

Barbieri, Stefano; Konrad, Kai A.; Malueg, David A.

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# Preemption contests between groups

Stefano Barbieri\*

Kai A. Konrad\*\*

and

David A. Malueg\*\*\*

*We consider a preemption game between competing groups; firms lobbying individually for their groups' interests provide an empirical example. Among symmetric groups, the first firm to take action bears an (unobserved) cost and wins the prize on behalf of its group. In equilibrium, the firm with the lowest cost takes action, but with delay. More competition and a smaller ratio of costs to benefits reduce delay. Firms in larger groups wait longer, but group action can occur earlier, as the probability of a low-cost firm is higher. Asymmetries in group size or strength of externalities also matter.*

## 1. Introduction

■ We study incentives and equilibrium behavior when members of a group can preempt other groups on behalf of their own group. In such a framework, group members' free-riding incentives inside the own group conflict with preemption motives that apply in the inter-group competition. Each player prefers that his group preempts the other groups, but that another member of his group is the preempting player who bears the costs of the preemptive action.

As an illustrative example, consider two industries, both of which can press a policy to implement trade regulations in their favor. The CEOs of the companies of these two industries may take action on behalf of their own groups: for example, when they talk to influential politicians, they can explain why such protection policies are necessary for their industry. Similar considerations apply to other lobbying contexts. Yackee (2019) surveys the general literature on the politics of rule making. She alludes to Yackee (2012) who finds that “off the public record” communication between lobbyists and regulators may influence regulatory rules at an early preproposal

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\* Tulane University; sbarbier@tulane.edu.

\*\* Max Planck Institute for Tax Law and Public Finance; kai.konrad@tax.mpg.de.

\*\*\* University of California, Riverside; david.malueg@ucr.edu.

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stage: early lobbying can have an agenda-setting role. As access to decision makers is a scarce resource and the mental readiness and receptiveness of politicians are limited, volunteering on behalf of one's own industry also has an opportunity cost: the CEO cannot use this opportunity to lobby for a more firm specific interest, for instance. In the absence of competition from other industries, a CEO may prefer to pitch for his or her own company interests and hope that other CEO colleagues will stand up for the interests of their industries. CEOs can meet politicians in different forums.<sup>1</sup> Company leaders can also be members of the entourage of prime ministers or governors on important state visits and travel with them in their government jets.<sup>2</sup> Influence attempts might rarely become public. An exception is "the memo...written by Robert E. Murray, a longtime Trump supporter who donated \$300,000 to the president's inauguration. In it, Mr. Murray, the head of Murray Energy, presented Mr. Trump with a wish list of environmental rollbacks just weeks after the inauguration." (Friedman, 2018.) This wish list, dated March 2017 and *organized in order of priority*, starts with the elimination of the Clean Power Plan, the overturning of the "endangerment finding for greenhouse gases," the elimination of tax credits for solar and wind energy producers, and the withdrawal from the Paris Climate Agreement. It is clear that the priorities identified by Mr. Murray benefit the entire fossil fuel industry, rather than just the coal plants owned by Murray Energy. From a theoretical point of view, the CEOs' problem describes a waiting game as analyzed in Bliss and Nalebuff (1984) and Fudenberg and Tirole (1986). The incentives change if there is competition between industries for trade policy favors, as we assume.

It is useful to relate our illustrative example to the approaches that describe lobbying in the literature.<sup>3</sup> A first branch of the literature starting with Olson (1965) focuses on the relationship between resources mobilized and policy influence. Grossman and Helpman (1994) suggests a menu-auction approach as a formal description of this allocation problem. A second branch of the literature considers the role of lobbies as experts and the problem of strategic information transmission (Austen-Smith and Wright, 1993; Cotton and Déllis, 2016) and distinguishes between buying access and issues of asymmetric information (Austen-Smith, 1995; Cotton, 2012). A third branch of the literature focuses on the use of informal relationships and connections to obtain access. The relationship between a firm's CEO and a politician may be at a personal level.<sup>4</sup> But it may also be the case that a firm's CEO decides to "hire" a lobbyist with the needed connections.<sup>5</sup>

An important aspect is the role of group mobilization. Olson (1965) emphasized the free-riding problems of interest groups. This topic is recurrent in the literature on rent-seeking between groups (Katz, Nitzan, and Rosenberg 1990, Esteban and Ray 2001). The theory of Grossman and Helpman (1994) elaborates on a framework that takes lobbies as unitary players. Martimort and Lefevre (2019) advance these theories focusing on the role of information asymmetries inside interest groups for whether and how they overcome free-riding incentives. Recently, researchers also found substantial evidence that illustrates that single large firms often take action directly and on their own (Bombardini and Trebbi 2012, Kim and Osgood 2019), particularly if firms in an industry are sufficiently heterogeneous, offering examples of special interests influencing policy outcomes, but without formation of a lobby.

Of course, the realities of lobbying are very complex, and no single theoretical model can capture all of its facets. We view our model as focusing on relationships and access in a

<sup>1</sup> An illustrative example is US President Donald Trump's Manufacturing Council and the Strategy & Policy Forum, both of which he founded early in 2017.

<sup>2</sup> For a description of such missions see, for instance, the report by Washington Governor Jay Inslee Staff (2017).

<sup>3</sup> Here we partially follow Rodrik (1995) and Groll and McKinley (2015). For general reviews on special interest group and lobbying activities, see Olson (1965), Grossman and Helpman (2001) and Hall and Deardorff (2006).

<sup>4</sup> This is common in Russia and other transition economies (see, e.g., Frye, 2002). Studies of politically connected firms are also common, and we refer to Kerr, Lincoln, and Mishra (2014), who list several.

<sup>5</sup> Connections to politicians are key and well-rewarded in lobbying, see, for example, Wise (2007), Bertrand, Bombardini, and Trebbi (2014), and Blanes-i-Vidal, Draca, and Fons-Rosen (2012).

framework in which large firms have an incentive to take *stand-alone* action, but with spillovers between them. We highlight three elements: preemption competition, timing as a means to resolve incomplete information, and free riding.

First, policy requests from different branches or industries may directly contradict or preclude each other. For instance, steel producers might like tariff protection on steel, but not on cars, whereas car producers might dislike tariff protection on steel. But even if the different requests do not interfere with each other directly, they can still be in resource rivalry to each other. For instance, politics cannot implement any number of trade protection measures without discrediting the country internationally, and politicians may also be limited in the number of bills they can press for. Furthermore, it may be easier to implement measures during the “honeymoon” period of a newly elected politician, who therefore has to prioritize possible policies.<sup>6</sup> Finally, and most importantly, the legislators’ resources may also be limited more generally and legislators have opportunity costs of legislating. Legislative effort might also be subject to opportunity costs such as the formation of costly voting coalitions (Glazer and McMillan 1990),<sup>7</sup> and Ellis and Groll (2019) state that: “It is well known that the evaluation and implementation of policy is an expensive process. Policymakers faced with finite budgets must therefore decide how to allocate their limited resources between activities associated with information acquisition or policy enactment.” If such (regulatory) capacity constraints or convex opportunity costs exist, the incentive to wait for one’s own CEO colleagues to become active faces the incentive to preempt colleagues from other industries.

A second key element of our model is timing: the CEO decides not only whether, but also when, to take action. In this context it is important to acknowledge the “power of the proposal maker” that emerges in many contexts and most notably in the contexts of voting (see, e.g., Harrington 1990) and bargaining.<sup>8</sup> It is also important to note that politicians may have pressure on their time unrelated to lobbying. So if a CEO delays action, the politician may end up not granting access at all because her time is otherwise occupied.<sup>9</sup> These constraints will be mapped by the assumption that the politician has time for exactly one initiative, namely, the first one brought to the politician’s attention.

The final key element we highlight is free riding within the set of firms that have interests that partially overlap. In our formal framework single firms or their CEOs may take individual lobbying action. Recent empirical research provides strong evidence that this happens. Ludema, Mayda, and Mishra (2018) study the political influence of individual firms on bills to suspend tariffs on US imports. Kim (2017) provides a list of cases in which single firms lobbied for a specific tariff bill in Table 4 of his article. See also the empirical work by Bombardini and Trebbi (2012) and Osgood (2017) for further more detailed evidence and discussion. In their survey, Kim and Osgood (2019) even write about a “prevalence of firm-level political activities around trade.”

This firm-level intervention is strongly in line with our model, as long as these activities have some positive spillovers to other firms in the same industry.<sup>10</sup> A literature about the prelegislative

<sup>6</sup> The honeymoon effect is well-studied in political science, see, for example, Buchler and Dominguez (2005) and references therein.

<sup>7</sup> The legislator’s time-opportunity-cost of legislating is the key element of the explanation for why legislation is broad-based and tends to address the new rather than amend old legislation in Glazer and McMillan (1992).

<sup>8</sup> Alternating bargaining in the Rubinstein tradition or alternating policy offers in a political economy context in the Baron-Ferejohn tradition typically allow for future proposals by other players. At the cost of complexity, our model could account for this explicitly.

<sup>9</sup> Ellis and Groll (2019) elaborate on the evidence of increasing resource pressure and declining capacity of legislators in the US.

<sup>10</sup> An informative anecdotal example is the letter by Electronic Theatre Controls, Inc. (ETC), (see US International Trade Commission, 2012) which expresses opposition to a tariff suspension bill. It alludes to the major disadvantages of this bill for *both* ETC and “other US-based manufacturers of similar products.”

process in which specific proposals emerge is informative in this context, as it suggests a channel for firm-level interventions.<sup>11</sup>

Lobbying by single firms describes a situation in which the firm that volunteers bears the cost of volunteering but might draft its interventions according to its individual interests, such that it has a larger gross benefit of its successful lobbying than would other firms in the same industry. This fits with our model as long as the net benefit of such volunteers falls short of the positive spillover that emerges to some set of member firms in the same industry. Furthermore, we assume that what matters for the success of a group of firms with aligned interests is whether a single player from this group decides to take action.<sup>12</sup> This creates incentives for an extreme form of free riding and raises the issue of which firm might push for its preferred trade policy.

Bombardini (2008) finds a positive correlation between firm size and individual activism and Kim and Osgood (2019) suggest that highly skewed distributions of firm sizes may give rise to industry associations dominated by large firms. The empirical observations relate to the volunteering/waiting game aspect inside each set of heterogeneous firms and map many of its elements well: if single firms take lobbying actions, their specific benefits and costs determine which (or rather “when” a) firm becomes active.

A second, equally important model element is the preemption competition between sets of firms (e.g., upstream and downstream producers) or competition for scarce legislative resources. Indeed, Gawande, Krishna, and Olarreaga (2012) emphasize the element of competition between opposing lobbying interests.<sup>13</sup> Finally, if a larger number of lobbying interests compete for the same legislator’s attention or time budget, or if the legislator’s constraints tighten such that there are more groups per policy favor that can be granted, this corresponds, in our setup, to an increase in the number of sets of firms that compete. As one might expect, this pushes individual firms to act sooner.

The problem here combines incentives in a waiting game with incentives in a preemption game: as in the trade-protection illustration, the waiting game emerges inside a group. A group is defined here as a set of firms whose interests are at least partially aligned, such that the lobbying action taken by an individual firm has some positive spillovers to these other firms. The preemption game applies to the relationship between these groups. Our analysis shows how the two motives affect the individual player’s decision making and addresses questions about how the timing of action depends on the intensity of competition between groups (number of groups), the general size of the groups overall, and the asymmetry between groups. Preemption games tilt the timing of actions such that players act earlier. Waiting games tilt the timing of actions such that players tend to delay action. With incomplete information, preemption games and waiting games tend to assign the timing of action in equilibrium to the low-cost players. As there are now multiple, partially countervailing incentives, we also ask whether this general insight remains valid.

The literature indicates that individual-firm level action is significant in the trade policy context, and preemption incentives and free-riding incentives often pose collective action problems that yield socially inefficient outcomes. It would not be surprising if, with the exception of informal and non-repeated contexts,<sup>14</sup> institutions emerge that overcome the intra-group collective action problem by making groups act as unitary players. Many of the empirical frameworks that

<sup>11</sup> As West (2009) reports: “Informal conversations and e-mail exchanges are almost ubiquitous forms of participation in proposal development.”

<sup>12</sup> For example, when analyzing individual firms who are proponents of tariff exemption bills, Ludema et al. (2018) state that in their dataset of bills the percentage of bills with more than one proponent is only 3%.

<sup>13</sup> They model this competition along the lines of the Grossman–Helpman model, but as is well known, this is only one of several competing approaches. We have argued above why being first with a policy proposal might be important for policy outcomes.

<sup>14</sup> Possible examples outside industrial organization can be found in the military context when individuals volunteer for suicide missions for their own troops, or when informal groups benefit from one of the members assuming special responsibility in a leadership position.

come to mind therefore assume that the intra-group free-riding problems are taken care of, with examples ranging from entry problems<sup>15</sup> to R&D competition<sup>16</sup> and *de facto* standard setting in industries with network effects<sup>17</sup> in industrial organization. Whereas the *de facto* incentives to institutionalize groups and coordinate their behavior internally may limit the number of examples in which the free-riding incentives and the preemption incentives are directly at work and face each other, this does not make the study of the game with free-riding inside purely non-cooperative groups less relevant. The study of the non-cooperative equilibrium is a benchmark that establishes the implicit or explicit default threat point or status-quo for collective bargaining that may take place inside a group. As is well-known from bargaining theory (see, e.g., the discussion in Binmore, Rubinstein, and Wolinsky 1986), the payoff characteristics of the non-cooperative equilibrium are relevant for the cooperative solution that might be reached, for the sustainability of such cooperation in infinitely repeated games, and for how the group members coordinate their actions and split the surplus from cooperation among themselves.

