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A primer about storage in bottom-up models of future energy systems.  
Fundamentals of storage operation and investment in competitive  
long-term equilibria

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**A PRIMER ABOUT STORAGE IN BOTTOM-UP MODELS OF FUTURE ENERGY SYSTEMS.**  
– FUNDAMENTALS OF STORAGE OPERATION AND INVESTMENT IN COMPETITIVE LONG-TERM EQUILIBRIA –

HEMF Working Paper No. 02/2020

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August 2020

UNIVERSITÄT  
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*Open-Minded*

A primer about storage in bottom-up models of future energy systems. Fundamentals of storage operation and investment in competitive long-term equilibria

by Benjamin Böcker and Christoph Weber

## Abstract

Growing shares of renewable energy sources (RES) in the electricity system increase the need for flexible balancing of supply-dependent feed-in of RES and time-varying demand. Besides flexible, conventional technologies and demand-responses, storage is an important option. Through the use of an analytical approach, this paper explores the implications of the short- and long-run electricity market equilibrium.

While conventional and renewable technologies have fixed positions in the supply stack depending on their operational costs, storage may shift in the supply stack over time. Hence, shifts in the supply stack only occur if the storage is either full or empty; otherwise, the so-called shadow price of storage is constant.

In a long-term partial electricity market equilibrium, this implies more complicated patterns of operational cash flows for storage technologies. These cash flow patterns will determine, in turn: how storage investments pay off, to what extent storage is part of an efficient energy system, and what power-to-energy ratio should storage have.

The implications for future sustainable energy systems with multiple storage technologies are illustrated in a stylized application of Lithium-Ion batteries and pump hydro storage as two possible storage technologies.

Keywords: energy storage, electricity market equilibrium, analytical approach, power system dynamics

JEL-Classification: Q41, Q43, D41, C68

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## Content

1	Introduction .....	1
2	Basic model .....	2
2.1	General problem formulation.....	3
2.2	Solution approach.....	6
3	Operation principles .....	7
3.1	Fundamentals.....	7
3.2	Water values and the specific role of storage.....	10
4	Investment Principles .....	11
5	Derivation of pricing rules and main operation modes for storage technologies .....	12
5.1	Main operation mode 1 (MOM 1) – flexible storage charging.....	15
5.2	Main operation mode 2 (MOM 2) – flexible storage discharging .....	16
6	Storage as part of the efficient portfolio – An application.....	17
6.1	Scenario assumptions.....	17
6.2	Main results .....	18
6.3	Storage operation and main operation modes .....	19
6.4	Paying back investment costs .....	24
7	Conclusions .....	25
	References.....	27
	Appendix .....	29
	Appendix A Lagrange-Function and first-order derivatives.....	29
	Appendix B Shadow price of the demand - Proof of Proposition 1 .....	30
	Appendix C Shadow price of the storage level (water-value) - Proof of Proposition 2 .....	35
	Appendix D Price levels and operation modes - Proof of Proposition 3.....	38
	Acknowledgment .....	42

# 1 Introduction

The political and societal aim of slowing global warming by reducing carbon emissions coupled with the limitations of fossil fuel resources induced a dramatic change in the global energy system. In terms of currently installed capacities in Germany, wind power and photovoltaic (PV) systems are already among the most important technologies, and their share is expected to increase further in the future. Aside from new challenges to the grid due to decentralized feed-in, the main problem remains the balancing of the intermittent renewable feed-in and energy demand.

Under the foreseen expansion of renewable energy sources (RES), the aggregated energy supply of the system will be less of a problem than matching the demand at each point in time. Today, conventional power plants are primarily used to compensate for the feed-in fluctuations to ensure the availability of electricity at demanded times. In future energy systems, conventional technologies are acting more and more as backup technology, or flexibility options, and therefore have to compete with other options like energy storage systems (ESSs). The advantage of ESSs is their ability to shift energy over time, which allows the increase of the share of usable RES feed-in for covering the demand. High but declining costs characterize most of the ESS technologies; however, pumped-hydro storage (PHS) is efficient and proven (Steffen, 2012) hence why its widespread rollout could be expected in the medium-term.

Consequently, ESSs are likely to be part of the efficient technology portfolio in future power systems, thus impacting the price formation in wholesale electricity markets. Today, the variable costs of the marginal price-setting conventional power plant – as described in offer-stack or merit-order models of electricity market price formation – drive the electricity price in competitive markets. In this case, the price is a monotonic (step-wise) function of the current demand. Nevertheless, if storage technologies are part of an efficient energy system, the price is no longer uniquely determined by current demand and is no longer explicitly given by the marginal costs of the last operating thermal plant.

This paper proposes an analytical approach to determine both the price formation and the efficient technology mix for energy systems with storage. The optimal generation mix is derived from a system planning perspective that is well-known to be equivalent to the competitive market equilibrium. Therefore, the paper contributes to a better understanding of the marginal value of storage and what long-term market equilibria, including full cost recovery for conventional, renewable, and storage technologies, look like. This paper builds and extends upon Steffen and Weber (2013) by including explicit volume limitations of storage, and by separating investment costs for charging and discharging capacity as well as storage volume. This technique provides

additional insights into the value of storage in future energy systems. Notably, it turns out that storage technologies do not occupy a fixed position in the offer stack. Rather, their position varies, as the opportunity costs for charging and discharging vary over time. Moreover, generalizing Lamont's analysis (2013), we do not limit storage cycling patterns to empty-full and full-empty half-cycles, and we include the possibility of multiple storage technologies. Furthermore, we provide a novel representation of storage charging and discharging as well as load shedding in a generalized supply stack along with a comprehensive analysis of main operation modes and dual variables (shadow prices).

*Section 2* discusses the developed analytical approach. Concerning the role of storage technologies, *Section 0* discusses their operation principles and investment pay-offs are investigated in *Section 4*. In *Section 5*, pricing rules, as well as main operation modes for storage technologies, are derived. *Section 6* provides a numerical application, and finally, *Section 7* concludes the paper.

## **2 Basic model**

Different approaches exist to analyze the benefits of new technology in a system context. As a first approximation, the analysis may be carried out in an individual investor perspective. In this approach, the technology is valued under a given system context and with corresponding prices. This valuation (see, e.g., analyzing arbitrage gains for storage at given market prices) is valid as long as the feedback from the use of the new technology on the prices and other system parameters is negligible (see, e.g. notably (Zafirakis, Chalvatzis, Baiocchi & Daskalakis, 2016; Brijs, Geth, Siddiqui, Hobbs & Belmans, 2016; Krishnamurthy, Uckun, Zhou, Thimmapuram & Botterud, 2017; Pozo. Contreras & Sauma, 2014; Bradbury, Pratson & Patino-Echeverri, 2014)). For the first unit of new technology, this is a good approximation, but with increasing investments into new technology, the feedback may no longer be neglected and has to be taken into account.

For this purpose, it is useful to switch to a system perspective to evaluate different technologies simultaneously in a system context (Stoft, 2002). The advantage is that the central interdependencies are considered. With an increasing number of technologies and influencing factors, the complexity of the model may increase rapidly. Hence, this basic idea is applied in a broad range of different approaches, such as investigating the combination of storage for a specific renewable technology. For example, Tuohy and M (2011) investigate the effects of implementing pumped hydro storage in an electricity system with very high wind penetration. The used unit commitment model includes uncertainty of wind feed-ins, and the economics of the system, including the pay-off of storage investment, are investigated. It is shown that storage partly avoids wind curtailment, and uncertainties increase its value.

More general model formulations are used to describe further interrelations like reserves or ramping constraints (Bruninx et al. 2016) as well as to analyze the effect of other exogenous drivers like political objectives (Böcker, Steffen & Weber, 2015).

In an even more general perspective, the variety of technologies available for power generation is taken into account. The operation of thermal power plants is thereby primarily driven by their short-run marginal cost, which in turn determines their position in the offer stack (the so-called “merit order”). The efficient portfolio of thermal generation assets is then obtained by considering the long-term capacity planning problem (also known as a peak-load-pricing problem: see, e.g., Boiteux 1960, 1960; Steffen & Weber, 2013), which is an extension of the merit-order model considering both investment and operational costs and the load restrictions.

System models are especially valuable in analyzing the interactions between different technologies in competitive markets. Mostly large-scale models are developed that are mainly used to provide detailed numerical estimates of optimal portfolios. Besides, more qualitative insights may be obtained from system models by interpreting the shadow prices (Lamont, 2013). Lamont (2013) provides a theoretical framework (including its application) to evaluate the marginal values of storage charging, discharging capacity and volume as well as their impact on the wholesale price. The objective is to investigate the effects of a storage system as an enabler for renewable investments and the use of baseload generation.

The present paper complements this work by considering a more general approach in discrete time resolution with multiple storage technologies. Notably, this is used to derive detailed analytical insights into the various operation modes of storage and how they reflect either the scarcity of charging or discharging opportunities. As a starting point, a stylized energy system model with its key restrictions is formulated in *Section 2.1*, and the solution approach based on the Lagrangian is sketched in *Section 2.2*.

## 2.1 General problem formulation

The optimal electricity system configuration is determined by minimizing the system costs  $\mathcal{C}$  of a technology portfolio, which can cover the demand  $D_t$  overtime  $t$ . Technologies that provide electricity by conversion of fossil or renewable energy sources are labeled generation technologies  $i$ . Besides generation technologies, storage technologies  $j$  are considered, which can shift energy over time. Additional restrictions like minimum generation limits or ramping constraints of technologies are neglected in this model.

The efficient capacities of generation technologies define the optimal system configuration  $K_i$  together with efficient discharging and charging capacities of storage  $K_j^+$  and  $K_j^-$  and efficient storage volume  $V_j$ . Additionally, the optimal generation schedules  $y_{i,t}$ , as well as the storage

discharging and charging schedule  $s_{j,t}^+$  and  $s_{j,t}^-$ , are decision variables. The corresponding objective function is then:

$$\begin{aligned} & \min_{K_i, K_j^+, K_j^-, V_j, y_{i,t}, s_{j,t}^+, s_{j,t}^-} C \\ C = & \sum_t \sum_i c_i^{op} \cdot \Delta t \cdot y_{i,t} + \sum_i c_i^{inv} \cdot K_i + \sum_j c_j^{inv,-} \cdot K_j^- + \sum_j c_j^{inv,+} \cdot K_j^+ \\ & + \sum_j c_j^{inv,V} \cdot V_j \end{aligned} \quad (1)$$

Main drivers for the composition of the efficient portfolio are the operational costs  $c_i^{op}$  (multiplied with the time step length  $\Delta t$  since these are energy-based)<sup>1</sup> and investment costs  $c_i^{inv}$  for generation technologies. Investment costs of storage systems do not only depend on the storage capacity (charging  $c_j^{inv,-}$  and discharging  $c_j^{inv,+}$ ) but also on the storage volume  $c_j^{inv,V}$ . We define the optimization problem for a representative period  $\{1, \dots, T\}$ , which is typically one year. Correspondingly, the investment costs are annualized costs, which are derived from the up-front costs through multiplication with an annuity factor reflecting both capital costs (interest rate) and lifetime.

The cost minimization is constrained by restrictions on the balance of supply and demand (see equation (2)), capacity constraints (see equation (3)), and storage filling levels (see equation (4) and (5)). For each of these constraints, the corresponding dual variable represents their shadow prices indicating the marginal decrease in system costs for an easing of the constraint by one unit. Since most constraints are defined as inequalities, complementary slackness conditions and Karush-Kuhn-Tucker conditions are expected to hold.

The primary restriction is the supply-demand balance  $R_{D,t}$  at all times (equation (2)). The demand  $D_t$  may be matched by generation  $y_{i,t}$  as well as storage discharging  $s_{j,t}^+$ . If the storage is charged  $s_{j,t}^-$ , the supplied energy has to increase.

$$R_{D,t}: \sum_i y_{i,t} + \sum_j s_{j,t}^+ = D_t + \sum_j s_{j,t}^- \perp \mu_t \quad (2)$$

The dual variable  $\mu_t$  thereby corresponds to the electricity price in the system.

The installed capacities restrict energy supply (see equation (3)).

$$\begin{aligned} R_{i,t}: y_{i,t} &\leq b_{i,t} \cdot K_i \perp v_{i,t} \geq 0 \\ R_{j,t}^+: s_{j,t}^+ &\leq K_j^+ \perp v_{j,t}^+ \geq 0 \\ R_{j,t}^-: s_{j,t}^- &\leq K_j^- \perp v_{j,t}^- \geq 0 \end{aligned} \quad (3)$$

---

<sup>1</sup> For renewable technologies, operation costs can be assumed as nearly zero.



While conventional and storage technologies can only operate within the limit of installed capacities, renewable energy sources like wind and solar are characterized by an uncontrollable power supply (in case of wind and solar highly weather-dependent) and curtailment is the only possible control. This dynamic can be captured by introducing a time-variable yield  $b_{i,t}$  with values between 0 and 1, which describes the available generation at time  $t$  as a fraction of the installed capacity  $K_i$ .<sup>2</sup> For conventional technologies  $b_{i,t}$  can be set to one<sup>3</sup>. For these restrictions, the shadow prices indicate the value of an additional capacity unit (or the capacity rent) for each generation technology  $v_{i,t}$  as well as charging  $v_{j,t}^-$  and discharging capacity  $v_{j,t}^+$  of the storage.

Storage technologies as part of the efficient portfolio furthermore require a storage level restriction ( $R_{x_{j,t}}$ , equation (4)). The change in storage level  $x_{j,t+1} - x_{j,t}$  corresponds to the difference of charging and discharging flows multiplied by the time increment  $\Delta t$ . In the general case, a charging efficiency  $\eta_j^-$  and a discharging efficiency  $\eta_j^+$  are thereby applied.

$$R_{x_{j,t}}: x_{j,t+1} - x_{j,t} = \left( \eta_j^- \cdot s_{j,t}^- - \frac{1}{\eta_j^+} \cdot s_{j,t}^+ \right) \cdot \Delta t \pm \lambda_{j,t} \quad (4)$$

In a long-term equilibrium, the terminal filling level of the storage must equal its initial filling level. Otherwise, the equilibrium would not be sustainable. However, any ex-ante fixation of the storage level may be inefficient. Therefore, the periodic boundary condition  $x_{j,T} = x_{j,0}$  is used.  $\lambda_{j,t}$  as the corresponding shadow price is also known as water value and represents the current marginal value of the content of the storage.

