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# **HOW TO SET BUDGET CAPS FOR COMPETITIVE GRANTS**

**Alessandro De Chiara  
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# How to Set Budget Caps for Competitive Grants

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## Abstract

We study how funding agencies should set budget caps for competitive grants. We show that budget caps influence the researchers' submission strategy and, in particular, whether they steer their project choice towards the agencies' favorite projects, and the level of funds they request. The welfare impact of alternative approaches depends on the level of competition, the cost of public funds and the social value of project implementation.

*Keywords:* Competitive Grants, Procurement of Innovation, Project Choice, Research Funding, Research Tournament.

*JEL Classification:* D8, O25, O30, O31, O38, L2.

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## 1. Introduction

National and international agencies routinely launch competitive calls for proposals for conducting research studies or implementing projects in specific areas of interest. When defining these funding opportunities, the agencies face a critical trade-off between the need to contain the spending and the goal of incentivizing the submission of valuable projects by potential recipients of the call (e.g. research institutions, private firms, individual researchers).

In this note, we study how funding agencies should set the budget caps for these competitive grants. The natural screening role that budget caps can play is typically overlooked, as they are often set to cover the expected implementation cost of the project or to exhaust the funds available to the granting institution. We find that these practical solutions are usually inefficient, even when an agency is fully informed about the implementation cost.

We develop a model in which a public agency can award a grant to one of many agents to conduct a pre-defined research project. Each agent can submit a budget proposal to carry out the project. An information asymmetry arises because the public agency cannot verify the projects available to the agents and preferences are misaligned. We show that the maximum amount of funding available, i.e. the budget cap, plays a prominent role in determining the agents' project choice, and, in particular, whether the agents steer their research towards the agency's favorite project. We discuss how the effect of steering on welfare depends on the level of competition, the cost of public funds and the social value of project implementation. The model and its insight also apply to procurement of innovation.

Our framework builds on Armstrong and Vickers (2010) who analyze project choice when the principal does not observe the projects available to the agent, and on Berkovitch and Israel (2004) who solve for the optimal mechanism in the same context. Both these papers consider a single-agent framework, whilst we analyze a competitive setting, as in De Chiara and Iossa (2018). Further, unlike these papers, we model a project submission game where the agents submit a funding proposal along with the project, within the budget cap set by the funding agency. By determining the budget cap for competitive research grants, our paper is also linked to De Fraja (2016), who studies the optimal allocation of funding among research institutions.

## 2. Model

A public agency wishes to implement a predefined research project, that we call  $A$ . For example,  $A$  may be a project for a new electrochemical energy storage technology.<sup>1</sup> To this purpose, she can award a grant to one of  $M$  (male) agents. Each agent may be capable of implementing project  $A$  and/or project  $B$  (e.g., a novel thermal storage technology). Specifically, an agent has only a project of type  $A$  (respectively,  $B$ ) with probability  $P_A \geq 0$  (resp.,  $P_B \geq 0$ ), and both projects  $A$  and  $B$  with probability  $P_{AB} = 1 - P_A - P_B \geq 0$ . The agency and the agents have misaligned preferences. Projects  $A$  and  $B$  give the agency utility  $u_H > 0$  and zero respectively. In contrast, project  $A$  gives zero private benefit to an agent, whereas project  $B$  yields  $\pi_H$ . Implementing any type of project costs  $k > 0$ , with  $\pi_H > k$ , but each euro of funding provided by the agency costs  $(1 + \lambda) > 0$ , where  $\lambda$  captures the shadow cost of public funds. Each agent privately knows her own available projects, whereas the probability distribution of the projects is publicly known. The type of submitted projects is perfectly verifiable. We assume that each agent can carry out only one project and has enough funding to cover the implementation cost. The agents' key decisions are whether to participate in the grant competition if they have project  $A$  but could also implement project  $B$  with their own funds, and which budget  $T_i$  (or bid) to request to carry out  $A$  if they participate in the call. The agency chooses the budget cap  $R$ , which is the maximum funds she is willing to pay for the implementation of project  $A$ . All players are risk neutral.

