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Financial Risk Meter based on expectiles

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IRTG 1792 Discussion Paper 2021-008

Financial Risk Meter based on Expectiles

Rui Ren [*]
Meng-Jou Lu [*2]
Yingxing Li [*3]
Wolfgang Karl Härdle [* *3 *4 *5 *6]

[*] Humboldt-Universität zu Berlin, Germany
[*2] Asia University, Taiwan
[*3] Xiamen University, China
[*4] Singapore Management University, Singapore
[*5] Charles University, Czech Republic
[*6] National Chiao Tung University, Taiwan

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Financial Risk Meter based on Expectiles*

Rui Ren† Meng-Jou Lu ‡ Yingxing Li § Wolfgang Karl Härdle ¶

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Abstract

The Financial Risk Meter (FRM) is an established mechanism that, based on conditional Value at Risk (VaR) ideas, yields insight into the dynamics of network risk. Originally, the FRM has been composed via Lasso based quantile regression, but we here extend it by incorporating the idea of expectiles, thus indicating not only the tail probability but rather the actual tail loss given a stress situation in the network. The expectile variant of the FRM enjoys several advantages: Firstly, the coherent and multivariate tail risk indicator conditional expectile-based VaR (CoEVaR) can be derived, which is sensitive to the magnitude of extreme losses. Next, FRM index is not restricted to an index compared to the quantile based FRM mechanisms, but can be expanded to a set of systemic tail risk indicators, which provide investors with numerous tools in terms of diverse risk preferences. The power of FRM also lies in displaying FRM distribution across various entities every day. Two distinct patterns can be discovered under high stress and during stable periods from the empirical results in the United States stock market. Furthermore, the framework is able to identify individual risk characteristics and capture spillover effects in a network.

Keywords: *expectiles, EVaR, CoEVaR, expectile lasso regression, network analysis, systemic risk, Financial Risk Meter*

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†School of Business and Economics, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin, Germany, co-corresponding author, E-mail: rui.ren@hu-berlin.de

‡Department of Finance, Asia University, 500 Lioufeng Rd., Wufeng, Taichung 41354, Taiwan, Email: mangrou@gmail.com

§W.I.S.E. - Wang Yanan Institute for Studies in Economics, Xiamen University, 422 Siming S Rd, 361005 Fujian, China, co-corresponding author, Email: yxli@xmu.edu.cn

¶Blockchain Research Center, Humboldt-Universität zu Berlin, Germany. Wang Yanan Institute for Studies in Economics, Xiamen University, China. Sim Kee Boon Institute for Financial Economics, Singapore Management University, Singapore. Faculty of Mathematics and Physics, Charles University, Czech Republic.National Chiao Tung University, Taiwan.

1 Introduction

The interdependencies and mutual liabilities between financial institutions constitute a complex systemic network that is crucial to understand for systemic risk management. The main concern is that coagulation of risk receivers may cause a severe economic crisis. This so-called systemic risk defines some trigger events (Schwarcz, 2008), such as a big economic shock or institutional distress, and causes spillover-effects that jeopardise the stability of the whole financial system. Previous researches have been conducted in analyzing this in the framework of financial institutions (Abbassi et al., 2017, Aldasoro and Alves, 2018, and Kreis and Leisen, 2018). Recently, regulators and supranational agencies pay increasing attention to the embedded consequences of tail events in risk management analysis.

A proper risk measure is essential to achieve fair investment decisions. A leading risk measure is Value at Risk (VaR) which is a tail event probability that is designed for studying one dimensional profit and loss situations. Adrian and Brunnermeier (2016) extended this to propose CoVaR which adequately accounts for financial institutions' dependence structure changes across time and varied market conditions. Moreover, the Tail Event NETwork (TENET) addressed by Härdle et al. (2016) further generalizes CoVaR by joining network dynamics. Mihoci et al. (2020) propose the Financial Risk Meter (FRM) system by lasso quantile regression. Ren et al. (2020) apply this FRM technically to portfolio selection and obtain results even convey the ongoing Covid crisis. However, the approaches mentioned above adopt the quantile based risk measure, which is not coherent and less sensitive to the magnitudes of the losses (Kuan et al., 2009).

This paper extends the existing quantiles based FRM techniques in four ways. Firstly, we consider the coherent risk measure and develop the conditional expectile-based Value at Risk (CoEVaR), which measures the tail event probability of a financial institution conditional on the distress of others in a network. Secondly, we adopt the lasso expectile regression to estimate CoEVaR under the high dimensional setting. Moreover, we gain the economic insights for the penalization term λ, which could be interpreted as the change of tail events in terms of the change of risk driver influence. Thirdly, the FRM in our discussion is not solely an index but its power lies in evaluating the risk contributions (RC) across various entities on each day by estimating the underlying distribution. Based on that, we can generate multiple systemic tail risk indicators, e.g. the average, weighted and different quantile level indices, which provide numerous tools for investors with diverse risk preferences. Fourthly, we detect the interdependencies across financial institutions and study the spillover effects. By a detailed study on the distribution of the individual coin's risk indicators, we are able to identify high "co-stress" entities. Those entities with larger outdegree centrality impacting other nodes are "risk emitters". Risk receivers are then those "activated" via spill-over effects.

We evaluate the performance of our approach via empirical analysis in the United States (US) stock market. We select the 100 biggest financial institutions based on their daily market capitalization from the US S&P500 Composite Index, and consider six macro state variables to implement our FRM algorithm. The non-parametric kernel density estimation method is implemented to approximate the RC distribution every day. Two obvious patterns can be discovered. Under high stress days, the densities are probably to have fat tails, whereas the shape of density is squeezed during a stable period. Moreover, one can acquire a group of FRM systemic indices from the RC distribution, though. we find that the weighted FRM might be more volatile than the unweighted one, implying

that large caps may not be treated as safe havens during the COVID crisis. Additionally, significant negative relation is found between a financial institution's CoEVaR and its price, especially under high stress, demonstrating that the downside risk indicator CoEVaR is more sensitive to the magnitude of extreme losses.

This paper further proposes modern techniques for analyzing the network effects on the tail events in three aspects. First, we concentrate on the network of 25 biggest financial institutions in US market due to their systemic relevance, and quantify their interconnectedness. In particular, we investigate the interconnections among financial institutions during the period of the COVID crisis. Interestingly, we discover the increasing interdependencies in 2020, which helps us to make a strategy for portfolio selection and risk management. Hernandez et al. (2020) attempt to analyse the topology of the tail event financial market network for investment purposes. It is also worth mentioning that our results confirm the existence of different market situations (pre-crisis, crisis, and post-crisis) and tail risk levels. Second, we propose to depict the network (or "adjacency matrix") via the expectile lasso regression coefficients β instead of the correlation coefficients Chen et al., 2019. As the network might reveal more information from tail events, our study is more capable to describe stress related structure. A positive adjacency matrix 'Average+' and a negative one 'Average-' allocate the tail event risk profiles into contagion and diversification baskets respectively. Third, we analyse the risk contribution from each company. In doing so, one may identify whether a company is a risk transmitter or a recipient. Then the supervisors can rank the systemic importance for each institution and measure the resulting connectedness in the financial system.

The remainder of this paper is organized as follows. In section 2 the methodology used to construct the expectile FRM is introduced. Section 3 presents the data, computation and visualization of the results. Section 4 conclude. All codes used to do the calculation in this paper is responsible via Quantlet.

2 Methodology

2.1 Traditional risk measures

Value at Risk (VaR) and Expected Shortfall are two widely applied risk measures. Let $X_{j,t}$ represents the log return of a financial institution j at time t. VaR compute the risk for a financial institution or company j as the opposite of the tail quantile $q_{j,t,\alpha}$, which satisfies

$$\mathrm{P}\left(X_{j,t} \leq q_{j,t,\alpha}\right) = \alpha. \tag{1}$$

VaR could be obtained by minimizing the asymmetric weighted absolute error $\min_b \mathsf{E}(|X_{j,t} - b|)$. Let $F(X_{j,t})$ denote the distribution of $X_{j,t}$. As the solution $q_{j,t,\alpha}$ satisfies

$$\alpha = \frac{\int_{-\infty}^{q_{j,t,\alpha}} dF(X_{j,t})}{\int_{-\infty}^{q_{j,t,\alpha}} dF(X_{j,t}) + \int_{q_{j,t,\alpha}}^{\infty} dF(X_{j,t})}, \tag{2}$$

it indicates that VaR depends only on the relative frequency of more extreme loss rather than the magnitude of the loss. Hence it might be less tail sensitive, i.e., two portfolios might have the same VaR, but with complete different tail shapes. It may not well reflect the risk exposure in terms of the size of potential loss. Moreover, VaR does not satisfy the

subadditivity and hence diversification may not reduce the risk measured by VaR, which is not so desirable.

In contrast, ES captures the average of all potential losses exceeding the VaR at a given confidence level, thus taking into account the magnitude of the potential loss. ES can be defined as

$$\mathrm{ES}_{j,t} = \mathsf{E}\left[X_{j,t} | X_{j,t} \leq q_{j,t,\alpha}\right], \tag{3}$$

and has been adopted as a new risk measure by Basel III in 2016. As discussed in Artzner et al. (1999), ES is a coherent risk measure as it satisfies translation invariant, subadditivity, positive homogeneity, monotonicity. However, ES only considers the conditional downside mean and might be too conservative.

2.2 Expectile based VaR

This motivates us to consider using the expectile based VaR to measure the risk, i.e. EVaR, by the opposite of the τth expectile for $X_{j,t}$. Denote the the τth expectile for company j at time t as $e_{j,t,\tau}$. As suggested in Newey and Powell (1987), $e_{j,t,\tau}$ minimizes the asymmetric weighted square error $\min_b \mathsf{E}\{\rho_\tau(X_{j,t} - b)\}$, where $\rho_\tau(u) = |\tau - \mathbf{I}\{u \leq 0\}||u|^2$. In contrast to equation (2), we now have

$$\tau = \frac{\int_{-\infty}^{e_{j,t,\tau}} |X_{j,t} - e_{j,t,\tau}| \, dF(X_{j,t})}{\int_{-\infty}^{\infty} |X_{j,t} - e_{j,t,\tau}| \, dF(X_{j,t})} \tag{4}$$

which calculates the ratio of deviation below the expectile to the overall deviation, and could be interpreted as the index of prudentiality (Kuan et al., 2009).

