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## Persuasion with costly precision

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**Septembre 2017**

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# Persuasion with costly precision\*

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## Abstract

We study a sender's optimal choice of precision when he selects from a class of random signals to persuade a receiver to accept his request, with higher costs associated with higher precision. The receiver observes both the precision and a random realization of the signal. We characterize plausible equilibria under *discretion*, where the sender privately observes favourable or unfavourable information before choosing precision. In addition, we characterize the optimal choice of precision under *commitment*, where he chooses it before observing his private information. We then analyze optimal persuasion by comparing the sender's payoff from discretion and that from commitment. We demonstrate that as the prior becomes more pessimistic, the sender is more likely to prefer discretion to commitment.

**Keywords:** persuasion, costly signals, precision, signalling, D1 equilibrium, strategic communication, optimal information provision.

**JEL codes:** D72, D82.

## 1 Introduction

Consider an election in which a political challenger hopes to unseat an incumbent politician. Let us focus on the quality of the candidate, over which the voters have general agreement. In a general election, the candidate's quality could mean the competence of the politician in terms of making and implementing policy. In a primary election, the candidate's quality could mean the ability of the politician to win a general election and advance his party's agenda. While the incumbent's quality is typically well known to the public, the quality of the challenger is *ex ante* unknown to voters, and may be the challenger's private information. The challenger may spend effort and money on activities to increase his exposure—visiting his district, making media appearances, buying campaign advertisements, etc. Higher exposure gives people a better idea about the challenger's quality but is more costly. Moreover, the candidate cannot perfectly control the ramifications of his speeches or interviews, or the information that voters actually internalize. Would a high level of exposure signal a higher quality? If he had the choice, would the challenger prefer to commit to a given level of exposure before learning his quality?

Alternatively, consider a newly elected political leader, who has had no previous office-holding experience. He, as well as the electorate, has an initial estimate but not precise knowledge of his governing competence. He has to make a decision about the level of transparency for his administration, with the goal of building his reputation and securing reelection.<sup>1</sup> Being more transparent gives people a better idea about the leader's competence, but it is more costly in terms of time (meeting with the press and constituents) and resources (fulfilling information requests) devoted to providing access to information.)

To study economic or political situations like the ones above, in this paper, we build a theoretical model in which a sender (“he”) attempts to persuade a receiver (“she”) to accept a request. We assume that there are two types of the sender, high

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<sup>1</sup>In a recent example, Justin Trudeau, the newly elected Canadian Prime Minister, pushed for more transparency in his new administration by lowering the fees for access to information requests and promised to make appropriate changes to Access to Information Act (even though as of the writing of this article, has not done so). This is a departure from the practices of his predecessor, Stephen Harper. See [5].

or low. Throughout the paper, we also refer to the sender’s type as his quality. Under full information, the receiver would accept the high type and reject the low type. To capture the essence of the strategic interactions, we assume that the sender may select from a class of signals that differ in their *precision*, namely, how well the signal realization reflects the sender’s type. We assume that it is more costly to send a more precise signal. Before making a decision, the receiver observes both the sender’s signal choice and the signal realization.

We study two versions of the game – *discretion*, in which the sender chooses precision *after* learning his type, and *commitment*,<sup>2</sup> in which he does so *before* that. We label the former *persuasive signalling*, because as the sender possesses private information about his type, his choice of precision could itself *signal* his type, and in the meantime the signal realization serves to *persuade*, i.e., to provide relevant information to the receiver’s decision. We then consider *optimal persuasion*, in which we compare the sender’s expected payoffs under discretion and commitment.

In our analysis, we adopt the D1 refinement of perfect Bayesian equilibria in the persuasive signalling game, i.e., the discretion case. When the common prior about the sender’s type is pessimistic or neutral, that is, when it is commonly known that ex ante the receiver prefers rejection of the sender or is indifferent between acceptance and rejection, there is a unique plausible equilibrium, which is semi-separating. In this equilibrium, the high type chooses a positive precision level and the low type mixes between zero precision and that chosen by the high type. Furthermore, the high type’s choice of precision level, the posterior about quality associated with that precision, and both types’ payoffs are uniquely determined, irrespective of the prior distribution of types. In particular, the posterior associated with the higher precision is optimistic, in that without observing the signal realization, the receiver would in

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<sup>2</sup>We do not allow the sender to commit to levels of precision contingent on his realized type. This is in contrast to the full commitment assumption of Kamenica and Gentzkow [26] and Gentzkow and Kamenica [18]. We believe our assumption of limited commitment is realistic in certain settings. For example, when laws and regulations regarding transparency are written, they cannot be made contingent on the performance of the administration. If a lobbying group commissions a truly independent study of the social impact of a policy, it usually cannot dictate how results will be released depending on his later knowledge of its true impact—because he will be inevitably tempted to choose the option that benefits his interests.

fact accept the sender. For all other combinations of precision and posterior, the high type can convince the receiver of his quality by either being marginally more or less precise. It follows that the expected payoffs of both types of the sender are fixed, regardless of the prior. This is reminiscent of the properties of the Riley [37] outcome in a standard Spence [40, 41] job market signalling model. We also show that in the optimistic case, there may exist multiple plausible equilibria, including one in which the sender always chooses zero precision.

Based on our equilibrium characterization of the discretion case, we investigate the optimal persuasion policy of the sender. When the common prior is optimistic, the sender is indifferent between commitment and discretion, providing no information in both cases, as it is the best plausible equilibrium for the sender under discretion. The more interesting comparisons come from the neutral and pessimistic cases. In both cases, the sender could prefer either commitment or discretion. However, fixing everything else, as the receiver’s payoff from rejection increases, the sender’s preference unambiguously moves from commitment to discretion. The key observation is that the sender’s expected payoff from discretion does not depend on the receiver’s default payoff from rejection, while that from commitment is decreasing in it.

Even though our setup is relatively stylized, our analysis and its conclusions have the potential of providing economic insight into a number of similar environments, which we discuss at the end of the paper.

**Related literature.** With our model, we aim at studying economic and political environments where information transmission by a possibly privately informed sender stems from both the means and the contents of communication. In this respect, our paper belongs to a recent literature that deals with similar contexts, in a variety of applications, including works by Li and Li [29], Gill and SgROI [20, 21], and Boleslavsky, Cotton, and Gurnani [4]. Li and Li [29] study two privately informed political candidates who can choose the accuracy of a costly public signal (campaign) about his own qualifications or his opponent’s. They interpret this choice as that between positive and negative campaigns. Gill and SgROI [20, 21] consider a privately-informed producer submitting to a test designed to provide public information about the type of his product, where he chooses the toughness of the test, or,



how likely the product is to pass the test. Boleslavsky, Cotton, and Gurnani [4] study a new-product producer’s timing decision on demonstrations to compete against an established product. In contrast to our model, the information consumers obtain through demonstrations are horizontal in nature, which reflects each consumer’s idiosyncratic preferences on the product’s characteristics.

Our model, as well as those cited above, can be viewed as an extension of Milgrom’s [30] persuasion game.<sup>3</sup> The difference between our model and the classic persuasion game is that provision of information is no longer costless but involves a cost that is increasing in the precision of information. If it were costless in our model, then “unravelling” –full revelation of information– would occur, as in Milgrom’s [30] setup.<sup>4</sup>

In a closely related paper, Perez-Richet [33] considers a setting that has a binary type space for the sender and a binary action space for the receiver, but signals are *costless* as assumed by Kamenica and Gentzkow [26] and the set of signals is potentially restricted. He focuses on pure strategy perfect Bayesian equilibria and shows that three different refinement concepts all select the high type optimal equilibrium among such equilibria.<sup>5</sup> In contrast, we identify the D1 equilibria, but also make a comparison between commitment to a *fixed* precision and discretion.<sup>6</sup>

Our paper is also related to the Bayesian persuasion model of Kamenica and Gentzkow [26], who assume signals to be costless and the sender can commit to which signal to choose from an unrestricted set, before learning any private information.

In addition to Perez-Richet [33], Alonso and C amara [1], Hedlund [23], and Rayo and Segal [36] also study Bayesian persuasion where the sender has private information. In particular, our result that discretion is better for the sender for pessimistic

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<sup>3</sup>See also Grossman [22] and Milgrom and Roberts [32]. Milgrom [31] provides a survey of the literature on persuasion games.

<sup>4</sup>Koessler and Renault [27] generalize the full-revelation result to multi-dimensional settings and Hedlund [24] to settings with costly information provision.

<sup>5</sup>In particular, this means that the high type would always be accepted if perfect revelation is available as a signal.

<sup>6</sup>In our model, if signals were costless, then commitment would always weakly dominate discretion, given that full revelation would occur under discretion, while full revelation would also be available to the sender under commitment.

priors under costly signals is in contrast to the result of Alonso and Cãmara [1], who observe that the sender cannot benefit from becoming privately informed if the sender can commit to any signal from an unrestricted set at no cost. Bayesian persuasion with costly signals are studied by Gentzkow and Kamenica [18], who identify assumptions under which the sender would benefit from persuasion even when signals are costly.<sup>7</sup>

Our analysis of the “persuasive signalling” game, where the sender chooses the precision of her signal under private information, is related to a few other papers in the signalling literature. Daley and Green [11] consider a signalling game where the receiver has an extra piece of information about the sender, which they call grades. Their setup is closely modelled after that of the classic Spence [40, 41] job market signalling model, in that the sender is paid a wage based on his expected productivity. Due to assumptions that are different from ours, they obtain a unique D1 equilibrium (pooling, semi-pooling, or separating) for all possible priors, while in ours, there could exist multiple D1 equilibria. In another related paper, Feltovich, Harbaugh, and To [14] analyse a model of signalling in which the receiver has access to additional noisy information about the sender, which is not controlled by the sender. They consider three types of senders and focus on the existence of counter-signalling equilibria under appropriate assumptions on the distributions of the extra piece of information.<sup>8</sup> Chung and Esö [10] introduce career concerns into the framework and also demonstrate that countersignalling occurs. Heufer [25] offers an information economic and evolutionary justification of social drinking, using a modified signalling game.

Finally, there are a number of recent papers of persuasion where the sender has private information, but uses different instruments to persuade the receiver, which include papers by Chakraborty and Harbaugh [6], Che, Dessein, and Kartik [7], and Perez-Richet and Prady [34].

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<sup>7</sup>Other theoretical works that are variations of Kamenica and Gentzkow’s [26] model include those of Kolotilin, Li, Mylovanov, and Zapechelnuk [28] (private information of the receiver), Taneva [42] and Wang [43] (multiple receivers), Gentzkow and Kamenica [19] (endogenous acquisition of information).

<sup>8</sup>The extension to three types and the analysis of counter-signalling equilibria in our “persuasive signalling” game is the object of analysis of a companion paper.

The rest of the paper is organized as follows: Section 2 introduces the model; Section 3 characterizes equilibria under discretion; Section 4 investigates the sender’s choice under commitment; Section 5 characterizes the sender’s optimal policy of persuasion; Section 6 concludes. The appendices contain proofs and omitted analytical details from the main text.

## 2 Model

A sender (“he”) wants to convince a receiver (“she”) to take a certain action. The sender has private information about a decision-relevant state, which we hereby refer to as the sender’s quality. As is standard in the persuasion literature, the receiver faces a binary action space. She chooses an action  $y$  from  $Y = \{0, 1\}$ , where action 1 is favoured by the sender and can be interpreted as the receiver’s acceptance of a request by the sender, and 0 her rejection of it. The quality of the sender,  $\theta$ , can be high or low. It is drawn from the set  $\Theta \equiv \{\theta_L, \theta_H\}$ , according to the distribution  $(1-p^0, p^0)$ , where  $p^0 \in (0, 1)$  is the probability of the high type. The receiver’s payoff is equal to the quality of the sender if he takes action 1, and to a default payoff  $\theta_M$  otherwise:

$$U^R(\theta, y) = y\theta + (1 - y)\theta_M,$$

where  $y \in Y = \{0, 1\}$ . We assume  $\theta_M \in (\theta_L, \theta_H)$ , which means that the receiver always rejects the low type and accepts the high type when the sender’s quality is known.

The sender chooses a normally distributed random signal with standard deviation  $\sigma \in M = \overline{\mathbb{R}}_+ \equiv \mathbb{R}_+ \cup \{+\infty\}$ :<sup>9</sup>

$$s(\theta, \sigma) = \theta + \sigma\varepsilon, \tag{1}$$

where  $\varepsilon$  is a random variable with the standard normal distribution  $N(0, 1)$ . By choosing a lower  $\sigma$ , the sender selects a higher precision for the random signal  $s(\theta, \sigma)$  with support  $S = \mathbb{R}$ , which is directly informative about the quality of the sender. Our choice of the normal signal structure allows us to isolate the precision component from the level component of the information transmitted.

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<sup>9</sup>Our analysis can be readily extended to the case in which  $M$  is a strict subset of  $\overline{\mathbb{R}}_+$ .

Throughout the paper, we will refer to the sender’s choice of  $\sigma$  as a “signal,” noting that it measures the standard deviation or, inversely, the precision,<sup>10</sup> of a stochastic distribution. The receiver makes a decision, after observing the sender’s signal choice,  $\sigma$ , and a realization of the signal,  $s$ .

The sender’s payoff is equal to the receiver’s action  $y$  minus the cost of the signal:

$$U^S(y, \sigma) = y - C(\sigma),$$

where  $C : M \rightarrow \mathbb{R}_+$  is the cost function. All aspects of the game are common knowledge except the sender’s quality.

Throughout the rest of the paper, we restrict our attention to the following cost function:

$$C(\sigma) = \gamma \frac{\theta_H - \theta_L}{\sigma} \equiv \gamma a, \tag{2}$$

where  $a$  as defined is a measure of the precision of the signal and introduced to simplify expressions, and  $\gamma > 0$  measures, up to scale, the marginal cost of precision.<sup>11</sup> Our assumption that a more precise signal is costlier is analogous to that of Hedlund [24], though he defines precision differently. We believe this assumption fits well with certain applications, for example, those that we mentioned in the introduction and those that we discuss in subsection 5.1.

We now turn to the *discretion* case, in which the sender privately learns his quality before making his choice of signal. We will investigate the *commitment* case in Section 4. We label the game in the discretion case as *persuasive signalling* because the sender may *signal* to the receiver his type through the way he *persuades* her.<sup>12</sup>

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<sup>10</sup>In statistics, “precision” typically refers to the inverse of the variance of the normal distribution (see, for example, DeGroot [12]), but we believe our use of it to refer to the inverse of the standard deviation does not create confusion.

<sup>11</sup> The full characterization of the set of Perfect Bayesian Equilibria, which is available upon request from the authors, requires only that the cost is nonincreasing in  $\sigma$ , i.e., nondecreasing in precision.

The specific cost function (2) is used only in the characterization of D1 equilibria.

<sup>12</sup>The feature of our model that the receiver observes both the precision of the distribution and a realization of this distribution is also present in the persuasion setting of Kamenica and Gentzkow [26], and Perez-Richet [33]. In the first, there is no information asymmetry between the sender and the receiver, so there is no potential for signalling. In the second, signalling is not costly.

Note that we assume no “single-crossing” condition *per se* on the cost of precision, as in standard signalling games.<sup>13</sup> However, as we show in Lemma 2, the two types derive different relative benefits from having a higher precision. In this sense, our model enables a form of endogenous “single-crossing” condition. This feature is common to Daley and Green [11] and is similar in spirit to results present in other works in the literature, for example, those of Sobel [38] and Fang [13].

## 2.1 Strategies and equilibrium concept

With a slight abuse of notation, a behavioural strategy for the sender is a function  $\mu^S : \Theta \rightarrow \Delta M$  such that  $\sum_{\sigma} \mu^S(\sigma|\theta) = 1$  for all  $\theta \in \Theta$ , where  $\mu^S(\sigma|\theta)$  is the probability that a sender of type  $\theta$  chooses signal  $\sigma$ . A behavioural strategy for the receiver is a function  $\mu^R : M \times S \rightarrow [0, 1]$ , where  $\mu^R(\sigma, s)$  is the probability that a receiver that observed signal  $\sigma$  and signal realization  $s$  takes action  $y = 1$ .

First, we consider the standard Perfect Bayesian Equilibrium.

**Definition 1.** A *Perfect Bayesian Equilibrium* of the persuasive signalling game consists of sender’s and receiver’s behavioural strategies,  $\mu^S$  and  $\mu^R$  respectively, and the system of posterior beliefs  $p(\theta|\sigma, s)$ , such that

- (i) For all  $\sigma \in M$  and  $s \in \mathbb{R}$ ,  $\mu^R$  maximizes the receiver’s expected payoff given beliefs  $p(\cdot|\sigma, s)$ ;
- (ii) For all  $\theta \in \Theta$  and  $\sigma \in M$ ,  $\mu^S(\sigma|\theta) > 0$  only if  $\sigma$  maximizes type- $\theta$  sender’s expected payoff, given the receiver’s strategy  $\mu^R$ ;
- (iii) The receiver’s posterior beliefs are computed using Bayes rule whenever applicable and are consistent with the sender’s behavioural strategy:<sup>14</sup>

$$p(\theta|\sigma, s) = \frac{p^0(\theta)\mu^R(\sigma|\theta)f(s|\theta, \sigma)}{f(s, \sigma)} = \frac{p^0(\theta)\mu^R(\sigma|\theta)f(s|\theta, \sigma)}{\sum_{\theta' \in \Theta} p^0(\theta')\mu^R(\sigma|\theta')f(s|\theta', \sigma)},$$

where  $f$  denotes the probability density functions of the signal.

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<sup>13</sup>See Sobel [39] for a definition. Feltovich, Harbaugh, and To [14] also assume single crossing on the cost of the signal.

<sup>14</sup>Since the joint distribution of  $\sigma$  and  $s$  is hybrid, the Bayes’ rule has to be applied by using the concept of “regular conditional probability distribution.”

Similar to standard signalling games, our persuasive signalling game allows for many perfect Bayesian equilibria. We adopt the solution concept of *D1 equilibria*, which places restrictions on the receiver’s beliefs when the sender chooses an action that is outside of the support of the sender’s equilibrium mixed strategies.<sup>15</sup> Since the sender can only control the standard deviation of the signal,  $\sigma$ , but not its actual realization, we apply the D1 refinement on interim posterior beliefs, i.e., the receiver’s beliefs about the sender’s type based only on his choice of  $\sigma$ , but not on the signal realization.

Consider a deviation to a signal,  $\tilde{\sigma}$ , which is chosen with zero probability in equilibrium. Let  $D(\theta, T, \tilde{\sigma})$  be the set of mixed-strategy best responses by the receiver to action  $\tilde{\sigma}$  and beliefs concentrated on  $T \subset \Theta$ , so that a sender of type  $\theta \in \Theta$  strictly prefers  $\tilde{\sigma}$  to his equilibrium signal. Let  $D^0(\theta, T, \tilde{\sigma})$  be the set of mixed-strategy best responses such that  $\theta$  is indifferent between his equilibrium strategy and  $\sigma$ .

**Definition 2.** An equilibrium satisfies *D1* if for any signal  $\tilde{\sigma}$  not chosen in equilibrium, whenever

$$D(\theta, \Theta, \tilde{\sigma}) \cup D^0(\theta, \Theta, \tilde{\sigma}) \subsetneq D(\theta', \Theta, \tilde{\sigma}) \text{ for some } \theta' \in \Theta, \quad (3)$$

the receiver assigns a zero posterior probability to  $\theta$ , that is  $p(\theta|\tilde{\sigma}) = 0$ .

For intuition, the D1 refinement can be understood as follows. If there are more circumstances under which one sender type finds it profitable to deviate to an off-equilibrium action than another, then the receiver should assign zero probability to the latter in her belief when she sees the action taken.

### 3 Equilibrium under discretion

Hereafter, we use the single word “equilibrium” to refer to “perfect Bayesian equilibrium,” and the phrase “D1 equilibrium” to refer to a perfect Bayesian equilibrium that satisfies the D1 criterion.

