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# **The Impact of Monetary Policy on Bond Returns: A Segmented Markets Approach**

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## **Abstract:**

This paper assesses the contribution of monetary policy to the dynamics of bond real returns. We assume that the monetary authority controls the short-term nominal interest rate. We then model exogenously the joint dynamics of the aggregate endowment and the monetary policy variable, and determine bond real returns endogenously. Market segmentation is introduced by permanently excluding a fraction of households from financial markets. When markets are segmented, monetary policy has a direct liquidity effect on the participants' consumption and marginal utility, on the stochastic discount factor, and on real returns. With full participation, however, real returns are determined by the aggregate endowment only, so monetary policy can affect them only indirectly. Data on bond returns strongly favor the segmented markets model over the full participation model. For maturities up to 2 years, the segmented markets model is able to replicate the sign and the size of the impulse response of bond returns to monetary policy shocks, it correctly predicts the sign of their autocorrelation, and it closely matches their volatility as a function of maturity.

**Keywords:** bond returns volatility; limited participation; segmented markets; monetary policy shocks.

**JEL Classification:** E44; E52; G12.

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# 1. Introduction

This paper assesses the contribution of monetary policy to the dynamics of bond real returns. We assume that the monetary authority controls the short-term nominal interest rate. We then model exogenously the joint dynamics of the aggregate endowment and the monetary policy variable, and determine bond real returns endogenously.

We adopt a heterogeneous agents variant of the limited participation framework, the *segmented markets* model, previously studied by Alvarez and Atkeson (1996), Alvarez, Lucas and Weber (2001), Lahiri, Singh and Vegh (2003), and Occhino (2004). The central feature is that a set of households are permanently excluded from financial markets.

When markets are segmented, monetary policy has a direct effect. Changes in the stance of monetary policy affect the distribution of cash balances and consumption expenditures across households. An increase in interest rates induces traders to hold more bonds, to lower their holdings of cash balances, and to reduce their purchases of consumption goods. The traders' marginal utility of consumption rises, lowering the stochastic discount factor, and increasing expected bond real returns. The smaller the economic weight of traders in the economy, the larger is this liquidity effect of monetary policy on bond real returns. With full participation, however, real returns are determined exclusively by the aggregate endowment, so monetary policy can affect them only indirectly.

We then take the full and segmented markets models to the data. Three empirical dimensions are explored: the response of bond returns to nominal interest rate shocks; the autocorrelation of bond returns; and the term structure of volatility. The evidence strongly favors the segmented markets model in each case.

The full participation model has incorrect predictions about the impact effect of monetary policy, with real returns rising after an increase in interest rates. Real returns fall in the segmented markets model and closely track the impulse responses in the data thereafter.

The segmented markets model also matches the declining positive autocorrelations and increasing volatilities of bond returns as time to maturity increases. The full participation model has negative autocorrelations and can only match the higher volatilities of longer term bond returns by overstating short-term bond volatility.

The paper is organized as follows: section 2 describes the economy and defines the equilibrium; section 3 explains the numerical solution method; section 4 presents and comments on the empirical

results; section 5 concludes.

## 2. Model

The model is a cash-in-advance endowment economy, with a large number of households and a monetary authority. Time is discrete and is indexed by  $t \geq 0$ . There is a single non-durable consumption good, money, and one-period nominal bonds, which are claims to one unit of money payable at the end of the period. Households are of two types, traders and non-traders. Let  $\omega > 0$  and  $\omega^* \geq 0$  be respectively the number of traders and non-traders. We will refer to the case where  $\omega^* = 0$  and  $\omega^* > 0$  respectively as the full participation model and the segmented markets model.

Households of the same type are identical in all respects. The crucial difference between the two types of households is that non-traders spend all their money purchasing consumption goods, while traders can purchase bonds as well.

Households start each period with cash balances from the previous period. Then, two markets meet in sequence, a bond market and a goods market.

In the bond market, the monetary authority sells one-period nominal bonds to the traders, at the bond price  $q_t > 0$ . The monetary authority announces the bond price, and stands ready to issue and sell any number of bonds to clear the market at that price. Open market operations are then conducted in terms of the short-term nominal interest rate  $i_t$  defined by

$$q_t \equiv \frac{1}{1 + i_t}, \quad (1)$$

while the bond supply and the money supply are determined endogenously. We assume that the interest rate is strictly positive, and the bond price is strictly less than one.

After the bond market, all households participate in the goods market. Each trader and each non-trader respectively receive constant fractions  $\Lambda > 0$  and  $\Lambda^* > 0$  of the exogenous stochastic aggregate endowment  $Y_t > 0$ , with  $\omega\Lambda + \omega^*\Lambda^* = 1$ . The endowment cannot be consumed directly, and must be sold in exchange of money at the price  $P_t > 0$ . Households can only consume goods purchased with money held before the goods market session. The money supply  $M_t$  is defined as the amount of dollars  $P_t Y_t$  spent in the goods market. Bonds are redeemed after the goods market closes.

