sufficiently convex to satisfy a regularity condition, needed to ensure that when religious clerics have a high political weight λ in the institutional structure, the policy problem associated to institutional design is well behaved, and provides a finite equilibrium provision of m.

[6] See Wintrobe (1998) for a model of loyalty to the leader affecting taxation levels (via its affect on tax avoidance) and Coşgel and Miceli (2009) for a model specifically focusing on the role of religious legitimacy on tax collection.

effort is

$$c(\alpha_c)\Phi(e_i), \text{ with } \Phi(e_i) = \frac{e_i^2}{2} \text{ and } c(\alpha_c) = 1 + \phi\alpha_c, \; i = Re, S. \tag{2}$$

The parameter $\phi > 0$ represents the degree of restrictiveness of religious prescriptions on economic activities.

The ruler has utility

$$U_r(m) = \tau E - C\,(m)\,.$$

Clerics derive utility $\alpha_c\, m$ from religious services, at effort cost $\Psi(\alpha_c)$. The utility of the clerics therefore is

$$U_c(m, \alpha_c) = m \cdot \alpha_c - \Psi(\alpha_c) - F(m).$$

Finally, the utility of individuals in civil society is

$$U_i(e_i) = e_i(1 - \tau_i^e) - c(\alpha_c)\Phi(e_i), \;\; i = Re, S.$$

increasing and convex in their argument. We assume the cost functions $C(.)$, $F(.)$ and $\Psi(.)$ are increasing and convex in their argument.[7] We denote $\bar{\tau}$ the maximum feasible tax rate which the government can impose.

The model captures the key parameters of two of the three prevailing theories of the "Long Divergence": religious legitimacy (θ) and religious proscriptions (ϕ). In Section 4.1, we will extend the model to account for the third theory: constraints on executive power (Blaydes and Chaney 2013). In what follows, we analyze how these parameters affect equilibrium outcomes and the joint dynamics of culture and institutions.

3 Equilibrium and dynamics

We study equilibrium in the society we have outlined in the previous section. We also study the joint dynamics of cultural values and institutions in this society, which we will define precisely and operationally.

At any time t society reaches an equilibrium of a game between the ruler, clerics, and civil society. At equilibrium, (we postulate that) policy is chosen to maximize a social welfare function which weights the political power of the ruler, the clerics, and the civil society. The policy choice and the choices of the (representative) cleric and the

[7] We also assume that $F'(m) < C'(m)$ for all $m > 0$ i.e., that the marginal cost of infrastructure maintenance is smaller than the marginal cost of building infrastructures.

(representative) member of each type of civil society are taken non-cooperatively with respect to each other. This is to model a policy choice environment which is plagued by lack of commitment, whereby the policy maker is not allowed to pick the policy in advance of the choices of the economic agents. Formally, the *societal equilibrium* is a Nash equilibrium of the simultaneous game between agents and the policy maker.

Institutional change is modeled as the outcome of commitment mechanism which, by delegating power across political groups in society, can affect future policies chosen to maximize social welfare (see Bisin and Verdier 2017). Cultural dynamics are modeled as purposeful inter-generational transmission through parental socialization and imitation of society at large (Bisin and Verdier 2001, 2017).

3.1 Equilibrium

The policy choice is the amount of religious infrastructures m. It is collectively chosen to maximize the social welfare function W, which encodes the relative power of the groups as weights. The relative power of the ruler is fixed (to $\frac{1}{2}$).[8] The power of the clerics and of civil society is, respectively, $\frac{\lambda}{2}$ and $\frac{1-\lambda}{2}$. The social welfare function to be maximized by the choice of policy m is then:

$$W = \frac{1}{2}U_r(m) + \frac{\lambda}{2}U_c(m, \alpha_c) + \frac{1-\lambda}{2}\left[qU_{Re}(e_{Re}) + (1-q)U_S\left(e_S\right)\right]. \tag{3}$$

The ruler, clerics, and citizens choose, respectively, τ, α_c, and e_i, $i = Re, S$ to maximize their utility.

The policy choice m, and the choices of τ, α_c, and e_i, $i = Re, S$, constitute a Nash equilibrium, denoted $\{\tau(\lambda), m(\lambda), \alpha_c(\lambda), e_S(\lambda), e_{Re}(\lambda)\}$.[9] At equilibrium, the optimal tax rate $\tau(\lambda)$ is equal to its maximum possible value $\overline{\tau}$. As the ruler has a higher political weight that the citizens, taxation is fully extractive. In order to simplify notation, we write τ instead of $\bar{\tau} = \tau(\lambda)$ in the rest of the paper. The comparative statics at equilibrium are summarized in the following Lemma.

[8] This is just for simplicity and concreteness. All that is needed is that the ruler has a large enough power with respect to the other members of society.

[9] The equilibrium is fully characterized in the Appendix. Since there is a complementarity between the provision of the religious good m and the investments of the clerics in religious infrastructures α_c, the uniqueness of the equilibrium is not guaranteed. Under mild conditions, however, the equilibrium is uniquely determined.

Lemma 1 ***Religious infrastructures:*** *The optimal investment in religious infrastructures, $m(\lambda)$, and the optimal effort of the clerics, $\alpha_c(\lambda)$, are increasing in λ and independent from θ and ϕ.*

When the weight of the clerics in social choice increases, so does the marginal benefits of provisioning the religious infrastructure m. In turn, the clerics increase their own effort in provisioning religious services $\alpha_c(\lambda)$. Since the weight of the clerics in social choice is $\frac{\lambda}{2}$, both $\alpha_c(\lambda)$ and $m(\lambda)$ increase with λ.

In the model, clerics do not derive utility from imposing proscriptions on economic activity nor from legitimizing the ruler. Hence, the investment in religious infrastructure $m(\lambda)$ and the provision of the religious services $\alpha_c(\lambda)$ are independent from θ and ϕ.

Lemma 2 ***Labor effort:*** *The effort of secular individuals $e_S(\lambda)$ is decreasing in λ and ϕ and is independent from θ. On the other hand, as long as $\theta \geq \frac{\phi(1-\tau)}{\tau}$, the effort of religious individuals $e_{Re}(\lambda)$ is increasing in λ and θ, and is decreasing in ϕ.*

When the efficiency of the clerics to legitimize the ruler θ increases, so does the effort of religious individuals who subjectively perceive a lower tax rate. By contrast, the efficiency of the legitimizing technology has no effect on the effort of secular individuals. An increase in the degree of restrictiveness of religious proscriptions, ϕ, leads to lower efforts from both the religious and secular individuals, as harsher proscriptions decrease individuals' labor productivity.

The political weight of the clerics affects the labor efforts through $\alpha_c(\lambda)$, the equilibrium effort of the clerics. While more effort from the clerics $\alpha_c(\lambda)$ makes secular individuals reduce their own labor effort—through costly regulations and prohibitions ϕ—when $\theta \geq \frac{\phi(1-\tau)}{\tau}$, clerics have the opposite effect on the labor effort of religious individuals e_{Re}. This is because when clerics provide more effort, the religious individuals perceive a lower tax rate. Despite the costly religious regulation, they increase their effort in the face of higher investments in religious infrastructures. In order to make this key difference between secular and religious individuals stark, we make the following Assumption:

Assumption 1 $\theta \geq \frac{\phi(1-\tau)}{\tau}$.

We denote the tax base as $E(\lambda) = qe_{Re}(\lambda) + (1-q)e_S(\lambda)$. From the two previous Lemmas, we deduce the following result:

Lemma 3 ***Tax base:*** *The tax base is increasing in q and θ, and is decreasing in ϕ. It increases with λ as long as $q \geq \frac{\phi(1-\tau)}{\tau\theta}$.*

While religious infrastructures increase the scope of religious proscriptions, they also positively affect the effort of the religious individuals under Assumption 1. Hence, when religious individuals are sufficiently numerous, the latter effect dominates, and the tax base $E(\lambda)$ increases with the effort of the clerics $\alpha_c(\lambda)$, so it increases with λ. Similarly, since θ positively affects the labor effort of religious individuals, it also positively affects the tax base. Religious proscriptions ϕ negatively affect the tax base, as they decrease labor efforts. The tax base increases with the fraction of religious q, who provide greater effort than their secular counterparts.

3.2 Institutional Dynamics

Institutional change at each time t is represented by the choice of the relative power to be delegated to clerics and civil society in the future; that is, the choice of λ_{t+1} from the point of view of the social welfare function with weight λ_t.[10] More formally, at any time t, given institutions λ_t, future institutions λ_{t+1} are designed as the solution to:

$$\max_{\lambda_{t+1}} \frac{1}{2} U_r(m(\lambda_{t+1})) + \frac{\lambda_t}{2} U_c(m(\lambda_{t+1}), \alpha_c(\lambda_{t+1})) + \frac{1-\lambda_t}{2} \left[q_t U_{Re}(e_{Re}(\lambda_{t+1})) + (1-q_t) U_S\left(e_S(\lambda_{t+1})\right)\right]. \quad (4)$$

Institutional change operates as a commitment mechanism by delegation which internalizes two externalities that are not taken into account by individual decisions in equilibrium. The first one relates to the fact that the provision of religious infrastructure m grants *legitimacy* to the ruler by reducing the subjectively perceived tax rate for religious individuals. The second is the fact that it also has a depressing effect on labor productivity. Hence, more provision of the religious good m not only affects the utility of the clerics, but also feeds back into the utility of both the ruler and the citizens. Solving the optimization problem (4), we obtain the following result:

[10] We assume that institutional design is myopic. That is, institutions are designed for the future as if they would never be designed anew in the future. This implies that an institutional structure does not internalize institutional "slippery slopes," whereby moving to a different structure of decision rights may in turn trigger subsequent institutional changes leading to undesirable outcomes from the point of view of the initial structure. See Bisin and Verdier (2017) for a discussion of how this issue can be accounted in the model.

Proposition 1 *The optimization problem (4) admits a unique solution* $\lambda_{t+1} \in [0,1]$. *The solution is characterized by a threshold* $\overline{q}(\lambda_t) \in [0,1]$ *such that,*

$$\lambda_{t+1} > \lambda_t \text{ (resp. } \leq\text{), if } q_t > \overline{q}(\lambda_t) \text{(resp. } \leq\text{).}$$

Furthermore, the threshold $\overline{q}(\lambda_t)$ *is decreasing in* θ *and increasing in* ϕ.

The uniqueness result follows from the convexity of the optimization problem. Whether more power is delegated to the clerics over time depends on the fraction of religious individuals q_t. If the religious are sufficiently numerous, then a larger weight to the clerics $\lambda_{t+1} > \lambda_t$ increases their effort $\alpha_c(\lambda_{t+1})$. This in turn increases both the utility of the ruler U_r—who benefits from a larger tax base (Lemma 3)—and the total welfare of the citizens $q_t U_{Re} + (1-q_t)U_S$. Civil society can also benefit from higher effort from the clerics—despite religious proscriptions—as religious individuals are better off when they perceive a lower tax rate.

Relative to the comparative static results, when the strength of religious proscriptions ϕ increases, so does the cost for the ruler of using religious legitimacy as a means of extracting resources from the population. The parameter space over which λ_t increases shrinks as $\overline{q}$ increases. On the other hand, when clerics are efficient at legitimizing the ruler, i.e. when θ increases, then delegating power to the clerics is more beneficial and $\overline{q}$ decreases.

3.3 Cultural dynamics

Cultural dynamics are modeled as purposeful inter-generational transmission (Bisin and Verdier 2001, 2017), through parental socialization and imitation of society at large. *Direct vertical* socialization to the parent's trait $i \in \{Re, S\}$ occurs with probability d_i. If a child from a family with trait i is not directly socialized, which occurs with probability $1-d_i$, he/she is *horizontally/obliquely* socialized by picking the trait of a role model chosen randomly in the population.[11] The probability P_{ij} that a child in group i is socialized to trait j writes as:

$$\begin{aligned} P_{ii} &= d_i + (1-d_i)q_i \\ P_{ij} &= (1-d_i)q_j; \end{aligned} \tag{5}$$

[11] See Bisin and Verdier (2011) for a survey of the economic literature on cultural transmission. Vertical, horizontal, and oblique transmission are the core mechanisms in the dual-inheritance theory of cultural evolution. For more, see Cavalli-Sforza and Feldman (1981), Boyd and Richerson (1985), and Henrich (2015).

with $q_{Re} = q$ and $q_S = 1 - q$. We assume that the probability of direct socialization d_i is the solution of a parental socialization problem in which: a) parents are paternalistic (i.e., imperfectly altuistic) and have a bias for children sharing their own cultural trait, b) such paternalistic bias writes as $\Delta V_i(\lambda_t) = V_{ii}(\lambda_t) - V_{ij}(\lambda_t)$, where $V_{ij}(\lambda_t) = U_i(e_j(\lambda_t))$ is the utility perceived by a type i parent of having a type j child, for $i, j \in \{Re, S\}$ and $j \neq i$, c) parents of type $i \in \{Re, S\}$ have socialization costs that are increasing and convex in d_i, d) religious infrastructures m_t may act as complementary inputs to the transmission effort d_{Re} of religious families in the socialization of children to the religious trait.

As shown in the appendix, in such a set-up, the dynamics of the proportion of the population with the religious trait is characterized by the following "cultural replicator" dynamics:

$$q_{t+1} - q_t = q_t(1 - q_t)\{d^*_{Re} - d^*_S\}. \tag{6}$$

where $d^*_{Re} = D_{Re}\left[(1 - q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t)\right]$ is the equilibrium socialization effort of a religious parent, and an increasing function both in $(1 - q_t)\Delta V_{Re}(\lambda_t)$ and $m(\lambda_t)$. Similarly $d^*_S = D_S\left[q_t \Delta V_S(\lambda_t)\right]$ is the equilibrium socialization effort of a secular parent, and an increasing function of $q_t \Delta V_S(\lambda_t)$. In equation (6), the term

$$D(q_t, \lambda_t) = d^*_{Re} - d^*_S = D_{Re}\left[(1 - q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V_S(\lambda_t)\right]$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. This term is frequency dependent (i.e., it depends on the state of the population q_t). It is also affected by the institutional environment λ_t, as this variable interacts with the process of parental cultural transmission both through paternalistic motivations $\Delta V_i(\lambda_t)$, and through the provision of religious infrastructures $m_t = m(\lambda_t)$ as a complementary input to religious family socialization. We deduce the following result:

Proposition 2 *There exists a threshold $q^*(\lambda_t)$ such that*

$$q_{t+1} < q_t \text{ (resp. } \geq) \text{ if } q_t > q^*(\lambda_t) \text{(resp. } \leq).$$

Furthermore, the threshold $q^*(\lambda_t)$ is increasing in θ and λ_t and decreasing in ϕ.

Because the process of cultural transmission (5) is characterized by cultural substitution between vertical and oblique transmission, the relative "cultural fitness" of the religious trait $D(q_t, \lambda_t)$ is decreasing in the frequency q_t of religious individuals in the population

(Bisin and Verdier 2001). Consequently, the proportion $q^*(\lambda_t)$ such that $D(q^*(\lambda_t), \lambda_t) = 0$ is the unique attractor of the cultural dynamics in (6). When the fraction of religious individuals q_t is above (resp. below) $q^*(\lambda_t)$, then it decreases (resp. increases) in order to converge in the direction of $q^*(\lambda_t)$.

The dependence of the threshold $q^*(\lambda_t)$ on the institutional environment λ_t and comparative statics on the parameters θ and ϕ depends on how the relative "cultural fitness" $D(q_t, \lambda_t)$ of the religious trait is affected by changes in such features.

An increase in the political weight of the clerics λ_t affects cultural transmission in two ways, through its effect on socialization incentives $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$ and through its effect on religious infrastructures, $m = m(\lambda_t)$. On the one hand, an increase in λ_t promotes the clerics' effort $\alpha_c(\lambda_t)$ and consequently leads to a lower perceived tax rate τ^e_{Re} by religious individuals. The labor effort choice of religious and secular individuals is therefore further apart and, consequently, the incentives of parents to socialize their children to their own cultural trait, $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$, are larger in both groups.[12] However when the socialization effort of religious parents is more sensitive to these incentives than the effort of secular parents, the religious trait is relatively more successfully transmitted than the secular trait, and $D(q_t, \lambda_t)$ is shifted up with an increase in λ_t. On the other hand, an increase in λ_t also increases the amount of religious infrastructures $m = m(\lambda_t)$. When such infrastructures enter as complementary inputs in the socialization process of the religious trait, then again religious parents tend to socialize more intensively than secular ones when m increases. The religious trait has consequently higher cultural fitness than the secular trait and again $D(q_t, \lambda_t)$ is shifted up with λ_t. In either situation, the diffusion of the religious trait is favored by an increase in λ_t, and $q^*(\lambda_t)$ becomes larger.

A change in the other parameters θ and ϕ affects the relative cultural fitness of the religious trait only through their induced changes on the paternalistic motives $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$. For instance, a higher efficiency of the clerics θ tends to widen the gap between the optimal work effort of a religious individual compared to that of a secular individual. As a consequence, an increase in θ shifts up both $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$. As mentioned above, when religious parents are more sensitive to paternalistic motives than secular parents, these shifts lead religious parents to socialize more intensively than secular parents, and religious values are passed from generation to generation with a higher intensity. This results in a higher value of $q^*(\lambda_t)$. Conversely, a higher value of religious

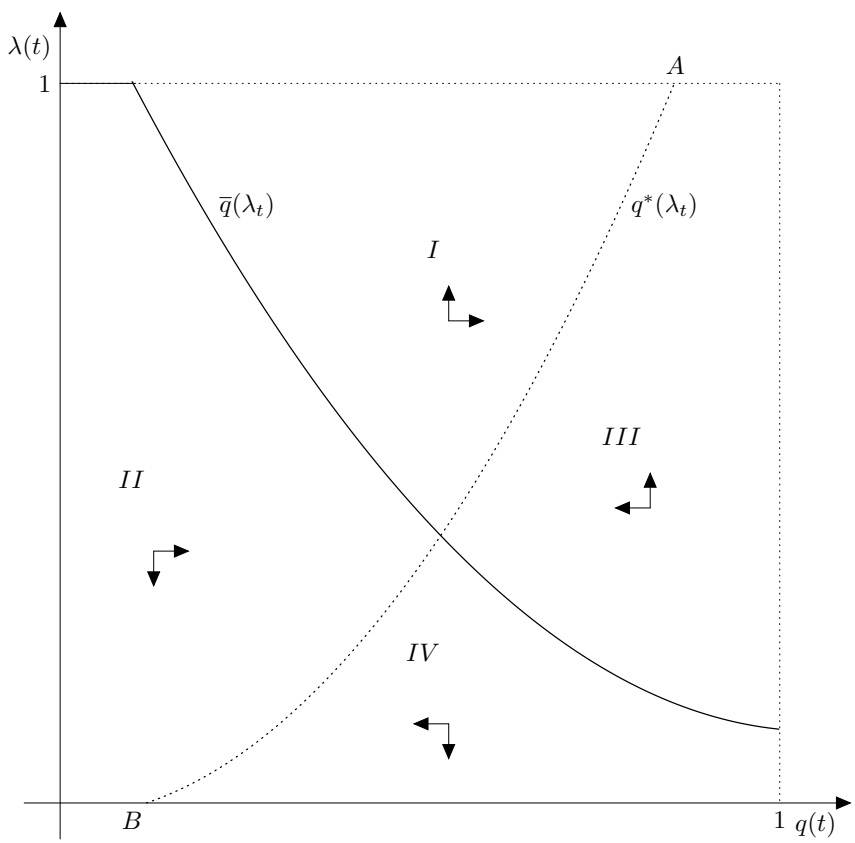
[12] Given the quadratic specification of the utility function $U_i(e_i)$, and substituting the optimal labor efforts in the utility of the citizens, one finds that $\Delta V_{re}(\lambda_t) = \Delta V_s(\lambda_t) = \frac{(\tau\theta\alpha_c(\lambda_t))^2}{2(1+\phi\alpha_c(\lambda_t))}$, which is increasing in λ_t.

proscriptions ϕ dampens the impact of work effort on economic outcomes. Consequently, behavioral differences induced by cultural traits are less relevant from a utility point of view. This in turn reduces the paternalistic motives $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$ of religious and secular parents. The effect of a change in proscriptions ϕ on cultural evolution is then qualitatively the opposite of that of a change in θ.

3.4 Joint dynamics of culture and institutions

Under the conditions of Propositions 1 and 2, we can represent the joint cultural and institutional dynamics in the phase diagram of Figure 1. The black line represents the threshold of the institutional dynamics $\overline{q}(\lambda_t)$. The dotted line represents the threshold $q^*(\lambda)$ associated with the cultural dynamics. The arrows in Figure 1 depict the joint dynamics of culture and institutions, given our results in Propositions 1 and 2.

Figure 1: Joint Dynamics of Culture and Institutions



The joint dynamics of culture and institutions in this society display two steady states. The first could be characterized as a *theocratic regime* represented by point A in Figure 1, where the ruler is legitimized by religion, the clerics have significant political power (λ is high), taxation is high (the tax rate τ is maximal and the tax base E is high), and the share of religious individuals in civil society is high (q is high). The second steady state, point B in Figure 1, could be characterized as a *secular regime* where the ruler is not legitimized by religion, clerics have little political power (λ is zero), taxation is limited (the tax rate τ is maximal but the tax base E is small), and civil society is secular (q is small). Two mechanisms characterize the dynamics.

