

Vasilev, Aleksandar

**Article — Accepted Manuscript (Postprint)**

## A Real-Business-Cycle model with human capital accumulation: Lessons for Bulgaria (1999-2018)

Macroeconomics and Finance in Emerging Market Economies

*Suggested Citation:* Vasilev, Aleksandar (2021) : A Real-Business-Cycle model with human capital accumulation: Lessons for Bulgaria (1999-2018), Macroeconomics and Finance in Emerging Market Economies, ISSN 1752-0851, Taylor and Francis, London, Iss. forthcoming

This Version is available at:

<https://hdl.handle.net/10419/234137>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# A Real-Business-Cycle model with human capital accumulation: Lessons for Bulgaria (1999-2018)

Aleksandar Vasilev\*

May 26, 2021

## Abstract

We introduce human capital accumulation into a real-business-cycle setup. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2018). We investigate the quantitative importance of the presence of skill acquisition for cyclical fluctuations in Bulgaria. After subjecting the model to a battery of tests, we find the quantitative effect of such a channel - aside from producing a moderate increase in the variability of hours - to be relatively small. In other words, government spending on education turns out to be an ineffective instrument when it comes to smoothing the cycle.

**Keywords:** business cycles, human capital, Bulgaria

*JEL Classification Codes:* E24, E32, H22, I21

---

\*Lecturer, Lincoln International Business School, UK. E-mail for correspondence: AVasilev@lincoln.ac.uk.

# 1 Introduction and Motivation

The standard real-business-cycle (RBC) model is based on the assumption that households chooses optimally not only their consumption path, but also their leisure, so hours worked are chosen optimally as well. In many macroeconomic applications that way of modeling human behavior is sufficient, as it captures the major trade-offs faced by the household, e.g. the consumption-leisure margin, as well as the current-future consumption dimension. An important shortcoming of this setup is the assumption of labor homogeneity, i.e., that each hour of work is remunerated at the same wage rate. In reality however, not all hours have the same productivity; in particular, more educated people are paid more. This is rationalized by the fact that the skill level attached to hours is what counts for productivity, not just the time spent working.

We take this aspect of life seriously, and introduce education as an alternative use of time in this paper. On an abstract level, education serves the role of a new technology, which is (i) costly in terms of time and resources, (ii) embodied in labor, or like a labor-biased technology, and (iii) its rate of growth is chosen optimally by the household. Indeed, a substantial amount of households' time is spent acquiring skills, which increase the return from working in the future, and thus higher consumption in the future.<sup>1</sup> Furthermore, human capital considerations could potentially introduce an important propagation mechanism for shocks in the model, by generating new trade-offs, and novel interactions in the model that could attenuate business cycle fluctuations. Given that physical capital accumulation motive is very important in the standard RBC model, investment in skills works in a similar way. We can thus think of knowledge accumulation in the same way as physical capital accumulation - with the important difference is that human capital is intangible.

These arguments suggest that the process of education may respond to the business cycle, and thus education may have important macroeconomic implications.<sup>2</sup> We take the

---

<sup>1</sup>This literature goes back to Mincer (1958), Schultz (1961, 1963), Becker (1962, 1964). In fact, as pointed in Kiker (1966), William Petty in 1676 was the first to provide an estimate of the stock of human capital in England. Uzawa (1965) and Lucas (1988) are classical papers in the endogenous growth literature.

<sup>2</sup>DeJong and Ingram (2001) demonstrate that a positive technology shock increases wages, increases the opportunity cost of leisure and education, and thus negatively affects the skill acquisition process. They record

education channel seriously, and incorporate human capital considerations as a new ingredient into a standard real-business-cycle (RBC) model with a government sector.<sup>3</sup> We argue that the analysis of such issues should be performed within a general equilibrium setup.

To obtain a quantitative assessment of the phenomenon, we calibrate the model for Bulgaria in the period 1999-2018, as Bulgaria provides a good testing case for the theory. In Bulgaria, according to UNESCO (2021), more than 98 percent of the population with at least primary education, 96 percent with at least lower-secondary education, 76 upper secondary, and 20 percent with at least Bachelors. Bulgaria is a former transition economy, and one of the relatively new EU member states. It is still developing, but the majority of the labor force is at least semi-skilled, which is a substantial difference from non-European developing economies. Therefore, some of the lessons learned can be useful for other developing countries in the region.

We then proceed to evaluate the effect of human capital accumulation as a potentially important transmission mechanism for business cycle fluctuations. To the best of our knowledge, this is the first study on the issue using modern macroeconomic modeling techniques, and thus an important contribution on the study of Bulgarian economy.<sup>4</sup> Unfortunately, after subjecting the model to a battery of tests, the quantitative effects turn out to be rather small. In other words, government discretionary spending on education would be an ineffective instrument when it comes to smoothing the cycle. This is an important and novel finding, as it comes in stark contrast with Keynesian aggregate demand prescriptions. However, the findings in this paper are model-specific, and valid for Bulgaria mostly. Thus, these findings are to be taken with a grain a salt, when blindly transported for another context.

---

a moderately-negative correlation (-0.31) between college enrollment and output in the US for the period 1970-1996. Other papers in the literature include He and Liu (2001), Dellas and Sakellaris (2003), Malley and Woitek (2009), as well as the references therein.

