

Neumärker, Karl Justus Bernhard; Pech, Gerald

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# The Role of Beliefs for the Sustainability of the Fiscal Constitution

K. J. Bernhard Neumärker<sup>†</sup> and Gerald Pech<sup>‡</sup>



<sup>†</sup> Saarland University, Department of Economics, Economic Theory, Bld. 31, Room 304, D - 66123 Saarbrücken and Ruhr-University Bochum, Department of Economics, GC 2/62, D-44780 Bochum, Email: [bernhard.k.neumaerker@ruhr-uni-bochum.de](mailto:bernhard.k.neumaerker@ruhr-uni-bochum.de)

<sup>‡</sup> May Wong-Smith Fellow, CRIEFF, St Salvator's College, University of St Andrews, St Andrews, KY16 9AL, Scotland, UK, Email: [gerald.pech@st-andrews.ac.uk](mailto:gerald.pech@st-andrews.ac.uk)

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# The role of beliefs for the sustainability of the fiscal constitution\*

K.J. Bernhard Neumärker and Gerald Pech<sup>†</sup>

February 19, 2003

## ABSTRACT

Why does the government not defect from the constitution? This article focuses on the dynamic restraints the government faces under the rule of law: violations against unconstitutional laws are not punished under the constitution. If a violating government cannot commit itself never to reinstall the constitution enforcing an unconstitutional law becomes difficult. Citizens' expectations to go unpunished when not complying may be self-fulfilling. Deriving the equilibrium of a global game we show that this mechanism is effectively deterring a government from defecting from a constitutionally permissible tax rate.

JEL codes: *K42, H26, E61, D7* Keywords: *tax evasion, global games, self-fulfilling expectations, dynamic policy restraints*

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## I. INTRODUCTION

*No one is bound to obey an unconstitutional law and no courts are bound to enforce it*  
(American Jurisprudence, Second Edition, Volume 16, Section 177).

Some constitutional frameworks explicitly or implicitly set a maximum rate at which the government can impose a tax. Such a constitutional restraint was introduced by proposition 13 in California in 1978. The amendment placed a limit of 1% of the 1875 market value to the amount of property taxes chargeable (Advisory Commission on Intergovernmental relations, Intergovernmental Perspective 5 No. 1, 1979). Eight other states followed this fiscal reform step. In Germany the constitutional court has lately interpreted the constitution as saying that the government must not claim more than half of the income of its citizens.

The wider question of why the government actually obeys such a restriction has, to our knowledge, never been answered in the literature. Clearly, there are counter checks and balances in the constitutional framework. But pointing to these devices does not answer the question of why the government should accept the mechanisms imposed by the constitution at all when it has the power to execute a policy at will.

Apart from the question of why constitutional restraints are not violated there is another, closely related observation: Governments rarely dare to openly violate the formal proceedings required by the constitution. In fact, even in flagrant cases of transgression, the government involved is usually trying to at least formally stay within the constitutional order. Recent examples include the dealings of President Mugabe of Zimbabwe and President Musharraf of Pakistan. Even the disempowerment of parliament in the German Third Reich was approved by a decision taken in parliament itself.

This suggests that there must be a special provision of the formal constitutional order that has a larger impact than material substance. In an attempt to find such a provision this article explores the consequences of one integral part of any constitutional order: the rule of law. Under the rule of law, breaking an unconstitutional law is with impunity. As long as there is any real chance that the unconstitutional government is succeeded by a constitutional one, this prospect discourages the obedience of the unconstitutional law. Economically, resistance against unconstitutional laws must increase the cost of enforcing the unconstitutional law. More than this: Once the enforcement of the unconstitutional law becomes sufficiently costly, the government has an incentive to return to the constitutional state in order to lower its cost, thereby making the expectations of impunity self-fulfilling. From this angle, it makes totally sense that governments strive to give their actions constitutional durability.

This paper builds the bridge between costly detection efforts and relief from prosecution in the field of taxation and tax evasion. This is the area of law which has probably been most thoroughly explored in terms of formal analysis. The idea is that no government is able to enforce its tax laws if everybody evades taxes and that everybody evades taxes if the government is predicted not to enforce the law.

For our formal analysis we assume that the government can effectively choose between a constitutional state in which an announced punishment is actually carried out and an unconstitutional state in which the punishment is only carried out when the government stays in the unconstitutional state. Consider the position of a government which has just violated the constitution, i.e. which has announced a tax rate which is too high. Citizens declare taxes and the government observes aggregate tax evasion and learns about its strength or its

preference for the unconstitutional state. If strength is low and enforcement is sufficiently costly, the government decides that it is better off under the constitution. Under common knowledge there are multiple equilibria for a range of preference parameter: If a sufficient share of citizens believe that the government returns to the constitution, this expectation is self-fulfilling. But there is another equilibrium where the government stays outside of the constitution and this is expected by the citizens. Our way to derive a unique critical value for the preference parameter is standard now in economics: We analyse a global game where citizens receive only a noisy signal of the government's strength and have to conclude whether sufficiently many citizens have received a signal which induces them to speculate on the government giving up its policy. From the possibility of a switch back a dynamic policy restraint results. This causes the government to be more reluctant to violate the constitution in the first place, so the rule of law is an endogenous enforcement mechanism. We extend our results to a stationary infinite horizon framework. While we get strong results on the effectiveness of the rule of law we find that in the case of tax laws there is some ambiguity because the constitution usually not only guarantees freedom from punishment of violating an unconstitutional law - thereby encouraging tax evasion in the unconstitutional state - but also relief of unjustified taxes - thereby encouraging tax payment. Our results hold, however, if the cost of detection sufficiently increases with tax evasion.

Our model differs from alternative models of collective action which rely on herding behavior (Banerjee, 1992) or on signalling by other agents (Lohmann, 1993) which are inappropriate in a tax evasion framework. It also differs from dynamic approaches to the social contract like that of Kotlikoff/Persson/Svennson (1988) where the dynamic constraint is,

effectively, based on a rules-versus discretion argument.

Our paper is organized as follows: In section 2 we lay out the structure of the game. In section 3 we set up the decision problems of the citizens and the government. Section 4 establishes multiplicity of equilibria, i.e. switching back and not switching back after a violation in the case of common knowledge. In section 5 we reconsider the decision to switch back in the case of imperfect information. Section 6 applies results from the theory of global games to our heterogeneous agent framework. Section 7 sets up the government's decision problem whether or not to violate the constitution in the first place and extends the model to an infinite horizon.

## II. OVERVIEW OF THE GAME STRUCTURE

Figure 1 depicts the basic game between subsequent governments and the citizens. Node  $c$  signifies a constitutional and node  $nc$  a unconstitutional state. In each node the government and the private sector take a decision. Decision variables of the government appear in brace-brackets with the information set indicated after the dash. The decision variables and the information set of the private sector are written in square brackets. If the decision has an effect on the game path, we indicate in round brackets next to an edge the values of the decision variables which are in accordance with this path.

