

Marekwica, Marcel; Maurer, Raimond H.

**Working Paper**

## How unobservable Bond Positions in Retirement Accounts affect Asset Allocation

Working Paper Series: Finance & Accounting, No. 176

**Provided in Cooperation with:**

Faculty of Economics and Business Administration, Goethe University  
Frankfurt

Suggested Citation: Marekwica, Marcel; Maurer, Raimond H. (2007) : How unobservable Bond Positions in Retirement Accounts affect Asset Allocation, Working Paper Series: Finance & Accounting, No. 176, Johann Wolfgang Goethe-Universität Frankfurt am Main, Fachbereich Wirtschaftswissenschaften, Frankfurt a. M.

This Version is available at:

<http://hdl.handle.net/10419/23440>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

**JOHANN WOLFGANG GOETHE-UNIVERSITÄT  
FRANKFURT AM MAIN**

**FACHBEREICH WIRTSCHAFTSWISSENSCHAFTEN**

**Marcel Marekwica / Raimond Maurer**

**How unobservable Bond Positions in Retirement  
Accounts affect Asset Allocation**

**No. 176  
Mai 2007**



**WORKING PAPER SERIES: FINANCE & ACCOUNTING**

**MARCEL MAREKWICA\* / RAIMOND MAURER†**

**HOW UNOBSERVABLE BOND POSITIONS IN RETIREMENT  
ACCOUNTS AFFECT ASSET ALLOCATION‡**

**No. 176  
Mai 2007**

**ISSN 1434-3401**

---

\* Johann Wolfgang Goethe University of Frankfurt, Department of Finance, Kettenhofweg 139 (Uni-PF 58), D-60054 Frankfurt, Germany, Phone: ++49-69-798-25130, Fax: ++49-69-798-25228, Email: marekwica@finance.uni-frankfurt.de

† Johann Wolfgang Goethe University of Frankfurt, Department of Finance, Kettenhofweg 139 (Uni-PF 58), D-60054 Frankfurt, Germany, Phone: ++49-69-798-25227, Fax: ++49-69-798-25228, Email: maurer@finance.uni-frankfurt.de

‡ We would like to thank Jörg Berns-Müller, Beate Breuer, Wolfram Horneff, Alexander Schaefer and Michael Stamos for helpful comments. Financial support from the Friedrich Naumann Foundation is gratefully acknowledged. None of the above should be held responsible for any errors that remain. Opinions are solely those of the authors and not of the institutions with whom the authors are affiliated. c2007 by Marcel Marekwica and Raimond Maurer. All rights reserved.

*Working Paper Series Finance and Accounting are intended to make research findings available to other researchers in preliminary form, to encourage discussion and suggestions for revision before final publication. Opinions are solely those of the authors*

# How Unobservable Bond Positions in Retirement Accounts Affect Asset Allocation

Marcel Marekwica

Raimond Maurer

First draft: March 2006

This version: May 2007

## **Marcel Marekwica**

Johann Wolfgang Goethe University of Frankfurt

Department of Finance

Kettenhofweg 139 (Uni-PF 58)

D-60054 Frankfurt am Main

Germany

Phone: +49-69-798-25130, Fax: +49-69-798-25228

Email: marekwica@finance.uni-frankfurt.de

## **Raimond Maurer**

Johann Wolfgang Goethe University of Frankfurt

Department of Finance

Kettenhofweg 139 (Uni-PF 58)

D-60054 Frankfurt am Main

Germany

Phone: +49-69-798-25227, Fax: +49-69-798-25228

Email: RMaurer@wiwi.uni-frankfurt.de

We would like to thank Jörg Berns-Müller, Beate Breuer, Wolfram Horneff, Alexander Schaefer and Michael Stamos for helpful comments. This research was conducted with support from the Friedrich Naumann Foundation and the Bundesverband Investment and Asset Management (BVI). None of the above should be held responsible for any errors that remain. Opinions are solely those of the authors and not of the institutions with whom the authors are affiliated. ©2007 by Marcel Marekwica and Raimond Maurer. All rights reserved.

# How Unobservable Bond Positions in Retirement Accounts Affect Asset Allocation

First draft: March 2006

This version: May 2007

## Abstract

Many tax-codes around the world allow for special taxable treatment of savings in retirement accounts. In particular, profits in retirement accounts are usually tax exempt which allow investors to increase an asset's return by holding it in such a retirement account. While the existing literature on asset location shows that risk-free bonds are usually the preferred asset to hold in a retirement account, we explain how the tax exemption of profits in retirement accounts affects private investors' asset allocation. We show that total final wealth can be decomposed into what the investor would have earned in a taxable account and what is due to the tax exemption of profits in the retirement account. The tax exemption of profits can thus be considered a tax-gift which is similar to an implicit bond holding. As this tax-gift's impact on total final wealth decreases over time, so does the investor's equity exposure.

**JEL Classification Codes:** G11, H24

**Key Words:** asset location, asset allocation, tax-deferred accounts, tax exempt accounts

# 1 Introduction

Profits from dividends, interest and capital gains are usually subject to taxation. In tax-sheltered retirement accounts, however, profits remain untaxed which allows the return of an asset to increase from its after-tax to its pre-tax return by holding it in such a tax-sheltered retirement account. This paper explains optimal asset allocation (i.e. which assets to hold) and asset location (i.e. in which account to hold these assets) for investors having the opportunity to invest in both taxable and retirement accounts. In particular, it explains why the equity exposure in both taxable as well as retirement wealth decreases with a decreasing investment horizon. It is most closely related to the recent literature on optimal asset location decisions including Shoven and Sialm (1998), Shoven and Sialm (2003), Dammon et al. (2004), Poterba et al. (2004), Huang (2006) and Garlappi and Huang (2006). These papers conclude that assets facing a high tax-burden should generally be located in retirement accounts, while assets facing a low tax-burden are better located in conventional taxable accounts. Garlappi and Huang (2006), however, show that this finding does not hold in general.

While, in general, the existing literature shows that risk-free bonds are the preferred asset to hold in the retirement account, we explain how the investor's equity exposure in both the taxable and the retirement account depends on the length of the remaining investment horizon and the fraction of total wealth in the retirement account. Due to the tax exemption of profits in the retirement account, total wealth contains a tax-gift which becomes bigger as the remaining investment horizon increases. As bonds are usually the preferred asset to hold in a retirement account the tax-gift can be considered an implicit bond holding that is not directly observable in security accounts. As this tax-gift decreases with decreasing length of the remaining investment horizon, so does the unobservable bond holding and the observable equity exposure.

The paper proceeds as follows. Section 2 reviews the related literature. Section 3 introduces the model and discusses the tax-effects of investments into tax-sheltered retirement accounts. Section 4 provides numerical evidence. Section 5 concludes.

## 2 Prior Studies

Dilnot (1995) categorizes retirement accounts according to the taxable treatment of contributions, profits and withdrawals. Even though theoretically, there are many combinations of taxable treatments, in practice there are only three main types of retirement accounts to be found. The so-called taxable accounts (TAs) are accounts in which contributions can only be

Taxation of	TA	TEA	TDA
Contribution	X	X	
Profits	X		
Withdrawal			X

Table 1: Taxable treatment in different accounts

made from after-income-tax dollars, and where profits are taxable and withdrawals are tax exempt. Tax-deferred accounts (TDAs) such as IRAs are characterized by the opportunity to contribute to them from pre-income-tax dollars, and withdrawals are taxable while profits are tax exempt. In tax exempt accounts (TEAs) like Roth-IRAs, contributions are made from after income tax dollars, profits are tax exempt, and withdrawals are usually tax-exempt. In many countries, different kinds of accounts coexist with different taxable treatment. Whereas TAs can be used for other investment objectives besides saving for retirement, TEAs and TDAs are pure retirement accounts. Early withdrawals from them are subject to a penalty tax and contributions to them are limited by law. As only TAs are flexible enough to be used as saving accounts for other investment objectives as well, only TEAs and TDAs are referred to as retirement accounts (RAs) in this article. Due to the differing taxable treatment of the funds in the various accounts, investing a pre-tax dollar in each of the separate types of accounts usually results in varying risk-return profiles of that dollar and different changes in the investor's wealth. Table 1 summarizes the taxable treatment of the three different types of accounts.

