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## Intertemporal Distribution, Sufficiency, and the Social Cost of Carbon

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# Intertemporal Distribution, Sufficiency, and the Social Cost of Carbon

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**Abstract:** We explore how the intertemporal distribution of well-being affects the social cost of carbon. In contrast to the literature that studies parameters of a particular social welfare function, such as the discount rate, we shift the focus and directly assume a parametric form for the intertemporal distribution of well-being. This has the advantage of avoiding explicit discounting choices, which has initiated much debate. Specifically, we consider a set of intertemporal distributions that reach a pre-specified steady-state level of “sufficient” well-being, or equivalently, after a pre-specified “end-of-growth horizon”. We numerically illustrate our results in DICE and find that the social cost of carbon increases over-proportionally with the sufficiency level of well-being. While the social cost of carbon in 2015 is US\$ 7 if the sufficiency level is four-fold the present level, it is US\$ 30 if the sufficiency level is 15-fold and US\$ 100 if the sufficiency level is 26-fold the present level. This shows in a transparent way how conceptions of intergenerational distributive justice drive the social cost of carbon.

**JEL-Classification:** Q01, Q54, H21, C61, D31, D91

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**Keywords:** Climate change, social cost of carbon, optimal tax, DICE, optimal growth, sustainability, social welfare function, discounting

# 1 Introduction

A major challenge for humankind is avoiding dangerous climate change. Economic studies of optimal climate policy typically use integrated assessment models (IAMs) to determine an optimal path of emission abatement (Stern 2007, Nordhaus 2008, Golosov et al. 2014). Many of these studies adopt an intertemporal discounted utilitarian social welfare function (SWF) and arrive at remarkably different estimates for the optimal tax rate on carbon emissions into the atmosphere, i.e. the social cost of carbon (Table 1). These differences are largely attributable to the specific parametrization of the SWF in terms of the so called “ethical parameters”, namely the social time preference rate ( $\rho$ ) and the intertemporal elasticity of substitution ( $1/\eta$ ). The specification of parameter values for  $\rho$  and  $\eta$  translates into specific assumptions about how well-being<sup>1</sup> ought to be intertemporally distributed.

Starting with Ramsey (1928), the long lasting economic and philosophical discussion on which intertemporal SWF should be applied mostly focuses on the “correct” parametrization of the SWF within the standard discounted utilitarian framework (Buchholz and Schymura 2011). Recently there is a growing literature developing alternative social welfare criteria (Asheim 2010, Zuber and Asheim 2012, Fleurbaey and Zuber 2015) although applications of these in well-known IAMs are still relatively rare (Botzen and van den Bergh 2014).

Instead of studying a particular SWF and restricting the analysis to specific ethical parameter values, one can also take a very different approach, which avoids explicit discounting choices: the intertemporal distribution of well-being can directly be specified in a parametric form. Recently this direct approach has been applied to study sustainable economic development in the light of anthropogenic climate change (Llavador et al. 2010,

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<sup>1</sup>In the studies presented in table 1 the level of well-being reduces to an index of consumption equivalents (or inclusive consumption), which abstracts from the relative price effects of other components of well-being on the social cost of carbon, like environmental quality (Stern and Persson 2008). We acknowledge this shortcoming, but stick to using inclusive consumption as a proxy for well-being as defined in Nordhaus and Sztorc (2013) and Nordhaus (2014) for our analysis.

Table 1: Selected estimates of the optimal carbon tax, quoted after [Goloso et al. \(2014\)](#)

Study	Parameter	Optimal tax
<a href="#">Nordhaus (2008)</a>	$\rho = 1.5\%, \eta = 2$	30 US\$/tC
<a href="#">Goloso et al. (2014)</a>	$\rho = 1.5\%, \eta = 1$	60 US\$/tC
<a href="#">Stern (2007)</a>	$\rho = 0.1\%, \eta = 1$	250 US\$/tC

[2011](#), [Roemer 2011](#)). However, these studies do not systematically address the question how the intertemporal distribution of well-being is related to the social cost of carbon.

In this paper we parametrize intertemporal paths of well-being that allow us to study the trade-off between the intertemporal distribution of human-well being and the present social cost of carbon. We choose a specific set of intertemporal distributions that is driven by five underlying assumptions, which mainly reflect a schedule of smoothly decreasing growth rates leading to a steady state with a pre-specified constant, “sufficient” level of well-being, or, equivalently, after a pre-specified “end-of-growth” horizon, resulting in an “s-shaped” intertemporal distribution of well-being. Due to, among others, the last global economic crisis, climate change and biodiversity loss, the debate on limits to economic growth pioneered by [Meadows et al. \(1972\)](#) has recently been intensified ([Turner 2008](#), [Victor 2010](#), [Antal and van den Bergh 2014](#)). In a recent questionnaire on public opinions on economic growth and environmental sustainability [Dreus and van den Bergh \(2016\)](#) find that two thirds of the respondents believe that growth in rich countries will stop at some future point in time. This is consistent with developing countries typically following an s-shaped course of economic development with high initial growth rates, which decrease in the course of time. Also the DICE model assumes that the growth rate will continuously decline down to zero.

Among the s-shaped set of development paths, we determine the one that minimizes the time until the pre-specified sufficient level of well-being is reached (i.e., the “end-of-growth horizon”). By varying the sufficiency level of well-being we can study how the desire for economic efficiency, growth and the resulting intertemporal distribution affects

the social cost of carbon. We quantitatively illustrate our results with the 2013 version of DICE (Nordhaus and Sztorc 2013, Nordhaus 2014), which is the most widespread and well-known IAM. The minimization of the end-of-growth horizon requires a full-fledged dynamic optimization, as it affects patterns of investment in human-made capital, as well as carbon emissions into the atmosphere, both of which have long-term consequences that fully have to be taken into account.

We believe that our approach to directly define intertemporal distributions of well-being has clear advantages over making specific discounting choices. For society and policy-makers it might be easier to agree on a certain intertemporal distribution of well-being than to argue on parameter values for a particular SWF. In a recent survey, Drupp et al. (2015) elicit expert opinion on the value of the long-term social discount rate. One of the responses to their open-ended question for comments was the following: *"Instead of imposing a SWF and calculate the corresponding optimum, it is 'better' to depict a set of feasible paths of consumption, production, temperature, income distribution, etc. and let the policy maker make a choice"* (Drupp et al. 2015, p.17). A similar metaphor has been proposed by Edenhofer and Minx (2014) who suggest economists to construct a feasible "map" of economic development that could be used by policy-makers to "navigate" among different policy options.

Such a "map" requires to parameterize a conceivable set of feasible paths of well-being. As discussed above, the set of s-shaped paths of intertemporal well-being is a particular sensible assumption. This is why we focus on this particular specification in this paper. For each efficient path the policy-maker will be able to obtain the associated social cost of carbon under optimal climate policy. For society our approach could lead to a better informed discussion on normative conceptions of intergenerational distributive justice, which crucially determine the social cost of carbon and are typically hidden in discounting choices within the standard discounted utilitarian model. It becomes very clear, for example, that the desire to attain a high level of well-being in the future, or equivalently to keep the global economy growing for a longer time horizon, substantially increases the social costs of carbon, because in the long-run growth of well-being requires to protect the future generations from adverse consequences of climate change.

The remainder of this paper is organized as follows: Section 2 first formally derives the condition prescribing the optimal intertemporal distribution of well-being when using a (discounted) utilitarian SWF, which is embodied in most IAMs like DICE. Second, we briefly sketch the relevant literature that uses this condition to capture social preferences with respect to intertemporal distributions in IAMs. Section 3 characterizes our approach of directly considering a specific functional form for the intertemporal distribution of well-being. Section 4 presents the numerical results of the dynamic optimization, before section 5 discusses our results.

## 2 Intertemporal Distributional Objectives embodied in a Social Welfare Function

The dominant approach to determine the social costs of carbon is to use a Social Welfare Function in a dynamic Integrated Assessment Model of climate and the economy (IAMs), such as DICE (Nordhaus 2014). In order to contrast our approach of directly specifying the intertemporal distribution of well-being with a functional form to this standard in the literature, we briefly describe the Social Welfare approach.

Most deterministic dynamic IAMs rank intertemporal paths of per capita consumption  $c_t$ , which they refer to as inclusive consumption capturing “well-being”, by means of the intertemporal social welfare function (SWF),

$$W_0(c_0, c_1, c_2, \dots) = \sum_{t=0}^T \frac{1}{(1 + \rho)^t} L_t \frac{c_t^{1-\eta}}{1 - \eta}, \quad (1)$$

which can be interpreted as the discounted Utilitarian objective function or as the utility function of a representative, infinitely-lived agent (ILA), weighted by population size  $L_t$ . We consider a discrete-time setting with  $t = 0, 1, 2, \dots, T$ . The parameters of the welfare function are the time preference rate,  $\rho$ , and the preference for consumption smoothing over time,  $\eta$ , with  $1/\eta$  being the constant intertemporal elasticity of substitution of consumption.

Maximizing (1) subject to the economic and climate constraints of the DICE model

(Nordhaus 2014) leads to the following condition (see appendix 1 for a derivation),

$$(1 + \rho) \left( 1 + \frac{c_t - c_{t-1}}{c_{t-1}} \right)^\eta = 1 + Y_{K_t} - \delta^K, \quad (2)$$

where  $Y_{K_t}$  denotes the marginal productivity of capital and  $\delta^K$  the proportional rate of capital depreciation. Equation (2) is the discrete-time version of the well-known Ramsey rule (Dasgupta 2008) and characterizes the intertemporal distribution of well-being that is optimal according to (1).

Much of the recent economic debate on the social costs of carbon focuses on how a society should choose the values for the discounting parameters of a SWF, i.e.  $\rho$  and  $\eta$ . Interpreting the SWF (1) as the utility function of a representative ILA, these parameters can be derived from observed behavior on markets reflecting opportunity costs of capital (Arrow et al. 1996, Buchholz and Schymura 2011). In this vein, Nordhaus (2008) argues that short-term time preferences should be in line with historical consumption choices. He thus uses the Ramsey equation (2) to determine  $\rho$  and  $\eta$  from inferred values of real market interest rates and the consumption growth rate.

