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An auction story: How simple bids struggle with uncertainty

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ABSTRACT

Short-term electricity markets are key to an efficient production by generation units. We develop a two-period model to assess different bidding formats to determine for each bidding format the optimal bidding strategy of competitive generators facing price-uncertainty. We compare the results for simple bidding, block bidding and multi-part bidding. We find that even under optimal simple and block bidding, generators face the risk of ex-post suboptimal solutions, whereas in multi-part bidding these do not occur. This points to efficiency gains of multi-part bidding in the presence of uncertainty in electricity markets.

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1. Introduction

Short-term electricity markets, especially day-ahead and intraday markets, were set up with the goal to efficiently match generation and demand of electricity. A key challenge for market operators and participants is to account for inter-temporal linkages across time, which is caused by the technical characteristics of generation units connected to the electricity grid (Elmaghraby and Oren, 1999). Examples of these inter-temporal technical limitations are start-up costs that occur once when a power plant is started up, ramping constraints that limit the rate of change of production, the maximum time a load can be shed without impacting its service quality (for example cooling houses) or the remaining electricity stored in a battery. All these limitations imply that actors cannot sell electricity in different time periods as fully independent products, and thus introduce some level of complementarity (or substitutability, depending of technology) of sales in consecutive hours. Traditionally, these inter-temporal linkages have received less attention in European intraday-markets, because (i) overall load profile followed repeated patterns and thus could already be considered in bids or even in product design, (ii) large and often integrated generators could optimize within their portfolio. This is changing with rising shares of intermittent renewable energy sources

and their stochastic variations, as well as the increasing role of smaller-scale generation and flexibility providers, so that increasing shares of generation or load need to be rescheduled in shorter time frames.

The market design to facilitate such adjustments can be informed by literature on auction theory. However, while most literature deals with auctions for single goods, the literature on multi-unit auctions is not as well developed (Klemperer, 2004). The most closely related strand are combinatorial auctions, which allow bidders to express the willingness to buy packages of goods (Cramton et al., 2007). In this paper, we concentrate on the specific case of multi-unit auctions which allow expressing complementarities via bidding multiple parameters, which exist in the praxis of electricity markets.

Various bidding designs exist to coordinate production of generators, usually distinguishing between simple bidding (single time period with one price component), block bidding (multi-time period with one price component), and multi-part bidding (often in the form of three-part bids), where generators submit bids describing their variable and start-up costs (and/or other technical and financial parameters). While multi-part bidding is used in practice in US day-ahead markets, such as PJM (2019), and similar bid structures (e.g. minimum income conditions) are implemented in the European common coupling algorithm Euphemia (NEMO Committee, 2019) and used in some European markets (such as on the Iberian peninsula), most of European markets rely on simple bids and (variations of) block bids only for the day-ahead market, and only allow for simple bids in the intraday time frame.

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We model these bidding formats to assess their performance in the presence of uncertainty, by building a simple two-period model of a price taker's optimal bids. In simple bidding, generators need to account for start-up costs in individually accepted bids, to exclude inefficient solutions, while not excluding too many profitable combinations where the price results in losses in one period, but is high enough in the other period as to make the combination of the two profitable. Due to this inherent trade-off in bidding, ex-post several inefficiencies in production are possible, for example, the unit is accepted for one period, although the price is not high enough to recover costs, or it is accepted for one period only, when it would have been efficient to dispatch the unit in two periods.

For block bidding, the unit can only be accepted jointly. However, under low prices below variable cost in one period and high prices in the other, it might have been more efficient to let the unit run for a single time period.

The advantages and disadvantages of these market designs have been discussed previously in literature. [Elmaghraby and Oren \(1999\)](#) compare energy only hourly auctions (termed vertical auctions) with horizontal auctions (defined by load slices) in an analytical model under certainty and conclude that, even under perfect information, no efficient outcome can be achieved under energy only hourly auctions. [Sioshansi et al. \(2010\)](#) compare multi-part bids with simple energy bids in a deterministic simulation study and find that energy-only simple bidding results in inefficiencies due to ignoring the inter-temporal linkages and non-convex generation costs, while multi-part bidding may result in some equity and incentive issues (which can partly be mitigated by uplift payments). [Sioshansi and Nicholson \(2011\)](#) explicitly look at market power issues with regard to multi-part (here two-part) and simple bidding in an analytic framework of a duopoly under deterministic demand levels (with a numerical quantification). The authors find that under both formats strategic bidding is taking place, leading in many cases to mixed-strategy equilibria. Finally, they find that energy only markets with simple bidding yield higher prices. [O'Neill et al. \(2005\)](#) emphasise that in markets with non-convexities, market-equilibria without multi-part bids might not exist. The authors present clearing mechanisms and how to interpret prices in mixed-integer problems to arrive at market clearing prices. [Reguant \(2014\)](#) analyses the usage of complementary bidding mechanisms (i.e. multi-part bids in the terminology of this paper) in the Spanish electricity market using a structural model and explains apparent mark-ups in simple bids that in part exist due to start-up costs; however, she does not address the question of what drives firms to use one or the other bidding format. In the corresponding thesis chapter ([Reguant, 2011](#)), an additional computational analysis is presented, which compares the welfare effects of simple bidding and multi-part bidding using a computational counter-factual analysis. Here, multi-part bidding leads to efficiency gains, even in the presence of market power, as well as lower prices. This is caused by increased volatility in the market if simple bids only are allowed, and the reduced coordination of start-up decisions.

While these papers discuss inefficiencies arising in equilibrium with and without market power, uncertainty may lead to further inefficiencies due to possible coordination issues. [Cramton \(2017\)](#) discusses qualitatively that the internalisation of start-up and other costs based on market expectations would often turn out to be wrong and distort the energy offer curve. Engineering literature on the other hand, uses stochastic optimisation to derive optimal bidding strategies under price uncertainty ([Conejo et al., 2002](#)). To our knowledge, however, the role uncertainty explicitly plays in reducing the efficiency of simple bidding as compared to multi-part bidding has not been discussed in literature using an analytical framework. In contrast, this paper concentrates on the effect that price uncertainty has in isolation from other effects on the bidding strategies of generators in the presence of start-up cost. While this inefficiency is similar in nature to the principal coordination issue of simple marginal prices if there are indivisibilities and other

non-convexities ([O'Neill et al., 2005](#)), in our model the inefficiency would cease to exist in case that there was no uncertainty. This is a relevant problem especially for short-term markets since in intra-day trading price uncertainty has very significant levels ([Neuhoff et al., 2016](#)), and may increase in future with higher levels of variable renewable energy sources.

2. Bidding format description & optimal bidding strategies

We are investigating bidding formats using a simplified analytical framework of a single, price-taking actor with a power plant exposed to uncertain power prices in two time periods. The power plant is participating in a uniform price auction in a large competitive market. The model consists of two stages, which are presented in [Fig. 1](#). In [Table A.1](#) (in [Appendix A](#)), all parameters and variables of the model can be found.

In the first stage, depending on the bidding format, the actor bids in order to maximize expected profit over both trading periods, given the price expectations, as well as the clearing rules of the bidding format.

In the second stage, the market is cleared according to the submitted bids, the clearing rule of the bidding formats (described in detail below) and the stochastic realisations of the prices in the two consecutive periods. The prices are independently uniformly distributed $U(0, p_{max})$. As a result of market clearing, bids of the actor are either accepted or not. If any of the bids are accepted the power plant needs to produce power and its owner receives the revenue of the accepted period (s) and incurs production costs. The power plant has fixed, positive start-up costs c_s , which occur if the power plant produces in one or two periods. Secondly, the plant has positive variable production costs of c_v , which occur in every period that the plant is producing. For simplicity, we assume that the capacity K of the power plant, as well as the length l of one trading period are 1, so prices and costs can be simply added to result in the profit. We assume that in case a bid has been accepted, there is no possibility for the power plant to not produce, by, for example, paying an imbalance price instead, or by re-trading electricity in a consecutive market. We relax the assumption of a single trading round, as well as the power plant size and period length in [Section 4](#). In the following, the three bidding formats and clearing rules of the auction are described.

2.1. Simple bids

In the simple bid format, the power plant can bid for each period separately with the bids b_1 and b_2 . If the realised prices p_1 and p_2 are equal or larger than the respective bid, the bid is accepted and the unit needs to produce with the associated costs. Bids can be individually accepted, so that a unit may produce in one period, but not the other.

Depending on the prices p_1 and p_2 the profit for the actor is thus:

$$\pi_{Simple}(b_1, b_2, p_1, p_2) = \begin{cases} p_1 + p_2 - c_s - 2c_v & \text{for } p_1 \geq b_1 \wedge p_2 \geq b_2 \\ p_1 - c_s - c_v & \text{for } p_1 \geq b_1 \wedge p_2 < b_2 \\ p_2 - c_s - c_v & \text{for } p_1 < b_1 \wedge p_2 \geq b_2 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The pay-offs for an example can be found in [Fig. 2](#), where the profit for the actor is depicted depending on realisations of p_1 and p_2 and for a bid $b = b_1 = b_2$ (marked with dotted lines in the figure, and corresponding to the optimal bid derived below). To the right of the vertical dotted line period 1 is accepted as $p_1 \geq b_1$, and above the horizontal dotted line the bid in period 2 is accepted as $p_2 \geq b_2$. Within the fourth quadrant in the top right, bids in both periods are accepted. In this example, the actor faces a risk of losing money for low levels of p_1 and p_2 that are just above the bid level b .

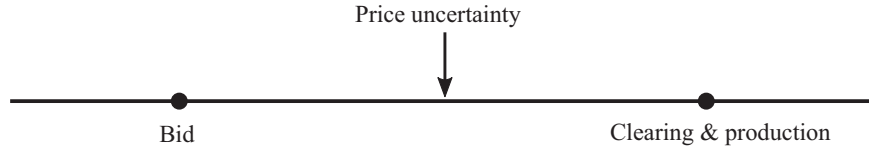


Fig. 1. Model time-line.