From a methodological point of view, our contribution brings together the problem of preemption and the problem of contributing to a group public good. Starting with Olson and Zeckhauser (1966), many aspects of the problem of contributing to a public good have been studied. Much of this literature assumes a group of uncoordinated players with overlapping interests and whose efforts add linearly to determine the aggregate group effort. Hirshleifer (1983) introduced alternatives to this technology—particularly the “best-shot” technology according to which only the largest contribution of a member of the group matters. Barbieri and Malueg (2014) use this group-contribution technology. The question of who volunteers has led to the study of the “volunteer’s dilemma” game (Diekmann, 1985). In this, such a group of players gains and enjoys a public good if at least one of its members makes a contribution, where the contribution has a predefined cost for the contributor, and all players choose simultaneously whether to contribute. A key issue in this volunteer’s dilemma is coordination. Lack of coordination may lead to inefficiency: multiple (or no) players may expend effort.<sup>18</sup> If players can choose when to contribute, then “delay” may serve as a natural coordination device (Bliss and Nalebuff, 1984, Fudenberg and Tirole, 1986). Players with a high contribution cost or with a low stake may be inclined to wait. This can lead to provision by only the player with the lowest cost, but with some delay.

The preemption game between such uncoordinated sets of players adds an element of competition to this dynamic volunteer’s game. It can be seen as a contest between multiple groups, where the members of each group individually decide on their contributions to the group’s contest effort. Contest theory has studied group contests under a variety of assumptions about how groups’ efforts determine the winner group, how group members’ efforts contribute to the aggregate group effort, and what players know about each other, inside the group and across groups. A first approach considered additivity of contributions.<sup>19</sup> In an important departure, Fu, Lu, and Pan (2015) allow for an allocation of tasks among the competing teams. Also more recently, Hirshleifer’s best-shot effort-aggregation technology has been applied in inter-group contests. This includes work by Barbieri, Malueg, and Topolyan (2014), Chowdhury, Lee, and Sheremeta

<sup>15</sup> See, for example, Argenziano and Schmidt-Dengler (2014), Boyarchenko and Levendorskii (2014), Bloch, Fabrizio, and Lippert (2015), Fudenberg, Gilbert, Stiglitz, and Tirole (1983), Mason and Weeds (2010), Nishide and Yagi (2016), and Ruiz-Aliseda (2016).

<sup>16</sup> See, for example, Tirole (1983), Hopenhayn and Squintani (2011, 2016), and Nishihara (2018).

<sup>17</sup> Similar problems might emerge for industries with network externalities (Katz and Shapiro, 1985; Farrell and Saloner, 1986). Preemptive entry might be costly for the preempting firm, but set a *de facto* standard that benefits a subset of firms who prefer this standard compared to one preferred by other firms.

<sup>18</sup> Many aspects of this game have been studied. See, for example, Diekmann (1993) and He, Wang, and Li (2014) for the role of players’ cost asymmetry, and Archetti (2009) and Peña and Noeldeke (2016) for considerations of group size. For a contribution focusing on timing, see Bergstrom (2017).

<sup>19</sup> Seminal contributions in this field are Katz, Nitzan, and Rosenberg (1990) and Esteban and Ray (2001).



(2013), and Barbieri and Malueg (2016).<sup>20</sup> The preemption game between groups relates to this literature on group contests. It describes a group contest with heterogeneous players, incomplete information, and a dynamic timing structure of the contribution decision. This richer dynamic structure plays a coordinating role. Unlike in the waiting games by Bliss and Nalebuff (1984) or Fudenberg and Tirole (1986), however, the timing decision is shaped by *two* factors: the free-riding incentives *and* the preemption threat by players of competing groups. Except for the case of bunching when several players volunteer immediately, for symmetric teams the only player who expends effort is the one with the lowest cost.<sup>21</sup>

We proceed in several steps. Section 2 provides the main analysis. First, we outline the key building blocks of the formal model. Then, we study the equilibrium and its properties, including welfare, in the parameter range in which the equilibrium is interior. Section 3 considers asymmetry between the competing groups and strategies, focusing on group size, strength of the externalities within groups, and the possibility that agents may be differentially informed about teammates and competitors. Section 4 discusses the results and concludes. All proofs are in the Appendix.

## 2. The formal framework

■ We first describe the formal framework that combines the problem of preemption between groups with the problems of free-riding and coordination within each group. Then we turn to the characterization of equilibrium and study its properties.

□ **Players, actions, and payoffs.** We define  $N$  as the set of all players  $i$  and  $\{N_1, N_2, \dots, N_K\}$  a partition of these players into  $K$  groups of identical size with  $n$  players in each group. A representative player is denoted by  $i$ . This player is further characterized by his cost of effort  $c_i$ . All players' cost parameters are drawn independently from the same atomless cumulative distribution function  $F$ . We assume  $F$  is continuous and differentiable on its support  $[\underline{c}, \bar{c}] \subset (0, \infty)$ . We denote the density of  $F$  by  $f$  and assume it to be strictly positive on  $(\underline{c}, \bar{c})$ . Each player  $i$  knows the value of his own  $c_i$  and knows the distribution from which all players' valuations are drawn, but not the values of other players' realized costs, neither for members of his own group nor for members of the other groups.<sup>22</sup>

Player  $i$ 's action is denoted by  $T_i$  and is chosen from the interval  $[0, \infty]$ . The action is the time until which player  $i$  waits to provide the public good to his own group ("grabs"), given that none of the other  $Kn - 1$  players grabbed prior to  $T_i$ . All players choose their  $T_i$  independently and simultaneously, based on the information of their own costs, the distribution  $F$ , and the rules of the game. Players cannot observe the actual choices of grabbing times  $T_i$ , except that, as time goes on, they observe whether another player has grabbed. If a player has not observed any of the other players grabbing prior to time  $T_i$ , then player  $i$  takes action at this point and the game ends.

The gross benefit for each member of a group is  $V$  if a member of the group grabs first. The payoff of player  $i$  with cost  $c$  is equal to  $(V - c)e^{-\rho T}$  if he is the player who grabs first and at time  $T$ , where  $\rho$  is the common discount rate.<sup>23</sup> Should several players, from one or more groups, choose simultaneously, a random mechanism selects one of these players to bear the cost and his

<sup>20</sup> This literature also considered asymmetric conflict, by which one group aggregates effort according to a best-shot technology and the other group aggregates according to a weakest-link technology (Clark and Konrad, 2007; Chowdhury and Topolyan, 2016).

<sup>21</sup> Related problems are discussed in Bonatti and Hörner (2011) and Sahuguet (2006). The latter looks at a group whose members share several tasks. Some of them are more burdensome than others. Players prefer not to take any of the costly tasks, but conditional on taking up one task, they prefer one with a lower cost.

<sup>22</sup> See the discussion in Section 3 for a possible way to relax this assumption.

<sup>23</sup> We interpret  $\rho$  as the sum of a standard time preference for money and an exogenous instantaneous probability that the game ends.

group receives the prize.<sup>24</sup> The payoff of player  $i$  is equal to  $Ve^{-\rho T}$  if a teammate grabs first at time  $T$ , and equal to zero if a player from another group grabs first.

We assume throughout that  $\bar{c} \leq V$ . In light of the empirical literature on single firms' trade policy activism, this might appear restrictive. This literature emphasizes the correlation of firms' trade-policy interest with firm size, such that a joint trade association would not represent them all well, because a large firm might benefit from successful lobbying for a reduction in trade barriers even if it bears the cost ( $c < V$ ), whereas this need not hold for small firms ( $c > V$ ) or even suffer from trade liberalization ( $V < 0$ ). However, as described before, the borders of what defines the set of firms forming a group might be drawn by whether its members have some positive externalities from its members' lobbying activities. And as described following Proposition 4 and in footnote 37,  $\bar{c} \leq V$  is a simplifying assumption and almost all of our results go through when  $\bar{c} > V$ . And the choice of zero as the payoff of members of a non-winning group is a normalization.<sup>25</sup>

Before we turn to the equilibrium analysis, we highlight that the reduced-form we study encompasses the empirical framework with other, more general frameworks, and we also describe limits of its generality.

First, the setup is a reduced form describing a situation in which the volunteer might have a considerable cost of provision, but also considerable private benefits, whereas the positive spillovers inside the group are smaller, and need not be homogenous. This is particularly relevant in the context of single firms that lobby for their favorite trade policy. To be more specific, consider the illustrative example on individual lobbying efforts in which the CEO uses personal access to decision makers or a single firm in an industry lobbies for a policy that gives this firm a benefit  $B_i + V$ , which is composed of the benefit  $V$  applying to each firm in this industry plus a possible additional firm-specific benefit  $B_i$ . However, the lobbying decision has opportunity cost of size  $C_i$ . In this case, the volunteering firm has a net benefit of  $(B_i + V) - C_i = V - (C_i - B_i)$ , whereas all bystanding firms of the same industry enjoy just  $V$ . A firm's idiosyncratic net cost  $C_i - B_i$  might be the firm's private information. For instance, a CEO's opportunity cost  $C_i$  is known to her, but unobserved by other CEOs and is the outcome of an independent draw from the same random distribution for all CEOs, and similar considerations might apply to the component  $B_i$ .<sup>26</sup> This problem reduces to the one described above if we define  $c_i \equiv C_i - B_i$ .

Second, our findings encompass frameworks in which the utility of grabbing and the utility of a free-riding member of the winning group are non-linear functions of the benefit of winning and the cost of grabbing. To see this, normalize the utility of a member of a losing group to zero:  $u(0, 0) = 0$ , let  $u(V, 0)$  be the utility of a free-riding member of the winner group and  $u(V, -c)$  the utility of the grabbing player with a grabbing cost  $c$ , such that  $u(V, 0) > u(V, -c) > u(0, 0) = 0$ . Redefining quantities as

$$\hat{c} \equiv u(V, 0) - u(V, -c), \text{ and } \hat{V} \equiv u(V, 0),$$

the utility of grabbing is simply  $\hat{V} - \hat{c}$ , that of free-riding and winning is  $\hat{V}$ , that of losing remains 0, and  $\hat{c}$  is a random variable that has a cumulative distribution function  $\hat{F}([u(V, 0) - u(V, -c)])$ . Structurally the resulting setup is equivalent to the one we consider, with  $(\hat{c}, \hat{V}, \hat{F})$  replacing  $(c, V, F)$ .

Third, we follow the assumption that is common in the waiting-game literature: grabbing time affects the payoff  $V$  and the grabbing cost  $c_i$  of players just through exponential discounting. The assumption may reasonably well apply to the CEOs' decision problems that motivate

<sup>24</sup> In a first-price auction with externalities, only the winning player pays a price and causes positive externalities for the members of his group. In the preemption game all group members bear some waiting costs, and cost heterogeneity drives the outcome. We thank a reviewer for this comparison.

<sup>25</sup> The analysis can easily be modified to assume that members of non-winning groups receive a non-zero loser prize and that the benefit of being a non-grabbing member of the winning group is some positive amount.

<sup>26</sup> The empirical work on firm lobbying suggests that size is one of the determinants of  $B_i$ , but it is clear that other, less well-observed factors also matter.



our analysis. The cost of volunteering might decline over time, due to technical progress. The cost could also increase if unsolved problems compound over time. Our assumption of time independence rests safely in the middle between these alternatives. We discuss some possible implications of time-dependent benefits or grabbing costs when we turn to a discussion of the welfare properties of preemption between groups.

□ **Properties of an interior equilibrium.** The equilibrium is characterized by players' decision to grab at a time  $T$  if none of the other players grabbed earlier, with  $T$  a function of the player's own cost of grabbing. The characterization focuses on symmetric equilibria, so we describe each player's strategy by a function  $T : [\underline{c}, \bar{c}] \rightarrow [0, \infty]$ , where  $T(\cdot)$  maps the player's own cost  $c$  to the conditional time of own grabbing,  $T(c)$ .

*The equilibrium characterization.* Standard incentive compatibility arguments imply that a player's optimal strategy is weakly increasing in  $c$ . We define a (symmetric) equilibrium as *interior* if the equilibrium strategy  $T$  is strictly increasing on  $[\underline{c}, \bar{c}]$ .<sup>27</sup> Given that  $F$  is atomless and  $T$  is strictly increasing, there are no ties—that is, more than one player grabbing at a given time. Hence, delay that does not change the probability of grabbing first is wasteful. It follows that  $T(\underline{c}) = 0$  and  $T$  is continuous. Because  $T$  is nondecreasing,  $T$  is differentiable almost everywhere. Our first proposition characterizes the interior symmetric equilibrium.

