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Doctoral Thesis

**R&D-based economic growth, directed technical change,
and environmental policy**

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PhD Thesis

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R&D-Based Economic Growth, Directed Technical Change, and Environmental Policy

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Introduction - English

Technological advances are not manna from heaven. They are largely the result of basic university research as well as research and development (R&D) efforts by profit seeking firms. It has long been recognized that long-run economic growth is driven by technological advances rather than the accumulation of physical capital. Thus R&D activities naturally take center stage when analyzing economic growth related issues. This explains the frequent use of R&D-based models in modern economic growth research.

The present thesis consists of three self-contained chapters, and R&D-based models constitute the keystone in each of them. Chapter 1 examines U.S. productivity growth through the lens of R&D-based models. The analysis focuses on the relationship between effective R&D efforts and the speed of productivity growth. Chapter 2 and 3 focus on environmental policy issues related to economic growth. In these analyses, the results are strongly affected by both the direction and the speed of technical change. The key issue is how environmental policy affects private agents' incentives to conduct different types of research.

Chapter 1 is inspired by the recent debate on U.S. productivity growth prospects. The main issue in this debate is expectations about the ability of future innovations to create productivity growth (see Gordon 2015; Brynjolfsson and McAfee 2014). Chapter 1 conducts a formal analysis of this issue by assessing the debate from an R&D-based model perspective. The empirical approach is inspired by Ha and Howitt (2007), who test different R&D-based model varieties using U.S. data.

When assessing U.S. productivity growth from an R&D-based model perspective, it is crucial to distinguish between model varieties. The main objective of Chapter 1 is therefore to test different R&D-based model varieties. The analysis is based on a general R&D-based model framework, nesting the model varieties of interest. This framework implies a cointegrating relationship between productivity, R&D intensity (defined as R&D expenditures as a share of GDP), and employment. Exploiting U.S. data for the period 1953-2014, this relationship is examined using both single-equation and cointegrated VAR models. The

empirical results provide clear evidence against the widely used fully endogenous variety, while the estimates are consistent with the semi-endogenous variety. Thus future economic growth research focusing on the very long run should build on the semi-endogenous variety. Forecasts based on the obtained empirical estimates indicate that U.S. productivity growth will continue at a historically slow pace.

Besides the empirical results, the analysis in Chapter 1 contributes to the existing literature in several ways. Most importantly, the analysis extends the theoretical framework developed by Ha and Howitt (2007) to allow for productivity gains from horizontal innovation. Another important contribution is the examination of a structural transition in U.S. research intensity in the late 1950s. It is shown that this transition strongly biases the results obtained by Ha and Howitt (2007).

Chapter 2 and 3 switch the focus toward environmental policy issues. In the long run, the effectiveness of an environmental policy is to a large extent determined by its ability to affect the direction of technical change. R&D-based models are, therefore, particularly useful when assessing the long-run impact of environmental policy.

Chapter 2 investigates how environmental policy affects long-run economic growth, when environmental policy also affects the direction of technical change. The analysis is based on an R&D-based model with two types of technology: one for production and one for pollution abatement. Production technologies are used to produce consumption goods, and pollution emission is an unavoidable by-product of this production. Pollution abatement technologies reduce pollution emission stemming from production. The government imposes a pollution emission tax to incentivize pollution abatement efforts. In contrast to previous studies (Hart 2004; Ricci 2007), production and pollution abatement technologies are embodied in separate intermediate good types. This allows for a more realistic representation of the innovation process. Chapter 2 is partly based on the author's master's thesis, but it extends the analysis substantially both theoretically and empirically.

The analysis shows that a tighter environmental policy increases the demand for pollution abatement technologies which incentivizes research in these technologies. However, the opposite holds for production technologies, as the policy change increases the user cost of these polluting technologies. Hence the analysis shows that a tighter environmental policy unambiguously reduces long-run economic growth. However, simulations based on U.S. data indicate that even large environmental policy reforms have small long-run economic growth effects. The calculated welfare effects are, nevertheless, relatively large, as even small changes

in a growth rate have large level effects in the long run. These results indicate that static models and models with exogenous technical change leave out an important welfare effect of environmental policy.

The analysis in Chapter 2 contributes to the literature in at least two ways. First, in contrast to the closest related literature, the analysis clearly shows that there is a trade-off between economic growth and a clean environment. Still, the numerical results indicate that even large environmental policy reforms have small long-run economic growth effects and large environmental impacts. These results might be especially interesting for developing countries, where the trade-off between economic growth and a clean environment is particularly pronounced. Second, the chapter develops a novel modeling strategy which seems especially appropriate when focusing on local toxic air pollution. The main results are derived analytically, and it is shown that the developed model is consistent with stylized facts presented in the chapter. Accordingly, the developed modeling strategy seems to foster both tractability and empirical relevance.

Chapter 3 investigates how to achieve a given climate goal - for instance, the two-degree temperature limit from the Paris Agreement 2015 - in an R&D-based model featuring both directed technical change *and* population growth. The latter aspect is neglected in previous studies. This seems problematic, as the United Nations (2017) expects the global population to grow by nearly 50 percent from 2017 to 2100. Surely these 3.6 billion additional people will have a significant impact on the environment.

The analysis highlights two potentially counteracting effects of population growth on pollution emission. The first effect is a neo-Malthusian effect: more people implies more production and thereby more pollution emission. The second effect is named the Simon effect after Julian Simon (1981, 1998): more people implies a larger idea creating capacity resulting in faster knowledge growth. The Simon effect might decrease pollution emission per worker provided that research focuses sufficiently on environmentally friendly production technologies.

The modeling strategy is inspired by Acemoglu et al. (2012). The model features two types of production technologies, polluting and non-polluting, and consumption goods are produced using a combination of these two technologies. Scientists develop new production technologies, and the direction of research is determined by the relative profitability of research in the two types of technology. It is shown that the model developed by Acemoglu et al. (2012) cannot match the observed decreasing trend in the global CO₂ intensity (CO₂

emissions as a share of GDP). This feature results from a strong path dependency of research efforts in their model. Specifically, research only targets the most advanced technology, polluting or non-polluting, under *laissez-faire*. In contrast, the model developed in Chapter 3 allows for simultaneous research in both types of technology. As a result, the model is able to match the decreasing trend in the global CO₂ intensity.

The analytical and numerical results show that population growth is a major burden on the environment. Hence population reducing policies are relevant environmental policy tools. Permanent subsidies can permanently and fully direct research toward non-polluting technologies. Yet such policies are typically insufficient to ensure a given climate goal unless they are combined with sufficiently strict population control policies. Finally, the analysis highlights the effectiveness of a pollution emission tax. An emission tax not only incentivize a less polluting input mix in the production process; it also increases the market size for non-polluting production technologies, and this directs research toward these technologies.

Introduction - Danish

Teknologiske fremskridt kommer ikke som manna fra himlen. De er primært resultatet af en aktiv forskningsindsats på universiteter og i profitsøgende virksomheder. Det har længe været anerkendt, at langsigtet økonomisk vækst er drevet af teknologiske fremskridt frem for akkumulation af fysisk kapital. Den samlede forsknings- og udviklingsindsats spiller derfor en central rolle i analyser af problemer relateret til langsigtet økonomisk vækst. Dette forklarer den hyppige brug af forsknings- og udviklingsbaserede (herfra F&U-baserede) vækstmodeller i moderne økonomisk vækstforskning.

Denne afhandling består af tre selvstændige artikler, og F&U-baserede vækstmodeller spiller en central rolle i hver af dem. Kapitel 1 undersøger produktivitetsvæksten i USA fra et F&U-baseret vækstmodelperspektiv. Analysen fokuserer på forholdet imellem den effektive forskningsindsats og den hastighed, hvormed produktiviteten øges. Kapitel 2 og 3 fokuserer på miljøspørgsmål relateret til økonomisk vækst. Resultaterne af disse analyser er stærkt påvirket af både retningen og hastigheden af den teknologiske udvikling. Omdrejningspunktet er, hvordan miljøpolitik påvirker private aktørers incitamenter til at foretage forskellige typer forskning.

Kapitel 1 er inspireret af den verserende debat om udsigterne for produktivitetsvæksten i USA. I sidste ende handler debatten om forventninger til fremtidige innovationers evne til at skabe vækst (se Gordon 2015; Brynjolfsson og McAfee 2014). Kapitel 1 foretager en formel analyse af USA's produktivitetsudvikling med udgangspunkt i F&U-baserede vækstmodeller. Den empiriske tilgang er inspireret af Ha og Howitt (2007), som tester forskellige F&U-baserede vækstmodeltyper på data fra USA.

Når produktivitetsudviklingen analyseres fra et F&U-baseret vækstmodelperspektiv, er det afgørende at skelne imellem forskellige modelvarianter. Det primære formål i Kapitel 1 er derfor at teste forskellige F&U-baserede vækstmodelvarianter. Analysen er baseret på en generel F&U-baseret vækstmodel, som indeholder de modelvarianter, der har interesse. Denne generelle model prædikerer, at produktiviteten, forskningsintensiteten (defineret som

F&U-udgifter som andel af BNP) og beskæftigelsen kointegrerer. Dette kointegrationsforhold undersøges på data fra USA ved brug af både simple enhedsrodtest og kointegrerede VAR modeller. De empiriske resultater fra disse undersøgelser er i modstrid med den fuldt endogene modelvariant. Resultaterne er derimod ikke i modstrid med den semi-endogene modelvariant. Dermed peger resultaterne på, at fremtidige undersøgelser med fokus på det meget lange sigt bør baseres på den semi-endogene modelvariant. Fremskrivninger baseret på de empiriske resultater indikerer, at produktivitetsvæksten i USA vil fortsætte med en historisk lav hastighed.

Udover de empiriske resultater har Kapitel 1 også adskillige andre bidrag til den eksisterende litteratur. Det vigtigste af disse bidrag er en modeludvidelse, som tillader produktivtetsgevinster fra horisontal innovation. Denne effekt er fraværende i studiet af Ha og Howitt (2007). Et andet vigtigt metodisk bidrag er undersøgelsen af en strukturel transition i USA's forskningsintensitet i slutningen af 1950'erne. Det vises, at Ha og Howitts resultater er stærkt påvirket af denne transition, og at deres empiriske resultater ikke er robuste, når denne transition tages i betragtning.

Kapitel 2 og 3 drejer fokus over på miljøpolitiske spørgsmål. På lang sigt er effektiviteten af en miljøpolitik i høj grad bestemt af dennes effekt på retningen for den teknologiske udvikling. F&U-baserede vækstmodeller er derfor specielt brugbare i vurderingen af de langsigtede effekter af forskellige miljøpolitikker.

Kapitel 2 undersøger, hvordan miljøpolitik påvirker den langsigtede økonomiske vækst, når miljøpolitik også påvirker retningen for den teknologiske udvikling. Analysen er baseret på en F&U-baseret vækstmodel med både produktionsteknologi og forureningsbekæmpelsesteknologi. Produktionsteknologierne bruges til at producere forbrugsgoder, og forureningsudslip er et uundgåeligt biprodukt af denne produktion. Forureningsbekæmpelsesteknologierne kan benyttes til at reducere forureningsudslippet fra produktionen. Regeringen pålægger en afgift på forureningsudslip, hvilket giver virksomhederne incitament til forureningsbekæmpelse. I modsætning til tidligere studier (Hart 2004; Ricci 2007) er produktions- og forureningsbekæmpelsesteknologier indeholdt i separate mellemprodukttyper. Dette muliggør en mere realistisk repræsentation af innovationsprocessen. Kapitel 2 bygger delvist på forfatterens kandidatspeciale, men analysen udvides betragteligt både teoretisk og empirisk.

Analysen viser utvetydigt, at en strengere miljøpolitik øger efterspørgslen efter forureningsbekæmpelsesteknologier, hvilket øger incitamentet til at forske i disse teknologier. Samtidig falder efterspørgslen efter produktionsteknologier, idet en strengere miljøpolitik øger omkost-

ningerne ved at benytte disse forurenende teknologier. Dermed reduceres incitamentet til at forske i produktionsteknologier. Analysen viser derfor entydigt, at en strengere miljøpolitik reducerer den langsigtede økonomiske vækst. Simulationer baseret på data fra USA indikerer, at selv store miljøpolitiske reformer har små langsigtede væksteffekter. De beregnede velfærdseffekter er ikke desto mindre betydelige, da selv små ændringer i en vækstrate har store niveaueffekter på lang sigt. Disse resultater indikerer, at statiske modeller og modeller med eksogene teknologiske fremskridt undlader en vigtig velfærdseffekt af miljøpolitiske ændringer.

Analysen i Kapitel 2 bidrager til den eksisterende litteratur på mindst to måder. I modsætning til Hart (2004) og Ricci (2007) viser analysen, at der er et klart trade-off imellem økonomisk vækst og et renere miljø. Men analysen viser også, at selv store miljøreformer har små langsigtede økonomiske væksteffekter. Disse resultater er specielt interessante for udviklingslande, hvor dilemmaet imellem økonomisk vækst og et renere miljø er særlig udtalt. Derudover udvikles en ny modelleringsstrategi, som synes særlig relevant, når der fokuseres på giftig lokal luftforurening. De primære resultater udledes analytisk, og det vises, at den udviklede model er konsistent med empiriske regulariteter, som præsenteres i kapitlet. Dermed synes denne nye modelleringsstrategi at fremme empirisk relevans, samtidig med at en relativt simpel modelramme kan fastholdes.

Kapitel 3 undersøger, hvordan en given klimamålsætning - f.eks. 2 graders-målsætningen fra Parisaftalen 2015 - kan opnås i en F&U-baseret vækstmodel, som indeholder både flere teknologityper og befolkningsvækst. Det sidstnævnte aspekt ignoreres i den tidligere litteratur på området. Dette er problematisk, idet FN forventer, at den globale befolkning vokser med næsten 50 pct. fra 2017 til 2100. Det synes åbenlyst, at disse 3,6 milliarder ekstra individer vil have en betydelig effekt på det globale forureningsudslip.

Analysen fremhæver to potentielt modsatrettede effekter af befolkningsvækst på det globale forureningsudslip. Den første effekt er en neo-malthusiansk effekt: flere mennesker medfører større produktion og dermed større forureningsudslip. Den anden effekt kaldes Simoneffekten efter Julian Simon (1981, 1998): flere mennesker medfører en større forskningskapacitet, hvilket resulterer i en hurtigere teknologisk udvikling. Simoneffekten kan reducere forureningsudslippet per arbejder, hvis en tilstrækkelig stor andel af den samlede forskningskapacitet fokuserer på udviklingen af miljøvenlige produktionsteknologier.

Modelleringsstrategien er inspireret af Acemoglu m.fl. (2012). Modellen indeholder to produktionsteknologityper: en forurenende og en ikke-forurenende. Forbrugsgoder kan pro-

duceres ved at kombinere disse to produktionsteknologityper. Forskere kan udvikle nye produktionsteknologier, og forskerne vil fokusere på den produktionsteknologitype, som er mest profitabel. Det vises, at den model, som udvikles af Acemoglu m.fl. (2012), ikke kan matche den observerede negative trend i den globale CO₂-intensitet (defineret som CO₂ emission som andel af BNP). Årsagen til dette problem kan findes i den måde, hvorpå Acemoglu m.fl. (2012) modellerer forskningssektoren. Mere specifikt prædikerer deres model, at der kun forskes i den mest avancerede teknologitype. I modsætning hertil tillader modellen, som udvikles i Kapitel 3, simultan forskning i begge teknologityper. Dette gør modellen i stand til at matche den observerede trend i den globale CO₂-intensitet.

De analytiske og numeriske resultater viser, at befolkningsvækst er en stor byrde for klimaet. Politiske tiltag, som reducerer befolkningsvæksten, er derfor relevante miljøpolitiske redskaber. Permanente subsidier kan fuldtud og permanent dirigere forskningsindsatsen imod miljøvenlige produktionsteknologier. Men sådanne subsidier til F&U kan typisk ikke sikre ambitiøse klimamålsætninger, medmindre de kombineres med befolkningsvækstreduserende tiltag. Endelig fremhæver analysen, at en skat på forureningsudslip er et effektivt miljøpolitisk instrument. En skat på forurening vil ikke blot øge incitamentet til at benytte mere miljøvenlige produktionsmetoder. Den vil også øge efterspørgslen efter miljøvenlige produktionsteknologier, hvilket øger incitamentet til at forske i denne teknologitype.

Chapter 1

Testing R&D-Based Endogenous Growth Models

Testing R&D-Based Endogenous Growth Models*

By Peter K. Kruse-Andersen

Abstract

This paper examines U.S. productivity growth through the lens of R&D-based models. A general R&D-based model, nesting both the fully and semi-endogenous varieties, is developed. The model implies a cointegrating relationship between productivity, research intensity, and employment. Exploiting U.S. data for the period 1953-2014, the relationship is examined using both single-equation and cointegrated VAR models. The results provide evidence against the widely used fully endogenous variety and in favor of the semi-endogenous variety. Forecasts based on the empirical estimates suggest a continuation of the recent U.S. productivity growth slowdown. Specifically, the annual growth rate of GDP per worker converges to less than 1.1 pct.

Keywords: Endogenous growth, semi-endogenous growth, total factor productivity (TFP), research and development (R&D), time series econometrics, cointegration

JEL Classification: C32, E24, O31, O41, O47, O51

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1 Introduction

Can we expect U.S. economic growth to continue at previous rates? Observers are divided. On the one side, Gordon (2012, 2016) argues that future innovations do not have the same potential to create productivity growth as innovations of the past. On the other side, Brynjolfsson and McAfee (2014) argue that we are entering a second machine age, where technologies like artificial intelligence and machine learning provide an escape from declining productivity growth rates.

R&D-based models represent state of the art within endogenous growth theory. These models provide a natural starting point for an empirical assessment of the debate. Within the class of R&D-based models, a key distinction is that between the fully and semi-endogenous varieties. The fully endogenous variety predicts that long-run exponential productivity growth is obtainable given a constant research intensity (defined as R&D expenditures as a share of GDP). In that sense, it is the relative R&D effort that matters for long-run productivity growth. Forecasts based on this variety predict a constant productivity growth rate, placing this variety on the optimistic side of the debate. In the semi-endogenous variety, productivity growth is tied to the absolute R&D effort and thereby the scale of the economy. The U.S. labor force is expected to grow significantly slower over the coming decades (Toossi 2016), implying slower growth in the scale of the U.S. economy. As a result, forecasts based on the semi-endogenous variety predict declining productivity growth. This variety thereby represents the more pessimistic side of the debate.

The present study tests R&D-based models using U.S. macro data for the period 1953-2014. The results provide clear evidence against the fully endogenous variety, while the results support the semi-endogenous variety. Thus the present study finds support for the more pessimistic economic growth view held by Gordon and others. As discussed below, these results coincide well with recent micro-level evidence presented by Bloom et al. (2017).

The results are obtained the following way. A general R&D-based model, nesting both the fully and semi-endogenous varieties, is developed. The theoretical model predicts a cointegrating relationship between multifactor productivity (MFP), research intensity, and employment. This relationship is examined using both single-equation and cointegrated vector autoregressive (VAR) models. The empirical results support R&D-based models in general. However, the widely used fully endogenous variety is clearly rejected, while the empirical results support the semi-endogenous variety. Forecasts based on the empirical

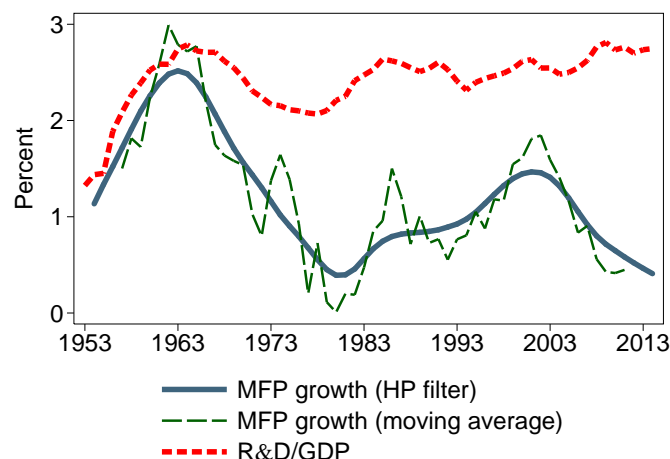


FIGURE 1: U.S. R&D expenditures as a share of GDP and smoothed U.S. MFP growth rate, 1953-2014. *Data sources:* U.S. Bureau of Economic Analysis (2016a), U.S. Bureau of Labor Statistics (2016), and the National Science Foundation.

Notes: The smoothing factor of the HP filter equals 100. The moving average is a 7-year centered moving average.

estimates suggest that the U.S. growth rate of GDP per worker converges to less than 1.1 pct. per year: a notable growth slowdown.

To understand the rejection of the fully endogenous variety, consider the basic trends in the U.S. economy since World War II shown in Figure 1. The figure shows that U.S. R&D expenditures were between 2 and 3 pct. of GDP for the period 1960-2014, while the smoothed U.S. MFP growth rate decreased from over 2 pct. in 1960 to less than 0.5 pct. in 2014. Hence the research intensity remained approximately constant, while the productivity growth rate declined. In contrast, the fully endogenous variety predicts that the productivity growth rate tracks the fraction of GDP spent on R&D (Aghion and Howitt 2005, p. 94-95).

Besides the empirical results, the present study contributes to the existing literature in four important ways. First, most previous studies build on the empirical strategy developed by Ha and Howitt (2006, 2007).¹ The present study extends the underlying theoretical framework of this strategy to allow for productivity gains from horizontal innovation. This methodological contribution turns out to be important. Second, Ha and Howitt (2007), among others, test the semi-endogenous variety under the restriction of no horizontal innovation. In contrast, the present study allows for horizontal innovation when testing this variety. Third, the present study identifies a structural transition in U.S. research intensity at the end of the 1950s (see Figure 1). Using simple unit root tests, it is shown that Ha and Howitt's results are strongly affected by this structural transition. The present study,

¹These studies include Madsen (2008), Madsen et al. (2010), Ang and Madsen (2011), and Kaila (2012). Other studies testing R&D-based growth models include Jones (1995b), Laincz and Peretto (2006), Ulku (2007), and Venturini (2012).

therefore, carefully takes this transition into account when estimating the parameters of interest. Fourth, in contrast to previous studies, the present study carefully identifies and address distinct macroeconomic events like the 1973 oil embargo which might otherwise bias the obtained results. Specification tests indicate that these distinct macroeconomic events must be taken into account to ensure well-specified econometric models.

The motivation for using a cointegrated VAR approach is the following. As emphasized by Ha and Howitt (2007), R&D-based models predict a cointegrating relationship between productivity, effective R&D input, and employment. Thus it seems natural to employ a cointegrated VAR model to estimate the parameters of interest. Furthermore, the cointegrated VAR approach has several advantages when testing R&D-based models: (i) potential mutual dependencies between the main variables are accommodated, (ii) the parameters are estimated by an efficient estimator, and (iii) the framework allows for a distinction between short and long-run relationships. The last point is especially important, as the focus of this paper is exclusively on the long-run relationship.

Moreover, it seems appropriate to use long time series when testing R&D-based models. Growth models are designed to explain macroeconomic tendencies over long time spans. Thus their predictions should be evaluated based on datasets covering long periods of time. In addition, the essential difference between the fully and semi-endogenous varieties lies in the time dimension, as the two varieties might produce indistinguishable patterns in the short run. Accordingly, cross-country and cross-industry data covering relatively short periods of time might not contain enough information to distinguish between the two model varieties.

The present study focuses on the U.S. economy. While economic growth in other countries over the last 50 years might have been driven mainly by catching-up effects, U.S. economic growth must have been driven mainly by knowledge expansion given its leading position. To the extent that R&D-based models are useful in a single-country setting, they should at least work in the U.S. case.

This paper proceeds as follows. First, the theoretical model is developed, and the implied cointegrating relationship is integrated into the empirical model (Section 2). Next, the data are presented (Section 3), and the theoretical model is tested using both single-equation and VAR models (Section 4). It is subsequently shown that the empirical results are robust to justifiable changes in the empirical approach (Section 5). The empirical estimates are then used to forecast U.S. MFP and GDP per worker figures (Section 6). The paper finishes with some concluding remarks (Section 7).

2 Theory

The identification strategy is as follows. A simple R&D-based model, nesting different R&D-based model varieties, is developed. The theoretical model predicts a cointegrating relationship between MFP, research intensity, and employment. This cointegrating relationship is estimated using a cointegrated VAR model. This strategy ensures that the main hypotheses distinguishing the different model varieties are tested within a coherent framework.

The theoretical model is designed to capture the long-run dynamics of the system, i.e. the long-run predictions of the different model varieties. The short-run dynamics of the empirical model are left unrestricted, allowing various adjustment processes to the cointegrating relationship.

2.1 Theoretical model

Time is continuous and indexed $t \geq 0$. Final goods are consumed, transformed into raw capital, or used as research input:

$$Y(t) = C(t) + J(t) + R(t), \quad (1)$$

where $Y(t)$ is aggregate output of final goods, $C(t)$ is aggregate consumption, $J(t)$ is aggregate capital investment, and $R(t)$ is aggregate investment in R&D.

Final goods are produced from labor and specialized capital goods:

$$Y(t) = Q(t)^{\eta+\alpha} \left(\frac{1}{Q(t)} \int_0^{Q(t)} m(i, t)^\alpha A(i, t) di \right) L(t)^{1-\alpha}, \quad 0 < \alpha < 1, \quad \eta \geq 0, \quad (2)$$

where $Q(t)$ is a measure of the specialized capital good varieties, $m(i, t)$ is the service of specialized capital good $i \in [0, Q(t)]$, $A(i, t)$ is the productivity associated with specialized capital good i , and $L(t)$ is aggregate labor input. The parameter η captures productivity gains from horizontal innovation, cf. (7) below.²

Following Aghion and Howitt (1998, p. 94-96), it requires $A(i, t)$ units of raw capital to produce specialized capital good i , i.e. it requires more raw capital to produce more advanced specialized capital goods. The market clearing condition for raw capital, $K(t)$,

²The production function from Aghion and Howitt (1998, p. 407) is nested as the special case $\eta = 0$, where horizontal innovation has no productivity-enhancing effects. Additionally, the production function from Romer (1990, p. 83) is nested as the special case $\eta = (1 - \alpha)$ and $A(i, t)$ equals a constant, so that there are no quality improvements of specialized capital goods.

amounts to:

$$K(t) = \int_0^{Q(t)} A(i, t) m(i, t) di. \quad (3)$$

The stock of raw capital evolves according to

$$\frac{dK(t)}{dt} \equiv \dot{K}(t) = J(t) - \delta K(t), \quad \delta > 0, \quad K(0) > 0 \text{ given}, \quad (4)$$

where δ is the capital depreciation rate.

The expansion of specialized capital good varieties has been motivated in different ways in the literature (see Young 1998; Aghion and Howitt 1998, ch. 12; Howitt 1999). The common feature of these mechanisms is that the range of varieties increases with the size of the labor input. Following Jones (1999), the range of specialized capital good varieties expands according to:

$$Q(t) = L(t)^\beta, \quad 0 \leq \beta \leq 1. \quad (5)$$

Let the technological level, $A(t)$, be defined as the average productivity associated with the specialized capital goods:

$$A(t) \equiv \frac{1}{Q(t)} \int_0^{Q(t)} A(i, t) di.$$

Investments in R&D increase the technological level:

$$\frac{dA(t)}{dt} \equiv \dot{A}(t) = \lambda A(t)^\phi \left(\frac{R(t)}{A(t)Q(t)} \right)^\sigma, \quad \lambda > 0, \quad \phi \leq 1, \quad 0 < \sigma < 1, \quad A(0) > 0 \text{ given}, \quad (6)$$

where $A(t)^\phi$ captures knowledge spillovers and the effective research input is given by $R(t)/(A(t)Q(t))$. Note that each unit of final goods spent on research becomes less effective over time, as it is spread over more specialized capital good varieties, and as the technological level becomes more advanced. Thus, the absolute amount of resources allocated to R&D must increase over time to keep the effective research input constant.

Specialized capital good varieties are produced under monopolistic competition, while final goods are produced under perfect competition. In equilibrium, all specialized capital good varieties are produced in the same quantity. Hence $m(i, t) = \bar{m}(t)$ for all i . It follows from (2) and (3) that: $Y(t) = A(t)Q(t)^{\eta+\alpha} \bar{m}(t)^\alpha L(t)^{1-\alpha}$ and $K(t) = A(t)Q(t) \bar{m}(t)$. These

two relations imply that:

$$Y(t) = A(t)^{1-\alpha} Q(t)^\eta K(t)^\alpha L(t)^{1-\alpha}. \quad (7)$$

Assuming a constant savings rate, a constant research intensity, and a constant labor input growth rate,³ the economy converges to a long-run equilibrium featuring a constant capital-output ratio (see Appendix A). To focus on the long-run equilibrium, the capital-output ratio is from now assumed constant: $K(t)/Y(t) = \kappa > 0$. This seems appropriate given that the U.S. capital-output ratio has been approximately constant since the 1950s (Jones 2015).

From (5), (6), and (7) it then follows that:

$$g_A(t) \equiv \frac{\dot{A}(t)}{A(t)} = \bar{\lambda} A(t)^{\phi-1} X(t)^\sigma L(t)^{\sigma(1-\beta+\beta\frac{\eta}{1-\alpha})}, \quad \bar{\lambda} \equiv \lambda \kappa^{\sigma\frac{\alpha}{1-\alpha}}, \quad X(t) \equiv \frac{R(t)}{Y(t)}, \quad (8)$$

where $g_A(t)$ is the technological growth rate and $X(t)$ is the *research intensity*. Hence the model describes the evolution of $A(t)$ given the paths of $X(t)$ and $L(t)$.

The parameters ϕ and β are crucial for the model's long-run predictions. If $\phi > 0$ it gets easier to innovate as the technological level increases: a standing-on-shoulders effect. In contrast, it becomes increasingly more difficult to innovate if $\phi < 0$: a fishing-out effect. According to (8), the technological growth rate is negatively affected by the technological level (state-dependence) unless $\phi = 1$. In this limiting case, the standing-on-shoulders effect is as strong as it can possibly be without leading to accelerating economic growth for a constant research intensity and labor input.

It appears from (8) that a larger labor input has two opposing effects on the technological growth rate. On the one hand, a larger labor input implies that more resources can be devoted to the creation of technical knowledge: a non-rival good. On the other hand, a larger labor input implies that more specialized capital good varieties are developed. As a result, research efforts are spread over a larger range of goods which dilutes the effect of R&D expenditures. The first effect dominates the second except for in the special case $\eta = 0$ and $\beta = 1$, where the two effects balance out. And the technological growth rate is only independent of the labor input (state-independence) in this special case.

Unfortunately the technological level, $A(t)$, cannot be observed empirically. Instead, an

³This case seems relevant as both investment in physical capital and R&D relative to GDP were approximately constant over the investigated period (Jones 2015), while the labor input grew at an approximately constant rate (see Section 3).

economist will observe the labor and capital inputs as well as the final output. From these measures and (7), the economist computes MFP as: $\tilde{A}(t) \equiv A(t)^{1-\alpha}Q(t)^\eta$. As an alternative productivity measure, the economist computes GDP (=output) per worker, $y(t)$, from the formula: $y(t) \equiv Y(t)/L(t) = \kappa^{\frac{\alpha}{1-\alpha}} \tilde{A}(t)^{\frac{1}{1-\alpha}}$. It then follows from (5), (7), and (8) that the growth rates of MFP and GDP per worker are given by

$$g_{\tilde{A}}(t) = \beta\eta g_L(t) + (1-\alpha) \underbrace{\bar{\lambda}A(t)^{\phi-1}X(t)^\sigma L(t)^{\sigma(1-\beta+\beta\frac{\eta}{1-\alpha})}}_{g_A(t)} \quad \text{and} \quad g_y(t) = \frac{g_{\tilde{A}}(t)}{1-\alpha},$$

where the growth rate of a variable $z(t)$ is denoted $g_z(t)$.

There are four growth model varieties nested in the model presented above. To simplify an examination of the model's long-run predictions, let the labor input grow at a non-negative constant rate, $g_L(t) = n \geq 0$, and let the research intensity remain constant over time, $X(t) = \bar{X} > 0$. The long-run predictions of the four growth model varieties are described below, while further documentation is provided in Appendix B. In the following exposition, productivity refers to both MFP and GDP per worker.

First-generation fully endogenous: $[n = 0, \phi = 1, \beta = 0, \eta = 0]$.

The productivity growth rate is increasing in research intensity, and productivity grows at a constant rate given a constant research intensity. The labor input is assumed constant, as a growing labor input implies an increasing economic growth rate. This variety essentially includes the models developed by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

Second-generation fully endogenous: $[n \geq 0, \phi = 1, \beta = 1, \eta = 0]$.

As in the previous model variety, the productivity growth rate is increasing in research intensity, and it grows at a constant rate given a constant research intensity. The labor input is allowed to grow, but the growth rate of the labor input has no direct effect on the productivity growth rate.⁴ This variety essentially includes the models developed by Aghion and Howitt (1998, ch. 12), Dinopoulos and Thompson (1998), Peretto (1998), Young (1998), and Howitt (1999).

Semi-endogenous: $[n \geq 0, \phi < 1, \beta < 1, \eta \geq 0]$ or $[n \geq 0, \phi < 1, \beta = 1, \eta > 0]$.

To maintain a constant productivity growth rate, the effective research input has to grow

⁴Typically, the growth rate of the labor input affects investment in R&D. The productivity growth rate is thereby indirectly affected by the growth rate of the labor input. But the productivity growth rate remains unaffected by changes in the labor input if the research intensity is constant.

exponentially over time. For a constant research intensity, this can only be achieved through an increase in the scale of the economy, represented by the labor input. Accordingly, the long-run productivity growth rate is proportional to the growth rate of the labor input. Policies that permanently increase the research intensity only increase the productivity growth rate temporarily. These temporary effects might last for a long time, depending on the parameter values. This variety essentially includes the models developed by Jones (1995a), Kortum (1997), Segerstrom (1998), and Li (2000).⁵

Less-than-exponential: $[n \geq 0, \phi < 1, \beta = 1, \eta = 0]$.

The labor input has no direct impact on productivity growth. The productivity growth rate decreases over time. But this decrease might be slow, depending on the value of ϕ . The model exhibits perpetual economic growth in the sense that GDP per worker approaches infinity for time approaching infinity. Indeed the model belongs to a class of growth models exhibiting quasi-arithmetic growth (see Groth et al. 2010).

Based on U.S. data presented in Section 3, the first-generation fully endogenous growth models are clearly rejected. The U.S. labor input more than doubled from 1960 to 2014, while the research intensity remained approximately constant. The economic growth rate declined, while first-generation fully endogenous growth models predict an increase. Hence the present study focuses on the last three model varieties.

2.2 Empirical model

The empirically relevant version of (8) is given by:

$$g_A(t) = \bar{\lambda} \left(\tilde{A}(t)^{\tilde{\phi}} X(t) L(t)^{\tilde{\beta}} \right)^{\sigma}, \quad \tilde{\phi} \equiv \frac{\phi - 1}{\sigma(1 - \alpha)}, \quad \tilde{\beta} \equiv 1 - \beta + \beta \frac{\eta}{1 - \alpha} \left(\frac{\sigma + 1 - \phi}{\sigma} \right). \quad (9)$$

When relating the model to data, time becomes discrete and a time dependent variable z is denoted z_t . The log discrete time version of (9) amounts to:

$$\ln A_{t+1} - \ln A_t \equiv \Delta \ln A_{t+1} = \gamma + \psi t + \sigma \left(\tilde{\phi} \ln \tilde{A}_t + \ln X_t + \tilde{\beta} \ln L_t \right), \quad (10)$$

where $\bar{\lambda}$ is allowed to change over time at a constant growth rate ψ . The inclusion of the trend ensures that the model is balanced in terms of the Johansen test. In addition, if knowledge

⁵If $n = 0$ the semi-endogenous variety exhibits quasi-arithmetic growth, and the model dynamics are similar to the less-than-exponential growth case.

spillovers from other countries become stronger or weaker over time in a systematic way, this would be captured by the trend. The hypothesis that $\bar{\lambda}$ is constant corresponds to the hypothesis $\psi = 0$. This hypothesis cannot be rejected by any of the cointegrated VAR models considered below, further strengthening the assumption that $\bar{\lambda}$ is constant over the investigated period.

Through the rest of this section, it is assumed that $\ln \tilde{A}_t$, $\ln X_t$, and $\ln L_t$ are $I(1)$ (integrated of order one). This assumption is backed up by empirical evidence in Section 4.1. As $\ln \tilde{A}_t$ and $\ln L_t$ are $I(1)$, $\ln A_t$ is $I(1)$ as well. Hence the left hand side of (10) is stationary. Consequently, the right hand side must be stationary as well. For the right hand side to be stationary, $\ln \tilde{A}_t$, $\ln X_t$, and $\ln L_t$ must cointegrate. When normalizing in terms of σ , the cointegrating relationship amounts to:

$$\tilde{\gamma} + \tilde{\psi}t + \tilde{\phi} \ln \tilde{A}_t + \ln X_t + \tilde{\beta} \ln L_t \sim I(0), \quad \tilde{\gamma} \equiv \frac{\gamma}{\sigma}, \quad \tilde{\psi} \equiv \frac{\psi}{\sigma}. \quad (11)$$

The cointegrating relationship given by (11) is the identifying relationship used to estimate the parameters of interest: $\tilde{\psi}$, $\tilde{\phi}$, and $\tilde{\beta}$. The parameter assumptions distinguishing the four model varieties discussed above are implicitly tested. In particular, the hypotheses associated with the parameter assumptions of these four model varieties are given by:

- First-generation fully endogenous: $\tilde{\psi} = 0$, $\tilde{\phi} = 0$, and $\tilde{\beta} = 1$.
- Second-generation fully endogenous: $\tilde{\psi} = 0$, $\tilde{\phi} = 0$, and $\tilde{\beta} = 0$.
- Semi-endogenous: $\tilde{\psi} = 0$, $\tilde{\phi} < 0$, and $\tilde{\beta} > 0$.
- Less-than-exponential growth: $\tilde{\psi} = 0$, $\tilde{\phi} < 0$, and $\tilde{\beta} = 0$.

At first glance, it is tempting to use a single-equation cointegration procedure. But a single-equation approach is problematic, as there might be feedback effects from MFP to the other two variables in the short run. In fact, all three variables might error correct, making the single-equation procedure invalid.

Therefore, a VAR model is employed and the cointegrating relationship (11) is exploited within this framework which has the benefits discussed above. The VAR model is given by:

$$\Delta Z_t = a(b', b_1) \begin{pmatrix} Z_{t-1} \\ t \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta Z_{t-i} + \Phi D_t + \chi + \nu_t, \quad \nu_t \sim IN_3[0, \Omega], \quad (12)$$

where $Z_t = (\ln \tilde{A}_t, \ln X_t, \ln L_t)'$; a and b are $(3 \times r)$ matrices and the rank of $\Pi \equiv ab'$ is $r \leq 3$; k is the number of lags; $\Gamma_1, \dots, \Gamma_{k-1}$ are matrices of parameters; b_1 and χ are vectors of constants; D_t is a matrix of dummies; Φ is a matrix of unrestricted coefficients; ν_t is a vector of error terms; and $IN_3[0, \Omega]$ is a 3-dimensional independent normal distribution with mean zero and covariance matrix Ω (symmetric and positive definite).

According to the theory, there is a single cointegrating relationship between the three variables given by (11). Consequently, a single cointegrating relationship is used if the hypothesis of a single cointegrating relationship cannot be rejected by a Johansen test at the 5 pct. level of significance. This turns out to be the case for all considered VAR models. The cointegrated VAR model used to estimate the parameters of interest is given by:

$$\begin{pmatrix} \Delta \ln \tilde{A}_t \\ \Delta \ln X_t \\ \Delta \ln L_t \end{pmatrix} = a \begin{pmatrix} \tilde{\phi} \\ 1 \\ \tilde{\beta} \\ \tilde{\psi} \end{pmatrix}' \begin{pmatrix} \ln A_{t-1} \\ \ln X_{t-1} \\ \ln L_{t-1} \\ t \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \begin{pmatrix} \Delta \ln A_{t-i} \\ \Delta \ln X_{t-i} \\ \Delta \ln L_{t-i} \end{pmatrix} + \Phi D_t + \chi + \nu_t, \quad (13)$$

where a captures short-run adjustments to the long-run relation, while the long-run relationship is captured by the vector: $(\tilde{\phi}, 1, \tilde{\beta}, \tilde{\psi})$. Based on the data presented below, the constant is unrestricted while the trend is restricted to the cointegrating space.

3 Data

The empirical analysis is based on U.S. data for the period 1953-2014. The length of the sample is restricted by the R&D expenditure data which are only available for the period 1953-2014 at the time of writing. It appears from (1) that Y_t equals GDP and R_t equals aggregate R&D expenditures. Research expenditure data are provided by the National Science Foundation and covers total U.S. R&D expenditures.⁶ The GDP data are obtained from the U.S. Bureau of Economic Analysis (2016a). The research intensity, X_t , is computed by dividing the R&D expenditures by GDP. The labor input, L_t , is measured by full-time equivalent employment (U.S. Bureau of Economic Analysis 2016b). There are several ways to compute the productivity measure, \tilde{A}_t . To ensure that the results are robust to different productivity measures, three generally accepted productivity measures are used: the private

⁶This study exploits the research expenditure data used by Boroush (2016). Values from 2014 are preliminary, but qualitatively similar estimation results can be obtained for the sample 1953-2013.

business sector multifactor productivity measure from the U.S. Bureau of Labor Statistics (2016), the business sector total factor productivity (TFP) measure from Fernald (2014a), and GDP per (full-time equivalent) worker. From here after, MFP refers to the multifactor productivity measure from the U.S. Bureau of Labor Statistics (2016), while TFP refers to the total factor productivity measure from Fernald (2014a). MFP and TFP data are indexed such that the 1953 value equals 100.

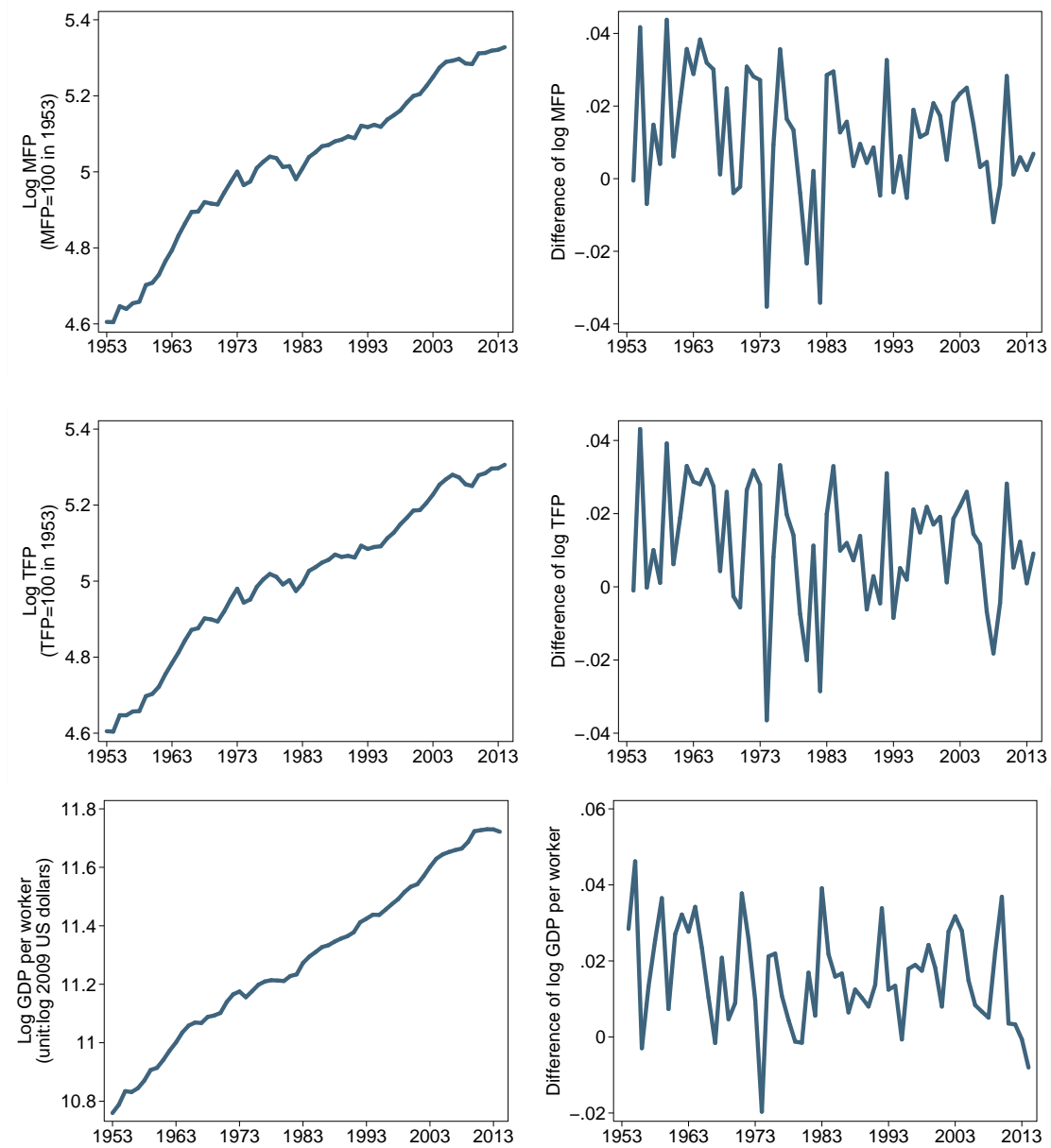


FIGURE 2: Productivity measures, \tilde{A}_t , (MFP, TFP, and GDP per worker) in logs: levels and differences, 1953-2014.