Other studies interpret the intertemporal SWF (1) as the (discounted) Utilitarian objective. According to this point of view, ethical considerations regarding intergenerational trade-offs of well-being should guide the choice of  $\rho$  and  $\eta$  (Arrow et al. 1996, Aldy et al. 2010), often implying a more long-term focus on climate impacts (Stern 2007, Gerlagh and Liski 2017). Already Ramsey (1928) and Pigou (1932) argued for a zero rate of time preference on ethical grounds. In this approach, the rate at which future well-being is discounted falls considerably below the opportunity cost of capital and thus these studies arrive at much higher estimates for the social cost of carbon. In that vein, (Stern 2007) uses a very small value of  $\rho = 0.001$ , merely to reflect a positive probability that humankind may become extinct at some future date, and calculates an optimal carbon tax, which exceeds the one recommended by Nordhaus (2008) by almost one order of magnitude.

In addition, an increasing number of studies consider declining discount rates, e.g. due to uncertainty about future discount rates (Arrow et al. 2013, 2014, Cropper et al. 2014), or when aggregating heterogeneous individual discount rates (Heal and Millner



2013, Quaas et al. 2017). There are also several studies that consider Epstein-Zin (Epstein and Zin 1989) preferences to disentangle risk aversion and time preference in order to more appropriately capture observed preferences over the distribution of income over time (Ackerman et al. 2013). Finally, some studies use surveys to elicit expert opinion on the discounting parameters of a SWF (Weitzman 2001, Drupp et al. 2015, Howard and Sylvan 2015). The most recent expert survey from Drupp et al. (2015) finds that only a minority of experts recommends discounting parameters in line with the Ramsey equation (2).

The diversity in views on the correct values for the discounting parameters  $\rho$  and  $\eta$  is one of the main drivers in explaining the broad range of estimates for the social cost of carbon today. Many studies, however, do not make explicit which distributional assumptions are embodied in their choice of  $\rho$  and  $\eta$ . Indeed, small differences in the specification of  $\rho$  and  $\eta$  may lead to substantially different values for the social cost of carbon (van den Bijgaart et al. 2016). We believe that most policy-makers, and probably even many scholars, will have difficulties in making up their opinion on the correct values for these parameters accurately enough that they can trust the resulting value for the social cost of carbon. Possibly policy-makers will not be able to recognize what actually drives a certain estimate for the social cost of carbon, which may damage the applicability of such a quantitative estimate.

We thus propose to shift the focus away from discounting choices of a particular SWF, but instead directly assume a functional form for the intertemporal distribution of well-being. This enables us to simulate a set of intertemporal distributions and quantitatively estimate each resulting social cost of carbon by using the DICE model. In the next section 3 we explicitly introduce the suggested functional form for the intertemporal distribution of well-being.

### 3 A Direct Specification of the Intertemporal Distribution

The main aspects in our specification of the intertemporal distribution of well-being are that (a) the present level of well-being,  $c_0$ , is kept fixed, and (b) that some steady-state level of well-being,  $c_{\max}$ , is obtained after a finite time of growth, i.e. after the “end-of-growth horizon”,  $\tau$ . This means, we impose the following assumptions.

**Assumption 1** (Given level of initial well-being). *The current generation’s well-being is kept fixed at the observed level  $c_0$ .*

**Assumption 2** (end-of-growth horizon/sufficiency). *Growth in well-being is maintained until  $\tau > 0$  such that after finite time  $\tau$  a steady-state with constant well-being  $c_{\max} > c_0$  is reached.*

In addition, we have to specify the development of well-being between the present,  $t = 0$ , and  $\tau > 0$ . To this end, we define the notions of a preference for “early growth” and a preference for “smooth growth”, which we capture by ‘preference’ functions  $h(\cdot)$  and  $f(\cdot)$ , respectively. Both apply to the period of growth only, as after  $\tau$  the economy then is in a steady state by definition, such that notions of “early growth” or “smooth growth” are meaningless after  $\tau$ .

In a sense, the notions of “early growth” and “smooth growth” capture the ideas of discounting and a preference for intertemporal consumption smoothing. The functional forms of  $h(\cdot)$  and  $f(\cdot)$  will then specify the preference for “early growth” and “smooth growth”. The difference between the setting considered here and the standard discounted utilitarian setting is that here we define the two separately, rather than having both aspects of intertemporal preferences built into one social welfare function.

**Definition 1** (early growth). *If there are two feasible streams  $c_t$  and  $c'_t$  with the same aggregate level of well-being over  $[0, T]$ ,  $\int_0^T (c_t - c'_t) dt = 0$ ,  $c_t$  is said to exhibit ‘early growth’ relative to  $c'_t$  if*

$$\int_0^\tau h(t - \tau) c_t dt > \int_0^\tau h(t - \tau) c'_t dt, \quad (3)$$

with  $h'(t - \tau) < 0$  (and without loss of generality  $h(0) = 0$ ) capturing preference for early growth.

The function  $h(\cdot)$  that is declining over time, similarly to a discount factor, gives a higher weight to well-being at early points in time compared to later points in time. The more strongly  $h(\cdot)$  declines over time, the ‘stronger’ is the preference for early growth.<sup>2</sup>

The second assumption is that society tends to prefer “smooth growth”, i.e. a consumption path that avoids strong fluctuations, defined as follows:

**Definition 2** (smooth growth). *If there are two feasible streams  $c_t$  and  $c'_t$  with the same aggregate level of well-being over  $[0, T]$ ,  $\int_0^T (c_t - c'_t) dt = 0$ ,  $c_t$  is said to have ‘smooth growth’ relative to  $c'_t$  if*

$$\int_0^T f(c_t) dt > \int_0^T f(c'_t) dt, \quad (4)$$

with  $f''(c) < 0$  capturing preference for smooth growth.

The curvature properties of the concave function  $f(\cdot)$  capture the preference for smoothing out consumption over time. The more concave the function  $f(\cdot)$  is, the stronger is the preference for smooth growth.<sup>3</sup> Based on these two definitions, we now state the assumptions on social preferences.

**Assumption 3** (early growth). *Consider two feasible streams  $c_t$  and  $c'_t$  with the same aggregate level of well-being over  $[0, T]$ , and same ‘smoothness’,  $\int_0^T (f(c_t) - f(c'_t)) dt = 0$ . If  $c_t$  exhibits ‘early growth’ relative to  $c'_t$ ,  $c_t$  is preferred over  $c'_t$ ,  $c_t \succ c'_t$ .*

**Assumption 4** (smooth growth). *Consider two feasible streams  $c_t$  and  $c'_t$  with the same aggregate consumption over  $[0, T]$ , and same ‘earliness’,  $\int_0^T h(t - \tau) (c_t - c'_t) dt = 0$ . If  $c_t$  exhibits ‘smooth growth’ relative to  $c'_t$ ,  $c_t$  is preferred over  $c'_t$ ,  $c_t \succ c'_t$ .*

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<sup>2</sup>Below we specify the preference for early growth by a quadratic function  $h(t - \tau) = (\tau - t)^2$ .

<sup>3</sup>Below we specify the preference for smooth growth by the Shannon entropy measure, i.e. we specify  $f(c) = -c \ln(c)$ .

To illustrate how these assumptions work, consider the function

$$c_t = \begin{cases} c_{\max}^{1-(1-\frac{t}{\theta})^2} \cdot c_0^{(1-\frac{t}{\theta})^2} & \text{for } t \leq \theta \\ c_{\max} & \text{for } t > \theta \end{cases}, \quad (5)$$

for  $c_t$ , which has the three parameters  $c_0$ ,  $c_{\max}$  and  $\theta$  (we will discuss this function in further detail below). If both  $c_t$  and  $c'_t$  have such a functional form, in Assumption 3 two of the parameters are constrained by the conditions on identical aggregate well-being and identical ‘smoothness’, while the differences in the third one can give rise to differences in ‘earliness’. Similarly, in Assumption 4 two of the parameters are constrained by the conditions on identical aggregate well-being and identical ‘earliness’, while the differences in the third one can give rise to differences in ‘smoothness’. This is illustrated in Figure 1, panels (a) and (b), where we use  $f(c) = -c \ln(c)$  to characterize the preference for smoothness and  $h(t - \tau) = (\tau - t)^2$  to characterize the preference for earliness (see footnotes 2 and 3). In panel (a), the two paths  $c_t$  and  $c'_t$  have the same aggregate well-being (4914), and the same ‘smoothness’ (-6199), but the path  $c_t$  exhibits more early growth compared to  $c'_t$  (15.3 vs. 11.0 millions). In panel (b), the two paths  $c_t$  and  $c'_t$  have the same aggregate well-being (4914), and the same ‘earliness’ (15.3 millions), but the path  $c_t$  exhibits more smooth growth compared to  $c'_t$  (-6199 vs. -6584).

We will see below that when the function  $h(\cdot)$  capturing the preference for early growth and the function  $f(\cdot)$  capturing the preference for smooth growth are specified, and given assumptions 1 and 2, the intertemporal distribution of well-being is specified except for the values of  $\tau$  and  $c_{\max}$ . These are fixed by imposing the following standard efficiency condition.

**Assumption 5** (efficiency). *Consider two feasible streams  $c_t$  and  $c'_t$  with the same ‘smoothness’,  $\int_0^\tau (f(c_t) - f(c'_t)) dt = 0$ , and the same ‘earliness’,  $\int_0^\tau h(t-\tau) (c_t - c'_t) dt = 0$ . If  $c_t$  exhibits a higher aggregate level of well-being than  $c'_t$ ,  $\int_0^T (c_t - c'_t) dt > 0$ ,  $c_t$  is preferred over  $c'_t$ ,  $c_t \succ c'_t$ .*

Given the above-made assumptions, the consumption path is characterized by the following theorem.

