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**Working Paper**

## Agnostic output gap estimation and decomposition in large cross-sections

KOF Working Papers, No. 467

**Provided in Cooperation with:**

KOF Swiss Economic Institute, ETH Zurich

*Suggested Citation:* Eckert, Florian; Sarferaz, Samad (2019) : Agnostic output gap estimation and decomposition in large cross-sections, KOF Working Papers, No. 467, ETH Zurich, KOF Swiss Economic Institute, Zurich,  
<https://doi.org/10.3929/ethz-b-000384365>

This Version is available at:

<https://hdl.handle.net/10419/235081>

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**Publication Date:**

2019-12

**Permanent Link:**

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Florian Eckert and Samad Sarferaz

**KOF Working Papers, No. 467, December 2019**

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# Agnostic Output Gap Estimation and Decomposition in Large Cross-Sections

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December 8, 2019

## Abstract

This paper uses a Bayesian non-stationary dynamic factor model to extract common trends and cycles from large datasets. An important but neglected feature of Bayesian statistics allows to treat stationary and non-stationary time series equally in terms of parameter estimation. Based on this feature we show how to extract common trends and cycles from the data by ex-post processing the posterior output and describe how to derive an agnostic output gap measure. We apply the procedure to a large panel of quarterly time series that covers 158 macroeconomic and financial series for the United States. We find that our derived output gap measure tracks the U.S. business cycle well, exhibiting a high correlation with alternative estimates of the output gap. Since the factors are extracted from a comprehensive dataset, the resulting output gap estimates are stable at the current edge and can be decomposed in a new and meaningful way.

**JEL Classification:** C11, C32, E32

**Keywords:** Non-Stationary Dynamic Factor Model, Potential Output Estimation, Output Gap Decomposition.

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# 1 Introduction

The ability to decompose economic development into its long-run trajectory and a cyclical component is of paramount importance to both fiscal and monetary policy makers. They rely on precise estimates of the output gap in order to determine the stance of their policies and to detect structural imbalances. The importance of reliable estimates for potential output has been, for example, echoed recently in the heated debate on secular stagnation. Since it is not observable, the estimation of potential output turns out to be quite challenging and the outcome depends largely on the underlying model assumptions.

This paper addresses these issues using a novel approach. It bypasses the assumptions usually imposed in the literature by exploiting an important feature of Bayesian statistics, in which stationary and non-stationary time series can be treated equally in terms of model parameter estimation (see Sims and Uhlig, 1991; Uhlig, 1994). This feature is key in extracting common trends and cycles in an ex-ante agnostic fashion. By extending the identification strategy of Aßmann et al. (2016) to the non-stationary case, we estimate a Bayesian non-stationary dynamic factor model that accounts for several common trends and cycles in a large cross section of macroeconomic data. The estimation procedure requires no ex-ante assumptions on the stochastic properties of the output gap and a minimum of restrictions on the parameter space. The identification strategy follows a decision theoretic approach via minimizing the posterior expected loss to disentangle the non-stationary factors into a common trend and one or more common cycles.

Besides the methodological innovation of identifying non-stationary factors, we also contribute to the literature on output gap estimation by presenting a procedure that uses a large set of economic variables to extract output gaps. The existing output gap literature can be broadly classified into statistical, relational and structural methods. Statistical methods decompose a univariate time series into a permanent and transitory component as in Beveridge and Nelson (1981) and Morley et al. (2003) or by removing the cyclical component using filtering techniques as in Hodrick and Prescott (1997) and Baxter and King (1999). Relational methods model the output gap as a function of well observable market outcomes such as inflation or the unemployment rate. Commonly used methods are unobserved components models such as Kuttner (1994) and Gerlach and Smets (1999), structural vector autoregressions as in Cochrane (1994) and Dupasquier et al. (1999) and multivariate Beveridge-Nelson decompositions as in Evans and Reichlin (1994). Structural methods model potential output as a function of production factors at normal degrees of capacity utilization and a smoothed measure of total factor productivity as in Nelson (1964) and Thurow and Taylor (1966).

All these methods have in common that they rely on strong, non-generic assumptions on the stochastic properties of the cycle or the underlying economic model. They often require the calibration of many parameters and the estimated output gaps are sensitive to changes in those parameters. The model proposed in this paper avoids these ex-ante restrictions in their entirety, following a more agnostic approach. Furthermore, Orphanides

and van Norden (2002) have pointed out that most output gap estimates are unreliable at the sample end due to model instabilities and data revisions. As Smets (2002) has shown, this decreases the usefulness of output gap estimates significantly for policy makers. Aastveit and Trovik (2014) attempt to overcome this issue by estimating a stationary dynamic factor model and then applying a Hodrick-Prescott filter on the projected GDP series. While this reduces the revisions at the current edge, the HP-filter itself has been heavily criticized by Hamilton (2018) for introducing spurious dynamic relations, inappropriate smoothing parameters and poor performance at the end of the time series. Our approach addresses these issues by directly linking GDP to common trends and cycles in a comprehensive set of macroeconomic and financial variables without resorting to pre-filtering techniques. The output gap resulting from our approach leads to stable results at the sample end and is less prone to data revisions. Moreover, the ability to extract several cycles from the data allows for an insightful decomposition of the historical output gap.

We apply our procedure to a large panel of macroeconomic data from the United States. The data spans the period from 1960 to 2019, consisting of 158 quarterly series from national accounts, industrial production, employment and wage statistics, flow of funds, interest rates and asset prices. The empirical findings suggest an output gap trajectory that is similar to alternative estimates from production function approaches. Our output gap measure is also less volatile than univariate statistical methods and draws a more accurate picture of the U.S. the business cycle, especially during the Great Recession, than relational methods. A decomposition of the output gap into several cyclical components reveals which of the components drive the fluctuations in the output gap. A first cyclical component moves at the business cycle frequency and is related most strongly to production and investment variables. A second cyclical component fluctuates at a lower frequency than the first one and loads strongly on labor market variables. A third component is dominated by monetary and financial variables, which provides insight on the impact of monetary policy on the output gap.

We further find that similar to other approaches, the significance of the agnostic output gap estimate in a Phillips curve regression has weakened over time. However, the cyclical components relating to production and monetary variables appear to have more explanatory power and it appears that in recent years the monetary cycle has become more important in explaining inflation. Moreover, a (quasi) real-time analysis shows that the proposed output gap estimate is less prone to revisions at the current edge.

The remainder of the paper is structured as follows. Section 2 presents the estimation and identification of the Bayesian non-stationary dynamic factor model and discusses the agnostic estimation and decomposition of output gaps. Section 3 presents the empirical results, contrasts the resulting output gap with several existing methods, discusses its decomposition and compares the results with regard to the stability of the estimates, robustness to data revisions and usefulness for policy makers. Section 4 concludes.

## 2 Econometric Framework

This section introduces the Bayesian non-stationary dynamic factor model, describes the identification procedure and shows how to derive the output gap in an agnostic fashion.<sup>1</sup>

### 2.1 Model and Estimation

The basic assumption underlying dynamic factor models is that a panel dataset can be characterized by one or more latent common components that capture the co-movements of the cross section and a variable-specific idiosyncratic component. These models imply that overall economic activity is driven by a small number of latent dynamic factors. To decompose the data into common trends and common cycles, we employ the following dynamic factor model

$$y_t = \Lambda f_t + e_t, \quad (1)$$

where  $y_t$  is the  $(N \times 1)$  vector of stationary and non-stationary time series in period  $t$  for  $t = 1, \dots, T$ ,  $f_t$  is a  $(q \times 1)$  vector of latent factors and  $\Lambda$  is an  $(N \times q)$  matrix of time-invariant factor loadings. The measurement errors  $e_t$  follow a first-order vector autoregressive process

$$e_t = \rho e_{t-1} + u_t, \quad e_t \sim \mathcal{N}(0, \Sigma). \quad (2)$$

The law of motion for the factors is given by

$$f_t = c + \phi_1 f_{t-1} + \dots + \phi_p f_{t-p} + v_t, \quad v_t \sim \mathcal{N}(0, Q), \quad (3)$$

where  $\phi_p$  is a  $(q \times q)$  matrix of autoregressive coefficients at lag  $p$ . The error term  $v_t$  is distributed normally with mean zero and covariance matrix  $Q$ . It is assumed that the majority of the cross-correlation in the data is accounted for by the dynamic factors. Both  $\rho$  and  $\Sigma$  are therefore restricted to be diagonal matrices.

$$\rho = \begin{bmatrix} \rho_1 & & \\ & \ddots & \\ & & \rho_N \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_1^2 & & \\ & \ddots & \\ & & \sigma_N^2 \end{bmatrix}.$$

The model parameters are estimated using a Gibbs sampling algorithm, consisting of the following blocks. In the first block we draw the factors conditional on model parameters and data using the algorithm by Chan and Jeliazkov (2009). In the second block we draw the model parameters from a Normal-Inverse Wishart distribution conditional on the factors and data. We cycle through 50k Gibbs iterations as burn-in and then save every

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<sup>1</sup>See Quah and Sargent (1993), Bai (2004) and Bai and Ng (2004) for non-Bayesian non-stationary dynamic factor models.

10th draw. Convergence of the sampler is checked by visual inspection using recursive mean plots and numerically by the equality of means test suggested by Geweke (1992). Appendix A.1 provides a detailed discussion of the estimation procedure.

A unique feature of this sampling process is the lack of identifying restrictions on the parameters. Only the error covariance matrix of the transition equation  $Q$  is assumed to be an identity matrix, which anchors the sampler. Because both  $\Lambda$  and  $f$  are unknown, there are infinite observationally equivalent possibilities to describe  $X$ . This is commonly referred to as scale, sign and rotational indeterminacy, which will be discussed in more detail in the following subsection. In order for the dynamic factors to track the trends and cycles affecting GDP and not some unrelated movement in the data, it is useful to shrink the error term  $\sigma_{gdp}^2$  and the corresponding autoregressive coefficient  $\rho_{gdp}$  towards zero. As a result of this prior shrinkage, the error term on GDP follows a very small white noise process and the factors reflect only relevant trends and cycles. Because the error term lies outside of the rotational indeterminacy, the unidentified parameter space is also not affected by these prior restrictions. In particular, there is no restriction of the factors to be orthogonal, which allows for interaction between trend and cycle.

## 2.2 Identification

A common issue in the estimation of dynamic factor models is that factors and the corresponding factor loadings cannot be uniquely identified. This can be shown by multiplying factors and loadings in equation (1) with an orthogonal matrix  $D$  leading to

$$y_t = \Lambda D D' f_t + e_t, \quad (4)$$

with  $D'D = DD' = I_q$ . This transformation yields an observationally equivalent model even though factors and loadings are different, which is referred to as the rotation problem (see Anderson and Rubin, 1956). The parameters in equation (3) are also affected by the rotational indeterminacy

$$\tilde{f}_t = \tilde{c} + D'\phi_1 D \tilde{f}_{t-1} + \dots + D'\phi_p D \tilde{f}_{t-p} + \tilde{v}_t,$$

where  $\tilde{f}_t = D'f_t$  and  $\tilde{c} = D'c$ . In order to uniquely identify the model, it is common to put constraints on the parameter space. Bai and Wang (2015) show that it is necessary to impose at least  $q^2$  restrictions on the factor loadings. Geweke and Zhou (1996) solve the identification problem by restricting the upper  $(q \times q)$  block of  $\Lambda$  to a positive lower triangular matrix and the variance of the factor innovations to an identity matrix. Bai and Wang (2015) restrict the upper  $(q \times q)$  block of  $\Lambda$  to be an identity matrix and show a convenient approach of imposing these linear restrictions. This allows the covariance matrix of the factor innovations to be unrestricted.

