Herrmann, Markus; Hibbeln, Martin

Seasonality in catastrophe bonds and market-implied catastrophe arrival frequencies

Journal of Risk and Insurance

Provided in Cooperation with:
John Wiley & Sons

Suggested Citation: Herrmann, Markus; Hibbeln, Martin (2021) : Seasonality in catastrophe bonds and market-implied catastrophe arrival frequencies, Journal of Risk and Insurance, ISSN 1539-6975, Wiley, Hoboken, NJ, Vol. 88, Iss. 3, pp. 785-818, https://doi.org/10.1111/jori.12335

Received: 17 October 2019 | Revised: 9 November 2020 | Accepted: 22 November 2020

DOI: 10.1111/jori.12335

ORIGINAL ARTICLE

Journal of Risk and Insurance

Seasonality in catastrophe bonds and market-implied catastrophe arrival frequencies

Markus Herrmann | **Martin Hibbeln**

Mercator School of Management, University of Duisburg-Essen, Duisburg, Germany

Correspondence
Martin Hibbeln, Mercator School of Management, University of Duisburg-Essen, Lotharstr. 65, 47057 Duisburg, Germany.
Email: martin.hibbeln@uni-due.de

Funding information
German Insurance Science Association (DVfVW)

Abstract

We develop a conceptual framework to model the seasonality in the probability of catastrophe bonds being triggered. This seasonality causes strong seasonal fluctuations in spreads. For example, the spread on a hurricane bond is highest at the start of the hurricane season and declines as time goes by without a hurricane. The spread is lowest at the end of the hurricane season assuming the bond was not triggered, and then gradually increases as the next hurricane season approaches. The model also implies that the magnitude of the seasonality effect increases with the expected loss and the approaching maturity of the bond. The model is supported by an empirical analysis that indicates that up to 47% of market fluctuations in the yield spreads on single-peril hurricane bonds can be explained by seasonality. In addition, we provide a method to obtain market-implied distributions of arrival frequencies from secondary market spreads.

KEYWORDS
alternative risk transfer, bond spreads, catastrophe arrival frequencies, seasonality, underwriting risk

JEL CLASSIFICATION
G12; G22

1 | INTRODUCTION

Catastrophe bonds ("cat bonds") are vehicles to transfer underwriting risk from sponsors, which are mostly insurance or reinsurance companies but sometimes also corporates or sovereigns, to capital markets.[1] The development of the cat bond market mirrors the growing demand for major natural catastrophe protection. Climate change and growing properties in coastal areas may have contributed to this demand. Although the main characteristic of cat bonds is fungibility of catastrophe risk on the secondary market, the knowledge of the secondary market of cat bonds is sparse. We want to reduce this gap by providing insights into one of the most important drivers of secondary market spreads: seasonality. Whereas for the vast majority of traditional corporate bonds there is no clear seasonality of default risk, the default risk of cat bonds fluctuates with the likelihood of qualifying events, for example, U.S. hurricanes mostly occur in summer or fall and do not occur in spring. Although seasonality clearly has an impact on cat bonds, the link between the seasonal nature of catastrophic events and cat bond spreads is unexplored in the scientific empirical literature.

A typical cat bond pays a flexible coupon that consists of a floating interest rate such as the LIBOR or a money market rate plus a fixed additional coupon—the risk premium or spread. While the fixed coupon of a bond remains unchanged, its implicit spread may fluctuate throughout its lifetime depending on its price on secondary markets. These current secondary market spreads are of utmost importance to investors and issuers alike: Investors purchase additional cat bonds if spreads are high enough to satisfy their risk appetite, whereas they may refrain from the purchase of new cat bonds on the primary markets if they do not offer the same or better rates as cat bonds on the secondary markets. Issuers sell additional cat bonds if spreads on the secondary market for similar risk are lower than rates for traditional reinsurance contracts.[2]

The empirical literature on cat bonds rarely investigates secondary market spreads. Braun (2016) establishes an econometric pricing model to estimate cat bond spreads on primary markets. Lane and Mahul (2008) investigate the influence of the expected loss, peril type, and the reinsurance cycle on cat bond spreads. They use secondary market data in form of one additional observation after issuance for each bond. Dieckmann (2010) uses secondary market data to investigate the change in reinsurance rates for existing bonds after hurricane Katrina. However, he abstracts from seasonality in windstorms by assuming constant exogenous parameters, which can distort empirical results. Braun et al. (2019) indirectly rely on secondary ILS data by determining common risk factors in ILS fund returns. Gürtler et al. (2016) use secondary market data to investigate the impact of hurricane Katrina and the default of Lehman Brothers on spreads; moreover, they study the impact of bond-specific factors and macroeconomic variables on cat bond spreads. They acknowledge seasonality effects on secondary markets but eliminate it by dropping all observations where the time to maturity deviates from a multiple of a full year, thereby loosing up to 75% of their quarterly observations.

We develop a conceptual framework to model the seasonality in the probability of trigger events in catastrophe bonds. This conceptual framework has two elements: A hazard rate

[1]Cat bonds have importance beyond the insurance sector: For example, developing countries issue cat bonds to receive payments required for reconstruction and to support the population in case of the occurrence of natural catastrophes. In 2018 the International Bank for Reconstruction and Development launched a series of cat bonds that protect Latin American countries from earthquake damages for a total volume of US$ 1360 m. FIFA issued a US$ 262 m cat bond to protect itself against the possible cancelation of the 2006 World Cup in Germany.

[2]Braun (2016) provides a detailed description of the structure of a cat bond.

model and a modeled seasonality measure. (1) Based on the hazard rate model, we illustrate the theoretical implications for cat bond spreads stemming from seasonal fluctuations in the probability of a cat bond being triggered. From this hazard rate model, we derive a set of hypotheses describing the seasonality on the cat bond market, for example, the general pattern and its increasing amplitude with respect to maturity and riskiness. (2) We derive a comprehensible measure to model the seasonal fluctuations in spreads. This measure transforms seasonally fluctuating arrival frequencies—that is, the distribution of the likelihood of peril events occurring across 1 year—into the time-varying expected loss of each individual cat bond.

We support this theoretical framework by analyzing fluctuations of secondary market cat bond spreads based on a data set that includes 386 seasonality-affected cat bonds issued between 2002 and 2017. This data set includes almost the entire cat bond universe. We acquire these spreads from yearly market reports from Lane Financials LLC. Spreads supplied in these market reports are quotes surveyed from dealers. These quotes from different dealers are then averaged across dealers do acquire spreads for individual bonds (Gürtler et al., 2016).[3] In addition, we show seasonality effects for spreads drawn from actual trading data as reported in the Trade Reporting and Compliance Engine (TRACE). To the best of our knowledge, we are the first to use TRACE data on cat bonds in a scientific paper; however, our main analyses rely on dealer quotes because the available timeframe for the TRACE data started only in 2015 and, given the low trading frequency for cat bonds, the number of observations is much smaller than in the quarterly Lane Financials LLC data set. To explain fluctuations on secondary markets, we use linear fixed effects regression models, thereby explaining the changes in spreads within each individual bond's observations. We use the relative distributions of arrival frequencies for hurricanes and European winter storms modeled by Applied Insurance Research (AIR) on a monthly basis. To obtain these distributions, we were in touch with a representative from AIR and used information provided in Poliquin and Lalonde (2012). Additionally, we provide a method to extract market-implied arrival frequencies from secondary market spreads, thereby offering an opportunity to access the additional information that investors possess.

We have three main results: First, we document how seasonality affects cat bond spreads. We find that spreads peak right before the risk season starts and reach their lowest point right after risk season ends; the amplitude of seasonal fluctuation increases as a bond nears maturity; in absolute terms, bonds with high expected loss (EL)[4] fluctuate more strongly than bonds with low EL; single-peril bonds fluctuate more strongly than multi-peril bonds. Second, the proposed "seasonality-adjusted EL" measure, which is based on the developed conceptual framework, captures seasonal fluctuations on cat bond spreads. It explains up to 47% of all secondary market fluctuations among cat bonds that are affected by seasonality (measured by adjusted within R^2). The results on the seasonality measure are strongly supported by the robustness check with TRACE data. Third, we are able to estimate the market-implied distributions of arrival frequencies from secondary market data. These market-implied distributions explain secondary market fluctuations as good as modeled distributions of arrival frequencies.

The remainder of this article is as follows: Section 2 provides an overview of related literature. In Section 3, we develop a conceptual framework to model the seasonality in the probability of catastrophe bonds being triggered and establish hypotheses on seasonality. Section 4 describes the data set. The econometric models are presented in Section 5.

[3]Yearly market reports from Lane Financials LLC are available at www.lanefinancialllc.com.

[4]The yearly EL can be taken from the cat bond prospectus. Our data source for the EL are yearly market reports from Lane Financials LLC.

Section 6 contains results on the hypotheses, the proposed seasonality measure, and a robustness check. Section 7 presents the methodology for the market-implied distribution and its empirical results. Section 8 concludes.

2 | LITERATURE

2.1 | Seasonality on financial markets

Seasonality effects on the general financial markets have been investigated thoroughly in the empirical literature. For stocks, Keim (1983) and Lakonishok and Smidt (1984) find depressed returns on Mondays and week-of-the-month patterns, while others (e.g., De Bondt & Thaler, 1987, Gultekin & Gultekin, 1983, Rozeff & Kinney, 1976) find abnormal returns for certain months of the year most prominently defining the "January effect." Jordan and Jordan (1991) and Schneeweis and Woolridge (1979) find that corporate bonds exhibit January, turn-of-the-year, and week-of-the-month effects. More recent literature relates the January effect to systematic risk and fluctuating risk aversion (Sun & Tong, 2010), to the returns of the momentum strategy (Yao, 2012) and to the returns of mutual funds (Vidal-García & Vidal, 2014). Overall, the established empirical literature on seasonality in bond or stock returns has not found strong evidence for the existence of abnormal returns of certain days of the week or certain months of the year. Generally, the magnitude of seasonality for stocks and bonds is small and unpronounced. Additionally, Zhang and Jacobsen (2013) find different monthly effects with reversing directions depending on selected sub samples from a 300-year long data set of UK stock returns. They conclude that monthly return patterns are due to selection bias, noise and data snooping but are no real effect.

For agricultural commodities, Black (1976) states that prices follow a seasonal pattern: Prices are high before harvest and low after harvest. The success of a harvest is closely related to external conditions such as sunshine, wind, and rainfall. Consequently, the price of these assets is related to climate and weather. For futures of concentrated orange juice, Roll (1984) finds clear empirical evidence for a seasonal pattern in relation to extreme weather events. Orange trees die during prolonged periods of below freezing temperature. In Florida, where most U.S. orange juice is produced, these extreme temperatures can only occur in the winter. Hence, the likelihood of below freezing temperature is an important risk factor in the pricing of orange juice futures during this time: Prices are high in autumn reflecting the probability of freezing temperatures during the winter season. "Each day thereafter that passes without a freeze should be accompanied by a slight price decline, a relief that winter is one day closer to being over" (Roll, 1984). For orange juice and other agricultural products, seasonal prices are supply-driven. For prices of other commodities, such as natural gas, which is typically used to heat houses during the winter, prices are instead demand-driven (see Gorton et al., 2013). The magnitude of seasonal fluctuations in commodities is alleviated by costs of storage (see Fama & French, 1987).

For the property and casualty insurance industry, Ammar (2020) identifies seasonal changes in the implied volatility smile of insurance stock options. Generally, the slope of the implied volatility smile is much steeper for insurance stock options than for the whole economy. However, outside of the hurricane season, the smile for insurance stock options becomes flatter than during the hurricane season. Ammar (2020) indicates that markets might demand more in-the-money and at-the-money options outside of the hurricane season because large drops in insurance stock prices are less likely.

2.2 | Seasonality on cat bond markets

As previous research has indicated, the EL is the primary driver of cat bond spreads (e.g., Braun, 2016, Galeotti et al., 2013, Gürtler et al., 2016, Lane & Mahul, 2008). As discussed by Lane (2000), the EL is the product of the probability of first loss (PFL) and the conditional expected loss (CEL).[5] The PFL in turn is some function of the arrival frequency λ_t of qualifying events. The EL is measured on a per-year basis.

$$\mathrm{EL}_t = \mathrm{PFL}(\lambda_t)\cdot\mathrm{CEL}. \tag{1}$$

Consider a bond, which triggers a default when certain predetermined parameters of a catastrophe are fulfilled. This can be an earthquake of a certain level on the Richter scale or a hurricane whose wind speed exceeds a certain threshold. This type of trigger is referred to as a parametric trigger.[6] The likelihood of qualifying events depends on two conditions: (a) an event needs to take place and (b) this event has to be of a magnitude large enough to set off the parametric trigger. For some perils, such as earthquakes, likelihood and severity of events are independent and identically distributed (i.i.d.). Within a calendar year, these events do not have seasons. Other events that depend on weather conditions are unevenly distributed; namely, European winter storms, North-American hurricanes, and Japanese cyclones.[7] For example, the likelihood of a hurricane is high between June and November while it is almost zero between December and May. In consequence, the EL of a cat bond can fluctuate substantially throughout a calendar year. This fluctuation is not represented in coupons: Cat bonds typically pay a fixed coupon above LIBOR or some other money market rate that does not adjust according to changes in underlying EL. This means contrary to the empirical asset literature on stocks and bonds, cat bond spreads are strongly affected by seasonality, but no existing study explicitly analyzes their seasonal patterns.[8] Overall, the seasonality of cat bonds follows a clear rationale: the uneven distribution of default risk.[9] As a consequence of seasonality, cat bond spreads are partially predictable. However, seasonal fluctuation in spreads stem from fluctuations in the EL, which means

[5]In a credit risk context, different terms are used for the elements of the EL. The PFL is equivalent to the probability of default (PD), the CEL is equivalent to the loss given default (LGD).