Complementarity. In regions I and IV of Figure 1, the ruler's option to rely on religious legitimacy to increase tax capacity induces a fundamental *complementarity of the dynamics of culture and institutions*. Committing to an institutional set-up delegating power to the clerics reinforces the incentives of the religious individuals to transmit their values inter-generationally. In turn, a predominance of religious individuals in civil society augments political incentives to commit and change the institutional set-up so as to empower the clerics. Complementarity operates to produce dynamics converging to the *theocratic regime*, as represented by point A in Figure 1 or to the *secular regime*, as represented by point B.

Transitory paths. In regions II and III of Figure 1, the dynamics are not characterized by a complementarity. In region II, for example, religious individuals are insufficiently numerous and λ_t decreases over time. At the same time, religious values grow: as the religious trait is not widespread, religious individuals invest more in direct socialization (Bisin and Verdier 2001). Depending on the speed of institutional change relative to cultural change, the joint dynamics can either reach region I or region IV. Region II is a transitory path to the theocratic equilibrium when the religious population grows fast despite the political weight of the clerics decreasing over time. In this case, religious individuals become sufficiently numerous at some point that the course of institutional change is reversed, and the political power of religious clerics starts to grow after a transitory period.

In region III, religious individuals are sufficiently numerous for the political power of the religious clerics to increase over time. But the religious population is too large, so secular individuals invest more in direct socialization. Again, depending on the speed of institutional change relative to cultural change, either region I or region IV could be reached by the joint dynamics. If the religious population decreases faster than religious

institutions grow, we can expect the joint dynamics to reach region IV. In this case, the religious population becomes so low after a transitory period that the political weight of the clerics decreases over time and equilibrium B is reached in the long-run.

Proposition 3 ***Joint dynamics of culture and institutions:*** *The likelihood of reaching the theocratic equilibrium is increasing in religious legitimacy θ and decreasing in the level of religious proscriptions ϕ.*

Proposition 3 combines the results established in our analysis of the institutional and cultural dynamics (Propositions 1 and 2). A higher efficiency of the clerics θ—by definition—decreases the subjectively perceived tax rate of the religious. As a consequence, religious parents have a higher willingness to transmit their cultural values intergenerationally. At the same time, clerics become more important in the institutional apparatus, as they increase social welfare by (i) lowering the perceived cost of effort and (ii) increasing the rents extracted by the ruler. Therefore, the complementarity between the spread of religious values and institutional changes delegating power to the clerics is reinforced when θ is higher.

At the opposite end, when religious proscriptions ϕ increase, the cost for the ruler from using religious legitimacy as a means of extraction increases. The threshold $\overline{q}(\lambda_t)$ also increases. Similarly, more religious proscriptions makes the religious trait less resilient, as the threshold $q^*(\lambda_t)$ associated with the cultural dynamics decreases. This explains why the complementarity between the spread of religious values and institutional changes delegating power to the clerics is weakened.

3.5 The historical stylized pattern

In the historical context we study—Western Europe and the Middle East over the period 1000–1800—the literature has proposed three key differences between the regions. Our model, so far, can account for two of these three differences. The first is that Muslim religious authorities had greater capacity to legitimate (i.e., higher θ) than their Christian counterparts. This was due to the environment in which the religions were born. Christianity was born in the Roman Empire and was in no position to legitimate the emperor. Early Christian doctrine is reflective of the low legitimating capacity of Christianity (Feldman 1997; Rubin 2011). For instance, Jesus famously said "Render unto Caesar the things which are Caesar's, and unto God the things that are God's" (Matthew 22:21). Meanwhile,

Islam formed conterminously with expanding empire, and there are numerous important Islamic dictates specifying the righteousness of following leaders who act in accordance with Islam (Hallaq 2005; Rubin 2011, 2017). Although early Islamic rulers claimed to have religious authority vested in themselves (Crone and Hinds 1986), after the religious establishment consolidated in the ninth century (Coşgel, Miceli and Ahmed 2009), and certainly after the rise of the madrasa system in the 11th century (Kuru 2019), religious authorities were the primary agents capable of determining whether rulers acted in accordance with Islam.

Second, economically-inhibitive religious proscriptions existed in both Christianity and Islam, but were more pervasive and *persisted* for much longer in the latter (i.e., ϕ is higher in Islam). For instance, Kuran (2005, 2011) cites how Islamic law regarding partnerships and inheritance combined to discourage long-lived or large business ventures. Partnerships would be split among numerous heirs upon the death of any partner, any of whom could dissolve the enterprise. Another well-known set of proscriptions are those related to usury, which existed in both Islam and Christianity, but persisted for much longer in the former (Noonan 1957; Rubin 2011, 2017). More generally, Islamic law covers numerous aspects of commercial life, but it was formulated in the first few centuries of Islam. While Islamic law did change over time to address economic exigencies, this change was slower than changes that occurred to secular law in Europe (Berman 1983; Hallaq 1984, 2005).

Our model describes how differences in these two key parameters, θ and ϕ, may have affected the institutional, cultural, and economic trajectories of the Middle East and Western Europe. These parameters affect the dynamics induced by the complementarity of culture, q, and institutions, λ. In one equilibrium, which as Proposition 3 states may arise from high θ or low ϕ, the dynamics are characterized by the reinforcement over time of religious legitimacy, power of the clerics, and diffusion of religion in civil society. In the historical context we are interested in, this was roughly the situation in Western Europe following the fall of the Roman Empire. The Germanic "follower kingdoms" were not initially ruled by Christians, although the Roman population had largely become Christianized in the fourth and fifth centuries. This provided strong incentive for Germanic rulers to either convert to Christianity or promote Christianity.[13] In other words, rulers seeking legitimacy in a high q environment chose to employ religious legitimacy as a key component of their right to rule (even if θ was not as high as it was in the Islamic Middle East).

[13] One of the most important cases is the conversion of the Frankish king Clovis (r. 481–509), who employed Christianity to legitimize his Frankish expansion into new territory (Tierney 1970, Rubin 2017, pp. 62–63).

The "high q, high λ" equilibrium characterized Western Europe until the 11th century, when the re-birth of commerce gave rise to independent cities and increased tensions between the religious and secular elite (Angelucci, Meraglia and Voigtländer 2020; Rubin 2011). The rebirth of commerce entailed that religious proscriptions (ϕ in our model), such as the ban on usury, were more economically harmful. In the absence of widespread trade prior to the Commercial Revolution, such proscriptions had little dampening effect on the economy. Yet, they became increasingly harmful as trade flourished (Rubin 2011). The increase in ϕ combined with the relatively low θ of Christian religious authorities encouraged rulers to break with the Church as a key means of legitimation. The most important event in this break was the Investiture Controversy (1075–1122), a conflict between various secular rulers and the papacy over the role of the former in religious affairs. The Investiture Controversy culminated in European rulers seeking alternative justifications for their rule (i.e., lowering λ) (Tierney 1988, pp. 33–95). They found these alternative justifications in the universities, where leading scholars provided justification for secular rule based on Aristotelian thought, while others helped codify various branches of secular law such as merchant law, feudal law, and manorial law (Berman 1983; Cantoni and Yuchtman 2014; Hollenbach and Pierskalla 2020). Indeed, Blaydes, Grimmer and McQueen (2018) find that it was precisely in this period that European political advice texts began to de-emphasize religious appeals. As a whole, these events helped place much of Western Europe on a path towards the more "secular" equilibrium described in our model, in which there is little role for religious authorities in legitimating political rule, with more political power resting in civil society.

The complementarity between culture and institutions was exacerbated by the Reformation, which further decreased the political power of the clerical elites. In England, Greif and Rubin (2020) find that following the Reformation, the political power of religious authorities dropped significantly and the law (as formed in Parliament) became a key source of royal legitimacy. In Germany, Cantoni, Dittmar and Yuchtman (2018) find that, following the Reformation, there was a massive reallocation of resources and education from religious to secular purposes. In other words, where the Reformation undermined the political power of the Church (i.e., lowered λ), less cultural capital was invested in religious pursuits.

In the Muslim Middle East, a "high q, high λ" equilibrium emerged a few centuries after the initial spread of Islam and persisted (with ebbs and flows) until the present day. Islam spread quickly, reaching Spain in the west and the Indian subcontinent in the east

within its first century. The "Islamic world" was not thoroughly Muslim for a century or two after the initial spread of Islam (i.e., it had low q), and it first spread along trade routes before spreading into other Muslim-controlled territory (Michalopoulos, Naghavi and Prarolo 2016, 2018). After the first Caliphate (632–661), whose rulers were companions of the Prophet Muhammad, the Sunni successor empires (the Umayyad Empire (661–750) and the Abbasid empire (750–1258)) employed Islamic religious authorities to legitimate rule, provide jurisprudence, and administer imperial rule.

In the context of our model, these early empires were in region II of Figure 1: they had relatively high λ but low q. In this region, either a secular or theocratic equilibrium can arise in the long run, depending on the relative speeds of institutional and cultural change. In the historical context, institutional change away from a strong clerical class was slow to arise. The reason was that, despite the initially low q, religious authorities provided stability and essential services. After the religious establishment consolidated in the eighth and ninth centuries, they were able to provide legitimacy by providing judgments and new interpretations of law that supported the state (Hallaq 2005; Rubin 2017). Over time, the Muslim population grew (i.e., q increased), and the religious elite were also able to provide legitimacy by associating the ruler's name with piety. As a result, a "theocratic equilibrium" emerged in the long run.

Two examples from two different periods and regions highlight the reinforcement of Muslim institutions and culture in a "high q, high λ" world. First, Chaney (2013) finds that Egyptian religious authorities were more secure in their rule (e.g., higher λ) when the Nile flooded or there was a drought. This is precisely when a ruler would most need religious legitimacy, both because the tax base would be lower and because there was a greater threat of revolt. Moreover, this was a period of increasing Islamization of the Egyptian population (i.e., q was increasing). Saleh (2018) finds evidence of massive conversions of lower socio-economic status Copts into Islam: by 1200, Muslims were 80% of the Egyptian population, and by 1500 they were over 90% of the population.[14] Combined, these two studies reveal a "high q, high λ" equilibrium, with cultural and institutional forces reinforcing each other.

A second example comes from the Arab provinces of the Ottoman Empire, where the population had largely converted to Islam centuries prior to Ottoman expansion (i.e., q was high). In the late 15th century, the Ottomans brought the religious establishment into the state, establishing the office of the Grand Mufti (chief religious jurist). This gave the

[14] Saleh (2018) argues that negative selection among Copts was due to the poll tax that non-Muslims had to pay; those that could not afford it simply converted to Islam.

Ottomans significant power to formulate controversial decisions in a manner consistent with Islam (Imber 1997). Meanwhile, the reinforcement of institutions and culture strengthened after the Ottomans conquered the Egyptian Mamluk Empire (in 1517) and took control over Mecca and Medina, the two holy cities of Islam. This further enhanced the capacity of clerics to confer legitimacy by associating the ruler with Islamic piety (e.g., mentioning the name of the legitimate ruler in each Friday sermon or supporting obedience to the ruler in judicial rulings) (Hallaq 2005, ch. 8). Thus, the high level of religious legitimacy (θ) provided by Muslim clerics resulted in a "high q, high λ" equilibrium for much of Ottoman history.

Hence, our model squares two of the leading theories of the "Long Divergence." It suggests that the diverging long-run paths of these economies of the two regions—"high q, high λ" in the Middle East and "low q, low λ" in Western Europe—were in part a result of the relatively high efficacy of religious legitimacy (θ) in the Islamic world. This meant that the two regions had different responses to religious proscriptions (ϕ). In Western Europe, once commerce revived in the 11th and 12th centuries, religious proscriptions were sufficiently economically damaging to push society on the path that ultimately resulted in a low q, low λ equilibrium. On the other hand, in the Islamic world such religious proscriptions may have been even more economically damaging at the time, given that the Islamic world was ahead of Europe. However, the relatively high θ in Middle Eastern societies meant that a high q, high λ equilibrium persisted *in spite of* the relatively high religious proscriptions (ϕ) of Islam. These insights therefore unify Kuran's theory emphasizing religious proscriptions with theories emphasizing religious legitimacy (Kuru 2019; Platteau 2017; Rubin 2017). Kuran's theory is not just that religious proscriptions existed in Islamic law, but that they *persisted* for so long after they were useful. Meanwhile, an emphasis on religious proscriptions reveals why legitimating arrangements changed over time (in Europe). More importantly, it sheds light on why the relative economic stagnation of the Middle East occurred *in spite of* the welfare-enhancing properties of religious legitimacy.

So far, our model does not account for the third major theory of the long divergence: Middle Eastern rulers had more unconstrained power relative to other elites. Blaydes and Chaney (2013) ascribe the relatively greater power of Middle Eastern rulers to their access to slave soldiers, which gave rulers access to coercive power without ceding political power. Meanwhile, weaker European rulers had greater incentive to negotiate with their economic (i.e., feudal) elites for revenue and military power, since they had little capacity to rule otherwise (Duby 1982). Throughout Europe, rulers also ceded power to urban burghers,

who had relative freedom from imperial rule (Angelucci, Meraglia and Voigtländer 2020; Mann 1986; Putnam, Leonardi and Nanetti 1994; Schulz 2020). More generally, this meant that Muslim rulers had less constraint on their power, which a large literature suggests is harmful for economic growth (Acemoglu and Robinson 2012; Acemoglu, Johnson and Robinson 2005*b*; North and Weingast 1989; North, Wallis and Weingast 2009; van Zanden, Buringh and Bosker 2012). Our model does not permit the ruler to share power with other elites that may constrain her, so it cannot speak to the conditions under which this occurs in a high q, high λ environment. In the next section, we extend the model to consider how political decentralization interacts with the various parameters of importance in our model (namely, θ and ϕ).

4 Extensions

In this section we extend and enrich the model by considering the emergence of political decentralization and technological progress. This will allow us to read more clearly the historical evidence through the theoretical implications of our model.

4.1 Religious legitimacy and political decentralization

Pre-modern states tended to have little fiscal capacity or capacity to provide law and order to regions far away from the capital. Administrative capacity tended to be quite weak in most parts of the world, meaning that rulers could not easily implement their desired policies (González de Lara, Greif and Jha 2008; Greif 2008; Ma and Rubin 2019). As such, there was a limit to the potential tax revenue available to rulers that was well below the optima on a Laffer curve (Besley and Persson 2009, 2010; Dincecco 2009; Johnson and Koyama 2017). This issue is (implicitly) central to the framework proposed by Blaydes and Chaney (2013). Without the capacity to collect revenue on their own, pre-modern rulers had to delegate tax collection to powerful people. Such powerful people could deter tax evasion via force and more easily assess just how much taxable surplus an individual or community had. The degree to which rulers had to delegate depended on their own power vis-à-vis other elites. According to Blaydes and Chaney (2013), Muslim rulers had to delegate less because they had access to slave soldiers. This meant they did not need local elites for military service or, oftentimes, tax collection. Meanwhile, the feudal arrangement

in medieval Europe was such that local taxes were collected by powerful people and in return rulers received military service and, occasionally, tax revenue.

In this section, we model the interactions between rulers and powerful elites. We consider a modified version of the previous model where political power is divided between three groups: the ruler, religious clerics, and a secular elite (i.e., feudal lords, parliament, or the military). We study the conditions under which the ruler decentralizes political institutions by sharing political power with the secular elite, who has the capacity to collect taxes.

We treat the secular elites as representatives of the citizenry. In terms of the distribution of power between groups, we assign the "ruling coalition" the combined weight of the ruler and the secular elites, $\frac{1}{2} + \frac{1-\lambda}{2} = 1 - \frac{\lambda}{2}$, in social welfare. This is similar to the baseline model, with the citizenry being replaced by the secular elites. In other words, if the ruler and the secular elites are the "ruling coalition" (as in North, Wallis and Weingast 2009), then $1 - \frac{\lambda}{2}$ is the total weight of the coalition. The clerics have weight $\frac{\lambda}{2}$ and citizens have no political power, that is, zero weight.[15]

The secular elite enforces tax compliance and it shares with the ruler the proceeds of tax collection. The share of the tax revenues accruing to the ruler vis-a-vis the secular elites is $\beta \in [0, 1]$. As a simple illustration, a regime where $\lambda = 1$ can be interpreted as a theocracy, while $\lambda = 0$ is a dictatorship when $\beta = 1$, and a republic when $\beta = 0$, as the ruler does not benefit from tax revenues. We denote $\alpha_l \in [0, \overline{\alpha}_l]$ the enforcement effort of the secular elites, with $\overline{\alpha}_l > 0$. Let $\mu\frac{\alpha_l^2}{2}$, $\mu > 0$, be a quadratic cost associated with this effort. The utility of the secular elites can be expressed as:

$$U_l(m, \alpha_l) = (1 - \beta)[\tau E - C(m)] - \mu\frac{\alpha_l^2}{2}. \tag{7}$$

Consider now the utility of the ruler. We assume the ruler faces a cost $\rho\alpha_l$ when letting the secular elite enforce tax compliance α_l. For instance, medieval European rulers provided feudal lords with lands to administer. Tax enforcement was accompanied with the hiring and building up of a force of violence by these lords. These elements suggest that the more the ruler cedes to lords the power of tax enforcement, the larger is the military power of the lords, which may eventually be turned against the ruler herself. The cost $\rho\alpha_l$ is a simple way to capture such threats. We maintain the assumption that the maintenance cost of

[15] This is a simplification to reduce the dimensionality of the dynamics of institutions while expanding the qualitative features of the narrative of the interactions between ruler, clerics, and citizens we analyzed in Section 3.

religious infrastructures paid by the clerics is $F(m)$. The utility of the ruler is then

$$U_r(m) = \beta(\tau E - C(m)) - \rho\alpha_l;$$

and the utility of the clerics is

$$U_c(m, \alpha_c) = m\alpha_c - F(m) - \psi(\alpha_c).$$

We assume that citizens do not necessarily comply with tax collection. Let $\epsilon(\alpha_l)$ be a measure of the capacity of tax enforcement on the part of the elites and c an (inverse) measure of the capacity of individuals to evade taxes; so that an individual who does not comply with tax collection faces an expected tax rate $c\epsilon(\alpha_l)$. Specifically, we assume that $\epsilon(\alpha_l)$ in increasing in α_l and c is drawn from a uniform distribution on a segment $[0, \overline{c}]$, with $\overline{c} > 0$.[16] The productivity of citizens is $\frac{1}{1+\phi\alpha_c}$, with ϕ being religious proscriptions. The utility of an individual belonging to type $i \in \{Re, S\}$ with an evasion capacity c is then:

$$U_i = \begin{cases} \frac{1-\tau_i}{1+\phi\alpha_c} & \text{if the individual complies, and} \\ \frac{1-c\epsilon(\alpha_l)}{1+\phi\alpha_c} & \text{otherwise,} \end{cases} \tag{8}$$

with $\tau_{Re} = \tau(1 - \theta\alpha_c)$ and $\tau_S = \tau$.

Equilibrium: At any time t, society reaches an equilibrium of the game between the ruler, clerics, secular elite, and civil society. The religious infrastructures m are collectively chosen to maximize social welfare:

$$\left(1 - \frac{\lambda}{2}\right)[U_r(m) + U_l(m, \alpha_l)] + \frac{\lambda}{2}U_c(m, \alpha_c)). \tag{9}$$

The clerics and the secular elite choose, respectively, α_c and α_l. We denote $\{m(\lambda), \alpha_c(\lambda), \alpha_l(\lambda, \beta)\}$ the equilibrium. Solving the equilibrium, we obtain the following results:

Lemma 4 ***Religious infrastructures:*** *The optimal investments in religious infrastructures $m(\lambda)$ and the optimal effort of the clerics $\alpha_c(\lambda)$ are increasing in λ, and independent from β, θ, and ϕ.*

[16] For analytical convenience, we assume $\epsilon(\alpha_l) = \frac{\epsilon_0}{1-\alpha_l}$; so that $\epsilon_0 \in (0, 1)$ is the enforcement level when the secular elites are not providing an effort ($\alpha_l = 0$). For simplicity, we also assume that the maximum enforcement level that the secular elite can undertake $\overline{\alpha}_l$ is less than $1 - \epsilon_0$, so that $\epsilon(\alpha_l)$ always lies in the interval $[\epsilon_0, 1]$.

Lemma 5 ***Tax Enforcement:*** *The optimal enforcement effort of the secular elite $\alpha_l(\lambda, \beta)$ is decreasing in β, λ, q, θ and ϕ.*

Lemma 4 is similar to Lemma 1 in the previous model and has the same intuition. Lemma 5 highlights several results. First, when the ruler receives a larger share of the tax revenues β, the secular elite invests less in enforcing tax collection. Second, since individuals subjectively perceive a lower tax rate when clerics provide more effort, they also comply more with taxation, reducing the need for the secular elite to supply their own enforcement effort. Additionally, more effort from the clerics implies more religious proscriptions, which depress citizens' labor productivity, and decreases the proceeds of the tax collection. This also decreases the effort provided by the secular elite in enforcing the tax collection. Hence for both reasons, the clerics' legitimizing effort α_c, and the secular elite tax enforcement effort α_l are strategic substitutes with respect to building up the tax base of society. Consequently, given that clerics provide more effort when they are more powerful (i.e. when λ is higher), the secular elite is conversely less willing to enforce the tax collection in such a case: (i.e $\alpha_l(\lambda, \beta)$ decreases with λ).

The same intuition explains both the effect of a higher frequency q of religious individuals and of more efficient clerics θ on the effort of the secular elite α_l. Finally, when the religious proscriptions ϕ get stronger, then the proceeds of the tax collection are reduced, so the secular elite also provides less tax enforcement effort.