However, there are two issues with identification techniques using ex-ante parameter restrictions. The first issue concerns the order dependence of variables in  $X$  and its impact on factor identification. Because the loadings are fixed on the first  $q$  variables, their correlation structure affects the factors. The second issue of multimodality arises because parameter restrictions may induce local modes, which leads to complications in sampling.<sup>2</sup> In order to avoid these issues, Abmann et al. (2016) propose an ex-post approach to identification. They restrict  $Q$  to be an identity matrix and stack the remaining parameters in a vector  $\Theta$  as

$$\Theta = \left( \text{vec}(\Lambda)', \text{vec}(\phi_1)', \dots, \text{vec}(\phi_p)', c', \text{diag}(\rho)', \text{diag}(\Sigma)' \right)'$$

To find a unique solution to the rotation problem, given by  $\Theta^*$ , they suggest to minimize a loss function of the following form

$$L_D(\Theta^*, H(D)\Theta) = (H(D)\Theta - \Theta^*)'(H(D)\Theta - \Theta^*), \quad (5)$$

where

$$H(D) = \begin{bmatrix} D' \otimes I_N & 0 & 0 & 0 \\ 0 & I_p(D' \otimes D') & 0 & 0 \\ 0 & 0 & D' & 0 \\ 0 & 0 & 0 & I_{2N} \end{bmatrix} \quad (6)$$

such that

$$H(D)\Theta = (\text{vec}(\Lambda D)', \text{vec}(D'\phi_1 D)', \dots, \text{vec}(D\phi_p D)', (D'c)', \text{diag}(\rho)', \text{diag}(\Sigma)')'.$$

Since the posterior distribution is approximated using the unconstrained Gibbs sampler, the minimization problem has to be solved for each draw  $\Theta^{(r)}$

$$\{\{D^{(r)}\}_{r=1}^R, \Theta^*\} = \arg \min \sum_{r=1}^R L_D(\Theta^*, H(D^{(r)})\Theta^{(r)}),$$

where  $D^{(r)'}D^{(r)} = D^{(r)}D^{(r)'} = I_q$  because  $D^{(r)}$  is orthogonal. Additionally,  $r = 1, \dots, R$  denotes a sample from the unconstrained posterior distribution. Because each sample  $\{H(D^{(r)})\Theta^{(r)}\}_{r=1}^R$  is assigned the same posterior probability, Abmann et al. (2016) refer to the unconstrained sample as orthogonally mixed. The loss function is minimized numerically by generating Givens rotation matrices that minimize

$$\frac{1}{R} \sum_{r=1}^R \left( H(D^{(r)})\Theta^{(r)} - \overline{H(D)\Theta} \right)' \left( H(D^{(r)})\Theta^{(r)} - \overline{H(D)\Theta} \right) \quad (7)$$

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<sup>2</sup>See Abmann et al. (2016) and Chan et al. (2017) for a more detailed discussion of these issues.

where

$$\overline{H(D)\Theta} = \frac{1}{R} \sum_{r=1}^R H(D^{(r)})\Theta^{(r)}.$$

The complete algorithm can be found in appendix A.2. The resulting sample  $\{D^{(r)}\theta^{(r)}\}_{r=1}^R$  is not subject to orthogonal mixing anymore and minimizes the loss function. After each draw from the joint posterior distribution has been rotated this way, the resulting parameter distributions describe a unique set of factors. Furthermore, they are not subject to the previously described issues of order dependence and multimodality. Appendix A.2 provides a detailed discussion of the identification procedure and the numerical optimization of rotation matrices.

### 2.3 Agnostic Output Gap Identification

Now we describe how to uncover a common trend and several common cycles from the data in an ex-ante agnostic fashion. We use the fact that a transformation of each draw by a single orthogonal matrix  $D_*$  in equation (7) results into the same loss (see Abmann et al., 2016). This can be illustrated by using the identities  $H(D_*)'H(D_*) = I_q$  and  $H(D_*)H(D^{(r)}) = H(D_*D^{(r)})$  and by rewriting the minimization problem in (7) as

$$\begin{aligned} & \frac{1}{R} \sum_{r=1}^R \left( H(D^{(r)})\Theta^{(r)} - \overline{H(D)\Theta} \right)' H(D_*)'H(D_*) \left( H(D^{(r)})\Theta^{(r)} - \overline{H(D)\Theta} \right) \\ &= \frac{1}{R} \sum_{r=1}^R \left( H(D_*D^{(r)})\Theta^{(r)} - \overline{H(D_*D)\Theta} \right)' \left( H(D_*D^{(r)})\Theta^{(r)} - \overline{H(D_*D)\Theta} \right). \end{aligned}$$

We find matrix  $D^*$  by defining a loss function that punishes deviations from the case of one common trend that follows a random walk with drift and one or more stationary common cycles. While it is an option for the loss function to target the parameters directly, it turns out to be more stable in practice to estimate the factors in a first step and afterwards target certain time series properties of the estimated factors. A simple approach is to regress all factors but the first one on a constant and a time index. The loss is then defined as the sum of the squared coefficients, which results in a single  $D^*$  that rotates each parameter draw.

Having solved the rotation problem, there is still no economic interpretation to sign and scale of the factors. In the simple case of two factors, GDP is simply a function of a trend and a cycle. It is therefore useful to change column signs and scale each column in  $\Lambda$  such that the loadings for GDP are equal to one. This observationally equivalent transformation allows us to express GDP as a function of a common trend and a common

cycle.<sup>3</sup> The output gap is then given by

$$\text{Output Gap}_t \approx X_{GDP,t} - \mathcal{T}_t = \mathcal{C}_t$$

The output gap can be derived directly from the cyclical factor as the time series in levels are on a logarithmic scale. Because the data has been standardized before the estimation, the factors need to be rescaled using mean and standard deviation of log GDP in order to have a meaningful interpretation.

In the case of more than one common cycle, the rotation problem is solved identically by making sure that the cyclical factors follow a stationary process. The loadings on the trend factor are again scaled such that the absolute coefficient on GDP is equal to one. Since it is ex-ante unclear whether a cyclical factor affects the output gap positively or negatively, the sign and scale indeterminacy can only be solved by finding the unique solution at which the sum of the cyclical factors is equal to the output gap determined in the case with a single cycle. An interpretation of the individual components of the output gap can be retrieved by looking at the factor loadings. In order to determine the appropriate number of cyclical factors, a useful measure is the Bayesian information criterion.

### 3 Empirical Results

In this section we present our empirical findings. We describe the dataset, compare our output gap estimates with alternative measures, discuss our output gap decomposition, contrast the sample end stability of our model with those of its competitors and test the explanatory power of the different output gap estimates for inflation.

#### 3.1 Data

In order to get meaningful trends and cycles, it is of crucial importance to include variables that correlate with either potential output, the output gap or both. The inclusion of indicators that are resilient to revisions such as survey data or industrial production helps to stabilize the estimates at the current edge. Because the cyclical position of the economy is usually reflected in most economic variables, it is beneficial to include a broad set of indicators. In the following case of the United States, the FRED-QD database is used to obtain a comprehensive representation of economic activity (McCracken and Ng, 2016)<sup>4</sup>. The data spans a period from 1961 Q1 to 2019 Q2 in quarterly frequency. The dataset includes time series from national accounts, industrial production, employment and wage statistics, interest rates and asset prices. The definition of the data is consistent over time or adjusted if necessary. We omit consumer and producer price indices since

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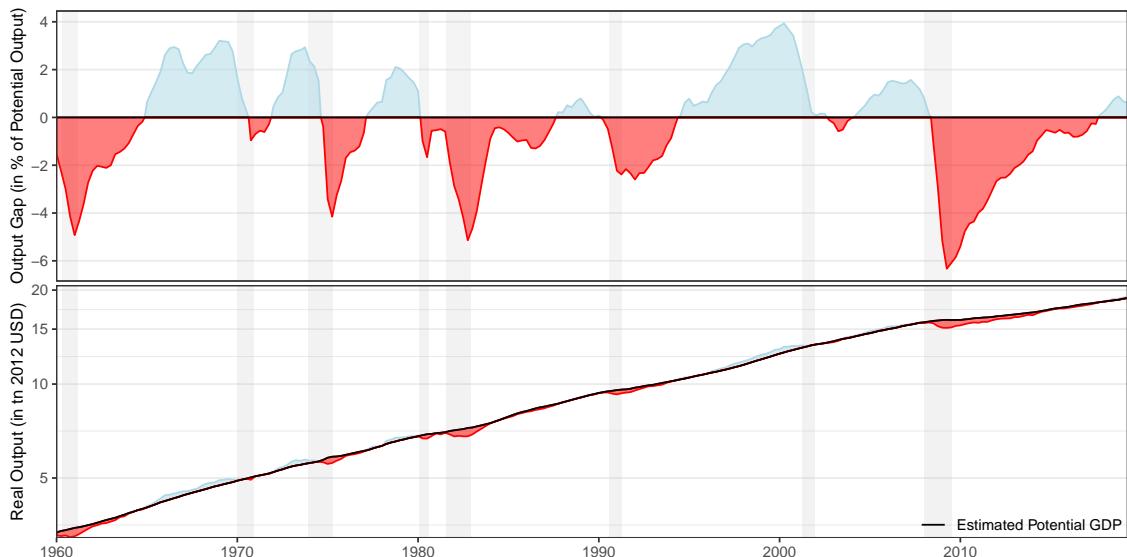
<sup>3</sup>The approximate identity has been imposed during the sampling procedure using prior restrictions on the measurement error of GDP. See appendix A.3 for a detailed analysis of the residuals for different model specifications.

<sup>4</sup><https://research.stlouisfed.org/econ/mccracken/fred-databases/>

their trend is not of interest in this analysis. The remaining 158 stock and flow variables are log-transformed to eliminate exponential growth and stabilize the variance over time. Ratios of stocks and flows or growth rates are not transformed. Then all series are standardized to have mean zero and variance one such that they have equal weight in the estimation of the dynamic factors. As a result, all series enter the model in log levels and most seem to be not stationary. The time series are classified into several groups according to their economic interpretation: Output and Production, Consumption and Income, Financial, Investment, Labor Market, and Monetary. A complete statement of the data used is available in appendix A.5.