At stage 0 in period  $t$ , the government is in the constitutional state and observes a state variable  $k_{t-1}$  which is the current realization of a preference indicator  $k$ . A positive value expresses a preference of the government for the unconstitutional state and a negative value indicates a preference for the constitutional state. This preference might be explained by

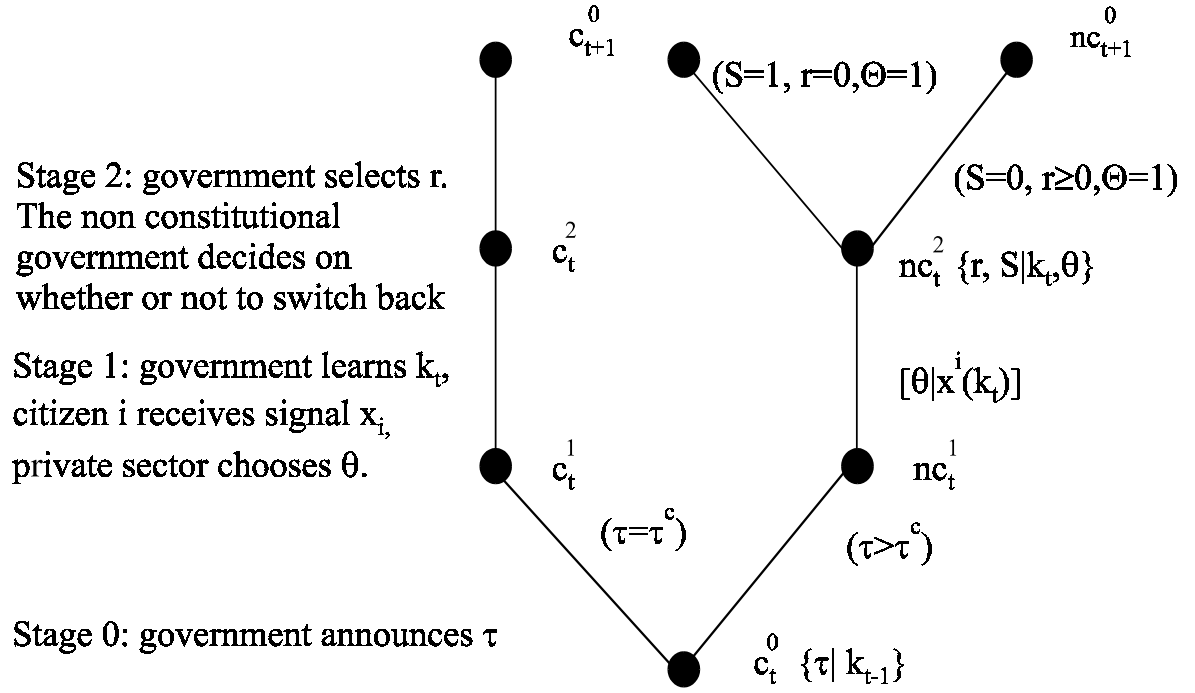


Figure 1: The game between government and citizens

extra benefits which are available to the government when it acts outside of the constitution. We may think of  $k_{t-1}$  as a value which the government carries over from the previous period, so we assume that  $k_{t-1}$  is common knowledge. For the decision to switch between constitutional and unconstitutional state the government compares the difference in expected revenue to the current realization of  $k$ .<sup>1</sup>

In our context, the constitution consists of a constitutional tax rate  $\tau^c$  and the rule of law. The decision to defect is taken at stage 0 when the government sets its actual tax rate. It stays within the constitution (i.e. moves to  $c_t^1$ ) if it selects the tax rate  $\tau = \tau^c$  (we assume that  $\tau^c$  is binding for any given expectation, i.e. the government always wants to tax as

<sup>1</sup>We treat  $k$  as a threshold value instead of a periodic utility. Alternatively one may think of a periodic benefit which is consumed immediately after the defection.



much as it can without changing the expectations of the citizens). The government violates the constitution by selecting a tax rate  $\tau$  in excess of  $\tau^c$ . In this case it moves to  $nc_t^1$ . The decision to violate is governed by the preference  $k_{t-1}$ .

In stage 1, before the government has a chance to reconsider its decision to violate the constitution it realizes a preference shock. Its current value  $k_t$  results from a stochastic process with  $k_t$  equally distributed over  $[k_{t-1} - \kappa, k_{t-1} + \kappa]$ . We assume that the preference shock occurs immediately after selecting the tax rate. Citizen  $i$  observes a noise signal  $x^i(k_t)$  which is equally distributed over  $[k_t - \varepsilon, k_t + \varepsilon]$ . Having updated their beliefs citizens decide how much taxes they declare.

In stage two, after having observed aggregate tax evasion,  $\theta$ , the government selects an appropriate detection probability  $r$  and a fine rate  $\Theta$  which support the tax policy and reconsiders its decision to violate the constitution.

If the government switches back to the constitution it has to give up revenue from fines and tax revenues as far as they result from taxing in excess of the constitutionally admissible tax rate. This means that the government which switches back always sets the detection effort equal to zero in order to save detection costs. The reverse is not necessarily true, i.e. a government which stubbornly continues outside the constitution might still decide that it is not worthwhile to pursue tax evaders. Once the government has switched back it is in the constitutional node  $c_{t+1}^0$ . For our two period exposition we assume that the government is committed to stick to the constitutional tax rate for one more period. For the infinite horizon extension of the model note that the next decision would be on a tax rate for  $t + 1$  with a preference value  $k_t$ . Clearly, given that the government was prepared to switch back

with  $k_t$  it also decides to stay in the constitution in the next round.

For the government which in stage 1 is in the constitutional state we assume that now is not the time to decide on its constitutional status. From a technical angle this makes sense because we had assumed that decisions on the tax rate are taken at stage 0, so there is the only place to violate the constitution. Materially, however, this assumption clearly matters: We effectively rule out that the government can wait until the citizens have confidently declared their taxes in order to subsequently renege on the tax rate. Such a tax would raise the same problem in our model as does a capital levy in models of capital taxation. With a limited time horizon the argument for the sustainability of the constitution would unravel, i.e. ex post the government always wants to impose a tax rate of 1. With an infinite planning horizon a tax rate below 1 can be shown to be sustainable if trust matters (see Chari and Kehoe, 1990). However, incorporating such an argument into our model would certainly overstretch the framework.

We proceed as follows: We consider the decision to switch back first. Imperfect observability of  $k$  on the side of the citizens gives rise to a global game. This fact enables us to derive a unique equilibrium value  $k_t^*$  such that the public forces the government to switch back if  $k_t < k_t^*$ . In a next step we show that for vanishing  $\varepsilon$  the game with prior  $k_{t-1}$  has the same equilibrium value as the game without prior. Finally we derive a critical value for  $k_{t-1}$  such that the government does not abandon the constitution if  $k_{t-1} < k_{t-1}^{**}$ . We can show that the switch back mechanism economically matters, i.e. that it increases the critical value: If the government realizes that it might be forced to switch back when it moves to the unconstitutional state  $nc_t^1$ , it is always more reluctant to do so, than if it could commit

to stay outside of the constitution forever.