Institutional Dynamics. We may now extend the main model with regards to the structure of political weights across society. The ruler can delegate power to the clerics λ and also constrain herself to share more revenues with the secular elites by decreasing her own fraction β of fiscal revenues.

Institutional change internalizes two types of externalities that are not taken into account by equilibrium individual decisions. First, as in the previous model, the religious provision m grants *legitimacy* to the ruler by reducing the subjectively perceived tax rate of religious individuals, while at the same time depressing labor productivity because of religious proscriptions. Second, institutions now also respond to the externality implied by the enforcement effort α_l of the secular elite on the fiscal revenue received by the ruler. By committing to share the proceeds of the tax collection, the ruler can indirectly induce a higher fiscal capacity for his own benefit.

Hence, given the current institutional structure (λ_t, β_t), future institutions $(\lambda_{t+1}, \beta_{t+1})$ are designed as the solution to:

$$\max_{\lambda_{t+1},\beta_{t+1}} \left(1 - \frac{\lambda_t}{2}\right) [U_r(m(\lambda_{t+1}), \alpha_l(\lambda_{t+1}, \beta_{t+1})) + U_l(m(\lambda_{t+1}), \alpha_l(\lambda_{t+1}, \beta_{t+1}))] + \frac{\lambda_t}{2} U_c(m(\lambda_{t+1}), \alpha_c(\lambda_{t+1}); \tag{10}$$

with $\{m(\lambda_{t+1}), \alpha_c(\lambda_{t+1}), \alpha_l(\lambda_{t+1}, \beta_{t+1})\}$ denoting the equilibrium of period t, as evaluated under an institutional set-up $(\lambda_{t+1}, \beta_{t+1})$. Solving this optimization problem, we deduce the following results which characterize the institutional dynamics:

Proposition 4 *When $C(m)$ and $F(m)$ are sufficiently convex, the optimization problem (10) admits a unique solution $(\lambda_{t+1}, \beta_{t+1}) \in [0,1]^2$ and:*

There exists a threshold $\overline{q}_d(\lambda_t) \in [0,1]$ such that if $q_t > \overline{q}_d(\lambda_t)$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$. Moreover $\overline{q}_d(\lambda_t)$ is decreasing in λ_t.

There exists a threshold $\tilde{q}_d(\lambda_t, \beta_t) \in [0,1]$ with $\tilde{q}_d(\lambda_t, 1) = 1$ such that if $q_t > \tilde{q}_d(\lambda_t, \beta_t)$, then $\beta_{t+1} > \beta_t$. Otherwise, $\beta_{t+1} \leq \beta_t$. Morever the threshold $\tilde{q}_d(\lambda_t, \beta_t)$ is decreasing in λ_t and increasing in β_t.

The uniqueness result follows from the convexity and the separability of the two dimensions of the optimization problem (10). As before, whether the ruler delegates more power to clerics over time depends on the fraction of religious individuals q_t. If the religious are sufficiently numerous, then more weight to the clerics $\lambda_{t+1} > \lambda_t$ increases their effort $\alpha_c(\lambda_{t+1})$. This will increase the utility of the ruler, who benefits from a larger tax base (Lemma 4). Second, when the religious are sufficiently numerous, the political weight of the secular elite relative to the ruler tends to decrease, $\beta_{t+1} > \beta_t$. As the ruler becomes more reliant on religious legitimacy to raise revenues, he also faces weaker incentives to delegate power to the secular elite and to build fiscal capacity.

Cultural Dynamics. As before, cultural evolution is driven by some process of intergenerational transmission emanating from paternalistic parents and oblique social role models. The formal features of the cultural dynamics need, however, to be amended to the new specification of citizens' preferences and the fact that parents now internalize that their children will have their own evasion capacity c. Again one may compute the

paternalistic motives ΔV_{Re} and ΔV_S to transmit the religious and the secular trait in this context. As shown in the appendix, due to the quadratic specification of the expected payoff functions, these simply write as function of the state variables λ, β, q such that $\Delta V_S = \Delta V_{Re} = \Delta V(\lambda, \beta, q)$.[17] The dynamics of the frequency of the religious trait is again characterized by the following "cultural replicator" dynamics:

$$q_{t+1} - q_t = q_t(1 - q_t)D(q_t, \lambda_t, \beta_t). \tag{11}$$

where again

$$\begin{aligned} D(\lambda_t, \beta_t, q_t) &= d^*_{Re} - d^*_S \\ &= D_{Re}\left[(1 - q_t)\Delta V(\lambda_t, \beta_t, q_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t, \beta_t, q_t)\right] \end{aligned}$$

is the relative "cultural fitness" of the religious trait in the population, and in general depends now on the three state variables λ, β, q. When the cultural substituability between vertical and oblique transmission is strong enough, the relative "cultural fitness" of the religious trait $D(\lambda_t, \beta_t, q_t)$ is decreasing in the frequency q_t of religious individuals in the population and we deduce the following result:

Proposition 5 *With strong enough cultural substitution between vertical and horizontal cultural transmission, there exists a unique threshold $q^*_d(\lambda_t, \beta_t)$ such that*

$$q_{t+1} < q_t \text{ (resp. } \geq) \text{ if } q_t > q^*_d(\lambda_t, \beta_t) \text{ (resp. } \leq) \ .$$

As before, the threshold $q^*_d(\lambda_t, \beta_t)$ is the unique attractor of the cultural dynamics (11). Hence, when the fraction of religious individuals q_t is above (resp. below) the threshold $q^*_d(\lambda_t, \beta_t)$, it tends to decrease (resp. increase).

Joint Dynamics. The joint dynamics of culture and institutions in this society are now three dimensional: the two institutional parameters λ_t, β_t and the cultural component q_t, evolve jointly, as characterized in Propositions 4 and 5. A full characterization of this dynamic system is difficult. Still one can get some insights on the forces behind the

[17] Because the equilibrium tax collection effort $\alpha_l(\lambda, \beta, q)$ of the secular elite enters into the paternalistic motives, we may note that $\Delta V(\lambda, \beta, q)$ now also depends on q and is actually an increasing function of q (see the appendix).

joint dynamics by investigating how the thresholds $\overline{q}_d(\lambda_t)$, $\tilde{q}_d(\lambda_t, \beta_t)$ and $q_d^*(\lambda_t, \beta_t)$ that characterize respectively the dynamics of λ_t, β_t and q_t are themselves affected by these state variables.

As in the benchmark model, there is again a fundamental complementarity between the dynamics of culture and institutions. To see that, note first that because $\overline{q}_d(\lambda_t)$ is decreasing in λ_t, from Proposition 4, the political weight of the religious clerics λ_t keeps increasing (resp. decreasing) over time as soon as it is above (resp. below) a threshold $\overline{\lambda}(q_t)$ defined by $\overline{q}_d(\lambda) = q_t$. A strong (resp. weak) clerics' institutional representation is reinforced (resp. weakened) over time. This feature creates a force towards an institutional steady state characterized by a *theocratic institutional regime* with $\lambda = 1$, or on the contrary a *secular institutional regime* with $\lambda = 0$. Also, given that the threshold $\overline{\lambda}(q_t)$ is decreasing in q_t, the reinforcing dynamics for the theocratic institutional regime are facilitated (resp. weakened) when the religious (resp. secular) trait is already well disseminated in society.

Conversely, from Proposition 5, $q_d^*(\lambda_t, \beta_t)$ is increasing in the institutional weight λ_t of the clerics. As before, a theocratic institutional regime with a high value of λ_t stimulates more religious infrastructures and reinforces the incentive of religious individuals to pass their values inter-generationally. Religious values are more widely diffused within a theocratic institutional regime, while secular values widely prevail under an secular institutional regime.

With respect to the dynamics of political centralization β_t, Proposition 4 reveals that β_t is more likely to increase as q_t and λ_t become larger. Indeed, as the threshold $\tilde{q}_d(\lambda_t, \beta_t)$ is increasing in λ_t and β_t, the condition for $\beta_{t+1} - \beta_t \gtrless 0$ rewrites as $\beta_t \lessgtr \widetilde{\beta}_d(\lambda_t, q_t)$ with $\widetilde{\beta}_d(\lambda_t, q_t)$ increasing both in λ_t and q_t. This feature underlines a force for the system to move in the direction of a steady state level of political centralization $\widetilde{\beta}_d^*$ that is increasing both in the level of institutional power λ of the clerics, and the extent q of religious values prevailing in the society. The more theocratic the state and the more diffused the religious values in the population, the larger the religious legitimacy enjoyed by the ruler, and the lower the need to empower the secular elite for fiscal consolidation.

Qualitatively, the previous discussion indicates that the joint dynamics of culture and institutions entails to the possibility of two steady states. The first is a *theocratic regime with political centralization*, where the ruler has a strong say on fiscal revenues (β is high) and is legitimized by religion, while the clerics have significant political power ($\lambda = 1$). Fiscal capacity is low, as the secular elite have minimal incentives to enforce tax collection. The share of religious individuals in civil society is also high (q is high). The second steady

state is a *secular regime with political decentralization.* The ruler is fiscally weak while the secular elite is strong (β is low). The clerics have little political power ($\lambda = 0$), while fiscal capacity is high given that secular elites have now strong incentives to enforce tax collection. At the same time, the share of religious individuals is low (q is low).

In the appendix, we show that the previous discussion can be made analytically more precise in the case where the threshold of the cultural dynamics $q_d^*(\lambda_t, \beta_t)$ does not depend on β_t. The dynamics of λ_t and q_t are then decoupled from the dynamics of β_t and follow the same pattern as in the benchmark model. Depending on the initial conditions (λ_0, q_0), (λ_t, q_t) converge towards *a theocratic regime* $(1, q_d^*(1))$ or a *secular regime* $(0, q_d^*(0))$. Associated with these dynamics, political centralization then converges towards strong state centralization with $\beta_1^* = \widetilde{\beta}_d(1, q_d^*(1))$, or weak state centralization $\beta_0^* = \widetilde{\beta}_d(0, q_d^*(0)) < \beta_1^*$.

As in the benchmark model, a ruler's option to rely on religious legitimacy induces a fundamental complementarity between the dynamics of culture and institutions. When a ruler relies more on religious legitimacy to raise revenues, she also faces increasingly lower incentives to delegate power to the secular elite and to consolidate fiscal capacity. As she becomes fiscally stronger relative to the secular elite, she also commits to an institutional set-up delegating more power to the clerics, leading to an increased diffusion of religious values in the society. In turn, the predominance of religious individuals augments the political incentives to bias the institutional structure towards both the clerics and the ruler. This dynamic complementarity between institutions and culture then operates to produce a process converging towards a *theocratic regime with political centralization.*

Alternatively, when a ruler relies less on religious legitimacy to raise revenues, she also faces stronger incentives to delegate power to the secular elite, who consequently consolidates fiscal capacity. As the ruler becomes more reliant on her secular elite to collect taxes, she accordingly faces lower incentives to commit to an institutional set-up where religious clerics are powerful. Both the political weight of the clerics and the value of passing religious values inter-generationally decrease. A lower predominance of religious individuals in society and a lower legitimacy to raise taxes directly further augments the political incentives to consolidate fiscal capacity by empowering the secular elite. Eventually, the joint dynamics of culture and institutions converge towards a *secular regime with political decentralization.*

4.2 The historical stylized pattern

This extension allows us to unify the three main theories of the "long divergence." It takes seriously the idea that rulers can be constrained by other powerful elites in society and searches for the conditions under which this is likely to happen. Importantly, it does so in the context of the previously-established framework in which religious legitimacy and religious proscriptions play a role in determining the joint evolution of institutions and culture. But how do our findings accord with the historical record?

We first consider the relationship between constraint on executive power and fiscal capacity. This relationship is central to the extension proposed in Section 4.1. There is a large literature claiming that states in which fiscal capacity and the "power of the purse" are held by groups outside of the central executive are able to collect more taxes due to greater constraints on executive power (Besley and Persson 2009; Dincecco 2009; Karaman and Pamuk 2013; Ma and Rubin 2019; North and Weingast 1989; Stasavage 2011). Our model adds additional insight to this literature by shedding light on the process through which political decentralization, as we define it, engenders cultural change (i.e., secularization) that reinforces the state's fiscal capacity. One of our primary insights is that rulers will only decentralize political authority when the returns from religious legitimacy (via taxation) are sufficiently low. This in turn triggers cultural change to a more secular society. On the contrary, when society is religious, the returns from religious legitimacy may be high even when religious proscriptions impinge on productive effort. In this case, culture and institutions evolve in tandem and society becomes more religious over time.

Section 4.1 highlights multiple reasons why European political institutions became decentralized in the medieval period. First, following the fall of the Western Roman Empire, European rulers had relatively little fiscal power relative to other elites. In the terms of our model, their initial level of β was low. This also follows from the framework of Blaydes and Chaney (2013). They argue that European rulers were weak relative to other elites because they lacked access to independent sources of military power, unlike Muslim rulers who could employ slave soldiers.

However, as we noted in the introduction, an explanation relying solely on executive constraint leaves a major question unanswered. If Muslim rulers were so strong relative to other elites, why should they have feared decentralizing some of their power to those "secular" elites, which could have yielded more tax revenue? Even as late as the early modern period, Ottoman tax collection was notoriously low (Karaman and Pamuk 2013).

Why did the Ottomans not give more power to local notables, who would have almost certainly had more capacity to collect taxes? These elites should not have been a threat to Muslim rulers. After all, rulers had slave soldiers and local elites did not.

Our model provides insights which help solve this puzzle. It suggests that the ruler's fiscal power relative to other elites (β) *interacted* with the greater legitimating capacity of Muslim religious authorities. Muslim rulers failed to decentralize political power not because they feared that other elites would become too strong. They did so because political decentralization would result in a weakening of the efficacy of religious legitimacy. Granting more power to secular authorities would have encouraged a cultural shift to a more secular state, yielding religious legitimacy less effective. Given the relative efficacy of religious legitimacy, this would not have been an optimal strategy for a Muslim ruler. This was exacerbated by access to slave soldiers, which gave the ruler more initial power vis-à-vis other elites. However, as the model indicates, this relative power (β) changes endogenously over time. Just because Muslim rulers had an initial advantage vis-à-vis other elites does not explain why it persisted.

The opposite is true in medieval Western Europe. The relatively weak initial power of rulers combined with the relatively weak legitimating capacity of the Church incentivized rulers to decentralize political power. This ultimately yielded a secular equilibrium in which religious proscriptions barely impinged on economic development.

These insights accord well with the historical record. Medieval European economic and political institutions were highly decentralized. Feudal institutions gave lords—secular lords as well as powerful bishops—great power over their local domains, and in return the lords provided military service and tax revenue to their sovereign (Duby 1982). Over the course of the late medieval and early modern periods, parliaments became the primary institution which bargained with European rulers (Angelucci, Meraglia and Voigtländer 2020; van Zanden, Buringh and Bosker 2012). Parliaments allowed the economic elite to gain representation at the political bargaining table, and they generally included three classes: the landed nobility, powerful churchmen, and commercial/urban elite. As warfare became more expensive, European rulers ceded more to these elites, who could provide them with revenue (Gennaioli and Voth 2015; North and Weingast 1989; Stasavage 2011; Tilly 1990). Ultimately, parliaments became the main tool for constraining rulers, which

resulted in a massive increase in fiscal capacity (Dincecco 2009; Johnson and Koyama 2017; North and Weingast 1989; Tilly 1990; van Zanden, Buringh and Bosker 2012).[18]

On the other hand, in the Middle East economic power was decentralized but *political power* remained centralized (Coşgel and Miceli 2005; Karaman and Pamuk 2013; Karaman 2009). For instance, at the height of Ottoman power in the fifteenth and early sixteenth centuries, the sultan derived two-thirds to three-quarters of his revenue through the *tımar* system, a military lease contract whereby the provincial cavalry collected agricultural taxes directly from the peasantry as remuneration for their military services to the state (Coşgel and Miceli 2005). The *tımar* system was similar to the tax collection system of feudal Europe, where local feudal lords controlled revenues in return for military service. However, a key difference between the two is that European feudal lords also had political power: their families ruled over their domains for generations, providing local law and order, collecting taxes, and representing them in parliament. On the other hand, *tımar* holders were rotated every few years *precisely* so that they would not acquire local political power. All political power remained vested in the sultan and key religious authorities, not *tımar* holders. Unlike European elites, who were ultimately able to constrain their rulers and receive concessions in return for revenue, *tımar* holders never organized collectively in any manner close to resembling a parliament, and Ottoman rulers remained relatively unconstrained (Balla and Johnson 2009). As a result, the economic elite rarely had any real political power in the Ottoman Empire (Pamuk 2004*a*,*b*). Meanwhile, religious legitimacy remained important (as discussed in Section 3.5), and as a result sultans ceded purview over commercial law to religious authorities, and the associated proscriptions dampening economic activity lasted for centuries (Kuran 2011).

This raises the question of how our model squares with Kuran's insight that religious proscriptions associated with Islamic law held the Middle East back. After all, Proposition 3 indicates that greater proscriptions should lead to a secular equilibrium, not a theocratic one. While it is true that in some instances, religious proscriptions can strengthen religious groups (Abramitzky 2008; Berman 2000; Iannaccone 1992), such groups are naturally limited in size. The mechanisms highlighted in this literature only work for smaller religious groups like Protestant sects or ultra-Orthodox Jews. These are not the groups of interest in the present paper. We are interested in the cultural evolution of the primary religious groups in medieval Western Europe and the Middle East. So how do we square the fact that

[18] For a theoretical treatment of the rise of state capacity and its affect on economic development, see Acemoglu (2005) and Besley and Persson (2009, 2010).

Islam had relatively strong religious proscriptions with the fact that the historical equilibrium was closer to what we describe as "theocratic"? This is where the other parameters in the model have bite. Our model suggests that the "theocratic" equilibrium that arose in the Islamic world arose *in spite of* the relatively strong religious proscriptions imposed by Islam. In a society with high θ, religious proscriptions were not enough to discourage rulers from decentralizing power and thereby weaken religious authorities. This insight is therefore consistent with Kuran, who focuses both on the religious proscriptions placed by Islamic law *and* their persistence over time. Once the theocratic equilibrium became established, religious proscriptions dampened economic activity in numerous unforeseeable ways, as Kuran shows in great detail.

4.3 Religious legitimacy and technological progress

Another key driver of the long divergence was technological and scientific progress. Although this is not highlighted in the central theories of the long divergence, nearly every theory of Britain's (and eventually Europe's) industrialization asks why Britain eventually became technologically advanced beginning of the 18th century (Allen 2009; Mokyr 1990, 2010, 2016). Although not all the advancements of the Industrial Revolution were science based—especially inventions in textile production—many were (including the quintessential invention of the period, the steam engine). That Europe pulled ahead in science and technology is puzzling: for centuries after the spread of Islam, the Middle East had a massive technological and scientific lead on Western Europe (Chaney 2016). What happened? Why was there a reversal of scientific and technological fortunes between the two regions?

In this section, we extend the model to account for the role that the various parameters of interest may have played in encouraging or stifling technological and scientific development. Again, we consider an extended version of the model where political power is divided between religious clerics and the ruler. But now, we study the conditions under which the ruler allows an endogenous technological choice or adoption of a scientific innovation, which is a source of productivity gains although it sometimes erodes religious beliefs.

More specifically, let the ruler and the clerics have political weights $1-\lambda$ and λ respectively. Let also the parameter $\alpha_I \in [0, \alpha_{\max}]$ denote a variable characterizing the technology level of the society. We assume that the level of technology is a policy instrument bounded by the knowledge frontier $\alpha_{\max}$.

Given that our primary interest is to study the joint evolution of culture, institutions, and technology, we consider again a reduced form model where the political power of the citizens is set to zero. The ruler now has utility

$$U_r = \tau E - C(m);$$

and religious clerics have utility

$$U_c(m, \alpha_c) = m\alpha_c - F(m) - \psi(\alpha_c).$$

We now consider religious legitimacy as a function of technology. Specifically, the religious legitimacy of the ruler, $\theta(\alpha_I) = \theta_0 - k\alpha_I$, is a decreasing function of the level of technology α_I.[19] In other words, adoption of innovative and sophisticated technologies erodes traditional religious beliefs where the ruler is seen as legitimate. This can be inherent to the process of innovative or scientific discoveries, which question the relationship between people and the natural world (Bénabou, Ticchi and Vindigni 2020; Mokyr 1990; Squicciarini 2020).[20] Finally, we assume that labor productivity is proportional to the technology level: $a = \alpha_I$.

As in the previous section, citizens do not necessarily comply with tax collection and differ in their (inverse) evasion capacity c. We fix now the taxation enforcement measure to $\epsilon_0 < 1$.

Equilibrium: At any time t, society reaches an equilibrium of the game between the ruler, the clerics, and civil society. Following the same line of reasoning as in the previous section (see the Appendix) the tax base of the economy is:

$$E = E(\alpha_I, \alpha_c, q_t) = \frac{\alpha_I}{1 + \phi\alpha_c}\{1 - \frac{\tau(1 - q_t\theta(\alpha_I) \cdot \alpha_c)}{\epsilon_0 \bar{c}}\}$$

[19] To avoid some cumbersome taxonomy, we assume that $k\alpha_{\max} < \theta_0 < 2k\alpha_{\max}$. The first inequality ensures that religious legitimacy can always be produced at any potential technological level. The second inequality ensures that maximum knowledge $\alpha_{\max}$ is sufficiently large not to always constrain the equilibrium technology choice by society.