EVaR has close relationship with VaR and ES. Jones (1994) shows that expectile has a one to one mapping with quantile. Taylor (2008) show that ES can be determined by EVaR via

$$\mathrm{ES}_{j,t} = \mathsf{E}\left[X_{j,t} | X_{j,t} \leq q_{j,t,\tilde{\alpha}}\right] = e_{j,t,\tau} + \frac{e_{j,t,\tau} - \mathsf{E}[X_{j,t}]}{1 - 2\tau} \frac{\tau}{\tilde{\alpha}}, \tag{5}$$

where $\tilde{\alpha}$ is the quantile level such that $e_{j,t,\tau} = q_{j,t,\tilde{\alpha}}$. Distinct from VaR, EVaR has the coherent property as indicated by Proposition below.

Proposition 1. *Define a general class of risk measures induced by a risk aversion function* $\phi(\alpha) \in L^1[0,1]$ *as* $M_\phi(X) = -\int_0^1 \phi(\alpha) F_X^-(\alpha) d\alpha$, *where* $F_X^{-1}(\alpha) = inf\{x | F_X(x) \geq \alpha\}$. *Let* $\tilde{\phi}_\tau(\alpha)$ *be the function that induces the EVaR, i.e.* $-e_{j,t,\tau}$, *for* $\tau \leq 0.5$.

Then $\tilde{\phi}_\tau(\alpha)$ *is non-negative, non-increasing and satisfies* $\int_0^1 \tilde{\phi}_\tau(\alpha) d\alpha = 1$. *Hence* $\tilde{\phi}_\tau(\alpha)$ *is admissible and EVaR is a coherent measure of risk.*

In fact, $\tilde{\phi}_\tau(\alpha)$ is non-negative and satisfies the normalization criterion for all τ,, but we require $\tau \leq 0.5$ to ensure the non-increasing property. Furthermore, EVaR can be interpreted as a weighted average of the conditional upside mean $E(X_{j,t} | X_{j,t} > e_{j,t,\tau})$ and the conditional downside mean $E(X_{j,t} | X_{j,t} \leq e_{j,t,\tau})$. As it balances between the cost of margin shortfall and the opportunity cost of overcharge, it is less conservative compared with ES. Above descriptions are based on the unconditional case. We shall also discuss conditional EVaR below.

2.3 Conditional EVaR

In reality, entity is not in an isolated environment. Hence it is necessary to explore the interdependency across different companies and detect the spillover effect in a network topology.

Adrian and Brunnermeier (2011) proposed the Conditional VaR (CoVaR) of company j given X_i as

$$P(X_{j,t} \leq CoVaR_{j|i,t,\tau}|X_{i,t}) = \alpha.$$

The CoVaR is estimated in two steps of linear quantile regression by assuming

$$\begin{aligned} X_{i,t} &= \tilde{a}_i + \tilde{\gamma}_i^\top M_{t-1} + \epsilon_{i,t} \\ X_{j,t} &= a_j + \gamma_j^\top M_{t-1} + \beta_j^\top X_{i,t} + \epsilon_{j,t}, \end{aligned} \tag{6}$$

Mimic the idea of CoVaR in Adrian and Brunnermeier (2011), we define the CoEVaR of a financial institution j given all other $X_{i,t}$ with $i \neq j$ at level $\tau \in (0,1)$ by solving the linear expectile regression instead. Take equation (6) as an example, we now minimize the loss function defined by

$$\min \left\{ \sum_{t=T-D+1}^{T} \rho_\tau \left(X_{j,t} - a_j - \gamma_j^\top M_{t-1} - \beta_j^\top X_{-j,t} \right) \right\} \tag{7}$$

where ρ_τ is the asymmetric loss function satisfying

$$\rho_\tau(u) = |\tau - \mathbf{I}\{u \leq 0\}||u|^2, \tag{8}$$

and we are using the most recent D observations for calculation. Clearly, the choice of D affects the estimate of the coefficients and the risk measure CoEVar. For notation simplicity, we supress the index D and leave the discussions on how to choose D for future study.

2.4 FRM lambda distribution

The approach described above is suitable when the number of institution p is not large. However, as the dimension p increases, the estimated coefficients might have lots of variability. Therefore, we recommend to include the Lasso penalty and consider the minimization criterion:

$$\min \left\{ \sum_{t=T-D+1}^{T} \rho_\tau \left(X_{j,t} - a_j - \gamma_j^\top M_{t-1} - \beta_j^\top X_{-j,t} \right) + \lambda_j \|\beta_j\|_1 \right\} \tag{9}$$

Note that the penalty term λ also has an economic interpretation and larger value of λ might indicate higher risk exposure. Define $Y_{j,t} = X_{j,t} - a_j - \gamma_j^\top M_{t-1}$ and $Y_j = (Y_{j,T-D+1}, \cdots, Y_{j,T})$, X_{-j} as the $D \times (p-1)$ matrix whose tth column collects all $X_{i,t}$ for $i \neq j$, and W be the diagonal matrix satisfying $W_{tt} = \tau$ if $Y_{j,t} - \beta_j^\top X_{-j,t} > 0$, and $W_{tt} = 1 - \tau$ if $Y_{j,t} - \beta_j^\top X_{-j,t} \leq 0$. Then the minimization criterion could be denoted as

$$f(\beta, \lambda) = (Y_j - X_{-j}\beta_j)^\top W (Y_j - X_{-j}\beta_j) + \lambda_j \|\beta_j\|_1.$$

Treating λ as a fixed value in the objective function of the penalized regression. Then

$$\frac{\partial f(\beta_j, \lambda_j)}{\partial \beta_j} = -X_{-j}^\top W(Y_j - X_{-j}\beta_j) + \lambda_j u(\beta_j)$$

where $u(\beta_j)$ is the $(p-1)\times 1$ vector formed by $u(\beta_{j,i})$, $\beta_{j,i}$ is the ith component of β_j and $u(\beta_{j,i}) = 1$ if $\beta_{j,i} > 0$, $u(\beta_{j,i}) = -1$ if $\beta_{j,i} < 0$ and $u(\beta_{j,i}) \in (-1, 1)$ if $\beta_{j,i} = 0$.

By first order condition, we have

$$X_{-j}^\top W(Y - X_{-j}\hat{\beta}_j) = \lambda_j u(\hat{\beta}_j)$$

Note that

$$\hat{\beta}_j^\top X_{-j}^\top W(Y - X_{-j}\hat{\beta}_j) = \lambda_j \hat{\beta}_j^\top u(\hat{\beta}_j) = \lambda_j \|\hat{\beta}_j\|_1$$

Hence

$$\lambda_j = \frac{\hat{\beta}_j^\top X_{-j}^\top W(Y - X_{-j}\hat{\beta}_j)}{\|\hat{\beta}_j\|_1}, \tag{10}$$

so it measures the change of tail events in terms of the change of risk driver influence.

In practice, we could select λ_j by minimizing the generalized approximate cross-validation (Yuan, 2006),

$$\lambda_{j,T}^* = \arg\min \mathrm{GACV} = \arg\min \frac{\sum_{t=T-D+1}^{T} \rho_\tau \left(X_{j,t} - a_j - \gamma_j^\top M_{t-1} - \beta_j^\top X_{-j,t}\right)}{k - df} \tag{11}$$

with df as a measure of the effective dimensionality of the fitted model. We include the subscribe T to emphasize that the optimal penalty also depends on the time period when the observations are taken. In practice we could update $\lambda_{j,T}^*$ daily when we have new available observations. We could extend the idea of FRM concept developed in (Mihoci et al., 2020; Ren et al., 2020), and extract useful information by summary statistics based on $\lambda_{j,T}^*$ across different j's. For example, we could define the unweighted expectile based FRM index as

$$\mathrm{FRM}_T = \frac{1}{p}\sum_{j=1}^{p} \lambda_{j,T}^* \tag{12}$$

or the weighted version

$$\mathrm{FRM}_T = \frac{1}{\sum_{j=1}^{p} \mathrm{Mktcap}_j} \sum_{j=1}^{p} \lambda_{j,T}^* \, \mathrm{Mktcap}_j \,. \tag{13}$$

Alternatively, FRM index can also be calculated as α quantile of cumulative density distribution F for $0 < \alpha < 1$:

$$F^{-1}(\alpha) = \inf\{\lambda_T : F(\lambda_T) \geq \alpha\}, \tag{14}$$

where F is the common distribution for $\lambda_{j,T}^*$ across j on trading day T. Thus we can have a group of FRM systemic indicators that provide numerous tools for investors with diverse risk preferences.

2.5 FRM network analysis

Borrowing the idea of TENET in Härdle et al. (2016), we arrange the expectile lasso regression coefficients β_j on trading day T into a matrix A_T and set its diagonal elements as 0.

$$A_T = \begin{pmatrix} \beta^T_{1,1} & \beta^T_{1,2} & \cdots & \beta^T_{1,p} \\ \beta^T_{2,1} & \beta^T_{2,2} & \cdots & \beta^T_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ \beta^T_{p,1} & \beta^T_{p,2} & \cdots & \beta^T_{p,p} \end{pmatrix} \tag{15}$$

Given the context of the coefficients being indicators for risk emission and spillover, we interpret it as an adjacency matrix for a directed network. The adjacency matrix representation, in turn, allows us to consider the interaction between the selected companies in the spirit of graph theory. In particular, network centrality is important as it implies the structure of graph and identifies key vertices. To describe the topology of the FRM network, we focus on the degree centrality, and the indegree and outdegree. Degree centrality is defined as

$$\tilde{D} = \sum_{j=1}^{p} \sum_{i=1}^{p} \mathbf{1}(\beta^T_{j,i}) \tag{16}$$

$$\mathbf{1}(\beta^T_{j,i}) = \begin{cases} 1 & \text{if } \beta^T_{j,i} \neq 0 \\ 0 & \text{if } \beta^T_{j,i} = 0 \end{cases},$$

which captures total connectedness in a graph. Indegree is the number of inflows meaning that how many other companies influence the node. Indegree of company j is defined as

$$Ind_j = \sum_{i=1}^{p} \mathbf{1}(\beta^T_{j,i}) \tag{17}$$

there company j can be regarded as a risk receiver in this situation. Similarly, outdegree is the number of out-going links implying that hot many other companies the node affects. Outdgree of company i is,

$$Outd_i = \sum_{j=1}^{p} \mathbf{1}(\beta^T_{j,i}) \tag{18}$$

there company i can be treated as a risk emitter.