2.1.1. Expected profits & optimal bidding

For simple bidding, since the two distributions are assumed to be identical, bidding in the two time periods is symmetric (a full derivation to show that the bids are identical can be found in the [Appendix B.1](#)) and we can simplify the derivation to an identical decision variable b for both periods. Given the probability of having a bid accepted $(1 - \frac{b}{p_{\max}})$ and the expected profit conditional on the acceptance of a bid $(\frac{p_{\max}+b}{2})$, the expected profit for a given bid level b is:

$$\begin{aligned} \mathbb{E}[\pi_{\text{Simple}}(b)] &= \int_0^{p_{\max}} \int_0^{p_{\max}} \rho(p_1) \cdot \rho(p_2) \cdot \pi_{\text{Simple}}(b, p_1, p_2) dp_1 dp_2 \\ &= \int_b^{p_{\max}} \int_b^{p_{\max}} \frac{p_1 + p_2 - c_s - 2c_v}{p_{\max}^2} dp_1 dp_2 + \int_0^b \int_b^{p_{\max}} \frac{p_1 - c_s - c_v}{p_{\max}^2} dp_1 dp_2 \\ &\quad + \int_b^{p_{\max}} \int_0^b \frac{p_2 - c_s - c_v}{p_{\max}^2} dp_1 dp_2 \\ &= \left(1 - \frac{b}{p_{\max}}\right)^2 (p_{\max} + b - c_s - 2c_v) \\ &\quad + \frac{2b}{p_{\max}} \left(1 - \frac{b}{p_{\max}}\right) \left(\frac{p_{\max} + b}{2} - c_s - c_v\right) \end{aligned} \quad (2)$$

Differentiating with respect to b and checking for sufficient and necessary optimality conditions (see [Appendix B.1](#)), the optimal bidding strategy (with $b^* = p_{\max}$ equivalent to no bidding) is:

$$b^* = \begin{cases} \frac{c_v \cdot p_{\max}}{p_{\max} - c_s} & \text{for } c_s + c_v < p_{\max} \\ \text{No bid} & \text{otherwise} \end{cases} \quad (3)$$

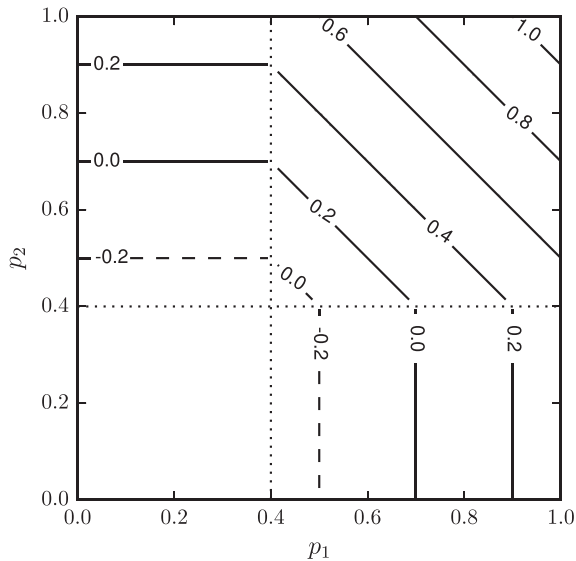


Fig. 2. Ex-post pay-offs for simple bidding example as solid isoprofit lines under optimal bids as dotted lines ($p_{\max} = 1$, $c_v = 0.2$, $c_s = 0.5$, $b^* = 0.4$).

Thus the optimal strategy is to adjust the bid above marginal short-term cost, but only by a share of start-up costs. Two points are notable about the solution: First, the optimal bid can only be equal to or lower than the simple sum of variable and fixed costs. This means that in case of start-up costs the actor is willing to run the risk of losing money in case only one time period is accepted rather than two, as the expected profitability of being accepted in two periods is outweighing the risk of only being accepted in a single period. Second, the solution is not equal to the simple heuristic of bidding variable costs plus the distributed fixed cost over the two time periods. The optimal bid may be both lower and higher than this heuristic.

With the optimal bids, the expected profit can be derived:

$$\mathbb{E}[\pi_{\text{Simple}}^*] = \begin{cases} \frac{(p_{\max} - c_s - c_v)^2}{p_{\max} - c_s} & \text{for } c_s + c_v < p_{\max} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

As anticipated the expected profit under optimal bidding is decreasing both with start-up costs c_s and variable costs c_v .

2.2. Block bids

In block bidding, the actor couples two bids, so that the two periods are either jointly accepted or rejected. Hence, if the block bid b_b is accepted, the unit is producing in both periods. Therefore the profit function is:

$$\pi_{\text{Block}}(b_b, p_1, p_2) = \begin{cases} p_1 + p_2 - c_s - 2c_v & \text{for } p_1 + p_2 \geq b_b \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

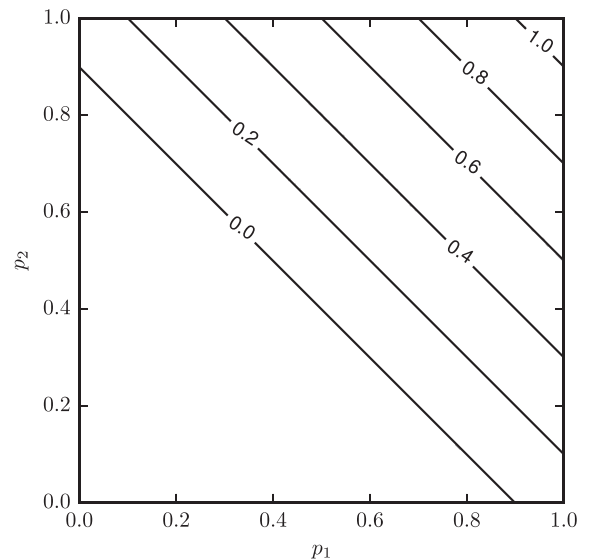


Fig. 3. Ex-post pay-offs for block bidding example as isoprofit lines under optimal block bid identical to 0.0-line ($p_{\max} = 1$, $c_v = 0.2$, $c_s = 0.5$, $b_b^* = 0.9$).

The pay-offs for realisations of p_1 and p_2 for an example can be found in Fig. 3, which corresponds to the optimal bidding strategy derived below.

2.2.1. Expected profits & optimal bidding

For block bidding the expected profit is:

$$\mathbb{E}[\pi_{Block}(b_b)] = \int_0^{p_{max}} \int_0^{p_{max}} \rho(p_1) \cdot \rho(p_2) \cdot \pi_{Block}(b_b, p_1, p_2) dp_1 dp_2 \quad (6)$$

Two cases can be distinguished. For $0 \leq b_b < p_{max}$ the integral needs to distinguish two areas:

$$\begin{aligned} \mathbb{E}[\pi_{Block}(b_b)] &= \int_{b_b}^{p_{max}} \int_0^{p_{max}} \frac{1}{p_{max}^2} (p_1 + p_2 - c_s - 2c_v) dp_1 dp_2 \\ &+ \int_0^{b_b} \int_{b_b-p_2}^{p_{max}} \frac{1}{p_{max}^2} (p_1 + p_2 - c_s - 2c_v) dp_1 dp_2 \\ &= p_{max} - c_s + \frac{c_s \cdot b_b^2}{2p_{max}^2} - 2c_v + \frac{c_v \cdot b_b^2}{p_{max}^2} - \frac{b_b^3}{3p_{max}^2} \end{aligned} \quad (7)$$

For $2p_{max} > b_b \geq p_{max}$, only one area needs to be considered:

$$\begin{aligned} \mathbb{E}[\pi_{Block}(b_b)] &= \int_{b_b-p_{max}}^{p_{max}} \int_{b_b-p_2}^{p_{max}} \frac{1}{p_{max}^2} (p_1 + p_2 - c_s - 2c_v) dp_1 dp_2 \\ &= \frac{1}{6p_{max}^2} (b_b - 2p_{max})^2 (2p_{max} + 2b_b - 3c_s - 6c_v) \end{aligned} \quad (8)$$

As it is not profitable to produce if the cost exceed the maximum revenue over two periods, the expected profit is:

$$\mathbb{E}[\pi_{Block}(b_b)] = \begin{cases} p_{max} - c_s + \frac{c_s \cdot b_b^2}{2p_{max}^2} - 2c_v + \frac{c_v \cdot b_b^2}{p_{max}^2} - \frac{b_b^3}{3p_{max}^2} & \text{for } 0 \leq b_b < p_{max} \\ \frac{1}{6p_{max}^2} (b_b - 2p_{max})^2 (-3c_s - 6c_v + 2p_{max} + 2b_b) & \text{for } p_{max} \leq b_b < 2p_{max} \\ 0 & \text{for } b_b \geq 2p_{max} \end{cases} \quad (9)$$

Differentiating the expected profit, and checking that the optimality conditions are fulfilled (see appendix) we find the optimal bid b_b^* to be:

$$b_b^* = \begin{cases} c_s + 2c_v & \text{for } c_s + 2c_v < 2p_{max} \\ \text{No bid} & \text{otherwise} \end{cases} \quad (10)$$

This is also intuitively correct, as a price taker under uniform bidding will bid the true cost for a joint delivery of two periods. In contrast to simple bidding, there is no trade-off between risking potential losses for a higher expected profit: as a joint bid for two periods is submitted there is no risk involved in only having one bid accepted. However, there are situations in which one period has a very high price covering both variable and fixed costs in a single period, and the other period has a very low price not covering variable costs (as will be discussed further in the Section 3).

Inserting the optimal bidding strategy into the expected profit function of block bidding yields the expected profit under optimal bidding:

$$\mathbb{E}[\pi_{Block}^*] = \begin{cases} p_{max} - c_s - 2c_v + \frac{(c_s + 2c_v)^3}{6p_{max}^2} & \text{for } c_s + 2c_v < p_{max} \\ \frac{1}{6p_{max}^2} (2p_{max} - c_s - 2c_v)^3 & \text{for } p_{max} \leq c_s + 2c_v < 2p_{max} \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

In the first case, the bid (which equals the cost of producing in two periods) could theoretically be covered by the prices in one period. The second case represents the setting where the bid is larger than the

highest reachable price in the market and the revenues from both periods are necessary.