*Proposition 1.* If  $\underline{c} \geq c_0 \equiv \frac{(K-1)n}{Kn-1}V$ , then the unique interior symmetric equilibrium strategy  $T$  satisfies

$$T(\underline{c}) = 0 \quad \text{and} \quad T'(c) = \frac{f(c)}{(1-F(c))\rho(V-c)} \left( c - \underbrace{\frac{(K-1)n}{Kn-1}V}_{\equiv c_0} \right) \quad \forall c \in (\underline{c}, \bar{c}). \quad (1)$$

The equilibrium strategy<sup>28</sup> in Proposition 1 follows intuitively from balancing the marginal cost of delay with its marginal benefit as follows. It is opportune to combine marginal costs and benefits into marginal changes in payoffs if the allocation is unaltered (i.e., a player is not preempted while delaying own grabbing by an instant of time) and if the allocation is altered (i.e., a player is preempted due to the choice of an additional delay). Consider a player  $i$  with cost  $c$  who plans to grab at date  $T(c)$  in equilibrium. The marginal decrease in payoff at time  $T(c)$  brought about by delaying slightly (i.e., by behaving as a player with a slightly higher cost) if the allocation is unaltered is

$$\rho(V-c)dT, \quad (2)$$

that is, the loss in the present (net) value of the prize. If the allocation is altered, that is, if player  $i$  is not the first to grab anymore because of the delay, then the marginal benefit in payoff at time  $T(c)$  is  $c - \frac{(K-1)n}{Kn-1}V = c - c_0$ , which is the saving in the cost of grabbing minus the loss from being preempted by another group. Now let  $h(\cdot)$  be the hazard rate function for  $F$ , that is,  $h(c) \equiv f(c)/(1-F(c))$ . The term  $h(\tilde{c})(Kn-1)$  is the hazard rate of the minimum cost of all other agents at  $\tilde{c}$ : if the cdf of the minimum cost of all other agents is  $G(\tilde{c}) \equiv 1 - (1-F(\tilde{c}))^{Kn-1}$ , then

$$g(\tilde{c}) \equiv G'(\tilde{c}) = (Kn-1)(1-F(\tilde{c}))^{Kn-2}f(\tilde{c}),$$

<sup>27</sup> It will follow from Lemma A1 in the characterization of equilibrium with low costs (in the Appendix) that an equilibrium strategy is interior if and only if it is strictly positive for all  $c > \underline{c}$ .

<sup>28</sup> Note that  $n \geq 2$  is necessary to get any delay in equilibrium. In the absence of possible preemption by another group ( $K = 1$ ), equation (1) reduces to equation (4) in Bliss and Nalebuff (1984) if we take into account that they assume  $\rho = 1$  and derive equilibrium for the game with  $n + 1$  agents.

so

$$\frac{g(\tilde{c})}{1 - G(\tilde{c})} = \frac{(Kn - 1)(1 - F(\tilde{c}))^{Kn-2} f(\tilde{c})}{(1 - F(\tilde{c}))^{Kn-1}} = (Kn - 1) \frac{f(\tilde{c})}{1 - F(\tilde{c})} = (Kn - 1)h(\tilde{c}).$$

Therefore,  $h(c)(Kn - 1)dc$  is the probability that by delaying slightly beyond  $T(c)$  player  $i$  is no longer the first to grab. Combining these observations, the expected marginal change in payoff at time  $T(c)$  brought about by delaying slightly when the allocation is altered turns out to be

$$h(c)(Kn - 1)dc(c - c_0). \quad (3)$$

Now setting (2) equal to (3) yields

$$\frac{dT}{dc} = \frac{h(c)(Kn - 1)}{\rho(V - c)}(c - c_0),$$

which is equivalent to (1).

Note that, for  $c > c_0$ ,  $dT/dc > 0$ : players sort themselves such that those with a higher grabbing cost choose to grab later. Thus, the equilibrium makes an efficient selection: only the player with the lowest cost grabs. Whereas Proposition 1 deals with a case of “sufficiently high costs,” where  $\underline{c} \geq c_0$ , in the Appendix we deal with the possibility of “low costs,” where  $\underline{c} < c_0$ .<sup>29</sup> The arguments developed there establish that the equilibrium described in Proposition 1 is the unique symmetric equilibrium if  $\underline{c} \geq c_0$ , without restricting attention to strictly increasing strategies. In the remainder of this section we maintain the assumption that  $\underline{c} \geq c_0$  and explore the properties of the interior equilibrium.

Because  $T(\underline{c}) = 0$ , we have

$$T(c) = \int_{\underline{c}}^c T'(y) dy, \quad \text{for all } c \in [\underline{c}, \bar{c}].$$

Further, (1) implies that  $\lim_{c \uparrow \bar{c}} T(c) = +\infty$ . Indeed, for any  $\tilde{c} \in (\underline{c}, \bar{c})$  we have

$$\begin{aligned} \lim_{\hat{c} \uparrow \tilde{c}} T(\hat{c}) - T(\tilde{c}) &= \lim_{\hat{c} \uparrow \tilde{c}} \int_{\tilde{c}}^{\hat{c}} T'(c) dc = \lim_{\hat{c} \uparrow \tilde{c}} \int_{\tilde{c}}^{\hat{c}} h(c) \frac{(Kn - 1)(c - c_0)}{\rho(V - c)} dc \\ &\geq \frac{(Kn - 1)(\tilde{c} - c_0)}{\rho(V - \tilde{c})} \lim_{\hat{c} \uparrow \tilde{c}} \int_{\tilde{c}}^{\hat{c}} h(c) dc \\ &= \frac{(Kn - 1)(\tilde{c} - c_0)}{\rho(V - \tilde{c})} \left[ -\lim_{\hat{c} \uparrow \tilde{c}} \log(1 - F(\hat{c})) + \log(1 - F(\tilde{c})) \right] \\ &= +\infty. \end{aligned}$$

Intuitively, equilibrium delay becomes unbounded for the highest-cost type because the rate of payoff change in (3), which captures the marginal net benefit of delay, becomes infinite at  $\bar{c}$ , by  $\lim_{c \uparrow \bar{c}} h(c) = +\infty$ , and this is true regardless of the form of  $F$ .<sup>30</sup>

Recalling from the proof of Proposition 1 that  $U(c^*, c)$ , see (A2), is the payoff to a player with cost  $c$  acting as if his cost were  $c^*$ , by the envelope theorem we can write the equilibrium utility  $U^E(c) \equiv U(c, c)$  as

$$U^E(c) = V - \underline{c} - \int_{\underline{c}}^c e^{-\rho T(y)} (1 - F(y))^{Kn-1} dy. \quad (4)$$

We next turn to the comparative-statics properties of the equilibrium. For this analysis it matters whether the number of group members affects the size of the benefit  $V$  each member of

<sup>29</sup> When costs lower than  $c_0$  are possible, those low-cost types of players will grab immediately, that is, there is partial bunching at  $T = 0$  and an interior equilibrium does not exist.

<sup>30</sup> To see this, let  $H(c) \equiv -\log(1 - F(c))$  and note that  $\lim_{c \uparrow \bar{c}} H(c) = +\infty$ , which, by  $\bar{c} < \infty$ , requires  $H'(\bar{c}) = +\infty$ . But  $H'(c) = h(c)$ ; hence,  $\lim_{c \uparrow \bar{c}} h(c) = +\infty$ .

the winning group enjoys. If a given benefit has to be divided between the group members, then  $V$  becomes a function of  $n$ . We concentrate here on the case in which the benefit  $V$  is non-rival inside the group: winning is a pure public good for the group, such that  $V$  is invariant in  $n$  and the aggregate benefit of the winner group increases linearly in  $n$ .

*Individual grabbing time.* First we examine how individual grabbing times change with model parameters. For comparative statics we include the dependence of  $T$  and  $U^E$  on  $K$  and  $n$  and implicitly assume changes in parameters continue to yield interior equilibria.<sup>31</sup>

*Proposition 2.* For a player of a given type  $c$ , the grabbing time  $T(c; K, n)$  is (linearly) decreasing in the number  $K$  of groups and it is (linearly) increasing in the size  $n$  of the groups. Consider now a new cost distribution  $\hat{F}$  obtained from  $F$  by a parallel rightward shift by  $\Delta > 0$  (i.e.,  $\hat{F}(y + \Delta) = F(y)$ ), and denote with  $\hat{T}$  and  $T$  the respective equilibrium strategies obtained from (1). Then  $\hat{T}(c + \Delta) > T(c)$  for any  $c \in (\underline{c}, \bar{c})$ .

According to Proposition 2 individual decision makers tend to grab earlier if  $K$ , the intensity in competition, is higher, and they grab less hastily if the group has more members. If there are more groups or smaller groups, the strict order of grabbing times is maintained but the whole function  $T(\cdot)$  shifts—for given behavior of other players, it turns out that each player has a higher preemption motive and a smaller free-riding incentive. The intuition can be gained from an individual player's partial optimal response to changes in  $n$  and  $K$ , as overall equilibrium effects qualitatively go in the same direction (although they may be attenuated). Delay becomes less attractive for increased  $K$ : if a player does not grab and is preempted, for higher  $K$  it is more likely that the preemptor is from a rival group and this ends up decreasing the payoff in (3). Similarly, delay becomes more attractive for increased  $n$ . For a player who is just indifferent between grabbing or waiting at some value  $T$ , it becomes more likely that another player grabs between  $T$  and  $T + dT$ . Moreover, it is more likely that that this other player belongs to one's own group, because  $(n - 1)/[(n - 1) + n(K - 1)]$  is increasing in  $n$ . Thus, waiting becomes strictly superior for a higher  $n$ .

The last part of Proposition 2 allows an interpretation of the consequences of a reduction in the private benefit of grabbing  $B$  described in Section 2. As discussed there, a decrease in the grabbing player's private gross benefit  $B$  by  $\Delta$  effects a  $\Delta$ -sized rightward shift of the cost distribution. Intuitively, if grabbing becomes less beneficial for everyone, the incentive to preempt decreases and agents grab later. But this does not simply result in a parallel rightward shift of the equilibrium strategy by  $\Delta$ : because the overall competitive pressure has decreased, there is also an equilibrium effect that pushes each cost type to delay further.

The dynamic structure of the preemption conflict between groups can be compared to the between-groups contest problem in Barbieri and Malueg (2016) in which all players choose their all-pay contest efforts in a static game with a best-shot contest technology. In contrast to Barbieri and Malueg (2016), it is interesting to note that the qualitative comparative static properties of the grabbing time in the preemption game require no assumptions regarding the shape of  $F$  (e.g., nothing is invoked about elasticity of  $F$ ).<sup>32</sup>

*Expected ending time.* Next, we calculate the expected time at which the game stops. Denote by  $ET^E$  the stopping time at which the first grabbing occurs when there are  $K$  groups of  $n$  players each and each player follows the strategy in Proposition 1. The following lemma relates  $ET^E$  to the model parameters.

<sup>31</sup> The condition for interior equilibrium was  $\underline{c} \geq c_0 \equiv (K - 1)nV/(Kn - 1)$ . Because  $c_0$  is decreasing in  $n$ , starting from an interior equilibrium a higher  $n$  yields an interior equilibrium, too. In contrast,  $c_0$  increases in  $K$ , with limit  $V$ . Consequently, *cet. par.*, as  $K$  becomes sufficiently large the symmetric equilibrium will not be interior.

<sup>32</sup> In general, elasticity of  $F$ , which played a crucial role in determining the direction of many comparative statics results in Barbieri and Malueg (2016), plays no role in our setup. See also footnote 35.

*Lemma 1.* Suppose the expected time at which grabbing first occurs,  $ET^E$ , is finite.<sup>33</sup> Then  $ET^E$  satisfies

$$K\rho ET^E + (K - 1) = \int_{\underline{c}}^{\bar{c}} \frac{n-1}{n} \frac{y}{V-y} d(1 - (1 - F(y))^{Kn}). \quad (5)$$

In what follows we assume  $ET^E$  is finite. From (5) the next proposition derives comparative statics for the expected duration of the game.

*Proposition 3.* The following comparative statics results hold for the expected time at which the game ends: The expected end occurs sooner (i) if the discount rate  $\rho$  is higher or if the gross benefit  $V$  for each group member is higher or (ii) if the number  $K$  of groups is higher. (iii) If  $\frac{y(1-F(y))^K}{V-y}$  is decreasing (increasing) in  $y$ , then  $ET^E$  increases (decreases) in  $n$ . (iv) A first-order stochastic dominance (FOSD) increase in the costs increases  $ET^E$ . (v) A second-order stochastic dominance (SOSD) increase in the risk of the overall minimum cost increases  $ET^E$ .

The unambiguous comparative static results (i) and (ii) in Proposition 3 are intuitively plausible: increases in  $\rho$  and  $V$  make grabbing more attractive and so do increases in  $K$ , as shown in Proposition 2. The possibility described in part (iii) that  $n$  may increase or decrease  $ET^E$  is also intuitive. An increase in  $n$  has two countervailing effects: each player with a given type  $c$  grabs later, but as the number of players increases, the probability distribution of the lowest cost type shifts. Indeed, the probability that the lowest realized cost, among those of  $Kn$  players, is higher than a given  $c$  becomes less and less as  $n$  increases, for all possible  $c$  inside the support. We identify a sufficient condition for either effect to dominate. The result in part (iv) is also intuitive. For instance, consider the rightward shift in the cost distribution analyzed in Proposition 2, which, recalling our discussion in Section 2, can be interpreted as a decrease in the private benefit of grabbing  $B$ . This rightward shift in costs increases  $T$  as described in Proposition 2, and therefore it increases  $ET^E$  as well. Part (iv) of Proposition 3 extends these consideration to FOSD increases in costs. The results in parts (iii) and (v) deserve more attention, which we provide by way of two examples after discussing welfare.<sup>34</sup>

The distribution of the cost of volunteering also matters for delay. Delay is, on average, larger if the *minimum* cost of volunteering is more dispersed. But, as we show in Section 2, delay can be shorter if the *individual* cost of volunteering is more dispersed.

It is instructive to apply these comparative statics to the main example on firm lobbying and the empirical regularities described in the Introduction. The equilibrium shows that firms with the smallest net cost of volunteering are active. As this is influenced by  $C_i$  as well as by  $B_i$ , for a given cost of lobbying, the firm that has a higher idiosyncratic benefit is more likely to lobby. Likewise, our result that grabbing time tends to infinity for firms with the largest realized cost, regardless of the details of the distribution, reinforces the result that firms with small idiosyncratic benefit never lobby. Furthermore, the results on the dispersion of the individual cost of volunteering nicely correspond with the idea that firm lobbying tends to be more relevant if the industry is more heterogeneous. Finally, increases in competition push agents to act sooner, which can be interpreted as saying that tighter demands on scarce legislative resources increase lobbying.

*Expected payoffs.* Finally, we describe the effects parameter changes have on expected utility.

