

Vasilev, Aleksandar

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## A model with knowledge externalities and educational policy

Aleksandar Vasilev, Lecturer, University of Lincoln, UK. E-mail: avasilev@lincoln.ac.uk

**Abstract:** We utilize a standard endogenous-growth model with knowledge spillovers (the “k-K” model). We characterize the optimal Balanced Growth Path (BGP), and compare it to path under market competition. In the presence of externalities, markets fail. One way to restore efficiency is to subsidize knowledge accumulation, and finance the subsidy by taxing final consumption.

**Keywords:** knowledge spillovers, externalities, educational subsidy

### Introduction

In this note we present a standard endogenous-growth model with knowledge externalities, as in Vasilev (2017). The main idea of such setups is that spillovers in knowledge are critical to accumulating growth. We derive the path that would be chosen by a Social Planner (SP), as then all external effects will be fully internalized, and hence the growth path is efficient. Then we proceed and decentralize the economy by allowing for market prices to determine allocations, and compute the growth path under competitive equilibrium. We show that the growth rate is lower in a market economy, as compared to the rate chosen by the SP, since agents fail to incorporate the social effects into their individual optimization problems. A well-formulated policy, such as a subsidy to knowledge, will be then showed to improve upon allocations.

### Model setup

The model setup consists of preferences, technology and resources, which are described in turn.

Preferences: There is a representative household, which is infinitely lived and maximizes the stream of discounted utility

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t), \quad (1)$$

where  $0 < \beta < 1$  is the discount factor, and  $c_t$  denotes consumption in period  $t$ .

Technology: the production technology is as follows: There are  $N > 0$  firms in the economy, and their number is held constant;  $k_{it}$  is knowledge (capital) of firm  $i$  in period  $t$ ,

$K_t = \sum_{i=1}^N k_{it}$  is aggregate knowledge (capital), and  $K_{-it} = \sum_{j \neq i}^N k_{jt}$  is aggregate knowledge of all firms other than firm  $i$ .

Each firm uses a production function of the form

$$y_{it} = k_{it}^{\alpha} K_{-it}^{1-\alpha}, \quad (2)$$

Where the individual firm’s ability to produce is influenced by the knowledge of others. In other words, there are positive complementarities (spillovers) between the two types of knowledge. Given that  $0 < \alpha < 1$ , there are decreasing returns to scale (DRS) in own knowledge, as the firm takes the knowledge of others as given. The production function is constant-returns-to-scale (CRS) in both inputs jointly.

Aggregate output is then

$$Y_t = \sum_{i=1}^N y_{it}, \quad (3)$$

Resource constraint is

$$c_t + \sum_{i=1}^N [k_{i,t+1} - k_{it}] = Y_t, \quad (4)$$

Where the second expression is aggregate investment; for simplicity we assume no depreciation of knowledge. Knowledge is also assumed to be irreversible, or

$$k_{i,t+1} \geq k_{it} \quad (5)$$

In other words, we cannot give up knowledge to gain additional consumption.

$$\text{Each firm starts with an initial endowment of knowledge: } k_{i0} = k_0 > 0. \quad (6)$$

The Social Planner (SP) maximizes (1) s.t (2)-(6). Noting that in equilibrium the solution must be symmetric (as all individual firms use the same production function), the equilibrium condition produces the following balanced growth path (BGP):

$$\frac{1}{\beta}(1 + g^{SP}) = (N - 1)\alpha(N - 1)^{1-\alpha} + (1 - \alpha)(N - 1)^{-\alpha} + 1 \quad (7)$$

Note that the SP understands that what firm N does, is good for everybody. When everybody accumulates more knowledge, it helps each individual firm. Note that N should be large, so that each firm is small (atomistic) relative to the aggregate. The result above will not hold if N=2, as then the two firms will be large relative to the aggregate, and thus there will be important strategic interactions and feedback effects.

