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THE VICTORY OF HOPE OVER ANGST? FUNDING, ASSET ALLOCATION, AND RISK-TAKING IN GERMAN PUBLIC SECTOR PENSION REFORM[§]

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Abstract

Wie in vielen anderen Ländern auch, beruhen die deutschen Beamtenpensionen traditionell auf einem umlage- und steuerfinanzierten System der Leistungszusage (defined benefit pension – DB). Aufgrund fehlender Rücklagen resultieren aus den Pensionsverspechen ungedeckte Verbindlichkeiten in Milliardenhöhe, die als solche jedoch nicht offiziell als Staatsverschuldung ausgewiesen werden. Das damit einhergehende Problem steigender Belastungen zukünftiger Haushalte wurde von der Politik erkannt, und erste Schritte in Richtung Kapitaldeckung wurden mit der Einführung der Versorgungsrücklagen sowie der Finanzierungsfonds vollzogen. Vor diesem Hintergrund evaluiert diese Studie die Chancen und Risiken, die mit dem Übergang zu einem (partiell) kapitalgedeckten Beamtenpensionssystems verbunden sind. Als Datengrundlage dient hierzu die vollständige Personalstandsstatistik des Landes Hessen, dessen Beamtenpopulation repräsentativ für den größten Teil des deutschen Beamtensystems ist.

Unter Verwendung eigens für diese Studie berechneter Beamtensterbetafeln werden zunächst die Pensionsansprüche der aktuellen Pensionäre sowie die bereits erdienten Anwartschaften der aktuell diensttuenden Beamten aktuariell bewertet. Auf Grundlage einer 50-Jahres-Prognose der Beamtenpopulationsentwicklung werden die zur Finanzierung der Pensionsversprechen benötigten Beiträge, d.h. der Beitragssatz in Bezug auf die Beamtengehälter, deterministisch bestimmt. Im Rahmen einer Monte Carlo-Studie und auf Basis eines stochastischen Barwert-Ansatzes wird sodann die Anlagestrategie für das Planvermögen bestimmt, die zu minimalem Crash-Risiko, gemessen als Conditional Value at Risk der gesamten Pensionskosten, führt. Abschließend wird aufgezeigt, welchen Freiraum der Pensionsplanmanager hinsichtlich der Wahl von Beitragssatz und Anlagestrategie hat, wenn er nur auf Einhaltung eines vorgegebenen Risikobudgets verpflichtet wurde.

1. Introduction

Public employee pension systems throughout the developed world have traditionally been of the pay-as-you-go (PAYGO) defined benefit (DB) variety, where pensioner payments are financed by taxes (contributions) levied on the working generation. But as the number of retirees rises relative to the working-age group, such systems have begun to face financial distress. This trend has been exacerbated in many countries, among them Germany, by high unemployment rates producing further deterioration of the contribution base. In the long run, public sector pension benefits will have to be cut or contributions increased, if the systems are to be maintained.

An alternative path sometimes offered to ease the crunch of paying for public employee pensions is to move toward funding: here, plan assets are gradually built up, invested, and enhanced returns devoted to partly defray civil servants' pension costs. In this study, we evaluate the impact of introducing partial prefunding, paired with a strategic investment policy for the German federal state of Hesse. The analysis assesses the impact of introducing a supplementary tax-sponsored pension fund whose contributions are invested in the capital market and used to relieve the state budget from (some) pension payments. Our model determines the expectation and the Conditional Value-at-Risk of economic pension costs using a stochastic simulation process for pension plan assets. This approach simultaneously determines the optimal contribution rate and asset allocation that controls the expected economic costs of providing the promised pensions, while at the same time controlling investment risk. Specifically, we offer answers to the following questions:

1. How can the plan be designed to control cash-flow shortfall risk, so as to mitigate the potential burden borne by future generations of taxpayers?

2. What is the optimal asset allocation for this fund as it is built up, to generate a maximum return while simultaneously restricting capital market and liability risk?

3. What are reasonable combinations of annual contribution rates and asset allocation to a state-managed pension fund, which will limit costs of providing promised public sector pensions?

We anticipate that this research will interest several sorts of policymaker groups. First, focusing on the German case, the state and Federal governments should find it relevant, as these entities face considerable public sector pension liabilities. Second, our findings will also be of interest to other European countries, as most have substantial underfunded defined benefit plans for civil servants.

In what follows, we first offer a brief description of the structure of civil servant pensions in Germany, focusing on their benefit formulas, their financing, and the resulting current as well as future plan obligations for taxpayers. Next, we turn to an analysis of the actuarial status of the Hesse civil servants' pension plan and evaluate how much would have to be contributed to fund this plan in a nonstochastic context. Subsequently we evaluate the asset-liability and decision-making process from the viewpoint of the plan sponsor, to determine sensible plan asset allocation behavior. A final section summarizes findings and implications.

2. Civil Servant Pensions in Germany

Whereas civil servant pensions are relatively well-funded in the United States (Hustead and Mitchell, 2001), the same is not the case in Germany. There, most civil servants have been promised an unfunded, non-contributory, tax-sponsored DB pension with benefits which are a function of salary and service. Over time, politicians and employees have gradually become aware of the cost of public pensions, particularly as the population has aged, and a few small reforms have been implemented to date. In 1996, for instance, the German state of Rhineland-Palatinate launched a financing fund for newly hired civil servants to cope with the increasing burden of future pension payments. The state currently pays 20-30 percent of covered payroll for active civil servants into that fund. No doubt due to politicians' risk aversion, the fund's investment portfolio has thus far been restricted to government bonds, thereby neglecting the opportunity to improve returns by investing in equity markets. Accordingly, while first steps toward funding German civil servants pensions have been taken, no public pension plan is fully funded and investment patterns in the few cases where the plans have assets are extraordinarily conservative.

2.1 Public Sector Pension Parameters

It is widely recognized that a pension plan represents a long-term contract between an employer and the plan participants. That is, workers give up current salary in exchange for future retirement benefits, either directly through salary deferral, or indirectly through foregone earnings (Husted and Mitchell, 2001). From the perspective of an employer, the structure of a retirement plan and its overall generosity can be an important means of attracting, recruiting, and retaining valued employees. Particularly when an employer must invest in specific training for the employee to do his job, it is in his interest to restrict worker mobility once trained so as to maximize returns on this investment (McGill et al., 2006). In this sense, traditional DB pensions have been recognized as suitable for workers in lifetime jobs (and less appropriate for mobile workers).

Originating in the Middle Ages, the German civil service system with its rights and duties was initially codified in the *Allgemeines Preussisches Landrecht* of 1794, an early Prussian constitution, and with some adaptations, the system is still in force today (Gillis, 1968). As in many other countries, a German civil servant traditionally commits his lifetime to public sector tasks. This civil servant then is promised a retirement annuity that depends on his age at retirement, his years of service in the public sector, and his final salary. In exchange for a noncontributory plan, civil servants "pay" indirectly by having significantly lower gross earnings than other public sector worker with comparable credentials.[1] Civil servants are also not included in the national social security system[2] nor covered by supplementary occupational pensions; rather, each state has a DB pension plan specific to that state.[3] As public employees are not covered by social security, they receive higher retirement benefits than their private sector counterparts who tend to receive both social security and occupational pension benefits (Heubeck and Rürup 2000).

In recent years, however, German public pension plan generosity has begun to be curtailed substantially. Until 1991, for instance, the DB pension formula provided accruals according to a nonlinear function. Thus, for service to 10 years, the replacement rate was 35 percent of final salary; for an additional 15 years of service, benefits rose by 2 percent per year. After 25 years of service, benefit accruals increased at only 1 percent annually, resulting in a maximum replacement rate of 75 percent of final salary after 35 years of service. Then in 1992, the benefit formula was transformed into a strictly linear function: benefit accrual was set at 1.875 percent of final salary per year of service, so that the traditional 75 percent replacement rate would be paid only after 40 years of service (rather than after 35 years as previously). In 2003, new legislation again brought benefit cuts for civil servant pensions. Over time, annual pension accruals will be gradually reduced to 1.79375 percent of final salary, providing retiring civil servants with a maximum replacement rate of 71.75 percent after 40 years of service. Current pensioners will also be affected in that their usual post-retirement benefit increases will be slightly reduced. After 8 rounds of pension increases, their nominal replacement rate will have declined to (a maximum of) 71.75 percent (though the nominal pension benefit will have increased to some extent).