<sup>33</sup> A sufficient condition is  $\bar{c} < V$ , or, if  $\bar{c} = V$ ,  $\lim_{c \uparrow \bar{c}} (1 - F(c))^{Kn-1} / (\bar{c} - c) < \infty$ . Details are available from the authors.

<sup>34</sup> These results extend the homologous ones in Theorems 4 and 5 of Bliss and Nalebuff (1984); however, their Theorem 5 characterizes only the behavior at the tails.

**Proposition 4.** (i) The expected payoff of a player with a given cost type  $c$  is higher if the size of all groups is larger. (ii) The expected payoff is constant with respect to changes in the number of groups.

Intuitively, the main, direct effect of an increase in  $n$  is to dilute the cost burden of grabbing among a larger number of players, while at the same time keeping each player's expected gross benefit constant at  $V/K$  in a symmetric equilibrium. As determined in Proposition 3, the expected time at which the game ends may increase, but this equilibrium effect is overwhelmed by the direct effect. The proof of the proposition shows that this holds strictly, not only *ex ante*, but also for all players, irrespective of their cost type, except for the type with the lowest possible cost realization who always grabs immediately. If there is only one group (as in Bliss and Nalebuff, 1984), this is easy to understand: each player faces a higher probability that another player from his group grabs first, which preserves the benefit, but probabilistically shifts the cost burden of grabbing. Proposition 4 shows that this effect carries over to a multigroup framework with preemption.

We assumed that for almost all possible cost types volunteering is better than not to volunteer if no one else takes action. What happens if the cost  $c$  exceeds the value  $V$  for some cost types? It is a dominant strategy for them never to grab ( $T = +\infty$ ). But even if such types exist, the differential equation in Proposition 1 continues to characterize the equilibrium strategy for lower cost types, including  $T(c) \rightarrow \infty$  as the cost approaches  $V$  from below. Similarly, Proposition 2 continues to hold for  $c < V$ . However, Proposition 3 fails to hold because there is now a positive probability that no one ever grabs, implying that the expected size of the delay becomes infinite. Finally, Proposition 4 continues to hold because players with  $c \in (V, \bar{c}]$  never take action, such that they all have the same expected utility equal to  $U^E(V)$ , and the analysis  $c \in (\underline{c}, V]$  continues to apply.

The payoff neutrality with respect to the number of groups is less intuitive. A larger number of groups makes it more likely that a single group is preempted. This reduces all players' expected payoffs. However, the increase in preemption pressure induces players of given types to grab earlier. This reduces wasteful delay. The proposition shows that the two effects just offset each other.

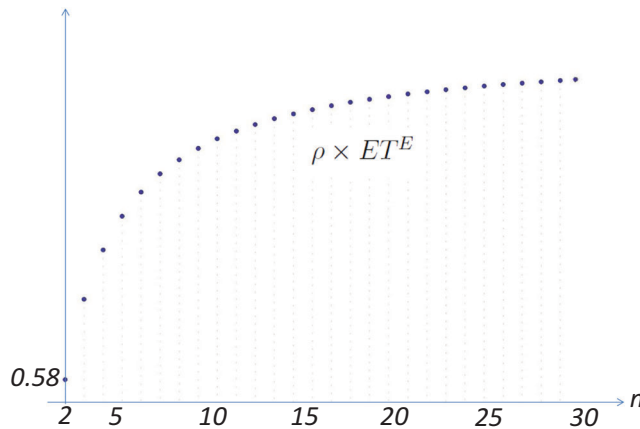
□ **Welfare.** A natural measure of welfare in the preemption game is the sum of payoffs of all players in all groups. A maximum  $nV - c_{i_{\min}}$  of this sum is reached in the context of *ex ante* symmetric groups of given size  $n$  if the player  $i_{\min}$  who has the lowest cost  $c_{i_{\min}}$  of all players takes action at  $T = 0$ . In comparison to this welfare benchmark, the welfare in equilibrium is  $[nV - c_{i_{\min}}]e^{-\rho T(c_{i_{\min}})}$ . In the symmetric case with groups of equal size it does not matter for welfare which group receives the public good. The player who has the smallest cost among all players in all groups should grab and does grab in the equilibrium. The delay of taking action by  $T(c_{i_{\min}})$  in the equilibrium constitutes the only efficiency loss. It discounts the sum of total net benefits.

Proposition 4 discusses payoffs if the number of agents in the model is allowed to change. An alternative comparison is what happens after reconstituting a given number of players into a smaller number of larger symmetric groups. By Proposition 2, we know that decreasing the number of groups (without changing team size) would increase individual grabbing times, and then increasing the number of players per group would further increase grabbing times. As individual grabbing times increase, the reorganization also implies the expected duration of the game increases. Finally, Proposition 4 implies that reorganizing the players into fewer larger groups strictly increases expected payoffs for all types but  $\underline{c}$ . As discussed, these conclusions depend strongly on the question whether the winner prize is a pure public good for all members of the group, or whether a change in group size affects the benefit  $V$  which each of the members receives.

It is important to note that our setup has a time-invariant cost of grabbing. As discussed above, there are reasons why the cost of grabbing may increase or decrease over time. We took

FIGURE 1

EFFECTS OF INCREASING  $n$  ON  $ET^E$ :  $V = K = 2$  AND  $F(c) = 2(c - \frac{3}{2}) \quad \forall c \in [\frac{3}{2}, 2]$  [Color figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]



the middle position, where these costs are constant, except for time discounting, and this is why an immediate action is always efficient here. However, the analysis is suggestive for what would happen if early action is inefficient due to a high social cost of early action. Free-riding incentives that cause a delay might then reduce the efficiency costs from hasty preemption behavior. At the same time, the fear that other groups might preempt a player's group can reduce the efficiency costs of free-riding. If there is a cost of excessive delay as well as a cost of premature action, the specific parametric situation would determine whether action is taken too early or too late from an efficiency perspective.

□ **Numerical examples.** We illustrate some of the comparative-statics results by way of numerical examples. The first illustrates a non-monotonicity result suggested by part (iii) of Proposition 3.

*Example 1* (The effect of  $n$  on expected stopping time). Let  $V = 2$  and assume costs are distributed according to  $F(c) = 2^t(c - \frac{3}{2})^t$ , for  $c \in [\frac{3}{2}, 2]$ .

First note that

$$\text{sign} \left[ \frac{d}{dy} \left( \frac{y(1 - F(y))^K}{V - y} \right) \right] = \text{sign}[V - Ky(V - y)h(y)]. \quad (6)$$

We begin with  $K = 2$ , in which case  $c_0 = \frac{n}{2n-1}$ . Thus, the symmetric equilibrium is interior for all  $n \geq 2$ . Now consider  $t = 1$ . Here, we see that

$$V - Ky(V - y)h(y) = 2(1 - y),$$

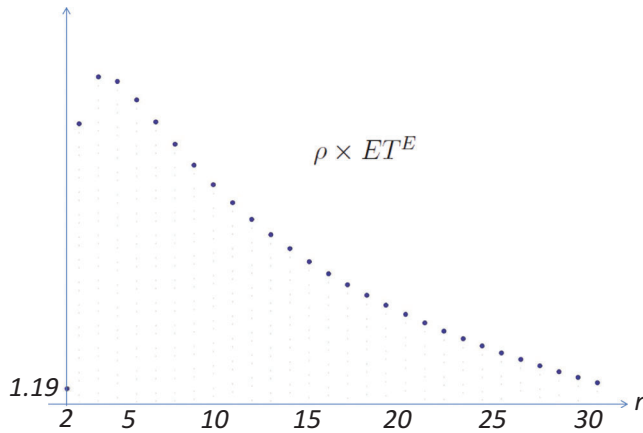
which is negative for the relevant range. Therefore, (6) and part (iii) of Proposition 3 imply  $ET^E$  is increasing in  $n$ ; Figure 1 depicts the relationship between  $\rho ET^E$  and  $n$  for  $n = 2, \dots, 30$ .

A pattern similar to that in Figure 1 holds for any  $t \leq 1$ . Thus, for  $t \leq 1$  we see that the free-riding effect is very strong and it overwhelms the presence of additional agents on each team, which would otherwise lead to a lower expected stopping time. In contrast, if  $t = 2$ , for example, we find that  $ET^E$  first increases and then decreases with  $n$ , as depicted in Figure 2. Here we see that  $ET^E$  increases for  $n$  going from 2 to 4, but further increases in team size decrease  $ET^E$  because the "order-statistic" effect of having a better distribution of the minimum cost eventually prevails. Proposition 3 implies that  $\frac{y(1-F(y))^K}{V-y}$  must be increasing for at least a range of  $y$ ; indeed,



FIGURE 2

EFFECTS OF INCREASING  $n$  ON  $ET^E$ :  $V = K = 2$  AND  $F(c) = 4(c - \frac{3}{2})^2 \quad \forall c \in [\frac{3}{2}, 2]$  [Color figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]



it is inverse U-shaped in  $y$ , which helps explain why  $ET^E$  is first increasing with  $n$ , then it turns decreasing and it stays so. As  $n$  grows, the distribution of the minimum costs is more and more concentrated toward lower values of  $c$ . Therefore, for  $n$  sufficiently large, the relevant part of  $\frac{y(1-F(y))^K}{V-y}$  is increasing and the result follows as for Proposition 3. A pattern similar to that in Figure 2 is displayed by all parameterizations with  $t > 1$ .<sup>35</sup> It is interesting to note that, with more competing teams, the switch in the direction of the relationship between  $ET^E$  and  $n$  occurs later. Indeed, by (6), if  $K$  increases, then  $\frac{y(1-F(y))^K}{V-y}$  becomes decreasing for a larger set of  $y$ . For example, if  $t = 2$  and  $K = 3$ , then  $ET^E$  remains increasing in  $n$  up to  $n = 7$ .<sup>36</sup>

We next illustrate the distributional effect described in part (v) of Proposition 3.

*Example 2* (The effect of SOSD changes in the cost distribution). Fix  $K = 2$ ,  $V = 3$ ,  $\rho = 1$ , and  $n = 3$ , and let costs be uniformly distributed on  $[\underline{c}, \bar{c}]$ .

Consider first  $\underline{c} = 2$  and  $\bar{c} = 3$ . Then, the average overall minimum cost is approximately 2.143 and (5) yields  $ET^E \approx 0.367$ . Consider now  $\underline{c} = 2.1$  and  $\bar{c} = 2.4$ . Then, the average minimum cost remains 2.143, approximately, and one can show that we have effected a SOSD reduction in risk of the cost distribution of the overall minimum cost. Using (5), we now have that  $ET^E$  decreases to approximately 0.336, and the direction of the change is in accordance with part (v) of Proposition 3.

If one instead performs a mean-preserving spread of the individual cost distribution, then the result can be different. Consider for instance a uniform cost distribution with  $\underline{c} = 2.1$  and  $\bar{c} = 2.9$ , which keeps the individual expected cost at 2.5, but is less risky than our initial uniform distribution on  $[2, 3]$ . Note that the average minimum cost *increases* to 2.214 for individual costs that are uniformly distributed on  $[2.1, 2.9]$ . And the direction of the comparative statics reverses, as  $ET^E$  increases to 0.465, approximately.

<sup>35</sup> In contrast with Barbieri and Malueg (2016), elasticity of  $F$  plays no role in determining the direction of our comparative statics. Indeed,  $F(c)$  is elastic for any  $t > 1$ , but Figure 2 shows an ambiguous effect of  $n$  on  $ET^E$ .

<sup>36</sup> One needs to be a little careful here because for  $K = 3$  and  $n = 2$ ,  $c_0 = 8/5 > 3/2$ , which implies we need to analyze the corner solution. But as explained in the characterization of the equilibrium with low costs in the Appendix, the comparative statics work out here, too. It is the case that for  $K = 3$  and  $n \geq 3$ , we have  $c_0 \leq 3/2$ .

### 3. Two asymmetric groups

■ The analysis of preemption between symmetric groups revealed the general trade-off between waiting and grabbing, and how it depends on the size of groups and their number. In many areas in which preemption between groups emerges, groups are asymmetric; for example, they may differ in size. In a grabbing competition between groups of different size the questions arise whether a larger group is more likely to preempt, and whether its members earn larger payoffs than the members of competing groups. Formally, we consider two groups of different size and label the first group “small” and indicate the number of players in the small group with  $n_s$ . Similarly, the second group is “large” and has  $n_l$  members, with  $n_l \geq n_s$ . In what follows, we restrict attention to interior equilibria in which all agents within a group adopt the same strategy, but these strategies are allowed to vary between groups. We label these strategies as  $T_s$  and  $T_l$ , respectively. Our focus is to characterize equilibrium, to identify systematic differences in behavior between members of the small and large groups, and to establish which group is most likely to win.

Beyond size issues, we are also interested in payoff differences between groups and among agents. As discussed in Section 2, we allow the grabbing agent to have a larger gross utility than teammates who benefit from the externality. Unlike our treatment in Section 2, we now carry two explicit separate values for each group, rather than reinterpreting  $c = C - B$ . Therefore, in the small group the grabbing agent now receives  $V_s^g$ , whereas the other members obtain  $V_s^e$ . Similarly, in the large group we now have  $V_l^g$  and  $V_l^e$ . Our objective is to determine how the strength of the externalities affects the probability of victory and utility.

It turns out that, at a minor notational cost but with great benefit for exposition, it is possible to consider also two different cdfs of costs  $F_s$  and  $F_l$ , with densities  $f_s$  and  $f_l$ , respectively. (Similarly, one could consider two different discount factors.) After characterizing equilibrium, in our comparative statics we will consider changes one at a time: First, we will assume that the only difference between groups is size, and then we will consider changes in the strength of the externalities within groups.