3 Empirical Results

3.1 Data description

To estimate the dynamic FRM in the US stock market, one selects financial institutions from the US SP500 Composite Index, and in addition, six lagged macro economic variables. From Bloomberg data services, we took macro prudential variables which capture low frequency variation in tail risk not directly related to the system risk exposure. Those macro economic variables carry information on the average and volatility of the risk factors. The macro prudential input is listed below

- ⊡ The returns of the REITs Index which is a capitalization-weighted index of Real Estate Investment Trusts to capture the general condition of real estate sector

- The SP 500 equity index returns describing the overall performance of the stock market
- The returns of Chicago Board Options VIX Index measuring SP 500 Index option implied volatility
- The change in the credit spread between Moody's Baa corporate bond yield and 10 year Treasury yield
- The change in the slope of the yield curve, measured by the spread between the 10 year Treasury yield and the 3 month Treasury bill rate
- The change in the 3 month Treasury bill rate to capture the time variation in the tails of asset returns.

FRM can be generalized to multiple situations. Risk factors might be yield curves, cryptocurrencies, credit default swap changes, etc. Accordingly, we can evaluate systemic risk in the cryptocurency markets, bond market, derivative market and so on.

3.2 FRM distribution and index family

We firstly select the 100 biggest financial institutions based on their daily market capitalization, and then attach macro economic variables to implement FRM algorithm in each rolling window. On any given trading day in consideration, we take the price returns over an estimation window 63 business days. FRM allows to deal with high-dimensional data in one experiment due to its embedded attributes. The expectile level τ is equal to 0.05 without mentioning in this section. We also perform a sensitivity analysis by varying τ to 0.01, 0.1, 0.25 and 0.5 respectively.

As described in Section 3, FRM is not solely an index but displays RC distribution on each trading day. Figure 3.1 shows the FRM distribution on different days from 1st June 2019 to January 2021 for $\tau = 0.05$. Blue line denotes the average index, and the red points represent the financial institutions with the largest lambda value on each day. The time interval covered the COVID-19 pandemic period. The sudden spread of COVID-19 caught most economies off guard and ill-prepared. Unlike the "usual" financial market crises such as government debt crises and housing market crises investors have witnessed over the last decades, the COVID-19 related market turmoil does not give people enough time to respond in time. Faced with such an unpredictable market set-up, an investor should look at the entire FRM distribution paying particular attention to tail event risk. As can be seen from the figure that the FRM indicator was at a high level from March to June 2020, demonstrating that financial institutions were influenced by the COVID-19 pandemic in a negative way. Berkshire Hathaway Inc (BRK.B) had the highest lambda reading in February 2020, indicating that it was in high stress. The realized price of BRK.B decreased obviously in March 2020 shown in the left part of Figure 3.5 followed by the FRM predicted results. It is widely accepted that relatively small caps are more volatile or easy to be effected faced with market stress, e.g. People's United Financial (PBCT), Northern Trust Corp (NTRS) and, SVB Financial Group (SIVB),. However, large financial institutions in terms of market capitalisation may also have the highest readings in individual $\lambda_{j,t}$, the high co-stress in tail risk scenarios, during the pandemic period, e.g. BRK.B, JP Morgan Chase & Co (JPM), Wells Fargo & Co (WFC), American Express Co (AXP), with the highest readings. A more detailed look will be taken in Section 4.4 in terms of network analysis.

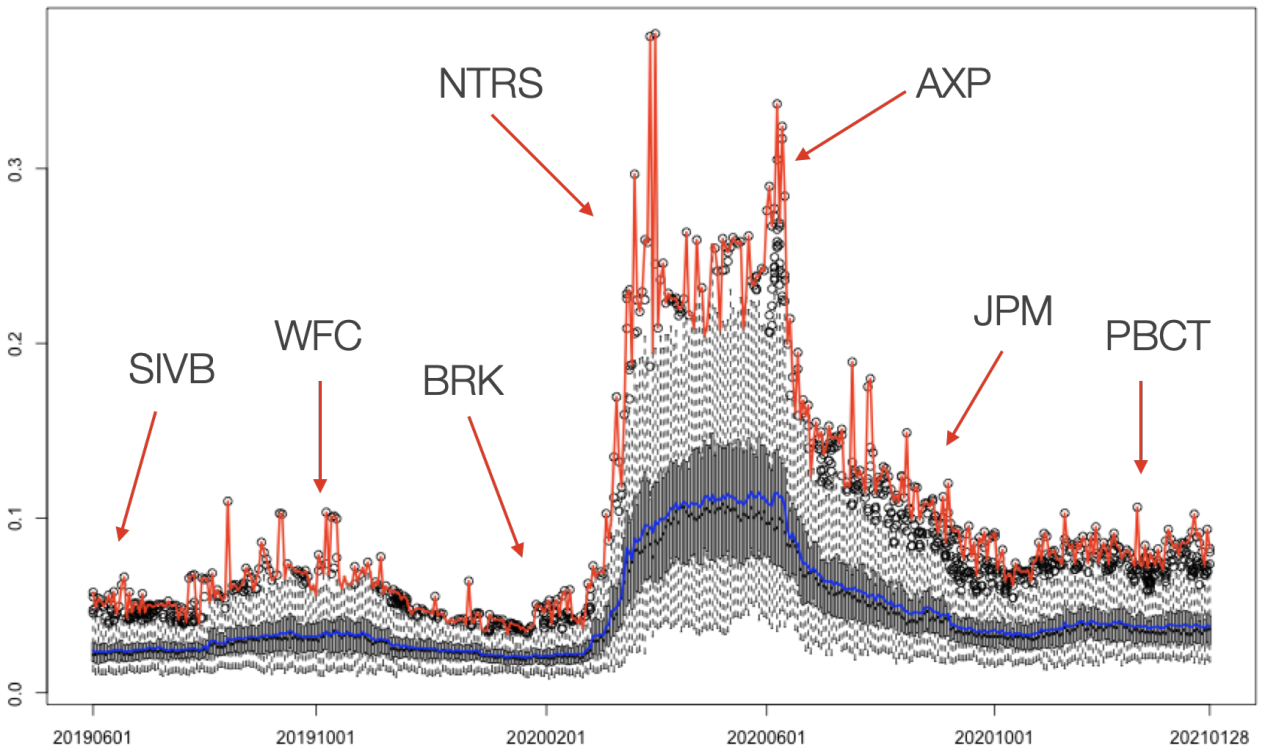

Figure 3.1: Boxplot of individrual λ distribution from June 2019 to January 2021 for $\tau = 0.05$. The average FRM index is colored in blue, and the red points represent the financial institutions with the largest RC value, e.g. BRK.B, JPM, Wells Fargo & Co (WFC), American Express Co (AXP), People's United Financial (PBCT), Northern Trust Corp (NTRS) and, SVB Financial Group (SIVB)

Systemic risk is not caused by normal market volatility, and cannot be avoided through diversification (Schwarcz, 2008). Chicago Board Options VIX Index, Systemic Risk Index (SRISK), and Google Trends are common systemic risk measures in US (Härdle et al., 2017). We compare FRM index with other systemic indicators or proxies by scaling them to the value between 0 and 1. The Systemic Risk Index (SRISK) is introduced to measure the systemic RC of a financial firm by Brownlees and Engle (2017). As a monthly risk indicator, SRISK needs to be transferred to daily data to compare with other indices. Weekly google trend indices can also be changed to daily time series by applying cubic interpolation. The first sub-figure in Figure 3.2 shows that the Google queries for "coronavirus" dramatically increase in March and April, implying that people began to be aware of the danger of the coronavirus in US. Due to its destructive power to the society, the searches for "financial crisis" remained at a high level from March to June. The implied volatility index also peaked in April. FRM index had a high level from April to July consistent with SRISK, demonstrating the coronavirus crisis had an bad impact on the financial market. Different from SRISK, FRM is daily time series index that can capture dynamics more quickly and detect risk in a network.

The second sub-figure of Figure 3.2 visualizes FRM index family explained in Section 2. The unweighted FRM index is in blue while the market capitalization weighted FRM index is painted in dark grey. Investors who are interested in large financial institutions should pay more attention to the weighted index. The weighted version is more volatile than the average one, implying that large caps are volatile during the COVID crisis and cannot be treated as safe havens. The quantile based FRM index in green is similar to the average expectile FRM index. The 0.25, 0.5, 0.7 and 0.9 quantile of lambda empirical density distribution are shown in black, orange, violet and red respectively. The multiple indices provide numerous tools for investors with diverse risk preferences.

Despite the above mentioned attributes, FRM can obtain RC distribution across financial institutions. The power of the FRM technology lies in displaying the FRM distribution on each day. Thus one may study time-varying probability density distributions and hence explore the dynamics of tail risks. The non-parametric kernel density estimation method is implemented to approximate the FRM distribution for each day. Three patterns can be discovered in Figure 3.3. When trading days are under high stress, the densities are probably to have fat tails e.g. on 15th April 2020, 13th May 2020 and 9th June 2020. The shape of density is squeezed during stable period e.g. 15th July 2019, 14th June 2020, 4th February 2020. When recovering from the high stress, the density is neither as fat as that at high risk nor as thin as that in healthy condition, e.g. 8th November 2019, 1st September 2020, 3rd November 2020 and 6th January 2021. Although the exact distributions of $\lambda_{j,T}$ series are still unknown, we may conclude that they are right-skewed distributions.