Although optimal asset allocation is a topic that has been intensively discussed in economic literature, research on optimal asset location is quite a recent field of research. It goes back to studies of Tepper and Affleck (1974), Black (1980) and Tepper (1981). They analyze optimal investment strategies of companies that run defined-benefit pension plans. If these companies do not face any short-selling restrictions and their gains are fully taxable at the moment of occurrence (i.e. there is no deferral option), they should hold bonds only in their defined-benefit pension plan, where the taxable treatment is similar to that of TEAs. Auerbach and King (1983) point out that this result also applies to investors having the opportunity to invest into a retirement account and a TA.

Shoven and Sialm (1998) and Shoven (1999) introduce the problem of asset location to household investment decisions and point out that each choice of an investment strategy for retirement saving does not only contain a choice about the assets to invest in (the so-called asset allocation problem), but also a choice about which of these assets to locate in a retirement account and which to locate in a taxable account (the so-called asset location problem). In their

studies, they analyze asset location and asset allocation decisions for simulated distributions of wealth. They arrive at the conclusion that due to tax-inefficiency of many actively managed mutual funds, these funds have their preferred location in a retirement account, while tax exempt bonds should be held in a taxable account.

Bodie and Crane (1997), Poterba and Samwick (2001) and Barber and Odean (2003) analyze asset location strategies used by private investors in practice. They report that investors do not realize the opportunities that TDAs offer to them. In particular, they often choose suboptimal asset location strategies. Contribution rates have been found to have an especially substantial impact on utility losses (Gomes et al. (2006)). Amromin (2002) and Bergstresser and Poterba (2004) report similar findings and further point out that many investors have considerable amounts of money in both accounts and hold significantly more stocks in their TDAs than in their TAs. Benartzi and Thaler (2001) point out that investors tend to follow naive  $\frac{1}{n}$ -diversification strategies in their retirement accounts and pension plans and thus the more stock funds offered to them by these plans the more stocks investors tend to hold.

According to Gale and Scholz (1994), the majority of investors in the US that contribute to an individual retirement account are either older than 59 or have substantial funds in a TA in addition to the funds in the IRA. Either of these cases makes the need for an early withdrawal - which is accompanied by a penalty tax - quite unlikely. This is why we put the focus of our analysis on the asset location and asset allocation decisions induced by the tax-effects and ignore early withdrawals.

A dollar invested into a risk-free asset in an RA results in a higher after-tax yield than a dollar invested into the same risk-free asset in a TA. Hence, the dollar in the RA is worth more than the dollar in the TA, a fact which has been pointed out by Dammon et al. (2004) and Poterba (2004). Dammon et al. (2004) call the difference between the value of the dollar in the TEA and the value of the dollar in the TA the "shadow price" of that dollar. This "shadow price" depends on the relative dividend of the assets, the relative capital gains of the assets, the level of tax-rates on these dividends and gains, and the length of the remaining investment horizon. Due to the penalty tax for early withdrawal and the maximum contribution limits for TEAs and TDAs, TAs are often used for retirement saving as well. Hence, private investors saving for retirement usually only locate some part of their retirement savings in a retirement account. Due to the difference in taxable treatment of assets between a taxable account and a retirement account, it is important to make an informed decision as to which asset to locate in the retirement account and which to locate in the taxable account.

The impact of tax-timing strategies on optimal asset location decisions with diversification



concerns is analyzed in Dammon et al. (2004) and Zaman (2005). In Zaman's numerical study, stocks tend to have their preferred location in the taxable account to use potential benefits from tax-timing and exploit the higher tax-burden of bonds. This result does not differ from that in Dammon et al. (2004), whose analysis contains only one risky asset. This finding suggests that it suffices to consider one risky asset in our numerical analysis.

Furthermore, (long-term) capital gains are often subject to a lower tax-rate than dividends and interest. This is why the returns on stocks that mainly consist of capital gains tend to be taxed at a lower rate than the returns on bonds mainly consisting of interest payments. For this reason, Shoven and Sialm (2003) argue that bonds should have their preferred location in a retirement account and stocks shall only be held there if no bonds are held in the taxable account at all. This argument seems to be very convincing at first sight and has been shown to be correct for investors that are not facing short-selling restrictions (see e.g. Dammon et al. (2004), Shoven and Sialm (2003) or the theoretical paper of Huang (2006)). However, Garlappi and Huang (2006) show that this does not necessarily hold if the investor is short-selling constrained.

If investors are facing high labor income risk and have limited liquid financial resources, some papers, among them Amromin (2003), Dammon et al. (2004) and Amromin (2005), argue that holding stocks in the retirement account and bonds in the taxable account can also be an efficient investment strategy when taking labor income shocks into account. They base their argument on the fact that due to the lower volatility of bonds, holding them in the taxable account reduces the probability of having to withdraw funds from the retirement account and to pay the penalty tax. The higher the probability and the order of magnitude of income shocks and the lower the liquid financial resources, the better the strategy to hold sufficient TA-wealth in bonds. Besides, under current law there seem to be tax arbitrage opportunities between savings in retirement accounts and mortgage payments (Amromin et al. (2007)). The literature on taxation and optimal portfolio choice is surveyed in Poterba (2002) and Campbell (2006).

While the focus of the existing literature is on the asset location decision, we concentrate on the impact of the tax-effects in retirement accounts for the investor's asset allocation decision.

## **3 Tax-Effects in Retirement Accounts**

### **3.1 Effects of Tax Deferral**

In this article it is assumed that dividends, interest, and capital gains in the TA are taxable at occurrence. That is, no matter if realized or not, capital gains are fully taxable, i.e. there is

no tax-timing option. Equivalently one could also assume the corresponding tax-rate to be the "effective tax-rate" as described in Constantinides (1983), which reflects the unmodeled optimal tax realization strategy in the risky asset. It is further assumed that the investor cannot go short and there is no need for an early withdrawal from a retirement account. This in particular implies that the investor does not have to pay the penalty tax for early withdrawal at any point in time. A market with  $n$  assets is considered. Assets can be traded without incurring any transaction costs.

Let  $\alpha_{l,i,t}$  denote the proportion of asset  $i$  in period  $t$  with location  $l \in \{R, T\}$ , in which  $R$  denotes the location in a retirement account (TDA or TEA) and  $T$  denotes a location in the taxable account. Let  $\boldsymbol{\alpha}_{l,t} := (\alpha_{l,1,t}, \dots, \alpha_{l,n,t})^\top$  be the vector of proportions of the  $n$  assets in period  $t$  with location  $l$ .  $0 \leq \tau_d < 1$  denotes the constant tax-rate on dividends and interest, and  $0 \leq \tau_g < 1$  the constant tax-rate on capital gains. In particular, it is assumed that short-term and long-term capital gains are subject to the same tax-rate.  $0 \leq d_{R,i,t}$  and  $-1 \leq g_{R,i,t}$  are the dividend or interest rate (dividend rate in this article) and the capital gains rate, respectively, for asset  $i$  in period  $t$  in the retirement account (and thus on a pre-tax basis).  $d_{T,i,t} := (1 - \tau_d)d_{R,i,t}$  and  $g_{T,i,t} := (1 - \tau_g)g_{R,i,t}$  denote the dividend rate and the capital gains rate of asset  $i$  in period  $t$  in the TA (and thus on an after-tax basis).  $\mathbf{d}_{l,t} := (d_{l,1,t}, \dots, d_{l,n,t})^\top$  denotes the vector of dividend rates in period  $t$  with location  $l$  and  $\mathbf{g}_{l,t} := (g_{l,1,t}, \dots, g_{l,n,t})^\top$  denotes the vector of capital gains rates in period  $t$  with location  $l$ .  $0 \leq \tau_{p,t} < 1$  denotes an exogenously given personal income tax-rate of the investor in period  $t$ .  $\mathbf{1}$  denotes a column vector of  $n$  ones. The vector of gross returns  $\mathbf{R}_{l,t}$  in period  $t$  for assets with location  $l$  is given by  $\mathbf{R}_{l,t} := \mathbf{1} + \mathbf{d}_{l,t} + \mathbf{g}_{l,t}$ .  $W_{T,t}$  denotes the wealth of some investor in a TA and  $W_{R,t}$  the wealth of that investor in a RA at the beginning of period  $t$ .