2.3. Multi-part bidding

For multi-part bidding, the actor can bid start-up cost b_s and variable cost b_v that are valid for both periods. Dependent on market prices the actor will be accepted in zero, one or two periods for which the profit function is:

$$\pi_{Multipart}(b_s, b_v, p_1, p_2) = \begin{cases} p_1 - c_s - c_v & \text{for } p_1 \geq b_s + b_v \wedge p_2 < b_v \\ p_2 - c_s - c_v & \text{for } p_2 \geq b_s + b_v \wedge p_1 < b_v \\ p_1 + p_2 - c_s - 2c_v & \text{for } p_1 + p_2 \geq b_s + 2b_v \wedge p_1 \geq b_v \leq p_2 \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

The pay-offs for realisations of p_1 and p_2 for an example can be found in Fig. 4. Visually the case differentiation between the dispatch of one and two units occurs as kinks in the isolines at variable cost levels.

2.3.1. Expected profits & optimal bidding

For multi-part bidding the expected profit is:

$$\mathbb{E}[\pi_{Multipart}] = \int_0^{p_{max}} \int_0^{p_{max}} \rho(p_1) \cdot \rho(p_2) \cdot \pi_{Multipart}(b_v, b_s, p_1, p_2) dp_1 dp_2 \quad (13)$$

As the multi-part algorithm is defined in such a way that units are only accepted in the auction if the prices exceed the variable costs in the respective period and if start-up costs are covered over either one or two periods (according to bids) potential profit is foregone if the actor bids either above or below the true costs. It is thus optimal to bid the truthful variable and start-up costs (i.e. $b_s^* = c_s$ and $b_v^* = c_v$). A formal derivation of this result can be found in the Appendix B.3).

By distinguishing three cases defined by combinations of c_s and c_v , which determine whether only jointly accepted bids or also individually accepted are possible solutions, we find the expected profit under optimal bidding to be:

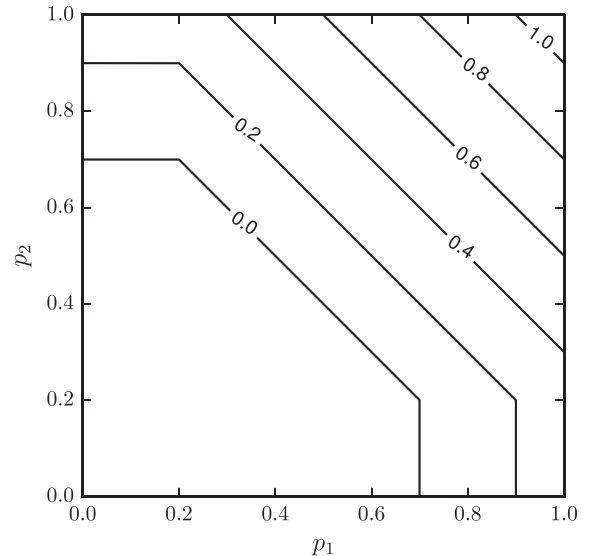


Fig. 4. Ex-post pay-offs for multi-part bidding example as isoprofit lines under optimal bidding ($p_{max} = 1$, $b_v^* = c_v = 0.2$, $b_s^* = c_s = 0.5$).

$$\mathbb{E}[\pi_{\text{Multipart}}^*] = \begin{cases} \frac{c_s^3}{6p_{\max}^2} + \frac{c_s^2 \cdot c_v}{p_{\max}^2} + \frac{c_s \cdot c_v^2}{p_{\max}^2} - c_s + \frac{c_v^2}{p_{\max}} - 2c_v + p_{\max} & \text{for } c_s + c_v < p_{\max} \\ \frac{1}{6p_{\max}^2} (2p_{\max} - c_s - 2c_v)^3 & \text{for } p_{\max} \leq c_s + 2c_v < 2p_{\max} \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

The case differentiation for the expected profit (at $c_s + c_v = p_{\max}$) occurs at the combination of start-up and variable costs where the dispatch in a single time period becomes inefficient, as start-up and variable cost exceed the maximum price level in the period. In that case, the expected profits of block and multi-part bidding are identical.

3. Inefficiencies & comparison of expected profits

3.1. Example equilibrium and inefficiencies

We show in the following that under price uncertainty, actors face suboptimal market outcomes under simple and block bidding compared to multi-part bidding. This reduced profitability may also constitute an inefficiency as the distorted bids may lead to an ex-post inefficient dispatch, i.e. other power plants may have been able to produce for lower cost at the realised equilibrium price levels. A simple example illustrates that reduced actor profitability can correspond to inefficient market outcomes. For this we assume a perfectly competitive market with infinitesimally small power plants (of identical capacities) with variable costs only (i.e. no start-up costs), uniformly distributed between zero and p_{\max} . In addition the already described power plant with both variable and start-up costs is also present in the market. The inelastic demand is independently uniformly distributed between zero and the total capacity of all power plants in the market. Thus in equilibrium, prices are uniformly distributed from zero to p_{\max} , corresponding to our model description in the previous section.

In the outcomes where the power plant with start-up costs faces suboptimal ex-post realised profits, it is either making a loss while producing (Cases A, C, D and E), or it would have been more profitable to produce (more) at the given prices (Case B).¹ The reduced profitability is equal to efficiency losses in market equilibrium, as for any given realised price p , the power plant with start-up costs either replaces or is replaced by the marginal power plant with variable costs p in the merit order. These inefficiencies fundamentally result from the existence of uncertainty regarding market prices under interdependency of the two time steps, introduced by start-up costs. In simple bidding, actors need to bid independently for two time periods, despite start-up costs introducing time interdependencies. In block bidding these interdependencies are considered, however acceptance in single time periods is ruled out.

It is important to note that the discussed inefficiencies are not identical (but similar in nature) to the principal coordination issue of simple marginal prices if there are indivisibilities and other non-convexities (O'Neill et al., 2005). In contrast, in our model the inefficiency would cease to exist in case that there was no uncertainty in at least one of the two periods, as the bid for the uncertain period could be adjusted taking into account the price of the certain period, or if the periods were perfectly correlated (cf. Appendix D).

The example in Fig. 5 shows three types of inefficiency of simple bidding as compared to multi-part bidding and two types of inefficiencies of block bidding as compared to multi-part bidding occurring in our example case. In the following, we are showing the inefficiency in all four cases by comparing the difference in profits between the profit functions of simple, block and multi-part bidding (as in the example case the reduced profitability corresponds to the loss in efficiency).

Areas A show the inefficiency of an ex-post overproduction. The power plant is producing in one time period, although the price in the single period is not sufficient to cover variable and start-up costs for a single period. No production is taking place under multi-part bidding, resulting in zero profit. As a result, profits under simple bidding are negative and smaller than under multi-part bidding. In our example this corresponds to an inefficiency as another power plant out of the merit-order (in our example with variable costs p_1 or respectively p_2) would be able to operate without a loss.

$$\Delta\pi_{\text{Simple-Multipart}}^A = \begin{cases} p_1 - c_s - c_v < 0 & \text{for } p_1 \geq b^* \wedge p_2 < b^* \wedge p_1 < c_s + c_v \wedge p_1 + p_2 < 2c_v + c_s \\ p_2 - c_s - c_v < 0 & \text{for } p_2 \geq b_2 \wedge p_1 < b^* \wedge p_2 < c_s + c_v \wedge p_1 + p_2 < 2c_v + c_s \end{cases} \quad (15)$$

In areas B we depict the inefficiency of an ex-post underproduction. The plant produces in only one time period, whereas a production in two time periods is taking place under multi-part bidding. The difference between the profit functions of simple and multi-part bidding can be expressed as²:

$$\Delta\pi_{\text{Simple-Multipart}}^B = \begin{cases} c_v - p_2 < 0 & \text{for } p_1 > b^* \wedge p_2 < b^* \wedge p_1 + p_2 \geq 2c_v + c_s \\ c_v - p_1 < 0 & \text{for } p_2 > b_2 \wedge p_1 < b^* \wedge p_1 + p_2 \geq 2c_v + c_s \end{cases} \quad (16)$$

As the profit under simple bidding is smaller than under multi-part bidding this represents an inefficiency in our example, as the actor's power plant could produce at lower costs in the non-accepted period than the marginal price setting power plant (in our example the plants with variable costs p_1 or p_2).

In area C we depict the inefficiency of an ex-post overproduction in two periods. The plant is dispatched in two time slots as compared to no dispatch at all in the efficient case (and as is the case under multi-part bidding). In our example case other power plants (with variable costs p_1 and p_2) would instead be more efficiently dispatched.

$$\Delta\pi_{\text{Simple-Multipart}}^C = \begin{cases} p_1 + p_2 - 2c_v - c_s < 0 & \text{for } p_1 \geq b^* \wedge p_2 \geq b^* \wedge p_1 + p_2 < 2c_v + c_s \end{cases} \quad (17)$$

The example in Fig. 5 shows two inefficiencies of block as compared to multi-part bidding. In the areas D the plant is not dispatched at all, although a dispatch in one period would be efficient as it is taking place under multi-part bidding, representing a missed profit, and an inefficiency in our example case as another more expensive power plant (with variable cost level p_1 or p_2 in the two cases) is dispatched:

$$\Delta\pi_{\text{Block-Multipart}}^D = \begin{cases} c_v + c_s - p_1 < 0 & \text{for } p_1 + p_2 < 2c_v + c_s = b_b^* \wedge p_1 > c_v + c_s \\ c_v + c_s - p_2 < 0 & \text{for } p_1 + p_2 < 2c_v + c_s = b_b^* \wedge p_2 > c_v + c_s \end{cases} \quad (18)$$

In the area E the plant is accepted for two periods under block bidding, although it would be more efficient to produce in a single period. This results in a reduced profit as the variable cost in the additional period exceeds the revenue. In our example it would be more efficient to dispatch the power plants with variable costs p_1 or p_2 , respectively.