The aggregate endowment  $Y_t$  and the nominal interest rate  $i_t$  are the only sources of uncertainty in the economy, and their joint dynamics is exogenously modeled as follows. Let  $\{\bar{Y}_t, \bar{i}_t\}_{t=0}^\infty$  be the non-stochastic steady state values of the aggregate endowment and the interest rate, and

let us assume that  $\bar{Y}_{t+1}/\bar{Y}_t = \alpha$  and  $\bar{i}_t = i$  are constant over time. We assume that  $\hat{z}_t \equiv [\log(Y_t) - \log(\bar{Y}_t), \log(i_t) - \log(i)]$  follows the AR(N) process,

$$\hat{z}_t = \sum_{n=1}^N \hat{z}_{t-n} B_n + \eta_t C, \quad (2)$$

where  $B_n$  and  $C$  are  $2 \times 2$  matrices,  $C$  is upper triangular,  $\eta_t$  is a  $1 \times 2$  vector of independently and identically distributed standard Gaussian shocks.

Each trader chooses consumption  $C_t$ , bonds  $B_t$ , and next-period cash balances  $A_{t+1}$  to solve

$$\max_{\{C_t > 0, B_t, A_{t+1} > 0\}_{t=0}^{\infty}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t) \right], \quad (3)$$

subject to

$$q_t B_t + P_t C_t \leq A_t, \quad (4)$$

$$A_{t+1} = A_t - q_t B_t - P_t C_t + P_t \Lambda Y_t + B_t,$$

given the traders' initial cash balances  $A_0 > 0$  in period zero.  $E_0$  is the expectation conditional on information available after  $\hat{z}_0$  has been revealed. The period utility function  $u(C) \equiv C^{1-\sigma}/(1-\sigma)$  is constant relative risk aversion, and the preferences discount factor satisfies  $\beta\alpha^{1-\sigma} \in (0, 1)$ .

Since the bond price  $q_t$  is strictly less than one for all  $t$ , holding idle cash balances is never optimal for traders, so the traders' cash-in-advance constraint always holds with equality. Then, the two constraints (4) in the problem (3) can be substituted with

$$\begin{aligned} q_t B_t + P_t C_t &= A_t, \\ A_{t+1} &= P_t \Lambda Y_t + B_t. \end{aligned} \quad (5)$$

Non-traders spend all their initial cash balances purchasing consumption goods. Under this assumption, the behavior of a non-trader is simply described by,

$$\begin{aligned} P_t C_t^* &= A_t^*, \\ A_{t+1}^* &= P_t \Lambda^* Y_t, \end{aligned} \quad (6)$$

given the non-traders' initial cash balances  $A_0^* > 0$  in period zero.

The economy is described by the traders' initial assets  $A_0$ , the non-traders initial assets  $A_0^*$ , the initial exogenous state  $[\hat{z}_0, \dots, \hat{z}_{-N+1}]$ , and the law of motion (2) for the exogenous state  $\hat{z}_t$ . Histories are made of the sequences of all possible realizations of the shocks  $\eta_t$ . An equilibrium is a set of contingent sequences  $\{C_t > 0, B_t, A_{t+1} > 0\}_{t=0}^{\infty}$  of consumption demand, bond demand and cash balances for traders,  $\{C_t^* > 0, A_{t+1}^* > 0\}_{t=0}^{\infty}$  of consumption demand and cash balances for non-traders, a contingent sequence  $\{D_t\}_{t=0}^{\infty}$  of bonds supplied by the monetary authority, and

a contingent sequence  $\{P_t > 0\}_{t=0}^{\infty}$  of prices such that the traders' contingent sequence solves the traders' optimization problem (3), the non-traders' contingent sequence satisfies the non-traders' equations (6), and the bond and goods market equilibrium conditions

$$\begin{aligned}\omega B_t &= D_t, \\ \omega C_t + \omega^* C_t^* &= Y_t.\end{aligned}\tag{7}$$

The necessary first-order conditions for the traders' optimization problem are

$$\begin{aligned}\beta^t u'(C_t) - \nu_t^1 P_t &= 0, \\ -q_t \nu_t^1 + \nu_t^2 &= 0, \\ -\nu_t^2 + E_t[\nu_{t+1}^1] &= 0,\end{aligned}\tag{8}$$

and the transversality condition is

$$\lim_{t \rightarrow \infty} E_0 [\nu_t^1 A_t] = 0,\tag{9}$$

where  $\nu_t^1$  and  $\nu_t^2$  are the Lagrange multipliers associated with the two constraints (5). From the first-order conditions, it follows that

$$\begin{aligned}\beta^t u'(C_t) &= \nu_t^1 P_t, \\ q_t \nu_t^1 &= E_t[\nu_{t+1}^1].\end{aligned}\tag{10}$$

The system describing the equilibrium is, then, made of the identity (1), the law of motion (2) for the exogenous state, the traders' first-order conditions (10), the traders' constraints (5), the non-traders' constraints (6), and the equilibrium conditions (7).

In this paper, we focus on the predictions of the model about bond real returns. All throughout, bond real returns are real holding period returns of discount real bonds, which are assets with a fixed real payoff at some fixed maturity date. To compute the price of any financial asset, we assume that only traders can participate in the bond market. From standard arguments, it follows that the equilibrium price  $Q_t$  of a one-period financial asset with nominal payoff  $\Pi_{t+1}$  and the equilibrium real price  $Q_t^*$  of a one-period financial asset with real payoff  $\Pi_{t+1}^*$  are given by

$$\begin{aligned}\nu_t^1 Q_t &= E_t[\nu_{t+1}^1 \Pi_{t+1}], \\ \nu_t^1 Q_t^* P_t &= E_t[\nu_{t+1}^1 \Pi_{t+1}^* P_{t+1}].\end{aligned}\tag{11}$$

Multi-period financial assets are priced in the same way, recursively.