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<sup>15</sup>See Fudenberg and Tirole [17] and Sobel [39] for a discussion of the refinement. The papers that contain the initial idea are those by Banks and Sobel [3], Cho and Kreps [8], and Cho and Sobel [9]. Recent applied work (see, for example, Ash, Morelli, and Van Weelden [2]; Fox and Van Weelden [15]; Fu and Li [16]; Rayo [35]; Daley and Green [11] ) has made wide use of the D1 refinement and its variations.

First, we solve the receiver's optimization problem. Given her belief on the sender's type,  $p(\cdot|\sigma, s)$ , which is conditional on the sender's signal  $\sigma$  and its realization  $s$ , she must choose the probability of accepting the sender (action 1) in order to maximize her expected utility

$$\max_{\mu^R \in [0,1]} EU^R = \max_{\mu^R \in [0,1]} \mu^R E_p[\theta|\sigma, s] + (1 - \mu^R)\theta_M,$$

where the expectation is taken with respect to the receiver's posterior about the sender's quality conditional on  $(\sigma, s)$ .

The solution to the receiver's problem is straightforward—it involves the receiver's accepting the sender ( $\mu^R(\sigma, s) = 1$ ) with probability one if his expected quality is better than the default option, i.e., if  $E_p[\theta|\sigma, s] > \theta_M$  and rejecting him with probability one if  $E_p[\theta|\sigma, s] < \theta_M$ . In case of indifference, any  $\mu^R(\sigma, s) \in [0, 1]$  constitutes a best-response. We assume that in this case the receiver accepts the sender with probability 1/2.

Let  $\pi_i(\sigma, p)$  be the probability that the receiver accepts the sender, as perceived by a sender of type  $i \in \{H, L\}$  who chooses signal  $\sigma$ , anticipating an interim belief by the receiver  $p$ . The problem for the sender is to choose signal(s)  $\sigma$  in order to maximize his expected payoff. That is, given the function  $\pi$ , a signal  $\sigma$  is chosen with positive probability if and only if:

$$\sigma \in \arg \max_{\sigma' \in M} U^S(\theta, \sigma') = \arg \max_{\sigma' \in M} \pi_i(\sigma', p) - C(\sigma') = \arg \max_{\sigma' \in M} E[\mu^R(\sigma', s)] - C(\sigma').$$

We assert in the following lemma that for any signal, the receiver's decision rule must have a cutoff property, wherein the receiver accepts the sender's request if and only if the signal exceeds a cutoff. This encompasses the trivial cases where the receiver always or never accepts the sender's request, where the cutoff is set to minus or plus infinity respectively. With a slight abuse of notation, we denote by  $p$  the receiver's interim posterior about the probability that the sender is of high type, given signal  $\sigma$ .

**Lemma 1.** *For any  $\sigma \in M$  and associated interim posterior  $p$ ,*

1. *there exists a cutoff value  $\bar{s} \in \bar{\mathbb{R}} \equiv \mathbb{R} \cup \{-\infty, +\infty\}$ , such that the receiver accepts the sender's request if and only if  $s > \bar{s}$ ;*

2.  $\bar{s}$  is decreasing in the interim posterior,  $p$ .

The proof of the above Lemma can be found in the appendix, but the key observation is that a higher signal realization is associated with a better posterior about the sender's quality, which gives the cutoff property of the receiver's optimal decision. In our particular context, the definition of  $\bar{s}$  is given by the following equation:

$$\frac{\theta_H p \phi\left(\frac{\bar{s}-\theta_H}{\sigma}\right) + \theta_L (1-p) \phi\left(\frac{\bar{s}-\theta_L}{\sigma}\right)}{p \phi\left(\frac{\bar{s}-\theta_H}{\sigma}\right) + (1-p) \phi\left(\frac{\bar{s}-\theta_L}{\sigma}\right)} = \theta_M, \quad (4)$$

which we may, using more compact notations, rearrange into

$$p(\theta_H - \theta_M)\phi(x) + (1-p)(\theta_L - \theta_M)\phi(x+a) = 0, \quad (5)$$

where

$$x = \frac{\bar{s} - \theta_H}{\sigma} \text{ and } a = \frac{\theta_H - \theta_L}{\sigma},$$

where  $x$  is the high type's cutoff point when normalized into standard normal distribution and  $a$ , as defined in (2), equals the difference between the normalized cutoff points of the high type and the low type. For compactness of notation, we frequently use notations like  $\bar{s}'$ ,  $x'$ , and  $a'$  defined for an alternative signal  $\sigma'$  and/or interim posterior  $p'$ .

The signal realization threshold  $\bar{s}$  is uniquely determined given signal  $\sigma$  and interim posterior  $p$ . The probability that a sender of type  $i$  is accepted by the receiver is therefore the probability type  $i$  generates a signal realization above  $\bar{s}$ . Hence, the acceptance probability function  $\pi_i : M \times \Delta\Theta \rightarrow [0, 1]$ ,  $i = H, L$  can be written

$$\begin{aligned} \pi_i(\sigma, p) &= Pr[s(\theta_i, \sigma) \geq \bar{s}(\sigma, p)], \\ &= 1 - \Phi\left(\frac{\bar{s}(\sigma, p) - \theta_i}{\sigma}\right). \end{aligned} \quad (6)$$

Clearly,  $\pi_i(\sigma, p)$  is non-decreasing in  $p$  and  $\pi_H(\sigma, p) > \pi_L(\sigma, p)$  for all  $\sigma \in (0, \infty)$  and  $p \in (0, 1)$ , that is, when the high type and low type choose the same level of



positive precision, the probability of the high type being accepted is strictly higher. It is convenient to write

$$\begin{aligned}\pi_H(\sigma, p) &= 1 - \Phi(x); \\ \pi_L(\sigma, p) &= 1 - \Phi(x + a).\end{aligned}\tag{7}$$

### 3.1 Types of D1 equilibria

In our setup, where the cost of precision does not depend on the type of the sender, there exists a unique separating (perfect Bayesian) equilibrium, yet it fails D1.<sup>16</sup> To see this, note that in a separating equilibria the probabilities of acceptance for types H and L are respectively 1 and 0. To satisfy incentive conditions, type L must choose  $\sigma_L = +\infty$  and type H must choose  $\sigma_H$  such that  $C(\sigma_H) = 1$ , so that both types get a payoff of 0. Now, consider any off-equilibrium signal choice  $\sigma > \sigma_H$ . For any best response  $\bar{s}$  of the receiver, type H is always more likely to benefit from deviation to  $\sigma$ . According to the D1 restriction on off-equilibrium beliefs, the receiver must then assign probability 1 to type H being the deviating type, who, given this belief, would then want to deviate. Therefore, the unique separating equilibrium in our model fails D1, which implies that a D1 equilibrium, if any, must involve some degree of pooling.

This result is in sharp contrast to a standard Spence [40, 41] job market signalling model. If we consider its variation closest to our model, without the single-crossing condition, then there exists a unique separating equilibrium where both types are indifferent between choosing high and low education. This equilibrium survives D1 because types H and L have the same preferences. By the same reasoning, any pooling equilibrium also satisfies D1. In our model, however, due to the additional signal realization the receiver observes, the D1 criterion eliminates separating equilibria, as well as many pooling equilibria, as we later show.

We first investigate when pooling on a particular level of precision, i.e., both the high and the low type choosing it with a positive probability, is possible in equilibrium. This includes pure pooling, where both types choose one particular

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<sup>16</sup>See Appendix C for the formal statement and proof of this result. Similar results have also been found by Daley and Green [11].

level of precision with probability one; or *semi-pooling*, where, in addition to both types pooling on a particular precision, at least one of the two types mixes between different levels of precision.

To facilitate discussion, it is helpful to define three categories of interim posterior beliefs,  $p$ , which will be used throughout the rest of the paper. Let  $E_p(\theta) = p\theta_H + (1-p)\theta_L$  be the sender's expected quality associated with belief  $p$ . We denote by  $\bar{p}$  the belief  $p$  that makes it equal to the default quality  $\theta_M$ , i.e.,  $E_{\bar{p}}(\theta) = \theta_M$ , which is equivalent to

$$\bar{p}(\theta_H - \theta_M) + (1 - \bar{p})(\theta_L - \theta_M) = 0. \quad (8)$$

If we consider the “weighted” relative likelihood of the low type, or the degree of pessimism

$$\lambda \equiv \frac{(1-p)(\theta_M - \theta_L)}{p(\theta_H - \theta_M)}, \quad (9)$$

then  $p = \bar{p}$  is equivalent to

$$\lambda = 1.$$

The condition for the receiver's optimal decision, (5), may therefore be rewritten as:

$$\phi(x) = \lambda\phi(x + a). \quad (10)$$

Note that (10) implies that when  $\lambda = 1$ ,  $x = -(x + a)$ .

**Definition 3.** We say that the receiver's belief  $p$  is:

1. *optimistic* if  $p > \bar{p}$  ( $\lambda < 1$ );
2. *neutral* if  $p = \bar{p}$  ( $\lambda = 1$ );
3. *pessimistic* if  $p < \bar{p}$  ( $\lambda > 1$ ).

In other words, without any additional information, in the pessimistic case, the receiver would have rejected the sender, while in the optimistic case, the receiver would have accepted the sender.

Figures 1, 2, and 3 illustrate the probabilities of acceptance for the two types and the cost function, in the three cases. Note that there is a symmetry between the

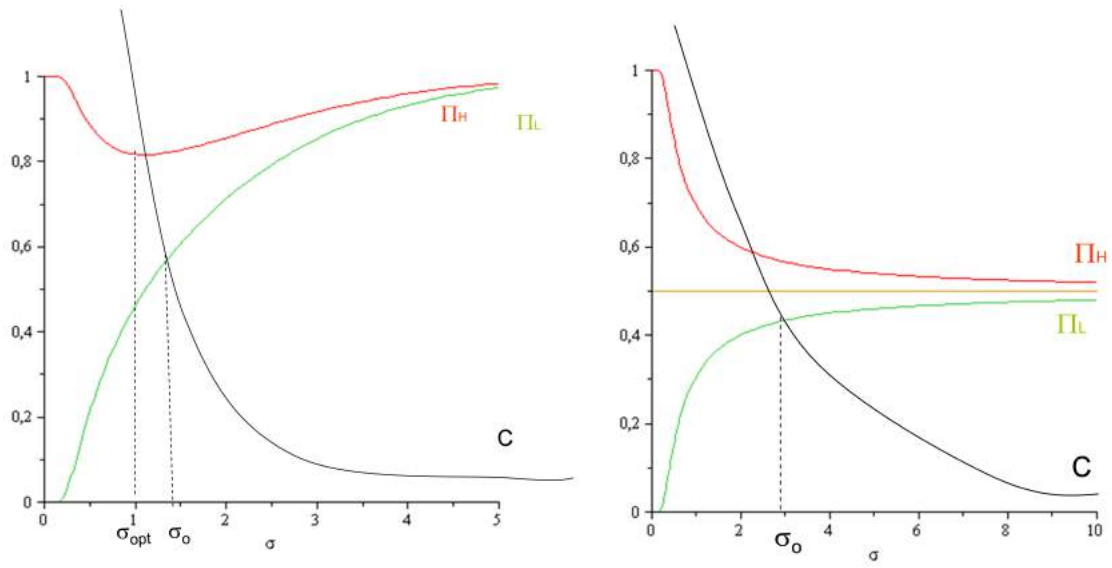


Figure 1: Sender's payoffs—optimistic case. Figure 2: Sender's payoffs—neutral case.

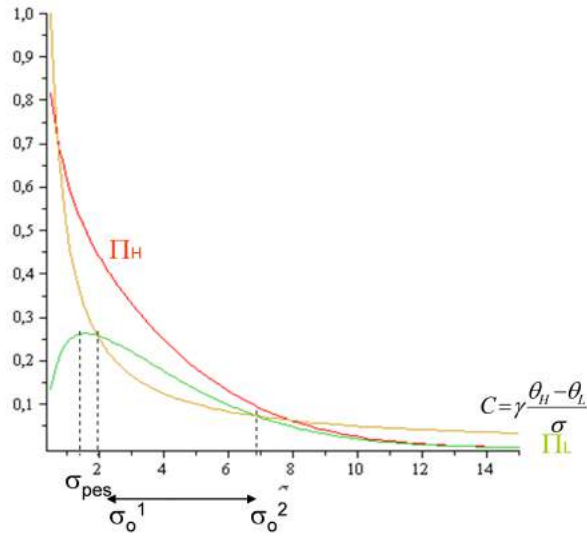


Figure 3: Sender's payoffs—pessimistic case.

optimistic case and the pessimistic case, in that the probability of acceptance for the high type in the optimistic case behaves exactly as the probability of rejection for the low type in the pessimistic case.

Two important properties of the probabilities of acceptance will be used in the proofs and are therefore summarized in the following Lemma.

**Lemma 2.** *The difference between the high and low type probability of acceptance  $\pi_H(\sigma, p) - \pi_L(\sigma, p)$  :*

1. *is decreasing in  $\sigma$  (increasing in  $a$ ) for any  $p$ ;*
2. *is increasing in  $p$  on  $[0, \bar{p}]$  and decreasing in  $p$  on  $[\bar{p}, 1]$ , and therefore maximized at  $\bar{p}$  for any  $\sigma$ , with its maximum value being  $2\Phi[(\theta_H - \theta_L)/(2\sigma)] - 1$ , or  $2\Phi(a/2) - 1$ , which is decreasing in  $\sigma$ .*

Part 1 of the lemma states that the payoff differential between the high type and the low type is increasing in the precision of the signal on which they pool. Part 2 states that their payoff differential increases when the receiver's interim posterior moves towards neutrality. In essence, this means that signals are more useful for differentiating the two types when they are more precise and when the receiver's belief is closer to being neutral.

Note that in equilibrium, a signal  $\sigma$  and interim posterior  $p$  can be part of a pooling or semi-pooling equilibrium only if the low type's associated payoff satisfies

$$\pi_L(\sigma, p) - C(\sigma) \geq 0,$$

which in turn also guarantees that the high type's payoff is non-negative.

In the neutral and optimistic cases, we have that  $\pi_L(\sigma, p) - C(\sigma) \geq 0$  for all  $\sigma \geq \sigma_0$ , where  $\sigma_0$  is the  $\sigma$  (see Figures 1 and 2) that satisfies

$$C(\sigma_0) = \pi_L(\sigma_0, p). \tag{11}$$

In the pessimistic case, the two curves  $\pi_L(\sigma, p)$  and  $C(\sigma)$  do not necessarily intersect. In addition, for general cost functions, when  $\sigma$  goes to infinity the relation between the probability of acceptance and the cost function is not clear. However,

for any cost function of the type in (2), in the pessimistic case, we can show that whenever  $\pi_L(\sigma, p)$  and  $C(\sigma)$  intersect, they intersect twice.<sup>17</sup> Let  $\sigma_0^1$  and  $\sigma_0^2$  denote the smaller and the larger of these two intersections. We have that  $\pi_L(\sigma, p) - C(\sigma) \geq 0$  if and only if  $\sigma \in [\sigma_0^1, \sigma_0^2]$ .<sup>18</sup>

Among semi-pooling equilibria, we concentrate on *semi-separating equilibria*. This is without loss of generality, as we will show that these are the only types of semi-pooling equilibria satisfying the D1 refinement.

**Definition 4.** A semi-pooling equilibrium of the Persuasive Signalling game is said to be *semi-separating* if at least one type chooses with a positive probability a signal that is not chosen by any other type.

It is straightforward to demonstrate semi-separating equilibria always exist. In particular, let the low type mix between  $+\infty$  and a  $\sigma < +\infty$ , such that  $\pi_L(\sigma, p) - C(\sigma) = 0$ . Note that as long as the low type chooses  $+\infty$  with a high enough probability, the interim posterior  $p$  associated with signal  $\sigma < +\infty$  is optimistic, and therefore there exists  $\sigma = \sigma_0(p)$  as defined by (11) so as to make the low type indifferent.

As is standard in signalling models, to best support an equilibrium, beliefs that a deviating sender is a low type may be set to one. However, such beliefs are not always plausible in the D1 sense. We develop a series of intermediate results that will lead to our main propositions on D1 equilibria. We begin by introducing some notations that are related to potential deviations from an equilibrium action. Let  $\sigma \in (0, +\infty)$  be a signal chosen by both types in equilibrium, with associated interim posterior  $p \in (0, 1)$ , and  $\sigma'$  be one chosen by neither. Denote by  $\hat{p}_i(\sigma')$  ( $i = H, L$ ) the  $p'$  that satisfies the following equality:

$$\pi_i(\sigma', p') - \pi_i(\sigma, p) = C(\sigma') - C(\sigma). \quad (12)$$

In other words,  $\hat{p}_H(\sigma')$  is the interim posterior associated with  $\sigma'$  for which the high type is indifferent between deviating to  $\sigma'$  and sticking to  $\sigma$ . Similarly,  $\hat{p}_L(\sigma')$  is

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<sup>17</sup>See Appendix B. With a more general cost function, it would still be the case that for any interim pessimistic posterior  $p$ , pooling on any signal  $\sigma$  can be supported as part of a PBE if and only if  $\pi_L(\sigma, p) - C(\sigma) \geq 0$ .

<sup>18</sup>See Appendix B.

the counterpart for the low type. For notational simplicity, we have suppressed its dependence on  $p$ . Type  $i$  ( $i = H, L$ ) would find it strictly profitable to deviate to  $\sigma'$  if its associated interim posterior is higher than the cutoff  $\widehat{p}_i(\sigma')$ . Note then that in our setting the D1 condition, (3), instead of being expressed in terms of sets of best responses, could be stated in terms of belief thresholds  $\widehat{p}_i(\sigma')$ . If there exists a  $\sigma'$  such that

$$\pi_H(\sigma', \widehat{p}_H(\sigma')) - \pi_H(\sigma, p) > \pi_L(\sigma', \widehat{p}_H(\sigma')) - \pi_L(\sigma, p); \quad (13)$$

then the combination  $(\sigma, p)$  cannot be part of a D1 equilibrium.

To rule out an outcome as D1 equilibrium, it therefore suffices to show that (13) holds for some  $\sigma'$ . Given our assumptions, it is straightforward to see that  $\pi_i(\sigma', \widehat{p}_H(\sigma'))$  ( $i = H, L$ ) is continuously differentiable in  $\sigma'$  for all  $\sigma' \in (0, +\infty)$  and  $p \in (0, 1)$ . When  $\sigma' = \sigma$ , both sides of (13) are equal to 0, since  $\widehat{p}_H(\sigma) = p$ . To ensure that there does not exist a  $\sigma'$  such that (13) holds, it is necessary that

$$\frac{d[\pi_H(\sigma', \widehat{p}_H(\sigma')) - \pi_L(\sigma', \widehat{p}_H(\sigma'))]}{d\sigma'} = 0 \text{ at } \sigma' = \sigma. \quad (14)$$

If not, then either a slight upward (if the expression in (14) is positive) or downward deviation (if it is negative) would disrupt the equilibrium.

**Lemma 3.** *Consider any signal  $\sigma$  chosen with positive probability by both types:*

1.  $\sigma$  can be part of a D1 equilibrium only if the associated interim posterior,  $p$ , is optimistic, or  $\lambda < 1$ .
2.  $\sigma$  and an optimistic interim posterior  $p$  survives as part of a D1 equilibrium if and only if

$$\gamma = \frac{\phi(x)}{1 - \lambda}. \quad (15)$$

The implications of the above lemma are striking: Part 1 states that in any D1 equilibrium, the pool of types that choose a particular signal can only be optimistic; Part 2 further pinpoints the unique interim posterior for any precision in any D1 equilibrium.