¹ Discrete start-up choices and other non-convexities can also lead to market results, where prices indicate profitable production opportunities while a unit is not dispatched and vice versa (leading to uplift payments or paradoxically rejected bids).

² For the first case the simplification is: $\Delta\pi = p_1 - c_v - c_s - (p_1 + p_2 - 2c_v - c_s) = c_v - p_2$, the second case is analogue to the first case.

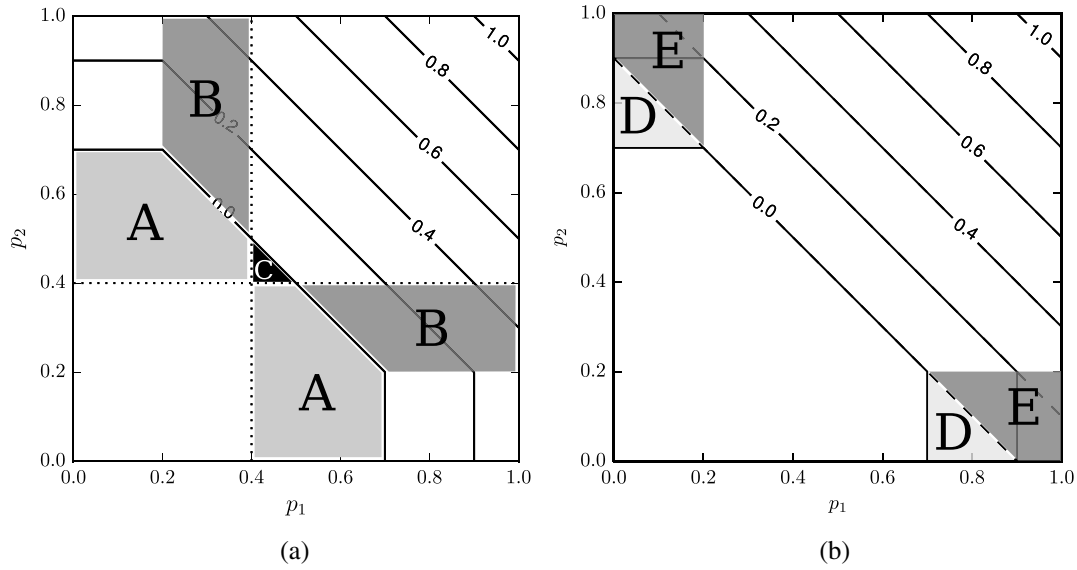


Fig. 5. Inefficiencies of simple bidding (a, bids as dotted lines) and block bidding (b, bid as dashed-solid line) as compared to multi-part bidding (solid 0.0-line) ($p_{max} = 1$, $c_v = 0.2$, $c_s = 0.5$, $b = 0.4$).

$$\Delta\pi_{Block-Multi-part}^E = \begin{cases} p_1 - c_v < 0 & \text{for } p_1 + p_2 \geq 2c_v + c_s = b_b^* \wedge p_1 < c_v \\ p_2 - c_v < 0 & \text{for } p_1 + p_2 \geq 2c_v + c_s = b_b^* \wedge p_2 < c_v \end{cases} \quad (19)$$

As a result for all combinations of variable costs c_v and start-up costs c_s with non-zero profits, multi-part expected profits are larger than either single bid or block bid profits (for proof see [Appendix C](#)), and in our example equilibrium multi-part bidding provides the more efficient market outcomes. Whether block or simple bidding is more profitable for a single actor depends on the combination of c_v and c_s relative to the price distribution.

3.2. Numerical evaluation

In the following, we show numerical evaluations for the profits and ratios of expected profits for all relevant combinations of c_v and c_s for two periods of 1 h each with uniform price expectations between 0 and 100 EUR/MWh. For illustration we compare these profit differences to typical ranges of c_v and c_s of common power plant types: coal, combined cycle gas turbines (CCGT) and open cycle gas turbines (OCGT), with variations on start-up costs based on whether it is a cold or hot start (assumptions can be found in [Table F.2](#), based on [Schröder et al. \(2013\)](#)). While this comparison is based on expected profits given fixed distributions in two hours and not a market equilibrium, it gives a first indication of the order of magnitude of efficiency losses under the different bidding formats.

[Fig. 6](#) compares the expected profits from multi-part bidding and simple bidding. We draw isolines that depict the combinations of start-up and variable costs that result in constant levels of expected profits. We do this for simple bidding in Subfigure (a), for multi-part bidding in Subfigure (b) and depict the ratio of expected profits of simple bidding and multi-part bidding in Subfigure (c). It is evident that if the only available format is simple bidding, producers with high start-up costs, such as cold coal power plants and CCGTs, have significantly lower expected profits as compared to the multi-part benchmark both in absolute and relative terms (at least for the example dispatch duration of 2 h).

[Fig. 7](#) compares the expected profits in a multi-part format with block bidding. The subfigures show the absolute profits of block bids

(Subfigure a) and the relative difference in comparison to multi-part bidding (Subfigure b). Under block bidding power plants with very high start-up costs have nearly as high expected profits using block bids, as in the multi-part bidding format, whereas power plants with lower start-up costs and high variable costs have significantly lower profits as compared to the multi-part case (and the simple bidding case).

As sometimes markets allow both block and simple bids, market participants will choose the optimal bid type according to their power plant type. [Fig. 8](#) shows the expected profits if the optimal choice (in expectation) is made between block and simple bids (Subfigure a). Where the isolines have a kink, block and simple bidding yield the same profits. Subfigure b shows the ratio between the optimal simple-block choice and multi-part bidding. Given the chosen period length and price distributions the biggest inefficiencies occur for OCGT plants and CCGT plants.

The results in this section are example quantifications for the specific case of a dispatch in two consecutive hours under high price uncertainty. It is important to stress that results depend on the conditions. Especially coal and CCGT power plants are usually not dispatched for two hours at a time, so that the corresponding inefficiency over longer time periods, and under lower uncertainty, will be lower. Furthermore, the assumed price distribution that results from our example market equilibrium will be different in practice. This is further elaborated in the discussion.

We do not assess the profitability of a combination of simple and block bids against multi-part bids due to the following reason: A combination of block bids and simple bids would lead to the risk of double selling the production volumes as soon as the simple bid and the block bid is accepted. Therefore, either an additional buy offer in form of a block bid or the simple bid could be used to offset the double buying (trading in several time periods is outside of the scope of this model).

4. Model extension: retrading for simple bidding

So far we did not model a second trading round where agents can re-adjust their position. This represents a limitation of the model, as a second trading round (and further trading rounds) can potentially reduce the inefficiencies identified in the paper for simple bidding:

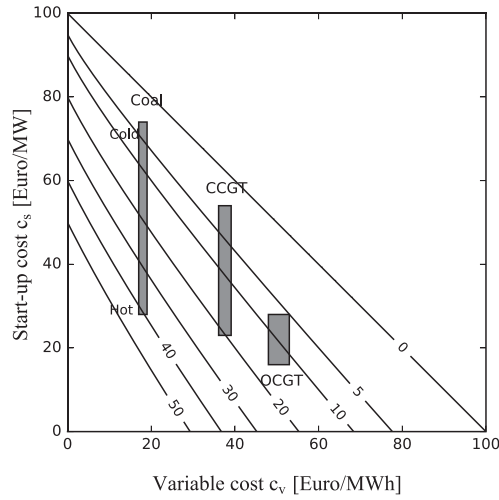
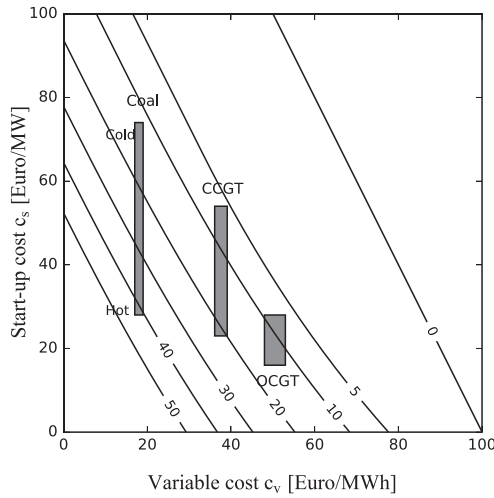
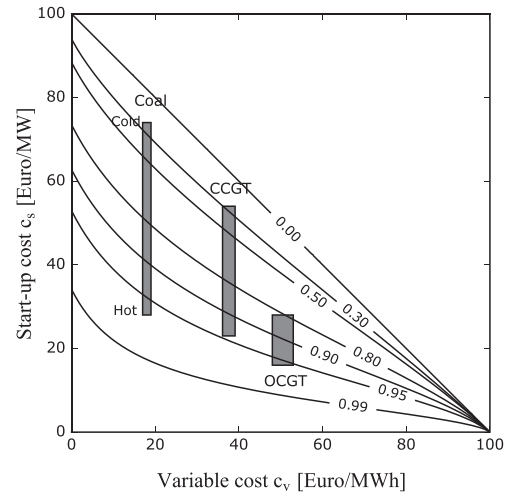
(a) Exp. profit simple bidding: $\mathbb{E}[\pi_{Simple}^*]$ (b) Exp. profit multi-part bidding: $\mathbb{E}[\pi_{Multipart}^*]$ (c) Ratio: $\mathbb{E}[\pi_{Simple}^*]/\mathbb{E}[\pi_{Multipart}^*]$

Fig. 6. Comparison multi-part and simple bidding profit.

- Case A: When the unit is accepted in one of two time periods, but ex-post it would have been more efficient to be accepted in none, the power plant operator aims to buy power back in order not to produce at all.
- Case B: When the unit is accepted in one of two time periods, but ex-post it would have been more efficient to be accepted in both periods, the actor aims to sell in the additional period.
- Case C: When the unit is accepted in both periods, but ex-post it would have been more efficient to be accepted in none, the actor tries to buy power back for both periods.