### 3. Solution

For convenience, variables are normalized as follows. As in Lucas (1990), nominal variables are normalized by aggregate cash balances available at the beginning of the period. Let  $\bar{A}_t \equiv \omega A_t +$

$\omega^* A_t^*$  be the initial aggregate cash balances. Then,  $y_t \equiv Y_t/\bar{Y}_t$ ,  $\nu_t \equiv u'(\omega)\nu_t^1 \bar{A}_t/\beta^t u'(\bar{Y}_t)\bar{Y}_t$ ,  $c_t \equiv \omega C_t/\bar{Y}_t$ ,  $b_t \equiv \omega B_t/\bar{A}_t$ ,  $a_t \equiv \omega A_t/\bar{A}_t$ ,  $c_t^* \equiv \omega^* C_t^*/\bar{Y}_t$ ,  $a_t^* \equiv \omega^* A_t^*/\bar{A}_t$ ,  $d_t \equiv D_t/\bar{A}_t$ ,  $\gamma_t \equiv \bar{A}_{t+1}/\bar{A}_t$ ,  $p_t \equiv P_t \bar{Y}_t/\bar{A}_t$ . Also, let us define  $\lambda \equiv \omega\Lambda = 1 - \omega^*\Lambda^*$  the traders' share of the aggregate endowment. Then,  $\lambda = 1$  in the full participation model, and  $\lambda \in (0, 1)$  in the segmented markets model.

The system describing the equilibrium can then be written as

$$q_t(1 + i_t) \equiv 1, \quad (12a)$$

$$u'(c_t) = \nu_t p_t, \quad (12b)$$

$$q_t \gamma_t \nu_t = \beta E_t[\nu_{t+1}] u'(\alpha) \alpha, \quad (12c)$$

$$q_t b_t + p_t c_t = a_t, \quad (12d)$$

$$\gamma_t a_{t+1} = p_t \lambda y_t + b_t, \quad (12e)$$

$$p_t c_t^* = a_t^*, \quad (12f)$$

$$\gamma_t a_{t+1}^* = p_t (1 - \lambda) y_t, \quad (12g)$$

$$b_t = d_t, \quad (12h)$$

$$c_t + c_t^* = y_t, \quad (12i)$$

$$a_t + a_t^* = 1, \quad (12j)$$

together with the law of motion (2) for the exogenous state. The transversality condition (9) can be written as

$$\lim_{t \rightarrow \infty} E_0 [\beta^t u'(\bar{Y}_t) \bar{Y}_t \nu_t a_t / u'(\omega) \omega] = 0, \quad (13)$$

and the asset pricing equations (11) as

$$\begin{aligned} \gamma_t \nu_t Q_t &= \beta E_t[\nu_{t+1} \Pi_{t+1}] u'(\alpha) \alpha, \\ \nu_t Q_t^* p_t &= \beta E_t[\nu_{t+1} \Pi_{t+1}^* p_{t+1}] u'(\alpha). \end{aligned} \quad (14)$$

It is convenient to derive an equivalent system as follows. From the households' budget constraints (12e) and (12g), it follows that

$$\begin{aligned} \gamma_t a_{t+1} + \gamma_t a_{t+1}^* &= p_t \lambda y_t + b_t + p_t (1 - \lambda) y_t, \\ q_t \gamma_t [a_{t+1} + a_{t+1}^*] &= q_t p_t y_t + q_t b_t. \end{aligned}$$

Then, using the households' cash-in-advance constraints (12d) and (12f), the bond price (12a), and

the goods market equilibrium condition (12j),

$$\begin{aligned} q_t \gamma_t [a_{t+1} + a_{t+1}^*] &= q_t p_t y_t + a_t - p_t c_t + a_t^* - p_t c_t^*, \\ q_t \gamma_t &= q_t p_t y_t + 1 - p_t c_t - p_t c_t^*, \\ q_t \gamma_t + (1 - q_t) p_t y_t &= 1, \end{aligned}$$

which we use in place of the traders' budget constraint (12e) in the previous system (12).

In the non-stochastic steady state, all normalized variables are constant over time, and  $y_t = 1$ . Since  $\beta \alpha^{1-\sigma} \in (0, 1)$ , the transversality condition (13) is satisfied in the non-stochastic steady state. After log-linearizing<sup>1</sup> the system around the non-stochastic steady state, we obtain

$$\begin{aligned} \hat{q}_t + \frac{i}{1+i} \hat{i}_t &\equiv 0, \\ -\sigma \hat{c}_t &= \hat{\nu}_t + \hat{p}_t, \\ \hat{q}_t + \hat{\gamma}_t + \hat{\nu}_t &= E_t[\hat{\nu}_{t+1}], \\ qb[\hat{q}_t + \hat{b}_t] + pc[\hat{p}_t + \hat{c}_t] &= a\hat{a}_t, \\ q\gamma[\hat{q}_t + \hat{\gamma}_t] + (1-q)py[-\frac{q}{1-q}\hat{q}_t + \hat{p}_t + \hat{y}_t] &= 0, \\ \hat{p}_t + \hat{c}_t^* &= \hat{a}_t^*, \\ \hat{\gamma}_t + \hat{a}_{t+1}^* &= \hat{p}_t + \hat{y}_t, \\ \hat{b}_t &= \hat{d}_t, \\ c\hat{c}_t + c^* \hat{c}_t^* &= y\hat{y}_t, \\ a\hat{a}_t + a^* \hat{a}_t^* &= 0, \end{aligned} \tag{15}$$

where the variables without the time subscript are the non-stochastic steady state values, while the variables with the hat are the percentage deviations from the steady state values.