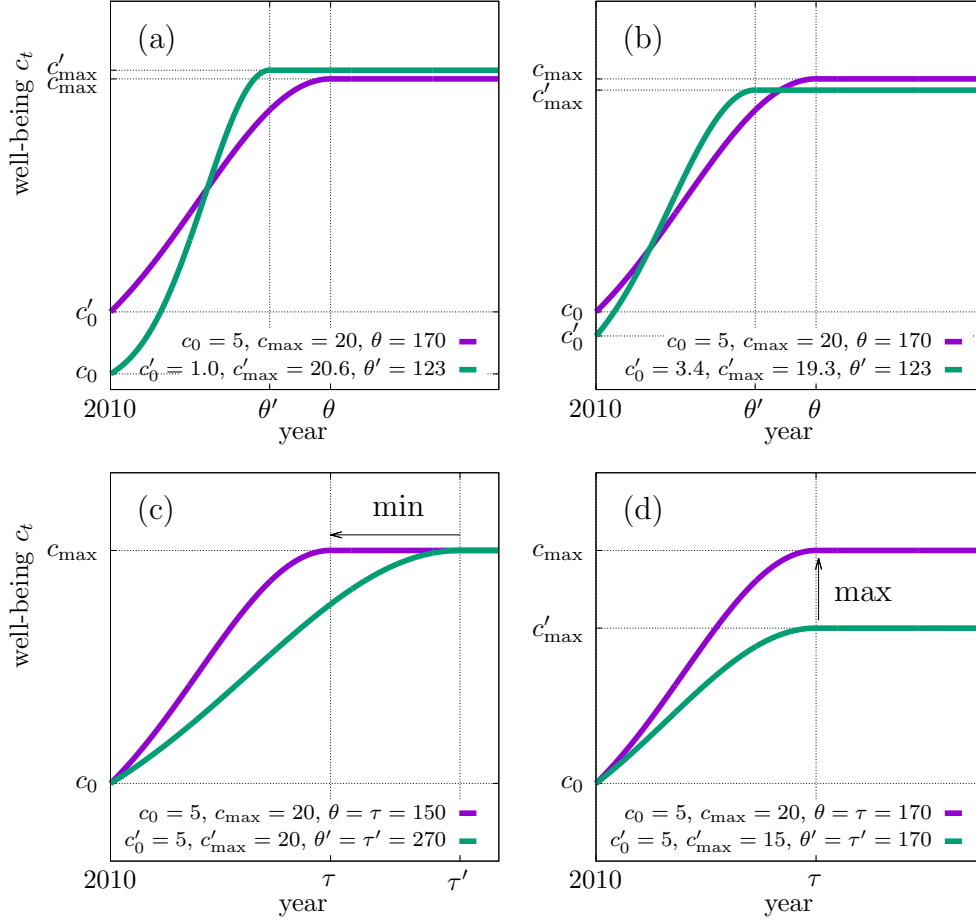


Figure 1: Panels (a) and (b) illustrate Assumptions 3 and 4. In panel (a), the two paths  $c_t$  and  $c'_t$  have the same aggregate well-being per capita and the same smoothness, but  $c_t$  exhibits more early growth compared to  $c'_t$ . In panel (b), the two paths  $c_t$  and  $c'_t$  have the same aggregate well-being and the same earliness, but  $c_t$  exhibits more smooth growth compared to  $c'_t$ . Panels (c) and (d) show well-being per capita  $c_t$  for varying end-of-growth horizon (panel c) and varying maximum consumption (panel d). In both panels, the shift from the lower to the upper curve illustrates optimization towards the Pareto-efficient path of well-being. All paths are constructed with the functional form (5), parameter specifications are given in the figures.

**Theorem 1.** *Under Assumptions 1–5, and for a pre-specified end-of-growth horizon  $\tau$ , well-being  $c_t$  is monotonic in time from  $c_0$  to  $c_\tau = c_{\max}$  according to*

$$c_t = f'^{-1} \left( f'(c_{\max}) \left( 1 + \frac{h(t - \tau)}{h(-\tau)} \left( \frac{f'(c_0)}{f'(c_{\max})} - 1 \right) \right) \right) \quad (6)$$

with

$$c_{\max} = \max \{ c_{\max} \in \mathbb{X} \mid c_t \text{ given by (6) for all } t < \tau \text{ and } c_t = c_{\max} \text{ for all } t \geq \tau \}. \quad (7)$$

where  $\mathbb{X}$  is the set of consumption paths that is feasible given the economic and climate constraints.

The proof of theorem 1 can be found in appendix 2. Smoothness and monotonic growth follow directly from assumptions 3 and 4. For a given end-of-growth-horizon  $\tau$ , assumptions 1–4 specify the intertemporal distribution of well-being only up to the free parameter  $c_{\max}$ . Efficiency requires that this level is chosen at the maximum feasible level, as stated in (7).

The higher the sufficiency level of steady-state well-being or the more we shift  $\tau$  into the future, the more evenly growth in well-being will be intertemporally distributed and thus, the more weight society puts on intergenerational distributive justice. Thus, changing  $c_{\max}$  or shifting  $\tau$  back and fourth in time enables us to study a set of intertemporal distributions, which can incorporate different conceptions of intergenerational distributive justice. If  $\tau$  is equal to zero, the result would be the maximin distribution (Solow 1974), i.e. an intertemporally constant well-being per capita level, which equals the level of the first, worst-off generation. By construction, however, maximin dismisses any investment into the well-being of future generations above the level of the present generation. Consequently, a strict application hinders economic growth leading to stagnation (Rawls 1971). Rawls (1971) proposed a two-stage model in order to assure distributive justice between generations. During a first accumulation phase generations would be required to adopt a positive net savings rate determined by a just savings principle resulting in positive growth rates of well-being. Once equal liberties and just institutions are implemented the net savings rate would fall to zero in the beginning of the second stage leading to a zero steady-state growth rate of well-being. Thus, Rawls' idea is

that economic growth is not desirable per se, but rather because it brings about just institutions and equal liberties. Once these are established generations should leave at least as much as they received from the generation before. In terms of our model, the first phase takes until  $\tau$ , while the second phase corresponds to a steady state at the sufficiency level of well-being  $c_{\max}$ .

Some growth in human well-being may be desirable for different reasons. For example parents may wish their children to have a higher quality of life compared to them or society wants to make an on-going development of humankind's achievements, like increasing average life expectancy, also possible in the far future (Llavador et al. 2011). In a series of recent papers Llavador, Roemer, and Silvestre (2010, 2011) and Roemer (2011) (LRS) study the implications of alternative distributions of intergenerational well-being on the first generations' well-being in a dynamic framework with an exogenous emissions scenario. By maximizing the initial level of well-being and maintaining a constant exogenous growth rate of well-being afterwards, they show that both intergenerational maximin (i.e. no growth) and a sustainable growth path are feasible and yield higher levels of well-being for the first generation than their reference value in 2000. Moreover, they find that in case of the sustainable growth path, the trade-off for the first generation in terms of consumption sacrifice is small compared to the prospect of sustained future growth in well-being.

We reflect these considerations by requiring efficiency, which means to maximize consumption subject to the technical and natural constraints. Here we consider these constraints as specified in the DICE model (Nordhaus 2014). Appendices 1 and 3 describe the details of the DICE model including functional forms.

The maximization problem in equation (7) states that the Pareto efficient path is obtained by choosing the path of well-being per capita and  $CO_2$  emissions in a way that results in the maximal steady state level of well-being per capita for a particular end-of-growth horizon. Straightforward calculations show that  $c_t$ , as given by (6) is monotonically increasing with  $c_{\max}$  and monotonically decreasing with  $\tau$ . Thus, efficiency can be equivalently obtained by maximizing  $c_{\max}$  for a given end-of-growth horizon  $\tau$ , or by minimizing  $\tau$  for a given sufficiency level of well-being  $c_{\max}$ . In the following, we

focus on the latter approach, and vary the pre-specified steady-state level of well-being per capita that could be interpreted as “sufficient” in the sense of Rawls (1971). We are particularly interested in the question how changes in  $c_{\max}$  affect the social cost of carbon (see section 4).

The exact functional forms of  $h(\cdot)$  and  $f(\cdot)$  determine the shape of the function that gives well-being as a function of time in the growth period  $0 < t < \tau$ . In order to apply our approach, we have to give the problem more structure and to specify the functional forms for  $h(\cdot)$  and  $f(\cdot)$ . In the following corollary to Theorem 1 we propose specific functional forms that lead to a growth function that we find particularly appealing.

**Corollary 1.** *Under Assumptions 1–5, and with the specification  $f(c) = -c \ln(c)$  and  $h(t - \tau) = (t - \tau)^2$ , the intertemporal distribution of well-being is given by (5) with  $\theta = \tau$  and the objective is to choose, for given values of  $c_0$  and  $\tau$ , the maximal feasible  $c_{\max}$  given the economic and climate constraints.*

The results follows after few steps of calculations when using the functional forms for  $h(\cdot)$  and  $f(\cdot)$  in (6). Corollary 1 specifies a particular preference for early growth, as captured by the particular specification of the function  $h(t - \tau) = (t - \tau)^2$ , i.e. the squared difference of the present point in time from the end-of-growth horizon  $\tau$ . It also specifies a particular preference for smooth growth, as captured by the particular specification of the function  $f(\cdot) = -c \ln(c)$ , which is the Shannon entropy of the consumption path – a “smoothness” measure commonly used in various applications.

As a result, we obtain the functional form for the intertemporal distribution of well-being results given in (5). At any point in time, well-being  $c_t$  is a weighted geometric average of initial well-being and the sufficient steady-state level of well-being  $c_{\max}$ , where the weight on  $c_{\max}$  increases over time, for  $t < \tau$ , according to  $(1 - t/\tau)^2$ . If  $c_{\max}$  is large compared to  $c_0$ , a sigmoid time path of well-being results.

Using  $g_c$  to denote the initial growth rate of  $c_t$ ,  $g_c \equiv \dot{c}_0/c_0$ , the functional form (5) can also be written as  $c_t = c_{\max} \exp\left(-\frac{g_0}{2\tau}(t - \tau)^2\right)$  for all  $t < \tau$ . For the growth rate of well-being we obtain

$$\frac{\dot{c}_t}{c_t} = \max\{g_c(1 - t/\tau), 0\}. \quad (8)$$



Thus, we have growth,  $\dot{c}_t > 0$ , for all  $t < \tau$  and  $c_t = c_{\max}$  thereafter. The growth rate is linearly decreasing from  $g^c$  to zero within the time horizon  $\tau$ , and remains zero after  $\tau$ . This reflects the stylized fact that during the course of economic development growth rates are typically relatively high first and decline thereafter. The linear decrease is a particular simple description of this process.<sup>4</sup>

Panels (c) and (d) in Figure 1 illustrate the optimization problems to maximize  $c_{\max}$  for a given value of  $\tau$  or to minimize  $\tau$  for a given value of  $c_{\max}$ . Both problems are equivalent.

A path of well-being described by (8) is feasible only if  $g^c$  is sufficiently small. Indeed, the longer the time horizon  $\tau$ , the smaller  $g^c$  must be to render the path (8) feasible given the economic and climate constraints imposed by the DICE model.

In order to connect our approach of directly specifying the intertemporal distribution of well-being to the standard setting that uses a discounted utilitarian SWF as in equation (1) we ask, for which time-varying intergenerational discount rates of well-being would the streams of well-being that correspond to the different specifications of  $c_{\max}$  turn out to be optimal if these discount rates were used in a SWF as given in equation (1)? The answer is that the path of well-being corresponding to a high value of  $c_{\max}$  is the optimal outcome if the discount rate is small, or, equivalently, the discount factor is high. Hence, we next derive the endogenous social discount rate of well-being. In other words, we determine the dual to the problem mainly considered in this paper, which would be to maximize the present value of consumption, applying the time path of social discount rates implied by the model considered here.