[6]The trigger types that are employed more frequently are "Indemnity" and "Index" triggers. The EL of these bonds depends on the likelihood of events and their severity in a similar fashion as bonds with parametric triggers. Please refer to Finken and Laux (2009) for a discussion on the benefits of index and parametric triggers and Braun (2016) for further discussion on indemnity triggers.

[7]There are only seven single-peril cyclone bonds in our data. This number is too low to separately model the Japanese cyclone season with panel data regression models. Hence, we abstract from modeling this seasonality. Nevertheless, the suggested methodology can be applied to Japanese cyclone bonds if more of these bonds are issued in the future. Additional events, whose likelihood and severity are not independent and identically distributed across a calendar year, are tornados, thunderstorms and hail. However, since cat bonds are usually created to cover extreme risk, we focus our model on large-scale seasonal perils: hurricanes and European winter storms.

[8]Only a few other financial securities, like industry loss warranties (ILWs), weather derivatives and some commodities, are also known for having such strong seasonal fluctuations. For a discussion on contract features and pricing of ILWs please refer to Gatzert and Schmeiser (2011); for a discussion on the pricing of weather derivatives please refer to Alaton et al. (2002) and Campbell et al. (2005). It appears plausible that ILWs and weather derivatives exposed to seasonal perils fluctuate in similar fashion as cat bonds. Hence, it may be worthwhile to apply the proposed methodology to model seasonality in ILWs, too.

[9]Seasonal fluctuations on cat bonds can occur in the absence of new information in the market. In other words, such seasonality is already contained in the current information set. On the contrary, announcements of loss events and hurricane forecasts bring new information to the cat bond market, which is conceptually related to ad-hoc profit warnings in the context of other financial securities.

TABLE 1 Modeled distribution of arrival frequencies

	U.S. hurricanes (%)	EU winter storms (%)
January	0.0	26.0
February	0.0	16.5
March	0.0	11.5
April	0.0	0.0
May	0.2	0.0
June	3.6	0.0
July	12.5	0.0
August	28.7	0.0
September	34.6	0.0
October	18.3	11.0
November	2.0	14.0
December	0.1	21.0
Total	100.0	100.0

Note: Distributions of arrival frequencies for U.S. hurricanes and EU winter storms as modeled by AIR. These numbers describe the relative share of arrival frequencies throughout a calendar year.

that spreads react to the seasonality of the underlying risk. Thus, fluctuations of cat bond spreads do not automatically allow for the creation of alpha, and they are not necessarily a violation of the efficient market hypothesis (see Fama, 1970).

Hainaut (2012) models seasonality in tornados through a double stochastic Poisson process whose arrival frequency fluctuates across a year following an Ornstein-Uhlenbeck process fitted to empirical data. We implement a variable that instead relies on the modelled arrival frequencies of AIR, which is one of the leading risk modeling firms. They use weather models and simulation methods to derive arrival frequencies for U.S. hurricanes and European winter storms from empirical data. We have data on the relative distribution of arrival frequencies of European winter storms and U.S. hurricanes on a monthly basis. Since we lack data on the severity, we assume the severity of a peril event (hurricane or European winter storm) to be i.i.d. for each time period within a year.

Table 1 illustrates the distributions of arrival frequencies for North American hurricanes and European winter storms as provided by AIR. Using these data, we assume that the distribution of arrival frequencies is constant between years.[10] The American hurricane season begins in June and ends in November, and most hurricanes occur in August and September. The European winter storm season begins in October and ends in March, and most winter storms occur in December and January. In months where the arrival frequency is zero, it is virtually impossible that a respective event can occur.

[10]If the seasonal pattern changed over time, time-varying arrival frequencies would be required to capture these changes. However, we have no information regarding such time-varying arrival frequencies. Moreover, it should be noticed that neither a general trend toward an increasing likelihood of natural catastrophes, nor cyclical event probabilities between years (e.g., due to El Niño), as discussed in Goldenberg et al. (2001), imply that the within-year distribution is time-varying, too.

2.3 | Other drivers of cat bond spreads

The cat bond literature considers additional factors that influence cat bond spreads.[11] We separate these factors into two groups: (a) time-invariant factors and (b) time-variant factors. While the factors of group (a) are very important for explaining cat bond spreads, we do not include them in our empirical analysis due to their time-invariant nature. Instead, we explain within-bond secondary market fluctuations through within transformations, thereby controlling for any observable and unobservable constant variables on bond level (see Section 5). On the contrary, we include variables of group (b) as control variables in our following empirical analyses. Nevertheless, we briefly describe the influencing factors of both groups (a) and (b) to provide a more complete picture of factors that influence cat bond spreads.

Group (a) includes bond specific properties like trigger type, peril types and locations, peril numbers, issue volume, rating, and sponsor. Bonds with indemnity trigger could exhibit higher spreads due to possible moral hazard (Cummins & Weiss, 2009), but there is no clear empirical evidence (Braun, 2016). Peril types and locations have been investigated thoroughly in the literature (e.g., Braun, 2016, Lane & Mahul, 2008, Papachristou, 2011). Cat bonds with more than one peril type[12] or peril location exhibit a spread premium due to increased complexity (Gürtler et al., 2016). Additionally, spreads are higher for peak peril types (hurricane) and locations (U.S.). For cat bonds with a large issue volume, spreads could be lower due to higher liquidity (Dieckmann, 2010), but empirical results are inconclusive (Braun, 2016, Gürtler et al., 2016). Cat bonds with better ratings have lower spreads (Braun, 2016, Gürtler et al., 2016). Spreads are lower for bonds sponsored by Swiss RE, which can be attributed to high sponsor reputation (Braun, 2016).

The time-variant variables of group (b) include variables that are bond specific like time to maturity, but mostly refer to conditions on the financial market, like corporate bond spreads, equity returns, and reinsurance prices. Concerning time to maturity, there is no empirical evidence that declining time to maturity leads to declining spreads due to increasing liquidity (Braun, 2016, Dieckmann, 2010, Gürtler et al., 2016). Cat bond spreads are positively related to corporate bond spreads and equity returns (Braun, 2016, Gürtler et al., 2016). Furthermore, while cat bonds are often considered zero-beta bonds, they have exposure to general financial market conditions through possible flight-to-quality effects in downturn scenarios (Gürtler et al., 2016). As a substitute for reinsurance, cat bond spreads increase during a hard reinsurance market (Braun, 2016, Gürtler et al., 2016, Lane & Mahul, 2008).

3 | CONCEPTUAL FRAMEWORK, MODELED SEASONALITY, AND HYPOTHESES

3.1 | Conceptual framework

We develop a conceptual framework to model the seasonality in the probability of catastrophe bonds being triggered based on a hazard rate model, and we suggest one comprehensible

[11]Please refer to Braun (2016) and Gürtler et al. (2016) for a thorough empirical investigation of many of these factors. While most of the empirical cat bond literature employs ordinary least square (OLS) or panel data regression models, Beer and Braun (2020) use Poisson intensities from a reduced form model to explain spreads.

[12]For a discussion on the pricing of multi-peril bonds relative to single-peril bonds please refer to Lane (2004).

measure of a seasonality-adjusted EL to model the seasonal fluctuations in secondary market cat bond spreads.

To keep it simple, we value a cat bond that does not pay any coupons, that is, a zero-coupon bond, which is repaid at time of maturity T. In case of a default, there is no repayment, that is, its CEL is 100%. Investors are risk neutral, the riskless interest rate is 0%, and there are no transaction costs. Under these assumptions the valuation of *nonseasonal* cat bond at time t is simple. The value equals the probability to survive until maturity multiplied by its face value. We model the survival probability through a hazard rate model that follows a Poisson process. The bond survives if the number of defaults until maturity $N(T)$ is zero:

$$P[N(T) = 0] = \exp(-\lambda_h(T - t)), \tag{2}$$

where λ_h denotes the (homogeneous) hazard rate. The value of the nondefaulted zero-coupon bond at time t equals:

$$V_t = \text{FV}\cdot P[N(T) = 0] = \text{FV}\cdot\exp(-\lambda_h(T - t)), \tag{3}$$

where FV denotes the face value of the bond. The economic intuition behind this valuation formula is the following: To a risk-neutral investor with a riskless interest rate of zero, a cat bond is worth its face value that is, paid out in case of survival multiplied by the probability the bond survives until maturity. The longer the maturity of the bond $(T-t)$ and the higher the hazard rate, the lower is the probability of its survival. For a nonseasonal peril, the hazard rate λ_h is constant throughout its maturity. The relation between the value and the spread s_t of a zero-coupon bond with continuous discounting is[13]

$$V_t = \frac{\text{FV}}{e^{s_t(T-t)}} \Leftrightarrow s_t = \frac{\ln\left(\frac{\text{FV}}{V_t}\right)}{T - t}. \tag{4}$$

We can insert Equation (3) into Equation (4) to obtain a formula for the spread of a nonseasonal cat bond based on the hazard rate model:

$$s_t = \lambda_h. \tag{5}$$

Now, we assume the zero-coupon cat bond is not exposed to a nonseasonal peril such as earthquakes but is exposed to a *seasonal* peril, that is, the hazard rate λ_h fluctuates seasonally. More precisely, we use an inhomogeneous Poisson process where the intensity function $\lambda_h(t)$ fluctuates within the year. We can determine the value of such a seasonal cat bond at time t, which is nondefaulted at that time, based on the survival probability from t to T of an inhomogeneous Poisson process as follows:

$$V_t = \text{FV}\cdot\exp\left(-\int_t^T \lambda_h(\tau)d\tau\right). \tag{6}$$

We can now insert Equation (6) into Equation (4) and solve for s_t to acquire the corresponding fluctuating spread:

$$s_t = \frac{\int_t^T \lambda_h(\tau)d\tau}{T - t}. \tag{7}$$

[13]More generally, the spread can be defined as the difference between the yield to maturity and the risk-free rate, but in our model we assumed a risk-free rate of zero.

With λ_0 as the total hazard rate for one calendar year $\int_0^1 \lambda_h(\tau)d\tau =: \lambda_0$, we can define the ratio $\lambda_h(t)/\lambda_0 =: \lambda(t)$as the density function of arrival frequencies. This results in:

$$s_t = \frac{\int_t^T \lambda_h(\tau)d\tau}{T-t} = \frac{\lambda_0 \cdot \int_t^T \lambda(\tau)d\tau}{T-t}. \tag{8}$$

For illustration purposes, we provide an example for pricing a nonseasonal cat bond and a seasonal cat bond whose hazard rate $\lambda_h(t)$ follows a cosine function in Appendix A. Based on this example, we illustrate the spreads of a seasonal cat bond in Figure 1.

First, we can observe that the spread of a seasonal cat bond fluctuates strongly. In Figure 1 the spread follows a clear pattern: Spreads peak a couple of months before the season reaches its peak. The spread reaches its bottom when the season fades out at the end of the year.