The system (15) together with the law of motion (2) for the exogenous state can be reduced to a four equation system in the two exogenous variables  $\hat{y}_t$  and  $\hat{i}_t$ , the endogenous state variable  $\hat{a}_t^*$ , and the control variable  $\hat{\nu}_t$ . With standard methods, we derive the linear system describing the equilibrium evolution of the three state variables  $\hat{y}_t$ ,  $\hat{i}_t$ , and  $\hat{a}_t^*$ , and linking all the other variables

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<sup>1</sup> We have solved the model with the alternative methodology described in Occhino (2004). The methodology consists in defining the recursive competitive equilibrium as functions solving a system of equations, devising an operator whose fixed point is an equilibrium, and iterating on the operator until convergence. The advantage of this approach is that it avoids linearizing the model. The disadvantage is that it requires keeping the number of state variables as low as possible, and it can be used only for the case  $N = 1$ . Although the two methodologies are very different, they yield similar results in the  $N = 1$  case.



to the three state variables<sup>2</sup>. Then, we derive the percentage deviation  $\hat{Q}_t$  of the price of a one-period financial asset as a function of the percentage deviation  $\hat{\Pi}_{t+1}$  of its nominal payoff and the percentage deviation  $\hat{Q}_t^*$  of the real price of a one-period financial asset as a function of the percentage deviation  $\hat{\Pi}_{t+1}^*$  of its real payoff from

$$\begin{aligned}\hat{\gamma}_t + \hat{\nu}_t + \hat{Q}_t &= E_t[\hat{\nu}_{t+1} + \hat{\Pi}_{t+1}], \\ \hat{\nu}_t + \hat{Q}_t^* + \hat{p}_t &= E_t[\hat{\nu}_{t+1} + \hat{\Pi}_{t+1}^* + \hat{p}_{t+1}].\end{aligned}\tag{16}$$

Multi-period financial assets are priced recursively.

To gain further insight, after using  $-\sigma\hat{c}_t = \hat{\nu}_t + \hat{p}_t$  from the previous system (15), the last equation can be written as

$$\hat{Q}_t^* = E_t[-\sigma(\hat{c}_{t+1} - \hat{c}_t) + \hat{\Pi}_{t+1}^*],\tag{17}$$

which is a familiar asset pricing equation relating the real price of a one-period financial asset to its real payoff and to the intertemporal marginal rate of substitution of the subset of households which participate in financial markets. In the specific case of a one-period real bond, the real payoff is constant, so  $\hat{\Pi}_{t+1}^* = 0$ , and the percentage deviation of its real price  $\hat{Q}_t^*$  is equal to minus the expectation of the relative risk aversion  $\sigma$  times the percentage deviation of the traders' consumption growth rate. Equivalently,

$$\hat{r}_t = \sigma E_t[\hat{c}_{t+1} - \hat{c}_t],\tag{18}$$

the deviation  $\hat{r}_t$  of the real interest rate from its steady state value is equal to the relative risk aversion  $\sigma$  times the expected percentage deviation of the consumption growth rate of the subset of households which participate in financial markets.

## 4. Results

### 4.1 Calibration

The key parameters in the model are the traders' share of the aggregate endowment  $\lambda$  and the relative risk aversion  $\sigma$ .  $\lambda$  is a measure of the traders' economic weight. For instance, in the case that all households receive the same endowment,  $\lambda$  is the percentage of traders, that is the

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<sup>2</sup> The solution method is based on the eigenvalue decomposition of the matrix describing the evolution of the state and control variables. Very small imaginary parts of the solution are dropped. As a check, the model has been solved using MATLAB files written by Chris Sims and Paul Klein available at <http://www.ssc.uwo.ca/economics/faculty/klein/>. Their solution method is based on the Schur decomposition of the matrix describing the evolution of the state and control variables. The two methods yield identical solutions.

ratio  $\omega/(\omega + \omega^*)$  of the number of traders to the total number of households. When  $\lambda$  is equal to 1, the economy is the benchmark full participation, representative agent, endowment economy with cash-in-advance constraints. The lower  $\lambda$ , the greater is the degree of market segmentation. Below, we show results for values of  $\lambda$  in the range between 0.01 and 1, and for values of  $\sigma$  in the range between 0.5 and 3. We consider  $\lambda = 0.1$  and  $\sigma = 2$  as benchmark values<sup>3</sup> for the segmented markets model.

To calibrate the other parameters, we use monthly data for the period 1970:01-1999:12 from CRSP and from the FRED II Database of the Federal Reserve Bank of St. Louis.