Consider the calculus of a type- $c$  player in the small group contemplating grabbing at  $T_s(c^*)$ . His expected utility is

$$U_s(c^*, c) = \int_{c^*}^{\bar{c}} (V_s^g - c) e^{-\rho T_s(c^*)} (1 - F_l(T_l^{-1}(T_s(c^*))))^{n_l} d[1 - (1 - F_s(x))^{n_s-1}] \\ + \int_{\underline{c}}^{c^*} V_s^e e^{-\rho T_s(x)} (1 - F_l(T_l^{-1}(T_s(x))))^{n_l} d[1 - (1 - F_s(x))^{n_s-1}], \quad (7)$$

which is entirely analogous to (A2): the first addendum of the payoff displayed in (7) captures the possibility that this player carries his group to victory, and the second corresponds to a teammate carrying the group to victory.

We now use a construction due to Amann and Leininger (1996). Define  $k(c) \equiv T_l^{-1}(T_s(c))$ , so that  $k'(c) = \frac{T_s'(c)}{T_l'(k(c))}$ . Then, the first-order condition  $\frac{\partial U_s(c^*, c)}{\partial c^*} = 0$  at  $c^* = c$  gives

$$\frac{(V_s^e - (V_s^g - c))(1 - F_l(k(c))(n_s - 1)f_s(c) - (V_s^g - c)(1 - F_s(c))n_l f_l(k(c))k'(c))}{(V_s^g - c)(1 - F_s(c))(1 - F_l(k(c)))\rho} = T_s'(c); \quad (8)$$

and, proceeding similarly for the large group we have that

$$\frac{(V_l^e - (V_l^g - k(c)))(1 - F_s(c)(n_l - 1)f_l(k(c))k'(c) - (V_l^g - k(c))(1 - F_l(k(c)))n_s f_s(c))}{(V_l^g - k(c))(1 - F_l(k(c)))(1 - F_s(c))\rho} = T_s'(c). \quad (9)$$

Equalizing (8) and (9) we obtain

$$\left( \frac{V_s^e(n_s - 1)}{V_s^g - c} + 1 \right) \frac{f_s(c)}{1 - F_s(c)} \left[ \left( \frac{V_l^e(n_l - 1)}{V_l^g - k(c)} + 1 \right) \frac{f_l(k(c))}{1 - F_l(k(c))} \right]^{-1} = k'(c), \quad (10)$$

an ordinary differential equation for  $k(c)$ . After solving (10) with the initial condition  $k(\underline{c}) = \underline{c}$ , we can substitute  $k(c)$  into (8) and obtain the equilibrium  $T_s$ . Because we are considering an interior equilibrium, we need to ensure that  $T_s$  is strictly increasing. Combining (9) and (10), we obtain

$$T'_s(c) = \frac{f_s(c)}{1 - F_s(c)} \frac{n_s}{\rho} \left[ \frac{(V_l^e - (V_l^g - k(c)))(n_l - 1)}{V_l^e(n_l - 1) + (V_l^g - k(c))} \times \frac{V_s^e(n_s - 1) + (V_s^g - c)}{n_s(V_s^g - c)} - 1 \right]; \quad (11)$$

therefore,  $T'_s(c) \geq 0$  is ensured if

$$\frac{(V_l^e - (V_l^g - k(c)))(n_l - 1)}{V_l^e(n_l - 1) + (V_l^g - k(c))} \times \frac{V_s^e(n_s - 1) + (V_s^g - c)}{n_s(V_s^g - c)} \geq 1, \quad \forall c. \quad (12)$$

Because the left-hand side of (12) is strictly increasing in  $c$  because  $n_s > 1$ , a necessary and sufficient condition for  $T_s$  to be strictly increasing is that (12) holds at  $\underline{c}$ . By  $k(\underline{c}) = \underline{c}$ , (12) boils down to

$$\frac{(V_l^e - (V_l^g - \underline{c}))(n_l - 1)}{V_l^e(n_l - 1) + (V_l^g - \underline{c})} \times \frac{V_s^e(n_s - 1) + (V_s^g - \underline{c})}{n_s(V_s^g - \underline{c})} \geq 1. \quad (13)$$

Because  $T_l(c) = T_s(k^{-1}(c))$ , (13) also ensures that  $T_l(c)$  is strictly increasing. Notice that if  $F_s = F_l = F$ ,  $n_l = n_s = n$ , and  $V_l^e = V_s^e = V_l^g = V_s^g = V$ , then  $k(c) = c$  solves (10) with initial condition  $k(\underline{c}) = \underline{c}$ . As expected, this means  $T_s = T_l = T$ . Further, substituting  $k(c) = c$  into (9), we obtain (1), and (13) simplifies to

$$\frac{\underline{c}(n - 1)}{1} \frac{1}{n(V - \underline{c})} \geq 1 \iff \underline{c}(2n - 1) \geq nV,$$

so we essentially recover Proposition 1 when the number of symmetric groups  $K$  equals two.

Although deriving equilibrium strategies proves analytically infeasible when groups are asymmetric, some of the comparative statics of interest can be deduced using (10) directly. The following result shows that, if values and distributions are identical, agents in the small group act more aggressively.

**Proposition 5.** Consider the interior equilibrium described by (8), (9), and (10) with initial condition  $k(\underline{c}) = \underline{c}$ . If  $V_l^e = V_s^e = V_l^g = V_s^g = V$ ,  $F_s = F_l = F$ , and  $n_l > n_s$ , then  $T_s(c) < T_l(c)$  for any  $c \in (\underline{c}, \bar{c})$ .

Intuitively, at  $\underline{c}$  it must hold that  $T_l(\underline{c}) = T_s(\underline{c}) = 0$ . In both groups, the players with the lowest possible cost grab immediately as any such player cannot hope to be preempted by a teammate. Moreover,  $T'_l(\underline{c}) > T'_s(\underline{c})$  holds. To see this, consider the incentives of a player with  $c = \underline{c} + \varepsilon$  for small positive  $\varepsilon$  if he waits for some marginal unit of time  $dt$ . If the large group is, for instance, twice as large as the small group, then compared to a member of the large group, a member of the small group would face twice as high a probability of being preempted by a member of the other group. Moreover, the chances that a teammate grabs in the interval  $[0, dt]$  is smaller for members of the small group; thus, waiting is less attractive for members of the small group. Similar arguments rule out any other crossings between  $T_s$  and  $T_l$  before  $\bar{c}$ .

Also, it is not the case that there exists a positive measure of cost types in the large group that never grab. In other words, any solution of (10) with initial condition  $k(\underline{c}) = \underline{c}$  ends up satisfying  $k(\bar{c}) = \bar{c}$ . The apparent contradiction is resolved by noticing that  $\lim_{c \uparrow \bar{c}} T_s(c) = +\infty$ , just as it happens in the symmetric model.<sup>37</sup> We collect these observations in the following result.

<sup>37</sup> Here too our results go through if  $\bar{c} > V$ . For types  $c \in (V, \bar{c}]$  it is a dominant strategy never to grab ( $T = +\infty$ ). For  $c < V$ , (8), (9), and (10) with initial condition  $k(\underline{c}) = \underline{c}$  continue to characterize equilibrium. Proposition 5 is unaffected for  $c < V$ . And now we have  $k(V) = V$  and  $T_s(c) \rightarrow \infty$  as  $c \uparrow V$ .

TABLE 1 *Ex Ante Winning Probability For The Small Group,  $V = 2, n_s = 2, F_l(c) = F_s(c) = F(c)$  on  $[3/2, 2]$* 

$n_l \downarrow$	$F(c) = 1 - (4 - 2c)^5$	$F(c) = 1 - (4 - 2c)^2$	$F(c) = 2c - 3$	$F(c) = (2c - 3)^2$	$F(c) = (2c - 3)^5$
2	0.5	0.5	0.5	0.5	0.5
3	0.541360	0.535435	0.526259	0.518673	0.513964
4	0.559099	0.550223	0.536429	0.523292	0.514499
10	0.587210	0.573159	0.551014	0.522003	0.499090
20	0.595656	0.579926	0.554979	0.516549	0.483045
30	0.598380	0.582096	0.556211	0.513305	0.473895
50	0.600526	0.583802	0.557166	0.509585	0.463137
100	0.602114	0.585062	0.557862	0.505393	0.450186
1000	0.603447	0.586120	0.558421	0.497527	0.419639

*Proposition 6.* Consider the interior equilibrium described by (8), (9), and (10) with initial condition  $k(\underline{c}) = \underline{c}$ . Any solution  $k(c)$  satisfies  $k(\bar{c}) = \bar{c}$ . And the corresponding  $T_s(c)$  derived from (11) with initial condition  $T_s(\underline{c}) = 0$  has  $\lim_{c \uparrow \bar{c}} T_s(c) = +\infty$ .

By Proposition 5, we see that a true trade-off emerges when we try to establish whether the small or the large group is more likely to win. Agents of a given cost type are less hesitant in the small group, but the distribution of the minimum cost in the large group is “better,” assuming the only difference between groups is size. We denote the small group’s probability of winning by  $P_s^w$ . By the definition of  $k$ , we have

$$P_s^w = \int_{\underline{c}}^{\bar{c}} (1 - F_l(k(c)))^{n_l} d[1 - (1 - F_s(c))^{n_s}]. \quad (14)$$

Table 1 describes  $P_s^w$  as we increase the size of the large group and hold the small group to two members. The calculations are performed for variations of the power distribution cdf of costs in Example 1.

We therefore see that either the large or the small group can be more likely to win.<sup>38</sup> It is worth noting that the large group can be less likely to win than the small group even if the prize is a pure public good. Thus, the size trade-off does not have a simple solution.

We now turn to the effects of asymmetries in payoffs, performing the analysis for changes in the values for the small group. However, our result applies equally well to the large group, *mutatis mutandis*. In particular, we analyze the effect of a stronger within-group externality. Two countervailing effects arise. On the one hand, agents benefit more when a teammate grabs. On the other hand, one might expect free-riding to worsen and reduce the probability of grabbing. Our next results resolves the trade-off between these two effects.

*Proposition 7.* Consider the interior equilibrium described by (8), (9), and (10) with initial condition  $k(\underline{c}) = \underline{c}$ . An increase in  $V_s^e$  reduces the probability of victory of the small group, but it increases the payoff of any cost type in the small group, except for  $c = \underline{c}$ .

Intuitively, consider an increase of  $V_s^e$  to  $V_s^e + \Delta$ . Let player  $i$  in the small group be indifferent whether to grab at time  $T_i$  for given  $V_s^e$ . Suppose for a moment that the increase in  $V_s^e$  to  $V_s^e + \Delta$  does not change the grabbing behavior of all other players. If the benefit of grabbing remains  $V_s^g$  but the benefit of waiting and winning increases from  $V_s^e$  to  $V_s^e + \Delta$ , the indifference for  $i$  no longer holds and player  $i$  prefers to wait a little longer. This partial effect is indicative for the equilibrium effect. Furthermore, an increase in  $V_s^e$  also means that the public good of winning is more valuable for each member of the small group if the member free-rides. The proposition explains that this increase in the group’s payoffs overcompensates the the equilibrium effect of decreased win probability.

<sup>38</sup> In the Appendix, Proposition A1 gives a technical condition sufficient for the group-size effect to be negative.

We conclude this section discussing the possibility that agents may be differentially informed about teammates and competitors. In many of the examples described in the Introduction, it makes sense to posit that being part of the same team may reveal more information about teammates than competitors. It turns out that the techniques developed in this section can be used to characterize equilibrium when agents are differentially informed about teammates and competitors.

An especially tractable framework has two *ex ante* symmetric groups. Within each group, and independently across groups, agents' values are independently drawn either from  $F_H$  or from  $F_L$ . Within each group, team members are aware of their individual costs and of whether teammates' costs are drawn from  $F_H$  or  $F_L$ , but they do not know which distribution is the "true" one for the other group. Therefore, groups are (potentially) asymmetric at the *interim* moment when the distribution of values becomes common knowledge among teammates. Thus, the tools developed here for asymmetric groups become relevant.

In a symmetric equilibrium, there will be one grabbing function  $T_H$ , which is optimal after observing  $F_H$ , and one grabbing function  $T_L$ , which is optimal after observing  $F_L$ . Characterization of  $T_H$  and  $T_L$  proceeds just as in earlier in this section,<sup>39</sup> with FOCs analogous to (8) and (9), and a differential equation for  $T_L^{-1}(T_H(c))$  similar to (10).

Although the resulting equations do not admit a closed-form solution, one can show that an interesting pattern emerges. Suppose that  $F_H(c)$  "hazard-rate dominates"  $F_L(c)$ , so that costs under  $F_H(c)$  tend to be higher than under  $F_L(c)$ . Then one can show that agents are more aggressive in grabbing if they know that within their group costs are derived from the "higher" distribution, rather than if they knew costs were drawn from the "lower" distribution. Intuitively, this occurs because knowing that costs within a group tend to be large decreases the incentive to free-ride on teammates, and therefore grabbing occurs sooner.

## 4. Conclusion

■ We have given a full characterization of symmetric equilibrium preemption between groups when each group member can volunteer to preempt other groups on behalf of his own group. The analysis is based on the assumption that the act of preemption is costly for the member who carries it out and members have private information about their own costs of the preemption activity.

For symmetric groups, the symmetric equilibrium exists and is unique. Other than for a possible non-degenerate set of lowest costs types who all may act immediately, we found that players wait some time before taking action. They wait longer the higher are their own costs of acting. At an interior equilibrium no player acts immediately; hence, the preemption task is carried out by the player with the lowest cost. A player of a given cost type also waits longer the larger the group and the smaller the number of rival groups. Overall, however, an increase in group size need not increase the expected equilibrium delay: although each player type plans to wait longer in this case, the random composition of the minimum cost type improves as the group size increases, and this makes it more likely that a group has a member with a very low cost who would take early preemption action.