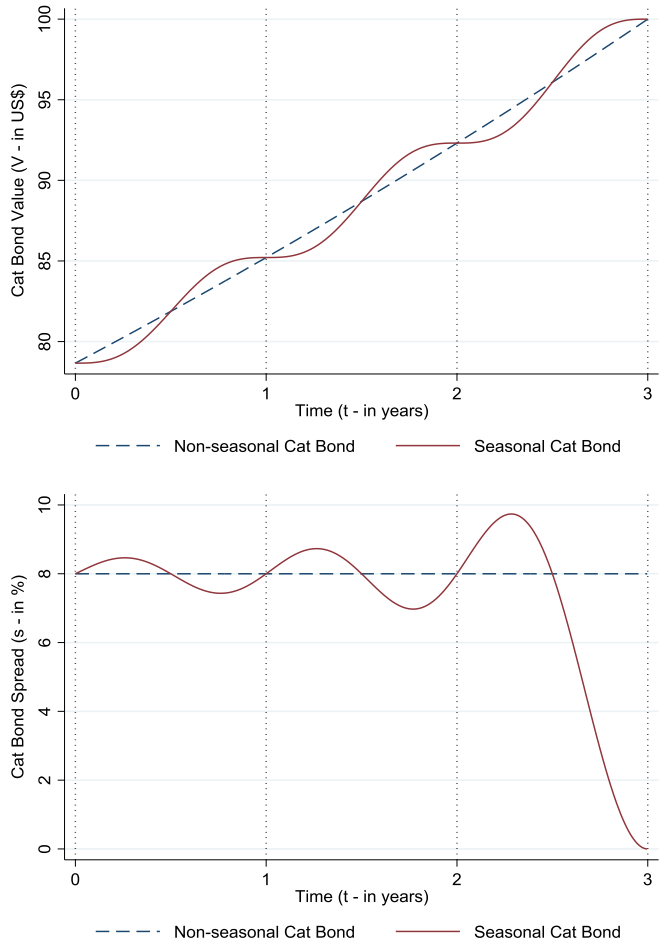

FIGURE 1 Value and spread of hypothetical cat bonds. Value and spread of hypothetical zero-coupon cat bonds with a hazard rate $\lambda_h = 8\%$ and a conditional expected loss CEL = 100% in the case without default. The cat bonds have a maturity of 3 years. The investors are risk-neutral and riskless interest rates are 0%. The nonseasonal cat bond has a hazard rate λ that is evenly distributed across a calendar year. The seasonal hazard rate $\lambda_h(t)$ for the seasonal cat bond follows a cosine function. This hazard rate $\lambda_h(t)$ for a seasonal bond is highest in the middle of the year and lowest at the end of the year. [Color figure can be viewed at wileyonlinelibrary.com]

3.2 | Modeled seasonality measure

We now establish a new seasonality measure utilizing the EL as the most important variable to explain cat bond spreads. In the hazard rate model, Equation (8) illustrates the seasonal fluctuation in spreads. We translate this formula into a new seasonality measure EL_t. The intuition behind this translation is as follows: The EL is related to the hazard rate λ_h and should fluctuate seasonally, accordingly. This "true" EL—the loss that investors expect at time t—thus, fluctuates with seasonal arrival frequencies. On the secondary market, investors do not price a cat bond according to the constant, yearly $EL_{initial}$ provided by the risk modeler, which can be taken from the offering circular, but rather evaluate the amount of remaining risk against the background of its remaining time to maturity. Considering that the EL is effectively the absolute amount of expected losses divided by the remaining time to maturity and the face value, we define EL_t as the relative expected loss on an annual basis which fluctuates depending on changes in the absolute amount of expected losses remaining and the decreasing time to maturity. Thus, we propose the following formula to create a seasonality-adjusted expected loss measure EL_t:

$$EL_t = \frac{\text{Remaining risk}_t}{\text{Remaining time}_t} = \frac{EL_{initial} \cdot \int_t^T \lambda(\tau) d\tau}{T - t}, \tag{9}$$

where t stands for the time of risk evaluation, T is the time of maturity and $\lambda(\tau)$ is the density function of arrival frequencies, which varies depending on the point in time τ. This seasonality-adjusted EL measure incorporates actual arrival frequencies. Since our data on spread is on quarterly basis, we generally aggregate monthly arrival frequencies to quarterly arrival frequencies, but the proposed formula can be used for arbitrary frequencies.

We obtain the modeled distributions of arrival frequencies λ for hurricanes and European winter storms from AIR. These distributions are on a monthly basis and exogenous to our model. As previously discussed, we do not have an exogenous distribution on the severity of peril events but instead we assume the severity of a peril event to be i.i.d. for each time period within a year. This could limit the accuracy of the proposed seasonality measure if the severity of peril events varies during different parts of the season. However, the proposed methodology could also be applied to severity if an exogenous distribution of severity is available. Such a model could either have two separate or a single seasonality measure for a combined distribution of arrival frequency and severity.

3.3 | Hypotheses

The theoretical model in Sections 3.1 and 3.2 implies that the spread of a seasonal cat bond fluctuates strongly. As illustrated in Figure 1, spreads peak a couple of months before the season reaches its peak, and reach their bottom when the season fades out at the end of the year. We expect a similar pattern for the real-world distribution of hurricanes and European winter storms and hypothesize the following:

H1: *Seasonality pattern: Cat bonds follow a seasonal pattern that expresses its highest spreads before risk season begins and its lowest spreads after risk season ends.*

Second, we can observe that the amplitude between seasonal peaks and bottoms increases as a bond approaches its maturity. The reason for the increasing seasonal amplitude of spreads when approaching maturity is as follows: While the amplitude of the seasonal value fluctuation remains almost constant throughout the cat bond's maturity (see e.g., Figure 1), its remaining time to maturity decreases. However, the bond's spread is more sensitive to changes in prices the closer it is to maturity. Hence, we hypothesize the following:

H2a: *Seasonality amplitude—maturity: The seasonal fluctuation of cat bonds increases with decreasing time to maturity.*

The theoretical model indicates that the amplitude of the seasonal fluctuation scales with the total hazard rate of one calendar year (see Equations 8 and 9). This total hazard rate λ_0 translates into the yearly EL of a Cat Bond. This EL is typically reported in the offering circular. Cat bonds have different yearly ELs. Some are very risky and have a high yearly EL of 15% while others have an EL below 1%. This could have an impact on the amplitude of seasonal fluctuation: Although the *relative* fluctuation of EL in seasonal cat bonds might be the same, the *absolute* fluctuation of EL might be larger for cat bonds that have a high yearly EL. Hence, in *absolute* terms, the spreads of cat bonds with a high yearly EL should fluctuate more strongly than the spreads of cat bonds with a low EL.

H2b: *Seasonality amplitude—EL: The absolute seasonal fluctuation of cat bonds with a high EL is larger than the seasonal fluctuation of cat bonds with a low EL.*

While our modeled seasonality measure from Section 3.2 simultaneously captures the three effects expressed in hypotheses H1, H2a, and H2b, we hypothesize two additional effects that influence the amplitude of seasonal fluctuations in cat bond spreads: Many cat bonds protect against more than one peril. Typically, these multi-peril bonds also protect against earthquakes that are not affected by seasonality; the arrival frequency of an earthquake is evenly distributed across a calendar year. These multi-peril bonds should express less pronounced seasonal fluctuation. The remaining fluctuation should be proportional to the distribution of its risk exposure between seasonal (e.g., wind or hurricane) and unseasonal perils (e.g., earthquake).

Consider a simple cat bond pricing model, where the spread (s_t) is the sum of some function $h(\cdot)$ of a bond's exposure to hurricane risk and some function $q(\cdot)$ of the same bond's exposure to earthquake risk. The weight ($w \in [0,1]$) determines how the bond's overall risk exposure is divided between hurricane and earthquake.

$$s_t = w \cdot h(\mathrm{EL}_t) + (1 - w) \cdot q(\mathrm{EL}_{\mathrm{initial}}), \tag{10}$$

with $\mathrm{EL}_t = \mathrm{EL}_{\mathrm{initial}} \cdot a_t$. $\mathrm{EL}_{\mathrm{inital}}$ is the constant yearly EL modelled by a risk modelling firm, which can be taken from the offering circular. The parameter a_t is a random variable that fluctuates with the U.S. hurricane season and is defined in such a way that $E(a_t) = 1$. As the model suggests, earthquakes are not exposed to seasonality. Therefore, the second summand does not contain a seasonally fluctuating EL. In this model, the bond's seasonal change in spread is proportional to the bonds weight w in hurricane exposure. If w is close to one, the spread fluctuates strongly with the U.S. hurricane season. When w is close to zero, the spread fluctuates only weakly with the U.S. hurricane season. From our data, we do not know a bond's specific weight w. However, we know that $0 < w < 1$ for any multi-peril bond. Therefore, a multi-peril bond exposed to some form of seasonality should fluctuate less than a single-peril bond that is affected by the same peril (in this case, U.S. hurricanes).

H2c: *Seasonality amplitude—multi-peril bonds: The seasonal fluctuation of multi-peril bonds is lower than the seasonal fluctuation of single-peril bonds.*

Similar to multi-peril and single-peril bonds, there may also be differences between peril types. U.S. hurricane bonds have very clear seasons: The arrival frequency of a hurricane is zero through the first half of a calendar year and varies throughout the second half. European wind bonds do not have a clear aggregate season: While the arrival frequencies for European winter storms fluctuates with a similar magnitude as hurricanes, European wind bonds often also protect against hail and thunderstorms that also occur outside of the winter storm season. In consequence, the arrival frequency of European wind perils is more evenly distributed across a calendar year. Hence, seasonality effects for European wind bonds should be less pronounced than seasonality effects for U.S. hurricane bonds.

H2d: *Seasonality amplitude—peril type: The seasonal fluctuation of North American hurricane bonds is higher than the seasonal fluctuation of European wind bonds.*

4 | DATA

Our initial data set consists of 587 cat bonds from 1996 to 2017. These bonds represent nearly the whole cat bond universe. We collected data from Artemis (hand-collected information on location and type) and Lane Financial LLC (EL, coupon, volume, maturity, and spread).[14] In additional robustness checks, we consider cat bonds pricing information from actual cat bond trades reported in TRACE. Our theoretical considerations and empirical analysis are based on (currently) non-defaulted bonds, which could default at any time in the future. Accordingly, we drop all bonds that were "distressed," which can mean a cat bond incurred a permanent loss after a trigger event or experienced a substantial temporary markdown.[15] We only mark a bond as "distressed" if it is reported as distressed in the Trade Notes of Lane Financials or part of the "Cat Bond Losses & Bonds At Risk" list on artemis.bm. Additionally, we dropped the following bonds: mortality risk bonds, bonds lacking crucial information such as EL, coupon, type or location and bonds lacking spreads—Lane Financial provides quarterly spreads from 2002 onwards. Furthermore, we drop all bonds whose perils are not affected by U.S. hurricanes or European winter storms. Ultimately, our final sample includes 386 bonds and 3947 quarterly observations from 2002 to 2017. The specific dates of the quarterly observations refer to 31st March for Q1, 30th June for Q2, 30th September for Q3, and 31st December for Q4.

Table 2 provides summary statistics on important variables. Eighty-nine percent of these cat bonds have exposure in North America, with Europe and Japan following at 31% and 11%, respectively. Concerning perils, hurricane is the most prominent whereas wind and earthquake have similar shares.[16] This means that the U.S. hurricane season is the most important season.

[14]Used information from Artemis can be acquired through the deal directory on www.artemis.bm. Used information from Lane Financial LLC can be acquired from annual reviews of the ILS markets, authored by Morton Lane and Roger Beckwith, provided on www.Lanefinancialllc.com.

[15]A temporary markdown occurs when a historic natural disaster threatens to trigger a bond, but the affected bond is ultimately cleared from a loss.

[16]Earthquake bonds in our sample stem from seasonality-affected multi-peril bonds that have some exposure to earthquakes.

TABLE 2 Cat bond specific information on 386 cat bonds

Variable	No. of bonds	Percentage (%)
Region		
North America	342	88.60
Europe	121	31.35
Japan	41	10.62
Other	3	0.78
Peril		
Hurricane	238	61.66
Wind	230	59.59
Earthquake	210	54.40
Peril number		
Single-peril	161	41.71
Multi-peril	225	58.29
Peril location		
Single-location	302	78.24
Multilocation	84	21.76
Peril number and peril location		
Single-peril and single-location	158	40.93
Multi-peril and/or multilocation	228	59.07
Rating		
AA	4	1.04
A	4	1.04
BBB	8	2.07
BB	162	41.97
B	101	26.17
NR	107	27.72

Note: For region and peril, the percentages of the categories exceed 100% because multi-peril and multilocation bonds have multiple peril types and locations, respectively. All other categories add up to 100%.

Region and Peril add up to more than 100% because multi-peril and multilocation bonds are included. Overall, more than half of the bonds are multi-peril bonds while less than a quarter are multilocation bonds. Table 3 provides summary statistics on the continuous variables $EL_{initial}$, the proposed seasonality measure EL_t for the hurricane season and the European winter storm season, spread as well as control variables. For each cat bond deal, an external risk modeling company provides a report of the underlying risk. It contains a distribution of modeled losses on a yearly basis. Hence, the mean of the loss distribution is the EL over 1 calendar year. During a cat bond's maturity, this $EL_{initial}$ stays constant over time.[17] While the $EL_{initial}$ can reach almost 15%, the median and the mean of $EL_{initial}$ are 1.67% and 2.63%,

[17]For an example on risk modeling and the resulting loss distribution please refer to Lane (2012). For bonds, which employ an indemnity trigger, the EL could change if the ceding insurance company, for example, underwrites more business. However, such cat bonds usually contain reset clauses that reset attachment and exhaustion points at regular time intervals to keep the EL constant in case the business of the insurance company has changed.