The standard retirement age for civil servants is currently 65, though early retirement is possible at age 63; for early retirement, a discount factor of 0.3 percentage

[1] For example, a single research assistant age 30 with no children employed under private law at a university earns an annual gross salary of about € 41,000. If that person is promoted to the rank of assistant professor, he will then be employed under civil servant rules, and his gross salary would shrink to about € 39,000.

[2] Civil servants are also exempt from unemployment insurance and the state pays a certain fraction of health expenses of civil servants and their families (ranging from 50% - 85%, depending on family status and number of children); see Börsch-Supan and Wilke (2003).

[3] While each state has its own pension plan, the benefit structure for civil servant pensions is nearly equal across the different states in Germany.

points per month of early retirement is applied. Because police work and fire fighting are physically demanding occupations, retirement benefits for public safety workers typically allow retirement at earlier ages, in part to maintain a younger workforce.[4] If an active or retired civil servant dies, the spouse is entitled to a survivorship benefit of 55 percent (formerly 60 percent) of the deceased civil servant's pension (in addition, orphans receive 20 percent and half-orphans 12 percent).

From the employee's perspective, the relatively more generous civil servant pension is partial compensation for their inflexibility and non-portability. For instance, if a civil servant were to leave his job for the private sector, he would forego a substantial part of his accrued pension benefits, losing over half of his pension accruals.[5] It is clear that this creates a strong disincentive for older civil servants to leave their public sector jobs.

2.2 Key Aspects of Hesse's Civil Servant Pension System

To illustrate the opportunities and risks of reform, we turn to an assessment of the civil servant pension offered to public workers employed by the German federal state of Hesse, one of 16 states that form the Federal Republic of Germany. Located in the southwestern center of Germany, Hesse's population of 6.1 million represents almost 8 percent of Germany's 83 million residents. Hesse's economic heart is the Rhine-Main area located around the 650.000 inhabitant city of Frankfurt, which is the center of the German financial and banking industry, domicile of Europe's second most important stock exchange and one of the world's largest airports. With a GDP of €204 billion, Hesse contributes about 9 percent to the German GDP.[6] As Hesse was part of the former West Germany, its population of civil servants may be seen as rather representative of the approximately 1.5 million active (about 4.5% of German workforce) and 900,000 retired civil servants in Germany as a whole.[7]

The dataset on which our study is based was provided by the Hessian Statistical Office. It contains anonymized demographic and economic data on virtually all active and retired civil servants in Hesse as of the beginning of 2004, including their age, sex,

[4] Another pathway to retirement for civil servants is disability, which we do not focus on in the present study. Disability payments are based on final salary and the replacement rate accrued until being disabled, reduced by 0.3 percentage points per month before 63 (not to exceed 10.8 percent). If disabled while on duty, payments are based on the salary projected to regular retirement. Subject to the regular maximum replacement rate, the civil servant is additionally credited 1/3 of the regular accrual he would have received up to the age of 60; the replacement rate is then increased by 20 percent. Altogether, the disability pension comes to at least 2/3 but cannot exceed 71.75 percent of the relevant salary.

[5] In this instance, the state will pay to the social security program an amount equal to the foregone employer contributions to social security for that employee.

[6] All data provided by the German Federal Statistical Office and the Hessian Statistical Office (2006).

[7] These numbers include only federal and state civil servants, but not the about 200,000 active and 500,000 retired civil servants worked for former state owned (but now privatized) enterprise, e.g. the German national railway and federal mail (Bundesministerium des Innern, 2005).

marital status, line of service (for active civil servants), and salary/pension payments.[8] In addition, to derive civil servant-specific mortality tables, we have sampled data on the number of living retirees at the beginning of each year as well as the number of those deceased each year for the period 1994-2004, by age and sex. Descriptive statistics on our sample appear in Table 1.

Table 1: Summary Statistics for Civil Servant and Retiree Pension Obligations

	Number	Female	Avg.Salary/ Pension	Avg. Age
Active Civil Servants	104,919	44.5%	€ 39,000	44.7
By Service Level				
Higher	28,946	37.0%	€ 46,000	47.7
Upper	63,843	49.3%	€ 38,000	45.6
Middle	11,609	38.4%	€ 31,000	40.3
Lower	503	12.5%	€ 26,000	43.7
Pensioners	60,418	47.2%	€ 29,000	71.8
By Origin of Claim				
Retired Civil Servants	43,065	28.0%	€ 33,000	69.6
Surviving Dependents	17,353	95.0%	€ 18,000	77.3

Notes: Authors' calculations using data provided by the State of Hesse for 2004.

The overall number of active civil servants in our sample for 2004 is 104,919, of which 45 percent are female. Salary payments to this workgroup amounted to an annual €4.26 billion or 33 percent of Hesse's annual state tax revenues. The higher service level includes mostly university professors and high school teachers, of which 37 percent are female and the average age is 47.7 years; annual salary levels average € 46,000. Some 61 percent of the sample is in the upper service level, which includes police inspectors and elementary and junior high school teachers. As many teachers are female, women make up 49.3 percent of employees in this group. With an average age of 45.6 years, these civil servants receive an average salary of € 38,000. The middle service level employs 11,609 workers, i.e. in lower police service, 38.4 percent of which are female. On average, these civil servants are 40.3 years old and earn € 31,000 per year. The lower service level is on average 43.7 years old and earns an average annual income of € 26.000.

Turning to retirees, pension payments for those in our sample amount to an overall € 1.76 billion or 14 percent of Hesse's annual tax revenues. In total, the number of pensioners is 60,418, of which 71 percent represent retired civil servants; the remaining 17,353 pensioners are surviving dependents. Most of the retired civil servants (72 percent) are male, but 95 percent of the dependents are female. Male retired civil servants are on average 70 years old and receive an average annual pension of € 34,500, whereas female retired civil servants average 68.6 years of age with an average annual pension of € 30,500. Among surviving dependents, females are on average 77.7 years old and earn an annual average pension income of € 18,500, while their male

[8] For the reason of anonymity, information on top-level civil servants as well as judges and state attorneys were omitted. Smaller cohorts, e.g. those within the lower service level, were grouped to larger units.

counterparts are only 68.8 years on average and receive mean dependents benefits of € 15,500 per year.

3. Valuing the Public Pension in a Non-Stochastic Framework

Next we turn to an analysis of the actuarial status of the Hesse civil servants' pension plan. In this section, we evaluate how much would have to be contributed to fund this plan in a nonstochastic context, which of course requires an actuarial evaluation of the plan's liabilities. In this section, accordingly, we combine information about the benefit formulas and anticipated retirement ages with demographic and economic assumptions. Specifically, we project how and when participants will leave active association with the plan and how long benefits will be paid after retirement. These assumptions must include anticipated mortality and disability rates, probabilities of retirement, and regular as well as survivor benefits. In what follows, we discuss these assumptions in detail for the civil servants covered by pensions in the state of Hesse.