Part 1 of the lemma states that any pooling associated with a pessimistic or neutral interim posterior cannot be part of a D1 equilibrium. The reason is that it is ruled out by deviation to a marginally more precise signal. Two effects drive this result. The first effect is due to the fact that, fixing any interim posterior, the difference between the high and the low type's probability of acceptance is increasing in the precision of the signal, by Part 1 of Lemma 2. Thus, according to the first effect, a deviation to a higher precision is more likely to benefit the high type. The second effect is zero for the neutral interim posterior but again favours the high type being the deviating type for a pessimistic interim posterior. In order to make the high type indifferent between adhering to the initial signal and deviating to a signal with marginally higher precision, hence higher cost, the receiver's belief at the latter must be marginally higher than the equilibrium interim posterior. However, since the interim posterior is pessimistic, an increase in the interim posterior would cause the difference between the high type's probability of acceptance and the low type's to become marginally larger, by Part 2 of Lemma 2. So, the combination of these two effects implies that the high type is more likely to benefit from a deviation to a more precise signal, and therefore such a deviation should be attributed to the high type with probability one, which eliminates pooling on a neutral or pessimistic interim posterior as part of any D1 equilibrium. If the prior is pessimistic or neutral, any D1 equilibrium must be semi-separating and the low type must mix between no precision and pooling with high type with a probability that leads to an optimistic interim posterior.

To understand Part 2 of Lemma 3, recall that if the D1 condition (3) selected against the low type, and probability one were attributed to the high type, then the high type would want to deviate. Therefore, we need to ensure that for any deviation  $\sigma'$ , the D1 condition fail. As explained earlier, in our setup, this is equivalent to showing that  $\widehat{p}_L(\sigma') \leq \widehat{p}_H(\sigma')$  or, as a sufficient condition, that for any  $p'$ ,  $\pi_H(\sigma', p') - \pi_L(\sigma', p') < \pi_H(\sigma, p) - \pi_L(\sigma, p)$ . For very high  $\sigma'$ , this is always the case, because by Part 1 of Lemma 2, the difference between  $\pi_H(\sigma', p') - \pi_L(\sigma', p')$  is smaller than the equilibrium difference. On the other hand, we can disregard very low  $\sigma'$ , for which the high type would never want to deviate because it is too costly. For intermediate

deviations it is important to define two levels of beliefs  $p'$ ,  $p_1$  and  $p_2$ , that satisfy:<sup>19</sup>

$$\pi_H(\sigma', p') - \pi_L(\sigma', p') = \pi_H(\sigma, p) - \pi_L(\sigma, p),$$

where  $p_1 < \bar{p} < p_2 < p$ . We provide here, with the support of Figure 4, an heuristic explanation of how to obtain condition (15) for a deviation  $\sigma' > \sigma$ . The two horizontal lines represent the equilibrium payoffs of the high and the low type, respectively. The two increasing curves are their respective payoffs corresponding to the deviation  $\sigma'$  plotted as a function of the interim belief. From Part 2 of Lemma 2, we know that the difference between these two curves is first increasing, reaching a maximum at  $\bar{p}$  and then decreasing. In the graph, the two curves are below the equilibrium payoffs both at  $p_1$  and at  $p_2$ . This means that the intersection between the high type's payoff at deviation  $\sigma'$  and his equilibrium payoff,  $\hat{p}_H(\sigma')$ , occurs to the right of  $p_2$ . Since  $\bar{p} < p_2$ , we have  $\pi_H(\sigma', \hat{p}_H(\sigma')) - \pi_L(\sigma', \hat{p}_H(\sigma')) < \pi_H(\sigma, p) - \pi_L(\sigma, p)$ . That is, when the high type is indifferent, the low type strictly wants to deviate. In this situation, the D1 condition (3) would not select against the low type and the deviation would not disrupt the proposed equilibrium on the basis of the D1 refinement. The analogous would hold when the two curves are above the equilibrium payoffs both at  $p_1$  and at  $p_2$ . Problems arise only when at  $p_1$  they are below the equilibrium payoff but at  $p_2$  they are above. In this case,  $\hat{p}_H(\sigma')$  would be in  $(p_1, p_2)$  and at  $\hat{p}_H(\sigma')$  the difference in the high and low type payoff would be higher than the equilibrium difference, implying that when the high type is indifferent between the equilibrium and  $\sigma'$ , the low type strictly prefers to not deviate, disrupting the proposed equilibrium as D1. To summarize, the signal-interim posterior pair  $(\sigma, p)$  survives as part of a D1 equilibrium if and only if we are not in the last situation. Condition (15) is obtained from considering marginal deviations to both higher and lower precision and using the cost function (2). It turns out that marginal deviations are all what matters to rule out that condition (3) selects against the low type. In particular, for marginally higher deviations we need  $\gamma \leq \phi(x)/(1 - \lambda)$ , and for marginally lower deviations we need  $\gamma \geq \phi(x)/(1 - \lambda)$ .

Our next lemma rules out pooling on more than one signal.

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<sup>19</sup>To simplify notation we have omitted the dependence of these two beliefs on  $\sigma'$ .



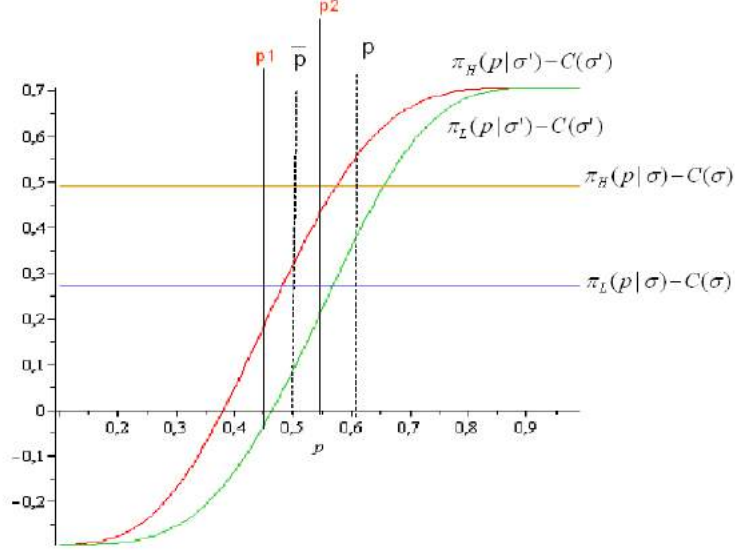


Figure 4: Example of D1 equilibrium.

**Lemma 4.** *There exist no D1 equilibria where the high and low types both mix between more than one signal.*

*Proof of Lemma 4:* Consider the situation where both the low type and the high type mix on two signals,  $\sigma$  and  $\sigma'$ , where  $\sigma < \sigma'$ , with respective interim posteriors  $p$  and  $p'$ . In order to satisfy the D1 criterion, by Lemma 3, it must be the case that both interim posteriors are optimistic  $\bar{p} < p' < p$  and that condition (15) holds for each of them. That is,

$$\gamma = \frac{\phi(x)}{1 - \lambda} = \frac{\phi(x')}{1 - \lambda'}.$$

From the fact that  $p > p'$  ( $\lambda < \lambda'$ ), we would need that  $\phi(x) > \phi(x')$ . Since both  $p$  and  $p'$  are optimistic,  $x$  and  $x'$  are both negative, this implies that  $x > x'$  and  $\Phi(x) > \Phi(x')$ , that is  $\pi_H(\sigma, p) < \pi_H(\sigma', p')$ . However, this turns out to be inconsistent with the conditions for mixing, because  $C(\sigma) > C(\sigma')$ . ■

The above Lemma rules out pooling on two different precision levels, because the D1 requirement and the assumption that the cost function is decreasing in precision are incompatible with each other.

### 3.2 Characterization of D1 equilibria

We start by characterising the set of D1 equilibria in the pessimistic and neutral cases using Lemma 3.

**Proposition 1.** *In the pessimistic and neutral case there exists a unique D1 equilibrium and it is semi-separating. Its (optimistic) interim posterior  $\tilde{p}$  and signal  $\tilde{\sigma}$ , are independent of the prior, and satisfy  $\tilde{\sigma} = \sigma_0(\tilde{p})$  and*

$$\gamma = \frac{\phi(x(\sigma_0(\tilde{p}), \tilde{p}))}{1 - \lambda(\tilde{p})}. \quad (16)$$

*Proof of Proposition 1.* We have discussed in the previous section that the unique separating equilibrium does not satisfy the D1 refinement. For analogous reasons no equilibrium in which the high type partially separates from the low type satisfies the D1 refinement. By Lemma 3, with pessimistic and neutral priors, no pooling equilibrium satisfies the D1 refinement. Finally, by Lemma 4, there is no D1 equilibrium where the two types pool on more than one signal. The only remaining possible D1 equilibrium is semi-separating: the high type chooses a signal  $\tilde{\sigma} < \infty$ , while the low type mixes between  $\tilde{\sigma}$  and  $+\infty$  and obtains a payoff of zero.

By Lemma 3,  $\tilde{\sigma}$  and its associated interim posterior,  $\tilde{p}$ , must satisfy condition (15), which becomes (16) in the semi-separating equilibrium. When  $p = \bar{p}$  we know from the Proof of Lemma 3 that the D1 condition (15) is not satisfied for low deviations, because  $\gamma < \frac{\phi(x)}{1-\lambda}$ .

As  $p$  increases, say to  $p'$  (by increasing the probability that low types choose  $+\infty$ ), from the zero payoff condition of the low type we have that  $\sigma_0$  decreases to  $\sigma'_0$ . This implies that  $C(\sigma_0)$  increases and  $x + a$  (which is inversely related to the probability that the low type is accepted) decreases. From the fact that  $\sigma'_0 < \sigma_0$ , we have  $a' > a$ , and  $x' + a' < x + a$ , which implies that  $x' < x$ . Given that  $p$  and  $p'$  are both optimistic, we have that  $|x| < |x'|$ , which in turns implies that  $\phi(x') < \phi(x)$ . Therefore, the numerator of the RHS of (16) is decreasing in  $p$ , while, from the definition of  $\lambda$ , the denominator of the RHS of (16) is increasing. It follows that the RHS of (16) is decreasing in  $p$ . When  $p$  goes to 1,  $x$  goes to  $-\infty$  and the RHS of (16) goes to zero. It follows that there is a unique  $\tilde{p} \in (\bar{p}, 1)$ , which does not depend

on the prior, for which (16) is satisfied. The unique D1 equilibrium is therefore the semi-separating equilibrium associated with this  $\tilde{p}$ , with the low type's mixing probabilities constructed appropriately. ■

The following corollary describes each type of sender's expected payoff in the D1 equilibrium.

**Corollary 1.1.** *Fix the cost function in (2). In the D1 equilibrium of the pessimistic and neutral cases, the high and low types' payoffs are independent of all parameters of the model except  $\gamma$ .*

*Proof of Corollary 1.1:* The three equations that uniquely determine the D1 equilibrium are:

$$\begin{aligned}\gamma &= \frac{\phi(x)}{1-\lambda}; \\ \phi(x) &= \lambda\phi(x+a); \\ 1 - \Phi(x+a) &= \gamma a.\end{aligned}$$

The only “free” parameter in the above equations is  $\gamma$ , the parameter of the cost function. Given  $\gamma$ , we can uniquely solve for  $x$  (negative),  $a$ , and  $\lambda$ . Given that the low type's payoff is 0 in any D1 equilibrium and that the high type's is

$$1 - \Phi(x) - \gamma a,$$

we conclude that neither type's payoff depends on any parameters of the model except  $\gamma$ . ■

The results of the above Proposition and Corollary indicate that our model provides a unique prediction in the neutral and pessimistic cases and the resulting precision choices are independent of the specific value of the prior. Furthermore, the cost parameter pins down the *payoffs* of each sender's type. This is reminiscent of the Riley outcome in the standard Spence signalling model.

We now consider the possible D1 equilibria in the optimistic case.

**Proposition 2.** *In the optimistic case:*

1. *pooling on no information is always a D1 equilibrium;*
2. *if  $p^0$  is below a certain threshold, there could be either one or two D1 pooling equilibria on a finite signal, while if it is above that threshold, the only D1 equilibrium is pooling on no information;*
3. *if  $p^0 < \tilde{p}$ , the semi-separating equilibrium that satisfies (16) is also D1.*

Intuitively, when the prior is optimistic, pooling on no information is a very robust equilibrium. In fact, both types obtain the highest possible payoff and therefore neither type would ever want to deviate. For very high priors, this is the only D1 equilibrium. When the prior is below  $\tilde{p}$ , the interim posterior associated with the semi-separating equilibrium above, it is straightforward to construct the same semi-separating equilibrium, which is D1. Finally, there exists a threshold below which there could be one or two additional pooling equilibria that are D1. Different from Daley and Green [11], our persuasive signalling game could have multiple D1 equilibria. For optimistic priors, not only there can be multiple D1 pooling equilibria but pooling and semi-separating equilibria can coexist.<sup>20</sup>

The above propositions have implications about what the receiver should believe in a D1 equilibrium depending on her prior belief and the signal chosen by the sender. When the sender chooses an uninformative signal, the receiver believes that the sender is of low quality if the equilibrium is semi-separating, which is always the case if the prior is not optimistic and can be the case if the prior is optimistic. The receiver believes that the sender is of high quality if the equilibrium is pooling on the uninformative signal, which only and always exists when the prior is optimistic. On the other hand, whenever the sender provides any informative signal to the receiver, the receiver believes that the sender's expected quality exceeds the default option's quality.

## 4 Commitment

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<sup>20</sup>In Appendix C we provide an alternative analysis of the D1 condition (15) that uses, as proposed by Daley and Green [11] the so called belief-indifference curves.

There are environments in which the sender may choose the precision of his signal before he learns his type. A newly elected politician can choose a policy of transparency up front. A manufacturer rolling out a new product can commit to granting access to the same reviewer or the same length of trial period regardless of the product to be rolled out.

We call this scenario “commitment,” as opposed to the “discretion” scenario considered in the previous section, in which the sender decides her precision level after she learns her type. Note that the commitment we consider is different from the case of full commitment considered by Kamenica and Gentzkow [26], where the sender is allowed to make a type-contingent signal choice. Our analysis therefore is more suitable for the contexts where there do not exist institutional or technological arrangements that allow type-contingent signal choices to be made. For example, it may be infeasible for a politician to sign a contract to advertise heavily on TV only when he finds out himself to be a strong candidate. More typically, a politician either commits to advertising buys at the beginning of a campaign (commitment in our terminology), makes advertising buys after getting better information about his strength (discretion).<sup>21</sup>

In the context of our setup, the sender’s choice of precision under commitment boils down to solving

$$\max_{\sigma \in M} p^0 \pi_H(\sigma, p^0) + (1 - p^0) \pi_L(\sigma, p^0) - C(\sigma), \quad (17)$$

where  $\pi_i(\sigma, p^0)$  is as defined by (5) (or equivalently, (10)) and (6) (or equivalently, (7)). Let  $\sigma^*$  be the solution to this problem. Note that it always exists given the continuity of the objective function and the compactness of the domain  $M$ . In addition, let  $u^*$  be the maximized payoff of the sender. Note that  $u^*$  is a function of  $p^0$ ,  $\theta_H$ ,  $\theta_M$ ,  $\theta_L$ , and the cost parameter  $\gamma$ , given the cost function  $C$  in (2). Equivalently, this

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<sup>21</sup>For example, in the Spring of 2016, Priorities USA, a super PAC supporting Hillary Clinton, committed to advertising spots in the swing states in the general election in the fall, before her even winning the Democratic nomination. In contrast, Marco Rubio increased his advertising heavily in his home state, Florida, right before the Republican primary there, after learning that he was a weaker candidate than expected from a string of losses in earlier states.

problem can also be stated as

$$\max_{a \in [0, \infty]} p^0 [1 - \Phi(x)] + (1 - p^0) [1 - \Phi(x + a)] - \gamma a, \quad (18)$$

For an interior solution to the optimization problem,  $a^* \in (0, +\infty)$ , the first order condition has to hold, which, using previously derived conditions, simplifies into

$$p^0 \phi(x) \frac{x + a}{a} + (1 - p^0) \phi(x + a) \frac{x}{a} - \gamma = 0, \quad (19)$$

which in the neutral case (because  $p^0 = \bar{p}$  and  $-x = x + a$ ) simplifies to

$$\frac{1}{2}(2\bar{p} - 1)\phi(x) - \gamma = 0. \quad (20)$$

We now turn to the characterization of the sender's optimal choice under Commitment.

**Proposition 3.** *Under Commitment, the sender's optimal choice of precision is as follows:*

1. *In the optimistic case, the sender commits to  $\sigma^* = +\infty$  (or  $a^* = 0$ ).*
2. *In the neutral case,*
  - (a) *the sender commits to  $\sigma^* = +\infty$  (or  $a^* = 0$ ) if  $\theta_M \leq (\theta_H + \theta_L)/2$  or if  $\theta_M > (\theta_H + \theta_L)/2$  and  $\gamma \geq (2\bar{p} - 1)\phi(0)/2$ ;*
  - (b) *the sender commits to a finite  $\sigma^*$  (or  $a^* > 0$ ) if  $\theta_M > (\theta_H + \theta_L)/2$  and  $\gamma < (2\bar{p} - 1)\phi(0)/2$ , which is the unique solution to (19).*
3. *In the pessimistic case,*
  - (a) *the sender commits to  $\sigma^* = +\infty$  (or  $a^* = 0$ ) if  $\theta_M \leq (\theta_H + \theta_L)/2$ ;*
  - (b) *the sender commits to a finite  $\sigma^*$  (or  $a^* > 0$ ) if  $\theta_M > (\theta_H + \theta_L)/2$  and  $\gamma$  is small enough such that (19) has a solution and such that the higher precision among the two solutions to (19) gives the sender a positive payoff.*

In the optimistic case, the sender would like to commit to no information provision and therefore zero costs, because the receiver would then accept the sender's request. In the neutral and pessimistic cases, if the default option lies to the left of the midpoint between high quality and low quality,  $(\theta_H + \theta_L)/2$ , the sender would again prefer to commit to no information revelation. Note that, regardless of signal precision, given the symmetry of the Normal distribution, if the signal realization is at the midpoint, the expected value of the sender's quality is equal to the ex ante expected quality, which is equal to the default quality in the neutral case and strictly less than the default quality in the pessimistic case. Thus, in order to be accepted, the sender must reach a signal realization above the midpoint, which in turn implies that the conditional probability of the low type being rejected by the receiver is always higher than (or equal to, in the neutral case) that of the high type being accepted. When the low type's quality is closer to the default quality, it means that it is more likely that the sender will be low type. Thus, it does not benefit the sender to provide information to the receiver. In contrast, if the default option is relatively close to high quality, and if the cost of providing information is relatively low, then, the sender would commit to a positive level of precision, because high quality is very likely, and it is more likely for the sender to benefit from information provision. Intuitively, if the sender anticipates a very likely but small improvement over the receiver's default option, he is willing to provide information; if the sender anticipates a very unlikely but large improvement, he is unwilling to do so.

## 5 Optimal persuasion

In certain scenarios, the sender, when faced with an option to commit to a level of precision, could either take the option or choose to "wait and see" instead. A political challenger may commit to a certain level of campaigning from the outset, or wait and increase intensity of campaigning after realizing that he himself is a more viable candidate. The new political leader may commit to a transparency policy at the beginning, or choose to disclose good news and hide bad news about his administration. To offer a potential explanation of such phenomena, we compare the sender's expected payoffs under discretion and commitment.

In the optimistic case, by Proposition 3,  $\sigma^* = +\infty$ , as the sender finds it optimal

to commit to sending a completely uninformative signal and always getting accepted. In the meantime, when the prior is optimistic, pooling on  $+\infty$  is also a D1 equilibrium in our model when the sender chooses a signal after learning her type. So, assuming the sender and receiver play this pooling equilibrium under discretion, which is the best from the sender's perspective, the sender is indifferent between commitment and discretion.<sup>22</sup>

Consider now the neutral and pessimistic cases. By Proposition 1, under discretion the only D1 equilibrium is semi-separating, where the high type chooses  $\tilde{\sigma} \in (0, +\infty)$ , while the low type mixes between  $+\infty$  and  $\tilde{\sigma}$ , where

$$\tilde{\sigma} = \sigma_0(\tilde{p}),$$

and  $\tilde{p}$  is defined by (16). Let  $\tilde{u}$  be the sender's ex ante expected payoff in this equilibrium. To see if the sender's expected payoff under commitment  $u^*$  is strictly higher than that under discretion  $\tilde{u}$ , we need to determine whether or not

$$\begin{aligned} u^* &\equiv p^0 [\pi_H(\sigma^*, p^0) - C(\sigma^*)] + (1 - p^0) [\pi_L(\sigma^*, p^0) - C(\sigma^*)] \\ &> p^0 [\pi_H(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma})] + (1 - p^0) [\pi_L(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma})] \equiv \tilde{u}. \end{aligned} \quad (21)$$

Since in the semi-separating equilibrium the low type's expected payoff is zero,  $\tilde{u}$  can be simplified into

$$\tilde{u} = p^0 [\pi_H(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma})].$$

By Corollary 1.1,  $\pi_H(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma})$  is dependent only on the cost function (or  $\gamma$ ). Therefore, as opposed to  $u^*$ ,  $\tilde{u}$  is a function of  $p^0$  and  $\gamma$  only.