Using  $\varphi_t$  to denote the Lagrangian multiplier for the constraint

$$c_t \geq \bar{c}_t = c_{\max}^{1 - \left(1 - \frac{t}{\tau}\right)^2} c_0^{\left(1 - \frac{t}{\tau}\right)^2} \quad (9)$$

derived from (5), everything else as in the derivation of efficient developments in ap-

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<sup>4</sup>If the growth rate of well-being does not linearly decrease, the dynamics of well-being per capita and the resulting social cost of carbon would depend on additional parameters that determine the curvature of the growth path. Imposing linearity here is a simplification, as it reduces the number of parameter values to be specified.

pendix 5, we find that the first-order conditions for this dynamic optimization problem are formally identical to (12), except that (12a) is replaced by  $\varphi_t = \lambda_t^K$  for all  $t$ .

Thus, we can interpret the expression  $\Delta(t; \tau) \equiv \frac{\varphi_t}{\varphi_0}$  as an intergenerational discount factor of well-being, where we explicitly note the dependency on  $\tau$ . The average yearly intergenerational discount rate  $\bar{r}$  to arrive at the intergenerational discount factor  $\Delta(t; \tau)$  for a specific time period  $t$  can then be defined as  $\bar{r} \equiv \Delta(t; \tau)^{-1/t} - 1$ . Furthermore, we define the time-dependent yearly intergenerational discount rate as  $r_t = \left( \frac{\Delta(t-1; \tau)}{\Delta(t; \tau)} \right) - 1$ .

## 4 Quantitative Results for the DICE Model

Economic growth comes at a cost, since emitting one additional ton of  $CO_2$  into the atmosphere causes future climate damage, which society *should* (in a normative sense) consider in form of the social cost of carbon. The social cost of carbon associated with a given intertemporal distribution of well-being at a particular point in time is measured by the shadow price of  $CO_2$  divided by the shadow price of aggregate well-being at this time. Within the optimal DICE framework that we consider here, the social cost of carbon at this point in time must correspond to the societal cost of abating an additional ton of  $CO_2$  at that time, for reasons of efficiency. Indeed Nordhaus (2014) uses the marginal abatement cost function to calculate the social cost of carbon (see appendix 3 for the exact formula).

We are interested in how the social cost of carbon today varies with the sufficiency level of steady-state well-being  $c_{\max}$  and the corresponding intertemporal distribution of well-being. Hence, we exogenously change  $\tau$  and evaluate the corresponding optimal social cost of carbon in 2015. The numerical dynamic optimization results presented in the following have been calculated using the Knitro solver (version 9.0.1) together with the AMPL optimization software. The programming code is provided in appendix 4.

Figure 2 shows the time paths for well-being per capita for different sufficiency levels of well-being, measured in multiples of the 2010 level of well-being,  $c_0$ ,  $c_{\max} \in \{2, 5, 10, 15, 18, 22, 26\} \times c_0$ , under minimized end-of-growth horizons. The higher the maximal level of well-being per capita the more the end-of-growth horizon is shifted

into the future for which  $c_{\max}$  can efficiently be reached. The sub-graph in figure 2 illustrates that future growth in well-being per capita can only be implemented at the expense of lower near term growth rates (2010-2030). The smaller the steady-state level of well-being, the higher are the initial levels of well-being per capita from 2010 to 2030. Thus, figure 2 nicely summarizes the intergenerational trade-off that our global society faces today: Achieving larger prosperity for future generations requires that the current generation sacrifices part of its well-being. Vice versa the current generation can be made better-off when the societal objective is to stop growing earlier implying a lower future prosperity. However, the absolute sacrifice that the current generation needs to make to enable larger future prosperity is relatively small compared to the possible absolute future gain in well-being.

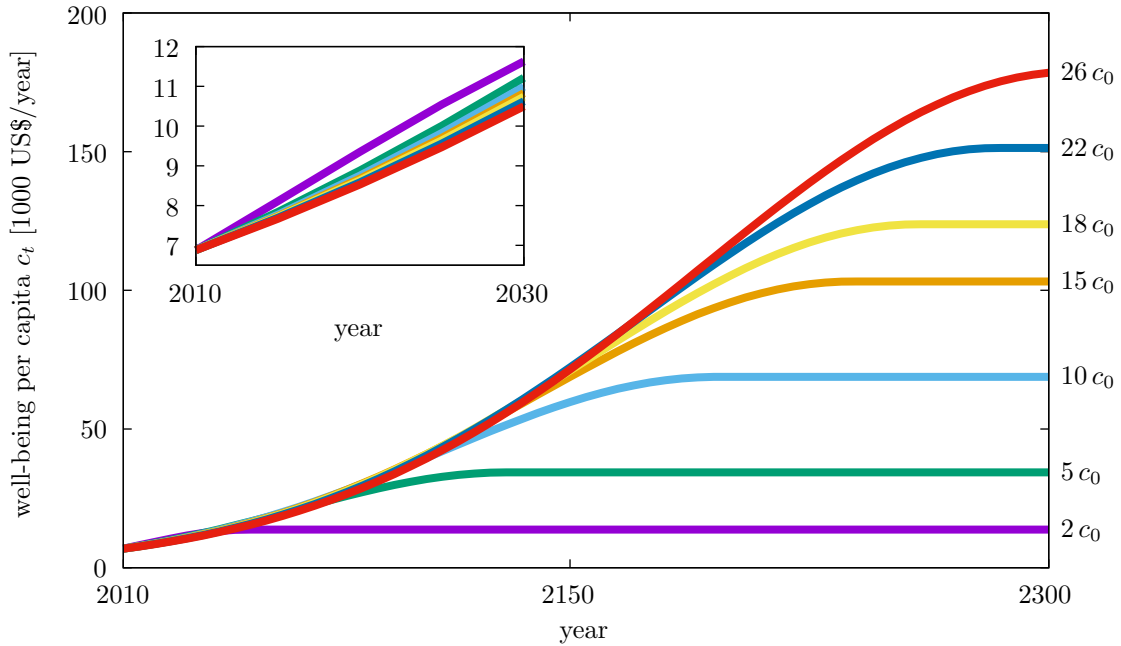


Figure 2: Time paths (2010-2300) of well-being per capita for varying sufficient steady-state levels of well-being  $c_{\max}$ , with optimized end-of-growth horizon  $\tau$ .

We are interested in how the level of sufficient steady state well-being drives the

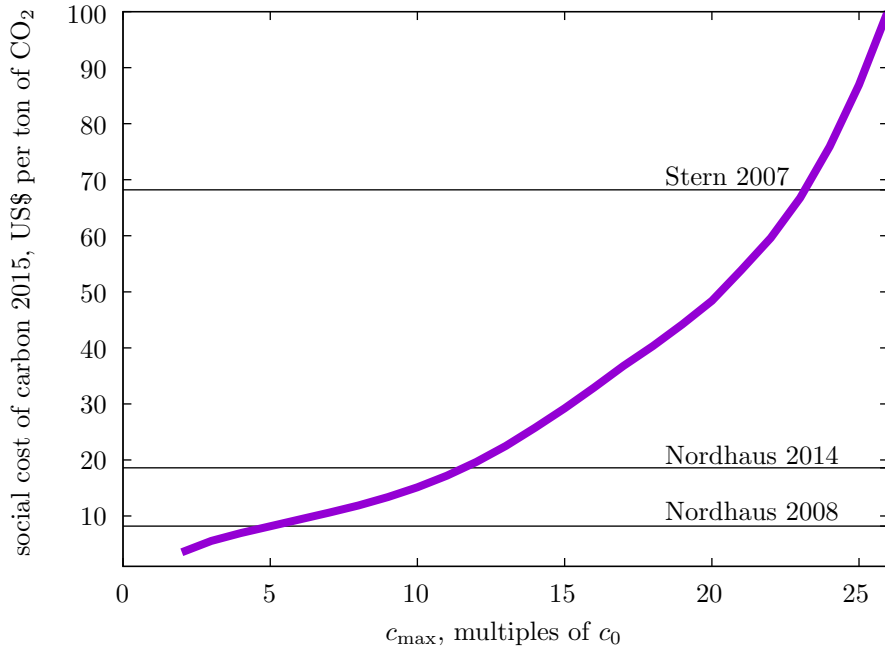


Figure 3: Social cost of carbon in 2015 as a function of the sufficient level of steady-state well-being  $c_{\max}$ , measured in multiples of the 2010 level  $c_0$ .

social cost of carbon today. By varying the steady state level of well-being  $c_{\max}$  that society would consider “sufficient”, we are able to directly assess how the distribution of growth between generations determines today’s cost of emitting an additional ton of  $CO_2$  into the atmosphere. Figure 3 shows the social cost of carbon in 2015 as a function of the sufficient steady-state levels of well-being  $c_{\max}$ . We find that the social cost of carbon in 2015 increases over-proportionally with the sufficiency level of well-being. While the social cost of carbon in 2015 is US\$ 7 if the sufficiency level is four-fold the present level, it is US\$ 30 if the sufficiency level is 15-fold and US\$ 100 if the sufficiency level is 26-fold the present level. The convex relationship between the sufficiency level of well-being and the social cost of carbon shows how strongly the social cost of carbon depends on the desired prospects for growth. When thinking of  $\tau$  as measuring for how many generations a positive growth rate is maintained, the trade-off between future growth in well-being and the social cost of carbon becomes the more severe the larger the number of generations for which a society wants to keep on growing and thus, the

more weight society puts on intergenerational distributive justice.

We can also compare the results of our approach with previous estimates of the social cost of carbon (cf. figure 3). The scenario of Nordhaus (2008), i.e. a social cost of carbon of US\$ 8.18 per ton of  $CO_2$  in 2015, is roughly equivalent to a steady-state level of well-being five times larger than the 2010 level. Increasing the sufficiency level of well-being to 11.5 times the 2010 level matches the social cost of carbon in 2013 DICE model being 18.6 US\$ per ton of  $CO_2$  (Nordhaus 2014). In turn, when growth ends at a level 23 times the 2010 level, the social cost of carbon in 2015 is US\$ 68.19, equivalent to the scenario of Stern (2007).