However, several effects may limit the effectiveness of sequential markets to reduce the discussed inefficiencies.

- As the time of delivery gets closer fewer flexibility options can be utilized, which leads to a steeper merit order curve (Henriot, 2014) and the missed utilisation of more efficient units.
- Secondly, in the case of continuous markets at a given point, the market usually only has limited liquidity, leading to price effects for

relatively small volumes of traded energy and increased uncertainty for actors that have open positions (Neuhoff et al., 2016).

Therefore, in order to approximate these effects, we consider the effect of the traded capacity on the price in a second trading round.

4.1. Case A: ex-post oversupply

This case describes the inefficiency area A, shown in Fig. 5, as in this area, the power plant is dispatched in one time period, although a dispatch is not efficient given the prices of the first auction round. Therefore, the power plant operator could buy the power back in a second auction round, in order not to need to produce.

Eq. (20) shows the inequality for which production would result in lower costs than re-trading in a second round, and uses the price sensitivity m (in $\text{EUR}/(\text{MWh} \cdot \text{MW})$), which depends on the traded capacity K and the length of the traded period l (the product is the traded energy in MWh). As long as this inequality holds, the actor will decide not to re-trade.

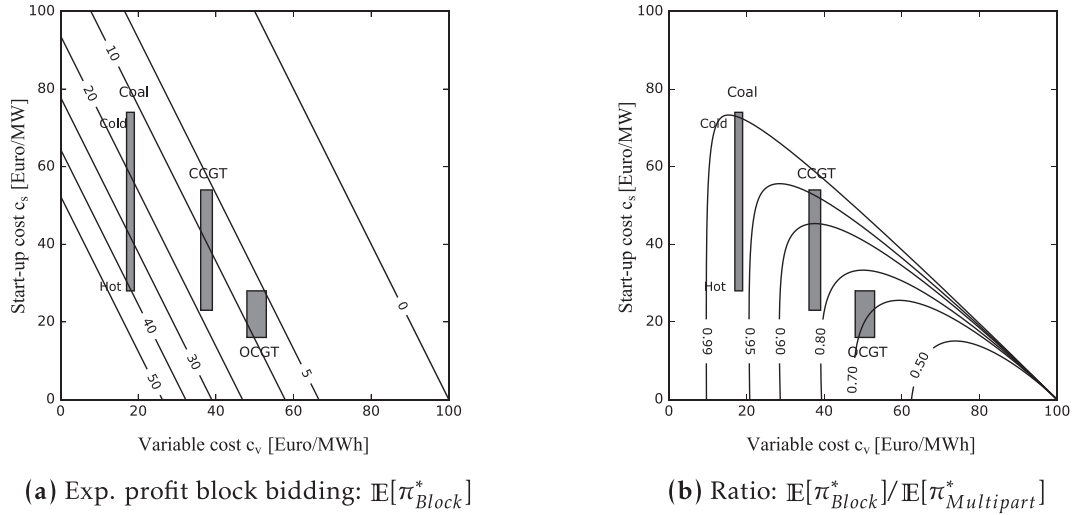


Fig. 7. Comparison multi-part and block bidding profit.

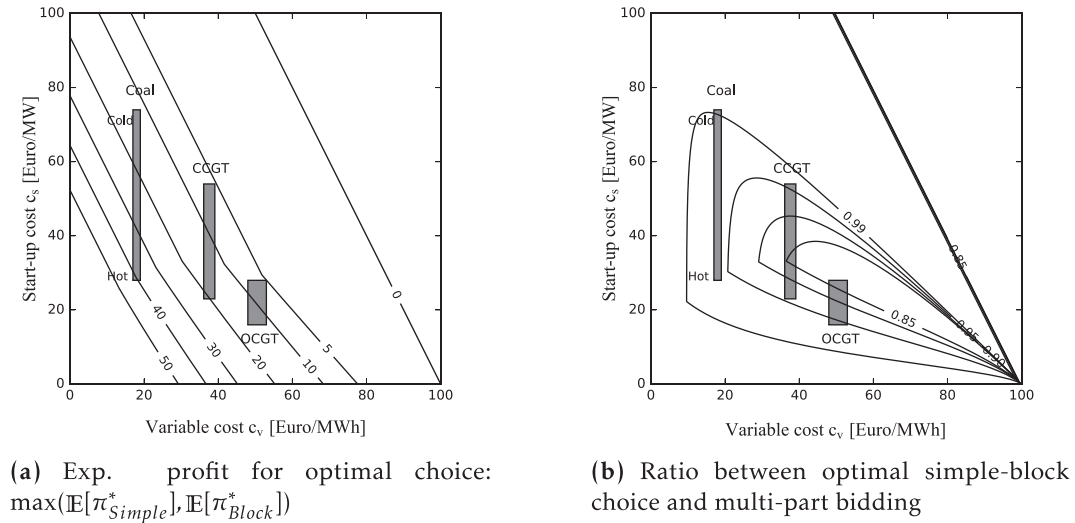


Fig. 8. Comparison of the multi-part profit to the optimal combination of simple and block bidding profit.

$$c_v \cdot K \cdot l + c_s \cdot K < (p_1^\alpha + m_1 \cdot K) \cdot K \cdot l \quad (20)$$

In order for this inequality to hold, m_1 must be bigger than a certain threshold for the price sensitivity.

$$m_1 > c_v + \frac{c_s - p_1^\alpha}{K} \quad (21)$$

To assess what the impact of different variable costs and start-up costs is on re-trading, we look at the situation of a just accepted bid ($p_1^\alpha = b^*$), which we hold constant, while varying either c_v or c_s . Reformulating Eq. (21) (see appendix for full documentation) towards c_v will lead to:

$$m_1 > \frac{(p_{max} - b^*)(b^* - c_v)}{b^* \cdot K} \quad (22)$$

Eq. (22) shows that for an increasing share of c_v in the production costs, the necessary m_1 for re-trading to be unprofitable is decreasing. This means that for power plants with

relatively high variable costs compared to their start-up costs (e.g. fast starting gas power plants) re-trading is only an option when the price elasticity of the market is very low, and prices do not increase significantly (above the actors cost level) when the actor tries to buy back power. Fast starting power plants with high variable cost are normally used at the upper end of the merit-order where a higher price elasticity can be observed.

Reformulating Eq. (21) (see appendix for full documentation) towards c_s will lead us to:

$$m_1 > \frac{(p_{max} - b^*) \cdot c_s}{K \cdot p_{max}} \quad (23)$$

This means that for power plants with relatively high start-up costs compared to their variable costs (e.g. lignite or nuclear power plants) re-trading is an option even when the price elasticity of the market is high. However, these power plants are normally very large (i.e. a large K) which will reduce m_1 again.

4.2. Case B: ex-post undersupply

This case describes the inefficiency area B, shown in Fig. 5, as in this area the plant is dispatched in only one time period, although a dispatch in two time periods would be efficient given the prices of the first auction round. Therefore, the power plant operator could have an intention to sell the electricity for the second time period in a second auction round. If Eq. (24) holds true re-trading is not profitable.

$$c_v \cdot K \cdot l + c_s \cdot K < 2c_v \cdot K \cdot l + c_s \cdot K - (p_2^\alpha - m_2 \cdot K) \cdot K \cdot l \quad (24)$$

Similarly to the case before, setting the price $p_2^\alpha = b^*$, and holding b^* constant while varying either c_v or c_s yields:

$$m_2 > \frac{b^* - c_v}{K} \quad (25)$$

This shows that for increasing shares of c_v in the total production cost, the necessary m_2 for re-trading to be unprofitable is decreasing. Hence, power plants with a high share of variable costs have no intention to re-trade as already a small price elasticity makes it unprofitable.

$$m_2 > \frac{b^* \cdot c_s}{K \cdot l \cdot p_{max}} \quad (26)$$

Power plants with a low share of variable cost and higher shares of start-up costs, on the other hand, will be willing to re-trade also during times of high price elasticity. However, these power plants are normally very large which will then reduce the necessary m_2 again.

4.3. Case C: ex-post oversupply in two periods

Eq. (27) holds in case re-trading to solve the inefficiency of Case C is not profitable.

$$c_v \cdot (2 \cdot K \cdot l) + c_s \cdot K < (p_1^\alpha + m_1 \cdot K) \cdot K \cdot l + (p_2^\alpha + m_2 \cdot K) \cdot K \cdot l \quad (27)$$

In order to compare the effect of c_v and c_s on the necessary price elasticity for the model to be correct, we reformulate Eq. (27) by assuming m_1 and m_2 to be equal and the bids to be just accepted in the two periods at $p_1^\alpha = p_2^\alpha = b^*$. Secondly, as done previously, we fix b^* , and vary only one of the two parameters at a time (see appendix for full documentation).

If b is fixed, and we vary c_v , we need to substitute c_s , which results in the following condition:

$$m > \frac{(p_{max} - 2b^*)(b^* - c_v)}{2 \cdot b^* \cdot K} \quad (28)$$

Thus, as in the previous cases, with an increasing variable cost share in the bidding price, the necessary price elasticity for re-trading to be unprofitable is decreasing.

If b is fixed, and we vary c_s , we need to substitute c_v , which results in the following condition:

$$m > \frac{c_s(p_{max} - 2b^*)}{2 \cdot K \cdot l \cdot p_{max}} \quad (29)$$

Thus, as previously, with increasing fixed costs, the necessary price elasticity in the market needs to be higher for electricity producers preferring not to re-trade.