TABLE 3 Summary statistics for $EL_{initial}$, spread, and control variables

	Obs.	Mean	*SD*	Min.	q25	q50	q75	Max.
$EL_{initial}$ (%)	386	2.63	2.45	0.00	1.12	1.67	3.41	14.75
EL_t-U.S. modeled (%)	3431	2.25	2.71	0.00	0.71	1.38	2.87	28.58
EL_t-EU modeled (%)	1248	2.53	3.03	0.00	0.74	1.45	3.62	28.73
Spread (%)	3947	7.38	4.95	0.64	4.13	5.99	9.23	43.69
Reins. index (points)	16	233	32	170	215	241	251	293
Corp. bond spreads (%)	60	4.53	2.59	1.22	1.77	4.00	5.48	14.79
Equity return 90 days (%)	60	1.9	7.7	−18.3	−1.1	2.1	6.3	18.5
Remaining maturity (months)	3947	20.4	13.24	0	9	20	30	98

Note: Summary statistics for the continuous variables expected loss at issue (EL_{inital}) on bond level and spread on observation level. $EL_{initial}$, as provided by risk modelers, is constant over time. Control variables are on a yearly basis (*Reinsurance Index*), quarterly basis (*Corporate bond spreads* and *Equity return 90 days*) and observation level (*Remaining maturity*).

respectively EL_t fluctuates between 0% and 29%. It is 0% when the bond has gone through all of its risk seasons but still has some time remaining until maturity. The maximum of 29% is roughly twice as large as the maximum for $EL_{initial}$. Quarterly spreads are taken from yearly market reports provided by Lane Financial LLC. For individual cat bonds, spreads can reach almost 44%. However, the median spread is 5.99% and the average spread is 7.38%.

In the empirical analyses, we also include time-variant control variables, namely reinsurance prices, corporate bond spreads, equity returns, and remaining time to maturity.[18] As a measure of reinsurance prices, we use the Guy Carpenter Global Property Rate-on-Line Index, which is on a yearly basis.[19] As corporate bond spreads, we use the Bank of America Merrill Lynch Option-Adjusted Spread indices of various rating classes, which are on a daily basis; concretely, we assign the corporate bond spread index with the same rating to the corresponding cat bonds. If a cat bond is not rated, we assign the BB corporate bond spread index because BB is the most common cat bond rating. For equity returns, we use the S&P500 performance index, which is on a daily basis.

5 | ECONOMETRIC MODEL

We are interested in explaining how secondary market spreads of each individual bond change due to seasonality after they were issued on primary markets. Therefore, we explain the variance of spreads within a group of observations on bond level. To do so, we use fixed effects regressions. A side effect is that we do not need any control variables that stay constant over time.[20] We use the following model for the spread s_{it}:

[18]For a detailed discussion of the underlying effects of these time-variant controls please refer to Section 2.3.

[19]Gürtler et al. (2016) use the Guy Carpenter Global Property Rate-on-Line Index. Braun (2016) uses the Lane Financial LLC Synthetic Rate-on-Line Index.

[20]Examples for such controls include the number of perils, the number of locations, peril type, peril location, trigger type, rating, volume or the constant yearly $EL_{initial}$.

$$s_{it} = \beta' X_{it} + \eta' C_{it} + \alpha_i + \varepsilon_{it}, \tag{11}$$

where i stands for the individual cat bond at time t; in our case, these are separate quarters. The vector X_{it} includes variables that fluctuate over time and are different on bond level. These variables are the seasonality measure from Section 3.2 or seasonal dummy variables defined in the subsequent section. The vector C_{it} contains all control variables that change over time. These time-variant variables are the remaining maturity, corporate bond spreads, equity returns and reinsurance prices. The error term is denoted by ε_{it}. The variable α_i is a bond-specific intercept that contains all variables of bond i that are constant over time. This variable disappears when within transformation is applied, that is, subtracting the mean of each variable from the respective variable in the model (e.g., $\ddot{s}_{it} = s_{it} - \bar{s}_i$):

$$\ddot{s}_{it} = \beta' \ddot{X}_{it} + \eta' \ddot{C}_{it} + \ddot{\varepsilon}_{it}. \tag{12}$$

Through the resulting fixed effect model, we estimate the coefficients in such a way that they capture differences to their bond-specific means. This way the model estimates the change of spreads within bonds across time while abstracting from differences between bonds. We measure the explanatory power based on the adjusted within R^2, which measures how much of the fluctuation of secondary market spreads around the individual mean can be explained by the seasonality variables.

6 | EMPIRICAL RESULTS

6.1 | Results for the hypotheses

We illustrate the seasonal fluctuation in spreads in two steps: In the first step, we create three different sets of seasonal dummy variables to test our hypotheses from Section 3.3. We use dummy variables because these effects would otherwise be hidden in the new seasonality measure. We employ: (a) interaction terms between the seasonal dummy variables with a cat bond's years to maturity, (b) interaction terms between the seasonal dummy variables with EL, and (c) separate dummy variables for European and North American seasons. In the second step, we use the new seasonality measure to capture these effects simultaneously and explain a large proportion of the secondary market fluctuation in cat bond spreads.

Data on spreads are available on a quarterly basis. The specific dates of our quarterly observations refer to 31st March for Q1, 30th June for Q2, 30th September for Q3 and 31st

TABLE 4 Definition of seasonal dummy variables

Quarter	North American hurricanes	European winter storms
Q1	No season U.S.	After season EU
Q2	Pre season U.S.	No season EU
Q3	High season U.S.	Pre season EU
Q4	After season U.S.	High season EU

Note: Seasonal dummy variables are on continental level. The seasonal dummy variables equal one in the corresponding quarter and zero otherwise. If a cat bond is unaffected by the U.S. or EU season, the respective dummy variables are zero.

December for Q4. Consequently, we define seasonal variables on this quarterly basis reflecting the respective seasonal states of these quarters. Our four seasonal variables are: *Pre season*, *High season*, *After season*, and *No season*.

Table 4 defines the set of seasonal dummy variables. U.S. seasonal dummies take the value "1" if they have some exposure to US hurricane or U.S. wind perils; EU seasonal dummies take the value "1" if they have some EU wind exposure. For both, U.S. and EU seasonal dummies, this includes multi-peril and multilocation bonds.

Table 5 shows a set of models with fixed effects estimation, which employ the seasonal dummy variables. The sample is limited to bonds that are affected by seasonality. Standard errors are clustered on bond level and robust to heteroscedasticity. *Pre season* is always the base category where spreads are expected to be highest. We test the seasonality pattern (H1) with model (1). We test for different seasonality amplitudes w.r.t. remaining maturity (H2a) and EL (H2b) based on models (2) and (3), respectively. With models (4), (5), and (6), we test the differences between multi-peril and single-peril bonds (H2c) as well as differences between U.S. and EU bonds (H2d).

To investigate the general pattern of seasonality (H1), we look at the seasonal dummies in model (1). Spreads are, on average, 1.85% points lower during *High season* than during *Pre season*. *After season*, spreads drop further to 2.31% points below *Pre season*. From *After season* to *No season*, spreads increase by 0.85% points, which is 1.46% points below *Pre season*. All differences from *Pre season* are statistically significant at the 0.1% level. The order of the seasons is in line with indicated theoretical arguments, lending support to H1.[21] Lane and Beckwith (2017) indicate the seasonal pattern can be reversed in large loss years to some extent. Normally, spreads decline during the season when no losses or only very few losses materialize. On the contrary, in large loss years with multiple distressed bonds, realized losses can cause drops in prices leading to jumps in spreads. However, this effect should not be pronounced in our analysis because we drop all distressed bonds.

To investigate the influence of a bond's approaching maturity (H2a), we use model (2). Again, *Pre season* serves as the base category. The interaction terms between *Ultimate year* and the seasonal dummies as well as *Penultimate year* and the seasonal dummies indicate whether the amplitude of seasonal fluctuation increases in the last 2 years of maturity. For these time variables, the time before the last 2 years of maturity serves as the base category.[22] All six interaction terms indicate increasing seasonal fluctuation as cat bonds near their maturity. Five of these interaction terms are significant at the 0.1% level, while the interaction term between *No season* and *Penultimate year* is significant at the 5% level. We exemplify the interpretation of the corresponding coefficients for *After season*: The coefficient of the interaction term between the *Penultimate year* and *After season* indicates that the amplitude of seasonal spread fluctuation increases by 0.45% points when a bond enters its penultimate year of maturity. In total, the amplitude between *Pre season* and *After season* is 1.62% points during this time. This effect is further amplified in the ultimate year of a bond's maturity: The coefficient of the interaction

[21]We have also applied the seasonality dummies for the U.S. and European seasons to a different sample of single-peril earthquake bonds. These bonds should not exhibit a seasonal fluctuation. We find small quarterly fluctuations, but this fluctuation appears negligible in size and of little explanatory power ($R^2 = 2\%$). This is substantially different, for example, from single-peril hurricane bonds, where seasonality can explain up to 47% of the spread variation (without considering additional control variables). Detailed results are available upon request.

[22]Most cat bonds have a maturity of 3 years. For these bonds, the base category is the first year. For all other bonds with a maturity of more than 3 years, the base category is a combination of all years before the ultimate and penultimate years.

TABLE 5 Impact of seasonality on spreads—test of hypotheses

	Spread					
	H1	**H2a**	**H2b**	**H2c**	**H2c**	**H2d**
Dependent variable	**US HU/wind**	**US HU/wind**	**US HU/wind**	**US HU/wind**	**US HU/wind**	**EU wind**
Test of hypothesis	**Full sample**	**Full sample**	**Full sample**	**Single-peril**	**Multi-peril**	**Single-peril**
Sample	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
High season U.S.	−1.846***	−0.979***	−1.291***	−2.665***	−1.835***	
	(−20.01)	(−14.28)	(−9.15)	(−10.96)	(−17.12)	
After season U.S.	−2.311***	−1.170***	−1.200***	−3.648***	−1.980***	
	(−17.92)	(−12.31)	(−6.83)	(−11.09)	(−12.65)	
No season U.S.	−1.457***	−0.449***	−0.477**	−2.293***	−1.063***	
	(−14.51)	(−5.64)	(−2.67)	(−9.65)	(−9.68)	
Penultimate year		0.448**				
		(2.72)				
Ultimate year		1.976***				
		(6.49)				
High season U.S. # Penultimate year		−0.394***				
		(−4.57)				
High season U.S. # Ultimate year		−2.356***				
		(−11.67)				
After season U.S. # Penultimate year		−0.450***				
		(−3.44)				
After season U.S. # Ultimate year		−3.035***				
		(−11.52)				
No season U.S. # Penultimate year		−0.208*				
		(−2.00)				
No season U.S. # Ultimate year		−2.899***				
		(−12.11)				
High season # EL			−0.230***			
			(−3.55)			
After season # EL			−0.457***			
			(−5.57)			
No season # EL			−0.404***			
			(−4.59)			
High season EU						0.014
						(0.14)
After season EU						−0.851***
						(−5.00)

(Continues)

TABLE 5 (Continued)

	Spread					
	H1	H2a	H2b	H2c	H2c	H2d
Dependent variable	US HU/wind	US HU/wind	US HU/wind	US HU/wind	US HU/wind	EU wind
Test of hypothesis	Full sample	Full sample	Full sample	Single-peril	Multi-peril	Single-peril
Sample	(1)	(2)	(3)	(4)	(5)	(6)
No season EU						−0.085
						(−0.45)
Reins. index	0.018***	0.019***	0.018***	0.013 + (1.95)	0.015**	0.013*
	(5.70)	(6.18)	(5.81)		(2.63)	(2.27)
Corp. bond spreads	0.486***	0.476***	0.482***	0.441***	0.541***	0.385***
	(17.31)	(17.91)	(17.47)	(10.81)	(13.42)	(5.28)
Equity price index	0.033***	0.031***	0.033***	0.038***	0.022**	0.041***
	(6.70)	(6.47)	(7.12)	(4.20)	(3.10)	(3.98)
Rem. maturity	0.054***	0.049***	0.055***	0.040**	0.072***	0.002
	(8.13)	(4.21)	(8.23)	(3.12)	(6.72)	(0.32)
Constant	1.613*	0.791	1.614*	3.736*	1.042	0.335
	(2.32)	(0.99)	(2.33)	(2.55)	(0.82)	(0.25)
Observations	3411	3411	3411	871	1421	483
Number of bonds	342	342	342	84	128	40
Within R^2	0.355	0.415	0.388	0.376	0.457	0.237
Adj. within R^2	0.354	0.413	0.386	0.370	0.455	0.225

Note: This table uses fixed effects regression models to determine the effect of seasonal factors on spreads (in %). All models, except for model (6), are limited to bonds that have exposure to U.S. storms (i.e., U.S. hurricane or U.S. wind bonds). *Pre season* is the base category for seasonal dummy variables. Models (1), (2), and (3) use all bonds with exposure to U.S. hurricanes. Model (4) uses single-peril/single-location U.S. hurricane bonds. Model (5) uses multi-peril bonds with some exposure to U.S. hurricanes; these bonds have exposure to U.S. hurricanes and other perils but do not cover other regions except North America. Model (6) uses single-peril/single-location EU wind bonds. *t*-values are shown in parentheses and heteroscedasticity robust standard errors are clustered at bond level. The symbols *, **, and *** indicate statistical significance at the 5%, 1%, and 0.1% levels, respectively.

term between *Ultimate year* and *After season* indicates that the amplitude increases by 3.04% points as compared to the years before the penultimate year. In total, the amplitude reaches 4.21% points.[23] Overall, this indicates the seasonal amplitude has increased in the penultimate year but even more strongly in the ultimate year of maturity. The coefficients of the remaining

[23]Modell (2) also includes dummy variables for the Penultimate year and Ultimate year. These coefficients are large and statistically significant. However, these coefficients must not be interpreted in such a way that the mean spread of a cat bond changes as it approaches its maturity. If one is to determine the mean change in spreads for a cat bond as it moves to its Penultimate year or Ultimate year implied by the dummy variables, he needs to include the mean coefficient of the respective interaction terms between the year dummies and seasonal dummies including a hypothetical coefficient of zero for the base category and add the year dummy. For the Penultimate year, this indicates an average shift in spreads of $(-0.394 - 0.450 - 0.208 - 0)/4 + 0.448 = 0.185\%$ points change in mean spreads. For the Ultimate year, this indicates an average shift in spreads of $(-2.356 - 3.035 - 2.899 - 0)/4 + 1.976 = -0.097\%$ points. Both of these shifts are close to zero.

interaction terms between *High season* and *No season* show the same effect with a similar magnitude. The results from model (2) strongly support hypothesis H2a that the seasonal fluctuation of cat bonds increases as their time to maturity decreases.