Each period is one month. The aggregate endowment growth rate  $\alpha - 1$  in the non-stochastic steady state is set equal to 0.0025, to match the 3.02% average yearly growth rate of real personal consumption expenditure (nondurable goods and services). The inverse of the gross real interest rate  $\beta u'(\alpha)$  in the non-stochastic steady state is set equal to 0.9939 to match the 7.34% average yearly real rate of return on the value-weighted total stock market index. The value of the preferences discount factor  $\beta$ , then, varies with the relative risk aversion  $\sigma$ .

To obtain the law of motion (2) for the exogenous state  $\hat{z}_t$ , we run a VAR with  $N$  lags of the linearly-detrended logarithm of real personal consumption expenditure and the logarithm of the effective federal funds rate. We set  $N = 12$  on the basis of the AIC, but we found the results were not very sensitive to this choice.

We now compare the predictions of the full participation model ( $\lambda = 1$ ) and the segmented markets model ( $\lambda \in (0, 1)$ ) on bond real returns dynamics with data. Bond real returns are real holding period returns of Treasury bonds with constant maturities. For ease of interpretation, we express rates in annual percentage points, and we multiply logarithms by 100.

## 4.2 Impulse response analysis

We begin with an impulse response analysis emphasizing the liquidity effect of monetary policy on bond real returns. We make the standard structural assumption that a monetary policy shock does not effect the aggregate endowment contemporaneously. We then estimate a tri-variate VAR system consisting of the detrended log consumption, the log federal funds rate, and the bond real return. We decompose the covariance matrix of the innovations using the Cholesky factorization, and we identify a contractionary monetary policy shock as a positive shock to the federal funds

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<sup>3</sup> In related work, Landon-Lane and Occhino (2004) estimate a segmented markets model with data on the money growth rate and the inflation rate, and obtain a maximum likelihood estimate of  $\lambda$  at 0.13.

rate equation. Figure 1 shows the impulse responses over a 24-month period to a 100 basis point increase in the federal funds rate.

[Insert Figure 1 Here]

The figure shows that, in the impact period of a contractionary monetary policy shock, bond returns decrease for maturities of 3 months and higher. The 3-month bond return falls by  $-0.33\%$ , the 1-year by  $-1.70\%$ , and the 2-year by  $-2.61\%$ . Returns stay negative for a few periods and then become positive for most of the following periods until the shock dissipates near the end of the 24-month horizon analyzed.

The full participation model mis-characterizes the bond return responses. 3-month, 1-year and 2-year bond returns *rise* in the impact period of a contractionary shock, and are *negative* for most of the following periods.

The decline of expected bond returns is due to a negative deviation of the aggregate consumption growth rate from its steady state value<sup>4</sup>. In the full participation model, the stochastic discount factor is a function of the aggregate consumption growth rate. The deviation of the real interest rate in (18), as well as the deviations of expected real returns, remain negative for most of the periods following a contractionary monetary policy shock.

The segmented markets model, however, correctly predicts the sign of the bond returns, although it tends to overstate the impact effect of the shock. The 3-month bond return falls by  $-1.08\%$  in the impact period, the 1-year and 2-year bond returns fall by  $-3.91\%$  and  $-5.18\%$ . After the first period, bond returns become quickly positive, and match closely their empirical counterparts.

With segmented markets, the stochastic discount factor is determined by the intertemporal marginal rate of substitution of households which participate in financial markets. As pointed out by Grossman and Weiss (1983), Lucas (1990), Alvarez and Atkeson (1996) and Occhino (2004) in different limited participation models, a contractionary monetary policy shock decreases the participants' cash balances and consumption expenditures, increases their expected consumption growth rate, and as in (18), increases the real interest rate and expected real returns. The segmented markets model then matches the positive returns in the data that follow the impact period of a contractionary monetary policy shock.

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<sup>4</sup> Christiano, Eichenbaum and Evans (1999) document that the deviation of the aggregate production growth rate is negative for about six quarters after a contractionary monetary policy shock.

To make a formal comparison between the full participation and the segmented markets models, we follow the design suggested by Canova (2001). We compute [16%, 84%] confidence bands for the empirical impulse responses using the Sims and Zha (1999) procedure. We then count the number of periods when the model impulse response is consistent with its' empirical counterpart.

The best fit for the segmented markets model is the 2-year bond return, for which the model response falls into the 68% confidence bands 15 periods out of 24. At that horizon, the full participation model response falls within the bands only 8 times. The comparison favors the segmented markets model for all four maturities. For the 1-month bond return, the count is 10 for segmented markets, and only 3 for full participation. At 3-months, the counts are 6 versus 1, and at 1-year, the counts are 9 versus 2. Summing over these four securities, the impulse responses fall within the bands 41.7% of the times for the segmented markets model and only 16.7% of the times for the full participation model. The Diebold-Mariano (1995) statistic for the number of impulse responses falling within the bands is 4.38 which clearly favors the segmented markets model.

### 4.3 Bond return autocorrelations

We now consider the autocorrelation structure of short-term bond real returns. Tables 1.1 and 1.2 display the first-order autocorrelation coefficients of the aggregate endowment, the nominal interest rate, and bond real returns with maturities 1 to 24 months in the model and in the data, for several values of the traders' share  $\lambda$  of the aggregate endowment and relative risk aversion  $\sigma$ .

[Insert Table 1.1 and 1.2 Here]

The autocorrelations of the logarithm of the aggregate endowment and the nominal interest rate approximately match the autocorrelations of the linearly detrended log consumption and the federal funds rate.