Additionally, the installed storage volume limits the time-dependent storage level.

$$R_{V_{j,t}}: x_{j,t} \leq V_j \pm v_{j,t}^V \geq 0 \quad (5)$$

Similar to the interpretation of the shadow price of the restriction in equation (3),  $v_{j,t}^V$  gives the marginal value of additional storage volume. All variables are furthermore assumed to be greater than zero.

As usual, for capacity planning approaches (starting with Boiteux, 1960) in the electricity industry, demand has been considered as price-inelastic so far. Empirical evidence indeed suggests that short-run price-elasticity for electricity is very low. Nevertheless, it is generally assumed that there is an upper bound to the willingness to pay for electricity. This value has also labeled the value of lost load (VoLL).

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<sup>2</sup> Note that only the time-variability of renewables is considered here. The forecast errors associated with their operation are neglected since this would lead to a stochastic long-term equilibrium that is hardly tractable analytically.

<sup>3</sup> Thereby unavailabilities resulting from planned maintenance and unplanned outages are neglected.

A straightforward approach for incorporating the possibility of demand reduction in the previously established framework is to define an additional “generation” source (i.e. load shedding), which allows reducing demand at very high variable cost (VoLL). It is also convenient to assume that this option has no (relevant) capacity limit and no investment costs since grid operators foresee it anyhow to ensure stable operations. In principle, it is also possible to include further (less costly) options of demand responses into the framework, yet this would not add substantial insight. Instead, many demand response options may be considered as storage since the unconsumed energy at some given point in time is frequently compensated by higher consumption later on (or on beforehand).

## 2.2 Solution approach

The set of *equations (1) to (5)* corresponds to a linear optimization problem that can be solved analytically using the Lagrange method, which is done in three steps. First, the Lagrangian is built by summing up the objective function and all restrictions multiplied with their Lagrange-Multiplier (shadow prices):

$$\begin{aligned}
L = \sum_t \bigg( & \sum_i c_i^{op} \cdot \Delta t \cdot y_{i,t} + \mu_t \cdot \left( D_t + \sum_j s_{j,t}^- - \sum_i y_{i,t} - \sum_j s_{j,t}^+ \right) \\
& + \sum_i v_{i,t} \cdot (y_{i,t} - b_{i,t} \cdot K_i) + \sum_j v_{j,t}^+ \cdot (s_{j,t}^+ - K_j^+) \\
& + \sum_j v_{j,t}^- \cdot (s_{j,t}^- - K_j^-) \\
& + \sum_j \lambda_{j,t} \cdot \left( x_{j,t+1} - x_{j,t} - \eta_j^- \cdot s_{j,t}^- \cdot \Delta t + \frac{1}{\eta_j^+} \cdot s_{j,t}^+ \cdot \Delta t \right) \\
& + \sum_j v_{j,t}^V \cdot (x_{j,t} - V_j) \bigg) + \sum_i c_i^{inv} \cdot K_i + \sum_j c_j^{inv,-} \cdot K_j^- \\
& + \sum_j c_j^{inv,+} \cdot K_j^+ + \sum_j c_j^{inv,V} \cdot V_j
\end{aligned} \tag{6}$$

Then the Lagrangian has to be differentiated concerning each primal and dual variable, and the derivatives have to be bounded by zero from below if the corresponding variable is bound to be positive. The obtained results for the Lagrange function and the first-order derivatives are summarized in Appendix A. In the last step, the variables are computed by solving the obtained system of inequalities (KKT-conditions). However, this step requires case distinctions since the inequality constraints may be binding or non-binding.

### 3 Operation principles

The solution of the previously defined linear optimization model defines the efficient portfolio, including the operation of different technologies over time. Subsequently, the fundamental properties of marginal technologies and electricity prices (shadow prices of demand) are first discussed in Section 3.1.

In Section 3.2, the particular role of storage technologies is emphasized by the investigation of the changes in the value of the storage contents.

#### 3.1 Fundamentals

With given capacities, operation at each moment in time may be characterized as follows:

**Proposition 1** *[dispatch of generation and storage technologies – control variables]*

*The operation of the generation and storage technologies, which may be considered as control variables, follows the following principles:*

*P 1.1. The dispatch of the generation technologies, corresponding to the choice of the control variables  $y_{i,t}$  is done in the order of increasing variable costs  $c_i^{op}$ .*

*P 1.2. The dispatch of storage discharging  $s_{j,t}^+$  is based on the same principle as  $\frac{1}{\eta_j^+} \lambda_{j,t}$  as corresponding variable (opportunity) cost.*

*P 1.3. Storage charging  $s_{j,t}^-$  is dispatched based on the variable (opportunity) value of charging in height of  $\eta_j^- \lambda_{j,t}$ .*

*Alternative decision variable and combined dispatchable supply stack:*

*P 1.4. Define the unused charging capacity of storage as an alternative control variable  $s_{j,t}^o = K_j^- - s_{j,t}^-$ , and split generation technologies into dispatchable  $i_d \in I_d$  and non-dispatchable (renewable energy)  $i_{RE} \in I_{RE}$  technologies as well as storage technologies into the unused charging  $j^o \in J^o$  and discharging  $j^+ \in J^+$  part., allowing the representation of all dispatchable controls as  $k \in \mathcal{K}$  with  $\mathcal{K} = I_d \cup J^o \cup J^+$  with their operation variable  $u_{k,t}$  such that  $u_{.,t} = [y_{.,t}' \ s_{.,t}^{o'} \ s_{.,t}^{+'}]'$ , with marginal costs  $\kappa_{k,t}$  and with capacities  $\tilde{K}_k$  such that  $\tilde{K}_k = [K_k' \ K_k^{-'} \ K_k^{+'}]'$ .*

*Moreover define a corresponding augmented residual demand (see Figure 1).<sup>4</sup>*

$$A_t = \max\{D_t - Y_{RE,t} + K_{total}^-, 0\} \quad (7)$$

---

<sup>4</sup> Note that the maximum operator in equation (7) is chosen to represent renewable curtailment in case of the augmented residual demand being negative, i.e. renewable feed-in exceeding the sum of demand and possible storage charging. This formulation implies that curtailment of fluctuating renewables is the ultimate control to prevent oversupply. It reflects thus the priority dispatch accorded to renewables in many legislations. Yet it also implies that this curtailment comes at no cost.

Then the supply-demand equilibrium restriction (see  $R_{D,t}$ , equation (2)) can be written as:

$$\sum_k u_{k,t} = A_t \perp \mu_t \quad (8)$$

and the optimal operation at any electricity price  $\mu_t$  is described through the merit-order of the combined dispatchable supply stack:

$$\begin{aligned} \text{Infra-marginal operation:} & \quad \text{If } \kappa_{k,t} < \mu_t, \text{ then } u_{k,t} = \tilde{K}_k \\ \text{Marginal operation:} & \quad \text{If } \kappa_{k,t} = \mu_t, \text{ then } u_{k,t} \in [0, \tilde{K}_k] \\ \text{Extra-marginal non-operation:} & \quad \text{If } \kappa_{k,t} > \mu_t, \text{ then } u_{k,t} = 0 \end{aligned} \quad (9)$$

For the proof, see Appendix B.

Based on proposition P1.4, we can now relate the *marginal costs*  $\kappa_{m,t}$  as follows to the type of marginal unit:

$$\begin{array}{ll} \text{marginal unit:} & \text{shadow price:} \\ \text{Conventional generation} & \kappa_{m,t} = c_m^{op} \cdot \Delta t, \quad m \in I_d \end{array} \quad (10)$$

$$\text{Storage (unused-)charging} \quad \kappa_{m,t} = \eta_m^- \cdot \lambda_{m,t} \cdot \Delta t, \quad m \in J^o \quad (11)$$

$$\text{Storage discharging} \quad \kappa_{m,t} = \frac{1}{\eta_m^+} \cdot \lambda_{m,t} \cdot \Delta t, \quad m \in J^+ \quad (12)$$

The combined dispatchable supply stack is then given by the increasing marginal costs<sup>5</sup> of the sorted controls  $k'$ . Figure 1 illustrates this for an example.

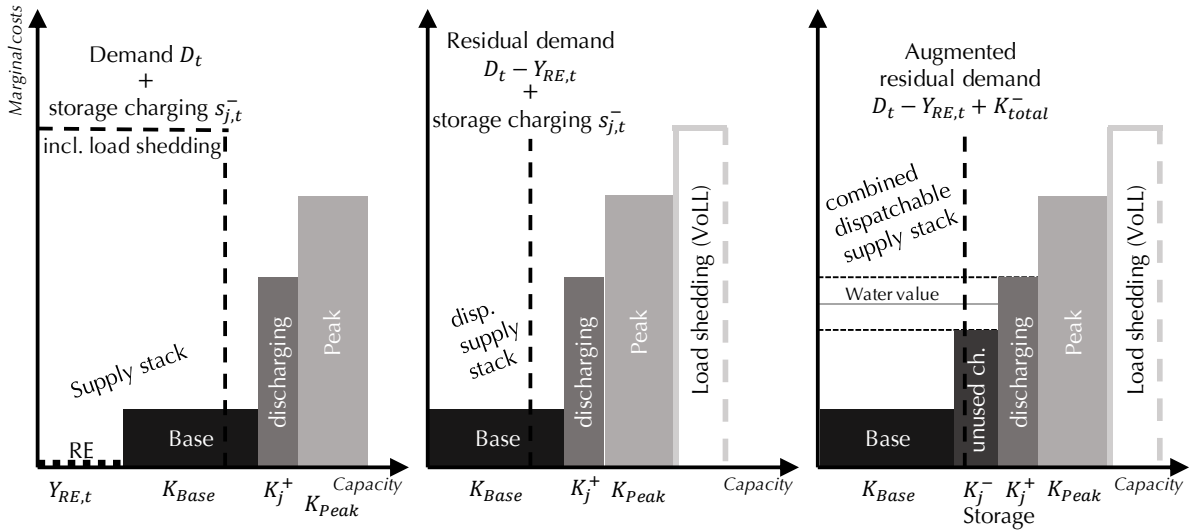


Figure 1: Exemplary demand and supply stack (merit-order) – modification from traditional merit order (left) to modified demand and supply stack including storage (right)

<sup>5</sup>  $\kappa_{k'-1,t} < \kappa_{k',t} < \kappa_{k'+1,t}$

In the left part of Figure 1, the traditional merit order with the total supply stack (incl. storage discharging) the inelastic demand curve (including load shedding as well as storage charging) is given.<sup>6</sup> In this setting, the demand, as well as the supply stack, are time-dependent. By introducing the residual demand as the difference between the demand and the intermittent feed-in (primarily renewables), only dispatchable generation technologies remain in the supply stack.

Additionally, load shedding is shifted from the demand to the supply side (center figure). Dispatchable charging is still part of the (residual) demand side. Under the assumption of P 1.4, all dispatchable controls (including unused-charging of storage) are forming the combined dispatchable supply stack. The position of the storage in the supply stack depends on the current value of the storage content (water value) and stays constant for a specified period (see following sections). In any case, the intersection between the augmented residual demand and the combined dispatchable supply stack yields the marginal technology and the price.

For the ordered supply stack, we may define the cumulative capacity  $K_{k,t}^c$  up to control  $k$  as follows:

$$K_{k,t}^c = \sum_{k' | \kappa_{k',t} \leq \kappa_{k,t}} K_{k'}, \quad K_{k,t}^{c-1} = \sum_{k' | \kappa_{k',t} < \kappa_{k,t}} K_{k'} \quad (13)$$

Additionally, we have defined here the cumulative capacity  $K_{k,t}^{c-1}$  of all technologies which are unequivocally preceding control  $k$  in the merit order. This separate definition is essential in case of ties, i.e. controls with identical marginal costs.

**Corollary 1** [marginal control]

*At any moment in time  $t$ <sup>7</sup>, there is a marginal control  $m$ , which sets the shadow price of demand  $\mu_t$  corresponding to its marginal costs  $\kappa_{m,t}$ , i.e.  $\mu_t = \kappa_{m,t}$ .*

*For any control  $k$ , precisely one of the following three possibilities holds at any time  $t$ :*

- a) *Infra-marginal technology:  $\kappa_{k,t} < \mu_t \wedge u_{k,t} = K_k \wedge K_{k,t}^c < A_t$*
- b) *Marginal technology:  $\kappa_{k,t} = \mu_t \wedge 0 \leq u_{k,t} \leq K_k \wedge K_{k,t}^{c-1} \leq A_t \leq K_{k,t}^c$*
- c) *Extra-marginal technology:  $\kappa_{k,t} > \mu_t \wedge u_{k,t} = 0 \wedge K_{k,t}^{c-1} > A_t$*

<sup>6</sup> Note that load shedding is not included in the formal problem formulation of section 2 in order to avoid further notational complications. Yet the extension is straightforward as illustrated in *Figure 1*. Effectively, load shedding is then equivalent to an additional (very expensive) conventional generation technology (see Stoft 2002).

<sup>7</sup> Except for a limited number of time steps, where another price is needed to ensure the matching of pay-offs with investment costs for each technology, see section 5, notably Proposition 3.

Note that this corollary does not preclude the case that there may be two marginal technologies at some point in time. For example, both conventional generation technology and storage discharging may be marginal. Similarly, also storage charging and conventional generation may simultaneously be marginal. We will come back to such cases in *Section 5*.

### 3.2 Water values and the specific role of storage

The previous results indicate that the marginal value drives the operation of storage  $\lambda_{j,t}$ , which is known by practitioners as “water value” or more generally “marginal storage value.” Inspection of the Lagrangian reveals the following properties:

**Proposition 2** *[storage value over time]*

*The shadow price  $\lambda_{j,t}$  describes the current position of discharging and charging options for storage  $j$  in the merit order. Its trajectory is derived from the following principles:*

*P 2.1. The marginal storage value remains constant ( $\lambda_{j,t+1} = \lambda_{j,t}$ ), if the storage operates strictly between its filling level bounds, i.e.,  $0 < x_{j,t} < V_j$  holds.*

*P 2.2. The marginal storage value increases ( $\lambda_{j,t+1} = \lambda_{j,t} + v_{j,t}^V$ ) only in time steps  $t$ , when the storage level reaches the upper bound ( $x_{j,t} = V_j$ ).*

*Note that the reverse implication is not valid, i.e., the water value may stay constant while the filling level reaches the upper bound.*

*P 2.3. The marginal storage value decreases ( $\lambda_{j,t+1} < \lambda_{j,t}$ ) only in time steps  $t$ , when the storage level reaches the lower bound ( $x_{j,t} = 0$ ).*

*Again, the reverse implication does not hold, i.e., the water value may stay constant while the lower bound of the storage level is reached.*

For the proof, see Appendix C.