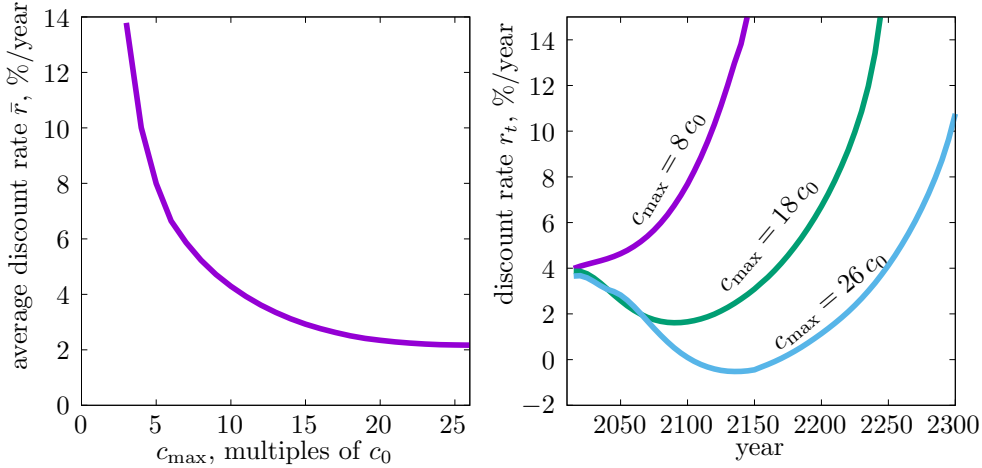


Figure 4: Left panel: Average endogenous intergenerational discount rate between 2010 and 2300 as a function of sufficient steady-state well-being  $c_{max}$ . Right panel: Endogenous intergenerational discount rate over time for three different levels of sufficient steady-state well-being.

The left panel in figure 4 sheds some further light on how our approach of directly specifying the intertemporal distribution of well-being relates to the standard setting using a discounted utilitarian SWF: It depicts the average yearly intergenerational discount rate<sup>5</sup> for well-being per capita  $\bar{r}(t; c_{max})$  as a function of  $c_{max}$ , which would result

<sup>5</sup>For the purpose of our analysis here we do not distinguish between different drivers of the intergenerational discount rate like the pure rate of time preference and the preference for intertemporal

in the respective discount factor in year 2100, i.e.  $\Delta(2100; c_{\max})$ . The function  $\bar{r}(t; c_{\max})$  is a decreasing function of  $c_{\max}$  reflecting that a higher level of steady-state well-being is equivalent to a smaller intergenerational discount rate. The right panel in figure 4 shows the implied intergenerational discount rates as a function of time for three different steady-state levels of well-being. Clearly, these implied discount rates are non-constant over time. They first increase only slowly (for a low level of  $c_{\max}$ ), or decrease (for higher levels of  $c_{\max}$ ), and may temporarily imply negative values (for high levels of  $c_{\max}$ ). Approaching the end-of-growth horizon  $\tau$ , the implied intergenerational discount rates increase strongly, reaching high values shortly before the steady state is reached.

Comparing the results shown in Figures 3 and 4, we find that for the steady-state level of well-being  $c_{\max} \approx 11.5$  that leads to a social cost of carbon similar to the value from Nordhaus (2014), the corresponding average intergenerational discount rate is about 4.2% per year, which is in line with the Nordhaus model.

In short, our analysis makes the intergenerational trade-off in the light of anthropogenic climate change very transparent: Increasing  $\tau$  and thereby letting  $g_t^c$  be more evenly intertemporally distributed, raises both the social cost of carbon in 2015 and the sacrifice in well-being for early generations.

## 5 Discussion and Conclusions

Our paper explores how alternative social objectives on the intertemporal distribution of well-being affect the social cost of carbon. Specifically, it opens a new perspective for the lively debate on which parametrization of an intertemporal SWF should be applied in climate economics, by shifting the focus directly on alternative specifications of the intertemporal distribution of well-being, instead of alternative specifications of an abstract welfare function.

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consumption smoothing. What matters in this paper is the possibility of choosing between different intertemporal distributions of well-being and acknowledging the implied social cost of carbon. The reason we make the link to discounting in figure 4, is to transparently show the way it is possible to switch from our approach back to the standard discounted utilitarian SWF setting.

Determining the time path of well-being that efficiently reaches a pre-specified constant steady-state level of well-being after a finite “end-of-growth” time horizon by means of dynamic optimization, we find that the social cost of carbon in 2015 is a convex function of this sufficiency level of steady-state well-being. The convex effect of the sufficiency level of well-being on the social cost of carbon is bounded above by the availability of a backstop technology. Nordhaus and Sztorc (2013) and Nordhaus (2014) assume that the initial backstop price is 344 US\$ for 100%  $CO_2$  removal in 2010 and then exponentially declines over time. Hence, the societal price of achieving a just intertemporal distribution in a carbon dependent economy also depends on the availability of mitigation technologies.

The level of well-being that is desired in steady state is related to the weight society puts on the well-being of future generations. In this respect, we find that a higher emphasis on long-run growth of well-being strongly raises the social cost of carbon today and increases the sacrifice in well-being for early generations. Consequently the earlier the point of zero growth is reached, implying a more uneven distribution of growth in well-being over time, the higher are the initial levels of per capita well-being from 2010 to 2030.

We acknowledge that for a policy-maker the specification of the steady-state level of well-being may be of similar complexity as the choice of the social discount rate within the standard approach of using a SWF. We believe, however, that a policy-maker could find it easier to choose a target steady-state level of well-being  $c_{\max}$  and understand the respective implications for intergenerational justice as opposed to understand the effect of the social discount rate being hidden in the SWF construct. Hence, although the policy-maker can be thought of as indirectly deciding on intergenerational discounting when choosing  $c_{\max}$ , the direct choice of an intertemporal distribution of well-being increases transparency for policy-makers and society.

By making the intergenerational trade-off in the light of anthropogenic climate change more transparent, the results show that normative conceptions of intergenerational distributive justice crucially determine the social cost of carbon. Therefore the knowledge of the overall societal goal in terms of the intertemporal distribution of

well-being should be the starting point of the integrated assessment of climate change. Policy-makers then choose among different feasible paths of well-being as opposed to discussing the “correct” parameters of the respective intertemporal SWF.

For the given modeling horizon of 300 years in DICE and an infinite end-of-growth horizon, the intertemporal distribution of well-being would be determined by the maximal growth rate of well-being per capita that can be sustained for every period leading to almost the maximal possible social cost of carbon of 344 US\$ in 2015. But what about the effect of extending the modeling horizon? Being an IAM, the DICE model is designed to estimate optimal carbon prices for a finite time horizon. Nevertheless, given that we are not aware of an appropriate time horizon to be considered in climate change analysis, it is important to understand that the average optimized growth rate of well-being per capita will decrease with increasing time-horizon. Moreover, as time approaches infinity, the economy will have paid for both climate damages and abatement costs. Thus, only the underlying one-sector growth model will be relevant from which it is well-known that the per capita growth rate equals the growth rate of total factor productivity, which equals zero as time approaches infinity.<sup>6</sup> In short, an infinite end-of-growth horizon  $\tau$  would be feasible within the DICE modeling horizon of 300 years, but not in general as growth will be zero in the limit, which makes sense for the DICE-economy, were growth is not decoupled from carbon-intensive production.

Naturally the analysis in the paper is subject to limitations. Obviously, as we use the DICE model, our quantitative results are subject to limitations build into the DICE model. This includes that we ignore uncertainty, endogenous population and directed technical change. To include, for example, uncertainty on the growth rate of well-being, would be a difficult task since the whole approach we propose in this paper relies on choosing among pathways of well-being per capita that are known with certainty. Nevertheless, this could be an interesting future research project. Incorporating uncertainty on climate variables in turn, would not change the reasoning behind our results, we

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<sup>6</sup>Let  $\bar{g}$  be the average growth rate of well-being per capita. Then we have  $\lim_{t \rightarrow \infty} g_0^A e^{-5t\delta^A} = \lim_{t \rightarrow \infty} \bar{g} = 0$ . See appendix 5 for detailed equations of the DICE model.



conjecture. Moreover, well-being is solely determined by an index of inclusive consumption per capita and hence, abstracts from relative price effects of other determinants of well-being. [Sterner and Persson \(2008\)](#) show that incorporating the relative price of environmental quality in the measurement of well-being increases optimal emission abatement in DICE. Hence, besides using a lower intergenerational discount rate, relative prices can be an additional argument for more stringent climate policies. We suspect, however, that considering relative prices would not change the qualitative insights of this paper as the relation between the intertemporal distribution of well-being and the social cost of carbon is independent from the way well-being is actually measured.

Furthermore, the set of pathways obtained by directly specifying our functional form for the intertemporal distribution of well-being with different growth horizons  $\tau$  is limited by its underlying assumptions. Although we believe that these assumptions are plausible and helpful to model the trade-off between the intergenerational distribution of well-being and the social cost of carbon, they are entirely normative and thus, subject to debate.

# Appendix 1: Efficient development of the climate-economy system

In the following we sketch the generic framework of the integrated climate-economy model according to the discrete-time DICE structure (Nordhaus and Sztorc 2013, Nordhaus 2014). Functional forms and parameter specifications can be found in appendix 2.

We use  $K_t$  to denote the economy's capital stock in period  $t$ , and  $Y(L_t, K_t, E_t, T_t^{AT}, t)$  to denote the production function. Output depends positively on labor  $L_t$ , which changes over time due to population growth, the capital stock  $K_t$ , and carbon emissions  $E_t$ . Output decreases with the global mean atmospheric temperature,  $T_t^{AT}$ , which determines climate damages. Productivity increases over time due to exogenous technical progress. Aggregate inclusive consumption<sup>7</sup> in period  $t$  is  $C_t = L_t c_t$ , where  $L_t$  is population size and  $c_t$  is per capita consumption. Using  $\delta^K$  to denote the proportional rate of capital depreciation, the national accounting equation reads

$$K_{t+1} = (1 - \delta^K) K_t + Y(L_t, K_t, E_t, T_t^{AT}, t) - L_t c_t \quad \text{accounting (10a)}$$

The atmospheric temperature develops according to

$$T_{t+1}^{AT} = T_t^{AT} + \chi^1 [F_{t+1} - \chi^2 T_t^{AT} - \chi^3 (T_t^{AT} - T_t^{LO})] \quad \text{atmospheric temperature (10b)}$$

where

$$F_{t+1} = \kappa \left[ \frac{\log(M_{t+1}^{AT}/M_{EQ}^{AT})}{\log 2} \right] + F_{t+1}^{EX} \quad (10c)$$

is radiative forcing, which depends on the atmospheric stock of Carbon,  $M_t^{AT}$ . Atmospheric temperature also reacts to the lower ocean temperature, which, in turn, develops according to

$$T_{t+1}^{LO} = T_t^{LO} + \chi^4 (T_t^{AT} - T_t^{LO}) \quad \text{lower ocean temperature (10d)}$$

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<sup>7</sup>Referred to as "consumption" from here on.