Bond returns data reveal a smooth decline of the first-order autocorrelation as a function of maturity. The 1-month autocorrelation is 0.48. At 1-year, the autocorrelation has fallen to 0.31; by 2-years, the autocorrelation is 0.25.

The full participation model is far from replicating these moments. There is a short-run positive autocorrelation of 0.03 at the 1-month horizon, but it is much smaller than in the data. The autocorrelations then turn negative until the 11-month returns. Here again, the bond return autocorrelations are simply inheriting through the stochastic discount factor the behavior of the aggregate consumption growth rate.

The segmented markets model, however, correctly predicts the sign of the autocorrelations, and their decline with maturity, although the predicted decline is faster than in the data. For our benchmark case of  $\sigma = 2$  and  $\lambda = 0.1$ , the 1-month autocorrelation is 0.64 compared with 0.48, while the 1-year autocorrelation is 0.10 compared with 0.31. Except for the 1-month maturity, the model autocorrelations are lower than in the data. Nonetheless, the Diebold-Mariano statistic, comparing the root mean squared errors of the full participation and the segmented markets models, is 6.64, strongly favoring the assumption of market segmentation.

#### 4.4 Bond market volatility

We now turn to the contribution of the liquidity effect of monetary policy to the volatility structure of bond real returns. Tables 2.1 and 2.2 display the standard deviations of the aggregate endowment, the nominal interest rate and bond real returns in the model and in the data, for several values of  $\lambda$  and  $\sigma$ .

[Insert Table 2.1 and 2.2 Here]

The most striking feature of the data is that the volatility of bond real returns is a steeply increasing function of maturity. The 1-month standard deviation is 3.67, the 1-year and 2-year are 8.47 and 13.01 respectively.

The full participation model ( $\lambda = 1$ ) is not able to replicate the volatility of bond returns. At the benchmark relative risk aversion  $\sigma = 2$ , it significantly under-predicts bond returns volatilities for maturities of 1-year and higher. When the relative risk aversion is higher, all volatilities increase, so the full participation model over-predicts bond return volatilities for short maturities. For instance, when  $\sigma = 3$ , the full participation model approximately matches the 1-year volatility, still under-predicts the 2-year volatility, and significantly over-predicts the volatility for maturities up to 6 months.

The segmented markets model accounts for the contribution of monetary policy to bond returns volatility. The model with  $\lambda = 0.1$  and  $\sigma = 2$  does an excellent job in predicting the volatility of bond returns as function of maturity. The Diebold-Mariano statistic for a root mean squared error loss function is 3.02 which favors the segmented markets model over the full participation model at the 99% confidence level.

The predicted bond return volatility in the segmented markets case derives from two sources. The first is the aggregate endowment volatility, which is common to full participation models.

The second is the volatility of the monetary policy variable, namely the nominal interest rate. The higher the relative risk aversion, the more effective the first source. The higher market segmentation (the lower  $\lambda$ ), the more effective the second source. Both increasing the risk aversion and increasing market segmentation increase bond returns volatility.

The model cannot replicate bond returns volatilities further along the yield curve. To fully explain the volatility of assets with longer maturities, we would need to introduce more persistent shocks. Bansal and Yaron (2003), for instance, introduce an additional stochastic component of the aggregate endowment growth rate with small volatility and large persistence.

## 5. Conclusion

In a segmented markets model, we have been able to account for the contribution of monetary policy to bond real returns. Data on Treasury bond returns strongly favor the segmented markets model over the full participation model. For maturities up to 2 years, the segmented markets model is able to replicate the sign and the size of the impulse response of bond returns to monetary policy shocks, it correctly predicts the sign of their autocorrelation, and it closely matches their volatility along the yield curve.

In future work, we plan to study the effect of endogenizing production. With real sector shocks, we hope to explain the impact of segmented markets on long term bonds and equities.

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Table 1.1						
Impact of Market Segmentation and Risk Aversion on Autocorrelations						
	$\sigma = 3$					Data
	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0.2$	$\lambda = 0.1$	$\lambda = 0.01$	
1-month	0.0297	0.7256	0.7371	0.6298	0.4434	0.4850
2-month	-0.2809	0.4408	0.6022	0.4500	0.1546	0.4838
3-month	-0.2099	0.2019	0.5199	0.4946	0.2419	0.4802
4-month	-0.1533	0.1015	0.3977	0.4569	0.2802	0.4726
5-month	-0.1904	0.0659	0.3106	0.3842	0.2247	0.4608
6-month	-0.1582	0.0470	0.2462	0.3333	0.2341	0.4455
7-month	-0.1999	0.0285	0.1833	0.2486	0.1639	0.4272
8-month	-0.1594	0.0260	0.1578	0.2255	0.1789	0.4063
9-month	-0.1006	0.0242	0.1367	0.2037	0.2014	0.3833
10-month	-0.1135	0.0170	0.1127	0.1668	0.1624	0.3589
11-month	0.0298	0.0116	0.0963	0.1485	0.1893	0.3334
1-year	0.0331	0.0162	0.0917	0.1427	0.1975	0.3074
2-year	-0.1592	0.0822	0.0680	0.0825	0.1000	0.2470
	$\sigma = 2$					Data
	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0.2$	$\lambda = 0.1$	$\lambda = 0.01$	
1-month	0.0297	0.6279	0.7123	0.6366	0.4507	0.4850
2-month	-0.2809	0.2384	0.5317	0.4530	0.1649	0.4838
3-month	-0.2099	0.0422	0.3945	0.4599	0.2520	0.4802
4-month	-0.1533	-0.0148	0.2683	0.3927	0.2882	0.4726
5-month	-0.1904	-0.0319	0.1923	0.3114	0.2318	0.4608
6-month	-0.1582	-0.0365	0.1448	0.2576	0.2385	0.4455
7-month	-0.1999	-0.0482	0.0997	0.1844	0.1667	0.4272
8-month	-0.1594	-0.0436	0.0852	0.1644	0.1795	0.4063
9-month	-0.1006	-0.0381	0.0744	0.1476	0.1987	0.3833
10-month	-0.1135	-0.0432	0.0590	0.1192	0.1595	0.3589
11-month	0.0298	-0.0403	0.0521	0.1079	0.1827	0.3334
1-year	0.0331	-0.0306	0.0518	0.1047	0.1895	0.3074
2-year	-0.1592	0.0315	0.0462	0.0641	0.0948	0.2470
					Model	Data
$\log(y)$					0.9719	0.9716
$i$					0.9871	0.9771
D-M				6.6374		