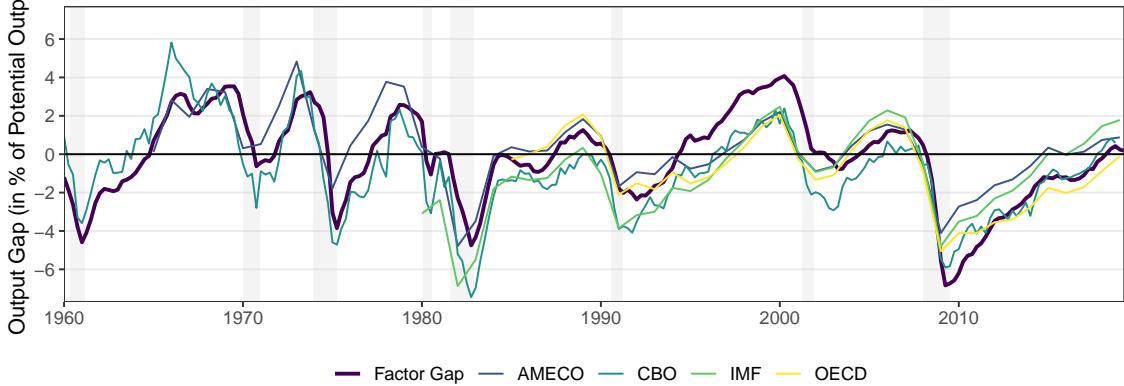
### 3.2 Output Gap Estimates

The output gap estimates resulting from the dynamic factor model are intuitive and require very little assumptions. Figure 1 shows the output gap resulting from the approach proposed in this paper, using one common trend and one common cycle.



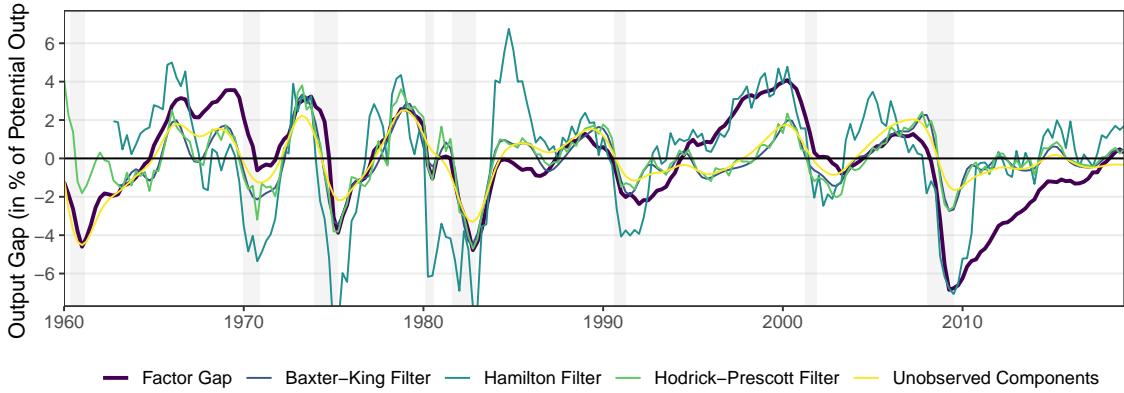
**Figure 1: Estimated Output Gap.** Blue and red areas indicate inflationary and deflationary output gaps. Shaded areas mark NBER recessions.

Figure 2 compares the resulting output gap to the production function estimates of several international institutions. Interestingly, the agnostic factor approach appears to be very similar to estimates from the European Comission (AMECO), Congressional Budget Office (CBO), International Monetary Fund (IMF) and the Organisation for Economic Co-operation and Development (OECD). Differences arise mostly from the fact that the output gap was not as negative after the recessions in the early 1980s and 1990s, but much more positive during the dot-com bubble. This could be attributed to the fact that many of the traditional output gap measure use less information from the tertiary sector than our approach.



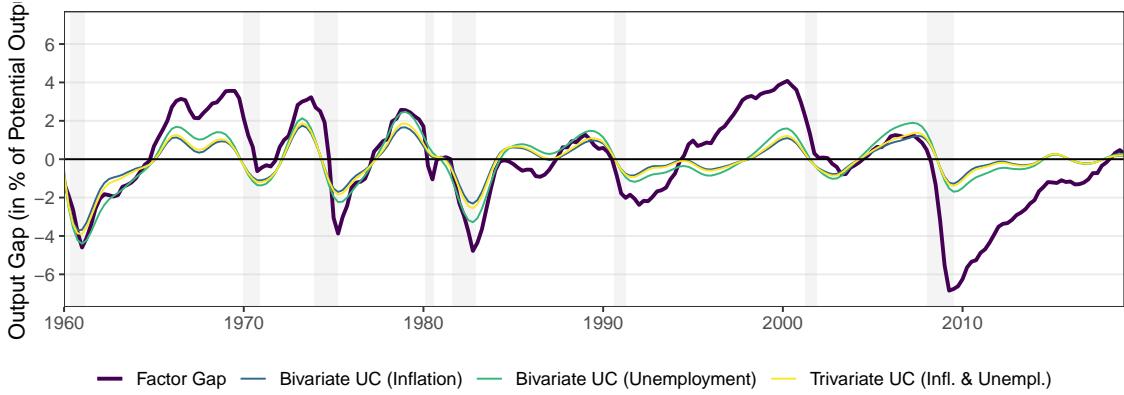
**Figure 2: Comparison of Output Gap with Structural Methods.** *Shaded areas mark recessions as determined by the NBER Business Cycle Dating Committee. Output gaps shown are estimated using production function approaches.*

Figure 3 compares the agnostic factor gap to univariate statistical methods such as the Baxter-King filter (Baxter and King, 1999), Hamilton filter (Hamilton, 2018), Hodrick-Prescott filter (Hodrick and Prescott, 1997) or an unobserved components model similar to (Kuttner, 1994). While the Baxter-King filter and the Hodrick-Prescott filter are smooth and lead to very similar results, the Hamilton filter is more volatile. However, it also captures the large inflationary gap during the dot-com bubble. All statistical methods have in common that they are not as persistent as the factor gap.



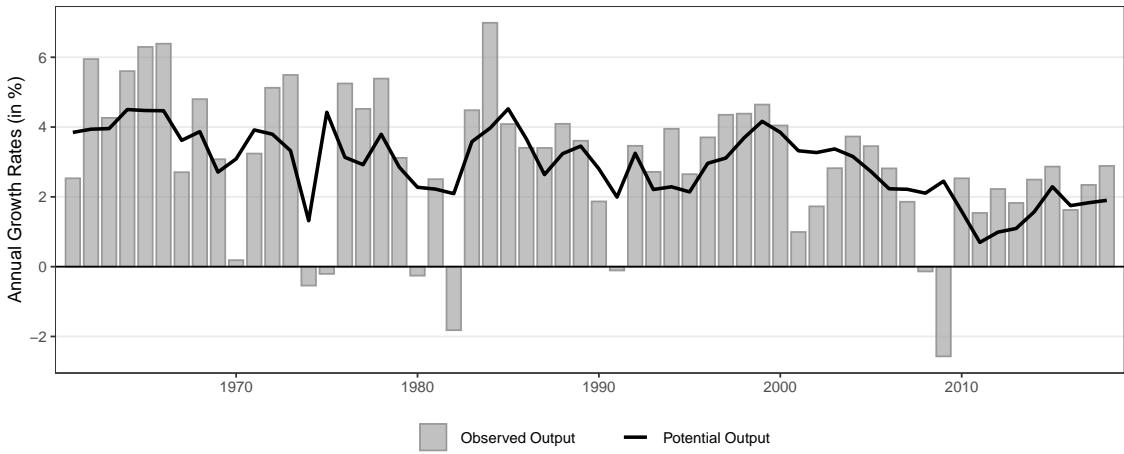
**Figure 3: Comparison of Output Gap with Statistical Methods.** *Shaded areas mark recessions as determined by the NBER Business Cycle Dating Committee. Output gaps shown are estimated using univariate statistical methods.*

Figure 4 compares the factor-based output gap to results from unobserved components models that either include a Phillips curve relationship between the output gap and core inflation, an Okun's law type relationship between the unemployment rate and the output gap, or both. However, there seems to be no large differences in the estimates.



**Figure 4: Comparison of Output Gap with Relational Methods.** *Shaded areas mark recessions as determined by the NBER Business Cycle Dating Committee. Output gaps shown are estimated using univariate relational methods.*

It is important to highlight that the common trend has a natural interpretation as potential output, which is an important measure in many applications. The factors allow for interaction between trend and cycle, which is crucial since a persistent shock to output is likely to affect potential output as well. Figure 5 further investigates the decline in potential growth rates. It shows that there is some impact of economic fluctuations on potential growth. Periods of weak output growth are likely to be followed by lower potential growth rates. While there is definitely a decrease in potential growth since the early 2000s, it is difficult to establish whether this weakening is a product of structural change that has been going on for a while or is a relatively recent phenomenon.

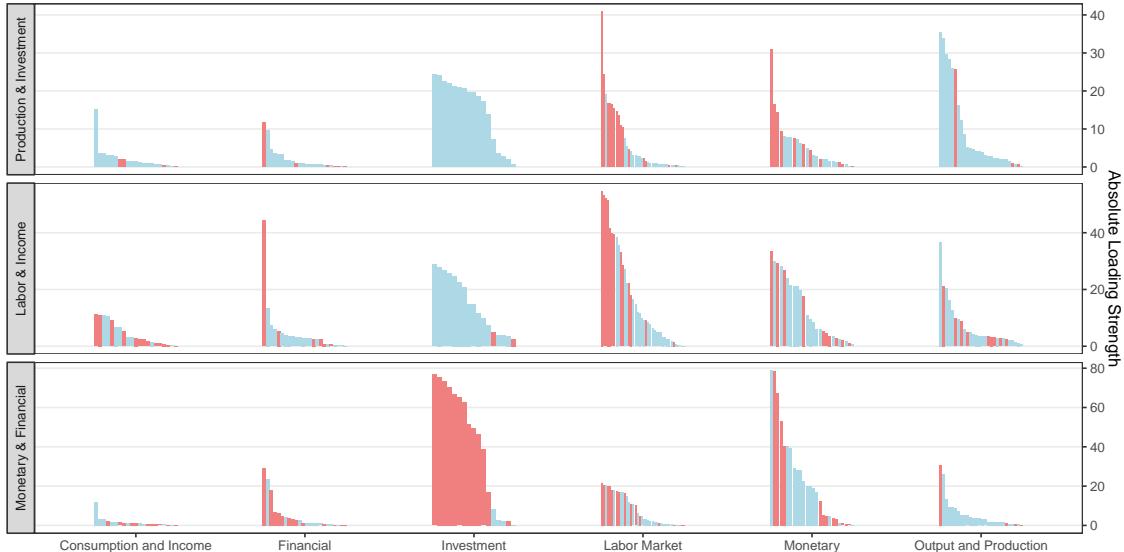


**Figure 5: Potential Growth.** *Figure shows annual GDP and potential GDP growth rates from 1960 to 2019.*

### 3.3 Output Gap Decomposition

The benefits of using a Bayesian non-stationary dynamic factor model does not end with estimating the output gap and potential output. Using more than one common cycle, it is possible to identify the fundamental factors that drive the output gap. We use the Bayesian information criterion to determine how many cycles to include in the models. Based on the lowest BIC, we select the model using three cycles for the output gap decomposition.<sup>5</sup>

It is useful to look at the factor loadings in order to find an economic interpretation for the cycles found in the data. Figure 6 shows sign and strength of the cyclical factors with respect to the economic groups described in section 3.1. It shows that all investment and production series load strongly on all factors, while series associated with consumption and income exhibit rather weak factor loadings. This result conforms with the general understanding that business cycles are mostly driven by investment activity as the economy operates below or above capacity. While for some groups such as investment the signs of the factor loadings are very homogenous, other groups such as labor or monetary have more mixed signs. This is because variables such as unemployment rate and employment growth react in opposite directions to the same economic development.