3.1 Mortality Patterns

German civil servants generally tend to enjoy lower mortality rates than the general population (apart from police and other special service units). For this reason, it is essential to derive mortality tables specific to this group, yet the state government does not currently maintain these. We therefore derive the relevant mortality tables using standard actuarial methods and personnel statistics provided by the Hessian statistical office. To this end, for each year from 1994-2004, we sampled the numbers $l_{x,t}$ of active and retired civil servants by sex which were alive of age *x* at the beginning of the respective year *t*, and out of this cohort we compute the number $d_{x,t}$ of those who died before reaching year *t+1*. By dividing the aggregate number of deaths by the aggregate number of lives covered, we derive the raw rate of mortality $\hat{q}_{x,1999}$ of a person aged *x* around the year 1999 (Wolfsdorf 1997, pp. 50ff., DAV 2004, p. 76):

$$\hat{q}_{x,1999} = \frac{\sum_{t=1994}^{2004} d_{x,t}}{\sum_{t=1994}^{2004} l_{x,t}}. \qquad (1)$$

To smooth erratic changes in mortality rates due to small samples, we employ the well-established method of Whittaker (1923) and Henderson (1924) to obtain graduated rates of mortality $q_{x,1999}$:

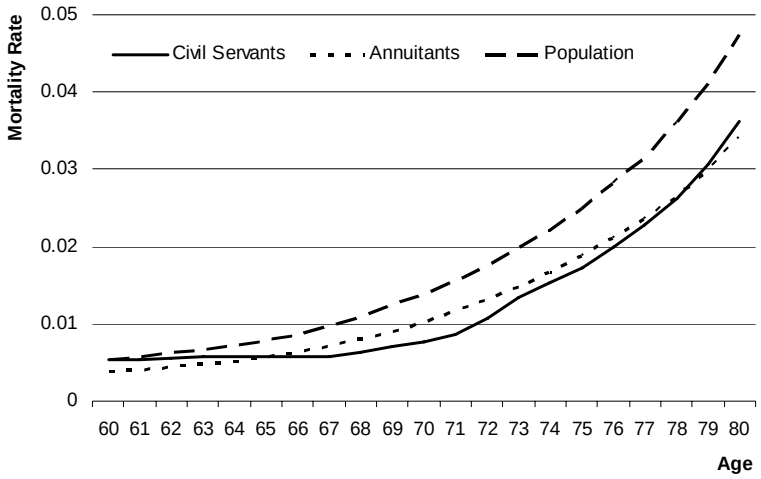
$$\min_{q_i} \sum_{i=0}^{T} x_i \left(\hat{q}_i - q_i\right)^2 + g \sum_{i=0}^{T-3} \left(\Delta^3 \hat{q}_i\right)^2, \qquad (2)$$

where x_i represents the size of the number of observations for cohort *i* relative to the over-all sample size, *g* balances the goodness-of-fit versus the level of smoothness, with $g = 0$ resulting in a perfect fit of raw and graduated mortality rates, and Δ represents the difference operator (Joseph 1952, p. 99).

On analyzing the derived mortality rates thus derived, it turns out that for ages 60 to 80, the results are quite close to the annuitant mortality rates published by the German Association of Actuaries (DAV 2004), supporting the notion that civil servants live longer than the average population (see Figure 1).[9]

We make use of this result when estimating mortality rates for those above the age of 80. Here, the usual approach would be to specify a law of mortality (e.g. Gompertz-Makeham), calibrate it based on empirical observations, and then extrapolate mortality rates for the very old (Milbrodt and Helbig 1999). Due to the limited number of observations in our sample, we instead use annuitant mortality rates for over age 80.

Figure 1: A Comparison of Mortality Rates for Civil Servants, the Total Population, and Annuitants



Notes: Authors' compilation of mortality rates for female Hessian civil servants; annuitants' mortality rates as derived by the German Association of Actuaries (DAV 2004); population mortality rates as stated by the German Federal Statistical Office (mortality table 2003/2005).

[9] Specifically, the mortality table we employ is the DAV 2006 R, Aggregattafel 2. Ordnung (see DAV 2004, p. 53ff.).

To account for future mortality improvements, we incorporate an exponential trend function to project future mortality rates base on those of 1999 (DAV 2004, p. 38):

$$q_{x,t} = q_{x,1999} \cdot \exp\left(-F(x) \cdot (t - 1999)\right). \quad (3)$$

For the age-depending trend factor *F(x)* we also rely on data provided by the German Association of Actuaries, using their unadjusted mid-term trend function (Zieltrend 2. Ordnung) (see DAV 2004, p. 56ff.). Employing this trend function, the rate of mortality of a female civil servant aged 65 will decrease by 40 percent until the year 2030.

3.2 Population Dynamics

When analyzing the transition to a funded pension system, it is necessary to account not only for current employees, but also to recognize that new employees will be hired in the future. Projecting civil service employment far into the future is, of course, quite complex, and a complete characterization of all possible evolutionary paths of the future workforce is not the primary focus of this study. We therefore take a simple yet sensible approach to describe the development of the future workforce, drawing on the current active population as a base. We forecast the evolution of age and salary for every existing civil servant assuming constant marital status over time. When a position becomes vacant, we assign to that job a new civil servant with a 50% chance of being male or female, abstracting from promotion from within. The new worker's age is assigned as the average age of new hires, accounting for average time spent on position-related education or other types of public service that will be credited as pensionable years in civil service. The worker's salary is assigned as the age-related remuneration for this position and marital status is that of the previous position holder.

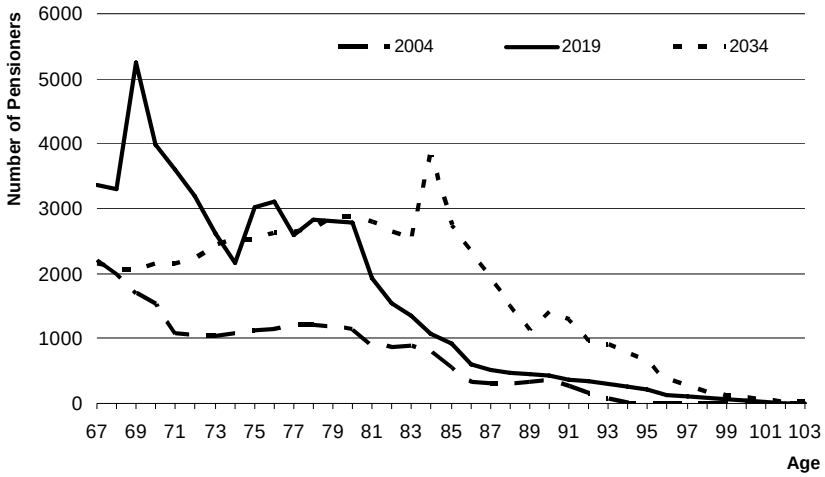
Employee turnover other than retirement can be assumed to be of minor significance. While there might be some fluctuation within the group of assistant professors that leave civil service after not receiving tenure and start a career in the private sector, most state employees remain in service once they become full civil servants.[10] We therefore do not account for employee turnover and instead assume that civil servants remain in service until retirement age. This latter is set at 67, anticipating that the latest changes to public pension regulations (i.e. the increase of regular retirement age from 65 to 67) is also required for civil servants. It is furthermore

[10] This is due to the fact that the pension benefits are not particularly portable when a civil servant moves to the private sector. As a result, observed low turnover is in line with the general aim of a generous DB-pension plan, i.e. to retain public sector employees in their jobs.

assumed that all civil servants reach this age with certainty and remain active until then.[11]

Retired civil servants are not modeled individually; rather they are represented by the expected cash flows that result from the indexed life annuities they receive according to the civil service pension benefit formula based on service years and final salary. For workers with a spouse, these annuities are indexed joint-and-survivor annuities where we assume that both partners are of same age and opposite sexes. Figure 2 shows the projected age distribution for old-age pensioners over the next 30 years.

Figure 2: Projected Age Distribution of Old-Age Pensioners



Notes: Authors' calculations using data provided by the State of Hesse.

As can be seen, the number of retirees and their average age is projected to rise substantially over the next decades. In particular, we find that a significant number of civil servants will retire from 2017 onward.

3.3 Economic Assumptions

The three central economic factors that have a major impact on the valuation of a pension plan are inflation, salary growth, and investment returns; all these are interrelated and must be considered simultaneously (Hustead and Mitchell, 2001). Although inflation in the Euro zone is currently low, it still is important for pension cash flows and their valuation. Accordingly, we base all our analysis on real returns and financial values, which simplifies the modeling by eliminating a stochastic factor and

[11] Provisions for early retirement and disability benefits, as well as dependents' benefits due to death in service, are therefore neglected.

also sheds light on the actual economic cost of providing real civil servant pensions as salary increases and pension benefits tend to be tied to the consumer price index.[12] We therefore assume that salaries and pensions are at least increased with inflation, i.e. the minimum growth rate of real wages is zero. (Below we also report an alternative set of results using a real wage growth rates of 1 percent, to further explore how these salary assumptions affect results.)