The first part of this proposition states that the “water value” remains constant as long as the reservoir content does not reach the upper or lower bound. The second part then states that marginal storage value may increase when the storage gets full, whereas the last part affirms that the value may decrease when the storage is empty. The key point is that changes in water value may only occur when the storage level reaches its bounds. Nevertheless, the opposite conclusion does not hold: Empty or full storage may occur without inducing a change in the water value, which may be the case when there are ties (similar marginal costs) in the merit order. This phenomenon will be discussed further in *Section 4*.

Contact points, defined as points where the marginal storage value changes, may consequently be used to separate different phases in storage operation. We label these phases with constant water values **half-cycles (HC)**.

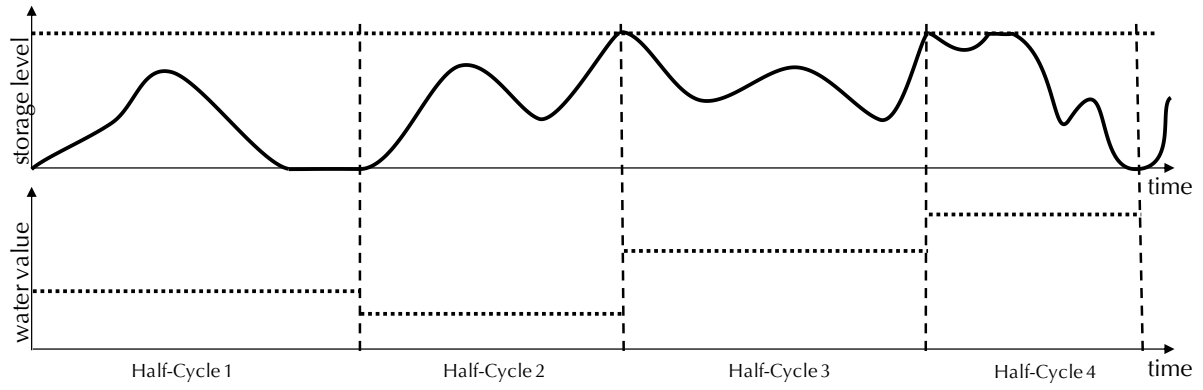


Figure 2: Half-cycles with exemplary storage filling levels and water values.

As shown in Figure 2, half-cycles may consist of an empty-full or a full-empty transition. However, also the other combinations of start and end states, i.e. empty-empty and full-full, can constitute a half cycle. When multiple generation technologies are present, an increase in the marginal value is not necessarily followed by a decrease at the next boundary. Instead, one increase in the water value may follow another (see Figure 2,  $HC2 \rightarrow HC3$  and  $HC3 \rightarrow HC4$ ), and correspondingly, the boundaries reached may repeatedly be the same.

While empty storage at the end of a half-cycle implies a decreasing water value, full storage inversely goes along with an increase in the water value: an immediate consequence of the trajectories of the dual variables, as discussed in Proposition 2.

Note that different storage technologies may have different half cycles and may operate differently at the same point in time. For example, large seasonal storage may be charged while at the same time small daily storage may be discharged.

## 4 Investment Principles

The optimal generation mix is derived from a long-term cost minimization perspective (see Section 2). However, using the shadow prices from the Lagrangian, the complementarity formulation (see, e.g., Gabriel, 2013), this planning optimum may also be reformulated and described as a competitive partial equilibrium. Consequently, each technology in the efficient portfolio will recover its full investment cost from operation margins in different time steps – otherwise it would not be built. We may summarize this as follows (see (A - 2) to (A - 5)):

$$\text{Generation technology:} \quad c_i^{inv} = \sum_t v_{i,t} \cdot b_{i,t} \perp K_i > 0 \quad (14)$$

$$\text{Storage charging:} \quad c_j^{inv,-} = \sum_t v_{j,t}^- \perp K_j^- > 0 \quad (15)$$

$$\text{Storage discharging:} \quad c_j^{inv,+} = \sum_t v_{j,t}^+ \perp K_j^+ > 0 \quad (16)$$

Storage volume: 
$$c_j^{inv,V} = \sum_t v_{j,t}^V \perp V_j > 0 \quad (17)$$

Note that excess returns are also excluded in this formulation, i.e. the sum of operation margins cannot be strictly superior to the annualized investment cost – in a competitive market this would induce market entry. Moreover, every single investment cost component for a (storage) technology has to be paid back by corresponding operation margins.

Typically, technologies generate operating margins for paying back their investment costs during time segments  $t$  when they are infra-marginal. During these time steps  $t$ , another technology sets the price  $\mu_t$  and the contribution margin  $v_{i,t}$  is equal to the difference between the price and the marginal costs of the infra-marginal technology; this does not only apply to generation technologies  $i$ , but also for storage charging and discharging (see equation (18), (19) and (20)). Storage volume generates its operation margins by the positive jumps in the shadow price of the storage level (see equation (21)) (Steffen und Weber 2013; Lamont 2013).

Conventional technology, see (A - 6): 
$$v_{i,t} = \begin{cases} \mu_t - c_i^{op} \cdot \Delta t & y_{i,t} = K_i \\ 0 & y_{i,t} < K_i \end{cases} \quad (18)$$

Storage charging, see (A - 7): 
$$v_{j,t}^- = \begin{cases} \lambda_{j,t} \cdot \eta_j^- \cdot \Delta t - \mu_t & s_{j,t}^- = K_j^- \\ 0 & s_{j,t}^- < K_j^- \end{cases} \quad (19)$$

Storage discharging, see (A - 8): 
$$v_{j,t}^+ = \begin{cases} \mu_t - \frac{1}{\eta_j^+} \lambda_{j,t} \cdot \Delta t & s_{j,t}^+ = K_j^+ \\ 0 & s_{j,t}^+ < K_j^+ \end{cases} \quad (20)$$

Storage volume, see (A - 9): 
$$v_{j,t}^V = \begin{cases} \lambda_{j,t} - \lambda_{j,t-1} & x_{j,t} = V_j \\ 0 & x_{j,t} < V_j \end{cases} \quad (21)$$

While conventional technologies and storage discharging are infra-marginal in times when the price exceeds their marginal (opportunity) costs, the opposite holds for storage charging. If unused-charging is considered instead of charging (as suggested in *Section 3.1*), this anomaly disappears. However, the operating margins are then less easily interpretable.

In the next section, we will show that additional levels of operating margins and, therefore prices have to occur to match the investment costs exactly.

## 5 Derivation of pricing rules and main operation modes for storage technologies

The principles mentioned above may now be used to derive some rules about shadow prices – both for demand and for storage content. Already in a system with only generation technologies, the fulfillment of integral investment payback equalities leads to some peculiarities in price formation in discrete time as indicated in the first part of the subsequent Proposition 3. In the



case of storage, the integral reservoir filling level constraint even leads to a priori unexpected price patterns and dominant operation modes as illustrated in the second part of Proposition 3. In order to prepare these statements, it is useful to reconsider the operation of the generation and storage units first. The left part of Figure 3 illustrates the operation of a generation unit  $i$  by indicating capacity bands against (augmented) residual load levels over time.

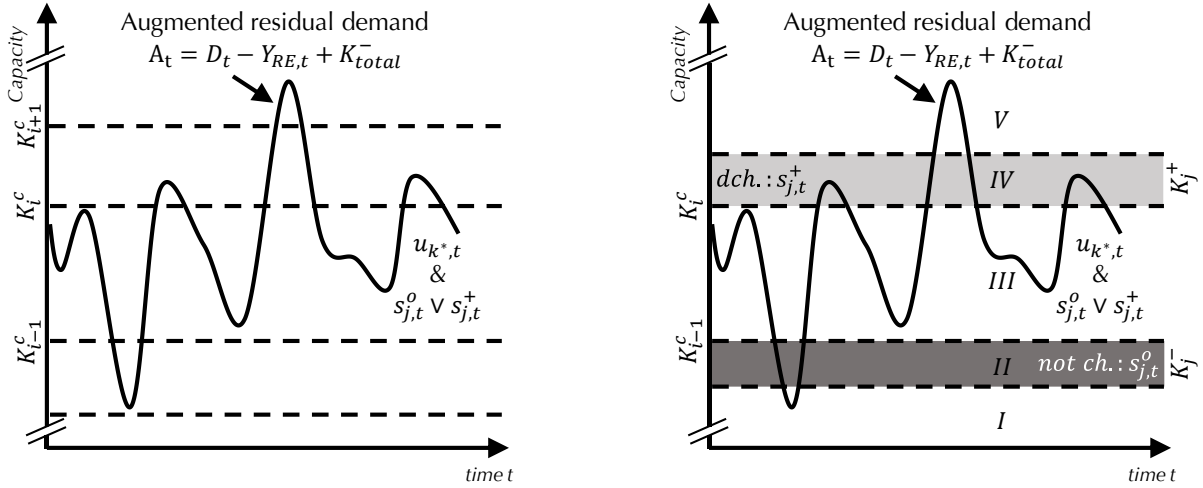


Figure 3: Main operation modes of generation technologies and storage technologies

As stated in proposition 1, the technology will not operate as long as  $A_t$  is below the lower capacity level  $K_{i-1}^c$ . Consequently, it will also not earn any operation margin. When  $A_t$  is inside the capacity band corresponding to technology  $i$ , i.e.  $A_t > K_{i-1}^c$  and  $A_t < K_i^c$ , then the technology is marginal and being the price-setting technology, it does not earn any operating margin. This may change for the case  $A_t = K_i^c$  but it will certainly whenever  $A_t$  exceeds  $K_i^c$ , because then another technology is price-setting.

In the case of storage technologies, similar considerations may be made, yet here five operation areas have a priori to be distinguished (*I to V*, see right part of Figure 3). And additionally, the volume constraint relevant for each half-cycle has to be fulfilled.

The five operation areas may be briefly characterized as follows:

*I Full charging (infra-marginal charging) & no discharging (extra-marginal discharging):*

$$s_{j,t}^- = K_j^- \text{ (resp. } s_{j,t}^o = 0) \wedge s_{j,t}^+ = 0$$

*II Partial charging (marginal (unused-)charging) & no discharging (extra-marginal discharging):*

$$K_j^- > s_{j,t}^- > 0 \text{ (resp. } 0 < s_{j,t}^o < K_j^-) \wedge s_{j,t}^+ = 0$$

*III No charging (extra-marginal charging) & no discharging (extra-marginal discharging):*

$$s_{j,t}^- = 0 \text{ (resp. } s_{j,t}^o = K_j^-) \wedge s_{j,t}^+ = 0$$

IV No charging (extra-marginal charging) & partial discharging (marginal discharging):

$$s_{j,t}^- = 0 \text{ (resp. } s_{j,t}^o = K_j^-) \wedge 0 < s_{j,t}^+ < K_j^+$$

V No charging (extra-marginal charging) & full discharging (infra-marginal):

$$s_{j,t}^- = 0 \text{ (resp. } s_{j,t}^o = K_j^-) \wedge s_{j,t}^+ = K_j^+$$

The critical question is then on the height of the shadow price for the storage content since it determines the position of the storage in the overall merit order and hence triggers the switch between the operation areas. As discussed in Section 3.2, this shadow price may change over time, yet it will stay constant within one half-cycle.

If the steps of the merit-order are all distinct, the storage charging and discharging quantities are entirely determined by the position in the merit order, even in the case of marginal charging or discharging. More precisely we get for marginal discharging  $s_{j,t}^+ = A_t - K_j^{c-1}$  and for marginal charging  $s_{j,t}^- = K_j^c - A_t$  ( $s_{j,t}^o = A_t - K_j^{c-1}$ ). At the same time, the storage volume constraints are binding at the beginning and the end of each half-cycle and hence the sum of all charging and discharging activities is prespecified. In general, this leads to an overdetermined equation system without solution— unless the assumption of distinct steps in the merit order is abandoned, leading to the following proposition:

**Proposition 3** [price levels and operation modes]

P 3.1. In a system with only generation technologies and load shedding, the prices correspond to the variable cost of one technology except for time steps where  $A_t = K_i^c$  holds for some technology  $i$ . For those time steps, the following equality holds:

$$\forall i \sum_{t|A_t=K_i^c} (\mu_t - c_i^{op} \cdot \Delta t) = (c_i^{inv} - c_{i+1}^{inv}) - T_{i+1} \cdot (c_{i+1}^{op} - c_i^{op}) \cdot \Delta t \quad (22)$$

Thereby  $T_{i+1}$  is the integer such that  $T_{i+1} = \text{card}(\{t|A_t > K_i^c\})$ . At the same time, it must satisfy the relations  $T_{i+1} \leq \frac{c_i^{inv} - c_{i+1}^{inv}}{(c_{i+1}^{op} - c_i^{op}) \cdot \Delta t}$  and  $T_{i+1} \geq \frac{c_i^{inv} - c_{i+1}^{inv}}{(c_{i+1}^{op} - c_i^{op}) \cdot \Delta t} - \text{card}(\{t|A_t = K_i^c\})$ .

P 3.2. In a system including generation and storage technologies as well as load shedding, the storage shadow values for a storage technology  $j$  with  $N_j$  half-cycles will align in up to  $N_j - 1$  half-cycles with the variable costs of some other (reference) control  $k^r$  (not necessarily the same in all half-cycles). Thereby the two main operation modes (MOM) may be distinguished:

MOM 1 – flexible charging of storage  $j$  in half-cycle  $n_j$ :

$$\exists k^r \in \mathcal{K} \setminus \{j\} \quad \forall t \in \{t_{n_j}^{start}, t_{n_j}^{end}\}, \quad \lambda_{j,t} = \frac{1}{\eta_j} \cdot \kappa_{k^r,t} \cdot \frac{1}{\Delta t} \quad (23)$$

MOM 2 – flexible discharging of storage  $j$  in half-cycle  $n_j$ :

$$\exists k^r \in \mathcal{K} \setminus \{j\} \quad \forall t \in \forall t \in \{t_{n_j}^{start}, t_{n_j}^{end}\}, \quad \lambda_{j,t} = \eta_j^+ \cdot \kappa_{k^r,t} \cdot \frac{1}{\Delta t} \quad (24)$$

P 3.3. In the system described in P 3.2, other shadow prices will occur in selected half-cycles and time steps in order to satisfy the investment payback equalities (18) to (21).