[20] Religious precepts are not always antithetical to scientific advancement. Indeed, White (1972, 1978) and Davids (2013) argue that certain medieval European technologies were complementary to the Church's interest. For the sake of this extension, we focus on technologies that are antithetical to the interests of religious authorities. Mokyr (1990) argues that this more often than not the case with new and disruptive technologies.

The policy choices, that is the religious infrastructure m and the technology level α_I are collectively chosen so as to maximize social welfare:

$$W = (1 - \lambda_t)U_r(m, \alpha_I, \alpha_c, q_t) + \lambda_t U_c(m, \alpha_c); \tag{12}$$

while the clerics choose α_c. Solving the equilibrium:

$$\alpha_c = m, \ -C'(m) + \lambda\alpha_c = 0, \tag{13}$$

$$\alpha_I\left(\alpha_c, q_t\right) = \min\left[\frac{\epsilon_0\bar{c} - \tau(1 - q_t\theta_0\alpha_c)}{2\tau q_t k\alpha_c}, \alpha_{\max}\right] \tag{14}$$

The optimal choice of technology reflects the trade-off on the tax base of an increase in labor productivity and the erosion of religious legitimacy provided by the clerics. It can also be seen that the optimal level of technology $\alpha_I\left(\alpha_c, q_t\right)$ is decreasing in q_t and in α_c. When the religious are more numerous and/or clerics undertake higher religious efforts, the ruler is more reliant on religious legitimacy to raise revenues. Consequently, he is also more reluctant to adopt innovative activities that may erode such legitimacy.

The solution to (13) and (14) provides the equilibrium values $m\left(\lambda_t\right)$, such that $C'(m) = \lambda_t m$, $\alpha_c\left(\lambda_t\right) = m\left(\lambda_t\right)$, and $\alpha_I\left(\lambda_t, q_t\right) = \alpha_I\left(m\left(\lambda\right), q_t\right)$.

Institutional Dynamics. We allow the ruler to delegate power to the clerics λ. Institutional change again internalizes the externality that is not taken into account by individual decisions in equilibrium. As in the benchmark model, the provision of religious infrastructures m grants legitimacy to the ruler by reducing the subjectively perceived tax rate of religious individuals, while at the same time depressing labor productivity because of increased religious proscriptions. As will be clear below, this interacts with the choice of optimal technology adopted by society.

More specifically, given institutions λ_t, future institutions λ_{t+1} are designed as the solution to:

$$\max_{\lambda_{t+1}}(1 - \lambda_t)\left[U_r(m(\lambda_{t+1}), \alpha_I(\lambda_{t+1}), \alpha_c\left(\lambda_{t+1}\right), q_t)\right] + \lambda_t U_c(m\left(\lambda_{t+1}\right), \alpha_c(\lambda_{t+1})) \tag{15}$$

with $\{m(\lambda_{t+1}), \alpha_c(\lambda_{t+1}), \alpha_I(\lambda_{t+1})\}$ the equilibrium of period $t+1$, as evaluated under the institutional set-up λ_t. Solving this optimization problem, we deduce that:

Proposition 6 *The optimization problem (15) admits a unique solution* $(\lambda_{t+1}) \in [0,1]$. *Furthermore, there exists a threshold* $\overline{q}_I(\lambda_t)$ *such that*

$$\lambda_{t+1} > \lambda_t \text{ (resp. } \leq) \text{ if } q_t > \overline{q}_I(\lambda_t) \text{ (resp. } \leq).$$

As in the previous extension, the uniqueness result follows from the convexity of the optimization problem (15). Whether the ruler delegates more power to the clerics over time depends again on the fraction of religious individuals q_t. If the religious are sufficiently numerous, then religious legitimacy matters relatively more than technology for the ruler's tax base. Consequently, more weight to the clerics $\lambda_{t+1} > \lambda_t$ is provided, as this increases their effort $\alpha_c(\lambda_{t+1})$. The ruler consequently benefits from a larger tax base.

Cultural Dynamics. As in the previous section, cultural dynamics are driven by intergenerational transmission decisions from the citizens. Following the steps of the previous section, we find the following result:

Proposition 7 *There exists a unique threshold* $q_I^*(\lambda_t)$ *such that*

$$q_{t+1} < q_t \text{ (resp. } \geq) \text{ if } q_t > q_I^*(\lambda_t) \text{ (resp. } \leq).$$

Furthermore, the threshold $q_I^*(\lambda_t)$ *is increasing in* λ_t.

The cultural dynamics are still as in (6) and the threshold value $q_I^*(\lambda_t)$ is their unique attractor. Hence, when the fraction of religious individuals q_t is above (resp. below) $q_I^*(\lambda_t)$, it tends to decrease (resp. increase).

Joint Dynamics. There are two steady states. In the *theocratic regime* equilibrium, the ruler is legitimized by religion. The clerics have significant power (λ is high) and religious beliefs are widespread (q is high). For both reasons, the technology level implemented in society is low, as this threatens the religious legitimacy generated in this theocratic state. Because, innovation adoption and scientific activity is limited, labor productivity is low, as are fiscal revenues despite extractive taxation. The second steady state is a *secular innovative regime* where a high level of technology close to the knowledge frontier is adopted. Clerics are weak, given that innovations limit their capacity to legitimate the ruler (λ is zero) and the share of religious individuals is low (q is low). Fiscal revenues can be substantial, given that a process of scientific innovation leads to an overall increase in labor productivity.

Complementarity. Again, a ruler's option to rely on religious legitimacy induces a fundamental complementarity of the dynamics of culture and institutions. Along the path towards a theocratic steady state, the ruler relies more on religious legitimacy to raise revenues. She also faces increasingly lower incentives to adopt efficient innovations that erode her legitimacy. The ruler then commits to an institutional set-up delegating an increasingly large share of power to the clerics, reinforcing the incentive of religious individuals to pass their values inter-generationally. In turn, this further decreases the incentive of the ruler to adopt innovative technologies. Labor productivity stays low, given that technology is limited. Finally, taxes are increasingly more extractive given that the population becomes more religious but labor productivity remains low.

On the other hand, as a ruler relies less on religious legitimacy to raise revenues, she also faces stronger incentives to adopt innovations that increase labor productivity and consequently the fiscal base. As the ruler becomes more reliant on innovative activities to raise revenues, her religious legitimacy erodes, so she faces less incentive to commit to an institutional set-up where the religious clerics are powerful. Both the political weight of the clerics and the value of passing religious values inter-generationally decrease. A lower predominance of religious individuals further augments the political incentives to commit and change the institutional set-up so as to adopt more efficient technologies, leading to a substantial increase over time in labor productivity and fiscal revenues. Eventually, the joint dynamics of culture and institutions converge to a *secular regime* where the implemented technology is not constrained by political forces, but only by the existing knowledge frontier.

4.4 The historical stylized pattern

One of the great mysteries of the long divergence is the reversal of fortunes between Middle Eastern and Western European science and technology. Data presented in Chaney (2016) reveals that not only were scientific topics among the most ubiquitous in the corpus of Islamic writings up through the 11th century, but up to that point the Islamic world well out-paced Europe in scientific output. At some point in the 11th and 12th centuries, however, a reversal of fortunes occurred. Islamic scientific production began to wane around the 12th century. This was not simply a matter of the Islamic world falling behind relative to Europe; it fell behind in *absolute* terms relative to what it had once been. At

the same time, scientific works became much more prevalent in Western Europe. By the end of the medieval period, Western Europe had a technological and scientific lead, and this would only grow in subsequent centuries. Can this reversal of fortunes be explained by our model?

Our model, along with the history overviewed in Section 3.5, suggests that the reversal of technological and scientific fortunes was a consequences of a changing equilibrium in which Muslim religious authorities became increasingly important for legitimating the state while European rulers sought alternative forms of legitimacy. In the Middle East, the 11th century saw the rise of the madrasa system (Chaney 2016; Kuru 2019). This institutionalized the political role that had increasingly been played by religious authorities since their consolidation under the Abbasids in the 9th and 10th centuries (Coşgel, Miceli and Ahmed 2009; Rubin 2017). In this equilibrium, as we describe in Section 4.3, religion played an important role in legitimating rule (λ was large), society was largely religious (q was large), and science and technology were impeded. As in Bénabou, Ticchi and Vindigni (2020), technological stagnation mutually benefited religious authorities and the state: the former lost power when alternative means of discovering truths or interpreting the world were present, and the latter was harmed when one of its key sources of legitimacy was undermined. In the context of Middle Eastern history, this logic sheds light on both why madrasas were allowed to thrive in spite of their negative effects on scientific production and why rulers throughout the Muslim world banned one of the most important technologies of the late medieval period: the printing press. Coşgel, Miceli and Rubin (2012) argues that the Ottomans banned the press for over 240 years after first hearing of it precisely because it threatened the religious establishment. By the 15th century, religious authorities across the Islamic world (not just in the Ottoman Empire) had set up high barriers to entry. The largest of these barriers was the years of training required to know various religious texts and interpretations of those texts. These barriers raised the status of the religious elite, further entrenching the "high-λ, high-q" equilibrium. The printing press threatened to undermine these barriers and the equilibrium they helped uphold. Had printing become widespread, a much larger share of the population would have had access to the great religious and non-religious texts of the Islamic world (and beyond). This would have undermined one of the very features that gave Muslim religious authorities the power to legitimate in the first place. Hence, as our model predicts, heavy restrictions were placed on this vastly important technology.

Muslim religious authorities had good reason to fear the spread of printing. They only needed to look to Europe, where the press helped facilitate one of the great movements against Church power in the history of Christianity: the Protestant Reformation (Boerner, Rubin and Severgnini 2020; Dittmar and Seabold 2020; Rubin 2014). Unlike Ottoman religious authorities, the Church was not able to stop the spread of the printing press. The reason why this was the case follows from the logic of the model. As noted in Section 3.5, the Church had already lost much of its legitimating power in Europe prior to the spread of printing. Alternative sources of legitimacy had emerged in the form of universities (which provided a theoretical justification for monarchical rule) and parliaments (which brought together elites who could legitimate rule in return for a seat at the political bargaining table). By 1200 or so, religious authorities had lost their monopoly over the printed word as well; book demand and supply was increasingly found in university towns and urban centers (Buringh and Van Zanden 2009). As a result, there was little the Church could have done to stop the spread of printing had it wanted to. By the mid-15th century, Europe was in a "low-λ, low-q" equilibrium. Our model suggests that this should also entail few restrictions on technology—at least those technologies that damage the capacity of religious authorities to legitimate. The history of printing suggests that this was the case.

The Christian world was hardly uniform in the degree to which religious legitimacy was part of the broader political equilibrium. This was especially true after the Reformation, which fundamentally undermined the role of religious authority in the ruling coalition (Rubin 2017). This had consequences for the spread of science and technology. Bénabou, Ticchi and Vindigni (2020) summarize many of the scientific and technological advances blocked or suppressed by the Church, including the works of Galileo, the Copernican Revolution, Newtonism, the Scientific Revolution, and technical education in schools. These restrictions were much more widely applied in Catholic areas than Protestant ones. According to Mokyr (2016), it was the "culture of growth" supported by the Republic of Letters that permitted the spread of the new, rational thinking of those like Bacon and Newton. While the Republic of Letters was a pan-European phenomenon, there was little resistance in the leading Protestant lands (England and the Dutch Republic). Meanwhile, even after the first wave of industrialization, the Church attempted to limit secular education and curriculum in schools (Squicciarini 2020).

In short, this extension helps explain both the technological and scientific reversal of fortunes between Western Europe and the Middle East as well as the the divergence

within Europe. In Protestant Europe, new inventions and scientific ideas were allowed to spread relatively unimpeded. This is what the model predicts would be the case in a "low-λ, low-q" equilibrium. The equilibrium in Catholic Europe was one of higher λ and q, and as a result some (though certainly not all) scientific and technological advances were suppressed. In the "high-λ, high-q" equilibrium that pervaded most of the medieval and early modern Middle East (at least, after the 11th century), scientific and technological advancements were even more restricted. Our model explains these outcomes not solely as reflecting the desires of religious authorities, but also their place in their society's broader political-economy and cultural equilibria.

5 Conclusion

In this paper we provide an explanation for an important historical phenomenon: the *long divergence* between Middle Eastern and Western European economies from the year 1000 C.E. to the beginning of the Industrial Revolution. We provide an explanation in terms of a model of institutional and cultural change. In doing so, we unify prevailing theories based on religious legitimacy, religious proscriptions, and decentralization of political power. In the process, our model resolves many puzzles left unaddressed in the literature.

In the context of the long divergence, the model centers on the power dynamics of rulers, clerics, and secular elites in framing institutions in a religious environment. It highlights three central historical features of these power dynamics: rulers derive *legitimacy* from the religious elites, religious authorities impose *proscriptions* that impinge on economic development, constraints on executive power and the decentralization of political power have a fundamental role in inducing economic growth. Most importantly, the model highlights how the institutions resulting from the power dynamics of rulers, clerics, and secular elites interact with the spread of culture (religious beliefs) in civil society. Political centralization interacts with religious legitimacy and religious proscriptions to determine its long-run economic and political paths. Citizens remain religious or not, in the face of religious proscriptions, depending on the feedback between religious institutions and cultural evolution. Religious legitimacy to the political system depends crucially on the prominence of religious values in society.

We intend this as an illustration of the explanatory power of a class of models centered on some simple general and yet minimal components: institutions as the relative political power of different groups in society to affect policy decisions, institutional change as

a mechanism to internalize externalities and other distortions characterizing the equilibrium. in society, and the cultural profile of values and preferences in society as evolving according to various socioeconomic incentives.[21] The interdependence between institutions and culture is a fundamental factor, jointly (along with technology) driving socio-economic change in these models. Our model reveals how such interdependence has played a central role in determining the long-run economic, religious, and institutional paths of societies.

[21] See Acemoglu, Egorov and Sonin (2021); Acemoglu, Johnson and Robinson (2005*a*); Bisin and Verdier (2021); Persson and Tabellini (2021) for surveys of this class of models.

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Appendices

A Mathematical Appendix

A.1 Proofs of Lemmas 1, 2 and 3

In order to prove the three Lemmas of the main text, we solve the equilibrium, where the ruler chooses the amount of religious infrastructures m so as to maximize the social welfare W, with $e_{Re}(\lambda)$

$$W = \frac{1}{2}U_R(m) + \frac{\lambda}{2}U_c(m, \alpha_c) + \frac{1-\lambda}{2}\left[qU_{Re}(e_{Re}) + (1-q)U_S\left(e_S\right)\right]. \tag{A.1}$$

The clerics and the individuals choose, respectively, α_c and $e_i,\ i = Re, S$ to maximize their utility. The equilibrium is denoted $\{\tau(\lambda), m(\lambda), \alpha_c(\lambda), e_S(\lambda), e_{Re}(\lambda)\}$. Since $\lambda \leq 1$, $\tau(\lambda)$ is equal to $\overline{\tau} \equiv \tau$ and the remaining first-order conditions are:

$$\begin{cases} -C'(m) - \lambda F'(m) + \lambda \cdot \alpha_c = 0 \\ m - \Psi'(\alpha_c) = 0 \\ (1-\tau_{Re}) - (1+\phi\alpha_c)e_{Re} = 0 \\ (1-\tau) - (1+\phi\alpha_c)e_S = 0, \end{cases} \tag{A.2}$$

or after substitution:

$$\begin{cases} C'(m) + \lambda F'(m) = \lambda\alpha_c \\ \Psi'(\alpha_c) = m \\ e_{Re} = \frac{1-\tau+\tau\theta\alpha_c}{1+\phi\alpha_c} \\ e_S = \frac{1-\tau}{1+\phi\alpha_c} \end{cases} \tag{A.3}$$

Assuming that the marginal cost functions $C'(.)$, $F'(.)$ and $\Psi'(.)$ are increasing convex functions (ie. $C'''(.) \geq 0, F'''(.) \geq 0$ and $\Psi'''(.) \geq 0$) with at least one of these cost derivatives strictly convex), and the limit condition $\lim_{x\to\infty} F''(x) > 1$, and $F''(0)\Psi''(0) < 1$, then the first two equations of (A.3) simply characterize a unique equilibrium couple $m(\lambda) > 0$ and $\alpha_c(\lambda) > 0$ when $\frac{C''(0)\Psi''(0)}{1-F''(0)\Psi''(0)} < \lambda$, while $m(\lambda) = \alpha_c(\lambda) = 0$ for $\lambda \leq \frac{C''(0)\Psi''(0)}{1-F''(0)\Psi''(0)}$.

Lemma 1: Differentiating the previous first-order conditions, it is easy to note that the optimal provision of religious infrastructure $m(\lambda) > 0$ and the effort of the clerics $\alpha_c(\lambda) > 0$ are both increasing in λ and independent from θ and ϕ. This concludes the proof of Lemma 1.

Lemma 2: The equilibrium production efforts are obtained as

$$\begin{cases} e_{Re}(\lambda) = \frac{1-\tau+\tau\theta\alpha_c(\lambda)}{1+\phi\alpha_c(\lambda)} \\ e_S(\lambda) = \frac{1-\tau}{1+\phi\alpha_c(\lambda)} \end{cases} \tag{A.4}$$

The equilibrium secular effort $e_S^*(\lambda)$ is decreasing in clerics activities α_c^* and thus, it is decreasing in λ. It is independent from ϕ and θ

Additionally, from the equation above, $e_{Re}(\lambda)$ increases with θ and decreases with ϕ. The effect of $\alpha_c(\lambda)$ on $e_{Re}(\beta, \lambda)$ is ambiguous. By deriving $e_{Re}(\lambda)$ with respect to α_c, we find that when $\theta > \frac{1-\tau}{\tau}\phi$, then $e_{Re}(\lambda)$ increases with $\alpha_c(\lambda)$, in which case $e_{Re}(\lambda)$ increases with λ. This concludes the proof of Lemma 2.

Lemma 3: The equilibrium tax base of the ruler writes as

$$E(\lambda) = q \cdot e_{Re}(\lambda) + (1-q) \cdot e_s(\lambda), \tag{A.5}$$

so

$$E(\lambda) = \frac{1-\tau+\tau\theta q \cdot \alpha_c(\lambda)}{1+\phi\alpha_c(\lambda)}. \tag{A.6}$$

By deriving the previous expression with respect to $\alpha_c(\lambda)$, we find that the tax base is increasing in the clerics' effort if and only if $q \geq \frac{1-\tau}{\tau\theta}\phi$. Hence, when the previous condition is satisfied, $E(\lambda)$ is increasing in λ. Finally, from (A.6), $E(\lambda)$ is increasing in q and θ, and decreasing in ϕ. This concludes the proof of Lemma 3.

A.2 Proof of Proposition 1

- First, we demonstrate that the optimization problem (4) rewritten below admits a unique solution $\lambda_{t+1} \in [0,1]$:

$$\max_{\lambda_{t+1}} \frac{1}{2}U_r(m(\lambda_{t+1})) + \frac{\lambda_t}{2}U_c(m(\lambda_{t+1}), \alpha_c(\lambda_{t+1}))+ \frac{1-\lambda_t}{2}\left[q_t U_{re}(e_{Re}(\lambda_{t+1})) + (1-q_t)U_s\left(e_s(\lambda_{t+1})\right)\right]. \quad \text{(A.7)}$$

In order to solve this maximization problem, we consider the following related optimization problem:

$$\max_m W = \frac{1}{2}\{U_r(m) + \lambda_t \widetilde{U}_c(m) + (1-\lambda_t)\left[q_t\widetilde{U}_{re}(m) + (1-q_t)\widetilde{U}_s(m)\right]\}, \quad \text{(A.8)}$$

with

$$\begin{cases} \widetilde{\alpha}_c(m) = \psi'^{-1}(m) \\ E(m) = \frac{1-\tau+\tau\theta q\widetilde{\alpha}_c(m)}{1+\phi\widetilde{\alpha}_c(m)} \\ U_r(m) = \tau E(m) - C(m) \\ \widetilde{U}_c(m) = \widetilde{\alpha}_c(m)m - \psi(\widetilde{\alpha}_c(m)) - F(m) \\ \widetilde{U}_{Re}(m) = \frac{1-\tau+\tau\theta\widetilde{\alpha}_c(m)}{1+\phi\widetilde{\alpha}_c(m)} \\ \widetilde{U}_S(m) = \frac{1-\tau}{1+\phi\widetilde{\alpha}_c(m)}. \end{cases} \quad \text{(A.9)}$$

In the optimization problem (A.8), the choice of the religious infrastructure m is made by a ruler able to commit to the provision of m, and therefore internalizing the two externalities detailed in the main text. We find that:

$$2\frac{dW}{dm} = \lambda_t\left[\widetilde{\alpha}_c(m) - F'(m)\right] - C'(m) + \tau E'(m) + (1-\lambda_t)[q\widetilde{U}'_{Re}(m) + (1-q)\widetilde{U}'_S(m)]. \quad \text{(A.10)}$$

When $C(.)$ and $F(.)$ are sufficiently convex, the function W is concave in m, and the previous optimization admits a unique solution $\tilde{m}(\lambda_t) \geq 0$.