Assumptions on investment returns are crucial as they influence the rate at which pension liabilities should be discounted. Due to their long-term character, pension liabilities have a long duration and therefore are sensitive to the discount rate selected. According the choice of the discount rate there is an ongoing debate, which Blake (2006, p. 77) defines as the difference between an actuarial versus an economic valuation of pension liabilities. Traditionally, actuaries choose the discount rate which reflects a reasonable projected expected return of the asset backing the pension liabilities. If the pension assets are partly invested in equities, the discount rate includes also an equity risk premium, which is from an ex ante perspective not realized. By contrast, many economists argue that the relevant number for discounting future pension payments is the riskless rate of interest rate reflecting the financing cost of the plan sponsor. In what follows, we follow the latter approach and discount liabilities at the real rate on (quasi-) risk-free long term government bonds. Our economic rationale is that this rate reflects the state's financing costs. We assume that this real risk free interest rate is in the base case 3 percent.[13] To check for the interest rate sensitivity of our analysis, we also explore results with a real interest rate of 1.5 percent.

3.4 Implied Pension Debt for Current Civil Servants

Our primary focus in this paper is to show how one can structure and fund public employee pension promises in the future. Nevertheless, it is worth knowing how large already-accrued pension claims might be. Table 2 summarizes both the implied pension debt for retired civil servants currently receiving old age and survivor benefits, as well as active workers' accrued pension liabilities (as of 2004), as well as future projected benefits. We offer these for two alternative discount rates and real rates of salary and pension increases.

[12] Federal law stipulates that salaries as well as pensions are adjusted for changes in the general economic and financial circumstances, which mainly means changes in consumer prices. Cost of living adjustments are not explicitly guaranteed on an annual basis, but in the past salaries and pensions have kept pace with inflation and may even rise in real terms due to rises in real average incomes outside the public sector.

[13] The difference between the average nominal par yield of long term German government bonds and the average inflation rate for the post-WWII period is about 4 percent. Inflation protected bonds in the Eurozone currently yield about 2 percent. This market is currently not well developed for government bonds (especially those with long durations) which supports the assumption of a real interest rate of 3%.

Table 2: Implied Pension Debt for Current Pensioners and Workers

		Current Pensioners		PBO Active Workers		Future Accumulations	
		Salary-/Pension Increase p.a.		Salary-/Pension Increase p.a.		Salary-/Pension Increase p.a.	
	Disc. Rate	0% (1)	1% (2)	0% (3)	1% (4)	0% (5)	1% (6)
				Panel 1: Total (billion €)			
(1)	1.5%	28.41	31.90	28.52	37.43	20.70	30.41
(2)	3%	24.30	26.97	19.63	25.21	12.14	17.36
				Panel 2: As % of Explicit Public Debt			
(3)	1.5%	96.5	108.3	96.9	127.1	70.3	103.3
(4)	3%	82.6	91.6	66.7	85.6	41.2	59.0

Notes: Authors' calculations using data provided by the State of Hesse for 2004.

Panel 1 of the table reports pension liabilities in terms of billions of '04 €, while Panel 2 relates the liabilities to the official level of state indebtedness. Pension debt is referred to as "implied" in the Table, as it is not formally reported as explicit government debt. The results show that explicit state debt is greatly understated by omission of the civil servant pension obligations. For instance, for the base case with a discount rate of 3 percent and zero real pension increases (Row 2, Column 1), the pension liability to current pensioners totals € 24.3 billion, or 82.6 percent of the official € 29.44 billion Hessian government debt reported in 2004. For active workers, we calculate the projected benefit obligation (PBO) defined as (Milevsky 2006, p. 173):

$$PBO = \sum_i \frac{1.79375 \cdot \tau_i \cdot S_{67,i} \cdot \overline{a}_{67,i}}{(1+r)^{67-Age_i}}, \qquad (4)$$

where (for each civil servant i of Age_i) τ_i is the current number of service years in 2004, $S_{67,i}$ is the (expected) salary at retirement age 67, $\overline{a}_{67,i}$ is the immediate pension annuity factor, and r is the discount rate. While there is an ongoing debate about whether the PBO in general is an adequate measure for DB pension liabilities (c.f. Bodie, 1990; Mittelstaedt and Regier 1993; Gold, 2004), we argue that employing the PBO is appropriate for German civil servants' pension liabilities. In addition to the inflation-related pay rises discussed above and promotions to higher-paid positions, which we do not include in this study, civil servants' salaries increase deterministically with age. As there is no turnover in the workforce, using the accumulated pension obligation (ABO) and therefore neglecting these pay increases would understate the already-accrued liabilities.

With the same economic assumptions, pension liabilities for current workers amount to € 19.63 billion, or 66.7 percent of the official debt (Rows 2 and 4, Column 3). Here we see that the implied pension debt for retired and currently active civil servants to date – representing already-accrued benefits – sums to € 43.9 billion, a sum

that is almost 150 percent of explicit state debt (summing the values in Rows 2 and 4, Columns 1 and 3). Not surprisingly, if the discount rate were cut to 1.5 percent holding real wages and benefits constant, accrued pension obligations would grow by € 13 billion, or by almost 30 percent.

As civil servant turnover is virtually negligible, it is safe to assume that virtually all workers retire from the system in the future. For this reason, Columns 5-6 of Table 2 show what new liabilities would result if the current system were kept in place for active civil servants; in other words, this represents future pension accumulations due to the replacement rate increasing over time. With no salary increases and an interest rate of 3 percent, these future pension accruals can be valued at € 12.1 billon or 41 percent of debt outstanding (Rows 2 and 4, Column 5). For active workers, then, the pension promise (both past and future) amounts to about € 31.8 billion, or about 108 percent of current explicit debt (summing Columns 3 and 5 in Rows 2 and 4 respectively); to this, the state's total obligation to retirees must be added coming to a total of 190 percent of explicit debt (summing Columns 1, 3, and 5, Row 4). In other words, the total obligation for both active and retired civil servants amounts to almost twice the current explicit state government debt.

This is of course a lower-bound estimate, as nonzero real salary increases are anticipated by most civil servants. If salaries were to rise in real terms by a single percentage point per year, this would elevate the expected present value of public pension commitments to a staggering € 69.5 billion, or 236 percent of explicit debt (summing Columns 2, 4, and 6, Rows 2 and 4 respectively). On the assumption that already-retired individuals' pensions might be financed separately, in what follows below we focus only on active workers, scheduled benefit formulas maintained into the future would cost € 42.6 billion, or 145 percent of explicit debt if workers receive one percent real annual salary growth. (Using a 1.5 percent discount rate boosts unfunded liabilities for current workers to € 67.8 billion, and the unfunded unrecognized debt to 230 percent of recognized state debt.)

3.5 Funding Future Civil Servants' Pension Benefits

Moving to fund the public DB pension plan requires that assets be built up and invested in the pension fund. Accordingly, a key responsibility of the plan sponsor is to figure out pension liability patterns, and then to specify how much must be contributed to pay for those liabilities. In other words, one must assign in a systematic and consistent manner the expected cost of pension accruals as each year of service passes which gives rise to that cost. One way to make this assignment or cost allocation is termed the *actuarial cost method*, which assigns to each fiscal or plan year the actuarial present value of the costs assumed to have accrued in that year, or the so-called "normal cost," on the view that actuarial assumptions are realized. Since civil servant pension

benefits in Hesse are determined as a percentage of final salary times years of service, we use the aggregate level percentage of payroll method to determine the normal cost. This means that the total projected cost allocated to the plan is expressed as a percentage of active members' payroll (McGill et al., 2005). Accordingly, we calculate the actuarial present value of future pension accruals based on future salary and service over the next 50 years (2004-53), starting with our initial population, and evolve it through time given the dynamics presented above. As of 2053, we conduct a discontinuance valuation, though of course this calculation could be extended further into the future quite naturally. The ratio of present value of pension liabilities to the present value of salary payments may be interpreted as the deterministic yearly contribution rate as a percentage of active civil servants' covered payroll needed to fund pension promises.[14]

In what follows, we assume that these contributions are paid by the state (i.e. the employer) at the beginning of each year and the returns on invested asset are determined by a fixed (i.e. non-stochastic) interest rate. In this case, contributions, in conjunction with the return on invested assets, would have to be sufficient to finance promised pension payments. Results are summarized in Table 3 for the 3 percent discount rate and zero salary/benefit growth rate, along with the two alternatives we focused on above (namely, a 1.5 percent discount rate and 1 percent real salary/pension growth). The first four columns of Table 3 show the present value of current workers' projected pension liabilities and salaries for various rates of annual increase, while Columns 5 and 6 report the ratio of the present value of pension costs to salaries on a contribution basis.