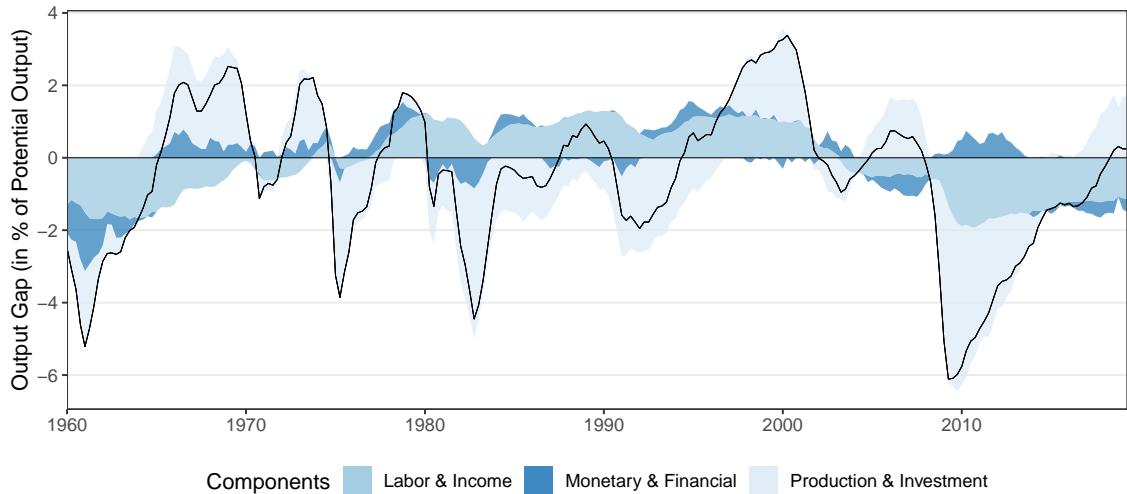


**Figure 6: Factor Loadings.** Blue bars indicate positive, red bars negative factor loadings.

The first factor loads strongly and positively on investment and output and production and loads negatively on most of the labor market and monetary variables. The first factor is dominated by investment and output and production. The second factor loads similarly on the aforementioned group of variables, but loads strongest the on labor market variables. The third factor loads strongly on monetary variables and negative on investment. Figure 7 shows the output gap as the sum of these fundamental driving forces. The factor that is dominated by monetary and financial variables is restrictive for much of the late 1970s

<sup>5</sup>See appendix A.4 for output gap decompositions under alternative specifications and appendix A.3 for a residual analysis of those different specifications.

and 1980s as a result of the Volcker deflation. It is then again very restrictive before the Great Recession as a result of quick monetary tightening before the Great Recession. The crisis years see a slightly positive contribution of the monetary cycle to the output gap, while becoming more restrictive in recent years.

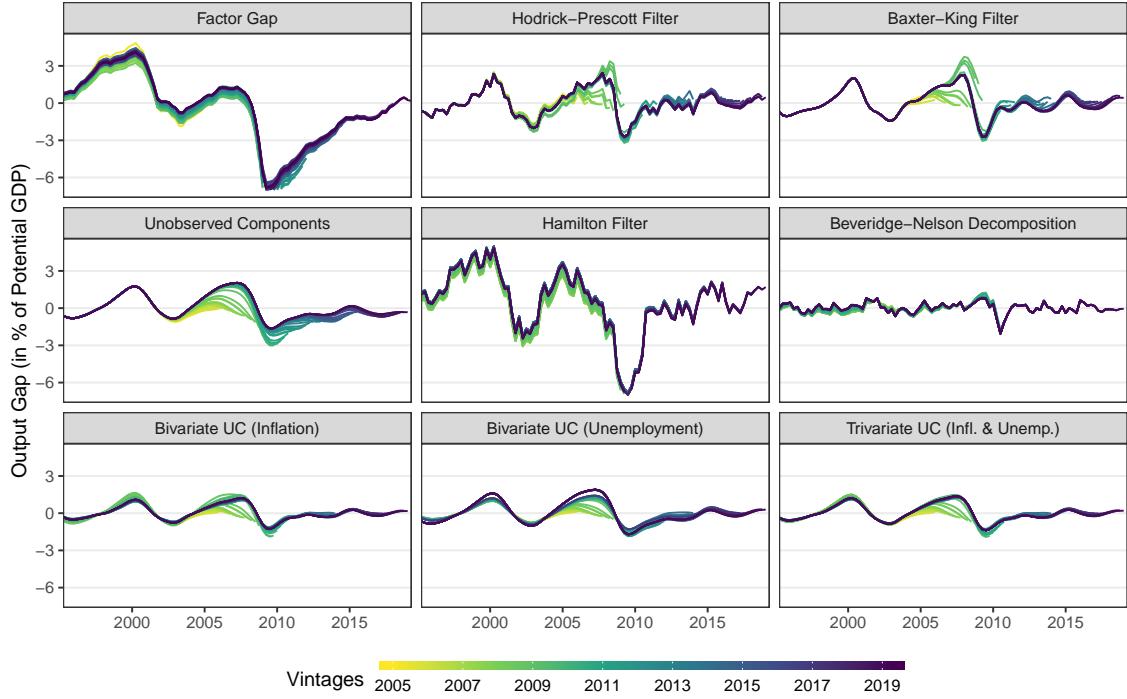


**Figure 7: Output Gap Decomposition.** Figure shows a decomposition of the stationary factor into three cyclical components.

More influential is the factor dominated by labor market variables, which is positive for much of the 1980s and 1990s, likely a result of lower taxes and high productivity during that period. This cycle provides also a negative and persistent contribution to the negative output gap after the Great Recession. This is reflected in the slow decline of unemployment and the subdued wage growth in recent years. The most volatile factor is the one dominated by production and investment, which contributes greatly to the fluctuations around potential output.

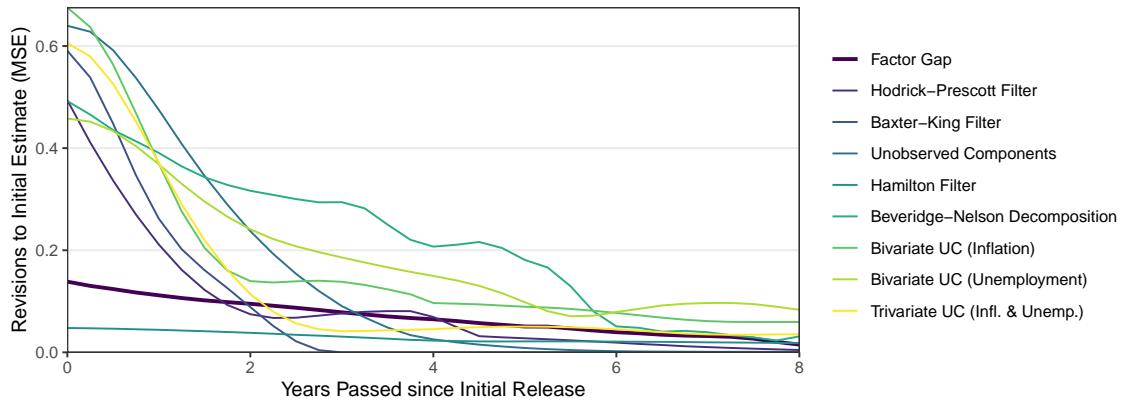
### 3.4 Stability and Robustness

Since potential output is unobservable, it is not straightforward to identify a single most appropriate measure for the output gap. In order for an output gap measure to be useful for policy makers, it should provide a good indication for the degree of capacity utilization in the economy. Furthermore, it should be stable over time and especially at the current edge. The robustness of the estimates at the current edge is tested using a pseudo-realtime backtest with an expanding windows from 2005 to 2019. The methods tested are mostly univariate with the exception of the multivariate unobserved components model, which uses industrial capacity utilization as an indicator. The dynamic factor model relies pseudo-realtime vintages of the full cross-section. Figure 8 shows the vintages obtained from the backtest. Especially the Hodrick-Prescott filter and the Baxter-King filter perform poorly during the Great Recession. The factor-based gap, the Hamilton filter and the unobserved components model on the other hand are fairly robust.



**Figure 8: Pseudo-Real Time Vintages.** Vintages from 2005 to 2019 are obtained from a pseudo-realtime experiment using an expanding window in quarterly frequency.

Figure 9 shows the mean squared revision of the output gap estimates by how much time has passed since the initial release. In order to account for the different scales of the output gap measures, revisions are scaled by the variance of corresponding output gap. It shows that all methods have only minor revisions to an output gap estimate a few years after it has been estimated for the first time. However, there are substantial differences in the quarters following the initial release. The Hodrick-Prescott Filter and the Baxter-King filter are quite unstable in the first few quarters after the initial release, but are not revised anymore after a couple of years. In the case of unobserved components models, the trivariate model appears to be more stable than the bivariate approaches. The Hamilton filter is the only approach being more robust than the factor-based gap.



**Figure 9: Output Gap Revisions.** Figure shows the mean squared revision of an output gap estimate scaled by its variance. Revisions are defined as the difference in the output gap. Quarterly pseudo-realtime vintages from 2005 to 2019 are used.

### 3.5 Explanatory Power for Inflation

In order to determine the usefulness of an output gap for policy makers, we test the explanatory power of various output gap estimates for core consumer price inflation. Following Coibion and Gorodnichenko (2015), we estimate an expectations-augmented Phillips curve given by

$$\pi_t - E_t \pi_{t+1} = \beta \mathcal{C}_t + \epsilon_t,$$

where  $x_t$  is the temporally disaggregated output gap,  $\pi$  denotes annual core inflation in quarterly frequency and  $E_t \pi_{t+1}$  denotes inflation expectations, given by the average inflation rate of the preceding year. Also in line with Coibion and Gorodnichenko (2015), the regression is estimated for the inflationary period from 1960 to 1984, the great moderation from 1985 to 2007 and the crisis and recovery years from 2008 to 2018. Table 1 shows the estimates for  $\beta$ . A common result in the literature and also visible here is the weakening Philips curve relation. For all methods, both strength and significance of the coefficients decrease rapidly during the great moderation. Interestingly, for the monetary and financial cycle the coefficient turns negative during the period of the great moderation.