3.3 FRM CoEVaR

CoEVaR can be estimated by implementing expectile based FRM illustrated in Section 2. Figure 3.4 displays the relationship between CoEVaR and price for four financial institutions namely BRK.B, WFC and VISA. There exists a significant negative correlation between BRK.B's CoEVaR and its price especially when BRK.B is at high risk which justify that BRK.B has the highest lambda reading in Figure 3.1. The data points are highlighted in red during high stress period from March to June 2020. The same results

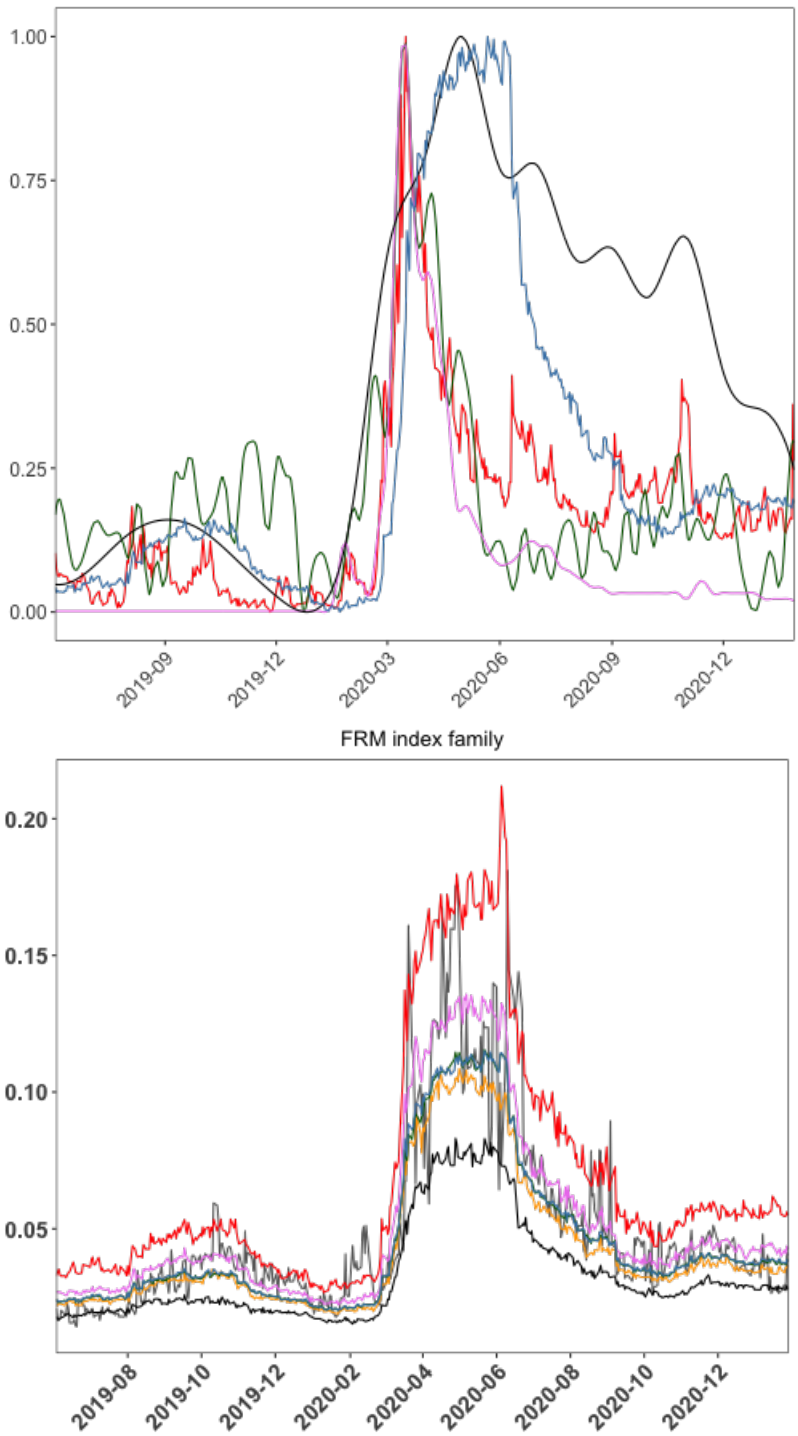

Figure 3.2: Normalized Expectile average FRM index, SRISK systemic risk index, Implied volatility index, Google trend index for the word "coronavirus",Google trend index for the word "financial crisis" are shown in the first figure. The second figure displays FRM index family from June 2019 to January 2021

hold for all four companies that significant negative relation exists between their respective CoEVaR and price. It illustrates that the downside risk indicator CoEVaR is more sensitive

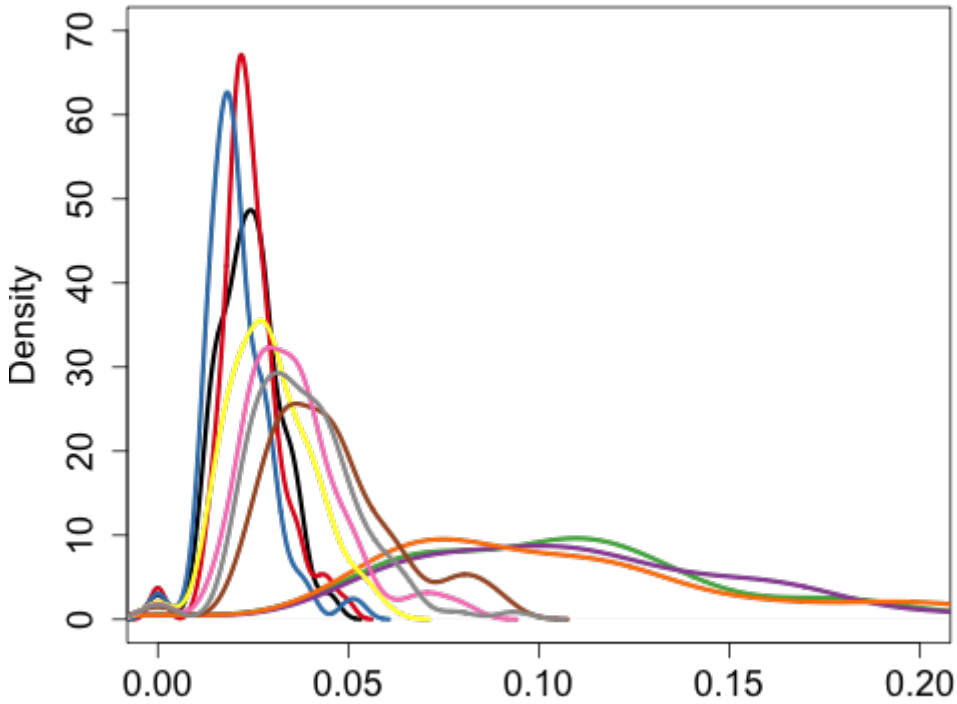

Figure 3.3: Three patterns of estimated λ pdf. Black, red and blue line represent the distribution on 15th July 2019, 14th June 2020, 4th February 2020. Green, purple and orange line show the distribution on 15th April 2020, 13th May 2020 and 9th June 2020 when the stock market is in high systemic risk. The remaining pointed lines illustrate the distribution on 8th November 2019, 1st September 2020, 3rd November 2020 and 6th January 2021 when recovering from the high stress.

to the magnitude of extreme losses consistent with the theoretical explanation from Kuan et al. (2009).

Table 3.1 shows the correlation between CoEVaR and price, the correlation between ES and price for nine financial institutions through whole sample, and the average over one hundred largest financial institutions in terms of market cap. Both two coherent risk measures have a negative correlation with asset price, however, the absolute value of CoEVaR is usually larger than that of ES, e.g. Bank of America Corp (BAC), SP Global Inc(SPG), AON insurance company shown in the table. The average value of correlation of ES is also smaller than CoEVaR concerning the absolute value, which demonstrates that CoEVaR is more accurate to indicate downside tail risk. Figure 3.5 visualizes the relation between CoEVaR and price trend for BRK.K and JPM. As can be seen from it, CoEVaR is at a high level when the price decreases for both companies, accordingly, can be treated as an alternative tail risk variable.

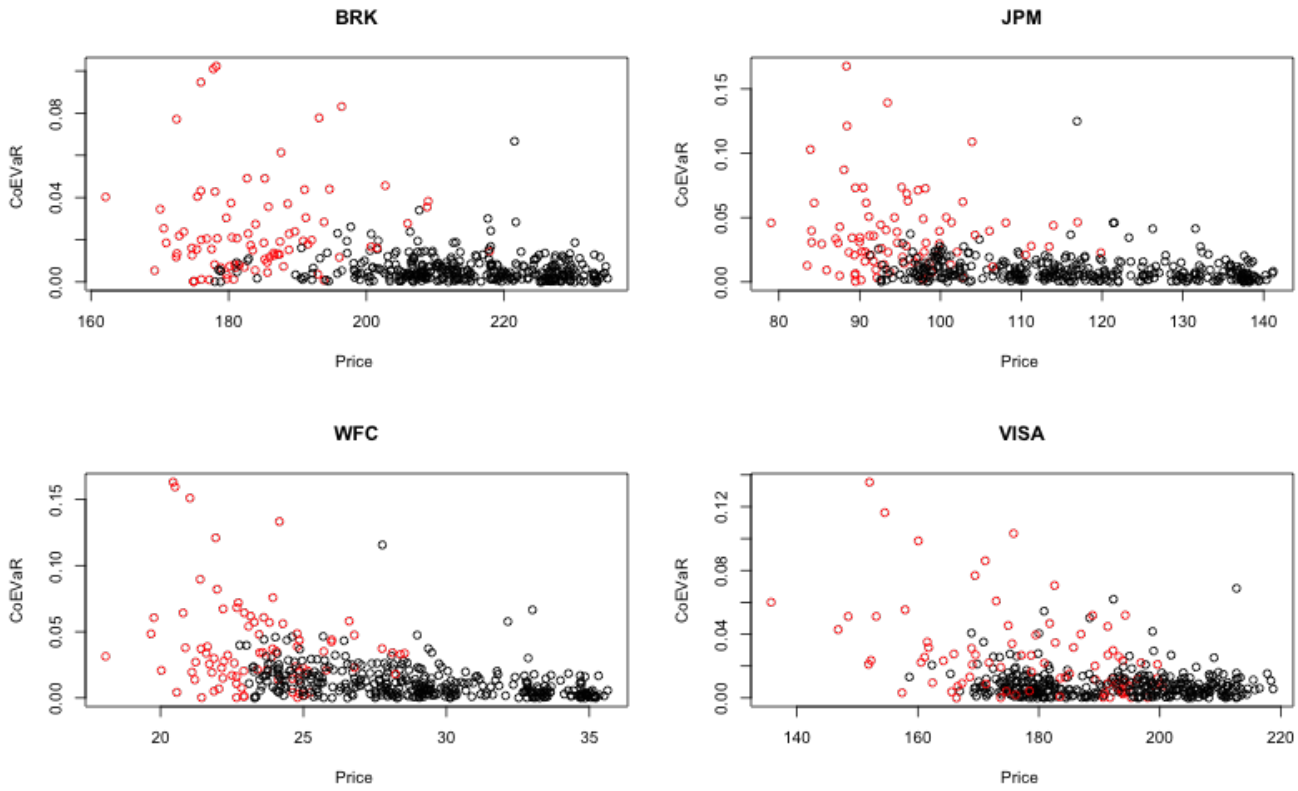

Figure 3.4: CoEVaR for BRK.B, JPM, WFC and VISA, with CoEVaR highlighted in red during the high stress period from March to June 2020, $\tau = 0.05$

In the next section, we plan to detect network behaviour estimated over a recent time frame, so as to minimize spillover effects. With the FRM technology, one can analyze in detail the network behaviour at various expectile levels.