Data sources: U.S. Bureau of Economic Analysis (2016a, 2016b), U.S. Bureau of Labor Statistics (2016), and Fernald (2014a).

The three productivity measures in logs are displayed in levels and in differences in Figure 2. All three measures grew fast during the 1950s and 1960s, while the growth rate was

reduced during the 1970s and 1980s. The growth rate increased again during the mid-1990s until the mid-2000s. Finally, the growth rate slowed down after the mid-2000s. And as noted by Fernald (2014b), the slowdown began several years prior to the financial crisis of 2007–2008.

Transitory drops in all three productivity measures occurred in 1974; a shock associated with the 1973 oil embargo. There are also large transitory drops in MFP and TFP in 1982 and 1993. These drops are associated with the 1980s and 1990s recessions. The large growth rates of 1955 and 1959 are associated with the recoveries after the 1953 and 1958 recessions.

The research intensity in logs is displayed in levels and in differences in Figure 3. The research intensity increased rapidly in the 1950s and remained at a permanently higher level after 1957, possibly indicating a structural transition following World War II. The research intensity increased especially fast from 1955 to 1956, creating an exceptionally large spike in the differenced data. This surge in research intensity was caused by large increases in R&D expenditures funded by the business sector and the federal government. The research intensity also grew fast from 1962 to 1963: a consequence of a historically large increase in federally funded R&D expenditures.

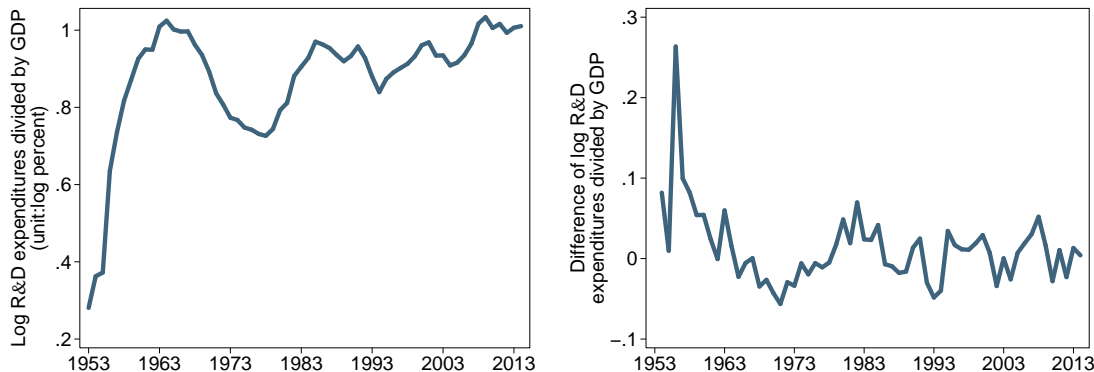


FIGURE 3: Research intensity, X_t , in logs: levels and differences, 1953-2014.

Data sources: U.S. Bureau of Economic Analysis (2016a) and the National Science Foundation.

Employment in logs is displayed in levels and in differences in Figure 4. Employment seems to increase at an approximately constant growth rate through the period. Yet, the generally smooth increase was permanently interrupted by a large drop in 2009 caused by the financial crisis of 2007–2008. In addition, employment seems temporarily affected by the same historical events as the productivity measures.

The data inspection indicates a structural transition in U.S. research intensity after World War II. When analyzing the data, one has to carefully take this transition into account (see Section 4.1). The inspection also indicate a large permanent effect of the financial crisis of

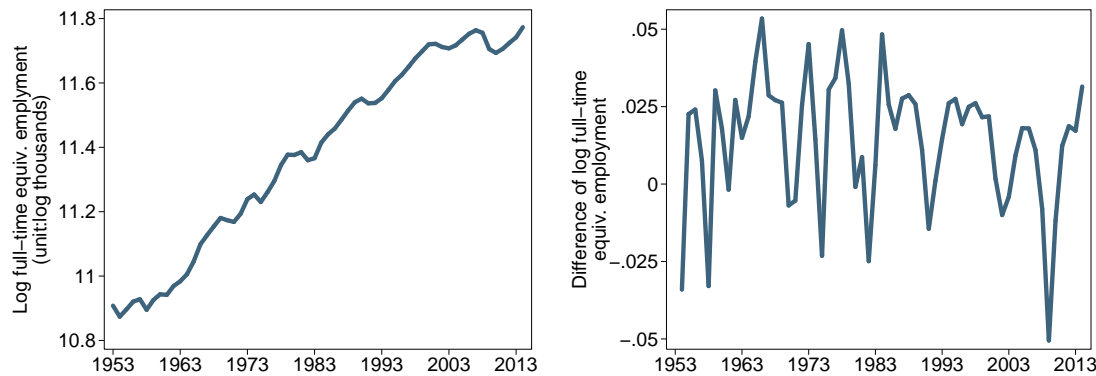


FIGURE 4: Full-time equivalent employment, L_t , in logs: levels and differences, 1953-2014.
Data source: U.S. Bureau of Economic Analysis (2016b).

2007–2008 on employment. In addition, the inspection indicates transitory effects on the productivity measures, coinciding with distinct macroeconomic events. Both the permanent and transitory effects caused by distinct events should be accounted for in the cointegrated VAR model analysis. Finally, employment and the productivity measures are clearly trending which motivates the deterministic terms in (12) and (13), see Juselius (2006, ch. 6).

4 Empirical Analysis

The first part of the empirical analysis indicates that the fully endogenous variety can be rejected using single-equation time series models. These models also document the structural transition in U.S. research intensity discussed above which motivates a restricted sample for the cointegrated VAR model analysis. In the second part, the cointegrating relationship (11) is estimated using (13). The results clearly support the semi-endogenous variety.

A 5 pct. level of significance is used throughout the paper. A two-sided alternative is used to evaluate the null hypotheses: $\tilde{\phi} = 0$ and $\tilde{\beta} = 0$. A one-sided alternative might, however, be considered more appropriate given the alternative hypotheses: $\tilde{\phi} < 0$ and $\tilde{\beta} > 0$. Yet, the distinction between one and two-sided alternatives is seldomly important.

4.1 Unit root tests

From (10) it follows that the second-generation fully endogenous growth models predict that: $\Delta \ln \tilde{A}_{t+1} = (1-\alpha)\gamma + (1-\alpha)\sigma \ln X_t$. Thus according to this variety, $\ln X_t$ must be stationary if $\ln \tilde{A}_t$ is an $I(1)$ process. The results obtained by Ha and Howitt (2007) indicate that this is the case. Yet as illustrated below, their findings seem strongly affected by a structural transition in the U.S. research intensity occurring in the 1950s.

Unit root tests for the main variables are conducted using augmented Dickey-Fuller tests. The tests are based on autoregressive (AR) models, where the lag length is determined by the Schwarz criterion. The models contain both a constant and an intercept. If the process contains a unit root, the trend feeds into the process as a quadratic term. Thus it might be more appropriate to test the joint hypothesis of a unit root and the exclusion of the trend component. This joint test is evaluated using a LR (likelihood ratio) statistic, whereas the standard Dickey-Fuller test is assessed based on a t-statistic.

TABLE 1: Augmented Dickey-Fuller test results

Sample: 1953-2014					
	$\ln \tilde{A}$			$\ln X$	$\ln L$
	MFP	TFP	GDPW		
t-statistic	-1.93	-2.03	-2.78	-3.95	-1.10
LR statistic	3.79	4.21	7.77	14.74	1.31
Unit root	Yes	Yes	Yes	No	Yes
AR(p) process	1	1	2	2	3
Sample: 1960-2014					
	$\ln \tilde{A}$			$\ln X$	$\ln L$
	MFP	TFP	GDPW		
t-statistic	-3.34	-3.21	-2.95	-2.26	-1.38
LR statistic	10.68	9.96	8.64	5.27	2.05
Unit root	Yes	Yes	Yes	Yes	Yes
AR(p) process	1	1	2	2	3

Notes : GDPW is GDP per worker. Bold indicates significance at the 5 pct. level. The models have been estimated with a trend and an intercept. The lag length is determined by the Schwarz criterion using the full sample. The 5 pct. critical values are -3.4 and 12.4 for the t-statistic and the LR statistic, respectively.

Table 1 reports unit root test results for the main variables. The results support the hypotheses that $\ln \tilde{A}_t$ and $\ln L_t$ are $I(1)$ for the periods 1953-2014 and 1960-2014. Using the whole sample (1953-2014), the test results clearly reject a unit root in $\ln X_t$, and thus the hypothesis that $\ln X_t$ is $I(1)$ is rejected. The stationarity of $\ln X_t$ coincides well with second-generation fully endogenous growth models. However, this stationarity result is not very robust. As shown in the lower part of Table 1, the hypothesis of a unit root in $\ln X_t$ cannot be rejected for the sample 1960-2014.⁷

In fact, backward recursive estimation indicates that $\ln X_t$ contains a unit root, when the estimated sample starts after 1956. Figure 5 shows backward recursively estimated t-

⁷Augmented Dickey-Fuller tests (without trends) clearly reject unit roots in $\Delta \ln \tilde{A}_t$ and $\Delta \ln L_t$ for both periods, further indicating that $\ln \tilde{A}_t$ and $\ln L_t$ are $I(1)$. Likewise, an augmented Dickey-Fuller test rejects a unit root in $\Delta \ln X_t$ for the period 1960-2014, indicating that $\ln X_t$ is $I(1)$ over this period.

statistics from AR(2) models with or without trends for $\ln X_t$ for samples ending in 2000 or 2014. In almost all cases, the hypothesis of a unit root in $\ln X_t$ cannot be rejected when the estimated sample starts after 1956. Similar patterns are obtained based on AR(1) and AR(3) models (see Figure 12 in Appendix C). Test statistics for samples ending in 2000 are included to illustrate the nonrobustness of the results obtained by Ha and Howitt (2007).

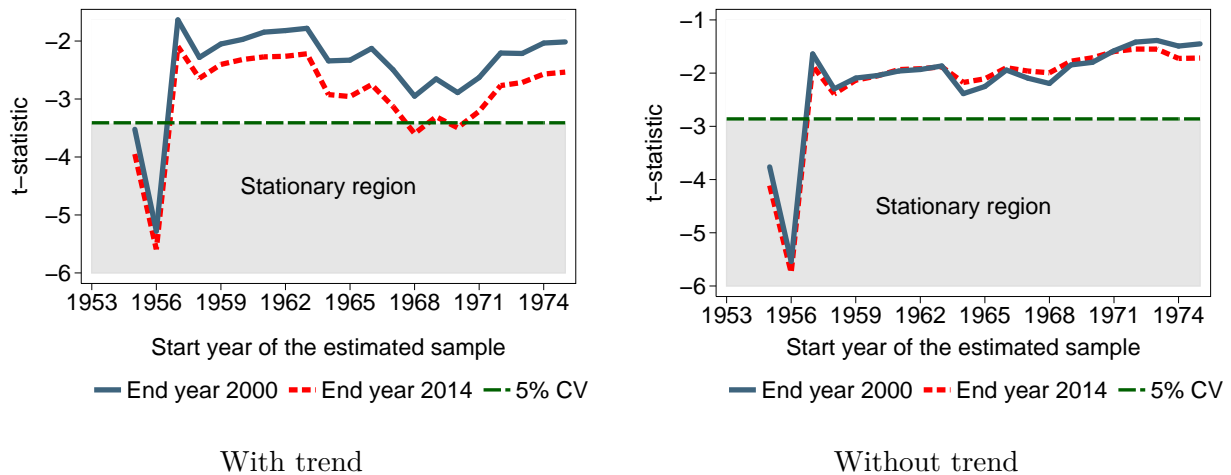


FIGURE 5: Backward recursive estimation of t-statistics for AR(2) models with and without a trend for $\ln X_t$ for estimated sample ending in 2000 or 2014.

Notes: The 5 pct. critical value (5% CV) is -3.41 with a trend and -2.86 without a trend.

The rejection of a unit root in $\ln X_t$ using the full sample is a consequence of the large permanent increase in the level of $\ln X_t$ occurring in the 1950s. The test procedures interpret this as strong mean reversion which makes the time series appear more stationary than it actually is. It seems natural to assume that the rapid growth in U.S. research intensity occurring in the 1950s was caused by a structural transition following World War II. To avoid potentially large effects from this unique structural change, the main empirical analysis concentrates on the period 1960-2014.

The unit root test results clearly provide evidence against the fully endogenous variety. Still, the single-equation approach has some clear limitations like the inability to estimate $\tilde{\phi}$ and $\tilde{\beta}$. These limitations motivate the cointegrated VAR approach taken in the next section.

4.2 Cointegrated VAR analysis

Overview

This section provides estimation results for the VAR and cointegrated VAR models presented in Section 2.2. Estimation results are provided for all three productivity measures for the period 1960-2014. Three results are worth emphasizing before diving into the details of

the estimation procedure. First, the trend can always be excluded from the cointegrating relationship (11). Second, when the trend is excluded, $\tilde{\phi}$ is estimated to be negative and significant, while the estimate of $\tilde{\beta}$ is positive and significant. Hence the results clearly favor the semi-endogenous variety. Third, the joint hypothesis $\tilde{\psi} = \tilde{\beta} = 0$ is accepted using MFP, borderline accepted using TFP, and rejected using GDP per worker, providing mixed evidence for the model of less-than-exponential growth.

Analysis

Estimating (12) using different samples, lag lengths, and dummies reveals some robust findings. First, the Schwarz criterion often suggests two lags (and sometimes three lags), while LR tests suggest three lags using a general-to-specific approach. Yet, the model with three lags is preferred as its estimates are more robust to various changes in the empirical approach. Second, Johansen tests never reject the hypothesis of a single cointegrating relationship which seems to support the general theoretical model. Johansen test results for all models are provided in Appendix C. Third, the models systematically detect outliers in 1963, 1974, 1982, 1993, and 2009. These outliers correspond to distinct events in the U.S. economic history as discussed in Section 3.

As the cointegrated VAR model (13) is formulated in terms of first differences, a transitory dummy is given by a vector $(0, \dots, 0, 1, -1, 0, \dots, 0)$, while a permanent dummy is given by a vector $(0, \dots, 0, 1, 0, \dots, 0)$, see Juselius (2006, ch. 6). Transitory dummies are included to account for transitory effects to the system caused by distinct events like the oil price shock of 1974 caused by the 1973 oil embargo. This oil price shock had a strong negative effect on MFP and TFP in 1974: both measures decreased more than 3 pct. These drops were followed by fast growth, indicating that the shock only had a transitory effect.

A permanent dummy for 2009 is included in all models to account for the permanent drop in employment caused by the financial crisis of 2007–2008 (see Figure 4). Following the literature, the remaining dummies are detected from the estimated residuals. A dummy is included in the year with the largest residual and estimated again. This process continues until the largest (normalized) residual is below ± 2.5 (see Hendry and Juselius 2001). Using this procedure, transitory dummies for 1963, 1974, 1982, and 1993 are included using the MFP and TFP measures, while transitory dummies for 1963, 1974, and 1982 are included for the GDP per worker measure.

Table 2 reports estimation results from a cointegrated VAR model with three lags, tran-

sitory dummies for 1963, 1974, 1982 and 1993, and a permanent dummy for 2009 for the sample 1960-2014 using the MFP measure. The rank test and WS columns are based on the VAR model (12). The rank test column indicates that the hypothesis of a single cointegrating relationship cannot be rejected using the Johansen test for cointegration. The WS column indicates that the VAR model is well specified. A model is defined as well specified if the hypotheses of normal errors, no autocorrelation, and no ARCH effects cannot be rejected using multivariate tests at the 5 pct. level. Specification test descriptions and test statistics for the three models estimated in this section are provided in Appendix C.

TABLE 2: Estimation results from a cointegrated VAR model using MFP, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
A1	55	1963, 1974, 1982, 1993	2009	3	+	+	-0.768 (-0.932)	1.493 (3.549)	-0.021 (-2.053)	
A1 (R1)							-3.459 (-3.752)	1.679 (3.018)	0.000 -	0.607
A1 (R2)							-0.814 (-3.386)	0.000 -	0.000 -	0.124
A1 (R3)							0.000 -	-0.453 (-2.660)	0.000 -	0.045

Notes: N denotes the number of observations, WS indicates whether a model is well specified ('+' if it is), and the rank test indicates whether the hypothesis of a single cointegrating relationship can be rejected by the Johansen test at the 5 pct. level ('+' if it cannot). The numbers in brackets are t-statistics. (R1) indicates that the trend coefficient is restricted to equal zero, (R2) indicates that the trend coefficient and $\tilde{\beta}$ are restricted to equal zero, and (R3) indicates that the trend coefficient and $\tilde{\phi}$ are restricted to equal zero. The p-values of the LR tests are adjusted for small sample size (Bartlett corrected p-values). All dummies are unrestricted.

The parameter estimates in Table 2 are based on the cointegrated VAR model (13). The unrestricted model, Model A1, estimates $\tilde{\phi}$ to be negative but insignificantly different from zero. The Wald test suggest that the trend is significant, but the LR test suggest that it can be excluded from the model. As the LR test performs better than the Wald test for finite samples (Haug 2002), one should prioritize the LR test results. In a model where the trend is excluded from the cointegrating relationship, Model A1 (R1), $\tilde{\phi}$ is estimated to be negative and significant, while the estimate of $\tilde{\beta}$ is positive and significant. Both estimates provide evidence against second-generation fully endogenous growth models. A LR test suggests that the employment variable can be excluded together with the trend, Model A1 (R2), which supports the case of less-than-exponential growth. In this case, $\tilde{\phi}$ remains negative and significant. Finally, the trend and the productivity variable cannot be excluded jointly, Model A1 (R3), and when they are, the employment variable becomes negative and significant which is inconsistent with the general model.

Table 3 provides results from a cointegrated VAR model using the TFP measure. It is clear from a comparison between Table 2 and 3 that the two models generate similar results both qualitatively and quantitatively. A notable exception is the LR test for restriction (R2), where the p-value is much lower using TFP compared to MFP. Thus the less-than-exponential growth case seems to demand too much of the data.⁸

TABLE 3: Estimation results from a cointegrated VAR model using TFP, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
B1	55	1963, 1974, 1982, 1993	2009	3	-	+	-0.464 (-0.622)	1.537 (4.010)	-0.025 (-2.758)	
B1 (R1)							-3.441 (-3.729)	1.637 (2.985)	0.000 -	0.268
B1 (R2)							-0.809 (-3.447)	0.000 -	0.000 -	0.063
B1 (R3)							0.000 -	-0.434 (-2.713)	0.000 -	0.020

Notes: See notes to Table 2.

Estimation results using the GDP per worker measure are provided in Table 4. The general patterns from Table 2 are repeated except that the model of less-than-exponential growth, Model C1 (R2), is clearly rejected. Note that: $\ln y_t = \text{constant} + 1/(1 - \alpha) \ln \tilde{A}_t$. Hence, this model estimates $(1 - \alpha)\tilde{\phi}$ instead of just $\tilde{\phi}$. The point estimate of $(1 - \alpha)\tilde{\phi}$ in Model C1 (R1) coincides surprisingly well with the point estimates of $\tilde{\phi}$ in Model A1 (R1) and Model B1 (R1) given that α equals about 0.34 empirically. The point estimate of $\tilde{\beta}$ in Model C1 (R1) is also close to the point estimates from Model A1 (R1) and Model B1 (R1).

A potential concern is the impact of the dummy variables. Yet, it turns out that the dummy variables are not crucial for the main conclusions. As the transitory dummies are added consecutively, three to four other models were in each case estimated before arriving at the final model. Estimation results for these other models as well as models without any dummies are reported in Table 7, 8, and 9 in Appendix C. It appears that the general conclusions can be reached from these models as well. In particular, $\tilde{\phi}$ is always estimated to be negative and significant when the trend is excluded. Likewise, $\tilde{\beta}$ is always positive and significant when the trend is excluded. Furthermore, the trend can always be excluded from the cointegrating relationship. Meanwhile, excluding the trend and productivity variable

⁸Model B1 does not appear well specified as the hypothesis of normally distributed errors is borderline rejected. The normal distribution of errors increases the efficiency of the estimator, but it is not crucial for its properties (see Johansen 1991).

TABLE 4: Estimation results from a cointegrated VAR model using GDP per worker, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$(1 - \alpha)\tilde{\phi}$	$\tilde{\beta}$	Trend	
C1	55	1963, 1974, 1982	2009	3	+	+	-0.233 (-0.215)	1.938 (4.693)	-0.034 (-1.609)	
C1 (R1)							-2.199 (-6.337)	1.603 (5.287)	0.000 -	0.369
C1 (R2)							-0.868 (-3.704)	0.000 -	0.000 -	0.006
C1 (R3)							0.000 -	-0.933 (-2.734)	0.000 -	0.002

Notes: See notes to Table 2.

jointly, or the trend and employment variable jointly, results in low p-values for the LR test.

All in all, the evidence seems to favor the semi-endogenous variety. The evidence supporting the model of less-than-exponential growth is fragile, indicating that this case demands too much of the data. Yet, the estimates clearly reject the hypothesis $\tilde{\phi} = 0$, providing further evidence against the fully endogenous variety.

The point estimates of $\tilde{\beta}$ are consistently above one, indicating positive productivity gains from horizontal innovation. This finding highlights the importance of the main methodological contribution of this paper: the introduction of productivity gains from horizontal innovation in the Ha and Howitt (2007) framework.

5 Robustness Checks

In this section, it is shown that the results from Section 4.2 are robust to justifiable changes in the empirical approach. In general, the patterns from Section 4.2 repeat: (i) the trend can be excluded, (ii) when the trend is excluded, the estimate of $\tilde{\phi}$ is negative and significant while the estimate of $\tilde{\beta}$ is positive and significant, (iii) when both the trend and the employment variable are excluded, the p-value of the LR test falls notably, (iv) when excluding both the trend and the employment variable the estimate of $\tilde{\phi}$ becomes larger but remains negative and significant, and (v) both the trend and productivity variable are not or borderline excludable, and $\tilde{\beta}$ becomes negative when they are excluded.

Full sample

When the full sample is employed, it is necessary to add a permanent dummy for 1956 to account for the large permanent shock hitting U.S. research intensity that year (see Figure 3). In addition, the data indicate that a transitory dummy should be added in 1959, where the economy was exiting the Eisenhower Recession. A multivariate test indicate that $\ln X_t$ is non-stationary for the sample 1953-2014 for a VAR model with three lags, transitory dummies for 1959, 1963, 1974, 1982 and 1993, and permanent dummies for 1956 and 2009.

Estimation results using the MFP measure are provided in Table 5. In general, the qualitative patterns from the main analysis are repeated. One difference being that the hypothesis $\tilde{\beta} = 0$ is only rejected using a one-sided alternative (which seems appropriate given the alternative hypothesis $\tilde{\beta} > 0$). Similar results can be obtained using the TFP and GDP per worker measures, see Table 10 and 11 in Appendix C.

TABLE 5: Estimation results from a cointegrated VAR model using MFP, sample 1953-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
A2	62	1959, 1963, 1974, 1982, 1993	1956, 2009	3	+	+	-3.249 (-2.041)	1.230 (1.309)	0.005 (0.265)	
A2 (R1)							-2.764 (-2.518)	1.284 (1.871)	0.000 -	0.944
A2 (R2)							-0.762 (-3.285)	0.000 -	0.000 -	0.600
A2 (R3)							0.000 -	-0.464 (-2.894)	0.000 -	0.403

Notes: See notes to Table 2.

Moving window estimation

It is important to verify that the start and end years of the sample are not essential for the results. To verify that a cointegrated VAR model with three lags systematically rejects the hypotheses $\tilde{\phi} = 0$ and $\tilde{\beta} = 0$, Model A1 (R1) is estimated for a 50-year rolling window starting with the sample 1957-2006 and ending with the sample 1965-2014. The moving window approach ensures comparability between estimates as the power is kept constant. The starting sample 1957-2006 is chosen such that the large permanent shock to U.S. research intensity in 1956 is avoided. The results are presented in Figure 6. The null hypothesis $\tilde{\phi} = 0$ is consistently rejected. Meanwhile, the point estimate of $\tilde{\beta}$ is systematically positive, while

the hypothesis $\tilde{\beta} = 0$ is rejected for most subsamples. In addition, all estimates for both $\tilde{\phi}$ and $\tilde{\beta}$ are consistent with the equivalent estimates for the period 1960-2014 (see Model A1 (R1) in Table 2). Finally, similar patterns can be obtained using Model B1 (R1) and C1 (R1), see Figure 13 and 14 in Appendix C.

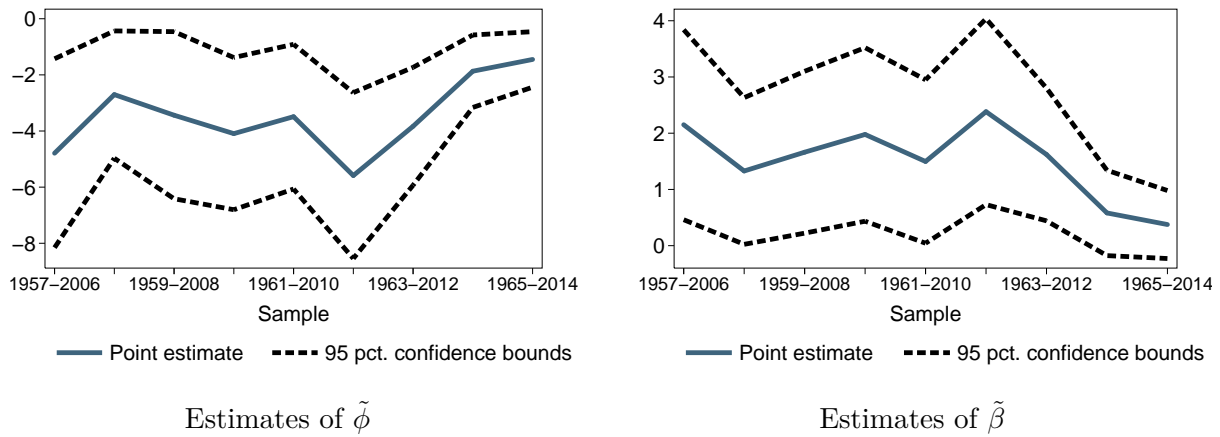


FIGURE 6: Estimates of $\tilde{\phi}$ and $\tilde{\beta}$ based on Model A1 (R1) for 50-year samples, (1957-2006)-(1965-2014).

Business sector R&D

R&D-based models usually focus on research activities funded by private investors and conducted by private firms. The role of the government is confined to subsidizing these private research activities. Accordingly, one might argue that these models should be tested using business sector R&D expenditure data.

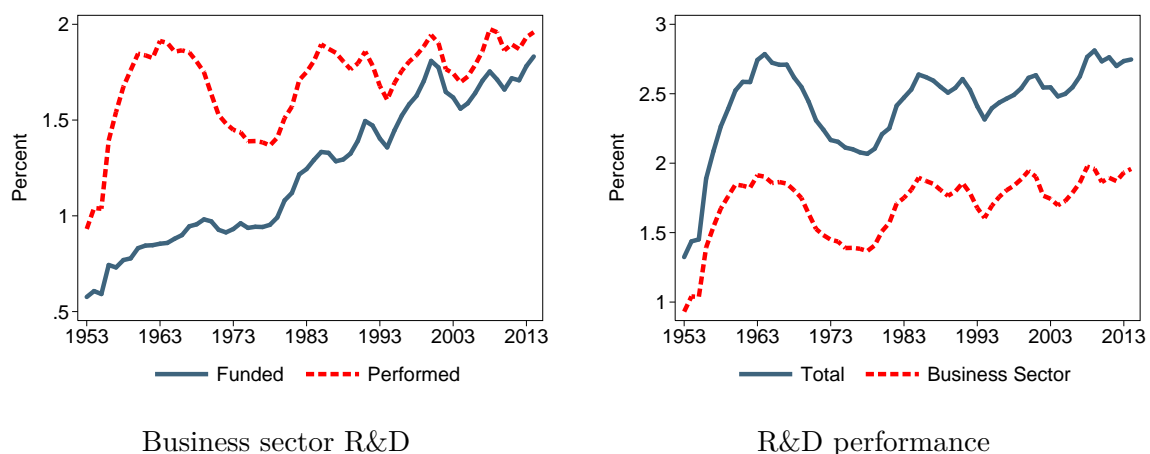


FIGURE 7: U.S. R&D expenditures as a share of GDP, 1953-2014.

Data sources: U.S. Bureau of Economic Analysis (2016a) and the National Science Foundation.

The left panel of Figure 7 shows U.S. R&D expenditures funded and performed by the business sector divided by GDP. U.S. R&D expenditures funded by the business sector has increased much faster than GDP since 1953. As productivity growth slowed down over

this period, the fully endogenous variety has no chance if R&D expenditures funded by the business sector is the correct measure to use. However, the correct measure appears to be R&D expenditures performed by the business sector.

Nevertheless, the variation in R&D expenditures performed by the business sector seems to mimic that of total R&D expenditures. This is clear from the right panel of Figure 7 which shows U.S. R&D expenditures performed in total and by the business sector divided by GDP. The two time series seem strongly correlated, and R&D expenditures performed by the business sector constitute roughly 70 pct. of total R&D expenditures in all years.

Given the high correlation between total R&D expenditures and R&D expenditures performed by the business sector, the obtained results using the business sector R&D expenditure data are similar to those obtained using total R&D expenditures. This is evident from the evidence provided in Table 6, and thus, the main results are robust to this alternative R&D expenditure measure. The same patterns can be obtained using the other two productivity measures, see Table 12 and 13 in Appendix C.

TABLE 6: Estimation results from a cointegrated VAR model using MFP and R&D expenditures performed by the business sector, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
A3	55	1963, 1974, 1982, 1993	2009	3	+	+	-2.305 (-1.784)	1.766 (2.582)	-0.014 (-0.823)	
A3 (R1)							-4.132 (-3.604)	1.948 (2.811)	0.000 -	0.819
A3 (R2)							-0.916 (-3.781)	0.000 -	0.000 -	0.198
A3 (R3)							0.000 -	-0.500 (-3.089)	0.000 -	0.065

Notes: See notes to Table 2.

Further considerations

Based on the theoretical model, the present study uses R&D expenditures over GDP to measure the research intensity. Alternatively, one could formulate a model where the research intensity is given by researchers over population. This alternative measure would, however, not take potential changes in the capital-labor intensity of research into account. In addition, this measure increased substantially through the investigated period (Jones 2015). Thus the fully endogenous variety has little chance if this alternative measure is employed.

One might argue that the LR tests should be conducted using bootstrap techniques given the small sample size and potential misspecifications (see Cavaliere et al. 2015; Boswijk et al. 2016). In this analysis, p-values based on wild bootstrapping are generally higher compared to their Bartlett corrected counterparts. Hence the model of less-than-exponential growth gains some additional support from these test results. Still, the p-values are reduced substantially when moving from restriction (R1) to restriction (R2).

Finally, it could be argued that the rank tests should be evaluated based on bootstrap methods (see Cavaliere et al. 2010a, 2010b). Yet rank tests based on wild bootstrapping confirm the previous finding: the hypothesis of a single cointegrating relationship cannot be rejected in any of the presented models.

6 Productivity Forecasts

This section provides U.S. productivity growth forecasts based on the theoretical model and empirical estimates. The simulation exercise is based on the empirical estimates from Model A1 (R1) and Model A1 (R2). The former represents the semi-endogenous case, while the latter represents the less-than-exponential growth case.

The log discrete time approximation of (8) amounts to:

$$\ln A_{t+1} = \bar{\gamma} + \phi \ln A_t + \sigma \ln X_t + \sigma \left(1 - \beta + \beta \frac{\eta}{1 - \alpha}\right) \ln L_t. \quad (14)$$

Given $\ln A_t$, the log of MFP, $\ln \tilde{A}_t$, can be computed using the formula:

$$\ln \tilde{A}_t = (1 - \alpha) \ln A_t + \beta \eta \ln L_t. \quad (15)$$

As the empirical results do not allow for a direct identification of α , β , η , $\bar{\gamma}$, ϕ , and σ , these values are determined in four steps. First, α and σ are set equal to 0.34 and 0.042, respectively. The former value is based on the U.S. capital share of income for the period 1960-2014 as measured by the U.S. Bureau of Labor Statistics, and the latter is taken from Venturini (2012). Second, it is assumed that β equals one which increases the comparability between the two cases examined. Additionally, it seems intuitive that the range of specialized capital good varieties is proportional to employment. Third, the parameters η and ϕ are computed from α , β and σ , and the empirical estimates of $\tilde{\phi}$ and $\tilde{\beta}$. Finally, $\bar{\gamma}$ is estimated by minimizing the sum of squared deviations between actual and simulated MFP values for

the period 1960-2014. The initial technological level, $\ln A_{1960}$, is computed from (15). Given $\ln A_{1960}$, $\{\ln X_t\}_{1960}^{2014}$, and $\{\ln L_t\}_{1960}^{2014}$ the predicted technological level is computed using (14), and the predicted MFP is computed using (15).

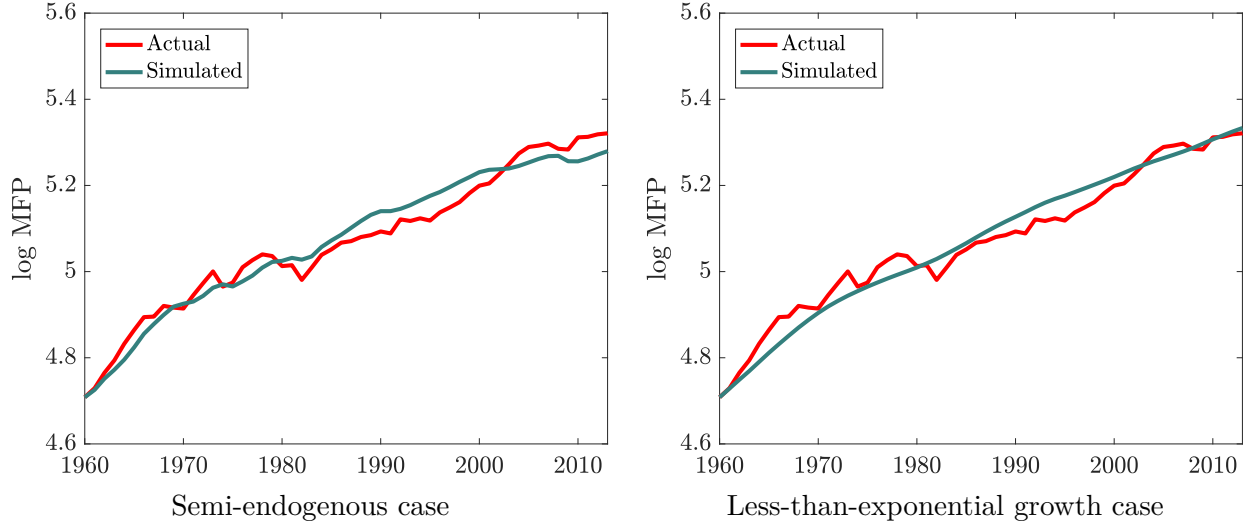


FIGURE 8: Actual and simulated U.S. MFP, 1960-2014.

Notes: All simulations are conducted using (14) and (15) as well as the actual 1960 log MFP value ($\ln \tilde{A}_{1960}$) and the actual time series for $\ln X_t$ and $\ln L_t$. The simulations are based on the empirical estimates from Model A1 (R1) (left panel) and Model A1 (R2) (right panel), the parameter values $\alpha = 0.34$, $\beta = 1$ and $\sigma = 0.042$, and estimated $\bar{\gamma}$ values.

The actual and simulated MFP values for the period 1960-2014 are shown in Figure 8. The left panel shows the semi-endogenous case where $\eta \approx 0.34$ and $\phi \approx 0.90$, while the right panel shows the less-than-exponential growth case where $\eta = 0$ and $\phi \approx 0.98$. In both cases, the simulated values seem to capture the long-run trend in MFP. Note that in both cases, ϕ is close to one which might explain why previous studies have been unable to reject the hypothesis $\phi = 1$.

To forecast MFP, assumptions on the future research intensity and employment are necessary. The research intensity is assumed to be at its 2014 level at all future dates (2.75 pct.). Employment is assumed to grow at its average (compound) growth rate from 1960 to 2014 (1.55 pct. per year). MFP growth forecasts based on the empirical estimates from Model A1 (R1) and Model A1 (R2) are shown in Figure 9. Actual MFP growth data are used for the period 2000-2014, while forecasted values are used for the period 2015-2060, and the entire series is smoothed using the HP filter. According to the semi-endogenous forecast, the MFP growth rate converges to around 0.75 pct. per year. The corresponding GDP per worker growth rate is around 1.1 pct. per year, i.e. a 0.4 percentage point decline compared to the average growth rate from 1960 to 2014. The less-than-exponential growth case is even more pessimistic. The MFP growth rate converges to zero which implies that the growth

rate of GDP per worker converges to zero as well. The convergence is, however, slow such that the growth rates of MFP and GDP per worker are around 0.3 and 0.5 pct. per year, respectively, in 2060.

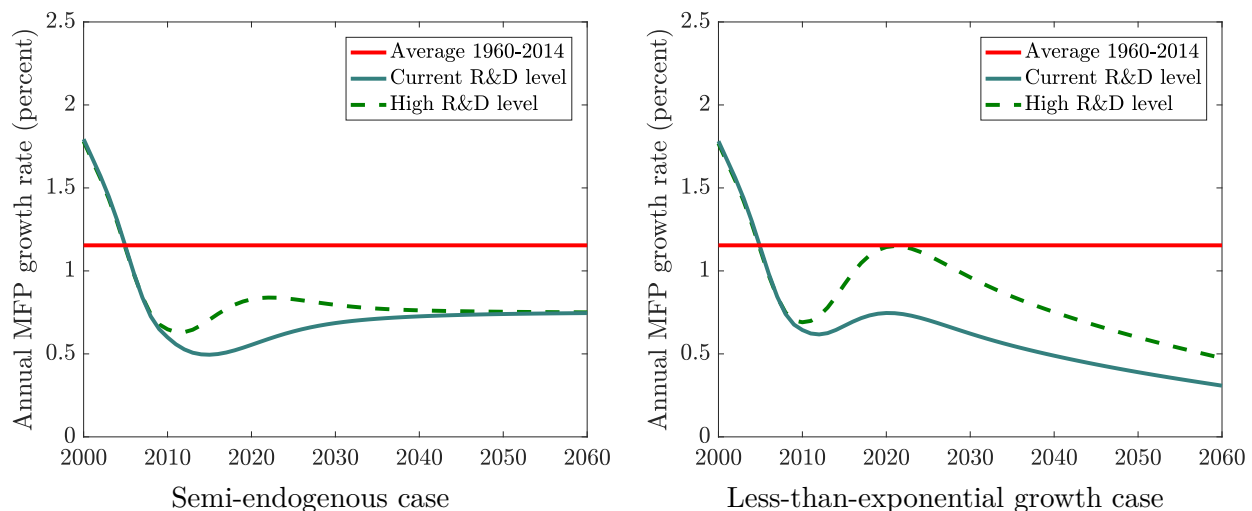


FIGURE 9: U.S. MFP growth forecast.

Notes: Actual values are used for the period 2000-2014 and forecasted values are used for the remaining period. The actual and forecasted MFP growth rates are smoothed using the HP filter with a smoothing factor of 100. All forecasts are conducted using (14) and (15) as well as actual 2014 MFP and employment values. The research intensity, X_t , is set equal to its 2014 value (2.75 pct.) for the period 2014-2060, while the research intensity is 0.5 percentage points higher in the high R&D scenario. The employment level is assumed to grow at 1.55 pct. per year after 2014. The simulations are based on the empirical estimates from Model A1 (R1) (left panel) and Model A1 (R2) (right panel), the parameter values $\alpha = 0.34$, $\beta = 1$ and $\sigma = 0.042$, and estimated $\bar{\gamma}$ values.

A natural policy response to these dismal MFP growth projections is to increase the research intensity. To check how an R&D boost affects future MFP growth, the MFP growth rate is simulated assuming a 0.5 percentage point increase in the research intensity compared to the 2014 level (3.25 pct.). From Figure 9, it appears that this policy affects the MFP growth rate much more in the less-than-exponential growth case. In the semi-endogenous case, a boost to R&D expenditures increases MFP growth notably over about 25 years. In contrast, the effect is strong through most of the period in the less-than-exponential growth case. As population growth does not affect productivity growth in the less-than-exponential growth case, this case demands a higher ϕ value compared to the semi-endogenous case to match historical U.S. productivity growth trends. As a large ϕ value implies slower convergence to the long-run growth rate, the impact from an R&D boost is larger in this case.

The U.S. Bureau of Labor Statistics expects the labor force to grow much slower over the coming decades compared to the period 1960-2014 (Toossi 2015, 2016). The semi-endogenous forecast shown in the left panel of Figure 9 should, therefore, be viewed as an upper bound.

To obtain a more realistic semi-endogenous forecast, future employment is computed from the labor force projections by Toossi (2016). Employment equals the projected labor force multiplied by the average ratio between the two variables from 1960 to 2014. This method seems appropriate, as the ratio between the two variables has been approximately constant at least since the 1960s.⁹

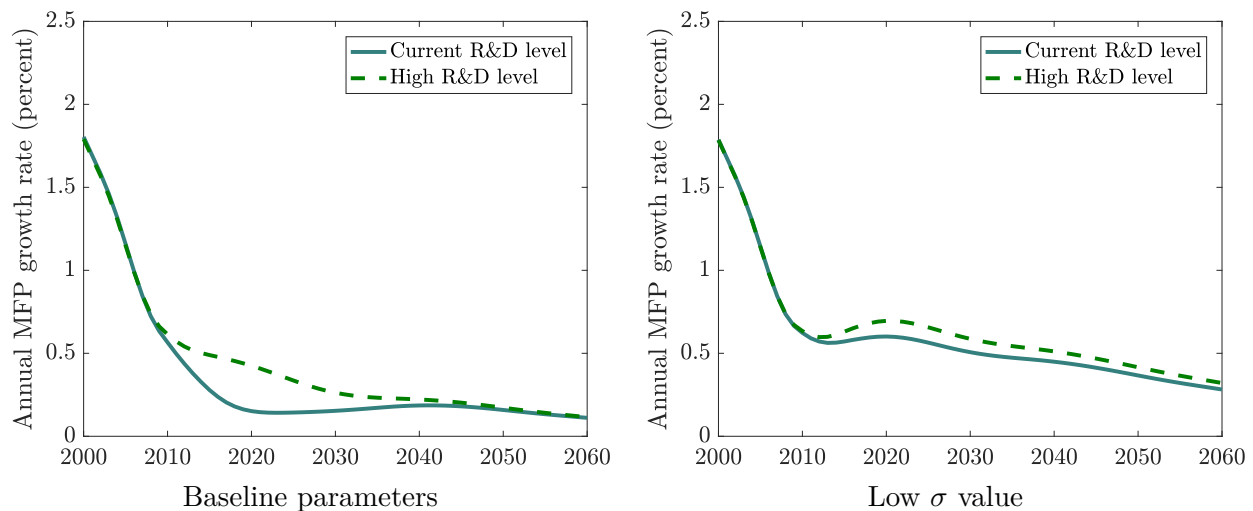


FIGURE 10: Semi-endogenous U.S. MFP growth forecast based on labor force projections from the U.S. Bureau of Labor Statistics.

Notes: Actual values are used for the period 2000-2014 and forecasted values are used for the remaining period. The actual and forecasted MFP growth rates are smoothed using the HP filter with a smoothing factor of 100. All forecasts are conducted using (14) and (15) as well as actual 2014 MFP and employment values. The research intensity, X_t , is set equal to its 2014 value (2.75 pct.) for the period 2014-2060, while the research intensity is 0.5 percentage points higher in the high R&D scenario. Employment is set equal to the labor force forecasts by Toossi (2016) multiplied by the average ratio between employment and the labor force for the period 1960-2014. The simulations are based on the empirical estimates from Model A1 (R1) (left panel), the parameter values $\alpha = 0.34$ and $\beta = 1$, and estimated $\bar{\gamma}$ values. The left and right panel show simulations for $\sigma = 0.042$ and $\sigma = 0.01$, respectively.

The resulting MFP forecast is shown in the left panel of Figure 10. The long-run annual growth rate of MFP converges fast to less than 0.2 pct., corresponding to a long-run annual growth rate of GDP per worker of less than 0.3 pct. Employment grows at an annual average rate of about 0.4 pct. from 2014 to 2060; a substantial reduction from an annual average growth rate of about 1.5 pct. from 1960 to 2014. The slow employment growth starts already after 2016 which explains the fast productivity growth slowdown. A 0.5 percentage point increase in research intensity reduces the speed of the productivity slowdown, but it does not stop it.

The forecasts are especially sensitive to the assumed σ value; a value associated with a high degree of uncertainty. Given α and $\tilde{\phi}$, a lower σ value implies a higher value of ϕ . Hence a forecast based on a lower σ value predicts a slower convergence, implying a more optimistic

⁹Historical labor force data are obtained from the U.S. Bureau of Labor Statistics (2017).

productivity growth forecast over the next several decades. Yet, the very long-run growth rate under exponential employment growth is barely affected.