The sequence of events is as follows. In stage 0, the agency announces  $R$ . In stage 1, each agent  $i$  who has a project  $A$  decides whether to submit it for implementation together with the budget request  $T_i$ .

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<sup>1</sup>E.g., see: <https://ec.europa.eu/research/participants/portal/desktop/en/opportunities/h2020/topics/fetproact-04-2019.html>.

In stage 2, the agency observes the submitted proposals and selects the lowest bidder, who receives the requested funding. Ties are broken randomly. In stage 3, the winning bidder - if any - receives the grant and project  $A$  is implemented. Each of the other agents decides whether to implement a project with their own funds. We look for the symmetric equilibrium which maximizes the agency's expected payoff.

### 3. Analysis

Consider an agent  $i$  who only has a project of type  $A$ , i.e.  $i$  is in state  $(A)$ . This agent is willing to implement  $A$  if  $T_i - k \geq 0$ , i.e., if the grant he receives if he wins the contest is larger than the implementation cost  $k$ . In contrast, an agent  $j$  who has both types of project, i.e.  $j$  is in state  $(AB)$ , can also decide to implement project  $B$ . Therefore,  $j$  will participate in the contest and submit a price bid  $T_j$  if

$$T_j - k \geq \pi_H - k.$$

In seeking the agency's optimal choice of  $R$ , we can restrict attention to two options. First, the agency can set a budget cap which induces only firms in state  $(A)$  to bid. This is achieved by setting  $R = k$ . By doing so, the agency does not give up any rent to agents of this type as their participation constraint binds. All agents in state  $(A)$  take part in the contest and bid  $T = k$ . Agents in state  $(AB)$  prefer not to participate. The agency's expected utility is:

$$\begin{aligned} U(R = k) &= \sum_{j=1}^M \binom{M}{j} (P_A)^j (1 - P_A)^{M-j} [u_H - k(1 + \lambda)] \\ &= (1 - (1 - P_A)^M) [u_H - k(1 + \lambda)]. \end{aligned}$$

If there is at least one agent in state  $(A)$ , the agency obtains  $u_H$  and pays the winning bid to the winner of the contest. Agents' aggregate payoff is

$$\Pi(R = k) = M(1 - P_A)[\pi_H - k].$$

Second, the agency can set a budget cap that induces agents in  $(AB)$  to participate. If so, the agency optimally chooses the minimum budget cap which induces the agents' participation in  $(AB)$ , without giving up any rent to this type of agents, i.e.  $R = \pi_H$ . As for the bidding behavior of agents in state  $(A)$ , there is no symmetric equilibrium in which they place positive probability on a specific price. Thus, the equilibrium is in mixed strategy. The implication is that now agents in state  $(A)$  earn a rent if they win the contest.

The following lemma characterizes the agents' equilibrium behavior under this option (proof in appendix).<sup>2</sup>

**Lemma 1.** *Let  $R = \pi_H$ ; each agent  $j$  in state  $(AB)$  takes part in the contest and bids  $T_j = \pi_H$ ;  $j$  implements project  $A$  if he wins and  $B$  otherwise; agents in state  $(A)$  take part in the contest and, in a symmetric equilibrium, the cumulative distribution function of an agent's bids is given by:*

$$F(T) = \begin{cases} 0 & \text{for } T \leq \phi \\ \frac{1 - (1 - P_A) \left( \frac{M-1}{T-k} \sqrt{\frac{\pi_H - k}{T-k}} \right)}{P_A} & \text{for } \phi < T < \pi_H \\ 1 & \text{for } T \geq \pi_H, \end{cases}$$

where

$$\phi = k + (\pi_H - k)(1 - P_A)^{M-1}.$$

<sup>2</sup>There is a close resemblance between the distribution function in the lemma and the price strategy followed by firms under Bertrand competition with an uncertain number of competitors (see Janssen and Rasmusen, 2002). There is also a link to the literature on price dispersion, e.g. see Varian (1980).