**Table 1. Expectations-Augmented Phillips Curve Coefficients**

	Entire Sample	Inflationary Period	Great Moderation	Crisis and Recovery
	1960 - 2018	1960 - 1984	1985 - 2007	2008 - 2018
Factor Gap	0.16*** (0.03)	0.38*** (0.06)	0.05* (0.02)	0.02 (0.02)
Labor & Income Cycle	0.01 (0.07)	0.1 (0.19)	-0.06 (0.04)	0.01 (0.04)
Monetary & Financial Cycle	0.68*** (0.19)	1.43*** (0.37)	-0.47** (0.14)	0.05 (0.15)
Output & Production Cycle	0.18*** (0.04)	0.35*** (0.08)	0.10*** (0.02)	0.03 (0.02)
<b>Structural Methods</b>				
AMECO			0.13*** (0.03)	0.05 (0.03)
CBO	0.14*** (0.03)	0.21*** (0.05)	0.08*** (0.02)	0.02 (0.02)
IMF			0.09*** (0.02)	0.03 (0.02)
OECD			0.13*** (0.02)	0.01 (0.02)
<b>Statistical Methods</b>				
Baxter-King Filter	0.3*** (0.04)	0.38*** (0.08)	0.12*** (0.04)	0.14* (0.05)
Beveridge-Nelson Decomposition	0.39*** (0.11)	0.53** (0.2)	0.04 (0.1)	0.17 (0.11)
Hamilton Filter	0.08*** (0.02)	0.11** (0.04)	0.02 (0.02)	0.07*** (0.02)
Hodrick-Prescott Filter	0.27*** (0.04)	0.34*** (0.08)	0.1** (0.03)	0.14** (0.05)
Unobserved Components	0.37*** (0.05)	0.52*** (0.09)	0.12** (0.04)	0.14 (0.07)
<b>Relational Methods</b>				
Bivariate UC (Inflation)	0.58*** (0.08)	0.75*** (0.14)	0.2** (0.06)	0.29** (0.1)
Bivariate UC (Unemployment)	0.39*** (0.05)	0.53*** (0.09)	0.13*** (0.04)	0.16* (0.07)
Trivariate UC (Infl.& Unempl.)	0.52*** (0.07)	0.69*** (0.13)	0.18** (0.05)	0.26** (0.09)

Notes: Standard errors in parentheses, significance indicated by \*\*\* p < 0.001, \*\* p < 0.01 and \* p < 0.05. Output gap estimates by AMECO, IMF and OECD do not cover the entire estimation period.

Table 2 shows the results from including all cyclical components of the factor gap in the expectations-augmented Phillips curve. It shows that there might be beneficial for the stability of the Phillips curve to include multiple cycles given that the coefficients are jointly significant for every estimation period. While the production cycle has always been significant, the labor and income cycle was so only during the inflationary period from 1960 to 1984. The monetary cycle on the other hand has been only significant since the Great Recession, indicating a certain impact of monetary policy.

**Table 2. Multiple Cycles in Expectations-Augmented Phillips Curve**

	Entire Sample	Inflationary Period	Great Moderation	Crisis and Recovery
	1960 - 2018	1960 - 1984	1985 - 2007	2008 - 2018
Labor & Income Cycle	0.05 (0.06)	0.57** (0.19)	0.00 (0.04)	-0.09 (0.05)
Monetary & Financial Cycle	0.75*** (0.18)	0.65 (0.43)	-0.28 (0.15)	0.76** (0.23)
Output & Production Cycle	0.18*** (0.03)	0.40*** (0.11)	0.08** (0.03)	0.16*** (0.04)
Adjusted $R^2$	0.15	0.25	0.18	0.21
$F$ -statistic	14.66***	10.83***	7.62***	4.96**

Notes: Standard errors in parentheses, significance indicated by \*\*\* p < 0.001, \*\* p < 0.01 and \* p < 0.05.

## 4 Conclusion

This paper proposes a novel approach to estimate output gaps using a Bayesian non-stationary dynamic factor model. It provides an efficient sampling algorithm and an identification method that avoids ex-ante restrictions on the parameter space. By ex-post processing the posterior output, it is possible to extract common trends and cycles from a large set of macroeconomic data. This allows for the estimation of the output gap and its decomposition into multiple cyclical components. The model requires a minimum of assumptions and is therefore far more generic than many existing methods.

We have applied the model to a comprehensive dataset for the United States, spanning the period from 1960 to 2019 in quarterly frequency. The estimated output gap measure is in line with alternative measures and in particular follows a trajectory that is similar to production function estimates. A decomposition of the output gap shows that two cyclical components explain most of the fluctuations. Both factors relate strongly to all groups of variables known to be important for business cycles. While one factor is at the business cycle frequency and dominated by production and investment variables, the other factor is at a lower frequency and additionally dominated by labor market variables. A third component is dominated by monetary and financial variables.

In contrast to many alternative methods, estimates of the output gap and potential output derived from the non-stationary factor model are relatively stable at the end of the sample. Furthermore, the cyclical components appear to have more explanatory power in a Phillips curve regression than the whole output gap.

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## A Appendix

### A.1 Posterior Simulation

This section describes the estimation of the dynamic factor model. In order to account for serial correlation in the measurement errors, both sides of equation (1) are multiplied by  $(1 - \rho L)$ , where  $L$  is the lag operator.

$$(1 - \rho L)y_t = (1 - \rho L)\Lambda f_t + (1 - \rho L)e_t$$

Following ?, the ‘quasi-differenced’ measurement equation is therefore given by

$$\tilde{y}_t = \Lambda f_t - \rho \Lambda f_{t-1} + u_t \quad (8)$$

where  $\tilde{y}_t = y_t - \rho y_{t-1}$  and  $u_t = e_t - \rho e_{t-1}$ . The likelihood function is therefore proportional to

$$\frac{1}{|\Sigma|^{T/2}} \exp \left[ -\frac{1}{2} \sum_{t=1}^T \left( (1 - \rho L)y_t - (1 - \rho L)\Lambda f_t \right) \Sigma^{-1} \left( (1 - \rho L)y_t - (1 - \rho L)\Lambda f_t \right) \right]$$

We assume normal-inverse gamma priors. All priors are chosen to be as diffuse as possible. Draws from the marginal distributions of the parameters are obtained by approximating the joint posterior distribution via Gibbs sampling from the conditional distributions.

Following Aßmann et al. (2016), convergence is checked using orthogonally invariant quantities such as the error variance  $\Sigma$  of the measurement equation, the product of loadings and factors and the determinants of the VAR coefficient matrices. As proposed by Geweke (1992), equality of the means of the first and last part of a Markov chain is tested as a convergence diagnostic. Convergence is assumed if the test rejects the null hypothesis of equality for fewer than 5% of all chains on a 5% confidence interval.

The following steps are repeated until convergence.

#### Step 1: Estimating factors conditional on model parameters and data

The latent states are sampled jointly using the efficient and sparse state smoothing and simulation algorithm by Chan and Jeliazkov (2009). In order to group the parameters in appropriate blocks, the measurement equation (8) is stacked over the  $T$  time periods.

$$\tilde{Y} = GF + U, \quad U \sim N(0, I_T \otimes \Sigma) \quad (9)$$

where

$$\begin{aligned} \tilde{Y}_{(N(T-1) \times 1)} &= \begin{bmatrix} \tilde{y}_2 \\ \vdots \\ \tilde{y}_T \end{bmatrix}, & G_{(N(T-1) \times qT)} &= \begin{bmatrix} -\rho \Lambda & \Lambda & & \\ & \ddots & & \\ & & \ddots & \\ & & & -\rho \Lambda & \Lambda \end{bmatrix}, & F_{(qT \times 1)} &= \begin{bmatrix} f_1 \\ \vdots \\ f_T \end{bmatrix} \end{aligned}$$

The state equation needs to be stacked correspondingly.

$$HF = C + V, \quad V \sim N(0, I_{T-1} \otimes Q) \quad (10)$$

where

$$\begin{matrix} H \\ (q(T-1) \times qT) \end{matrix} = \begin{bmatrix} -\phi_1 & I_q & & & \\ \vdots & \ddots & I_q & & \\ -\phi_p & \ddots & -\phi_1 & I_q & \\ & \ddots & \vdots & -\phi_1 & I_q \\ & & -\phi_p & \dots & -\phi_1 & I_q \end{bmatrix}, \quad \begin{matrix} C \\ (q(T-1) \times 1) \end{matrix} = \begin{bmatrix} c \\ \vdots \\ c \end{bmatrix}$$

This setup implies an improper diffuse prior distribution on the initial states of the latent factors, which gets rid of the need to estimate them according to ?. The precision matrix  $K$  is then given by  $H' S^{-1} H$  and the conditional posterior distribution of the factors is normal.

$$F \sim N(\hat{F}, P^{-1}) \quad (11)$$

where the precision  $P$  and the mean  $\hat{F}$  are given by

$$\begin{aligned} P &= K + G'(I_T \otimes \Sigma^{-1})G \\ \hat{F} &= P^{-1}(H'C + G'(I_T \otimes \Sigma^{-1})\tilde{Y}) \end{aligned}$$

This algorithm is computationally very efficient if block-banded matrices and sparse matrix algorithms are used (Chan and Jeliazkov, 2009). It is even faster to compute the banded Cholesky factor of  $P$  and solve for  $\hat{F}$  by forward- and backward substitution.

### Step 2: Estimating model parameters conditional on factors and data

In a second step, the model parameters are generated conditional on the factors and the observed data. The factor loadings  $\Lambda$  are obtained via vectorization of equation (8) and stacking the quasi-differenced factors according into a sparse matrix  $Z$ .

$$\begin{matrix} Z \\ (N(T-1) \times 2N) \end{matrix} = \begin{bmatrix} f_2 \otimes I_q \\ \vdots \\ f_T \otimes I_q \end{bmatrix} - \begin{bmatrix} f_1 \otimes \rho \\ \vdots \\ f_{T-1} \otimes \rho \end{bmatrix}$$

Using  $\lambda = \text{vec}(\Lambda)$ , the conditional posterior distribution of the factor loadings is given by

$$\lambda \sim \mathcal{N}(b_1, B_1)$$

where  $B_1 = (Z'(I_{T-1} \otimes \Sigma^{-1})Z + B_0^{-1})^{-1}$  and  $b_1 = B_1(Z'(I_{T-1} \otimes \Sigma^{-1})\tilde{Y} + B_0^{-1}b_0)$ . The prior distribution is chosen as uninformative as possible in order to impose no assumptions

on the factors and to ensure that the likelihood is invariant to any rotations.

The covariance matrix of the measurement errors  $\Sigma$  can be retrieved conveniently from  $\tilde{Y} - Z\lambda$ . Assuming that a sufficiently large number of dynamic factors accounts for the cross-correlation in the data,  $\Sigma$  is assumed to be a diagonal matrix. Defining  $u_i$  as a vector of length  $T$  containing the errors from the  $i$ th variable, the diagonal elements  $\sigma_i^2$  are drawn equation-by-equation from an inverse Gamma distribution.

$$\sigma_i^2 \sim \mathcal{IG}(\alpha_{1,i}, \delta_{1,i}) \quad (12)$$

where  $\alpha_{1,i} = \alpha_{0,i} + T$  and  $\delta_{1,i} = \delta_{1,i} + u_i u_i'$ . Although the priors can be completely uninformative in order to estimate common trends and cycles, it is sometimes useful to shrink certain errors towards zero. We will do so in the case of errors on GDP such that the factors closely fit the actual observations of the time series we are interested in.