3.4 Network analysis

Chen et al., 2019 address that pairwise similarities don't reveal equal severity. The same conclusions also made by Härdle et al., 2016 and Hautsch et al., 2015. In our paper, the adjacency matrix is generated from (15). It allows us to consider the interaction between the selected financial institutions and measure spillover effects. Figure 3.6 shows the corresponding active links. In particular, the bank BAC has a very high influence on Citi Bank (the orange point) in 2019. Note that the negative shocks from BAC will affect Citi Bank which also influences other financial institutions. Thus, these domino effects cause severe consequences in the financial industry.

The impact of the COVID-19 pandemic exhibits severe economic consequences. The global economy is projected to decline by 3% in real GDP for 2020. Rizwan et al., 2020 summarize the reasons that the COVID-19 crisis leads to the banking system's elevated systemic risk vulnerabilities in several ways. Liquidity risk is raised due to economic slowdowns, because financial institutions reduce the capital markets due to potential credit rating downgrades. Finally, a dramatic decline in intermediation business can jeopardize

Table 3.1: Correlation between CoEVaR and price, correlation between ES and price for nine financial institutions, and the average over 100 biggest financial institutions, $\tau = 0.05$

	Name	CoEVaR	ES
1	BRK.B	-0.65005	-0.40298
2	JPM	-0.62657	-0.36886
3	BAC	-0.51304	-0.43982
4	WFC	-0.46234	-0.34474
5	AXP	-0.68849	-0.43284
6	SPG	-0.69701	-0.45473
7	AON	-0.42143	-0.3463
8	SIVB	-0.44544	-0.34779
9	PBCT	-0.64351	-0.46023
	...		
10	Average	-0.47739	-0.30704

the ability to finance operations and funding costs of financial institutions (Ari et al., 2020). These risks may spread like a contagion through interconnected financial institutions. As can be seen in Figure 3.6, we take the annual average $\{\beta^T_{j,i}\}$ of the largest market capital 25 companies. Hence, the overwhelming numbers of positive interdependences may trigger the systemic risk among US firms in 2020. This result is consistent with the views of Rizwan et al., 2020.

Systemic risk is induced by positive interdependencies, whereas the negative ones are benefiting from risk diversification. Therefore, the instability indeed is caused by positive rather than negative interdependence, suggesting an asymmetric impact. In order to have a close look, we separate the positive and negative entries into two groups. A sequence of snapshots can be merged into the averages of adjacency from 2016 to 2020 as shown in Figure 3.7. One can observe from the Average+ that the risk contagion emerges geographically, while the Average- illustrates the major contributors of risk diversification in the US financial institutions. The other implication from Figure 3.7 is that the numbers of positive entries are overwhelming over the negative ones is that the practitioners and regulators should pay higher attention to the impact of systemic risk.

The network in Figure 3.8 visualizes one of the adjacency matrices of Figure 3.6. A highly connected Average+ network corresponds to a global contagion, whereas the sparse Average- network reveals scarce risk diversification. Interestingly, there are four financial institutions, Mastercard Inc. (4), WFC (6), Equinix (17), and Morgan Stanley (12) isolated in Average- network. That means they can't diverse the risk as the systemic risk is induced. In particular, as can be seen in Average+, Mastercard Inc. (4) has high influence on other financial institutions in the positive network, However, it doesn't have any connections in Average-. As Mastercard's profit suffers negative shock severely, it is easy to trigger the instability of financial system. If the stock return of Mastercard is dropping dramatically, it will also make its connected financial institutions suffering negative stock returns. Mastercard doesn't have significant negative interdependencies with others that may help for risk diversification. Thus, the systemic risk is easy to be induced by this kind of company.

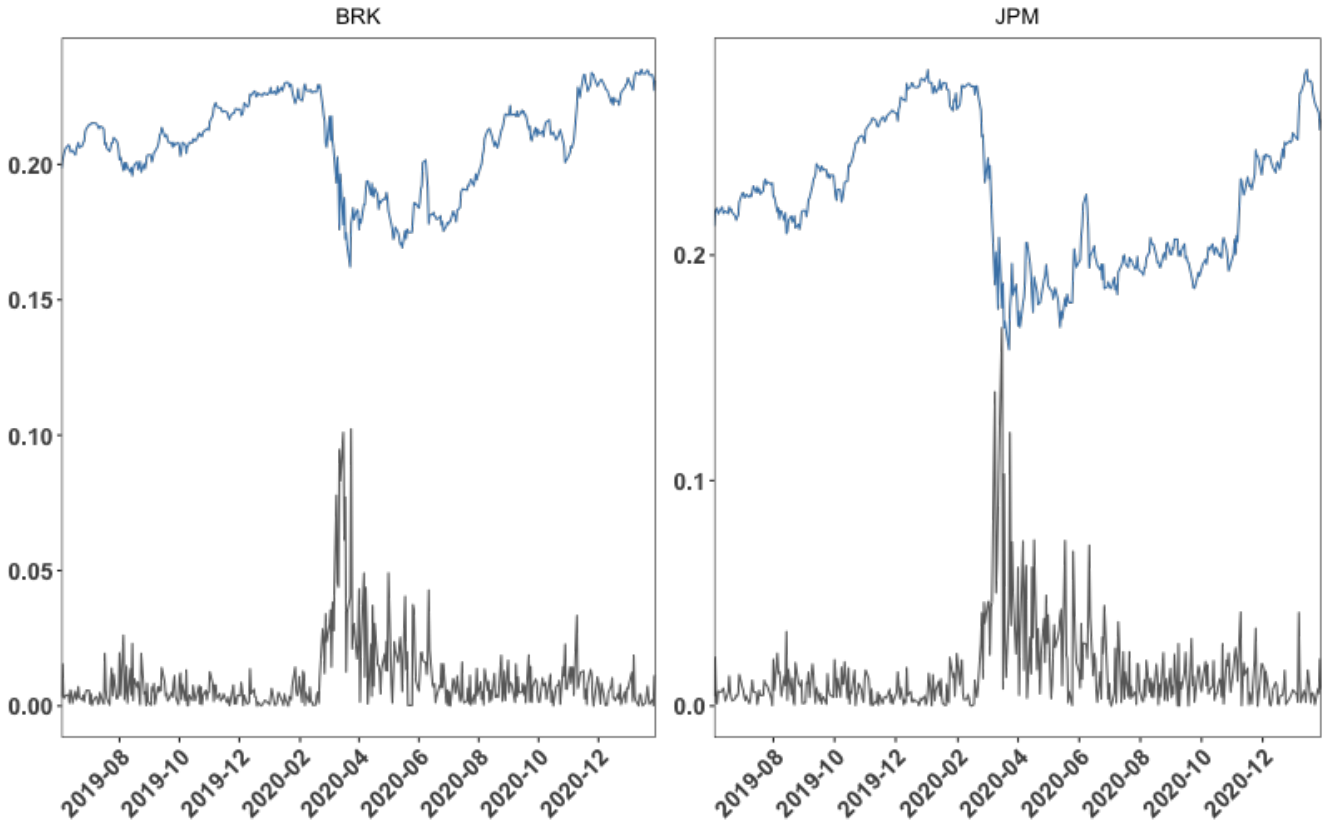

Figure 3.5: CoEVaR and price trend for BRK.B and JPM, $\tau = 0.05$

The analysis of co-movements in tail event scenarios is possible through a deeper analysis of the τ, which is the level of expectile, equal to 0.01, 0.05, 0.1, 0.25, and 0.5 with the date 20200203. From Figure 3.9 to 3.13, the columns show the marginal return contribution by respective US companies. For example, in Figure 3.9, at a $\tau = 0.01$, the stock returns of JPM are explained by 2 network nodes, specifically BAC and Citi Bank (Citi) in the third row. Moreover, JPM itself influences two financial institutions (third column). In the Figure 3.11, as $\tau = 0.1$, the stock return of JPM are best explained by 3 stocks, and it contributes marginal return to 2 financial institutions, Citi and Financial Services Group (PNC). This contrasts with WFC, its stock returns explained by three and two stocks, respectively, but itself emitting risk to other companies, seven and five respectively for $\tau = 0.01$ and $\tau = 0.1$. WFC is more likely to create spillover to the entire network.