Each bid yields the same expected payoff to an agent in state ( $A$ ), who now obtains more than the cost of implementing the project. To determine an agent's expected payoff, we can consider the payoff the agent would obtain in state ( $A$ ) when he submits the limit bid  $T = \pi_H$ . In that case, the agent wins the contest only if there is no competitor in state ( $A$ ). We obtain:

$$\begin{aligned}\Pi_i(R = \pi_H) &= P_A(1 - P_A)^{M-1}[\pi_H - k] + (1 - P_A)[\pi_H - k] \\ &= (1 - P_A)[1 + P_A(1 - P_A)^{M-2}][\pi_H - k].\end{aligned}$$

Agents' expected aggregate payoff is obtained by simply summing up each agent's expected payoff:

$$\Pi(R = \pi_H) = M(1 - P_A)(1 + P_A(1 - P_A)^{M-2})[\pi_H - k].$$

To determine the agency's expected utility, we need to pin down the expected grant given to an agent in state ( $A$ ). Let  $j \geq 1$  denote the number of agents in state ( $A$ ). The expected winning bid is given by the expected value of the first order statistic of  $j$  draws from  $F$ :

$$\int_{\phi}^{\pi_H} j[1 - F(T)]^{j-1} T f(T) dT,$$

where  $f(\cdot)$  denotes the density function. Thus, the agency's expected utility when  $R = \pi_H$  is given by:

$$\begin{aligned}U(R = \pi_H) &= (1 - (1 - P_A)^M) u_H \\ &\quad - \sum_{j=1}^M \binom{M}{j} P_A^j (1 - P_A)^{M-j} \left( \int_{\phi}^{\pi_H} j[1 - F(T)]^{j-1} T f(T) dT \right) (1 + \lambda) \\ &\quad + ((1 - P_A)^M - P_B^M) [u_H - \pi_H(1 + \lambda)].\end{aligned}$$

Setting  $R = \pi_H$  induces agents in state ( $AB$ ) to give up implementing their preferred project ( $B$ ) and submit project  $A$  to the agency, instead. This *steering effect* increases the probability that the agency obtains project  $A$ . Despite maximizing her own utility, a necessary condition for the agency to prefer setting this high budget cap is that *steering* the agents who are in state ( $AB$ ) towards the implementation of project  $A$  is socially desirable, namely  $u_H - \pi_H(1 + \lambda) \geq 0$ . However, this condition is not sufficient. This steering effect must be valuable enough so as to outweigh the higher expected cost of implementing project  $A$ , due to the rent that is given up to agents who are in state ( $A$ ). Let  $C$  denote the expected differential cost of obtaining project  $A$  from agents in state ( $A$ ) when the agency sets  $R = \pi_H$  instead of  $R = k$ :

$$C \equiv \sum_{j=1}^M \binom{M}{j} (P_A)^j (1 - P_A)^{M-j} \left( \int_{\phi}^{\pi_H} j[1 - F(T)]^{j-1} T f(T) dT - k \right) (1 + \lambda).$$

The agency's optimal choice of the budget cap is provided in the following proposition.

**Proposition 1.** *The public agency optimally sets  $R = \pi_H$  instead of  $R = k$  if and only if this inequality holds:*

$$((1 - P_A)^M - P_B^M) [u_H - \pi_H(1 + \lambda)] \geq C. \quad (1)$$

#### 4. Discussion

*Competition effect.* Stiffer competition as captured by an increase in the number of agents  $M$  has ambiguous effects on condition (1). Steering becomes less beneficial when  $M$  increases as it becomes more likely that at least one agent is in state ( $A$ ). Taking the derivative of the left-hand side of (1) with respect to  $M$ , we obtain:<sup>3</sup>

$$((1 - P_A)^M \ln[1 - P_A] - P_B^M \ln[P_B]) [u_H - \pi_H(1 + \lambda)] \leq 0,$$

<sup>3</sup>For computational reasons, here we treat  $M$  as a continuous variable.

since  $1 - P_A = P_{AB} + P_B \geq P_B$ . As for the expected cost of steering, the probability that the agency overpays an agent in (A) with a high budget cap goes up - to see this, note that the derivative of  $1 - (1 - P_A)^M$  increases in  $M$ . However, fiercer competition induces the agents in state (A) to bid more aggressively, thereby reducing the expected rent.<sup>4</sup> Irrespective of the budget cap, the agency is always better off when competition is fiercer as both  $U(R = k)$  and  $U(R = \pi_H)$  are increasing in  $M$ .