In order to get the autoregressive coefficients  $\rho$ , the serially correlated errors can be retrieved equation by equation from  $Y - f\Lambda'$ .  $Y$  is defined as a  $(T \times N)$  matrix containing the data and  $f$  an  $(T \times q)$  matrix of factors. Assuming  $e_i$  to be a vector of length  $T$  containing the errors from the  $i$ th variable, the conditional posterior distribution of the diagonal elements of  $\rho$  is given by

$$\rho_i \sim \mathcal{N}(p_{1,i}, P_{1,i})$$

where  $P_{1,i} = (\sigma_i^{-2} e_i' e_i + P_{0,i}^{-1})^{-1}$  and  $p_{1,i} = P_{1,i}(\sigma_i^{-2} e_i' e_i L + P_{0,i}^{-1} p_{0,i})$ . In order to enforce stationarity, it is necessary to impose a truncated normal prior by discarding draws that are greater or equal than unity. Since we are interested in keeping the errors on GDP small and inconspicuous, we also shrink this particular autoregressive coefficient towards zero.

The coefficients  $\phi_1, \dots, \phi_p$  of the vector autoregression are obtained by stacking the factors according to

$$F_+ = \begin{bmatrix} f'_{p+1} \\ \vdots \\ f'_T \end{bmatrix}, \quad F_l = \begin{bmatrix} 1 & f'_p & \dots & f'_1 \\ \vdots & & \ddots & \\ 1 & f'_{T-1} & \dots & f'_{T-p} \end{bmatrix} \quad (13)$$

Defining  $\Phi = \text{vec}(c, \phi_1, \dots, \phi_p)'$ , the conditional posterior is given by

$$\Phi \sim \mathcal{N}(a_1, A_1) \quad (14)$$

where  $A_1 = Q \otimes (F_l' F_l)^{-1}$  and  $a_1 = (F_l' F_l)^{-1} F_l' F_+$ . Since  $Q$  is restricted to be an identity matrix  $I_q$ , factors and loadings are scaled during the sampling process.

## A.2 Identification Algorithm

Since the loss function is globally convex in  $\Theta^*$ , all minima of the loss function can be characterized as

$$\frac{1}{R} \sum_{r=1}^R \left( H(D^{(r)})\Theta^{(r)} - \overline{H(D)\Theta} \right)' \left( H(D^{(r)})\Theta^{(r)} - \overline{H(D)\Theta} \right) \quad (15)$$

where

$$\overline{H(D)\Theta} = \frac{1}{R} \sum_{r=1}^R H(D^{(r)})\Theta^{(r)}.$$

It is necessary to initialize a vector  $\Theta^*$ , for which we can use the last draw from the unconstrained sampler. Then the minimization problem has to be solved numerically in two repeating steps.

(1) The minimization problem has to be solved for each draw  $r = 1, \dots, R$ .

$$\begin{aligned} D^{(r)} &= \arg \min_D L_D \left( \Theta^*, H(D) \Theta^{(r)} \right) \\ &= \arg \min_D \left( H(D) \Theta^{(r)} - \Theta^* \right)' \left( H(D) \Theta^{(r)} - \Theta^* \right) \end{aligned}$$

For the numerical optimization, we can factorize the orthogonal matrices  $D^{(r)}$  into a reflection matrix  $Z$  and  $q(q-1)/2$  Givens rotation matrices. [?](#) have parametrized each Givens rotation matrix  $G_{i,j}$  with an angle  $\gamma_{(i,j)} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$  and the reflection matrix  $Z$  to be a diagonal matrix with entries either 1 or -1. Abmann et al. (2016) show that the numerical optimization is easier to handle if the angular domain is  $\gamma_{(i,j)} \in [-\pi, \pi]$  and only a single entry of the reflection matrix  $Z$  is allowed to vary between -1 and 1. This allows to split the numerical optimization into two subproblems.

$$D = \begin{cases} D_+ = Z_+ \prod_{(i,j) \in \{1, \dots, q\}, j > i} G_{i,j,q} & \text{if } \det(D) = 1 \\ D_- = Z_- \prod_{(i,j) \in \{1, \dots, q\}, j > i} G_{i,j,q} & \text{if } \det(D) = -1 \end{cases}$$

where

$$Z_+ = \begin{pmatrix} I_{q-1} & 0 \\ 0 & 1 \end{pmatrix}, \quad Z_- = \begin{pmatrix} I_{q-1} & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$G_{i,j,q} = \begin{pmatrix} g_{1,1} & \dots & g_{1,q} \\ \vdots & \ddots & \vdots \\ g_{q,1} & \dots & g_{q,q} \end{pmatrix}$$

The entries  $g_{r,s}$  in each givens rotation matrix  $G_{i,j}$  are parametrized with an angle  $\gamma_{(i,j)} \in [-\pi, \pi]$ .

$$g_{r,s} = \begin{cases} 1, & \text{for } i \neq r = s \neq j \\ \cos(\gamma_{(i,j)}), & \text{for } r = s = i \text{ and } r = s = j \\ -\sin(\gamma_{(i,j)}), & \text{for } r = j, s = i \\ \sin(\gamma_{(i,j)}), & \text{for } r = i, s = j \\ 0, & \text{else} \end{cases}$$

This leads to  $q(q-1)/2$  Givens matrices  $G_{i,j,q}$  of the following form, where each matrix is parametrized with an angle  $\gamma_{(i,j)} \in [-\pi, \pi]$ .

$$G_{i,j,q} = \begin{pmatrix} I & 0 & 0 & 0 & 0 \\ 0 & \cos(\gamma_{(i,j)}) & 0 & -\sin(\gamma_{(i,j)}) & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & \sin(\gamma_{(i,j)}) & 0 & \cos(\gamma_{(i,j)}) & 0 \\ 0 & 0 & 0 & 0 & I \end{pmatrix} \begin{matrix} i \\ & j \end{matrix}$$

The numerical optimization procedure therefore has to minimize the loss function over  $q(q-1)/2$  parameters  $\gamma_{(i,j)}$  for both  $Z_+$  and  $Z_-$ . From the two resulting orthogonal matrices  $D_+$  and  $D_-$ , we select the one as  $D^{(r)}$  that provides the smaller loss.

$$D^{(r)} = \arg \min_{D_+, D_-} \left\{ L_D \left( \Theta^*, H(D_+) \Theta^{(r)} \right), L_D \left( \Theta^*, H(D_-) \Theta^{(r)} \right) \right\}$$

This procedure yields a different rotation matrix  $D^{(r)}$  for each draw from the unconstrained Gibbs sampler.

- (2) Choose  $\Theta^* = \overline{H(D)\Theta}$  and return to step (1) in order to minimize the loss function for each draw again.

Following Aßmann et al. (2016), convergence is assumed if the sum of squared deviations between two successive  $\Theta^*$  is lower than  $10^{-9}$ .