5. Discussion

The analytic model is based on a set of simplifications necessary for tractability and clarity of economic effects we would like to demonstrate. The presented model only depicts the bidding behaviour of a

single actor with variable and start-up costs in detail, and the shown efficiency losses are based on a simplified competitive market equilibrium where the other power plants have (and bid) exclusively variable costs. In this case, any incorporation of start-up costs in the simple bids (or the limitation on block bids) will lead to inefficient dispatch of the power plant in the discussed cases. In more general market equilibria and realistic assumptions, this effect may not always occur for all types of power plants, or in the order of magnitude discussed in this paper. The structure of costs of power plants (and market power of participants), their place in the merit order and the nature of expected price distribution will decide whether the distortion of simple bids is strong enough to lead to an inefficient dispatch in some realised market outcomes. In markets with many power plants with start up costs, all market participants will adjust their bids, so energy prices will on average be higher even if market power is mitigated (Sioshansi and Nicholson, 2011) (all auction designs are similarly vulnerable to abuse of market power according to idem). An inefficient shift in the merit order may nonetheless occur between power plants of different cost structures, and the higher electricity prices will by itself lead to short-run efficiency losses if there is elastic demand present in the market; these may be avoided under multi-part bidding depending on whether make-whole payments are necessary in the market and how they are allocated (Sioshansi et al., 2010). Further limitations to our analysis are listed below.

We treat the distributions in the two time periods as completely independent, however often prices (and demand levels) are correlated between hours. The effect we discuss will cease to exist if the prices in the two periods are perfectly correlated, as there is no risk involved to have an inefficient dispatch (either the plant is efficiently dispatched in two periods if the single price covers both start-up and variable costs for two periods, or not. See Appendix D for formal proof). For partial correlations the effect is expected to be in between the two extremes.

Secondly, we only consider two time slots, rather than 24 hourly or even more quarterly products in our analysis. Usually commitment to longer periods is possible, which reduces the significance of start-up costs; however, it also implies additional interactions across multiple hours that could increase the relevance of the effect.

Thirdly, we model in a sensitivity analysis (cf. Section 4) a second auction round and could identify a mitigation of some of the effects. Additional auction rounds thus might further mitigate inefficiency of simple and block bidding as compared to multi-part bidding. However, a second effect might partially off-set this opportunity. Market parties may find it increasingly costly to close open positions in subsequent markets, as these usually have limited depth and exhibit a steepening merit order curve as fewer units can respond closer to real-time.

Finally, we do not model a portfolio of generators and the possibility for generators to make a generic bid and then dispatch those of their units which can fulfil the bids at the lowest cost (portfolio based bidding is still common in Western European power markets). This may also reduce the inefficiencies of simple bids discussed in this paper for those market players that have a large portfolio, whereas smaller market participants would be more exposed to this sort of inefficiency.

6. Conclusions

Several bidding formats exist to clear electricity markets, with simple bids and block bidding on the one side and multi-part bidding, which explicitly lets generators state cost components and technical constraints in their bids, on the other side. While these bidding formats have been discussed in the literature, this discussion has mostly focused on the direct coordination problem due to indivisibility, other non-convexities which exists both in deterministic and uncertain settings as well as incentive compatibility of bidding formats and their performance under market power. The suitability of different bidding formats to coordinate the clearing of electricity markets in the presence of

uncertainty as well as complementarities across several hours has been less well analysed in economic literature.

We derive optimal bidding strategies for each of the bidding formats simple bidding, block bidding and multi-part bidding using a two-period model of a price taker in a simultaneously cleared uniform price auction. We show in the analytic model that in the presence of start-up costs and price uncertainty in electricity markets, generators face ex-post suboptimal market outcomes under simple and block bidding, while this doesn't occur in multi-part bidding. The question whether simple or block bidding is more profitable depends on whether start-up costs are high as compared to variable costs, in which case block bids are more profitable for a generator. Using an example case we demonstrate that these single-actor suboptimal outcomes can also lead to efficiency losses in market equilibrium.

With uncertainty about auction clearing prices, simple or block bidding formats do not allow producers to specify their willingness to produce as a function of the outcome in the adjacent hour for which they face complementarities in their production function (for example due to start-up costs). Using numerical examples of different power plant types we show that this effect could occur at significant levels given common electricity market prices and power plant characteristics.

Our results point to potential inefficiencies in electricity markets in the absence of multi-part bids especially in the presence of uncertainty.

Appendix A. Parameters and variables

Table A.1

Description of parameters and variables.

Parameters		Variables	
p_h	Price in trading period h	b_h	Simple bid for period h
p_{max}	Maximum price	b	Symmetric simple bid
c_s	Start-up cost	b_b	Block bid
c_v	Variable cost	b_s	Multi-part bid for start-up cost
K	Capacity of power plant	b_v	Multi-part bid for variable cost
l	Length of trading period	π_{simple}	Profit from simple bidding

Appendix B. Derivations of expected profits & optimal bidding

B.1. Simple bidding

B.1.1. Simple bidding with symmetric bids

The expected profit for a given bid level b is:

$$\begin{aligned} \mathbb{E}[\pi_{Simple}(b)] &= \left(1 - \frac{b}{p_{max}}\right)^2 (p_{max} + b - c_s - 2c_v) \\ &+ \frac{2b}{p_{max}} \left(1 - \frac{b}{p_{max}}\right) \left(\frac{p_{max} + b}{2} - c_s - c_v\right) \end{aligned} \quad (B.1)$$

In order to find stationary points, we differentiate by b :

$$\frac{d}{db} \mathbb{E}[\pi_{Simple}(b)] = \frac{1}{p_{max}^2} (2c_s \cdot b + 2c_v \cdot p_{max} - 2p_{max} \cdot b) = 0 \quad (B.2)$$

Solving for b we find a stationary point, which is optimal for $b < p_{max}$.³

$$b^* = \frac{c_v \cdot p_{max}}{p_{max} - c_s} \quad (B.3)$$

The probability functions for prices are 0, except for $0 < p \leq p_{max}$. To completely describe the strategy space of the generator, we can limit the bids to $0 < b \leq p_{max}$. Therefore, we test the conditions on c_v and c_s , that can be derived by imposing the limits on the bids (this doesn't mean that c_v or c_s need to obey those limits. However, outside these limits the generator shouldn't bid).

These may be mitigated to the extent that further trading rounds or continuous trading allows generators to re-adjust their production schedules. In an extension of the model, we assess how a further trading round can reduce the inefficiency. While subsequent markets may help to alleviate the problem, this might be offset by declining liquidity and depth, as well as the steepening merit order curve over time. Further research should address this question under general equilibrium conditions and investigate how uncertainty and further trading rounds impact the performance of bidding formats in terms of incentive compatibility and under market power.

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³ Second derivative: $\frac{d^2}{db^2} \mathbb{E}[\pi_{Simple}(b^*)] = \frac{1}{p_{max}^2} (2c_s - 2p_{max}) < 0$, for $c_s < p_{max}$

From $b < = p_{max}$

$$p_{max} > \frac{c_v \cdot p_{max}}{p_{max} - c_s} + c_s < p_{max} \quad (B.4)$$

B.1.2. Proof that identical price distribution leads to symmetric bidding

For two separate bids the expected profit given bids b_1 and b_2 is:

$$\begin{aligned} \mathbb{E}[\pi_{Simple}(b_1, b_2)] &= \frac{b_1}{p_{max}} \left(-\frac{b_2}{p_{max}} + 1 \right) \left(\frac{b_2}{2} - c_s - c_v + \frac{p_{max}}{2} \right) \\ &\quad + \frac{b_2}{p_{max}} \left(-\frac{b_1}{p_{max}} + 1 \right) \left(\frac{b_1}{2} - c_s - c_v + \frac{p_{max}}{2} \right) \\ &\quad + \left(-\frac{b_1}{p_{max}} + 1 \right) \left(-\frac{b_2}{p_{max}} + 1 \right) \left(\frac{b_1}{2} + \frac{b_2}{2} - c_s - 2c_v + p_{max} \right) \end{aligned} \quad (B.5)$$

Taking the gradient and finding stationary points

$$\nabla \mathbb{E}[\pi_{Simple}(b_1, b_2)] = \left(\frac{1}{p_{max}^2} (-b_1 \cdot p_{max} + b_2 \cdot c_s + c_v \cdot p_{max}), \frac{1}{p_{max}^2} (b_1 \cdot c_s - b_2 \cdot p_{max} + c_v \cdot p_{max}) \right) = (0, 0) \quad (B.6)$$

We can solve for b_1 and b_2 :

$$b_1 = \frac{c_v \cdot p_{max}}{p_{max} - c_s}, \quad b_2 = \frac{c_v \cdot p_{max}}{p_{max} - c_s} \quad (B.7)$$

Checking for optimality (second order derivatives should be negative):

$$\frac{d^2}{db_1^2} \mathbb{E}[\pi_{Simple}] = \frac{d^2}{db_2^2} \mathbb{E}[\pi_{Simple}] = -\frac{1}{p_{max}} < 0 \quad (B.8)$$

Second optimality condition:

$$\frac{d^2}{db_1^2} \mathbb{E}[\pi_{Simple}] \cdot \frac{d^2}{db_2^2} \mathbb{E}[\pi_{Simple}] - \left(\frac{d^2}{db_1 db_2} \mathbb{E}[\pi_{Simple}] \right)^2 = \frac{1}{p_{max}^4} (-c_s^2 + p_{max}^2) > 0 \quad (B.9)$$

Since p_{max} and c_s are positive, it implies that the bids are optimal if $c_s < p_{max}$. The solution is identical to the simple version in [Section 2](#).