Tax-deferred accounts allow deferring income taxation until withdrawal. That means contributions to such tax-deferred accounts are made from pretax-income. This is in contrast to taxable accounts and tax exempt accounts, where contributions can only be made from after-tax dollars. Investors facing a high marginal income tax rate at the time of contribution, but expecting a lower personal income tax-rate at the time of withdrawal can lower their expected relative income tax burden by an investment in a TDA.

Assume the investor is initially endowed with  $W_{T,0}$  dollars in a TA and  $W_{R,0}$  dollars in a TEA. For simplicity, she is not allowed to shift funds between the two accounts and maximizes utility over a  $T$ -period investment horizon from total final wealth  $W_T := W_{T,T} + W_{R,T}$ .

The investment decision of the investor can be decomposed into two parts. On the one hand, the investor has to decide which assets to hold. This problem is known as the asset allocation

problem. On the other hand, she has to decide which of the assets to hold in the retirement account and which in the taxable account. This problem is known as the asset location problem. In the remainder of this paper an asset allocation for a  $T$ -period investment horizon is defined to be a tuple  $(\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_T)$  of vectors  $\boldsymbol{\alpha}_t$  ( $t \in \mathbb{N}_T := \{n \in \mathbb{N} | n \leq T\}$ ), which contain the proportions of the assets relative to total wealth  $W_t$ . An asset location for a  $T$ -period investment horizon with investment opportunities in a retirement account and a TA is a tuple  $(L_1, \dots, L_T)$  of tuples  $L_t = (\boldsymbol{\alpha}_{R,t}, \boldsymbol{\alpha}_{T,t})$  ( $t \in \mathbb{N}_T$ ), such that  $\boldsymbol{\alpha}_t = \frac{W_{T,t-1}}{W_{t-1}} \boldsymbol{\alpha}_{T,t} + \frac{W_{R,t-1}}{W_{t-1}} \boldsymbol{\alpha}_{R,t}$  ( $t \in \mathbb{N}_T$ ). When returns are stochastic,  $W_{T,t}$ ,  $W_{R,t}$ , and  $W_t$  are not known before the beginning of period  $t$ , which is why  $\boldsymbol{\alpha}_{T,t}$ ,  $\boldsymbol{\alpha}_{R,t}$ , and  $\boldsymbol{\alpha}_t$  also cannot be determined before the end of period  $t - 1$ , that is at the beginning of period  $t$ . The tuple  $(I_1, \dots, I_T)$  of tuples  $I_t = (\boldsymbol{\alpha}_t, \boldsymbol{\alpha}_{R,t}, \boldsymbol{\alpha}_{T,t})$  ( $t \in \mathbb{N}_T$ ) is called an investment strategy. While the asset allocation  $(\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_T)$  provides information about proportions of the asset relative to total wealth, the asset location  $(L_1, \dots, L_T)$  provides information about the accounts in which these assets are held and with which proportions.

If one pre-income-tax dollar at the end of period 0 is invested in a TDA for a  $T$ -period investment horizon and  $x \cdot y$  denotes the Euclidian scalar product of two  $n$ -dimensional vectors  $x$  and  $y$ , total final wealth after income tax is given by

$$\prod_{t=1}^T \left( \boldsymbol{\alpha}_{R,t} \cdot (\mathbf{1} + \mathbf{g}_{R,t} + \mathbf{d}_{R,t}) \right) (1 - \tau_{p,T}) = \prod_{t=1}^T \left( \boldsymbol{\alpha}_{R,t} \cdot \mathbf{R}_{T,t} \right) (1 - \tau_{p,T}). \quad (1)$$

If one invests one such dollar for a  $T$ -period investment horizon in a TEA, income taxation is already due at the time of contribution and one ends up with a final wealth after income tax of

$$(1 - \tau_{p,0}) \prod_{t=1}^T \left( \boldsymbol{\alpha}_{R,t} \cdot (\mathbf{1} + \mathbf{g}_{R,t} + \mathbf{d}_{R,t}) \right) = (1 - \tau_{p,0}) \prod_{t=1}^T \left( \boldsymbol{\alpha}_{R,t} \cdot \mathbf{R}_{T,t} \right). \quad (2)$$

The only difference in total final after income tax wealth of an investment in a TDA and a TEA is the point in time at which the investor has to pay the income tax. As  $\tau_{p,0}$  might differ from  $\tau_{p,T}$ , an investment in a TDA might result in different final wealth than an investment in a TEA. An investment in a TDA offers the opportunity to face an income tax-rate at withdrawal  $\tau_{p,T}$  that is lower than the income tax-rate at contribution  $\tau_{p,0}$ , but bears the risk that  $\tau_{p,T}$  is higher than  $\tau_{p,0}$ . If contributions to a TEA are made from income that has already been subject to income taxation, the factor  $1 - \tau_{p,0}$  is to be omitted.

If one invests one pre-income-tax dollar in a TA for a  $T$ -period investment horizon, total

final after income tax wealth is given by

$$(1 - \tau_{p,0}) \prod_{t=1}^T \left( \boldsymbol{\alpha}_{T,t} \cdot (\mathbf{1} + (1 - \tau_g) \mathbf{g}_{R,t} + (1 - \tau_d) \mathbf{d}_{R,t}) \right) = (1 - \tau_{p,0}) \prod_{t=1}^T (\boldsymbol{\alpha}_{T,t} \cdot \mathbf{R}_{T,t}). \quad (3)$$

Total final wealth after income tax of an investment in a TA and a TEA with the same proportions of the assets  $\boldsymbol{\alpha}_{R,t} = \boldsymbol{\alpha}_{T,t} \forall t \in \mathbb{N}_T$  only differs in the rates of return. In the TA, dividend rates and capital gains rates shrink towards zero by the factor  $1 - \tau_d$  and  $1 - \tau_g$ , respectively. If one invests in assets whose dividend rates and capital gains rates cannot become negative, an investment in a TEA is at least as good as an investment into a TA. If  $\tau_{p,0} \geq \tau_{p,T}$ , this also holds for a comparison between an investment in a TA and a TDA. If however  $\tau_{p,T} > \tau_{p,0}$ , an investment in a TA does not necessarily dominate an investment in a TDA, because for investment horizons of sufficient length, the tax exemption of profits in the TDA can outweigh the tax-burden of the higher income tax-rate.

### 3.2 Effects of Tax Exemption of Profits in Retirement Accounts

Saving for retirement is usually a process lasting several decades, where the effect of tax exemption on profits in TDAs and TEAs becomes of increasing importance due to the compounding of interest and the length of the investment horizon. When combined with the effect of shrinking returns on total final wealth, the tax exemption of profits in retirement accounts can be considered a public contribution to private retirement saving. This contribution does not come directly in the form of a payment to the retirement account, but indirectly as what can be called a tax-gift. For an investor who can invest in a TA and a TEA total final wealth is given by

$$W_T = W_{T,0} \prod_{t=1}^T (\boldsymbol{\alpha}_{T,t} \cdot \mathbf{R}_{T,t}) + W_{R,0} \prod_{t=1}^T (\boldsymbol{\alpha}_{R,t} \cdot \mathbf{R}_{R,t}). \quad (4)$$

For positive returns and  $\boldsymbol{\alpha}_{T,t} = \boldsymbol{\alpha}_{R,t}$  ( $t \in \mathbb{N}_T$ ), growth of wealth in the TEA is higher than in the TA due to the tax exemption of profits. Thus, the longer the investment horizon the stronger the impact of wealth in the TEA on total final wealth. The tax advantage that results from the tax exemption of profits in the TEA in period  $t$  is given by

$$T_t := \boldsymbol{\alpha}_{R,t} \cdot (\mathbf{R}_{R,t} - \mathbf{R}_{T,t}) = \boldsymbol{\alpha}_{R,t} \cdot (\tau_d \mathbf{d}_{R,t} + \tau_g \mathbf{g}_{R,t}). \quad (5)$$