The carbon cycle is modeled by the following three-box model which stocks of carbon in the atmosphere,  $M_t^{AT}$ , the upper ocean,  $M_t^{UP}$ , and the lower ocean,  $M_t^{LO}$ . Carbon emissions enter into the atmosphere. The entire carbon cycle is described by

$$M_{t+1}^{AT} = E_{t+1} + \phi^{11} M_t^{AT} + \phi^{21} M_t^{UP} \quad \text{atmospheric carbon stock} \quad (10e)$$

$$M_{t+1}^{UP} = \phi^{12} M_t^{AT} + \phi^{22} M_t^{UP} + \phi^{32} M_t^{LO} \quad \text{upper ocean carbon stock} \quad (10f)$$

$$M_{t+1}^{LO} = \phi^{23} M_t^{UP} + \phi^{33} M_t^{LO} \quad \text{lower ocean carbon stock} \quad (10g)$$

Given the initial states of capital,  $K_0$ , resource,  $S_0$ , carbon stocks,  $M_0^{AT}$ ,  $M_0^{UP}$ ,  $M_0^{LO}$ , and temperatures,  $T_0^{AT}$  and  $T_0^{LO}$ , and given population and technology developments, the set of equations (10) define all feasible consumption/emission paths. The question is which among all feasible paths should be chosen. The answer to this question determines the social cost of carbon, i.e. the shadow price of carbon emissions into the atmosphere.

One natural restriction would be to choose only among the dynamically efficient consumption/emissions paths. Assuming that well-being depends only on consumption, a feasible path  $c_t = (c_0, c_1, \dots)$  is dynamically efficient if no other feasible path  $c'_t = (c'_0, c'_1, \dots)$  exists with  $c'_t \geq c_t$  for all  $t$  and  $c'_t > c_t$  for at least one period  $t$ . The dynamically efficient investment/emission path is found by maximizing per capita consumption  $c_{t_m}$  at one period in time  $t_m$ , keeping consumption at all other points in time at some pre-specified feasible minimum levels,  $c_t \geq \bar{c}_t$  for all  $t \neq t_m$ . Defining  $\pi_{t_m} = 1$  and  $\bar{c}_{t_m} = 0$ , the Lagrangian for this optimization problem can be compactly written as

$$\begin{aligned} L = & \sum_{t=0}^{\infty} \pi_t L_t (c_t - \bar{c}_t) + \lambda_t^K \left( (1 - \delta^K) K_t + Y(L_t, K_t, E_t, T_t^{AT}, t) - L_t c_t - K_{t+1} \right) \\ & + \lambda_t^{TAT} \left( T_t^{AT} + \chi^1 \left[ \kappa \left[ \frac{\log(M_{t+1}^{AT}/M_{EQ}^{AT})}{\log 2} \right] + F_{t+1}^{EX} - \chi^2 T_t^{AT} - \chi^3 (T_t^{AT} - T_t^{LO}) \right] - T_{t+1}^{AT} \right) \\ & + \lambda_t^{TLO} (T_t^{LO} + \chi^4 (T_t^{AT} - T_t^{LO}) - T_{t+1}^{LO}) \\ & + \lambda_t^{MAT} (E_{t+1} + \phi^{11} M_t^{AT} + \phi^{21} M_t^{UP} - M_{t+1}^{AT}) \\ & + \lambda_t^{MUP} (\phi^{12} M_t^{AT} + \phi^{22} M_t^{UP} + \phi^{32} M_t^{LO} - M_{t+1}^{UP}) \\ & + \lambda_t^{MLO} (\phi^{23} M_t^{UP} + \phi^{33} M_t^{LO} - M_{t+1}^{LO}), \quad (11) \end{aligned}$$

where  $\pi_t$  is the Lagrangian multiplier for the constraint  $c_t \geq \bar{c}_t$ ;  $\lambda_t^K$  for the capital accumulation constraint (10a);  $\lambda_t^{TAT}$  for atmospheric temperature (10b);  $\lambda_t^{TLO}$  for the

temperature of the lower ocean (10d); and  $\lambda_t^{MAT}$ ,  $\lambda_t^{MUP}$ , and  $\lambda_t^{MLO}$  for the carbon stocks in the atmosphere, upper and lower ocean, respectively.

The first-order conditions describing an efficient development of the climate-economy system can be written as follows. The conditions for the dynamically efficient consumption and emission levels are

$$\frac{\partial L}{\partial C_t} = 0 \quad \Leftrightarrow \quad \pi_t = \lambda_t^K \quad (12a)$$

$$\frac{\partial L}{\partial E_t} = 0 \quad \Leftrightarrow \quad \lambda_t^K Y_{E_t} = -\lambda_{t-1}^{MAT} \quad (12b)$$

and the condition for the efficient intertemporal allocation of capital is

$$\frac{\partial L}{\partial K_t} = 0 \quad \Leftrightarrow \quad \lambda_t^K (1 - \delta^K + Y_{K_t}) = \lambda_{t-1}^K \quad (12c)$$

The conditions for the efficient temperature dynamics capture the coupled dynamics of the two temperature boxes, as well as damage caused by the atmospheric temperature on production output,

$$\frac{\partial L}{\partial T_t^{AT}} = 0 \quad \Leftrightarrow \quad \lambda_{t-1}^{TAT} = \lambda_t^{TAT} (1 - \chi^1 (\chi^2 + \chi^3)) + \lambda_t^{TLO} \chi^4 + \lambda_t^K Y_{T_t^{AT}} \quad (12d)$$

$$\frac{\partial L}{\partial T_t^{LO}} = 0 \quad \Leftrightarrow \quad \lambda_{t-1}^{TLO} = \lambda_t^{TAT} \chi^1 \chi^3 + \lambda_t^{TLO} (1 - \chi^4). \quad (12e)$$

Finally, the efficient dynamics of the carbon cycle are characterized by

$$\frac{\partial L}{\partial M_t^{AT}} = 0 \quad \Leftrightarrow \quad \lambda_{t-1}^{MAT} = \lambda_t^{MAT} \phi^{11} + \lambda_t^{MUP} \phi^{12} + \lambda_{t-1}^{TAT} \frac{\chi^1 \kappa}{\log 2} \frac{1}{M_t^{AT}} \quad (12f)$$

$$\frac{\partial L}{\partial M_t^{UP}} = 0 \quad \Leftrightarrow \quad \lambda_{t-1}^{MUP} = \lambda_t^{MAT} \phi^{21} + \lambda_t^{MUP} \phi^{22} + \lambda_t^{MLO} \phi^{23} \quad (12g)$$

$$\frac{\partial L}{\partial M_t^{LO}} = 0 \quad \Leftrightarrow \quad \lambda_{t-1}^{MLO} = \lambda_t^{MUP} \phi^{32} + \lambda_t^{MLO} \phi^{33}, \quad (12h)$$

capturing the dynamics of carbon flows between the three boxes and the effect of atmospheric carbon on the atmospheric temperature. Conditions (12) together with (10) characterize any Pareto-efficient dynamic path. The initial social cost of carbon, measured in units of consumption, along the Pareto-efficient path are given by the ratio of

the Lagrangian multiplier of atmospheric carbon,  $\lambda_0^{MAT}$ , and of consumption at  $t = 0$ ,  $\lambda_0^K$ . Clearly, the pre-specified consumption levels  $\bar{c}_t$  will have a major influence on the social cost of carbon. Thus, a central question is how to distribute per-capita consumption over time, i.e. which among the many Pareto-efficient paths to choose.

Maximizing (1) subject to (10) leads to conditions (12), but with (12a) replaced by

$$(1 + \rho)^{-t} c_t^{-\eta} = \lambda_t^K \quad \stackrel{(12c)}{\Leftrightarrow} \quad (1 + \rho) \left( 1 + \frac{c_t - c_{t-1}}{c_{t-1}} \right)^\eta = 1 + Y_{K_t} - \delta^K, \quad (13)$$

which is the discrete-time version of the well-known Ramsey rule (Dasgupta 2008). This condition characterizes the intertemporal distribution of consumption that is optimal according to (1).

## Appendix 2: Proof of Theorem 1

In general there are trade-offs between Assumptions 3, 4, and 5, i.e. the preferences for early growth, smooth growth, and efficiency. An intertemporal distribution of well-being  $c_t$ ,  $t \in 0, T$  with some finite  $T > \tau$  that is compatible with all three Assumptions 3–5 is obtained by maximizing one of these three objectives while keeping the other two at some minimum levels. Additionally imposing 1 and 2, a path of well-being that is compatible with Assumptions 1–5 is characterized by the solution of the following optimization problem.

$$\max_{\{c_t\}_0^\tau; c_{\max} \in \mathbb{X}} \int_0^T c_t dt \quad \text{subject to} \quad (14)$$

$$\int_0^\tau f(c_t) dt \geq \sigma \quad (\text{Lagrangian multiplier } \lambda) \quad (15)$$

$$\int_0^\tau h(t - \tau) c_t dt \geq \epsilon \quad (\text{Lagrangian multiplier } \mu) \quad (16)$$

$$c_t = c_{\max} \quad \text{for all } t \geq \tau \quad (\text{Lagrangian multiplier } \nu_t), \quad (17)$$

and  $c_0$  given, and where  $\sigma$  and  $\epsilon$  are levels of ‘smoothness’ and ‘earliness’ fixed at feasible levels. In this formulation, the maximization (14) captures assumption 5 of efficiency, i.e. to maximize aggregate well-being over time while not reducing smoothness or earliness

of growth below the respective given levels: constraint (15) captures the preference for smooth growth (assumption 4), constraint (16) captures the preference for early growth (assumption 3), constraint (17) the assumption that a sufficient level of well-being is maintained after the end-of-growth horizon  $\tau$  (assumption 2), and finally the initial condition captures assumption 1.