The right panel of Figure 10 shows a forecast based on a σ value of 0.01 (instead of the baseline value 0.042). The implied value of ϕ is 0.97, resulting in a slow productivity growth deceleration. The MFP and GDP per worker growth rates are around 0.3 and 0.4 pct. per year in 2060. A boost to R&D has little effect due to the low σ value, but the effect lasts for a long time due to the high ϕ value.

7 Concluding Remarks

The results obtained above seems to support the more pessimistic view on U.S. productivity growth prospects held by Gordon (2012, 2015) and others. The empirical estimates reflect that spillovers in research are weaker than assumed in the fully endogenous variety. This result coincides with recent micro-level evidence. Bloom et al. (2017) show that research productivity seems to be declining in various industries, products, and firms. Specifically, it requires increasingly more efficient research input to maintain exponential growth; a finding consistent with the spillover assumption of the semi-endogenous variety.

The results obtained above support R&D-based models in general. Yet the fully endogenous variety is clearly rejected by the empirical evidence. In contrast, the empirical results support the semi-endogenous variety, while there seems to be mixed support for the less-than-exponential growth model. Thus the results suggest that future economic growth research focusing on the very long run should build on the semi-endogenous variety. Still, the fully endogenous variety may be a useful first approximation within a limited time frame given its relative dynamic simplicity.

Naturally, the analysis conducted above has some important limitations. As emphasized by Dalgaard and Kreiner (2003), the functional form of the knowledge production function remains unknown. Yet, the sensible empirical results indicate that the functional form assumed in the present study is a good approximation. Additionally, knowledge spillovers between the U.S. and other countries were largely ignored. However, if these spillovers were systematic, they should be picked up by either the trend or the constant in the empirical model. Furthermore, the obtained results are based on relatively few observations. Given the data available today, the results appear robust. It is, however, not unthinkable that some of the conclusions reached in this paper might be challenged when another decade of

data become available. Still, the employed dataset contains a lot of information on long-run economic growth, as it stretches over a long and relatively stable period in U.S. economic history. Finally, the theoretical model focuses on the long-run relationship between the main variables. Transitional dynamics are thereby indirectly assumed to play a minor role which is motivated by the approximately constant capital-output ratio observed over the investigated period. It is important to emphasize that the empirical model does not restrict the short-run dynamics, and thus adjustments to the long-run relationship might take various forms.

8 Appendix

A Equilibrium Stability

In this appendix, it is shown that with a constant savings rate, a constant research intensity, and a constant labor input growth rate, the long-run equilibrium is asymptotically stable and features a constant capital-output ratio. Only the semi-endogenous case ($\beta < 1$, $\phi < 1$, $\eta > 0$) is considered here, but the same properties hold for the other model varieties.

Let s_k denote the constant savings rate in physical capital, and let \bar{X} denote the constant research intensity. To ensure positive consumption: $s_k + \bar{X} < 1$. The constant population growth rate is denoted $n > 0$ which implies that: $L(t) = L(0)e^{nt}$.

Consider the normalization: $\bar{z}(t) \equiv Z(t)/(A(t)L(t)^{1+\frac{\beta\eta}{1-\alpha}})$. From (4), (5), (7), and (8), it follows that the economy is described by the system:

$$\dot{\bar{k}}(t) = s_k \bar{k}(t)^\alpha - [\delta + \tilde{n} + g_A(t)] \bar{k}(t) \quad \text{and} \quad g_A(t) = \lambda A(t)^{\phi-1} \bar{X}^\sigma \bar{k}(t)^{\alpha\sigma} L(t)^{\sigma(1-\beta+\frac{\beta\eta}{1-\alpha})},$$

where $\tilde{n} \equiv (1 + \beta\eta/(1-\alpha))n$ and $g_A(t) \equiv \dot{A}(t)/A(t)$. In the steady state equilibrium:

$$\bar{k}^* = \left(\frac{s_k}{\delta + \tilde{n} + g_A^*} \right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad g_A^* = \frac{\sigma \left(1 - \beta + \frac{\beta\eta}{1-\alpha} \right)}{1 - \phi} n.$$

In this equilibrium, the capital-output ratio is constant: $K(t)/Y(t) = s_k/(\delta + \tilde{n} + g_A^*)$.

To show that the steady state equilibrium (\bar{k}^*, g_A^*) is asymptotically stable, the above system is reformulated into an autonomous system. Consider the normalization: $\tilde{z}(t) \equiv Z(t)/(A^*(t)L(t)^{1+\frac{\beta\eta}{1-\alpha}})$, where $A^*(t) = A^*(0)e^{g_A^*t}$. It follows directly that: $\tilde{k}(t) = \bar{k}(t)a(t)$, where $a(t) \equiv A(t)/A^*(t)$. In the steady state equilibrium, $\bar{k}(t)$ and $a(t)$ are constant. Hence $\tilde{k}(t)$ must be constant as well. The system is rewritten in autonomous form:

$$\dot{\tilde{k}}(t) = s_k a(t)^{1-\alpha} \tilde{k}(t)^\alpha - \chi \tilde{k}(t) \quad \text{and} \quad \dot{a}(t) = \tilde{\lambda} a(t)^{\phi-\alpha\sigma} \tilde{k}(t)^{\alpha\sigma} - g_A^* a(t),$$

where $\chi \equiv \delta + \tilde{n} + g_A^*$ and $\tilde{\lambda} \equiv \lambda \bar{X}^\sigma A^*(0)^{\phi-1} L(0)^{\sigma(1-\beta+\frac{\beta\eta}{1-\alpha})}$.

Define the functions: $\dot{\tilde{k}}(t) = g(\tilde{k}(t), a(t))$ and $\dot{a}(t) = f(\tilde{k}(t), a(t))$. In addition, define the matrix:

$$B \equiv \begin{pmatrix} \frac{\partial g(\tilde{k}^*, a^*)}{\partial \tilde{k}(t)} & \frac{\partial g(\tilde{k}^*, a^*)}{\partial a(t)} \\ \frac{\partial f(\tilde{k}^*, a^*)}{\partial \tilde{k}(t)} & \frac{\partial f(\tilde{k}^*, a^*)}{\partial a(t)} \end{pmatrix} = \begin{pmatrix} \alpha s_k (a^*)^{1-\alpha} (\tilde{k}^*)^{\alpha-1} - \chi & (1-\alpha) s_k (a^*)^{-\alpha} (\tilde{k}^*)^\alpha \\ \alpha \sigma \tilde{\lambda} (a^*)^{\phi-\alpha\sigma} (\tilde{k}^*)^{\alpha\sigma-1} & (\phi - \alpha\sigma) \tilde{\lambda} (a^*)^{\phi-\alpha\sigma-1} (\tilde{k}^*)^{\alpha\sigma} - g_A^* \end{pmatrix}.$$

Using that $(a^*)^{\phi-\alpha\sigma-1}(\tilde{k}^*)^{\alpha\sigma} = g_A^*/\tilde{\lambda}$ and $(\tilde{k}^*/a^*)^{\alpha-1} = \chi/s_k$:

$$B = \begin{pmatrix} (\alpha-1)\chi & (1-\alpha)s_k \left(\frac{\tilde{k}^*}{a^*}\right)^\alpha \\ \alpha\sigma g_A^* \left(\frac{\tilde{k}^*}{a^*}\right)^{-1} & (\phi-\alpha\sigma-1)g_A^* \end{pmatrix}.$$

The trace and determinant of B are given by:

$$\text{tr}(B) = -(1-\alpha)\chi - (1-\phi+\alpha\sigma)g_A^* < 0 \quad \text{and} \quad |B| = (1-\phi)(1-\alpha)\chi g_A^* > 0.$$

Thus the equilibrium point (\tilde{k}^*, a^*) is locally asymptotically stable.

Figure 11 shows the phase diagram of the system. For the open set \mathfrak{R}_+^2 , the two curves only intercept in the stable equilibrium (\tilde{k}^*, a^*) . Hence this equilibrium is unique and globally asymptotically stable.

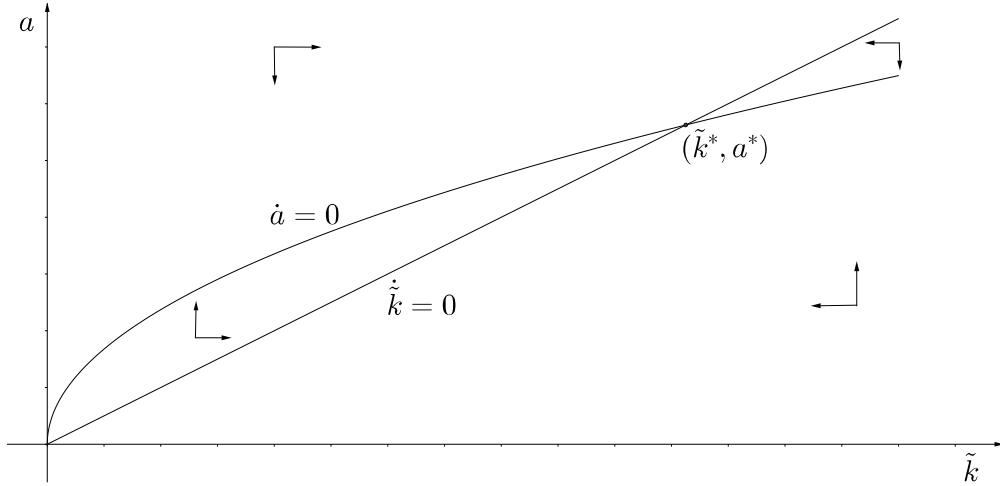


FIGURE 11: Phase diagram.

B The Four Growth Model Varieties

First-generation fully endogenous: $[n = 0, \phi = 1, \beta = 0, \eta = 0]$.

The growth rates of MFP and GDP per worker are given by

$$g_{\bar{A}}(t) = (1-\alpha)\bar{\lambda}\bar{X}^\sigma L^\sigma \quad \text{and} \quad g_y(t) = \bar{\lambda}\bar{X}^\sigma L^\sigma.$$

Increasing the research intensity, \bar{X} , increases the growth rates of MFP and GDP per worker. The labor input is assumed constant, but if the labor input was allowed to grow, the growth rates of MFP and GDP per worker would also grow. Economic growth would continuously accelerate which is clearly inconsistent with empirical evidence.

Second-generation fully endogenous: $[n \geq 0, \phi = 1, \beta = 1, \eta = 0]$.

The growth rates of MFP and GDP per worker are unaffected by the labor input, but otherwise the relations from the first-generation fully endogenous growth models carry through:

$$g_{\bar{A}}(t) = (1 - \alpha)\bar{\lambda}\bar{X}^\sigma \quad \text{and} \quad g_y(t) = \bar{\lambda}\bar{X}^\sigma.$$

Increasing the research intensity increases the growth rates of MFP and GDP per worker, but changes in the labor input have no effects on the growth rates. In fact, the growth rates are completely unaffected by the two state variables: $A(t)$ and $L(t)$.

Semi-endogenous: $[n \geq 0, \phi < 1, \beta < 1, \eta \geq 0]$ or $[n \geq 0, \phi < 1, \beta = 1, \eta > 0]$.

The technological growth rate evolves according to the differential equation:

$$G_A(t) \equiv \frac{\dot{g}_A(t)}{g_A(t)} = (\phi - 1)g_A(t) + \sigma \left(1 - \beta + \beta \frac{\eta}{1 - \alpha} \right) n.$$

If $G_A(t)$ is positive, then $g_A(t)$ increases over time which reduces $G_A(t)$. Likewise, if $G_A(t)$ is negative, then $g_A(t)$ decreases over time which increases $G_A(t)$. As a consequence, $G_A(t)$ converges to zero regardless of the initial value of $g_A(t)$. Accordingly,

$$g_A(t) \xrightarrow{t \rightarrow \infty} \sigma \left(1 - \beta + \beta \frac{\eta}{1 - \alpha} \right) \left(\frac{n}{1 - \phi} \right).$$

It follows that the long-run growth rates of MFP and GDP per worker are given by

$$g_{\bar{A}}(t) = \left\{ \beta\eta + (1 - \alpha)\sigma \left(1 - \beta + \beta \frac{\eta}{1 - \alpha} \right) \left(\frac{1}{1 - \phi} \right) \right\} n \quad \text{and} \quad g_y(t) = \frac{g_{\bar{A}}(t)}{1 - \alpha}.$$

Thus the long-run growth rates of MFP and GDP per worker are proportional to the growth rate of the labor input. Still, both growth rates increase temporarily if the research intensity is increased. Over time both growth rates converge to their long-run values, but they might converge slowly.

Less-than-exponential: $[n \geq 0, \phi < 1, \beta = 1, \eta = 0]$.

The growth rates of MFP and GDP per worker approach zero as time goes to infinity. This is clear from the following expressions:

$$g_{\bar{A}}(t) = (1 - \alpha)\bar{\lambda}A(t)^{\phi-1}\bar{X}^\sigma \quad \text{and} \quad g_y(t) = \bar{\lambda}A(t)^{\phi-1}\bar{X}^\sigma.$$

To determine the path of $A(t)$, define the variable: $B(t) \equiv A(t)^{1-\phi}$. Differentiating $B(t)$ with respect to time yields: $\dot{B}(t) = (1-\phi)\bar{\lambda}\bar{X}^\sigma$. The solution to this differential equation is given by:

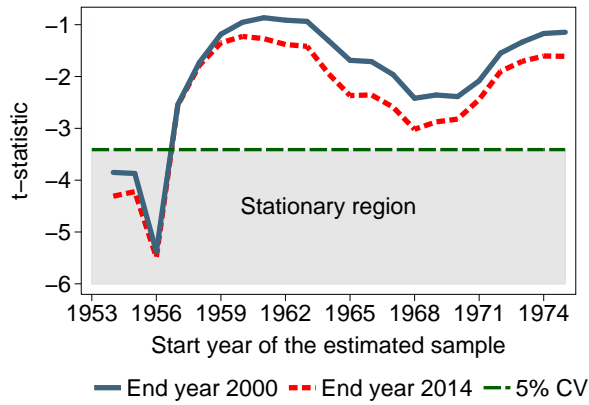
$$B(t) = B(0) + (1-\phi)\bar{\lambda}\bar{X}^\sigma t.$$

The technological level is, therefore, determined by:

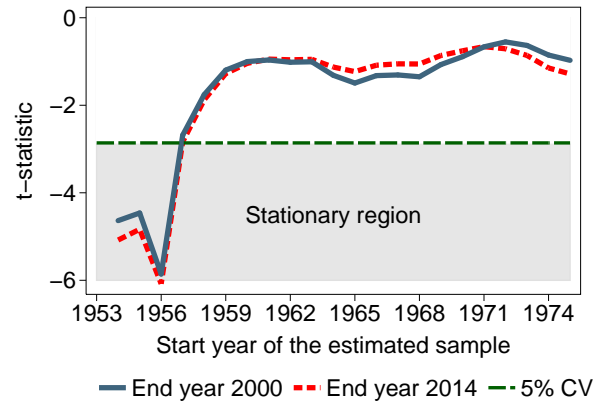
$$A(t) = A(0) \left[1 + (1-\phi)\bar{\lambda}\bar{X}^\sigma A(0)^{\phi-1} t \right]^{\frac{1}{1-\phi}}.$$

It follows from the expression that the model does feature perpetual growth, as MFP and GDP per worker go to infinity as time goes to infinity. Stagnation and exponential growth are given by the two limiting cases: $\phi \rightarrow -\infty$ and $\phi \rightarrow 1$. See Groth et al. (2010) for a thorough discussion of models featuring less-than-exponential growth.

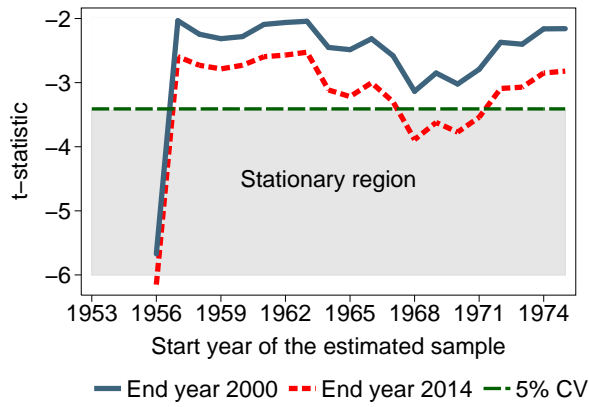
C Additional Empirical Results



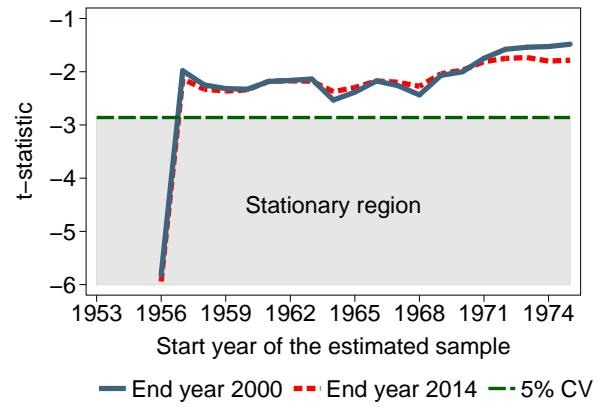
AR(1) model with trend



AR(1) model without trend



AR(3) model with trend



AR(3) model without trend

FIGURE 12: Backward recursive estimation of t-statistics for AR(1) and AR(3) models with and without a trend for $\ln X_t$ for estimated samples ending in 2000 or 2014.

Notes: The 5 pct. critical value (5% CV) is -3.41 with a trend and -2.86 without a trend.

TABLE 7: Estimation results from a cointegrated VAR model using MFP with different dummy variables, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
A1.0	55			3	-	+	4.460 (5.461)	1.028 (2.337)	-0.060 (-5.718)	
A1.0 (R1)							-2.998 (-3.216)	1.327 (2.335)	0.000 -	0.093
A1.0 (R2)							-0.900 (-3.720)	0.000 -	0.000 -	0.068
A1.0 (R3)							0.000 -	-0.495 (-2.973)	0.000 -	0.024
A1.1	55		2009	3	-	+	8.043 (5.117)	0.813 (0.972)	-0.089 (-4.421)	
A1.1 (R1)							-3.490 (-3.607)	1.620 (2.758)	0.000 -	0.289
A1.1 (R2)							-0.905 (-3.720)	0.000 -	0.000 -	0.122
A1.1 (R3)							0.000 -	-0.502 (-2.974)	0.000 -	0.045
A1.2	55	1974	2009	3	-	+	13.693 (4.880)	0.152 (0.103)	-0.129 (-3.607)	
A1.2 (R1)							-3.712 (-3.853)	1.776 (3.040)	0.000 -	0.423
A1.2 (R2)							-0.876 (-3.610)	0.000 -	0.000 -	0.112
A1.2 (R3)							0.000 -	-0.480 (-2.833)	0.000 -	0.040
A1.3	55	1974, 1982	2009	3	-	+	56.800 (4.329)	-4.484 (-0.654)	-0.441 (-2.645)	
A1.3 (R1)							-3.756 (-3.879)	1.825 (3.115)	0.000 -	0.699
A1.3 (R2)							-0.851 (-3.453)	0.000 -	0.000 -	0.142
A1.3 (R3)							0.000 -	-0.464 (-2.688)	0.000 -	0.055
A1.4	55	1974, 1982, 1993	2009	3	+	+	-1.469 (-1.547)	1.585 (3.242)	-0.016 (-1.371)	
A1.4 (R1)							-3.657 (-3.865)	1.785 (3.122)	0.000 -	0.770
A1.4 (R2)							-0.828 (-3.407)	0.000 -	0.000 -	0.128
A1.4 (R3)							0.000 -	-0.455 (-2.663)	0.000 -	0.047
A1	55	1963, 1972, 1982, 1993	2009	3	+	+	-0.768 (-0.932)	1.493 (3.549)	-0.021 (-2.053)	
A1 (R1)							-3.459 (-3.752)	1.679 (3.018)	0.000 -	0.607
A1 (R2)							-0.814 (-3.386)	0.000 -	0.000 -	0.124
A1 (R3)							0.000 -	-0.453 (-2.660)	0.000 -	0.045

Notes: See notes to Table 2.

TABLE 8: Estimation results from a cointegrated VAR model using TFP with different dummy variables, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
B1.0	55			3	-	+	4.505 (4.438)	1.081 (1.990)	-0.060 (-4.789)	
B1.0 (R1)							-2.632 (-2.831)	1.107 (1.984)	0.000 -	0.407
B1.0 (R2)							-0.859 (-3.657)	0.000 -	0.000 -	0.290
B1.0 (R3)							0.000 -	-0.460 (-2.993)	0.000 -	0.123
B1.1	55		2009	3	-	+	-5.956 (-2.873)	1.547 (1.396)	0.022 (0.847)	
B1.1 (R1)							-3.306 (-3.244)	1.473 (2.417)	0.000 -	0.860
B1.1 (R2)							-0.929 (-3.707)	0.000 -	0.000 -	0.307
B1.1 (R3)							0.000 -	-0.511 (-3.043)	0.000 -	0.127
B1.2	55	1974	2009	3	-	+	-2.993 (-2.299)	1.643 (2.376)	-0.005 (-0.313)	
B1.2 (R1)							-3.642 (-3.541)	1.682 (2.738)	0.000 -	0.937
B1.2 (R2)							-0.921 (-3.629)	0.000 -	0.000 -	0.200
B1.2 (R3)							0.000 -	-0.500 (-2.919)	0.000 -	0.073
B1.3	55	1974, 1982	2009	3	-	+	-0.838 (-1.018)	1.566 (3.614)	-0.022 (-2.205)	
B1.3 (R1)							-3.718 (-3.760)	1.762 (2.991)	0.000 -	0.497
B1.3 (R2)							-0.864 (-3.505)	0.000 -	0.000 -	0.100
B1.3 (R3)							0.000 -	-0.459 (-2.757)	0.000 -	0.033
B1.4	55	1974, 1982, 1993	2009	3	-	+	-0.482 (-0.634)	1.558 (3.964)	-0.025 (-2.717)	
B1.4 (R1)							-3.625 (-3.760)	1.730 (3.019)	0.000 -	0.302
B1.4 (R2)							-0.831 (-3.431)	0.000 -	0.000 -	0.068
B1.4 (R3)							0.000 -	-0.443 (-2.702)	0.000 -	0.023
B1	55	1963, 1974, 1982, 1993	2009	3	-	+	-0.464 (-0.622)	1.537 (4.010)	-0.025 (-2.758)	
B1 (R1)							-3.441 (-3.729)	1.637 (2.985)	0.000 -	0.268
B1 (R2)							-0.809 (-3.447)	0.000 -	0.000 -	0.063
B1 (R3)							0.000 -	-0.434 (-2.713)	0.000 -	0.020

Notes: See notes to Table 2.

TABLE 9: Estimation results from a cointegrated VAR model using GDP per worker with different dummy variables, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$(1 - \alpha)\tilde{\phi}$	$\tilde{\beta}$	Trend	
C1.0	55			3	-	+	0.924 (0.949)	1.916 (4.980)	-0.050 (-2.621)	
C1.0 (R1)							-1.968 (-5.501)	1.381 (4.398)	0.000	0.243
C1.0 (R2)							-0.860 (-3.886)	0.000	0.000	0.015
C1.0 (R3)							0.000	-1.035 (-2.935)	0.000	0.004
C1.1	55		2009	3	-	+	-0.431 (-0.384)	1.869 (4.323)	-0.030 (-1.373)	
C1.1 (R1)							-2.172 (-6.097)	1.561 (5.007)	0.000	0.478
C1.1 (R2)							-0.884 (-3.891)	0.000	0.000	0.011
C1.1 (R3)							0.000	-0.966 (-2.928)	0.000	0.003
C1.2	55	1982	2009	3	-	+	-0.358 (-0.333)	1.884 (4.541)	-0.031 (-1.484)	
C1.2 (R1)							-2.162 (-6.330)	1.568 (5.240)	0.000	0.420
C1.2 (R2)							-0.869 (-3.848)	0.000	0.000	0.007
C1.2 (R3)							0.000	-0.953 (-2.864)	0.000	0.002
C1.3	55	1963, 1982	2009	3	+	+	-0.177 (-0.166)	1.884 (4.627)	-0.033 (-1.630)	
C1.3 (R1)							-2.110 (-6.183)	1.524 (5.108)	0.000	0.367
C1.3 (R2)							-0.844 (-3.838)	0.000	0.000	0.008
C1.3 (R3)							0.000	-0.942 (-2.875)	0.000	0.002
C1	55	1963, 1974, 1982	2009	3	+	+	-0.233 (-0.215)	1.938 (4.693)	-0.034 (-1.609)	
C1 (R1)							-2.199 (-6.337)	1.603 (5.287)	0.000	0.369
C1 (R2)							-0.868 (-3.704)	0.000	0.000	0.006
C1 (R3)							0.000	-0.933 (-2.734)	0.000	0.002
C1.4	55	1963, 1974, 1982, 1993	2009	3	+	+	-0.394 (-0.358)	1.927 (4.632)	-0.031 (-1.467)	
C1.4 (R1)							-2.197 (-6.436)	1.607 (5.390)	0.000	0.388
C1.4 (R2)							-0.871 (-3.704)	0.000	0.000	0.006
C1.4 (R3)							0.000	-0.970 (-2.741)	0.000	0.002

Notes: See notes to Table 2.

TABLE 10: Estimation results from a cointegrated VAR model using TFP, sample 1953-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
B2	62	1959, 1963, 1974, 1982, 1993	1956, 2009	3	+	+	-1.249 (-1.043)	1.565 (2.241)	-0.019 (-1.286)	
B2 (R1)							-2.781 (-2.349)	1.266 (1.741)	0.000 -	0.672
B2 (R2)							-0.774 (-3.199)	0.000 -	0.000 -	0.557
B2 (R3)							0.000 -	-0.463 (-2.849)	0.000 -	0.375

Notes: See notes to Table 2.

TABLE 11: Estimation results from a cointegrated VAR model using GDP per worker, sample 1953-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$(1 - \alpha)\tilde{\phi}$	$\tilde{\beta}$	Trend	
C2	62	1959, 1963, 1974, 1982, 1993	1956, 2009	3	+	+	0.337 (0.264)	2.444 (5.024)	-0.051 (-2.102)	
C2 (R1)							-2.588 (-5.553)	1.947 (4.795)	0.000 -	0.252
C2 (R2)							-0.646 (-3.481)	0.000 -	0.000 -	0.013
C2 (R3)							0.000 -	-0.587 (-2.822)	0.000 -	0.007

Notes: See notes to Table 2.

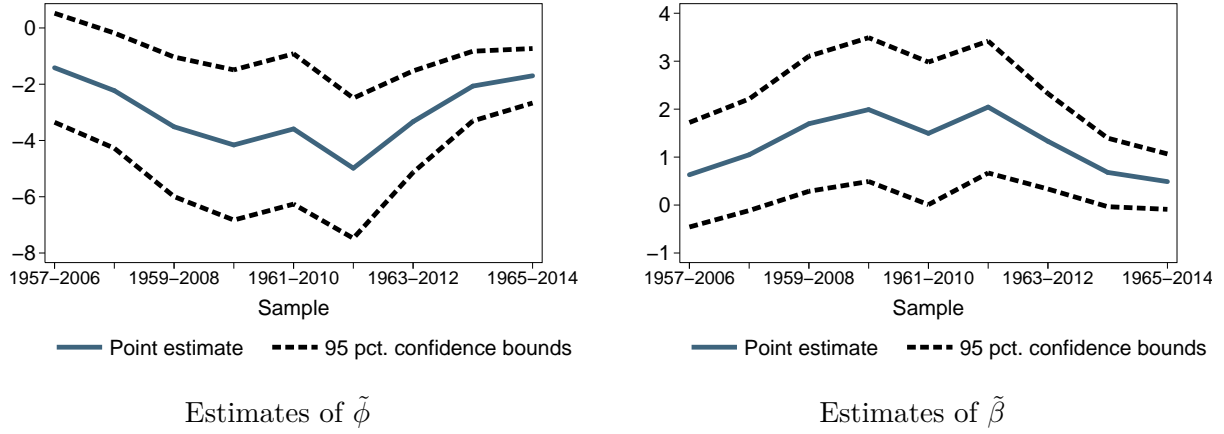


FIGURE 13: Estimates of $\tilde{\phi}$ and $\tilde{\beta}$ based on Model B1 (R1) for 50-year samples, (1957-2006)-(1965-2014).

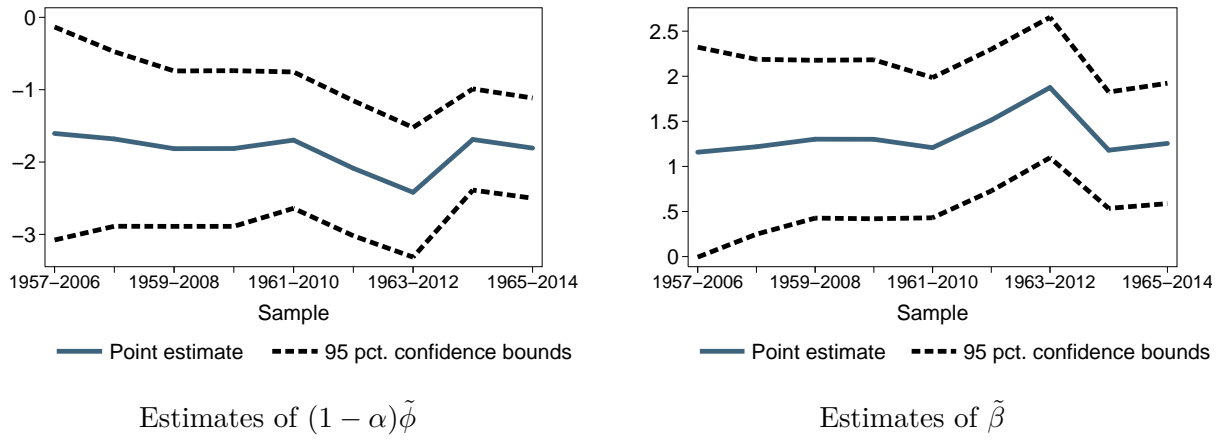


FIGURE 14: Estimates of $(1 - \alpha)\tilde{\phi}$ and $\tilde{\beta}$ based on Model C1 (R1) for 50-year samples, (1957-2006)-(1965-2014).

TABLE 12: Estimation results from a cointegrated VAR model using TFP and R&D expenditures performed by the business sector, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
B3	55	1963, 1972, 1982, 1993	2009	3	+	+	-0.762 (-0.784)	1.723 (3.345)	-0.027 (-2.269)	
B3 (R1)							-4.297 (-3.648)	1.995 (2.846)	0.000 -	0.360
B3 (R2)							-0.937 (-3.830)	0.000 -	0.000 -	0.101
B3 (R3)							0.000 -	-0.494 (-3.111)	0.000 -	0.028

Notes: See notes to Table 2.

TABLE 13: Estimation results from a cointegrated VAR model using GDP per worker and R&D expenditures performed by the business sector, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$(1 - \alpha)\tilde{\phi}$	$\tilde{\beta}$	Trend	
C3	55	1963, 1974, 1982, 1993	2009	3	+	+	-0.886 (-0.655)	1.990 (-3.882)	-0.027 (-1.045)	
C3 (R1)							-2.492 (-6.016)	1.736 (4.785)	0.000 -	0.545
C3 (R2)							-0.832 (-4.422)	0.000 -	0.000 -	0.010
C3 (R3)							0.000 -	-0.854 (-3.180)	0.000 -	0.002

Notes: See notes to Table 2.

Johansen test results

The hypothesis of a single cointegrating relationship is accepted if it cannot be rejected by the Johansen test at the 5 pct. level of significance. The critical values are simulated using CATS in WinRATS 9.0 (5,000 replications and a random walk length of 400). See Juselius (2006, ch. 8) for further details. The test results are provided in Table 14. The hypothesis of a single cointegrating relationship is clearly accepted at the 5 pct. level for all nine models.

TABLE 14: Johansen test results for a single cointegrating relationship

Model	Trace	Trace*	CV ₉₅	p-value	p-value*
Model A1	17.0	14.7	25.2	0.39	0.58
Model B1	12.7	11.0	25.3	0.74	0.86
Model C1	15.9	12.1	25.0	0.46	0.77
Model A2	13.4	10.0	26.4	0.72	0.92
Model B2	10.6	8.0	26.2	0.89	0.98
Model C2	15.3	9.4	25.8	0.57	0.94
Model A3	16.3	14.2	25.4	0.45	0.62
Model B3	12.7	10.9	25.1	0.74	0.86
Model C3	14.0	11.3	25.3	0.63	0.84

Notes: * indicates correction for small sample size, and CV₉₅ is the 95 pct. critical level.

Specification tests

A model is considered well specified if the hypotheses of normal errors, no autocorrelation, and no ARCH effects cannot be rejected at the 5 pct. level of significance using multivariate tests. This appendix provides both multivariate and univariate test results. The univariate results are useful when diagnosing the source of misspecification. All test results are obtained from CATS in WinRATS 9.0. See Juselius (2006, ch. 4) for further details about the test procedures.

Tests for no autocorrelation: LM tests for first and second-order autocorrelation.

Tests for no ARCH effects: Multivariate LM tests for first and second-order ARCH effects, and univariate tests for ARCH effect.

Normality: Univariate and multivariate tests for normality.

TABLE 15: Model A1: Multivariate Statistics

Autocorrelation		Normality	ARCH	
LM(1)	LM(2)		LM(1)	LM(2)
13.4	8.1	7.4	19.0	48.5
[0.15]	[0.53]	[0.29]	[0.99]	[0.99]

Notes: p-values in brackets.

TABLE 16: Model A1: Univariate Statistics

Variable	ARCH	Normality	R ²
$\Delta \ln \tilde{A}$	0.64 [0.89]	1.60 [0.45]	0.68
$\Delta \ln X$	1.49 [0.69]	6.92 [0.03]	0.60
$\Delta \ln L$	2.57 [0.46]	2.18 [0.34]	0.69

Notes: p-values in brackets.

TABLE 17: Model B1: Multivariate Statistics

Autocorrelation		Normality	ARCH	
LM(1)	LM(2)		LM(1)	LM(2)
16.2	6.8	12.7	42.0	67.5
[0.06]	[0.66]	[0.05]	[0.23]	[0.63]

Notes: p-values in brackets.

TABLE 18: Model B1: Univariate Statistics

Variable	ARCH	Normality	R ²
$\Delta \ln \tilde{A}$	1.21 [0.75]	2.60 [0.27]	0.65
$\Delta \ln X$	1.05 [0.79]	8.03 [0.02]	0.60
$\Delta \ln L$	2.19 [0.54]	3.94 [0.14]	0.72

Notes: p-values in brackets.

TABLE 19: Model C1: Multivariate Statistics

Autocorrelation		Normality	ARCH	
LM(1)	LM(2)		LM(1)	LM(2)
10.3	7.7	4.3	23.9	61.5
[0.33]	[0.57]	[0.64]	[0.94]	[0.81]

Notes: p-values in brackets.

TABLE 20: Model C1: Univariate Statistics

Variable	ARCH	Normality	R ²
$\Delta \ln \tilde{A}$	1.99 [0.57]	1.20 [0.55]	0.79
$\Delta \ln X$	1.29 [0.73]	2.98 [0.23]	0.57
$\Delta \ln L$	4.88 [0.18]	2.62 [0.27]	0.71

Notes: p-values in brackets.

Chapter 2

Directed Technical Change and Economic Growth Effects of Environmental Policy

Directed Technical Change and Economic Growth Effects of Environmental Policy*

By Peter K. Kruse-Andersen

Abstract

A Schumpeterian growth model is developed to investigate how environmental policy affects economic growth, when environmental policy also affects the direction of technical change. In contrast to previous models, production and pollution abatement technologies are embodied in separate intermediate good types. A set of stylized facts related to pollution emission, environmental policy, and pollution abatement expenditures is presented, and the model is designed to match these stylized facts. It is shown analytically that a tightening of the environmental policy directs research efforts toward pollution abatement technologies and away from production technologies. Hence economic growth is reduced unambiguously. Simulations indicate that even large environmental policy reforms have small economic growth effects. Yet, these economic growth effects have relatively large welfare effects, indicating that static models and models with exogenous technological change leave out an important welfare effect.

Keywords: Directed technical change, endogenous growth, pollution, environmental policy, Schumpeterian growth model

JEL Classification: O30, O41, O44, Q55, Q58

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1 Introduction

Extensive empirical work provides evidence suggesting that a tighter environmental policy stimulates environmental innovation (e.g., Brunnermeier and Cohen 2003; Johnstone and Labonne 2006; Popp 2006; Arimura et al. 2007; Ambec et al. 2011; Haščič et al. 2012; Aghion et al. 2016). But, does this stimulation come at the expense of other types of research? If so, environmental policy might have a negative effect on economic growth.

In this paper, a Schumpeterian growth model is developed to investigate how a tighter environmental policy affects economic growth, when environmental policy also affects the direction of research efforts. The model is constructed such that it matches several stylized facts concerning pollution emission, environmental policy, and pollution abatement expenditures. It is shown analytically that a tighter environmental policy unambiguously reduces the economic growth rate as well as the growth rate of pollution emission. However, simulations based on U.S. data indicate that even large environmental policy reforms barely affect the long-run economic growth rate. This finding appears remarkably robust to changes in parameter values and estimation targets. The simulations, nevertheless, indicate that the economic growth effects constitute a large share of the overall welfare effects of environmental policy changes, as even small changes in growth rates have large level effects in the long run. Thus, the results indicate that static models and exogenous growth models (like the DICE model) leave out an important welfare effect of environmental policy.

Besides the policy implications, this analysis also contributes to the literature on directed technical change and the environment by developing a novel modeling strategy. A Schumpeterian growth model is developed, where production and pollution abatement technologies are embodied in separate intermediate good types. The R&D sector is bifurcated into two subsectors: one for production technologies and one for pollution abatement technologies. In contrast, Hart (2004, 2007) and Ricci (2007a) assume that production and pollution abatement technologies are embodied in the same intermediate goods. In their works, intermediate goods can be improved along two dimensions: productivity and environmental friendliness. As more environmentally friendly intermediates are less productive, R&D firms face a design trade-off when attempting to develop higher intermediate good qualities.¹

To illustrate the difference between the two modeling strategies, imagine a firm obtaining a patent on a new quality of a certain machine type. If production and pollution abatement

¹Acemoglu et al. (2012, p. 146) also briefly consider a model, where production and pollution abatement technologies are embodied in the same intermediate goods.

technologies are embodied in the same inventions (as assumed by Hart [2004, 2007] and Ricci [2007a]), the new machine quality is both more productive and emits less pollution per output unit compared to previous qualities. If the two technologies are embodied in separate inventions (as in the present study), one firm would obtain a patent on a more productive machine, while another firm would obtain a patent on a catalyst or filter, that could be implemented into the machine to reduce pollution emission. Hence, innovation arrivals of pollution abatement technologies are detached from innovation arrivals of production technologies. If a firm develops both production and pollution abatement technologies, these are developed in separate R&D units. One unit can be successful during a certain time interval while the other fails. If the two technologies are embodied in the same inventions, the firm has only one R&D unit. Either the firm develops a more productive and more environmentally friendly machine quality, or no innovation occurs.

Separating the two technology types results in a more realistic representation of the innovation process for at least two reasons. First, it seems natural to assume that the innovation arrivals of production and pollution abatement technologies are independent. Certainly, it is possible to invent a better catalyst or filter without also inventing a more productive machine. Second, to a large extent, private firms are only willing to conduct research, when the developed ideas can be protected. As patents are granted very specific components rather than entire systems, it seems appropriate to assume that production and pollution abatement technologies are developed separately.

Additionally, the framework developed in the present study seems to foster tractability and empirical relevance. The policy implications for economic growth are derived analytically, and they are unambiguous. In contrast, Ricci (2007a) derives the economic growth implications numerically, and Hart (2004) finds ambiguous economic growth effects. The model presented below matches several stylized facts concerning pollution emission, environmental policy, and pollution abatement expenditures. In contrast to many other models in the literature (e.g., Acemoglu et al. 2012), research occurs simultaneously in both R&D subsectors which seems like the more empirically relevant case (see Dechezleprêtre et al. 2014; Noailly and Smeets 2015).

The model developed in the present study also has the advantage that it clearly illustrates how environmental policy influences the intersectoral labor allocation between production and research as well as the intrasectoral labor allocations of these two sectors. In contrast, some studies have definitively shut down intersectoral labor allocation effects. Acemoglu

et al. (2012) assume a constant labor input in both production and research. Thus, environmental policy can only affect the intrasectoral labor allocations. As the trade-off between production and research is eliminated, their model does not qualify as a (complete) endogenous growth model (Pottier et al. 2014). In the present study, it is shown that environmental policy affects the intersectoral labor allocation except for in a knife-edge case. This intersectoral effect is important when investigating economic growth effects of environmental policy changes, as the labor reallocation between production and research implies intertemporal changes in the production capacity.

Besides the works by Hart (2004, 2007) and Ricci (2007a), the present study relates to several strands of literature. First, it is related to a large body of literature investigating how environmental policy affects economic growth (e.g., Gradus and Smulders 1993; Bovenberg and de Mooij 1997; Nielsen et al. 1995; Hettich 1998; Schou 2002; Karydas and Zhang 2017). These studies are typically novel in their identification of channels through which environmental policy might enhance growth, but they do not feature directed technical change.²

Second, the methodology used in the present study is closely related to works by Smulders and de Nooij (2003), Brock and Taylor (2010), and André and Smulders (2014). These works develop growth models which match certain stylized facts to answer environmentally related questions. The main goal of the present study is not to explain the stylized facts presented below. Rather, the model is designed to be consistent with the stylized facts to ensure that it is empirically relevant. Other models in the literature have been developed with little or no regards to empirical tendencies, and it is therefore difficult to assess the usefulness of their policy implications.

Third, the present study relates to a growing body of literature investigating how environmental policy affects the direction of technological change in endogenous growth models.³ In these works, it is usually assumed that output is produced using a constant elasticity of substitution production technology with two input types: clean and dirty. Pollution emission is an unavoidable by-product associated with dirty input use while no pollution emission comes from clean input use. Environmental policy can then skew incentives such that it becomes relatively more attractive to conduct research in clean input technologies.

The modeling strategy developed in the present study might be a useful alternative to

²See Ricci (2007b) for a survey on channels through which environmental policy affects economic growth.

³E.g., Saint-Paul (2002), Smulders and de Nooij (2003), Grimaud and Rougé (2008), Acemoglu et al. (2012), André and Smulders (2014), Hemous (2015), and Van den Bijgaart (2017).

the clean-dirty-input approach. Specifically, the modeling strategy developed in the present study seems appropriate when considering pollutants for which emissions can be substantially reduced by end-of-pipe technologies.⁴ These include toxic air pollutants like SO_2 and NO_x which cause adverse health effects (see Hocking 2005; WHO 2006). In the next section, it is shown that U.S. coal, oil and gas consumption increased substantially after 1970, while emissions of several toxic air pollutants peaked around this date. Thus when considering these pollutants, it seems appropriate to focus on pollution abatement efforts rather than input substitution.⁵

Given the focus on toxic pollutants, the results obtained in the present study are especially interesting for developing countries. In these countries, economic growth is necessary to lift a large share of the population out of poverty (UN 2009). But rapid economic growth is often associated with rapid growth in toxic air pollution.⁶ Accordingly, the trade-off between economic growth and a clean environment is particularly pronounced for these countries.

The paper proceeds as follows. The next section presents stylized facts related to pollution emission, environmental policy, and pollution abatement expenditures. Section 3 presents the model, and the policy implications are derived analytically in Section 4. Section 5 relates the model to the stylized facts. Natural model extensions are discussed in Section 6, and Section 7 provides a quantitative analysis. Section 8 provides a discussion and suggests directions for future research.

2 Stylized Facts

Some of the stylized facts presented in this section serve as motivation for model assumptions made in the subsequent section. The remaining facts serve as a first empirical test of the model.⁷

⁴End-of-pipe technologies should be interpreted broadly as any pollution abatement technique reducing pollution emission without reducing the polluting input. This includes cleaning the polluting input before use (e.g., coal washing) and treatment of by-products before release to the atmosphere.

⁵This claim is further supported by empirical evidence from West Germany and the Netherlands which suggests that end-of-pipe technologies were the most important determinant for the substantial SO_2 emission reductions in these two countries over the period 1980-1990 (de Bruyn 1997, 1999).

⁶For instance, China has experienced fast economic growth since the 1970s. As a consequence, Chinese coal consumption increased rapidly which increased pollution emission (Zheng and Kahn 2017). In 2013, more than 99 pct. of the Chinese population was exposed to $\text{PM}_{2.5}$ levels above the WHO guidelines (Brauer et al. 2015).

⁷Some of the stylized facts presented below are to my knowledge new to the literature. The rest have been presented in either Brock and Taylor (2005, 2010) or Botta and Kozluk (2014).