Table 3: Projected Benefit Obligations (PBO) and Contribution Rates: Deterministic Model

		PV Pension Liabilities (in € bn)		PV Salaries (in € bn)		PV Pensions/PV Salaries (in %)	
		Salary-/Pension Increase p.a.		Salary-/Pension Increase p.a.		Salary-/Pension Increase p.a.	
	Disc. Rate	0% (1)	1% (2)	0% (3)	1% (4)	0% (5)	1% (6)
(1)	1.5%	44.8	79.8	149.3	186.6	30.0	42.8
(2)	3%	20.8	34.6	111.5	135.3	18.7	25.6

Notes: Authors' calculations using data provided by the State of Hesse for 2004.

Focusing as a benchmark on the results with the 3 percent discount rate and no real salary or pension increases, the present value of future pension liabilities incurred until 2053 amounts to € 20.8 billion (Row 2, Column 1), while salary payments over the same horizon have a present value of € 111.5 billion (Row 2, Column 3). Accordingly, the ratio of present values and therefore the average required contribution rate is 18.7 percent of payroll each year into the future (Row 2, Column 5). If real salaries and

[14] To be clear, here we set aside the pension obligation already accrued by retirees, on the presumption that these will have to be handled with some other financing mechanism.

pensions instead were to rise by 1 percent per year, the implied contribution rate would be closer to 26 percent of payroll every year. Not surprisingly, the results are also sensitive to the discount rate, since when a lower discount rate is applied, this increases both the present value of pension liabilities and salary payments. But since pension liabilities are of longer duration than salary payments, contribution rates rise when discount rates fall. If the discount rate were 1.5 percent, for instance, contribution rates vary between 30 and 42.8 percent, while they range between 18.7 and 25.6 percent for the 3 percent interest rate. Inasmuch as the contribution rate of 18.7 percent of payroll per year amount corresponds rather well to the contribution rates adopted to enhance the funding in Rhineland-Palatinate's new civil servant plan, we shall use this contribution target below in our further analysis.

4. Stochastic Investment Returns and Pension Plan Funding

A major concern of plan sponsors running a DB plan is the inherent uncertainty of the capital market returns earned on assets backing the pension liabilities. On the one hand, good investment performance can help the plan sponsor meet his benefit promises and reduce required contribution. On the other hand, if assets fall short of plan liabilities, supplementary contributions by the plan sponsor might be required to fill the gap. The gap itself, of course, may be used as a funding target for plan liabilities specified in a solvency plan. Also crucial is the plan's asset allocation policy, namely how it weights the portfolio in terms of equity and bonds, as this influences the DB pension system's risk and return profile. In what follows, we evaluate the asset-liability and decision-making process from the viewpoint of the plan sponsor, to determine a sensible plan asset allocation.

4.1 The Pension Manager's Objectives and Asset/Liability Modeling

Let us assume that the objective of moving to a funded DB pension scheme is to minimize the worst-case total cost of running the plan over a future long-term time horizon.[15] In our case, the actuarial projection of plan liabilities uses a year-by-year PBO valuation over a 50 year horizon with mortality rates and populations dynamics described in the previous section. In this framework, at the beginning of every period *t*, the sponsor endows the pension plan with funds in form of regular contributions RC_t, determined by a fixed contribution rate *CR* related to the salary payments of active civil servants in *t*. Pension payments due at time *t* are made out of the fund, and remaining plan assets are invested in the capital markets.

At the end of every period, the plan's funding situation is scrutinized. If the funding ratio, defined as the ratio of current plan assets to the current projected benefit

[15] As described previously, we set aside benefits already promised to today's retirees as well as those accrued by currently active civil servants.

obligation, were to drop below 90 percent, then a solvency rule requires supplementary contributions SC_t must be made to the plan, so as to return to a funding ratio of 100 percent. If the funding ratio ever exceeds 120 percent, the rate of regular contributions (*CR*) can be reduced by 50 percent. The plan sponsor will benefit from a contribution holiday (i.e. $RC = 0$) when the funding ratio rises above 150 percent. Depending on the scenario under investigation, the plan sponsor may be able to withdraw excess funds when the funding ratio exceeds 180 percent. After 50 years, the plan is assumed to be terminated by a (hypothetical) hard freeze, i.e. at that time all accrued liabilities are transferred to a private insurer together with assets to fund them.

The aim of the investment policy is to generate a sufficiently high return that helps reduce overall pension plan costs, while at the same time controlling for capital market fluctuations that might result in substantial worst-case risks. In our model, the investment universe comprises two broad asset classes: a global equity index fund, and a European government bond index fund. By assuming that the state invests only in index funds, we can ensure that the state cannot systematically influence prices. The stochastic and dynamics of the evolution portfolio's value over time are governed by a multiplicative random walk with drift. The serially independent and identically normal distributed one-period log-returns are given by:

$$R_{P,t} = \mu_P + \sigma_P Z_t, \tag{5}$$

where $Z_t \sim N(0,1)$. Under the assumption of a static asset allocation over time, the portfolio is continuously rebalanced to maintain the original investment weights. In this case, the expected portfolio log-return μ_P and the portfolio return standard deviation σ_P can be derived as (Feldstein et al. 2001):

$$\begin{aligned} \sigma_P^2 &= x^2\sigma_E^2 + (1-x)^2\sigma_B^2 + 2x(1-x)\sigma_E\sigma_B\rho_{E,B} \\ \mu_P &= x\left(\mu_E + 0.5\sigma_E^2\right) + (1-x)\left(\mu_B + 0.5\sigma_B^2\right) - 0.5\sigma_P^2 \end{aligned}, \tag{6}$$

where x represents the weight of equities in the portfolio, μ_E (μ_B) is the expected log-return, σ_E (σ_B) the return standard deviation of equities (bonds) and ρ_{EB} the coefficient of correlation between both asset classes.

The asset model parameters we employ for the simulation are given in Table 4. For equities, these are estimated from the MSCI World Equities total return index (1974 – 2003). For bonds, our estimates are based on the JP Morgan European Government Bonds total return index (1988 – 2003), which we augment with the REXP German Government Bonds total return index for the period 1974 – 1987 due to unavailability

of the former. All index data are provided by DataStream. To incorporate estimation risk in the critical estimation of expected asset returns, we increase the estimated return standard deviations by the standard errors of the mean return estimations (Barry 1974; Klein and Bawa 1976; this corresponds to the two-step estimation procedure used in Feldstein et al., 2001). Financial integration in Europe, the creation of the European Monetary Union, and the accompanying Maastricht convergence criteria, have reduced interest rates in the Euro zone, leading to lower expected future returns on European government bonds. We therefore reduce the historically estimated return expectation for the European bonds by 1 percent. Finally, we subtract the equivalent of 30 basis points from the annual portfolio return to reflect administrative expenses.

Table 4: Simulation Model Parameters for Stochastic Asset Case

	Expected log return	Standard deviation	Correlations: Global equities	Correlations: European bonds
Global equities	7.1%	20.2%	1	
European bonds	4.5%	6.7%	0.14	1

Notes: Authors' calculations.

The optimal investment and contribution policy for the partially funded public plan is obtained from running a Monte Carlo simulation with 10,000 iterations over the 50-year projection horizon. To identify the optimal investment and contribution policy for the pension plan, we assume that the objective of the plan sponsor is to minimize the worst-case cost of running the plan.[16] Accordingly, we specify the probability distribution of the stochastic present value of total pension cost *TPC* and identify the asset allocation and regular contribution rate fixed at the beginning of the projection horizon[17] that minimizes the Conditional Value at Risk at the 5 percent level.