Moreover, as τ decreases, in extreme situations, the companies have more influence on others. As outlined, in order to minimize spillover effects among those US companies, network behaviour estimated over a recent time frame can be observed. With the FRM technology, we analyze in detail the network behaviour at various expectile levels where ES and EVaR can be derived from. In Figure 3.2, the systemic risk is increasing from March, and decreasing in June 2020. Having closer look at 20200203 and 20200622 in Figure 3.14, we find that there are lots of interdependencies on 20200203 rather than 20200622. Starting with February 2020, the interconnections among US financial institutions are very intense that induces higher risk in the market. Once risk managers realize this high systemic risk, they start to operate risk management of their portfolios. Hence, this might reduce the systemic risk of the entire market. In Figure 3.14, we can investigate the connections

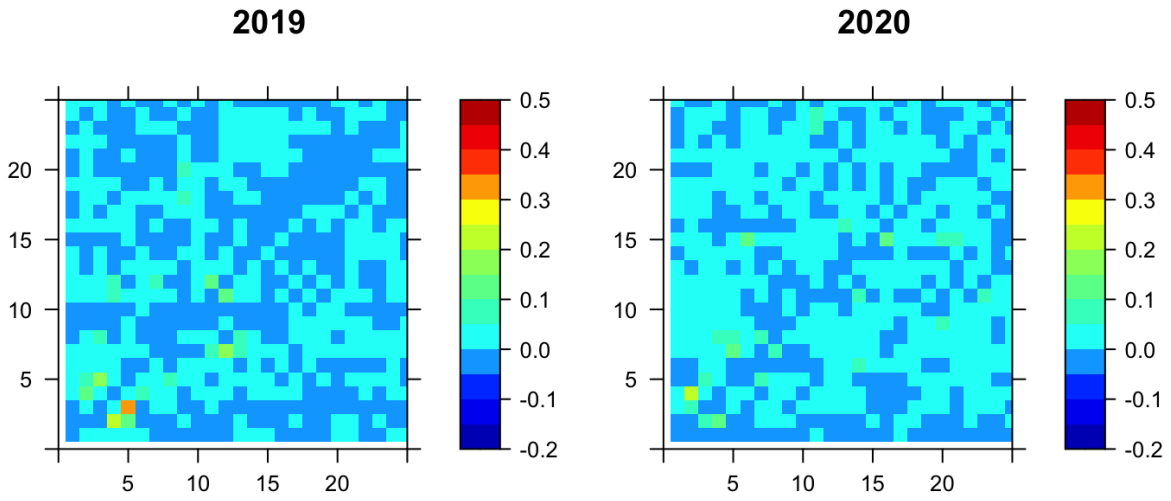

Figure 3.6: The Largest 25 Companies Adjacency Matrix
Financial institutions' names are in Figure 3.16

between each company when the systemic risk is rising. Therefore, the practitioners and risk managers should be cautious of those companies with high interdependencies.

In order to investigate the influence of connections, the network index is estimated from eq. (17) and (18). The average network index shows the risk level which each company emits. It reaches the bottom in March 2020, and then starts to rise sharply, peaking in December. As can be seen, the blue line in Figure 3.2 is also appeared to be increased in March 2020. This suggests increased RC of each financial institution may lead to higher aggregated systemic risk in the market indicated in Figure 3.2. It is worth mentioning that the contribution of each company is various with the levels of τ . According to Figure 3.15, the average RC of each company is shifting down entirely as the level of τ is getting larger. This is consistent with the point that we make, an asymmetric impact from the company in different market conditions. In a hectic situation, the average RC from each financial institution is entirely larger than the one in quiet situation.

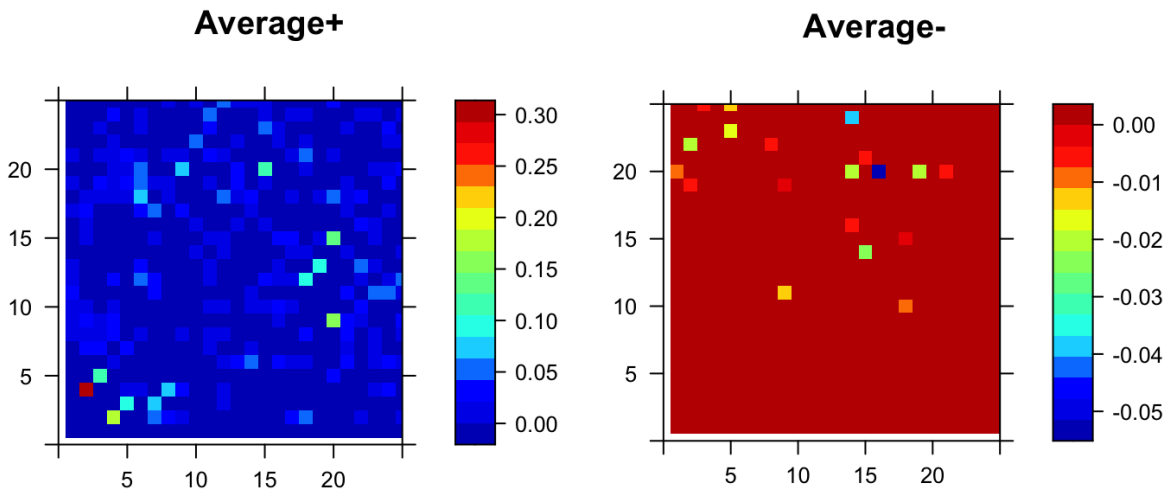

Figure 3.7: Adjacency matrices Average+ and Average-
The average values are estimated from 2016 to 2020.
Financial institutions' name is in Figure 3.16.

4 Conclusions

In this study, we propose the FRM method based on expectiles instead of quantiles that provides more general tools for practitioners and risk managers with diverse risk preferences. With this extension, the penalty term λ which is estimated from the expectile regression captures the change of tail events in terms of the change of risk driver influence. Systemic risk depends on the interdependence and the joint dynamics of financial institutions in stress situations. We employ the positive and negative network factors to detect risk propagation and diversification. This analytic decomposition is able to identify the source of systemic vulnerabilities. The network analysis with adjacency information also allows us to quantify the risk contribution of each financial institution with various expectile levels. The average risk contribution is concentrating on lower expectiles, whereas the lower risk is on higher expectiles. However, more network analysis methods, i.e. eigenvalue centrality can be applied in the future research.

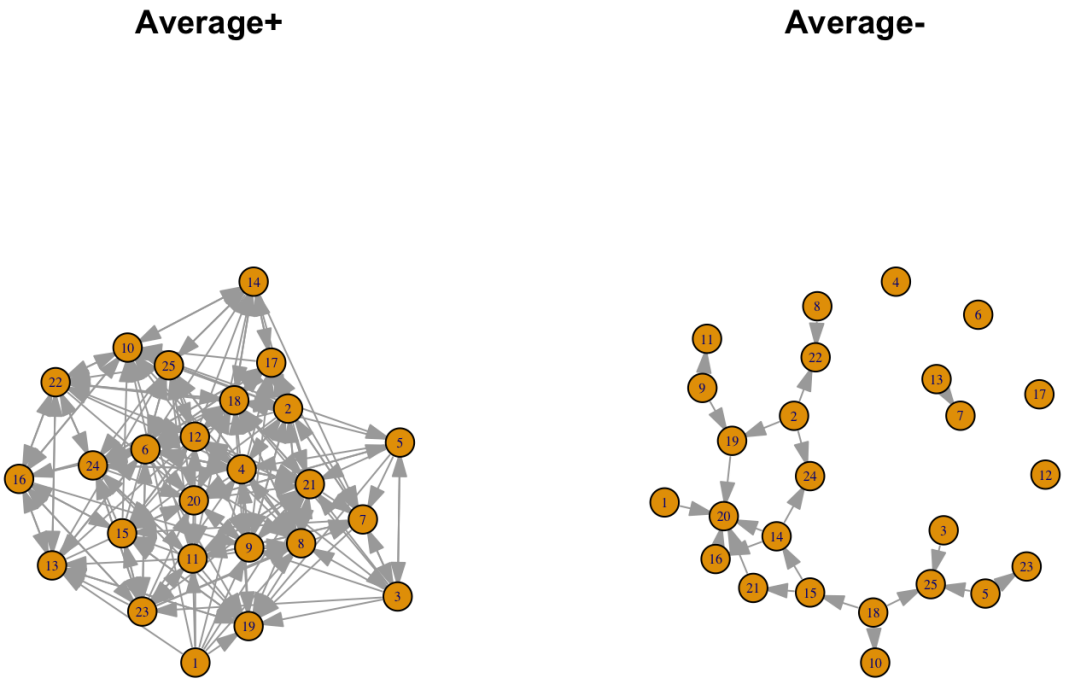

Figure 3.8: Network visualization for Average+ and Average-

In a nutshell, our model highlights the systemic importance for each financial institution and allows investigating the joint dynamics in the financial system. The methodologies and techniques we propose are tailored to describe the systemic risk in the financial system.