Note, that the process of information revelation is crucial for the model. If the government would already know the precise value of its true character in  $nc_t^2$ , the citizens could infer the true character of the government from the tax rate selected. As is well known from the analysis of global games, making  $k$  common knowledge destroys the uniqueness result on  $k^*$ , but does not destroy the self-fulfilling property of beliefs. Furthermore, because in  $c_t^0$  the value of  $k_{t-1}$  is common knowledge, the government has no superior knowledge of its character, so we can ignore any aspects arising from active signalling behavior of the government.

For the tax rate determination stage  $nc_t^1$ , we assume that the government aims to maximize the expected revenue in the unconstitutional state. This implies that there is some myopia on the side of the government. Alternative assumptions would be that the government maximizes the probability of staying in the unconstitutional state or that the government maximizes its expected revenue over both states. As will become obvious, all formulations result in the same tax policy if the constitutional tax rate is 0. However, for the sake of simplicity we stick with the first goal and note, that this does not affect the basic results of our paper.

### III. THE BASIC MODEL OF TAX EVASION

In this section we set out the basic model of tax evasion which is a variant of Kolm (1973). We normalize gross income and population size to one, denominate  $y^i$  the income declared for tax purposes by citizen  $i$ ,  $r$  the probability of detection and  $\tau$  the tax rate. A fine is

levied at a linear rate  $\Theta$  on non declared income  $(1 - y^i)$ . It satisfies the (period by period) non-bankruptcy condition  $\Theta(1 - y^i) + \tau y^i \leq 1$ .<sup>2</sup>

### A. The Citizen's Problem

A citizen  $i$  has to decide on declaring or evading taxes before she knows precisely what kind of government she faces. Her expected (period by period) utility is linear in disposable income:

$$EU^i = 1 - ((1 - P^i)\tau + P^i\tau^c - \eta^0) y^i - (1 - P^i) r^E \Theta(1 - y^i) + \eta^i y^i. \quad (1)$$

$P^i$  is her subjective probability that the government does not enforce the policy  $(\tau, \Theta, r)$  and varies across citizens. Below, we are going to show how  $P^i$  depends on the observation of a signal of  $k_t$ . In the constitutional state,  $\tau \equiv \tau^c$  and the announced policy is actually carried out, so we have  $(1 - P^i) \equiv 1$ . In the unconstitutional state, the announced tax rate  $\tau$  differs from the constitutionally admissible tax rate  $\tau^c$  and the government may wish to switch back to the constitution. Here, the citizen's expected tax rate on declared income  $y^i$  is  $(1 - P^i)\tau + P^i\tau^c$ .  $(1 - P^i)r^E\Theta$  is the expected charge on non declared income  $(1 - y^i)$ .  $r^E$  is the detection probability which the citizen expects to face in the case where the government enforces its policy. The decision of the government on  $r$  depends entirely on the amount of tax evaders which it faces ex post. Here we assume that the citizen correctly anticipates the amount of tax evaders when he makes his own decision to evade.<sup>3</sup> As we are going to see, the value of  $\Theta$  is always 1.  $\eta^i$  is the citizen's preference for paying taxes. We assume

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<sup>2</sup>As all citizens - including the marginal citizen who does so by assumption - realise a corner solution, the restraint is, effectively,  $\Theta \leq 1$ .

<sup>3</sup>In part 1 of the appendix we show that this assumption is justified if the noise in the signal  $x^i$  vanishes.

that  $\eta^i \geq 0$  for all  $i$ . The empirical literature (see e.g. Slemrod, 1992) reports that people often declare taxes when it seems irrational in monetary terms to do so. Alternatively,  $\eta^i$  can be interpreted as an individual risk premium for the risky option tax evasion. Let  $G(i) = \int (l|\eta^k \geq \eta^i)dl$  be the share of citizens who are more risk averse than  $i$ . Utility is linear in  $y^i$  so the citizen's tax evasion decision takes him to a corner solution and we have the following

**Lemma 1** *Unless a citizen is indifferent to the tax evasion problem either she declares her whole income or nothing at all. Breaking indifference by assuming that she evades, a citizen evades taxes if*

$$\Phi^i \equiv \tau - r^E\Theta + \frac{P^i\tau^c - \eta^i}{(1 - P^i)} \geq 0. \quad (2)$$

**Proof.** Follows from (1) by linearity of  $U^i$  in  $y^i$ . ■

For given  $\tau^c$  and  $\tau$  the share of citizens evading taxes  $\theta \in [0, 1]$  depends on the expected detection probability  $r^E$ , the fine  $\Theta$  and their belief,  $P^i$ . (2) is equivalent to stating that the utility differential from evading all taxes instead of declaring his total income

$$u^i \equiv U^i(y^i = 0) - U^i(y^i = 1) = (1 - P^i)(\tau - Er\Theta) + P^i\tau^c - \eta^i \quad (3)$$

is positive. As  $P^i$  increases, the cost of paying taxes increases proportionately to  $\tau^c$ , the tax rate applying after the switch back. As  $P^i$  decreases, the cost of paying taxes increases by  $\tau - r^E\Theta$ , the price differential which applies if the policy is enforced. For  $P = 1$ , everybody evades taxes provided that assumption 1 holds for the most good willing (or risk averse) citizen:

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In this case he is only uncertain about whether the government is weak enough to give in.

**Assumption 1**  $\bar{\eta} < \tau^c$ .

This assumption seems well defensible: It claims that the constitutional tax is so high so that nobody voluntarily pays it in the absence of detection efforts. Note that, from our argument above,  $u^i$  is increasing in  $P^i$  only if

$$\tau - r^E \Theta < \tau^c. \quad (4)$$

The reason for this ambivalence is that the constitution not only promises to abandon the fine - which encourages tax evasion - but also to drop the unconstitutional share of taxes - which encourages tax declaration. Once it is accepted that unconstitutional taxes should be relieved under the constitution, tax evasion is sensitive to  $P$  only if the former effect outweighs the latter. As we are going to show, the government chooses its instruments such that (4) holds almost surely if the preference shock gets small (see corollary 1 below).

## B. The Government's Problem

In this section we deal with the government's problem of choosing its instruments  $(\tau, r, \Theta) \in [0, 1]^3$ . Formally we treat this choice problem as preceding the decision of whether or not to switch back, although we know that if the government actually switches back it might want to correct the detection probability chosen. From the timing of decisions outlined in figure 1, the government determines  $r$  and  $\Theta$  in the last stage and the tax rate  $\tau$  in the first stage. We solve the problem backwards, starting with the selection of a detection policy for a given tax policy. In that stage of the game, the government knows its own character  $k_t$  and the actual share of tax evaders,  $\theta$ .