<sup>3</sup>In what is to follow, we will use the words "education", "training", and "skill acquisition" interchangeably, as we take an abstract view of accumulation of knowledge, and thus will not distinguish between formal or informal education, generalized vs specialized training, learning-by-doing, etc.

<sup>4</sup>Relative to Alessandrini et al. (2015), Kim and Lee (2007), and Perli and Sakellaris (2007), this study has a different focus.

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized competitive equilibrium system, Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables, and compared the simulated second moments of theoretical variables against their empirical counterparts. Section 6 concludes the paper.

## 2 Model Description

There is a representative household, which derives utility out of consumption and leisure. The time available to the household can be spent working, in education, or as leisure. The government taxes consumption spending, and levies a common proportional ("flat") tax on labor and capital income to finance purchases of government consumption goods, and government transfers. On the production side, there is a representative firm, which hires labor and capital to produce a homogeneous final good, which could be used for consumption, investment, or government purchases.

### 2.1 Households

There is a representative household, which maximizes its expected utility function as in Guvenen and Kuruscu (2006):

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln(1 - h_t - e_t) \right\} \quad (2.1)$$

where  $E_0$  denotes household's expectations as of period 0,  $c_t$  denotes household's private consumption in period  $t$ ,  $h_t$  are hours worked in period  $t$ ,  $e_t$  are hours spent in education,  $0 < \beta < 1$  is the discount factor,  $0 < \gamma < 1$  is the relative weight that the household attaches to leisure.<sup>5</sup>

The household starts with an initial stock of physical capital  $k_0 > 0$ , and has to decide how much to add to it in the form of new investment,  $i_t^k$ . The law of motion for physical

---

<sup>5</sup>Torres (2013) refers to hours worked in this context as "raw labor."

capital is

$$k_{t+1} = i_t^k + (1 - \delta^k)k_t \quad (2.2)$$

and  $0 < \delta^k < 1$  is the depreciation rate of physical capital. Next, the real interest rate is  $r_t$ , hence the before-tax capital income of the household in period  $t$  equals  $r_t k_t$ .

In addition, the household begins with an initial endowments of skills,  $s_0 > 0$ , and can choose to invest in human capital,  $i_t^s$ , where the law of motion for knowledge accumulation is as follows:

$$s_{t+1} = i_t^s + (1 - \delta^s)s_t, \quad (2.3)$$

and  $0 < \delta^s < 1$  is the depreciation rate of human capital. As in Ben-Porath (1967) and Trostel (1993), we will assume that process of investment in human capital is based on both the current stock of knowledge, and time spent in education, or:

$$i_t^s = B_t e_t^\theta s_t^{1-\theta}, \quad (2.4)$$

where  $0 < \theta, 1 - \theta < 1$  captures the marginal return to education and existing human capital, respectively, and  $B_t$  reflects the efficiency of new human capital production function.<sup>6</sup> Note that increasing human capital requires not only more time spent in education, but is also conditional on the current stock of human capital.

Next, the household can generate labor income by supplying efficiency hours  $s_t h_t$  to the representative firm at the wage rate of  $w_t$  per efficiency hour, so pre-tax labor income equals  $w_t s_t h_t$ .<sup>7</sup> Lastly, the household owns the firm in the economy and has a legal claim on all the firm's profit,  $\pi_t$ .

The household's problem can be now simplified to

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln(1 - h_t - e_t) \right\} \quad (2.5)$$

---

<sup>6</sup>Here we model investment in human capital as a production function using time, as in Mincer (1958), Heckman (1976) and Haley (1976). Alternatively, as shown in Vasilev (2015a), the process could be modelled as taking real resources, and more specifically, being part of final uses of output.

<sup>7</sup>As pointed out in Torres (2013), the formulation of efficiency hours in this way allows us to discuss skill acquisition without the need for different agent types, based on skill level.

s.t.

$$(1 + \tau^c)c_t + k_{t+1} - (1 - \delta^k)k_t = (1 - \tau^y)[r_t k_t + \pi_t + w_t s_t h_t] + g_t^t \quad (2.6)$$

and

$$s_{t+1} = i_t^s + (1 - \delta^s)s_t, \quad (2.7)$$

where where  $\tau^c$  is the tax on consumption,  $\tau^y$  is the proportional income tax rate ( $0 < \tau^c, \tau^y < 1$ ), and  $g_t^t$  denotes government transfers. The household takes the tax rates  $\{\tau^c, \tau^y\}_{t=0}^\infty$ , government consumption and transfers,  $\{g_t^c, g_t^t\}_{t=0}^\infty$ , profit  $\{\pi_t\}_{t=0}^\infty$ , the realized technology processes  $\{A_t, B_t\}_{t=0}^\infty$ , prices  $\{w_t, r_t\}_{t=0}^\infty$ , and chooses  $\{c_t, h_t, k_{t+1}, s_{t+1}\}_{t=0}^\infty$  to maximize its utility subject to the budget constraint.<sup>8</sup>