**Lemma 5.** *In order for the sender to prefer commitment to discretion, there must exist a pooling equilibrium under the prior  $p^0$ .*

*Proof of Lemma 5:* Condition (21) holds only if

$$\pi_H(\sigma^*, p^0) - C(\sigma^*) > \pi_H(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma})$$

or

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<sup>22</sup>The sender would strictly prefer commitment to discretion if other D1 equilibria were played under discretion.



$$\pi_L(\sigma^*, p^0) - C(\sigma^*) > \pi_L(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma}) = 0.$$

However, if only the first inequality was holding, condition (3) would hold against the low type in the semi-separating equilibrium defined in Proposition 1. This would contradict the fact that the semi-separating equilibrium is D1. Therefore, given that  $\pi_L(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma}) = 0$ ,  $\sigma^*$  must be a pooling equilibrium under the prior  $p^0$ . ■

The proof utilizes the natural consequence of the D1 equilibrium under discretion. If the sender can commit to a precision level that gives a higher expected payoff than that under discretion, it must be that the low type is better off under commitment. If not, the high type would need to be better off. However, this would mean the existence of a signal that breaks the D1 equilibrium under discretion, which is a contradiction.

The intuition of the above result is that when it is common knowledge that the sender's quality is very likely to be low, such that there does not exist a pooling equilibrium under discretion, the sender does not want to commit to an information disclosure policy. By so doing, it would be very unlikely that she could reap the benefit of information revelation, because the receiver would set the acceptance bar very high due to the very pessimistic prior.

Now, let us consider the neutral case. By assumption, the sender is accepted with probability  $1/2$ , if no information is provided by the sender. An immediate observation is therefore that as long as the probability of the high type,  $p^0$ , is less than or equal to  $1/2$ , the sender prefers commitment to discretion, because in the unique separating D1 equilibrium under discretion, the low type receives a zero payoff, yet the high type receives a payoff less than one. In fact, Proposition 3 states that the sender will commit to no information provision when  $p^0 \leq 1/2$ .

We now present the main results of this section, which are illustrated in Figure 5.

**Proposition 4.** *In deciding an optimal persuasion policy, the sender's payoff comparison between commitment and discretion is as follows:*

1. *When the prior is optimistic, the sender is indifferent between commitment and discretion.*

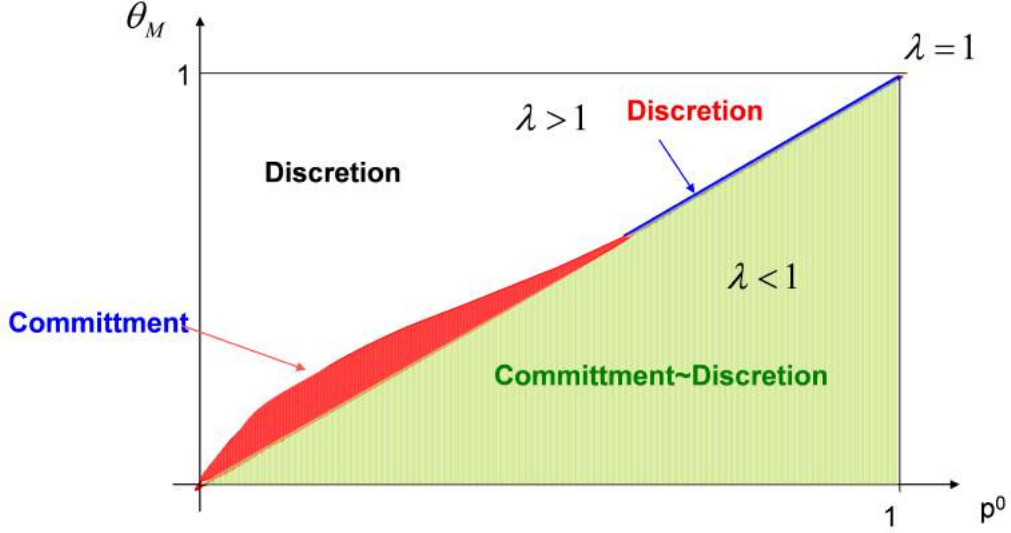


Figure 5: Optimal persuasion: comparison between commitment and discretion. The horizontal axis is the prior probability of  $H$  and the vertical axis is the receiver's default payoff.

2. When the prior is neutral, the sender prefers commitment to discretion if and only if the probability of high type,  $p^0 = \bar{p}$ , is lower than some  $\bar{p}^c \in (1/2, 1]$ .
3. When the prior is pessimistic, the sender prefers discretion to commitment if the prior  $p^0$  is lower than some  $p^c \in (0, \bar{p})$ .
4. When the prior is neutral or pessimistic, fixing  $p^0$ , there exists a cutoff  $\theta_M^* \in [\bar{\theta}_M(p^0), \theta_H]$ , such that the sender prefers commitment to discretion if and only if  $\theta_M \leq \theta_M^*$ , where  $\bar{\theta}_M(p^0)$  is the  $\theta_M$  that makes the receiver neutral:

$$\frac{(1 - p^0)(\theta_M - \theta_L)}{p^0(\theta_H - \theta_M)} = 1.$$

Let us discuss below the implications of the above proposition.

In the optimistic case, as we have already discussed, pooling on no information provision is both the best D1 equilibrium in the persuasive signalling game and

the sender’s optimal choice under commitment. The sender is therefore indifferent between discretion and commitment, assuming the sender-optimal D1 equilibrium will be played under discretion.

In the neutral case, as the prior,  $p^0$ , increases—which implies that, fixing  $\theta_H$  and  $\theta_L$ , the default payoff,  $\theta_M$ , increases to maintain neutrality—the sender’s expected payoff under discretion increases linearly, while the one under commitment is nonlinear in  $p^0$ .<sup>23</sup> However, it can be shown that the sender prefers discretion to commitment if and only if the prior  $p^0$  is above a certain threshold, if at all. Even though the neutral case is knife-edge, this pattern is still interesting. This means that when the receiver is ex ante indifferent between acceptance and rejection, the sender prefers discretion to commitment if and only if his quality is very likely to be better than the default but the quality difference is relatively small.

In the pessimistic case, when the prior is very low, the sender strictly prefers discretion to commitment owing to Lemma 5 and the fact that there does not exist any pooling equilibrium for very low priors. The most interesting result on optimal persuasion in our setup is in the neutral and pessimistic cases, fixing the prior  $p^0$ : as the default payoff of the receiver,  $\theta_M$ , increases, which in a sense makes her more pessimistic about the sender’s quality, the sender becomes more inclined to prefer discretion. This result is implied by two facts. First, the sender’s expected payoff under discretion is independent of  $\theta_M$  by Corollary 1.1. Second, an envelope argument demonstrates that the sender’s expected payoff under commitment is nonincreasing in the receiver’s default payoff, because for any precision choice by the sender, his probability of acceptance is nonincreasing in the receiver’s default payoff. Therefore, as the receiver’s default payoff improves, the sender’s optimal persuasion policy unequivocally moves from commitment to discretion.

**Remark.** The comparison above crucially depends on our modelling assumptions, in particular, that precision is costly and that the sender does not have full commitment. In fact our “commitment” scenario is one where the sender is allowed to commit to the *same* precision level regardless of his type, but not make precision contingent on his realized quality. When signals are costless and the sender has full commitment, as Kamenica and Gentzkow [26] assume, Alonso and Câmara [1] show

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<sup>23</sup>Alternatively, one can think of shifting both  $\theta_H$  and  $\theta_L$  down by the same amount.

that the sender would never benefit from learning his type before persuasion takes place. Even if we dropped just the costly precision assumption, then unravelling (full revelation) would occur in the persuasive signalling game. The sender would prefer commitment because he could commit to perfect precision and replicate the outcome under discretion.

## 5.1 Applications and discussion

There are many political and economic situations that broadly fit the descriptions of our model. We first flesh out details of the two leading examples introduced at the beginning of the paper, and briefly discuss other applications.

1. **Political campaigning by a new challenger.** Consider the example provided in the introduction where a political challenger hopes to unseat an incumbent. The challenger's quality  $\theta \in \Theta$  is potentially his private information and the incumbent has commonly known quality  $\theta_M$ . Quality is an office-specific characteristic that positively affects voters' utilities. The challenger (the sender) can use a political campaign to affect voters' (the receiver) assessment about his quality. In political campaigns, the signal realization  $s$  is the outcome of the candidate's effort and money spent on activities to increase his exposure—attending town hall meetings, making visits to one's district, making media appearances, buying campaign advertisements, etc. More exposure (lower  $\sigma$ ) corresponds to voters getting better information about the challenger. In this setting, the receiver is the representative voter who must choose whether to elect the new challenger or re-elect the incumbent.<sup>24</sup>

A potential interpretation of the optimal persuasion question we study is whether a challenger without political experience would prefer to acquire ex-

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<sup>24</sup>The assumption of a representative voter is without loss of generality in a context where voters have common value preferences (the quality of the elected politician) and observe the same information. It also applies to situations where voters have different political preferences, the candidates' political positions are commonly known, and a voter's utility derived from a candidate's political position and that from his quality is additively separable. In this case, it is sufficient to redefine the incumbent's quality as the sum of his quality and the incumbent's positional (dis)advantage over the challenger for the median voter.

perience and therefore knowledge of his own quality before running for office. The political challenger, who faces an uphill battle to win an election because the incumbent is perceived to be on average more suitable for the position, may choose his campaign strategy after conducting polls, or he could commit to a campaign strategy up front. In our model, the challenger's expected payoff under discretion will not be dependent on how strong his incumbent opponent is. However, his expected payoff under commitment will deteriorate as the incumbent becomes stronger. So, as the incumbent becomes stronger, or, alternatively, the challenger becomes weaker, the challenger is more likely to choose discretion.

**2. Choice of transparency policy by a newly elected political leader.**

Consider a newly elected political leader, who has had no previous experience of holding office. He has no private information about his governing competence  $\theta \in \Theta$ . He has to make a decision about the level of transparency for his administration, with the goal of building his reputation and securing re-election against a future challenger with expected competence  $\theta_M$ . Being more transparent (higher precision of the signal  $s$ ) gives voters (the receiver) a better idea about the leader's competence, but it is more costly in terms of time (meeting with the press and constituents) and resources (fulfilling information requests, making data and records easily available to the public) devoted to providing access to information. He could decide the level of transparency at the beginning of his term, or do so later and be more transparent only when he privately learns good news about his competence. Our model predicts that as the policy initiatives he plans to undertake become worse for the public, he would more likely prefer to announce how open his administration will be about his policies after he learns more about their public support.

Other applications of our model include:

1. A lobbying group who wants to persuade a benevolent legislator to support a policy that favours its interests through commissioning expert studies. The precision of the signal is reflected in the quality of experts, or the number of independent studies to commission.

2. A producer with a new product wants to convince a buyer to switch to it from a well established product through a free trial.<sup>25</sup> The precision of the signal is reflected in the length of the trial period.
3. An entrepreneur wants to convince potential investors to participate in its IPO (initial public offering) through an independent auditor’s approval of the financials of his business. The precision of the signal is reflected in the prestige of the auditing firm.<sup>26</sup>

We may take our results to the leading examples of our paper. The interpretation for the other settings are analogous. In our first leading example, the political challenger may choose his campaign strategy after conducting polls, or he could commit to a campaign strategy up front. This could be viewed both as comparisons of different institutional scenarios or optimal choice by the political challenger. Proposition 2 tells us that, under discretion, a political challenger who *ex ante* has a higher expected quality than the incumbent (optimistic case) is likely to devote minimal campaign effort or resources to persuade voters of his quality.<sup>27</sup> This is true also in the commitment scenario as Proposition 3 shows, which implies a strong challenger is indifferent between commitment and discretion (Proposition 4). In contrast, if, perhaps more typically, the political challenger faces an uphill battle to win an election against a stronger incumbent, then in the discretion case, as Proposition 1 shows, he will devote effort and money to campaigning even if he is worse than the incumbent. Corollary 1.1 further tells us that the challenger’s expected payoff under discretion will not be dependent on how strong his incumbent opponent is. Under commitment, as Proposition 3 shows, the challenger either commits to no information provision or a positive precision, depending on how likely he is better than the incumbent. If there is no information provision, his expected payoff does not change with the incumbent’s quality, while if his optimal precision choice is positive, his expected

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<sup>25</sup>This is prevalent in subscription-based products and services, like antivirus software products, Amazon Prime, Apple Music, and most “software as a service” (SAAS) products.

<sup>26</sup>Weber and Willenborg [44] provide empirical evidence that larger and more costly auditors typically provide better prediction about the post-IPO performance of companies.

<sup>27</sup>Note that, however, it is necessarily not the *only* plausible outcome in the D1 sense, unless the challenger is very strong.

payoff under commitment will deteriorate as the incumbent becomes stronger. So, as the incumbent becomes stronger, or, alternatively, the challenger becomes weaker, the challenger is more likely to choose discretion.<sup>28</sup> Our results can also be applied to the case of a newly elected leader, who can commit to a level of transparency after coming into office, or choose it later on at his discretion. Our model predicts that as the policy initiatives he plans to undertake become worse for the public, he would more likely prefer to announce how open his administration will be about his policies after he learns more about their public support.

## 6 Conclusion

We have considered a model of persuasion between a privately informed sender and a receiver, where (1) the sender aims to convince a receiver to take one of two possible actions; (2) the sender chooses the precision of the information to be provided to the receiver; (3) the receiver observes both the level of precision and a resulting random realization of information. This setting potentially encompasses a variety of political and economic situations, which, as alluded to previously, may include an upstart political candidate's intensity of campaigning, a new political leader's policy on transparency, a lobbying group's commissioned studies, a new product manufacturer's length of free trial period, and an IPO firm's choice of auditor.

We analyze two versions of the model: discretion, where the sender chooses his precision before observing his quality, and commitment, where he does so before learning about his quality.

Under discretion, using the standard D1 equilibrium selection criterion, we show that in equilibrium, whenever any informative signal is chosen by the sender, it must be an indication to the receiver that the sender's information is optimistic, i.e., the receiver before observing the realization of the signal would accept the sender's request. In the pessimistic and neutral cases the equilibrium is unique and must be semi-separating: the low type mixes between no information provision and the level

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<sup>28</sup>We abstract from the possibility that the politician/sender influences the electorate/receiver through lying or misrepresenting facts, as is arguably the case in many controversial policy debates. We do so in order to highlight the role that information provision/persuasion plays in signalling the sender's quality.

of precision offered by the high type. In such an equilibrium, the level of precision and the associated belief of the receiver is independent of the prior. Furthermore, the high and low types' payoffs depend only on the cost of precision, but not on the particular prior or how the default option compares with the two types. In the optimistic case, pooling on no information provision is always a D1 equilibrium, while there may also exist other D1 equilibria.

Under commitment, we show that the sender would commit to no information provision in the optimistic case, and would commit to a positive precision in the pessimistic and neutral cases when cost of precision is low and when the sender offers a relatively high probability of a small improvement over receiver's default option.

Using the above characterization results, we also provide an analysis of the sender's optimal policy of persuasion, namely, whether the sender would like to commit to a level of precision before learning his type (commitment), or choose it after (discretion). We show that, when the prior about the sender's quality is optimistic, the sender is indifferent between commitment and discretion as the best outcome for the sender in both cases is to not provide information. However, interesting comparisons arise in the neutral and pessimistic cases. In both cases, the sender could either prefer commitment or discretion. We show that the sender strictly prefers discretion to commitment when the prior is so low that there does not exist a pooling (perfect Bayesian) equilibrium. Furthermore, as the default option of the receiver improves, the sender's preference unambiguously move from commitment to discretion. This is driven by the fact that the sender's expected payoff from discretion is independent of the receiver's default option, while his commitment payoff is decreasing in it. This implication is potentially testable.

We believe that, through our admittedly stylized model, we have provided fresh insights on the choice of information provision strategies by privately informed economic and political players. We show that the level of information provision (e.g., the length of free trial offered by a software producer) has the potential to signal quality, but only in a semi-separating equilibrium. Furthermore, the sender prefers to choose an information provision policy before learning his quality only if he is relatively confident about his quality vis-à-vis the receiver's default option. General-



izations of these results or empirical tests of these results are the natural next steps for further research.

## 7 Appendix A: proofs

*Proof of Lemma 1.* Note that

1. For any  $\sigma \in M$ ,  $\theta$ , and  $\theta'$ , where  $\theta < \theta'$ ,  $f(\cdot|\theta', \sigma)$  dominates  $f(\cdot|\theta, \sigma)$  in the sense of monotone likelihood ratio condition.

This follows directly from the fact that

$$\frac{f(s|\theta', \sigma)}{f(s|\theta, \sigma)} = \exp \left[ \frac{2s(\theta' - \theta) - (\theta'^2 - \theta^2)}{2\sigma^2} \right],$$

which is clearly strictly increasing in  $s$  given that  $\theta' > \theta$ .

2. For any  $\sigma \in M$ , interim posterior  $p(\cdot|\sigma) \in \text{int}(\Delta\Theta)$  and signal realizations  $s$  and  $s'$ , where  $s < s'$ , the posterior  $p(\cdot|\sigma, s')$  dominates  $p(\cdot|\sigma, s)$ . In other words,  $\frac{p(\theta|\sigma, s')}{p(\theta|\sigma, s)}$  is increasing in  $\theta$ , or equivalently, for any  $\theta, \theta' \in \Theta$  and  $\theta < \theta'$ ,

$$\frac{p(\theta'|\sigma, s')}{p(\theta|\sigma, s')} > \frac{p(\theta'|\sigma, s)}{p(\theta|\sigma, s)}.$$

To see that this is the case, re-arrange the above expression as

$$\frac{p(\theta'|\sigma, s')}{p(\theta|\sigma, s')} > \frac{p(\theta'|\sigma, s)}{p(\theta|\sigma, s)}.$$

We now only have to show that

$$\frac{p(\theta'|\sigma, s)}{p(\theta|\sigma, s)} = \frac{p(\theta'|\sigma)f(s|\theta', \sigma)/f(s)}{p(\theta|\sigma)f(s|\theta, \sigma)/f(s)},$$

is strictly increasing in  $s$ , which is the case by Point 1.

3. The result of the lemma then follows, using the fact that dominance in terms of monotone likelihood ratio condition implies first order stochastic dominance, which in turns implies that  $E_p(\theta|\sigma, s)$  is a strictly increasing function of  $s$  and  $p$  [in fact the same holds for  $E_p(v(\theta)|\sigma, s)$  where  $v$  is a strictly increasing function]. ■

In order to prove most of the remaining results of the paper some preliminary notation and results are essential.