There is a detection technology which is represented by the cost function  $C(r, \theta)$ . We assume that  $C$  is convex with  $C(0, 0) = 0$ ,  $C_r > 0$ ,  $C_{rr} > 0$ ,  $C_\theta > 0$ ,  $C_{\theta\theta} > 0$ ,  $C_{r\theta} > 0$ . The last assumption is necessary for the game to exhibit supermodularity: as more citizens evade taxes, the government's cost increases, making a switch back more likely.<sup>4</sup> Furthermore, we demand that  $C(., 1) > 1$ , i.e. if everybody evades taxes, detection becomes prohibitively costly.<sup>5</sup>

Formally, the problem in the last stage is to maximize revenue from taxing evaders at rate  $\Theta r$  net of detection costs. Tax payers are taxed at the predetermined rate  $\hat{\tau}$ :

$$V = \theta\Theta r + (1 - \theta)\hat{\tau} - C(r, \theta) \text{ s.t. } \theta = \hat{\theta}$$

For a given government income from fines it is always better to increase  $\Theta$  and reduce the (costly) detection effort so it is immediate that, in an optimum, the no bankruptcy condition for the citizen is binding, i.e.  $\Theta \equiv 1$ . This is in line with Kolm's (1973) polemic principle, which says that in an optimum tax evaders should be hung with a probability approaching zero.

Because  $C(., 1) > 1$  and  $C_{\theta\theta} > 0$  there is uniquely one level of tax evasion  $\bar{\theta}$ , where the government gives up detection efforts. The ex post optimal detection policy given the share of evaders is

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<sup>4</sup>Assumption 2 below establishes a sufficient condition.

<sup>5</sup>The assumption that the government cannot detect any tax evaders when confronted with complete disobedience is clearly restrictive, especially, when the government does not act under the constitution and is, therefore, free to adopt discriminatory measures. For our results it would be sufficient that it is prohibitively costly to extend detection beyond some critical value which is "not too far" above the detection level under the constitution.

$$C_r(r^*, \theta) \leq \theta \Rightarrow r^* = \text{Min}[C_r^{-1}(r^*, \theta), 1] \text{ for } \theta \leq \bar{\theta} \quad (5)$$

$$r = 0 \text{ else}$$

Note, that  $r$  is lower if it is chosen subsequently rather than simultaneously with  $\tau$ . The reason for this is that, when selected ex post,  $r$  does no longer contribute to deterring tax evaders.

The problem of selecting the tax rate is more complex, at least in the case where the government is in the unconstitutional state. In the constitutional state where the government has selected a tax rate of  $\tau = \tau^c$ , we have  $P = 0$  for all citizens. That is, the government always enforces its policy. Now suppose that the government picks  $\tau > \tau^c$  and enters the unconstitutional state. When selecting the tax rate the government does not know its future determination  $k_t$  to see it through, nor does it know the realization of tax evasion,  $\theta$ . As a first step however, assume that a certain realization of  $k_t$  has taken place. Now citizens make their decision on evasion knowing  $\tau$  and a private signal of the true character of the government,  $x^i(k_t)$ . Having received the signal, each citizen holds a certain belief  $P^i$  that the government is going to switch back. A citizen of type  $\eta^i$  evades taxes if  $\Phi^i(P^i, \tau - r^E, \tau^c, \eta^i) \geq 0$ . Therefore, the mass of tax evaders is

$$\theta(\tilde{P}, \tau - r^E, \tau^c | k_t) = \int_{i|\Phi^i \geq 0} di \quad (6)$$

where  $\tilde{P}$  is the distribution of beliefs  $P^i$  of the citizens. We assume that  $P^i$  and  $\eta^i$  are stochastically independent. From (2) it is immediate that  $\frac{\partial \theta}{\partial(\tau-r)} > 0$  and  $\frac{\partial \theta}{\partial P} > 0$ . We interpret  $\frac{\partial \theta}{\partial P}$  as the reaction to a decrease of trust. Formally,  $\partial \tilde{P}$  is a change from  $\tilde{P}^0$  to  $\tilde{P}^1$



where the cumulative distribution of  $\tilde{P}^0$  stochastically dominates the cumulative distribution of  $\tilde{P}^1$ .<sup>6</sup>

The relationship between  $\tau$  and  $r$  is rather subtle in this model. Selecting a tax rate such that  $\theta = 0$  would obviously result in a situation where  $r = 0$ , so the tax rate could be at most  $\tau = \underline{\eta}$ . Although this would be sustainable as a taxation-detection equilibrium, it can never be optimal to implement this policy because a slight increase would tax everybody and only induce a vanishing crowding out effect.

With heterogeneity and a given  $k_t$  there is a unique pair  $(\theta, r)$  for any tax rate selection  $\tau$  such that given  $\tilde{P}$  only individuals with  $\Phi^i < 0$  declare taxes.<sup>7</sup> The taxation-detection equilibrium is determined by (5) and the inverse function of the ex ante correspondence (6):

$$r = \theta^{-1}(\hat{\tau}, \tilde{P}, \tau^c)$$

where we have used the fact that citizens correctly anticipate the share of evaders and, therefore,  $r^E = r$ . This system can be uniquely solved<sup>8</sup> to yield

$$\frac{dr}{d\tau} = \frac{\theta_\tau^{-1}}{\frac{1}{C_{rr}} + \theta_\tau^{-1}} \text{ for } \theta \leq \bar{\theta} \quad (7)$$

Noting that  $\theta_\tau^{-1} > 0$  for  $\theta < 1$  we find that  $0 < \frac{dr}{d\tau} < 1$  for all efficient levels of taxation. For  $\theta > \bar{\theta}$ , we have  $\frac{dr}{d\tau} = 0$ . When the government gives up detection efforts at  $\theta = \bar{\theta}$  it

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<sup>6</sup>Let  $H(P^k) = \int (i|P^i \leq P^k) di$ . Stochastic dominance implies that  $H(P^k|\tilde{P}^0) \geq H(P^k|\tilde{P}^1)$  for all  $P^k$  or, equivalently, that  $\tilde{P}^1$  results from a flattening of  $\tilde{P}^0$ .

<sup>7</sup>In the absence of heterogeneity we would have to postulate that the private sector plays a mixed strategy in the case of certainty where  $P^i$  is 0 for everybody: for any tax rate, the private sector plays tax evasion with a probability  $\theta$  such that  $r(\theta(\tau - r^*)) = r^*$ . This equilibrium is, however, not very convincing in that the private sector is indifferent toward switching to any other mixed strategy  $\theta'$ .

<sup>8</sup>see part 2 of the appendix.

switches from a high detection probability to a zero detection probability which drives the share of tax evaders up to 1 with revenues falling from  $(1 - \bar{\theta})\tau$ .