The first-order optimality conditions as as follows:

$$c_t : \frac{1}{c_t} = \lambda_t(1 + \tau^c) \quad (2.8)$$

$$h_t : \frac{\gamma}{1 - h_t - e_t} = \lambda_t(1 - \tau^y)w_t \quad (2.9)$$

$$e_t : \frac{\gamma}{1 - h_t - e_t} = \mu_t B_t \theta e_t^{\theta-1} s_t^{1-\theta} \quad (2.10)$$

$$k_{t+1} : \lambda_t = \beta E_t \lambda_{t+1} \left[ 1 + (1 - \tau^y)r_{t+1} - \delta \right] \quad (2.11)$$

$$s_{t+1} : \mu_t = \beta E_t \mu_{t+1} \left[ 1 - \delta + (1 - \theta)B_t e_{t+1}^\theta s_{t+1}^{-\theta} \right] + \beta E_t \lambda_{t+1} w_{t+1} h_{t+1} \quad (2.12)$$

$$TVC : \lim_{t \rightarrow \infty} \beta^t \lambda_t k_{t+1} = 0 \quad (2.13)$$

where  $\lambda_t$  is the Lagrangean multiplier attached to household's budget constraint in period  $t$ , and  $\mu_t$  is the Lagrangean multiplier attached to the human capital accumulation equation. The interpretation of the first-order conditions above is as follows: the first one states that for each household, the marginal utility of consumption equals the marginal utility of wealth, corrected for the consumption tax rate. The second and the third equations state that when choosing labor supply optimally, at the margin, each hour spent by the household working or in education should balance the benefit from doing so in terms of additional income

---

<sup>8</sup>Note that by choosing  $k_{t+1}$ ,  $s_{t+1}$ ,  $e_t$ , and given  $B_t$ , the household is implicitly setting investment levels  $i_t^k$  and  $i_t^s$  optimally.

generated, and the cost measured in terms of lower utility of leisure. The fourth equation is the so-called "Euler condition," which describes how the household chooses to allocate physical capital over time, taking into consideration the progressivity of capital taxation, which acts as an inflation factor, further decreasing the after-tax return to capital. Similarly, the fifth equation describes the optimal accumulation of skills over time. The last condition is called the "transversality condition" (TVC): it states that at the end of the horizon, the value of physical capital should be zero.

## 2.2 Firm problem

There is a representative firm in the economy, which produces a homogeneous product. The price of output is normalized to unity. The production technology is Cobb-Douglas and uses both physical capital,  $k_t$ , and efficiency labor hours,  $s_t h_t$ , to maximize static profit

$$\Pi_t = A_t k_t^\alpha (s_t h_t)^{1-\alpha} - r_t k_t - w_t s_t h_t, \quad (2.14)$$

where  $A_t$  denotes the level of technology in period  $t$ . Since the firm rents the capital from households, the problem of the firm is a sequence of static profit maximizing problems. In equilibrium, there are no profits, and each input is priced according to its marginal product, *i.e.*:

$$k_t : \alpha \frac{y_t}{k_t} = r_t, \quad (2.15)$$

$$s_t h_t : (1 - \alpha) \frac{y_t}{s_t h_t} = w_t. \quad (2.16)$$

In equilibrium, given that the inputs of production are paid their marginal products,  $\pi_t = 0$ ,  $\forall t$ .

## 2.3 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption, in order to finance spending on wasteful government purchases, and government transfers. The government budget constraint is as follows:

$$g_t^c + g_t^t = \tau^c c_t + \tau^y [w_t s_t h_t + r_t k_t + \pi_t] \quad (2.17)$$



Consumption tax, income tax rate and government consumption-to-output ratio would be chosen to match the average share in data, and government transfers would be determined residually.

## 2.4 Exogenous stochastic processes

The exogenous processes for total factor productivity,  $A_t$ , and human capital production,  $B_t$ , will follow AR(1) processes in natural logarithms:

$$\ln A_{t+1} = (1 - \rho_A) \ln A + \rho_A \ln A_t + \epsilon_{t+1}^A \quad (2.18)$$

$$\ln B_{t+1} = (1 - \rho_B) \ln B + \rho_B \ln B_t + \epsilon_{t+1}^B, \quad (2.19)$$

where  $A, B$  are the steady-state values of the two processes,  $0 < \rho_B, \rho_A < 1$  are the respective persistence parameters, and the productivity innovations and changes to institutional quality are drawn from the following distributions:  $\epsilon_t^A \sim i.i.dN(0, \sigma_A^2)$  and  $\epsilon_t^B \sim i.i.dN(0, \sigma_B^2)$ , respectively. the government budget is always balanced.

## 2.5 Dynamic Competitive Equilibrium (DCE)

For the given processes followed by the two technologies  $\{A_t, B_t\}_{t=0}^\infty$ , the two tax rates  $\{\tau^c, \tau^y\}_{t=0}^\infty$ , and initial physical and human capital stock  $\{k_0, s_0\}$ , the decentralized dynamic competitive equilibrium is a list of sequences  $\{c_t, i_t^k, i_t^s, k_t, s_t, h_t, e_t\}_{t=0}^\infty$  for the household, a sequence of government purchases and transfers  $\{g_t^c, g_t^t\}_{t=0}^\infty$ , and input prices  $\{w_t, r_t\}_{t=0}^\infty$  such that (i) the household maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) government budget is balanced in each period; (iv) all markets clear.