2.1 Pollution emissions and intensities

Following Brock and Taylor (2005, 2010), the present study focuses on air pollution, and in particular, the air pollutants CO_2 , CO , NO_x , SO_2 or SO_x , and VOC (volatile organic compounds). As shown in the left panel of Figure 1, U.S. pollution intensities - defined as emissions over GDP - decreased almost monotonically for all five air pollutants over the period 1940-2014. The right panel of Figure 1 shows that U.S. pollution emissions did not exhibit the same clear trend. Instead, emissions for the four pollutants CO , NO_x , SO_2 , and VOC increased until around 1970. After 1973, CO , SO_2 , and VOC emissions decreased rapidly. The exception is CO_2 emissions which increased systematically over the period, while NO_x emissions did not decrease notably before the late 1990s. As income increased with an approximately constant growth rate throughout the period, it has been hypothesized that there exists an inverted U-shaped relationship between income and pollution emission. A relationship often referred to as the environmental Kuznets curve (see Stern 2014). These tendencies lead to the first stylized fact.

Stylized Fact 1. *Pollution emissions might increase or decrease while income increases. Pollution intensities decrease with income.*

A model consistent with Stylized Fact 1 predicts that pollution emissions grow slower than income.

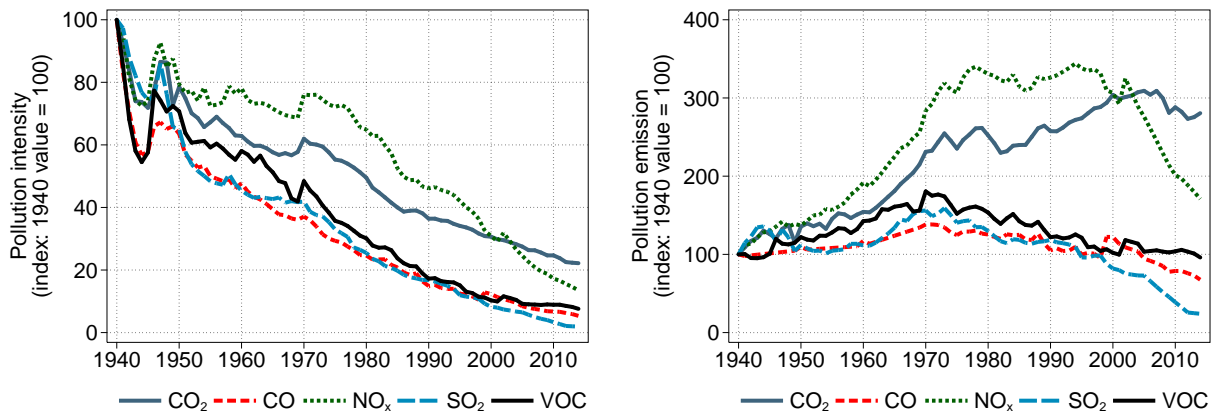


FIGURE 1: U.S. pollution emissions and intensities, 1940-2014.

Data sources: U.S. Environmental Protection Agency (2000, 2017), U.S. Bureau of Economic Analysis (2017), and Boden et al. (2017).

Notes: Pollution intensity defined as emission divided by GDP. There is a data break between 1998 and 1999 for CO , NO_x , SO_2 , and VOC. Data for the period 1940-1998 are obtained from the U.S. Environmental Protection Agency (2000), and the remaining data are obtained from the U.S. Environmental Protection Agency (2017).

The model presented in the next section focuses on end-of-pipe technologies. To motivate

this modeling strategy, consider the post World War II U.S. fossil fuel consumption shown in Figure 2. The figure shows a generally positive trend in U.S. fossil fuel consumption over the period 1949-2014. The consumption of all three fossil fuel types increased substantially from 1970 to 2014. Meanwhile, the pollution emissions of CO, NO_x, SO₂, and VOC decreased notably, as shown in Figure 1. These patterns reflect that end-of-pipe technologies might substantially reduce emissions of these pollutants. It is, for instance, technically possible to install pollution abatement systems capable of removing over 90 pct. of the SO₂ emissions associated with fossil fuel combustion (Roy and Sardar 2015). Thus it seems inappropriate to focus on the input side, when modeling the development of these air pollutants. These considerations lead to the following stylized fact.

Stylized Fact 2. *Fossil fuel consumption might increase, while emissions of CO, NO_x, SO₂, and VOC decrease.*

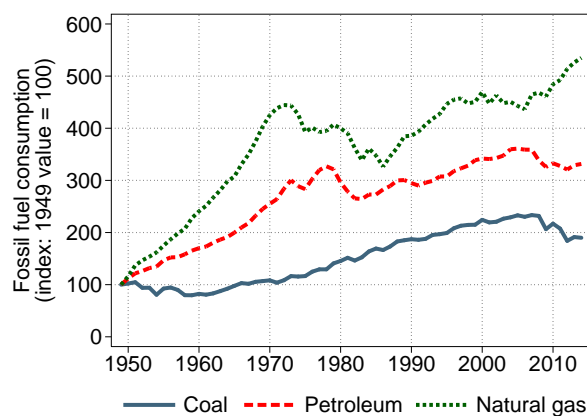


FIGURE 2: U.S. fossil fuel consumption, 1949-2014.

Data source: U.S. Energy Information Administration (2017).

Notes: Consumption is measured in either volume (petroleum and natural gas) or weight (coal).

Generally, OECD countries experienced a decline in the pollution intensities of CO₂, CO, NO_x, SO_x, and VOC during the period 1990-2012.⁸ In fact, it seems like pollution intensities are decreasing over time and with environmental policy stringency. Let environmental policy stringency be measured by the economy-wide environmental policy stringency (EPS) index described by Botta and Kozluk (2014). The EPS index is defined from zero to six, where zero is when environmental policy is nonexistent and six is a very stringent environmental policy.

Table 1 shows log transformed pollution intensities regressed on time and/or the EPS index for the OECD countries over the period 1990-2012. The regression results indicate

⁸Disregarding Mexico due to insufficient data, the exceptions are Portugal, Spain, and Turkey for CO₂; Iceland for SO₂; and Chile for VOC.

TABLE 1: Log transformed pollution intensities regressed on time and/or the EPS index for OECD countries, 1990-2012

	ln(CO ₂ /GDP)			ln(CO/GDP)			ln(NO _x /GDP)			ln(SO _x /GDP)			ln(VOC/GDP)		
Time	-0.02*	-0.02*	-0.07*	-0.07*	-0.07*	-0.05*	-0.04*	-0.10*	-0.08*	-0.06*	-0.06*	-0.06*	-0.06*	-0.06*	-0.06*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
EPS	-0.14*	-0.04*		-0.43*	0.04*		-0.31*	-0.05*		-0.64*	-0.08*		-0.39*	-0.01	-0.01
	(0.00)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)		(0.02)	(0.04)		(0.01)	(0.01)	(0.01)
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	594	510	510	594	510	510	594	510	510	594	510	510	594	510	510
R ²	0.70	0.63	0.73	0.91	0.68	0.93	0.84	0.71	0.86	0.78	0.63	0.77	0.88	0.69	0.89
Adjusted R ²			0.72			0.93			0.85			0.76			0.89

Data sources: OECD (2014a, 2014b, 2014c) and Botta and Kozluk (2014).

Notes: Regressions conducted using the OLS estimator. Standard errors in brackets. (*) indicates significance at the 5 pct. level. Environmental policy stringency is measured using the EPS index presented by Botta and Kozluk (2014). Pollution intensity defined as emission divided by GDP (unit: tons of emission per 1,000 U.S. 2005-dollars).

that both time and environmental policy stringency are negatively correlated with pollution intensities. The coefficients for both time and the EPS index are negative and significant at the five pct. level in 14 out of 15 regressions. Note that in most cases the EPS coefficient is significant when controlling for time. This indicates that the negative correlation between pollution intensities and the EPS index is not only caused by the fact that both measures are correlated with time. This leads to the following stylized fact.

Stylized Fact 3. *Pollution intensities decrease over time and with environmental policy stringency.*

2.2 Environmental policy

Figure 3 depicts the evolution of the EPS index over time. The left panel shows that the EPS values for all individual OECD countries (for which data are available) increased from 1995 to 2012. The right panel shows that the average EPS value in the OECD increased systematically through the period 1990-2012. This evidence leads to the following stylized fact.

Stylized Fact 4. *Environmental policy stringency increases over time.*

Figure 4 depicts tax revenues from environmentally related taxes as share of GDP for some of the largest OECD countries over the period 1994-2012. The tax revenues from environmentally related taxes as share of GDP remain remarkably constant over the period despite many economic events, e.g. business cycles, policy changes, and the Financial Crisis of 2008. This evidence is summarized in the following stylized fact.

Stylized Fact 5. *The tax revenue from environmentally related taxes is approximately a constant share of GDP for long periods of time.*

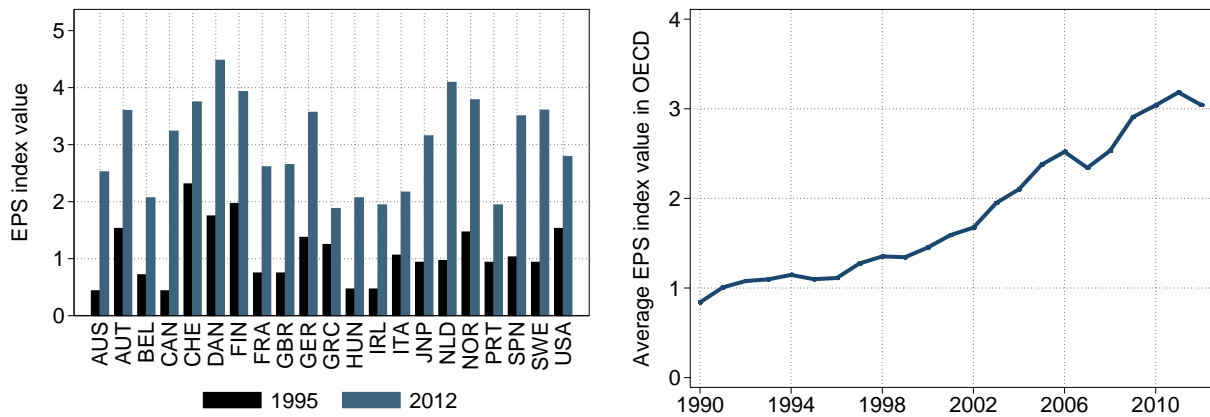


FIGURE 3: The economy-wide environmental policy stringency (EPS) index.

Data source: Botta and Kozluk (2014).

Notes: Environmental policy stringency is measured using the EPS index presented by Botta and Kozluk (2014). Due to missing data, the 1990 value is used instead of the 1995 value for IRL.

The theoretical model focuses on environmental taxes. As the tax revenues from environmentally related taxes seem to be a constant share of GDP, it is assumed that the government adjusts environmental tax rates to ensure this relation. Using this policy rule, environmental tax rates must increase over time as pollution intensities decrease. Higher environmental tax rates translate into a stricter environmental policy, and thus, the EPS index value increases over time.

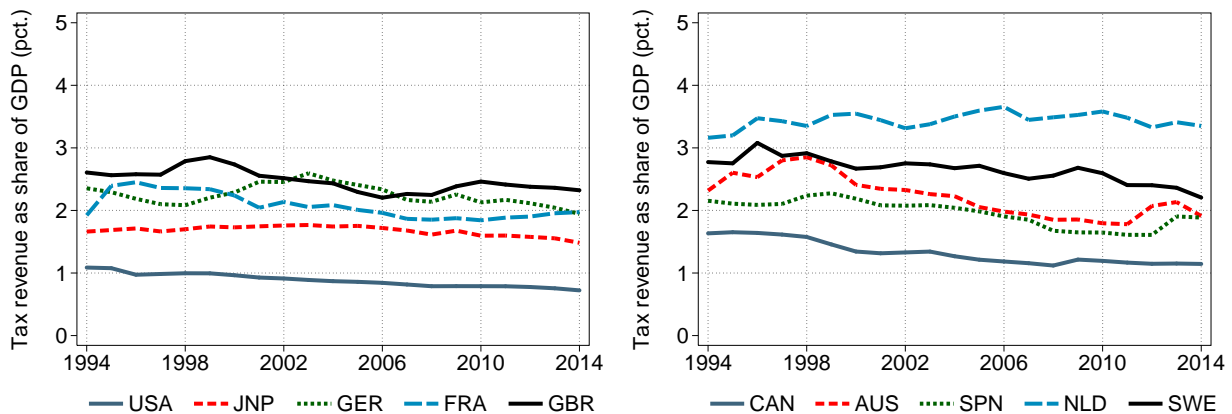


FIGURE 4: Revenues from environmentally related taxes, 1994-2014.

Data source: OECD (2017).

2.3 Pollution abatement expenditures

An important component in most growth models designed to answer environmentally related questions is the pollution abatement expenditures. Data on this subject are relatively scarce. The present study focuses on the U.S. case for which the longest time series are available. The left panel of Figure 5 shows that aggregate U.S. pollution abatement expenditures were

approximately a constant share of GDP over the period 1975-1994. The measure increased notably from 1972 to 1975. This increase can probably be attributed to the Clean Air Act of 1970 which changed U.S. air pollution policy substantially.⁹ The evidence presented in the left panel of Figure 5 together with the evidence from Figure 1 lead to the following stylized fact.

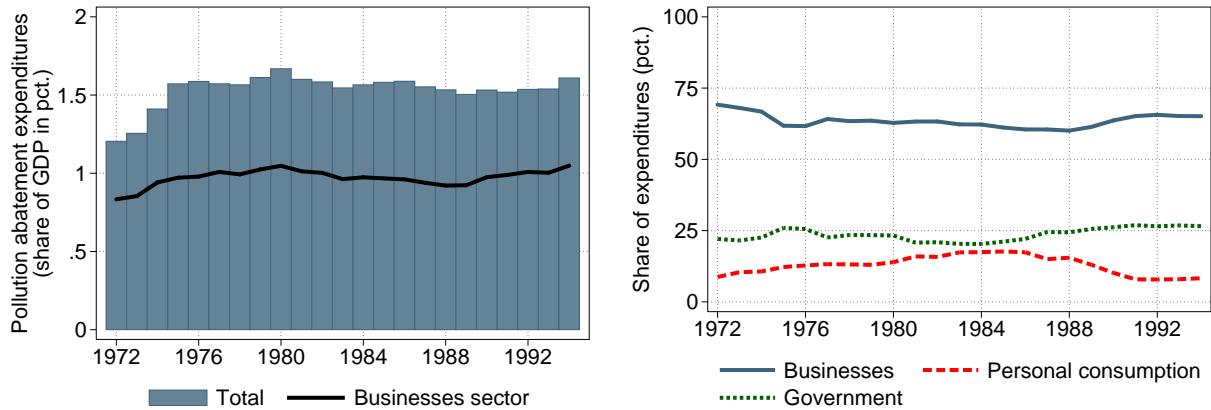


FIGURE 5: U.S. pollution abatement expenditures, 1972-1994.

Data sources: U.S. Bureau of Economic Analysis (2017) and Vogan (1996).

Stylized Fact 6. *Pollution abatement expenditures can be a roughly constant share of GDP while pollution emissions decrease.*

Stylized Fact 6 provides strong evidence against models, where pollution abatement expenditures must take up an increasing share of economic output over time to reduce pollution emission. This model class includes the model developed by Stokey (1998).

The left panel of Figure 5 also shows that pollution abatement expenditures were roughly a constant share of GDP for the business sector during the period. The right panel of Figure 5 depicts pollution abatement expenditure shares by sector. The business sector had a share of over 60 pct. for the whole period, whereas each of the other two sectors never had a share above 30 pct. This evidence leads to the final stylized fact.

Stylized Fact 7. *Pollution abatement expenditures fall primarily on the business sector. In addition, the pollution abatement expenditures of the business sector can be a roughly constant share of GDP while pollution emissions decrease.*

Stylized Fact 6 and 7 have two important implications. First, a theoretical model should allow for decreasing pollution emissions and intensities, when aggregate and business sector

⁹The Air Act Amendments of 1977 and 1990 added major amendments to the Clean Air Act of 1970, but it seems like they did not have a large effect on the pollution abatement expenditures. For more information about the Clean Air Act, see Davidson and Norbeck (2012).

pollution abatement expenditures are some constant share of GDP. Second, as the business sector accounts for most of the aggregate pollution abatement expenditures, it seems natural to focus on this sector in theoretical work.

3 The Model

The model is an extension of the Schumpeterian growth model presented by Aghion and Howitt (1998, p. 85-92). There are two sectors: a manufacturing sector and a research sector. First, the manufacturing sector is presented and the market equilibrium of this sector derived, given an exogenous supply of manufacturing workers. Thereafter, the research sector is presented, and the equilibrium labor allocation between the two sectors is determined.

3.1 Manufacturing sector

The manufacturing sector is subdivided into two sectors: the final goods sector and the intermediate goods sector. In the final goods sector, firms produce consumption goods using intermediate goods. These intermediate goods are referred to as production intermediates. Production of consumption goods pollute, and the firms can limit the pollution emission stemming from production using another type of intermediate goods referred to as (pollution) abatement intermediates. In the intermediate goods sector, monopolists produce different intermediate good varieties using labor. The monopolists are divided into two subsectors: one for each intermediate good type.

Final goods sector

A mass of firms indexed $j \in [0, N_t]$ produce homogeneous final goods using the technology:

$$Y_{jt} = \max \left(0; \int_0^1 (x_{ijt}^Y)^\alpha A_{it}^Y di - FA_t^Y \right), \quad i \in [0, 1], \quad 0 < \alpha < 1, \quad F > 0, \quad (1)$$

where Y_{jt} is final good output, x_{ijt}^Y is the amount of production intermediate i , A_{it}^Y is the productivity associated with production intermediate i , and $A_t^Y \equiv \int_0^1 A_{it}^Y di$ is the average productivity associated with production intermediates. The term FA_t^Y is a quasi-fixed cost associated with production. The quasi-fixed cost reflects that positive output requires a minimum input, and this minimum increases, as the production process becomes more ad-

vanced.¹⁰ The production function implies decreasing returns to scale on the firm level after some level of intermediate good input. A low α implies strong decreasing returns to scale and a low substitutability between intermediate good varieties.

All final good output is consumed, and aggregate output, Y_t , equals aggregate consumption, C_t :

$$Y_t = \int_0^{N_t} Y_{jt} dj = C_t. \quad (2)$$

Pollution emission is an unavoidable by-product of production. Following Stokey (1998), pollution emission from each final good firm, P_{jt} , is given by

$$P_{jt} = Y_{jt} z_{jt}^\psi, \quad 0 < \psi < 1, \quad (3)$$

where $z_{jt} \in [0, 1]$ is a pollution intensity index, where higher values imply a dirtier production process. The parameter ψ is the elasticity of pollution emission with respect to the pollution intensity index.

The pollution intensity index is reduced through pollution abatement efforts according to

$$z_{jt} = \min \left(1; Y_{jt}^{\tilde{\beta}} Z_{jt}^{-\tilde{\chi}} \right), \quad 0 < \tilde{\beta} < 1, \quad 0 < \tilde{\chi} < 1,$$

where Z_{jt} measures the effective pollution abatement effort by firm j . Without a minimum pollution abatement effort, pollution emission is proportional to output. If the pollution abatement effort is above some threshold value, it limits the pollution emission stemming from production. And this threshold value increases with the scale of production. The parameter $\tilde{\chi}$ reflects the effectiveness of abatement efforts in reducing pollution emission, while the parameter $\tilde{\beta}$ reflects how strongly the production scale reduces the impact of a given abatement effort.

The effective pollution abatement effort is given by:

$$Z_{jt} = \int_0^1 (x_{hjt}^Z)^\mu A_{ht}^Z dh, \quad h \in [0, 1], \quad 0 < \mu < 1, \quad (4)$$

where x_{hjt}^Z is the amount of abatement intermediate h , and A_{ht}^Z measures the effectiveness associated with abatement intermediate h . For later use, the average effectiveness of abate-

¹⁰The quasi-fixed cost ensures a market equilibrium with a finite and time constant mass of final good firms.

ment intermediates is denoted $A_t^Z \equiv \int_0^1 A_{ht}^Z dh$. The function (4) implies decreasing returns to scale on the firm level. A low μ implies strong decreasing returns to scale and a low substitutability between intermediate good varieties. To ensure that firms conduct pollution abatement in equilibrium, pollution abatement must be sufficiently effective. Thus it is assumed that: $\tilde{\beta} < \tilde{\chi}\mu$.

The pollution function developed in the present study differs from Stokey's framework in two important ways. First, in Stokey's framework, output is proportional to the pollution intensity index, and firms can without cost regulate this index. Pollution abatement is thereby directly linked to output losses. In contrast, the present study assumes that the pollution intensity index depends on costly abatement efforts. Second, the present study assumes that pollution abatement requires specialized pollution abatement technologies. These technologies can be improved through research efforts, allowing the model to match Stylized Fact 6.

A pollution tax incentivize pollution abatement. Motivated by Stylized Fact 5, the government adjusts the pollution tax rate, such that the implied tax revenue remains a constant share of output:

$$P_t \tau_t = \phi Y_t, \quad 0 < \phi < 1, \quad (5)$$

where $P_t = \int_0^{N_t} P_{jt} dj$ is aggregate pollution emission, $\tau_t > 0$ is the pollution tax rate, and ϕ is the tax revenue from environmentally related taxes as share of output. The parameter ϕ is referred to as the *environmental tax revenue parameter*. The environmental tax revenue parameter reflects the technology-adjusted environmental policy stringency. If technological improvements allow firms to reduce their pollution intensity, the government increases the pollution tax rate, leaving the tax revenue per unit of output unchanged. Through the entire analysis, the environmental policy is assumed to be tight enough to ensure that $z_{jt} < 1$. Without this assumption, pollution emission is proportional to output which is inconsistent with Stylized Fact 1.¹¹

Final good firms operate under perfect competition. They maximize the flow of profits, π_{jt}^f , given by:

$$\pi_{jt}^f = Y_{jt} - \int_0^1 x_{ijt}^Y p_{it}^Y di - \int_0^1 x_{hjt}^Z p_{ht}^Z dh - P_{jt} \tau_t, \quad (6)$$

¹¹In Appendix A, it is shown how tight the environmental policy must be to ensure that an equilibrium with $z_{jt} < 1 \forall j \in [0, N_t]$ exists.

where p_{it}^Y is the price of production intermediate i , and p_{ht}^Z is the price of abatement intermediate h . All prices are in terms of final goods. Each firm j maximizes π_{jt}^f with respect to $(x_{ijt}^Y)_{i \in [0,1]}$ and $(x_{hjt}^Z)_{h \in [0,1]}$ taking $(p_{it}^Y)_{i \in [0,1]}$, $(p_{ht}^Z)_{h \in [0,1]}$, and τ_t as given.

The first-order conditions imply that

$$\begin{aligned} p_{it}^Y &= \alpha \left(1 - \beta \frac{P_{jt}}{Y_{jt}} \tau_t \right) (x_{ijt}^Y)^{\alpha-1} A_{it}^Y, \quad \forall i \in [0,1], \quad \beta \equiv 1 + \psi \tilde{\beta} \quad \text{and} \\ p_{ht}^Z &= \chi \mu \left(\frac{P_{jt}}{Z_{jt}} \right) \tau_t (x_{hjt}^Z)^{\mu-1} A_{ht}^Z, \quad \forall h \in [0,1], \quad \chi \equiv \psi \tilde{\chi}. \end{aligned}$$

It is shown in Appendix B, that the economy is in a symmetric equilibrium, where all final good firms behave similarly. Hence the first-order conditions imply that

$$p_{it}^Y = \alpha(1 - \beta\phi) (x_{ijt}^Y)^{\alpha-1} A_{it}^Y \quad \text{and} \quad (7)$$

$$p_{ht}^Z = \chi \mu \phi \left(\frac{\hat{Y}_t}{\hat{Z}_t} \right) (x_{hjt}^Z)^{\mu-1} A_{ht}^Z, \quad (8)$$

where hats denote per final good firm quantities (e.g., \hat{Y}_t denotes final good output per final good firm).

Intermediate goods sector

All intermediate goods are produced by labor. Given the technological design, one unit of labor can produce A_t^Y/A_{it}^Y units of intermediate $i \in [0,1]$ and A_t^Z/A_{ht}^Z units of intermediate $h \in [0,1]$. Hence it requires more labor to produce relatively more advanced intermediate good varieties.¹² Intermediate good producers operate under monopolistic competition, as each monopolist has the exclusive right to produce the highest existing quality of a certain intermediate good.

Monopolist $i \in [0,1]$ faces the inverse demand function given by (7) when maximizing profits. The first-order condition implies:

$$x_{it}^Y = \left(\frac{\alpha^2(1 - \phi\beta)}{w_t} A_t^Y \right)^{\frac{1}{1-\alpha}} \equiv \hat{x}_t^Y, \quad (9)$$

where w_t is the wage rate, and \hat{x}_t^Y is the amount of each production intermediate used by each final good firm.

¹²This assumption is not crucial, but it reduces the complexity of the model. Assuming that one unit of labor can produce one unit of any intermediate good yields similar policy implications (see Kruse-Andersen 2016).

It follows from (7) and (9) that: $p_{it}^Y = (1/\alpha)(A_{it}^Y/A_t^Y)w_t$. Hence the price of intermediate $i \in [0, 1]$ equals the marginal cost of production multiplied by a constant markup. The markup is decreasing in α as higher values of α implies higher substitutability between production intermediates which reduces the market power of the monopolists. Using the expression for the price and (9), the equilibrium profit of monopolist i is obtained:

$$\tilde{\pi}_{it}^Y = \left(\frac{1-\alpha}{\alpha}\right) w_t \left(\frac{A_{it}^Y}{A_t^Y}\right) \hat{x}_t^Y N_t. \quad (10)$$

Monopolist $h \in [0, 1]$ faces the inverse demand function given by (8) when maximizing profits. The first-order condition implies:

$$x_{ht}^Z = \left(\frac{\chi\mu^2\phi\hat{Y}_t}{\hat{Z}_t w_t} A_t^Z\right)^{\frac{1}{1-\mu}} \equiv \hat{x}_t^Z, \quad (11)$$

where \hat{x}_t^Z is the amount of each abatement intermediate used by each final good firm.

It follows from (4) and (11) that (see Appendix C):

$$\hat{x}_t^Z = \chi\mu^2\phi\frac{\hat{Y}_t}{w_t}. \quad (12)$$

From (8) and (12) it follows that: $p_{ht}^Z = (1/\mu)(A_{ht}^Z/A_t^Z)w_t$. Hence the price of intermediate $h \in [0, 1]$ equals the marginal cost of production multiplied by a constant markup. A higher value of μ implies higher substitutability between abatement intermediates which reduces the market power of the monopolists and thereby the markup. Using the expression for the price and (12), the equilibrium profit of monopolist h is obtained:

$$\tilde{\pi}_{ht}^Z = \left(\frac{1-\mu}{\mu}\right) w_t \left(\frac{A_{ht}^Z}{A_t^Z}\right) \hat{x}_t^Z N_t. \quad (13)$$

Equilibrium in the manufacturing sector

Firms enter the final goods sector until profits are driven to zero, implying that:

$$\pi_{jt}^f = 0, \quad \forall j \in [0, N_t]. \quad (14)$$

An expression for N_t is derived from (5), (6), (9), (12), (14), and the equilibrium inter-

mediate good prices (see Appendix D):

$$N_t = \frac{1 - \alpha(1 - \phi\beta) - \chi\mu\phi - \phi}{\alpha(1 - \phi\beta)F} \frac{Y_t}{A_t^Y}. \quad (15)$$

To ensure a positive mass of final good firms: $1 - \alpha(1 - \phi\beta) - \chi\mu\phi - \phi > 0$ and $1 > \phi\beta$.

The manufacturing clearing condition requires that:

$$L_t = L_t^Y + L_t^Z, \quad L_t^Y = \int_0^{N_t} \left(\int_0^1 x_{ijt}^Y \frac{A_{it}^Y}{A_t^Y} di \right) dj, \quad L_t^Z = \int_0^{N_t} \left(\int_0^1 x_{hjt}^Z \frac{A_{ht}^Z}{A_t^Z} dh \right) dj, \quad (16)$$

where L_t is labor supply in manufacturing, while L_t^Y and L_t^Z are manufacturing labor producing production and abatement intermediates, respectively.

As shown in Appendix E, the wage rate is derived from (9), (12), (15), and (16):

$$w_t = \Omega \frac{Y_t}{L_t}, \quad \Omega \equiv \alpha(1 - \phi - \chi\mu\phi) + \chi\mu^2\phi. \quad (17)$$

Since $L_t^Y = N_t \hat{x}_t^Y$, $L_t^Z = N_t \hat{x}_t^Z$, and $Y_t = N_t \hat{Y}_t$, it follows from (9), (12), and (17) that

$$L_t^Y = \frac{\alpha(1 - \phi - \chi\mu\phi)}{\Omega} L_t \quad \text{and} \quad L_t^Z = \frac{\chi\mu^2\phi}{\Omega} L_t. \quad (18)$$

Intuitively, the fraction of labor devoted to pollution abatement is large, if the environmental policy is tight (large ϕ), or if pollution abatement is effective (large χ and/or large μ). Meanwhile, the fraction of labor devoted to production is large, if production intermediates exhibit low decreasing returns (large α).

3.2 Research sector

Research labs conduct research to invent new and higher intermediate good qualities. If a lab successfully develops a new intermediate good quality, it receives a patent of infinite duration on that intermediate good quality and becomes a monopolist in the intermediate goods sector. This monopoly is destroyed if another lab invents a higher quality of the same intermediate good. Hence the innovation process is associated with creative destruction.

The labs hire research labor to invent new intermediate good qualities. Let the amounts of research labor developing higher qualities of intermediate good $i \in [0, 1]$ and $h \in [0, 1]$ be denoted n_{it}^Y and n_{ht}^Z , respectively. Innovations arrive randomly following Poisson processes, and the associated Poisson arrival rates are λn_{it}^Y and ηn_{ht}^Z , respectively.

Whenever a lab successfully invents a higher intermediate good quality, the technological level of that quality is given by the leading-edge technology of its technology type: $\bar{A}_t^Y \equiv \max_{i \in [0,1]} (A_{it}^Y)$ or $\bar{A}_t^Z \equiv \max_{h \in [0,1]} (A_{ht}^Z)$. New inventions increase the leading-edge technology through positive knowledge spillover effects. As shown by Aghion and Howitt (1998, ch. 3), the leading-edge technology is proportional to the average technological level. Thus the technological development can be expressed in terms of the average technological levels which evolve according to (details available in Appendix F):

$$\dot{A}_t^Y = A_t^Y \lambda \gamma n_t^Y, \quad A_0^Y > 0 \text{ given}, \quad \lambda > 0, \quad \gamma > 0 \quad \text{and} \quad (19)$$

$$\dot{A}_t^Z = A_t^Z \eta \xi n_t^Z, \quad A_0^Z > 0 \text{ given}, \quad \eta > 0, \quad \xi > 0, \quad (20)$$

where $n_t^Y \equiv \int_0^1 n_{it}^Y di$, $n_t^Z \equiv \int_0^1 n_{ht}^Z dh$, the dots denote derivatives with respect to time, and γA_t^Y and ξA_t^Z capture the sizes of the knowledge spillover effects.¹³

There is free entry, and all research firms take the wage rate as given. As the probability per time unit of inventing a new intermediate good quality is proportional to the labor input, firms enter the research sector until the following research arbitrage conditions are fulfilled:

$$w_t \geq \lambda \bar{V}_{it}^Y \quad \text{for} \quad n_{it}^Y \geq 0 \quad \text{and} \quad w_t = \lambda \bar{V}_{it}^Y \quad \text{if} \quad n_{it}^Y > 0; \quad \text{and} \quad (21)$$

$$w_t \geq \eta \bar{V}_{ht}^Z \quad \text{for} \quad n_{ht}^Z \geq 0 \quad \text{and} \quad w_t = \eta \bar{V}_{ht}^Z \quad \text{if} \quad n_{ht}^Z > 0; \quad (22)$$

where \bar{V}_{it}^Y and \bar{V}_{ht}^Z denote the values of inventing new qualities of intermediate good $i \in [0, 1]$ and $h \in [0, 1]$, respectively. The research arbitrage conditions ensure that in equilibrium, the cost of having a researcher working for one unit of time equals the expected value of his/her work.

The value of a patent is the net present value of the expected future profit stream associated with the monopoly on that intermediate good quality:

$$V_{it}^Y = \int_t^\infty \tilde{\pi}_{is}^Y e^{-\int_t^s (r_u + n_{iu}^Y \lambda) du} ds \quad \text{and} \quad V_{ht}^Z = \int_t^\infty \tilde{\pi}_{hs}^Z e^{-\int_t^s (r_u + n_{hu}^Z \eta) du} ds, \quad (23)$$

¹³The knife-edge assumptions in (19) and (20) imply that the model belongs to the fully endogenous variety (see Chapter 1). Given the results obtained in Chapter 1, the long-run growth effects should be viewed as a first approximation for a limited time period. Using a fully endogenous growth model reduces the dynamic complexity considerably. A semi-endogenous growth model would yield similar results for the dynamic motion over several decades as long as the knowledge spillover effects are strong. This seems like the empirically plausible case given the results from Chapter 1. Additionally, the strong scale effect highlighted by Jones (1995a, 2005) can be eliminated following the modeling strategy of Aghion and Howitt (1998, p. 407-415) and Howitt (1999) without changing the main results.

where r_t is the (risk-free) real interest rate. The instantaneous profit streams are discounted by modified interest rates which reflect the opportunity cost of investing in research firms (the real interest rate) and the risk of losing the monopoly position (the Poisson intensities).

As new intermediate good qualities of a certain type invented at the same point in time are associated with the same technological level (\bar{A}_t^Y or \bar{A}_t^Z), the profit streams associated with monopolies on any of these qualities are the same. It then follows from (21), (22), and (23) that the same amount of research is conducted on each intermediate good, given the type. Hence, $n_t^Y \equiv \int_0^1 n_{it}^Y di = n_{it}^Y$ and $n_t^Z \equiv \int_0^1 n_{ht}^Z dh = n_{ht}^Z$ in equilibrium.

3.3 Equilibrium

The equilibrium labor allocation between manufacturing and research essentially reflects consumption and saving decisions in the economy. Consumer saving finances research firms. These investments increase the financial wealth of the consumers. Due to idiosyncratic risk for research firms, consumers are able to diversify risk away and obtain the risk free real interest rate on their savings.

A representative household solves the problem:

$$\begin{aligned} & \max_{(C_t)_{t=0}^{\infty}} \int_0^{\infty} (\ln(C_t) - \kappa E(P_t)) e^{-\rho t} dt, \quad E'(P_t) > 0, \quad \rho > 0, \quad \kappa \geq 0, \\ & \text{st. } \dot{\tilde{W}}_t = w_t \bar{L} + r_t \tilde{W}_t + I_t - C_t, \quad \tilde{W}_0 > 0 \text{ given, } \lim_{t \rightarrow \infty} \tilde{W}_t e^{-\int_0^t r_s ds} \geq 0, \quad C_t \geq 0, \end{aligned}$$

where \tilde{W}_t is financial wealth, $\bar{L} > 0$ is aggregate labor supply, and I_t is a lump-sum government transfer. The government keeps a balanced budget by transferring the entire tax revenue to the representative household at all moments in time: $I_t = \phi Y_t$. The financial wealth of the representative household is the net present value of all existing intermediate producers: $\tilde{W}_t = \int_0^1 V_{it}^Y di + \int_0^1 V_{ht}^Z dh$.

Optimal behavior requires:

$$\lim_{t \rightarrow \infty} \tilde{W}_t e^{-\int_0^t r_s ds} = 0 \quad \text{and} \quad g_{Y,t} = g_{C,t} \equiv \frac{\dot{C}_t}{C_t} = r_t - \rho, \quad (24)$$

where $g_{X,t}$ denotes the growth rate of a variable X_t .

To close the model, the following labor market clearing condition is imposed:

$$\bar{L} = L_t^Y + L_t^Z + n_t^Y + n_t^Z. \quad (25)$$

Define a balanced growth path as a path where the variables C_t , Y_t , Y_{jt} , A_t^Y , w_t , A_t^Z , P_t , and P_{jt} all grow at constant rates, while N_t , L_t^Y , L_t^Z , n_t^Y , and n_t^Z are constant. According to the following proposition, the economy is always on a balanced growth path.

Proposition 1. *Assuming that the parameters are such that $L_0 > 0$, $n_0^Y > 0$, and $n_0^Z > 0$, then the economy is always on a balanced growth path, where the variables N_t , L_t^Y , L_t^Z , n_t^Y , and n_t^Z are constant over time; C_t , Y_t , Y_{jt} , A_t^Y , and w_t all grow at the constant rate $g_{AY} \equiv \dot{A}_t^Y/A_t^Y = n^Y \lambda \gamma$; A_t^Z grows at the constant rate $g_{AZ} \equiv \dot{A}_t^Z/A_t^Z = n^Z \eta \xi$; and P_t and P_{jt} grow at the constant rate $g_P \equiv \dot{P}_t/P_t = \beta n^Y \lambda \gamma - \chi n^Z \eta \xi$.*

Proof. See Appendix G. □

Since the labor allocation is time invariant according to Proposition 1, the model is solved under this assumption for expositional simplicity. The proof of Proposition 1 in Appendix G shows how to solve the model without assuming a time invariant labor allocation.

From the research arbitrage conditions (21) and (22), as well as (10), (13), and (17), it follows that (see Appendix H):

$$n^Y = \frac{(1-\alpha)(1-\phi-\chi\mu\phi)}{\Omega} L - \frac{\rho}{\lambda(1+\gamma)} \quad \text{and} \quad n^Z = \frac{(1-\mu)\chi\mu\phi}{\Omega} L - \frac{\rho}{\eta(1+\xi)}.$$

The labor market allocation is determined from these two expressions and the labor market clearing condition (25):

$$L = \frac{\alpha(1-\phi-\mu\chi\phi) + \chi\mu^2\phi}{1-\phi} \Gamma, \quad \Gamma \equiv \bar{L} + \frac{\rho}{\lambda(1+\gamma)} + \frac{\rho}{\eta(1+\xi)}, \quad (26)$$

$$n^Y = \frac{(1-\alpha)(1-\phi-\chi\mu\phi)}{1-\phi} \Gamma - \frac{\rho}{\lambda(1+\gamma)}, \quad \text{and} \quad (27)$$

$$n^Z = \frac{(1-\mu)\chi\mu\phi}{1-\phi} \Gamma - \frac{\rho}{\eta(1+\xi)}. \quad (28)$$

The parameter conditions ensuring $n^Y > 0$ and $n^Z > 0$ follow directly from (27) and (28).

4 Policy Implications

To study how a tightening of the environmental policy affects economic growth, an unexpected increase in the environmental tax revenue parameter, ϕ , is analyzed. In reality, the government possesses many other environmental policy instruments besides environmental

taxes. For the mechanisms described in this model, what matters is that pollution emission becomes more expensive, when the environmental policy is tightened.

Consider the following proposition.

Proposition 2. *If $L > 0$, $n^Y > 0$, and $n^Z > 0$, then:*

(i) $\frac{dL^Y}{d\phi} < 0$ and $\frac{dL^Z}{d\phi} > 0$; (ii) $\frac{dn^Y}{d\phi} < 0$ and $\frac{dn^Z}{d\phi} > 0$; and (iii) $\frac{dL}{d\phi} \geq 0$ for $\alpha \leq \mu$.

Proof. See Appendix I. □

The proposition highlights three labor allocation effects of a tightening of the environmental policy. The first effect is a *production direction effect*: reallocation of labor used for manufacturing. A tightening of the environmental policy increases the price of pollution emission. This causes the demand for abatement intermediates to increase while the demand for production intermediates decreases. To ensure that supply equals demand, manufacturing labor must be reallocated from production to pollution abatement.

The second effect is a *research direction effect*: reallocation of labor used for research. As the demand for abatement intermediates increases, so does the value of a patent on a new abatement technology relative to the cost of conducting research (the wage rate). The opposite holds for new production technologies. Labor is reallocated from research in production technologies to research in abatement technologies due to research arbitrage. This reallocation of labor affects the value of patents through the expected duration of a monopoly position.

The final effect is a *labor force allocation effect*: reallocation of labor between manufacturing and research. Labor is reallocated from manufacturing to research and vice versa, depending on the relative sizes of α and μ . If $\alpha < \mu$ the total profit in the intermediate goods sector decreases, when the environmental policy is tightened, given the labor allocation between manufacturing and research. The reason is that intermediate goods with the average price $(1/\alpha)w_t$ are substituted for intermediate goods with the relatively lower average price $(1/\mu)w_t$. As total profit in the intermediate goods sector is reduced, the incentive to conduct research is diminished. Labor then shifts from research to manufacturing until the research arbitrage conditions are fulfilled.

The labor allocation effects described above govern the effects on output growth and pollution emission growth. The following proposition follows almost immediately from Proposition 2:

Proposition 3. *If $L > 0$, $n^Y > 0$, and $n^Z > 0$, then: $\frac{dg_Y}{d\phi} < 0$ and $\frac{dg_P}{d\phi} < 0$.*

According to Proposition 3, a tightening of the environmental policy unambiguously decreases the growth rates of output and pollution emission.

A natural question is then: why can the potential inflow of labor from manufacturing to research never be strong enough to increase the growth rate of output? When a tightening of the environmental policy causes labor to flow from manufacturing to research, the reason is that profits in the intermediate goods sector have increased. However, this overall increase is entirely driven by the abatement intermediate producers, as profits for production intermediate producers decrease unambiguously due to the production direction effect. Hence the labor inflow from manufacturing to research is directed entirely to research in abatement technologies. In contrast, labor allocated to research in production technologies is reduced unambiguously.

As in other models featuring directed technical change, the relative profit for producers of production and pollution abatement technologies is determined by a *price effect* and a *market size effect*. The relative profit between production and abatement intermediate producers is:

$$\frac{\pi_t^Y}{\pi_t^Z} = \frac{\int_0^1 \pi_{it}^Y di}{\int_0^1 \pi_{ht}^Z dh} = \underbrace{\left(\frac{1/\alpha - 1}{1/\mu - 1} \right)}_{\text{Price effect (equilibrium)}} \times \underbrace{\left(\frac{\alpha(1 - \phi - \chi\mu\phi)}{\chi\mu^2\phi} \right)}_{\text{Market size effect (equilibrium)}}.$$

The size of the *price effect* is determined by the relative size of the two markups. Hence, the price effect is determined by technology and market power. Both are unaffected by environmental policy. Thus, the relative profit is only directly affected by environmental policy through the *market size effect* which is simply a different formulation of the production direction effect. The price effect still affects how strong the relative profit reacts to changes in the environmental policy, as the markups ($1/\alpha$ and $1/\mu$) determine the strength of the labor force allocation effect.

Meanwhile, the research direction effect ensures that both production and pollution abatement technologies are developed simultaneously. If, for instance, the value of a patent on a new abatement technology becomes larger, more labor is allocated to research in abatement technologies. As a result, the expected duration of a monopoly position associated with a new abatement technology is reduced. This ensures that in equilibrium, it is equally attractive to conduct research in both types of technology. This effect is eliminated in models with one-period patents, and it might cause research to permanently focus only on one type of technology.

5 Stylized facts revisited

Motivated by Stylized Fact 2 and 7, the model focuses on end-of-pipe technologies in the business sector, while Stylized Fact 5 motivates the policy function (5). Now consider the predicted evolutions in pollution emission and intensity. Define the pollution intensity as: $\mathcal{P}_t \equiv P_t/Y_t$.¹⁴ It follows directly from Proposition 1 that the growth rates of income, pollution emission, and pollution intensity are given by: $\dot{Y}_t/Y_t = n^Y \lambda \gamma$, $\dot{P}_t/P_t = \beta n^Y \lambda \gamma - \chi n^Z \eta \xi$, and $\dot{\mathcal{P}}_t/\mathcal{P}_t = (\beta - 1)n^Y \lambda \gamma - \chi n^Z \eta \xi$. The model allows for a decreasing pollution intensity, while income increases. This matches Stylized Fact 1 and 3. Meanwhile, pollution emission might increase and decrease with income even when the pollution intensity decreases with income. It depends on the environmental tax revenue parameter, ϕ , which affects n^Y positively and n^Z negatively. According to the evidence presented in Figure 1, pollution emission increased until 1970 and then decreased for most pollutants. For the model to match this pattern, an increase in ϕ around 1970 is needed. That is, according to the model, U.S. environmental policy was tightened around 1970 which fits well with the implementation of the Clean Air Act of 1970.

Next, the environmental policy stringency measured by the EPS index corresponds to some monotone transformation of the pollution tax rate, τ_t . Consider the government's tax rule expressed in terms of the pollution intensity: $\phi = \mathcal{P}_t \tau_t$. As the pollution intensity decreases, the pollution tax rate must increase. This matches both Stylized Fact 3 and 4: the EPS index is negatively correlated with the pollution intensity and the EPS index increases over time.

Finally, aggregate and business sector pollution abatement expenditures are the total expenditures on pollution abatement for the final goods sector, as this is the only sector that pollutes. The pollution abatement expenditures are given by the cost of purchasing abatement intermediates for all final good firms: $N_t \int_0^1 \hat{x}_t^Z p_{ht}^Z dh = \chi \mu \phi Y_t$. Hence, aggregate and business sector pollution abatement expenditures are a constant share of output which fits well with the data presented in Figure 5. Also, the model matches Stylized Fact 6 and 7, as aggregate and business sector pollution abatement expenditures can be a constant share of output, while pollution emission decreases (if ϕ is sufficiently large).¹⁵

¹⁴As the output-GDP ratio remains constant over time, the stylized facts can be evaluated using output (see Appendix J).