To investigate the influence of a bond's EL on amplitude (H2b), in model (3) we interact the seasonal dummy variables with the individual yearly EL of each bond. The coefficient of the interaction term between *After season* and EL of −0.46 indicates that the absolute difference in spreads between *Pre season* and *After season* increases by 0.46% points for each 1% point increase in EL. The coefficient is statistically significant at the 0.1% level. These results support hypothesis H2b that the absolute amplitude increases with the EL of a cat bond.[24]

We use models (4) and (5) to compare single-peril and multi-peril bonds (H2c). Model (4) is limited to single-peril U.S. hurricane bonds whereas model (5) is limited to multi-peril U.S. hurricane bonds whose other perils are exclusively located in North America. For single-peril bonds, the *After season* coefficient is much higher than for multi-peril bonds. In model (4), a single-peril bond has, on average, a 3.65% points higher spread right after risk season compared to right before risk season. In model (5) for a multi-peril bond, this difference is only 1.98% points. This means that single-peril bonds fluctuate more strongly with seasonality variables than multi-peril bonds, which supports hypothesis H2c.

To investigate differences between North American and European seasons (H2d), model (6) contains the seasonal dummies for the European season. Its sample contains single-peril wind bonds exclusively located in Europe. Generally, the coefficients for the North American season in model (4) are larger than the coefficients for the European season in model (6). Additionally, the European season is less clear: We see a difference between *Pre season* and *After season* as expected but spreads in *High season* and *No season* are almost on the same level as *Pre season*. The likely reason is that most North American bonds contribute capacity towards hurricanes while European wind bonds are not only triggered by European winter storms but also by other wind perils such as hail or thunderstorms.[25] In consequence, European wind bonds are less susceptible to the European winter storm season. Overall, the results indicate that the amplitude of U.S. hurricane bonds is larger than the amplitude of European wind bonds, which supports hypothesis H2d.

In summary, all hypotheses from Section 3.3 are supported by our results. A model that uses the complete set of dummy variables to combine the effects illustrated in Table 5 on a sample of all seasonality-affected cat bonds yields an adj. within R^2 of 0.301 without control variables and 0.447 when controls are included.[26] This means that 30% of all secondary market within fluctuation in spreads can be explained by seasonal dummy variables and the additional interaction terms between EL and respective year dummies until maturity.

[24]Please note that the seasonal dummy variables must not be interpreted individually in model (3): The coefficient for *After season* is now smaller (in absolute terms) than the coefficient for *High season*. However, this does not mean that spreads are smallest during *High season*. The average EL in the sample is 2.3%, which results in $-1.291\% - 0.230 * 2.3\% = -1.820\%$ for *High season* and $-1.200\% - 0.457 * 2.3\% = -2.251\%$ for *After season*.

[25]In Section 7 we determine the implied distribution for the observable seasonal fluctuation of European wind bonds. In this implied distribution, we observe substantial amounts of arrival frequencies in July, August, and September.

[26]Detailed results are available upon request.

6.2 | Results for the modeled seasonality measure

Table 6 illustrates the effect of the new seasonality measure, the seasonality-adjusted expected loss (EL_t), on secondary market spreads utilizing the exogenous arrival frequencies from AIR.[27] The spread compensates the investor for the EL (corresponding to the actuarially fair premium) and, additionally, for uncertainty in payoffs. Hence, the estimated EL coefficient, the EL multiple, is usually >1 in the empirical cat bond literature. If the coefficient is much larger than 1, investors demand a higher compensation for each unit of EL. In model (1), the coefficient of 1.135 for the U.S. hurricane EL_t indicates that a one-percentage point change in EL_t leads to more than a one-percentage point change in spreads of the same sign.[28] The coefficient is highly significant at the 0.1% level. Concerning differences between U.S. hurricane and EU wind bonds, the coefficient for the European winter storm season is less than half the size of the coefficient for the hurricane season. The smaller coefficient for the European winter storm season lends further support to hypothesis H2d that U.S. hurricane bonds fluctuate more strongly than European wind bonds. Overall, the seasonality measure EL_t explains a large part of the fluctuation on secondary markets: The adjusted within R^2 of 0.326 shows that almost a third of all secondary market fluctuation of seasonality-affected bonds can be explained by the proposed seasonality measure. For comparison, a model that combines seasonal dummy variables and interaction terms to reflect maturity effects and the EL (H2a and H2b) only yields an adjusted within R^2 of 0.301.[29] This means that the proposed seasonality measure captures the different seasonality effects mentioned above and leads to a better model fit.[30]

Models (2) and (3) split the sample from model (1) into two separate subsamples: Model (2) contains single-peril bonds, while model (3) contains multi-peril bonds. For single-peril bonds, the coefficients for both seasons are larger than the coefficients for multi-peril bonds, lending further support to hypothesis H2c, which suggests that single-peril cat bonds are more strongly

[27]Although data from Lane Financial LLC are quarterly, we also utilize the within-quarter variation of arrival frequencies from AIR through within quarter issue and maturity dates. However, the use of this within-quarter distribution remains limited. We fully exploit the monthly distribution of exogenous arrival frequencies from AIR in the robustness check based on TRACE data where we have the specific dates of real trades and interpolate between months to obtain a daily distribution of arrival frequencies.

[28]Galeotti et al. (2013) investigate different functional relationships between the spread and the EL and find a linear relationship to be most appropriate, which confirms that the risk premium can be described as a (constant) multiple of the EL. On primary markets, Braun (2016) reports a multiplier between the expected loss and spread of 2.210 (between estimation), which is double the amount we see on the secondary market (within estimation). The difference between the coefficient of the seasonality measure EL_t and the established coefficient in the literature is likely due to two effects: First, the multiple of spread and EL had a tendency to decrease since the inception of the cat bond market so that the multiplier could have declined over time; our sample ends 2017 while the sample of Braun (2016) ends 2009. A univariate OLS regression of spread on EL at issuance for our data set reveals a multiplier of 1.884 (between), confirming that the multiplier has indeed declined; results are available upon request. Second, the sample includes multi-peril bonds that contain risk, which are either not seasonality-affected (like earthquakes) or other wind perils whose seasonality we do not model (such as tornados or severe thunderstorms). For example, a cat bond that insures against earthquakes and hurricanes in equal shares fluctuates at the same pro-rata share with the hurricane season. To account for this effect, we repeat model (1) for the subsample of single-peril and single-location bonds in model (2). This analysis reveals a coefficient of 1.734 (within estimation). Ultimately, we acquire two coefficients that align quite well after we accounted for these two effects (1.884 between vs. 1.734 within).

[29]The model combines the interaction terms between EL and seasonal dummy variables, as well as the ultimate and penultimate years of maturity from models (2) and (3) in Table 5 using the full sample. Detailed results for this model are available upon request.

[30]We compare these two models using the Bayesian information criterion (BIC). For the model with the new seasonality measures, we acquire a BIC value of 15837 whereas the model with the set of dummy variables has a BIC value of 16182. The difference in BIC values clearly exceeds 10, which is strong evidence for a better model fit based on the new seasonality measure (see Raftery, 1995).

TABLE 6 Impact of seasonality on spreads—seasonality-adjusted EL using the modeled seasonality measure

	Spread					
Dependent variable	**Full sample**	**Single-peril**	**Multi-peril**	**Full sample**	**Single-peril**	**Multi-peril**
Sample	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
EL_t-U.S. modeled	1.135***	1.734***	0.931***	1.093***	1.731***	0.858***
	(15.25)	(13.36)	(12.75)	(15.30)	(13.91)	(13.27)
EL_t-EU modeled	0.461***	0.736**	0.419***	0.445***	0.777***	0.390***
	(5.79)	(2.77)	(5.32)	(5.83)	(3.42)	(5.12)
Reins. index				0.020***	0.017***	0.022***
				(7.47)	(4.78)	(6.06)
Corp. bond spreads				0.413***	0.342***	0.473***
				(20.59)	(12.08)	(18.02)
Equity returns				0.024***	0.022***	0.024***
				(5.63)	(3.54)	(4.38)
Rem. maturity				0.030***	0.014*	0.044***
				(5.66)	(2.24)	(6.12)
Constant	4.795***	4.795***	3.559***	5.463***	3.559***	5.463***
	(30.03)	(30.03)	(15.30)	(28.84)	(15.30)	(28.84)
Observations	3947	1573	2374	3947	1573	2374
Number of bonds	386	154	232	386	154	232
Within R^2	0.326	0.466	0.282	0.494	0.576	0.509
Adj. within R^2	0.326	0.466	0.282	0.494	0.574	0.508

Note: This table uses fixed effects regression models to determine the effect of seasonal factors on spreads (in %) using modeled arrival frequencies from AIR. The sample is limited to bonds that are affected by seasonality. Model (1) introduces the seasonality-adjusted EL for the full seasonality sample. Models (2) and (3) use only single-peril or multi-peril seasonality bonds, respectively. Models (4)–(6) include control variables. *t*-values are shown in parentheses and heteroscedasticity robust standard errors are clustered at bond level. The symbols *, **, and *** indicate statistical significance at the 5%, 1%, and 0.1% levels, respectively.

affected by seasonality than multi-peril bonds. The adjusted within R^2 of 0.466 in model (2) indicates that almost half of all secondary market fluctuation in single-peril bonds can be accredited to seasonality. With an adjusted within R^2 of 0.282 in model (3), this share is much lower for multi-peril bonds due to exposure to other nonseasonal perils. Overall, results hold for both single-peril and multi-peril bonds.

Throughout Table 6 the coefficients for the European winter storm season are below 1. However, this does not mean the seasonal fluctuations in European winter storms are not sufficiently reflected in spreads. Similar to the dampening effect of earthquake exposure in multi-peril bonds, the coefficient for the European wind bonds can also drop below 1 if wind bonds have a relatively strong exposure to other wind perils that occur outside of the European winter storm season. These other wind perils can be thunderstorms or hail which typically occur in the summer and not during the winter storm season. Section 7 shows that a large proportion of risk from July to September is implied in the trading activity of European wind bonds lifting the coefficient for single-peril European wind bonds to above 1.

Models (4)–(6) additionally include time-variant control variables. We find that the coefficients for the seasonality measures remain almost unchanged after the inclusion of controls. The estimated coefficients of these controls are consistent with the empirical cat bond literature (see Gürtler et al., 2016 and Braun, 2016), and almost all of them are significant at the 0.1% level.[31] Comparing the explanatory power of Models (1) and (4), an additional 17% of changes in spreads can be explained through corporate bond spread, equity returns, and reinsurance rates.[32]

6.3 | Robustness check: TRACE data

In this robustness check we repeat the analyses from Table 6 with a separate daily data set from the Trade Reporting and Compliance Engine (TRACE). The results for the TRACE data set strongly support the previous results on seasonality. Instead of only relying on dealer quotes, we also use real trading data from TRACE. In general, TRACE contains the specific trading dates, the traded volume, and real prices net of accrued interest for over-the-counter trades. From January 2015 to March 2017, we acquired a data set that includes 134 bonds with 1537 daily prices. We drop all distressed bonds and use only seasonality-affected bonds to analyze seasonality effects, resulting in a TRACE data set with 1069 daily observations of 61 seasonality-affected bonds.