The Lagrangian for the optimization problem is  $L = \int_0^T c_t dt + \lambda (\sigma - \int_0^\tau f(c_t) dt) + \mu (\epsilon - \int_0^\tau h(t - \tau) c_t dt)$  and the first-order conditions for choosing  $c_t$  efficiently for all  $t < \tau$  read

$$1 - \mu h(t - \tau) - \lambda f'(c_t) = 0. \quad (18)$$

Note that  $\mu$  and  $\lambda$  are time-invariant. Their values depend on the reference values  $\sigma$  for smooth growth and  $\epsilon$  for early growth. Rearranging we obtain

$$\Leftrightarrow c_t = f'^{-1} \left( \frac{1 - \mu h(t - \tau)}{\lambda} \right). \quad (19)$$

Using the boundary condition  $c_\tau = c_{\max}$  we obtain

$$c_{\max} = f'^{-1} \left( \frac{1}{\lambda} \right) \Leftrightarrow \lambda = \frac{1}{f'(c_{\max})} \quad (20)$$

The initial condition yields

$$c_0 = f'^{-1} \left( \frac{1 - \mu h(-\tau)}{\lambda} \right) \Leftrightarrow \mu = \frac{1}{h(-\tau)} \left( 1 - \frac{f'(c_0)}{f'(c_{\max})} \right), \quad (21)$$

where we have used the expression for  $\lambda$  derived above. Using (20) and (21) in (19) we obtain (6). For  $t \geq \tau$ , the first-order condition for the efficient level of  $c_t$  is  $1 = \nu_t$ . The derivative of the Lagrangian with respect to  $c_{\max}$  is equal to  $\nu_t = 1 > 0$ . Thus,  $c_{\max}$  is to be chosen at the maximum feasible level as specified in the theorem. Note that the given assumptions characterize the preferred paths of well-being in a rather restrictive way. Given  $c_0$  and  $\tau$  (Assumptions 1 and 2), and choosing  $c_{\max}$  at the maximum feasible level (Assumption 5) fully determines the ‘earliness’ and ‘smoothness’ of the path – there are no further degrees of freedom. Alternatively one could choose earliness or smoothness

at the maximum feasible levels, but then the other of the two as well as aggregate consumption would be fixed.

As the last step of the proof, we verify that the path of well-being characterized by (6) is monotonic in time. This is the case, as  $f'^{-1}(\cdot)$  is a decreasing function (due to the assumed concavity of  $f(\cdot)$ ) and as

$$\begin{aligned} \frac{d}{dt} \left( f'(c_{\max}) \left( 1 + \frac{h(t-\tau)}{h(-\tau)} \left( \frac{f'(c_0)}{f'(c_{\max})} - 1 \right) \right) \right) \\ = \frac{h'(t-\tau)}{h(-\tau)} (f'(c_0) - f'(c_{\max})) \stackrel{\leq}{\geq} 0 \quad \Leftrightarrow \quad c_0 \stackrel{\leq}{\geq} c_{\max}. \quad (22) \end{aligned}$$

## Appendix 3: DICE 2013R Model

Table 2: DICE 2013R, Data (1)

Parameter	Unity	Equation
<b>Preferences</b>		
Time preference rate / year	-	$\rho = 0.015$
Elasticity of marginal utility of consumption	-	$\eta = 1.45$
<b>Population and technology</b>		
Capital elasticity	-	$\gamma = 0.3$
Initial world population	Millions	$L_0 = 6838$
Population growth rate	-	$g^L = 0.134$
Depreciation rate of capital / year	-	$\delta^K = 0.1$
Initial world gross output	Trillions 2005 US\$	$Y_0^{Gross} = 63.69$
Initial capital	Trillion 2005 US\$	$K_0 = 135$
Initial level of total factor productivity (TFP)	-	$A_0 = 3.8$
Initial growth rate of TFP / period	-	$g_0^A = 0.079$
Decline rate of TFP / period	-	$\delta^A = 0.006$



Table 3: DICE 2013R, Data (2)

Parameter	Unity	Equation
<b>Emissions</b>		
Initial industrial emissions	Gigatons CO <sub>2</sub>	$E_0^{Ind} = 33.61$
Initial emissions control rate	-	$\mu_0 = 0.039$
Emissions control rate 2010-2150	-	$\mu = [0, 1]$
Emissions control rate 2150-2310	-	$\mu = [0, 1.2]$
Initial land emissions	Gigatons CO <sub>2</sub>	$E_0^{Land} = 3.3$
Initial cumulative emissions	Gigatons CO <sub>2</sub>	$E_0^{Cum} = 90$
Initial carbon intensity	Kilograms CO <sub>2</sub>	$\sigma_0 = \frac{E_0^{Ind}}{Y^{Gross}(1-\mu_0)}$
Initial growth of carbon intensity	-	$g_0^\sigma = -0.01$
Decline rate of carbon intensity / period	-	$\delta^\sigma = -0.001$
<b>Carbon cycle</b>		
Initial concentration atmosphere	Gigatons carbon	$M_0^{AT} = 830.4$
Initial concentration upper oceans	Gigatons carbon	$M_0^{UP} = 1527$
Initial concentration deep oceans	Gigatons carbon	$M_0^{LO} = 10010$
Equilibrium concentration atmosphere	Gigatons carbon	$M_{EQ}^{AT} = 588$
Equilibrium concentration upper oceans	Gigatons carbon	$M_{EQ}^{UP} = 1350$
Equilibrium concentration deep oceans	Gigatons carbon	$M_{EQ}^{UP} = 10000$
Flow atmosphere to atmosphere	-	$\phi_{11} = 1 - \phi_{12}$
Flow upper oceans to atmosphere	-	$\phi_{21} = \phi_{12} \frac{M_{EQ}^{AT}}{M_{EQ}^{UP}}$
Flow atmosphere to upper oceans	-	$\phi_{12} = 0.088$
Flow upper oceans to upper oceans	-	$\phi_{22} = 1 - \phi_{21} - \phi_{23}$
Flow deep oceans to upper oceans	-	$\phi_{32} = \phi_{23} \frac{M_{EQ}^{UP}}{M_{EQ}^{LO}}$
Flow upper oceans to deep oceans	-	$\phi_{23} = 0.0025$
Flow deep oceans to deep oceans	-	$\phi_{11} = 1 - \phi_{12}$

Table 4: DICE 2013R, Data (2)

Parameter	Unity	Equation
<b>Climate model</b>		
Equilibrium climate sensitivity	°C increase /doubling of CO <sub>2</sub>	$\nu = 2.9$
Forcing due to equilibrium CO <sub>2</sub> doubling	Watts / square meter	$\kappa = 3.8$
2010 forcing of non-CO <sub>2</sub> GHG	Watts / square meter	$F_0^{EX} = 0.25$
2100 forcing of non-CO <sub>2</sub> GHG	Watts / square meter	$F_{18}^{EX} = 0.7$
Initial atmospheric temperature change	°C from 1750	$T_0^{AT} = 0.8$
Initial deep oceans temperature change	°C from 1750	$T_0^{LO} = 0.0068$
Speed of adjustment atmospheric temperature	-	$\chi_1 = 0.098$
Equilibrium forcing /doubling of CO <sub>2</sub>	-	$\chi_2 = \kappa/\nu$
Heat loss from atmosphere to deep oceans	-	$\chi_3 = 0.088$
Heat gain of deep oceans	-	$\chi_4 = 0.025$
<b>Climate change abatement costs</b>		
Damage quadratic term	-	$\psi = 0.00267$
Initial abatement costs	Trillions 2005 US\$	$\Lambda_0 = 0$
Exponent of abatement cost function	-	$\Theta = 2.8$
Initial backstop price	2005 US\$	$p_0^{Back} = 344$
Decline rate of backstop price / period	-	$g^{Back}$

Table 5: DICE 2013R, Exogenous equations

Parameter	Unity	Equation
Total population	Millions	$L_t = L_{t-1} \left( \frac{10500}{L_{t-1}} \right)^{g^L}$
Total factor productivity (TFP)	-	$A_t = \frac{A_{t-1}}{1-g_{t-1}^A}$
Growth rate TFP per period	-	$g_t^A = g_0^A e^{-5t\delta^A}$
Total factor productivity (TFP)	-	$A_t = \frac{A_{t-1}}{1-g_{t-1}^A}$
Carbon intensity	Kilograms CO <sub>2</sub> / output	$\sigma_t = \sigma_{t-1} e^{-5g_{t-1}^\sigma}$
Growth rate of carbon intensity / period	-	$g_t^\sigma = \frac{g_{t-1}^\sigma}{(1+\delta^\sigma)^5}$
External forcing	Watts / square meter	$F_t^{EX} = F_0^{EX} + \frac{1}{18} (F_{18}^{EX} - F_0^{EX}) (t + 2)$
Backstop price	2005 US\$ / ton of CO <sub>2</sub>	$p_t^{Back} = p_{t-1}^{Back} (1 - g^{Back})$
Adjusted cost factor for backstop	2005 US\$ / ton of CO <sub>2</sub>	$\hat{p}_t^{Back} = \frac{p_t^{Back} \sigma_t}{1000 \theta}$

Table 6: DICE 2013R, Endogenous equations

Parameter	Unity	Equation
Welfare under Nordhaus	-	$W_0(c_0, c_1, c_2, \dots) = \sum_{t=0}^{60} \frac{1}{(1+\rho)^{5t}} L_t \frac{c_t^{1-\eta}}{1-\eta}$
Total emissions	Gigatons CO <sub>2</sub>	$E_t = E_t^{Ind} + E_t^{Land}$
Carbon concentration atmosphere	Gigatons carbon	$M_t^{AT} = \frac{5}{3.666} E_t + \phi^{11} M_{t-1}^{AT} + \phi^{21} M_{t-1}^{UP}$
Carbon concentration upper oceans	Gigatons carbon	$M_t^{UP} = \phi^{12} M_{t-1}^{AT} + \phi^{22} M_{t-1}^{UP} + \phi^{32} M_{t-1}^{LO}$
Carbon concentration lower oceans	Gigatons carbon	$M_t^{LO} = \phi^{23} M_{t-1}^{UP} + \phi^{33} M_{t-1}^{LO}$
Total radiative forcing	Watts / square meter	$F_t = \kappa \left[ \frac{\log(M_t^{AT}/M_{EQ}^{AT})}{\log 2} \right] + F_t^{EX}$
Atmospheric temperature change	°C from 1750	$T_t^{AT} = T_{t-1}^{AT} + \chi [F_t - \chi^2 T_{t-1}^{AT} - \chi^3 (T_{t-1}^{AT} - T_{t-1}^{LO})]$
Upper ocean temperature change	°C from 1750	$T_t^{LO} = T_{t-1}^{LO} + \chi^4 (T_{t-1}^{AT} - T_{t-1}^{LO})$
Capital	Trillions 2005 US\$	$K_t = (1 - \delta^K)^5 K_{t-1} + 5I_{t-1}$
Gross output	Trillions 2005 US\$	$Y^{Gross} = A_t \left( \frac{L_t}{1000} \right)^{1-\gamma} K_t^\gamma$
Industrial emissions	Gigatons CO <sub>2</sub>	$E_t^{Ind} = \sigma_t (1 - \mu_t) Y^{Gross}$
Cumulative emissions	Gigatons carbon	$E_t^{Cum} = E_{t-1}^{Cum} + \frac{5}{3.666} E_{t-1}^{Ind} \leq 6000$
Abatement costs	Trillions 2005 US\$	$\Lambda_t = Y_t^{Gross} \hat{p}_t^{Back} \mu_t^\Theta$
Damage fraction	-	$\Omega_t = \Psi(T_t^{AT})^2$
Net output	Trillions 2005 US\$	$Y_t = [Y_t^{Gross} (1 - \Omega_t)] - \Lambda_t$
Consumption	Trillions 2005 US\$	$C_t = Y_t - I_t$
Consumption per capita	Thousands 2005 US\$	$c_t = \frac{1000 C_t}{L_t}$
Social cost of carbon	2005 US\$ / ton of carbon	$p_t^c = p_t^{Back} \mu_t^{\Theta-1}$