Proof see Appendix D

The two main operation modes distinguished in P 3.2 are discussed in the subsequent sections. In MOM 1 (see Section 5.1), storage (unused-)charging and the use of the reference control  $k^r$  are at the same cost level in the merit order. Hence the system optimizer (or the market equilibrium) is indifferent between (unused-)charging and the reference control, as long as other constraints like the energy balance of the storage are fulfilled. Hence charging is at least partly flexible in time while storage discharging patterns are clearly defined (constrained). MOM 2 (see Section 5.2), on the contrary, has predetermined storage charging and partly flexible discharging patterns. Flexible thereby means that multiple optimal solutions exist with differences in capacity usage over time.

## 5.1 Main operation mode 1 (MOM 1) – flexible storage charging

Put somewhat differently, MOM 1 corresponds to half-cycles with given storage discharging pattern and a surplus of chargeable energy through some control  $k^r$ . This surplus makes charging energy not scarce and drives the water value to the level of marginal cost of  $k^r$ , divided by the charging efficiency  $\eta_j^-$  (see equation (11) and Figure 4). Note that the control  $k^r$  will typically be a generation technology with constant marginal cost over the entire optimization horizon. However, it could also be another storage that is larger (in terms of its volume-to-capacity ratio) so that the marginal cost remains constant over the half-cycle of storage  $j$ .

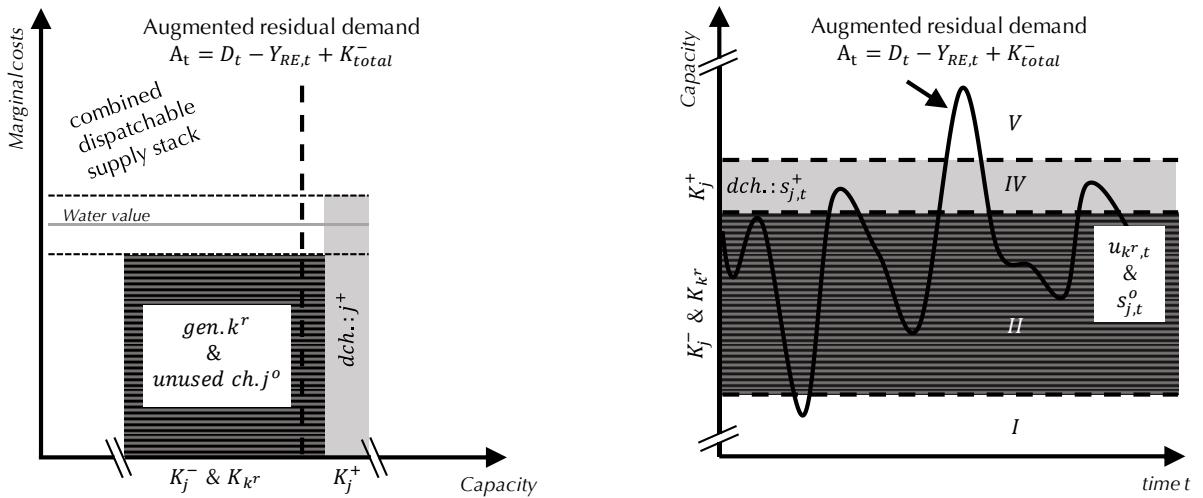


Figure 4: Combined dispatchable supply stack for MOM 1

During states II in MOM 1, (unused-)charging with control  $k^r$  is not constrained by available energy, which implies indifferences of capacity usage over time, meaning that during state II in MOM 1 the half cycle control  $k^r$  and storage charging  $j^-$  indifferently set the price to  $\kappa_{k^r,t}$  (see equation (23)).

$$\mu_t = \kappa_{k^r,t} = \lambda_{j,t} \cdot \eta_j^- \cdot \Delta t \quad (25)$$

Therefore temporarily, marginal (unused-)charging  $s_{j,t}^o$  and marginal operation  $u_{k^r,t}$  (see equation (8)) are in a tie position, and only the sum is well defined:

$$u_{k^r,t} + s_{j,t}^o = A_t - K_{k^r}^{c-1} \quad (26)$$

Discharging is extra-marginal in this state II as indicated before:  $s_{j,t}^+ = 0$ .

For state IV, discharging is marginal, and the price will then align on the efficiency-adjusted water-value:

$$\mu_t = \lambda_{j,t} \cdot \frac{1}{\eta_j^+} \cdot \Delta t = \kappa_{k^r,t} \cdot \frac{1}{\eta_j^- \cdot \eta_j^+} \quad (27)$$

i.e., the price in state IV corresponds to the costs of the reference technology-adjusted by the cycling efficiency of the storage.

## 5.2 Main operation mode 2 (MOM 2) – flexible storage discharging

MOM 2 corresponds to half-cycles with limited storage charging, and through discharging, the storage  $j$  is able to substitute generation of  $k^r$  partly. In comparison to MOM 1, storage discharging is flexible in time although restricted by available energy. Correspondingly the water value gets up to the cost level of the next marginal technology  $k^r$ , corrected by the discharging efficiency  $\eta_j^-$  (see equation (12) and Figure 5).

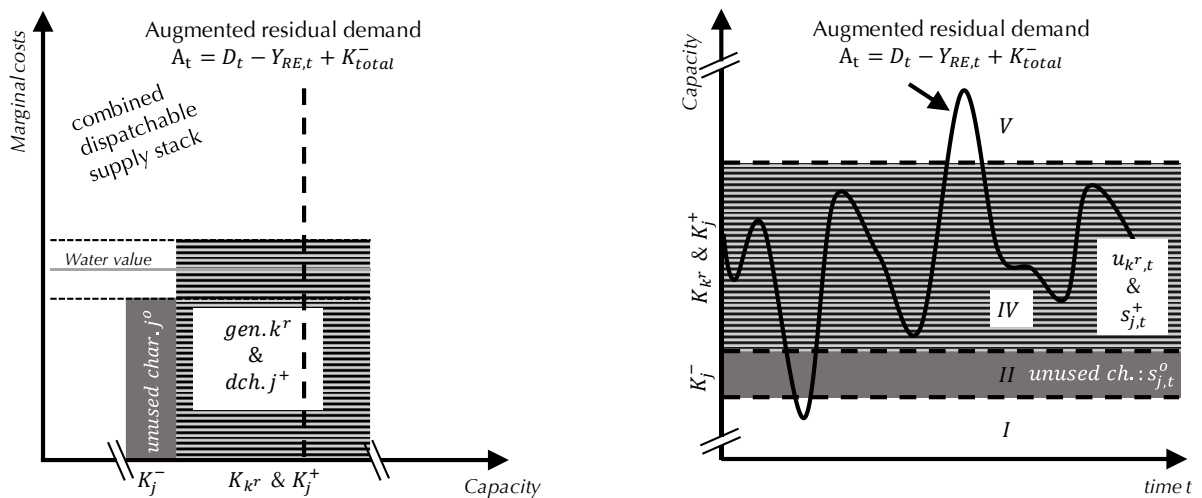


Figure 5: Combined dispatchable supply stack for MOM 2

During states IV in MOM 2, discharging is a perfect substitute for the use of control  $k^r$  (except for the energy restriction), which implies indifferences in discharging capacity usage over time. Correspondingly, marginal discharging  $s_{j,t}^+$  and marginal operation  $u_{k^r,t}$  are in a tie position during state IV in MOM 2, and only their sum is well defined (see equation (8)).

$$u_{k^r,t} + s_{j,t}^+ = A_t - K_{k^r}^{c-1} \quad (28)$$

Half cycle control  $k^r$  and storage discharging  $j^+$  hence both set the price to  $\kappa_{k^r,t}$  (see equation (24)).

$$\mu_t = \kappa_{k^r,t} = \frac{\lambda_{j,t}}{\eta_j^+} \cdot \Delta t \quad (29)$$

The corresponding marginal charging in state II then occurs at a price:

$$\mu_t = \lambda_{j,t} \cdot \eta_j^- \cdot \Delta t = \kappa_{k^r,t} \cdot \eta_j^- \cdot \eta_j^+ \quad (30)$$

i.e., the price in state II in MOM 2 is again determined by the costs of the reference technology-adjusted by the cycling efficiency of the storage.

## 6 Storage as part of the efficient portfolio – An application

The previous sections have provided analytical insights in the drivers for storage operation and investment, notably on the changing role of storage over time within efficient technology portfolios. In the subsequent application, the operation and investment characteristics are discussed for a discretized and numerically solved optimization problem. The intention is now to provide illustrations of the described properties in a stylized but not entirely artificial example.

After describing the scenario assumptions (see *Section 6.1*), the main results are summarized for Lithium-Ion (Li-Ion) and Pumped Hydro Storage (PHS) as part of the efficient portfolio (see *Section 6.2*). Thereby scenarios with only one of the two technologies are considered as well as both technologies in combination. *Section 6.3* highlights main operation modes by analyzing specific examples as well as illustrating the effects on the merit order. To conclude, payback of the investment costs is shown in *Section 6.4*.

### 6.1 Scenario assumptions

This application serves to illustrate the analytics using a case study for Germany for the year 2040 (see, e.g., Böcker et al., 2015). Besides four conventional technologies, two storage technologies are considered as possible elements of the efficient portfolio (see Table 1).

Table 1: Main input parameter<sup>8</sup>

	Capacity costs	Volume costs	Technical lifetime	Efficiency	Operational costs	Fuel costs
	k€/MW	k€/MWh	years	%	€/MWh	€/MWh
Lignite	1,500	0	40	49%	37.9	8.2
Hard coal	1,200	0	40	51%	50.6	23.9
CCGT	700	0	30	62%	63.4	50.5
OCCGT	400	0	25	41%	95.9	76.3
VoLL	0	0	--	--	10.000	--
PHS	840	20	50	80%	--	--
Li-Ion	100	150	20	90%	--	--

In this application, renewable capacities are set exogenously (see Table 2), so that the given residual demand has to be covered by the efficient portfolio of conventional and storage technologies. The time series for demand and renewable feed-in are taken from historical observations in 2011 and scaled to the installed capacities. Furthermore, it is assumed that no penalty for renewable curtailment has to be paid. The operational costs for fossil technologies are derived from the fuel prices given in Table 1 and a CO<sub>2</sub> price set to 40 €/tCO<sub>2</sub>. The charging efficiency is set to the cycle efficiency of the storage, and the discharging efficiency is correspondingly set to one. As shown in Table 1, load shedding is included as an additional technology with the operational costs of 10€/kWh (VoLL).

Table 2: Installed renewable capacities

Wind onshore	60.1 GW
Wind offshore	54.0 GW
Photovoltaic	74.5 GW

Transmission capacities to neighboring countries are not taken into account, nor are internal grid restrictions.

## 6.2 Main results

The application shows that with the given cost structure in this greenfield approach, hard coal is not part of the efficient portfolio in 2040. In the scenario without storage, the efficient portfolio includes 81 GW of conventional generation, including 43% Lignite, while lost load sums up to 7.3 GWh, corresponding to roughly 0.0015 % of total demand (see Table 3).

<sup>8</sup> based on data by (Kost et al. 2013; Fürstenwerth und Waldmann 2014; IEA 2013; Sterner und Stadler 2014) and own analyses

If the option Li-Ion can be chosen, the total capacity rises to 82 GW and storage with a capacity of 7 GW, and a volume of 27 GWh is found to be efficient. The storage is then discharged in about four full load hours. The storage capacity replaces gas-fired generation, especially OCGT, but also CCGT. The lost load decreases to less than 2 GWh.

In contrast, the PHS scenario leads to an efficient storage capacity of about 11 GW and 259 GWh, i.e. the storage configuration has a power-to-energy ratio of 1:23. This results from the fact that storage volume is cheaper in case of PHS whereas charging and discharging capacities are cheaper for Li-Ion batteries. Consequently, the investment into PHS storage substitutes for various conventional technologies, not only gas technologies.

Table 3: Efficient capacities and system costs with and without Li-Ion and PHS storage

	No storage	Li-Ion	PHS	Li-Ion & PHS
Lignite	34.8 GW	34.9 GW	33.2 GW	33.2 GW
Hard Coal	0 GW	0 GW	0 GW	0 GW
CCGT	17.7 GW	15.8 GW	13.3 GW	13.2 GW
OCGT	28.2 GW	24.3 GW	23.1 GW	22.0 GW
Li-Ion	0 GW	7.1 GW (27.4 GWh)	0 GW	4.2 GW (12.9 GWh)
PHS	0 GW	0 GW	11.1 GW (259.3 GWh)	9.1 GW (239.8 GWh)
Lost Load	(max) 2.8 GW (sum) 7.3 GWh	(max) 1.3 GW (sum) 1.8 GWh	(max) 2.8 GW (sum) 7.3 GWh	(max) 1.7 GW (sum) 3.0 GWh
Total Capacity	80.7 GW	82.1 GW	80.7 GW	81.7 GW
System Cost	16.94 bn€	16.77 bn€	16.70bn€	16.64 bn€

In an energy system with PHS, additional Li-Ion storage will lead to a decrease in efficient PHS capacity by 2 GW. Lignite and CCGT stay almost constant; only the efficient capacity of OCGT peakers decreases by 1 GW.

### 6.3 Storage operation and main operation modes

As indicated in Table 4, Li-Ion storage has significantly more half-cycles (such as changes in the marginal storage value) than pumped hydro storage. Even for one storage technology, the length of the half cycles varies strongly, for example, for PHS the longest half-cycle encompasses more than 400 hours for a mean of approximately 85. Moreover, these half-cycles correspond to somewhat different storage values as indicated in Figure 9.

Table 4: Half Cycles for Li-Ion and PHS storage

		Li-Ion	PHS	Li-Ion & PHS
Li-Ion	num	388	---	455
	max duration [h]	140	---	115
	min duration [h]	3	---	3
	mean duration [h]	20	---	17
PHS	num	---	103	96
	max duration [h]	---	404	378
	min duration [h]	---	3	6
	mean duration [h]	---	82	87

Subsequently, we focus on the application case of PHS as sole storage technology for further insights into the main storage operation modes. In the following two figures, the main operation modes (MOM 1 in Figure 6 and MOM 2 in Figure 7) are illustrated for the case that the marginal storage value aligns with the marginal cost of lignite.