*Project value and private benefits.* An increase in  $u_H$  makes it more likely that the agency will prefer  $R = \pi_H$ , whereas an increase in  $\pi_H$  has ambiguous effects: although the steering effect shrinks, the effect on the bidding behavior is inconclusive.<sup>5</sup> Our qualitative results continue to hold even when the agency enjoys some benefits from the implementation of project B, provided that  $u_H$  is large enough. However, the more the agency benefits from B, the more difficult it is to satisfy condition (1), and thus more likely it is that  $R = K$  is preferred.

*Shadow cost of public funds.* Qualitatively, a reduction in  $\lambda$  has the same impact as an increase in  $u_H$ . By increasing the net value for the implementation of project A, it makes it more likely that the agency will prefer  $R = \pi_H$ .

*Social welfare.* The agents' aggregate payoff is weakly increasing in the budget cap. Since the agency overlooks the agents' private benefits when setting the budget cap, steering will be induced too rarely. Therefore, from a social welfare perspective, the agency chooses  $R = \pi_H$  too rarely.

*Less conflicting preferences.* If project A gives private benefits  $\pi_L < \pi_H$  to an agent, setting  $R = k$  is never efficient. The agency should choose either  $R = \pi_H - \pi_L$ , or  $R = k - \pi_L$ .

## 5. Conclusions

Budget caps affect agents' project choice and the level of funds they request. Setting budget caps equal to project implementation costs is generally suboptimal, even when these costs are known. Higher budget caps steer researchers' towards the agency's favorite projects at the cost of increasing agents' rents.

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## Appendix - For Online Publication

### Proof of Lemma 1

First, notice that the agency must set  $R \geq \pi_H$  if she wants to induce participation of agents in state (AB). However, setting a budget cap higher than  $\pi_H$  would generate an (extra) rent for agents in state (A)

<sup>4</sup>Specifically, the effect of a marginal increase in  $M$  on the expected value of the first order statistic of  $j$  draws from  $F$  is:

$$- \frac{\partial \phi}{\partial M} j \phi f(\phi) - \int_{\phi}^{\pi_H} j(j-1)[1-F(T)]^{j-2} T f(T) \frac{\partial F}{\partial M} dT$$

$$+ \int_{\phi}^{\pi_H} j[1-F(T)]^{j-1} T \frac{\partial f}{\partial M} dT.$$

Note that  $\frac{\partial \phi}{\partial M}$  is negative, meaning that the support of the equilibrium bids shifts to the left;  $\frac{\partial f}{\partial M}$  is negative, whereas  $\frac{\partial F}{\partial M}$  is positive. Hence, the second and the third term in the above expression are negative, which imply that the agents bid more aggressively for a fixed support.

<sup>5</sup>While  $\frac{\partial F}{\partial \pi_H} < 0$ , the sign of  $\frac{\partial f}{\partial \pi_H}$  is ambiguous.

and  $(AB)$ . Hence,  $R = \pi_H$  and agents in state  $(AB)$  will participate submitting a price bid  $T = \pi_H$ . In the rest of the proof, we focus on the price-bidding behavior of agents in state  $(A)$  and we restrict attention to symmetric equilibria where each agent in state  $(A)$  follows the same bid strategy.

Agents in  $(A)$  will never submit bids below  $k$ , for if they win with such a bid, they will not be willing to implement the project. In addition, they will not submit bids above  $\pi_H$  or their bids will be discarded. Furthermore, there is no equilibrium in which all agents submit  $T = k$  as they would get no surplus whereas an agent could secure a payoff  $(1 - P_A)^{M-1}[\pi_H - \epsilon - k] > 0$  by submitting a bid arbitrarily close to  $\pi_H$ .