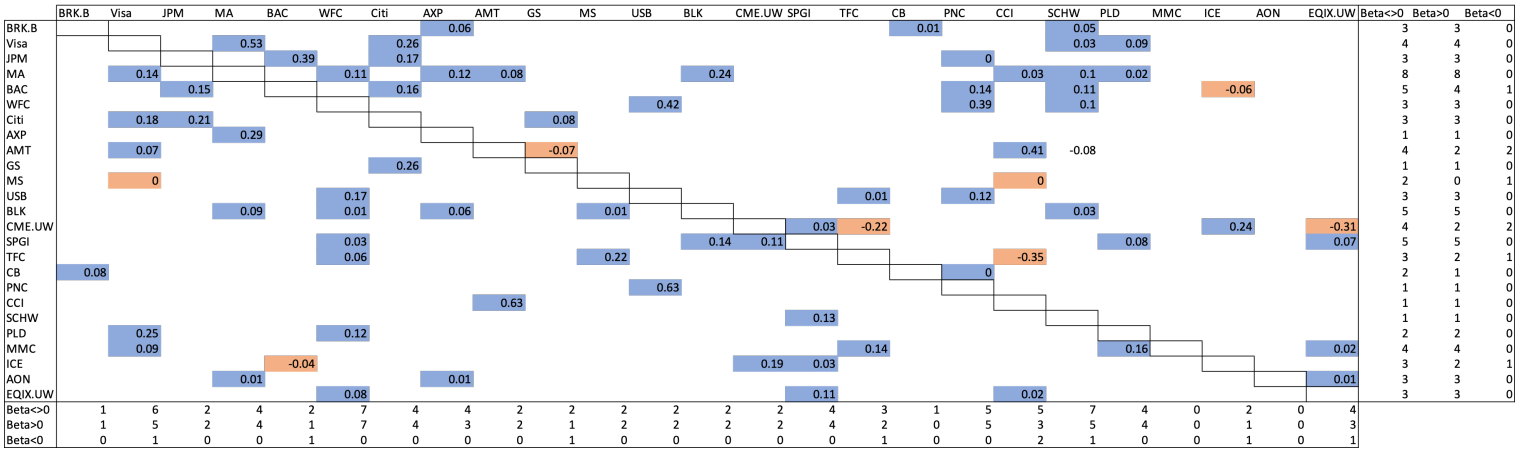

Figure 3.9: Network on 20200203 for $\tau = 0.01$

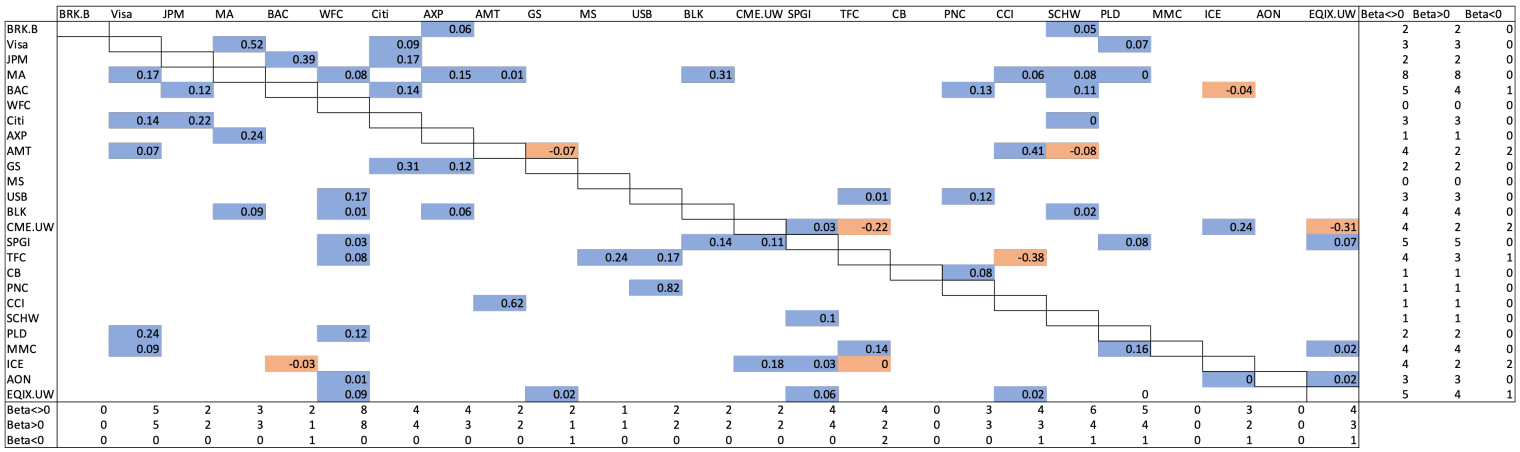

Figure 3.10: Network on 20200203 for $\tau = 0.05$

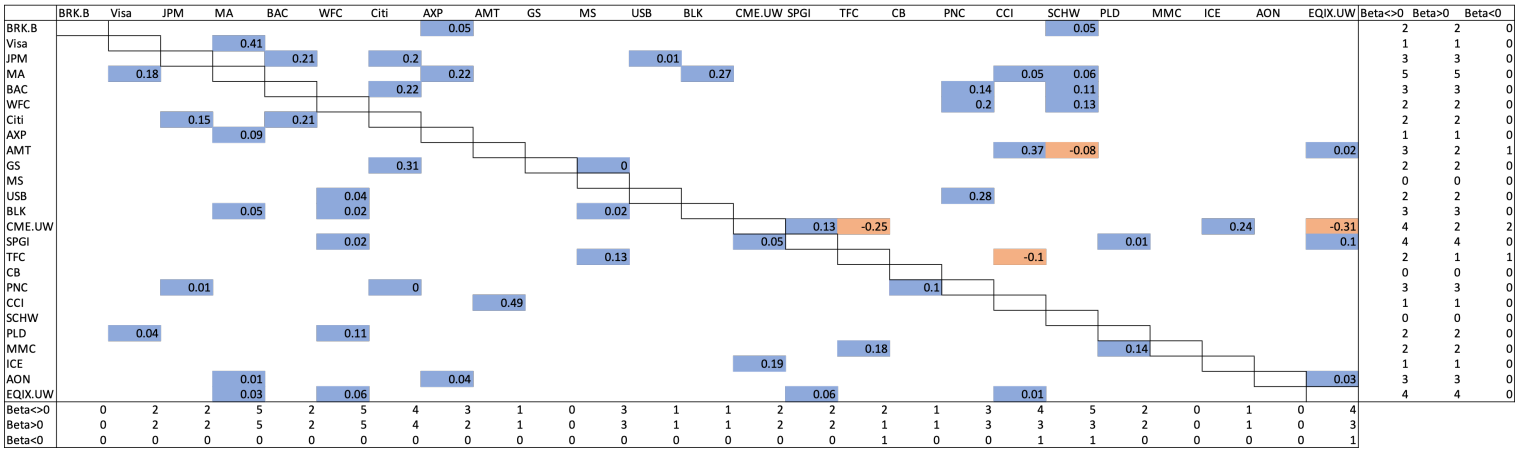

Figure 3.11: Network on 20200203 for $\tau = 0.1$

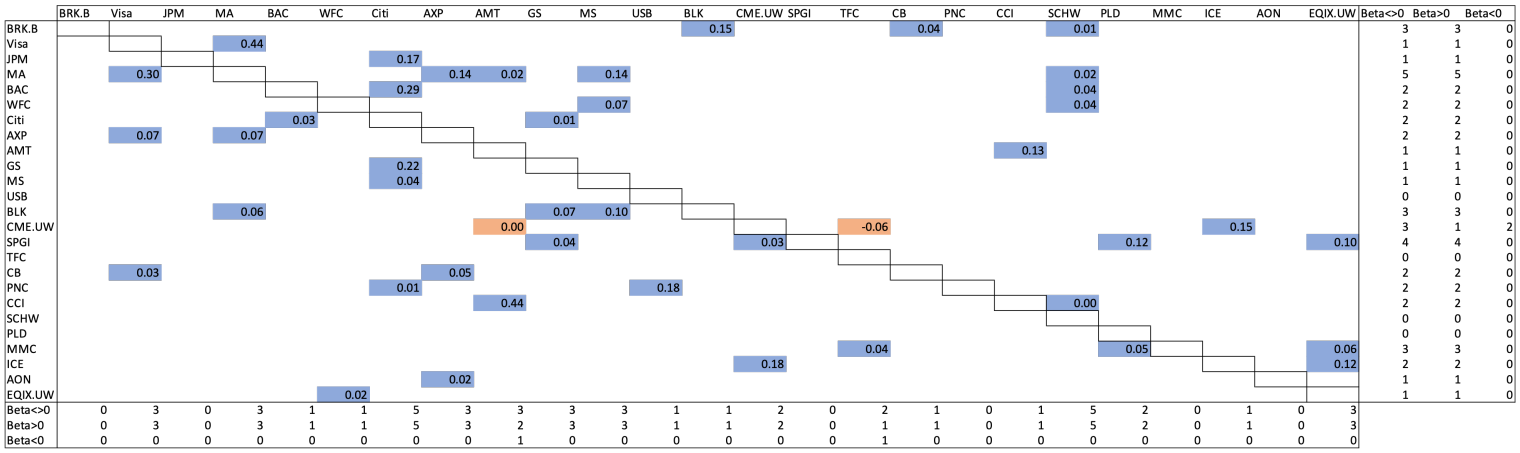

Figure 3.12: Network on 20200203 for $\tau = 0.25$

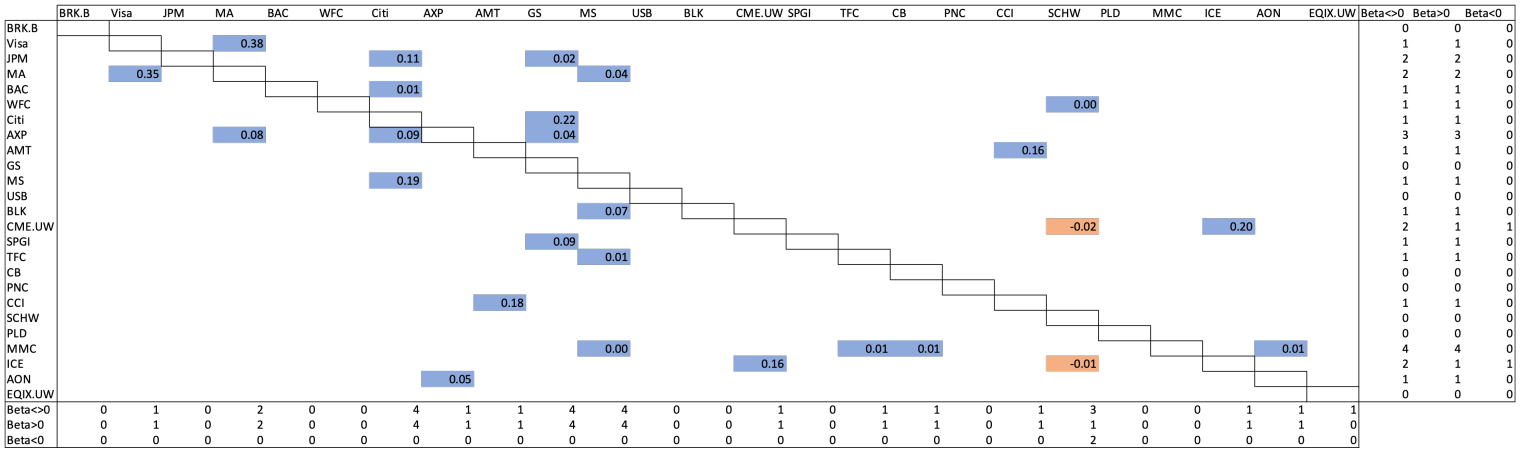

Figure 3.13: Network on 20200203 for $\tau = 0.5$

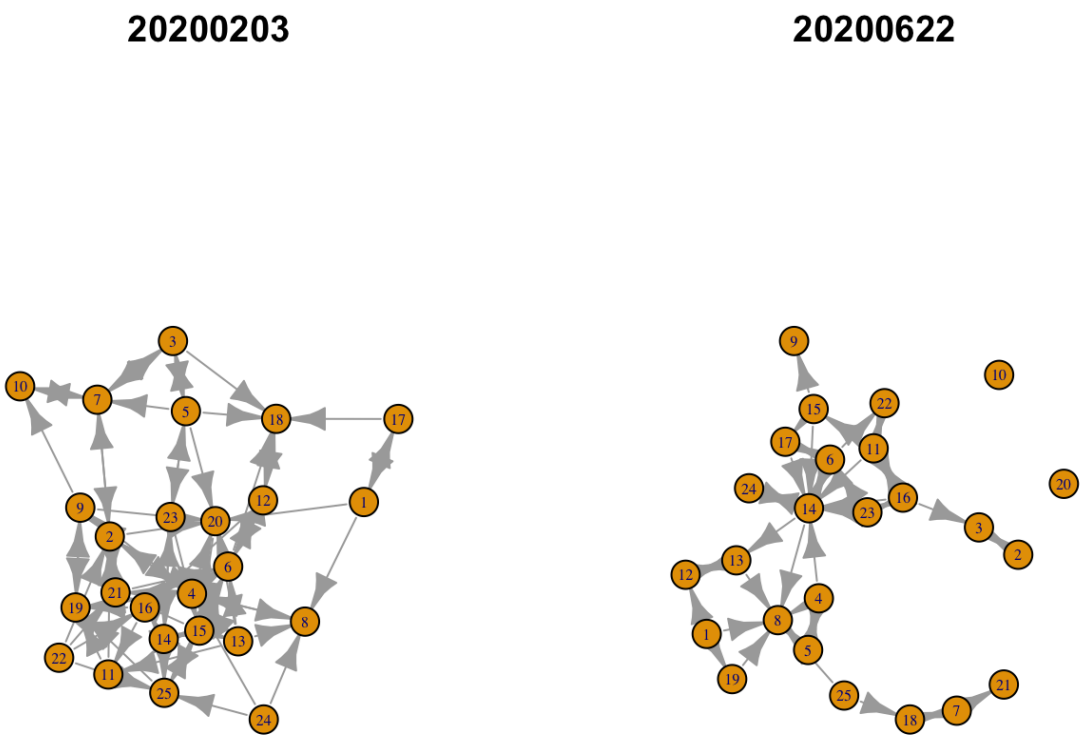

Figure 3.14: Network visualization for 20200203 and 20200622 at $\tau = 0.01$

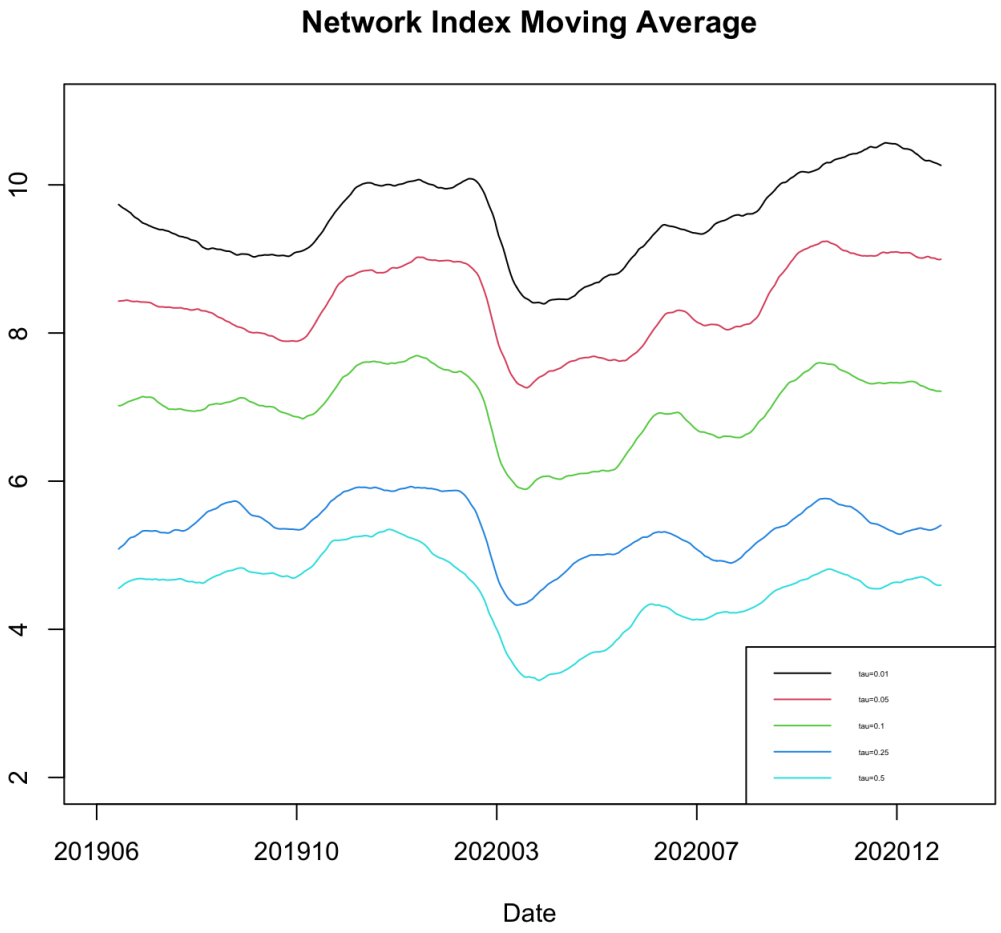

Figure 3.15: Network Index Moving Average
The value is 20 days moving average at
$\tau = 0.01, 0.05, 0.1, 0.25, 0.5$ from 20190603 to 20210128.

No.	Ticker	Name
1	BRK.B.UN.Equity	Berkshire Hathaway Inc
2	V.UN.Equity	Visa Inc
3	JPM.UN.Equity	JPMorgan Chase & Co
4	MA.UN.Equity	Mastercard Inc
5	BAC.UN.Equity	Bank of America Corp
6	WFC.UN.Equity	Wells Fargo & Co
7	AMT.UN.Equity	American Tower Corp
8	C.UN.Equity	Citigroup Inc
9	AXP.UN.Equity	American Express Co
10	BLK.UN.Equity	BlackRock Inc
11	SPGI.UN.Equity	S&P Global Inc
12	MS.UN.Equity	Morgan Stanley
13	GS.UN.Equity	Goldman Sachs Group Inc/The
14	PLD.UN.Equity	Prologis Inc
15	CCI.UN.Equity	Crown Castle International Corp
16	CME.UW.Equity	CME Group Inc
17	EQIX.UW.Equity	Equinix Inc
18	CB.UN.Equity	Chubb Ltd
19	USB.UN.Equity	US Bancorp
20	MMC.UN.Equity	Marsh & McLennan Cos Inc
21	TFC.UN.Equity	Truist Financial Corp
22	ICE.UN.Equity	Intercontinental Exchange Inc
23	MCO.UN.Equity	Moody's Corp
24	SCHW.UN.Equity	Charles Schwab Corp/The
25	PNC.UN.Equity	PNC Financial Services Group Inc/The

Figure 3.16: The financial institutions list

References

Abbassi, P., C. Brownlees, C. Hans, and N. Podlich (2017): "Credit risk interconnectedness: What does the market really know?" *Journal of Financial Stability*, 29, 1–12.

Adrian, T. and M. K. Brunnermeier (2016): "CoVaR," *The American Economic Review*, 106, 1705–41.

Aldasoro, I. and I. Alves (2018): "Multiplex interbank networks and systemic importance: An application to European data," *Journal of Financial Stability*, 35, 17–37.