In the first stage, the government does not know its character  $k_t$ . However, the distribution of beliefs,  $\tilde{P}$ , depends on the realization of  $k_t$ . Therefore, the government has to treat  $\tilde{P}(k_t)$  as a stochastic variable. We had assumed that the government sets the tax rate to maximize its revenue in the unconstitutional state, i.e. if it does not switch back.<sup>9</sup> Therefore, for its tax selection problem, the government takes expectations over all possible realizations of  $k_t \geq k_t^*$ , uses  $\theta = \theta(\tau, Er, \tilde{P}(k_t), \tau^c)$  and solves:

$$\underset{\tau}{Max} \quad EV = E[\theta r(\tau, \theta) + (1 - \theta)\tau - C(r(\tau, \theta), \theta)] \quad (8)$$

$$\text{s.t. } \tau \leq 1. \quad (9)$$

Define the Lagrangian  $L = EV + \lambda(1 - \tau - s)$ , for some  $s \geq 0$ . The FOC's are

$$\frac{\partial L}{\partial \tau} = E[(1 - \theta) - (\tau - r + C_\theta)(\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial r} \frac{dr}{d\tau}) + (\theta - C_r)\frac{dr}{d\tau}] - \lambda = 0 \quad (10)$$

$$\frac{\partial L}{\partial \lambda} = 1 - \tau - s = 0 \text{ and } s = 0, \lambda > 0 \text{ or } s > 0 \text{ and } \lambda = 0. \quad (11)$$

(11) are the Kuhn-Tucker conditions which say that either (a)  $\lambda = 0$  and  $\tau < 1$  or (b)  $\tau = 1$  and  $\lambda > 0$ . For  $k_t \geq k_t^*$  we either have  $\theta(k_t) - C_r(\theta(k_t)) = 0$  or  $dr = 0$ , so taking expectations the last term in (10) equals zero. Using  $E[\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial r} \frac{dr}{d\tau}] = E[\frac{\partial \theta}{\partial \tau}(1 - \frac{dr}{d\tau})] > 0$  in (10) we observe that the first term is positive and the second factor in the second term is positive so that for  $\lambda = 0$  the expression on the left-hand side assumes zero only if  $\tau - Er + C_\theta > 0$ .

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<sup>9</sup>This assumption basically spares us discussing another non continuity when solving the government's problem.

Note that for calculating  $Er$  we take expectations over all possible realizations  $r^E(\theta(k_t))$ .

Because the government never chooses  $\tau$  such that the expected value of tax evaders is  $E\theta = 1$ , we know that<sup>10</sup>

$$\tau - Er < \bar{\eta}. \quad (12)$$

Together with assumption 1 this yields  $\tau - Er < \tau^c$ . In view of condition (4) which has to be fulfilled for citizens to respond positively to a change in  $P$  we find:

**Corollary 1** *The government sets  $\tau$  such that it fulfills  $\tau - Er < \tau^c$  ex ante. For  $\kappa \rightarrow 0$  the government picks  $\tau$  such that  $\tau - r^E < \tau^c$  is almost always true ex post.*

This means that at least in the case where the preference shock vanishes citizens are almost surely able to successfully coordinate their evasion strategies.

(10) yields

$$\tau = \frac{(1 - E\theta)}{E \frac{\partial \theta}{\partial \tau} (1 - \frac{dr}{d\tau})} + (Er - EC_\theta) - \lambda \quad (13)$$

If the restraint  $\tau \leq 1$  is not binding ( $\lambda = 0$ ) and increasing taxes increases expected receipts by  $(1 - E\theta)$  but also reduces government income due to an increase in tax evaders. A higher expected detection rate reduces tax evasion and allows to levy a higher tax rate. Finally, adverse effects of higher taxes run via increasing tax evasion and thereby detection

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<sup>10</sup>Note that ex post it must always be true that  $\tau - r^E(\theta(k_t)) < \bar{\eta}$  for all  $k_t$ . Suppose that not. Then there is one  $\hat{k}$  such that  $\theta = 1$  for  $k_t < \hat{k}$ . Now let  $k_{t-1}$  assume a high enough value such that the government knows that the private sector is going to end up holding beliefs of  $P^i = 0$  for all  $i$ . In that case,  $\tau - r^E(\theta(k_t)) < \bar{\eta}$  because otherwise  $\theta = 1$  for all realisations of  $k_t$ . Now for all smaller priors  $k_{t-1}$  the belief of the public in the switch back is increased, i.e.  $d\tilde{P} > 0$ . But because  $\frac{\partial \theta}{\partial P} > 0$  the condition  $\tau - r^E(\theta(k_t)) < \bar{\eta}$  must also be fulfilled for these priors. Finally, integrating over all possible  $k$ 's yields (12).

costs. Observe, that for  $\theta = 0$  we would have  $r \rightarrow 0$  (from (5)) and, therefore,  $dr/d\tau \rightarrow 1$  (from (7)), so  $\tau$  would be 1, defeating any equilibrium of that kind.

#### IV. MULTIPLE EQUILIBRIA UNDER COMMON KNOWLEDGE

Now assume that we are in the unconstitutional state and the parameter  $k_t$  can be perfectly observed by the citizens. We construct a Nash equilibrium in the following way: Given the strategies of the other citizens and the government, no citizen wishes to change his own strategies. Furthermore, given the strategies of the citizen, the government wishes to carry out its policy. It is immediate from the definition of an equilibrium that all citizens follow the same strategy (i.e. evade or not evade). We are showing that if  $\tau^c$  is sufficiently small, there is a range  $k_t \in (k^L, k^H)$  where either all citizens evade taxes and the government switches back or all citizens pay taxes and the government stays in the unconstitutional state.

Consider the best government policy if  $P = 1$  for all citizens. From (2) and because  $\bar{\eta} < \tau^c$  from assumption 1 we have  $\theta = 1$  (5). Because  $C(., 1) > 1$  the government does not undertake any detection effort. Resulting revenue  $V(1) = 0$ .

Now consider the case  $P = 0$  for all citizens. Solving the government's problem backwards yields  $C_r(r, \underline{\theta}) = \underline{\theta}$ . As we have seen, having no tax evaders cannot be an equilibrium, so  $\underline{\theta} > 0$ . Because  $\tau = r(\underline{\theta}) + \underline{\eta}$  would result in a mass of tax evaders of zero, it must be that the government sets

$$\tau^0 > r(\underline{\theta}) + \underline{\eta}. \quad (14)$$

Because enforcement is actually carried out (i.e.  $\theta \leq \bar{\theta}$ ) resulting revenues are  $V(0) =$

$(1 - \underline{\theta})\tau^0 + \underline{\theta}r(\theta^0) - C(r(\underline{\theta}), \underline{\theta}) > 0$  so  $V(0) > V(1)$  is true. Furthermore  $\tau^0 > \tau^c$  because we had assumed that  $\tau^c$  is actually binding.