Note that $\alpha_c(\lambda) = \widetilde{\alpha}_c(m(\lambda))$, $U_i(e_i(\lambda)) = \widetilde{U}_i(m(\lambda))$ for $i = \{Re, S\}$, and $U_c(m(\lambda), \alpha_c(\lambda)) = \widetilde{U}_c(m(\lambda))$. Given that $\tilde{m}(\lambda_t)$ maximizes the social welfare when the externalities are internalized, the solution λ_{t+1} of the optimization problem (4), should be such as to induce an

equilibrium choice $m(\lambda_{t+1})$ as close to $\tilde{m}(\lambda_t)$ as possible:

$$\lambda_{t+1} = \begin{cases} \lambda \text{ s.t } m(\lambda) = \tilde{m}(\lambda_t) & \text{if } \tilde{m}(\lambda_t) \in (m(0), m(1)) \\ 1 & \text{if } \tilde{m}(\lambda_t) > m(1) \\ 0 & \text{if } \tilde{m}(\lambda_t) < m(0). \end{cases} \tag{A.11}$$

When the clerics have power λ_{t+1} given by (A.11), institutions are designed for $t+1$ so as to induce a choice $m(\lambda_{t+1})$ in that period that maximizes the social welfare of period t. Given that $m(\lambda)$ is increasing in λ,this solution λ_{t+1} of problem (4) is unique and the institutional dynamics are well defined.

- In the second step of the proof, we demonstrate that there exists a threshold $\overline{q}(\lambda_t)$ such that if $q_t > \overline{q}$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$.

In order to demonstrate this claim, we first show the following intermediary result:

Lemma 6 *$\lambda_{t+1} > \lambda_t$ if and only if $\tilde{m}(\lambda_t) > m(\lambda_t)$.*

Proof: Indeed, $\tilde{m}(\lambda_t) > m(\lambda_t)$ means that if the ruler had the capacity to commit, in period t, to provide religious infrastructures m, then he would chooses a level $\tilde{m}(\lambda_t)$ strictly above what he actually provides in equilibrium. Since $m(.)$ is an increasing function (Lemma 1), we deduce that λ_{t+1} is such that $\lambda_{t+1} > \lambda_t$.[22]**QED.**

Lemma 7 *$\tilde{m}(\lambda_t) > m(\lambda_t)$ if and only if $q > \overline{q}(\lambda_t)$, with:*

$$\overline{q}(\lambda_t) = \frac{1}{\tau\theta} \frac{\phi(1-\tau)\left[\tau + (1-\lambda_t)\frac{1-\tau}{2}\right] + \lambda_t F'(m(\lambda_t))}{\tau + (1-\lambda_t)\left[1 - \tau + \tau\theta\alpha_c^*(\lambda_t)\left(1 + \frac{\phi}{2}\alpha_c^*(\lambda_t)\right)\right]} \tag{A.12}$$

Proof: From the proof of Lemma 1 above, the first-order condition associated with the determination of $m(\lambda)$ is:

$$\lambda_t \widetilde{\alpha}_c(m) - C'(m) = 0, \tag{A.13}$$

given that $\widetilde{\alpha}_c(m) = \psi'^{-1}(m)$.

[22]When an interior solution exists, λ_{t+1} solves $\tilde{m}(\lambda_t) = m(\lambda_{t+1})$. Hence, if $\tilde{m}(\lambda_t) > m(\lambda_t)$ then $\lambda_{t+1} > \lambda_t$.

The first order condition for the determination of $\tilde{m}(\lambda_t)$ writes as $\frac{dW}{dm} = 0$, with

$$\frac{dW}{dm} = \frac{1}{2}\left[\lambda_t\left[\widetilde{\alpha}_c(m) - F'(m)\right] - C'(m) + \tau E'(m) + (1-\lambda_t)[q\widetilde{U}'_{Re}(m) + (1-q)\widetilde{U}'_S(m)]\right]. \tag{A.14}$$

Consider the expression

$$H(m) = \tau \cdot E'(m) + (1-\lambda)[q\widetilde{U}'_{Re}(m) + (1-q)\widetilde{U}'_S(m)]\}.$$

Given the two FOCs above, we deduce that $\tilde{m}(\lambda_t) > m(\lambda_t)$ if and only if $H(m(\lambda_t)) > 0$. We show that condition $H(m(\lambda_t)) > 0$ is equivalent to a condition over the possible values of q.

$$\begin{aligned} E'(m) &= q \cdot \frac{de_{Re}}{dm} + (1-q) \cdot \frac{de_S}{dm} && \text{(A.15)} \\ &= \frac{q\tau\theta - (1-\tau)\phi}{\left[1+\phi\widetilde{\alpha}_c(m)\right]^2}\frac{d\widetilde{\alpha}_c(m))}{dm}, && \text{(A.16)} \end{aligned}$$

$$\begin{aligned} U'_{Re}(m) &= e_{Re}(m)\left[\theta\tau - \phi \cdot \frac{e_{Re}(m)}{2}\right]\frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.17)} \\ &= \frac{1-\tau+\tau\theta\widetilde{\alpha}_c(m)}{1+\phi\widetilde{\alpha}_c(m)}\left[\theta\tau - \phi \cdot \frac{1}{2}\frac{1-\tau+\tau\theta\widetilde{\alpha}_c(m)}{1+\phi\widetilde{\alpha}_c(m)}\right]\frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.18)} \end{aligned}$$

and

$$\begin{aligned} U'_s(m) &= -\frac{\phi\left(e_S(m)\right)^2}{2}\frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.19)} \\ &= -\phi \cdot \frac{1}{2}\left[\frac{1-\tau}{1+\phi\widetilde{\alpha}_c(m)}\right]^2 \cdot \frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.20)} \end{aligned}$$

Thus,

$$2\frac{dW}{dm} = \lambda\widetilde{\alpha}_c(m) - C'(m) + H(m) - \lambda F'(m),$$

with

$$\left[1+\phi\widetilde{\alpha}_c(m)\right]^2 \frac{H(m)}{\frac{d\widetilde{\alpha}_c(m)}{dm}} = \tau \cdot (q\tau\theta - (1-\tau)\phi) + (1-\lambda)G(m)$$

and

$$\begin{aligned} G(m) &= q(1-\tau+\tau\theta\widetilde{\alpha}_c(m))\left[\theta\tau\left(1+\phi\widetilde{\alpha}_c(m)\right)-\frac{\phi}{2}(1-\tau+\tau\theta\widetilde{\alpha}_c(m))\right] \\ &\quad -(1-q)\frac{\phi}{2}\left[1-\tau\right]^2 \\ &= q\tau\theta\left[(1-\tau)+\tau\theta\widetilde{\alpha}_c(m)\left(1+\frac{\phi}{2}\widetilde{\alpha}_c(m)\right)\right]-\frac{\phi}{2}\left[1-\tau\right]^2 \end{aligned}$$

Then the condition $H(m)-\lambda F'(m)\geq 0$ writes as

$$\tau\cdot(q\tau\theta-(1-\tau)\phi)+(1-\lambda)\left[\begin{array}{c} q\tau\theta\left[(1-\tau)+\tau\theta\widetilde{\alpha}_c(m)\left(1+\frac{\phi}{2}\widetilde{\alpha}_c(m)\right)\right] \\ -\frac{\phi}{2}\left[1-\tau\right]^2 \end{array}\right]-\lambda F'(m)\geq 0$$

or using $\alpha_c(\lambda)=\widetilde{\alpha}_c(m(\lambda))=\Psi'^{-1}(m(\lambda))$ and rearranging terms:

$$\overline{q}\left(\lambda\right)=\frac{1}{\tau\theta}\frac{\phi\left(1-\tau\right)\left[\tau+(1-\lambda)\frac{1-\tau}{2}\right]+\lambda F'(m(\lambda))}{\tau+(1-\lambda)\left[1-\tau+\tau\theta\alpha_c(\lambda)\left(1+\frac{\phi}{2}\alpha_c(\lambda)\right)\right]}, \tag{A.21}$$

and $\alpha_c(\lambda)$ is an increasing function of λ. We conclude that $\tilde{m}(\lambda_t)>m(\lambda_t)$ if and only if $q>\overline{q}(\lambda_t)$.**QED.**

Combining the results established in Lemmas 6 and 7, it follows that $\lambda_{t+1}>\lambda_t$ if and only if $q>\overline{q}(\lambda_t)$.

Finally, from (A.21), we deduce that $\overline{q}(\lambda_t)$ is decreasing in θ and ϕ. This concludes the proof of the first point of Proposition 1.

A.3 Proof of Proposition 2

As mentioned in the main text, cultural dynamics are modeled as purposeful inter-generational transmission (Bisin and Verdier (2001), Bisin and Verdier (2017)), through parental socialization and imitation of society at large. *Direct vertical* socialization to the parent's trait $i\in\{Re,S\}$ occurs with probability d_i. If a child from a family with trait i is not directly socialized, which occurs with probability $1-d_i$, he/she is *horizontally/obliquely* socialized by picking the trait of a role model chosen randomly in the population. The probability

P_{ij} that a child in group i is socialized to trait j writes as:

$$\begin{cases} P_{ii} = d_i + (1 - d_i)q_i \\ P_{ij} = (1 - d_i)q_j \end{cases} \tag{A.22}$$

with $q_{Re} = q_t$ and $q_S = 1 - q_t$. Let $V_{ij}(\lambda_t) = U_i(e_j(\lambda_t))$ denote the utility to a cultural trait i parent of a type j child, with $i, j \in \{Re, S\}$. We denote the paternalistic bias of a parent of type i as $\Delta V_i(\lambda_t) = V_{ii}(\lambda_t) - V_{ij}(\lambda_t)$, for $j \neq i$. The socialization cost $h_{Re}(d_{Re}, m)$ of a parent of type Re (respectively S) is assumed to be a smooth function with $\frac{\partial h_{Re}(d_{Re},m)}{\partial d_{Re}} \geq 0$; $\frac{\partial^2 h_{Re}(d_{Re},m)}{\partial d_{Re}^2} > 0$ (ie. $h_{Re}(d_{Re}, m)$ is increasing convex in d_{Re}) and the Inada conditions $h_{Re}(0, m) = \frac{\partial h_{Re}(0,m)}{\partial d_{Re}} = 0$, $\lim_{d\to 1} h_{Re}(d, m) = \lim_{d\to 1} \frac{\partial h_{Re}(d,m)}{\partial d_{Re}} = +\infty$ Similarly, the socialization cost $h_S(d_S)$ of a parent of type S satifies $h'_S(d_S) \geq 0$; $h''_S(d_S) > 0$ (ie. $h_S(d_S)$ is increasing convex in d_S (ie.), and $h_S(0) = h'_S(0) = 0$, $\lim_{d\to 1} h_S(d) = \lim_{d\to 1} h'_S(d) = +\infty$.

Furthermore, to reflect the fact religious infrastructures may enter as a complementary input to parental effort for transmission of the religious trait, we assume that $\frac{\partial h_{Re}(d_{Re},m)}{\partial m} \leq 0$ and $\frac{\partial^2 h_{Re}(d_{Re},m)}{\partial d_{Re}\partial m} \leq 0$, (ie. m affects negatively the cost and the marginal cost of socialization of religious parents). Following Bisin and Verdier (2001), direct socialization d^*_{Re} of religious parents is the solution to the following socialization problem:

$$\max_{d_{Re}} -h_{Re}(d_{Re}, m_t) + P_{ReRe} \cdot V_{ReRe}(\lambda_t) + P_{ReS} \cdot V_{Re\ S}(\lambda_t), \tag{A.23}$$

while direct socialization d^*_S of secular parents is the solution to the following socialization problem:

$$\max_{d_S} -h_S(d_S) + P_{SS} \cdot V_{SS}(\lambda_t) + P_{SRe} \cdot V_{SRe}(\lambda_t), \tag{A.24}$$

The FOCs of the previous programs determine the optimal socialization efforts as:

$$\frac{\partial h_{Re}(d^*_{Re}, m_t)}{\partial d_{Re}} = (1 - q_t)\Delta V_{Re}(\lambda_t) \text{ and } h'_S(d^*_S) = q_t \Delta V_s(\lambda_t)$$

which can be rewritten as $d^*_{Re}(q_t, \lambda_t) = D_{Re}((1 - q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t))$ and $d^*_S(q_t, \lambda_t) = D_S(q_t \Delta V_S(\lambda_t))$.

Note that by the Inada conditions on $h_{Re}(\cdot,\cdot)$, $d^*_{Re} \in [0,1]$, and $D_{Re}(0,m) = 0$. As well $D_{Re}(\cdot,\cdot)$ is an increasing function of both arguments $(1-q_t)\Delta V_{Re}(\lambda_t)$ and m, as we have:

$$\frac{\partial d^*_{Re}}{\partial(1-q_t)\Delta V_{Re}(\lambda_t))} = \frac{1}{\frac{\partial^2 h_{Re}}{\partial d^2_{Re}}} > 0 \text{ and } \frac{\partial d^*_{Re}}{\partial m_t} = -\frac{\frac{\partial^2 h_{Re}}{\partial d_{Re}\partial m}}{\frac{\partial^2 h_{Re}}{\partial d^2_{Re}}} > 0$$

Similarly the Inada conditions on $h_S(\cdot)$ ensure that $d^*_S \in [0,1]$, $D_S(0) = 0$. As well $d^*_S = D_S(q_t\Delta V_S(\lambda_t))$ is an increasing function of $q_t\Delta V_S(\lambda_t)$ as

$$\frac{\partial d^*_S}{\partial(q_t\Delta V_S(\lambda_t))} = \frac{1}{h''_S} > 0$$

Using the Law of Large Numbers, one easily obtains the intergenerational evolution of the frequency of the religious trait q_t in the population as

$$q_{t+1} = q_t \cdot P_{ReRe} + (1-q_t)\cdot P_{SRe}$$

or after substitution of (A.22) and the values of d^*_{Re} and d^*_S,

$$q_{t+1} - q_t = q_t(1-q_t)\{d^*_{Re}(q_t,\lambda_t) - d^*_S(q_t,\lambda_t)\}. \tag{A.25}$$

As mentioned in the main text, in equation (A.25), the term

$$\begin{aligned} D(q_t,\lambda_t) &= d^*_{Re}(q_t,\lambda_t) - d^*_S(q_t,\lambda_t) \\ &= D_{Re}[(1-q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t)] - D_S[q_t\Delta V_S(\lambda_t)] \end{aligned}$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. This term is frequency dependent (ie. depends on the state of the population q_t). Moreover simple inspection shows that $D(q_t,\lambda_t)$ is a decreasing function of q_t, with $D(0,\lambda_t) = D_{Re}[\Delta V_{Re}(\lambda_t), m(\lambda_t)] > 0$ and $D(1,\lambda_t) = -D_S[\Delta V_S(\lambda_t)] < 0$. From this it follows that there exists a unique threshold $q^*(\lambda_t) \in (0,1)$ such that

$$D(q^*(\lambda_t),\lambda_t) = 0 \tag{A.26}$$

Inspection of equation (A.25) and the fact that $D(q_t,\lambda_t)$ is a decreasing function of q_t provides immediately that $q_{t+1} < q_t$ if and only if $q_t > q^*(\lambda_t)$, proving therefore proposition in the main text. **QED.**

A.4 Comparative statics on the cultural threshold $q^*(\lambda_t)$

The relative "cultural fitness" of the religious trait $D(q_t, \lambda_t)$ is affected by the institutional environment λ_t, as this variable interacts with the process of parental cultural transmission both through paternalistic motivations $\Delta V_i(\lambda_t)$, and through the provision of religious infrastructures $m_t = m(\lambda_t)$ as a complementary input to religious family socialization. Therefore the dependence of the threshold $q^*(\lambda_t)$ on the institutional environment λ_t and the comparative statics on the parameters θ and ϕ depends on how the relative "cultural fitness" $D(q_t, \lambda_t)$ of the religious trait is affected by changes in such features.

It is first useful to note that with the quadratic specification for the utility functions $U_i(.)$ of workers, the paternalistic motives $.\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$ are equal and take a simple form. Indeed we have:

$$\begin{cases} V_{ReRe}(\lambda) = \frac{(1-\tau_{Re})^2}{2(1+\phi\alpha_c(\lambda))} \\ V_{Re\ S}(\lambda) = (1-\tau_{Re})\frac{1-\tau}{1+\phi\alpha_c(\lambda)} - \frac{1}{2}(1+\phi\alpha_c(\lambda))\frac{(1-\tau)^2}{(1+\phi\alpha_c(\lambda))^2}. \end{cases} \tag{A.27}$$

Hence,

$$\Delta V_{Re}(\lambda) = V_{ReRe}(\lambda) - V_{Re\ S}(\lambda) = \frac{(\tau\theta\alpha_c(\lambda))^2}{2(1+\phi\alpha_c(\lambda))}. \tag{A.28}$$

Similarly, we find that

$$\begin{cases} V_{SS}(\lambda) = \frac{(1-\tau)^2}{2(1+\phi\alpha_c(\lambda))} \\ V_{SRe}(\lambda) = (1-\tau)\frac{1-\tau_{Re}}{1+\phi\alpha_c(\lambda)} - \frac{1}{2}(1+\phi\alpha_c(\lambda))\frac{(1-\tau_{Re})^2}{(1+\phi\alpha_c(\lambda))^2}. \end{cases}$$

and

$$\Delta V_S(\lambda) = V_{SS}(\lambda) - V_{SRe}(\lambda) = \frac{(\tau\theta\alpha_c(\lambda))^2}{2(1+\phi\alpha_c(\lambda))} \tag{A.29}$$

Thus posing $\Delta V(\lambda) = \frac{(\tau\theta\alpha_c(\lambda))^2}{2(1+\phi\alpha_c(\lambda))}$, we get $\Delta V_{Re}(\lambda) = \Delta V_S(\lambda) = \Delta V(\lambda)$ and the relative "cultural fitness" of the religious trait $D(q_t, \lambda_t)$ rewrites as:

$$D(q_t, \lambda_t) = D_{Re}\left[(1-q_t)\Delta V(\lambda_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t)\right]$$

Now, considering the functions $D_{Re}(x, y)$ and $D_S(z)$ that respectively characterize the optimal socialization behavior of religious parents as

$$d^*_{Re}(q_t, \lambda_t) = D_{Re}\left[(1-q_t)\Delta V(\lambda_t), m(\lambda_t)\right], \text{and} \quad d^*_S(q_t, \lambda_t) = D_S(q_t \Delta V_S(\lambda_t))$$

define the sensitivity of parents' socialization to paternalistic motives by the following elasticities:

$$\epsilon_{Re}(q,\lambda) = \frac{\partial D_{Re}(x,y)}{\partial x} \cdot \frac{x}{D_{Re}} \text{ and } \epsilon_S(q,\lambda) = \frac{\partial D_S}{\partial z} \cdot \frac{z}{D_{Re}}\Bigg|$$

evaluated respectively at $x = (1-q)\Delta V(\lambda)$ and $y = m(\lambda)$, and $z = q\Delta V_S(\lambda)$.

Differentiation of (A.26) then provides with $d^*(\lambda_t) = d^*_{Re}(q^*(\lambda_t),\lambda_t) = d^*_S(q^*(\lambda_t),\lambda_t)$

$$q^{*\prime}(\lambda_t) = \frac{\left[\epsilon_{Re}(q^*,\lambda_t) - \epsilon_S(q^*,\lambda_t)\right] d^*(\lambda_t) \cdot \frac{\Delta V'(\lambda_t)}{\Delta V(\lambda_t)} + \frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)}{-\frac{\partial D}{\partial q}(q^*(\lambda_t),\lambda_t)} \tag{A.30}$$

Given that $\frac{\partial D}{\partial q}(q^*(\lambda_t),\lambda_t) < 0$, $\frac{\partial q^*}{\partial \lambda_t}$ has the sign of the numerator. This numerator is composed of two terms reflecting the two channels through which the institutional environment λ_t affects cultural transmission. The first term $K(\lambda_t) = \left[\epsilon_{Re}(q^*,\lambda_t) - \epsilon_S(q^*,\lambda_t)\right] d^*(\lambda_t) \cdot \frac{\Delta V'(\lambda_t)}{\Delta V(\lambda_t)}$ is the paternalistic motive channel. As $\Delta V'(\lambda_t) > 0$, both types of parents increase the intensity of socialization to their own traits. The sign of $K(\lambda_t)$ depends on the relative sensitivity of parents' socialization to paternalistic motives. It is positive when $\epsilon_{Re}(q^*,\lambda_t) > \epsilon_S(q^*,\lambda_t)$, namely when the socialization rate of religious parents d^*_{Re} is more sensitive to paternalistic motives than the one of secular parents d^*_S.

The second term $\frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)$ is positive. It reflects the fact that by promoting religious infrastructures that enter as complementary inputs in the socialization process of the religious trait, an increase in the clerics weight λ_t makes the religious trait to be relatively more successfully transmitted than the secular trait.

From this discussion it follows that when religious parents' socialization efforts are more sensitive to paternalistic motives than secular parents (ie. $\epsilon_{Re}(q,\lambda_t) > \epsilon_S(q,\lambda_t)$) , and (or) when religious infrastructures are strong enough complementary inputs to socialization to the religious trait, then the numerator of (A.71) is positive and $q^*(\lambda_t)$ is increasing in λ_t.

As can be seen from (A.28) and (A.29), a change in the other parameters θ (the efficiency of the clerics) and ϕ (the restrictiveness of religious proscriptions) affects the relative cultural fitness of the religious trait only through their induced changes on the paternalistic motive $\Delta V(\lambda_t)$, with $\Delta V(\lambda)$ increasing in θ,and decreasing ϕ. It follows that

$$\frac{\partial q^*(\lambda_t)}{\partial \theta} = \frac{K(\lambda_t) \cdot \frac{\partial \Delta V(\lambda_t)}{\partial \theta}}{-\frac{\partial D}{\partial q}(q^*(\lambda_t),\lambda_t)} \text{ and } \frac{\partial q^*(\lambda_t)}{\partial \theta} = \frac{K(\lambda_t) \cdot \frac{\partial \Delta V(\lambda_t)}{\partial \phi}}{-\frac{\partial D}{\partial q}(q^*(\lambda_t),\lambda_t)}$$

When religious parents are more sensitive to paternalistic motives than secular parents, one has $K(\lambda_t) > 0$ and a positive shift in θ (negative shift of ϕ) leads to a higher value of $q^*(\lambda_t)$. This provides the comparative statics discussion on $q^*(\lambda_t)$ in the main text. **QED**.