¹⁵Abatement research expenditures are not part of the aggregate pollution abatement expenditures (Vogan 1996).

6 Model Extensions

The assumption of logarithmic preferences in the instantaneous utility function might be considered problematic, as extensive empirical evidence suggests that the degree of relative risk aversion is above one (Attanasio and Weber 1993; Okubo 2011; Havránek 2015). To relax this assumption, consider the model from Section 3 with the instantaneous utility function

$$u(C_t, P_t) = \frac{C_t^{1-\theta}}{1-\theta} - \kappa E(P_t), \quad \theta > 1, \quad \kappa \geq 0, \quad E'(P_t) > 0, \quad (29)$$

where θ is the degree of relative risk aversion. Most of the derivations from Section 3 are still valid. The real interest rate is now given by: $r = \theta g_C + \rho$. In Appendix K, it is shown that the environmental policy implications with respect to the economic growth rate as well as the growth rate of pollution emission are unchanged by this extension.

Furthermore, the absence of physical capital might seem unappealing, as accumulation of physical capital usually plays a major role in economic growth models. In Appendix L, a similar model with physical capital is sketched. In this model, labor becomes an input in the final goods sector, and it is assumed that final good firms operate under monopolistic competition a la Dixit and Stiglitz (1977).¹⁶ Also, intermediate goods are produced using physical capital instead of labor. The long-run economic growth implications are unchanged.

7 Simulations

This section investigates the quantitative environmental policy implications of the model. It is shown that even large changes in the environmental policy stringency have relative small effects on the economic growth rate. Additionally, it is shown that these relatively small economic growth effects can have large welfare effects as even small changes in growth rates have large level effects in the long run.

The simulations are based on the model with CRRA utility from Section 6. Section 7.1 estimates the model using U.S. data. The quantitative effects of a tighter environmental policy are investigated in Section 7.2. Finally, Section 7.3 provides a number of sensitivity checks.

¹⁶The monopolistic element is necessary to ensure nonnegative profits for final good firms.

7.1 Estimation

The estimation procedure is divided into two steps. First, parameters present in other growth models are assigned values from previous papers: $\alpha = 0.4$ and $\lambda = 0.5$ (from Ricci 2007a), $\rho = 0.02$ (from Groth and Ricci 2011), and $\theta = 1.5$ (consistent with Havránek 2015). Assuming that it is equally difficult to invent new production and abatement intermediate good qualities: $\eta = \lambda = 0.5$. Total labor supply is normalized to one and the parameter F (which only has a strong impact on the variable N) is set to 0.1. The parameter μ is set to 0.4 such that the markups in the intermediate goods sector are the same. Since $\tilde{\beta} < \mu\tilde{\chi}$, there must be a substantial difference between $\tilde{\beta}$ and $\tilde{\chi}$. Thus $\tilde{\beta}$ is set to 0.2 and $\tilde{\chi}$ is set to 0.8. The parameter ψ is set to 0.9 to ensure effective pollution abatement.

Second, the model is estimated using three estimation targets. The first target is the U.S. aggregate pollution abatement expenditures which constituted about 1.61 pct. of GDP in 1994. In terms of the model, these expenditures equals $\chi\mu\phi$ times the output-GDP ratio.¹⁷ The last two targets are the average U.S. growth rates of real GDP and pollution emission over the period 1970-2012. The former is computed directly from national accounts data and equals 2.85 pct. per year. Unfortunately, it is difficult to operationalize the pollution emission variable, as it is unclear how to construct a single pollution emission measure. As in Section 2, the focus is on air pollution. A pollution emission index is constructed by indexing the pollution emissions of CO, NO_x, SO₂, and VOC to 100 in 1970 and taking the unweighted average of these indexes. Computing the pollution emission growth rate from this pollution emission index yields -2.21 pct. per year.¹⁸

The model is estimated to match these three estimation targets by minimizing the sum of squared deviations between predicted and actual target values with respect to γ , ξ , and ϕ . The estimated environmental tax revenue parameter, ϕ , equals 5.86 pct. This measure quantifies the technology-adjusted U.S. environmental policy stringency. The estimated value γ (≈ 0.1) is much lower than the estimated value ξ (≈ 19). This difference is intuitive. Since ϕ is relatively small, the market for abatement technologies is also relatively small. The model therefore predicts that only a small fraction of the researchers (about 1.4 pct.) develop new pollution abatement technologies. But since pollution emission growth has been

¹⁷GDP is derived in Appendix J.

¹⁸The index is computed using data obtained only from the EPA website. The index could also be computed using shadow prices as relative weights. However, the results are not sensitive to changes in the pollution growth rate used to calibrate the model. As shown in Section 7.3, putting all weight on either the fastest (VOC) or the slowest (SO₂) growing pollutant does not change the main conclusions of the exercise, indicating that the main conclusions can be obtained using any weighted average of the five growth rates.

negative while GDP growth has been positive, pollution abatement research must have been very effective. And since the arrival rate parameters are assumed equal, the effectiveness must come through the spillover parameter ξ . This result aligns well with recent empirical evidence from patent citation data which suggests that knowledge spillovers from environmentally friendly technologies are large compared to other technologies (Dechezleprêtre et al. 2014).

7.2 Simulation results

Growth effects

Figure 6 depicts changes in the growth rates of output and pollution emission as functions of the environmental tax revenue parameter, ϕ . The simulations are conducted for different μ values, as the qualitative policy analysis highlighted the importance of this parameter.

The left panel of Figure 6 shows that the output growth rate is only weakly affected by the environmental tax revenue parameter. Consider a tax reform which increases the revenue from the pollution tax by one percentage point (pp) of output (an increase in ϕ of one pp): a large environmental tax reform. This reform would decrease the growth rate of output by around 0.01 pp for all three simulations. Meanwhile, this reform would have a huge effect on the pollution emission growth rate which would be reduced by between 1.5 and 2 pp.

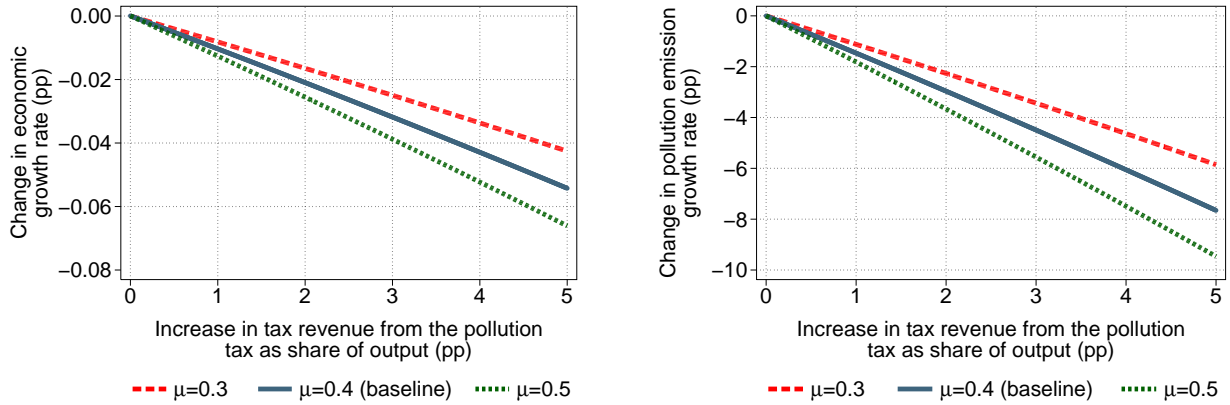


FIGURE 6: Changes in the growth rates of output and pollution emission as functions of increases in the tax revenue from the pollution tax as share of output.

The intuition is intimately linked to the market size for pollution abatement technologies. As the model matches the U.S. pollution abatement expenditures as share of GDP - which is a small number - the market for pollution abatement technologies is relatively small. This implies a relatively low estimate of ϕ . Thus small increases in ϕ have relatively large effects on the market size for pollution abatement technologies, while the effect on the market

size for production technologies is relatively small. This leads to a large relative change in the research input for pollution abatement technologies and a small relative change in the research input for production technologies. Consequently, the pollution emission growth rate reacts much more on environmental policy changes compared to the economic growth rate.

Additionally, since U.S. pollution emission has decreased notably during the last plus 40 years, pollution abatement research must have had a strong impact on pollution emission despite the relatively small input. Consequently, ξ becomes relatively large when estimating the model, implying effective pollution abatement research. The opposite is true for research in production technologies which results in a relatively low value of γ . As a consequence, the economic growth rate reacts relatively less on marginal changes in research efforts compared to the pollution emission growth rate.

Level effects

Another issue is the level effects of environmental policy changes, i.e. the effect given the technological level. Figure 7 shows changes in central variables as functions of the environmental tax revenue parameter given the technological level $(A_t^Y, A_t^Z) = (1, 1)$. The simulation results indicate that a tighter environmental policy: (i) reduces profits for production intermediate producers and increases profits for abatement intermediate producers, (ii) reduces output and the wage rate, and (iii) reduces pollution emission and intensity. All effects are amplified by increases in μ which follows the intuition from Section 4.

Note that profits for abatement intermediate producers are strongly affected by environmental policy changes. Intuitively, the market for pollution abatement technologies only exists due to the pollution tax. Since the environmental tax revenue parameter is relatively small, even small changes in this parameter have relatively large effects on the market size for pollution abatement technologies. And since profits for abatement intermediate producers are proportional to the market size for pollution abatement technologies, these profits react strongly on environmental policy changes. Profits for production intermediate producers react much less on environmental policy changes. The reason is that the market size for abatement technologies remains relatively small, and thus environmental policy changes have a relatively small impact on the overall labor allocation in manufacturing.

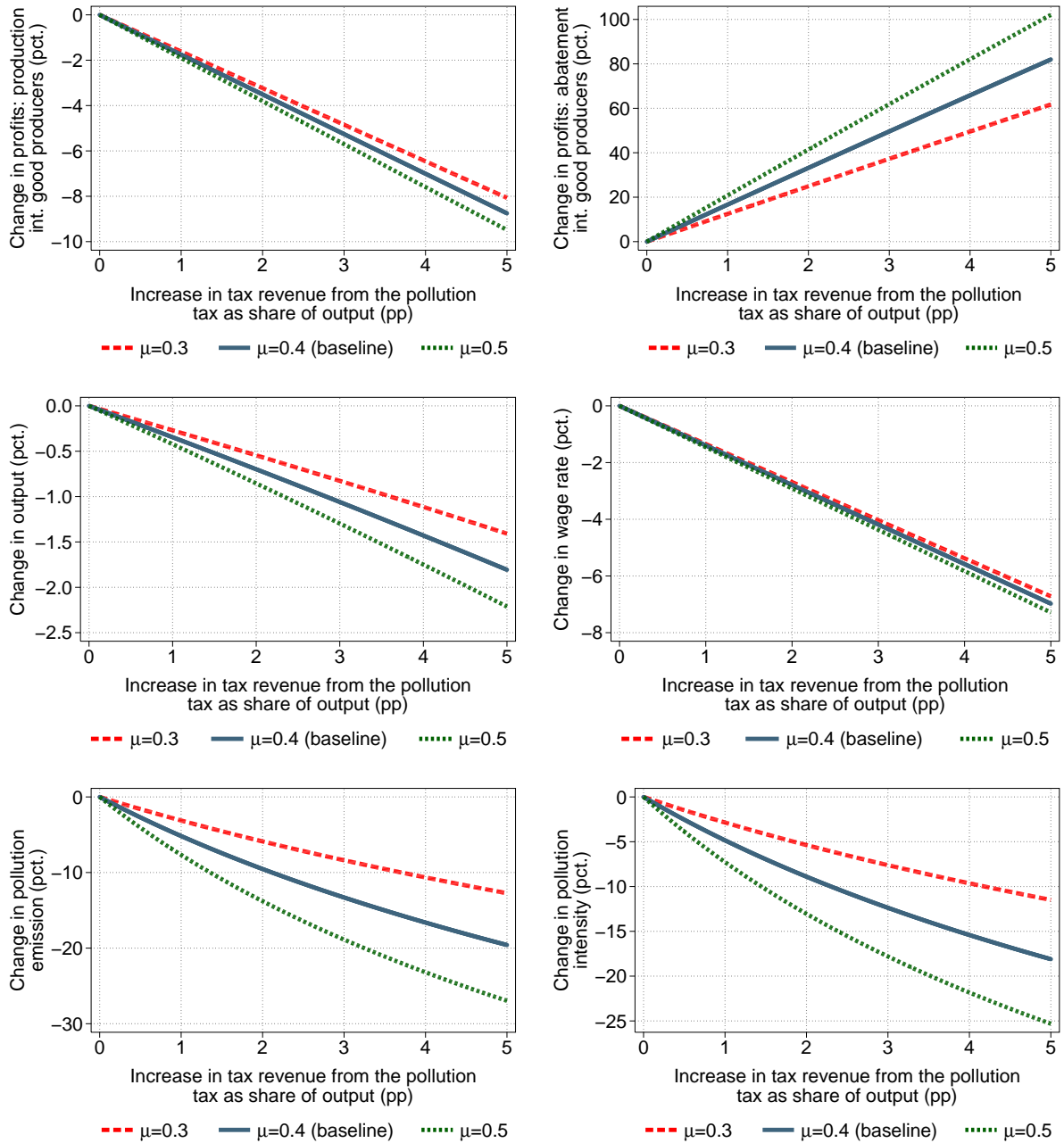


FIGURE 7: Static relative changes in central variables as functions of increases in the tax revenue from the pollution tax as share of output.

Welfare effects

Despite the numerically small economic growth effects of environmental policies, the welfare implications might be substantial as even small changes in a growth rate have large level effects in the long run. The left panel of Figure 8 shows the output level loss 50 years after an environmental policy change for the baseline estimation. About 58 pct. of the output level loss is caused by the economic growth effect.

Another way to evaluate the welfare cost of environmental policy reforms is to consider how much the current consumption level must change to keep utility constant. This figure is

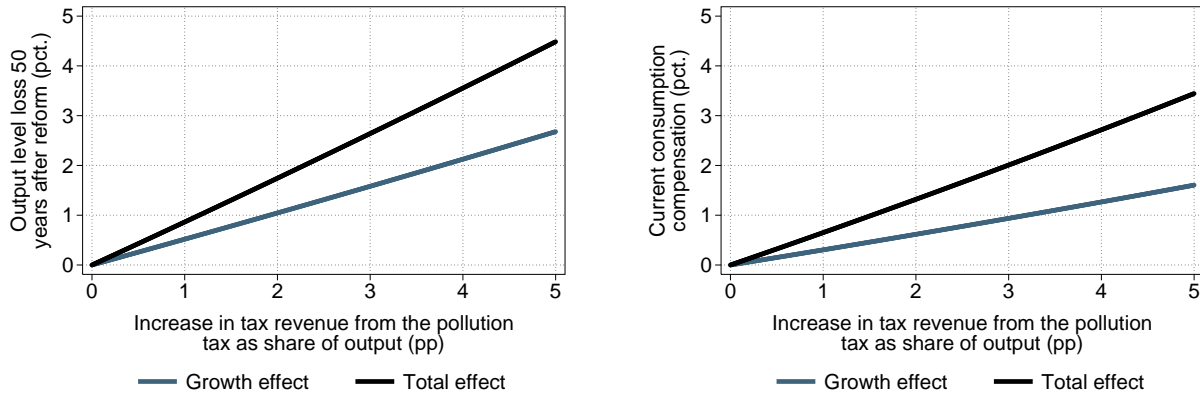


FIGURE 8: Output level loss 50 years after environmental tax reform and current consumption compensation as functions of increases in the tax revenue from the pollution tax as share of output. *Notes:* The current consumption compensation measures how much the current consumption level must increase to keep utility constant.

referred to as the current consumption compensation (CCC). For this welfare measure, the pollution element in the instantaneous utility function matters. To avoid further assumptions on the functional form of the instantaneous utility function, it is assumed that $\kappa = 0$. Accordingly, the computed CCC should be viewed as an upper bound. Details on how the CCC is computed are reported in Appendix M.

The right panel of Figure 8 shows the CCC divided into a growth effect and a total effect. The welfare loss stemming from the growth effect accounts for about 46 pct. of the total CCC. The relative impact of the growth effect is larger for the output level loss measure compared to the CCC measure. At first glance, this seems counter intuitive as the former is computed based on a finite horizon, while the latter is computed based on an infinite horizon. The reason is that due to discounting, consumption closer in time to the implementation of the reform gains relatively more weight. Thus the initial consumption level reduction caused by an environmental tax reform is relatively more important for the CCC measure.

These computations indicate that static models and exogenous growth models leave out a substantial part of the welfare effects of environmental policy reforms. The growth effect accounts for more than 50 pct. of the output level loss 50 years later, while it accounts for about 46 pct. of the welfare loss measured by the CCC. Thus, despite the small effect on the economic growth rate, it is important to take the growth effects into account when evaluating the welfare effects of environmental policy reforms. The relatively large welfare effects from seemingly small reductions in the economic growth rate are intuitive. When the economic growth rate is reduced, consumption is reduced at all future dates. Despite consumption discounting, these consumption losses have a relatively large present value.

7.3 Sensitivity analysis

It is natural to wonder how sensitive the obtained results are to changes in the baseline parameter values and estimation targets. Table 2 shows reductions in the economic growth rate from a one percentage point increase in the environmental tax revenue parameter for simulations, where one parameter assumption or estimation target deviates from its baseline value. Note that the freedom to change parameter assumptions are limited by the parameter restrictions mentioned above.

TABLE 2: Economic growth rate reductions associated with a one percentage point increase in the tax revenue from the pollution tax as share of output, ϕ , for single parameter or estimation target changes relative to baseline

Deviating value	α	$\tilde{\beta}$	$\tilde{\chi}$	ψ	$\lambda = \eta$	μ	θ	ρ	F	g_P	PAE
New value	0.20	0.10	0.60	0.60	0.25	0.30	1.00	0.10	0.01	-4.14%	0.805%
Growth reduction (pp)	0.010	0.010	0.008	0.007	0.011	0.008	0.010	0.010	0.010	0.010	0.010
Change from baseline (pp)	0.000	0.000	-0.002	-0.003	0.001	-0.002	0.000	0.000	0.000	0.000	-0.001
Deviating value	α	$\tilde{\beta}$	$\tilde{\chi}$	ψ	$\lambda = \eta$	μ	θ	ρ	F	g_P	PAE
New value	0.60	0.30	0.90	0.99	0.75	0.60	2.50	0.03	1.00	-1.55%	4.00%
Growth reduction (pp)	0.011	0.010	0.012	0.011	0.010	0.015	0.011	0.011	0.010	0.010	0.013
Change from baseline (pp)	0.000	0.000	0.001	0.001	0.000	0.005	0.001	0.000	0.000	0.000	0.003

Notes: The model is first estimated using the procedure described in Section 7.1. The tax revenue parameter is then increased by 1 pp. The baseline parameter assumptions are: $\alpha = 0.4$, $\tilde{\beta} = 0.2$, $\tilde{\chi} = 0.8$, $\psi = 0.9$, $\lambda = \eta = 0.5$, $\mu = 0.4$, $\theta = 1.5$, $\rho = 0.02$, and $F = 0.1$. The baseline growth targets are: $g_Y = 2.85\%$ and $g_P = -2.21\%$. The baseline pollution abatement expenditure (PAE) target is 1.61%.

The results shown in Table 2 indicate that the main conclusions from Section 7.2 are robust to a variety of individual parameter and estimation target changes. None of the simulations predict large growth rate reductions, and the growth rate reductions are close to that of the baseline simulation.

8 Discussion

It is somewhat surprising that the economic growth effects of environmental policy changes are small in the simulation exercises, as several model assumptions amplify these effects. Starting with technology, there are three assumptions worth emphasizing. First, pollution has no negative effects on production.¹⁹ As a result, environmental policy cannot boost productivity through a reduction in pollution emission. Second, abatement technologies have

¹⁹Pollution emission can, for example, reduce production through its effect on labor supply (due to short-term effects on health, see Hanna and Olival [2015]) and labor productivity (Zivin and Neidell 2012).

no productivity-enhancing effects, as they can only reduce pollution emission. In reality, they might improve productivity through a decrease in resource use (see Porter and van der Linde 1995). Third, knowledge spillover effects between production and abatement technologies are ignored. Introducing such spillover effects would reduce the effect on economic growth from changes in the labor allocation within the R&D sector.

Two assumptions related to allocation mechanisms also amplify the economic growth effects of environmental policy changes. First, the tax revenue from the pollution tax is transferred to the representative household. Alternatively, this tax revenue could finance subsidies to productivity-enhancing research activities and thereby reduce the negative economic growth effect of a tighter environmental policy. Second, the representative household takes pollution emission as given and has additive preferences concerning consumption and pollution emission. If, instead, consumption and a clean environment are compliments, environmental policies reducing pollution emission increase the savings rate and thus the economic growth rate (see Mohtadi 1996).

Given these considerations, it appears puzzling that economic growth effects of large environmental policy reforms are almost absent in the simulation exercises. Intuitively, the results can be explained the following way. As U.S. pollution abatement expenditures only constitute a small fraction of GDP, the market for pollution abatement technologies is small. Hence, environmental policy reforms that substantially increase the market size for pollution abatement technologies, only reallocates a small fraction of the economy's total resources. Consequently, even large environmental policy changes have a small impact on productivity-enhancing research.

In the light of this analysis, several directions for future research appear fruitful. The simulation exercise conducted above illustrates how a technology-adjusted environmental policy stringency measure can be estimated. Since the measure takes the technological level of a country into account, it makes the environmental policy stringency between countries more comparable which might be useful to policy makers. Future studies might estimate technology-adjusted environmental policy stringency measures across countries based on the framework developed above. Additionally, the model developed in the present study might be useful when studying how pollution emissions interact with economic growth in developing countries. Pollution levels in developing countries are often high (WHO 2006), and it is therefore likely that a tighter environmental policy could boost economic growth in these countries through positive health effects.

9 Appendix

A Positive abatement effort and environmental policy stringency

In this appendix, it is shown that if the environmental policy is sufficiently tight, firms do not have an incentive to deviate from a symmetric equilibrium with positive abatement efforts.

It is shown in Appendix B that if firms abate pollution emission, all firms behave similarly. Thus if a firm deviates from a symmetric equilibrium with positive abatement, that firm would not conduct any pollution abatement. To show that no firms will deviate if the environmental policy is tight enough, profits for the deviating firm is compared to that of a non-deviating firm which is zero.

Let the deviating firm be indexed \bar{j} . The deviating firm has an infinitesimally small effect on equilibrium prices and quantities, and thus, these are computed as in Section 3.

The deviating firm solves the problem:

$$\max_{(x_{i\bar{j}t}^Y)_{i \in [0,1]}} (1 - \tau_t)Y_{\bar{j}t} - \int_0^1 x_{i\bar{j}t}^Y p_{it}^Y di.$$

The first-order conditions imply:

$$p_{it}^Y = (1 - \tau_t)\alpha (x_{i\bar{j}t}^Y)^{\alpha-1} A_{it}^Y.$$

Since all other firms abate pollution emission: $p_{it}^Y = (1/\alpha)w_t(A_{it}^Y/A_t^Y)$. The demand for production intermediates for the deviating firm is therefore:

$$x_{i\bar{j}t}^Y = \left(\frac{(1 - \tau_t)\alpha^2 A_t^Y}{w_t} \right)^{\frac{1}{1-\alpha}} \equiv \hat{x}_{\bar{j}t}^Y.$$

It follows that:

$$\hat{x}_{\bar{j}t}^Y = \frac{(1 - \tau_t)\alpha^2}{w_t} (Y_{\bar{j}t} + F A_t^Y).$$

Profits are therefore given by:

$$\pi_{\bar{j}t}^f = (1 - \tau_t) \left[(1 - \alpha)Y_{\bar{j}t} - \alpha F A_t^Y \right].$$

It follows that

$$\pi_{jt}^f < 0 \quad \Leftrightarrow \quad (1 - \alpha)Y_{jt} > \alpha F A_t^Y.$$

Substituting in the expression for \hat{x}_{jt}^Y :

$$\tau_t > 1 - \left(\frac{1}{\alpha^2}\right) \left(\frac{w_t}{A_t^Y}\right) \left(\frac{F}{1 - \alpha}\right)^{\frac{1-\alpha}{\alpha}}.$$

The inequality is rewritten as

$$\tau_t > 1 - (1 - \phi\beta) \left(\frac{(1 - \alpha)(1 - \alpha(1 - \phi\beta) - \chi\mu\phi - \phi)}{1 - \chi\mu\phi - \phi} \right)^{\frac{1-\alpha}{\alpha}}, \quad (30)$$

where it is used that

$$w_t = \alpha^2(1 - \beta\phi) \left(\frac{1 - \alpha(1 - \phi\beta) - \chi\mu\phi - \phi}{(1 - \chi\mu\phi - \phi)F} \right)^{\frac{1-\alpha}{\alpha}} A_t^Y.$$

The tax rate is rewritten as

$$\begin{aligned} \tau_t &= \phi \frac{Y_t}{P_t} = \phi \frac{\hat{Y}_t}{\hat{P}_t} \\ &= \phi \left(\frac{\chi\mu^2\phi L}{\Omega} \right)^{\tilde{\chi}\mu\psi} \left(\frac{\alpha(1 - \beta\phi)F}{1 - \alpha(1 - \beta\phi) - \chi\mu\phi - \phi} \right)^{-\tilde{\beta}\psi} (A_t^Y)^{-\tilde{\beta}\psi} (A_t^Z)^{\tilde{\chi}\psi}, \end{aligned}$$

where L is given by (26). Thus if (30) holds, firms do not have an incentive to deviate. In the empirically relevant case, where parameters are such that the pollution intensity decreases over time, τ_t increases over time. In that case, the inequality holds at all points in time if it holds initially - which is assumed to be the case.

B Symmetric equilibrium

The problem for monopolist i in the intermediate goods sector is:

$$\max_{x_{ijt}^Y} \int_0^{N_t} p_{it}^Y x_{ijt}^Y - w_t \left(\frac{A_{it}^Y}{A_t^Y} \right) x_{ijt}^Y dj.$$

Substituting in the inverse demand function:

$$\max_{x_{ijt}^Y} \int_0^{N_t} \alpha (x_{ijt}^Y)^\alpha A_{it}^Y \left(1 - \beta \tau_t \frac{P_{jt}}{Y_{jt}} \right) - w_t \left(\frac{A_{it}^Y}{A_t^Y} \right) x_{ijt}^Y \, dj.$$

The first-order condition for monopolist i is rewritten as

$$x_{ijt}^Y = \left(\frac{\alpha^2 \left(1 - \beta \tau_t \frac{P_{jt}}{Y_{jt}} \right) A_t^Y}{w_t} \right)^{\frac{1}{1-\alpha}} \equiv \hat{x}_{jt}^Y.$$

Thus all monopolists supply the same amount to a given final good producer. The implied price of intermediate good i is: $p_{it}^Y = (1/\alpha)w_t(A_{it}^Y/A_t^Y)$.

It follows that

$$Y_{jt} + F A_t^Y = \left(\frac{\alpha^2 \left(1 - \beta \tau_t \frac{P_{jt}}{Y_{jt}} \right) A_t^Y}{w_t} \right)^{-1} \hat{x}_{jt}^Y A_t^Y.$$

The problem for monopolist h in the intermediate goods sector is:

$$\max_{x_{hjt}^Z} \int_0^{N_t} p_{ht}^Z x_{hjt}^Z - w_t \left(\frac{A_{ht}^Z}{A_t^Z} \right) x_{hjt}^Z \, dj.$$

Substituting in the inverse demand function:

$$\max_{x_{hjt}^Z} \int_0^{N_t} \chi \mu \tau_t \left(\frac{P_{jt}}{Z_{jt}} \right) (x_{hjt}^Z)^\mu - w_t \left(\frac{A_{ht}^Z}{A_t^Z} \right) x_{hjt}^Z \, dj.$$

The first-order condition for monopolist i is rewritten as

$$x_{hjt}^Z = \left(\frac{\chi \mu^2 \tau_t P_{jt}}{w_t Z_{jt}} A_t^Z \right)^{\frac{1}{1-\mu}} \equiv \hat{x}_{jt}^Z.$$

Thus all monopolists supply the same amount to a given final good producer. The implied price of intermediate good h is: $p_{ht}^Z = (1/\mu)w_t(A_{ht}^Z/A_t^Z)$.

Using the expression for \hat{x}_{jt}^Z and (4):

$$Z_{jt} = \left(\frac{\chi \mu^2 \tau}{w_t} \right)^{\frac{\mu}{1+\chi\mu}} Y_{jt}^{\frac{\beta\mu}{1+\chi\mu}} (A_t^Z)^{\frac{1}{1+\chi\mu}}.$$

The pollution emission and the pollution intensity, $\mathcal{P}_{jt} \equiv P_{jt}/Y_{jt}$, of firm j can then be

expressed as functions of Y_{jt} :

$$P_{jt}(Y_{jt}) = \left(\frac{\chi\mu^2\tau}{w_t} \right)^{-\frac{\chi\mu}{1+\chi\mu}} Y_{jt}^{\frac{\beta}{1+\chi\mu}} (A_t^Z)^{\frac{-\chi}{1+\chi\mu}} \quad \text{and} \quad \mathcal{P}_{jt}(Y_{jt}) = \left(\frac{\chi\mu^2\tau}{w_t} \right)^{-\frac{\chi\mu}{1+\chi\mu}} Y_{jt}^{\frac{\beta}{1+\chi\mu}-1} (A_t^Z)^{\frac{-\chi}{1+\chi\mu}}.$$

Since $\beta < 1 + \chi\mu$ it follow that $P'_{jt}(Y_{jt}) > 0$, $P''_{jt}(Y_{jt}) < 0$, $\mathcal{P}'_{jt}(Y_{jt}) < 0$, and $\mathcal{P}''_{jt}(Y_{jt}) > 0$.

Profits for firm j amounts to:

$$\begin{aligned} \pi_{jt}^f(Y_{jt}) &= Y_{jt} - \alpha(1 - \beta\tau_t\mathcal{P}_{jt}(Y_{jt}))(FA_t^Y + Y_{jt}) - \chi\mu\tau_tP_{jt}(Y_{jt}) - \tau_tP_{jt}(Y_{jt}) \\ &= (1 - \alpha)Y_{jt} + (\alpha\beta - \chi\mu - 1)\tau_tP_{jt}(Y_{jt}) - \alpha FA_t^Y + \alpha\beta F\tau_tA_t^Y\mathcal{P}_{jt}(Y_{jt}). \end{aligned}$$

Taking the first-order condition of the profit function yields:

$$(1 - \alpha) = (1 + \chi\mu - \alpha\beta)\tau_tP'_{jt}(Y_{jt}) - \alpha\beta F\tau_tA_t^Y\mathcal{P}'_{jt}(Y_{jt}).$$

The left-hand side is a positive constant. Since $\alpha\beta < 1 + \chi\mu$ the right-hand side approaches infinity for Y_{jt} approaching zero. The right-hand side is strictly decreasing in Y_{jt} and approaches zero for Y_{jt} approaching infinity. Thus there is a unique equilibrium. As only one Y_{jt} is consistent with profit maximization, all firms produce the same quantity of final output. As pollution emission and intensity for firm j are strictly monotone functions of Y_{jt} , the unique value of Y_{jt} implies unique pollution emission and intensity levels for all firms. As a consequence, all firms also conduct the same level of abatement and purchase the same amount of intermediate goods. Accordingly, the economy is always in a symmetric equilibrium, where all final good firms behave similarly.

C Deriving (12)

Substituting (11) into (4) yields

$$\hat{Z}_t = \underbrace{\left(\frac{\chi\mu^2\phi\hat{Y}_t}{\hat{Z}_tw_t} A_t^Z \right)^{\frac{\mu}{1-\mu}}}_{(\hat{x}_t^Z)^\mu} \int_0^1 A_{ht}^Z dh = \left(\frac{\chi\mu^2\phi\hat{Y}_t}{\hat{Z}_tw_t} A_t^Z \right)^{\frac{\mu}{1-\mu}} A_t^Z \Leftrightarrow \hat{Z}_t = \left(\frac{\chi\mu^2\phi\hat{Y}_t}{w_t} \right)^\mu A_t^Z.$$

Substituting the expression into (11):

$$\hat{x}_t^Z = \left(\left(\frac{\chi\mu^2\phi\hat{Y}_t}{w_t} \right) A_t^Z \underbrace{\left(\frac{\chi\mu^2\phi\hat{Y}_t}{w_t} \right)^{-\mu} \frac{1}{A_t^Z}}_{1/\hat{Z}_t} \right)^{\frac{1}{1-\mu}} = \chi\mu^2\phi \frac{\hat{Y}_t}{w_t}.$$

D Deriving (15)

The expression is derived from equilibrium condition (14). To evaluate this expression, each element is first derived as functions of \hat{Y}_t .

It follows from (1) and (9) that:

$$\hat{x}_t^Y = \frac{\alpha^2(1-\beta\phi)}{w_t} (\hat{Y}_t + F A_t^Y).$$

Using the above expression and the expression for p_t^Y :

$$\int_0^1 \hat{x}_t^Y p_t^Y di = \alpha(1-\beta\phi) (\hat{Y}_t + F A_t^Y).$$

From (12) and the expression for p_t^Z :

$$\int_0^1 \hat{x}_t^Z p_t^Z dh = \chi\mu\phi\hat{Y}_t.$$

Using that $Y_t = \hat{Y}_t N_t$ and substituting the above expressions and (5) into (6):

$$\pi_{jt}^f = (1 - \alpha(1 - \beta\phi) - \chi\mu\phi - \phi) \frac{Y_t}{N_t} - \alpha(1 - \beta\phi) F A_t^Y.$$

Using (14):

$$N_t = \frac{1 - \alpha(1 - \beta\phi) - \chi\mu\phi - \phi}{\alpha(1 - \beta\phi)F} \frac{Y_t}{A_t^Y}.$$

E Deriving (17)

Since all intermediate goods of a given type are used in the same quantity, it follows from (16) that: $L_t = N_t \hat{x}_t^Y + N_t \hat{x}_t^Z$. It follows from (1) and (9) that:

$$\hat{x}_t^Y = \frac{\alpha^2(1-\beta\phi)}{w_t} (\hat{Y}_t + F A_t^Y).$$

Substituting this expression and (12) into (16):

$$L_t = \left(\alpha^2(1 - \beta\phi) + \chi\mu^2\phi \right) \frac{Y_t}{w_t} + \frac{\alpha^2(1 - \beta\phi)}{w_t} FN_t A_t^Y.$$

From (15) it follows that:

$$FN_t A_t^Y = \frac{1 - \alpha(1 - \phi\beta) - \chi\mu\phi - \phi}{\alpha(1 - \phi\beta)} Y_t.$$

Substituting this expression into the expression for L_t :

$$L_t = \left(\alpha(1 - \phi - \chi\mu\phi) + \chi\mu^2\phi \right) \frac{Y_t}{w_t}.$$

The expression is clearly equivalent to (17).

F Technological development

Define the relative productivity variables: $a_{it}^Y \equiv A_{it}^Y / \bar{A}_t^Y$ and $a_{ht}^Z \equiv A_{ht}^Z / \bar{A}_t^Z$. Given that intermediate goods are ranked according to their relative productivity, Aghion and Howitt (1998, ch. 3) show that the relative productivities converge in distribution to:

$$i = \left(a_{it}^Y \right)^{\frac{1}{\gamma}} \quad \text{and} \quad h = \left(a_{ht}^Z \right)^{\frac{1}{\xi}}.$$

It is assumed that the productivities are distributed like this from the outset. It then follows that $\bar{a}_t^Y \equiv \bar{A}_t^Y / A_t^Y$ and $\bar{a}_t^Z \equiv \bar{A}_t^Z / A_t^Z$ are time invariant and given by $\bar{a}^Y = 1 + \gamma$ and $\bar{a}^Z = 1 + \xi$. That is, the average technological level grows at the same rate as the leading-edge technology. The technological development can, therefore, be expressed by the two differential equations (19) and (20).

G Proof of Proposition 1

Proof. As $C_t = Y_t$ it follows from (24) that: $Y_s = Y_t e^{\int_t^s (r_u - \rho) du}$, where $s \geq t$. From (19) and (20) it follows that: $A_s^Y = A_t^Y e^{\lambda \gamma \int_t^s n_u^Y du}$ and $A_s^Z = A_t^Z e^{\eta \xi \int_t^s n_u^Z du}$. These expressions are used to evaluate the value of new patents in each R&D subsector:

$$\begin{aligned}\bar{V}_t^Y &= \int_t^\infty (1 - \alpha)(1 - \phi - \chi \mu \phi) \bar{a}^Y Y_t e^{-\int_t^s (r_u + n_u^Y \lambda) du} ds \quad \text{and} \\ \bar{V}_t^Z &= \int_t^\infty (1 - \mu) \chi \mu \phi \bar{a}^Z Y_t e^{-\int_t^s (r_u + n_u^Z \eta - \lambda \gamma n_u^Y + \eta \xi n_u^Z) du} ds,\end{aligned}$$

where $\bar{a}^Y \equiv \bar{A}_t^Y / A_t^Y$ and $\bar{a}^Z \equiv \bar{A}_t^Z / A_t^Z$. In Appendix F, it is shown that: $\bar{a}^Y = 1 + \gamma$ and $\bar{a}^Z = 1 + \xi$.

Dividing the expression for \bar{V}_t^Y on both sides with w_t :

$$v_t^Y = \frac{(1 - \alpha)(1 - \phi - \chi \mu \phi)(1 + \gamma)}{\Omega} L_t \int_t^\infty e^{-\int_t^s \rho + n_u^Y \lambda (1 + \gamma) du} ds, \quad v_t^Y \equiv \frac{\bar{V}_t^Y}{w_t}.$$

Taking the derivative with respect to time yields:

$$\begin{aligned}0 &= g_{L,t} v_t^Y + \frac{(1 - \alpha)(1 - \phi - \chi \mu \phi)(1 + \gamma)}{\Omega} L_t \frac{d}{dt} \left(\int_t^\infty e^{M(s,t)} ds \right), \\ M(s, t) &\equiv - \int_t^s \rho + n_u^Y \lambda (1 + \gamma) du,\end{aligned} \tag{31}$$

where it is used that $v_t^Y = 1/\lambda$ according to (21). The main difficulty is to differentiate the integral terms. Employing Leibniz's integral rule

$$\begin{aligned}\frac{d}{dt} \left(\int_t^b e^{M(s,t)} ds \right) &= \int_t^b \frac{\partial e^{M(s,t)}}{\partial t} ds + e^{M(b,t)} \frac{db}{dt} - e^{M(t,t)} \frac{dt}{dt} \\ &= \int_t^b \frac{\partial e^{M(s,t)}}{\partial M(s,t)} \frac{\partial M(s,t)}{\partial t} ds - 1 \\ &= \int_t^b e^{M(s,t)} \frac{\partial M(s,t)}{\partial t} ds - 1,\end{aligned}$$

where $b \gg t$. The partial derivative of $M(s, t)$ with respect to t is given by

$$\frac{\partial M(s, t)}{\partial t} = \frac{\partial}{\partial t} \left(- \int_t^s \rho + n_u^Y \lambda (1 + \gamma) du \right) = \rho + n_t^Y \lambda (1 + \gamma),$$

where the last equality follows from the formula: $\frac{\partial}{\partial a} \left(\int_a^b f(x) dx \right) = -f(a)$. Thus,

$$\frac{d}{dt} \left(\int_t^b e^{M(s,t)} ds \right) = \left(\rho + n_t^Y \lambda (1 + \gamma) \right) \int_t^b e^{M(s,t)} ds - 1.$$

Plotting the expression into (31) yields:

$$0 = \left(g_{L,t} + \rho + n_t^Y \lambda (1 + \gamma) \right) v_t^Y - \frac{(1 - \alpha)(1 - \phi - \chi\mu\phi)(1 + \gamma)}{\Omega} L_t.$$

Accordingly,

$$v_t^Y = \frac{(1 - \alpha)(1 - \phi - \chi\mu\phi)(1 + \gamma) L_t}{\Omega (g_{L,t} + \rho + n_t^Y \lambda (1 + \gamma))}.$$

Employing the research arbitrage equation (21):

$$1 = \lambda \frac{(1 - \alpha)(1 - \phi - \chi\mu\phi)(1 + \gamma) L_t}{\Omega (g_{L,t} + \rho + n_t^Y \lambda (1 + \gamma))}.$$

The expression is reformulated to:

$$n_t^Y = \frac{(1 - \alpha)(1 - \phi - \chi\mu\phi)L_t}{\Omega} - \frac{\rho}{\lambda(1 + \gamma)} - \frac{g_{L,t}}{\lambda(1 + \gamma)}.$$

The same procedure is conducted on the expression for \bar{V}_t^Z . This results in the expression

$$n_t^Z = \frac{(1 - \mu)\chi\mu\phi L_t}{\Omega} - \frac{\rho}{\eta(1 + \xi)} - \frac{g_{L,t}}{\eta(1 + \xi)}.$$

Employing the labor market clearing condition:

$$L_t = \left(\Gamma + \left(\frac{1}{\lambda(1 + \gamma)} + \frac{1}{\eta(1 + \xi)} \right) g_{L,t} \right) \frac{\Omega}{1 - \phi}.$$

This expression implies that

$$g_{L,t} = \left(\frac{1}{\lambda(1 + \gamma)} + \frac{1}{\eta(1 + \xi)} \right)^{-1} \left(\frac{1 - \phi}{\Omega} L_t - \Gamma \right).$$

If $L_0(1 - \phi)/\Omega > \Gamma$ then $g_{L,0} > 0$ and both $g_{L,t}$ and L_t increases over time. At some point, t' , $n_{t'}^Y = 0$ and/or $n_{t'}^Z = 0$. It is easy to verify that $g_{L,t'} > 0$ at this point. If, for instance,

$n_{t'}^Y = 0$ then:

$$\begin{aligned} \frac{g_{L,t'}}{\lambda(1+\gamma)} &> \frac{(1-\alpha)(1-\phi-\chi\mu\phi)L_{t'}}{\Omega} - \frac{\rho}{\lambda(1+\gamma)} \\ &> \frac{(1-\alpha)(1-\phi-\chi\mu\phi)L_0}{\Omega} - \frac{\rho}{\lambda(1+\gamma)} > 0. \end{aligned}$$

A positive $g_{L,t'}$ implies that $g_{L,t'}$ continues to be positive and increasing. It then follows that $L_{t''} = \bar{L}$ for some $t'' > 0$. This is inconsistent with the research arbitrage equations. In the absence of research, it must be the case that:

$$1 > \lambda \frac{(1-\alpha)(1-\phi-\chi\mu\phi)(1+\gamma)L_{t''}}{\Omega\rho} \quad \text{and} \quad 1 > \eta \frac{(1-\mu)\chi\mu\phi(1+\xi)L_{t''}}{\Omega\rho}.$$

At time $t = 0$ research is conducted, implying that:

$$1 = \lambda \frac{(1-\alpha)(1-\phi-\chi\mu\phi)(1+\gamma)L_0}{\Omega(g_{L,0} + \rho + n_0^Y\lambda(1+\gamma))} \quad \text{and} \quad 1 = \eta \frac{(1-\mu)\chi\mu\phi(1+\xi)L_0}{\Omega(g_{L,0} + \rho + n_0^Z\eta(1+\xi))}.$$

Since $L_0 < L_{t''}$, $n_0^Y > 0$, and $n_0^Z > 0$, it must be the case that $g_{L,0} < 0$. This is, however, inconsistent with $L_0 < L_{t''}$. Thus $g_{L,0} > 0$ leads to a contradiction.

If $L_0(1-\phi)/\Omega < \Gamma$ then $g_{L,0} < 0$ and both $g_{L,t}$ and L_t decrease over time. At some point, t' , it must be the case that $L_{t'} = 0$ which is inconsistent with positive research according to the above expressions. Thus $g_{L,0} < 0$ leads to a contradiction.

Accordingly, it must be the case that $L_0(1-\phi)/\Omega = \Gamma$ then $g_{L,0} = 0$. It follows immediately that $L_t = L$, $n_t^Y = n^Y$, and $n_t^Z = n^Z$, where the allocations are given by (26), (27), and (28). It follows from (18) that $L_t^Y = L^Y$ and $L_t^Z = L^Z$. The constant growth rates of A_t^Y and A_t^Z are given directly by (19) and (20). From (15) it follows that \hat{Y}_t grows at the same rate as A_t^Y . From (1), (2), (15), and (17):

$$N = \alpha(1-\phi-\alpha(1-\phi\beta)-\chi\mu\phi)^{\frac{1}{\alpha}}(1-\phi-\chi\mu\phi)^{\frac{\alpha-1}{\alpha}}\frac{L}{\Omega}.$$

Thus N is constant over time. It is straightforward to show that this implies that C_t , Y_t and w_t grow at the same rate as A_t^Y . This implies that \hat{Z}_t grows at the same rate as A_t^Z . Pollution emission per firm then grows by the rate: $g_P = \beta n^Y \lambda \gamma - \chi n^Z \eta \xi$. This is also true for aggregate pollution emission since N is constant over time. \square

H Deriving research inputs

Evaluating the research arbitrage condition (21):

$$\begin{aligned} w_t &= \lambda(1 - \alpha)(1 - \phi - \chi\mu\phi) \int_t^\infty \frac{\bar{A}_t^Y}{A_s^Y} Y_s e^{-\int_t^s r + n^Y \lambda du} ds \\ &= \lambda(1 - \alpha)(1 - \phi - \chi\mu\phi) \frac{\bar{A}_t^Y}{A_t^Y} Y_t \int_t^\infty e^{-\int_t^s r + n^Y \lambda du} ds, \end{aligned}$$

where it is used that A_t^Y and Y_t grow at the same rate.