From (4), which defines the threshold  $\bar{s}$ , by the Implicit Function Theorem, we have

$$\frac{\partial \bar{s}}{\partial \sigma} = \frac{p(\theta_H - \theta_M)x^2\phi(x) + (1-p)(\theta_L - \theta_M)(x+a)^2\phi(x+a)}{D},$$

where the term  $D = p(\theta_H - \theta_M)x\phi(x) + (1-p)(\theta_L - \theta_M)(x+a)\phi(x+a)$ , which, using (5), is equal to  $-ap(\theta_H - \theta_M)\phi(x)$  and  $a(1-p)(\theta_L - \theta_M)\phi(x+a)$ . Since  $a > 0$ , the term  $D$  is always negative. The expression for  $\partial \bar{s} / \partial \sigma$  can be simplified into

$$\frac{\partial \bar{s}}{\partial \sigma} = \frac{-x^2}{a} + \frac{(x+a)^2}{a} = 2x + a. \quad (22)$$

Given the definition of  $x$  and  $a$  in (5), we can conclude

$$\begin{aligned} \frac{\partial x}{\partial \sigma} &= \frac{\partial \bar{s} / \partial \sigma \cdot \sigma - (\bar{s} - \theta_H)}{\sigma^2} = \frac{x+a}{\sigma}, \\ \frac{\partial(x+a)}{\partial \sigma} &= \frac{x+a}{\sigma} + \frac{\partial a}{\partial \sigma} = \frac{x}{\sigma}. \end{aligned}$$

Alternatively, from (10), we may obtain explicit expressions for  $x$  and  $x+a$ :

$$x = \frac{\ln \lambda}{a} - \frac{a}{2} = \frac{\sigma \ln \lambda}{\theta_H - \theta_L} - \frac{\theta_H - \theta_L}{2\sigma}, \quad (23)$$

$$x+a = \frac{\ln \lambda}{a} + \frac{a}{2} = \frac{\sigma \ln \lambda}{\theta_H - \theta_L} + \frac{\theta_H - \theta_L}{2\sigma}, \quad (24)$$

and, in fact,

$$\bar{s} = \frac{\sigma^2 \ln \lambda}{\theta_H - \theta_L} + \frac{\theta_H + \theta_L}{2}. \quad (25)$$

Notice that independent of the prior  $p$ , when  $\sigma$  goes to 0,  $\bar{s}$  converges to  $(\theta_H + \theta_L)/2$ . From (22) and (23) we can also obtain the following explicit expressions:

$$\frac{\partial \bar{s}}{\partial \sigma} = \frac{2\sigma \ln \lambda}{\theta_H - \theta_L}, \quad (26)$$

$$\frac{\partial x}{\partial \sigma} = \frac{x+a}{\sigma} = \left[ \frac{\ln \lambda}{a} + \frac{a}{2} \right] / \sigma = \frac{\ln \lambda}{\theta_H - \theta_L} + \frac{\theta_H - \theta_L}{2\sigma^2}. \quad (27)$$

In the neutral and pessimistic cases, where  $\ln \lambda \geq 0$ , this is always positive ( $x$  increasing in  $\sigma$ ). In the optimistic case,  $x$  is increasing in  $\sigma$  if and only if

$$\sigma < \frac{\theta_H - \theta_L}{\sqrt{2(-\ln \lambda)}} \equiv \sigma_{opt}(p).$$

It follows that  $x + a = 0$  at  $\sigma = \sigma_{opt}(p)$ .

$$\frac{\partial(x+a)}{\partial\sigma} = \frac{\partial x}{\partial\sigma} + \frac{\partial a}{\partial\sigma} = \frac{1}{2} \left[ \frac{\ln \lambda}{a} - \frac{a}{2} \right] = \frac{\ln \lambda}{(\theta_H - \theta_L)} - \frac{\theta_H - \theta_L}{2\sigma^2} = \frac{x}{\sigma}. \quad (28)$$

In the optimistic and neutral cases this is always negative ( $x+a$  decreasing). In the pessimistic case, this is negative ( $x+a$  decreasing) if and only if

$$\sigma < \frac{\theta_H - \theta_L}{\sqrt{2 \ln \lambda}} \equiv \sigma_{pes}(p)$$

and positive ( $x+a$  increasing) otherwise. It follows that in the pessimistic case,  $x = 0$  at  $\sigma = \sigma_{pes}(p)$ .

The inspection of (23), (24), and (25) leads to the following Lemma.

**Lemma 6.** *The threshold  $\bar{s}$  and the terms  $x$  and  $a$ , have the following properties:*

	$\bar{s}$	$x$	$ x $ vs $ x+a $	$\partial\bar{s}/\partial\sigma$
if $\lambda < 1$	$< (\theta_H + \theta_L)/2$	$< -a/2$	$ x  >  x+a $	$< 0$
if $\lambda = 1$	$= (\theta_H + \theta_L)/2$	$= -a/2$	$ x  =  x+a $	$= 0$
if $\lambda > 1$	$> (\theta_H + \theta_L)/2$	$> -a/2$	$ x  <  x+a $	$> 0$

Using (7) on Page 13 and (27) and (28) above, the derivatives of acceptance probabilities for the low type and the high type with respect to  $\sigma$  are:

$$\begin{aligned} \frac{\partial\pi_L(\sigma, p)}{\partial\sigma} &= -\phi(x+a) \frac{x}{\sigma}, \\ \frac{\partial\pi_H(\sigma, p)}{\partial\sigma} &= -\phi(x) \frac{x+a}{\sigma}. \end{aligned}$$

The above results allow us to characterise the patterns of the probability of acceptance of each type in the following two Lemmas.

**Lemma 7.** *The pattern of the probability of acceptance of the low type, can be described as follows:*

1. For any interim posterior  $p$ :  $\pi_L(0, p) = 0$ .
2. When  $p \geq \bar{p}$ ,  $\pi_L(\sigma, p)$  is strictly increasing in  $\sigma$ .
3. When  $p < \bar{p}$ ,  $\pi_L(\sigma, p)$  is strictly increasing in  $\sigma$  for  $\sigma < \sigma_{pes}$ , and strictly decreasing for  $\sigma > \sigma_{pes}$ .
4. The limit  $\lim_{\sigma \rightarrow \infty} \pi_L(\sigma, p)$  is 1 when  $p > \bar{p}$ ,  $1/2$  when  $p = \bar{p}$ , and 0 when  $p < \bar{p}$ .

Proof of Lemma 7:

1. For any interim posterior,  $\lim_{\sigma \rightarrow 0} (x+a) = 0$ . Therefore  $\lim_{\sigma \rightarrow 0} 1 - \Phi(x+a) = 0$ .
2. When  $p \geq \bar{p}$ ,  $x < 0$ , therefore  $\frac{\partial \pi_L(\sigma, p)}{\partial \sigma} - \phi(x+a) \frac{x}{\sigma} > 0$ .
3. When  $p < \bar{p}$ , we have shown below (28) that  $x < 0$  if  $\sigma < \sigma_{pes}$  and  $x > 0$  if  $\sigma > \sigma_{pes}$ . The sign of  $\frac{\partial \pi_L(\sigma, p)}{\partial \sigma}$  follows.
4. When  $p > \bar{p}$ ,  $\lim_{\sigma \rightarrow +\infty} (x+a) = -\infty$ ; when  $p = \bar{p}$ ,  $\lim_{\sigma \rightarrow +\infty} (x+a) = \lim_{\sigma \rightarrow +\infty} a/2 = 0$ ; when  $p < \bar{p}$ ,  $\lim_{\sigma \rightarrow +\infty} (x+a) = +\infty$ . The result follows.

**Lemma 8.** *The pattern of the probability of acceptance of the high type, can be described as follows:*

1. For any interim posterior:  $\pi_H(0, p) = 1$ .
2. When  $p > \bar{p}$ ,  $\pi_H(\sigma, p)$  is strictly decreasing in  $\sigma$  for  $\sigma < \sigma_{opt}$  and strictly increasing for  $\sigma > \sigma_{opt}$ .
3. When  $p \leq \bar{p}$ ,  $\pi_H(\sigma, p)$  is strictly decreasing in  $\sigma$ .
4. The limit  $\lim_{\sigma \rightarrow +\infty} \pi_H(\sigma, p)$  is 1 when  $p > \bar{p}$ ,  $1/2$  when  $p = \bar{p}$ , and 0 when  $p < \bar{p}$ .

Proof of Lemma 8:

1. For any interim posterior,  $\lim_{\sigma \rightarrow 0} x = -\infty$ . Therefore  $\lim_{\sigma \rightarrow 0} 1 - \Phi(x) = 1$ .

2. When  $p > \bar{p}$ , we have shown below (27) that  $x + a$  is positive if  $\sigma < \sigma_{opt}$  and negative if  $\sigma > \sigma_{opt}$ . The result follows from the expression of  $\frac{\partial \pi_H(\sigma, p)}{\partial \sigma} = -\phi(x) \frac{x+a}{\sigma}$ .
3. When  $p \leq \bar{p}$ ,  $x + a$  is positive. The sign of  $\frac{\partial \pi_H(\sigma, p)}{\partial \sigma}$  follows.
4. When  $p > \bar{p}$ ,  $\lim_{\sigma \rightarrow +\infty} x = -\infty$ ; when  $p = \bar{p}$ ,  $\lim_{\sigma \rightarrow +\infty} x = \lim_{\sigma \rightarrow +\infty} -a/2 = 0$ ; when  $p < \bar{p}$ ,  $\lim_{\sigma \rightarrow +\infty} x = +\infty$ . The results follow from the expression of  $\pi_H(\sigma, p) = 1 - \Phi(x)$ .

*Proof of Lemma 2:* 1. Take the derivative with respect to  $\sigma$  of the difference in the probability of being accepted for the high and the low type:

$$\begin{aligned} \frac{\partial \pi_H}{\partial \sigma} - \frac{\partial \pi_L}{\partial \sigma} &= \frac{\phi(x)}{\sigma} \left[ x - \frac{\partial \bar{s}}{\partial \sigma} \right] - \frac{\phi(x+a)}{\sigma} \left[ (x+a) - \frac{\partial \bar{s}}{\partial \sigma} \right] \\ &= -\frac{\phi(x)}{\sigma}(x+a) + \frac{\phi(x+a)}{\sigma}x < 0, \end{aligned} \quad (29)$$

where we have used (22).

In the neutral case,  $\phi(x) = \phi(x+a)$  and  $x+a = -x$  so

$$\frac{\partial \pi_H}{\partial \sigma} - \frac{\partial \pi_L}{\partial \sigma} = -\frac{\phi(x)a}{\sigma} < 0.$$

In the optimistic case,  $x < 0$  and  $x+a$  can be positive or negative but  $|x| > |x+a|$ . When  $x+a > 0$ , (29) is clearly negative. It is also negative when  $x+a < 0$ , because  $\phi(x) < \phi(x+a)$  and the positive term has a smaller absolute value. In the pessimistic case,  $x+a > 0$  and  $x$  can be positive or negative but  $|x| < |x+a|$ . Clearly, (29) is negative when  $x < 0$ . It is also negative when  $x > 0$ , because  $\phi(x) > \phi(x+a)$  and the positive term has a smaller absolute value.

2. Consider now the derivative with respect to  $p$  of the difference in the probability of being accepted for the high and the low type:

$$\frac{\partial \pi_H}{\partial p} - \frac{\partial \pi_L}{\partial p} = -[\phi(x) - \phi(x+a)] \overbrace{\frac{\partial x}{\partial p}}^{-}$$

Note that  $0 \leq |x| < x + a$  when  $p$  is pessimistic and  $x < 0 \leq |x + a|$  when  $p$  is optimistic. Therefore, the above expression is positive when  $p \in [0, \bar{p}]$ , negative when  $p \in [\bar{p}, 1]$ , and is zero if and only if  $p = \bar{p}$ , which implies  $x = -(x + a)$  and  $\bar{s}(\sigma, \bar{p}) = (\theta_H + \theta_L)/2$ . The maximum value of  $\pi_H(\sigma, p) - \pi_L(\sigma, p)$  is therefore reached at  $p = \bar{p}$  and is equal to

$$\left[1 - \Phi\left(\frac{-a}{2\sigma}\right)\right] - \left[1 - \Phi\left(\frac{a}{2\sigma}\right)\right] = 2\Phi\left(\frac{a}{2\sigma}\right) - 1,$$

which is decreasing in  $\sigma$ . ■

*Proof of Lemma 3:*

Part 1: Take a pooling equilibrium on  $+\infty$ . Consider first the optimistic case. The sender is always accepted. Independent of his type he obtains a payoff of 1. A deviation to any finite signal would provide a strictly lower payoff to either type. So no type would want to deviate for any belief.

Consider now the neutral case, where both types are accepted with probability  $1/2$  (the argument would work for any other tie-breaking rule except the one that calls for acceptance with probability one) and obtain the same payoff. Any deviation  $\tilde{\sigma} < \infty$  would more likely benefit the high type, because for any belief  $p \in (0, 1)$  he gets a strictly higher probability of acceptance. So, there are deviations, those with very low cost, where condition (3) holds with the high type wanting to deviate in more cases. If probability one is given to the high type, then he would indeed want to deviate. The same argument applies to the pessimistic case.

Now, we argue that pooling on any  $\sigma \in (0, +\infty)$  cannot be associated with an interim posterior that is pessimistic or neutral.

PESSIMISTIC CASE: Let  $\sigma_i^M = \arg \max_{\sigma} \pi_i(\sigma, p) - C(\sigma)$ ,  $i = H, L$ , i.e., the signal that maximizes type  $i$ 's expected payoff. It can be shown that  $\sigma_H^M < \sigma_L^M < \sigma_0^2$ . Assume the probability of acceptance of the low type,  $\pi_L(\sigma, p)$ , and the cost,  $C(\sigma)$ , intersect, so that there exist pooling equilibria.

First, we consider the case  $\sigma_H^M > \sigma_0^1$ . This implies that  $\sigma_H^M$  can be supported as a pooling equilibrium, because  $\sigma_H^M < \sigma_L^M < \sigma_0^2$ .

*Step 1:* Consider any pooling equilibrium  $\sigma \in (\sigma_H^M, \sigma_L^M]$ . We want to show that it

does not satisfy D1. Take a deviation  $\tilde{\sigma} \in [\sigma_H^M, \sigma)$ . Since

$$\frac{\partial(\pi_H(\sigma, p) - \pi_L(\sigma, p))}{\partial\sigma} < 0,$$

we have that

$$\begin{aligned} & [\pi_H(\tilde{\sigma}, p) - \pi_L(\tilde{\sigma}, p)] - [\pi_H(\sigma, p) - \pi_L(\sigma, p)] \\ &= [\pi_H(\tilde{\sigma}, p) - \pi_H(\sigma, p)] - [\pi_L(\tilde{\sigma}, p) - \pi_L(\sigma, p)] \\ &> 0. \end{aligned}$$

That is, at  $p$ , whenever L (weakly) wants to deviate, H strictly does. For a deviation  $\tilde{\sigma}$  in  $[\sigma_H^M, \sigma)$ , at  $p$  the high type wants to deviate but the low type does not. It must be that  $\hat{p}_H(\tilde{\sigma}) < \hat{p}_L(\tilde{\sigma})$ . The D1 condition would select against the low type, and therefore the high type would want to deviate to  $\tilde{\sigma}$  because  $\pi_H(\tilde{\sigma}, p) < 1$ .

*Step 2:* Consider any pooling equilibrium  $\sigma \in (\sigma_L^M, \sigma_0^2]$ . We want to show that it does not satisfy D1. Consider a deviation  $\tilde{\sigma} < \sigma$ . As in *Step 1* we have that at interim posterior  $p$ , whenever L (weakly) wants to deviate, H strictly does. Now, let  $\sigma'' < \sigma_L^M$  be the signal such that at  $p$  the low type is getting the equilibrium payoff. That is:  $\pi_L(\sigma'', p) - C(\sigma'') = \pi_L(\sigma, p) - C(\sigma)$ . Consider the deviation  $\tilde{\sigma} = \sigma''$ . From the argument above and the definition of  $\sigma''$  we know that at  $p$ , while the low type is indifferent, the high type strictly prefers to deviate. This means that at this specific deviation,  $\hat{p}_L(\sigma'') = p$  and that  $\hat{p}_H(\sigma'') < p$ . Therefore, the equilibrium does not satisfy D1.

*Step 3:* We are left to check that no signal  $\sigma \in [\sigma_0^1, \sigma_H^M]$  satisfies D1.

Observe that

$$\begin{aligned} & \frac{d[\pi_H(\sigma', \hat{p}_H) - \pi_L(\sigma', \hat{p}_H)]}{d\sigma'} \\ &= \frac{\partial[\pi_H(\sigma', \hat{p}_H) - \pi_L(\sigma', \hat{p}_H)]}{\partial\hat{p}_H} \cdot \frac{\partial\hat{p}_H}{\partial\sigma'} + \frac{\partial[\pi_H(\sigma', \hat{p}_H) - \pi_L(\sigma', \hat{p}_H)]}{\partial\sigma'}. \end{aligned} \quad (30)$$

We want to determine the sign of the expression in (30) at  $\sigma' = \sigma$  ( $\hat{p}_H(\sigma') = p$ ).

The first part of the first term is positive because at  $p' = \hat{p}_H(\sigma') = p < \bar{p}$ ,  $\pi_H(\sigma', p') - \pi_L(\sigma', p')$  is increasing in the second argument  $p'$ . The second part of the

first term is negative because, using the Implicit Function Theorem on (12):

$$\frac{\partial \widehat{p}_H(\sigma')}{\partial \sigma'} = - \frac{\frac{\partial [\pi_H(\sigma', \widehat{p}_H) - C(\sigma')]}{\partial \sigma'}}{\frac{\partial \pi_H(\sigma', \widehat{p}_H)}{\partial \widehat{p}_H}}.$$

For slightly lower deviations  $\sigma' < \sigma$ , the numerator is positive because  $\pi_H(\sigma, p) - C(\sigma)$  is increasing in  $\sigma$  until  $\sigma_H^M$  and we are considering equilibria  $\sigma \leq \sigma_H^M$ . The denominator is also positive, because for any fixed signal, the probability of acceptance for the high type is increasing in the receiver's interim posterior.

The second term is always negative, because for any fixed interim posterior,  $\pi_H(\sigma, p) - \pi_L(\sigma, p)$  is decreasing in the signal  $\sigma$ , or increasing in the precision. It follows that for  $\varepsilon$  small enough, if  $\sigma' \in (\sigma - \varepsilon, \sigma)$ , then

$$\frac{d[\pi_H(\sigma', \widehat{p}_H) - \pi_L(\sigma', \widehat{p}_H)]}{d\sigma'} < 0.$$

This means that (14) is violated. So, the pooling equilibrium fails the D1 criterion. In particular, a marginally more precise signal with  $\sigma' < \sigma$  would be viewed as coming from the high type for sure, which in turn would induce the high type to deviate. To conclude, no equilibrium  $\sigma \in [\sigma_0^1, \sigma_H^M]$  would survive D1.

Second, consider the case where  $\sigma_H^M < \sigma_0^1$  ( $\sigma_H^M$  is not a pooling equilibrium but  $\sigma_0^1$  is well defined). Suppose the equilibrium is such that  $\sigma \leq \sigma_L^M$  [recall that  $\sigma_L^M$  is the point where the difference  $\pi_L(\sigma, p) - C(\sigma)$  is maximized, and that  $\sigma_L^M > \sigma_H^M$ ]. Take a deviation  $\sigma' < \sigma$ . At the interim posterior  $p' = p$ , the low type does not want to deviate [his payoff decreases when going towards lower deviations] but the high type does (his payoff increases going towards  $\sigma_H^M$ ), implying that the D1 condition selects against the low type. So the receiver should believe with probability one that the deviation comes from the high type, thereby disrupting the equilibrium. A similar argument holds when  $\sigma > \sigma_L^M$ .

NEUTRAL CASE: the proof for the neutral case is analogous. The only difference is that we know that at  $p = \bar{p}$  the difference between the high and low types' probabilities of acceptance is maximized for any signal  $\sigma$ . Therefore, for marginal deviations from the equilibrium signal  $\sigma$  ( $\sigma' \rightarrow \sigma$ ), the first term in (30) is zero. Therefore,

$$\frac{d[\pi_H(\sigma', \widehat{p}_H) - \pi_L(\sigma', \widehat{p}_H)]}{d\sigma'} < 0.$$



Again, a slightly lower deviation would be viewed as coming from the high type for sure, so no signal  $\sigma$  survives D1.