## 3 Data and Model Calibration

To characterize business cycle fluctuations in Bulgaria, we will focus on the period following the introduction of the currency board (1999-2018). Quarterly data on output, consumption and investment was collected from National Statistical Institute (2019), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2019). The calibration strategy described in this section follows a long-established tradition in modern

macroeconomics: first, as in Vasilev (2016), the discount factor,  $\beta = 0.982$ , is set to match the steady-state capital-to-output ratio in Bulgaria,  $k/y = 13.964$ , in the steady-state Euler equation. The labor share parameter,  $1 - \alpha = 0.571$ , is obtained as in Vasilev (2017d), and equals the average value of labor income in aggregate output over the period 1999-2018. This value is slightly higher as compared to other studies on developed economies, due to the overaccumulation of physical capital, which was part of the ideology of the totalitarian regime, which was in place until 1989. Next, the average labor and capital income tax rate was set to  $\tau^y = 0.1$ . This is the average effective tax rate on income between 1999-2007, when Bulgaria used progressive income taxation, and equal to the proportional income tax rate introduced as of 2008. Similarly, the average tax rate on consumption is set to its value over the period,  $\tau^c = 0.2$ . Due to the lack of data, the share of labor in the skill investment function is set to  $\theta = 0.76$  as in Malley and Woitek (2009).<sup>9</sup>

Next, the relative weight attached to the utility out of leisure in the household's utility function,  $\gamma$ , is calibrated to match that in steady-state consumers would supply one-third of their time endowment to working. This is in line with the estimates for Bulgaria (Vasilev 2017a) as well over the period studied. Next, the depreciation rate of physical and human capital in Bulgaria,  $\delta^k = 0.013$  and  $\delta^s = 0.002$  were taken from Vasilev (2016). Using NSI (2019) data on the total expenditure on education as a proxy for investment in skills, we obtain that the share of human capital in output is  $i^s/y = 0.172$ . Finally, the process followed by the TFP process is estimated from the detrended series by running an AR(1) regression and saving the residuals. Due to the lack of data, we use the same values for the process followed by the productivity of human capital. Table 1 below summarizes the values of all model parameters used in the paper.

## 4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are

---

<sup>9</sup>Torres (2013) also uses a  $\theta = 0.76$  in their simulation. We experimented with different values of the parameter, and found that our findings remain robust.

Table 1: Model Parameters

Parameter	Value	Description	Method
$\beta$	0.982	Discount factor	Calibrated
$\alpha$	0.429	Capital Share	Data average
$1 - \alpha$	0.571	Labor Share	Calibrated
$\gamma$	0.873	Relative weight attached to leisure	Calibrated
$\delta^k$	0.013	Depreciation rate on physical capital	Data average
$\delta^s$	0.013	Depreciation rate on physical capital	Data average
$\theta$	0.760	Share of labor, skills investment function	Set
$\tau^y$	0.100	Average tax rate on income	Data average
$\tau^c$	0.200	VAT/consumption tax rate	Data average
$A$	0.604	Steady-state value of TFP process	Calibrated
$B$	1.000	Steady-state of human capital productivity	Set
$\rho_A$	0.701	AR(1) persistence coefficient, TFP process	Estimated
$\sigma_A$	0.044	st. error, TFP process	Estimated
$\rho_B$	0.701	AR(1) persistence coefficient, human capital process	Set
$\sigma_B$	0.044	st. error, human capital process	Set

reported in Table 2 below. The steady-state level of output was normalized to unity (hence the level of technology  $A$  differs from one, which is usually the normalization done in other studies), which greatly simplified the computations. Next, the model matches consumption-to-output and government purchases ratios by construction; The investment ratios are also closely approximated, despite the closed-economy assumption and the absence of foreign trade sector.<sup>10</sup> The shares of income are also identical to those in data, which is an artefact of the assumptions imposed on functional form of the aggregate production function. The after-tax return, where  $\bar{r} = (1 - \tau^y)r - \delta$  is also relatively well-captured by the model. Lastly, given the absence of debt, and the fact that transfers were chosen residually to balance the government budget constraint, the result along this dimension is understandably not so close to the average ratio in data.

<sup>10</sup>Note also that investment in human capital is an intangible investment in this setup, and thus does not count as a final use of output.

Table 2: Data Averages and Long-run Solution

Variable	Description	Data	Model
$y$	Steady-state output	N/A	1.000
$c/y$	Consumption-to-output ratio	0.648	0.674
$i^k/y$	Physical capital investment-to-output ratio	0.201	0.175
$i^s/y$	Human capital investment-to-output ratio	0.172	0.172
$k/y$	Physical capital-to-output ratio	13.96	13.96
$s/y$	Human capital-to-output ratio	N/A	30.44
$g^c/y$	Government consumption-to-output ratio	0.151	0.151
$wsh/y$	Efficiency-labor income-to-output ratio	0.571	0.571
$rk/y$	Capital income-to-output ratio	0.429	0.429
$h$	Share of time spent working	0.333	0.333
$\bar{r}$	After-tax net return on capital	0.014	0.016

## 5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity, and to the human capital investment process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts.