Define the relative productivity variables: $a_{it}^Y \equiv A_{it}^Y / \bar{A}_t^Y$ and $a_{ht}^Z \equiv A_{ht}^Z / \bar{A}_t^Z$. Given that intermediate goods are ranked according to their relative productivity, it follows that in the long run (see Aghion and Howitt 1998, ch. 3):

$$i = \left(a_{it}^Y\right)^{\frac{1}{\gamma}} \quad \text{and} \quad h = \left(a_{ht}^Z\right)^{\frac{1}{\xi}}.$$

Assuming that the productivities are distributed like this from the outset, $\bar{a}_t^Y \equiv \bar{A}_t^Y / A_t^Y$ and $\bar{a}_t^Z \equiv \bar{A}_t^Z / A_t^Z$ are time invariant, and they are given by: $\bar{a}^Y = 1 + \gamma$ and $\bar{a}^Z = 1 + \xi$.

Substituting in (17), it follows that

$$\begin{aligned} 1 &= \lambda \frac{(1 - \alpha)(1 - \phi - \chi\mu\phi)(1 + \gamma)L}{\Omega(\rho + n^Y \lambda(1 + \gamma))} \Leftrightarrow \\ n^Y &= \frac{(1 - \alpha)(1 - \phi - \chi\mu\phi)}{\Omega} L - \frac{\rho}{\lambda(1 + \gamma)}. \end{aligned}$$

The expression for n^Z is derived in the same way.

I Proof of Proposition 2

Proof. It follows directly from (26) that

$$\frac{dL}{d\phi} = \frac{\chi\mu(\mu - \alpha)}{(1 - \phi)^2} \Gamma \gtrless 0 \quad \text{for} \quad \alpha \gtrless \mu.$$

From (27) and (28):

$$\frac{dn^Y}{d\phi} = -\frac{(1 - \alpha)\chi\mu}{(1 - \phi)^2} \Gamma < 0 \quad \text{and} \quad \frac{dn^Z}{d\phi} = \frac{(1 - \mu)\chi\mu}{(1 - \phi)^2} \Gamma > 0.$$

From (18) and (26) it follows that

$$L^Y = \frac{\alpha(1 - \phi - \chi\mu\phi)}{(1 - \phi)}\Gamma \quad \text{and} \quad L^Z = \frac{\chi\mu^2\phi}{(1 - \phi)}\Gamma.$$

Accordingly,

$$\frac{dL^Y}{d\phi} = -\frac{\alpha\chi\mu}{(1 - \phi)^2} < 0 \quad \text{and} \quad \frac{dL^Z}{d\phi} = \frac{\chi\mu^2}{(1 - \phi)^2} > 0.$$

□

J Computing GDP

In the end, the representative household receives the entire income of the economy: wage income, profits from the intermediate goods sector, and government transfers. The sum of these incomes constitute GDP from the income side. From the user side, GDP equals the sum of aggregate consumption and saving. Accordingly:

$$GDP_t = C_t + \dot{\tilde{W}}_t = \bar{L}w_t + r_t\tilde{W}_t + I_t.$$

The financial wealth of the representative household is given by:

$$\begin{aligned} \tilde{W}_t &= \left(\frac{(1 - \alpha)(1 - \phi - \chi\mu\phi)}{\lambda(1 + \gamma)n^Y + \rho} + \frac{(1 - \mu)\chi\mu\phi}{\eta(1 + \xi)n^Z + \rho} \right) Y_t \Rightarrow \\ \dot{\tilde{W}}_t &= \left(\frac{(1 - \alpha)(1 - \phi - \chi\mu\phi)}{\lambda(1 + \gamma)n^Y + \rho} + \frac{(1 - \mu)\chi\mu\phi}{\eta(1 + \xi)n^Z + \rho} \right) \lambda\gamma n^Y Y_t \\ &= \left(\frac{1 - \phi}{\Gamma} \right) \left(\frac{1}{\lambda(1 + \gamma)} + \frac{1}{\eta(1 + \xi)} \right) \lambda\gamma n^Y Y_t. \end{aligned}$$

GDP is, therefore, given by

$$GDP_t = \left\{ 1 + \left(\frac{1 - \phi}{\Gamma} \right) \left(\frac{1}{\lambda(1 + \gamma)} + \frac{1}{\eta(1 + \xi)} \right) \lambda\gamma n^Y \right\} Y_t.$$

Hence the GDP-output ratio is constant over time.

K Model with CRRA utility

The real interest rate is now given by: $r_t = \theta g_C + \rho$. Thus when solving the research arbitrage equations, the research labor allocations are given by

$$\begin{aligned} n^Y &= \frac{(1-\alpha)(1-\phi-\chi\mu\phi)}{\Omega} \frac{1}{\Sigma_1} L - \frac{\rho}{\lambda(1+\theta\gamma)} \quad \text{and} \\ n^Z &= \frac{(1-\mu)\chi\mu\phi}{\Omega} L - \Sigma_2 n^Y - \frac{\rho}{\eta(1+\xi)}, \end{aligned}$$

where $\Sigma_1 \equiv (1+\theta\gamma)/(1+\gamma)$ and $\Sigma_2 \equiv (\theta-1)(\lambda/\eta)\gamma/(1+\xi)$.

From the labor market clearing condition it follows that

$$\begin{aligned} L &= \tilde{\Gamma} \left(1 + \frac{(1-\alpha)(1-\phi-\chi\mu\phi)}{\Omega} \frac{1-\Sigma_2}{\Sigma_1} + \frac{(1-\mu)\chi\mu\phi}{\Omega} \right)^{-1}, \quad \text{where} \\ \tilde{\Gamma} &\equiv \bar{L} + \rho \left(\frac{1-\Sigma_2}{\lambda(1+\theta\gamma)} + \frac{1}{\eta(1+\xi)} \right). \end{aligned}$$

According to the following proposition, the qualitative policy implications are the same as for the model presented in Section 3.

Proposition 4. *Assuming that parameters are such that n^Y , n^Z , and L are positive:*

$$\begin{aligned} (i) \quad & \frac{dL}{d\phi} \gtrless 0 \quad \text{for} \quad \frac{\alpha}{1-\alpha} \gtrless \left(\frac{\mu}{1-\mu} \right) \left(\frac{1-\Sigma_2}{\Sigma_1} \right); \\ (ii) \quad & \frac{dn^Y}{d\phi} < 0 \quad \text{and} \quad \frac{dn^Z}{d\phi} > 0; \quad \text{and} \\ (iii) \quad & \frac{dg_Y}{d\phi} < 0 \quad \text{and} \quad \frac{dg_P}{d\phi} < 0. \end{aligned}$$

Proof. Differentiating the expression for L with respect to ϕ yields

$$\frac{dL}{d\phi} = \left(\frac{-L}{1 + \frac{(1-\alpha)(1-\phi-\chi\mu\phi)}{\Omega} \frac{1-\Sigma_2}{\Sigma_1} + \frac{(1-\mu)\chi\mu\phi}{\Omega}} \right) \left(\frac{(1-\mu)\chi\mu\alpha}{\Omega^2} - \frac{(1-\alpha)\chi\mu^2}{\Omega^2} \frac{1-\Sigma_2}{\Sigma_1} \right).$$

The expression is positive if

$$\frac{(1-\mu)\chi\mu\alpha}{\Omega^2} < \frac{(1-\alpha)\chi\mu^2}{\Omega^2} \frac{1-\Sigma_2}{\Sigma_1} \quad \Leftrightarrow \quad \frac{\alpha}{1-\alpha} < \left(\frac{\mu}{1-\mu} \right) \left(\frac{1-\Sigma_2}{\Sigma_1} \right).$$

The expression is negative for the opposite inequality. Hence, (i) in Proposition 4 is proved.

The derivative of n^Y with respect to ϕ amounts to

$$\frac{dn^Y}{d\phi} = \frac{1-\alpha}{\Sigma_1} \left(\frac{(1-\phi-\chi\mu\phi)}{\Omega} \frac{dL}{d\phi} - \frac{\chi\mu^2}{\Omega^2} L \right).$$

If the derivative of L with respect to ϕ is negative or zero, it follows immediately that the above expression is negative. It then follows from the labor market clearing condition that n^Z is increasing in ϕ .

Assume that the derivative of L with respect to ϕ is positive. Then it is not straightforward to evaluate the sign of the derivative of n^Y with respect to ϕ . Instead, consider the following expression obtained from the expressions for n^Y and n^Z :

$$\left(\frac{n^Z}{L} \right) = \left(\frac{(1-\mu)\chi\mu\phi}{\Omega} - \frac{(1-\alpha)(1-\phi-\chi\mu\phi)}{\Omega} \frac{\Sigma_2}{\Sigma_1} \right) - \frac{\rho}{\eta(1+\xi)\Sigma_1} \left(\frac{1}{L} \right).$$

The derivative of (n^Z/L) with respect to ϕ amounts to

$$\frac{d\left(\frac{n^Z}{L}\right)}{d\phi} = \frac{(1-\mu)\chi\mu\alpha}{\Omega^2} + \frac{(1-\alpha)\chi\mu^2}{\Omega^2} \left(\frac{\Sigma_2}{\Sigma_1} \right) + \frac{\rho}{\eta(1+\xi)\Sigma_1} \left(\frac{1}{L} \right)^2 \frac{dL}{d\phi}.$$

As the derivative of L with respect to ϕ is positive, this derivative is also positive. It is straightforward to show that this implies that the derivative of n^Z with respect to ϕ is positive, and then it follows from the labor market clearing condition that n^Y is decreasing in ϕ . This finishes the proof of (ii), and the proof of (iii) follows directly from (ii), (19), and (20). \square

L Model with physical capital

This appendix sketches a model closely related to the model presented in Section 3, but with a production process which incorporates physical capital. Relations similar to those presented in Section 3 will not be repeated.

In the final goods sector, firms operate under monopolistic competition a la Dixit and Stiglitz (1977). Firms produce using the production technology

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 (x_{ijt}^Y)^\alpha A_{it}^Y di,$$

where L_{jt} is labor input for firm j . Final goods are aggregated as follows:

$$Y_t = \left(\int_0^{N_t} Y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1.$$

Final goods can either be consumed or transformed into (physical) capital:

$$Y_t = C_t + \bar{I}_t + N_t F A_t^Y,$$

where \bar{I}_t is final goods used for investment in capital, and $F A_t^Y$ is a quasi-fixed cost paid by each final good firm to operate.

Consumers face the intra-temporal problem of maximizing Y_t with respect to $(Y_{jt})_{j \in [0, N_t]}$ given prices $(m_{jt})_{j \in [0, N_t]}$ subject to a given budget constraint. The standard solution is

$$Y_{jt} = Y_t \left(\frac{m_{jt}}{M_t} \right)^{-\epsilon}, \quad M_t \equiv \left(\int_0^{N_t} m_{jt}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}, \quad (32)$$

where M_t is the ideal price index, and thus, the price of final goods. Set M_t equal to one, such that all prices are in terms of final goods.

Following Aghion and Howitt (1998, p. 93-99), it is assumed that a monopolist needs $A_{it}^Y x_{ijt}^Y$ and $A_{ht}^Z x_{hjt}^Z$ units of capital to produce x_{ijt}^Y and x_{hjt}^Z units of intermediate good i and h , respectively. The idea is that it requires more capital to produce more advanced intermediate goods.

Capital evolves according to the differential equation: $\dot{K}_t = \bar{I}_t - \delta K_t$, where K_t is aggregate capital, \bar{I}_t is aggregate investment in capital, and $\delta > 0$ is the rate of depreciation. The monopolists rent capital from the representative household at the price $R_t = r_t + \delta$.

The labor and capital market clearing conditions are given by

$$\bar{L} = \int_0^{N_t} L_{jt} dj + n_t^Y + n_t^Z \quad \text{and} \quad K_t = \int_0^{N_t} \left(\int_0^1 x_{ijt}^Y A_{it}^Y di + \int_0^1 x_{hjt}^Z A_{ht}^Z dh \right) dj.$$

In a steady state equilibrium with a fixed labor allocation and a constant capital-output

ratio, the labor allocations are given by

$$\begin{aligned} L &= \frac{(1-\alpha)(1-1/\epsilon-\beta\phi)}{(1+\alpha)(1-\alpha)(1-1/\epsilon-\beta\phi) + (1-\mu)\chi\mu\phi} \left(\bar{L} + \frac{\rho}{\eta(1+\xi)} + \frac{\rho}{\lambda(1+\gamma)} \right), \\ n^Y &= \alpha L - \frac{\rho}{\lambda(1+\gamma)}, \quad \text{and} \\ n^Z &= \frac{(1-\mu)\chi\mu\phi}{(1-\alpha)(1-1/\epsilon-\beta\phi)} L - \frac{\rho}{\eta(1+\xi)}. \end{aligned}$$

It is easy to verify that a tighter environmental policy leads to a higher n^Z and a lower n^Y . Thus a tighter environmental policy reduces the economic growth rate.

M Current consumption compensation

In a steady state with $\kappa = 0$, welfare is given by:

$$U = \int_0^\infty \frac{C_t^{1-\theta}}{1-\theta} e^{-\rho t} dt = \frac{C_0^{1-\theta}}{(1-\theta)(\rho - (1-\theta)g_Y)}.$$

The current consumption level that makes the representative household indifferent between the old and new economic growth rate is the \tilde{C}_0 that solves the equality:

$$\frac{C_0^{1-\theta}}{(1-\theta)(\rho - (1-\theta)g_Y)} = \frac{\tilde{C}_0^{1-\theta}}{(1-\theta)(\rho - (1-\theta)\tilde{g}_Y)},$$

where \tilde{g}_Y is the economic growth rate after the reform. Using this expression, it is straightforward to compute \tilde{C}_0/C_0 which is the relative increase in current consumption that makes the representative household indifferent between the two growth rates. That is, the CCC stemming from the economic growth effect of the reform. The reform also reduces the initial consumption level. The total current consumption compensation is obtained by adding this level effect to the aforementioned growth effect.

Chapter 3

Directed Technical Change, Environmental Sustainability, and Population Growth

Directed Technical Change, Environmental Sustainability, and Population Growth*

By Peter K. Kruse-Andersen

Abstract

Population growth has two potentially counteracting effects on pollution emission: (i) more people implies more production and thereby more emission, and (ii) more people implies a larger research capacity which might reduce the emission intensity of production, depending on the direction of research. This paper investigates how to achieve a given climate goal in the presence of these two effects. A growth model featuring both directed technical change and population growth is developed. Consistent with empirical evidence, the model features simultaneous research in polluting and non-polluting technologies in the absence of major policy interventions. Both analytical and numerical results indicate that population growth is a burden on the environment, even when all research efforts are directed toward non-polluting technologies. Thus research subsidies alone are ineffectual environmental policy tools. Instead, the analysis highlights the effectiveness of pollution taxes and population control policies.

Keywords: Directed technical change, endogenous growth, environmental policy, environmental sustainability, climate change, population growth

JEL Classification: J11, O30, O41, Q54, Q55, Q58

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1 Introduction

Empirical studies find that a tighter environmental policy stimulates research in environmentally friendly technologies (e.g., Popp 2006; Haščič et al. 2012; Aghion et al. 2016). Yet many economic studies on climate change assume that technological change is governed by exogenous processes (e.g., Nordhaus and Sztorc 2013). These studies, therefore, neglect a core mechanism in climate change mitigation. In contrast, a recent strand of literature develops micro-founded growth models, where the direction of technical change is affected by environmental policies. However, this literature has neglected another core factor: population growth.¹ This seems problematic as the global population size is expected to increase by 48 pct. from 2017 to 2100 (United Nations 2017, medium variant). Moreover, empirical studies find that population growth affects CO₂ emission substantially more than income per capita growth (e.g., Liddle 2015; Casey and Galor 2017).

The present study fills this gap in the literature by developing a micro-founded growth model featuring both directed technical change and population growth. In this framework, population growth has two potentially counteracting effects on pollution emission, and a core issue thus becomes the relative strength of these effects. The first effect is a *neo-Malthusian effect*: given the technological level, a larger population leads to a larger production and thus more pollution emission. The second effect works through knowledge creation. Given the non-rival nature of knowledge, a larger population permits faster knowledge creation, as more resources can be allocated to research. If knowledge creation is directed toward environmentally friendly technologies, population growth has a negative effect on pollution emission. In the words of Julian Simon:

It is your mind that matters economically, as much as or more than your mouth or hands. The most important economic effect of population size and growth is the contribution of additional people to our stock of useful knowledge. And this contribution is great enough in the long run to overcome all the costs of population growth (Simon 1998, p. 367).

Given Simon's strong statement, it seems appropriate to name the second effect: the *Simon*

¹The studies mentioned include Saint-Paul (2002), Hart (2004), Ricci (2007a), Grimaud and Rougé (2008), Acemoglu et al. (2012), André and Smulders (2014), Hemous (2015), Daubanes et al. (2016), Hassler et al. (2016), Van den Bijgaart (2017), and Fried (forthcoming).

effect.² Note that both effects are scale effects, and that the Simon effect might increase or decrease pollution emission depending on the direction of research.

The present study examines how to achieve a given climate goal in a growth model featuring both the neo-Malthusian effect and the Simon effect. Consider the Paris Agreement. The goal of the agreement is to ensure that the global temperature increase does not exceed 2 degree Celsius. Such a policy goal can be translated into a CO₂ concentration limit. The present study investigates which environmental policies that are able to ensure that such a limit remains unviolated.

The modeling strategy of the present study is closely related to that of Acemoglu, Aghion, Bursztyn, and Hemous (2012), hereafter AABH, but it differs in two important ways. First, AABH assume a constant population size, whereas the model developed in the present study allows for population growth. The introduction of population growth matters for both the quantitative and qualitative policy implications.

Second, in the model developed by AABH, research only targets the most advanced technology, polluting or non-polluting, under *laissez-faire*, implying a strong path dependency of research efforts. This strong path dependency is not only implausible given evidence of simultaneous research in environmentally and non-environmentally friendly technologies. It also leads to a wrong prediction concerning the global CO₂ intensity trend. This point is discussed further in Section 2. Additionally, AABH assume that knowledge spillovers in research are as strong as they can possibly be without implying accelerating economic growth. If population growth is introduced directly into their framework, the long-run economic growth rate increases with the population size: an implausible feature often referred to as the strong scale effect (see Jones 2005). To avoid both the strong path dependency and the strong scale effect, the present study relaxes the knowledge spillover assumptions in research. This modeling strategy is motivated by the empirical evidence from Chapter 1 and Bloom et al. (2017), and it ensures that the model is able to match the global CO₂ intensity trend.

The present study finds that population growth is a major burden on the environment. This is confirmed both analytically and numerically. Analytical results based on exponential population growth show that the neo-Malthusian effect always dominates the Simon effect, in the long run, even when all research efforts are directed toward environmentally friendly technologies. This finding is intimately linked to the qualitative calibration procedure, and,

²The positive effects of population size on innovation have long been recognized. William Petty was probably the first to realize this in 1682 (Petty 1899, p. 474). But, the relationship has also been recognized in modern economic research (e.g., Kuznets 1960; Simon 1977; Simon 1981; Kremer 1993; Jones 1995).

in particular, to the departure from the strong path dependency. Weaker spillover effects imply a lower research productivity and thereby a weaker Simon effect. The neo-Malthusian effect always dominates the Simon effect in the long run, when spillovers are weak enough to break the strong path dependency.

The analytical policy implications follow directly from this result. First, subsidies can direct research efforts toward environmentally friendly technologies which strengthens the Simon effect. But since the neo-Malthusian effect always dominates the Simon effect in the long run, research subsidies cannot ensure environmental sustainability. In contrast, AABH find that even temporary research subsidies might ensure environmental sustainability. The results differ partly because the present study incorporates population growth and thereby the neo-Malthusian effect. In fact, the present study finds that permanent research subsidies can ensure environmental sustainability in the absence of population growth. Second, a tax on pollution emission can both direct research efforts toward environmentally friendly technologies and increase the incentive to use a more environmentally friendly input mix in the production process. Together, this production input mix effect and the Simon effect can dominate the neo-Malthusian effect if the pollution penalization increases sufficiently fast. Hence a pollution tax can ensure environmental sustainability. Third, due to the positive net contribution of population growth to pollution emission, environmental sustainability requires a less stringent environmental tax policy for a lower population growth rate. Hence population control policies may be useful in conjunction with a pollution tax policy.

Simulations based on different population projections from the United Nations confirm the general policy implications of the analytical analysis. One important finding is that the Simon effect actually dominates the neo-Malthusian effect within this century if research is directed fully toward environmentally friendly technologies. But even in this case, reducing the population growth rate weakens the neo-Malthusian effect substantially more than the Simon effect. Population growth therefore continues to be a burden on the environment. In addition, it turns out that directing research efforts toward environmentally friendly technologies cannot ensure that the two-degree temperature limit from the Paris Agreement remains unviolated under the baseline population growth scenario. Hence staying below a two-degree temperature increase involves either population control policies, a pollution tax, or both.

Besides the aforementioned literature, this paper is related to a strand of literature inves-

investigating how population growth relates to natural resource and pollution emission issues.³ Some studies (e.g., Nordhaus 2002; Popp 2004; Gerlagh 2008) implement directed technical change features into integrated assessment models. These studies feature both a directed technical change mechanism and population growth, but the directed technical change mechanism is not micro-founded. Thus standard problems arise when assessing the policy implications of these highly aggregated models. The model developed by Bretschger (2013) has a flavor similar to the model developed in the present study. Population growth stimulates knowledge creation, but it also increases the scale of the economy, leaving less non-renewable resources per worker. Population growth thereby has two counteracting effects on productivity growth. In contrast to the model developed below, Bretschger's model only features one type of technology. Policy makers can therefore only affect the speed and not the direction of technical change. Casey (2017) develops an endogenous growth model, where research efforts can improve productivity and energy efficiency of capital goods. Energy taxes incentivize energy efficiency research at the expense of productivity increasing research. Even though the population size is allowed to grow, Casey's study differs notably from the present study, as the resources available for research are completely independent of the population size. Consequently, the scale of the economy has no impact on technological development which completely eliminates the Simon effect.

The present study proceeds as follows. Section 2 motivates the departure from the strong path dependency highlighted by AABH. Section 3 presents the model, and Section 4 investigates the long-run policy implications of the model. Section 5 provides simulations for the period 2014-2100 based on population projections from the United Nations. Finally, the analysis is discussed in Section 6.

2 Avoiding the Lock-in Equilibrium

A key feature of the model developed by AABH is the strong path dependency of research. In their model, production of consumption goods requires polluting and non-polluting inputs. The technologies associated with the production of these inputs can be improved through research efforts. In their setup, research only focuses on the most advanced technology under laissez-faire. Consequently, the model dynamics are completely path depended. As the technology associated with the polluting input is initially more advanced, research is locked

³E.g., Schou (2002), Shi (2003), Asheim et al. (2007), Bréchet and Lambrecht (2009), Schäfer (2014), Peretto and Valente (2015), and Dietz et al. (2016).

to this technology from the outset. Environmental policy can then skew incentives such that research is redirected toward the technology associated with the non-polluting input. When this technology is sufficiently advanced, the market economy continues to develop it even without policy interventions. Accordingly, a temporary subsidy can permanently change the direction of technological development which ensures environmental sustainability under some additional assumptions.

Essentially AABH highlight the consequences of a *lock-in equilibrium* in research. As research focuses only on the most advanced technology, research is locked to this technology under *laissez-faire*. This lock-in equilibrium is not empirically plausible. Empirical evidence suggests that both environmentally and non-environmentally friendly technologies are developed today (e.g., Dechezleprêtre et al. 2014; Noailly and Smeets 2015). And historical evidence shows that both types of technology have been developed continuously in the past. Hydropower has, for instance, been used since ancient times. And after the introduction of hydroelectric production via turbines in the 19th century, hydropower played an increasingly important role in modern industrialization (Narbel et al. 2014, p. 172-177).⁴

In addition, because of the lock-in equilibrium, the model developed by AABH cannot match the evolution in the global CO₂ intensity, defined as (anthropogenic) CO₂ emission divided by GDP. Consider the identity:

$$\text{CO}_2 \text{ emission} \equiv \underbrace{\text{CO}_2 \text{ emission/GDP}}_{\text{CO}_2 \text{ intensity}} \times \underbrace{\text{GDP/Population}}_{\text{GDP per capita}} \times \text{Population}.$$

The identity decomposes global CO₂ emission into three factors: (i) CO₂ intensity, (ii) GDP per capita, and (iii) population size. The left panel of Figure 1 shows that global (anthropogenic) CO₂ emission more than tripled from 1960 to 2014. The decomposition shown in the right panel of Figure 1 indicates that the increase was caused by economic and population growth, while a reduction in the CO₂ intensity had a dampening effect.

Theoretical models designed to assess climate change issues should be able to match the empirical tendencies presented in Figure 1. The model developed by AABH features GDP per capita growth, but it does not allow for population growth. Additionally, due to the lock-in equilibrium, their model predicts that the pollution intensity increases and converges

⁴Another example is wind power. Prior to the Industrial Revolution, wind power was a major source of energy in Europe. Despite the fact that wind power was to a large degree substituted by new fossil fuel technologies after the Industrial Revolution, the development of windmills continued. A particularly significant development was the introduction of scientific testing and evaluation of windmills in the 18th century (Manwell et al. 2009, ch. 1).

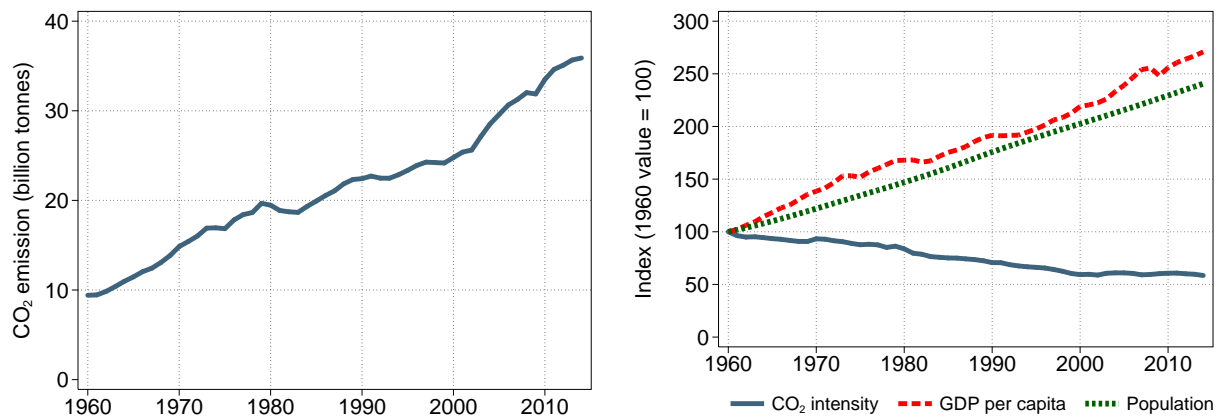


FIGURE 1: Global CO₂ emission, CO₂ intensity, GDP per capita, and population size, 1960-2014.

Data sources: Boden et al. (2015), United Nations (2017), and World Bank (2016).

Notes: CO₂ emissions are emissions from fossil fuels and cement production. Carbon in tonnes is converted to CO₂ in tonnes using the conversion factor 3.664. GDP is in market prices, constant 2010 U.S. dollars.

to a constant which is clearly inconsistent with the empirical evidence from Figure 1. This is shown formally in Appendix A. In contrast, the model developed in the present study can replicate the empirical tendencies presented in Figure 1, but it requires a departure from the lock-in equilibrium.

To avoid the lock-in equilibrium, the present study relaxes the spillover assumptions in research. If the spillover effects in research are sufficiently weak, it becomes increasingly less attractive to research in a specific technology, the more advanced this technology becomes. Thus if one type of technology becomes sufficiently advanced, researchers switch their focus to another technology. In the end, all technologies are developed continuously, and the lock-in equilibrium is avoided.⁵

3 The Model

The model features a growing labor force, and labor has two potential uses: manufacturing and research. In the manufacturing sector, consumption goods are produced from polluting and non-polluting intermediate goods. These inputs are in turn produced by intermediate good specific machines. The machines are produced by labor, and thus manufacturing labor is indirectly devoted to either polluting or non-polluting intermediate good production. In the research sector, scientists develop new machine varieties for either polluting or non-

⁵Daubanes et al. (2016) investigate the Green Paradox hypothesis within an AABH-style model. As in the present study, Daubanes et al. relax the spillover assumptions in research to ensure simultaneous development of both polluting and non-polluting technologies. Yet, they assume a constant population size which eliminates the scale effects investigated in the present study.

polluting intermediate good production. The increase in machine varieties causes labor productivity in manufacturing to grow. The direction of technical change is determined by the relative profitability of research in the two machine types.

3.1 Structure

Time is discrete and denoted $t \geq 0$. Consumers spend their entire income each period on consumption goods, and utility is strictly increasing in consumption. Each consumer supplies one unit of labor inelastically, and aggregate labor supply, \bar{L}_t , evolves according to:

$$\bar{L}_t = (1 + n)^t \bar{L}_0, \quad n \geq 0, \quad \bar{L}_0 > 0,$$

where n is the constant population growth rate.⁶

Consumption goods are produced using the production technology:

$$C_t = \left[(Y_t^c)^{\frac{\epsilon-1}{\epsilon}} + (Y_t^d)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 0, \quad \epsilon \neq 1, \quad (1)$$

where C_t is aggregate consumption, Y_t^c measures "clean" intermediate good input, and Y_t^d measures "dirty" intermediate good input. Use of dirty intermediate goods causes pollution emission, while the use of clean intermediate goods does not. Aggregate pollution emission equals Y_t^d . The parameter ϵ is the (constant) elasticity of substitution between clean and dirty intermediate goods. When $\epsilon > 1$ the two intermediate good types are gross substitutes, and when $\epsilon < 1$ they are gross complements.⁷

The interpretation of clean and dirty intermediate goods is rather broad. Clean (dirty) intermediate goods are inputs in the production process which reduce (increase) the pollution intensity of production. To show this formally, consider the pollution intensity of the manufacturing sector:

$$\frac{Y_t^d}{C_t} = \left[1 + \left(\frac{Y_t^c}{Y_t^d} \right)^{\frac{\epsilon-1}{\epsilon}} \right]^{-\frac{\epsilon}{\epsilon-1}}.$$

⁶Assuming a constant population growth rate simplifies the analytical analysis considerably. When the model is simulated in Section 5, population projection data from the United Nations are used directly to measure \bar{L}_t . Simulations based on the average projected population growth rate over the period 2014-2100 provide similar results.

⁷When $\epsilon > 1$ the demand for clean (dirty) intermediate goods increases if the price of dirty (clean) intermediate goods increases which implies that the two intermediate goods are gross substitutes. The opposite is true when $\epsilon < 1$ which implies that the two intermediate goods are gross complements.

It follows directly that the pollution intensity increases in Y_t^d and decreases in Y_t^c .

Pollution emission increases the pollution stock, E_t . Policy makers set a climate goal in terms of the pollution stock (e.g. a CO₂ concentration limit): $0 < \bar{E} < \infty$. *Environmental sustainability* is obtained if $E_t < \bar{E}$ for all $t \geq 0$. The pollution stock evolves according to:

$$E_{t+1} = f(Z_t), \quad Z_t = (Y_{t-\bar{v}}^d, Y_{t-\bar{v}+1}^d, \dots, Y_t^d)', \quad E_0 \geq 0,$$

where \bar{v} is a positive integer, $0 \leq f_{Y_v^d} < \infty$ for $v = \{t - \bar{v}, \dots, t - 1\}$, $0 < f_{Y_t^d} < \infty$, and $\lim_{Y_t^d \rightarrow \infty} f(Z_t) > \bar{E}$. This formulation of the pollution stock is very general and allows present emissions to affect the pollution stock many periods later.⁸ In this setup, environmental sustainability is ensured if \bar{E} is sufficiently large and Y_t^d decreases in the long run at a constant growth rate. Correspondingly, environmental sustainability is not obtained if Y_t^d , in the long run, grows at a constant positive rate.⁹

Clean and dirty intermediate goods are produced by machines. Machines used to construct clean and dirty intermediate goods are indexed i and h , respectively. The production of clean and dirty intermediate goods are given by

$$Y_t^j = A \left((N_t^j)^{\alpha-1+\alpha\psi} \int_0^{N_t^j} (x_{kt}^j)^\alpha dk \right)^{\frac{1}{\alpha}}, \quad A > 0, \quad 0 < \alpha < 1, \quad \psi > 0, \quad (2)$$

where x_{kt}^j denotes the quantity of machine $k \in \{i, h\}$ used in subsector $j \in \{c, d\}$, N_t^j measures the varieties of machines in subsector j such that $i \in [0, N_t^c]$ and $h \in [0, N_t^d]$, and $1/(1 - \alpha)$ is the elasticity of substitution between machine varieties.¹⁰ The parameter ψ reflects the productivity gains associated with machine varieties, cf. (11) below. Holding the total machine input constant, more varieties results in more intermediate good output. Accordingly, the machine variety measures are referred to as the technological levels. The factor N_t^j to the power of $(\alpha - 1 + \alpha\psi)$ ensures that an arbitrary parameter link between the elasticity of substitution and the productivity gains from machine varieties is broken (cf. Alvarez-Pelaez and Groth 2005). Breaking this parameter link eases the interpretation of

⁸This law of motion for the pollution stock largely follows Van den Bijgaart (2017). The formulation includes the functional forms used by AABH and Pottier et al. (2014). The same results can also be obtained using the law of motion from Golosov et al. (2014).

⁹Let Y_t^d decrease over time at a constant rate. Then $Y_t^d \rightarrow 0$ for $t \rightarrow \infty$. Hence E_t cannot increase systematically. Given that \bar{E} is sufficiently large, $E_t < \bar{E}$ for all $t \geq 0$ implying environmental sustainability. If Y_t^d increases at a constant positive rate, then $f(Z_t) > \bar{E}$ for some $t' \in t$, as $Y_t^d \rightarrow \infty$ for $t \rightarrow \infty$.

¹⁰These machines are better interpreted as intermediate goods, as they depreciate fully after use. The word "machines" is used to clearly distinguish them from clean and dirty intermediate goods.

the obtained results without complicating the math notably.

Machines are produced one-for-one by labor input such that

$$L_t = L_t^c + L_t^d, \quad L_t^c = \int_0^{N_t^c} x_{it}^c di, \quad \text{and} \quad L_t^d = \int_0^{N_t^d} x_{ht}^d dh, \quad (3)$$

where L_t measures labor input in manufacturing, and L_t^j measures labor input used to produce machines for subsector j .

The R&D sector is bifurcated into two subsectors: one for each machine type. In both subsectors, scientists conduct research to invent new machine varieties a la Romer (1990). Scientists can switch between R&D subsectors at the beginning of period t' . There is a standing-on-shoulders effect in research such that a scientist in R&D subsector $j \in \{c, d\}$ starts $N_{t'}^j$ projects in the beginning of period t' . A project can either fail or succeed. For each successful project, the scientist develops a new machine variety which can be produced and used from the beginning of period $t' + 1$. The success probability for each project is: $\eta^j (N_{t'}^j)^{-\phi^j} < 1$, where $0 < \eta^j < 1$, and $\phi^j > 0$. The success probability decreases with the technological level, as the easiest ideas are invented first: a fishing-out effect. The parameters ϕ^c and ϕ^d measures the strengths of the fishing-out effects in the two R&D subsectors. The inclusion of fishing-out effects is motivated by the empirical evidence from Chapter 1 and Bloom et al. (2017). The inclusion of fishing-out effects allows the model to eliminate both the implausible lock-in equilibrium discussed in Section 2 and the strong scale effect discussed by Jones (2005).

The structure described above leads to the following evolutions in the technological levels:

$$N_{t+1}^j = \left(1 + \eta^j (N_t^j)^{-\phi^j} s_t^j\right) N_t^j, \quad N_0^j \geq 1, \quad 0 < \eta^j < 1, \quad \phi^j > 0, \quad (4)$$

where s_t^j measures scientist input in R&D subsector $j \in \{c, d\}$. The number of machine varieties in subsector j in period $t' + 1$ equals the number of varieties in the subsector in period t' plus the varieties developed through period t' . The latter is given by the total number of projects (number of scientists times the number of projects per scientist), $s_t^j N_t^j$, times the success probability per project, $\eta^j (N_t^j)^{-\phi^j}$.

The market clearing condition for scientist input requires that:

$$s_t = s_t^c + s_t^d. \quad (5)$$

Finally, for reasons outside the model, a constant fraction ω of the population is allocated to research and the remaining fraction, $(1 - \omega)$, is allocated to manufacturing. Thus,

$$L_t = (1 - \omega)\bar{L}_t \quad \text{and} \quad s_t = \omega\bar{L}_t, \quad 0 < \omega < 1. \quad (6)$$

3.2 The market economy

In the market economy, consumption goods and intermediate goods are produced under perfect competition and the labor market is perfectly competitive. New machine varieties are produced under monopolistic competition, while old machine varieties are produced under perfect competition.

The first-order conditions of the representative firm producing consumption goods imply

$$\left(\frac{Y_t^c}{Y_t^d}\right) = \left(\frac{p_t^c}{p_t^d}\right)^{-\epsilon}, \quad (7)$$

where p_t^c and p_t^d denote the prices of clean and dirty intermediate goods, respectively. The consumption good is numéraire, implying that

$$\left[(p_t^c)^{1-\epsilon} + (p_t^d)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = 1.$$

The first-order conditions of the two representative intermediate good producers give the implicit demand functions:

$$A^\alpha \left(N_t^j\right)^{\alpha-1+\alpha\psi} \left(Y_t^j\right)^{1-\alpha} \left(x_{kt}^j\right)^{\alpha-1} p_t^j = p_{kt}^j, \quad (8)$$

where p_{kt}^j denote the price of machine $k \in \{i, h\}$ used in subsector $j \in \{c, d\}$.

If a scientist develops a new machine variety in period t' , he/she receives a one-period patent on that machine variety. Hence the scientist becomes a monopolist of that machine variety in period $t' + 1$. Thereafter, the machine variety is produced under perfect competition. To correct the market failure associated with the monopoly power on new machine varieties, the government pays a share $(1 - \alpha)$ of the production costs for new machine producers. This subsidy is financed through lump-sum taxes.

The monopolists maximize profits subject to (8). The supply of machines - both new and

old varieties - is given by

$$x_{kt}^j = \left(\frac{A^\alpha (N_t^j)^{\alpha-1+\alpha\psi} p_t^j}{w_t} \right)^{\frac{1}{1-\alpha}} Y_t^j \equiv \hat{x}_t^j, \quad (9)$$

where w_t is the wage rate. It then follows that the price of any machine equals the wage rate. Per-period profits for new machine producers are then given by

$$\pi_{kt}^j = (1 - \alpha) w_t \hat{x}_t^j \equiv \hat{\pi}_t^j, \quad (10)$$

where π_{it}^c and π_{ht}^d denote profits for monopolist $i \in (N_{t-1}^c, N_t^c]$ and $h \in (N_{t-1}^d, N_t^d]$, respectively. Meanwhile, old machine producers obtain zero profits.

It follows from (2), (3), and (9) that

$$Y_t^c = A (N_t^c)^\psi L_t^c \quad \text{and} \quad Y_t^d = A (N_t^d)^\psi L_t^d. \quad (11)$$

From (3), (7), (9), and (11) it follows that

$$\left(\frac{p_t^c}{p_t^d} \right) = \left(\frac{N_t^c}{N_t^d} \right)^{-\psi}, \quad \left(\frac{Y_t^c}{Y_t^d} \right) = \left(\frac{N_t^c}{N_t^d} \right)^{\epsilon\psi}, \quad \text{and} \quad \left(\frac{L_t^c}{L_t^d} \right) = \left(\frac{N_t^c}{N_t^d} \right)^{(\epsilon-1)\psi}. \quad (12)$$

The supply of consumption and intermediate goods are computed from (1), (3), (6), (11), and (12):

$$Y_t^c = A \left[(N_t^c)^{-(\epsilon-1)\psi} + (N_t^d)^{-(\epsilon-1)\psi} \right]^{-1} (N_t^c)^\psi (N_t^d)^{-(\epsilon-1)\psi} (1 - \omega) \bar{L}_t, \quad (13)$$

$$Y_t^d = A \left[(N_t^c)^{-(\epsilon-1)\psi} + (N_t^d)^{-(\epsilon-1)\psi} \right]^{-1} (N_t^d)^\psi (N_t^c)^{-(\epsilon-1)\psi} (1 - \omega) \bar{L}_t, \quad \text{and} \quad (14)$$

$$C_t = A \left[(N_t^c)^{-(\epsilon-1)\psi} + (N_t^d)^{-(\epsilon-1)\psi} \right]^{\frac{1}{\epsilon-1}} (N_t^c)^\psi (N_t^d)^\psi (1 - \omega) \bar{L}_t. \quad (15)$$

The struggle between the neo-Malthusian effect and the Simon effect is already apparent from the expression of Y_t^d . Assume that all scientists work in the clean R&D subsector. In this case, N_t^c increases while N_t^d remains constant. If $\epsilon > 1$, pollution emission per capita, Y_t^d / \bar{L}_t , unambiguously decreases in the long run. Meanwhile, the evolution of aggregate pollution emission depends on how fast the population grows compared to how fast pollution emission per capita decreases.

Let $\tilde{\pi}_t^c$ and $\tilde{\pi}_t^d$ denote the expected discounted profits for scientists conducting research

in the clean and dirty R&D subsector, respectively. These discounted profits consist of four factors: (i) the discount rate, (ii) the number of projects, (iii) the success probability per project, and (iv) the value per obtained patent. Accordingly,

$$\tilde{\pi}_t^j = \underbrace{\frac{1}{(1+r_{t+1})}}_{\text{Discount rate}} \times \underbrace{\binom{N_t^j}{}}_{\text{Projects per scientist}} \times \underbrace{\eta^j (N_t^j)^{-\phi^j}}_{\text{Success probability per project}} \times \underbrace{\hat{\pi}_t^j}_{\text{Profit per one-period patent}}$$

where r_{t+1} is the real interest rate.

The ratio of expected discounted profits guides the scientists' decisions, as the scientists maximize expected discounted profits. The (expected discounted) profit ratio amounts to

$$\left(\frac{\tilde{\pi}_t^c}{\tilde{\pi}_t^d} \right) = \underbrace{\left(\frac{\eta^c (N_t^c)^{-\phi^c}}{\eta^d (N_t^d)^{-\phi^d}} \right)}_{\text{Success probability effect}} \times \underbrace{\left(\frac{\hat{x}_{t+1}^c}{\hat{x}_{t+1}^d} \right)}_{\text{Market size effect}} \times \underbrace{\left(\frac{N_t^c}{N_t^d} \right)}_{\text{Standing-on-shoulders effect}}. \quad (16)$$

Three effects determine the profit ratio. The *success probability effect* has not been emphasized in the previous literature, where it is state independent. Scientists are prone to research in a subsector, the greater the success probability per project. Due to fishing-out effects, the success probability per project decreases with the technological level. The *market size effect* reflects that innovation is directed toward the relatively largest subsector.¹¹ Finally, the *standing-on-shoulders effect* reflects that researchers can start more projects in the technologically forward subsector. The success probability effect drags innovation toward the technologically backward subsector, while the market size effect and the standing-on-shoulders effect drag innovation toward the technologically forward subsector.

The success probability effect turns out to be crucial. If the success probability effect is sufficiently strong, it can dominate the market size effect and the standing-on-shoulders effect, such that the implausible lock-in equilibrium is avoided. This requires relatively strong fishing-out effects represented by ϕ^c and ϕ^d .

The equilibrium profit ratio is derived from (3), (4), (5), (6), (12), and (16):

$$F(s_t^c, \bar{L}_t, N_t^c, N_t^d) \equiv \left(\frac{\tilde{\pi}_t^c}{\tilde{\pi}_t^d} \right) = \left(\frac{\eta^c}{\eta^d} \right) \times \left(\frac{1 + \eta^c (N_t^c)^{-\phi^c} s_t^c}{1 + \eta^d (N_t^d)^{-\phi^d} (\omega \bar{L}_t - s_t^c)} \right)^{(\epsilon-1)\psi-1} \times \frac{(N_t^c)^{(\epsilon-1)\psi-\phi^c}}{(N_t^d)^{(\epsilon-1)\psi-\phi^d}}.$$

The profit ratio is determined by the labor input in the clean R&D subsector given the state

¹¹The market size effect emphasized here essentially includes both the market size effect and the price effect emphasized by AABH. This is clear from (9). But since there is no parameter link between α and ψ , the market size effect emphasized in the present study incorporates an additional technological component, cf. (9).

variables \bar{L}_t , N_t^c , and N_t^d . Now consider the following lemma.