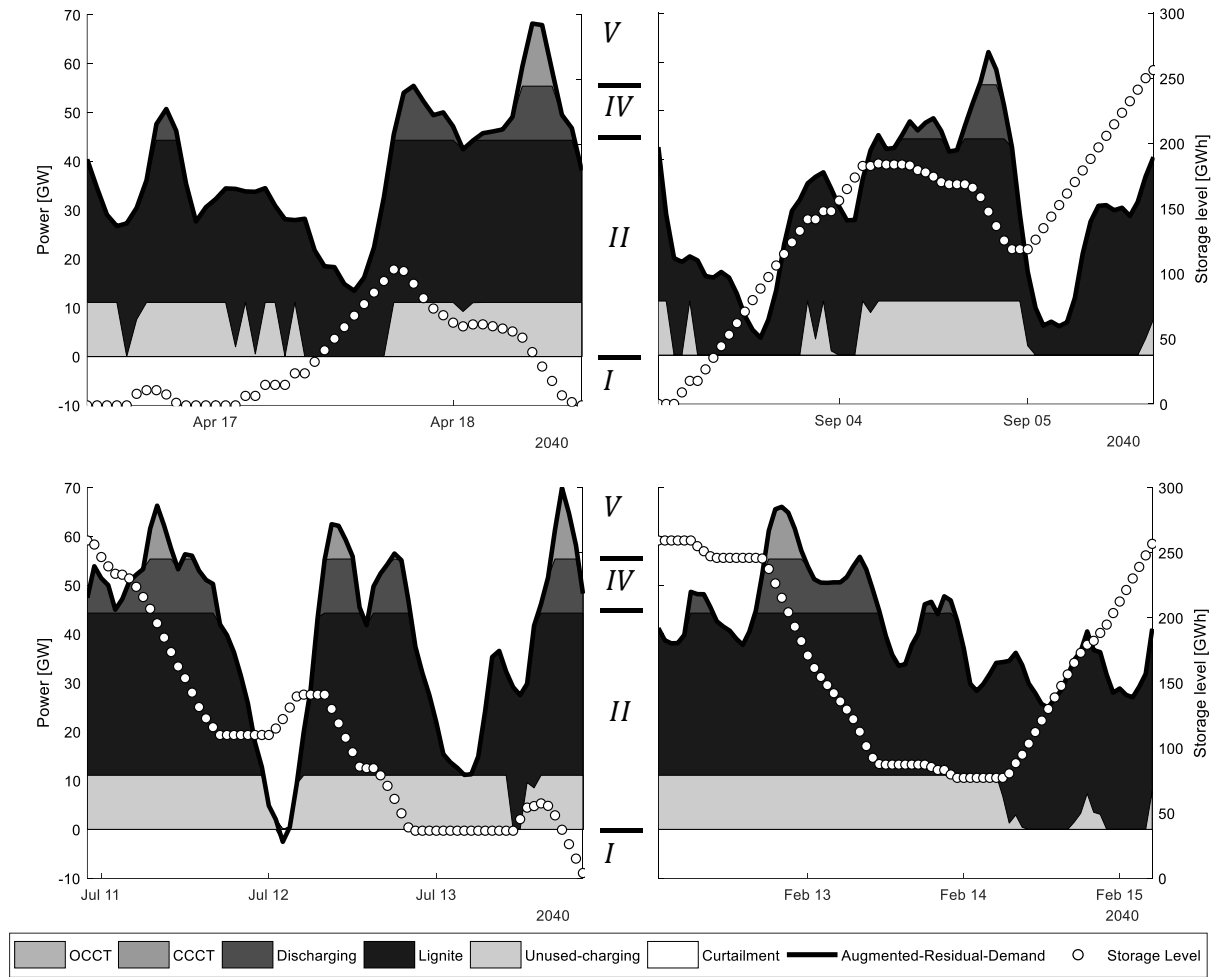


Figure 6: Exemplary energy supply (left axis) and storage level (right axis) during half cycles of PHS in MOM 1 aligned on lignite as marginal technology. Start and end filling levels combinations as follows: top left: empty-empty; top right: empty-full; bottom left: full-empty; bottom right: full-full, bottom right.



Each of the four subfigures shows a different type of half-cycle, i.e. with another combination of start and end filling levels (full or empty). As deduced in *Section 5*, the operation of storage in a selected half cycle (with a corresponding marginal technology) has some flexibility due to indifferences during state II in MOM 1 – this is observable through the fluctuations between storage unused-charging and lignite in Figure 6. In MOM 2, this flexibility is observable during state IV (see Figure 7), where storage discharging and lignite production alternate frequently. On the contrary, the optimal operation pattern is fully predetermined by the augmented demand  $A_t$  during the other states.

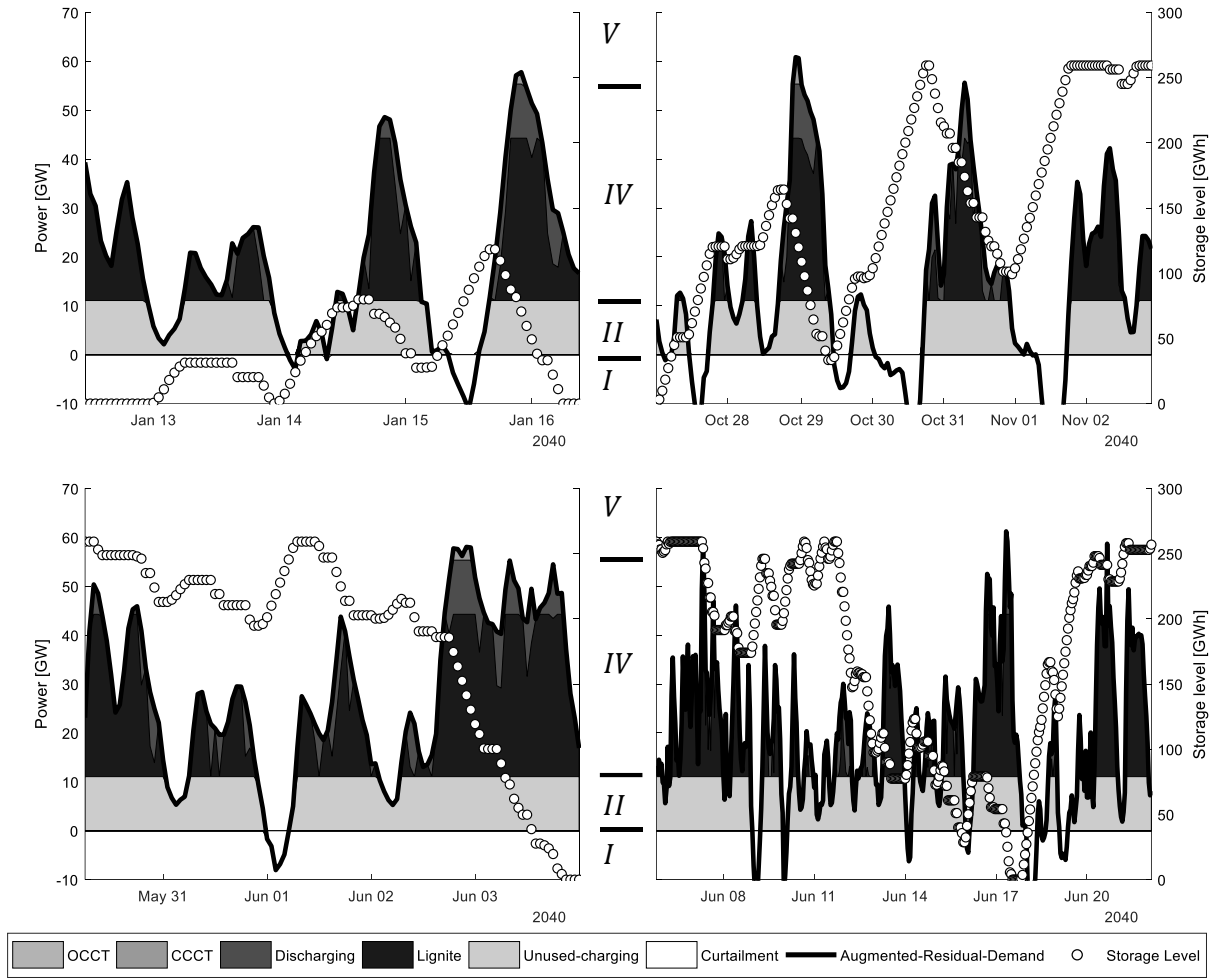


Figure 7: Exemplary energy supply (left axis) and storage level (right axis) during half cycles of PHS in MOM 2 aligned on lignite as marginal technology. Start and end filling levels combinations as follows: top left: empty-empty; top right: empty-full; bottom left: full-empty; bottom right.

When augmented residual demand gets negative, this indicates that the excess renewable feed-in is larger than the storage charging capacity (state I). Then charging occurs at full rate (since (unused-)charging is extra-marginal), and additionally, renewable curtailment occurs. When (unused-)charging is marginal (state II), the use of the charging is entirely determined in MOM 2 (see Figure 7). On the contrary, the operation is flexible (and thus numerically fluctuating, see

Figure 6) in MOM 1 due to the indifference between storage (unused-)charging and lignite. State III is empty in both MOMs since there is no generation technology with marginal costs close to lignite, which would fall between the marginal costs of (unused-)charging and discharging. In state IV, discharging is used to complement  $A_t$  in MOM 1. In MOM 2, a flexible use of lignite and discharging is on the contrary observed (see Figure 7). However, during state V, when discharging gets inframarginal, discharging and lignite entirely operate in both MOMs.

In both figures, we furthermore find examples illustrating that reaching the lower or upper limit of the storage filling level is not a sufficient condition for changes in the storage shadow value and hence the start of another half cycle. Instead, we see, such as in the bottom left of Figure 7, that the storage is full again around 5 a.m. on 1<sup>st</sup> June, yet before and after the marginal storage value corresponds to the marginal cost of lignite in MOM 2.

Switching between operation modes over time leads to a profile of irregular steps for the water value (illustrated in Figure 8). Six primary levels can be distinguished with roughly 100 changes in the investigated year (see half-cycles Table 4) and accumulation in the medium range. One additional level may be observed in mid-October (see Figure 8), which results from the discretization of the model and the condition to refinance PHS volume-based investment costs (see equation (17) and Proposition 2).

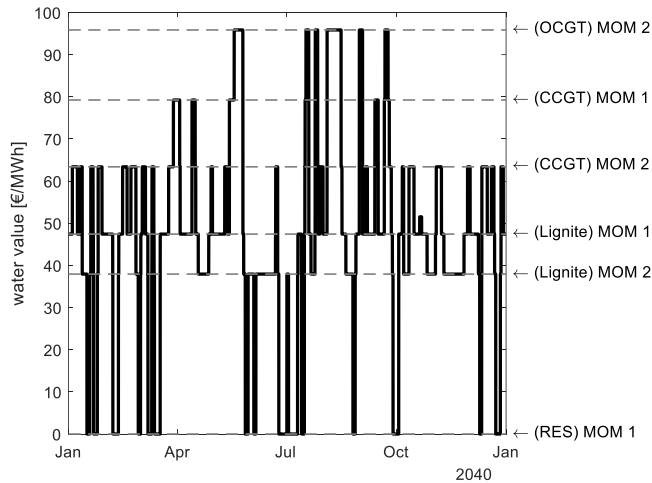


Figure 8: Water values (marginal storage value) of PHS over time with the corresponding MOM and the related generation technology (in parentheses).

Figure 9 summarizes the obtained electricity prices in the model (marked as cycles) as a function of the augmented residual demand and plotted against the supply stack of the corresponding MOM. In contrast to conventional electricity systems with stable supply stacks (merit-order), the combined dispatchable supply stack (see Proposition 1) changes between half-cycles depending on the relevant operation mode; therefore, six different supply stacks are shown in Figure 9. The functional relationship between residual demand and marginal costs is hence not stable over

time. As expected, the computed electricity price is a monotonic function of augmented residual demand within each MOM. For every single value of residual demand, a fixed marginal control is used, and the marginal costs are correspondingly fixed. However, across different MOMs, different price levels may correspond to the same residual demand level. For example, at an augmented residual load of 50 GW, prices may vary between the cost of lignite and the cost of charging the PHS that is discharging at the marginal cost of an OCGT.

Besides the three levels of marginal costs for the three conventional technologies and the zero marginal cost level for cases with the excess supply by renewables, five further price levels are hence observed due to storage discharging and unused-charging (hatched areas in Figure 9). The price level corresponding to load shedding has thereby been omitted in the graphs since it occurs very rarely and would require another scale.

In the MOM 1 (upper part of Figure 9), there is always an excess of chargeable energy through a generation technology  $k^r$  (indicated in parentheses in Figure 9), which is tantamount to indifference between both controls. This indifference during charging sets the water value in the MOM 1 and during discharging the storage sets the price above the respective marginal generation costs of  $k^r$  (with the reciprocal of the cycling efficiency as multiplier). In case of RES, this price remains zero. During MOM 1, storage technologies are hence similar to a new generation technology in the supply stack.

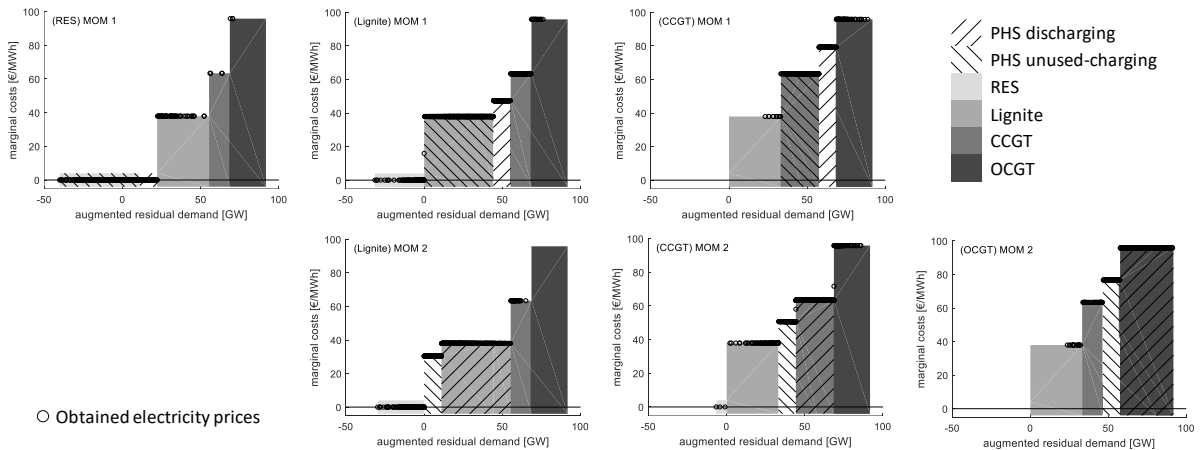


Figure 9: Merit-Order (marginal costs to cover augmented residual demand) for different MOM's.