Part 2: In our set up type  $i$  would want to deviate to  $\sigma'$  if and only if the associated belief  $p' > \hat{p}_i(\sigma')$ . If condition (3) selected against the low type,  $\hat{p}_H(\sigma') < \hat{p}_L(\sigma')$ , the high type would want to deviate. We want to find a necessary and sufficient condition for condition (3) not holding against the low type, which is equivalent to there existing best responses (sequentially rational given some beliefs) by the receiver that make it preferable for the low type to deviate but not for the high type. In particular we want to show that condition D1 in (3) fails to select against the low type ( $\hat{p}_H(\sigma') \geq \hat{p}_L(\sigma')$ ) if and only if

$$\gamma = \frac{\phi(x)}{1 - \lambda}.$$

We proceed in several steps.

*Step 1:* Given  $\sigma$ , let  $\hat{\sigma} > \sigma$  be the value of sigma such that the maximum difference between  $\pi_H$  and  $\pi_L$  is equal to the equilibrium difference, i.e.,

$$\pi_H(\bar{p}, \hat{\sigma}) - \pi_L(\bar{p}, \hat{\sigma}) = \pi_H(\sigma, p) - \pi_L(\sigma, p),$$

that is,

$$2\Phi\left(\frac{\theta_H - \theta_L}{2\hat{\sigma}}\right) - 1 = \pi_H(\sigma, p) - \pi_L(\sigma, p).$$

Note that  $\hat{\sigma}$  is well defined because, given  $p > \bar{p}$ ,  $\pi_H(\sigma, p) - \pi_L(\sigma, p)$  is lower than its maximum  $\pi_H(\sigma, \bar{p}) - \pi_L(\sigma, \bar{p})$ . Since  $\pi_H(\sigma, p) - \pi_L(\sigma, p)$  is continuous, decreasing in  $\sigma$ , and goes to 0 as  $\sigma$  goes to infinity, the results follow.

For any deviation  $\sigma' \geq \hat{\sigma}$  the D1 condition would not hold against the low type. In fact, for any such  $\sigma'$ ,  $\pi_H(\sigma', \bar{p}) - \pi_L(\sigma', \bar{p}) \leq \pi_H(\sigma, p) - \pi_L(\sigma, p)$  implying that for any  $p'$ ,  $\pi_H(\sigma', p') - \pi_L(\sigma', p') \leq \pi_H(\sigma, p) - \pi_L(\sigma, p)$ . Whenever the high type wants to deviate, the low type also does.

*Step 2:* We consider deviations  $\sigma' < \hat{\sigma}$ . For any such deviation  $\sigma'$ , let  $p_1(\sigma')$  and  $p_2(\sigma')$  respectively be the beliefs  $p'$  that satisfy

$$\pi_H(\sigma', p') - \pi_L(\sigma', p') = \pi_H(\sigma, p) - \pi_L(\sigma, p) \tag{31}$$

The existence of  $p_1$  and  $p_2$ , imposing  $p_1 < p_2$  without loss of generality, is ensured by the fact that  $\sigma' < \hat{\sigma}(\sigma)$ , which implies that  $\pi_H(\sigma', \bar{p}) - \pi(\sigma', \bar{p}) > \pi_H(\sigma, p) - \pi_L(\sigma, p)$ . In addition, we must have  $p_1 < \bar{p} < p_2 < p$ .

Given the definitions of  $p_1$  and  $p_2$ , at these beliefs, either both high and low types receive a greater payoff than their equilibrium payoffs, or neither does.

Consider first the case  $p' = p_1$ . If both types strictly prefer to deviate to  $\sigma'$ ,  $\hat{p}_L$  and  $\hat{p}_H$  are both lower than  $p_1$ . Since the difference  $\pi'_H - \pi'_L$  is increasing in  $p'$  up to  $\bar{p}$  and decreasing afterwards, it means that  $\hat{p}_L < \hat{p}_H$ . If both types are indifferent, then clearly,  $\hat{p}_L = \hat{p}_H = p_1$ .

Now, consider the case  $p' = p_2$ . If both types strictly prefer to stick to  $\sigma$ , then  $\hat{p}_L$  and  $\hat{p}_H$  are both higher than  $p_2$ . Again, using the property of the difference  $\pi'_H - \pi'_L$  (specifically, that it is decreasing in  $p'$  when  $p' > \bar{p}$ ), we have  $\hat{p}_L < \hat{p}_H$ . If both types are indifferent, then clearly,  $\hat{p}_L = \hat{p}_H = p_2$ .

Finally, if at  $p_1$  both low and high types receive a strictly lower payoff than in equilibrium, and at  $p_2$  they both receive a higher payoff than in equilibrium, then the D1 condition (3) would hold against the low type. Again, this is implied by the property of the difference  $\pi'_H - \pi'_L$ . If  $\hat{p}_H \in (p_1, \bar{p})$ , then at  $p' = \hat{p}_H$  the low type does not want to deviate because  $\pi_H(\sigma', \hat{p}_H) - \pi_L(\sigma', \hat{p}_H) > \pi_H(\sigma, p) - \pi_L(\sigma, p)$  so it must be  $\hat{p}_H < \hat{p}_L$ . The argument is analogous when  $\hat{p}_H \in (\bar{p}, p_2)$ .

Based on the above observations, the necessary and sufficient conditions for (3) not to hold against the low type is that either

$$\pi_H(\sigma', p_2) - C(\sigma') \leq \pi_H(\sigma, p) - C(\sigma), \quad (32)$$

which means that at  $p_2$  neither type strictly wants to deviate, or,

$$\pi_H(\sigma', p_1) - C(\sigma') \geq \pi_H(\sigma, p) - C(\sigma), \quad (33)$$

which means that at  $p_1$  both types weakly want to deviate.

*Step 3:* Now, we further develop conditions (32) and (33), in light of the cost function (2).

Consider first the condition on  $p_2$ , (32). Notice that when  $\sigma' \rightarrow \sigma$ , the LHS and RHS of (32) become equal, because  $p_2(\sigma') = p$ . We consider the derivative of the LHS. Take first:

$$\begin{aligned}\frac{d\pi_H(\sigma', p_2(\sigma'))}{d\sigma'} &= -\phi(x') \frac{dx'}{d\sigma'} = -\phi(x') \left[ \frac{\partial x'}{\partial p_2} \frac{\partial p_2}{\partial \sigma'} + \frac{\partial x'}{\partial \sigma'} \right] \\ &= \left[ \frac{\phi(x')\phi(x'+a')}{\phi(x'+a') - \phi(x')} \right] \frac{da'}{d\sigma'} < 0,\end{aligned}$$

where

$$x' = \frac{\bar{s}(p_2, \sigma') - \theta_H}{\sigma'}, a' = \frac{\theta_H - \theta_L}{\sigma'},$$

and where we have used from (31):

$$\frac{\partial p_2}{\partial \sigma'} = -\frac{-\phi(x') \frac{\partial x'}{\partial \sigma'} + \phi(x'+a') \left[ \frac{\partial x'}{\partial \sigma'} + \frac{\partial a'}{\partial \sigma'} \right]}{-\phi(x') \frac{\partial x'}{\partial p_2} + \phi(x'+a') \frac{\partial x'}{\partial p_2}}. \quad (34)$$

We then have that

$$\begin{aligned}\frac{d\pi_H(\sigma', p_2)}{d\sigma'} - \frac{dC(\sigma')}{d\sigma'} &= \left[ \frac{\phi(x')\phi(x'+a')}{\phi(x'+a') - \phi(x')} - \gamma \right] \frac{da'}{d\sigma'}\end{aligned}$$

Recall that  $p_2$  is optimistic and that in the optimistic case  $x' < 0$  and  $|x' + a'| < |x'|$ , so  $\phi(x' + a') - \phi(x') > 0$ .

First, consider higher deviations, that is  $\sigma' \in (\sigma, \hat{\sigma})$ . Since  $da'/d\sigma' < 0$ , we have that for any of such deviations  $\sigma'$

$$\frac{d\pi_H(\sigma', p_2)}{d\sigma'} - \frac{dC(\sigma')}{d\sigma'} \leq 0$$

if and only if

$$\gamma \leq \frac{\phi(x')\phi(x'+a')}{\phi(x'+a') - \phi(x')} = \frac{\phi(x')}{1 - \phi(x')/\phi(x'+a')} = \frac{\phi(x')}{1 - \lambda_2}, \quad (35)$$

where we have used the fact

$$\frac{\phi(x')}{\phi(x'+a')} = \frac{1 - p_2}{p_2} \cdot \frac{\theta_H - \theta_M}{\theta_M - \theta_L} \equiv \lambda_2$$

from (5) and (9).

Since we have that

$$\frac{dx'}{d\sigma'} = \frac{\phi(x' + a')}{\phi(x' + a') - \phi(x')} > 0$$

and  $\phi(x)$  increases in  $x$  for  $x < 0$ , we have that the numerator of (35) is increasing in  $x'$ . The denominator is decreasing in  $x'$  because  $\partial\lambda/\partial p < 0$  and  $\partial p_2/\partial\sigma' < 0$ .<sup>29</sup> Therefore, for upper deviations  $\sigma' \in (\sigma, \hat{\sigma})$ , the RHS of (35) is increasing in  $x'$ . It follows that to satisfy (32) it is sufficient that condition (35) holds locally:

$$\gamma \leq \frac{\phi(x)}{1 - \lambda}. \quad (36)$$

Actually this is also necessary. In fact, if for  $\sigma' \rightarrow \sigma$  we had

$$\frac{\partial\pi_H(\sigma', p_2)}{\partial\sigma'} - \frac{\partial C(\sigma')}{\partial\sigma'} > 0,$$

(32) would not hold. So, it must be that (36) holds.

We now consider the condition on  $p_1$ , (33), and claim that we can disregard it. If we take the derivative of the LHS of (33) we have the same algebra as above but now the sign is the opposite because  $p_1$  is pessimistic, which in turn implies that  $|x' + a'| > |x'|$  and thereby  $\phi(x' + a') - \phi(x') < 0$ . This implies that

$$\frac{\partial\pi_H(\sigma', p_1)}{\partial\sigma'} > 0,$$

and

$$\frac{\partial\pi_H(\sigma', p_1)}{\partial\sigma'} - \frac{dC(\sigma')}{d\sigma'} > 0.$$

By definition of  $\hat{\sigma}$ , we have that  $\pi_H(\hat{\sigma}, \bar{p}) - \pi_L(\hat{\sigma}, \bar{p}) = \pi_H(\sigma, p) - \pi_L(\sigma, p)$  and therefore  $p_1(\hat{\sigma}) = \bar{p}$ . When instead  $\sigma' \rightarrow \sigma$  we have that  $p_1(\sigma') < \bar{p}$ . Since when  $\sigma' \rightarrow \sigma$  (32) holds with equality and  $p_1 < p_2$ , (33) cannot hold. It is now important to notice that

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<sup>29</sup>The expression (34) can be simplified as

$$\frac{\partial p_2}{\partial\sigma'} = \frac{\phi(x' + a')x' - \phi(x')(x' + a')}{[\phi(x') - \phi(x' + a')] \cdot \partial x' / \partial p_2}.$$

The denominator is the product of two negative terms and is therefore positive. The numerator is negative, for all  $x' > |x' + a'|$ .

conditions (32) and (33) are mutually exclusive. In fact we have just shown that for marginal higher deviations,  $\sigma' \rightarrow \sigma$ , (33) cannot hold. In order for D1 condition not to apply, it is therefore necessary that (32) holds for such marginal deviations, that is  $\gamma \leq \frac{\phi(x)}{1-\lambda}$ . However, above we have shown that if this is the case, (32) also holds for non marginal higher deviations. But then, (33) can never hold and it is therefore irrelevant.

*Second*, consider a deviation  $\sigma' < \sigma$ . We may restrict to deviations  $\sigma' \in [\underline{\sigma}, \sigma]$  where  $\underline{\sigma}$  is such that if a deviation to  $\underline{\sigma}$  was considered to come from a high type for sure, the sender would get the same payoff from deviation as the high type gets in equilibrium, i.e.  $1 - C(\underline{\sigma}) = \pi_H(\sigma, p) - C(\sigma)$ . This is because for  $\sigma' < \underline{\sigma}$  the high type would not want to deviate for any beliefs of the receiver. Following similar reasoning to that of the case  $\sigma' > \sigma$ , in order for D1 condition not to hold against the low type for  $\sigma' \in [\underline{\sigma}, \sigma]$ , we need that (32) holds. Thus, since for  $\sigma' = \sigma$  equation (32) holds with equality, for lower deviations  $\sigma' \rightarrow \sigma$  we would need the following expression to be positive

$$\frac{d[\pi_H(\sigma', p_2) - C(\sigma')]}{d\sigma'} = \left[ \frac{\phi(x(\sigma', p_2))}{1 - \lambda_2} - \gamma \right] \frac{d\sigma'}{d\sigma'}. \quad (37)$$

The algebra for (37) is the same as for higher deviations. However, we now want it to be positive, that is,  $\gamma \geq \frac{\phi(x)}{1-\lambda}$ . In other words to have compatibility the conditions on high and low deviations, we want (37) to have a global maximum at  $\sigma' = \sigma$ . Therefore, necessary and sufficient condition for  $(\sigma, p)$  to survive as part of a D1 equilibrium, is that

$$\gamma = \frac{\phi(x)}{1 - \lambda}.$$

*Proof of Proposition 2:*

Part 1. The statement follows from the proof of Part 1 of Lemma 3.

Part 2. To prove part 2, one has to examine condition (15). On the one hand, its LHS is constant. On the other hand, fixing  $p$ , the RHS first increases in  $\sigma$ , until it reaches a maximum at  $\sigma_{opt}$ , and then decreases towards zero as  $\sigma$  approaches infinity. This is illustrated in Figure 6. As  $p$  increases, the RHS of (15) shifts down, and  $\sigma_{opt}$  moves right. Let  $\sigma_\gamma^1(p)$  and  $\sigma_\gamma^2(p)$  be the two intersections between the LHS and RHS of (15), when they do intersect. Given prior  $p$ , a pooling equilibrium on a

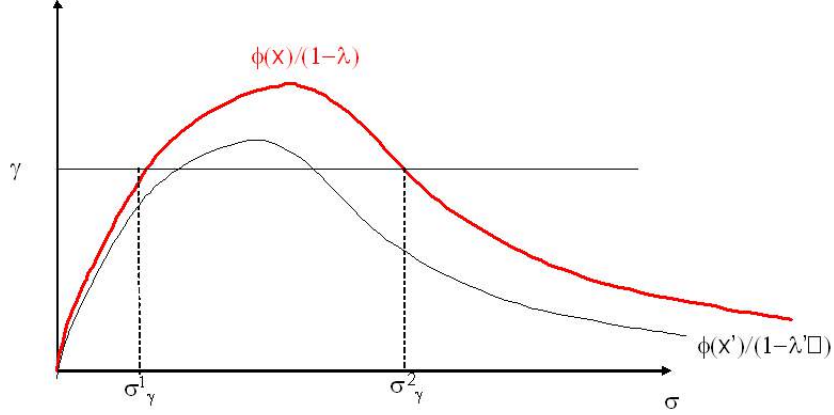


Figure 6: D1 equilibria with optimistic prior.

signal  $\sigma < \infty$  satisfies the D1 criterion if and only if  $\sigma = \sigma_\gamma^1(p)$  or  $\sigma_\gamma^2(p)$ . It follows that the signals  $\sigma_\gamma^1$  and  $\sigma_\gamma^2$  are D1 pooling equilibria if and only if they are pooling equilibria, which in the optimistic case is equivalent to that they are greater than  $\sigma_0(p)$ .

When  $p$  is very close to 1 so that the LHS and RHS of (15) do not intersect, pooling on  $+\infty$  is the unique D1 equilibrium. We need to see under which conditions  $\sigma_\gamma^1$  and  $\sigma_\gamma^2$  exist and are pooling equilibria.

Let  $p_{1/2}$  be the interim posterior belief such that the signal at which the low type is getting zero payoff equals the signal at which the probability of acceptance of the high type is at its minimum,

$$\sigma_0(p_{1/2}) = \sigma_{opt}(p_{1/2}).$$

Notice that, since at  $\sigma_{opt}(p)$ ,  $x + a = 0$ ,  $\pi_L(p, \sigma_{opt}(p)) = 1/2$  for all  $p < 1$ . In the meantime,  $\sigma_0(p)$  is defined by  $\pi_L(p, \sigma_0) = C(\sigma_0)$ . We show that  $p_{1/2}$  is well defined. When  $p \rightarrow \bar{p}$  or  $\lambda \rightarrow 1$ ,  $\sigma_0(p) < \sigma_{opt}(p)$ . This is because in the limit,  $x + a = a/2$  and  $\pi_L(\sigma, p) = 1 - \Phi(a/2)$ . As  $p \rightarrow \bar{p}$ , while  $\sigma_0$  is defined by  $1 - \Phi(a/2) = C(\sigma_0)$ ,  $\sigma_{opt}$  must satisfy  $1 - \Phi(a/2) = 1/2$ , and therefore  $\sigma_0(p) < \sigma_{opt}(p)$ , as the latter approaches  $\infty$ . When  $p \rightarrow 1$  or  $\lambda \rightarrow 0$ ,  $\sigma_0(p) > \sigma_{opt}(p)$ . This is because as  $p$ ,  $\pi_L(p, \sigma_0(p))$  converges to 1, which implies that when  $p \rightarrow 1$  we have that  $\pi_L(p, \sigma_0(p)) - \pi_L(p, \sigma_{opt}(p)) > 0$ .

This in turn implies  $\sigma_0(p) > \sigma_{opt}(p)$ . Since  $\pi_L(p, \sigma_0(p))$  is increasing in  $p$ , we have that, for any  $\gamma$ , there is a unique  $p_{1/2}$  such that: if  $p < p_{1/2}$  then  $\sigma_0(p) < \sigma_{opt}(p)$  and if  $p > p_{1/2}$  then  $\sigma_0(p) > \sigma_{opt}(p)$ .

We also know from Proposition 1 that for any fixed  $\gamma$ , there exists a  $\tilde{p}$  that satisfies (16), i.e.,

$$\gamma = \frac{\phi(x(\sigma_0(\tilde{p}), \tilde{p}))}{1 - \lambda(\tilde{p})}.$$

We need to see whether  $\sigma_0(\tilde{p}) = \sigma_\gamma^1(\tilde{p})$  or  $\sigma_0(\tilde{p}) = \sigma_\gamma^2(\tilde{p})$ .

From the proof of Proposition 1, we know that  $\phi(x(\sigma_0(p), p)/[1 - \lambda(p)])$  is decreasing in  $p$ , that  $\gamma < \phi(x(\sigma_0(p), p)/[1 - \lambda(p)])$  at  $p = \tilde{p}$  and  $\gamma > \phi(x(\sigma_0(p), p)/[1 - \lambda(p)])$  at  $p = 1$ .

If  $\tilde{p} \geq p_{1/2}$  and therefore  $\sigma_0(\tilde{p}) > \sigma_{opt}(\tilde{p})$ , then  $\sigma_0(\tilde{p}) = \sigma_\gamma^2(\tilde{p})$ . So at  $p = \tilde{p}$ , there exist two D1 equilibria:  $\sigma_\gamma^2$  and  $+\infty$ . For  $p$  higher than  $\tilde{p}$ , pooling on  $+\infty$  is the only D1 equilibrium. This is because for higher  $p$ , the whole curve  $\phi(x(\sigma, p))/[1 - \lambda(p)]$  shifts down and to the left,  $\sigma_{opt}(p) < \sigma_0(p) < \sigma_0(\tilde{p})$ , and  $\phi(x(\sigma_0(p), p)/[1 - \lambda(p)]) < \gamma$ , implying that  $\sigma_0(p)$  is to the right of  $\sigma_\gamma^2(p)$ . Conversely, for  $p$  lower than  $\tilde{p}$ , the whole curve  $\phi(x(\sigma, p))/[1 - \lambda(p)]$  shifts up and to the right,  $\sigma_0(p) > \sigma_0(\tilde{p})$ , and  $\phi(x(\sigma_0(p), p)/[1 - \lambda(p)]) > \gamma$ , implying that  $\sigma_0(p)$  is to the left of  $\sigma_\gamma^2(p)$ , which means that  $\sigma_\gamma^2(p)$  is a D1 equilibrium.