Lemma 1. *Assume that $(\epsilon - 1)\psi < 1$:*

1. *If $1 \leq F(\omega\bar{L}_t, \cdot)$ then $(s_t^c, s_t^d) = (\omega\bar{L}_t, 0)$ is a unique equilibrium in the R&D sector.*
2. *If $F(0, \cdot) \leq 1$ then $(s_t^c, s_t^d) = (0, \omega\bar{L}_t)$ is a unique equilibrium in the R&D sector.*
3. *If $F(\omega\bar{L}_t, \cdot) < 1 < F(0, \cdot)$ then $(s_t^c, s_t^d) = (s_t^*, \omega\bar{L}_t - s_t^*)$ is a unique equilibrium in the R&D sector, where s_t^* is the unique solution to $F(s_t^*, \cdot) = 1$.*

Proof. See Appendix B. □

The assumption $(\epsilon - 1)\psi < 1$ ensures a unique equilibrium, as the profit ratio is strictly decreasing in s_t^c . Without this parameter restriction, there might be multiple equilibria, and coordination among the scientists becomes important. The parameter assumption is, however, not important for the obtained results.¹²

3.3 Qualitative calibration

To focus the analysis on empirically plausible cases, this section restricts certain parameter values further. The global population size has been increasing since 1 million BC (Kremer 1993), and the United Nations (2017) expects that the global population size continues to grow at least until 2100. Thus a positive population growth rate, $n > 0$, is assumed through most of this analysis. Based on empirical evidence presented by Papageorgiou et al. (2017), clean and dirty intermediate goods are assumed gross substitutes, $\epsilon > 1$. This assumption remains controversial, and the results presented below therefore represent the optimistic case, where it is relatively easy to substitute between polluting and non-polluting production technologies. Additionally, the parameter restriction $(\epsilon - 1)\psi < 1$ is imposed to ensure a unique equilibrium in the R&D sector.

The following parameter restriction summarizes these assumptions:

Parameter Restriction 1. $n > 0$, $\epsilon > 1$, and $(\epsilon - 1)\psi < 1$.

The model dynamics are strongly affected by the initial conditions and the parameter values: ϵ , ψ , ϕ^c , and ϕ^d . While the initial conditions matter quantitatively, the long-run

¹²The same policy implications can be obtained in a model with $(\epsilon - 1)\psi \geq 1$, where it is assumed that all scientists work in a specific R&D subsector in case of multiple equilibria. The reason is that the parameter restriction ensuring an escape from the lock-in equilibrium is unchanged, and that parameter restriction implies that the neo-Malthusian effect always dominates the Simon effect in the long run.

qualitative behavior of the profit ratio is determined by the parameter values. Consider the following lemma.

Lemma 2. *Assuming that Parameter Restriction 1 holds, the profit ratio will, in the long run, equal one or fluctuate around one under laissez-faire if $(\epsilon - 1)\psi < \phi^c$ and $(\epsilon - 1)\psi < \phi^d$.*

Proof. See Appendix B. □

If the conditions from Lemma 2 are not fulfilled, the profit ratio might go to infinity or converge to zero depending on the initial conditions, and thus the model features the implausible lock-in equilibrium discussed above.

What is the empirically plausible case? Consider the equilibrium pollution intensity of the manufacturing sector:

$$\frac{Y_t^d}{C_t} = \left[(N_t^c)^{-(\epsilon-1)\psi} + (N_t^d)^{-(\epsilon-1)\psi} \right]^{-\frac{\epsilon}{\epsilon-1}} (N_t^c)^{-\epsilon\psi}.$$

When research is locked to the dirty R&D subsector, the pollution intensity converges to a constant from below under Parameter Restriction 1. This is clearly at odds with the decreasing pollution intensity shown in the right panel of Figure 1.¹³ Thus research cannot be locked to the dirty R&D subsector under laissez-faire. Research cannot be locked to the clean R&D subsector either, as this would imply a decreasing CO₂ emission per capita, cf. (14). This prediction is clearly counterfactual, as the global CO₂ emission per capita increased from around 3 tonnes in 1960 to around 5 tonnes in 2014. Thus research must, in the long run, be conducted in both R&D subsectors in the absence of severe policy interventions. To ensure this, the following parameter restriction is imposed based on Lemma 2.

Parameter Restriction 2. $(\epsilon - 1)\psi < \phi^c$ and $(\epsilon - 1)\psi < \phi^d$.

This parameter restriction is crucial for the policy implications which is discussed further in the subsequent section. The restriction also allows the model to match the empirical patterns from Figure 1 (see Section 5).

4 Policy Implications

This section investigates which policies that are able to ensure environmental sustainability. Here, the long-run evolution in pollution emission is key. To understand the core mechanisms

¹³If $\epsilon < 1$, the pollution intensity approaches infinity, as the dirty technological level approaches infinity.

governing the policy implications, this section first investigates the relative strengths of the neo-Malthusian effect and the Simon effect. First consider the following definition:

Definition 1. *Research has **clean bias** if it is permanently and fully directed toward the clean R&D subsector. Likewise, research has **dirty bias** if it is permanently and fully directed toward the dirty R&D subsector.*

The full strength of the Simon effect is obtained when research has clean bias. Thus if the neo-Malthusian effect dominates the Simon effect in this case, clean bias research is not sufficient to ensure environmental sustainability.

Now consider the following decomposition of pollution emission:

$$Y_t^d = \frac{Y_t^{d,\text{neo}} \times Y_t^{d,\text{Simon}}}{Y_0^d}, \quad Y_t^{d,\text{neo}} = \left(\frac{Y_0^d}{\bar{L}_0} \right) \bar{L}_t, \quad \text{and} \quad Y_t^{d,\text{Simon}} = \left(\frac{Y_t^d}{\bar{L}_t} \right) \bar{L}_0,$$

where $Y_t^{d,\text{neo}}$ is pollution emission holding technology constant, and $Y_t^{d,\text{Simon}}$ is pollution emission with a constant population size, but technological development as under population growth. The neo-Malthusian effect and the Simon effect becomes apparent when computing the growth factor of pollution emission:

$$(1 + g_{Y^d,t}) = \underbrace{(1 + g_{Y^{d,\text{neo}},t})}_{\text{neo-Malthusian effect}} \times \underbrace{(1 + g_{Y^{d,\text{Simon}},t})}_{\text{Simon effect}},$$

where the growth factor of a variable V is denoted $(1 + g_{V,t})$.

To formally investigate the long-run relative strengths of the two effects under clean bias, the following lemma states the long-run technological growth rates when research is biased:

Lemma 3. *Assume that Parameter Restriction 1 holds. If research has clean bias, the long-run growth factor of N_t^c denoted $(1 + g_{N^c})$ equals $(1 + n)^{\frac{1}{\phi^c}}$, while the long-run growth factor of N_t^d denoted $(1 + g_{N^d})$ equals zero. Likewise, if research has dirty bias $(1 + g_{N^c})$ equals zero and $(1 + g_{N^d})$ equals $(1 + n)^{\frac{1}{\phi^d}}$.*

Proof. See Appendix B. □

From Lemma 3 it follows that if research has clean bias, the long-run growth factor of pollution emission amounts to

$$(1 + g_{Y^d,t}) = \underbrace{(1 + n)}_{\text{neo-Malthusian effect}} \times \underbrace{\frac{Y_{t+1}^d}{Y_t^d} (1 + n)^{-1}}_{\text{Simon effect}} \xrightarrow{t \rightarrow \infty} \underbrace{(1 + n)}_{\text{neo-Malthusian effect}} \times \underbrace{(1 + n)^{-\frac{(\epsilon-1)\psi}{\phi^c}}}_{\text{Simon effect}}.$$

The expression illustrates the struggle between the neo-Malthusian effect and the Simon effect. The neo-Malthusian effect directly affects pollution emission through an increase in the amount of workers in manufacturing (effect on extensive margin). The Simon effect comes into play as a higher population growth rate leads to a faster increase in the research capacity of the economy and thereby a faster development of clean technologies. Since there is no development of dirty technologies due to clean bias research, the relative use of dirty technologies decreases, implying less pollution emission per worker (effect on intensive margin). This effect is weakened by the fishing-out effects in the clean R&D subsector, as stronger fishing-out effects imply slower technological development. In contrast, the effect is amplified by a higher elasticity of substitution between clean and dirty technologies, ϵ , as this implies less costly input substitution, and thereby larger changes in the input mix for changes in the relative technological level, N_t^c/N_t^d . Additionally, the Simon effect is strengthened by the productivity gains from machine varieties, represented by ψ . Larger productivity gains from machine varieties imply larger productivity gains from research, and thus, a faster change in the relative productivity of clean and dirty technologies.

The following proposition formalizes these considerations:

Proposition 1. *Assuming that Parameter Restriction 1 holds and that research has clean bias, then aggregate pollution emission:*

- (i) *decreases in the long run at a constant rate if and only if $(\epsilon - 1)\psi > \phi^c$,*
- (ii) *increases in the long run at a constant rate if and only if $(\epsilon - 1)\psi < \phi^c$, and*
- (iii) *remains constant in the long run if and only if $(\epsilon - 1)\psi = \phi^c$.*

Proof. See Appendix B. □

It follows from Proposition 1 that in the empirically plausible case where Parameter Restriction 2 holds (see Section 3.3), the neo-Malthusian effect always dominates the Simon effect in the long run even when research has clean bias. Note that this result holds for all values of ω , implying that it does not rest on the assumption of a fixed intersectoral labor allocation.

Intuitively, to avoid the implausible lock-in equilibrium, the fishing-out effects represented by ϕ^c and ϕ^d must be relatively strong. Strong fishing-out effects imply low research productivity which implies a weak Simon effect. In fact, the requirement ensuring a departure from the lock-in equilibrium implies that the Simon effect is weaker than the neo-Malthusian effect in the long run, even when research has clean bias.

4.1 Research subsidies

The research subsidy considered is a subsidy to profits for new clean machine producers which ensures that research has clean bias. The subsidy is combined with a profit tax for new dirty machine producers to reduce the cost of the subsidy. The profit tax would not provide any revenue since research has clean bias, but it reduces the cost of the subsidy, as it makes the clean R&D subsector relatively more attractive.

A temporary subsidy can ensure that research has clean bias for as long as it lasts. But when it expires, it follows from Lemma 2 that in the empirically plausible case, research is conducted in both R&D subsectors in the long run. Hence a temporary subsidy cannot ensure clean bias. In contrast, a permanent research subsidy can ensure clean bias. But in the empirically plausible case where Parameter Restriction 2 holds, Proposition 1 implies that these subsidies cannot ensure environmental sustainability.

These results are summarized in the following proposition:

Proposition 2. *Assuming that Parameter Restriction 1 and 2 hold:*

- (i) *a temporary subsidy to the clean R&D subsector cannot permanently direct all research toward the clean R&D subsector.*
- (ii) *neither a temporary nor a permanent research subsidy to the clean R&D subsector can ensure environmental sustainability.*

Proof. See Appendix B. □

Intuitively, research subsidies cannot ensure environmental sustainability, as the neo-Malthusian effect always dominates the Simon effect in the long run. Thus even though all research efforts aim at reducing pollution emission per worker, aggregate pollution emission still grows, in the long run, due to a growing workforce. To ensure environmental sustainability, it is necessary to use an additional policy instrument to tip the balance. In the next section, it is shown that a tax on pollution emission can do just that.

The policy implications summarized in Proposition 2 contrast to previous results. AABH find that an environmental disaster can be avoided using a *temporary* subsidy to clean research. The results obtained in the present study differ for two reasons. First, the introduction of population growth implies that aggregate pollution emission might increase despite decreasing pollution emission per worker caused by clean bias research. In fact, a *permanent* research subsidy can ensure environmental sustainability in the absence of population growth, cf. Section 4.3 below. Second, the parameter restrictions ensuring that the

implausible lock-in equilibrium is avoided implies that the profit ratio is attracted to one under laissez-faire. Hence, in contrast to AABH, only a permanent research subsidy can ensure clean bias research.

4.2 A pollution tax

The government imposes a tax, $\tilde{\tau}_t$, per unit of pollution emission. The tax is paid by the consumption good producers, and the tax revenue is transferred lump-sum to the consumers and/or used to finance research subsidies. The price for purchasing and using a dirty intermediate good becomes: $p_t^d + \tilde{\tau}_t$. This price is rewritten as: $p_t^d \tau_t$, where $\tau_t \equiv 1 + \tilde{\tau}_t/p_t^d$. The variable τ_t is referred to as the *pollution penalty*, as it reflects the penalty (introduced by the pollution tax) associated with dirty intermediate good use.

When the pollution penalty is introduced, dirty intermediate good use amounts to:

$$Y_t^d = A \left[\left(N_t^c \right)^{-(\epsilon-1)\psi} \tau_t^{-\epsilon} + \left(N_t^d \right)^{-(\epsilon-1)\psi} \right]^{-1} \left(N_t^c \right)^{-(\epsilon-1)\psi} \tau_t^{-\epsilon} \left(N_t^d \right)^{\psi} (1 - \omega) \bar{L}_t.$$

The pollution penalty reduces the incentive to use dirty intermediate goods in the production process: a *production input mix effect*. This effect is amplified by the elasticity of substitution, as a higher elasticity implies a lower cost of input substitution.

First consider a combination of a permanent research subsidy ensuring clean bias and a constant pollution penalty. In this case, the pollution emission tax would increase over time at the same rate as p_t^d which increases over time since research has clean bias. Environmental sustainability is not obtained in this case which is formally stated in the following proposition.

Proposition 3. *Assuming that Parameter Restriction 1 and 2 hold, and that research has clean bias, a constant pollution penalty, $\tau_t = \tau$, cannot ensure environmental sustainability.*

Proof. See Appendix B. □

The constant pollution penalty affects the level but not the long-run growth rate of dirty intermediate good use. As the long-run growth rate of pollution emission is positive under Parameter Restriction 2, environmental sustainability is not obtained.

To ensure environmental sustainability, the pollution penalty must increase over time. The following policy rule is considered:

$$\tau_t = (1 + g_\tau)^t \tau_0, \quad g_\tau > 0, \quad \tau_0 > 0, \quad (17)$$

where g_τ is the constant growth rate of the pollution penalty.

A tax on pollution emission can permanently direct research toward the clean R&D sub-sector if the pollution penalty is sufficiently large initially and grows sufficiently fast. In this case, the pollution tax also ensures environmental sustainability given that \bar{E} is sufficiently large. This result is summarized in the following proposition.

Proposition 4. *Assuming that Parameter Restriction 1 and 2 hold, a pollution tax can ensure both clean bias and environmental sustainability if it is sufficiently large initially, the pollution penalty grows by a constant factor above $(1+n)^{\frac{\phi^c - (\epsilon-1)\psi}{\epsilon\phi^c}}$, and \bar{E} is sufficiently large.*

Proof. See Appendix B. □

To understand Proposition 4, consider the long-run growth factor of pollution emission under clean bias:

$$(1 + g_{Y^d,t}) \xrightarrow{t \rightarrow \infty} \underbrace{(1+n)}_{\text{Neo-Malthusian effect}} \times \underbrace{(1+n)^{-\frac{(\epsilon-1)\psi}{\phi^c}}}_{\text{Simon effect}} \times \underbrace{(1+g_\tau)^{-\epsilon}}_{\text{Production input mix effect}}.$$

Pollution emission decreases in the long run if the Simon effect and the production input mix effect dominate the neo-Malthusian effect. As the neo-Malthusian effect dominates the Simon effect, this requires a sufficiently fast increase in the pollution penalty. Both the neo-Malthusian effect and the Simon effect are amplified by faster population growth. Hence environmental sustainability requires a lower pollution penalty growth rate, when the population growth rate is reduced.

4.3 Population control policies

Since the neo-Malthusian effect dominates the Simon effect in the long run, the above findings suggest that the environmental sustainability problem is largely caused by population growth. Thus intuitively it should be easier to ensure environmental sustainability in the absence of population growth. This intuition is confirmed by the following proposition.

Proposition 5. *Assuming that Parameter Restriction 2 holds, $n = 0$, $\epsilon > 1$, and $(\epsilon-1)\psi < 1$, then environmental sustainability can be obtained using a permanent research subsidy ensuring that research has clean bias if \bar{E} is sufficiently large.*

Proof. See Appendix B. □

Even with a population growth rate of zero, there is still perpetual growth in the clean technological level. The growth rate decreases over time, but the clean technological level approaches infinity as time approaches infinity (see Groth et al. [2010] for a thorough examination of less-than-exponential growth). Since the dirty technological level is constant given clean bias, pollution emission per worker decreases over time, while the amount of workers remains constant. Thus aggregate pollution emission decreases in the long run which ensures environmental sustainability if \bar{E} is sufficiently large.

4.4 Robustness

In Appendix C, it is shown how the intersectoral labor allocation can be endogenized without changing the environmental policy implications. This supplementary model is based on an overlapping generations model with two generations: young and old. Only the young work, and they save for retirement by investing in R&D firms. Appendix D shows how the policy implications are robust to the introduction of stepping-on-toes effects in research. These stepping-on-toes effects slowdown technological development and work much like the fishing-out effects in the long run. Finally, Appendix E presents a model with knowledge spillovers between the two R&D subsectors. In this model, there might exist a parameter space in which the lock-in equilibrium can be avoided, while the Simon effect dominates the neo-Malthusian effect in the long run under clean bias. However, the above findings carry through outside this parameter space.

5 Simulations

The analytical analysis conducted above has two limitations. First, the results are based on the asymptotic properties of the model. Keeping the global temperature increase below 2 degrees Celsius requires substantial action within this century, and thus, it might be insufficient to only consider these asymptotic policy implications. Second, the analytical results are derived assuming exponential population growth which is at odds with expected future population growth patterns. To confront these matters, this section provides a quantitative analysis for the period 2014-2100, based on population projections from the United Nations.

5.1 Model adjustments

The model from Section 3 is adjusted to improve its empirical relevance. First, global population size data are used directly such that \bar{L}_t equals the estimated or projected global population size in billions. The population size data from the HYDE database (Goldewijk et al. 2010, 2011) are used for years prior to 1950, and population estimates and projections from the United Nations are used thereafter (United Nations 2017). Further information on the population size data is provided in Appendix F. The expected decreasing population growth rate is thereby taken into account in the following forecasts. Yet, simulations based on the average expected population growth rate from 2014 to 2100 provide similar results.

The second adjustment is that a carbon cycle replaces the general law of motion for the pollution stock. The idea is to construct a simple system capturing the main aspects of the carbon cycle. Following Nordhaus (2016), the climate system is given by

$$\begin{bmatrix} E_{t+1} \\ O_{t+1} \\ Q_{t+1} \end{bmatrix} = \begin{bmatrix} E_t \\ O_t \\ Q_t \end{bmatrix} \Pi + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \mu \zeta Y_t^d,$$

where E_t , O_t , and Q_t are the CO₂ concentrations (above the pre-industrial level) in the atmosphere, upper oceans, and deep oceans, respectively. Emitting one model unit of CO₂ corresponds to $\zeta > 0$ gigatonne (GT) CO₂ emission which increases the atmospheric CO₂ concentration by $\mu > 0$ parts per million (ppm). Π is a 3×3 transition matrix specifying the CO₂ transfers between the three reservoirs: E_t , O_t , and Q_t . There are no direct transfers between the atmosphere and deep oceans.

Finally, it is necessary to put more structure on consumer preferences to compute GDP which is the sum of value added in manufacturing and research. Value added in manufacturing equals the sum of value added by machine producers, intermediate good producers, and consumption good producers which amount to the value of aggregate consumption, C_t . Scientists in the research sector generate valuable assets: patents. Value added by the R&D sector is therefore the net present value of all patents obtained within the period. Accordingly, GDP amounts to:

$$GDP_t = C_t + \left(N_{t+1}^c - N_t^c \right) \frac{\pi_{t+1}^c}{1 + r_{t+1}} + \left(N_{t+1}^d - N_t^d \right) \frac{\pi_{t+1}^d}{1 + r_{t+1}},$$

where $(N_{t+1}^j - N_t^j)$ is the number of new patents obtained in R&D subsector j , and the present value of a subsector j patent is given by $\pi_{t+1}^j / (1 + r_{t+1})$.

To compute GDP, one needs to know how consumers intertemporally value consumption. Let consumer preferences be represented by the welfare function:

$$\sum_{t=0}^{t_{\max}} \frac{c_t^{1-\theta}}{1-\theta} \bar{L}_t \beta^t, \quad c_t \equiv C_t / \bar{L}_t, \quad 0 < \beta < 1, \quad \theta > 0, \quad \theta \neq 1.$$

The implied real interest rate is: $r_{t+1} = \beta^{-1} (c_{t+1}/c_t)^\theta - 1$.

5.2 Calibration and estimation

The estimation is based on model simulations for the period 1744-2014 using period lengths of one year. The estimation procedure consists of two steps. First, the parameters ϕ^c and ϕ^d are estimated by minimizing the sum of squared deviations between actual and simulated average growth rates of GDP and CO₂ emission for the period 1960-2014. Hence the model is estimated such that it matches the patterns shown in Figure 1. The initial technological level is set to $(N_0^c, N_0^d) = (1, 1)$, and the baseline parameter values are: $A = 1$, $\alpha = 0.4$, $\beta = 0.985$, $\psi = 2$, $\eta^c = \eta^d = 0.7$, $\omega = 0.02$, and $\theta = 1.5$.¹⁴ The parameter values are set such that the average annual growth rate of GDP per capita over the period 2014-2100 is approximately 1.9 pct. This calibration target largely follows Nordhaus (2016, p. 6). The simulation results appear robust to changes in the baseline parameter values as long as the estimation and calibration targets are held constant.

The dynamic system is strongly affected by ϵ , ϕ^c , and ϕ^d . To ensure that GDP and CO₂ emission grow sufficiently fast, ϕ^c and ϕ^d must be sufficiently small. On the other hand, they must be large enough compared to ϵ such that the lock-in equilibrium is avoided. An ϵ value equal to or above 1.5 results in a poor fit. Thus the simulations presented below are based on $\epsilon = 1.4$. The simulation results represent the optimistic case or upper bound, as more aggressive environmental policies are needed to reach a given climate target for lower ϵ values. The estimated values of ϕ^c and ϕ^d are around 0.90 and 1.00, respectively. These estimates coincide well with recent micro-level evidence, indicating that spillovers from clean innovations are relatively larger than spillovers from dirty innovations (Dechezleprêtre et al. 2014). The simulated average GDP and CO₂ emission growth rates for the period 1960-2014 deviates from the actual average growth rates by less than 0.05 basis points, indicating a good fit on this account.

¹⁴The discount factor, β , is taken from Golosov et al. (2014). The relative risk aversion parameter, θ , is consistent with estimates from Havránek (2015). UNESCO (2015) finds that almost 2 pct. of global GDP was allocated to R&D in 2013 which motivates the choice of ω .

Second, the transition matrix, Π , is estimated by minimizing the sum of squared deviations between the actual and predicted CO_2 concentrations in the atmosphere from 1744 to 2014 given the simulated CO_2 emissions. Further information on the CO_2 concentration data is provided in Appendix G. To minimize the number of estimated parameters, it is assumed that 0.147 pct. of the CO_2 in the deep oceans are transferred back to the upper oceans, and that 0.7 pct. of the CO_2 in the upper oceans are transported to the deep oceans.¹⁵ The parameter ζ is computed such that simulated pollution emission equals actual pollution emission in 2014 measured in GT CO_2 . One GT CO_2 increases the CO_2 concentration in the atmosphere by 0.128 ppm. Thus μ is set equal to this value.¹⁶ The pre-industrial CO_2 concentration is set to 280 ppm. Consistent with the above definitions, the initial CO_2 concentrations in the three reservoirs are set to zero in 1744. The resulting transition matrix implies slow CO_2 transfers between the three reservoirs. Actual and simulated CO_2 concentration paths are shown in Appendix I.

5.3 Simulation results

Laissez-faire

Figure 2 shows projected CO_2 concentration paths for a laissez-faire economy under different population scenarios. The baseline scenario is based on the medium variant of the United Nations population projections. In this scenario, the CO_2 concentration in 2100 is just above 800 ppm: slightly below the DICE-2016R baseline scenario (see Nordhaus 2016). The left panel of Figure 2 shows CO_2 concentration paths for the low, medium, and high variants of the United Nations population projections. There is a substantial difference between the concentration levels in 2100 for these three scenarios, highlighting the importance of population growth.

The right panel of Figure 2 shows the concentration path in the absence of population growth. In this case, the concentration in 2100 is reduced substantially compared to the baseline. If the concentration limit is 500 ppm, then over half of the necessary concentration reduction from the baseline is obtained by keeping the population size constant. Still, even in the absence of population growth, the concentration in 2100 is far above the range, 425-520 ppm, in which the two-degree temperature increase is avoided with a reasonably high

¹⁵Both numbers are taken directly from DICE-2016R.

¹⁶One GT CO_2 equals 1/3.667 GT carbon and one GT carbon increases the atmospheric concentration of CO_2 by 1/2.130 ppm, implying that 0.128 GT CO_2 increases the atmospheric concentration of CO_2 by one ppm. The conversion factors are taken from Clark (1982, p. 467).

probability.¹⁷

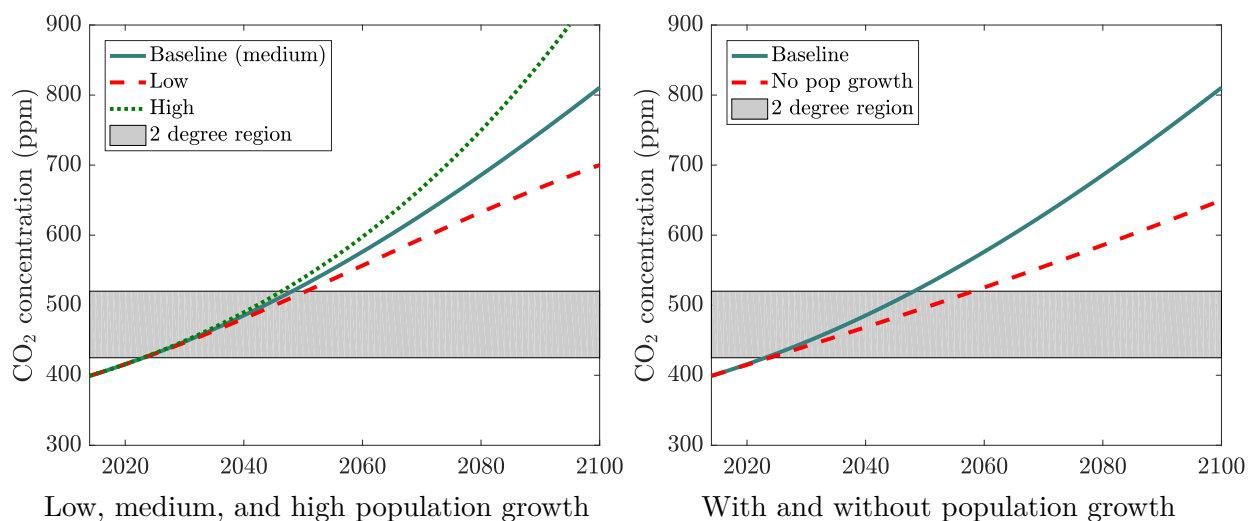


FIGURE 2: Projected CO₂ concentration paths for a laissez-faire economy under different population scenarios, 2014-2100.

Notes: The low, medium/baseline, and high population growth scenarios are based on the low, medium, and high population projections from the United Nations (2017).

Environmental policies

The baseline scenario indicates that substantial environmental policies are needed to ensure that the temperature increase remains below 2 degrees Celsius within this century. The analytical analysis highlights the effectiveness of a CO₂ tax in regards to climate change mitigation. The remaining part of this section examines: (i) how a CO₂ tax is efficiently implemented to reach a given CO₂ concentration target in 2100, and (ii) how efficient this instrument is compared to relevant alternatives.

For comparability with the analytical results, consider a tax policy following the tax rule (17). Following this rule, the (constrained) optimal environmental policy is computed by maximizing welfare with respect to the initial pollution penalty and the pollution penalty growth rate under the constraint of a specific CO₂ concentration in 2100. The implied CO₂ tax rate is derived from the pollution penalty and the equilibrium price of dirty intermediate goods.

Figure 3 shows optimal CO₂ tax rate paths for the two CO₂ concentration targets 500 and 550 ppm under different population scenarios. A 500 ppm concentration ensures that the two-degree limit is met with a reasonable probability. In contrast, there is little or no chance

¹⁷The range is based on IPCC (2014, Table 6.3). The range is also affected by the evolutions in other greenhouse gases which are treated as exogenous. There is a 32 to 84 pct. probability of exceeding the two-degree temperature limit within this range. In the context of the present analysis, the range illustrates how difficult it is to ensure that the two-degree temperature limit remains unviolated with a reasonably high probability. The specific limit values should be interpreted with caution.

of staying below a two-degree temperature increase for a 550 ppm concentration. However, a three-degree temperature increase is highly unlikely for a 550 ppm concentration.

As shown in the left panel of Figure 3, the tax rate for the 500 ppm target increases very fast under the baseline population scenario, indicating that this target might be politically unfeasible at this stage. However, the tax rate increases much slower under low or no population growth. If the concentration target is set to 550 ppm, the tax rate path is substantially lower. Again, lower population growth implies lower CO₂ tax rate paths.

The optimal policies push the bulk of consumption per capita losses toward the end of the period due to consumption smoothing and discounting. Thus the environmental policies are gradually tightened. This is reflected by the CO₂ tax rate paths shown in Figure 3. If climate change damages are introduced, the optimal environmental policies become initially more tight to reduce damages at the beginning of the period, where per capita consumption has a relatively higher value. However, the CO₂ tax still grows over time, pushing consumption losses toward the end of the period, if climate change damages are modeled as in Golosov et al. (2014). This is shown in Figure 9 in Appendix H.

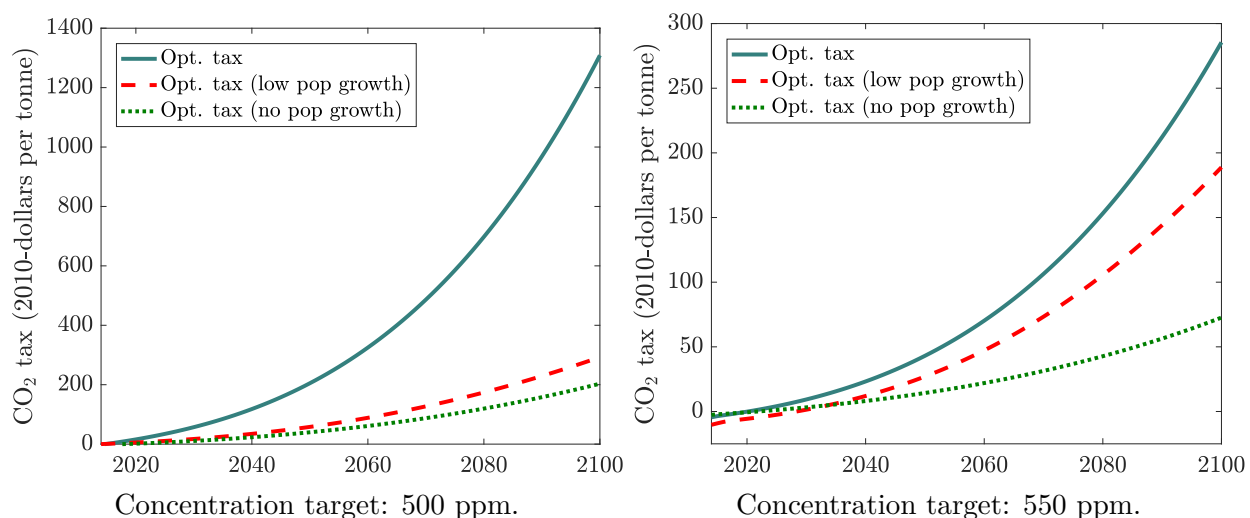


FIGURE 3: Optimal CO₂ tax rate paths under different population scenarios and CO₂ concentration targets, 2014-2100.

Notes: The tax policies follow the simple tax rule (17). The policies are optimal in the sense that they minimize the welfare loss of the representative household while achieving the CO₂ concentration target.

For the baseline population scenario, there is a large difference between the tax rate paths for the 500 and 550 ppm targets. This difference is mainly driven by one circumstance: concentration targets above 522 ppm are reachable without redirecting all scientists to the clean R&D subsector through the entire period. This is shown in the left panel of Figure 4.

After this 522 ppm threshold, additional concentration level reductions are driven entirely by the production input mix effect, as the effect from redirecting research toward the clean

R&D subsector has been fully exploited. Thus the environmental policy stringency, associated with more ambitious climate targets, starts to increase much faster after the 522 ppm threshold (see Figure 12 in Appendix I). This explains the substantial difference between the two baseline tax rate paths for the 500 and 550 ppm targets shown in Figure 3. As a consequence, the value of consumption per capita losses increases much faster after the 522 ppm threshold. This is clear from the right panel of Figure 4. The 522 ppm threshold might therefore be interesting for policy makers.

Reducing the population growth rate also reduces the threshold value. Under low population growth, the threshold (490 ppm) is within the aforementioned two-degree range, further illustrating the importance of population growth. This also explains why the tax rate paths shown in the left panel of Figure 3 are so sensitive to changes in the population scenario.

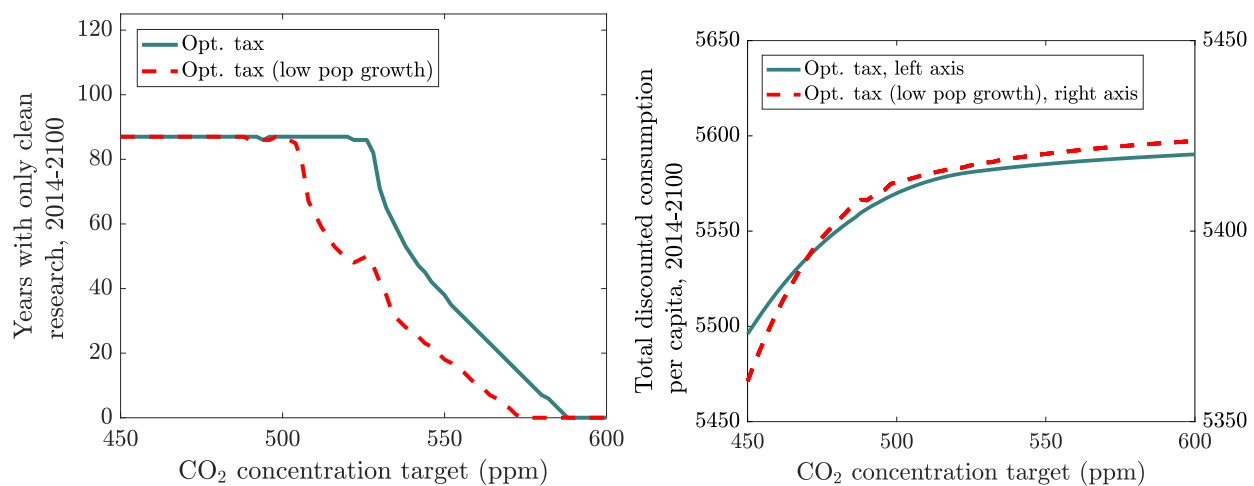


FIGURE 4: *Left panel:* years with only clean research over the period 2014-2100 for optimal tax policies as a function of the CO₂ concentration target in 2100. *Right panel:* total discounted consumption per capita over the period 2014-2100 for optimal tax policies as a function of the CO₂ concentration target in 2100.

CO₂ concentration paths for different policies are shown in Figure 5. The left panel of Figure 5 shows that a subsidy ensuring clean bias research results in a CO₂ concentration of 538 ppm in 2100. In the absence of population growth, the subsidy results in a CO₂ concentration of 497 ppm in 2100. Thus seen from a sustainability perspective, a combined subsidy and population control policy is a relevant alternative to a pure tax policy.

Emissions actually decrease over time under the two subsidy policies shown in the left panel of Figure 5. Hence the neo-Malthusian effect does not dominate the Simon effect over the relevant time period. It is, however, clear from the right panel of Figure 5 that reducing the population growth rate lowers the CO₂ concentration in 2100, indicating that the neo-Malthusian effect is weakened more than the Simon effect from population growth

reductions. Thus population growth continues to be a burden on the environment even when research has clean bias.

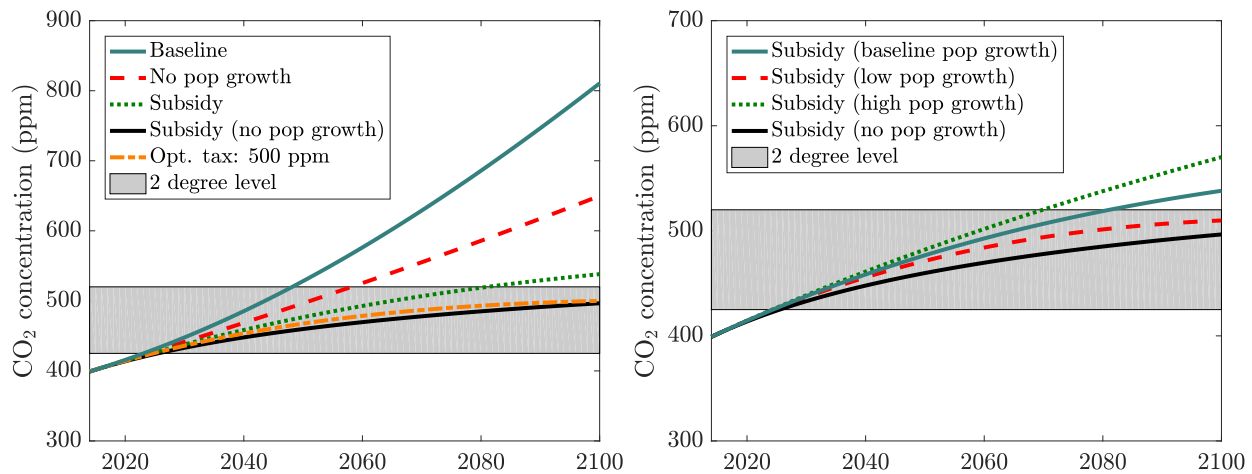


FIGURE 5: CO₂ concentration paths under different policies, 2014-2100.

Notes: All subsidy policies ensure that research has clean bias.

Welfare considerations

From a welfare perspective, the tax policies seem to be preferable vis-à-vis subsidy policies. Under baseline population growth, a tax policy can achieve the same CO₂ concentration in 2100 as a subsidy policy (538 ppm), while leading to a lower welfare loss. Intuitively, the tax policy postpones consumption losses, reducing the present value of these losses (see Figure 13 in Appendix I).

Figure 6 shows per capita consumption losses associated with the optimal tax policies relative to the baseline scenario. The left panel shows that under the baseline population scenario, the policies reduce consumption per capita relatively more toward the end of the period, and the loss is between zero and 2.1 pct. in all years. The CO₂ tax reduces the production efficiency through a distortion of intermediate good prices. It also distorts the allocation of scientists within the R&D sector, and this reduces productivity gains from research. But it does not affect the labor inputs in the two sectors. This explains the relatively small per capita consumption losses. The losses are, however, substantially larger for lower values of ϵ , as substitution away from dirty technologies becomes more costly.

The right panel of Figure 6 shows that the per capita consumption losses are notably higher, losses of over 20 pct. in 2100, with a low population growth rate. As population growth drives research input growth, a lower population growth rate significantly slows technological development, resulting in large consumption per capita losses.

Introducing climate damages substantially change the consumption loss profiles. This is shown in Figure 10 in Appendix H. In general, the tax policies increase relative consumption per capita notably under the baseline population scenario. Under low population growth, the tax policies still result in substantial relative consumption losses. But these relative losses are smaller than those obtained without climate change damages.

Assuming that population control policies are feasible and costless, the results shown in Figure 6 indicate that a pure tax policy is preferable compared to a combined tax and population control policy. The same holds for a comparison between a pure tax policy and a combined subsidy and population control policy (see Figure 13 in Appendix I). The optimal tax policies are derived from the welfare function presented above. Thus, the way the population size enters the welfare function matters for the optimal CO₂ tax rate paths. This might be considered problematic, due to the lack of general agreement about the way the population size should enter the welfare function (see IPCC [2014, ch. 3] for a discussion). However, since a population growth slowdown reduces research input growth, it also reduces consumption per capita growth. Hence a welfare function with less weight on the population size leads to similar policy implications. Nevertheless, it seems likely that the consumption loss figures associated with population control policies are exaggerated. This potential exaggeration results from the lack of regional heterogeneity with respect to population growth prospects and research intensities. This point is discussed further in Section 6.

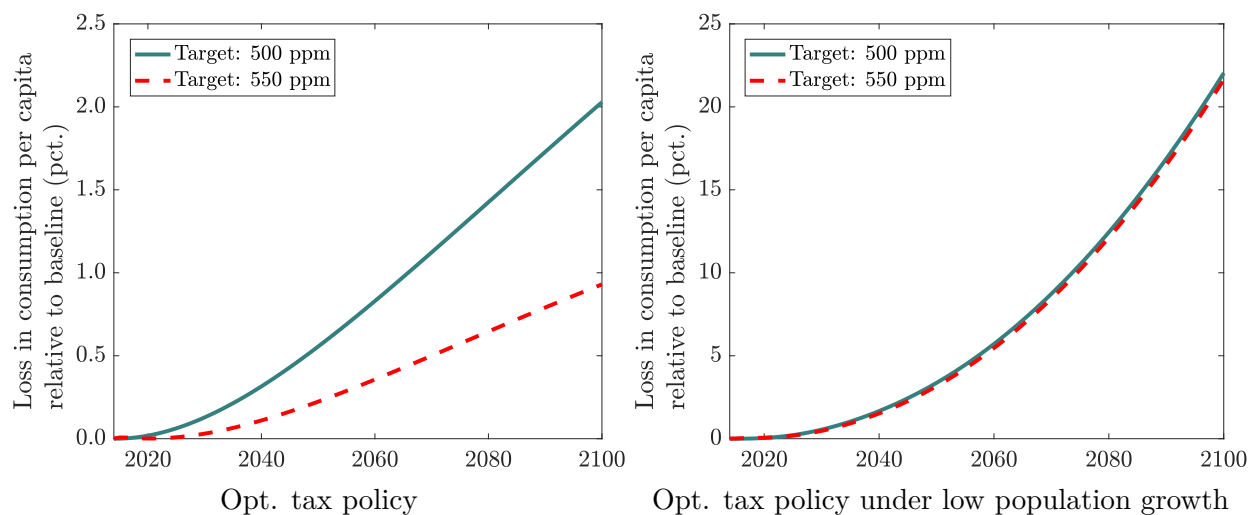


FIGURE 6: Consumption per capita losses associated with optimal tax policies relative to the baseline scenario, 2014-2100.

6 Discussion

A central assumption is the fixed allocation of labor between manufacturing and research. The assumption simplifies the analysis substantially, and it increases the focus on the intrasectoral labor allocations which are essential for environmental sustainability. A supplementary model with similar policy implications allowing for intersectoral labor mobility is presented in Appendix C. The assumption can also be motivated the following way. Given a highly skewed distribution of research skill, reallocating labor from manufacturing to research might have little effect on the effective research input (see Jaimovich and Rebelo 2017). Thus the central question continues to be how the effective research input is allocated within the R&D sector. Finally, one might argue that subsidies, ensuring that research has clean bias, change the profitability of research substantially over time. This increases the incentive for workers to move from manufacturing to research, calling the fixed intersectoral labor allocation into question. However, as mentioned in Section 4.1, the research subsidy is combined with an inactive profit tax on new dirty machine producers. The government can almost fully control the profitability of research using these two instruments. Hence the subsidy policy might be designed such that the incentive to conduct research is kept constant over time.

The concentration threshold from Section 5.3 is partly a consequence of the fixed sectoral labor allocation. In this semi-endogenous framework, the growth rates of the technological levels are driven by the research input growth rates. Due to the fixed sectoral labor allocation, policies cannot temporarily boost research input growth above the population growth rate which creates the threshold illustrated in Section 5. Without the fixed sectoral labor allocation, environmental policy could temporarily boost the research input growth rate above the maximum long-run rate through a combination of taxes and subsidies. But as argued above, a highly skewed distribution of research skill limits the potential scope of such policies. Hence the concentration threshold remains relevant even though the present framework might exaggerate the sharpness of it.

The fossil fuel input causing CO₂ emission is not explicitly modeled. As a consequence, the environmental policy actions necessary to obtain a given climate target might be exaggerated. In particular, if fossil fuels are scarce, the price of fossil fuels might increase over time due to scarcity rents. Consistency between fossil fuel use and pollution emission requires that the fossil fuel input is proportional to pollution emission. Introducing fossil fuels into

the framework developed above therefore corresponds to an additional pollution tax. This natural pollution penalty would reduce the pollution emission tax rate necessary to obtain a given climate target. However, as emphasized by Gerlagh (2011), global warming issues arise from the abundance of fossil fuels.¹⁸ Thus mechanisms like scarcity rents might be of little importance within the relevant time frame of global warming. This is further backed-up by the lack of empirical evidence supporting Hotelling's rule for fossil fuels (see Hart and Spiro 2011). Thus it seems naive to rely on scarcity rents to play a major role in climate change mitigation over the next 80 years. Note also that the Green Paradox (see Sinn 2008) would not necessarily arise in this framework, as the demand for fossil fuels is limited by the demand for dirty intermediate goods. In fact, the present framework corresponds to a case, where the fossil fuel price is fixed at zero.