TRACE provides clean bond prices but no spreads. However, the TRACE prices allow us to determine spreads if we know each cat bond's individual cash flow: Cat bonds are Floating Rate Notes and usually pay a fixed coupon over a riskless interest rate. For the riskless interest rate, we use the United States Treasury yield curve. Following Fabozzi (2005), we use the forward rates determined from the United States Treasury yield curve to proxy unknown future spot rates. We have acquired the fixed coupons over the riskless interest rate from the Lane Financial data set and cross-checked them with data from Thomson Reuters DataStream. Compared to regular floating rate notes, cat bonds have a unique property: The fixed coupon above the riskless rate does not necessarily stay fixed throughout a cat bond's maturity. A cat bond's maturity is usually a bit longer than the length of the underlying reinsurance contract between the sponsor and the special purpose vehicle (e.g., a 3-year cat bond usually has a maturity of 3 years and a few extra days or months). The time, when this reinsurance contract is in effect, is often referred to as the "risk period" of a cat bond. Outside of the risk period, a cat bond does not have exposure to the underlying insurance risk. During this time, a cat bond usually pays a much smaller fixed spread above the riskless rate to reflect the lower default risk. The specific end date of the risk period and the reduced spread above the riskless rate are unavailable to us. However, we believe the risk season should mimic reinsurance contracts and

[31]The coefficient of *Reinsurance index* indicates that a 1-point change in the Guy-carpenter rate-on-Line index is associated with a 0.02% point change in spreads, which confirms the results of Gürtler et al. (2016). A 1% point change in *Corp. bond spreads* is associated with a 0.41% point change in cat bond spreads; this effect is twice as large as Gürtler et al. (2016) and 1.5 times as large as in Braun (2016). A 1% point change in *Equity returns* is associated with 0.02% points change in cat bond spreads, which is slightly larger than in Gürtler et al. (2016). Finally, a 1-month decrease in remaining maturity is associated with a 0.03% point decline in cat bond spreads, which is in line with Gürtler et al. (2016). When comparing our coefficient of *Rem. maturity* to the coefficient of *TTM* in Gürtler et al. (2016), please note that *Rem. Maturity* is formatted in months while *TTM* is formatted in years.

[32]Estimated seasonality coefficients are also robust to unknown covariates expressed by quarter fixed effects, year fixed effects and year-quarter fixed effects. Detailed results are available upon request.

end on the last day of a month close to the cat bond's maturity date. Therefore, we assume the risk period to end on the last day of the month before its maturity. We assume a cat bond to pay a reduced spread above the riskless rate of 0.5% from the first day of the cat bond's month of maturity up to its day of maturity.[33] Consequently, this period of reduced coupons cannot be longer than 31 days.

To determine the necessary seasonality variable, we have used the methodology from Section 3.2. This time we have 365 steps per year corresponding to calendar days, accounting for the daily nature of the TRACE data set. We again employ the monthly distributions of U.S. hurricanes and European winter storms from AIR. We turn this monthly distribution into a daily distribution by interpolating between the midpoints of each month in a linear fashion. Hence, we gain a seasonality variable in a daily frequency.

Table 7 contains results on seasonality for the complete seasonality-affected TRACE sample as well as single-peril and multi-peril subsamples. For the complete TRACE sample in model

TABLE 7 Robustness check—impact of seasonality on spreads based on TRACE data

	Spread					
Dependent variable	**Full sample**	**Single-peril**	**Multi-peril**	**Full sample**	**Single-peril**	**Multi-peril**
Sample	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
EL_t-U.S. modeled	1.588***	2.013***	1.232***	1.341***	1.633***	1.136***
	(7.65)	(6.05)	(7.93)	(7.54)	(5.06)	(6.63)
EL_t-EU modeled	0.320		0.031	0.424		0.223
	(0.92)		(0.08)	(1.55)		(0.77)
Reins. index				0.031***	0.029	0.034***
				(3.46)	(1.50)	(4.56)
Corp. bond spreads				0.196*	0.126	0.190*
				(2.59)	(1.26)	(2.30)
Equity price index				−0.015	−0.011	−0.022*
				(−1.50)	(−0.53)	(−2.65)
Rem. maturity				0.036*	0.037	0.026*
				(2.52)	(1.29)	(2.14)
Constant	1.890***	1.219**	2.651***	−4.958**	−4.923	−4.774***
	(5.60)	(2.89)	(8.42)	(−2.99)	(−1.54)	(−3.53)
Observations	1069	454	615	1069	454	615
Number of bonds	91	30	61	91	30	61
Within R^2	0.529	0.618	0.472	0.643	0.696	0.600
Adj. within R^2	0.528	0.617	0.471	0.641	0.693	0.596

Note: Similar to Table 6, this table uses fixed effects regression models to determine the effect of seasonal factors on spreads (in %) using modeled arrival frequencies from AIR, but based on TRACE prices from FINRA. Model (1) introduces the seasonality-adjusted EL for the full seasonality sample. Models (2) and (3) use only single-peril or multi-peril seasonality bonds, respectively. Models (4)–(6) include control variables. *t*-values are shown in parentheses and heteroscedasticity robust standard errors are clustered at bond level. The symbols *, **, and *** indicate statistical significance at the 5%, 1%, and 0.1% levels, respectively.

(1), the coefficient for the U.S. season is highly statistically significant at the 0.1% level, while the coefficient for the European winter storm season is not statistically significant. Generally, only few European wind bonds were issued in the given period. Additionally, European bonds are often not included in TRACE. Thus, the coefficients for the European season remain insignificant due to the limited sample size. For the complete sample, 53% of the secondary market fluctuation can be explained through the seasonality measure. For the single-peril subsample in model (2), explanatory power increases to almost 62%. The coefficient for the U.S. season is larger than in model (1) and highly statistically significant at the 0.1% level. The coefficient for the European winter storm season is omitted because no single-peril European wind bond trades were reported in TRACE from January 2015 to March 2017. For multi-peril bonds in model (3), the explanatory power declines to 47%. The coefficient for the U.S. season remains highly statistically significant at the 0.1% level while its size has declined to a bit more than half its previous level. As compared to results for the Lane Financials data set in Table 6, coefficients and explanatory power have mostly increased. The increases in coefficients and explanatory power could be attributed to the different timeframes of both data sets as well as a more detailed modelling of seasonality through daily data. Although the seasonal coefficients slightly decrease, results hold after the inclusion of controls in Models (4)–(6). Overall, the results on the TRACE data set strongly support the results from earlier sections and highlight their reliability. Statistical significance remains high and explanatory power increases. In addition to results from the quarterly Lane Financials data, the daily TRACE data exemplifies the methodology's flexibility towards changes in data frequency.

7 | MARKET-IMPLIED SEASONALITY

7.1 | Methodology for the market-implied seasonality measure

To create the seasonality measure described in Section 3.2, we employed a modeled distribution of arrival frequency shares provided by AIR, which were exogenous to our model. However, it is also possible to deduct a market-implied distribution of arrival frequency shares from secondary market data. Through this channel, the investors' opinion on seasonality and their distribution of arrival frequencies can be extracted from the data. Investors may have additional information, which the risk modelers have not provided. For the remainder of the paper, the externally modelled distribution of arrival frequencies from AIR is named *modeled* arrival frequency as opposed to the *market-implied* arrival frequency derived from secondary market trading. Again, severity is assumed to be i.i.d. for each period across a year.

To estimate the distribution of market-implied arrival frequencies, we create an econometric model that estimates arrival frequencies in such a way that it explains observed secondary market spreads as accurately as possible. To derive such a model, we utilize the seasonality measure from Section 3.2 to "reverse-engineer" the market-implied distribution of arrival frequencies. Thus, $\lambda_{im,\tau}$ denotes the estimates for the *implied* distribution of arrival frequencies:

[33]The risk period of Cranberry Re 2017-1, for example, ends June 30, 2020, while the bond is repaid on July 13, 2020. Between June 30, 2020 and July 13, 2020, this bond pays a coupon of 0.5% above the riskless rate.

$$\mathrm{EL}_{it} = \frac{\mathrm{EL}_{\mathrm{initial},i}}{T_i - t} \sum_{\tau=t}^{T_i} \lambda_{im,\tau}, \tag{13}$$

where we have used a discretized version of (9). We define $H_{it} := \mathrm{EL}_{\mathrm{initial},i}/(T_i - t)$ to simplify Equation (13):

$$\mathrm{EL}_{it} = H_{it} \sum_{\tau=t}^{T_i} \lambda_{im,\tau}. \tag{14}$$

Consider a regression model that relates the spread s of bond i at time t to the fluctuating EL of bond i at time t, the vector of time-variate controls C_{it} and constant bond properties α_i:

$$\begin{aligned} s_{it} &= \beta_1 \mathrm{EL}_{it} + \eta' C_{it} + \alpha_i + \varepsilon_{it} \\ &= \beta_1 H_{it} (\lambda_{im,t} + \lambda_{im,t+1} + \lambda_{im,t+2} + \ldots + \lambda_{im,T_i}) + \eta' C_{it} + \alpha_i + \varepsilon_{it}. \end{aligned} \tag{15}$$

A straightforward approach is to estimate every separate λ_{im} simultaneously. However, this approach appears infeasible because the number of separate λ_{im} increases linearly with the length of the data set. Consider the given data that ranges from 2002 to 2017 containing 15 years of spreads. In case of monthly estimation this results in estimating 180 separate parameters λ. Instead, it is possible to simplify the finite series by assuming a static seasonal pattern, that is, seasons do not change from one year to the next year:

$$\lambda_{im,\tau} \equiv \lambda_{im,\tau+\kappa}, \tag{16}$$

where κ corresponds to the number of periods reflecting 1 year. Thus, different λ_{im} take on repeated values in cycles of 1 full year. Therefore, we define the following variables as the static shares of arrival frequencies of the yearlong season: γ_1, γ_2, ..., γ_τ. These variables are the distribution of the arrival frequencies that we estimate from secondary market data. Since seasons repeat on a yearly basis, each $\lambda_{im,\tau}$ can be matched with a single γ that refers to the same seasonal period within each year. Consider the case of monthly arrival frequencies as explicitly provided in Table 1. For monthly data, the number of yearly steps is $\kappa = 12$. Then, γ_1 refers to the arrival frequency in January, γ_2 refers to the arrival frequency in February and so on. In consequence, γ_1 is equal to all λ_{im} that reflect January data. Therefore, the finite series from Equation (15) can be shortened to κ summands:

$$s_{it} = \beta_1 H_{it} (\gamma_1 m_{1,it} + \gamma_2 m_{2,it} + \ldots + \gamma_\kappa m_{\kappa,it}) + \eta' C_{it} + \alpha_i + \varepsilon_{it}, \tag{17}$$

where m_{it} describes how many times each arrival frequency γ is included in the remaining maturity of bond i at time t. To obtain a model equation that we can estimate, we define

$$\delta_j := \gamma_j \beta_1 \text{ and } d_{j,it} := m_{j,it} H_{it}, \tag{18}$$

which simplifies (17) to

$$s_{it} = \delta_1 d_{1,it} + \delta_2 d_{2,it} + \ldots + \delta_\kappa d_{\kappa,it} + \eta' C_{it} + \alpha_i + \varepsilon_{it}. \tag{19}$$

In principle, we can estimate the unknown parameters δ with standard OLS regression. The required values d_j can be determined with Formula (18). However, since the estimated arrival frequencies represent shares of peril events occurring for specific parts of a calendar year (e.g., days, months, or quarters), we apply two natural constraints to the parameters δ. First, it is impossible that less than 0% or more than 100% of all peril events occur at a specific day, month, or quarter.

Thus, all parameters δ have to be larger than 0% and smaller than 100%. Second, the sum of all parameters δ has to equal 100%. Therefore, we formalize the following constraints:

$$\delta_1, ..., \delta_\kappa \geq 0 \text{ and } \sum_{j=1}^{\kappa} \delta_j = 1. \tag{20}$$

To implement these constraints, we use a nonlinear model following Gould et al. (2010). For this purpose, we replace the parameters δ in (19) with the following terms:

$$\delta_1 = \frac{1}{q}, \quad \delta_2 = \frac{\exp(\vartheta_2)}{q}, ..., \delta_\kappa = \frac{\exp(\vartheta_\kappa)}{q}, \tag{21}$$

with $q = 1 + \exp(\vartheta_2) + \exp(\vartheta_3) + \cdots + \exp(\vartheta_\kappa)$. Applying (21) to (19) and applying fixed-effects transformations yields the following model:

$$\ddot{s}_{it} = \frac{1}{q}\ddot{d}_{1,it} + \frac{\exp(\vartheta_2)}{q}\ddot{d}_{2,it} + ... + \frac{\exp(\vartheta_\kappa)}{q}\ddot{d}_{\kappa,it} + \eta'\ddot{C}_{it} + \varepsilon_{it}. \tag{22}$$

We estimate the nonlinear model and its parameters ϑ and η with maximum-likelihood estimation. After we have estimated the parameters ϑ, we reapply Formula (21) to obtain the estimates for the market-implied distribution of arrival frequency shares δ under the proposed constraints.