### 5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology and human capital.<sup>11</sup> The impulse response functions (IRFs) are

<sup>11</sup>The shock to the human capital investment function is to be understood as some exogenous change in the quality of the education system, or a change in the preference for investment in knowledge. Alternatively, the shock might reflect the obsolescence of past knowledge, the increase in the demand for entrepreneurial

presented in Fig. 1 and Fig. 2, respectively. As a result of the one-time unexpected positive shock to total factor productivity, output increases upon impact. This expands the availability of resources in the economy, so used of output - consumption, investment, and government consumption also increase contemporaneously. The impulse responses are almost identical to the responses in a model with no human capital, so the quantitative effect of human capital channel is quite small.

The novelty is that upon impact of the technology shock the time devoted to education, falls. Furthermore, there is an inter-temporal substitution between working time and education, as investment in education today yields a higher return to labor in the future. In contrast, an increase in hours today, holding the level of human capital fixed, brings a higher return today. Time spent in education is thus countercyclical.<sup>12</sup> In turn, the reduction of educational activities decreases the stock of human capital.

The increase in productivity increases the after-tax return on the two factors of production, labor and capital. The representative households then respond to the incentives contained in prices and start accumulating capital, and supplies more hours worked. In turn, the increase in capital input feeds back in output through the production function and that further adds to the positive effect of the technology shock. In the labor market, the wage rate increases, and the household increases its hours worked. In turn, the increase in total hours further increases output, again indirectly. Overall, the reduction in human capital is compensated by the increase in labor in physical capital in the production of aggregate output.

Over time, as capital is being accumulated, its after-tax marginal product starts to decrease, which lowers the households' incentives to save. As a result, physical capital stock eventually returns to its steady-state, and exhibits a hump-shaped dynamics over its transition path. The rest of the model variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

In the second scenario, presented in Fig. 2 below, we present simulations for a 1%-surprise skills (as compared to academic knowledge), etc.

---

<sup>12</sup>This is in line with DeJong and Ingram (2001), as well as anecdotal evidence that in good times people work more, and return to school during bad time.

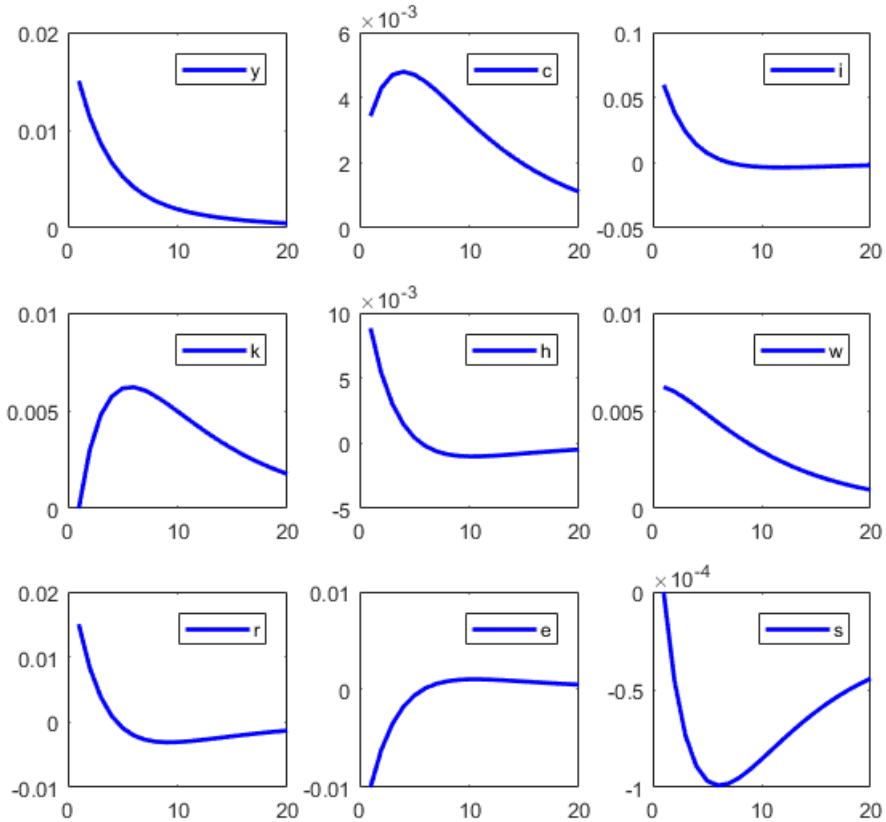


Figure 1: Impulse Responses to a 1% surprise innovation in total factor productivity

innovation to the productivity of human capital investment. As expected, the time dedicated to education is now higher, and that leads to higher stock of human capital over time. The quantitative effect is small, though. In addition, it is more than compensated by the fall in hours worked, and the decrease in capital accumulation. The scarcity of labor and capital causes an increase in their prices, but the effect again is rather small. Over time, as the effect of the shock dies, physical capital stock eventually returns to its steady-state, and exhibits a hump-shaped dynamics over its transition path. The rest of the model variables return to their old steady-states in a monotone fashion. Overall, the quantitative effect of this shock is small, and thus not a primary force behind the observed business cycles in Bulgaria.