The primary limitation of the present study is the treatment of the world economy as a single entity. The main issue is that research intensities and future population growth rates are generally negatively correlated. Developed regions tend to have high R&D intensities and low population growth prospects, while the opposite holds for developing regions. Africa is the most striking example. The United Nations expects the global population to increase by 3.6 billion from 2017 to 2100. Of these 3.6 billion extra individuals, 3.2 are expected to be Africans (United Nations 2017). Meanwhile only a tiny fraction of the global R&D investments are made in Africa (UNESCO 2015). Reducing population growth in developing regions is therefore likely to have a small impact on frontier technological development. Hence such population policies might be more effective than predicted above both in terms of consumption per capita losses and CO₂ emission reductions. On the other hand, the CO₂ emission per capita is relatively low in developing regions, reducing the climate impact of such policies. In addition, future climate change mitigation depends heavily on the ability of developing regions to adopt existing environmentally friendly production technologies. To investigate these issues further, the next natural step is to develop a two-region version of the model presented above.

¹⁸Stabilizing the global temperature increase within the 21st century requires a substantial decrease in CO₂ emissions from the world's energy sector (IPCC 2014, ch. 7). Seen from this perspective, fossil fuel reserves seem abundant. Based on known reservoirs that can be recovered in the future under existing economic and operating conditions, BP (2016) finds that the global coal reserves will last more than 110 years if future extraction continues at the 2015 level. The abundance of coal is further supported by Rogner (1997) and Rogner et al. (2012), who take changes in available technologies and yet unproven reserves into account.

7 Appendix

A Pollution Intensity in Acemoglu et al. (2012)

In this appendix, it is shown that the pollution intensity increases over time under laissez-faire in the model developed by Acemoglu et al. (2012, p. 134-141). All references in this appendix are references to Acemoglu et al. (2012).

There is no savings in the model, and thus GDP equals aggregate consumption. From (8):

$$GDP_t = C_t = Y_t - \psi \left(\int_0^1 x_{it}^c di + \int_0^1 x_{ht}^d dh \right),$$

where C_t is aggregate consumption, Y_t is aggregate output of final goods, x_{it}^c is clean machine i , and x_{ht}^d is dirty machine h . It takes ψ units of final goods to produce one unit of any machine.

The demand for machine $k \in \{i, h\}$ is given by (see page 160):

$$x_{kt}^j = \left(\frac{\alpha^2 p_t^j}{\psi} \right)^{\frac{1}{1-\alpha}} L_t^j A_{kt}^j \Leftrightarrow \left(\frac{x_{kt}^j}{A_{kt}^j} \right) = \left(\frac{\alpha^2 p_t^j}{\psi} \right)^{\frac{1}{1-\alpha}} L_t^j \equiv \hat{x}_t^j,$$

where $j \in \{c, d\}$, p_t^j is the price of intermediate j , L_t^j is labor input in sector j , and A_{kt}^j is the productivity associated with machine k .

The amount of final goods used to produce machines for a given sector is given by:

$$\psi \int_0^1 x_{kt}^j dk = \psi \int_0^1 \left(\frac{x_{kt}^j}{A_{kt}^j} \right) A_{kt}^j dk = \psi \hat{x}_t^j A_t^j, \quad A_t^j \equiv \int_0^1 A_{kt}^j dk,$$

where A_t^j is the average productivity associated with the machines in sector j . This amount can be expressed as a function of labor and intermediate input as well as the technological level of the sector:

$$Y_t^j = (L_t^j)^{1-\alpha} \int_0^1 (x_{kt}^j)^\alpha (A_{kt}^j)^{1-\alpha} dk = (L_t^j)^{1-\alpha} (\hat{x}_t^j)^\alpha A_t^j \Leftrightarrow \psi \int_0^1 x_{kt}^j dk = \psi (Y_t^j)^{\frac{1}{\alpha}} (A_t^j)^{\frac{\alpha-1}{\alpha}} (L_t^j)^{\frac{\alpha-1}{\alpha}},$$

where Y_t^j is intermediate good j .

It follows from (7), (19), and (A.5), that in equilibrium:

$$L_t^c = (A_t^d)^\varphi [(A_t^c)^\varphi + (A_t^d)^\varphi]^{-1}, \quad L_t^d = (A_t^c)^\varphi [(A_t^c)^\varphi + (A_t^d)^\varphi]^{-1},$$

$$Y_t^c = (A_t^c)(A_t^d)^{\alpha+\varphi} [(A_t^c)^\varphi + (A_t^d)^\varphi]^{-\frac{\alpha+\varphi}{\varphi}}, \quad \text{and} \quad Y_t^d = (A_t^c)^{\alpha+\varphi} (A_t^d) [(A_t^c)^\varphi + (A_t^d)^\varphi]^{-\frac{\alpha+\varphi}{\varphi}},$$

where $\varphi \equiv (1 - \epsilon)(1 - \alpha) < 0$.

It then follows that:

$$\psi \int_0^1 x_{it}^c di = \psi (A_t^c)(A_t^d)^{1+\varphi} [(A_t^c)^\varphi + (A_t^d)^\varphi]^{-\frac{1+\varphi}{\varphi}} \quad \text{and}$$

$$\psi \int_0^1 x_{ht}^d dh = \psi (A_t^c)^{1+\varphi} (A_t^d) [(A_t^c)^\varphi + (A_t^d)^\varphi]^{-\frac{1+\varphi}{\varphi}}.$$

Using the above and (19), it is straightforward to show that:

$$\psi \left(\int_0^1 x_{it}^c di + \int_0^1 x_{ht}^d dh \right) = \psi (A_t^c)(A_t^d) [(A_t^c)^\varphi + (A_t^d)^\varphi]^{-\frac{1}{\varphi}} = \psi Y_t.$$

Thus, aggregate consumption/GDP amounts to:

$$C_t = (1 - \psi)Y_t = (1 - \psi)(A_t^c)(A_t^d) [(A_t^c)^\varphi + (A_t^d)^\varphi]^{-\frac{1}{\varphi}}.$$

As a result, the pollution intensity is given by:

$$\left(\frac{Y_t^d}{C_t} \right) = \left(\frac{1}{1 - \psi} \right) [(A_t^c)^\varphi + (A_t^d)^\varphi]^{-\frac{\epsilon}{\epsilon-1}} (A_t^c)^{\alpha+\varphi-1}.$$

If research is conducted in the dirty sector only, then $A_t^d \rightarrow \infty$ for $t \rightarrow \infty$, while A_t^c remains constant over time. In this case:

$$\left(\frac{Y_t^d}{C_t} \right) \uparrow \left(\frac{1}{1 - \psi} \right) \quad \text{for } t \rightarrow \infty.$$

That is, the pollution intensity increases over time and converges to a positive constant which is inconsistent with empirical evidence. Note that research is directed toward the dirty sector from the outset due to Assumption 1. If research was directed toward the clean sector from the outset, the pollution intensity would decrease over time and approach zero which is compatible with the empirical evidence. The policy implications would, however, change substantially.

B Proofs

Proof of Lemma 1

Proof. It can easily be verified that:

$$\frac{\partial F(s_t^c, \cdot)}{\partial s_t^c} \gtrless 0 \quad \text{if} \quad (\epsilon - 1)\psi \gtrless 1.$$

Consider the case $(\epsilon - 1)\psi < 1$. In this case $F(\cdot)$ is strictly monotonically decreasing in s_t^c . It follows that: $F(0, \cdot) > F(\omega \bar{L}_t, \cdot)$. If $F(0, \cdot) > F(\omega \bar{L}_t, \cdot) > 1$ then $s_t^c = \omega \bar{L}_t$, as the profit ratio is greater than one for any value of s_t^c . If $1 > F(0, \cdot) > F(\omega \bar{L}_t, \cdot)$ then $s_t^c = 0$, as the profit ratio is less than one for any value of s_t^c . If $F(0, \cdot) > 1 > F(\omega \bar{L}_t, \cdot)$ then $s_t^c = s_t^*$, where s_t^* is the unique solution to $F(s_t^*, \cdot) = 1$. \square

Proof of Lemma 2

Proof. Consider the change in the growth rate of N_t^j given that research is permanently directed toward subsector j :

$$G_t^j \equiv \frac{g_{N^j, t+1}}{g_{N^j, t}} = (1 + n)(1 + g_{N^j, t})^{-\phi^j}, \quad g_{N^j, t} \equiv \frac{N_{t+1}^j - N_t^j}{N_t^j}.$$

It follows that if G_t^j is initially greater than one, then G_t^j converges to one from above, while G_t^j converges to one from below if G_t^j is initially below one. Accordingly,

$$(1 + g_{N^j, t}) \xrightarrow[t \rightarrow \infty]{} (1 + n)^{\frac{1}{\phi^j}}.$$

Consider the growth factor of the profit ratio when research is permanently directed towards the clean R&D subsector:

$$\frac{F(\omega \bar{L}_{t+1}, \bar{L}_{t+1}, N_{t+1}^c, N_t^d)}{F(\omega \bar{L}_t, \bar{L}_t, N_t^c, N_t^d)} = \left(\frac{(1 + g_{N^c, t+1})}{(1 + g_{N^c, t})} \right)^{(\epsilon-1)\psi-1} \times (1 + g_{N^c, t})^{(\epsilon-1)\psi-\phi^c}.$$

Given the long-run evolution in $(1 + g_{N^c, t})$, it follows that if research is permanently directed toward the clean R&D subsector then:

$$\frac{F(\omega \bar{L}_{t+1}, \bar{L}_{t+1}, N_{t+1}^c, N_t^d)}{F(\omega \bar{L}_t, \bar{L}_t, N_t^c, N_t^d)} \xrightarrow[t \rightarrow \infty]{} (1 + n)^{\frac{(\epsilon-1)\psi-\phi^c}{\phi^c}}.$$

When $(\epsilon - 1)\psi < \phi^c$ the growth factor is less than one, and the profit ratio converges to zero. Thus, the profit ratio cannot stay above one permanently.

Now consider the growth factor of the profit ratio when research is permanently directed towards the dirty R&D subsector:

$$\frac{F(0, \bar{L}_{t+1}, N_t^c, N_{t+1}^d)}{F(0, \bar{L}_t, N_t^c, N_t^d)} = \left(\frac{(1 + g_{N^d, t+1})}{(1 + g_{N^d, t})} \right)^{1 - (\epsilon - 1)\psi} \times (1 + g_{N^d, t})^{\phi^d - (\epsilon - 1)\psi}.$$

Given the long-run evolution in $(1 + g_{N^d, t})$, it follows that if research is permanently directed toward the dirty R&D subsector then:

$$\frac{F(0, \bar{L}_{t+1}, N_t^c, N_{t+1}^d)}{F(0, \bar{L}_t, N_t^c, N_t^d)} \xrightarrow{t \rightarrow \infty} (1 + n)^{\frac{\phi^d - (\epsilon - 1)\psi}{\phi^d}}.$$

When $(\epsilon - 1)\psi < \phi^d$ the growth factor is above one, and the profit ratio increases over time and approaches infinity for time approaching infinity. Thus, the profit ratio cannot stay below one permanently.

As the profit ratio cannot stay permanently above or below one, the profit ratio must equal one or fluctuate around one in the long run. \square

Proof of Lemma 3

Proof. The growth rate of N_t^j is given by: $g_{N^j, t} = \eta^j s_t^j (N_t^j)^{-\phi^j}$. If all researchers are permanently directed toward sector j , then $(s_{t+1}^j / s_t^j) = (1 + n)$ and

$$G_t^j \equiv \frac{g_{N^j, t+1}}{g_{N^j, t}} = (1 + n)(1 + g_{N^j, t})^{-\phi^j}.$$

If $G_t^j > 1$, then $(1 + n)(1 + g_{N^j, t})^{-\phi^j} > (1 + n)(1 + g_{N^j, t+1})^{-\phi^j}$. Thus, G_t^j decreases over time. If $G_t^j < 1$, then $(1 + n)(1 + g_{N^j, t})^{-\phi^j} < (1 + n)(1 + g_{N^j, t+1})^{-\phi^j}$. Thus, G_t^j increases over time. As a result, G_t^j converges to one. Hence, in the long run:

$$(1 + n)(1 + g_{N^j, t})^{-\phi^j} = 1 \quad \Leftrightarrow \quad (1 + g_{N^j, t}) = (1 + n)^{\frac{1}{\phi^j}}.$$

\square

Proof of Proposition 1

Proof. Consider the ratio, Y_{t+1}^d/Y_t^d , when research has clean bias:

$$\frac{Y_{t+1}^d}{Y_t^d} = \Xi(N_t^c, N_{t+1}^c, N^d)(1 + g_{N^c,t})^{-(\epsilon-1)\psi}(1+n), \quad \text{where}$$

$$\Xi(N_t^c, N_{t+1}^c, N^d) \equiv \frac{(N_t^c)^{-(\epsilon-1)\psi} + (N^d)^{-(\epsilon-1)\psi}}{(N_{t+1}^c)^{-(\epsilon-1)\psi} + (N^d)^{-(\epsilon-1)\psi}}.$$

Clearly, $\Xi(N_t^c, N_{t+1}^c, N^d)$ approaches one when N_t^c and N_{t+1}^c become large. It follows from Lemma 3 that when research has clean bias: $(1 + g_{N^c,t}) \rightarrow (1+n)^{\frac{1}{\phi^c}}$. Thus,

$$\frac{Y_{t+1}^d}{Y_t^d} \xrightarrow{t \rightarrow \infty} (1+n)^{\frac{\phi^c - (\epsilon-1)\psi}{\phi^c}} \begin{cases} < 1 & \text{if } (\epsilon-1)\psi > \phi^c \\ = 1 & \text{if } (\epsilon-1)\psi = \phi^c \\ > 1 & \text{if } (\epsilon-1)\psi < \phi^c \end{cases}.$$

□

Proof of Proposition 2

Proof. (i) It follows directly from Lemma 2 that without environmental policies, the profit ratio equals or fluctuates around one in the long run. As this property is independent of the technological level, a temporary subsidy cannot permanently direct research toward the clean R&D subsector.

(ii) If research has clean bias due to a permanent research subsidy, it follows from Proposition 1 that aggregate pollution emission approaches infinity for time approaching infinity. Hence for some $\bar{t} \in t$ it must hold that $f(Z_{\bar{t}}) > \bar{E}$. Accordingly, environmental sustainability is not obtained.

Under clean bias a variable V is denoted \bar{V} , while the same variable is denoted \tilde{V} when research is only temporarily directed toward the clean R&D subsector. If it can be proven that $\tilde{Y}_{t_0+T}^d > \bar{Y}_{t_0+T}^d$ for large values of T , where t_0 is the expiration date for the temporary subsidy, then a temporary subsidy cannot ensure environmental sustainability.

If research is temporarily directed toward the clean R&D subsector by a temporary subsidy, it follows from Proposition 2 that research is conducted in both R&D subsectors in the long run. Hence if T is large, it must hold that: $\bar{N}_{t_0+T}^c > \tilde{N}_{t_0+T}^c$ and $\bar{N}_{t_0+T}^d < \tilde{N}_{t_0+T}^d$. These

two inequalities are now used to show that:

$$\tilde{Y}_{t_0+T}^d > \bar{Y}_{t_0+T}^d \Leftrightarrow \left[1 + \left(\frac{\tilde{N}_{t_0+T}^c}{\tilde{N}_{t_0+T}^d} \right)^{(\epsilon-1)\psi} \right]^{-1} (\tilde{N}_{t_0+T}^d)^\psi > \left[1 + \left(\frac{\bar{N}_{t_0+T}^c}{\bar{N}_{t_0+T}^d} \right)^{(\epsilon-1)\psi} \right]^{-1} (\bar{N}_{t_0+T}^d)^\psi.$$

The statement is clearly true since

$$\left[1 + \left(\frac{\tilde{N}_{t_0+T}^c}{\tilde{N}_{t_0+T}^d} \right)^{(\epsilon-1)\psi} \right]^{-1} > \left[1 + \left(\frac{\bar{N}_{t_0+T}^c}{\bar{N}_{t_0+T}^d} \right)^{(\epsilon-1)\psi} \right]^{-1} \Leftrightarrow \frac{\bar{N}_{t_0+T}^c}{\bar{N}_{t_0+T}^d} > \frac{\tilde{N}_{t_0+T}^d}{\tilde{N}_{t_0+T}^c}.$$

Hence the pollution emission level is, in the long run, larger under a temporary research subsidy compared to a situation with clean bias. Thus at some point $\bar{t} \in t$ it holds that $f(Z_{\bar{t}}) > \bar{E}$, and environmental sustainability is not obtained. \square

Proof of Proposition 3

Proof. When research has clean bias:

$$\begin{aligned} \frac{Y_{t+1}^d}{Y_t^d} &= \Xi(\cdot) (1 + g_{N^c,t})^{-(\epsilon-1)\psi} (1 + n), \quad \text{where} \\ \Xi(\cdot) &\equiv \frac{(N_t^c)^{-(\epsilon-1)\psi} \tau^{-\epsilon} + (N^d)^{-(\epsilon-1)\psi}}{(N_{t+1}^c)^{-(\epsilon-1)\psi} \tau^{-\epsilon} + (N^d)^{-(\epsilon-1)\psi}}. \end{aligned}$$

From the dynamics of N_t^c it follows that: $\Xi(\cdot) \rightarrow 1$ for $t \rightarrow \infty$. From Lemma 3, it then follows that

$$\frac{Y_{t+1}^d}{Y_t^d} \xrightarrow[t \rightarrow \infty]{} (1 + n)^{\frac{\phi^c - (\epsilon-1)\psi}{\phi^c}} > 1.$$

As pollution emission grows at a positive rate in the long run, it must hold that $f(Z_{\bar{t}}) > \bar{E}$ for some $\bar{t} \in t$. Hence environmental sustainability is not obtained. \square

Proof of Proposition 4

Proof. The profit ratio is given by

$$F(s_t^c, \cdot) = \tau_t^\epsilon \times \left(\frac{\eta^c}{\eta^d} \right) \times \left(\frac{1 + \eta^c (N_t^c)^{-\phi^c} s_t^c}{1 + \eta^d (N_t^d)^{-\phi^d} (\omega \bar{L}_t - s_t^c)} \right)^{(\epsilon-1)\psi-1} \times \frac{(N_t^c)^{(\epsilon-1)\psi-\phi^c}}{(N_t^d)^{(\epsilon-1)\psi-\phi^d}}.$$

If τ_0 is sufficiently large, research is directed toward the clean R&D subsector from the outset. In the long run, the profit ratio can only stay above one, if the pollution penalty grows sufficiently fast. Consider the long-run growth rate of the profit ratio under clean bias:

$$\frac{F(\omega L_{t+1}, \cdot)}{F(\omega L_t, \cdot)} \xrightarrow{t \rightarrow \infty} (1+n)^{\frac{(\epsilon-1)\psi - \phi^c}{\phi^c}} (1+g_\tau)^\epsilon.$$

The profit ratio grows at a positive rate, in the long run, if and only if:

$$(1+g_\tau)^\epsilon (1+n)^{\frac{(\epsilon-1)\psi - \phi^c}{\phi^c}} > 1 \quad \Leftrightarrow \quad (1+g_\tau) > (1+n)^{\frac{\phi^c - (\epsilon-1)\psi}{\epsilon\phi^c}}.$$

Thus if the above condition is fulfilled, the pollution penalty ensures clean bias if it is sufficiently large initially.

When research has clean bias:

$$\begin{aligned} \frac{Y_{t+1}^d}{Y_t^d} &= \Xi(\cdot) (1+g_\tau)^{-\epsilon} (1+g_{N^c,t})^{-(\epsilon-1)\psi} (1+n), \quad \text{where} \\ \Xi(\cdot) &\equiv \frac{(N_t^c)^{-(\epsilon-1)\psi} \tau_t^{-\epsilon} + (N^d)^{-(\epsilon-1)\psi}}{(N_{t+1}^c)^{-(\epsilon-1)\psi} \tau_t^{-\epsilon} + (N^d)^{-(\epsilon-1)\psi}}. \end{aligned}$$

From the dynamics of N_t^c and τ_t it follows that: $\Xi(\cdot) \rightarrow 1$ for $t \rightarrow \infty$. From Lemma 3, it then follows that

$$\frac{Y_{t+1}^d}{Y_t^d} \xrightarrow{t \rightarrow \infty} (1+n)^{\frac{\phi^c - (\epsilon-1)\psi}{\phi^c}} (1+g_\tau)^{-\epsilon}.$$

Environmental sustainability is ensured if \bar{E} is sufficiently large and pollution emission decreases at a constant rate in the long run. The latter condition is fulfilled when:

$$(1+n)^{\frac{\phi^c - (\epsilon-1)\psi}{\phi^c}} (1+g_\tau)^{-\epsilon} < 1 \quad \Leftrightarrow \quad (1+g_\tau) > (1+n)^{\frac{\phi^c - (\epsilon-1)\psi}{\epsilon\phi^c}}.$$

□

Proof of Proposition 5

Proof. Clean bias research can be obtained through a permanent research subsidy. If $\phi^c = 1$, then $N_{t+1}^c - N_t^c = \eta^c \omega \bar{L}$. Hence $N_t^c \rightarrow \infty$ for $t \rightarrow \infty$. If $\phi^c < 1$, then $N_{t+1}^c - N_t^c = \eta^c \omega \bar{L} (N_t^c)^{1-\phi^c}$. Hence $N_t^c \rightarrow \infty$ for $t \rightarrow \infty$.

If $\phi^c > 1$, it is less clear how N_t^c behaves asymptotically, as the absolute change in N_t^c decreases over time. Consider the function:

$$N_{t+1}^c - N_t^c = \eta^c \omega \bar{L} (N_t^c)^{1-\phi^c} - \delta, \quad 0 < \delta < \eta^c \omega \bar{L} (N_0^c)^{1-\phi^c}.$$

In this case, N_t^c converges to the constant N^* , where

$$N^* = \left(\frac{\eta^c \omega \bar{L}}{\delta} \right)^{\frac{1}{\phi^c - 1}}.$$

It then follows directly that $N^* \rightarrow \infty$ for $\delta \rightarrow 0$. Hence in the limiting case where δ equals zero: $N_t^c \rightarrow \infty$ for $t \rightarrow \infty$.

Under clean bias:

$$Y_t^d = A \left[(N_t^c)^{-(\epsilon-1)\psi} + (N^d)^{-(\epsilon-1)\psi} \right]^{-1} (N_t^c)^{-(\epsilon-1)\psi} (N^d)^\psi (1-\omega) \bar{L}.$$

Clearly, $Y_t^d \rightarrow 0$ for $N_t^c \rightarrow \infty$, and thus environmental sustainability is obtained if \bar{E} is sufficiently large. \square

C An OLG approach

The model builds on the overlapping generations framework, and thus saving is introduced into the model. It is shown that the assumption of a constant intersectoral labor allocation can be motivated in this framework. As a consequence, all the main results from Section 3 carries through in this alternative model specification.

There are at all times two generations present in the economy: the young and the old. The young work, consume and save, while the old consume their entire savings. In period t , there are \bar{L}_t young and $\bar{L}_t/(1+n)$ old. The young solve the problem:

$$\begin{aligned} \max_{c_t^y, c_{t+1}^o} U_t &= \ln c_t^y + \ln G_t + \beta (\ln c_{t+1}^o + \ln G_{t+1}) \\ \text{st. } c_t^y &= (1 - \tau^w)w_t + \bar{\pi}_t - b_t, \quad c_{t+1}^o = (1 + r_{t+1})b_t, \quad (c_t^y, c_{t+1}^o, b_t) \geq 0, \\ 0 &< \beta < 1, \quad 0 < \tau^w < 1, \end{aligned}$$

where U_t is welfare, c_t^y is consumption as young, c_{t+1}^o is consumption as old, G_t is a public good provided by the government, β is the discount factor, τ^w is a wage tax, b_t is saving,

$\bar{\pi}_t$ is profits from old machine firms, and r_{t+1} is the real interest rate. The saving of the young is used to finance research. Hence the old generation owns patents on new machine varieties which generates the return on savings. The generational setup implies long time periods which eases consumption smoothing. This motivates the low intertemporal elasticity of substitution implied by the per-period utility function.

Consumption goods can either be consumed as private goods or as a public good. Clearing on the market for consumption goods requires that:

$$C_t = c_t^y \bar{L}_t + \bar{L}_{t-1} c_t^o + G_t.$$

The government keeps a balanced budget such that G_t equals the residual of the government's spending on subsidies and the government's income from taxes. From a technical point of view, the public good is added for tractability purposes. One can think of the public good as a way that the government can transfer resources back to the households without distorting relative prices and trade-offs. In that way, the public good works like a lump-sum transfer in the neoclassical growth model framework.

Otherwise the model from Section 3 is unchanged. Optimizing behavior imply

$$b_t = \left(\frac{\beta}{1 + \beta} \right) (1 - \tau^w) w_t.$$

As all saving are used to finance R&D expenditures:

$$b_t \bar{L}_t = w_t s_t \quad \Leftrightarrow \quad s_t = \tilde{\omega} \bar{L}_t, \quad \tilde{\omega} \equiv \left(\frac{\beta}{1 + \beta} \right) (1 - \tau^w).$$

This model predicts that a constant fraction, $\tilde{\omega}$, of the workforce are employed in the R&D sector. Since $\tilde{\omega}$ is unaffected by the environmental policies discussed in Section 3, the policy implications of this model version are identical to those in Section 4.

D Stepping-on-toes effects

Due to the non-rival nature of knowledge, doubling the research input might not double the output. Scientists working in different research labs might come up with the same ideas, implying decreasing returns to knowledge creation. This is often referred to as stepping-on-

toes effects, and empirical evidence suggests that these effects are important.¹⁹

To model this feature, it is assumed that a scientist can start $N_t^j(1 + s_t^j)^{-\chi}$ projects in R&D subsector $j \in \{c, d\}$ where $0 \leq \chi < 1$. The expected discounted profits for scientists conducting research in R&D subsector j amounts to:

$$\tilde{\pi}_t^j = \underbrace{\frac{1}{(1 + r_{t+1})}}_{\text{Discount rate}} \times \underbrace{\left(N_t^j\right) (1 + s_t^j)^{-\chi}}_{\text{Projects per scientist}} \times \underbrace{\eta^j \left(N_t^j\right)^{-\phi^j}}_{\text{Success probability per project}} \times \underbrace{\hat{\pi}_t^j}_{\text{Profit per one-period patent}}$$

The resulting profit ratio is given by:

$$F(s_t^c, \bar{L}_t, N_t^c, N_t^d) = \left(\frac{1 + s_t^c}{1 + \omega \bar{L}_t - s_t^c} \right)^{-\chi} \times \left(\frac{\eta^c}{\eta^d} \right) \times \left(\frac{1 + \eta^c (N_t^c)^{-\phi^c} (1 + s_t^c)^{-\chi} s_t^c}{1 + \eta^d (N_t^d)^{-\phi^d} (1 + \omega \bar{L}_t - s_t^c)^{-\chi} (\omega \bar{L}_t - s_t^c)} \right)^{(\epsilon-1)\psi-1} \times \frac{(N_t^c)^{(\epsilon-1)\psi-\phi^c}}{(N_t^d)^{(\epsilon-1)\psi-\phi^d}}.$$

It is assumed that $(\epsilon - 1)\psi < 1$ such that the profit ratio is strictly decreasing in s_t^c .

The long-run growth factor of N_t^j under j bias is given by:

$$(1 + g_{N^j,t}) = 1 + \eta^j \left(N_t^j\right)^{-\phi^j} (1 + s_t^j)^{-\chi} s_t^j \xrightarrow{t \rightarrow \infty} (1 + n)^{\frac{1-\chi}{\phi^j}}.$$

To avoid the lock-in equilibrium, Parameter Restriction 2 is substituted by:

Parameter Restriction 3. $(1 - \chi)(\epsilon - 1)\psi < \phi^c$ and $(1 - \chi)(\epsilon - 1)\psi < \phi^d$.

If research has clean bias:

$$\frac{Y_{t+1}^d}{Y_t^d} \xrightarrow{t \rightarrow \infty} (1 + n)^{\frac{\phi^c - (1-\chi)(\epsilon-1)\psi}{\phi^c}}.$$

Thus the neo-Malthusian effect dominates the Simon effect even when research has clean bias. From here, it is relatively easy to see that this model has the same qualitative policy implications, as the model presented in Section 3.

¹⁹Using manufacturing industry data, Venturini (2012) finds substantial stepping-on-toes effects. Macro estimates obtained in Chapter 1 also indicate that stepping-on-toes effects are important.

E Knowledge spillovers between R&D subsectors

The basic model features no knowledge spillovers between the two R&D subsectors. To assess the role of such spillovers, consider the following evolutions in the technological levels:

$$N_{t+1}^j = \left(1 + \eta^j (N_t^f)^\gamma (N_t^j)^{-\phi^j - \varphi} s_t^j\right) N_t^j, \quad 0 < \varphi < 1, \quad 0 < \gamma < 1, \quad j \in \{c, d\}, \quad (18)$$

where $0 < (1 - \varphi - \phi^j + \gamma) < 1$, N_t^f is the technological level of the other R&D subsector, and γ reflects the usefulness of subsector f knowledge for subsector j scientists. A scientist in R&D subsector j starts $(N_t^j)^{1-\varphi} (N_t^f)^\gamma$ new projects in period t . Each with the success probability $\eta^j (N_t^j)^{-\phi^j}$.

The intersubsector spillovers drag the profit ratio toward one, since technological advances in one R&D subsector increases the productivity of scientists in the other. This is clear from the equilibrium profit ratio:

$$F(\cdot) = \left(\frac{\eta^c}{\eta^d}\right) \times \left(\frac{1 + \eta^c (N_t^d)^\gamma (N_t^c)^{-\phi^c - \varphi} s_t^c}{1 + \eta^d (N_t^c)^\gamma (N_t^d)^{-\phi^d - \varphi} (\omega \bar{L}_t - s_t^c)}\right)^{(\epsilon-1)\psi-1} \times \frac{(N_t^c)^{(\epsilon-1)\psi - \phi^c - \varphi - \gamma}}{(N_t^d)^{(\epsilon-1)\psi - \phi^d - \varphi - \gamma}}.$$

To avoid the lock-in equilibrium, Parameter Restriction 2 is substituted by:

Parameter Restriction 4. $(\epsilon - 1)\psi < \phi^c + \varphi + \gamma$ and $(\epsilon - 1)\psi < \phi^d + \varphi + \gamma$.

If research has clean bias:

$$\frac{Y_{t+1}^d}{Y_t^d} \xrightarrow{t \rightarrow \infty} (1 + n)^{\frac{\phi^c + \varphi - (\epsilon-1)\psi}{\phi^c + \varphi}}.$$

Thus given that $\phi^c + \varphi < (\epsilon - 1)\psi < \phi^c + \varphi + \gamma$, the Simon effect dominates the neo-Malthusian effect in the long run under clean bias. The intuition is the following. The intersubsector spillovers reduce research productivity under clean bias, but they also increase the research productivity of the stagnant R&D subsector. Both effects reduce the incentive to permanently research in only one R&D subsector. The former effect is similar to the fishing-out effect, while the latter effect is substantially different, as it does not affect the research productivity when research has clean or dirty bias. Intersubsector spillovers thereby create a parameter space in which avoiding the lock-in equilibrium under laissez-faire is consistent with a dominating Simon effect under clean bias. Within this parameter space, permanent research subsidies can ensure environmental sustainability. The policy implications outside this parameter space are unchanged.

F Population data

The population dataset covers the period 1744-2014. It is constructed using two sources. Population data for the period 1744-1949 are taken from the History Database of the Global Environment (HYDE), see Goldewijk et al. (2010, 2011). The remaining data are taken from United Nations (2017). Due to missing observations, the data series is interpolated using a 10th degree polynomial. The left panel of Figure 7 shows fitted and actual values. Using different standard interpolation methods yield similar results.

The population growth projections are taken from United Nations (2017). The low, medium, and high population projection variants are shown in the right panel of Figure 7. The medium variant is used as the baseline population growth scenario.

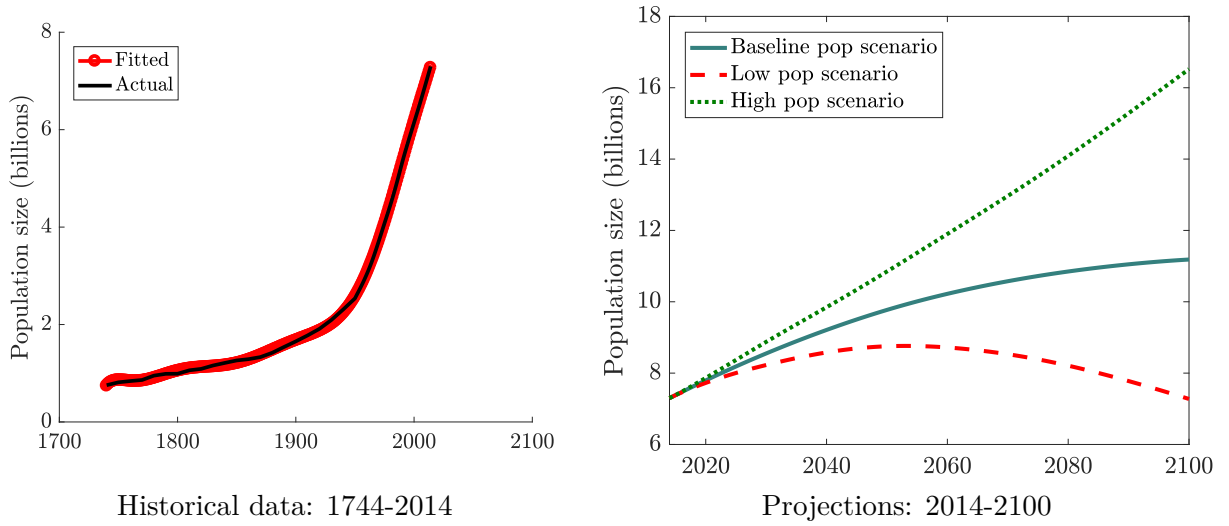


FIGURE 7: Population size data.

G CO₂ concentration data

The CO₂ concentration dataset covers the period 1744-2014. It is constructed from two sources. Concentration data for the period 1744-1953 are taken from Neftel et al. (1994), and concentration data for the period 1959-2014 are taken from Mauna Loa, Hawaii Observatory (2016). Due to missing observations, the data series is interpolated using a 10th degree polynomial. Figure 8 shows fitted and actual values. Using different standard interpolation methods yield similar results.

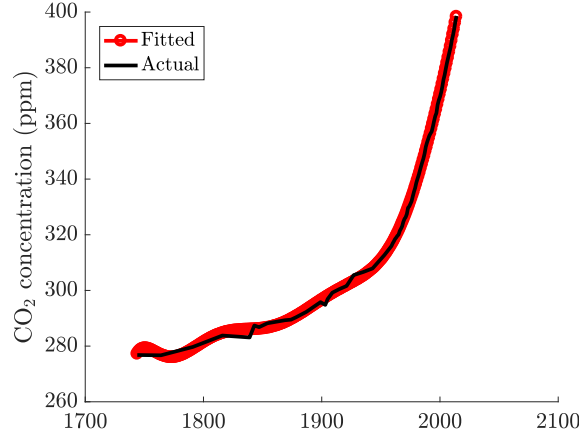


FIGURE 8: CO₂ concentration, 1744-2014.

H Climate damages

Let consumption net of damages, C_t^* , be given by:

$$C_t^* = C_t e^{-\sigma E_t}.$$

The parameter σ is set to $1.1292 \cdot 10^{-4}$. This value is taken directly from Golosov et al. (2014) and adjusted to account for different measurement units used for the atmospheric CO₂ concentration. The optimal tax rate paths are shown in Figure 9, while the relative per capita consumption losses are shown in Figure 10.

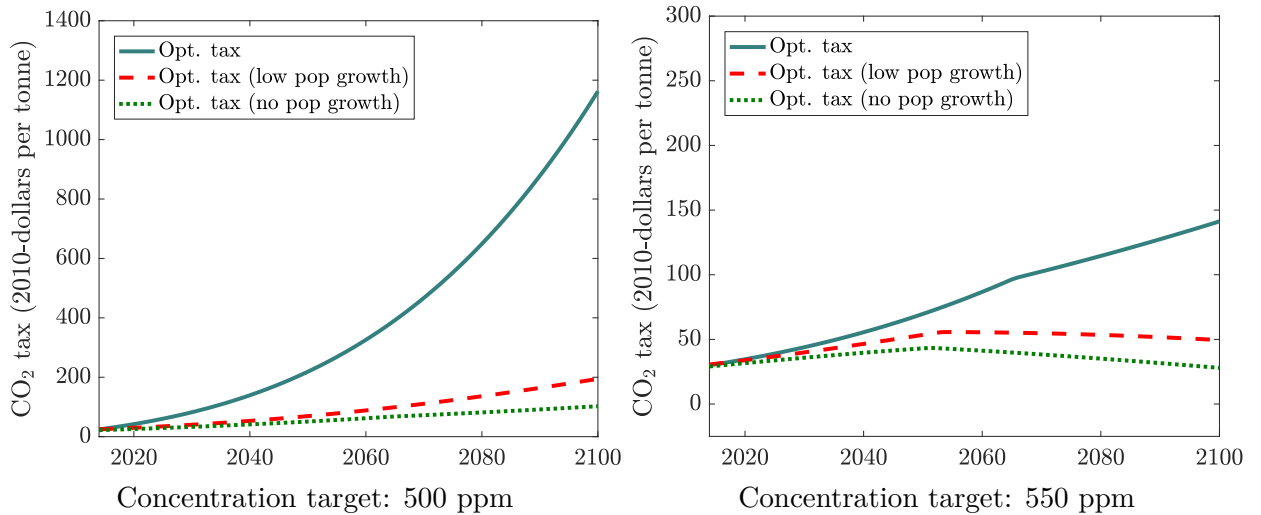


FIGURE 9: Optimal CO₂ tax rate paths in the presence of climate change damages under different population scenarios and CO₂ concentration targets, 2014-2100.

Notes: The tax policies follow the simple tax rule (17). The policies are optimal in the sense that they minimize the welfare loss of the representative household while achieving the CO₂ concentration target. The damage function is modeled as in Golosov et al. (2014).

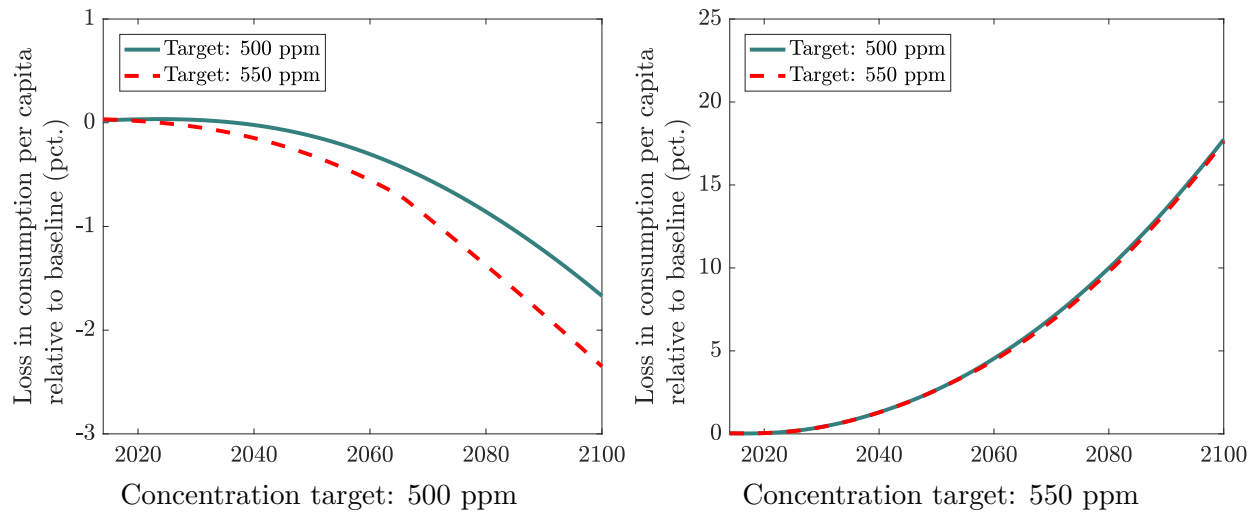


FIGURE 10: Consumption per capita losses associated with optimal tax policies in the presence of climate change damages relative to the baseline scenario, 2014-2100.

I Other simulation results

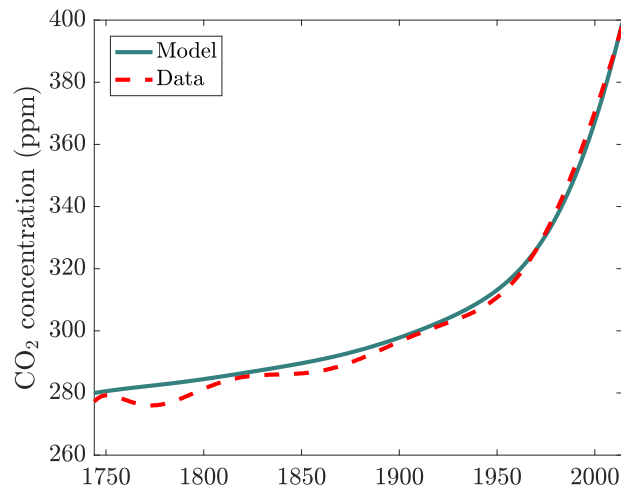


FIGURE 11: Actual and simulated CO₂ concentration paths, 1744-2014.

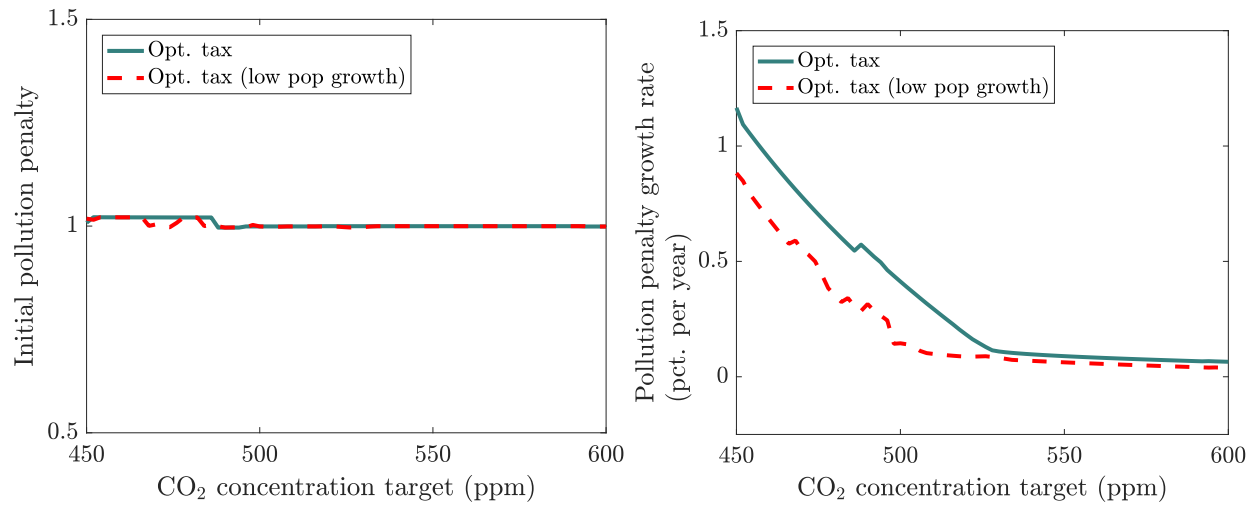


FIGURE 12: Optimal environmental policy as function of the concentration target, 2014-2100.

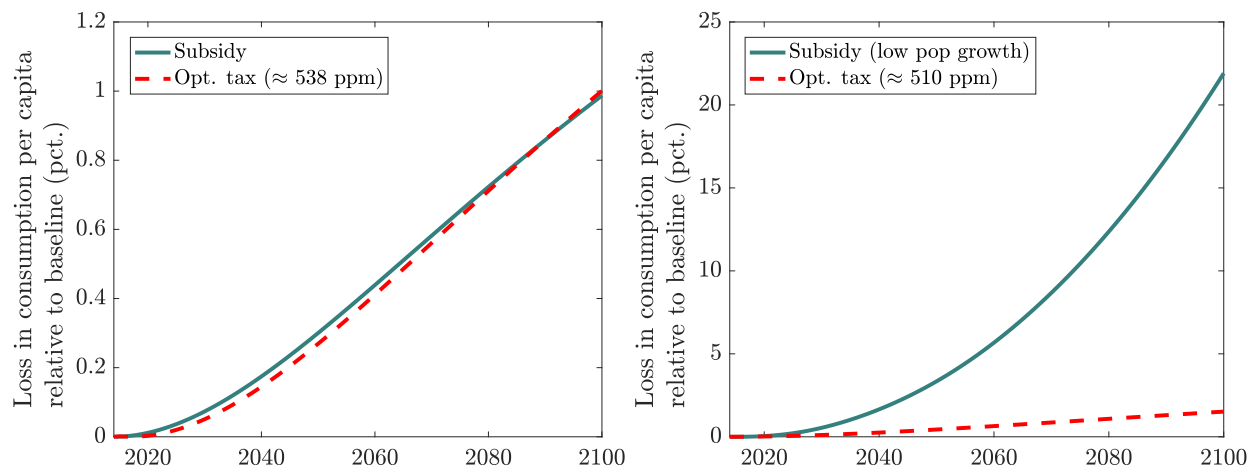


FIGURE 13: Comparison between consumption per capita losses relative to the baseline scenario for different policies, 2014-2100.

Notes: The tax policies follow the simple tax rule (17). The policies are optimal in the sense that they minimize the welfare loss of the representative household, while achieving the same CO_2 concentration in 2100, as the subsidy policy they are compared to. All subsidy policies ensure clean bias.

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