$T_t$  can be interpreted as a relative tax-gift in period  $t$  that is paid for each dollar invested into a TEA. Besides the vector of pre-tax and after-tax returns on the assets and the tax-rates on gains and dividends, the relative tax-gift depends in particular on the choice of the proportions of the assets  $\alpha_{R,t}$  in the TEA in period  $t$ . Equation (4) can be rewritten as

$$W_T = (W_{T,0} + W_{R,0}) \prod_{t=1}^T (\alpha_t \cdot \mathbf{R}_{T,t}) + \sum_{t=1}^T W_{R,0} \prod_{j=1}^{t-1} (\alpha_{R,j} \cdot \mathbf{R}_{R,j}) T_t \prod_{j=t+1}^T (\alpha_j \cdot \mathbf{R}_{T,j}). \quad (6)$$

Total final wealth after income tax  $W_T$  at the end of period  $T$ , thus consists of two components. The first summand is total final wealth an investor would have attained without having had the opportunity to invest in a TEA and is thus driven to invest her entire initial wealth  $W_{T,0} + W_{R,0}$  in a TA. The second

$$G_T := \sum_{t=1}^T W_{R,0} \prod_{j=1}^{t-1} (\alpha_{R,j} \cdot \mathbf{R}_{R,j}) T_t \prod_{j=t+1}^T (\alpha_j \cdot \mathbf{R}_{T,j}) \quad (7)$$

is the amount of total final wealth  $W_T$  that results from the tax exemption of profits as well as interest and compound interest on the profits in the TEA. It can be interpreted as a public tax-gift for private retirement saving in terms of non-levied taxes on gains and dividends as well as interest and compound interest and will be referred to as total *tax-gift*.

Especially for long investment horizons this tax-gift can be a substantial fraction of the investor's total final wealth. To demonstrate the power of this tax-gift consider an investor who is initially endowed with \$ 5,000 in both her taxable and her retirement account and can only invest into one risk-free asset with a pre-tax return of 6%. If the investor's investment horizon is 40 years and the tax-rate applicable to the return of the asset is 36%, i.e. her after-tax return is 3.84%, her total final wealth is \$ 74,000 and her tax-gift is \$ 28,857, which is about 39% of total final wealth. The tax-gift thus has a tremendous impact on total final wealth.

Equation (7) can be further decomposed into

$$\begin{aligned} G_T &= \sum_{\substack{t=1 \\ t \neq k}}^T W_{R,0} \prod_{\substack{j=1 \\ j \neq k}}^{t-1} (\alpha_{R,j} \cdot \mathbf{R}_{R,j}) T_t \prod_{j=t+1}^T (\alpha_j \cdot \mathbf{R}_{T,j}) \\ &\quad + W_{R,0} \prod_{j=1}^{k-1} (\alpha_{R,j} \cdot \mathbf{R}_{R,j}) T_k \prod_{j=k+1}^T (\alpha_j \cdot \mathbf{R}_{T,j}) \\ &\quad + \sum_{t=k+1}^T W_{R,0} \prod_{\substack{j=1 \\ j \neq k}}^{t-1} (\alpha_{R,j} \cdot \mathbf{R}_{R,j}) (\alpha_{R,k} \cdot \mathbf{R}_{R,k} - 1) T_t \prod_{j=t+1}^T (\alpha_j \cdot \mathbf{R}_{T,j}). \end{aligned} \quad (8)$$

According to Equation (8), the total tax-gift contained in total final wealth  $W_T$  can be decomposed into three summands. The part of  $G_T$  that is independent from the investment decision in period  $k$  and only depends on the tax-effects of the other periods is given by the first summand. The absolute change in total final wealth that results from the second summand

$$W_{R,0} \prod_{j=1}^{k-1} (\alpha_{R,j} \cdot \mathbf{R}_{R,j}) T_k \prod_{j=k+1}^T (\alpha_j \cdot \mathbf{R}_{T,j}) \quad (9)$$

describes the change in total final wealth that results from the relative tax-gift in period  $k$  and is called the *direct tax-effect* of the investment decision in period  $k$ , or just the direct tax-effect in this article. It depends on growth of TEA-wealth until the end of period  $k - 1$  and therefore also on the proportions of the assets in the TEA  $\alpha_{R,j}$  ( $j \in \mathbb{N}_{k-1}$ ) until period  $k - 1$ , the relative tax-gift  $T_k$ , and the after-tax growth rate  $\prod_{j=k+1}^T (\alpha_j \cdot \mathbf{R}_{T,j})$  until the end of the investment horizon.  $W_{R,0} \prod_{j=1}^{k-1} (\alpha_{R,j} \cdot \mathbf{R}_{R,j}) T_k$  describes the *tax-gift* in period  $k$  which then grows by the after-tax growth rate  $\prod_{j=k+1}^t (\alpha_j \cdot \mathbf{R}_{T,j})$  until the end of the investment horizon. The absolute change in total final wealth that results from the third summand

$$\sum_{t=k+1}^T W_{R,0} \prod_{\substack{j=1 \\ j \neq k}}^{t-1} (\alpha_{R,j} \cdot \mathbf{R}_{R,j}) (\alpha_{R,k} \cdot \mathbf{R}_{R,k} - 1) T_t \prod_{j=t+1}^T (\alpha_j \cdot \mathbf{R}_{T,j}) \quad (10)$$

is the change in total final wealth that results from the change of period  $k$  retirement wealth and is called the *indirect tax-effect* of the investment decision in period  $k$  or just the indirect tax-effect in the preceding. Each summand in (10) can be interpreted as follows:  $W_{R,0} \prod_{\substack{j=1 \\ j \neq k}}^{t-1} (\alpha_{R,j} \cdot \mathbf{R}_{R,j}) (\alpha_{R,k} \cdot \mathbf{R}_{R,k} - 1) T_t$  is the effect on period  $t$  total wealth, which results from the choice of  $\alpha_{R,k}$  in period  $k$  and the increase of the tax exempt basis in that period. It then grows with after-tax return  $\prod_{j=t+1}^T (\alpha_j \cdot \mathbf{R}_{T,j})$  until the end of period  $T$  to the indirect tax-effect.

The direct tax-effect from Equation (9) has a singular effect, as it only has an impact on total wealth in one single period and then grows with an after income tax return until the end of the investment horizon. The expression for the indirect tax-effect in Equation (10), however, has more than one summand for  $k \leq T - 2$ . Due to the change in the tax exempt basis, it has an impact on all future periods. This is why the impact on total final wealth from the indirect tax-effect in period  $k$  can be quantified by the direct tax-effects of all future periods on that part of TEA wealth that results from the investment decision in period  $k$ .

Both the direct and the indirect tax-effect are not necessarily an advantage to the investor.

If the total return on TEA-wealth in period  $t$  is negative, the relative tax-gift  $T_t$  can become negative. This is because contrary to an investment in a TA, the treasury does not participate in these losses via the taxation of dividends and gains. If  $T_t$  becomes negative, so does the direct tax-effect. As with the direct tax-effect, negative gains that outweigh potential interest or dividends can lead to a negative return on TEA wealth, which causes a relative reduction in the tax exempt basis of forthcoming periods by  $(\alpha_{R,t} \cdot \mathbf{R}_{R,t} - 1) < 0$ , and shows a negative indirect tax-effect.