If  $\tilde{p} < p_{1/2}$ , we have  $\sigma_0(\tilde{p}) < \sigma_{opt}(\tilde{p})$ , which implies  $\sigma_0(\tilde{p}) = \sigma_\gamma^1(\tilde{p})$ . At  $\tilde{p}$ , there are three D1 equilibria:  $\sigma_\gamma^1$ ,  $\sigma_\gamma^2$  and  $+\infty$ . Let  $p_\gamma$  be the maximum prior such that  $\sigma_\gamma^1$  and  $\sigma_\gamma^2$  are well defined. In other words,  $p_\gamma$  satisfies  $\gamma = \phi(x(\sigma_{opt}(p), p)/[1 - \lambda(p)])$ . Clearly, for priors  $p > p_\gamma$ ,  $+\infty$  is the only D1 pooling equilibrium. If  $p \in (\tilde{p}, p_\gamma)$ , there continue to exist three D1 equilibria:  $\sigma_\gamma^1(p)$ ,  $\sigma_\gamma^2(p)$ , and  $+\infty$ . This is because higher  $p$ , have lower  $\sigma_0(p)$  and  $\gamma > \phi(x)/(1 - \lambda)$ , implying that  $\sigma_0(p) < \sigma_\gamma^1(p)$  and, therefore, that  $\sigma_\gamma^1(p)$  and  $\sigma_\gamma^2(p)$  are both pooling equilibria. Conversely, for  $p$  lower than  $\tilde{p}$ , as before  $\sigma_0(p)$  is higher and  $\gamma < \phi(x)/(1 - \lambda)$ . However, now this implies that  $\sigma_0(p) \in [\sigma_\gamma^1(p), \sigma_\gamma^2(p)]$  and therefore only  $\sigma_\gamma^2(p)$  and  $+\infty$  are D1 equilibria.

Part 3 The statement follows from Proposition 1.

*Proof of Proposition 3.* 1. In the optimistic case, if the precision is  $a = 0$ , then the sender's expected payoff is 1, the maximum value he can achieve. Any

precision  $a > 0$  will result in a payoff strictly less. So  $a = 0$  is the unique optimal precision level.

For notational convenience, let  $u(a)$  be the expected payoff of the sender from committing to a precision  $a$ . Note that  $u(0) = 0$  in the pessimistic case and  $u(0) = 1/2$  in the neutral case by our tie-breaking assumption. On the other hand,  $u(\infty) = -\infty$ , so  $a = \infty$  is never optimal. Nevertheless, it is useful to analyze the behaviour of  $u$  as  $a \rightarrow \infty$ .

To find the optimal precision in the neutral and pessimistic cases, we need to first investigate the existence of a solution to the first order condition  $u'(a) = 0$ , as expressed by (19). After that, we will compare the sender's payoff at these critical points with his expected payoff at  $a = 0$ .

We now proceed to the analysis of the neutral and pessimistic cases.

2. In the neutral case, the derivative of the sender's expected payoff with respect to precision  $a$ ,  $u'(a)$ , is the LHS of (20), reproduced below.

$$\frac{1}{2}(2\bar{p} - 1)\phi(x) - \gamma = 0.$$

Using the fact that  $x = -a/2$  in the neutral case, we can see  $u''(a) < 0$  for all  $a \in [0, \infty)$ . Therefore, (20) is both necessary and sufficient for an  $a > 0$  to maximize the sender's expected payoff. An inspection of (20) will give the desired conclusions.

3. Using (9), (10), and (27), we may write

$$u'(a) = p\phi(x) \left[ \frac{x+a}{a} + \frac{\theta_H - \theta_M}{\theta_M - \theta_L} \cdot \frac{x}{a} \right] - \gamma. \quad (38)$$

Note that, as we do in the main text, the dependence of  $x$  on  $a$  is suppressed. Using the definitions  $a = (\theta_H - \theta_L)/\sigma$  and  $x = (\bar{s} - \theta_L)/\sigma$  (recall that  $\bar{s}$  is the cutoff signal realization for acceptance), we obtain

$$u'(a) = p\phi(x) \frac{\theta_M - \bar{s}}{\theta_M - \theta_L} - \gamma. \quad (39)$$



In the pessimistic cases, using Lemma 6,  $\bar{s}$ , the cutoff signal realization for acceptance, is decreasing in  $a$  (or increasing in  $\sigma$ ). Furthermore, using (26), when  $a = 0$  ( $\sigma = \infty$ ),  $\bar{s} = \infty$ ; when  $a = \infty$  ( $\sigma = 0$ ),  $\bar{s} = (\theta_H + \theta_L)/2$ . Therefore, whenever  $\theta_M \leq (\theta_H + \theta_L)/2$ , (39) is always negative and therefore the optimal precision is  $a = 0$ , which proves (a).

To prove (b), let  $a_M$  be the precision level such that  $\bar{s} = \theta_M$ . For all  $a \leq a_M$ ,  $\bar{s} > \theta_M$ , so  $u'(a) < 0$ . For  $a > a_M$ , using (27) and (39), we have

$$\begin{aligned} u''(a) &= \frac{p}{\theta_M - \theta_L} \left[ -x\phi(x) \frac{-(x+a)}{a} (\theta_M - \bar{s}) + \phi(x) [-(2x+a)] \cdot \left[ -\frac{\sigma}{a} \right] \right], \\ &= \frac{p\phi(x)}{a(\theta_M - \theta_L)} \left[ 1 - \frac{x+a}{2x+a} \cdot \frac{-x(\theta_M - \bar{s})}{\sigma} \right]. \end{aligned} \quad (40)$$

Note that

$$\begin{aligned} \frac{d}{da} \left( \frac{x+a}{2x+a} \right) &= \frac{-x(2x+a) - (x+a) \cdot [-(2x+a)]}{(2x+a)^2}, \\ &= \frac{a}{2x+a}, \\ &> 0. \end{aligned}$$

In addition,

$$\frac{-x(\theta_M - \bar{s})}{\sigma} = \frac{\theta_H - \bar{s}}{\sigma} \cdot \frac{\theta_M - \bar{s}}{\sigma},$$

which increases with  $a$  for  $a > a_M$  as  $\bar{s}$  is decreasing in  $a$  (hence  $\bar{s} < \theta_M < \theta_H$  for  $a > a_M$ ) and  $\sigma$  is inversely related to  $a$  (recall that  $a \equiv (\theta_H - \theta_L)/\sigma$ ). The above two remarks imply that  $u''$  is decreasing in  $a$ .

Given that  $u'(a_M) < 0$ ,  $u''(a_M) > 0$ ,  $\lim_{a \rightarrow \infty} u'(a) = -\gamma$  and  $u''$  is decreasing in  $a$  for  $a > a_M$ , we conclude that generically (except the value of  $\gamma$  such that  $u'(a) = 0$  and  $u''(a) = 0$  have the same solution), the function  $u$  has zero or two critical points. When it does have two critical points, the right one is a local maximum point. Evaluating  $u$  at this point and comparing its value with zero will tell us whether the optimal precision is 0 or interior. Note that by the Envelope Theorem, at the interior critical point, the sender's expected payoff is decreasing in  $\gamma$ . Therefore, we conclude that an interior optimal precision is the solution only if  $\gamma$  is small enough. ■

*Proof of Proposition 4:*

Part 1. We have already provided the argument in the main text. Both under commitment and in the best equilibrium under discretion the sender would choose to not provide any information.

Part 2. The payoff of the sender under commitment can be written as

$$u^* = \bar{p}[1 - \Phi(x)] + (1 - \bar{p})\Phi(x) - \gamma a,$$

where  $x = -a/2$  must satisfy (20). Indeed, using (20) and the fact that  $x = -a/2$ , we obtain

$$u^* = \bar{p} + (2\bar{p} - 1) [\phi(x)x - \Phi(x)],$$

On the other hand, the discretion payoff is

$$\tilde{u} = \bar{p} [\pi_H(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma})].$$

By Corollary 1.1,  $\pi_H(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma})$  is dependent only on the cost function (or  $\gamma$ ). Therefore,

$$\frac{u^* - \tilde{u}}{\bar{p}} = 1 - [\pi_H(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma})] + \frac{2\bar{p} - 1}{\bar{p}} [\phi(x)x - \Phi(x)].$$

Note that the first two terms do not depend on  $\bar{p}$ . So, as  $\bar{p}$  varies, the change in  $(u^* - \tilde{u})/\bar{p}$  is fully accounted for the change in the last term. Let us investigate how then it changes with  $\bar{p} \in [1/2, 1]$ . Note that its derivative with respect to  $\bar{p}$  can be written as

$$\begin{aligned} \frac{d(u^* - \tilde{u})/\bar{p}}{d\bar{p}} &= \frac{1}{\bar{p}^2} [\phi(x)x - \Phi(x)] + \frac{2\bar{p} - 1}{\bar{p}} [-x^2\phi(x)] \frac{\partial x}{\partial \bar{p}}, \\ &= \frac{\phi(x)x - \Phi(x) + 2\bar{p}(-x)\phi(x)}{\bar{p}^2}, \\ &= \frac{\gamma a - \Phi(x)}{\bar{p}^2}, \end{aligned}$$

where in the second equality sign we have used  $\partial x/\partial \bar{p}$  from (20)

$$\frac{\partial x}{\partial \bar{p}} = -\frac{2\phi(x)}{-x\phi(x)} = \frac{2}{(2\bar{p} - 1)x},$$

and in the third we have used (20) directly and the fact that in the neutral case  $x = -a/2$ . Note that  $\Phi(x)$  is the low type's probability of acceptance and  $\gamma a$  is the cost of signalling. This implies that  $(u^* - \tilde{u})/\bar{p}$  is increasing if and only if the optimal choice  $x$  under commitment is not a pooling equilibrium (in the perfect Bayesian sense). Note that by Lemma 5, if the optimal  $x$  under commitment is not a pooling equilibrium, then discretion is strictly better, i.e.,  $u^* < \tilde{u}$ . Consequently,  $(u^* - \tilde{u})/\bar{p}$  is always weakly decreasing in  $\bar{p}$  whenever commitment is weakly better than discretion. This implies that  $(u^* - \tilde{u})/\bar{p}$ , as a function of  $\bar{p}$ , can cross the horizontal axis at most once, and if at all, from above (from the left hand side), given that at  $\bar{p} = 1/2$ ,  $u^* > \tilde{u}$ . So, there exists a cutoff  $\bar{p}^c \in (1/2, 1]$ , such that the sender prefers commitment to discretion if and only if  $\bar{p} \leq \bar{p}^c$  (if  $\bar{p}^c = 1$ , then the sender always prefers commitment to discretion when the prior is neutral, which could occur when  $\gamma$  is relatively large).

Part 3. The argument is due to Lemma 5.

Part 4. We may first write (17) as

$$\max_{\sigma \in M} p^0 [1 - \Phi(x)] + (1 - p^0) [1 - \Phi(x + a)] - C(\sigma).$$

Note that in the receiver's optimality condition (10),  $\theta_M$  enters only through  $\lambda$ —an increase in  $\theta_M$  makes  $\lambda$  bigger or makes the receiver more pessimistic. Now, applying the implicit function theorem to (10), we obtain

$$\frac{\partial x}{\partial \lambda} = -\frac{\phi(x + a)}{-x\phi(x) + \lambda(x + a)\phi(x + a)} = \frac{1}{\lambda a} > 0,$$

where we have applied (10) to the denominator of the intermediate expression. This simply means that as the receiver becomes more pessimistic, the probability of acceptance goes down for both the high type and the low type.

Now applying the Envelope Theorem to the sender's optimal commitment prob-

lem above (ignoring how the optimal  $\sigma$  depend on  $\theta_M$ ), we have

$$\begin{aligned}\frac{\partial u^*}{\partial \theta_M} &= \frac{\partial u^*}{\partial \theta_M}, \\ &= \frac{\partial u^*}{\partial x} \cdot \frac{\partial x}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial \theta_M}, \\ &= [-p^0 \phi(x) - (1 - p^0) \phi(x + a)] \cdot \frac{1}{\lambda a} \cdot \frac{\partial \lambda}{\partial \theta_M}, \\ &< 0.\end{aligned}$$

On the other hand, the discretion payoff is equal to

$$\tilde{u} = p^0 [\pi_H(\tilde{\sigma}, \tilde{p}) - C(\tilde{\sigma})],$$

which is constant, fixing  $p^0$ . Therefore, as we increase  $\theta_M$ , discretion is more likely to dominate commitment, and there exists a cutoff  $\theta_M^* \in [\bar{\theta}_M(p^0), \theta_H]$ , such that the sender prefers commitment to discretion if and only if  $\theta_M \leq \theta_M^*$ , where  $\bar{\theta}_M(p^0)$  is the  $\theta_M$  that makes the receiver neutral.

## 8 Appendix B: results on the probabilities of acceptance

In this appendix, we derive properties of the probabilities of acceptance for the low type and the high type in all three cases: optimistic, neutral, and pessimistic.

### 8.1 Low type

To see how the expected payoff of the low type varies with  $\sigma$ , fixing belief  $p$ , note that

$$\frac{\partial \pi_L(\sigma, p)}{\partial \sigma} - \frac{\partial C}{\partial \sigma} = -\frac{x}{\sigma} \phi(x + a) + \gamma \frac{a}{\sigma}, \quad (41)$$

which is positive if and only if:

$$\gamma > \gamma_L \equiv \frac{x}{a} \phi(x + a). \quad (42)$$

In the *neutral and optimistic cases*,  $\pi_L(0, p) < C(0)$  while  $\pi_L(\infty, p) > C(\infty) = 0$ . In addition, while  $\pi_L$  is always increasing in  $\sigma$ ,  $C$  is always decreasing. Therefore, the difference between the two is always increasing, for any cost function satisfying the standard assumptions. When we restrict to cost functions of the type in (2), we can

see this also by noticing that since  $x < 0$ , condition (42) is satisfied for all  $\gamma > 0$ . We have that  $\pi_L(\sigma, p)$  and  $C(\sigma)$  intersect once and only once.

In the *pessimistic case*, without further assumptions, there is no guarantee that the two curves cross. We now show that in the pessimistic case, for the cost function in (2), whenever  $\pi_L(\sigma, p)$  and  $C(\sigma)$  intersect, they intersect twice. Observe that when  $\sigma$  approaches zero,  $\pi_L(\sigma, p)$  is below  $C(\sigma)$ . When  $\sigma \rightarrow \infty$ , while  $\pi_L(\sigma, p)$  and  $C(\sigma)$  converge to 0, as well as  $\partial\pi_L/\partial\sigma$  and  $\partial C/\partial\sigma$ , we can show that  $\pi_L(\sigma, p)$  is also below  $C(\sigma)$ . To see this, consider the ratio

$$\frac{\partial\pi_L(\sigma, p)/\partial\sigma}{\partial C/\partial\sigma} = \frac{-x\phi(x+a)/\sigma}{-\gamma a/\sigma} = \frac{x}{a} \cdot \frac{\phi(x+a)}{\gamma}.$$

Recall that  $x/a = \ln \lambda/a^2 - 1/2 = \sigma^2 \ln \lambda / (\theta_H - \theta_L)^2 - 1/2$ . When  $\sigma \rightarrow \infty$ ,  $x/a \rightarrow \infty$  at the speed of  $\sigma^2$ . The variable  $(x+a)$  goes to infinity at the same speed as  $\sigma$ . So  $\phi(x+a)$  goes to zero at a faster rate than  $\sigma^2$  and therefore  $\frac{\partial\pi_L}{\partial\sigma} / \frac{\partial C}{\partial\sigma} \rightarrow 0$ . It could therefore be that the two curves never cross and  $\pi_L(\sigma, p)$  is always below  $C(\sigma)$  or, if they cross, they must cross an even number of times. In other words, if the two curve cross, since both converge to zero as sigma grows large, the difference cannot be always increasing. At the critical points, the following FOC must be satisfied:

$$\frac{\partial\pi_L}{\partial\sigma} - \frac{\partial C}{\partial\sigma} = -\frac{x}{\sigma}\phi(x+a) + \gamma\frac{a}{\sigma} = 0.$$

that is, from (42)

$$\gamma = \phi(x+a)\frac{x}{a} = \gamma_L. \quad (43)$$

For  $\sigma < \sigma_{pes}$  (the point of max of  $\pi_L$ , and at which  $x = 0$ ),  $x < 0$ . Clearly  $\gamma_L < \gamma$ , for all  $\gamma > 0$ .<sup>30</sup>In fact, for  $x < 0$ ,  $\gamma_L$  is increasing in  $\sigma$  ( $x+a$  is positive and decreasing, meaning that  $\phi(x+a)$  is increasing, and  $\frac{x}{a}$  is increasing). We can restrict our attention to what happens for  $x > 0$ ,  $\sigma > \sigma_{pes}$ . We take the total derivative with respect to  $\sigma$  and obtain  $\frac{\partial\gamma_L}{\partial\sigma} = -\frac{\phi(x+a)}{\sigma a}[(x+a)(x^2-1)+x]$ . We have that  $\partial\gamma_L/\partial\sigma = 0$  if and only if

$$x^2 - 1 = \frac{x}{x+a}.$$

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<sup>30</sup>That is,  $\partial\pi_L/\partial\sigma - \partial C/\partial\sigma > 0$ .

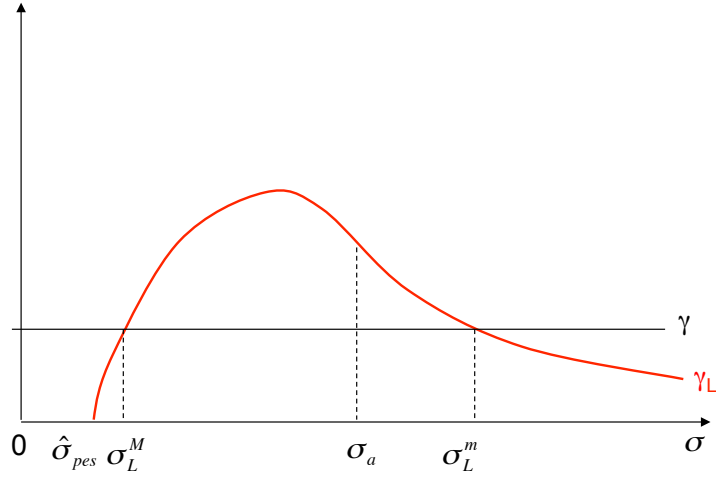


Figure 7: Maximum and minimum of  $\pi_L - C$ , pessimistic prior.

We know that  $x$  is increasing in  $\sigma$  so we can analyze the above expression in terms of  $x$ : The LHS is increasing and convex in  $x$ . When  $x = 0$  it equals  $-1$ . It equals 0 at  $x = 1$  and equals 1 at  $x = \sqrt{2}$ . The RHS is increasing and concave in  $x$ .<sup>31</sup> When  $x = 0$  it equals 0 and it is always less than 1. It follows that a unique intersection exists between the two curves at some  $\sigma \in (\sigma_a, \sigma_b)$ , where  $\sigma_a = \frac{(\theta_H - \theta_L)}{2 \ln \lambda} (1 + \sqrt{1 + 2 \ln \lambda}) = \frac{(\theta_H - \theta_L)}{\sqrt{1 + 2 \ln \lambda} - 1}$  is such that  $x(\sigma_a) = 1$  and  $\sigma_b = \frac{(\theta_H - \theta_L) \sqrt{2}}{2 \ln \lambda} [1 + \sqrt{1 + \ln \lambda}]$  is such that  $x(\sigma_b) = \sqrt{2}$ .