B.2. Block bidding

B.2.1. Finding extrema, for $0 \leq b_b < p_{max}$

$$\frac{d}{db_b} \mathbb{E}[\pi_{Block}(b_b)] = -\frac{b_b^2}{p_{max}^2} + \frac{b_b \cdot c_s}{p_{max}^2} + \frac{2b_b}{p_{max}^2} c_v = 0 \Rightarrow \quad (B.10)$$

$$b_b = \{0, c_s + 2c_v\} \quad (B.11)$$

Testing for optimality:

$$\frac{d^2}{db_b^2} \mathbb{E}[\pi_{Block}(b_b)] = -\frac{2b_b}{p_{max}^2} + \frac{c_s}{p_{max}^2} + \frac{2c_v}{p_{max}^2} \quad (B.12)$$

$$\frac{d^2}{db_b^2} \mathbb{E}[\pi_{Block}(0)] = \frac{c_s}{p_{max}^2} + \frac{2c_v}{p_{max}^2} > 0 \quad (\text{for } c_v > 0 \wedge p_{max} > 0) \quad (B.13)$$

\Rightarrow The stationary point 0 is a minimum.

$$\frac{d^2}{db_b^2} \mathbb{E}[\pi_{Block}(c_s + 2c_v)] = -\frac{1}{p_{max}^2} (c_s + 2c_v) < 0 \quad (\text{for } c_v > 0 \wedge p_{max} > 0) \quad (B.14)$$

\Rightarrow The stationary point $c_s + 2c_v$ is an optimum.

As the function is of degree 3, it has a maximum of two stationary points. As the bid for the maximum is larger than the minimum ($c_s + 2c_v > 0$), the function is strictly decreasing after the maximum (making it global for the investigated interval).

B.2.2. Finding extrema for $p_{\max} \leq b_b < 2p_{\max}$

$$\frac{d}{db_b} \mathbb{E}[\pi_{\text{Block}}(b_b)] = \frac{(b_b - 2p_{\max})^2}{3p_{\max}^2} + \frac{1}{6p_{\max}^2} (2b_b - 4p_{\max})(2b_b - 3c_s - 6c_v + 2p_{\max}) = 0 \quad (\text{B.15})$$

$$b_b^* = \{2p_{\max}, c_s + 2c_v\} \quad (\text{B.16})$$

Testing for optimality:

$$\frac{d^2}{db_b^2} \mathbb{E}[\pi_{\text{Block}}(b_b)] = \frac{1}{3p_{\max}^2} (4b_b - 8p_{\max}) + \frac{1}{3p_{\max}^2} (2b_b - 3c_s - 6c_v + 2p_{\max}) \quad (\text{B.17})$$

Testing optimality for first stationary point:

$$\frac{d^2}{db_b^2} \mathbb{E}[\pi_{\text{Block}}(2p_{\max})] = \frac{1}{p_{\max}^2} (-c_s - 2c_v + 2p_{\max}) \quad (\text{B.18})$$

$2p_{\max}$ is a minimum for $c_s + 2c_v < 2p_{\max}$.

Testing second stationary point:

$$\frac{d^2}{db_b^2} \mathbb{E}[\pi_{\text{Block}}(c_s + 2c_v)] = \frac{1}{p_{\max}^2} (c_s + 2c_v - 2p_{\max}) \quad (\text{B.19})$$

is negative for $c_s + 2c_v < 2p_{\max} \Rightarrow$ is an optimum.

As the function is of degree 3 and the minimum point is at the upper limit of the investigated interval ($2p_{\max}$), $c_s + 2c_v$ is the absolute maximum in the interval $[p_{\max}, 2p_{\max}]$.

B.3. Multi-part bidding

For multi-part bidding the expected profit is ($\mathbb{E}[\pi_{\text{Multipart}}] = \mathbb{E}[\pi_M]$):

$$\mathbb{E}[\pi_M] = \int_0^{p_{\max}} \int_0^{p_{\max}} \rho(p_1) \cdot \rho(p_2) \cdot \pi_{\text{Multipart}}(b_v, b_s, p_1, p_2) dp_1 dp_2 \quad (\text{B.20})$$

Three cases can be differentiated for the expected profit:

1. Either single period and the two-period dispatch can be successful (If $0 \leq c_v + c_s < p_{\max}$).
2. Only two period dispatch can be successful (If $c_v + c_s \geq p_{\max}$ and $2c_v + c_s < 2p_{\max}$).
3. No dispatch can be successful (If $2c_v + c_s > 2p_{\max}$).

B.3.1. Finding optima for $0 \leq c_v + c_s < p_{\max}$

$$\begin{aligned} \mathbb{E}[\pi_M] &= \int_0^{b_v} \int_{b_s+b_v}^{p_{\max}} \frac{1}{p_{\max}^2} (-c_s - c_v + p_1) dp_1 dp_2 + \int_{b_s+b_v}^{p_{\max}} \int_0^{b_v} \frac{1}{p_{\max}^2} (-c_s - c_v + p_2) dp_1 dp_2 \\ &+ \int_{b_s+2b_v-p_1}^{p_{\max}} \int_{b_s+2b_v-p_2}^{p_{\max}} \frac{1}{p_{\max}^2} (-c_s - 2c_v + p_1 + p_2) dp_1 dp_2 \\ &= \int_0^{b_v} \int_{b_s+b_v}^{p_{\max}} \frac{1}{p_{\max}^2} (-c_s - c_v + p_1) dp_1 dp_2 + \int_{b_s+b_v}^{p_{\max}} \int_0^{b_v} \frac{1}{p_{\max}^2} (-c_s - c_v + p_2) dp_1 dp_2 \\ &+ \int_{b_v}^{p_{\max}} \int_{b_v}^{p_{\max}} \frac{1}{p_{\max}^2} (-c_s - 2c_v + p_1 + p_2) dp_1 dp_2 - \int_{b_v}^{b_s+b_v} \int_{b_v}^{b_s+2b_v-p_2} \frac{1}{p_{\max}^2} (-c_s - 2c_v + p_1 + p_2) dp_1 dp_2 \\ &= p_{\max} - c_s - 2c_v + \frac{1}{p_{\max}^2} \left(-\frac{b_s^3}{3} - 2b_s^2 \cdot b_v + \frac{b_s^2 \cdot c_s}{2} + b_s^2 \cdot c_v - 2b_s \cdot b_v^2 + 2b_s \cdot b_v \cdot c_s + 2b_s \cdot b_v \cdot c_v + b_v^2 \cdot c_s - b_v^2 \cdot p_{\max} + 2b_v \cdot c_v \cdot p_{\max} \right) \end{aligned} \quad (\text{B.21})$$

To find the stationary points:

$$\begin{aligned} \nabla(\mathbb{E}[\pi_M])(b_v, b_s) &= \begin{bmatrix} \frac{\partial \mathbb{E}[\pi_M](b_v, b_s)}{\partial b_v} \\ \frac{\partial \mathbb{E}[\pi_M](b_v, b_s)}{\partial b_s} \end{bmatrix} \\ &= \begin{bmatrix} \frac{2}{p_{\max}^2}(-b_s^2 - 2b_s \cdot b_v + b_s \cdot c_s + b_s \cdot c_v + b_v \cdot c_s - b_v \cdot p_{\max} + c_v \cdot p_{\max}) \\ \frac{1}{p_{\max}^2}(-b_s^2 - 4b_s \cdot b_v + b_s \cdot c_s + 2b_s \cdot c_v - 2b_v^2 + 2b_v \cdot c_s + 2b_v \cdot c_v) \end{bmatrix} \\ &= 0 \end{aligned} \quad (\text{B.22})$$

Bidding truthfully ($b_s = c_s$, $b_v = c_v$) is the stationary point fulfilling the assumptions (only positive bidding and $c_v + c_s < p_{\max}$).

The Hessian matrix is:

$$H(\mathbb{E}[\pi_M])(b_v, b_s) = \begin{bmatrix} \frac{1}{p_{\max}^2}(-4b_s + 2c_s - 2p_{\max}) & \frac{1}{p_{\max}^2}(-4b_s - 4b_v + 2c_s + 2c_v) \\ \frac{1}{p_{\max}^2}(-4b_s - 4b_v + 2c_s + 2c_v) & \frac{1}{p_{\max}^2}(-2b_s - 4b_v + c_s + 2c_v) \end{bmatrix} \quad (\text{B.23})$$

The determinant at the stationary point is:

$$D(c_v, c_s) = -\frac{1}{p_{\max}^4}(2c_s^2 + 4c_s c_v - 2c_s p_{\max} + 4c_v^2 - 4c_v p_{\max}) > 0 \quad \text{for : } c_s + c_v < p_{\max} \quad (\text{B.24})$$

and as $\frac{\partial^2 \mathbb{E}[\pi_M](b_v, b_s)}{\partial b_v^2} < 0$, the stationary point is a maximum. The expected maximum profit in this case is thus:

$$\mathbb{E}[\pi_M^*] = p_{\max} - 2c_v - c_s + \frac{c_s^3}{6p_{\max}^2} + \frac{c_s^2 \cdot c_v}{p_{\max}^2} + \frac{c_s \cdot c_v^2}{p_{\max}^2} + \frac{c_v^2}{p_{\max}} \quad \text{for : } 0 \leq c_v + c_s < p_{\max} \quad (\text{B.25})$$

B.3.2. Finding optima for $2p_{\max} > 2c_v + c_s$ and $c_v + c_s \geq p_{\max}$

In this case only producing in the two periods is economically sensible (as the revenue in a single period would be insufficient to cover costs). This case is thus identical to the case of block bidding, and as the optimal bid under block bidding is $b_b = c_s + 2c_v$ this also corresponds to truthful bidding under multi-part bidding (in the case of $2p_{\max} > c_v + c_s > p_{\max}$), and the expected profit is:

$$\mathbb{E}[\pi_M^*] = \frac{1}{6p_{\max}^2}(2p_{\max} - c_s - 2c_v)^3 \quad \text{for : } 2p_{\max} > 2 \cdot c_v + c_s \text{ and } c_v + c_s \geq p_{\max} \quad (\text{B.26})$$

Appendix C. Proof that expected multi-part profit is larger than simple bid and block bid profit

C.1. Simple bid

Subtracting the expected multipart profit by the expected simple part profit we obtain:

$$\begin{aligned} \mathbb{E}[\pi_{\text{Multipart}}^*] - \mathbb{E}[\pi_{\text{Simple}}^*] &= \\ &\begin{cases} \frac{c_s^2}{p_{\max}^2(c_s - p_{\max})} \left(\frac{c_s^2}{6} + c_s \cdot c_v - \frac{c_s \cdot p_{\max}}{6} + c_v^2 - c_v \cdot p_{\max} \right) & \text{for } c_s + c_v < p_{\max} \\ \frac{1}{6p_{\max}^2}(2p_{\max} - c_s - 2c_v)^3 & \text{for } p_{\max} \leq c_s + c_v \wedge \\ & c_s + 2c_v < 2p_{\max} \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad (\text{C.1})$$