- **Example with constant elasticity socialization cost functions:**

Consider the following socialization cost functions:

$$\begin{cases} h_{Re}(d) = \frac{d^{1+\eta_{re}}}{1+\eta_{re}} \cdot \frac{1}{m^{\gamma}} \text{ and} \\ h_s(d) = \frac{d^{1+\eta_s}}{1+\eta_s}, \end{cases} \tag{A.31}$$

with $\eta_s \geq \eta_{re} > 0$ and $\gamma > 0$**.** The optimal socialization efforts are such that:

$$\begin{cases} d^*_{Re}(q_t, \lambda_t) = ((1-q_t)\Delta V(\lambda_t))^{\frac{1}{\eta_{re}}} \cdot m\,(\lambda_t)^{\frac{\gamma}{\eta_{re}}} \\ d^*_S(q_t, \lambda_t) = (q_t \Delta V(\lambda_t))^{\frac{1}{\eta_s}}. \end{cases} \tag{A.32}$$

and in this constant elasticity specification $\epsilon_{Re}(q,\lambda) - \epsilon_S(q,\lambda) = \frac{1}{\eta_{re}} - \frac{1}{\eta_s} \geq 0$. Rewriting the cultural dynamics equation (6), we deduce that:

$$q_{t+1} - q_t = q_t(1-q_t)\{((1-q_t)\Delta V(\lambda_t))^{\frac{1}{\eta_{re}}} \cdot m\,(\lambda_t)^{\frac{\gamma}{\eta_{re}}} - (q_t \Delta V(\lambda_t))^{\frac{1}{\eta_s}}\}, \tag{A.33}$$

which admits two unstable steady states $q = 0$ and $q = 1$, and a unique interior attractor, which we denote $q^*(\lambda_t)$ such that:

$$\frac{q^*(\lambda_t)^{\frac{1}{\eta_s}}}{(1-q^*(\lambda_t))^{\frac{1}{\eta_{re}}}} = \Delta V(\lambda_t)^{\frac{\eta_S - \eta_{re}}{\eta_S \eta_{re}}} \cdot m\,(\lambda_t)^{\frac{\gamma}{\eta_{re}}} \tag{A.34}$$

given that $\eta_S \geq \eta_{re}$, we deduce that $q^*(\lambda_t)$ is increasing in θ, λ_t, and decreasing in ϕ.

A.5 Proof of Proposition 3

The likelihood of reaching the theocratic equilibrium is increasing in θ: From Proposition 1, $\overline{q}(.)$ is decreasing in θ. From Proposition 2, $q^*(.)$ is increasing in θ. Hence, the measure of parameters for which there is a complementarity between the spread of religious values and an increase in the political weight of the clerics is larger. This explains why the likelihood of reaching the theocratic equilibrium increases.

The likelihood of reaching the theocratic equilibrium is decreasing in ϕ: From Proposition 1, $\overline{q}(.)$ is increasing in ϕ. From Proposition 2, $q^*(.)$ is decreasing in ϕ. Hence, the measure of parameters for which there is a complementarity between the spread of religious values and an increase in the political weight of the clerics is lower.

A.6 Proof of Lemmas 4 and 5:

In order to prove the two Lemmas, we first derive the tax base E. Since an individual of type $i \in \{re, s\}$ complies only when

$$\frac{1-\tau_i}{1+\phi\alpha_c} > \frac{1-c\epsilon}{1+\phi\alpha_c}, \tag{A.35}$$

with $\epsilon = \frac{\epsilon_0}{1-\alpha_l}$, the fraction of individuals of type i that comply is:

$$\int_{\tau/\epsilon}^{\overline{c}} \frac{dc}{\overline{c}} = 1 - \frac{\tau_i(1-\alpha_l)}{\epsilon_0\overline{c}}. \tag{A.36}$$

Summing the taxes that are collected in the two cultural groups, we find that the tax base is:

$$E = \frac{1}{1+\phi\alpha_c}\{1 - \frac{\tau(1-q\theta\alpha_c)(1-\alpha_l)}{\epsilon_0\overline{c}}\}. \tag{A.37}$$

We are now able to solve the equilibrium. As a matter of simplification, we assume throughout the extension that $\psi(\alpha_c)$ is quadratic with $\psi(\alpha_c) = \alpha_c^2/2$.

The first-order conditions associated with the determination of $m(\lambda)$, $\alpha_l(\lambda, \beta, q)$, and $\alpha_c(\lambda)$ are respectively:

$$\begin{cases} -(1-\frac{\lambda}{2})C'(m) + \frac{\lambda}{2}(\alpha_c - F'(m)) = 0, \\ -\alpha_l + (1-\beta)\tau\frac{\partial E}{\partial \alpha_l} \leq 0, \text{ and} \\ m - \alpha_c = 0. \end{cases} \tag{A.38}$$

The equilibrium is unique, when the marginal cost functions $F'(.)$ and $C'(.)$ are strictly increasing convex functions and $\lim_{m\to\infty} F''(m) > 1 > F''(0) + C''(0)$. Typically $m(\lambda) = \alpha_c(\lambda) = 0$ when $\lambda \leq 2\frac{C''(0)}{C''(0)+1-F''(0)}$, and $m(\lambda) = \alpha_c(\lambda) > 0$ is the positive solution of

$$(1-\frac{\lambda}{2})C'(m) + \frac{\lambda}{2}F'(m) = \frac{\lambda}{2}m, \tag{A.39}$$

when $\lambda > 2\frac{C''(0)}{C''(0)+1-F''(0)}$. From this, we deduce that $m(\lambda)$ and $\alpha_c(\lambda)$ are increasing in λ, when $F'(m) < C'(m)$ and is independent from β, θ, and ϕ. This concludes the proof of Lemma 4.

Substituting (A.37) in the the second FOC above, we find

$$\alpha_l(\lambda, \beta, q) = \begin{cases} (1-\beta)\frac{\tau^2(1-q\theta\alpha_c(\lambda))}{(1+\phi\alpha_c(\lambda))\overline{c}\epsilon_0} \text{ if } (1-\beta)\frac{\tau^2(1-q\theta\alpha_c(\lambda))}{(1+\phi\alpha_c(\lambda))\overline{c}\epsilon_0} < \overline{\alpha}_l \text{ and} \\ \overline{\alpha}_l \text{ otherwise.} \end{cases} \tag{A.40}$$

We deduce that $\alpha_l(\lambda, \beta, q)$ is decreasing in β, λ q, θ and ϕ. This concludes the proof of Lemma 5.

A.7 Proof of Proposition 4

As in the related proof of Proposition 1, we first demonstrate that the optimization, problem (10) – rewritten below – admits a unique solution $(\lambda_{t+1}, \beta_{t+1}) \in [0,1]^2$:

$$\max_{(\lambda_{t+1},\beta_{t+1})} (1-\frac{\lambda_t}{2})\{U_r(m(\lambda_{t+1}), \alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t)) + U_l(m(\lambda_{t+1}), \alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t))\} + \frac{\lambda_t}{2}U_c(m(\lambda_{t+1}), \alpha_c(\lambda_{t+1})),$$

In order to solve this maximization problem, we solve the following related optimization problem:

$$\max_{m,\alpha_l} W(m, \alpha_l, \lambda_t) = (1-\frac{\lambda_t}{2})\{U_r(m, \alpha_l) + U_l(m, \alpha_l)\} + \frac{\lambda_t}{2}U_c(m). \tag{A.41}$$

The solution, denoted $(\tilde{m}(\lambda_t, q_t), \tilde{\alpha}_l(\lambda_t, q_t))$,[23] maximizes the social welfare when the externalities are internalized, with

$$\begin{aligned} U_c(m) &= m\alpha_c(m) - \psi(\alpha_c(m)) - F(m) = \frac{1}{2}m^2 - F(m) \\ U_r(m, \alpha_l) &= \beta_t(\tau E(m, \alpha_l, q_t) - C(m)) - \rho\alpha_l \\ U_l(m, \alpha_l) &= (1-\beta_t)(\tau E(m, \alpha_l, q_t) - C(m)) - \frac{\alpha_l^2}{2} \end{aligned}$$

[23] making now explicit the dependence on the state variables (λ_t, q_t).

and

$$E(m,\alpha_l,q_t)=\frac{1}{1+\phi m}\{1-\frac{\tau(1-q_t\theta m)(1-\alpha_l)}{\epsilon_0\bar{c}}\}. \tag{A.42}$$

The previous optimization problem can be rewritten:

$$\max_{m,\alpha_l} W\left(m,\alpha_l,\lambda_t\right)=(1-\frac{\lambda_t}{2})\{\tau E(m,\alpha_l,q_t)-C(m)-\rho\alpha_l-\frac{\alpha_l^2}{2}\}+\frac{\lambda_t}{2}\{\frac{1}{2}m^2-F(m)\}, \tag{A.43}$$

In this optimization problem, the choices of both the religious provision m and of the effort of the secular elite α_l are made by a ruler who can commit, and hence that internalizes the externalities detailed in the main text. We find that the solution $(\tilde{m}(\lambda_t,q_t),\tilde{\alpha}_l(\lambda_t,q_t))$ of (A.41) solves the following equations:

$$\begin{cases} \frac{\partial W}{\partial m}=\frac{\lambda_t}{2}(m-F'(m))-(1-\frac{\lambda_t}{2})C'(m)+(1-\frac{\lambda_t}{2})\{\frac{-\phi}{(1+m\phi)^2}[1-\frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0\bar{c}}] \\ \qquad +\frac{1}{1+m\phi}\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0\bar{c}}\}=0, \\ \text{and} \\ \frac{\partial W}{\partial \alpha_l}=-\alpha_l-\rho+\frac{\tau^2(1-q_t\theta m)}{\bar{c}\epsilon_0(1+\phi m)}=0. \end{cases} \tag{A.44}$$

We deduce the following lemma which characterizes the solution $(\tilde{m}(\lambda_t,q_t),\tilde{\alpha}_l(\lambda_t,q_t))$ of (A.41)

Lemma 8 *the solution $(\tilde{\alpha}_l(\lambda_t,q_t),\tilde{m}(\lambda_t,q_t))$ is uniquely determined when $C(.)$, and $F\left(m\right)$ are sufficiently convex (ie $W\left(m,\alpha_l,\lambda_t\right)$ is concave in m,α_l).*

Proof: Specifically, it is a simple matter to see that

$$\begin{aligned} \frac{\partial^2 W}{\partial m^2} &= \frac{\lambda_t}{2}(1-F''(m))-(1-\frac{\lambda_t}{2})C''(m) \\ &\quad +(1-\frac{\lambda_t}{2})\frac{2\phi}{(1+m\phi)^2}\left[\begin{array}{c} -[\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0\bar{c}}] \\ +\frac{\phi}{(1+m\phi)}[1-\frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0\bar{c}}] \end{array}\right] \\ &< \frac{\lambda_t}{2}(1-F''(m))-(1-\frac{\lambda_t}{2})C''(m)+(1-\frac{\lambda_t}{2})\frac{2\phi^2}{(1+m\phi)^3}<0 \end{aligned}$$

when $F''(m)>1$ and $C''(m)>2\phi^2$, while:

$$\frac{\partial^2 W}{\partial \alpha_l^2}=-1<0 \text{ and } \frac{\partial^2 W}{\partial m\partial\alpha_l}=-\frac{\tau^2}{\bar{c}\epsilon_0}\frac{q_t\theta+\phi}{(\phi m+1)^2}<0 \tag{A.45}$$

Therefore the Hessian of $W(m, \alpha_l, \lambda_t)$ is given by:

$$\begin{aligned}
\Delta &= \frac{\partial^2 W}{\partial m^2} \cdot \frac{\partial^2 W}{\partial \alpha_l^2} - \left(\frac{\partial^2 W}{\partial m \partial \alpha_l}\right)^2 \\
&= \left[\frac{\lambda_t}{2}(F''(m) - 1) + (1 - \frac{\lambda_t}{2}) \left[\begin{array}{c} C''(m) + \frac{2\phi}{(1+m\phi)^2}\{[\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0 \bar{c}}] \\ -\frac{\phi}{(1+m\phi)}[1 - \frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0 \bar{c}}]\} \end{array} \right]\right] \\
&\quad -\frac{\tau^4}{(\bar{c}\epsilon_0)^2} \frac{(q_t\theta + \phi)^2}{(\phi m + 1)^4} \\
&> \left[\frac{\lambda_t}{2}(F''(m) - 1) + (1 - \frac{\lambda_t}{2}) \left[C''(m) - \frac{2\phi^2}{(1 + m\phi)^3}]\right]\right] \\
&\quad -\frac{\tau^4}{(\bar{c}\epsilon_0)^2} \frac{(q_t\theta + \phi)^2}{(\phi m + 1)^4}
\end{aligned}$$

and $\Delta > 0$ when $F''(m) > 1 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$ and $C''(m) > 2\phi^2 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$. Therefore $W(m, \alpha_l, \lambda_t)$ is concave in m, α_l when $C(.)$, and $F(m)$ are sufficiently convex. (ie. when $F''(m) > 1 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$ and $C''(m) > 2\phi^2 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$) **QED**.

Now consider $(\widetilde{m}^0(q_t), \widetilde{\alpha}_l^0(q_t)) = \arg\max_{m,\alpha_l} W(m, \alpha_l, 0)$ and $\widetilde{m}^1 = \arg\max_{m,\alpha_l} W(m, \alpha_l, 1)$. $\widetilde{m}^0$ respectively the optimal level of religious infrastructure of (A.41)when the secular elite (and the ruler) have full political power (ie. $\lambda = 0$), and when the society is in a theocracy (the religious clerics weight is $\lambda = 1$). It is reasonable to make the following assumption:[24]

$$\text{Assumption } \mathbf{M}\text{: } \widetilde{m}^0(q_t) < \widetilde{m}^1 \quad \text{for all} \quad q_t \in [0, 1]$$

namely that the clerics group always wish to have a higher level of religious infrastructures than the secular fraction of society (ruler and secular elite). We have then the following result:

Lemma 9 *Under assumption M, $\tilde{m}(\lambda_t, q_t)$ is increasing in λ_t and q_t.and $\widetilde{\alpha}_l(\lambda_t, q_t)$ is decreasing in λ_t and q_t.*

[24] A sufficient condition for assumption M to be satisfied is :

$$\frac{\tau\theta}{\epsilon_0 \bar{c}} < C'(\widetilde{m}^1)$$

where $\widetilde{m}^1$ is determined by the condition $\widetilde{m}^1 = \Phi'(\widetilde{m}^1)$.

Proof: Partial differentiation yields:

$$\frac{\partial W}{\partial m \partial \lambda} = \frac{m - F'(m)}{2} + \frac{C'(m)}{2} \tag{A.46}$$

$$-\frac{1}{2}\left[\begin{array}{c} \frac{-\phi}{(1+m\phi)^2}[1 - \frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0 \bar{c}}] \\ +\frac{1}{1+m\phi}\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0 \bar{c}} \end{array}\right] \tag{A.47}$$

$$\frac{\partial W}{\partial m \partial q} = (1 - \frac{\lambda_t}{2})\frac{1}{1+m\phi}\frac{\tau\theta(1-\alpha_l)}{\epsilon_0 \bar{c}}\{\frac{1}{(1+m\phi)}\} > 0$$

and

$$\frac{\partial^2 W}{\partial \alpha_l \partial \lambda} = 0 \text{ and } \frac{\partial^2 W}{\partial \alpha_l \partial q} = -\frac{\tau^2 \theta m}{\bar{c}\epsilon_0(1+\phi m)} < 0 \tag{A.48}$$

Substitution of the FOC (A.44) into (A.46),one obtains when evaluated at the optimal point $\tilde{m}, \tilde{\alpha}_l$:

$$\left(\frac{\partial W}{\partial m \partial \lambda}\right) = \frac{1}{(1 - \frac{\lambda_t}{2})}(\tilde{m} - F'(\tilde{m})) \tag{A.49}$$

which is positive as long as $\tilde{m}(\lambda_t, q_t) \leq \widetilde{m}^1$. Moreover differentiation of the FOC in (A.44), provides

$$\begin{aligned}
\begin{pmatrix} d\tilde{m} \\ d\tilde{\alpha}_l \end{pmatrix} &= \frac{1}{\Delta}\begin{pmatrix} \frac{\partial^2 W}{\partial \alpha_l^2} & -\frac{\partial^2 W}{\partial m \partial \alpha_l} \\ -\frac{\partial^2 W}{\partial m \partial \alpha_l} & \frac{\partial^2 W}{\partial m^2} \end{pmatrix}\begin{pmatrix} -\frac{\partial^2 W}{\partial m \partial \lambda}d\lambda_t - \frac{\partial^2 W}{\partial m \partial q}dq_t \\ -\frac{\partial^2 W}{\partial \alpha_l \partial \lambda}d\lambda_t - \frac{\partial^2 W}{\partial \alpha_l \partial q}dq_t \end{pmatrix} \\
&= \frac{1}{\Delta}\begin{pmatrix} -\frac{\partial^2 W}{\partial \alpha_l^2}\frac{\partial^2 W}{\partial m \partial \lambda}d\lambda_t + \left(-\frac{\partial^2 W}{\partial \alpha_l^2}\frac{\partial^2 W}{\partial m \partial q} + \frac{\partial^2 W}{\partial m \partial \alpha_l}\frac{\partial^2 W}{\partial \alpha_l \partial q}\right)dq_t \\ \frac{\partial^2 W}{\partial m \partial \alpha_l}\frac{\partial^2 W}{\partial m \partial \lambda}d\lambda_t + \left(-\frac{\partial^2 W}{\partial m^2}\frac{\partial^2 W}{\partial \alpha_l \partial q} + \frac{\partial^2 W}{\partial m \partial \alpha_l}\frac{\partial^2 W}{\partial m \partial q}\right)dq_t \end{pmatrix}
\end{aligned}$$

with all derivatives evaluated at $\tilde{m}, \tilde{\alpha}_l$. Hence using (A.45), (A.46) and (A.48), one gets

$$\frac{\partial \tilde{m}}{\partial \lambda_t} = \frac{1}{\Delta} \cdot \frac{\partial^2 W}{\partial m \partial \lambda}$$

the sign of which is the same as the sign of $\frac{\partial^2 W}{\partial m \partial \lambda}$. Now under assumption M, one can see from (A.49) that $\tilde{m}(\lambda_t, q_t)$ is increasing in λ_t as long as $\tilde{m}(\lambda_t, q_t) < \widetilde{m}^1$. Note first that $\tilde{m}(1, q_t) = \widetilde{m}^1$. Suppose then that there exists a value $\lambda < 1$ such that $\tilde{m}(\lambda, q_t) = \widetilde{m}^1$. From (A.44), and noting that

$$W(m, \alpha_l, \lambda) = \lambda W(m, \alpha_l, 1) + (1 - \lambda)W(m, \alpha_l, 0)$$

at this point $\tilde{m}(\lambda, q_t)$, $\widetilde{\alpha}_l(\lambda, q_t)$, one should have

$$\left(\frac{\partial W}{\partial m}\right)_{\tilde{m}^1, \tilde{\alpha}_l^1} = \lambda \frac{\partial W(m, \alpha_l, 1)}{\partial m} + (1-\lambda)\frac{\partial W(m, \alpha_l, 0)}{\partial m} = 0$$

But $\tilde{m}(\lambda, q_t) = \widetilde{m}^1 = \arg\max_{m,\alpha_l} W(m, \alpha_l, 1)$, implies that $\left(\frac{\partial W(m,\alpha_l,1)}{\partial m}\right) = 0$ at such point. Hence to satisfy the previous equation, we should also have $\frac{\partial W(m,\alpha_l,0)}{\partial m} = 0$, which in turn implies that $\tilde{m}(\lambda, q_t) = \widetilde{m}^0(q_t)$, a contradiction with assumption M. From this we conclude that $\tilde{m}(\lambda, q_t) < \widetilde{m}^1$ for all $\lambda < 1$ or $\tilde{m}(\lambda, q_t) > \widetilde{m}^1$ for all $\lambda < 1$. The only case consistent with assumption M is obviously that $\tilde{m}(\lambda, q_t) < \widetilde{m}^1$ for all $\lambda < 1$. From this we conclude that under assumption M, $\frac{\partial^2 W}{\partial m \partial \lambda}$ evaluated at $\tilde{m}(\lambda, q_t)$, $\widetilde{\alpha}_l(\lambda, q_t)$ is positive and therefore $\frac{\partial \tilde{m}}{\partial \lambda_t} > 0$ (ie. religious infrastructures $\tilde{m}(\lambda_t, q_t)$ is increasing in the clerics' political weight λ_t).

Similarly, using (A.45), (A.46) and (A.48), we have:

$$\frac{\partial \tilde{\alpha}_l}{\partial \lambda_t} = \frac{1}{\Delta} \cdot \underbrace{\frac{\partial^2 W}{\partial m \partial \alpha_l}}_{-} \frac{\partial^2 W}{\partial m \partial \lambda}$$

Hence $\frac{\partial \tilde{\alpha}_l}{\partial \lambda_t} < 0$ under assumption M (ie. the tax enforcement effort of the secular elite $\widetilde{\alpha}_l(\lambda_t, q_t)$ is decreasing in the clerics' weight λ_t).