Autocorrelation coefficients of the aggregate endowment, the nominal interest rate and bond real returns in the model and in the data.  $\lambda$  is the traders' share of the aggregate endowment,  $\sigma$  is the relative risk aversion. D-M is the Diebold-Mariano statistic comparing the full participation and the segmented markets models in the benchmark case  $\sigma = 2$  and  $\lambda = 0.1$ . The statistic has an asymptotic normal distribution.



Table 1.2						
Impact of Market Segmentation and Risk Aversion on Autocorrelations						
	$\sigma = 1$					Data
	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0.2$	$\lambda = 0.1$	$\lambda = 0.01$	
1-month	0.0297	0.4549	0.6642	0.6516	0.4724	0.4850
2-month	-0.2809	-0.0501	0.3584	0.4369	0.1956	0.4838
3-month	-0.2099	-0.1508	0.1619	0.3465	0.2802	0.4802
4-month	-0.1533	-0.1624	0.0689	0.2412	0.3087	0.4726
5-month	-0.1904	-0.1660	0.0255	0.1648	0.2492	0.4608
6-month	-0.1582	-0.1609	0.0057	0.1222	0.2471	0.4455
7-month	-0.1999	-0.1692	-0.0148	0.0765	0.1717	0.4272
8-month	-0.1594	-0.1599	-0.0168	0.0656	0.1782	0.4063
9-month	-0.1006	-0.1494	-0.0169	0.0582	0.1890	0.3833
10-month	-0.1135	-0.1554	-0.0232	0.0432	0.1502	0.3589
11-month	0.0298	-0.1426	-0.0206	0.0412	0.1649	0.3334
1-year	0.0331	-0.1295	-0.0170	0.0417	0.1684	0.3074
2-year	-0.1592	-0.1050	-0.0122	0.0272	0.0821	0.2470
	$\sigma = 0.5$					Data
	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0.2$	$\lambda = 0.1$	$\lambda = 0.01$	
1-month	0.0297	0.2965	0.5852	0.6507	0.5082	0.4850
2-month	-0.2809	-0.2373	0.1326	0.3457	0.2462	0.4838
3-month	-0.2099	-0.2729	-0.0353	0.1697	0.3204	0.4802
4-month	-0.1533	-0.2695	-0.0798	0.0757	0.3292	0.4726
5-month	-0.1904	-0.2720	-0.0971	0.0273	0.2618	0.4608
6-month	-0.1582	-0.2653	-0.1013	0.0058	0.2435	0.4455
7-month	-0.1999	-0.2740	-0.1097	-0.0161	0.1660	0.4272
8-month	-0.1594	-0.2636	-0.1067	-0.0185	0.1633	0.4063
9-month	-0.1006	-0.2529	-0.1029	-0.0188	0.1636	0.3833
10-month	-0.1135	-0.2610	-0.1063	-0.0254	0.1282	0.3589
11-month	0.0298	-0.2430	-0.0998	-0.0221	0.1339	0.3334
1-year	0.0331	-0.2321	-0.0959	-0.0197	0.1342	0.3074
2-year	-0.1592	-0.2322	-0.0964	-0.0228	0.0643	0.2470
					Model	Data
$\log(y)$					0.9719	0.9716
$i$					0.9871	0.9771

Autocorrelation coefficients of the aggregate endowment, the nominal interest rate and bond real returns in the model and in the data.  $\lambda$  is the traders' share of the aggregate endowment,  $\sigma$  is the relative risk aversion.