Now consider the decision of the government to switch back. The pay offs in the case where the government switches back is  $T_t^{nc,c}(P) = \tau^c\theta$  in the current period because the government sacrifices the unconstitutional part of the tax and does not undertake detection efforts. In the second period, income is  $V^c$ . If the government stays outside of the constitution, it has the unconstitutional income which is either  $V(P = 1)$  or  $V(P = 0)$ . The government switches back if

$$\Delta = V_{t+1}^c + T_t^{nc,c}(P) - V_t^{nc}(P) - V_{t+1}^{nc}(P) - k_t > 0. \quad (15)$$

Now, there is some  $k^L$  such the government even switches back if  $P = 0$  provided  $k_t < k^L$  where

$$k^L = V^c + T^{nc,c}(P = 0) - 2V^{nc}(P = 0) \quad (16)$$

Furthermore, there is  $k^H$  such that the government does not even switch back if  $P = 1$  as long as  $k_t \geq k^H$  and

$$k^H = V^c + T^{nc,c}(P = 1) - 2V^{nc}(P = 1) \quad (17)$$

From (17) and (16), there are parameter ranges where the government's strategy does not depend on the expectations of the citizens. Finally we have to show that  $k^L < k^H$ :

**Lemma 2** *There is  $\tau^c < \bar{\tau}^{c,H}$  such that under certainty the government does not switch back if  $P = 1$  and  $k_t \geq k^H$  where  $k^H > k^L$ .*

**Proof.** Using (16) and (17),  $k^H > k^L$  is  $T^{nc,c}(P = 0) - T^{nc,c}(P = 1) < 2(V^{nc}(P = 0) - V^{nc}(P = 1))$ .

Using  $T^{nc,c}(P = 1) = 0$ ,  $T^{nc,c}(P = 0) = \tau^c$  this implies that  $\frac{1}{2}\tau^{c,H} = V^{nc}(P = 0) - V^{nc}(P = 1)$ . ■

It might be surprising that lemma (2) places a maximum condition on the constitutional tax rate. Note, however that we are not interested in, say the maximum value of  $k_t$  which is still compatible with a switch back if  $P = 1$ , but only in the effect of tax evasion on the values of  $k_t$  which might be compatible with a switch back if  $P$  switches from 0 to 1. If the constitutional tax rate is rather high (i.e. only weakly binding), the government is rather easily convinced to switch back both for  $P = 1$  and  $P = 0$ . However, tax evasion might even be counterproductive in the sense that the government is more easily persuaded to switch back where it can keep most of its income (i.e. where  $(1 - \theta)$  is high and taxed at  $\tau^c$ ).

In the case where  $k_t \in (k^L, k^H)$  beliefs of the citizens are self-fulfilling and we have multiple equilibria:

**Proposition 1** *In the case of certainty, if  $\tau^c < \bar{\tau}^{c,H}$  and  $k_t \in (k^L, k^H)$ , there are two equilibria: one, where no-one declares income for tax purposes and one, where everybody declares income for tax purposes.*

**Proof.** Lemma 2 and the definitions of (17) and (16) imply that the government does not switch back if  $P = 0$  and  $k \geq k^L$  and it switches back if  $P = 1$  and  $k < k^H$ . Now citizens' strategies are  $\theta = 1$  if  $P = 1$  and  $\theta = 0$  if  $P = 0$ .

## V. THE SWITCH BACK CONDITION UNDER INCOMPLETE INFORMATION

Because multiplicity of equilibria is unsatisfying we are going to introduce uncertainty of the private agents over the fundamental  $k_t$  when they decide on tax evasion. This enables us to apply the results of the literature on global games to our game. In order to have a global game we need a pair  $\underline{k}, \bar{k}$  where there is a dominant strategy if citizens know that either  $k < \underline{k}$  or  $k > \bar{k}$  and strategies of the citizens have to be strategic complements, i.e. if more citizens evade taxes there are incentives for everybody else to evade taxes.

There are two problem which prevent us from directly using the results established so far to conclude that we have a global game.

First, proposition 1 cannot be directly translated: The government now sets its tax rate under uncertainty. Therefore, the tax rate is ex post wrong in the case where citizens agree that  $k_t$  is in the dominance range with  $P = 0$ . Therefore,  $V^{nc}(\hat{\tau}) < V^{nc}(\tau(P = 0))$  and the government has to be relatively more stubborn to stay in the unconstitutional state, so  $\underline{k} > k^L$ . In the case where  $k_t$  is in the dominance range where everybody believes in a switch back, the tax rate does not matter because  $\theta = 1$ . Therefore,  $\bar{k} = k^H$ . It is straightforward to see that the strategies which citizens play under certainty are dominant for  $k < \underline{k}$  and for  $k > \bar{k} = k^H$ .

Secondly, although we know that a sufficient share of tax evaders forces the government into a switch back, strategies might not be strategic complements throughout the range of unstable  $ks$ .

If the criterion  $\Delta$  increases in  $\theta$ , both problems are solved given that agents behave regularly (i.e. (4) is also fulfilled). Rewriting (15) using  $T^{nc,c}(P) = \tau^c(1 - \theta(P, \hat{\tau}, r))$  and

$V^{nc}(P) = \tau(1 - \theta(P, \hat{\tau}, r)) + r\theta(P, \hat{\tau}, r) - C(r, \theta(P, \hat{\tau}, r))$  we can immediately see that a sufficient condition for

$$\frac{\partial \Delta}{\partial \theta} > 0 \text{ for all } \theta \quad (18)$$

to be true is that the following assumption holds:

**Assumption 2**  $C(r, \theta)$  is sufficiently steep to satisfy

$$2(\hat{\tau} - r + C_\theta) > \tau^c \text{ for all } \theta(\hat{\tau}, r, \tilde{P}, \tau^c) \quad (19)$$

This condition is somewhat awkward because it is not a condition on the primitives of the model. However, from (13) we know that ex ante the condition  $\tau - Er + EC_\theta + \lambda = \frac{(1-E\theta)}{E\frac{\partial \theta}{\partial \tau}(1-\frac{dr}{d\tau})}$  holds. For  $C_{rr} \rightarrow \infty$  we have  $\frac{dr}{d\tau} \rightarrow 1$  so that  $\tau - Er + EC_\theta + \lambda$  is arbitrarily large. Therefore, there is always a cost function  $C$  which satisfies (19).

Assumption 2 is also fulfilled if  $\tau^c$  is sufficiently low. This condition is reminiscent of the fact that a lower  $\tau^c$  makes the switch back income less dependent on tax evasion. Tax evasion may actually frustrate the government from switching back, because a higher evasion means a loss in switch back income. This argument is the same as the one discussed in connection with lemma 2.

**Corollary 2** *Given that assumption 2 is fulfilled, we have  $\bar{k} > \underline{k}$  and tax avoidance strategies are strategic complements.*

**Proof.** The first part is immediate from the definitions (16) and (17) and from (15) in connection with (18). As, from (18) it is clear that the switch back becomes more likely with more evaders, strategic complementarity follows trivially with (4). ■



## VI. THE GAME WITH IMPERFECT INFORMATION OF $k$

In this part of the paper we derive a unique equilibrium for the game between the government and the citizens, using the results on global games by Carlsson/van Damme (1983), Morris/Shin (1998, 1999, 2000) and Frankel/Morris and Pauzner (2002). These papers show that multiplicity of equilibria is removed if the assumption of common knowledge is abandoned and replaced by the assumption, that agents observe - and know that the other agents observe - a fuzzy signal of the true state of the system. Our model incorporates imperfect observability of  $k_t$  on the side of the citizens. Corollary 2 shows that there are dominance areas - i.e. citizens have a dominating strategy if they know for sure that  $k_t < \underline{k}$  and  $k_t > \bar{k}$ . As the corollary also shows that tax avoidance strategies are complements in the unstable region of  $k_t$ , this can be used to iteratively eliminate dominated strategies moving from  $\underline{k}$  upwards and from  $\bar{k}$  downwards.