### 3.3 Generalization to TDAs

The argument for the case of an investment opportunity set with a TEA and a TA can be generalized to the case of an investment opportunity set with a TDA and a TA as follows: Initial wealth  $W_{R,0}$  which has been invested into a TEA at the end of period 0 must have been made from after income tax dollars. As an investment into a TDA is made from pre income-tax dollars, instead of investing  $W_{R,0}$  after income tax dollars into a TEA, one can invest  $\frac{W_{R,0}}{1-\tau_{p,0}}$  pre income-tax dollars in a TDA. As wealth in a TDA is still subject to income taxation at the end of the investment horizon at rate  $\tau_{p,T}$ , the investor can only consider  $W'_{R,t} := W_{R,t} \frac{1-\tau_{p,T}}{1-\tau_{p,0}}$  her effective wealth, as the remainder falls to the treasury at the end of period  $t$ . If  $\tau_{p,T} < \tau_{p,0}$ , then  $W'_{R,t} > W_{R,t}$  and an investment in a TDA results in a higher final wealth than an investment into a TEA due to the lower burdening with income tax. As  $W'_{R,t}$  and  $W_{R,t}$  only differ by the constant factor  $\frac{1-\tau_{p,T}}{1-\tau_{p,0}}$ , the problem of finding the optimal investment strategy with an investment opportunity set with both a TEA and a TA, is equivalent to the problem of with a TDA and a TA. Hence, it suffices to consider an investor with the opportunity to invest into both a TA and a TEA. As before,  $W_{R,t}$  denotes the wealth in the RA if the retirement account is a TEA and the effective wealth in the RA if the retirement account is a TDA.

### 3.4 Optimal Asset Location Decisions

According to the seminal work of Huang (2006), investors that do not face short-selling constraints should invest their entire retirement-wealth into the asset  $a$  with the highest effective tax-rate

$$\tau_a := \frac{(1 + (1 - \tau_d) d_B) (x_a - 1)}{(1 + (1 - \tau_d) d_B) x_a - 1} \quad (11)$$

in which  $x_a := \frac{1}{1-\tau_g} + \frac{d_a(\tau_d-\tau_g)-\tau_g}{(1-\tau_g)(1+(1-\tau_d)d_B)}$  is the replication cost in the TA of one dollar of asset  $a$  in the RA. Furthermore, in this case, the asset location and asset allocation problem are independent from each other, as each dollar in asset  $a$  in the RA can be replicated by  $\frac{1}{1-\tau_g}$  dollars of that asset and  $\frac{d_a(\tau_d-\tau_g)-\tau_g}{(1-\tau_g)(1+(1-\tau_d)d_B)}$  dollars of the risk-free asset in the TA. In particular, in such a setting the asset allocation and asset location decision are independent from each other.

If, however, the investor is not allowed to go short, it is no longer necessarily optimal for him to hold the asset with the highest effective tax-rate in the RA as shown in Garlappi and Huang (2006). This is due to the fact that the investor cannot replicate the tax-deferred portfolio in the taxable account if this required to go short. Garlappi and Huang (2006) have also pointed out that holding stocks in the RA can help smooth the ratio of the relative tax-gift  $T_t$  times  $W_{R,t}$  to total wealth  $W_t$ . They call this ratio the tax-subsidy. This tax-subsidy can be interpreted as that part of growth in total wealth that results from the relative tax-gift in period  $t$ . This growth in total wealth can be smoothed by constructing portfolios in the RA and the TA that have similar weights in the two assets.

Smoothing these extra growth-rates results in less volatile distributions of final wealth, which is desirable for risk-averse investors. However, Garlappi and Huang (2006) assume the return of the risky asset to be binomially distributed. In this case, it is possible to smooth the tax-subsidy in such a way that it takes the same value, independent from the realization of the return from the risky asset. If, however, the return on the risky asset  $S$  has a more complex distribution and can take more than two realizations, it is no longer possible to keep the tax-subsidy at the same level independent from the realization of the return. Nevertheless, in such a case, it is still possible to dampen the impact of the total tax-gift, which, according to Equation (7), is ceteris paribus best attained when only holding bonds in the RA. As one can see from Equation (6), in addition to the risks due to the volatility of growth of total wealth after-tax, the volatility of the total tax-gift is a second source of risk to which total final wealth is exposed when holding stocks in a retirement account.

For a given asset allocation  $\alpha_t$  ( $t \in \mathbb{N}_T \setminus \{k\}$ ), the decision to locate stocks instead of bonds in the retirement account in period  $k$  has three effects on the risk-return profile of total final wealth given everything else as equal. First, it has an impact on the direct tax-effect whose sign is depending on the absolute tax-burden of stocks and bonds. Second, due to the higher expected pre-tax return on stocks the expected indirect tax-effect increases. Third, if the pre- and the after-tax returns of  $S$  and  $B$  are not negatively correlated, the volatility of total final wealth increases as both the direct and the indirect tax-effect are subject to higher volatility. While



according to the first and third effect, bonds should be preferably located in the retirement account, the second effect suggests that stocks could be preferably held in the retirement account as well.

However, the changes of the risk-return profiles of stocks and bonds when shifting them from the TA to the RA suggest that bonds should still be the preferred asset to hold in the RA. Shifting bonds from the TA to the RA increases their return. However, shifting stocks from the TA to the RA does not only increase their expected return, but also its volatility. Moreover, bonds usually come with a higher effective tax-rate than stocks.

According to Equation (6) total final wealth consists of two components: total final wealth the investor would have attained without the opportunity of investing into a retirement account and a tax-gift that results from the tax exemption of profits in these accounts. As the impact of the tax-gift decreases with a decreasing investment horizon, its impact is stronger, the longer the remaining investment horizon. Due to the fact that investors usually prefer bonds in their retirement account, the properties of the tax-gift are more similar to those of bonds than to those of stocks. The tax-gift can thus be regarded an implicit bond position that is not directly observable in the investor's security accounts. The higher the unobservable implicit bond position, the higher the observable equity position. This is why for longer remaining investment horizons investors will have a higher observable bond position than for shorter remaining investment horizons. Hence, besides decreasing future labor income, decreasing unobservable bond holding in retirement accounts are another reason for decreasing equity exposure over the life cycle.

Due to the risks associated with the indirect tax-effect, the preference for bonds in the retirement account, and the shadow price of one dollar in the retirement account, an investor with a high fraction of her wealth in a retirement account will hold a lower total fraction of stocks than an investor with a high fraction of her wealth in a taxable account.

## 4 Numerical Evidence

For the numerical analysis, a short-selling constrained investor who can invest in a market with a risky asset  $S$  and a risk-free asset  $B$  is considered. The characteristics of  $S$  and  $B$  are similar to those of stocks and bonds, respectively. The capital gains rate of  $S$  is lognormally distributed with an expected gain of  $\mu_{g,S} = 0.09$  and a standard deviation of  $\sigma_{g,S} = 0.20$ . Asset  $S$  has a constant dividend rate of  $d_S = 0.02$ . Asset  $B$  has a certain interest-rate of  $d_B = 0.06$  and no capital gains. An investment horizon of  $T = 40$  years is considered which is a realistic

investment horizon for retirement saving. The tax-rates on dividends and gains are assumed to be  $\tau_d = 0.36$  and  $\tau_g = 0.2$ , respectively.<sup>1</sup> This parameter choice follows Dammon et al. (2004). For these parameter values, the effective tax-rates for stocks and bonds are  $\tau_S = 0.16$  and  $\tau_B = 0.36$ , respectively. Thus, investors that are not short-selling constrained in their taxable accounts should hold their entire retirement wealth in bonds. Even though we consider short-selling constraint investors, this result remains valid.