More formally, total pension costs are calculated as the sum of regular contributions RC_t and supplementary contributions SC_t made by the plan sponsor in period *t*. Depending on the set-up, these plan costs may or may not be reduced by the withdrawals W_t of excess funds. All payments into or withdrawals from the plan are discounted at the fixed real interest rate *r*, reflecting the government's financing cost. Thus, the optimization problem is specified by:

[16] See also Albrecht et al. (2006).

[17] We deliberately do not consider a strategy whereby the investment weights and contribution rates are optimized dynamically over time, e.g. by using a dynamic optimization framework. While from a theoretical vantage point this might yield better results, here we argue that political decisionmakers may be unable to implement this in practice.

$$\min_{x} CVaR_{5\%}\left(TPC = \sum_{t=0}^{T} \frac{RC_t + SC_t(1+\xi_1) - W_t(1-\xi_2)}{(1+r)^t} \right) \tag{7}$$

The Conditional Value at Risk (CVaR) at the α percent confidence level is defined as the expected total pension cost under the condition that its realization is greater than the Value at Risk (VaR) for that level, i.e.:

$$CVaR_{\alpha\%}(TPC) = E(TPC \mid TCP > VaR_{\alpha\%}(TPC)) \tag{8}$$

CVaR has significant advantages as a measure of risk over the commonly-used VaR measure, defined as $P(TPC > VaR_\alpha) = \alpha$, i.e. the costs that will not be exceeded with a given probability of (1- α) percent. First, CVAR not only concentrates on a specified percentile of a loss distribution, but it also accounts for the extent of the loss in the distributional tails beyond this percentile. Therefore, the Conditional Value at Risk is a coherent risk measure with respect to the axioms developed by Artzner et al. (1997, 1999). Second, from the perspective of numerical portfolios optimization, CVaR is better behaved than VaR because of its convexity with respect to decision variables (Rockafellar and Uryasev 2002).

In principle, pension benefits ought to be financed by the regular contributions to the plan so that supplementary contributions should be required only as a last resort (e.g. in case of a severe capital market downturn). If a plan sponsor must be forced to make supplementary contributions often, this indicates that the rate of regular contributions is likely to be insufficient. To encourage adequate regular contributions, we therefore introduce a penalty for supplementary contributions represented by the parameter ξ_1. This is structured such that to finance an asset shortfall of one Euro, the sum of (1+ ξ_1) Euros must be accounted for as a plan cost. At the same time, it seems reasonable to restrict the pension plan from being used as a "hedge fund" investment account that can become hugely overfunded (which might happen if the sponsor were to short government bonds to create excess revenues by cashing in on the equity premium). For this reason, we levy a withdrawal penalty on the state pension plan of ξ_2, so withdrawing one Euro from the plan means only (1- ξ_2) Euros are credited. The term ξ can also be interpreted as the cost of additional financing, and it therefore counters a common perception that public funds paid into the civil servant pension plan are "free" money.

4.2 Optimal Pension Fund Asset Allocation with a Fixed Contribution Rate

Next we draw on our results above to derive the optimal asset allocation for plan assets when the contribution rate is fixed at a given ratio of projected benefit obligation to the present value of projected future salaries. Table 5 summarizes the results for the same real discount rate of 3 percent, no real salary or pension increases, and a 20 percent penalty factor for supplementary contributions ξ_1 and withdrawals ξ_2. The fixed contribution rate is set at 18.7% since in Table 3 this resulted in a deterministic PBO of € 20.82 billion; this is a useful benchmark against we can measure the risk/return profile of various investment strategies.

Key findings are provided for three asset allocation policies, namely a 100 percent bond portfolio, a 100 percent equity portfolio, and the endogenously determined optimal portfolio which minimizes the CVaR. Results reported in Panel 1 of Table 5 display the related portfolio weights in equity and bond investments assuming a static asset allocation (Rows 1 and 2), the expected present value of total pension costs (Row 3), and the 5% Conditional Value at Risk (Row 4). Panel 2 contains, for the same investment policies, the expectation and the 5% Conditional Value at Risk of discounted supplementary contributions (Rows 5 and 6), as well as the discounted final-period withdrawals credited to the objective function (Rows 7 and 8). Additionally, two ways to dispose of pension surplus are modeled. Columns 1-3 permit the pension manager to withdraw plan assets exceeding 180 percent of the PBO (reduced by the penalty factor); these would be credited to the objective function as reducing pension costs. By contrast, in Columns 4-6, the excess assets are not credited, i.e. $\xi_2 = 100\%$, which reduces the plan manager's incentive to over-endow the pension fund by leaning on the excess of the equity premium over the assumed discount rate.

Table 5: Risk of Alternative Asset Allocation Patterns for Active Workers, Assuming Fixed Contribution Rate

	Fixed contribution rate: 18.7% Deterministic PBO: € 20.82 bn Real Discount Rate: 3%	Withdrawals credited 100% Bonds (1)	 100% Equities (2)	 Cost min. Asset Mix (3)	Withdrawals NOT credited 100% Bonds (4)	 100% Equities (5)	 Cost min. Asset Mix (6)
	Panel 1						
(1)	Equity weight	0%	100%	30%	0%	100%	24%
(2)	Bond weight	100%	0%	70%	100%	0%	76%
(3)	Expected pension costs (€ bn)	11.74	-12.90	3.03	17.60	15.04	14.96
(4)	5%-CVaR pension costs (€ bn)	23.92	28.74	19.70	25.50	35.56	22.70
	Panel 2						
(5)	Exp. Suppl. Contributions (€ bn)	1.18	6.61	0.79	1.18	6.61	0.70
(6)	5%-CVaR Suppl. Contrib. (€ bn)	5.95	21.97	4.94	5.95	21.97	4.50
(7)	Exp. Withdrawals (€ bn)	5.93	32.30	11.80	0.00	0.00	0.00
(8)	5%-CVaR Withdrawals (€ bn)	0.18	4.35	1.28	0.00	0.00	0.00

Notes: Authors' calculations using data provided by the State of Hesse. Contribution rate in % of salaries, real salary-/pension increase 0%. Supplementary contributions required in case of funding ratio (i.e. *fund assets/PBO*) below 90% to restore funding ratio of 100%. Contribution rate reduced by 50% (100%) in case of funding ratio above 120% (150%). Withdrawal of funds in excess of 180% of liabilities. Opportunity costs of supplementary contributions addressed by accounting for an agio of ξ_1 = 20%. Withdrawals credited to objective function in case *Withdrawals credited* (ξ_2 = 20%), lost in case *Withdrawals NOT credited* (ξ_2 = 100%).

Before evaluating the optimal asset investment policy, we assess two polar cases, namely a 100 percent bond investment case, and a 100 percent equity case. When the fund is fully invested in bonds, total expected pension costs for active employees come to € 11.74 billion (Row 3, Column 1) and the CVaR is valued at € 23.92 billion or about € 2.1 billion higher than the deterministic PBO benchmark (Row 4, Column 1). On top of regular pension contributions, taxpayers must anticipate making another € 1.18 billion in supplementary contributions, with a 5% CVaR of this amount about five times that, at € 5.95 billion (Rows 5 and 6, Column 1). When expected withdrawals can be credited, albeit with a penalty, the managers can anticipate a value of almost € 6 billion, while virtually no funds will be withdrawn in the worst case (Rows 7 and 8, Column 1).

By comparison, if the plan were to invest fully in equity (which we recognize is highly unlikely in the case of a public pension), there is enormous upside as well as downside potential. For instance, the expected value of withdrawals comes to an impressive € 32.3 billion (Row 7, Column 2), resulting in a negative expected total pension costs of - € 12.9 billion (Row 3, Column 2). What this means is that "on average", after a start-up phase, the plan manager could expect to have contribution holidays if he invested the entire fund in equities, and withdrawals would be sufficient to recover more monies than were paid into the plan. However, this impressive upside potential comes at the price of substantially enhanced capital market risk that could easily drive the funding situation to unacceptable levels. The CVaR of supplementary contributions comes to € 21.97 billion (Column 2, Row 6) leading to a CVaR of total pension costs of € 28.74 billion (Column 2, Row 4), which substantially exceeds the expected costs calculated under the deterministic PBO benchmark.