From (18) and because  $\frac{\partial \Delta}{\partial k_t} < 0$  there exists a strictly increasing function  $\phi(k_t)$  for  $k_t \in (\underline{k}, \bar{k})$  where upon increasing  $k_t$ ,  $\Delta$  switches from a positive to a negative value for a given share of evaders  $\theta$ . Let  $\phi(k_t)$  be the critical mass of evaders  $\theta$ . From the definition of  $\underline{k}$  and  $\bar{k}$  we know that  $\phi(\bar{k}) \geq \bar{\theta}$  and  $\phi(\underline{k}) \leq \underline{\theta}$ .

We assume that the signal  $x^i$  of  $k_t$  which citizen  $i$  receives is equally distributed over  $(k_t - \varepsilon, k_t + \varepsilon)$ . Because our argument builds upon the iterative elimination of dominated strategies as laid out in the appendix we know that strategies for agent  $i$  take the form: evade taxes if  $x^i \leq \xi^i$  for some cut off point  $\xi^i$ . This is, citizens evade taxes if they observe a government which is weaker than some threshold.<sup>11</sup> For the moment, assume that the

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<sup>11</sup>That strategies take this form emerges endogeneously in this model when we iterate from the boundaries.

distribution of cut off points is exogenously given. They are distributed according to  $\tilde{\xi}$  with  $f(\xi) : \xi(i) \rightarrow \mathfrak{R}^+$ . If  $\theta$  is the true state, then the probability that  $x \leq \xi$  is given by

$$W(\tilde{\xi}|k_t) = \int_{x=k_t-\varepsilon}^{x=k_t+\varepsilon} \frac{1}{2\varepsilon} \int_{\xi=x}^{\xi=\infty} f(\xi) d\xi dx. \quad (20)$$

$W(\tilde{\xi}|k_t)$  is the share of citizens who have received a signal falling short of their individual cut off point  $\xi$  given that  $\xi$  is distributed according to  $f$ . The term on the r.h.s. gives the probability that  $\xi$  is higher than the signal in the interval  $[\theta - \varepsilon, \theta + \varepsilon]$ . Now, if the true state is  $k_t$ , then the government defects if  $W(\tilde{\xi}|k_t) \geq \phi(k_t)$  with probability one. The maximum  $k_t$  for which the government switches back is uniquely<sup>12</sup> given by

$$\hat{k}_t = \text{Max}\{k_t | W(\tilde{\xi}|k_t) \geq \phi(k_t)\}. \quad (21)$$

Now, the probability which an agent who receives the message  $x^i$  assigns to the event that the government defects is

$$\psi^i(W(k_t, \tilde{\xi}^{-i}), \phi(\theta) | x^i) = \int_{x^i-\varepsilon}^{\hat{k}_t} \frac{1}{2\varepsilon} dk_t \quad (22)$$

where  $\frac{1}{2\varepsilon}$  is the density of the distribution of  $k_t$  and  $\tilde{\xi}^{-i}$  is the distribution of  $\xi$  without the agent (which coincides with  $\tilde{\xi}$  because the agent is atomic).  $\psi$  is attained by integrating over all  $k_t$  which are in accordance with a defection of the government and sets them in relation to all  $k_t$  which are possible from the observation (which has the measure 1). Let  $\xi^i$  be the highest signal  $x^i$  which elicits the reaction of a citizen, i.e. for which  $\psi^i(W(k_t, \tilde{\xi}^{-i}), \phi(k_t) | x^i)$  is satisfied (2) as an equality:

$$\xi^i = \text{Max}\{x | \int_{x-\varepsilon}^{\hat{k}_t} \frac{1}{2\varepsilon} dk_t \geq P^{i*}\} \quad (23)$$

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<sup>12</sup>See part 3 of the appendix

where  $P^{i*}$  is the critical value in (3). A immediate consequence of (23) is that  $\xi^i \sim i$  (or  $\xi^i \sim 1/\eta^i$ ).

In the appendix we show that the following proposition holds:

**Proposition 2** *In the case of uncertainty over  $k_t$  there is a unique equilibrium point  $k_t^*$  supported by a distribution of  $\tilde{\xi}$  such that  $\underline{k} < k_t^* < \bar{k}$  and the government switches back if  $k_t \leq k_t^*$ .*

**Proof.** See appendix ■

## VII. THE DECISION TO DEFECT FROM THE CONSTITUTION

Up to this point we have considered the case of a government which has already defected from the constitution. We have shown that depending on the realization of  $k_t$ , the government is forced to switch back by the reaction of the citizens. What we want to show now is that the switch back mechanism is capable of deterring the government from defecting at the outset.

For this we model the stage in  $c_t^0$  in figure 1 when the government is in the constitutional state. The preference of the government at this stage is  $k_{t-1}$  which is common knowledge. This is important, because in this case the decision of the government on whether to defect or not does not reveal anything to the citizen and is not subject to strategic considerations.

Recall that  $k_t$  is equally distributed over  $(k_{t-1} - \kappa, k_{t-1} + \kappa)$  and let the form of this process be common knowledge. Furthermore assume that the error term in the signal is vanishing and that any prior which is capable of generating the original equilibrium point  $k_t^*$  is contained in  $(\underline{k}, \bar{k})$ . For that case we show in the appendix:

**Lemma 3** Consider the game with a prior of  $k_t, k_{t-1}$ . Let  $\varepsilon \rightarrow 0$  and  $(k_t^* - \kappa, k_t^* + \kappa) \subset (\underline{k}, \bar{k})$ .

Then the equilibrium point in the game with a prior,  $k_t^{*'}$  and the equilibrium point in the game without a prior,  $k_t^*$ , coincide.

The government has to decide on whether it defects from the constitution, in which case it is in the unconstitutional stage in period  $t$ , or whether it stays with the constitution, in which case it receives an income of  $V^c$  in period  $t$ . Our only restriction on the timing is that the government can choose the constitution only by announcing a constitutional tax rate - in which case it is restricted to carry out taxation accordingly. For the decision in period  $t - 1$  this means that it may as well defect one period later. Let  $w$  be the density function of  $k$  conditional on the observation  $k_{t-1}$ . The decision criterion becomes

$$V^c \geq \int_{k_t < k_t^{*'}} w(k_t | k_{t-1}) \tau^c \theta(\tilde{P}(k_t)) dk_t + \int_{k_t \geq k_t^{*'}} w(k_t | k_{t-1}) V^{nc}(\tilde{P}(k_t)) dk_t + k_{t-1}. \quad (24)$$