7.2 | Results for the market-implied seasonality measure

In the previous section, we have established a methodology to estimate market-implied arrival frequencies from secondary market spreads. While the proposed methodology allows for

TABLE 8 Months of maturity for seasonality-affected cat bonds

	Single-peril U.S. hurricane	Single-peril EU wind	Single-peril and multi-peril
January	9	8	78
February	4	0	12
March	3	3	21
April	3	8	39
May	21	3	52
June	18	12	102
July	5	1	16
August	0	0	2
September	0	1	2
October	0	0	2
November	3	0	6
December	18	4	54
Total	84	40	386

Note: Number of single-peril U.S. hurricane bonds and single-peril EU wind bonds that mature in the respective months of a calendar year. The third column reports the same numbers for all seasonality-affected bonds; in addition to single-peril hurricane and wind bonds, these bonds also include multi-peril bonds.

different lengths of time steps (e.g., days, months, quarters), the specific length of a possible time step is limited by the frequency of available data. We have obtained quarterly spreads; however, it is possible to estimate a monthly distribution of arrival frequencies from quarterly data exploiting the specific dates of maturity. The estimation of each specific month is only possible if a sufficient number of bonds mature during this month. For example, U.S. hurricane bonds are typically tailored in such a way that they cover an entire hurricane season. Therefore, the estimation of monthly arrival frequencies within the hurricane season is difficult.

To estimate the market-implied distribution of arrival frequencies of U.S. hurricanes and European winter storms, we use single-peril/single-location subsamples of U.S. hurricane and European wind bonds, respectively. Additionally, we estimate the implied distribution of arrival frequencies of the aggregate cat bond market with a subsample of all seasonality-affected cat bonds that also contains multi-peril bonds. Table 8 contains the number of bonds that mature in the specific calendar months for the subsamples. The data confirm that no U.S. hurricane bonds mature in August through October. For EU wind bonds we observe that no bonds mature in February, August, October, and November, and for the aggregate seasonality-affected cat bond market, only few bonds mature August through October. To cope with these shortcomings, we combine these months with other months that offer a higher availability of data. For example, in U.S. hurricane bonds, we use a single parameter for October and November to obtain an average across these 2 months. We apply this method to all months that have too little data available. When combining months, we follow three rules: (a) We combine

TABLE 9 Market-implied distributions of arrival frequencies

	U.S. hurricanes		EU winter storms		Cat bond market	
	Estimate	**CI [95%]**	**Estimate**	**CI [95%]**	**Estimate**	**CI [95%]**
January	1.6%**	[0.6%; 2.6%]	11.5%***	[8.5%; 14.4%]	6.9%***	[5.5%; 8.2%]
February	0%		11.5%***	[8.5%; 14.4%]	0%	
March	0%		11.5%***	[8.5%; 14.4%]	0%	
April	0%		0%		0%	
May	0%		0%		0%	
June	0%		0%		0%	
July	25.7%***	[24.7%; 26.8%]	12.3%***	[8.7%; 15.8%]	22.4%***	[21.8%; 23.0%]
August	25.7%***	[24.7%; 26.8%]	12.3%***	[8.7%; 15.8%]	22.4%***	[21.8%; 23.0%]
September	25.7%***	[24.7%; 26.8%]	12.3%***	[8.7%; 15.8%]	22.4%***	[21.8%; 23.0%]
October	10.6%***	[9.1%; 12.1%]	9.6%***	[6.7%; 13.8%]	13.0%***	[12.2%; 13.7%]
November	10.6%***	[9.1%; 12.1%]	9.6%***	[6.7%; 13.8%]	13.0%***	[12.2%; 13.7%]
December	0%		9.6%***	[6.7%; 13.8%]	0%	
Total	100.0%		100.0%		100%	

Note: Market-implied arrival frequencies for U.S. hurricanes, EU winter storms and the seasonality-affected cat bond market as derived from secondary market data. The shares of arrival frequency were estimated taking the control variables used in previous tables into account, namely *Reinsurance index, Corporate bond spreads, Equity returns*, and *Rem. maturity*. The symbols *, **, and *** indicate statistical significance at the 5%, 1%, and 0.1% levels, respectively.

months where only two bonds or fewer mature, (b) we only combine months adjacent to each other, and (c) we do not combine months across quarters. Following these rules, we combine the following months: For U.S. hurricanes, we combine July/August/September and October/November. For EU Wind bonds, we aggregate January/February/March, July/August/September, and October/November/December. For the complete seasonality-affected cat bond market, we combine July/August/September and October/November.

The estimated market-implied distribution of arrival frequencies and the corresponding 95% confidence intervals are shown in Table 9. These arrival frequencies were estimated with the same control variables as in the previous models. The arrival frequencies for U.S. hurricanes and European winter storms mostly reflect the general pattern of the modeled distribution of arrival frequencies of AIR (see Table 1). For U.S. hurricanes, July, August, and September are the peak months for the hurricane season where investor trading implies hurricane arrival frequencies of 25.7% for each month. The season fades out during October and November with 10.6% each. The investor trading indicates no more hurricanes in December. All of these coefficients are statistically significant at the 0.1% level. For EU winter storms, January, February, and March are peak months for the winter storm season where investor trading implies winter storm arrival frequencies of 11.5% for each of these months. The other large portion of the winter storm season is reflected in the market-implied shares of arrival frequencies for October, November, and December with 9.6% each. Surprisingly, in July, August, and September, the market-implied share of arrival frequency deviates from the modeled share of arrival frequencies from AIR. In these months the shares of arrival frequencies are 12.3% each. The likely reason is that many EU wind bonds do not only insure against winter storms but also against other perils such as hail or severe thunderstorms, which typically occur in the summer. Thus, the market-implied arrival frequencies in July, August, and September can probably be attributed to the hail and thunderstorm season. All coefficients are highly statistically significant at the 0.1% level.

For the complete cat bond market, which includes all single-peril and multi-peril bonds, the U.S. hurricane season is the predominant seasonality factor. Investor trading indicates that most seasonal peril events occur from July through November. This also indicates that multi-peril bonds, whose distribution of risk among peril types is unknown to us, are also predominantly affected by U.S. hurricanes. The shares of peril events in January indicate the presence of the winter storm seasons. The lack of market-implied arrival frequency in February can be attributed to a lack of single-peril EU wind bonds that mature in February. If the additional multi-peril bonds, which mature in February, do not contain a substantial amount of winter storm risk, the model estimates 0% for this month. All coefficients are highly statistically significant at the 0.1% level.

As previously mentioned, the market-implied distributions of arrival frequencies are in line with the modeled distributions, while also picking up parts of the European hail and thunderstorm season. The results presented in Table 10 indicate how well the market-implied distributions explain the data by comparing them to models that use the modeled distributions. The subsamples are the same samples that were used to estimate the market-implied arrival frequencies. For U.S. hurricane bonds in model (1), the seasonal variable that uses market-implied U.S. hurricane arrival frequencies and time-variate controls explain 68% of all secondary market fluctuation. Comparing models (1) and (2), the coefficient for the U.S. hurricane season is almost the same for market-implied and modeled distributions of arrival frequencies. In both models, the coefficient is large and highly statistically significant at the 0.1% level. For European wind bonds in model (3), the seasonal variable that uses the market-implied

TABLE 10 Comparison of market-implied and modeled seasonality measures

	Spread				
	US hurricane		EU wind		
Dependent variable	Single-peril		Single-peril		Cat bond market
Sample	(1)	(2)	(3)	(4)	(5)
EL_t-U.S. market-implied	1.820***				
	(15.33)				
EL_t-U.S. modeled		1.802***			
		(13.87)			
EL_t-EU market-implied			1.247*		
			(2.62)		
EL_t-EU modeled				0.767**	
				(3.26)	
EL_t-World market-implied					1.124***
					(15.32)
Reins. index	0.021***	0.022***	0.014*	0.014*	0.020***
	(4.14)	(4.00)	(2.51)	(2.64)	(7.36)
Corp. bond spreads	0.334***	0.329***	0.419***	0.404***	0.413***
	(12.20)	(11.72)	(5.80)	(5.75)	(19.13)
Equity returns	0.014*	0.013 +	0.050***	0.043***	0.027***
	(2.03)	(1.81)	(5.20)	(3.75)	(5.98)
Rem. maturity	0.022 +	0.022 +	−0.007	0.003	0.032***
	(1.96)	(1.98)	(−1.06)	(0.49)	(5.71)
Constant	−3.364**	−3.549**	−2.050	−1.689	−2.234***
	(−2.89)	(−2.83)	(−1.11)	(−1.08)	(−3.58)
Observations	871	871	483	483	3947
Number of bonds	84	84	40	40	386
Within R^2	0.679	0.666	0.304	0.294	0.458
Adj. within R^2	0.677	0.664	0.297	0.287	0.457

Note: This table uses fixed effects regression models to determine the effect of seasonal factors on spreads (in %) using the market-implied distribution of arrival frequencies derived from secondary market data as well as our modelled seasonality measure. Models (1) and (2) use a subsample that is limited to single-peril/single-location hurricane bonds. Models (3) and (4) are limited to single-peril/single-location European wind bonds. Model (5) uses all single-peril and multi-peril cat bonds that are affected by U.S. hurricane or European wind perils. *t*-values are shown in parentheses and heteroscedasticity robust standard errors are clustered at bond level. The symbols *, **, and *** indicate statistical significance at the 5%, 1%, and 0.1% levels, respectively.

distribution of arrival frequencies and time-variate controls explain roughly 30% of secondary market fluctuation. This is likely a consequence of a less pronounced season, because generally, EU wind bonds also insure against other perils that are not exclusively European winter storms. The coefficient is significant at the 1% level. Comparing models (3) and (4), the distribution of market-implied arrival frequencies for European winter storms yields a higher explanatory power than the modeled distribution of arrival frequencies. Additionally, the coefficient for the

market-implied distribution in model (3) is larger than the coefficient for the modeled distribution in model (4). As previously mentioned, we believe the market-implied shares of arrival frequencies for European winter storms in Table 9 partially contain the hail and thunderstorm season as indicated by the high arrival frequency in quarter 3; this separate subseason is not reflected in the modeled distribution. On the contrary, the coefficients and the explanatory power for the U.S. hurricane season are almost the same for market-implied and modeled distributions. For the complete market in model (5), the seasonal variable that uses the market-implied distribution of arrival frequencies and time-variant controls explain 46% of secondary market fluctuation. The coefficient is significant at the 0.1% level.

Generally, the results for market-implied arrival frequencies for U.S. hurricanes and the EU winter storms are in line with previous results from Section 6. The results indicate that the market-implied distribution of arrival frequencies can be a valuable alternative to modeled arrival frequencies for modeling cat bond seasonality, particularly in the case that modeled arrival frequencies are unavailable. However, the distributions of arrival frequencies of specific perils depend on the availability of specific cat bond types. Single-peril hurricane bonds allow for the estimation of market-implied shares of hurricane arrival frequencies while European wind bonds mix European winter storms with thunderstorms and hail.

8 | CONCLUSION

Seasonal fluctuations are a major driver of cat bond spreads on secondary markets. We investigate their patterns for U.S. hurricane and European winter storm bonds. We propose a conceptual framework to model seasonality. This framework includes a hazard rate model to illustrate theoretical implications of seasonality on cat bonds and a new seasonality measure. This measure integrates the distribution of peril arrival frequency in econometric cat bond pricing models capturing theoretical implications from the hazard rate model.

Empirically, the seasonal pattern in spreads reflects the seasonal pattern in arrival frequencies: Spreads peak before risk season starts, hit their bottom after risk season ends, and adjust in between. The spreads' amplitude in seasonal fluctuation increases as a cat bond nears its maturity. Additionally, risky bonds with a high EL express stronger seasonal fluctuation than bonds with a low EL. Single-peril bonds express a stronger seasonal fluctuation than multi-peril bonds that have only some exposure to seasonal perils. Similarly, cat bonds that have a clear season (e.g., U.S. hurricane bonds) fluctuate more strongly than cat bonds whose season is less pronounced (e.g., European wind bonds).

The new seasonality measure captures these effects and explains a large fraction of secondary market fluctuations in seasonality-affected cat bonds. In addition, we provide a comprehensive method to deduct the market-implied distribution of peril arrival frequencies from observable secondary market spreads. Seasonal variables that use the market-implied instead of the modeled distribution of arrival frequencies explain secondary market spreads even slightly better as modeled seasonal variables. Generating the market-implied distributions of arrival frequencies offers an alternative if modeled distributions of arrival frequencies are unavailable; additionally, this method can be used to deduct the aggregate opinion of investors on arrival frequencies. We model arrival frequency and assume severity to be i.i.d. for each peril event during a calendar year. However, the methodology can be easily expanded to include two separate seasonality measures for arrival frequency and severity or a combination of both if a distribution of severity is available. On the contrary, the market-implied measure does not

require the i.i.d. assumption but solves for the time-varying EL-multiple that best fits the data. The results for both seasonality measures are similar, which indicates that our modelling assumption is rather unproblematic.