In Column 3 we depict the optimal investment strategy given the contribution rate, which in Rows 1 and 2 consists of 30 percent equities and 70 percent bonds. This lowers expected pension costs for active employees to only € 3.03 billion (Row 3, Column 3), much lower than the € 20.82 billion benchmark of required funding in the deterministic case. This low level of costs can be attributed to the considerable expected benefit of investing in the capital market. The fund is paid 18.7 percent of payroll from the outset for each active employee, but actual pension cash flows are initially small. Accordingly, a pension fund invested 30 percent in equities and 70 percent in bonds yields an expected gross return of 6.24 percent per year, which is double the 3 percent threshold for which the benchmark contribution rate of 18.7 percent was derived. With a return volatility of 8.17 percent, the pension fund faces rather moderate capital market risk, and as a result, the plan would be expected to accumulate considerable assets rather quickly. The possibility of contribution holidays as well as withdrawals steadily increases through time, while the risk of supplementary contributions required by the solvency rule diminishes. Compared to the deterministic case with a present value of payments or PBO of € 20.82 billion, this funding and investment policy mix reduces expected costs substantially. The worst case exposure is also well-controlled. For instance, the 5% Conditional Value at Risk of total pension costs (i.e. average costs in the 5 percent worst cases) only comes to € 19.7 billion (Row 4, Column 3) well below the benchmark. Expected supplementary contributions are also rather low, with a present value of € 790 million (Row 5, Column 3) and in the worst case – again represented by the 5 percent Conditional Value at Risk – they amount to € 4.94 billion, roughly 25 percent of the deterministic PBO (Row 6, Column 3). Giving managers credit for withdrawals can offer major cost-cutting potential for the plan. In the worst-case scenario, plan withdrawals would be only € 1.28 billion, but in expectation they amount to € 11.8 billion (Rows 7 and 8, Column 3).

If excess withdrawals are not credited to the account, managerial incentives to overendow the plan are greatly reduced. Here, the only incentive for risking supplementary contributions caused by underfunding is the opportunity to enjoy reduced contribution rates or even contribution holidays if the funding ratio should exceed 180 percent. Columns 4-6 of Table 5 show that expected pension costs and the CVaR amounts now exceed comparable values in Columns 1-2 when either extreme investment strategy is selected, while the supplementary contributions are the same, By contrast, the optimal investment strategy now requires that only 24 percent of the funds must be held in equities (Row 1, Column 6), which is 6 percentage points less than when withdrawals are credited. Clearly, having a capped upside potential makes return-driven investment less attractive. At the same time, not rewarding risk taking curtails worries about having to make supplementary contributions. The impact on total plan costs given the optimal investment strategy can again be seen in Panel 1. If withdrawals are not credited, the worst-case costs rise to an overall € 22.7 billion (Row 4, Column 6)

with expected plan costs of € 14.96 billion (Row 3, Column 6). This is an increase of almost € 12 billion compared to the case where withdrawals are permitted, roughly the value of expected withdrawals if they are allowed.

In sum, allowing the public pension plan to partially prefund benefits and engage in an optimal investment policy could be expected to mitigate the economic cost of providing the pension promise, while at the same time minimizing the consequences of capital market volatility.

4.3 Simultaneously Optimizing Contribution Rates and Pension Fund Asset Allocation

In the case of public pension plans, it is often thought that current policymakers might prefer relatively low contribution rates to hold the line on current fiscal expenditures. On the other hand, if regular contributions are too low, this could require high future supplementary contributions if the plan becomes underfunded. Accordingly, a balance must be found across these interests. Therefore we next analyze how changes in the contribution rates might affect key pension outcomes, in addition to asset allocation.

Given a contribution rate of 18.7 percent, we showed above that the optimal asset allocation is 30 percent in equities and 70 percent in bonds when pension asset withdrawals are credited to the plan manager's objective function. We also show that the worst-case total pension costs measured by the 5% CVaR are € 19.7 billion, compared to the deterministic PBO of € 20.82 billion (at the benchmark discount rate of 3 percent). Yet there is some room for the pension plan manager to tap the full risk budget, if he can change both the asset allocation and the regular contribution rate. Accordingly, in what follows, we seek the optimal combination of a fixed contribution rate and asset allocation which fully bails out the given risk budget specified by the deterministic PBO.

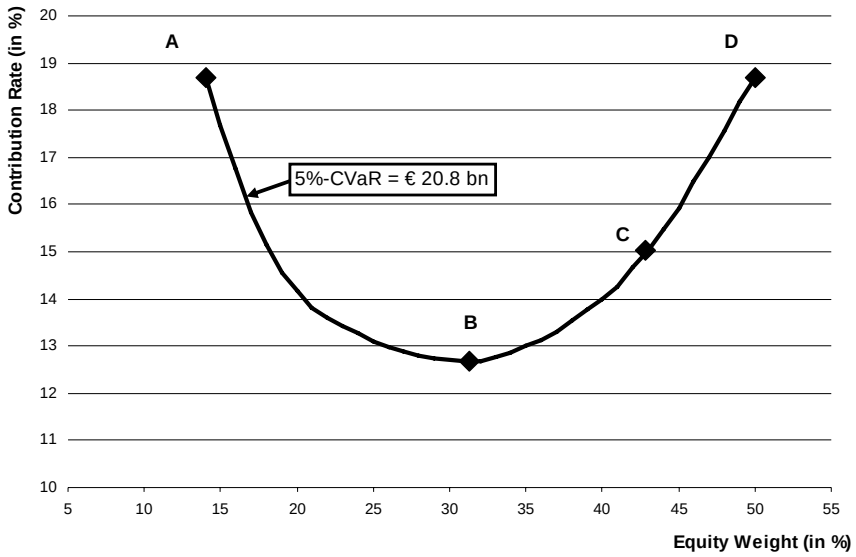
To obtain this solution, we vary the equity weights and the contribution rate (between 0 and 18.7 percent) to identify the equity/contribution pair that provides an overall worst-case pension cost equal to the PBO. From a formal perspective, we use the same objective function as equation (7) but we optimize over the investment weight in equities *x* as well as the rate of regular contributions *CR*:

$$\underset{x,CR}{CVaR_{5\%}}\left(TPC = \sum_{t=0}^{T} \frac{RC_t + SC_t\left(1+\xi_1\right) - W_t\left(1-\xi_2\right)}{\left(1+r\right)^t} \right) = PBO \qquad (9)$$

Results are depicted in Figure 3, assuming a 3 percent discount rate and permitting final-period excess funds to be credited. Here we observe the structural interrelation between the rate of regular contributions (*CR*) and the optimal equity exposure for a given risk budget measured by the 5% Conditional Value at Risk of total pension costs. Specifically, we present the "Iso-CVaR" curve for a value of € 20.82 billion in the space spanned by the fraction in equity and the regular contribution rate. This curve is U-shaped. This means the pension plan manager can reduce regular payments up to a minimum level of about 12.7 percent when simultaneously choosing an equity exposure of about 32 percent (Point B in Figure 3). If a higher regular contribution rate were preferred, the equity exposure can either be reduced or increased. For example, a contribution rate of 18.7 percent is consistent with the predetermined PBO risk budget for either an equity weight of about 14 percent (Point A) or 50 percent (Point D). This can be explained as follows: a portfolio heavier in equity is expected to generate relatively higher returns, which offers the opportunity of future contributions holidays and possible withdrawals. Yet the relatively high volatility of asset returns might also cause pension asset shortfalls, leading to a relatively high risk of supplementary contributions that naturally offsets the possibility of contribution holidays and withdrawals. By contrast, a more bond-oriented investment policy provides more stable expected investment returns over time, and therefore a relatively low risk of supplementary contributions. On the other hand, future taxpayers will have little chance to cut plan financing costs.