In the MOM 2 storage charging is limited while storage discharging partly substitutes generation of  $k^r$ , leading to the supply stacks displayed in the lower part of Figure 9. In these cases the water value is set by marginal discharging while indifference with marginal generation from  $k^r$  occurs. Up to the installed capacity of  $K_{k^r}$ , discharging and generation substitute each other (within the energy limits of the storage) and beyond  $K_{k^r}$ , the storage has to discharge to meet the augmented residual demand.

Figure 9 illustrates the changes in the supply stack due to the main operation modes but does not show which operation margins help to pay back the investment costs of the technologies. As stated in Proposition 3, some additional price-levels occur - these have been omitted for clarity in Figure 9 but will be discussed subsequently.

## 6.4 Paying back investment costs

In the competitive market equilibrium, not only demand will be served in (almost) every hour, but also the investment costs for the different technologies have to be balanced by corresponding operating margins (zero excess profit condition). Without storage, lignite can generate operating margins (or capacity rents) while CCGT and OCGT set the price due to the difference in marginal costs as well as during periods of load shedding (see Figure 10, top left). CCGT collects capacity rents while OCGT or load shedding sets the price. The peak-technology OCGT is not able to generate capacity rents during regular operation; they only occur in those hours where load shedding occurs.<sup>9</sup>

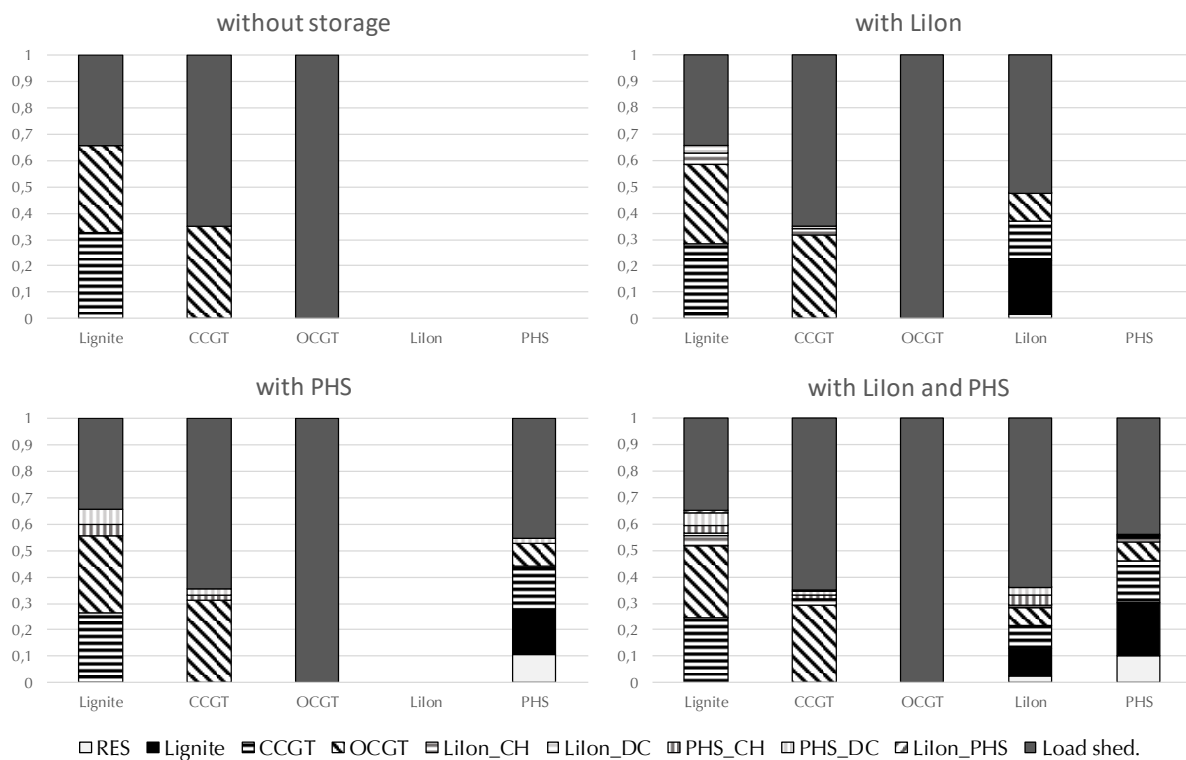


Figure 10: Paying back investment costs - capacity rents earned differentiated by marginal technology.

When storage is included in the energy system, conventional technologies generate somewhat lower capacity rents in cases where other conventional technologies set the price. However, additional revenues compensate this loss in hours where storage sets the price. In comparison to

<sup>9</sup> If such situations are prevented through regulation, the well-known missing-money problem occurs Newbery 2016.

the conventional technologies, storage can generate capacity rents by exploiting a wide variety of price-spreads; even when renewables drive the price to zero – then the storage may recover capacity rents when charging.

## 7 Conclusions

The paper provides analytical insights into the optimality of storage investment and usage in power systems with large shares of renewables. It, therefore, complements large-scale optimization models, which allow a detailed assessment of specific scenarios, by highlighting the main driving forces and trade-offs determining the implementation of storage in a competitive environment.

The analysis provides a unified framework that incorporates storage operation into the supply stack – yet emphasizing its time-variable place in the merit order. The place in the supply stack is driven by changing marginal storage values. However, these so-called “water values” remain constant as long as storage levels do not reach the boundaries. During such “half-cycles,” the marginal storage value, moreover, frequently aligns with the marginal costs of generation technologies, giving rise to two main operation modes of storage. Within these main operation modes, either charging or discharging is determined by the storage energy balance and therefore follows a given pattern, while the reverse operation has at least partly some flexibility, given indifferences in operation with some generation technology.

The water value changes with the changing scarcity of supply over time and hence increases in periods with high residual demand and decreases in periods with abundant renewable supply. If residual demand decreases, it is notably more efficient to “hand over” empty storage to the next half-cycle, allowing charging during lower prices, which means a cheaper technology sets the price, and therefore the water value decreases. If the need for energy increases (due to higher residual demand), the value of the storage content increases as well due to the increasing marginal costs of the marginal generation technology, moving the storage at the same time more to the right of the supply stack.

Overall, this not only leads to more complex price patterns with obvious tendencies towards peak shaving and valley filling. It also induces more diverse patterns of operation margins, which, in turn, allow to pay back all components of the original investment cost.

At the same time the developed analytical framework is sufficiently compact that it may serve as basis for further generalizations in future work. On the one hand, the framework may be transposed to a setting in continuous time, which should enable the use of more analytical tools and avoid some of the nasty details of integer mathematics. On the other hand, uncertainties in feed-in and demand may be considered in another step to address a further crucial challenge of

renewables. The combination of both extensions could hopefully path the way to a sound analytical treatment of a theoretically very intriguing and at the same time, practically very relevant problem: the generation adequacy problem in the presence of fluctuating renewables and storage. Solving this problem is a prerequisite to transforming the dream of a 100 % renewable electricity system into reality based on a solid foundation in terms of security of supply.

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## Appendix

### Appendix A Lagrange-Function and first-order derivatives

Lagrange function:

$$\begin{aligned}
 L = & \sum_t \left( \sum_i c_i^{op} \cdot \Delta t \cdot y_{i,t} + \mu_t \cdot \left( D_t + \sum_j s_{j,t}^- - \sum_i y_{i,t} - \sum_j s_{j,t}^+ \right) \right. \\
 & + \sum_i v_{i,t} \cdot (y_{i,t} - b_{i,t} \cdot K_i) + \sum_j v_{j,t}^+ \cdot (s_{j,t}^+ - K_j^+) \\
 & + \sum_j v_{j,t}^- \cdot (s_{j,t}^- - K_j^-) \\
 & + \sum_j \lambda_{j,t} \cdot \left( x_{j,t+1} - x_{j,t} - \eta_j^- \cdot s_{j,t}^- \cdot \Delta t + \frac{1}{\eta_j^+} \cdot s_{j,t}^+ \cdot \Delta t \right) \\
 & + \left. \sum_j v_{j,t}^V \cdot (x_{j,t} - V_j) \right) + \sum_i c_i^{inv} \cdot K_i + \sum_j c_j^{inv,-} \cdot K_j^- \\
 & + \sum_j c_j^{inv,+} \cdot K_j^+ + \sum_j c_j^{inv,V} \cdot V_j
 \end{aligned} \tag{A - 1}$$

**Derivatives with respect to primal variables:**

Conventional capacity (differentiation with respect to  $K_i$ ):

$$\frac{\partial L}{\partial K_i} = c_i^{inv} - \sum_t v_{i,t} \cdot b_{i,t} \geq 0 \quad \perp K_i \geq 0 \tag{A - 2}$$

Storage charging capacity (differentiation with respect to  $K_j^-$ ):

$$\frac{\partial L}{\partial K_j^-} = c_j^{inv,-} - \sum_t v_{j,t}^- \geq 0 \quad \perp K_j^- \geq 0 \tag{A - 3}$$

Storage discharging capacity (differentiation with respect to  $K_j^+$ ):

$$\frac{\partial L}{\partial K_j^+} = c_j^{inv,+} - \sum_t v_{j,t}^+ \geq 0 \quad \perp K_j^+ \geq 0 \tag{A - 4}$$

Storage volume (differentiation with respect to  $V_j$ ):

$$\frac{\partial L}{\partial V_j} = c_j^{inv,V} - \sum_t v_{j,t}^V \geq 0 \quad \perp V_j \geq 0 \tag{A - 5}$$

Generation (differentiation with respect to  $y_{i,t}$ ):

$$\frac{\partial L}{\partial y_{i,t}} = c_i^{op} \cdot \Delta t - \mu_t + v_{i,t} \geq 0 \quad \perp y_{i,t} \geq 0 \tag{A - 6}$$

Storage charging (differentiation with respect to  $s_{j,t}^-$ ):

$$\frac{\partial L}{\partial s_{j,t}^-} = \mu_t + v_{j,t}^- - \lambda_{j,t} \cdot \eta_j^- \cdot \Delta t \geq 0 \perp s_{j,t}^- \geq 0 \quad (\text{A - 7})$$

Storage discharging (differentiation with respect to  $s_{j,t}^+$ ):

$$\frac{\partial L}{\partial s_{j,t}^+} = -\mu_t + v_{j,t}^+ + \lambda_{j,t} \cdot \frac{1}{\eta_j^+} \cdot \Delta t \geq 0 \perp s_{j,t}^+ \geq 0 \quad (\text{A - 8})$$

Storage level (differentiation with respect to  $x_{j,t}$ ):

$$\frac{\partial L}{\partial x_{j,t}} = \lambda_{j,t-1} - \lambda_{j,t} + v_{j,t}^V \geq 0 \perp x_{j,t} \geq 0 \quad (\text{A - 9})$$

### Derivatives with respect to dual variables:

Wholesale price (differentiation with respect to  $\mu_t$ ):

$$\frac{\partial L}{\partial \mu_t} = D_t + \sum_j s_{j,t}^- - \sum_i y_{i,t} - \sum_j s_{j,t}^+ = 0 \perp \mu_t \quad (\text{A - 10})$$

Water value (differentiation with respect to  $\lambda_{j,t}$ ):

$$\frac{\partial L}{\partial \lambda_{j,t}} = x_{j,t+1} - x_{j,t} - \eta_j^- \cdot s_{j,t}^- \cdot \Delta t + \frac{1}{\eta_j^+} \cdot s_{j,t}^+ \cdot \Delta t \leq 0 \perp \lambda_{j,t} \geq 0 \quad (\text{A - 11})$$

Generation capacity value, generation (differentiation with respect to  $v_{i,t}$ ):

$$\frac{\partial L}{\partial v_{i,t}} = y_{i,t} - b_{i,t} \cdot K_i \leq 0 \perp v_{i,t} \geq 0 \quad (\text{A - 12})$$

Storage discharging capacity value (differentiation with respect to  $v_{j,t}^+$ ):

$$\frac{\partial L}{\partial v_{j,t}^+} = s_{j,t}^+ - K_j^+ \leq 0 \perp v_{j,t}^+ \geq 0 \quad (\text{A - 13})$$

Storage charging capacity value (differentiation with respect to  $v_{j,t}^-$ ):

$$\frac{\partial L}{\partial v_{j,t}^-} = s_{j,t}^- - K_j^- \leq 0 \perp v_{j,t}^- \geq 0 \quad (\text{A - 14})$$

Storage volume value (differentiation with respect to  $v_{j,t}^V$ ):

$$\frac{\partial L}{\partial v_{j,t}^V} = x_{j,t} - V_j \leq 0 \perp v_{j,t}^V \geq 0 \quad (\text{A - 15})$$

## Appendix B Shadow price of the demand - Proof of Proposition 1

**Proposition 1** [dispatch of generation and storage technologies – control variable]

The operation of the generation and storage technologies, which may be considered as control variables, follows the following principles.

*P 1.1* The dispatch of the generation technologies, corresponding to the choice of the control variables  $y_{i,t}$  is done in the order of increasing variable costs  $c_i^{op} \cdot \Delta t$ .

P 1.2 The dispatch of storage discharging  $s_{j,t}^+$  is based on the same principle with  $\frac{1}{\eta_j^+} \lambda_{j,t} \cdot \Delta t$  as corresponding variable (opportunity) cost.

P 1.3 Storage charging  $s_{j,t}^-$  is done based on the variable (opportunity) value of charging in height of  $\eta_j^- \lambda_{j,t} \cdot \Delta t$ .

Alternative decision variable and combined dispatchable supply stack

P 1.4 Define the not-operation of storage (unused charging) as an alternative control variable  $s_{j,t}^0 = K_j^- - s_{j,t}^-$  and split generation technologies into dispatchable  $i_d \in I_d$  and non-dispatchable (renewable energy)  $i_{RE} \in I_{RE}$  technologies as well as storage technologies into the not-charging  $j^0 \in J^0$  and discharging  $j^+ \in J^+$  part.