Our proposed seasonality measure provides a comprehensive method to academics, insurance companies, and investors to model seasonality and explain secondary market fluctuation. For academics, the proposed seasonality measure offers an opportunity to model and control for seasonality in future secondary market research, thereby avoiding a large loss of observations. The methodology for deducting the market-implied distribution of arrival frequencies from secondary market spreads offers an opportunity to extract information from market participants concerning these perils. For practitioners, the illustrated seasonality effects could improve market transparency. When insurance companies decide between a cat bond and a traditional reinsurance contract to transfer risk, they may turn to secondary cat bond markets to project possible spreads for a cat bond placement. In this context, seasonality is an important factor because it causes large fluctuations on the secondary markets. (Re-)insures should not over- or under-estimate spreads due to reasons of seasonality. As investors, specialized ILS funds must value their cat bond portfolio correctly at investor entry or exit;[34] however, trading is very infrequent creating extended periods without any market valuation. Modelling seasonal fluctuation could allow for a fair valuation of cat bonds when a proper market valuation is lacking. Overall, we provide new insights on the impact of seasonality on secondary market spreads of cat bonds that could advance the markets maturity and further facilitate its growth.

Predictable spread movements on secondary cat bond markets raise the question of a potential trading strategy exploiting these predictable seasonal fluctuations. Such a trading strategy would only offer abnormal returns if these seasonal, predictable price movements implied mispricing. However, we believe the seasonal fluctuations follow a risk related rationale. Mispricing would only occur when the seasonal fluctuation is too strong or not strong enough, that is, a unit of change in EL_t is not priced properly. However, if we use the primary market as an indicator for "correct" valuation of a unit of EL, we cannot identify such a mispricing: In Section 6 the coefficient for EL_t does not significantly deviate from the EL coefficient on the primary market. Additionally, to make a potential trading strategy profitable, the associated transaction costs need to be lower than the exploitable mispricing, so that these deviations would need to be rather large. Although analyzing a potential mispricing on the secondary cat bond market is beyond the scope of this paper, it would be a highly interesting topic for future research.

ACKNOWLEDGEMENTS

The authors thank the participants of the World Risk and Insurance Economics Congress 2020, the Annual Meeting of the German Finance Association 2019, the Annual Seminar of the European Group of Risk and Insurance Economists 2019, the Annual Meeting of the Western Risk and Insurance Association 2019, the Annual Congress of the German Insurance Science Association 2019, the RGS Doctoral Conference in Economics 2019, the Brown Bag Seminar in Duisburg 2019, and the Frankfurt Insurance Research Workshop 2018 for helpful comments. Additionally, the authors thank the editors and two anonymous referees for advice and helpful

[34]Aon Securities (2019) reports that in 59% of all ILS volume outstanding was held by specialized catastrophe funds in 2019.

comments. We are especially grateful for financial support from the German Insurance Science Association (DVfVW). Open Access funding enabled and organized by Projekt DEAL.

ORCID

Markus Herrmann http://orcid.org/0000-0001-8893-9685
Martin Hibbeln http://orcid.org/0000-0003-2350-9830

REFERENCES

Alaton, P., Djehiche, B., & Stillberger, D. (2002). On modelling and pricing weather derivatives. *Applied Mathematical Finance*, *9*(1), 1–20.

Ammar, S. B. (2020). Catastrophe risk and the implied volatility smile. *Journal of Risk and Insurance*, *87*(2), 381–405.

Aon Securities (2019). ILS Annual Report 2019.

Beer, S., & Braun, A. (2020). *Market-consistent valuation of natural catastrophe risk*. Working Paper.

Black, F. (1976). The pricing of commodity contracts. *Journal of Financial Economics*, *3*(1-2), 167–179.

De Bondt, W. F. M., & Thaler, R. H. (1987). Further evidence on investor overreaction and stock market seasonality. *The Journal of Finance*, *42*(3), 557–581.

Braun, A. (2016). Pricing in the primary market for cat bonds—New empirical evidence. *Journal of Risk and Insurance*, *83*(4), 811–847.

Braun, A., Ammar, S. B., & Eling M. (2019). Asset pricing and extreme event risk: Common factors in ILS fund returns. *Journal of Banking & Finance*, *102*, 59–78.

Campbell Sean, D., & Diebold, F. X. (2005). Weather forecasting for weather derivatives. *Journal of the American Statistical Association*, *100*(469), 6–16.

Cummins, J. D., & Weiss, M. A. (2009). Convergence of insurance and financial markets. *Journal of Risk and Insurance*, *76*(3), 493–545.

Dieckmann, S. (2010). *By force of nature—Explaining the yield spread on catastrophe bonds*. Working Paper.

Fabozzi, F. J. (2005). *The handbook of fixed income securities* (7th ed.). McGraw-Hill.

Fama, E. F. (1970). Efficient capital markets. *The Journal of Finance*, *25*(2), 383–417.

Fama, E. F., & French, K. R. (1987). Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage. *Journal of Business*, *60*(1), 55–73.

Finken, S., & Laux, C. (2009). Catastrophe bonds and reinsurance—The competitive effect of information-insensitive triggers. *Journal of Risk and Insurance*, *76*(3), 579–605.

Galeotti, M., Gürtler, M., & Winkelvos, C. (2013). Accuracy of premium calculation models for cat bonds—An empirical analysis. *The Journal of Risk and Insurance*, *80*(2), 401–421.

Gatzert, N., & Schmeiser, H. (2011). Industry loss warranties: Contract features, pricing, and central demand factors. *The Journal of Risk Finance*, *13*(1), 13–31.

Goldenberg, S. B., Landsea, C. W., Mestas-Nuñez, A. M., & Gray, W. M. (2001). The recent increase in atlantic hurricane activity: Causes and implications. *Science*, *293*(5529), 474–479.

Gorton, G. B., Hayashi, F., & Rouwenhorst, K. G. (2013). The fundamentals of commodity futures returns. *Review of Finance*, *17*(1), 35–105.

Gould, W., Pitblado, J., & Poi, B. (2010). *Maximum likelihood estimation with stata* (4th ed.). Stata Press.

Gultekin, M. N., & Gultekin, N. B. (1983). Stock market seasonality—international evidence. *Journal of Financial Economics*, *12*(4), 469–481.

Gürtler, M., Hibbeln, M., & Winkelvos, C. (2016). The impact of the financial crisis and natural catastrophes on CAT bonds. *Journal of Risk and Insurance*, *83*(3), 579–612.

Hainaut, D. (2012). Seasonality modelling for catastrophe bond pricing. *Bulletin Francais d'Actuariat*, *12*, 129–150.

Jordan, S. D., & Jordan, B. D. (1991). Seasonality in daily bond returns. *Journal of Financial and Quantitative Analysis*, *26*(2), 269–285.

Keim, D. B. (1983). Size-related anomalies and stock return seasonality—further empirical evidence. *Journal of Financial Economics*, *12*(1), 13–32.

Lakonishok, J., & Smidt, S. (1984). Volume and turn-of-the-year behavior. *Journal of Financial Economics*, *13*(3), 435–455.
Lane, M. N. (2000). Pricing risk transfer transactions. *ASTIN Bulletin: The Journal of the International Actuarial Association*, *30*(2), 259–293.
Lane, M. N. (2004). Arbitrage algebra and the price of multi-peril ILS. *The Journal of Risk Finance*, *5*(2), 45–51.
Lane, M. N., & Beckwith, R. (2017). *Annual review and commentary for the four quarters, Q2 2016 to Q1 2017*. Lane Financials LLC.
Lane, M. N., & Mahul, O. (2008). *Catastrophe risk pricing—An empirical analysis*. Policy Research Working Paper, The World Bank (4765).
Papachristou, D. (2011). Statistical analysis of the spreads of catastrophe bonds at the time of issue. *ASTIN Bulletin: The Journal of the IAA*, *41*(1), 251–277.
Poliquin, B., & Lalonde, D. (2012). Role of catastrophe risk modelling in ILS. In M. Lane (Ed.), *Alternative (Re) insurance strategies* (2nd ed.). Risk Books.
Raftery, A. E. (1995). Bayesian model selection in social research. *Sociological Methodology*, *25*, 111–163.
Roll, R. (1984). Orange juice and weather. *American Economic Review*, *74*(5), 861–880.
Rozeff, M. S., & Kinney, W. R. (1976). Capital market seasonality—The case of stock returns. *Journal of Financial Economics*, *3*(4), 379–402.
Schneeweis, T., & Woolridge, J. R. (1979). Capital market seasonality—The case of bond returns. *Journal of Financial and Quantitative Analysis*, *14*(5), 939–958.
Sun, Q., & Tong, W. H. S. (2010). Risk and the January effect. *Journal of Banking & Finance*, *34*(5), 965–974.
Swiss, R. E. (2013). Insurance-linked securities market update.
Vidal-García, J., & Vidal, M. (2014). Seasonality and idiosyncratic risk in mutual fund performance. *European Journal of Operational Research*, *233*(3), 613–624.
Yao, Y. (2012). Momentum, contrarian, and the January seasonality. *Journal of Banking & Finance*, *36*(10), 2757–2769.
Zhang, C. Y., & Jacobsen, B.(2013). Are monthly seasonals real? A three century perspective. *Review of Finance*, *17*(5), 1743–1785.

How to cite this article: Herrmann M, Hibbeln M. Seasonality in catastrophe bonds and market-implied catastrophe arrival frequencies. *J Risk Insur*. 2021;88:785–818. https://doi.org/10.1111/jori.12335

APPENDIX A

For illustration purposes, we model a seasonal zero-coupon cat bond where $\lambda_h(t)$ follows a cosine function that peaks in the middle of the year and is zero at the turn of the year:

$$\lambda_h(t) = \lambda_0 \cdot (1 - \cos(2\pi t)), \tag{A1}$$

where λ_0 is the total hazard rate for one calendar year: $\int_0^1 \lambda_h(\tau)\ d\tau = \lambda_0$. The value of such a cat bond is:

$$\begin{aligned} V_t &= FV \cdot \exp\left(-\lambda_0 \cdot \int_t^T (1 - \cos(2\pi\tau))d\tau\right) \\ &= FV \cdot \exp\left(-\lambda_0 \cdot \left(T - t - \tfrac{1}{2\pi}(\sin(2\pi T) - \sin(2\pi t))\right)\right) \end{aligned} \tag{A2}$$

and the spread equals:

$$s_t = \frac{\lambda_0 \cdot \left(T - t - \frac{1}{2\pi}(\sin(2\pi T) - \sin(2\pi t))\right)}{T - t}. \tag{A3}$$

Figure 1 illustrates values and spreads of a hypothetical nonseasonal and a seasonal zero-coupon cat bond. They have a maturity of 3 years, a hazard rate λ_h (and λ_0, respectively) of 8% and a CEL of 100%. We use Equations (3) and (5) to determine the value and spread of the nonseasonal cat bond and Equations (6) and (7) accordingly for the seasonal cat bond.

The value of the nonseasonal cat bond in Figure 1 (see Section 3.1) increases almost linearly in the case without default. In the case of default, the value would immediately jump to zero and remain at this value as we assumed a CEL of 100%. The value of the seasonal cat bond also increases over time, but it fluctuates depending on the seasonal state. At the turn of the year the value of the seasonal cat bond increases only slowly because the hazard rate is low, which means the probability that the bond survives until maturity increases relatively slowly. In the middle of a calendar year, the value of a seasonal cat bond increases strongly in the case without default because the hazard rate is high, so that the probability the bond survives until maturity increases relatively quickly.

The spread of a nonseasonal cat bond in Figure 1 is constant throughout its maturity, while the spread of a seasonal cat bond fluctuates strongly. At maturity, the spread approaches zero because the season ends on the same day as the bond matures. We can derive two main observations from this figure: A general seasonal pattern and an increasing amplitude as a bond nears its maturity. First, regarding the seasonal pattern, spreads peak a couple of months before the season reaches its peak in the middle of the year. They reach their bottom a couple of months before the season fades out at the end of the year. Second, the amplitude between seasonal peaks and bottoms increases as a bond approaches its maturity. From the first to the second year, the amplitude between the maximum and minimum in spreads increases from 1.12% points to 1.90% points, and increases further in the ultimate year.[35] The reason for the increasing seasonal amplitude of spreads when approaching maturity is as follows: While the amplitude of the seasonal value fluctuation remains almost constant throughout the cat bond's maturity, its remaining time to maturity decreases. However, the bond's spread is more sensitive to changes in prices the closer it is to maturity.

[35]In the last year spreads approach zero as the bond approaches its maturity and its last season fades out.