## Appendix 4: AMPL Programming Code

```
# PARAMETERS

# modelling horizon
param T:=60;

# population and technology
param gamma:=0.3; # capital elasticity in production function
param L0:=6838; # initial world population (millions)
param gL:=0.134; # initial growth rate of world population per period
such that popasynt=10500

param L {t in 0..T}>=0;
let L[0]:=L0;
let {t in 1..T} L[t]:=L[t-1]*((10500/L[t-1])^gL);

param deltaK:=0.1; #depreciation rate on capital per year
param Qgross0:=63.69; #initial world gross output (trillions 2005 USD)
param K0:=135; #initial capital value (trillions 2005 USD)
param A0:=3.8; #initial level of total factor productivity (TFP)
param gA0:=0.079; #initial growth rate for TFP per period
param deltaA:=0.006; #decline rate of TFP per period

param gA {t in 0..T}>=0; # growth rate for TFP per period
let {t in 0..T} gA[t]:=gA0*exp(-deltaA*5*(t));

param A {t in 0..T}>=0; # TFP
let A[0]:=A0;
let {t in 1..T} A[t]:=A[t-1]/(1-gA[t-1]);
```

```

# emission parameters
param gsigma0:=-0.01; #initial growth rate of sigma (coninuous per year)
param deltasigma:=-0.001; #decline rate of decarbonization per period
param ELand0:=3.3; # initial land emissions(GtCO2)
param deltaLand:=0.2; #decline rate of land emissions per period
param EInd0:=33.61; # initial industrial emissions(GtCO2)
param Ecum0:=90; #initial cumulative emissions (GtC)
param mu0:=0.039; # initial emissions control rate
param Lambda0:=0; # initial abatement costs(trillions 2005 USD)
param sigma0:=EInd0/(Qgross0*(1-mu0)); #initial carbon intensity(kgCO2 per output)

param gsigma {t in 0..T}; # growth rate of carbon intensity per period
let gsigma[0]:=gsigma0;
let {t in 1..T} gsigma[t]:=gsigma[t-1]*((1+deltasigma)^5);

param sigma {t in 0..T}>=0; # carbon intensity(kgCO2 per output of 2005 USD)
let sigma[0]:=sigma0;
let {t in 1..T} sigma[t]:=sigma[t-1]*exp(gsigma[t-1]*5);

param ELand {t in 0..T}>=0; # land emissions per period (GtCO2)
let ELand[0]:=ELand0;
let {t in 1..T} ELand[t]:=ELand [t-1]*(1-deltaLand);

# carbon cycle
param MAT0=830.4; # initial concentration in atmosphere(GtC)
param MUP0:=1527; # initial concentration in upper ocean/biosphere(GtC)
param MLO0:=10010; # initial concentration in deep oceans(GtC)
param MATEQ:=588; # equilibrium concentration in atmosphere
(preindustrial atmospheric carbon) (GtC)
param MUPEQ:=1350; # equilibrium concentration in upper ocean/biosphere (GtC)

```

```

param MLOEQ:=10000; # equilibrium concentration in deep oceans(GtC)

# flow parameters (carbon cycle transition matrix)
param phi12:=0.088;
param phi23:=0.00250;
param phi11=1-phi12;
param phi21=phi12*MATEQ/MUPEQ;
param phi22=1-phi21-phi23;
param phi32=phi23*MUPEQ/MLOEQ;
param phi33=1-phi32;

# climate model parameters
param nu:=2.9; # equilibrium climate sensitivity (°C per doubling CO2)
param kappa:=3.8; # forcing of equilibrium CO2 doubling (Wm-2)
param Fex0:=0.25; # 2010 forcing of non-CO2 GHG (Wm-2)
param Fex18:=0.70; # 2100 forcing of non-CO2 GHG (Wm-2)
param Fex {t in 0..T}=Fex0+1/18*(Fex18-Fex0)*(2+t); # external forcing (Wm-2)
param TL00:=0.0068; # initial temperature change of upper ocean/biosphere
(°C from 1750)
param TAT0:=0.80; # initial atmospheric temperature change (°C from 1750)
param xi1:=0.098; # speed of adjustment parameter for atmospheric temperature
param xi2=kappa/nu; # climate model parameter
param xi3:=0.088; # coefficient of heat loss from atmosphere
to upper oceans/biosphere
param xi4:=0.025; # coefficient of heat gain by deep oceans

# climate damage parameters
param Psi:=0.00267; # damage quadratic term

```

```

# abatement cost
param Theta:=2.8; # exponent of abatement cost function
param pback0:=344; # initial backstop price(2005 USD per tCO2)
param gback:=0.025; # decline rate of backstop price per period

param pback {t in 0..T}>=0; # backstop price(2005 USD per tCO2)
let pback[0]:=pback0;
let {t in 1..T} pback[t]:=pback[t-1]*(1-gback);

param phead {t in 0..T}=pback[t]*sigma[t]/Theta/1000;
# adjusted cost factor for backstop

# VARIABLES # Upper and lower bounds for stability according to Nordhaus (2013)

# capital(trillions 2005 USD)
var K {t in 0..T}>=1;

# Gross output(trillions 2005 USD)
var Qgross {t in 0..T}=A[t]*((L[t]/1000)^(1-gamma))*(K[t]^gamma);

# carbon atmosphere (GtC)
var MAT {t in 0..T}>=10;

# carbon upper ocean (GtC)
var MUP {t in 0..T}>=100;

# carbon lower ocean (GtC)
var MLO {t in 0..T}>=1000;

# total radiative forcing (Wm-2)

```



```

var F {t in 0..T}=kappa*((log(MAT[t]/MATEQ))/log(2))+Fex[t];

# atmospheric temperature change (°C from 1750)
var TAT {t in 0..T}>=0, <=40;

# ocean temperature change (°C from 1750)
var TLO {t in 0..T}>=-1, <=20;

# damage fraction
var Omega {t in 0..T}=Psi*(TAT[t])^2;

# damages(trillions 2005 USD)
var damage {t in 0..T}=Omega[t]*Qgross[t];

# emission control rate
var mu {t in 0..T}>=0;

# abatement costs as fraction of output(trillions 2005 USD)
var Lambda {t in 0..T}=Qgross[t]*phead[t]*(mu[t]^Theta);

# industrial emissions (GtCO2)
var EInd {t in 0..T}=sigma[t]*Qgross[t]*(1-mu[t]);

# total emissions (GtCO2)
var E {t in 0..T}=EInd[t]+ELand[t];

# maximum cumulative extraction fossil fuels (GtC)
var Ecum {t in 0..T}<=6000;

# Marginal cost of abatement (social cost of carbon)(2005 USD per tCO2)

```

```

var cprice {t in 0..T}=pback[t]*mu[t]^(Theta-1);

# output net of damages and abatement(trillions 2005 USD)
var Q {t in 0..T}=(Qgross[t]*(1-Omega[t]))-Lambda[t];

# per capita consumption (1000s 2005 USD)
param cmax default 178.88;
param c0 default 6.88;
var tau >=0;
var c {t in 0..T} = cmax^(1-(1-t/tau)^2)*c0^((1-t/tau)^2);

# aggregate consumption (trillions 2005 USD)
var C {t in 0..T} = L[t]*c[t]/1000;

# Investment(trillions 2005 USD)
var I {t in 0..T}>=0;

# OBJECTIVE FUNCTION

minimize objective_function: tau;

subject to constr_accounting {t in 0..T}: c[t]=1000*(Q[t]-I[t])/L[t];
#subject to constr_accounting {t in 0..T}: C[t]=Q[t]-I[t];
#subject to constr_emissions {t in 0..T}: E[t]=EInd[t]+ELand[t];
subject to constr_capital_dynamics {t in 1..T}: K[t]=(1-deltaK)^5*K[t-1]+5*I[t-1];
subject to constr_cumulativeemissions {t in 1..T}:
Ecum[t]=Ecum[t-1]+(EInd[t-1]*5/3.666);
subject to constr_atmosphere {t in 1..T}:
MAT[t]=E[t]*(5/3.666)+phi11*MAT[t-1]+phi21*MUP[t-1];
subject to constr_upper_ocean {t in 1..T}:

```

```

MUP[t]=phi12*MAT[t-1]+phi22*MUP[t-1]+phi32*MLO[t-1];
subject to constr_lower_ocean {t in 1..T}:
MLO[t]=phi23*MUP[t-1]+phi33*MLO[t-1];
subject to constr_atmospheric_temp {t in 1..T}:
TAT[t]=TAT[t-1]+xi1*((F[t]-xi2*TAT[t-1])-(xi3*(TAT[t-1]-TLO[t-1])));
subject to constr_ocean_temp {t in 1..T}:
TLO[t]=TLO[t-1]+xi4*(TAT[t-1]-TLO[t-1]);

# Initial conditions
subject to initial_capital: K[0] = K0;
subject to initial_Ecum: Ecum[0]=Ecum0;
subject to initial_MAT: MAT[0]=MAT0;
subject to initial_MUP: MUP[0]=MUP0;
subject to initial_MLO: MLO[0]=MLO0;
subject to initial_TLO: TLO[0]=TLO0;
subject to initial_TAT: TAT[0]=TAT0;
subject to control1 {t in 1..28}: mu[t]<=1;
subject to control2 {t in 29..T}: mu[t]<=1.2;

```

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