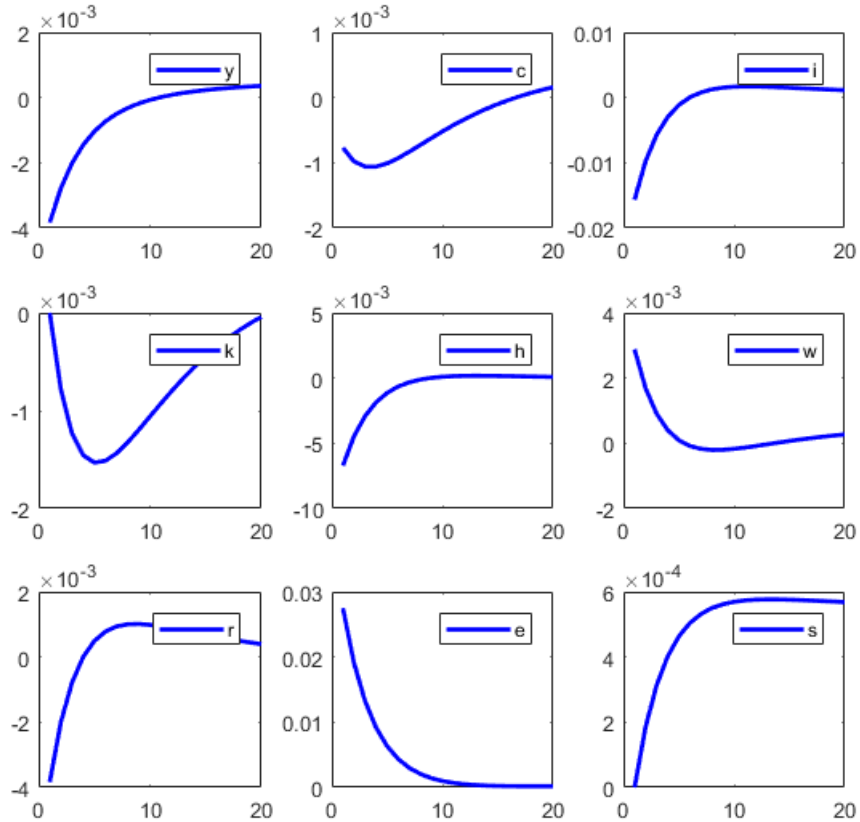


Figure 2: Impulse Responses to a 1% surprise innovation in human capital investment productivity

## 5.2 Simulation and moment-matching

As in Vasilev (2017e), we will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at quarterly frequency. We present the results for several specifications of the current model, with isolated TFP and productivity of the investment in human capital function, as well as their combined effect. We also compare the results against the benchmark model - with no human capital, and driven by TFP shocks only. In addition, to minimize the sample error, the simulated moments are averaged out

over the computer-generated draws. As in Vasilev (2016, 2017b, 2017c), both models match quite well the absolute volatility of output and investment. By construction, government consumption in the model varies as much as output. In addition, the predicted consumption and investment volatilities in all the models are too high. Still, the models are qualitatively consistent with the stylized fact that consumption generally varies less than output, while investment is more volatile than output. The model with human capital shocks only produces smoother consumption and investment series, but the quantitative effect is rather small.<sup>13</sup> Overall, the models are almost indistinguishable from one another, which could be interpreted that educational policy is an ineffective instrument to smooth business cycles.

Table 3: Business Cycle Moments

	Data	Model ( $A_t$ shocks only)	Model ( $B_t$ shocks only)	Model (both shocks)	Benchmark RBC model
$\sigma_y$	0.05	0.05	0.05	0.05	0.05
$\sigma_c/\sigma_y$	0.55	0.81	0.80	0.81	0.82
$\sigma_i/\sigma_y$	1.77	2.38	2.32	2.38	2.35
$\sigma_g/\sigma_y$	1.21	1.00	1.00	1.00	1.00
$\sigma_h/\sigma_y$	0.63	0.32	1.04	0.42	0.28
$\sigma_w/\sigma_y$	0.83	0.84	0.39	0.81	0.86
$corr(c, y)$	0.85	0.89	0.90	0.89	0.90
$corr(i, y)$	0.61	0.83	0.85	0.83	0.83
$corr(g, y)$	0.31	1.00	1.00	1.00	1.00
$corr(h, y)$	0.49	0.61	0.90	0.59	0.59
$corr(w, y)$	-0.01	0.95	0.00	0.89	0.96

With respect to the labor market variables, the variability of employment predicted by the model with human capital is lower than that in data, but the variability of wages in the model is very close to that in data. This is yet another confirmation that the perfectly-competitive assumption, e.g. Vasilev (2009), as well as the benchmark calibration here, does

<sup>13</sup>The only major effect is that in the absence of TFP shocks, wages become acyclical. When TPF shocks are allowed, this effect disappears.



not describe very well the dynamics of labor market variables. Next, in terms of contemporaneous correlations, the model systematically over-predicts the pro-cyclicality of the main aggregate variables - consumption, investment, and government consumption. This, however, is a common limitation of this class of models. Along the labor market dimension, the contemporaneous correlation of employment with output is too low. With respect to wages, the model predicts strong cyclicity, while wages in data are acyclical. This shortcoming is well-known in the literature and an artifact of the wage being equal to the labor productivity in the model.