### A.3 Residual Analysis

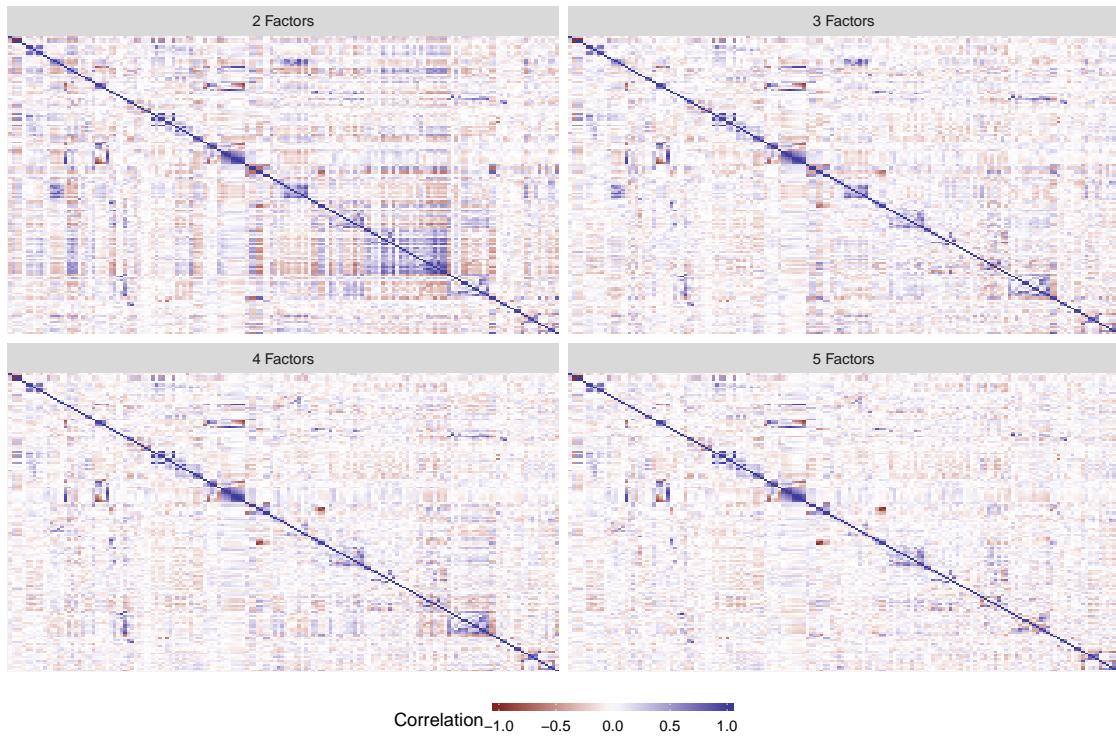


Figure 10: Correlation Matrices for Several Model Specifications

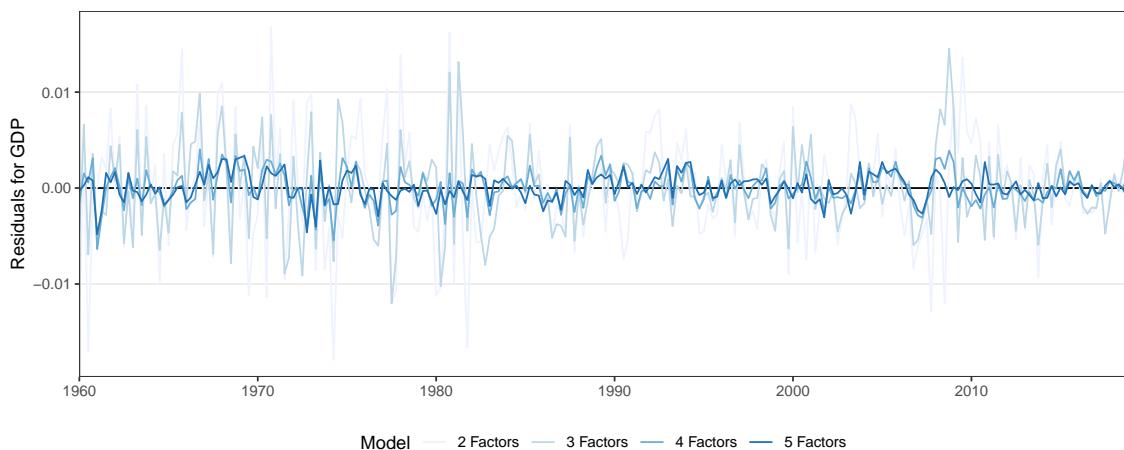
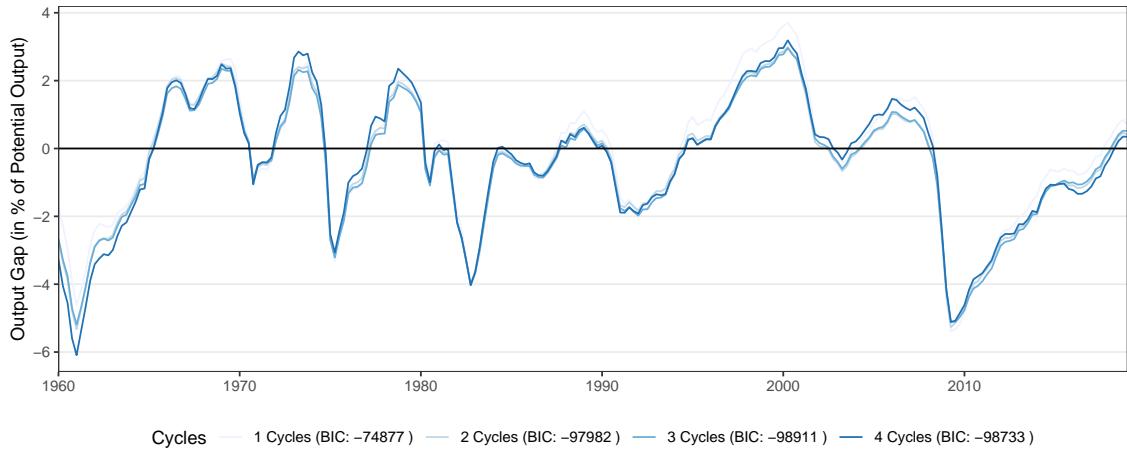


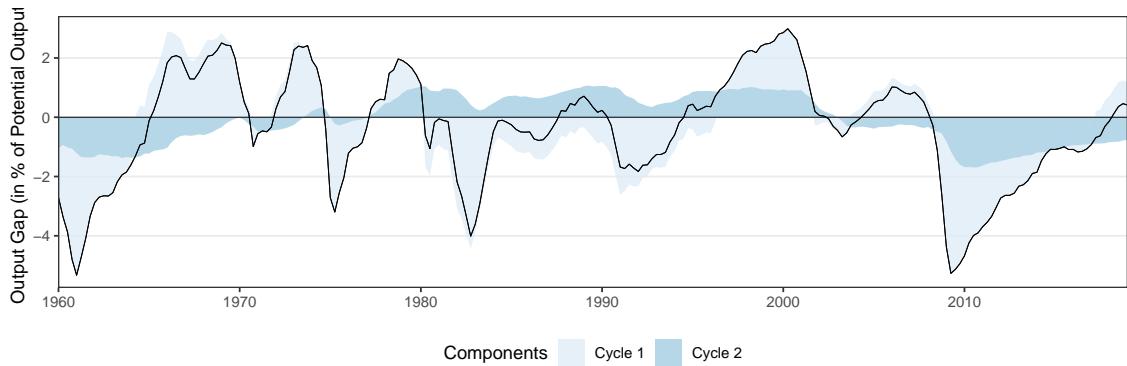
Figure 11: Residuals for GDP for Several Model Specifications