With externalities, the competitive equilibrium (CE) allocation will not be equivalent to the Social Planner (SP) allocation – with externalities CE is not efficient, as the First Welfare Theorem (FWT) does not hold any more. We now turn now to the competitive equilibrium in the kK model.

1. Consumer Problem: The consumer now take prices and firms' profit as given, and maximizes (1) s.t the budget constraint

$$\sum_{t=0}^{\infty} p_t c_t = \sum_{t=0}^{\infty} p_t \sum_{i=1}^N \pi_{it}, \quad (8)$$

Where  $\pi_{it}$  denotes firm  $i$ 's profit.

2. Firm's problem: Each firm maximizes dynamic profit by taking prices, and others' knowledge as given, or

$$\max \sum_{t=0}^{\infty} p_t [k_{it}^{\alpha} K_{-it}^{1-\alpha} - k_{it-1} + k_{it}] \quad (9)$$

s.t  $k_{it+1} \geq k_{it}, k_{i0} > 0$ .

3. Markets clear

In competitive equilibrium, the BGP is now:

$$\frac{1}{\beta}(1 + g^{CE}) = (1 - \alpha)(N - 1)^{-\alpha} + 1 \quad (10)$$

Therefore,  $g^{CE} \neq g^{SP}$ . The firms fail to internalize the positive externality, as they consider only individual benefits, and ignore social benefits. This differs substantially from the SP case, where spillover effects are taken into consideration. In fact, we can show  $g^{SP} > g^{CE}$  because of the positive externality (complementarity).

This result is an example of market failure, so there is room for government policies that could potentially improve upon allocative efficiency. One way to increase the growth rate is to subsidize knowledge accumulation at rate  $s$ . The way this subsidy is going to be financed is via consumption taxation, which is the way most of government expenditure is financed in Europe. The other alternative is to raise the funds through lump-sum taxation. As we will show below, both methods lead to the same outcome. The setup is now amended to

1. Consumer Problem: The consumer now take prices and firms' profit as given, and maximizes (1) s.t the budget constraint

$$\sum_{t=0}^{\infty} (1 + \tau^c) p_t c_t = \sum_{t=0}^{\infty} p_t [\sum_{i=1}^N \pi_{it} - T_t], \quad (11)$$

Where  $\pi_{it}$  denotes firm  $i$ 's profit.

2. Firm's problem (same as before): Each firm maximizes dynamic profit by taking prices, and others' knowledge as given, or

$$\max \sum_{t=0}^{\infty} p_t [k_{it}^{\alpha} K_{-it}^{1-\alpha} - k_{it-1} + k_{it}] \quad (9)$$

s.t  $k_{it+1} \geq k_{it}, k_{i0} > 0$ .

3. Government

$$T_t + \tau^c p_t c_t = s \sum_{i=1}^N (k_{it+1} - k_{it})$$

4. Markets clear

After some algebra, the BGP with the subsidy in place is now the same as the one in the SP setup:

$$\frac{1}{\beta} (1 + g^{CE}) = (N - 1)\alpha(N - 1)^{1-\alpha} + (1 - \alpha)(N - 1)^{-\alpha} + 1$$

Therefore, we can raise the education subsidy to end up at the first-best (SP) solution.

## Conclusions

Using a standard endogenous-growth model with knowledge spillovers as in Vasilev (2017), we show that the growth rate in a market economy is suboptimal, as individual agents fail to incorporate social gains in their decision-making. This proves that markets on their own do not always produce efficient outcomes, especially in the presence of knowledge externalities. This leaves room for government intervention as a way of improving allocative efficiency. We demonstrate that a well-targeted subsidy to knowledge accumulation can offset the effect of unexploited synergies, and an efficient growth rate in the economy can be achieved.

## References

Vasilev, A. (2017) "Notes on Endogenous Growth Models," *EconStor Research Reports 149876*, ZBW - Leibniz Information Centre for Economics.