The investor is assumed to maximize her utility of total final wealth  $W_T$  from a CRRA-utility function with a parameter of risk aversion of  $\gamma \in [0, \infty)$

$$U(W_T) := \begin{cases} \frac{W_T^{1-\gamma}}{1-\gamma} & \text{for } \gamma \neq 1 \\ \ln(W_T) & \text{for } \gamma = 1. \end{cases} \quad (12)$$

Her optimization problem is

$$\max_{\boldsymbol{\alpha}_T, \boldsymbol{\alpha}_R} \mathbb{E}[U[W_T]] \quad (13)$$

s.t.

$$W_t = W_{R,t} + W_{T,t} \quad (14)$$

$$W_{R,t+1} = W_{R,t} (\boldsymbol{\alpha}_{R,t} \cdot \mathbf{R}_{R,t}) \quad (15)$$

$$W_{T,t+1} = W_{T,t} (\boldsymbol{\alpha}_{T,t} \cdot \mathbf{R}_{T,t}) \quad (16)$$

$$0 \leq \boldsymbol{\alpha}_{R,t}, \boldsymbol{\alpha}_{T,t} \leq 1 \quad (17)$$

in which  $\boldsymbol{\alpha}_T := (\alpha_{T,1}, \dots, \alpha_{T,t})$  and  $\boldsymbol{\alpha}_R := (\alpha_{R,1}, \dots, \alpha_{R,t})$ . Normalizing by  $W_t$  and taking into account that  $U$  is homogeneous in wealth implies that the optimization problem is equivalent

---

<sup>1</sup>Since capital gains that are realized short-term are taxable at the investor's personal income tax-rate which corresponds to the tax-rate on dividends, choosing  $\tau_g$  to be smaller than  $\tau_d$  implies the assumption that the long-term capital gains rate applies to the investor's capital gains.

to the solution of

$$\max_{\boldsymbol{\alpha}_T, \boldsymbol{\alpha}_R} \mathbb{E} \left[ U \left[ \frac{W_T}{W_t} \right] \right] \quad (18)$$

s.t.

$$1 = \frac{W_{R,t}}{W_t} + \frac{W_{T,t}}{W_t} \quad (19)$$

$$\frac{W_{R,t+1}}{W_t} = \frac{W_{R,t}}{W_t} (\boldsymbol{\alpha}_{R,t} \cdot \mathbf{R}_{R,t}) \quad (20)$$

$$\frac{W_{T,t+1}}{W_t} = \frac{W_{T,t}}{W_t} (\boldsymbol{\alpha}_{T,t} \cdot \mathbf{R}_{T,t}) \quad (21)$$

$$0 \leq \boldsymbol{\alpha}_{R,t}, \boldsymbol{\alpha}_{T,t} \leq 1. \quad (22)$$

If  $V_t(X_t)$  denotes the investor's utility as a function of her states variables  $X_t$  at time  $t$ , it holds for an investor with risk-aversion of  $\gamma \neq 1$  that

$$V_t(X_t) := \max_{\boldsymbol{\alpha}_T, \boldsymbol{\alpha}_R} \mathbb{E} \left[ U \left[ \frac{W_{R,t}}{W_t} \right] \right] \quad (23)$$

$$= \max_{\boldsymbol{\alpha}_T, \boldsymbol{\alpha}_R} \mathbb{E} \left[ V_{t+1}(X_{t+1}) \cdot \left( \frac{W_{R,t}}{W_t} (\boldsymbol{\alpha}_{R,t} \cdot \mathbf{R}_{R,t}) + \left( 1 - \frac{W_{R,t}}{W_t} \right) (\boldsymbol{\alpha}_{T,t} \cdot \mathbf{R}_{T,t}) \right)^{1-\gamma} \right]. \quad (24)$$

The solution of this problem can thus be computed numerically using backward induction. For the optimization procedure one state-variable (the percentage of wealth in the RA relative to total wealth  $X_t = \frac{W_{R,t}}{W_t}$  at the beginning of period  $t$ ) is sufficient. The grid is spanned with 101 grid points that are equally distributed on  $[0, 1]$ . The integral of the expectation in Equation (24) is computed using Gaussian quadrature.

If the investor would only have the opportunity to invest into one account, she would hold the same fraction of stocks in all periods according to the classical result of Merton (1969) and Samuelson (1969). All derivations from a constant fraction of stocks thus have to be driven by the different taxable treatment of the assets in the two accounts.

Figure 1 shows the optimal equity proportion for an investor with risk-aversion of  $\gamma = 3$  as a function of time passed since the initial investment and her fraction of retirement wealth to total wealth. The upper left and the upper right graphs show her optimal fraction of stocks in the TA and the RA, respectively. If her fraction of wealth in the retirement account is zero or one, the investor is in the one-account world, there is no asset location decision, and in line with Merton (1969) and Samuelson (1969) she holds the same fraction of stocks independent from the remaining investment horizon. If, however, the investor is endowed with both taxable and retirement wealth, she has to decide, which assets to hold in the retirement account and

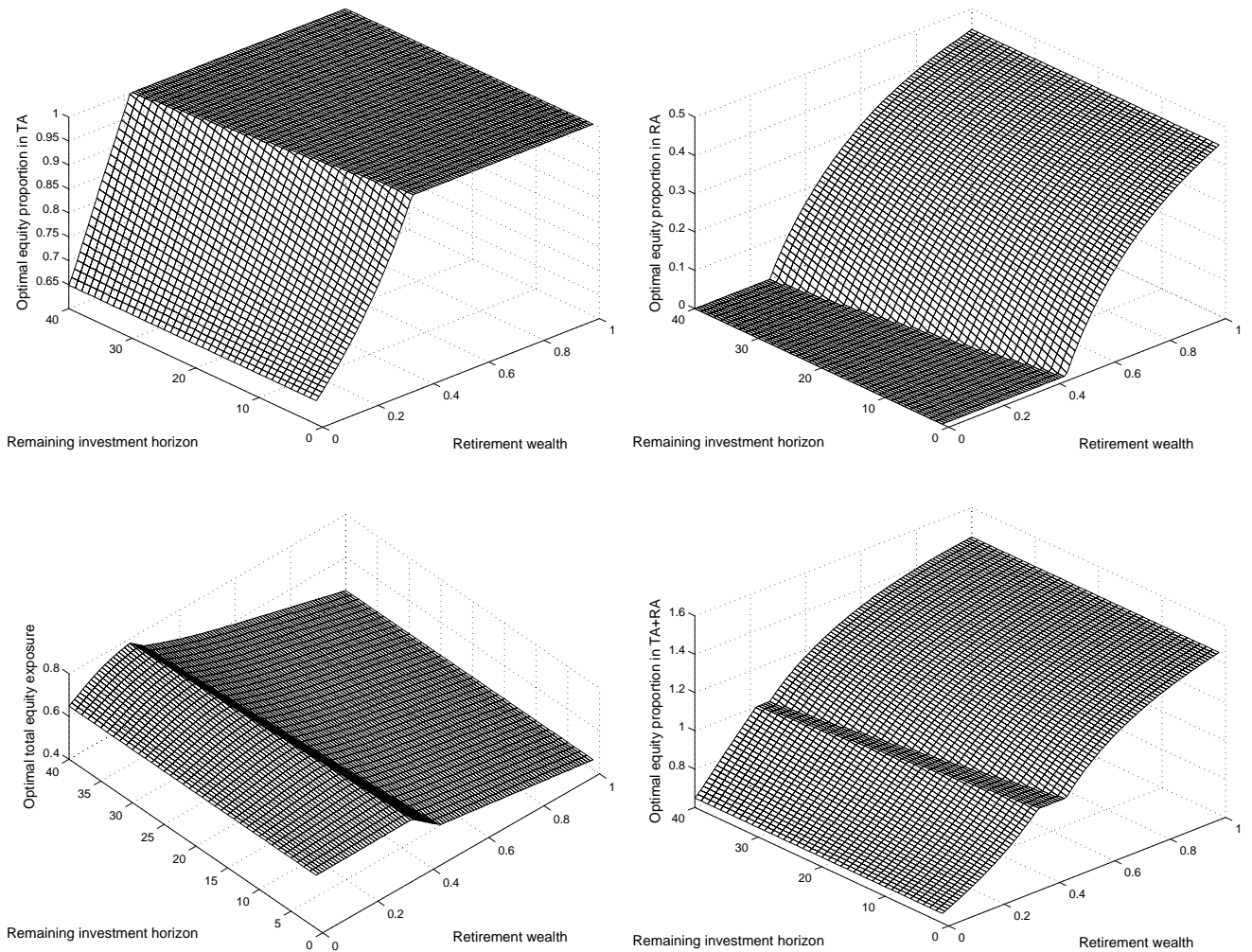


Figure 1: This Figure shows the optimal asset location and asset allocation strategy as a function of the investor's fraction of retirement wealth to total wealth (Retirement Wealth) and her remaining investment horizon. The upper left graph shows her optimal fraction of stocks in the TA, the upper right graph depicts her optimal fraction of stocks in the RA. The lower left graph shows the optimal overall equity exposure, the lower right graph shows the sum of the optimal relative equity exposure in the TA and the RA.

which in the taxable account. Confirming the findings of the recent literature on optimal asset location, we find the investor to prefer bonds in the retirement account and stocks in the taxable account.