Finally, substituting (A.45), (A.46) and (A.48),we obtain

$$\frac{\partial \tilde{m}}{\partial q_t} = \frac{1}{\Delta}\left(\underbrace{\frac{\partial^2 W}{\partial m \partial q}}_{+} + \underbrace{\frac{\partial^2 W}{\partial m \partial \alpha_l}}_{-} \cdot \underbrace{\frac{\partial^2 W}{\partial \alpha_l \partial q}}_{-}\right) > 0$$

$$\frac{\partial \tilde{\alpha}_l}{\partial q_t} = \frac{1}{\Delta}\left(-\underbrace{\frac{\partial^2 W}{\partial m^2}}_{+} \cdot \underbrace{\frac{\partial^2 W}{\partial \alpha_l \partial q}}_{-} + \underbrace{\frac{\partial^2 W}{\partial m \partial \alpha_l}}_{-} \cdot \underbrace{\frac{\partial^2 W}{\partial m \partial q}}_{+}\right) < 0$$

QED.

In order to simplify the problem, we make the following assumption on the higher bound $\overline{\alpha}_l$:

$$\textbf{Assumption A: } \overline{\alpha}_l < \frac{\tau^2}{1+\phi m(1)} \frac{1-\theta m(1)}{\epsilon_0 \overline{c}}$$

Before going further with the proof, we establish this intermediary result:

Lemma 10 *Under Assumption A, $\alpha_l(\lambda, \beta = 0) = \overline{\alpha}_l$ for any $(\lambda, q) \in [0,1]^2$.*

Proof: In order to prove Lemma 10, we need to write the first-order derivative of the utility of the secular elites with respect to α_l is:

$$\frac{\partial U_l}{\partial \alpha_l} = -\alpha_l + (1-\beta)\frac{\tau^2(1-q\theta m)}{(1+\phi m)\epsilon_0 \overline{c}}. \tag{A.50}$$

Hence, when $\beta = 0$, under Assumption A, $\frac{\partial U_l}{\partial \alpha_l} > 0$ for any $\alpha_l \in [0, \overline{\alpha}_l]$ and for any $(\lambda, q) \in [0,1]^2$, so $\alpha_l(\lambda, \beta = 0, q) = \overline{\alpha}_l$ for any $(\lambda, q) \in [0,1]^2$. This concludes the proof of the Lemma. **QED.**

Since $(\tilde{\alpha}_l(\lambda_t, q_t), \tilde{m}(\lambda_t, q_t))$ maximizes the social welfare when the externalities are internalized, $(\lambda_{t+1}, \beta_{t+1})$ solves the optimization problem (10) when:

$$\begin{cases} \tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1}), \text{ and} \\ \tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t) \end{cases} \tag{A.51}$$

Indeed, when the clerics and the ruler have power λ_{t+1} and β_{t+1}, institutions are designed for $t+1$ so as to induce a choice $m(\lambda_{t+1})$ and $\alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t)$ in that period that maximizes the social welfare under the institutional framework of period t. It remains to be proven that the solution $(\lambda_{t+1}, \beta_{t+1})$ of the system (A.51) is unique. Consider the following system with two unknown variables x and y:

$$\begin{cases} \tilde{m}(\lambda_t, q_t) = m(x), \text{ and} \\ \tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(x, y, q_t), \end{cases} \tag{A.52}$$

Consider first the case where an interior solution exists. Since the function $m(.)$ is increasing in its argument, from Lemma 4, there exists a unique value $x(\lambda_t, q_t) \in [0,1]$ such that $\tilde{m}(\lambda_t, q_t) = m(x)$. Substituting $x(\lambda_t, q_t)$ in the second equation, we find:

$$\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(x(\lambda_t, q_t), y, q_t), \tag{A.53}$$

By definition, $\tilde{\alpha}_l(\lambda_t, q_t) \in [0, \overline{\alpha}_l]$. Furthermore, as $\alpha_l(x(\lambda_t, q_t), y, q_t)$ is decreasing in y from Lemma 5, under Assumption A, $\alpha_l(x(\lambda_t, q_t), 1, q_t) = 0 \leq \alpha_l(x(\lambda_t, q_t), y, q_t) \leq \alpha_l(x(\lambda_t, q_t), 0, q_t) = \overline{\alpha}_l$. Hence, applying the theorem of intermediate values, there exists a single vector $(x(\lambda_t, q_t), y(\lambda_t, q_t)) \in [0, 1]^2$ such that (A.51) holds. We have demonstrated that the system (A.51) admits a unique interior solution, when this solution exists.

An interior solution does not always exists, as it can be that $\tilde{m}(\lambda_t, q_t) > m(\lambda_{t+1})$ or $\tilde{m}(\lambda_t, q_t) < m(\lambda_{t+1})$ for any $\lambda_{t+1} \in [0, 1]$. In these two cases, there is a single solution $(\lambda_{t+1}, \beta_{t+1})$ to the optimization problem (10), which is the unique vector such that $(m(\lambda_{t+1}), \alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t))$ maximizes (A.41). Indeed, when $\tilde{m}(\lambda_t, q_t) > m(\lambda_{t+1})$, then $\lambda_{t+1} = 1$, and β_{t+1} solves

$$\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(1, \beta_{t+1}, q_t) \tag{A.54}$$

for $\beta_{t+1} \in [0, 1]$. As $\alpha_l(1, \beta_{t+1}, q_t)$ is decreasing in β_{t+1} from Lemma 5, under Assumption **??**, $\alpha_l(1, 1, q_t) = 0 \leq \alpha_l(1, \beta_{t+1}, q_t) \leq \alpha_l(1, 0, q_t) = \overline{\alpha}_l$. Applying the theorem of intermediate values, there exists a single $\beta_{t+1} \in [0, 1]$ such that $\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(1, \beta_{t+1}, q_t)$

The reasoning is similar when $\tilde{m}(\lambda_t, q_t) < m(\lambda_{t+1})$ for any $\lambda_{t+1} \in [0, 1]$: $\lambda_{t+1} = 0$ and there is a unique solution $\beta_{t+1} \in [0, 1]$ to the equation $\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(0, \beta_{t+1}, q_t)$. From this we conclude that the optimization problem (10) admits a unique solution $(\lambda_{t+1}, \beta_{t+1})$.

We are now going to demonstrate that there exists a threshold $\overline{q}_d(\lambda_t)$ such that if $q_t > \overline{q}_d(\lambda_t)$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$. In order to demonstrate this claim, we will show the following intermediary result:

Lemma 11 *$\lambda_{t+1} > \lambda_t$ if and only if $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$.*

Proof: Indeed, $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ means that in (A.41), the ruler would want to commit to a provision level $\tilde{m}(\lambda_t, q_t)$ strictly above what is provided in equilibrium. Since $m(.)$ is increasing in λ (Lemma 4), we deduce that when the political weight λ_{t+1}, that decentralizes $\tilde{m}(\lambda_t, q_t)$ is such that $\tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1})$, one has that $\lambda_{t+1} > \lambda_t$. A similar reasoning can be applied for the corners when $\lambda_{t+1} = 1$ when $\tilde{m}(\lambda_t, q_t) > m(1)$ or $\lambda_{t+1} = 0$ when $\tilde{m}(\lambda_t, q_t) < m(0)$. **QED.**

Lemma 12 *$\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if $q > \overline{q}_d(\lambda_t)$, with $\overline{q_d}(\lambda_t)$ is defined as the threshold the value of $q \in [0, 1]$ such that*

$$q = \max\left[\min\left[\frac{\phi}{\theta}\{\frac{1}{\tau(1 - \tilde{\alpha}_l(\lambda_t, q))} - \frac{1}{\epsilon_0 \overline{c}}\}, 1\right], 0\right]. \tag{A.55}$$

Proof: Given that $\tilde{m}(\lambda_t, q_t)$ is increasing in q_t, the condition $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ is equivalent to $q_t > \overline{q}_d(\lambda_t) \in [0,1]$ with $\overline{q}_d(\lambda_t)$ defined such

$$\begin{aligned}
\tilde{m}(\lambda_t, \overline{q}_d(\lambda_t)) &= m(\lambda_t) \text{ when } \tilde{m}(\lambda_t, 0) \leq m(\lambda_t) \leq \tilde{m}(\lambda_t, 1) \\
\overline{q}_d(\lambda_t) &= 0 \quad \text{when } \tilde{m}(\lambda_t, 0) > m(\lambda_t) \\
\overline{q}_d(\lambda_t) &= 1 \quad \text{when } \tilde{m}(\lambda_t, 1) < m(\lambda_t)
\end{aligned}$$

More specifically, the first-order condition associated with the determination of $m(\lambda)$ is:

$$\frac{\lambda}{2}(m - F'(m)) - (1 - \frac{\lambda}{2})C'(m) = 0. \tag{A.56}$$

The first-order condition for the determination of $\tilde{m}(\lambda, q)$ writes as

$$\frac{\lambda}{2}(m - F'(m)) - (1 - \frac{\lambda}{2})C'(m) + (1 - \frac{\lambda}{2}) \left[\begin{array}{c} \frac{-\phi}{(1+m\phi)^2}[1 - \frac{\tau(1-q\theta m)(1-\tilde{\alpha}_l(\lambda,q))}{\epsilon_0 \overline{c}}] \\ + \frac{1}{1+m\phi} \frac{\tau q \theta (1-\tilde{\alpha}_l(\lambda,q))}{\epsilon_0 \overline{c}} \end{array} \right] = 0. \tag{A.57}$$

Given the two FOCs above, we deduce that $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if:

$$(1 - \frac{\lambda_t}{2})\{\frac{-\phi}{(1+m\phi)^2}[1 - \frac{\tau(1 - q_t\theta m)(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}] + \frac{1}{1+m\phi}\frac{\tau q_t \theta (1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}\} > 0, \tag{A.58}$$

or

$$\begin{aligned}
\phi[1 - \frac{\tau(1 - q_t\theta m)(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}] &< (1 + m\phi)\frac{\tau q_t \theta (1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}} \\
\phi \left[1 - \frac{\tau(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}\right] &< \frac{\tau q_t \theta (1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}} \\
\frac{\phi}{\theta}\left[\frac{\epsilon_0 \overline{c}}{\tau(1 - \tilde{\alpha}_l(\lambda_t, q_t))} - 1\right] &< q_t
\end{aligned}$$

which rewrites

$$q_t > \frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1 - \tilde{\alpha}_l(\lambda_t, q_t))} - 1\}. \tag{A.59}$$

Denote $\Sigma(\lambda, q)$ the function

$$\Sigma(\lambda, q) = q - \frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1 - \tilde{\alpha}_l(\lambda, q))} - 1\}$$

Given that $\tilde{\alpha}_l(\lambda, q)$ is a decreasing function of q, $\Sigma(\lambda, q)$ is an increasing function of q. Now condition (A.59)is equivalent to $q_t > \overline{q_d}(\lambda_t)$ with

$$\begin{aligned}
\overline{q_d}(\lambda_t) &= 0 \text{ when } \Sigma(\lambda_t, 0) = -\frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1-\tilde{\alpha}_l(\lambda_t, 0))} - 1\} > 0 \\
\overline{q_d}(\lambda_t) &= 1 \text{ when } \Sigma(\lambda_t, 1) = 1 - \frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1-\tilde{\alpha}_l(\lambda_t, 1)} - 1\} < 0 \\
\overline{q_d}(\lambda_t) &= q \in (0,1) \text{ such that } \Sigma(\lambda_t, q) = 0 \text{ otherwise}
\end{aligned}$$

Compactly, $\overline{q_d}(\lambda_t)$ is defined as the threshold the value of $q \in [0,1]$ such that

$$q = \max\left[\min\left[\frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1-\tilde{\alpha}_l(\lambda_t, q))} - 1\}, 1\right], 0\right]. \tag{A.60}$$

and $\tilde{m}(\lambda_t) > m(\lambda_t)$ if and only if $q > \overline{q}_d(\lambda_t)$. We deduce that $\overline{q}_d(\lambda_t)$ is increasing in ϕ and decreasing in θ and λ_t. Combining the results established in Lemma 11 and Lemma 12, we get that $\lambda_{t+1} > \lambda_t$ if and only if $q > \overline{q}_d(\lambda_t)$. **QED.**

Finally, we demonstrate that there exists a threshold $\tilde{q}_d(\lambda_t, \beta_t)$ such that if $q_t > \tilde{q}_d(\lambda_t, \beta_t)$, then $\beta_{t+1} > \beta_t$. Otherwise, $\beta_{t+1} \leq \beta_t$. In order to demonstrate this claim, we proceed in two steps. First, we show the following result:

Lemma 13 *$\beta_{t+1} > \beta_t$ if and only if $\tilde{\alpha}_l(\lambda_t, \beta_t) < \alpha_l(\lambda_{t+1}, \beta_t)$, with*

$$\lambda_{t+1} = \begin{cases} \lambda \ s.t \ m(\lambda) = \tilde{m}(\lambda_t) & \text{if } \tilde{m}(\lambda_t) \in (m(0), m(1)) \\ 1 & \text{if } \tilde{m}(\lambda_t) > m(1) \\ 0 & \text{if } \tilde{m}(\lambda_t) < m(0). \end{cases} \tag{A.61}$$

Proof: Indeed, $\tilde{\alpha}_l(\lambda_t, q_t) < \alpha_l(\lambda_{t+1}, \beta_t, q_t)$ means that – given that the clerics have an optimal weight λ_{t+1} – if the ruler could, he would wish the secular elite to provide a lower enforcement effort. Since $\alpha_l(\lambda_{t+1}, q_t, .)$ is a decreasing function of β_t, the ruler increases his own political weight β_t, so that the secular elite provides less effort: $\beta_{t+1} > \beta_t$. **QED.**

Lemma 14 *There exists a threshold $\tilde{q}_d(\lambda_t, \beta_t) \in [0,1]$ such that $\tilde{\alpha}_l(\lambda_t, q_t) < \alpha_l(\lambda_{t+1}, \beta_t, q_t)$ if and only if $q > \tilde{q}_d(\lambda_t, \beta_t)$, with $\tilde{q}_d(\lambda_t, 1) = 1$ and λ_{t+1} given in (A.61).*

Proof: The first-order condition associated with the determination of $\tilde{\alpha}_l(\lambda_t, q_t)$ is:

$$-\tilde{\alpha}_l(\lambda_t, q_t) - \rho + \frac{\tau^2(1 - q_t\theta\alpha_c(\tilde{m}(\lambda_t, q_t)))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(\tilde{m}(\lambda_t, q_t))} = 0 \tag{A.62}$$

Given that $\tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1})$, this rewrites as

$$-\tilde{\alpha}_l(\lambda_t, q_t) - \rho + \frac{\tau^2(1 - q_t\theta\alpha_c(m(\lambda_{t+1}))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(m(\lambda_{t+1}))} = 0 \tag{A.63}$$

The first-order condition associated with the determination of $\alpha_l(\lambda_{t+1}, \beta_t, q_t)$ is:

$$-\alpha_l(\lambda_{t+1}, \beta_t, q_t) + (1 - \beta_t)\frac{\tau^2(1 - q_t\theta\alpha_c(m(\lambda_{t+1}))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(m(\lambda_{t+1}))} = 0 \tag{A.64}$$

Hence, the inequality $\tilde{\alpha}_l(\lambda_t, q_t) < \alpha_l(\lambda_{t+1}, \beta_t, q_t)$ is verified when

$$\rho > \beta_t\frac{\tau^2(1 - q_t\theta\alpha_c(\tilde{m}(\lambda_t, q_t))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(\tilde{m}(\lambda_t, q_t))}, \tag{A.65}$$

Now the RHS of (A.65) is decreasing in q_t as $\tilde{m}(\lambda_t, q_t)$ is an increasing function of q_t so there exists a unique threshold $\tilde{q}_d(\lambda_t, \beta_t)$ such that if $q > \tilde{q}_d(\lambda_t, \beta_t)$, then (A.65) is satisfied. Otherwise, it is not satisfied. Moreover given that the RHS of (A.65) is decreasing in λ_t (as $\tilde{m}(\lambda_t, q_t)$ and $\alpha_c(\tilde{m}(\lambda_t, q_t)$ are increasing in λ_t), and increasing in β_t, it follows that the threshold $\tilde{q}_d(\lambda_t, \beta_t)$ is decreasing in λ_t and increasing in β_t. **QED.**

Combining the results established in Lemmas 12 and 14, we have demonstrated that $\beta_{t+1} > \beta_t$ if and only if $q > \tilde{q}_d(\lambda_t, \beta_t)$.

Summarizing, we have demonstrated the followings:

- The optimization problem (10) admits a unique solution $(\lambda_{t+1}, \beta_{t+1}) \in [0, 1]^2$.
- there exists a threshold $\tilde{q}_d(\lambda_t, \beta_t)$ such that if $q > \tilde{q}_d(\lambda_t, \beta_t)$ then $\beta_{t+1} > \beta_t$. Otherwise, $\beta_{t+1} \leq \beta_t$.
- There exists a threshold $\overline{q}_d(\lambda_t)$ such that if $q_t > \overline{q}_d(\lambda_t)$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$.
- $\tilde{q}_d(\lambda_t, \beta_t)$ is decreasing in λ_t and increasing in β_t. and $\overline{q}_d(\lambda_t)$ is decreasing in λ_t.

Finally, $\tilde{q}_d(\lambda_t, 1) = 1$ because in equilibrium, the secular elite provides no effort, $\alpha_l(\lambda_t, 0, q_t) = 0$ and have zero utility. Hence, an epsilon increase in their political weight $1 - \beta_t$ will increase the social welfare by increasing both the utility of the ruler, and of the secular elite. This concludes the proof of Proposition 4. **QED.**

A.8 Proof of Proposition 5

As in the proof of Proposition 2, we first deduce from the maximization program (A.23) that $d^*_{Re} = D_{Re}((1 - q_t)\Delta V_{Re}, m)$ with $D_{Re}(0, m) = 0$, and $D_{Re}(\cdot, \cdot)$ an increasing function of both arguments $(1 - q_t)\Delta V_{Re}$ and m. Also from (A.24) $d^*_S = D_S(q_t \Delta V_S)$ is an increasing function of $q_t \Delta V_S$.

Parents do not know the realization of their children's capacity c to escape taxation when cultural transmission occurs. Consequently, the paternalistic motives have to be amended to involve expectations of the induced utilities with respect such capacity c. More precisely we have:

$$\begin{cases} V_{ReRe}(\lambda, \beta, q) = \frac{(1-\tau_{Re})}{1+\phi\alpha_c(\lambda)} \int_{\tau_{Re}/\epsilon}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau_{Re}/\epsilon} \frac{(1-c\epsilon)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}} \\ V_{Re\ S}(\lambda, \beta, q) = \frac{(1-\tau_{Re})}{(1+\phi\alpha_c(\lambda))} \int_{\tau/\epsilon}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau/\epsilon} \frac{(1-c\epsilon)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}}, \end{cases} \tag{A.66}$$

with $\epsilon = \epsilon_0/(1 - \alpha_l(\lambda, \beta, q))$. Hence,

$$\Delta V_{Re}(\lambda, \beta, q) = \frac{(\tau\theta\alpha_c(\lambda))^2(1 - \alpha_l(\lambda, q, \beta))}{2\overline{c}\epsilon_0(1 + \phi\alpha_c(\lambda))}. \tag{A.67}$$

Similarly, we find that

$$\Delta V_S(\lambda, \beta, q) = \Delta V_{Re}(\lambda, \beta, q) = \Delta V(\lambda, \beta, q) = \frac{(\tau\theta\alpha_c(\lambda))^2(1 - \alpha_l(\lambda, \beta, q))}{2\overline{c}\epsilon_0(1 + \phi\alpha_c(\lambda))}. \tag{A.68}$$

Again the result that $\Delta V_s(\lambda, \beta, q) = \Delta V_{re}(\lambda, \beta, q)$ follows from the quadratic specification of the expected payoff functions. Note as well that because $\alpha_l(\lambda, \beta, q)$ depends on q (ie. is a decreasing function in q), $\Delta V(\lambda, \beta, q)$ also depends on q and is an increasing function of q

Now, the cultural dynamics write as

$$q_{t+1} - q_t = q_t(1 - q_t)D(\lambda_t, \beta_t, q_t). \tag{A.69}$$

with

$$D(\lambda_t, \beta_t, q_t) = d^*_{Re} - d^*_S = D_{Re}\left[(1-q_t)\Delta V(\lambda_t, \beta_t, q_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t, \beta_t, q_t)\right]$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. Again simple inspection shows

$$D(\lambda_t, \beta_t, 0) = D_{Re}\left[\Delta V(\lambda_t, \beta_t, 0), m(\lambda_t)\right] > 0$$

and

$$D(\lambda_t, \beta_t, 1) = -D_S\left[\Delta V(\lambda_t, \beta_t, 1)\right] < 0$$

From this it follows that there exists a threshold $q^*_d(\lambda_t, \beta_t) \in (0,1)$ such that

$$D(\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t)) = 0 \tag{A.70}$$

Compared to the benchmark model, $D(\lambda_t, \beta_t, q_t)$ may not be always decreasing function in q_t, as $\Delta V(\lambda_t, \beta_t, q_t)$ is increasing in q_t and the uniqueness of the threshold $q^*_d(\lambda_t, \beta_t)$ is not necessarily ensured. When however $(1-q)\Delta V(\lambda, \beta, q)$ is a decreasing function of q,[25] simple inspection shows that $D(\lambda_t, \beta_t, q_t)$ is a decreasing function of q_t and that $q_{t+1} < q_t$ if and only if $q_t > q^*_d(\lambda_t, \beta_t)$, as stated in proposition 5.