We now argue that there is no symmetric equilibrium in which all agents in  $(A)$  submit the same bid  $\tilde{T} \in (k, \pi_H)$ . With a bid equal to  $\tilde{T}$  agent  $i$  in state  $(A)$  expects to get:

$$\Pi_i(\tilde{T}, \tilde{T}|AA) = \sum_{j=0}^{M-1} \frac{1}{j+1} \binom{M-1}{j} P_A^j (1 - P_A)^{M-1-j} [\tilde{T} - k].$$

An agent in  $(A)$  could profitably deviate by submitting  $\tilde{T} - \epsilon$ , with  $\tilde{T} - k > \epsilon > 0$ , as he would obtain  $\tilde{T} - \epsilon - k > 0$  with probability one.

We now claim that there cannot be point masses in the equilibrium bid strategy. Suppose that an agent bids  $T \in (k, \pi_H)$  with some positive probability. Akin to the argument put forward above, another agent can gain by placing zero weight on  $T$  and positive weight on  $T - \epsilon$ .

Moreover, the agents randomize over a connected support. Let  $f(T)$  be the probability that an agent in  $(A)$  submits price  $T$ . It cannot be that  $f(\hat{T}) = 0$  for  $\hat{T} \in (T_1, T_2)$ , with  $T_2 > T_1$  and  $f(T_1) > 0, f(T_2) > 0$ . In each instance in which a bid  $T_1$  wins, also  $\hat{T}$  wins but yields a strictly higher payoff. Therefore,  $T_1$  would not be part of the equilibrium strategy and  $f(T_1) = 0$ .

Hence, the equilibrium in mixed strategy is described by the cumulative distribution function of price bids  $F(T)$ . To pin down this c.d.f. we need the expected payoff of an agent  $i$  who is in state  $(A)$  when other agents adhere to the same strategy. The probability that exactly  $M - 1 - j$  rivals are in state  $(A)$  is:

$$\binom{M-1}{j} P_A^{M-1-j} (1 - P_A)^j.$$

In this instance,  $i$ 's expected payoff if he bids  $T \in (k, \pi_L)$  is given by  $T - k$  times the probability that each of the  $M - 1 - j$  rivals submit more than  $T$ , i.e.,  $((1 - F(T))^{M-1-j} [T - k]$ . Firm  $i$ 's expected payoff from submitting  $T$  is

$$\begin{aligned} \Pi_i(T, F(T)|AA) &= \sum_{j=0}^{M-1} \binom{M-1}{j} (1 - P_A)^j [P_A (1 - F(T))]^{M-1-j} [T - k] \\ &= [1 - P_A F(T)]^{M-1} [T - k]. \end{aligned}$$

Agent  $i$  must obtain the same expected payoff from each price in which  $f(T) > 0$ , or else it would be better off submitting those prices which are associated with a higher expected payoff. To find the equilibrium prices, we take the derivative of the above expected payoff with respect to  $T$  and we set it equal to zero:

$$[1 - P_A F(T)] = (M - 1) P_A f(T) [T - k].$$

This is a differential equation whose solution yields the expression reported in the statement of the lemma. To see this, consider that the above equation can be rewritten as:

$$\frac{-P_A f(T)}{1 - P_A F(T)} = -\frac{1}{(M - 1)[T - k]}.$$

Adopting this change of variables  $y = F(T)$  and  $dy = f(T)dT$  and integrating

$$\begin{aligned} \int_{y_0}^y \frac{-P_A dy}{1 - P_A y} &= -\frac{1}{(M - 1)} \int_{T_0}^T \frac{1}{T - k} dT \\ \Leftrightarrow \ln[1 - P_A y] - \ln[1 - P_A y_0] &= \frac{1}{(M - 1)} (\ln[T_0 - k] - \ln[T - k]). \end{aligned}$$



Because at  $T_0 = \pi_H$  it holds that  $y_0 = 1$ ,

$$\ln[1 - P_A y] = \ln \left[ (1 - P_A) \left( \frac{\pi_H - k}{T - k} \right)^{\frac{1}{M-1}} \right],$$

from which it is immediate to recover the expression reported in the lemma once variable  $y$  is transformed back into  $F(T)$ .

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