This allows representing all dispatchable controls as  $k \in \mathcal{K}$  with  $\mathcal{K} = I_d \cup J^0 \cup J^+$  with their operation variable  $u_{k,t}$  with  $u_{k,t} = [y_{k,t} \quad s_{k,t}^0 \quad s_{k,t}^+]'$  and marginal costs  $\kappa_{k,t}$  in a merit-order as a combined dispatchable supply stack and define a corresponding augmented residual demand (see Figure 1)

$$A_t = \max\{D_t - Y_{RE,t} + K_{total}^-, 0\} \quad (7)$$

. Then the supply-demand equilibrium restriction (see  $R_D$ , equation (2)) can be written as:

$$\sum_k u_{k,t} = A_t \quad (8)$$

and the optimal operation at any electricity price  $\mu_t$  is described through the merit-order of the combined dispatchable supply stack:

$$\begin{aligned} \text{Inframarginal operation:} & \quad \text{If } \kappa_{k,t} < \mu_t, \text{ then } u_{k,t} = K_k \\ \text{Marginal operation:} & \quad \text{If } \kappa_{k,t} = \mu_t, \text{ then } u_{k,t} \in [0 K_k] \\ \text{Extramarginal non-operation:} & \quad \text{If } \kappa_{k,t} > \mu_t, \text{ then } u_{k,t} = 0 \end{aligned} \quad (9)$$

### **Proof of Proposition 1.**

Regarding P 1.1: The dispatch of generation technologies is determined by the complementarity condition (or Karush-Kuhn-Tucker condition) related to the production variable  $y_{i,t}$ :

$$\frac{\partial L}{\partial y_{i,t}} = c_i^{op} \cdot \Delta t + v_{i,t} - \mu_t \geq 0 \perp y_{i,t} \geq 0 \quad \text{see (A - 6)}$$

and the complementarity condition corresponding to the dual variable representing the capacity rent  $v_{i,t}$  for a conventional generation technology:

$$\frac{\partial L}{\partial v_{i,t}} = y_{i,t} - K_i \leq 0 \perp v_{i,t} \geq 0 \quad \text{see (A - 12)}$$

These constraints allow for three combinations of  $y_{i,t}$  and  $v_{i,t}$ , depending on the shadow price of demand  $\mu_t$  and its relation to the variable operation cost:

- $c_i^{op} \cdot \Delta t < \mu_t$ : This implies that  $v_{i,t} > 0$  in order to fulfill the first part of (A - 6). This, in turn, implies according to (A - 12) that  $y_{i,t} = K_i$ .
  - Technology  $i$  operates at its capacity limit and is infra-marginal
- $c_i^{op} \cdot \Delta t = \mu_t$ : Then necessarily  $v_{i,t} = 0$  because otherwise, the second part of (A - 6) and the first part of (A - 12) lead to a contradiction.  $y_{i,t}$  is then not determined by these two complementarity conditions. If there is only one technology fulfilling equality, the production  $y_{i,t}$  is determined through the supply-demand balance (A - 10).
  - Technology  $i$  is the (or at least one) marginal technology and may operate in the range  $[0 K_i]$ .
- $c_i^{op} \cdot \Delta t > \mu_t$ : Then necessarily  $y_{i,t} = 0$  given the fact that  $v_{i,t}$  is non-negative by construction and the complementarity condition in (A - 6). (A - 12) then implies  $v_{i,t} = 0$ .
  - Technology  $i$  is not running and is extra-marginal.

With increasing shadow price  $\mu_t$ , the generation units with the lowest marginal cost will hence operate first. Then the others will follow in the order of increasing operation costs, i.e., according to the “merit order.”

Regarding P 1.2: The complementarity condition determines the dispatch of storage discharging with respect to the charging variable  $s_{j,t}^+$

$$\frac{\partial L}{\partial s_{j,t}^+} = \frac{1}{\eta_j^+} \lambda_{j,t} \cdot \Delta t + v_{j,t}^+ - \mu_t \geq 0 \perp s_{j,t}^+ \geq 0 \quad \text{see (A - 8)}$$

Similar to the generation technologies, the KKT condition with respect to the capacity constraint and the corresponding dual variable (capacity rent)  $v_{j,t}^+$  also has to be considered:

$$\frac{\partial L}{\partial v_{j,t}^+} = s_{j,t}^+ - K_j^+ \leq 0 \perp v_{j,t}^+ \geq 0 \quad \text{see (A - 13)}$$

These equations are strictly analogous to those for the generation technologies, only the operational cost  $c_i^{op} \cdot \Delta t$  has been replaced by the term  $\frac{1}{\eta_j^+} \lambda_{j,t} \cdot \Delta t$  and the nomenclature of the primal and dual variable is different ( $s_{j,t}^+$  and  $v_{j,t}^+$  instead of  $y_{i,t}$  and  $v_{i,t}$ ). Hence the same operation rules apply.

Regarding P 1.3: The dispatch of storage charging is similarly determined by the complementary condition related to the charging variable  $s_{j,t}^-$

$$\frac{\partial L}{\partial s_{j,t}^-} = v_{j,t}^- - \lambda_{j,t} \cdot \eta_j^- \cdot \Delta t + \mu_t \geq 0 \perp s_{j,t}^- \geq 0 \quad \text{see (A - 7)}$$

The complementarity constraint with respect to the dual capacity rent variable  $v_{j,t}^-$  also has to be considered:

$$\frac{\partial L}{\partial v_{j,t}^-} = s_{j,t}^- - K_j^- \leq 0 \perp v_{j,t}^- \geq 0 \quad \text{see (A - 13)}$$

$$\frac{\partial L}{\partial v_{j,t}^-} = s_{j,t}^- - K_j^- \leq 0 \perp v_{j,t}^- \geq 0 \quad \text{see (A - 13)}$$

Again, these complementarity conditions define three operation modes, yet with reversed inequalities regarding the shadow price of demand  $\mu_t$ .

- $\lambda_{j,t} \cdot \eta_j^- \cdot \Delta t > \mu_t$ : This implies that  $v_{j,t}^- > 0$  in order to fulfill the first part of (A - 7). This, in turn, implies according to (A - 14) that  $s_{j,t}^- = K_j^-$ .
  - Charging of storage  $j$  operates at its capacity limit and is infra-marginal
- $\lambda_{j,t} \cdot \eta_j^- \cdot \Delta t = \mu_t$ : Then necessarily  $v_{j,t}^- = 0$  because otherwise, the second part of (A - 7) and the first part of (A - 14) lead to a contradiction.  $s_{j,t}^-$  is then not determined by these two complementarity conditions. If there is only one technology fulfilling equality, the charging  $s_{j,t}^-$  is determined through the supply-demand balance (A - 10).
  - Charging of storage  $j$  is the (or at least one) marginal technology and may operate in the range  $[0 K_i]$ .
- $\lambda_{j,t} \cdot \eta_j^- \cdot \Delta t < \mu_t$ : Then necessarily  $s_{j,t}^- = 0$  given the fact that  $v_{j,t}^-$  is non-negative by construction and the complementarity condition in (A - 7). (A - 14) then implies  $v_{j,t}^- = 0$ .
  - No charging of storage  $j$  occurs, and the charging is hence extra-marginal.

In contrast to storage discharging, storage charging is hence an extra-marginal operation, if the current electricity price  $\mu_t$  is above the marginal value ( $\lambda_{j,t} \cdot \eta_j^- \cdot \Delta t$ ) of the storage content.

Regarding P 1.4: Introducing an alternative decision variable  $s_{j,t}^o$  for not-operation of storage charging

$$s_{j,t}^o = K_j^- - s_{j,t}^- \quad (\text{B - 1})$$

as well as the split of generation technologies into supply dependent  $i_d$  and renewable-based  $i_{RE}$

$$\sum_i y_{i,t} = \sum_{i_d} y_{i_d,t} + \sum_{i_{RE}} y_{i_{RE},t} \quad (\text{B - 2})$$

and defining the aggregated renewable feed-in as  $Y_{RE,t}$

$$\sum_{i_{RE}} y_{i_{RE},t} = Y_{RE,t} \quad (\text{B - 3})$$

allows redefining the supply stack as well as the demand.

The supply-demand equilibrium restriction

$$\sum_i y_{i,t} + \sum_j s_{j,t}^+ = D_t + \sum_j s_{j,t}^- \quad \perp \mu_t \geq 0 \quad \text{see (2)}$$

may then be rewritten replacing corresponding terms according to (B - 1) to (B - 3):

$$\begin{aligned} \left( \sum_{i_d} y_{i_d,t} + Y_{RE,t} \right) + \sum_j s_{j,t}^+ &= D_t + \sum_j (K_j^- - s_{j,t}^o) \quad \perp \mu_t \geq 0 \\ \sum_{i_d} y_{i_d,t} + \sum_j s_{j,t}^+ + \sum_j s_{j,t}^o &= D_t - Y_{RE,t} + \sum_j K_j^- \quad \perp \mu_t \geq 0 \end{aligned} \quad (\text{B - 4})$$

With the following definitions of total charging capacity  $K_{total}^-$

$$\sum_j K_j^- = K_{total}^- \quad (\text{B - 5})$$

and augmented demand  $A_t$

$$A_t = \max\{D_t - Y_{RE,t} + K_{total}^-, 0\} \quad (7)$$

the final version of the modified supply-demand balance is obtained:

$$\sum_{i_d} y_{i_d,t} + \sum_j s_{j,t}^+ + \sum_j s_{j,t}^o = A_t \quad \perp \mu_t \geq 0 \quad (\text{B - 6})$$

The left-hand side of this equality consists throughout of non-negative terms. Therefore, the maximum operator in the definition of  $A_t$  is necessary to prevent infeasibilities in case of a negative residual load that exceeds the absorbing capacity of storage charging.

Collecting the variables on the left-hand side in one control vector  $u_{.,t}$

$$u_{.,t} = \begin{bmatrix} y_{.,t} \\ s_{.,t}^o \\ s_{.,t}^+ \end{bmatrix} \quad (\text{B - 7})$$

yields the following compact formulation of the demand-supply equilibrium

$$\sum_{k \in \mathcal{K}} u_{k,t} = A_t \quad (\text{B - 8})$$

The operation of the controls  $u_{k,t}$  according to the principle of a merit-order have already been demonstrated above for the controls  $y_{i,t}$  and  $s_{j,t}^+$ . They remain unchanged since the derivatives of the modified Lagrangian

$$\begin{aligned}
L^m = & \sum_t \left( \sum_{i_d} c_{i_d}^{op} \cdot \Delta t \cdot y_{i_d,t} + \mu_t \cdot \left( A_t - \sum_j s_{j,t}^o - \sum_{i_d} y_{i_d,t} - \sum_j s_{j,t}^+ \right) + \sum_i v_{i,t} \cdot (y_{i,t} - b_{i,t} \cdot K_i) \right. \\
& + \sum_j v_{j,t}^+ \cdot (s_{j,t}^+ - K_j^+) + \sum_j v_{j,t}^o \cdot (s_{j,t}^o - K_j^-) \\
& + \sum_j \lambda_{j,t} \cdot \left( x_{j,t+1} - x_{j,t} - \eta_j^- \cdot (K_j^- - s_{j,t}^o) \cdot \Delta t + \frac{1}{\eta_j^+} \cdot s_{j,t}^+ \cdot \Delta t \right) \\
& + \sum_j v_{j,t}^V \cdot (x_{j,t} - V_j) + \left. \sum_i c_i^{inv} \cdot K_i + \sum_j c_j^{inv,-} \cdot K_j^- + \sum_j c_j^{inv,+} \cdot K_j^+ \right. \\
& + \left. \sum_j c_j^{inv,V} \cdot V_j \right)
\end{aligned}$$

with respect to  $y_{i,t}$  and  $s_{j,t}^+$  are the same as for the original Lagrangian.

Yet the (unused-)charging variable  $s_{j,t}^o$  is now determined by the complementary slackness conditions with respect to  $s_{j,t}^o$ :

$$\frac{\partial L^m}{\partial s_{j,t}^o} = \lambda_{j,t} \cdot \eta_j^- \cdot \Delta t + v_{j,t}^o - \mu_t \geq 0 \perp s_{j,t}^o \geq 0 \quad (\text{B - 9})$$

and with respect to  $v_{j,t}^o$ :

$$\frac{\partial L^m}{\partial v_{j,t}^o} = s_{j,t}^o - K_j^- \leq 0 \perp v_{j,t}^o \geq 0 \quad (\text{B - 10})$$

These are now similar to those derived for conventional generation technologies, except that the variable (opportunity) costs of unused-charging in time step  $t$  are given by  $\lambda_{j,t} \cdot \eta_j^- \cdot \Delta t$ . Hence unused-charging is also part of the supply merit order.

## Appendix C Shadow price of the storage level (water-value) - Proof of Proposition 2

### **Proposition 2** [storage value over time]

The shadow price  $\lambda_{j,t}$  describes the current position of discharging and charging options for storage  $j$  in the merit order. Its trajectory is derived from the following principles:

*P 2.1* The marginal storage value remains constant ( $\lambda_{j,t+1} = \lambda_{j,t}$ ), if the storage operates strictly between its filling level bounds, i.e.,  $0 < x_{j,t} < V_j$  holds.

P 2.2 The marginal storage value increases ( $\lambda_{j,t+1} = \lambda_{j,t} + v_{j,t}^V$ ) only in times  $t$ , when the storage level reaches the upper bound ( $x_{j,t} = V_j$ ).

Note that the reverse implication is not valid, i.e. the water value may stay constant while the filling level reaches the upper bound.

P 2.3 The marginal storage value decreases ( $\lambda_{j,t+1} < \lambda_{j,t}$ ) only in times  $t$ , when the storage level reaches the lower bound ( $x_{j,t} = 0$ ).

Again, the reverse implication does not hold, i.e., the water value may stay constant while the lower bound of the storage level is reached.

### **Proof of Proposition 2.**

We analyze possible changes in the water value based on the complementarity condition with respect to storage level  $x_{j,t}$ :

$$\frac{\partial L}{\partial x_{j,t}} = \lambda_{j,t-1} + v_{j,t}^V - \lambda_{j,t} \geq 0 \perp x_{j,t} \geq 0 \quad \text{see (A - 9)}$$

Moreover, the complementarity condition resulting from the restriction related to the installed storage volume:

$$\frac{\partial L}{\partial v_{j,t}^V} = x_{j,t} - V_j \leq 0 \perp v_{j,t}^V \geq 0 \quad \text{see (A - 15)}$$

According to the former, decreases in the marginal storage value require the storage to be empty, i.e.  $x_{j,t} = 0$ , since  $v_{j,t}^V$  is non-negative by construction:

$$\lambda_{j,t} < \lambda_{j,t-1} \Rightarrow x_{j,t} = 0 \quad (\text{C - 1})$$

(Proposition P 2.3)

Conversely, the first part of (A - 9) implies that the marginal storage value may only increase through a strictly positive  $v_{j,t}^V$ . According to (A - 15) this may only occur when the storage is full.