#### A.4 Alternative Decompositions

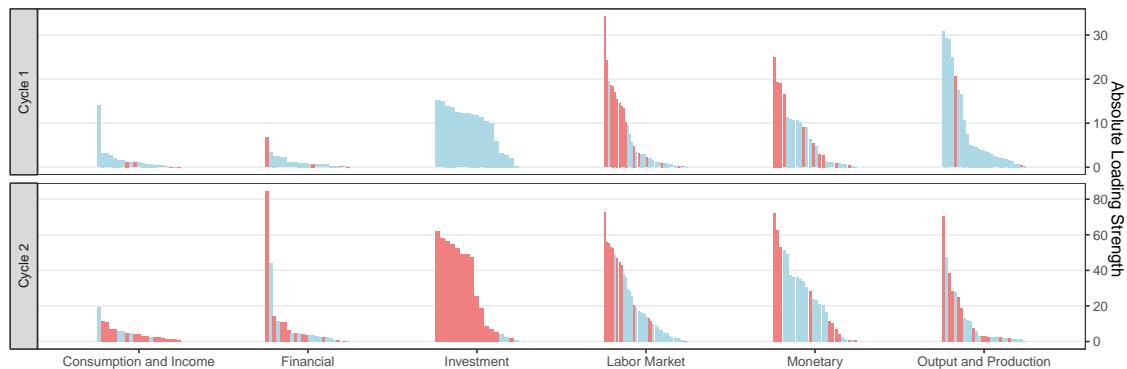


**Figure 12: Output Gaps from Multiple Cyclical Components.** *Bayesian information criterion in parenthesis.*

#### Decomposition with 1 Trend and 2 Cycles

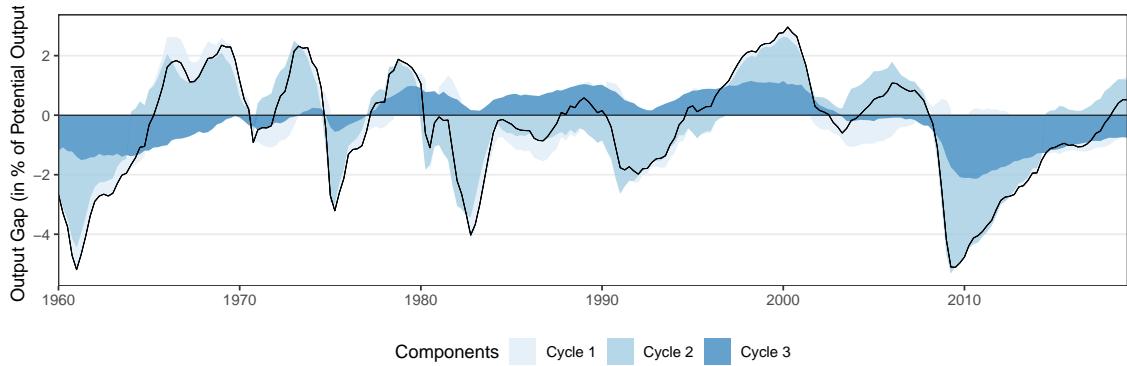


**Figure 13: Output Gap Decomposition with 2 Cycles**

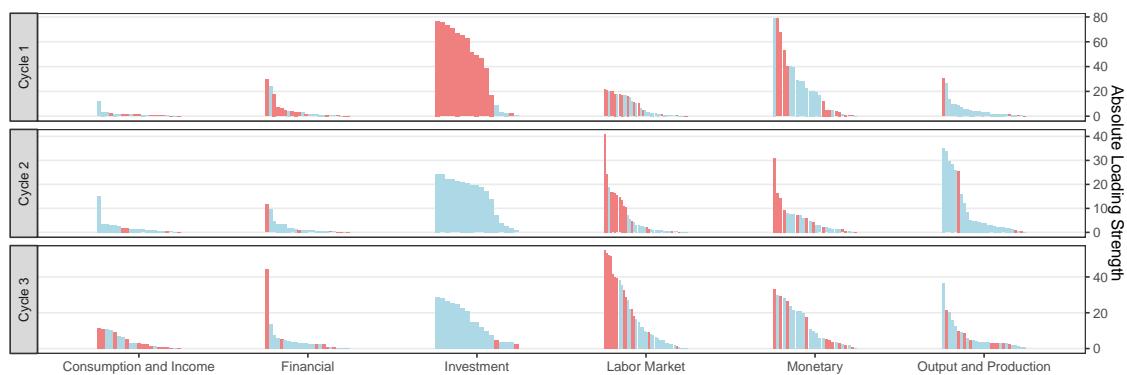


**Figure 14: Factor Loadings with 2 Cycles**

#### Decomposition with 1 Trend and 3 Cycles

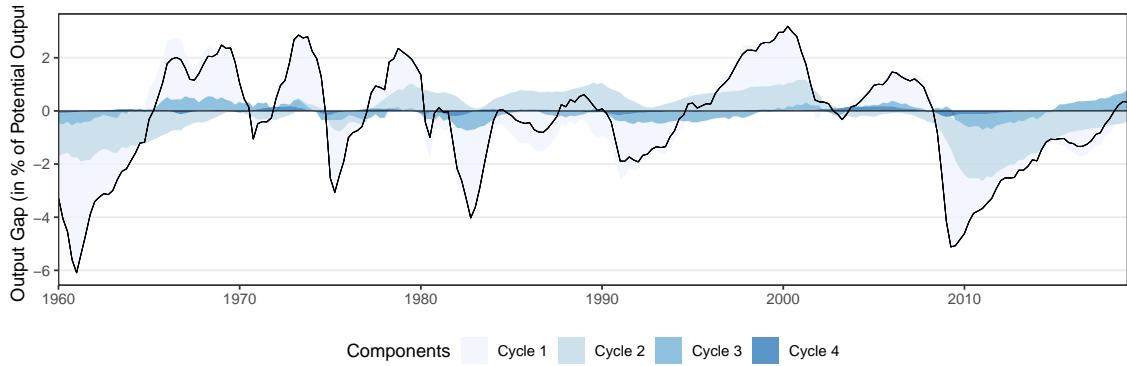


**Figure 15: Output Gap Decomposition with 3 Cycles**

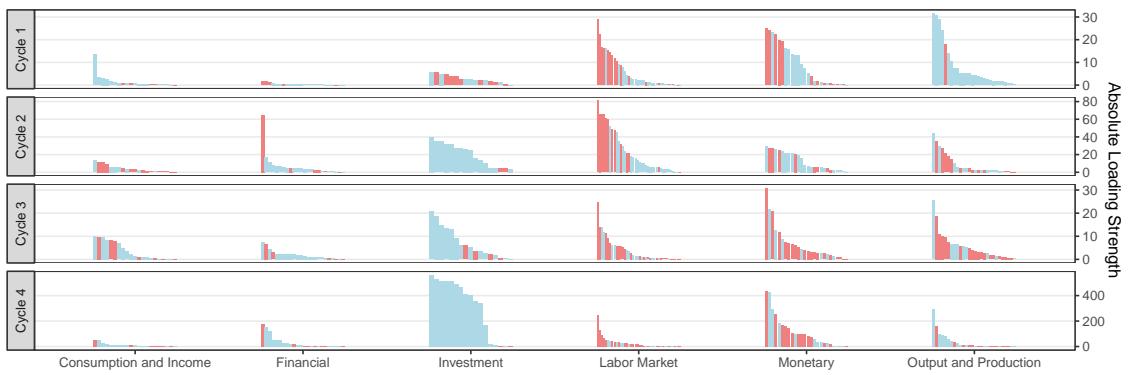


**Figure 16: Factor Loadings with 3 Cycles**

#### Decomposition with 1 Trend and 4 Cycles



**Figure 17: Output Gap Decomposition with 4 Cycles**



**Figure 18: Factor Loadings with 4 Cycles**

## A.5 Data

**Table 3. Data Description**

Description	Key	Order
<b>Consumption and Income</b>		
A823RL1Q225SBEA	Real Government Consumption Expenditures and Gross Investment: Federal	0
AMDMNOx	Average Hourly Earnings of Production and Nonsupervisory Employees: Construction	1
AMDMUOx	Average Hourly Earnings of Production and Nonsupervisory Employees: Manufacturing	1
BUSINVx	Total Business Inventories	1
CES0600000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing	1
CMRMTSPLx	Real Manufacturing and Trade Industries Sales	1
COMPRNFB	Nonfarm Business Sector: Real Compensation Per Hour	1
DPIC96	Real Disposable Personal Income	1
FGRECEPTx	Federal Government Current Receipts	1
GCEC1	Real Government Consumption Expenditures and Gross Investment	1
OPHNFB	Nonfarm Business Sector: Real Output Per Hour of All Persons	1
OPHPBS	Business Sector: Real Output Per Hour of All Persons	1
PCDGx	Personal Consumption Expenditures: Durable Goods	1
PCECC96	Real Personal Consumption Expenditures	1
PCESVx	Personal Consumption Expenditures: Services	1
PCNDx	Personal Consumption Expenditures: Nondurable Goods	1
RCPHBS	Business Sector: Real Compensation Per Hour	1
RSAFSx	Advance Retail Sales: Retail and Food Services, Total	1
SLCEx	State and Local Consumption Expenditures & Gross Investment	1
ULCBS	Business Sector: Unit Labor Cost	1
ULCNFB	Nonfarm Business Sector: Unit Labor Cost	1
UNLPNBS	Nonfarm Business Sector: Unit Nonlabor Payments	1
<b>Financial</b>		
BUSLOANSx	Commercial and Industrial Loans, All Commercial Banks	1
CNCFx	Corporate Net Cash Flow with IVA	1
CONSUMERx	Consumer Loans at All Commercial Banks	1
HNOREMQ027Sx	Households and nonprofit organizations; real estate at market value, Level	1
LIABPIx	Nonfinancial corporate business; debt securities; liability, Level	1
NIKKEI225	Nikkei Stock Average, Nikkei 225	1
NONREVSLx	Total Nonrevolving Credit Owned and Securitized, Outstanding	1
NWPIx	Net Worth as a Percentage of Disposable Personal Income	1
REALLNx	Real Estate Loans, All Commercial Banks	1
S.P..indust	S&P's Common Stock Price Index: Industrials	1
S.P.500	S&P's Common Stock Price Index: Composite	1
S.P.div.yield	S&P's Composite Common Stock: Dividend Yield	1
S.P.PE.ratio	PE ratio S&P's Composite Common Stock: Price-Earnings Ratio S&P	0
TABSHNOx	Households and Nonprofit Organizations; Total Assets, Level	1
TABSNNBx	Nonfinancial noncorporate business; total assets, Level	1

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TFAABSHNOx	Households and Nonprofit Organizations; Total Financial Assets, Level	1
TLBSHNOx	Households and Nonprofit Organizations; Total Liabilities, Level	1
TLBSNNBx	Nonfinancial noncorporate business; total liabilities, Level	1
TLBSNNCBx	Nonfinancial Corporate Business; Total Liabilities, Level	1
TNWBSHNOx	Households and Nonprofit Organizations; Net Worth, Level	1
TNWBSNNBx	Nonfinancial Noncorporate Business; Proprietors' Equity in Noncorporate Business (Net Worth), Level	1
TNWMVBSNNCBx	Nonfinancial Corporate Business; Net Worth, Level	1
TOTALSLx	Total Consumer Credit Owned and Securitized, Outstanding	1
TTAABSNNCBx	Nonfinancial Corporate Business; Nonfinancial Assets, Level	1
<hr/>		
<b>Investment</b>		
FPIx	Fixed Private Investment	1
GPDIC1	Real Gross Private Domestic Investment	1
HOUST	Housing Starts: Total: New Privately Owned Housing Units Started	1
HOUST5F	Privately Owned Housing Starts: 5-Unit Structures or More	1
HOUSTMW	Housing Starts in Midwest Census Region	1
HOUSTNE	Housing Starts in Northeast Census Region	1
HOUSTS	Housing Starts in South Census Region	1
HOUSTW	Housing Starts in West Census Region	1
INVEST	Gross Private Domestic Investment	1
PERMIT	New Private Housing Units Authorized by Building Permits	1
PERMITMW	New Private Housing Units Authorized by Building Permits in the Midwest Census Region	1
PERMITNE	New Private Housing Units Authorized by Building Permits in the Northeast Census Region	1
PERMITS	New Private Housing Units Authorized by Building Permits in the South Census Region	1
PERMITW	New Private Housing Units Authorized by Building Permits in the West Census Region	1
PNFIx	Private Nonresidential Fixed Investment	1
PRFIx	Private Residential Fixed Investment	1
Y033RC1Q027SBEAx	Gross Private Domestic Investment: Fixed Investment: Nonresidential: Equipment	1
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<b>Labor Market</b>		
CE16OV	Civilian Employment Level	1
CES9091000001	All Employees: Government: Federal	1
CES9092000001	All Employees: Government: State Government	1
CES9093000001	All Employees: Government: Local Government	1
CIVPART	Civilian Labor Force Participation Rate	1
CLAIMSx	4-Week Moving Average of Initial Claims	0
DMANEMP	All Employees: Durable Goods	1
HOABS	Business Sector: Hours of All Persons	1
HOANBS	Nonfarm Business Sector: Hours of All Persons	1
HWIURATIOx	Nonfarm Vacancies to Unemployment	0
LNS12032194	Employment Level: Part-Time for Economic Reasons, All Industries	1
LNS14000012	Unemployment Rate: 16 to 19 years	0
LNS14000025	Unemployment Rate: 20 years and over, Men	0

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LNS14000026	Unemployment Rate: 20 years and over, Women	0
MANEMP	All Employees: Manufacturing	1
NDMANEMP	All Employees: Nondurable goods	1
PAYEMS	All Employees: Total Nonfarm Payrolls	1
SRVPRD	All Employees: Service-Providing Industries	1
UEMP15T26	Number of Civilians Unemployed for 15 to 26 Weeks	1
UEMP27OV	Number of Civilians Unemployed for 27 Weeks and Over	1
UEMP5TO14	Number of Civilians Unemployed for 5 to 14 Weeks	1
UEMPLT5	Number of Civilians Unemployed for Less Than 5 Weeks	1
UEMPMEAN	Average (Mean) Duration of Unemployment	0
UNRATE	Civilian Unemployment Rate	0
UNRATELTx	Civilian Unemployment Rate Long Term	0
UNRATESTx	Civilian Unemployment Rate Short Term	0
USCONS	All Employees: Construction	1
USEHS	All Employees: Education and Health Services	1
USFIRE	All Employees: Financial Activities	1
USGOOD	All Employees: Goods-Producing Industries	1
USGOVT	All Employees: Government	1
USINFO	All Employees: Information Services	1
USLAH	All Employees: Leisure and Hospitality	1
USMINE	All Employees: Mining and logging	1
USPBS	All Employees: Professional and Business Services	1
USPRIV	All Employees: Total Private Industries	1
USSERV	All Employees: Other Services	1
USTPU	All Employees: Trade, Transportation and Utilities	1
USTRADE	All Employees: Retail Trade	1
USWTRADE	All Employees: Wholesale Trade	1
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<b>Monetary</b>		
AAA	Moody's Seasoned Aaa Corporate Bond Yield	0
AAAFFM	Moody's Seasoned Aaa Corporate Bond Minus Federal Funds Rate	0
BAA	Moody's Seasoned Baa Corporate Bond Yield	0
BAA10YM	Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity	0
CP3M	3-Month Commercial Paper Rate	0
CPF3MTB3Mx	Spread CP3M TB3M	0
DTCOLNVHFNM	Consumer Motor Vehicle Loans Owned by Finance Companies, Outstanding	1
DTCTHFNM	Total Consumer Loans and Leases Owned and Securitized by Finance Companies, Outstanding	1
EXCAUSx	Canada / U.S. Foreign Exchange Rate	1
EXJPUSx	Japan / U.S. Foreign Exchange Rate	1
EXSZUSx	Switzerland / U.S. Foreign Exchange Rate	1
EXUSUKx	U.S. / U.K. Foreign Exchange Rate	1
FEDFUNDS	Effective Federal Funds Rate	0
GS1	1-Year Treasury Constant Maturity Rate	0
GS10	10-Year Treasury Constant Maturity Rate	0
GS10TB3Mx	Yield Curve 10Y 3M	0
GS1TB3Mx	Yield Curve 1Y 3M	0

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GS5	5-Year Treasury Constant Maturity Rate	0
M1REAL	Real M1 Money Stock	1
M2REAL	Real M2 Money Stock	1
MZMREAL	Real MZM Money Stock	1
T5YFFM	5-Year Treasury Constant Maturity Minus Federal Funds Rate	0
TB3MS	3-Month Treasury Bill: Secondary Market Rate	0
TB3SMFFM	3-Month Treasury Bill Minus Federal Funds Rate	0
TB6M3Mx	Yield Curve 6M 3M	0
TB6MS	6-Month Treasury Bill: Secondary Market Rate	0
<b>Output and Production</b>		
A014RE1Q156NBEA	Shares of gross domestic product: Gross private domestic investment: Change in private inventories	0
AWHMAN	Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing	1
AWOTMAN	Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing	1
B020RE1Q156NBEA	Shares of gross domestic product: Exports of goods and services	0
B021RE1Q156NBEA	Shares of gross domestic product: Imports of goods and services	0
CES0600000007	Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing	0
CUMFNS	Capacity Utilization: Manufacturing	0
EXPGSC1	Real Exports of Goods and Services	1
GDPC1	Real Gross Domestic Product	1
HWIx	Weekly Hours Worked	1
IMPGSC1	Real Exports of Goods and Services	1
INDPRO	Industrial Production Index	1
IPB51110SQ	Industrial Production: Durable Goods: Automotive products	1
IPB51220SQ	Industrial Production: Consumer energy products	1
IPB51222S	Industrial Production: Residential utilities	1
IPBUSEQ	Industrial Production: Business Equipment	1
IPCONGD	Industrial Production: Consumer Goods	1
IPDCONGD	Industrial Production: Durable Consumer Goods	1
IPDMAT	Industrial Production: Durable Materials	1
IPFINAL	Industrial Production: Final Products (Market Group)	1
IPFUELS	Industrial Production: Fuels	1
IPMANSICS	Industrial Production: Manufacturing (SIC)	1
IPMAT	Industrial Production: Materials	1
IPNCONGD	Industrial Production: Nondurable Consumer Goods	1
IPNMAT	Industrial Production: Nondurable Materials	1
ISRATIOx	Total Business: Inventories to Sales Ratio	0
OUTBS	Business Sector: Real Output	1
OUTNFB	Nonfarm Business Sector: Real Output	1
UMCSENTx	University of Michigan: Consumer Sentiment	0

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