In the next subsection, as in Vasilev (2015c), we investigate the dynamic correlation between labor market variables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

### 5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients empirical ACFs and CCFs at different leads and lags are presented in Table 4 below against the averaged simulated AFCs and CCFs. As in Canova (2007), this is used as a goodness-of-fit measure. For the sake of economizing space, here we present the results for the model with both shocks at play.<sup>14</sup>

As seen from Table 4 above, the model compares relatively well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model. Still, the model with human capital accumulation generates too much persistence in output and both employment and unemployment, and is subject to the criticism in Nelson and Plosser (1982), Cogley and Nason (1995) and Rotemberg and Woodford (1996b), who argue that the RBC class of models do not have a strong internal propagation mechanism besides the strong persistence in the TFP process. In those models,

---

<sup>14</sup>The results for the other specifications are qualitatively similar, and are available upon request from the author.

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(n_t, n_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(n_t, n_{t-k})$	1.000	0.950	0.889	0.819
	(s.e.)	(0.000)	(0.031)	(0.059)	(0.085)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$	1.000	0.956	0.904	0.843
	(s.e.)	(0.000)	(0.027)	(0.052)	(0.076)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$	1.000	0.953	0.898	0.833
	(s.e.)	(0.000)	(0.030)	(0.058)	(0.084)
Data	$corr(c_t, c_{t-k})$	1.000	0.971	0.952	0.913
Model	$corr(c_t, c_{t-k})$	1.000	0.960	0.912	0.857
	(s.e.)	(0.000)	(0.025)	(0.047)	(0.069)
Data	$corr(i_t, i_{t-k})$	1.000	0.810	0.722	0.594
Model	$corr(i_t, i_{t-k})$	1.000	0.952	0.893	0.825
	(s.e.)	(0.000)	(0.030)	(0.058)	(0.083)
Data	$corr(w_t, w_{t-k})$	1.000	0.760	0.783	0.554
Model	$corr(w_t, w_{t-k})$	1.000	0.958	0.908	0.851
	(s.e.)	(0.000)	(0.026)	(0.050)	(0.072)

e.g. Vasilev (2009), and in the current one, labor market is modelled in the Walrasian market-clearing spirit, and output and employment persistence is low. Next, as seen from Table 5 below, over the business cycle, in data labor productivity leads employment. The model, however, cannot account for this fact. As in the standard RBC model a technology shock can be regarded as a factor shifting the labor demand curve, while holding the labor supply curve constant. Therefore, the effect between employment and labor productivity is only a contemporaneous one.

Table 5: Dynamic correlations for Bulgarian data and the model economy

		k						
Method	Statistic	-3	-2	-1	0	1	2	3
Data	$corr(h_t, w_{t-k})$	0.355	0.452	0.447	0.328	-0.040	-0.390	-0.57
Model	$corr(h_t, w_{t-k})$	0.022	0.027	0.031	0.229	-0.008	-0.022	-0.030
	(s.e.)	(0.339)	(0.295)	(0.242)	(0.434)	(0.211)	(0.253)	(0.288)

## 6 Conclusions

We introduce human capital accumulation into a real-business-cycle setup augmented with a detailed government sector. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2018). We investigate the quantitative importance of the presence of skill acquisition for cyclical fluctuations in Bulgaria. After subjecting the model to a battery of tests, we find the quantitative effect of such a channel - aside from producing a moderate increase in the variability of hours - to be relatively small. In other words, government discretionary spending on education would be an ineffective instrument when it comes to smoothing the cycle. This is an important and novel finding, as it comes in stark contrast with Keynesian aggregate demand prescriptions. Still, the findings in this paper are model-specific, and valid for Bulgaria mostly. Therefore, these findings are to be taken with a grain a salt, when blindly transported for another context.

**Conflict of interest statement:** On behalf of all authors, the corresponding author states that there is no conflict of interest.

## References

Diana Alessandrini, D., Kosempel, S. and Stengos, T. (2015) "The business cycle human capital accumulation nexus and its effect on hours worked volatility," *Journal of Economic Dynamics and Control* 51: 356-377.

Becker, G. (1962) "Investment in human capital: A theoretical analysis," *Journal of Political Economy* 70(1): 9-49.

Becker, G. (1964) *Human capital*. Columbia University Press: New York, US.

Ben-Porath, Y. (1964) "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy* 75(4): 352-365.

Bulgarian National Bank. 2019. Bulgarian National Bank Statistics. Available on-line at [www.bnb.bg](http://www.bnb.bg). Accessed on July 21, 2019.

Canova, Fabio. 2007. *Methods for Applied Macroeconomic Research*. Princeton University Press: Princeton, NJ.

Cogley, Timothy and James Nason. 1995. "Output dynamics in Real-Business-Cycles," *American Economic Review* 85(3): 492-511.

DeJong, D. and Ingram, B. (2001) "The cyclical behavior of skill acquisition," *Review of Economic Dynamics* 4(3): 536-561.

Dellas and Sakellaris, P. (2003) "On the cyclicity of the demand for education: Theory and evidence," *Oxford Economic Papers* 55(1): 148-172.

Guvenen, F. and Kuruscu, B. (2006) "Understanding wage inequality: Ben-Porath meets skill-biased technological change," *Federal Reserve Bank of Minneapolis Discussion Paper* 44, Minnesota, US.