We have just shown that  $\gamma_L$  initially ( $\sigma = 0$ ) is below  $\gamma$ , then it increases and reaches a maximum at some  $\sigma \in (\sigma_{pes}, \sigma_a)$ . Afterwards it decreases and converges to zero ( $(x + a) \rightarrow x \rightarrow \infty, x/a \rightarrow \infty$  but at a lower rate than  $\phi(x + a) \rightarrow 0$ ). The following Figure 7 represents this situation:

Between the two levels of  $\sigma$  that satisfy (43), the smallest  $\sigma_L^M$  corresponds to a maximum and the second  $\sigma_L^m$  to a minimum. Once again, if  $\gamma > \max \gamma_L \equiv \bar{\gamma}_L$  we are sure that the probability of acceptance and the cost do not intersect. If,  $\gamma < \bar{\gamma}_L$  we know that the difference between the two curves reach a local maximum and a local

<sup>31</sup>Notice that  $\frac{da}{dx} = -a/(x + a) < 0$ , and  $\frac{dx/(x+a)}{dx} = \frac{(x+a) - x(1+da/dx)}{(x+a)^2} = \frac{x+a-x+xa/(x+a)}{(x+a)^2} = \frac{a(x+a)+xa}{(x+a)^3} = \frac{2ax+a^2}{(x+a)^3} > 0$ . This can be rewritten as  $\frac{dx/(x+a)}{dx} = \frac{(x+a)^2 - x^2}{(x+a)^3}$  therefore,  $\frac{d^2x/(x+a)}{d^2x} = \frac{-2ax + [-a^2 - 4ax]x}{(x+a)^5} < 0 \forall x > 0$ .

minimum but we cannot be sure they intersect. For this we need  $\gamma$  to be small enough. A sufficient condition for the two curves to cross is that  $\gamma < \gamma^p$ , where  $\gamma^p = \frac{1-\Phi(\sqrt{2\ln\lambda})}{\sqrt{2\ln\lambda}}$  is the value of  $\gamma$  such that  $\pi_L(\sigma, p)$  and  $C(\sigma)$  intersect exactly at  $\hat{\sigma}_{pes}$ .

## 8.2 High type

For the high type,

$$\frac{\partial\pi_H(\sigma, p)}{\partial\sigma} - \frac{\partial C}{\partial\sigma} = \frac{-(x+a)\phi(x)}{\sigma} + \gamma\frac{a}{\sigma},$$

which is positive if and only if:

$$\gamma > \phi(x) \frac{x+a}{a} = \phi(x) \left[ \frac{x}{a} + 1 \right] \equiv \gamma_H \quad (44)$$

In the *neutral case*, by Lemma 6,  $x+a = a/2$  and  $x = -a/2$ . Condition (44) is satisfied iff  $\gamma > \max_{\sigma} \frac{\phi(a/2)}{2} = \phi(0)/2$ . Since  $\pi_H(0, p) < C(0)$ , we are sure there exists a unique intersection between the two curves as long as  $\pi_H(\infty, p) > C(\infty) = 0$ , which is certainly the case if as a tie-breaking rule we assume that  $\pi_H(\infty, p) = 1/2$ .

In the *optimistic case*,  $x+a$  is first positive then negative. In addition, similar as above,  $\pi_H(0, p) < C(0)$  and  $\pi_H(\infty, p) = 1 > C(\infty) = 0$ . Since  $C' < 0$ , we know that an intersection between  $\pi_H(\sigma, p)$  and  $C(\sigma)$  exists. We now find conditions under which  $\frac{\partial\pi_H(\sigma, p)}{\partial\sigma} - \frac{\partial C}{\partial\sigma} > 0$ , which assures that such intersection is unique.

When  $(x+a) \leq 0$ , which happens when  $\sigma \geq \sigma_{opt}(p)$ ,  $\frac{\partial\pi_H(\sigma, p)}{\partial\sigma} - \frac{\partial C}{\partial\sigma} > 0$  for all  $\gamma > 0$ . When  $(x+a) > 0$ , which happens when  $\sigma < \sigma_{opt}(p)$  (recall that at  $\sigma_{opt}(p)$ ,  $\pi_H(\sigma, p)$  reaches a minimum and  $x+a = 0$ ) we need to impose  $\gamma > \max_{\sigma} \gamma_H = \bar{\gamma}_H$ . Since  $\gamma_H(\sigma)$  is differentiable, the FOC characterizes its critical points:

$$\begin{aligned} \frac{\partial\gamma_H}{\partial\sigma} &= -x\phi(x) \left[ \frac{x}{a} + 1 \right] \frac{\partial x}{\partial\sigma} + \frac{\phi(x)}{a^2} \left[ \frac{\partial x}{\partial\sigma} a - x \frac{\partial a}{\partial\sigma} \right] \\ &= \phi(x) (-x) \left( \frac{\partial x}{\partial\sigma} \left[ \frac{x}{a} + 1 - \frac{1}{ax} \right] - \frac{1}{a\sigma} \right) \\ &= \phi(x) (-x) \left( \frac{x+a}{\sigma} \left[ \frac{x+a}{a} - \frac{1}{ax} \right] - \frac{1}{a\sigma} \right) \\ &= \frac{-\phi(x)}{\theta_H - \theta_L} \left( (x+a)^2 x - (x+a) - x \right) = 0. \end{aligned} \quad (45)$$

We want to find a solution to  $((x+a)^2 x - (x+a) - x) = 0$ . We re-write this expression in terms of  $(x+a)$  only. Let  $y = (x+a)$ , we want to find the solution to

$$\begin{aligned} y^2(y-a) - y - (y-a) &= 0, \\ (y^2 - 1)(y-a) - y &= 0, \end{aligned}$$

which we may rearrange into

$$y^2 - 1 = \frac{y}{y-a} \quad (46)$$

We are interested in intersections at some  $y > 0$ ,  $y < a/2$ .

The LSH starts at -1 and is increasing, it crosses the x-axis at  $y = 1$ . Since at  $y = a/2$ , the RHS is  $-1$ , the intersection must be at some  $y < 1$ . Notice that  $\frac{da}{dy} = \frac{-a}{x}$ , therefore  $\frac{dy/(y-a)}{dy} = \frac{-a}{(y-a)^2} [1 + y/x] < 0$ . This is because

$$\frac{da}{dy} = \frac{da/d\sigma}{dy/d\sigma} = \frac{-a/\sigma}{\frac{dx}{d\sigma} + \frac{da}{d\sigma}} = \frac{-a/\sigma}{\frac{x+a}{\sigma} - \frac{a}{\sigma}} = \frac{-a}{x},$$

$$\frac{dy/(y-a)}{dy} = \frac{da/d\sigma}{dy/d\sigma} = \frac{(y-a) - y[1 - da/dy]}{(y-a)^2} = \frac{-a - ya/x}{(y-a)^2} = \frac{-a}{(y-a)^2} [1 + y/x],$$

and

$$1 + \frac{x+a}{x} > 0,$$

for all  $|x+a| < |x|$ . It follows that there is a unique intersection of (46) and therefore a unique point that satisfies the FOC (45). In other words  $\bar{\gamma}_H$  is well defined. When  $\gamma > \bar{\gamma}_H$ , we know that the difference between the two curves is always increasing, and, therefore, there is a unique intersection between  $\pi_H(\sigma, p)$  and  $C(\sigma)$ . Since  $x \leq -a/2$  and therefore  $(x+a) \leq a/2$ , we have that  $\bar{\gamma}_H \leq \phi(0)/2$ , which is therefore sufficient condition for  $\pi_H(\sigma, p) - C(\sigma)$  to be increasing in all its domain.

In the *pessimistic case*,  $\partial\pi_H(\sigma, p)/\partial\sigma < 0$ . Without further assumptions, there is no guarantee that  $\pi_H(\sigma, p)$  crosses  $C(\sigma)$ . It should be noticed that when  $\sigma \rightarrow \infty$ , both  $\partial\pi_H(\sigma, p)/\partial\sigma$  and  $\partial C/\partial\sigma$  converge to zero. To know which one converges faster we look at

$$\frac{\partial\pi_H(\sigma, p)/\partial\sigma}{\partial C/\partial\sigma} = \frac{-(x+a)\phi(x)/\sigma}{\gamma a/\sigma} = \frac{(x+a)\phi(x)}{\gamma a},$$



where using the explicit expressions for  $x$  and  $a$  we have  $\frac{x+a}{a} = \frac{\ln \lambda}{a^2} + \frac{1}{2} = \frac{\sigma^2 \ln \lambda}{(\theta_H - \theta_L)^2} + \frac{1}{2}$ . Therefore when  $\sigma \rightarrow \infty$  we have that  $\frac{x+a}{a} \rightarrow \infty$  at the speed of  $\sigma^2$ . The variable  $x$  goes to infinity at the same speed as  $\sigma$ . So  $\phi(x)$  goes to zero at a higher rate than  $\sigma^2$  and therefore  $\frac{\partial \pi_H(\sigma, p)}{\partial \sigma} / \frac{\partial C}{\partial \sigma} \rightarrow 0$ . It follows that when  $\sigma$  is very large,  $\pi_H(\sigma, p)$  is below  $C(\sigma)$ . Since  $\pi_H(0, p) < C(0)$  and  $\lim_{\sigma \rightarrow \infty} \pi_H(\sigma, p) - C(\sigma) < 0$ , if  $\pi_H(\sigma, p)$  and  $C$  intersects, they must cross an even number of times. In addition, since both converge to zero as sigma grows large, if the two curves intersect, the difference cannot be always increasing, there must be a point to the right of the first intersection where the difference achieves a maximum. It could also achieve other local critical points. At the critical points, the following FOC must be satisfied:

$$\gamma = \phi(x) \frac{x+a}{a} \equiv \gamma_H. \quad (47)$$

The problem now is, for fixed  $\gamma$  to find  $\sigma$  where (47) is satisfied. Using the explicit expressions for  $x$ ,  $x+a$ , and  $a$ , The RHS can be rewritten as

$$\phi \left( \frac{\sigma \ln \lambda}{\theta_H - \theta_L} + -\frac{\theta_H - \theta_L}{\sigma} \right) \left( \frac{\sigma^2 \ln \lambda}{(\theta_H - \theta_L)^2} + \frac{1}{2} \right).$$

It goes to zero both for  $\sigma \rightarrow 0$  and for  $\sigma \rightarrow \infty$ . In the pessimistic case, it is always positive and, in addition, we have showed in the treatment of the optimistic case that it has a unique maximum  $\bar{\gamma}_H = \max_{\sigma} \gamma_H$ . As it is also represented in Figure 8 below, it follows that if there exist  $\sigma$  that satisfy (47) there must be two of them.

If  $\bar{\gamma}_H$  exists, it must be at a value of  $\sigma > \sigma_{pes}$  (recall that at  $\sigma_{pes}$ ,  $x = 0$ ). This is because, as long as  $x < 0$ , both  $\phi(x)$  and  $\frac{x+a}{a}$  increase in  $\sigma$ <sup>32</sup>, so the point where the first order condition is satisfied must be at some  $x > 0$ .

The above considerations also imply that between the two values of  $\sigma$  satisfying (47), the smallest  $\sigma_H^M$  must correspond to a maximum and the biggest  $\sigma_H^m$  to a minimum. This is because  $\pi_H(\sigma, p) - C$  is initially increasing (reaching a maximum) and then decreasing.

It follows that if  $\gamma > \bar{\gamma}_H$ ,  $\frac{\partial \pi_H(\sigma, p)}{\partial \sigma} - \frac{\partial C}{\partial \sigma}$  is always positive, implying that  $\pi_H(\sigma, p)$  is always below  $C(\sigma)$ . If  $\gamma < \bar{\gamma}_H$ , we know that  $\pi_H(\sigma, p) - C(\sigma)$  reaches a maximum and a minimum but this is not enough to assure that they intersect. What we can

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<sup>32</sup>  $\frac{d(x+a)/a}{d\sigma} = \frac{d(\sigma^2 \ln \lambda / (\theta_H - \theta_L)^2 + 1/2)}{d\sigma} = 2\sigma \ln \lambda / (\theta_H - \theta_L)^2$ .

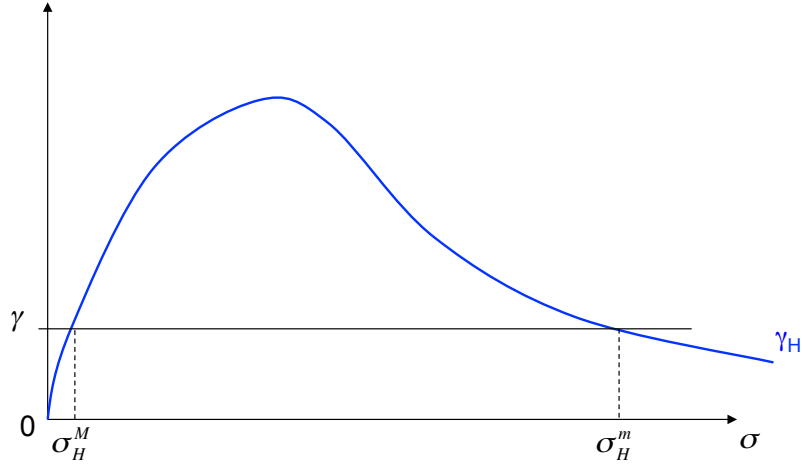


Figure 8: Maximum and minimum of  $\pi_H - C$ , pessimistic prior.

say is that they will intersect for  $\gamma$  low enough. Since  $\pi_H(\sigma, p) > \pi_L(\sigma, p) \forall \sigma$ , if the curve  $\pi_L(\sigma, p)$  intersects  $C(\sigma)$  so will the curve  $\pi_H(\sigma, p)$ .

In the pessimistic case it is useful to compare condition (47) with condition (43). Recalling that in the pessimistic case  $|x| < |x+a|$ , we have that when  $x > 0$ ,

$$\frac{\gamma_H}{\gamma_L} = \frac{\phi(x) \frac{(x+a)}{a}}{\phi(x+a) \frac{x}{a}} = \frac{\overbrace{\phi(x)}^{>1} \overbrace{x+a}^{>1}}{\phi(x+a) x} > 1.$$

So  $\bar{\gamma}_H > \bar{\gamma}_L$  meaning that in order to be sure that there is a maximum between the probability of acceptance for each type and the cost, we only need to impose that  $\gamma < \bar{\gamma}_L$ .<sup>33</sup>

## 9 Appendix C (online)

### SEPARATING EQUILIBRIA

**Lemma 9.** *In the persuasive signalling game*

<sup>33</sup>Notice that  $\gamma_H > \gamma_L$  also when  $x < 0$ , because  $\phi(x)(x+a) > \phi(x+a)x$ .

1. *there exists a unique fully separating equilibrium, where the low type chooses signal  $\sigma_L^S = +\infty$  such that  $C(\sigma_L^S) = 0$  and the high type chooses signal  $\sigma_H^S$  such that  $C(\sigma_H^S) = 1$ ;*
2. *this separating equilibrium is not D1.*

When the two sender types separate, each type chooses a different level of precision. Therefore, the receiver uses only the choice of precision but not the signal realization to infer the quality of the sender. Given our assumption that  $\theta_M \in (\theta_L, \theta_H)$ , it follows that, in a separating equilibrium, the probabilities of acceptance for types  $H$  and  $L$  are respectively 1 and 0. Given that the cost of signalling is strictly increasing in the precision, a separating equilibrium must be in pure strategies. In addition, since the absence of a single-crossing condition means that each type can mimic each other and get the exact payoff of the other type, the incentive compatibility conditions of each type imply that each type must receive the same payoff in a separating equilibrium. This implies the results of part 1 and 2 of the proposition.

The optimal choice for a low type, who is accepted with zero probability is to set precision at its lowest, i.e.,  $\sigma_L^S = +\infty$ , whose cost is assumed to be zero. To prevent deviation by the low type, the high type would have to choose the  $\sigma_H^S$  that gives him a payoff of zero.

For any best responses of the receiver ( $\bar{\sigma}$ ) when she observes a level of precision off the equilibrium path, the high type is always more likely to find it profitable to deviate to that level of precision. The D1 criterion then requires the receiver to believe that the deviating sender is of the high type with probability one. Given the belief, the high type would indeed find it profitable to deviate. Part 2 of Proposition 9 sets our model apart from standard signalling models. The separating equilibrium in our Persuasive Signalling model survives the Intuitive Criterion but fails D1.<sup>34</sup> This is in sharp contrast to a standard Spence [40, 41] job market signalling model, where, if the single-crossing condition is assumed, there are multiple separating equilibria. With two types of the sender, a unique equilibrium survives the Intuitive Criterion,

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<sup>34</sup>In fact, it also fails the weaker refinement of Divinity as long as there exists a finite  $\sigma$  such that  $\pi_H(\sigma, p) - C(\sigma) > 0$ .

while with more than two, a unique D1 equilibrium exists. In the meantime, no pooling equilibrium survives even the intuitive criterion, which is weaker than D1. Also in a Spence job market signalling model without single-crossing condition, there exists a unique separating equilibrium, in which both the high type and the low type are indifferent between choosing high education and low education and it survives D1 because the high type and the low type have exactly the same payoff function. By the same reasoning, any pooling equilibrium also satisfies D1.

#### A DIFFERENT LOOK AT THE D1 CONDITION (15)

Another way to look at the results above on the characterization of D1 equilibria is to analyse, as Daley and Green [11] do, the so-called belief-indifference curves. To facilitate the comparison, it is best to consider indifference curves in the  $(a, 1/\lambda)$  space, where the variable on the horizontal axis corresponds to what we call the precision of the signal, and the one of the vertical axis is just a positive transformation of the interim posterior. On the same curve then one can read the precision, and interim-belief locus that give the same expected payoff to the sender of a given type, knowing that the receiver uses this belief to calculate the acceptance threshold according to (4).

These curves are illustrated in Figure 9, where we have drawn the zero-payoff indifference curve for the low type and different indifferent curves for the high type. The D1-condition in (15) basically says that for a signal  $\sigma$  and associated interim belief  $p$  to be part of a D1 equilibrium it must be that at the corresponding  $(a, 1/\lambda)$  the indifference curves of the high and low types are tangent. We illustrate this point for the semi-separating equilibrium, which is unique and provides a zero payoff to the low type. From the figure we can see that  $(\tilde{a}, 1/\tilde{\lambda})$  is a D1 equilibrium because for any deviation  $a' \neq \tilde{a}$  the set of beliefs for which the high type would want to deviate (beliefs above its indifference curve) is included in the set of beliefs for which the low type would want to deviate. For the same reason, a point like  $(a_1, 1/\lambda_1)$ , where the tangency condition is not satisfied, for local deviations  $a' < a_1$  the high type would want to deviate more often than the low type, and therefore would not survive the D1-refinement.

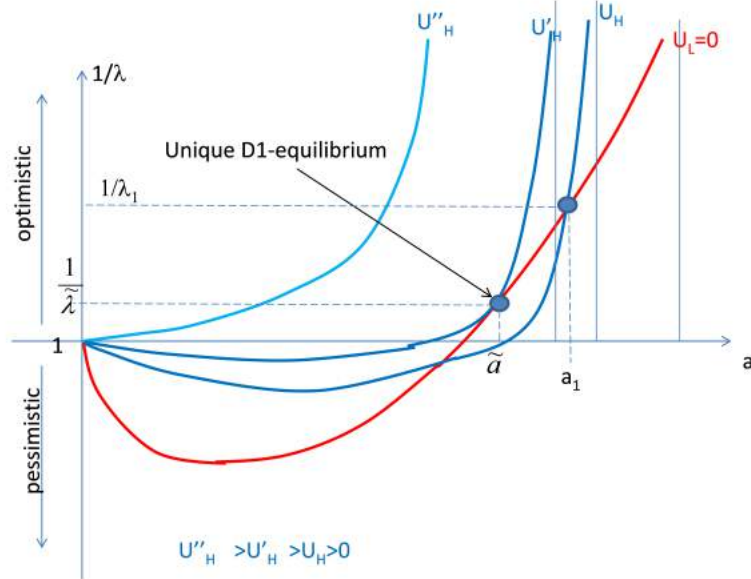


Figure 9: Belief-indifference curves: the horizontal axis corresponds to the precision of the signal and the vertical axis measures the level of optimism of the interim posterior.

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