Table 2.1						
Impact of Market Segmentation and Risk Aversion on Volatilities						
	$\sigma = 3$					Data
	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0.2$	$\lambda = 0.1$	$\lambda = 0.01$	
1-month	6.4168	5.3974	4.1326	3.4020	2.8154	3.6679
2-month	7.6663	5.9082	4.3074	3.6148	3.1624	3.9051
3-month	7.0517	7.2337	4.6186	3.6135	3.0055	4.1642
4-month	7.0226	8.5698	5.2559	3.8686	3.0005	4.4809
5-month	7.4876	9.3295	5.8439	4.1834	3.1505	4.8545
6-month	7.6704	10.0353	6.5598	4.5934	3.2381	5.2278
7-month	7.6219	10.5914	7.3787	5.2223	3.6332	5.7439
8-month	7.8148	10.9524	8.0249	5.6529	3.7316	6.2455
9-month	7.8900	11.4091	8.7610	6.1868	3.8704	6.7755
10-month	7.5804	11.7927	9.5251	6.8527	4.3019	7.3269
11-month	7.5991	12.7189	10.5958	7.7164	4.6392	7.8926
1-year	8.2558	12.9822	11.1212	8.1345	4.7580	8.4653
2-year	10.2395	10.7096	13.8110	12.2110	8.0916	13.0127
	$\sigma = 2$					Data
	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0.2$	$\lambda = 0.1$	$\lambda = 0.01$	
1-month	4.2778	4.6673	3.9877	3.4189	2.8293	3.6679
2-month	5.1109	5.3514	4.2217	3.6364	3.1714	3.9051
3-month	4.7011	6.7076	4.7261	3.7255	3.0192	4.1642
4-month	4.6817	7.8242	5.5369	4.0986	3.0233	4.4809
5-month	4.9918	8.3238	6.2417	4.5188	3.1810	4.8545
6-month	5.1136	8.7330	7.0202	5.0334	3.2834	5.2278
7-month	5.0813	9.0669	7.8790	5.7584	3.6962	5.7439
8-month	5.2099	9.1783	8.4936	6.2522	3.8112	6.2455
9-month	5.2600	9.3855	9.1579	6.8302	3.9704	6.7755
10-month	5.0536	9.5990	9.8528	7.5339	4.4202	7.3269
11-month	5.0661	10.2226	10.7702	8.3940	4.7822	7.8926
1-year	5.5039	10.2441	11.1355	8.7922	4.9147	8.4653
2-year	6.8263	8.1288	12.3435	12.2838	8.3553	13.0127
					Model	Data
$\log(y)$					1.5460	1.5307
$i$					2.7392	3.2341
D-M				3.0163		

Standard deviations of the aggregate endowment, the nominal interest rate and bond real returns in the model and in the data.  $\lambda$  is the traders' share of the aggregate endowment,  $\sigma$  is the relative risk aversion. D-M is the Diebold-Mariano statistic comparing the full participation and the segmented markets models in the benchmark case  $\sigma = 2$  and  $\lambda = 0.1$ . The statistic has an asymptotic normal distribution.

Table 2.2						
Impact of Market Segmentation and Risk Aversion on Volatilities						
	$\sigma = 1$					Data
	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0.2$	$\lambda = 0.1$	$\lambda = 0.01$	
1-month	2.1389	3.6948	3.8601	3.4964	2.8730	3.6679
2-month	2.5554	4.5692	4.2797	3.7567	3.2019	3.9051
3-month	2.3506	5.6629	5.1962	4.1210	3.0658	4.1642
4-month	2.3409	6.2596	6.2420	4.7848	3.0996	4.4809
5-month	2.4959	6.3640	7.0256	5.4281	3.2832	4.8545
6-month	2.5568	6.4326	7.7576	6.1255	3.4311	5.2778
7-month	2.5406	6.5531	8.5069	6.9800	3.8950	5.7439
8-month	2.6049	6.4756	8.9198	7.5337	4.0587	6.2455
9-month	2.6300	6.5011	9.3335	8.1236	4.2753	6.7755
10-month	2.5268	6.6077	9.7868	8.8116	4.7743	7.3269
11-month	2.5330	6.9178	10.3463	9.5754	5.2009	7.8926
1-year	2.7519	6.7769	10.3929	9.8577	5.3697	8.4653
2-year	3.4132	5.5916	9.9322	11.7073	9.0569	13.0127
	$\sigma = 0.5$					Data
	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0.2$	$\lambda = 0.1$	$\lambda = 0.01$	
1-month	1.0695	2.7318	3.7236	3.6140	2.9509	3.6779
2-month	1.2777	3.5553	4.4106	4.0115	3.2594	3.9051
3-month	1.1753	4.1894	5.6016	4.7947	3.1629	4.1642
4-month	1.1704	4.3853	6.5848	5.7526	3.2638	4.4809
5-month	1.2479	4.3286	7.1597	6.5376	3.5075	4.8545
6-month	1.2784	4.3063	7.6056	7.2779	3.7470	5.2778
7-month	1.2703	4.3716	8.0433	8.0799	4.3035	5.7439
8-month	1.3025	4.2846	8.1553	8.5189	4.5553	6.2455
9-month	1.3150	4.2827	8.2862	8.9494	4.8682	6.7755
10-month	1.2634	4.3572	8.4893	9.4477	5.4441	7.3269
11-month	1.2665	4.5041	8.7435	9.9573	5.9663	7.8926
1-year	1.3760	4.3625	8.5915	10.0058	6.1893	8.4653
2-year	1.7066	3.8243	7.9445	10.1933	10.1097	13.0127
					Model	Data
$\log(y)$					1.5460	1.5307
$i$					2.7392	3.2341

Standard deviations of the aggregate endowment, the nominal interest rate and bond real returns in the model and in the data.  $\lambda$  is the traders' share of the aggregate endowment,  $\sigma$  is the relative risk aversion.

**Figure 1:** Impulse Responses to Monetary Shock