We have also analyzed differences in group size, concentrating on the case of two groups. We find that members with a given cost type choose to act earlier if they belong to the smaller group. The larger group has a "better" order statistic for the lowest-cost member, but has stronger free-riding incentives. Consequently, depending on parameter values, either group may be more likely to win. These results are important for the role of group size as a competitive factor, showing no clear overall advantage for the small or the large group.

The described conflict between free-riding incentives and the fear of preemption should play a role in many contexts. We refer to empirical findings and anecdotal evidence that this problem is of immediate empirical relevance when single firms from different industries decide

<sup>39</sup> The details of the derivation are available from the authors upon request.

if and when they want to engage in stand-alone lobbying to further their group's interest. In this context, the analysis makes clear predictions as to which company will decide to act and when, depending on its own costs and advantages, group sizes, cost distributions, and the competition for influence on political decision-makers.

## Appendix

*Proof of Proposition 1.* If the strategy  $T$  is strictly increasing, then a player with value  $\underline{c}$  knows he will grab first, preempting all others. Therefore, he will choose  $T(\underline{c}) = 0$  because if  $T(\underline{c})$  were larger than zero, then grabbing the prize an instant sooner would be profitable deviation as it would reduce wasteful delay. Similarly, given that  $T$  is strictly increasing it must be continuous, for otherwise some types just above the point of discontinuity could increase their payoffs by reducing their grabbing time to avoid wasteful delay.

Now suppose  $T(\cdot)$  is continuous and strictly increasing. Consider the calculus of a single player  $i$  contemplating grabbing at  $T(c^*)$ . We first determine the probability that someone *else* will take action before date  $t$ . This probability depends on the minimum realized cost among the other  $Kn - 1$  players. The cumulative distribution function of the minimum cost among them, denoted by  $c_{\min}$ , is given by

$$\Pr(c_{\min} \leq x) = 1 - \Pr(\text{all other } Kn - 1 \text{ costs exceed } x) = 1 - (1 - F(x))^{Kn-1},$$

for which the associated density function is

$$(Kn - 1)(1 - F(x))^{Kn-2} f(x). \quad (\text{A1})$$

Moreover, if the game stops before  $T(c^*)$ , then the probability that  $i$ 's team has won is  $\frac{n-1}{Kn-1}$  because players are acting symmetrically. Following Bliss and Nalebuff (1984), we can now write the payoff to a player with cost  $c$  (a "type- $c$  player") acting as if his cost were  $c^*$  as

$$\begin{aligned} U(c^*, c) &= (V - c)e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-1} + \frac{n-1}{Kn-1} V \int_{\underline{c}}^{c^*} e^{-\rho T(x)} (Kn - 1)(1 - F(x))^{Kn-2} f(x) dx \\ &= (V - c)e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-1} + (n-1)V \int_{\underline{c}}^{c^*} e^{-\rho T(x)} (1 - F(x))^{Kn-2} f(x) dx. \end{aligned} \quad (\text{A2})$$

The first addendum of the payoff displayed in (A2) captures the possibility that this player carries his group to victory, whereas the second corresponds to a teammate carrying the group to victory. The type- $c$  player's first-order condition for choosing  $c^*$  is

$$\begin{aligned} 0 &= \frac{\partial U(c^*, c)}{\partial c^*} = -(\rho T'(c^*)(V - c)e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-1} \\ &\quad - (Kn - 1)(V - c)e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-2} f(c^*) \\ &\quad + (n - 1)V e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-2} f(c^*)) \\ &= e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-2} \\ &\quad \times \{V(n - 1)f(c^*) - (Kn - 1)(V - c)f(c^*) - \rho(V - c)(1 - F(c^*))T'(c^*)\} \\ &= e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-2} \\ &\quad \times \{(Kn - 1)c - (K - 1)nV\}f(c^*) - \rho(V - c)(1 - F(c^*))T'(c^*)\}. \end{aligned} \quad (\text{A3})$$

Where  $T(c) > 0$ , it must be that the first-order condition holds at  $c^* = c$ , so (A3) implies

$$T'(c) = \frac{f(c)}{1 - F(c)} \times \frac{(Kn - 1)c - (K - 1)nV}{\rho(V - c)}. \quad (\text{A4})$$

Note that the monotonicity requirement for  $T(c)$  requires that  $\frac{(Kn-1)c - (K-1)nV}{\rho(V-c)}$  is positive. This defines a lower bound  $c_0(K, n)$  for values of  $c$ . This lower bound is a function of  $K$  and  $n$  as stated in Proposition 1. Note further that  $T(c)$  as described in (1) identifies the global best response: equations (A3) and (A4) imply

$$\begin{aligned} \frac{\partial U(c^*, c)}{\partial c^*} &= e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-2} f(c^*) \\ &\quad \times \underbrace{\left\{ (Kn - 1)c - (K - 1)nV - \left( \frac{V - c}{V - c^*} \right) [(Kn - 1)c^* - (K - 1)nV] \right\}}_{\equiv \varphi(c^*; c)}. \end{aligned}$$



Now observe that  $\varphi(c; c) = 0$  and

$$\frac{\partial \varphi(c^*; c)}{\partial c^*} = -(V - c) \frac{(n-1)V}{(V - c^*)^2} < 0,$$

so  $U(c^*, c)$  is strictly quasi-concave in  $c^*$  with a maximum at  $c^* = c$ . Thus,  $\varphi(c; c) = 0$  yields the best response in (1).  $\square$

*Proof of Proposition 2.* Consider first an increase in  $K$ . We have, using (1),

$$\frac{\partial T'(c; K, n)}{\partial K} = \frac{h(c)n}{\rho(V - c)}(c - V) = -\frac{h(c)n}{\rho} < 0. \quad (\text{A5})$$

We now have

$$\begin{aligned} T(c; K, n) - T(c; K + 1, n) &= \int_{K+1}^K \frac{\partial T(c; k, n)}{\partial k} dk \\ &= \int_{K+1}^K \left( \int_{\underline{c}}^c \frac{\partial T'(y; k, n)}{\partial k} dy \right) dk \\ &= \int_{K+1}^K \left( \int_{\underline{c}}^c -\frac{h(y)n}{\rho} dy \right) dk \\ &= -\frac{n}{\rho} \int_{K+1}^K \log(1 - F(c)) dk \\ &= -\frac{n}{\rho} \log(1 - F(c)) \\ &\equiv \Delta_n(c), \end{aligned}$$

where it is clear that  $\Delta_n(c) > 0$  for all  $c \in (\underline{c}, \bar{c})$  and  $\Delta_n(c)$  is independent of  $K$ . By induction,  $T(c; K + 1, n) = T(c; 2, n) - (K - 1)\Delta_n(c)$ .

Consider next an increase in  $n$ . We have

$$\begin{aligned} T(c; K, n + 1) - T(c; K, n) &= \int_{\underline{c}}^c \frac{h(z)[(K(n + 1) - 1)z - (K - 1)(n + 1)V - ((Kn - 1)z - (K - 1)nV)]}{\rho(V - z)} dz \\ &= \int_{\underline{c}}^c \frac{h(z)}{\rho(V - z)} [Kz - (K - 1)V] dz \equiv \Delta_K(c). \end{aligned}$$

Then, for  $z > \underline{c}$ ,

$$z > \underline{c} \geq c_0 = \frac{(K - 1)n}{Kn - 1}V \geq \frac{K - 1}{K}V,$$

implying  $\Delta_K(c) > 0$  for all  $c > \underline{c}$ . Consequently,  $T(c; n + 1) > T(c; n)$  for all  $c \in (\underline{c}, \bar{c}]$ . That is, as the number of players per team increases, free-riding is more pervasive, with all types above  $\underline{c}$  grabbing at a later dates. Further, because  $\Delta_K(c)$  is independent of  $n$ , we have  $T(c; K, n + 1) = T(c; K, 2) + (n - 1)\Delta_K(c)$ .

Consider now a cost distribution  $\hat{F}(y + \Delta) = F(y)$  for  $y \in (\underline{c}, \bar{c})$  that describes a parallel right-shift of the distribution of cost types by some  $\Delta$ . An interior equilibrium now has  $\hat{T}(\underline{c} + \Delta) = 0$ , and it has

$$\hat{T}'(y + \Delta) = \frac{\hat{f}(y + \Delta)}{1 - \hat{F}(y + \Delta)} \times \frac{(Kn - 1)(y + \Delta) - (K - 1)nV}{\rho(V - (y + \Delta))}$$

for  $y \in (\underline{c}, \bar{c})$ , which can be written equivalently as

$$\begin{aligned} \hat{T}'(y + \Delta) &= \frac{f(y)}{1 - F(y)} \times \frac{(Kn - 1)(y + \Delta) - (K - 1)nV}{\rho(V - (y + \Delta))} \\ &> \frac{f(y)}{1 - F(y)} \times \frac{(Kn - 1)y - (K - 1)nV}{\rho(V - y)} \\ &= T'(y). \end{aligned}$$

The inequality uses

$$\frac{\partial \frac{(Kn-1)c-(K-1)nV}{\rho(V-c)}}{\partial c} = V \frac{n-1}{\rho(V-c)^2} > 0.$$

This implies: for all corresponding types  $y \in (\underline{c}, \bar{c})$  for  $F$  and  $y + \Delta \in (\underline{c} + \Delta, \bar{c} + \Delta)$  for  $\hat{F}$  the respective types for  $\hat{F}$  grab later than for  $F$ .  $\square$

*Proof of Lemma 1.* The expected time at which the game stops equals

$$\begin{aligned}
 ET^E &\equiv \int_{\underline{c}}^{\bar{c}} T(c) d(1 - (1 - F(c))^{Kn}) \\
 &= \int_{\underline{c}}^{\bar{c}} \left( \int_{\underline{c}}^c T'(y) dy \right) d(1 - (1 - F(c))^{Kn}) \\
 &= \int_{\underline{c}}^{\bar{c}} T'(y) \left( \int_y^{\bar{c}} d(1 - (1 - F(c))^{Kn}) \right) dy \\
 &= \int_{\underline{c}}^{\bar{c}} T'(y) (1 - F(y))^{Kn} dy \\
 &= \int_{\underline{c}}^{\bar{c}} \frac{f(y)}{1 - F(y)} \times \frac{(Kn - 1)y - (K - 1)nV}{\rho(V - y)} (1 - F(y))^{Kn} dy \\
 &= \int_{\underline{c}}^{\bar{c}} \frac{1}{K\rho} \left( \frac{n-1}{n} \frac{y}{V-y} - (K-1) \right) d(1 - (1 - F(y))^{Kn}), \tag{A6}
 \end{aligned}$$

where the third equality follows from interchanging the order of integration, and the fifth uses the strategy in (1). Equation (A6) can be rewritten as

$$K\rho ET^E + (K - 1) = \int_{\underline{c}}^{\bar{c}} \frac{n-1}{n} \frac{y}{V-y} d(1 - (1 - F(y))^{Kn}).$$

$\square$

*Proof of Proposition 3.*

- (i) From (5) it is immediate that an increase in the discount rate  $\rho$  or in the value of the gross benefit  $V$  decrease  $ET^E$ .
- (ii) The result on an increase in the number of groups follows immediately from the comparative statics of  $T(\cdot)$  in Proposition 2 by which increasing  $K$  leads each type to grab sooner, which, even without there being more players would result in the expected stopping time decreasing.
- (iii) Consider an increase in  $n$ . We can rewrite the right-hand side of (5) as

$$\int_{\underline{c}}^{\bar{c}} \frac{y(1 - F(y))^K}{V - y} d(1 - (1 - F(y))^{K(n-1)}).$$

We see that the probability distribution in this equation is that of the minimum cost out of  $K(n-1)$  independent realizations. As  $n$  increases, this probability distribution decreases in the sense of first-order stochastic dominance; therefore, if  $\frac{y(1-F(y))^K}{V-y}$  is decreasing (increasing) in  $y$ , then  $ET^E$  increases (decreases) in  $n$ .

- (iv) and (v) Consider equation (5), here reproduced:

$$K\rho ET^E + (K - 1) = \int_{\underline{c}}^{\bar{c}} \frac{n-1}{n} \frac{y}{V-y} d(1 - (1 - F(y))^{Kn}).$$

A FOSD increase in individual cost effects a FOSD increase in the minimum cost, which is distributed with cdf  $1 - (1 - F(y))^{Kn}$ . The results follow because the function  $\frac{y}{V-y}$  at the right-hand side of (5) is increasing and convex for  $y \in (0, V)$ .