Hence

$$\lambda_{j,t} > \lambda_{j,t-1} \Rightarrow x_{j,t} = V_j \quad (\text{C - 2})$$

(Proposition P 2.2)

In this case

$$\lambda_{j,t} = \lambda_{j,t-1} + v_{j,t}^V \quad (\text{C - 3})$$

By inversion of the arguments we may conclude from (C - 1) and (C - 2):

$$x_{j,t} > 0 \Rightarrow \lambda_{j,t} \geq \lambda_{j,t-1} \quad (\text{C - 4})$$



$$x_{j,t} < V_j \Rightarrow \lambda_{j,t} \leq \lambda_{j,t-1}$$

Combining both reverse implications, we obtain:

$$0 < x_{j,t} < V_j \Rightarrow \lambda_{j,t} = \lambda_{j,t-1} \quad (\text{C - 5})$$

(Proposition P 2.1)

## Appendix D Price levels and operation modes - Proof of Proposition 3

### Proposition 3 [price levels and operation modes]

P 3.1 In a system with only generation technologies and load shedding, the prices correspond to the variable cost of one technology except for those time steps where  $A_t = K_i^c$  holds for some technology  $i$ . In those time steps, the following equality holds:

$$\forall i \quad \sum_{t|A_t=K_i^c} (\mu_t - c_i^{op} \cdot \Delta t) = (c_i^{inv} - c_{i+1}^{inv}) - T_{i+1} \cdot (c_{i+1}^{op} - c_i^{op}) \cdot \Delta t \quad (22)$$

Thereby  $T_{i+1}$  is the integer such that  $T_{i+1} = \text{card}(\{t|A_t > K_i^c\})$ . At the same time, it must satisfy the relations  $T_{i+1} \leq \frac{c_i^{inv} - c_{i+1}^{inv}}{(c_{i+1}^{op} - c_i^{op}) \cdot \Delta t}$  and  $T_{i+1} \geq \frac{c_i^{inv} - c_{i+1}^{inv}}{(c_{i+1}^{op} - c_i^{op}) \cdot \Delta t} - \text{card}(\{t|A_t = K_i^c\})$ .

P 3.2 In a system including generation and storage technologies as well as load shedding, the storage shadow values for a storage technology  $j$  with  $N_j$  half-cycles will align in up to  $N_j - 1$  half-cycles with the variable costs of some other control  $k^r$  (not necessarily the same in all half-cycles). Thereby the two main operation modes (MOM) may be distinguished:

MOM 1 – flexible charging of storage  $j$  in half-cycle  $n$ :

$$\exists k^r \in \mathcal{K} \setminus \{j\} \quad \forall t \in T_{j,n}, \quad \lambda_{j,t} = \frac{1}{\eta_j} \cdot \kappa_{k^r,t} \cdot \frac{1}{\Delta t} \quad (23)$$

MOM 2 – flexible discharging of storage  $j$  in half-cycle  $n$ :

$$\exists k^r \in \mathcal{K} \setminus \{j\} \quad \forall t \in T_{j,n}, \quad \lambda_{j,t} = \eta_j^+ \cdot \kappa_{k^r,t} \cdot \frac{1}{\Delta t} \quad (24)$$

P 3.3 In the system described in P 3.2; other shadow prices will occur in selected half-cycles and time steps in order to satisfy the investment payback equalities (18) to (21)

#### Regarding P3.1:

We consider two conventional generation technologies  $i$  and  $i + 1$  that are part of the optimal generation mix (i.e.  $K_i > 0$  and  $K_{i+1} > 0$ ) and occupy successive positions in the merit order. If the two technologies do not have identical characteristics, this implies  $c_i^{inv} > c_{i+1}^{inv}$  and  $c_i^{op} < c_{i+1}^{op}$  (see (Sunderkötter und Weber 2012)).

From the complementarity condition (A – 2), we then know:

$$c_i^{inv} - \sum_t v_{i,t} = 0 \quad (\text{D} - 1)$$

Moreover, according to the complementarity condition (A – 6), strictly positive capacity rents are given by:

$$v_{i,t} = \mu_t - c_i^{op} \cdot \Delta t \quad (\text{D} - 2)$$

They occur for those  $t$  when:

$$A_t \geq K_i^c \quad (\text{D - 3})$$

Taken together, this yields:

$$c_i^{inv} - \sum_{t|A_t \geq K_i^c} (\mu_t - c_i^{op} \cdot \Delta t) = 0 \quad (\text{D - 4})$$

And analogously for the next technology in the supply stack:

$$c_{i+1}^{inv} - \sum_{t|A_t \geq K_{i+1}^c} (\mu_t - c_{i+1}^{op} \cdot \Delta t) = 0 \quad (\text{D - 5})$$

Taking differences, we obtain:

$$\begin{aligned} c_i^{inv} - c_{i+1}^{inv} + \sum_{t|A_t \geq K_{i+1}^c} (c_i^{op} - c_{i+1}^{op}) \cdot \Delta t + \sum_{t|K_i^c < A_t < K_{i+1}^c} (c_i^{op} - c_{i+1}^{op}) \cdot \Delta t \\ + \sum_{t|A_t = K_i^c} (c_i^{op} \cdot \Delta t - \mu_t) = 0 \end{aligned} \quad (\text{D - 6})$$

Thereby the first sum results from the elimination of  $\mu_t$  when taking the difference between the two original sums for those  $t$  included in both of them. The second sum contains terms included only in the sum for technology  $i$  where the price is set by the variable costs of technology  $i + 1$  since technology  $i + 1$  is marginal (given that  $K_i^c < A_t < K_{i+1}^c$ ). However, in the third sum, technology  $i + 1$  is not operating, but technology  $i$  is used up to its capacity limit. Hence capacity rents may occur for technology  $i$  while  $\mu_t$  is not determined by the marginal cost of the next technology.

Rearranging terms yields:

$$c_i^{inv} - c_{i+1}^{inv} = \sum_{t|A_t > K_i^c} (c_{i+1}^{op} - c_i^{op}) \cdot \Delta t + \sum_{t|A_t = K_i^c} (\mu_t - c_i^{op} \cdot \Delta t) \quad (\text{D - 7})$$

The first sum on the right side of (D - 7) yields an integer multiple of  $(c_{i+1}^{op} - c_i^{op}) \cdot \Delta t$ . Thereby the multiplier is given by:

$$T_{i+1} = \text{card}(\{t|A_t > K_i^c\}) \quad (\text{D - 8})$$

At the same time, the first sum cannot be larger than the left-hand side since the second sum is non-negative by construction. Hence

$$T_{i+1} \leq \frac{c_i^{inv} - c_{i+1}^{inv}}{(c_{i+1}^{op} - c_i^{op}) \cdot \Delta t} \quad (\text{D - 9})$$

On the other hand,  $\mu_t$  in the second sum cannot exceed  $c_{i+1}^{op}$ , since then technology  $i + 1$  would be infra-marginal instead of extra-marginal. Consequently

$$T_{i+1} \geq \frac{c_i^{inv} - c_{i+1}^{inv}}{(c_{i+1}^{op} - c_i^{op}) \cdot \Delta t} - \text{card}(\{t|A_t = K_i^c\}) \quad (\text{D - 10})$$

Regarding P3.2:

We start with the assumption that within each of  $N_j$  half-cycles for storage  $j$ , the controls  $k_{.,t}$  are characterized by different marginal (opportunity) costs  $\kappa_{k,t}$ .

Then the operation of storage  $j$  may be summarized as follows:

$$s_{j,t}^+ = \mathbf{1}_{K_{j^+,t}^{c-1} < A_t \leq K_{j^+,t}^c} \cdot (A_t - K_{j^+,t}^{c-1}) + \mathbf{1}_{A_t > K_{j^+,t}^c} \cdot K_j^+ \quad (\text{D} - 11)$$

$$s_{j,t}^- = \mathbf{1}_{K_{j^0,t}^{c-1} \leq A_t < K_{j^0,t}^c} \cdot (K_{j^0,t}^c - A_t) + \mathbf{1}_{A_t < K_{j^0,t}^{c-1}} \cdot K_j^- \quad (\text{D} - 12)$$

Summing the storage level changes (see equation (4)) overall time steps within one half cycle  $n_j$  we get:

$$\begin{aligned} x_{j,t_{n_j}^{\text{end}}} - x_{j,t_{n_j}^{\text{start}}} &= \sum_{t|t \geq t_{n_j}^{\text{start}} \wedge t < t_{n_j}^{\text{end}}} \left( \eta_j^- \cdot s_{j,t}^- - \frac{1}{\eta_j^+} \cdot s_{j,t}^+ \right) \cdot \Delta t \\ &= \sum_{t|t \geq t_{n_j}^{\text{start}} \wedge t < t_{n_j}^{\text{end}}} \left( \eta_j^- \cdot \left( \mathbf{1}_{K_{j^+,t}^{c-1} < A_t \leq K_{j^+,t}^c} \cdot (A_t - K_{j^+,t}^{c-1}) + \mathbf{1}_{A_t > K_{j^+,t}^c} \cdot K_j^+ \right) - \frac{1}{\eta_j^+} \cdot \left( \mathbf{1}_{K_{j^0,t}^{c-1} \leq A_t < K_{j^0,t}^c} \cdot (K_{j^0,t}^c - A_t) + \mathbf{1}_{A_t < K_{j^0,t}^{c-1}} \cdot K_j^- \right) \right) \cdot \Delta t \end{aligned} \quad (\text{D} - 13)$$

On the right-hand side, all terms are determined, if the capacities  $K_k$ , including notably  $K_j^-$  and  $K_j^+$ , are fixed. On the left-hand side, we have

$$x_{j,t_{n_j}^{\text{end}}} - x_{j,t_{n_j}^{\text{start}}} = B_{n_j} \cdot V_j \quad (\text{D} - 14)$$

There  $B_{n_j}$  is equal to 1 for an empty-full half-cycle and equal to -1 for a full-empty half-cycle. For the other half-cycles (empty-empty and full-full),  $B_{n_j}$  is equal to zero. With  $N_j$  half-cycles, we have hence  $N_j$  equations for at most three specific decision variables, namely  $V_j$ ,  $K_j^-$  and  $K_j^+$ . Moreover,  $K_j^-$  and  $K_j^+$  are subject to constraints similar to (D – 7), hence the effective number of degrees of freedom related to the equations of type (D – 13) is 1. Consequently, (up to)  $N_j - 1$  half-cycles are not subject to a restriction of type (D – 13); this is the case when the marginal storage value  $\lambda_{j,t}$  aligns with the marginal cost of another (generation) technology according to MOM 1 or MOM 2 as defined in equations (23) and (24). Then only the sum of storage activity and conventional generation is well defined as indicated in equations (26) and (28). Correspondingly the marginal charging activities in MOM 1 may be complemented by conventional generation  $u_{k^r,t}$  as required to meet the energy restriction of the half-cycle:

$$\begin{aligned}
x_{j,t_{n_j}^{end}} - x_{j,t_{n_j}^{start}} &= \sum_{t|t \geq t_{n_j}^{start} \wedge t < t_{n_j}^{end}} \left( \eta_j^- \right. \\
&\quad \cdot \left( \mathbf{1}_{K_{j^+,t}^{c-1} < A_t \leq K_{j^+,t}^c} \cdot (A_t - K_{j^+,t}^{c-1}) + \mathbf{1}_{A_t > K_{j^+,t}^c} \cdot K_{j^+,t}^c \right) - \frac{1}{\eta_j^+} \\
&\quad \cdot \left( \mathbf{1}_{K_{k^r,t}^{c-1} \leq A_t < K_{k^r,t}^c} \cdot (K_{k^r,t}^c - (A_t - u_{k^r,t})) + \mathbf{1}_{A_t < K_{j^o,t}^{c-1}} \cdot K_{j^o,t}^{c-1} \right) \cdot \Delta t
\end{aligned} \tag{D - 15}$$

Similarly, the marginal discharging activities in MOM 2 may be complemented by conventional generation  $u_{k^r,t}$  to meet the energy restriction of the half-cycle:

$$\begin{aligned}
x_{j,t_{n_j}^{end}} - x_{j,t_{n_j}^{start}} &= \sum_{t|t \geq t_{n_j}^{start} \wedge t < t_{n_j}^{end}} \left( \eta_j^- \right. \\
&\quad \cdot \left( \mathbf{1}_{K_{j^+,t}^{c-1} < A_t \leq K_{j^+,t}^c} \cdot (A_t - K_{j^+,t}^{c-1} - u_{k^r,t}) + \mathbf{1}_{A_t > K_{j^+,t}^c} \cdot K_{j^+,t}^c \right) \\
&\quad - \frac{1}{\eta_j^+} \cdot \left( \mathbf{1}_{K_{k^r,t}^{c-1} \leq A_t < K_{k^r,t}^c} \cdot (K_{k^r,t}^c - A_t) + \mathbf{1}_{A_t < K_{j^o,t}^{c-1}} \cdot K_{j^o,t}^{c-1} \right) \cdot \Delta t
\end{aligned} \tag{D - 16}$$

### Regarding P3.3:

Part P3.2 of this proposition indicates that the prices in the combined system will align in most cases, namely for MOM 1 and MOM 2, on the marginal generation costs of conventional generation or the corresponding complementary cost levels of storage (see equations (25),(27),(29) and (30)). As in the case of P3.1, the matching of discrete operation margin levels in infra-marginal times with pre-specified investment cost requires then additional price levels. However, this is much more cumbersome in notation due to the time-varying position of storage in the supply stack. That has two implications: 1) in each supply stack there are additional cost levels (see P3.2). 2) the supply-stack changes with each new half-cycle of each storage technology. Consequently, when the investment pay-back equality similar to (D – 4) is stated for the general technology mix, this looks much more intriguing even if limited to two storage technologies:

$$c_k^{inv} - \sum_{n_1=1}^{N_1} \sum_{n_2=1}^{N_2} \sum_{t| \left( t \in \{t_{n_1}^{start}, \dots, t_{n_1}^{end}\} \right) \wedge t \in \{t_{n_2}^{start}, \dots, t_{n_2}^{end}\} \wedge A_t \geq K_{k,t}^c} (\mu_t - \kappa_{k,t}) = 0 \tag{D - 17}$$

Yet the underlying logic for evaluation remains analogous to P3.1: for all  $A_t$  that do not correspond to a capacity limit  $K_{k,t}^c$  in the corresponding supply stack, the prices  $\mu_t$  will strictly align to the variable cost  $\kappa_{m,t}$  of some marginal control  $m$  since this is control is not scarce.

Consequently (D – 17) requires integer multiples of predefined operation margins to be matched to also predefined investment cost. This will usually not be possible unless additional price levels occur in those time steps when  $A_t = K_{k,t}^c$ , i.e. when the capacities of the marginal technology are fully used. In the absence of ties or other ambiguities in  $A_t$  and if not by chance the investment cost is matched just by integer multiples of standard operation margins; there will be as many  $\mu_t$  different from marginal costs of controls as there are controls.

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