For more insight into these interrelations, we analyze four specific combinations of contribution rates and equity weights depicted in Figure 3, namely the minimum return volatility combination (Point A), the minimum regular contribution combination (Point B), the maximum return combination (Point D), and an intermediate setting (Point C). Table 6 summarizes the results. Consistent with our objective function (8), all strategies have similar worst-case pension costs of € 20.8 billion equal to the deterministic PBO benchmark (Row 5). On the other hand, the different funding and investment policies generate substantial differences with regard to the risk of supplementary contributions and the opportunities for possible withdrawals. Starting with an 18.7 contribution, and a 14 percent equity weight (the minimum return volatility case in Column 1), the risk of supplementary contributions is low due to sufficient regular funding and stable investment returns over time. Supplementary contributions amount to only € 710 million in expectation, and € 4.37 billion in the worst case (Rows 6 and 7, Column 1). This, however, comes at the price of meager possible withdrawals (Rows 8 and 9) and a lower chance of contribution holidays.

Figure 3: Iso-CVaR Curve of Total Pension Costs



Notes: Authors' calculations using data provided by the State of Hesse.

Next we examine what would happen if the hope was to make only a minimum regular contribution – an option that might be favored by politicians with a relatively near-term time horizon. Column 2 sets the contribution rate at only 12.7 percent and then invests 32 percent of the fund assets in equities (Rows 1 and 2); the results show that this approach leads to considerably higher risks for future generations, compared to the better funded case. For instance, the worst-case supplementary contributions required to offset possible underfunding amount to € 10.27 billion (Row 7), which is more than 2 times the sum of the minimum volatility case (Column 1). Worst-case supplementary contributions are about 25 percent higher, compared to the maximum return case (Column 4), while at the same time expected withdrawals only amount to about 60 percent (Row 8).

Table 6: Asset Allocation and Regular Contributions for Given Risk Budget

	Deterministic PBO: € 20.82 bn Real Discount Rate: 3%	Min. Return Volatility (1)	Minimum Reg. Contr. (2)	Intermediate (3)	Maximum Return (4)
	Panel 1				
(1)	Contribution Rate	18.7%	12.7%	15%	18.7%
(2)	Equity weight	14%	32%	43%	50%
(3)	Bond weight	86%	68%	57%	50%
(4)	Expected pension costs (€ bn)	7.63	3.98	0.46	-2.23
(5)	5%-CVaR pension costs (€ bn)	20.8	20.8	20.8	20.8
	Panel 2				
(6)	Exp. Suppl. Contributions (€ bn)	0.71	3.67	2.77	1.45
(7)	5%-CVaR Suppl. Contrib. (€ bn)	4.37	10.27	9.70	7.83
(8)	Exp. Withdrawals (€ bn)	8.31	10.54	13.91	16.88
(9)	5%-CVaR Withdrawals (€ bn)	0.84	0.51	0.92	1.62

Notes: Authors' calculations using data provided by the State of Hesse. Contribution rate in % of salaries, real salary-/pension increase 0%. Supplementary contributions required in case of funding ratio (i.e. *fund assets/PBO*) below 90% to restore funding ratio of 100%. Contribution rate reduced by 50% (100%) in case of funding ratio above 120% (150%). Withdrawal of funds in excess of 180% of liabilities. Opportunity costs of supplementary contributions addressed by accounting for an agio of ξ_1 = 20%. Withdrawals credited to objective function (ξ_2 = 20%).

Alternatively, we examine the results from maintaining a regular contribution rate of 18.7 percent but increasing the equity exposure to 50 percent (Column 4). Naturally this results in a quite different risk-/return profile for future generations. Here, the worst-case supplementary contributions almost double the benchmark, rising from € 4.37 to € 7.83 billion (Row 7, Columns 1 versus 4). Of course this tactic does offer future taxpayers a respectable chance of participating in the capital markets and profiting from contribution holidays. As can be seen from Row 8, the expected present value of withdrawals under this high equity scenario comes to € 16.88 billion (Column 4), which is more than twice the amount in the minimum return volatility case.

Column 3 shows an intermediate case, which seeks to balance the interest of both groups. Here current taxpayers only have to pay regular contributions of 15 percent, and the portfolio is held 43 percent in equities. This allows future generations to benefit from possible contribution holidays and withdrawals, while providing an acceptable level of risk of supplementary contributions resulting from underfunding.

4.4 Further Results

To verify the robustness of our results, we have repeated the analyses for a variety of other parameter settings. Reducing the discount rate from 3 to 1.5 percent and cutting expected asset returns by the same 1.5 percent results in a deterministic PBO of € 44.8 billion and a corresponding contribution rate of 30 percent (Table 3, Row 1, Columns 1 and 5). Based on these and crediting fund withdrawals, the optimal equity weight comes to 26 percent with expected total pension costs of € 18.47 billion. Worst-case pension costs exceed the deterministic PBO by only € 1.7 billion or 4 percent.

Increasing the penalty factors for supplementary contributions and withdrawals to $\xi_1 = \xi_2 = 0.5$ also leads to an optimal equity weight of 26 percent. Expected and worst-case pension costs increase from € 3.03 billion and € 19.7 billion (Table 5, Column 3, Rows 3 and 4) to € 8.42 billion and € 21.44 billion respectively.

Allowing for contribution holidays already to be taken when the funding ratio exceeds 120 percent and postponing the withdrawal of excess funds until the final period results in expected pension costs of € 2.49 billion and € 19.6 billion in the worst case. This requires 29 percent of plan funds being held in equities and 71 percent in bonds.

5. Conclusions

As in many other nations, German civil servants are covered by unfunded defined benefit pensions. This substantial underfunding represents a substantial liability that is not, to date, recognized as explicit taxpayer obligations. For the German state of Hesse, we have conservatively estimated this liability for current pensioners of € 24 billion in present value, and amounts already accrued by current workers adds another € 20 billion (with a discount rate of 3 percent and zero real salary/pension increases). The total of € 44 billion is massive, amounting to almost 150 percent of explicit state debt. When we then also account for future pension accruals for active workers under current formulas, new obligations can be valued at € 12 billon or an additional 41 percent of outstanding debt. In other words, the total obligation of € 56 billion for both active and retired civil servants amounts to almost twice the current explicit state government debt. A one percent rise in salaries and benefits elevates the expected present value of public pension commitments to a staggering € 70 billion, or 236 percent of explicit debt.

Next we analyze ways to move to a better funded pension plan for Hesse's civil servants, narrowing the focus to active employees (on the assumption that already-retired workers' benefits would have to be paid from other sources). We show that annual contribution rates of around 19 percent of payroll would be consistent with a traditional actuarial valuation (assuming a discount rate of 3 percent). Given this, we study alternative investment policies using a stochastic asset/liability framework that minimizes the expected economic costs of providing the promised pensions, while at the same time controlling possibly adverse capital market outcomes. We illustrate that the optimal pension fund investment strategy given this contribution rate consists of 30 percent equities and 70 percent in bonds. It is interesting that this is virtually identical to the 31/69 stock/bond allocation for US public sector pension plans in 2007 (Wilshire Consulting, 2007). Compared to the deterministic case, this funding and investment policy mix reduces expected costs substantially, and the worst-case exposure is also well-controlled. Finally, we show that the pension plan manager can better tap the risk

budget if he is allowed to alter both his asset allocation and the plan's regular contribution rate. In one interesting case, current taxpayers are asked to pay regular contributions of only 15 percent, and the portfolio is held 43 percent in equities. This allows future generations to benefit from possible contribution holidays and withdrawals, while providing an acceptable level of risk of supplementary contributions resulting from underfunding.

Our research should be of broad interest to employees as well as taxpayers, insofar as many civil servant pensions in Europe are underfunded or even totally unfunded. For the German case, it would be useful to assess all state and Federal government public sector pension liabilities following methodology such as that presented here. Furthermore, we have shown that under plausible assumptions, as the shortfalls are gradually funded, the assets could be sensibly invested to reduce expected burdens on future workers and retirees. In ongoing work, we are evaluating additional aspects of the optimal investment scenario.

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