$\square$

*Proof of Proposition 4.*

- (i) Considering (4), we see that both  $e^{-\rho T(y;n)}$  and  $(1 - F(y))^{Kn-1}$  decrease with  $n$ , so  $U^E(c)$  strictly increases for  $c > \underline{c}$ , and the larger  $c$ , the larger the increase.
- (ii) Constancy of  $U^E(c)$  with respect to changes in  $K$  follows from the fact that the integrand in (4) is constant with respect to  $K$ . To see this, for notational convenience we begin by rewriting (4) as applied to cost type  $c'$  as follows:

$$U^E(c') = V - \underline{c} - \int_{\underline{c}}^{c'} e^{-\rho T(c)} (1 - F(c))^{Kn-1} dc.$$

Then, we take the natural log of the integrand to get

$$-\rho T(c; K) + (Kn - 1) \log(1 - F(c)),$$

and finally we differentiate with respect to  $K$  to get

$$-\rho \frac{\partial T(c; K)}{\partial K} + n \log(1 - F(c)).$$

Now we use  $\frac{\partial T(c; K)}{\partial K} = \int_{\underline{c}}^c \frac{\partial T'(y; K)}{\partial K} dy$  and substitute for  $\frac{\partial T'(y; K)}{\partial K}$  from (A5) to obtain

$$\begin{aligned} -\rho \int_{\underline{c}}^c \frac{-h(y)n}{\rho} dy + n \log(1 - F(c)) &= n \left( \log(1 - F(c)) + \int_{\underline{c}}^c h(c) dc \right) \\ &= n \left( \log(1 - F(c)) + \int_{\underline{c}}^c \frac{f(y)}{1 - F(y)} dy \right) \\ &= 0. \end{aligned}$$

Because the log of the integrand is constant with respect to  $K$ , so too is the integrand itself.  $\square$

*Proof of Proposition 5.* We prove  $T_s(c) < T_l(c)$  by showing the equivalent relation  $k(c) < c$  for all  $c \in (\underline{c}, \bar{c})$ . To see this, note that, at  $c = \underline{c}$ , (10) implies  $k'(\underline{c}) < 1$ , so in a sufficiently small right-neighborhood of  $\underline{c}$  we have  $k(c) < c$ . Consider now by contradiction the smallest cost  $\tilde{c}$  at which  $k(\tilde{c}) = \tilde{c}$ . Because  $k(c)$  starts smaller than  $c$ , this requires  $k'(\tilde{c}) \geq 1$ . But (10) and  $k(\tilde{c}) = \tilde{c}$  imply the contradiction  $k'(\tilde{c}) < 1$ .  $\square$

*Proof of Proposition 6.* Assume by contradiction that  $k(\bar{c}) < \bar{c}$ . Consider an arbitrary  $\tilde{c} \in (\underline{c}, \bar{c})$ . Using (10), we have

$$k(\bar{c}) - k(\tilde{c}) = \int_{\tilde{c}}^{\bar{c}} \frac{f_s(c)}{1 - F_s(c)} \cdot \frac{\frac{V_s^e(n_s - 1)}{V_s^g - c} + 1}{\frac{f_l(k(c))}{1 - F_l(k(c))} \left( \frac{V_l^e(n_l - 1)}{V_l^g - k(c)} + 1 \right)} dc.$$

Under the contradiction hypothesis, the second factor in the integrand displayed above is bounded below by a number  $b > 0$ . Therefore, we obtain

$$k(\bar{c}) - k(\tilde{c}) \geq b \int_{\tilde{c}}^{\bar{c}} \frac{f_s(c)}{1 - F_s(c)} dc = -b \left[ \lim_{c \uparrow \bar{c}} \log(1 - F_s(c)) - \log(1 - F_s(\tilde{c})) \right] = +\infty,$$

and we have a contradiction to  $k(\bar{c}) < \bar{c}$ . Similarly, assume by contradiction that  $T(\bar{c}) < \infty$ . Using (11) we have

$$T_s(\bar{c}) - T_s(\tilde{c}) = \int_{\tilde{c}}^{\bar{c}} \frac{f_s(c)}{1 - F_s(c)} \frac{n_s}{\rho} \cdot \left[ \frac{(V_l^e - (V_l^g - k(c)))(n_l - 1)}{V_l^e(n_l - 1) + (V_l^g - k(c))} \frac{V_s^e(n_s - 1) + (V_s^g - c)}{n_s(V_s^g - c)} - 1 \right] dc.$$

The term in brackets in the integrand above is bounded below by a number  $d > 0$ . This is because, by (13), (12) holds with strict inequality away from  $\underline{c}$ . Therefore, with similar steps as before, we obtain

$$T_s(\bar{c}) - T_s(\tilde{c}) > d \cdot \frac{n_s}{\rho} \int_{\tilde{c}}^{\bar{c}} \frac{f_s(c)}{1 - F_s(c)} dc = +\infty,$$

a contradiction.  $\square$

*Proof of Proposition 7.* We consider two values for the externality,  $V_s^e$  and  $\tilde{V}_s^e$ , with  $\tilde{V}_s^e > V_s^e$ . The corresponding equilibrium strategies are indicated with  $(T_s(c), T_l(c), k(c))$  and  $(\tilde{T}_s(c), \tilde{T}_l(c), \tilde{k}(c))$ . Using (14), we prove that the probability of victory of the small group decreases by showing that  $k(c) < \tilde{k}(c)$  for any  $c \in (\underline{c}, \bar{c})$ . To see this, note that, at  $c = \underline{c}$ , (10) and  $k(\underline{c}) = \tilde{k}(\underline{c}) = \underline{c}$  imply  $k'(\underline{c}) < \tilde{k}'(\underline{c})$ , so in a right-neighborhood of  $\underline{c}$  we have  $k(c) < \tilde{k}(c)$ . Consider now by contradiction the smallest cost  $z$  at which  $k(z) = \tilde{k}(z)$ . Because  $k(c)$  starts smaller than  $\tilde{k}(c)$ , this requires  $k'(z) \geq \tilde{k}'(z)$ . But (10) and  $k(z) = \tilde{k}(z)$  imply the contradiction  $k'(\tilde{c}) < \tilde{k}'(\tilde{c})$ . We can now see that  $T_s(c) < \tilde{T}_s(c)$  for any  $c \in (\underline{c}, \bar{c})$ : we know  $T_s(\underline{c}) = \tilde{T}_s(\underline{c})$  and, by (11),  $\tilde{T}_s'(c) > T_s'(c)$ , because the term in brackets in (11) is increasing in  $k(c)$  and in  $V_s^e$ . Now consider the equilibrium utility  $U_s^E(c) \equiv U_s(c, c)$ . Using the envelope theorem and (7), we have

$$U^E(c) = V_s^g - \underline{c} - \int_{\underline{c}}^c e^{-\rho T_s(y)} (1 - F(k(y)))^{n_l} (1 - F_s(y))^{n_s - 1} dy.$$

But because  $k(c) < \tilde{k}(c)$  and  $T_s(c) < \tilde{T}_s(c)$  for any  $c \in (\underline{c}, \bar{c})$ , we conclude that an increase in  $V_s^e$  increases the utility of all cost types in the small group.  $\square$

*Proposition A1* (A sufficient condition for a negative group-size effect). Consider an interior equilibrium described by (8), (9), and (10) with initial condition  $k(\underline{c}) = \underline{c}$ . Let  $V_l^e = V_s^e = V_l^g = V_s^g = V$ ,  $F_s = F_l = F$ , and  $n_l > n_s$ . Define the function  $\tilde{k}(c)$  as the solution to

$$(1 - F(\tilde{k}(c)))^{n_l} = (1 - F(c))^{n_s}. \quad (\text{A7})$$

If,  $\forall c \in [\underline{c}, \bar{c}]$ , we have

$$\tilde{k}(c) > \frac{V n_l (n_s - 1) c}{V n_s (n_l - 1) - (n_l - n_s) c}, \quad (\text{A8})$$

then  $P_s^w > 0.5$ .<sup>40</sup>

*Proof of Proposition A1.* Proposition (A1) is stated for  $V_l^e = V_s^e = V_l^g = V_s^g = V$  and  $F_s = F_l = F$ . Therefore, we begin by restating the differential equation for  $k$  in (10) under these assumptions. Further, to clarify the dependence on  $n_l$ , we include it, when needed, as an argument of the relevant functions. Recalling that  $h(\cdot)$  denotes the hazard rate of  $F$ , we obtain

$$k'(c; n_l) = \frac{V n_s - c}{V - c} h(c) \left[ \frac{V n_l - k(c; n_l)}{V - k(c; n_l)} h(k(c; n_l)) \right]^{-1}. \quad (\text{A9})$$

Note also that, if  $n_l$  were equal to  $n_s$ , then the only solution to (A9) with  $k(\underline{c}; n_s) = \underline{c}$  would be  $k(c; n_s) = c$ . Furthermore, in this case the probability of victory of the “small” group would be 0.5. The strategy of proof is to show that  $(1 - F(k(c; n_l)))^{n_l} > (1 - F(k(c; n_s)))^{n_s}$ , for all  $c \in (\underline{c}, \bar{c})$ , so that (14) ends up implying  $P_s^w > 0.5$ .

Using (A9), we obtain

$$\frac{d}{dc} (1 - F(k(c; n_l)))^{n_l} = -\frac{V n_s - c}{V - c} (1 - F(k(c; n_l)))^{n_l} \left[ \frac{n_l (V - k(c; n_l))}{V n_l - k(c; n_l)} \right]; \quad (\text{A10})$$

similarly,

$$\frac{d}{dc} (1 - F(k(c; n_s)))^{n_s} = -\frac{V n_s - c}{V - c} (1 - F(k(c; n_s)))^{n_s} \left[ \frac{n_s (V - k(c; n_s))}{V n_s - k(c; n_s)} \right]. \quad (\text{A11})$$

Note that, by  $k(\underline{c}; n_l) = \underline{c} = k(\underline{c}; n_s)$ , (A10) yields

$$\left. \frac{d}{dc} (1 - F(k(c; n_l)))^{n_l} \right|_{c=\underline{c}} = -\frac{V n_s - \underline{c}}{V - \underline{c}} \left[ \frac{n_l (V - \underline{c})}{V n_l - \underline{c}} \right];$$

whereas (A11) gives

$$\left. \frac{d}{dc} (1 - F(k(c; n_s)))^{n_s} \right|_{c=\underline{c}} = -\frac{V n_s - \underline{c}}{V - \underline{c}} \left[ \frac{n_s (V - \underline{c})}{V n_s - \underline{c}} \right].$$

Hence, by

$$\frac{n_l (V - \underline{c})}{V n_l - \underline{c}} < \frac{n_s (V - \underline{c})}{V n_s - \underline{c}},$$

using (A10) and (A11) we obtain

$$0 > \left. \frac{d}{dc} (1 - F(k(c; n_l)))^{n_l} \right|_{c=\underline{c}} > \left. \frac{d}{dc} (1 - F(k(c; n_s)))^{n_s} \right|_{c=\underline{c}}.$$

So, we see that  $(1 - F(k(c; n_l)))^{n_l}$  starts above  $(1 - F(k(c; n_s)))^{n_s}$  in a right-neighborhood of  $\underline{c}$ . Then, the proposition is proven if there is no intersection in  $(\underline{c}, \bar{c})$  between  $(1 - F(k(c; n_l)))^{n_l}$  and  $(1 - F(k(c; n_s)))^{n_s}$ .

Proceeding by contradiction, suppose there exists  $\tilde{c} \in (\underline{c}, \bar{c})$  such that

$$(1 - F(k(\tilde{c}; n_l)))^{n_l} = (1 - F(k(\tilde{c}; n_s)))^{n_s}. \quad (\text{A12})$$

As  $(1 - F(k(c; n_l)))^{n_l}$  starts above  $(1 - F(k(c; n_s)))^{n_s}$  in a right-neighborhood of  $\underline{c}$ , we must have that for at least one  $\tilde{c}$  that satisfies (A12), the following also holds:

$$\left. \frac{d}{dc} (1 - F(k(c; n_l)))^{n_l} \right|_{c=\tilde{c}} \leq \left. \frac{d}{dc} (1 - F(k(c; n_s)))^{n_s} \right|_{c=\tilde{c}}. \quad (\text{A13})$$

But using (A10), (A11), and  $k(\tilde{c}; n_s) = \tilde{c}$ , equation (A13) requires

$$\frac{n_s (V - \tilde{c})}{V n_s - \tilde{c}} \leq \frac{n_l (V - k(\tilde{c}; n_l))}{V n_l - k(\tilde{c}; n_l)}. \quad (\text{A14})$$

Note also that (A12), which can be equivalently restated as  $(1 - F(k(\tilde{c}; n_l)))^{n_l} = (1 - F(\tilde{c}))^{n_s}$ , can be concatenated to the definition of  $\tilde{k}$  in (A7) to give

$$(1 - F(k(\tilde{c}; n_l)))^{n_l} = (1 - F(\tilde{c}))^{n_s} = (1 - F(\tilde{k}(\tilde{c})))^{n_s},$$

<sup>40</sup> Consider for example  $n_s = 2$ ,  $n_l = 4$ , and  $F(c) = 2c - 3$  on  $[3/2, 2]$ . Then (A7) yields  $\tilde{k}(c) = 2 - \frac{\sqrt{4-2c}}{2}$ , and one can check numerically that (A8) is satisfied if and only if  $V \geq V^* \approx 2.21634$ .

which implies  $k(\tilde{c}; n_l) = \tilde{k}(\tilde{c})$ . Hence, (A14) implies

$$\frac{n_s(V - \tilde{c})}{Vn_s - \tilde{c}} \leq \frac{n_l(V - k(\tilde{c}; n_l))}{Vn_l - k(\tilde{c}; n_l)} = \frac{n_l(V - \tilde{k}(\tilde{c}))}{Vn_l - \tilde{k}(\tilde{c})}. \quad (\text{A15})$$

But the extremes of equation (A15) are directly contradicted by the hypothesis in (A8).  $\square$

*Characterization of the equilibrium with low costs.* Proposition 1 provided the symmetric equilibrium strategy under the assumption that  $\underline{c} \geq c_0 \equiv \frac{(K-1)n}{Kn-1}V$ . The strategy there fails to be weakly increasing if  $\underline{c} < c_0$  because (1) would imply  $T$  is strictly decreasing for  $c \in [\underline{c}, c_0)$ . To maintain weak monotonicity of the equilibrium strategy, we now investigate the possibility that an equilibrium strategy has a “flat spot,” that is, an interval over which it is constant. The following lemma shows that if a symmetric equilibrium strategy  $T$  has a flat spot, then it must occur at 0, which implies that, for some  $\hat{c} \geq \underline{c}$ ,  $T(c) = 0$  on  $[\underline{c}, \hat{c}]$  and  $T$  is strictly increasing for  $c > \hat{c}$ . Moreover, the strategy  $T$  must be continuous.<sup>41</sup>

*Lemma A1.* Suppose  $T$  is a symmetric equilibrium strategy. Then  $T$  is continuous and if  $T$  is constant on  $[\tilde{c}_l, \tilde{c}_h]$  and  $\underline{c} \leq \tilde{c}_l < \tilde{c}_h \leq \tilde{c}$ , then  $T(c) = 0$  for all  $c \in [\tilde{c}_l, \tilde{c}_h]$ . Further,  $\tilde{c}_h \leq c_0$ .