Haley, W. (1976) "Estimation of the Earnings Profile from Optimal Human Capital Accumulation," *Econometrica* 44(6): 1223-1238.

Heckman, J.J. (1976) "A Life-cycle Model of Earnings, Learning, and Consumption," *Jour-*

*nal of Political Economy* 84(4): 11-44.

He, H. and Liu, Z. (2007) "Investment-specific technological change, skill accumulation, and wage inequality," *Review of Economic Dynamics* 11(2): 314-334.

Hodrick, Robert and Edward Prescott. 1980. "Post-war US business cycles: An empirical investigation." *Unpublished manuscript* (Carnegie-Mellon University, Pittsburgh, PA).

Kiker, B. (1966) "The historical roots of the concept of human capital," *Journal of Political Economy* 74(5): 481-499.

Kim, D., and Lee, C. (2007) "On-the-job human capital accumulation in a real business cycle model: Implications for intertemporal substitution elasticity and labor hoarding," *Review of Economic Dynamics*, 10 (3): 494-518.

Lucas, R. (1988) "On the mechanics of economic development," *Journal of Monetary Economics* 22: 3-42.

Malley, J. and Woitek, U. (2009) "Productivity shocks and aggregate cycles in an estimated endogenous growth model," *CESifo Working Paper Series* 2672.

Mincer, J. (1958) "Investment in Human Capital and Personal Income Distribution," *Journal of Political Economy* 66(4): 281-302.

National Statistical Institute. 2019. Aggregate Statistical Indicators. Available on-line at [www.nsi.bg](http://www.nsi.bg). Accessed on July 21, 2019.

Nelson C., Plosser C. (1982) "Trends and random walk in macroeconomic time series," *Journal of Monetary Economics* 10:139162.

Perli, R., and Sakellaris, P. (1998) "Human capital formation and business cycle persis-

tence,” *Journal of Monetary Economics* 42(1): 67-92.

Rotemberg, Julio and Michael Woodford. 1996b. ”Real-Business-Cycle Models and the Forecastable Movements in Output, Hours, and Consumption,” *American Economic Review*, 86: 71-89.

Schultz, T. (1961) ”Investment in human capital,” *American Economic Review* 51(1): 1-17.

Schultz, T. (1963) *The economic value of education*. Columbia University Press: NY, US.

Torres, J. (2013) *Introduction to Dynamic Macroeconomics General Equilibrium Models*, Vernon Press: Malaga, Spain.

Trostel, P. (1993) ”The effect of taxation on human capital,” *Journal of Political Economy* 101(2): 327-350.

UNESCO (2021) *SDG 4 Data Explorer: Bulgaria*. Available on-line at <http://uis.unesco.org/>. Last accessed on March 20, 2021.

Uzawa, H. (1965) ”Optimum technical change in an aggregate model of economic growth,” *International Economic Review* 6: 18-31.

Vasilev, A. (2017a) ”Business Cycle Accounting: Bulgaria after the introduction of the currency board arrangement (1999-2014), *European Journal of Comparative Economics*, 14(2): 197-219.

Vasilev, A. (2017b) ”A Real-Business-Cycle model with efficiency wages and a government sector: the case of Bulgaria,” *Central European Journal of Economics and Econometrics*, 9(4): 359-377.

Vasilev, A. (2017c) "A Real-Business-Cycle model with reciprocity in labor relations and fiscal policy: the case of Bulgaria," *Bulgarian Economic Papers* BEP 03-2017, Center for Economic Theories and Policies, Sofia University St. Kliment Ohridski, Faculty of Economics and Business Administration, Sofia, Bulgaria.

Vasilev, A. (2017d) "VAT Evasion in Bulgaria: A General-Equilibrium Approach," *Review of Economics and Institutions*, 8(2): 2-17.

Vasilev, A. (2017e). "On the Cost of Opportunistic Behavior in the Public Sector: A General-Equilibrium Approach," *Journal of Public Economic Theory* 19(2): 565-582.

Vasilev, A. (2016a). "Progressive taxation and (in)stability in an endogenous growth model with human capital accumulation," *Journal of Economics and Econometrics* 59(2): 1-15.

Vasilev, A. (2016) "Search and matching frictions and business cycle fluctuations in Bulgaria," *Bulgarian Economic Papers* BEP 03-2016, Center for Economic Theories and Policies, Sofia University St. Kliment Ohridski, Faculty of Economics and Business Administration, Sofia, Bulgaria.

Vasilev, A. (2015a) "Welfare effects of flat income tax reform: the case of Bulgaria," *Eastern European Economics* 53(2): 205-220.

Vasilev, A. (2015b) "Welfare gains from the adoption of proportional taxation in a general-equilibrium model with a grey economy: the case of Bulgaria's 2008 flat tax reform," *Economic Change and Restructuring*, 48(2): 169-185.

Vasilev, A. (2015c). "Macroeconomic Effects of Public-Sector Unions," *LABOUR* 29(2): 101-126.

Vasilev, A. (2009) "Business cycles in Bulgaria and the Baltic countries: an RBC approach," *International Journal of Computational Economics and Econometrics*, 1(2): 148-170.