In such a case, defining again the sensitivity of parents' socialization to paternalistic motives by the following elasticities:

$$\epsilon_{Re} = \frac{\partial D_{Re}(x,y)}{\partial x} \cdot \frac{x}{D_{Re}} \text{ and } \epsilon_S = \frac{\partial D_S}{\partial z} \cdot \frac{z}{D_{Re}}\Bigg|$$

evaluated respectively at $x = (1-q)\Delta V(\lambda, \beta, q)$ and $y = m(\lambda)$, and $z = q\Delta V(\lambda, \beta, q)$, we obtain

$$\frac{\partial q^*_d(\lambda_t, \beta_t)}{\partial \lambda} = \frac{\left[\epsilon_{Re} - \epsilon_S\right] d^*\left(\lambda_t, \beta_t\right) \cdot \frac{\Delta V'_\lambda(\lambda_t, \beta_t)}{\Delta V(\lambda_t, \beta_t)} + \frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)}{-\frac{\partial D}{\partial q}(\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t))} \tag{A.71}$$

with $d^*\left(\lambda_t, \beta_t\right) = d^*_{Re}\left((\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t)\right) = d^*_S\left((\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t)\right)$, the equilibrium commun socialization rate at the threshold $q^*_d(\lambda_t, \beta_t)$. Again the numerator is composed of two terms reflecting the two channels through which the institutional environment λ_t affects cultural

[25] This is ensured when $1 > \frac{\tau^2}{\bar{c}\epsilon_0} \max\left(\frac{\theta}{\phi}, 1\right)$

transmission. The first term $K(\lambda_t) = [\epsilon_{Re} - \epsilon_S]\, d^*(\lambda_t, \beta_t) \cdot \frac{\Delta V'_\lambda(\lambda_t,\beta_t)}{\Delta V(\lambda_t,\beta_t)}$ is the paternalistic motive channel. As $\Delta V'_\lambda(\lambda_t, \beta_t) > 0$, the sign of $K(\lambda_t)$ depends on the relative sensitivity of parents' socialization to paternalistic motives. It is positive when $\epsilon_{Re} > \epsilon_S$, namely when the socialization rate of religious parents d^*_{Re} is more sensitive to paternalistic motives than the one of secular parents d^*_S. The second positive term $\frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)$ reflects the positive effect of promoting religious infrastructures as complementary inputs in the transmission process of the religious trait.

As in the benchmark model, it follows again that when religious parents' socialization efforts are more sensitive to paternalistic motives than secular parents (ie. $\epsilon_{Re} > \epsilon_S$) , and (or) when religious infrastructures are strong enough complementary inputs to socialization to the religious trait, then the numerator of (A.71) is positive and $q^*_d(\lambda_t, \beta_t)$ is increasing in λ_t.

- **Example with constant elasticity socialization cost functions**

Consider the following socialization cost functions:

$$\begin{cases} h_{Re}(d) = \frac{d^{1+\eta_{re}}}{1+\eta_{re}} \cdot \frac{1}{\chi(m)} \text{ and} \\ h_s(d) = \frac{d^{1+\eta_s}}{1+\eta_s}, \end{cases} \tag{A.72}$$

with $\eta_s \geq \eta_{re} > 0$ and $\chi'(m) > 0$. The optimal socialization efforts are such that:

$$\begin{cases} d^*_{Re}(q_t, \lambda_t) = ((1-q_t)\Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_{re}}} \cdot [\chi(\lambda_t)]^{\frac{1}{\eta_{re}}} \\ d^*_S(q_t, \lambda_t) = (q_t \Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_s}}. \end{cases} \tag{A.73}$$

and in this constant elasticity specification $\epsilon_{Re} - \epsilon_S = \frac{1}{\eta_{re}} - \frac{1}{\eta_s} \geq 0$. Cultural dynamics are described as:

$$q_{t+1} - q_t = q_t(1-q_t)\{((1-q_t)\Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_{re}}} \cdot [\chi(\lambda_t)]^{\frac{1}{\eta_{re}}} - (q_t \Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_s}}\}, \tag{A.74}$$

which admits two unstable steady states $q = 0$ and $q = 1$, and in general a unique interior attractor, which we denote $q_d^*(\lambda_t, \beta_t)$ such that:

$$\frac{q_d^*(\lambda_t, \beta_t)^{\frac{1}{\eta_s}}}{(1 - q_d^*(\lambda_t, \beta_t))^{\frac{1}{\eta_{re}}}} = [\frac{(\tau\theta\alpha_c(\lambda_t))^2(1 - \alpha_l(\lambda_t, \beta_t, q_d^*(\lambda_t, \beta_t))}{2\overline{c}\epsilon_0(1 + \phi\alpha_c(\lambda_t))}]^{\frac{\eta_S - \eta_{re}}{\eta_S \eta_{re}}} \cdot [\chi(\lambda_t)]^{\frac{1}{\eta_{re}}} \tag{A.75}$$

From the last equation, and given that $\eta_S > \eta_{re}$, we deduce that $q_d^*(\lambda_t, \beta_t)$ is increasing in θ, λ_t and β_t and decreasing in ϕ. This concludes the proof of Proposition 5.

- **Joint dynamics with $q_d^*(\lambda_t, \beta_t)$ independent from β_t.**

Consider the case where the socialization cost functions of religious and secular parents are given by the following form

$$h_{Re}(d, m) = \frac{d^{1+\eta}}{1+\eta}\frac{1}{\chi(m)},\ h_s(d) = \frac{d^{1+\eta}}{1+\eta} \quad \text{with } \eta > 0$$

from (A.75), it is immediate that the threshold $q_d^*(\lambda_t, \beta_t)$ is given by:

$$q_d^*(\lambda_t, \beta_t) = q_d^*(\lambda_t) = \frac{[\chi(\lambda_t)]}{1 + [\chi(\lambda_t)]}$$

and is therefore independent from β_t. In such a case the dynamics of λ_t and q_t are such that: $\lambda_{t+1} > \lambda_t$ if and only if $q_t > \overline{q}_d(\lambda_t)$, and $q_{t+1} > q_t$ if and only if $q_t < q_d^*(\lambda_t)$ They are then decoupled from the dynamics of β_t and follow the same pattern as in the benchmark model. Consequently, depending on the initial conditions (λ_0, q_0), (λ_t, q_t) converge towards *a theocratic regime* $(1, q_d^*(1))$ or a *secular regime* $(0, q_d^*(0))$. Associated to these dynamics, the dynamics of political centralization then converges towards strong state centralization with $\beta_1^* = \widetilde{\beta}_d(1, q_d^*(1))$, or weak state centralization $\beta_0^* = \widetilde{\beta}_d(0, q_d^*(0)) < \beta_1^*$. **QED.**

A.9 Proof of Proposition 6

We consider that the policymaker chooses the amount of religious infrastructures m, and level of technology $\alpha_I \in [0, \alpha_{\max}]$ to maximize

$$W(m, \alpha_I, \alpha_c, \lambda, q) = (1 - \lambda)[U_r(m, \alpha_I, q)] + \lambda U_c(m, \alpha_c)$$

while the cleric maximizes $U_c(m, \alpha_c)$ with respect to α_c with

$$\begin{aligned} U_r(m, \alpha_I, q) &= \tau E(\alpha_I, \alpha_c, q) - C(m) \\ U_c(\alpha_c, m) &= m\alpha_c - \frac{\alpha_c^2}{2} - F(m) \end{aligned}$$

(we assume for convenience that the cost of the religious infrastructures $C(m)$ is paid as a lump-sum cost by all segments of society) with

$$E(\alpha_I, \alpha_c, q) = \frac{\alpha_I}{1 + \phi\alpha_c}\{1 - \frac{\tau(1 - q_t\theta\alpha_c)}{\epsilon_0\bar{c}}\}$$

where religious legitimacy is decreasing in the innovation effort: $\theta = \theta(\alpha_I) = \theta_0 - k\alpha_I$. We assume $k\alpha_{\max} < \theta_0 < 2k\alpha_{\max}$ Given the institutional framework λ, one immediately gets

$$\alpha_c = m, \ -(1 - \lambda)C'(m) + \lambda(\alpha_c - F'(m)) = 0$$

and α_I determined by the FOC:

$$\alpha_I(\alpha_c, q) = \min\left[\frac{1 - \frac{\tau(1 - q\theta_0\alpha_c)}{\epsilon_0\bar{c}}}{\frac{2\tau qk\alpha_c}{\epsilon_0\bar{c}}}, \alpha_{\max}\right]$$

This gives the equilibrium values $m(\lambda)$ such that $(1 - \lambda)C'(m) + \lambda F'(m) = \lambda m$. (As usual we assume that $C'''(m) > 0$, $F'''(m) > 0$, $C'(0) = F'(0) = 0$ and $\lim_{m\to\infty} F''(m) > 1 > F''(0)$ to ensure the existence of a unique equilibrium for all $\lambda \in [0, 1]$. Note that $m(\lambda) > 0$ if and only if $\lambda > \frac{C''(0)}{C''(0)+1-F''(0)}$). This provides also $\alpha_c(\lambda) = m(\lambda)$, and $\alpha_I(\lambda, q) = \alpha_I(m(\lambda), q)$. When positive, it is also easy to see that $m(\lambda)$ is an increasing function of λ (as we assume that $C'(m) > F'(m)$).

As in the related proofs of Propositions 1 and 4, we first demonstrate that the optimization problem (15) admits a unique solution $\lambda_{t+1} \in [0, 1]$:

$$\max_{\lambda_{t+1}}(1 - \lambda_t)\left[U_r(m(\lambda_{t+1}), \alpha_I(\lambda_{t+1}), q_t)\right] + \lambda_t U_c(m(\lambda_{t+1}), \alpha_c(\lambda_{t+1}))) \tag{A.76}$$

In order to solve this maximization problem, we solve the following related optimization problem:

$$\max_{m,\alpha_I} \widetilde{W}(m, \alpha_I, \lambda_t, q_t) = (1 - \lambda_t)\left[U_r(m, \alpha_I, q_t)\right] + \lambda_t U_c(m), \tag{A.77}$$

where the solution, denoted $(\tilde{m}(\lambda_t, q_t), \widetilde{\alpha}_I(\lambda_t, q_t))$ maximizes the social welfare when the externalities are internalized, so given that $U_c(m) = U_c(m, \alpha_c(m)) = \frac{1}{2}m^2 - F(m)$, as $\alpha_c(m) = m$. $U_r(m, \alpha_I, q_t) = \tau E(m, \alpha_I, q_t) - C(m)$, with

$$E(m, \alpha_I, q_t) = \frac{\alpha_I}{1 + \phi m}\{1 - \frac{\tau(1 - q_t\left[\theta_0 - k\alpha_I\right] m)}{\epsilon_0 \overline{c}}\}. \tag{A.78}$$

We also assume that in the previous optimization problem, the choices of both the religious provision m and of the effort of the innovators α_I are made by a ruler who has a policy commitment capacity, internalizing the externalities associated with the policy choice problem described in the main text. We find that $(\tilde{m}(\lambda_t, q_t), \widetilde{\alpha}_I(\lambda_t, q_t))$ solves the following equations:

$$\begin{cases} \frac{\partial \widetilde{W}}{\partial m} = \lambda_t \left(m - F'(m)\right) - (1 - \lambda_t)C'(m) + (1 - \lambda_t)\frac{\alpha_I}{1+\phi m}\left[\frac{-\phi}{1+\phi m}\left[1 - \frac{\tau(1-q[[\theta_0 - k\alpha_I]]m)}{\epsilon_0 \overline{c}}\right] + \frac{\tau q[\theta_0 - k\alpha_I]}{\epsilon_0 \overline{c}}\right] = 0, \\ \frac{\partial \widetilde{W}}{\partial \alpha_I} = \frac{(1-\lambda_t)}{1+\phi m}\{\alpha_I\left[1 - \frac{\tau(1-q_t\theta m)}{\epsilon_0 \overline{c}}\right] - \frac{k\alpha_I \tau q_t m}{\epsilon_0 \overline{c}}\} = 0. \end{cases} \tag{A.79}$$

From the second FOC equation we again get the optimal level of technology:

$$\alpha_I\left(m, q_t\right) = \min\left[\frac{1 - \frac{\tau(1-q_t\theta_0 m)}{\epsilon_0 \overline{c}}}{\frac{2\tau q_t k m}{\epsilon_0 \overline{c}}}, \alpha_{\max}\right]$$

which rewrites as

$$\begin{aligned} \alpha_I\left(m, q_t\right) &= \frac{\frac{\epsilon_0 \overline{c}}{\tau} - 1}{2kq_t m} + \frac{\theta_0}{2k} = \alpha_I^{op}\left(m, q_t\right) \quad \text{when} \quad \frac{A}{q_t} \leq m \\ &= \alpha_{\max} \quad \text{when} \quad \frac{A}{q_t} \geq m \end{aligned}$$

with

$$A = \frac{\frac{\epsilon_0 \overline{c}}{\tau} - 1}{2k\alpha_{\max} - \theta_0} > 0$$

Note that $\alpha_I\left(m, q\right)$ is decreasing in q_t and m. Now the characterization of $\tilde{m}(\lambda_t, q_t)$ is obtained from

$$\Theta\left(m\right) = \frac{\partial \widetilde{W}}{\partial m}\left(m, \alpha_I\left(m, q_t\right), \lambda_t, q_t\right) \leq 0 \quad \text{and} \quad m \geq 0$$

When $C(m)$ and $F(m)$ are sufficiently convex, $\Theta(m)$ is decreasing in m. Moreover given that

$$\Theta(0) = (1-\lambda_t)\alpha_{\max}\left[-\phi\left[1-\frac{\tau}{\epsilon_0\overline{c}}\right]+\frac{\tau q_t\left[\theta_0-k\alpha_{\max}\right]}{\epsilon_0\overline{c}}\right]$$

we have $\Theta(0) > 0$ when

$$q_t > \overline{q} = \frac{\phi}{\left[\theta_0-k\alpha_{\max}\right]}\left[\frac{\epsilon_0\overline{c}}{\tau}-1\right]$$

Thus $\tilde{m}(\lambda_t, q_t) = 0$ for $q_t \leq \overline{q}$ and $\tilde{m}(\lambda_t, q_t) > 0$ for $q_t > \overline{q}$. Substitution provides $\widetilde{\alpha}_I(\lambda_t, q_t) = \alpha_I(\tilde{m}(\lambda_t, q_t), q_t)$.

Moreover as

$$\begin{aligned}\frac{\partial^2\widetilde{W}}{\partial m\partial q} &= (1-\lambda_t)\frac{\alpha_I}{1+\phi m}\left[\frac{-\phi}{1+\phi m}\left[\frac{\tau\left[\left[\theta_0-k\alpha_I\right]\right]m}{\epsilon_0\overline{c}}\right]+\frac{\tau\left[\theta_0-k\alpha_I\right]}{\epsilon_0\overline{c}}\right]\\ &= (1-\lambda_t)\frac{\alpha_I}{\left[1+\phi m\right]^2}\frac{\tau\left[\theta_0-k\alpha_I\right]}{\epsilon_0\overline{c}} > 0\end{aligned}$$

Then $\tilde{m}(\lambda_t, q_t)$ is increasing in q_t. As well $\tilde{m}(\lambda_t, q_t) \geq m(\lambda_t)$ if and only if

$$\begin{array}{c}\frac{-\phi}{1+\phi m(\lambda_t)}\left[1-\frac{\tau(1-q_t[[\theta_0-k\alpha_I(m(\lambda_t),q_t)]]m)}{\epsilon_0\overline{c}}\right]\\ +\frac{\tau q_t[\theta_0-k\alpha_I(m(\lambda_t),q_t)]}{\epsilon_0\overline{c}}\end{array} \geq 0$$

or

$$\phi\left[\frac{\epsilon_0\overline{c}}{\tau}-1\right] \leq q_t\left[\theta_0-k\alpha_I\left(m\left(\lambda_t\right), q_t\right)\right] \tag{A.80}$$

$q_t\left[\theta_0-k\alpha_I\left(m\left(\lambda_t\right), q_t\right)\right]$ is an increasing function of q_t and decreasing function of λ_t. Condition (A.80) can be rewritten as a threshold condition $q_t \geq \overline{q}_I(\lambda_t)$ for $\overline{q}_I(\lambda_t) \in (0,1]$ with $\overline{q}_I(\lambda_t)$ is a decreasing function of λ_t.

Summarizing we get $\tilde{m}(\lambda_t, q_t) \geq m(\lambda_t)$ if and only if $q_t \geq \overline{q}_I(\lambda_t)$ for $\overline{q}_I(\lambda_t) \in (0,1]$.

Since $(\tilde{m}(\lambda_t, q_t), \tilde{\alpha}_I(\lambda_t, q_t))$ maximizes the social welfare when the externalities are internalized, λ_{t+1} solves the optimization problem (15) when:

$$\begin{cases}\tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1}), \text{ and}\\ \tilde{\alpha}_I(\lambda_t, q_t) = \alpha_I\left(m(\lambda_{t+1}), q_t\right)\end{cases} \tag{A.81}$$

Given the first equality, it is immediate to see that the second equality is automatically satisfied from the definition of $\alpha_I(m, q_t)$. Given this the institutional dynamics of λ_t is uniquely determined. Observe as well that $\tilde{m}(\lambda_t, q_t) \geq m(\lambda_t)$ if and only if $q_t \geq \overline{q}_I(\lambda_t)$.

This can be rewritten as $m(\lambda_{t+1}) \geq m(\lambda_t)$ if and only if $q_t \geq \overline{q}_I(\lambda_t)$. Given the fact that $m(\lambda)$ is increasing in λ, we deduce the following result:

$$\lambda_{t+1} \geq \lambda_t \text{ if and only if } q_t \geq \overline{q}_I(\lambda_t)$$

This concludes the proof of Proposition 6.

A.10 Proof of Proposition 7

The paternalistic motives have to be amended to take into account the fact that productivity is optimally determined by the endogenous choice of technology: More precisely we have:

$$\begin{cases} V_{ReRe}(\lambda, q) = \frac{(1-\tau_{Re})\alpha_I(\lambda,q)}{1+\phi\alpha_c(\lambda)} \int_{\tau_{Re}/\epsilon_0}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau_{Re}/\epsilon_0} \frac{(1-c\epsilon_0)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}} \\ V_{Re\ S}(\lambda, q) = \frac{(1-\tau_{Re})\alpha_I(\lambda,q)}{(1+\phi\alpha_c(\lambda))} \int_{\tau/\epsilon_0}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau/\epsilon_0} \frac{(1-c\epsilon_0)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}}, \end{cases} \tag{A.82}$$

Hence,

$$\Delta V_{Re}(\lambda, \beta, q) = \frac{(\tau\theta\alpha_c(\lambda))^2 \alpha_I(\lambda, q)}{2\overline{c}\epsilon_0(1+\phi\alpha_c(\lambda))}. \tag{A.83}$$

Similarly, we find that

$$\Delta V_S(\lambda, \beta, q) = \Delta V_{Re}(\lambda, \beta, q) = \Delta V(\lambda, \beta, q) = \frac{(\tau\theta\alpha_c(\lambda))^2 \alpha_I(\lambda, q)}{2\overline{c}\epsilon_0(1+\phi\alpha_c(\lambda))}. \tag{A.84}$$

Again the result that $\Delta V_s(\lambda, \beta, q) = \Delta V_{re}(\lambda, \beta, q)$ follows from the quadratic specification of the expected payoff functions. Note as well that because $\alpha_I(\lambda, q))$ depends on q (ie. is a decreasing function in q), $\Delta V(\lambda, \beta, q)$ also depends on q and is decreasing function of q

Now, the cultural dynamics write as

$$q_{t+1} - q_t = q_t(1-q_t)D(\lambda_t, q_t). \tag{A.85}$$

with

$$D(\lambda_t, q_t) = d^*_{Re} - d^*_S = D_{Re}\left[(1-q_t)\Delta V(\lambda_t, q_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t, q_t)\right]$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. Again simple inspection shows

$$D(\lambda_t, 0) = D_{Re}\left[\Delta V(\lambda_t, 0), m(\lambda_t)\right] > 0$$

and

$$D(\lambda_t, 1) = -D_S\left[\Delta V(\lambda_t, 1)\right] < 0$$

From this it follows that there exists a threshold $q_I^*(\lambda_t) \in (0, 1)$ such that

$$D(\lambda_t, q_I^*(\lambda_t)) = 0 \tag{A.86}$$

Compared to the benchmark model, $D(\lambda_t, q_t)$ may not be always decreasing function in q_t, as $\Delta V(\lambda_t, q_t)$ is decreasing in q_t and the uniqueness of the threshold $q_d^*(\lambda_t)$ is not necessarily ensured. When however $q\Delta V(\lambda, q)$ is increasing function of q,[26] simple inspection shows that $D(\lambda_t, q_t)$ is a decreasing function of q_t and that $q_{t+1} < q_t$ if and only if $q_t > q_I^*(\lambda_t, \beta_t)$, as stated in proposition 7. **QED.**

[26] This is ensured when $1 > \frac{\tau^2}{\bar{c}\epsilon_0} \max\left(\frac{\theta}{\phi}, 1\right)$