Ari, A., S. Chen, and L. Ratnovski (2020): "COVID-19 and non-performing loans: lessons from past crises," *Available at SSRN 3632272*.

Brownlees, C. and R. F. Engle (2017): "SRISK: A conditional capital shortfall measure of systemic risk," *The Review of Financial Studies*, 30, 48–79.

Chen, C. Y.-H., W. K. Härdle, and Y. Okhrin (2019): "Tail event driven networks of SIFIs," *Journal of Econometrics*, 208, 282–298.

Härdle, W. K., W. Wang, and L. Yu (2016): "TENET: Tail-event driven network risk," *Journal of Econometrics*, 192, 499–513.

Härdle, W. K., W. Wang, and L. Zboňáková (2017): "Time varying quantile Lasso," in *Applied Quantitative Finance*, Springer, 331–353.

Hautsch, N., J. Schaumburg, and M. Schienle (2015): "Financial network systemic risk contributions," *Review of Finance*, 19, 685–738.

Hernandez, J. A., S. H. Kang, S. J. H. Shahzad, and S.-M. Yoon (2020): "Spillovers and diversification potential of bank equity returns from developed and emerging America," *The North American Journal of Economics and Finance*, 54, 101219.

Kreis, Y. and D. P. Leisen (2018): "Systemic risk in a structural model of bank default linkages," *Journal of Financial Stability*, 39, 221–236.

Kuan, C.-M., J.-H. Yeh, and Y.-C. Hsu (2009): "Assessing value at risk with care, the conditional autoregressive expectile models," *Journal of Econometrics*, 150, 261–270.

Mihoci, A., M. Althof, C. Y.-H. Chen, and W. K. Härdle (2020): "FRM financial risk meter," in *The Econometrics of Networks*, Emerald Publishing Limited.

Ren, R., M. Althof, and W. K. Härdle (2020): "Tail Risk Network Effects in the Cryptocurrency Market during the COVID-19 Crisis," *Available at SSRN 3753421*.

Rizwan, M. S., G. Ahmad, and D. Ashraf (2020): "Systemic risk: The impact of COVID-19," *Finance Research Letters*, 36, 101682.

Schwarcz, S. L. (2008): "Systemic risk," *Geo. Lj*, 97, 193.

Taylor, J. W. (2008): "Estimating value at risk and expected shortfall using expectiles," *Journal of Financial Econometrics*, 6, 231–252.

Yuan, M. (2006): "GACV for quantile smoothing splines," *Computational Statistics & Data Analysis*, 50, 813–829.

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