The lower right graph, which depicts the sum of the investor's optimal fraction of stocks in the TA and the RA, shows that the investor only holds stocks in the retirement account if her taxable wealth is entirely invested in stocks. With an increasing fraction of wealth in the retirement account, the investor first increases her fraction of stocks in the TA until it has attained 100% as can be seen from the upper left graph. However, increasing her fraction of retirement wealth further, does not cause the investor to immediately increase her fraction of stocks in the retirement account - as can be seen from the lower right graph. For small increases

in retirement wealth, she keeps her fraction of stocks in the retirement account at 0% (*plateau effect*), which reflects the fact that bonds are preferred in the RA as they come with a more advantageous risk-return profile in the retirement account. Furthermore, if only risk-free bonds are held in the RA, there is no additional source of risk from the tax-effect. The size of the plateau effect increases with a short-term investment horizon. This is due to the fact that for short investment horizons the probability of facing a negative tax-effect when being invested into equities in the retirement account is significantly higher than for long investment horizons.

If the fraction of retirement wealth increases even further, the investor increases the fraction of stocks in the retirement account. This reflects her desire not to get too heavily invested into bonds. However, the increase in the fraction of stocks in the retirement account is slower than in the taxable account. If the investor does not have any retirement wealth, her optimal equity exposure is about 70%, if she does not have any taxable wealth her optimal equity exposure is only about 50%. This is due to the fact that bonds have a more advantageous risk-return profile in the retirement account.

The investor's fraction of stocks in both the taxable account and the retirement account decreases with decreasing length of the remaining investment horizon. This is due to the fact that with decreasing investment horizon, the expected tax-gift decreases. As the investor prefers bonds in the retirement account for lower levels of retirement wealth, the tax-gift can be considered a certain income stream which is similar to a risk-free bond. If the investor has a higher retirement wealth, the tax-gift is still more similar to a risk-free bond than to a stock as the investor's fraction of stocks in the RA remains below 50%, despite the associated risk. This does not necessarily hold for investors with even lower risk-aversion which is why in that case the tax-gift can become more similar to an implicit stock-holding. Due to the absence of this implicit bond holding, the investor increases her explicit bond holding and thus decreases her equity exposure for short investment horizons.

The lower left graph depicts the fraction of stocks relative to total wealth. For low levels of retirement wealth, the stock fraction is more prevalent. This is due to sharp increases in equity exposure in the taxable account. This is explained by the fact that one dollar of bonds in the retirement account has a higher impact on total final wealth than one dollar in the taxable account due to the tax-effect. In particular, the indirect tax-effect has a tremendous impact for long investment horizons. As the tax-effect diminishes with decreasing investment horizon, the increase in the total fraction of stocks decreases with increasing time passed since the initial investment.

For higher levels of retirement wealth, the total fraction of stocks first rapidly decreases due

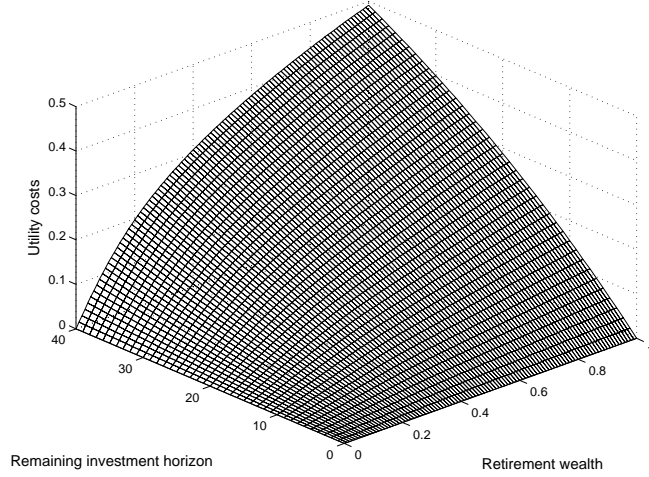


Figure 2: This Figure shows the increase in taxable wealth an investor who does not have retirement wealth needs to be compensated with to attain the same level of expected utility as an investor who is endowed with the same total wealth of which some is invested in a retirement account (utility costs) as a function of this investor’s fraction of total wealth in the retirement account (retirement wealth) and the remaining investment horizon.

to the plateau effect and the investor’s aversion against holding stocks in the retirement account. For even higher levels of retirement wealth, the total fraction of stocks decreases even further, a finding noted by Dammon et al. (2004). Consequently, one dollar of stocks in the retirement account has a higher impact on total final wealth than one dollar of stocks in the taxable account. However, the slope of the decrease is somewhat lower as the investor starts holding stocks in her retirement account. Nevertheless, as bonds come with a more advantageous risk-profile in the retirement account, this increase is not strong enough to prevent the total fraction of stocks from decreasing as retirement wealth increases. Our obtained results remain valid when varying the investor’s risk-aversion parameter  $\gamma$ .

Since retirement accounts allow for earning pre-tax returns while taxable accounts only allow for earning after-tax returns, one dollar in a retirement account is worth more than one dollar of taxable wealth. We analyze the value of retirement wealth by computing the increase in taxable wealth an investor with some given total wealth which is entirely held in a taxable accounts needs to be compensated with to attain the same level of expected utility as an investor who is holding the same amount of total wealth of which some part of is invested in a retirement account.

Figure 2 shows the increase in taxable wealth an investor who is initially endowed with only taxable wealth needs to be compensated with to attain the same level of expected utility as an investor who is endowed with the same total wealth of which some part is held in a retirement account (utility costs). The investor’s utility costs are increasing both in her retirement wealth

and her remaining investment horizon. Each dollar in the retirement account provides the investor with some tax-gift, which is why the investor's utility costs increase in her retirement wealth. With increasing length of the investment horizon the impact of the tax-gift increases. Consequently, the investor's utility costs increase in the length of her remaining investment horizon.

However, the investor's utility costs are concave in her retirement wealth. This is due to the fact that with increasing retirement wealth the investor increases her fraction of stocks in the retirement account for not getting too heavily invested into bonds. Since bonds are the preferred asset to hold in the retirement account, each dollar of bonds that is held in the retirement account provides the investor with a higher advantage than a dollar of stocks that is held in the retirement account. Consequently, her marginal utility gains decrease when she holds stocks in her retirement account.

## 5 Conclusion

In this paper, the field of asset location decisions for retirement savers having the opportunity to invest into both a retirement account and a taxable account is outlined.

Confirming recent findings in economic literature that bonds are the preferred asset to hold in the retirement account, we show that the investor only holds stocks in the retirement account if her taxable wealth is entirely invested into equity. We further show that the investor does not increase her equity exposure in the retirement account immediately with increasing retirement wealth, but prefers to hold only bonds in her retirement account for small increases in retirement wealth which is another indication for her preference for bonds in the retirement account. If, however, the investor's retirement wealth is substantial, she also holds some stocks in her retirement account in order to prevent from investing too heavily in bonds.

While the literature on optimal asset location concludes in (almost) one voice that bonds are the preferred asset to hold in retirement accounts, this paper focuses on the relation of asset location and asset allocation and shows that besides the locational preference of bonds in retirement accounts, the opportunity to invest in a retirement account also has an impact on asset allocation. It is argued that the different taxable treatment of capital gains and dividends in taxable as well as tax-sheltered retirement accounts has an impact on asset allocation. Compared to the benchmark of a constant equity proportion in a one-account problem, the investor's equity proportion depends on both her fraction of total wealth in the retirement account and the length of the remaining investment horizon. The longer the remaining investment horizon,

the higher the investor's equity exposure in both taxable and retirement account. This finding is due to the fact that total final wealth can be decomposed into what the investor would have attained in the absence of a tax-deferred investment vehicle and a tax-gift resulting from the tax exemption of profits in retirement accounts. As the properties of the tax-gift are more similar to those of risk-free bonds than to those of stocks and its impact is decreasing with decreasing remaining investment horizon, the investor is endowed with some implicit bond holding that is unobservable in her security account. As the impact of the tax-gift on total final wealth is stronger the longer the remaining investment horizon, the investor's equity exposure is higher the longer the remaining investment horizon.