For the first case ($c_v + c_s < p_{\max}$):

$$\frac{c_s^2}{p_{\max}^2(c_s - p_{\max})} \left(\frac{c_s^2}{6} + c_s \cdot c_v - \frac{c_s \cdot p_{\max}}{6} + c_v^2 - c_v \cdot p_{\max} \right) \geq 0 \quad (\text{C.2})$$

The first factor is strictly smaller than 0, since $c_s < p_{\max}$ (follows from $p_{\max} > c_s + c_v$). Therefore we can simplify to:

$$\frac{c_s^2}{6} + c_s \cdot c_v - \frac{c_s \cdot p_{\max}}{6} + c_v^2 - c_v \cdot p_{\max} \leq 0 \quad (\text{C.3})$$

$$\frac{c_s^2}{6} - \frac{c_s \cdot p_{max}}{6} + c_v(c_s + c_v - p_{max}) \leq 0 \quad (C.4)$$

Since $c_v(c_v + c_s - p_{max}) < 0$, it is sufficient to show that

$$\frac{c_s^2}{6} - \frac{c_s \cdot p_{max}}{6} \leq 0 \Leftrightarrow \frac{c_s}{6}(c_s - p_{max}) \leq 0 \quad (C.5)$$

Since $c_s > 0$ and $c_s \leq p_{max}$, q.e.d.

For the second case $\frac{1}{6p_{max}^2}(2p_{max} - c_s - 2c_v)^3 > 0$ follows directly from the definition of the piecewise function $p_{max} \leq c_s + c_v \wedge c_s + 2c_v < 2p_{max}$.

C.2. Block bid

Subtracting the expected multipart profit by the expected block bid profit we get:

$$\begin{aligned} \mathbb{E}[\pi_{Multipart}^*] - \mathbb{E}[\pi_{Block}^*] = \\ \begin{cases} \frac{c_v^2}{p_{max}^2} \left(p_{max} - c_s - \frac{4c_v}{3} \right) & \text{for } c_s + 2c_v < p_{max} \\ \frac{1}{3p_{max}^2} (c_s + c_v - p_{max})^2 (c_s + 4c_v - p_{max}) & \text{for } c_s + c_v < p_{max} \wedge \\ & p_{max} \leq c_s + 2c_v < 2p_{max} \\ 0 & \text{otherwise} \end{cases} \quad (C.6) \end{aligned}$$

For the first case, since $c_s + 2c_v < p_{max}$ it follows that:

$$\frac{c_v^2}{p_{max}^2} \left(p_{max} - c_s - \frac{4c_v}{3} \right) > 0 \quad (C.7)$$

For the second case:

$$\frac{1}{3p_{max}^2} (c_s + c_v - p_{max})^2 (c_s + 4c_v - p_{max}) \geq 0 \quad (C.8)$$

Since the first two factors are quadratic in nature (and thus positive) it is sufficient if:

$$(c_s + 4c_v - p_{max}) \geq 0 \quad (C.9)$$

Which is true, since per function definition $c_s + 2c_v \geq p_{max}$ and $c_s + 4c_v \geq p_{max}$.

Appendix D. Proof of equivalence of bidding formats under perfect correlation of prices

In case prices in period 1 and 2 are perfectly correlated ($p = p_1 = p_2$), the profit functions under simple, block and multi-part bidding simplify, as the cases with different prices in the two periods cannot occur any more.

For simple-bidding the profit function, defined in Eq. (1), simplifies to (with symmetric bidding):

$$\pi_{Simple}(b, p) = \begin{cases} 2p - c_s - 2c_v & \text{for } p \geq b \\ 0 & \text{otherwise} \end{cases} \quad (D.1)$$

For block bidding Eq. (5) simplifies to:

$$\pi_{Block}(b_b, p) = \begin{cases} 2p - c_s - 2c_v & \text{for } 2p \geq b_b \\ 0 & \text{otherwise} \end{cases} \quad (D.2)$$

And for multi-part bidding Eq. (12) simplifies to:

$$\pi_{Multipart}(b_s, b_v, p) = \begin{cases} 2p - c_s - 2c_v & \text{for } 2p \geq b_s + 2b_v \\ 0 & \text{otherwise} \end{cases} \quad (D.3)$$

As the only difference in the decision problem is now the name of the bidding variable ($2b^* = b_s^* + 2b_v^*$, with all possible combinations of b_s and b_v), the solution to the problem and expected profits are identical. We derive the optimal solution and expected profit as an example here for simple bidding:

$$\begin{aligned}\mathbb{E}[\pi_{\text{Simple,Correlated}}(b)] &= \int_b^{p_{\max}} \frac{2p - c_s - 2c_v}{p_{\max}} dp \\ &= \frac{(p_{\max} - b)(b - c_s - 2c_v + p_{\max})}{p_{\max}}\end{aligned}\quad (\text{D.4})$$

To find the stationary point:

$$\frac{d\mathbb{E}[\pi_{\text{Simple,Correlated}}(b)]}{db} = \frac{-2b + c_s + 2c_v}{p_{\max}} = 0 \quad (\text{D.5})$$

Solving for b:

$$b^* = \frac{c_s}{2} + c_v \quad (\text{D.6})$$

As the second derivative is strictly negative, this is the bid with the maximum profit, and corresponds to the expected solution of the sum of variable and start-up costs over two periods.

The expected profit under optimal bidding is then:

$$\mathbb{E}[\pi_{\text{Simple}}^*] = \begin{cases} \frac{(2p_{\max} - c_s - 2c_v)^2}{4p_{\max}} & \text{for } c_s + 2c_v < p_{\max} \\ 0 & \text{otherwise} \end{cases} \quad (\text{D.7})$$

Appendix E. A second trading round

E.1. Case A: Ex-post oversupply

Eq. (20) shows the inequality for which production would result in lower costs than re-trading in a second round. As long as this inequality holds, the actor will prefer to produce the electricity rather than re-trade

$$c_v \cdot K \cdot l + c_s \cdot K < (p_1^\alpha + m_1 \cdot K) \cdot K \cdot l \quad (\text{E.1})$$

In order for this inequality to hold, m_1 must be bigger than a certain threshold (21).

$$m_1 > c_v + \frac{\frac{c_s}{l} - p_1^\alpha}{K} \quad (\text{E.2})$$

In the worst case, the market price equals the bid of the producer $p_1^\alpha = b^*$, meaning the producer got just accepted at the lowest price possible. A market price lower than the bid of the producer is not relevant as then the producer would not be accepted at all. Therefore, we can reformulate Eq. (E.2) and result at (E.3)

$$m_1 > c_v + \frac{\frac{c_s}{l} - b^*}{K} \quad (\text{E.3})$$

In order to test the influence of c_s and c_v on the necessary m_1 for the actor not to wish to re-trade, we vary the share of c_s and c_v on the total production cost and leave b constant. We rearrange the equation for c_v and c_s .

$$b^* = \frac{c_v \cdot p_{\max}}{p_{\max} - c_s/l} \quad (\text{E.4})$$

$$\begin{aligned}c_s &= \frac{(b^* - c_v) \cdot l \cdot p_{\max}}{b^*} \\ c_v &= \frac{\left(p_{\max} - \frac{c_s}{l}\right) \cdot b^*}{p_{\max}}\end{aligned} \quad (\text{E.5})$$

We now replace c_s in Eq. (E.2)

$$m_1 > \frac{(p_{\max} - b^*)(b^* - c_v)}{b^* \cdot K} \quad (\text{E.6})$$

We now replace c_v in Eq. (E.2)

$$m_1 > \frac{(p_{\max} - b^*) \cdot c_s}{K \cdot p_{\max}} \quad (\text{E.7})$$

E.2. Case B: Ex-post undersupply

Analogue to Case A.

E.3. Case C: Ex-post oversupply in two periods

Eq. (E.8) shows the inequality for which production would result in lower costs than re-trading in a second round.

$$c_v \cdot (K \cdot l + K \cdot l) + c_s \cdot K < (p_1^\alpha + m_1 \cdot K) \cdot K \cdot l + (p_2^\alpha + m_2 \cdot K) \cdot K \cdot l \quad (\text{E.8})$$

In order for this inequality to hold $m_1 + m_2$ must be bigger than a certain threshold.

$$m_1 + m_2 > 2c_v + \frac{\frac{c_s}{l} - p_1^\alpha - p_2^\alpha}{K} \quad (\text{E.9})$$

If we simplify the case, by assuming that in both periods the offer has just been accepted with the identical bid b^* , and that the price elasticity m_1 and m_2 are identical for both periods, the inequality simplifies to:

$$m > \frac{2c_v \cdot l + c_s - 2b^* \cdot l}{2K \cdot l} \quad (\text{E.10})$$

The remaining derivations are then analogue to Case A.

Appendix F. Variable and start-up costs of power plants

Table F.2

Power plant example costs.

	$c_{v, \min}$	$c_{v, \max}$	$c_{s, \text{hot}}$	$c_{s, \text{cold}}$
	Euro/MWh	Euro/MWh	Euro/MW	Euro/MW
OCGT	48.34	52.99	16	28
CCGT	36.10	38.91	23	54
Coal	16.90	18.71	28	74

Appendix G. Code & numerical figures

The code to create the figures is published as a digital appendix at <https://doi.org/10.5281/zenodo.1463730>. Jupyter Notebook (v5.6.0) was used, running Python (v3.5.5), with the Sympy (v1.2), Numpy (v1.15.0) and Matplotlib (v2.2.3) packages. Inkscape was used to create Fig. 1 and add the shaded areas in Fig. 5.

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