*Proof.* Suppose  $T$  is a symmetric equilibrium strategy for which types  $c \in [\tilde{c}_l, \tilde{c}_h]$  grab at  $\tilde{T} > 0$ , that is, a strategy with a strictly positive flat spot:  $T(c) = \tilde{T}$  for all  $c \in [\tilde{c}_l, \tilde{c}_h]$ . We establish two facts to show this cannot be an equilibrium strategy. First, under the equilibrium conjecture, type  $\tilde{c}_h$  must prefer a contribution of  $\tilde{T}$  to one of  $\tilde{T} + \varepsilon$ . As  $\varepsilon \downarrow 0$ , this will imply  $\tilde{c}_h \leq c_0$ . Second, type  $\tilde{c}_l$  must prefer a contribution of  $\tilde{T}$  to one of  $\tilde{T} - \varepsilon$ . As  $\varepsilon \downarrow 0$ , this will imply  $\tilde{c}_l \geq c_0$ . Therefore, a flat spot at  $\tilde{T} > 0$  cannot exist in equilibrium.

Consider first  $\tilde{c}_h$ . The utility of one agent in group 1 with cost realization equal to  $\tilde{c}_h$  that contributes  $\tilde{T}$  is

$$\frac{n-1}{Kn-1}V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)} d(1 - (1 - F(x)^{Kn-1})) + (1 - F(\tilde{c}_l)^{Kn-1})e^{-\rho \tilde{T}} U_I(\tilde{c}_h), \quad (\text{A16})$$

where  $U_I(c)$  represents the payoff of a type  $c$  agent if the minimum cost of all other agents is above  $\tilde{c}_l$ , that is, conditional on all other agents having cost above  $\tilde{c}_l$ .

Now the logic behind (A16) is this. The first addendum is the expected payoff if the minimum cost of all other agents is below  $\tilde{c}_l$ : the average present value of  $V$  multiplied by the probability that one of the other group-1 agents wins, calculated under symmetry. The second addendum is the product of the probability that the minimum cost of all other agents is above  $\tilde{c}_l$ , multiplied by the present value of  $U_I(\tilde{c}_h)$ , with  $U_I(c)$  defined as

$$U_I(c) = VS_1(n, K) + (V - c)S_2(n, K),$$

where

$$S_1(n, K) = \sum_{j=1}^{n-1} \sum_{k=0}^{(K-1)n} \binom{n-1}{j} \binom{(K-1)n}{k} p^j (1-p)^{n-j-1} p^k (1-p)^{(K-1)n-k} \frac{j}{1+j+k},$$

$$S_2(n, K) = \sum_{j=0}^{n-1} \sum_{k=0}^{(K-1)n} \binom{n-1}{j} \binom{(K-1)n}{k} p^j (1-p)^{n-j-1} p^k (1-p)^{(K-1)n-k} \frac{1}{1+j+k},$$

and

$$p \equiv \frac{F(\tilde{c}_h) - F(\tilde{c}_l)}{1 - F(\tilde{c}_l)}. \quad (\text{A17})$$

The payoff  $U_I$  can be understood as follows. Here  $j$  indexes other group 1 players and  $k$  indexes group 2 players. Beginning with  $S_2$ , if there are  $j$  other players in group 1 bidding  $\tilde{T}$  and  $k$  players in the other  $K-1$  groups bidding  $\tilde{T}$  (as well as this player of interest in group 1), then the player of interest in group 1 is selected with probability  $\frac{1}{1+j+k}$ , in which case he earns payoff  $V - c$ ; but (moving to  $S_1$ ) if one of the other group-1 players is selected, which happens with probability  $\frac{j}{1+j+k}$ , then he gets the benefit  $V$  without incurring any cost. And, of course, the probability of this configuration of other players bidding  $\tilde{T}$  is

$$\binom{n-1}{j} p^j (1-p)^{n-j-1} \binom{(K-1)n}{k} p^k (1-p)^{(K-1)n-k}.$$

<sup>41</sup> The proof is not just an adaptation of the discrete-gain versus marginal-loss comparison familiar from standard auction theory because, by grabbing an instant sooner and breaking a tie, an agent increases discretely both his benefit and cost.

With a similar logic, the limit for  $\varepsilon \downarrow 0$  of the utility of one agent in group 1 with cost  $\tilde{c}_h$  that contributes  $\tilde{T} + \varepsilon$  is

$$\frac{n-1}{Kn-1} V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)} d(1 - (1 - F(x)^{Kn-1})) + (1 - F(\tilde{c}_l)^{Kn-1}) e^{-\rho \tilde{T}} U_R(\tilde{c}_h), \quad (\text{A18})$$

where  $U_R(\tilde{c}_h)$  is this player's payoff "from the right":

$$U_R(c) = VS_3(n, K) + (V - c)(1 - p)^{Kn-1},$$

where

$$S_3(n, K) = \sum_{j=1}^{n-1} \sum_{k=0}^{(K-1)n} \binom{n-1}{j} \binom{(K-1)n}{k} p^j (1-p)^{n-j-1} p^k (1-p)^{(K-1)n-k} \frac{j}{j+k}$$

and  $p$  is again given by (A17).

Using properties of a binomial distribution, one can establish

$$S_1(n, K) = \frac{n-1}{Kn-1} \left( 1 - \frac{1 - (1-p)^{Kn}}{pKn} \right) \quad (\text{A19})$$

$$S_2(n, K) = \frac{1 - (1-p)^{Kn}}{pKn} \quad (\text{A20})$$

$$S_3(n, K) = \frac{n-1}{Kn-1} (1 - (1-p)^{Kn-1}). \quad (\text{A21})$$

Because utility in (A16) must be at least as large as the one in (A18), we have

$$VS_1 + (V - \tilde{c}_h)S_2 = U_l(\tilde{c}_h) \geq U_R(\tilde{c}_h) = VS_3 + (V - \tilde{c}_h)(1 - p)^{Kn-1}, \quad (\text{A22})$$

and the extremes of (A22) imply  $V(S_1 + S_2 - S_3 - (1-p)^{Kn-1}) \geq \tilde{c}_h(S_2 - (1-p)^{Kn-1})$ , and therefore, substituting from (A19)–(A21), we have  $\tilde{c}_h \leq c_0$ .<sup>42</sup>

Moving now to  $\tilde{c}_l$ , the utility of one agent in group 1 with cost realization equal to  $\tilde{c}_l$  that contributes  $\tilde{T}$  is

$$\frac{n-1}{Kn-1} V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)} d(1 - (1 - F(x)^{Kn-1})) + (1 - F(\tilde{c}_l)^{Kn-1}) e^{-\rho \tilde{T}} U_l(\tilde{c}_l), \quad (\text{A23})$$

whereas the limit for  $\varepsilon \downarrow 0$  of the utility of one agent in group 1 with cost  $\tilde{c}_l$  that contributes  $\tilde{T} - \varepsilon$  is

$$\frac{n-1}{Kn-1} V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)} d(1 - (1 - F(x)^{Kn-1})) + (1 - F(\tilde{c}_l)^{Kn-1}) e^{-\rho \tilde{T}} (V - \tilde{c}_l). \quad (\text{A24})$$

Because utility in (A23) must be at least as large as the one in (A24), we have

$$VS_1 + (V - \tilde{c}_l)S_2 = U_l(\tilde{c}_l) \geq V - \tilde{c}_l,$$

and the extremes of the above-displayed equation imply  $\tilde{c}_l(1 - S_2) \geq V(1 - S_1 - S_2)$ , or  $\tilde{c}_l \geq c_0$ . Thus, we obtain  $\tilde{c}_l = \tilde{c}_h$ , so a flat spot at  $\tilde{T} > 0$  is impossible in equilibrium.

To see that  $T$  is continuous note that any discontinuity must be a jump discontinuity. If such a jump occurs at  $c'$ , then for sufficiently small  $\delta > 0$  the types in  $(c', c' + \delta)$  will find it strictly profitable to decrease their grabbing times discretely to avoid wasteful delay (there is no chance of a tie because there are no flat spots at positive times).

Finally, note that the logic leading to (A22) remains valid even if  $\tilde{T} = 0$ . Therefore, even if  $T$  is flat at zero for  $c \in [\underline{c}, \tilde{c}_h]$ , we obtain  $\tilde{c}_h \leq c_0$ .  $\square$

From Lemma A1 we now see that if an equilibrium strategy has a flat spot it must be over an interval of the form  $[\underline{c}, \tilde{c}_h]$ , where  $T$  takes on value 0, and  $\tilde{c}_h \leq c_0$ . Furthermore, for any  $c$  where  $T$  is strictly increasing, the equilibrium analysis is precisely as for an interior equilibrium, thus requiring  $c \geq c_0$ .

This reasoning has two implications. First, the equilibrium in Proposition 1 is unique among all symmetric ones, without focusing only on strictly increasing strategies. Second, we have the following:

<sup>42</sup> We note that  $S_2 - (1-p)^{Kn-1} > 0$  is equivalent to  $1 > (1-p)^{Kn-1}(1-p+pKn) \equiv \psi(p)$ . This latter inequality is satisfied because  $\psi(0) = 1$  and  $\psi'(p) < 0$ . Therefore, if a flat spot exists, then  $p > 0$  and  $S_2 - (1-p)^{Kn-1} > 0$ .



*Proposition A2* (Equilibrium when preemption costs may be low). If  $\underline{c} < c_0$ , then the unique symmetric equilibrium strategy  $T$  satisfies  $T(c) = 0$  for  $c \in [\underline{c}, c_0]$  and

$$T'(c) = \frac{f(c)}{(1 - F(c))} \frac{(Kn - 1)}{\rho(V - c)}(c - c_0) \quad \forall c \in (c_0, \bar{c}). \quad (\text{A25})$$

Therefore, a flat spot can (and indeed does) appear in the symmetric equilibrium strategy if costs are “low.” Once  $c_0$  is established as the point at which  $T$  begins increasing, the comparative statics described in Section 2 can be seen to hold generally.

*Proposition A3.* For each  $c \in [\underline{c}, \bar{c}]$ , the equilibrium strategy  $T(c; K, n)$  is weakly decreasing in  $K$  and weakly increasing in  $n$ . Furthermore,  $c_0(K, n + 1) < c_0(K, n) < c_0(K + 1, n)$ , and if  $\underline{c} < c_0(K, n) \equiv \frac{(K-1)n}{K\hat{n}-1}V$ , then

$$T(c; K, n + 1) > T(c; K, n) \quad \forall c \in (c_0(K, n + 1), \bar{c})$$

and

$$T(c; K, n) > T(c; K + 1, n) \quad \forall c \in (c_0(K, n), \bar{c}).$$

Moreover, increasing  $K$  also decreases  $ET^E$ .

*Proof.* First consider the effect of increasing  $n$ . Because  $c_0(K, n + 1) < c_0(K, n)$ , it follows that  $T(c; K, n + 1) = T(c; K, n) = 0$  for  $c \leq c_0(K, n)$  and  $T(c; K, n + 1) > T(c; K, n)$  for  $c \in (c_0(K, n + 1), c_0(K, n)]$ . Finally,  $T(c; K, n + 1) > T(c; K, n)$  for  $c \in (c_0(K, n), \bar{c}]$  because  $T(c_0(K, n); K, n + 1) > T(c_0(K, n); K, n)$  and  $T'$  increases with  $n$  on  $(c_0(K, n), \bar{c}]$ . To see this latter property, note that

$$\begin{aligned} \frac{\partial T'(c; K, \hat{n})}{\partial \hat{n}} &= \frac{h(c)K}{\rho(V - c)} \left( c - \frac{K - 1}{K}V \right) \\ &> \frac{h(c)K}{\rho(V - c)} \left( c - \frac{(K - 1)\hat{n}}{K\hat{n} - 1}V \right) \quad (\text{because } K \geq 2) \\ &> 0, \end{aligned}$$

where the second inequality follows because  $T(c; K, \hat{n}) > 0$  implies  $c > c_0(K, \hat{n}) \equiv \frac{(K-1)\hat{n}}{K\hat{n}-1}V$ .

One similarly shows that an increase in  $K$  reduces  $T$ . Analogously to the proof of the effect of increasing  $n$ , here we use the fact that increasing  $K$  increases  $c_0(K, n)$  and decreases  $T'$  (see (A5)). Moreover, because individual grabbing strategies decrease with an increase in  $K$  and because increasing  $K$  increases the number of players, it follows immediately that  $ET^E$  also decreases with  $K$ .  $\square$

From Proposition A3 we see that an increase in the number of teams, by reducing individual players' grabbing times, has the effect of decreasing the expected time at which the game ends. Indeed, because  $c_0(K, n) \rightarrow V$  as  $K \rightarrow \infty$ , everyone grabs almost instantly and  $ET^E \rightarrow 0$ . Not surprisingly, as the effect of increasing  $n$  on the expected duration of the contest was ambiguous for interior equilibria, so too is it ambiguous for corner equilibria. Surprisingly, however, whereas increasing  $n$  had the effect of increasing interim payoffs at interior equilibria, examples show the effect is ambiguous for corner equilibria.

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