## References

- AMROMIN, G. 2002. Portfolio allocation choices in taxable and tax-deferred accounts: An empirical analysis of tax efficiency. mimeo, University of Chicago Department of Economics.
- AMROMIN, G. 2003. Household portfolio choices in taxable and tax-deferred accounts: Another puzzle? *European Finance Review* 7:547–582.
- AMROMIN, G. 2005. Precautionary savings motives and tax efficiency of household portfolios: An empirical analysis. Finance and Economics Discussion Series, Federal Reserve Board, Washington, D.C.
- AMROMIN, G., HUANG, J., AND SIALM, C. 2007. The tradeoff between mortgage prepayments and tax-deferred retirement savings. *Journal of Public Economics* forthcoming.
- AUERBACH, A. AND KING, M. 1983. Taxation, portfolio choice, and debt-equity ratios: A general equilibrium model. *The Quarterly Journal of Economics* 98:587–610.
- BARBER, B. AND ODEAN, T. 2003. Are individual investors tax savvy? evidence from retail and discount brokerage accounts. *Journal of Public Economics* 88:419–442.
- BENARTZI, S. AND THALER, R. 2001. Naive diversification strategies in defined contribution saving plans. *American Economic Review* 91:79–98.
- BERGSTRESSER, D. AND POTERBA, J. 2004. Asset allocation and asset location: Household evidence from the survey of consumer finance. *Journal of Public Economics* 88:1893–1915.
- BLACK, F. 1980. The tax consequences of long-run pension policy. *Financial Analysts Journal* 36:21–28.



- BODIE, Z. AND CRANE, D. 1997. Personal investing: Advice, theory, and evidence. *Financial Analysts Journal* 53:13–28.
- CAMPBELL, J. 2006. Household finance. *Journal of Finance* 61:1553–1604.
- CONSTANTINIDES, G. 1983. Capital market equilibrium with personal taxes. *Econometrica* 51:611–636.
- DAMMON, R., SPATT, C., AND ZHANG, H. 2004. Optimal asset location and allocation with taxable and tax-deferred investing. *Journal of Finance* 59:999–1037.
- DILNOT, A. 1995. The taxation of private pensions. In Z. Bodie, O. Mitchell, and J. Turner (eds.), *Securing Employer Based Pensions: An international Perspective*. University of Pennsylvania Press.
- GALE, W. AND SCHOLZ, J. 1994. IRAs and household saving. *The American Economic Review* 84:1233–1260.
- GARLAPPI, L. AND HUANG, J. 2006. Are stocks desirable in tax-deferred accounts? *Journal of Public Economics* 90:2257–2283.
- GOMES, F., MICHAELIDES, A., AND POLKOVNICHENKO, V. 2006. Optimal savings with taxable and tax-deferred accounts. mimeo.
- HUANG, J. 2006. Taxable and tax-deferred investing: A tax arbitrage approach. mimeo.
- MERTON, R. 1969. Lifetime portfolio selection under uncertainty: The continuous time case. *Review of Economics and Statistics* 51:247–257.
- POTERBA, J. 2002. Taxation, risk-taking, and household portfolio behavior. In A. Auerbach and M. Feldstein (eds.), *Handbook of Public Economics Vol. 3*, chapter 17. Elsevier, North-Holland, Amsterdam.
- POTERBA, J. 2004. Valuing assets in retirement saving accounts. *National Tax Journal* 57:489–512.
- POTERBA, J. AND SAMWICK, A. 2001. Household portfolio allocation over the life cycle. In S. Ogura, T. Tachibanaki, and D. Wise (eds.), *Aging Issues in the US and Japan*. University of Chicago Press, Chicago.

- POTERBA, J., SHOVEN, J., AND SIALM, C. 2004. Asset location for retirement savers. *In* G. Gale, J. Shoven, and M. Warshawsky (eds.), *Private Pensions and Public Policies*. Brooking Institute, Washington, D.C.
- SAMUELSON, P. 1969. Lifetime portfolio selection under uncertainty: the discrete time case. *Review of Economics and Statistics* 51:239–246.
- SHOVEN, J. 1999. The location and allocation of assets in pension and conventional saving accounts. NBER Working Paper 7007.
- SHOVEN, J. AND SIALM, C. 1998. Long run asset allocation for retirement savings. *Journal of Private Portfolio Management* 1:13–26.
- SHOVEN, J. AND SIALM, C. 2003. Asset location in tax-deferred and conventional savings accounts. *Journal of Public Economics* 88:23–38.
- TEPPER, I. 1981. Taxation and corporate pension policy. *Journal of Finance* 36:1–13.
- TEPPER, I. AND AFFLECK, A. 1974. Pension plan liabilities and corporate financial strategies. *Journal of Finance* 29:1549–1564.
- ZAMAN, A. 2005. Tax-deferred investing with diversification concern. Sobey School of Business, St. Mary's University, Halifax, working paper.

## Working Paper Series: Finance & Accounting

- No.175: **Reint Gropp / Marco Lo Duca / Jukka Vesala**, Cross-Border Bank Contagion in Europe, Februar 2007
- No.174: **Wolfram J. Horneff / Raimond Maurer / Michael Stamos**, Optimal Gradual Annuitization: Quantifying the Costs of Switching to Annuities, Februar 2007
- No.173: **Reint Gropp / Arjan Kadareja**, Stale information, shocks and volatility, Februar 2007
- No.172: **Michael H. Grote / Florian Kube / Michael Küchen**, „Home, sweet home“- Die Entwicklung des Handelsvolumens deutscher Aktien im Ausland, Dezember 2006
- No.171: **Christina E. Bannier/ Dennis N. Hänsel**, Determinants of Banks' Engagement in Loan Securitization, October 2006
- No.170: **Marcel Marekwica/ Raimond Maurer**, Proof that it is not always optimal to locate bonds in a tax-deferred Account, September 2006
- No.169: **Marcel Marekwica/ Raimond Maurer/ Steffen Sebastian**, Asset Meltdown – Fact or Fiction?, August 2006
- No.168: **Steffen Sebastian/ Marcel Tyrell**, Open-End Real Estate Funds: Danger or Diamond?, May 2006
- No.167: **Grigori Feiguine**, Rentenreform in Russland – heutiger Stand und Entwicklungsperspektiven im internationalen Vergleich, May 2006
- No.166: **Tatiana Sedash**, Mutual Fund Investing – One of the Main Ways of Saving for Retirement in Russia, May 2006
- No.165: **Christina E. Bannier (geb. Metz)/ Falko Fecht/ Marcel Tyrell**, Open-End Real Estate Funds in Germany – Genesis and Crises, May 2006
- No.164: **Jan Pieter Krahen**, Die Stabilität von Finanzmärkten: Wie kann die Wirtschaftspolitik Vertrauen schaffen?, May 2006
- No.163: **Patrick Behr/ André Güttler**, Does the Stock Market React to Unsolicited Ratings?, April 2006
- No.162: **Reinhard H. Schmidt**, Stakeholderorientierung, Systemhaftigkeit und Stabilität der Corporate Governance in Deutschland, March 2006
- No.161: **Michael H. Grote/ Marc Ueber**, Home biased? A Spatial Analysis of the Domestic Merging Behavior of US Firms, February 2006
- No.160: **Christina E. Bannier (geb. Metz)/ Marcel Tyrell**, Modelling the Role of Credit Rating Agencies - Do They Spark off a Virtuous Circle?, November 2005
- No.159: **Rolf Reichardt**, Kapitalmarktorientierte Risikosteuerung in Banken: Marktwertsteuerung statt Marktzinsmethode, October 2005
- No.158: **Baris Serifsoy**, Business Models and Stock Exchange Performance - Empirical Evidence, July 2005

**For a complete list of working papers please visit**

**[www.finance.uni-frankfurt.de](http://www.finance.uni-frankfurt.de)**