The left hand side of this inequality is the expected pay off for the government in  $t$  if it chooses the unconstitutional state. The right hand side gives the expected utility from violating the constitution. In the case where  $k_t < k_t^{*'}$ , the government switches back and receives  $T^{nc,c}$ . In the other case, it stays in the unconstitutional state.  $w$  is the conditional distribution of  $k_t$  after the observation of  $k_{t-1}$ . For any  $k_t$  the state of the government induces a probability distribution on the side of the citizens,  $\tilde{P}(k_t)$ . Because citizens only receive a signal  $x \sim (k_t - \varepsilon, k_t + \varepsilon)$ , some citizens (but at most  $\phi(k_t^{*'})$ ) do (erroneously) evade taxes even if  $k_t \geq k_t^{*'}$

**Lemma 4** There is a unique switching point  $k_{t-1}^{**}$  such that the government does not violate the constitution if  $k_{t-1} < k_{t-1}^{**}$

**Proof.** Note that  $V^c < V^{nc}(\tilde{P}(k_t + \kappa)) = V^{nc}(P = 0)$  and that  $\tau^c \theta(\tilde{P}(k_t)) < V^c$ . Furthermore,  $\int_{k_t < k_t^*} w(k_t | k_{t-1}) dk_t = \frac{k_t^* - k_{t-1} + \kappa}{2\kappa}$  decreases from 1 to 0 as  $k_{t-1}$  goes from  $k^* - \kappa$  to  $k^* + \kappa$ . The r.h.s. of (24) increases monotonically as  $k_{t-1}$  increases. ■

Now it is straightforward to show that the switch back mechanism economically matters. Observe, that without the switch back mechanism, the condition for the government to defect from the constitution would read

$$V^c \geq V^{nc}(P = 0) + k_{t-1}. \quad (25)$$

Let  $k_{t-1}^0$  be the value for which (25) is binding. In order to evaluate (24), note that  $V^{nc}(\tilde{P}(k_t)) \leq V^{nc}(P = 0)$  for  $k_t \geq k_t^*$ . Now it is immediate that (25) is less binding than (24) and that  $k_{t-1}^{**} > k_{t-1}^0$  and we have the following proposition:

**Proposition 3** *For  $k_t^* \geq k_{t-1}^0 - \kappa$  the switch back mechanism is substantial: The critical value from which on the government defects without the switch back mechanism,  $k_{t-1}^0$ , is smaller than  $k_{t-1}^{**}$ .*

**Proof.** For any  $k_{t-1} \in (k_t^* - \kappa, k_t^* + \kappa)$  there is some probability that a switch back takes place which tends to make the r.h.s. of (24) smaller than (25), so the critical value for  $k_{t-1}$  increases. For  $k_{t-1}$  greater than  $k_t^* + \kappa$  the probability of a switch back would be 1. ■

It is easy to see that  $k_{t-1}^0 \in (\underline{k}, \bar{k})$ . Our model is too general, however, to relate  $k^*$  in an unambiguous way to  $k^0 - \kappa$ . That, in turn, would be necessary to relate  $k^0$  to  $k^{**}$ . There might be cases where the punishment after the defection is sufficiently weak so nothing follows from the rule of law. But as we have shown in Neumaerker/Pech, 2001 for a case in which expectations are Laplace some  $k^* > 0$  can be supported whereas  $k^0$  is always smaller

than zero. It is also worth noting that there is always an effect on outcomes if  $\kappa$  is sufficiently large.<sup>13</sup>

### VIII. EXTENSION TO THE INFINITE HORIZON CASE

We can now extend our results to an infinite horizon game. All we have to do is to endogenize the decisions in the nodes at  $t+1$  and note that the problem is stationary. In order to proceed we have to agree, however, that the government always loses the illegal fines and revenue of the last year whenever it decides to switch back. A stricter condition would make it increasingly difficult for a government ever to switch back after a couple of periods have elapsed outside of the constitution. Consider figure 1 again. As the government moves to stage 0 in  $t+1$  the prior  $k_t$  becomes common knowledge. Suppose government has to decide over a tax rate in  $c_{t+1}^0$ . First assume that  $k^* \leq k^{**}$  in which case a government which has switched back wants to stay with the constitution in  $t+1$ . This is the configuration shown in the upper part of figure 2. If  $k^* > k^0 - \kappa$ ,  $k^{**}$  undoubtedly exceeds  $k^0$ , so the intertemporal constraint is operative. Now let  $k^*$  increase. This gives rise to the case where  $k^* > k^{**}$  as shown in the lower part of figure 2. For  $k^*$  to be greater than  $k^{**}$  it is necessary, however, that  $k^{**} > k^0$ . This is because by proposition 3,  $k^{**} = k^0$  only if  $k^* \leq k^0 - \kappa$ . Now there are cases where the government switches back only to return to the unconstitutional state again. Obviously this cannot be an equilibrium at the stage where the switch back decision is taken. For  $k^* > k^{**}$ , the switch is not credible which means that  $k^{**}$  effectively constrains  $k^*$  in the infinite horizon case.

Therefore, we can restrict attention to cases where  $k^* \leq k^{**}$ . Here, any situation in

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<sup>13</sup>This is, provided that the condition in corollary 1 is also fulfilled ex post.

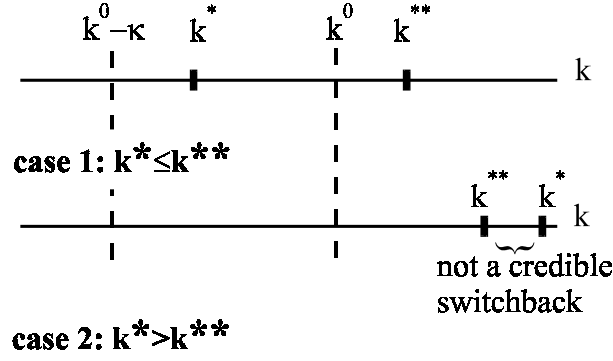


Figure 2: Ex ante and ex post switching points in the infinite horizon case

$c_{t+1}^0$  with  $k_{t-1} < k^{**}$  is stable. Suppose the government is already in  $nc_{t+1}^0$ , i.e. it has not switched back. Here,  $k_t$  is greater than the critical value  $k^*$  and the government picks the unconstitutional tax again. It is striking that there are situations with  $k_t \in (k^*, k^{**})$  where the government has not switched back although it would want to stay in the constitutional branch had it not violated before. This might even include cases ( $k^* < k^0$ ) where the government would actually stay with the constitution under the orthodox rule (these cases, however, vanish if  $\kappa$  gets small).

Legality seems to impose a cost at least in the latter case and it seems to unduly prevent the government from returning to the legal order in the other case. On second thought, however, this inflexibility of legality is a blessing. The critical element why the rule of law protects the constitution in the infinite horizon case is that the government can never expect to change from the unconstitutional to the constitutional branch of the game tree without a sacrifice.<sup>14</sup> If it could slip back into the constitutional state just by lowering the tax rate

<sup>14</sup>The point is that a flawed law cannot be corrected without consequence, but it has to be annulled.

This in turn has the consequence that once the government has moved to  $nc_{t+1}^0$  it can only return to the constitutional state by granting an amnesty for tax evaders.

while going on to collect last year's fines, the legality mechanism would be defeated. It is, in the end, the cost imposed by its inability to renegotiate easily on its constitutional status which deters the government from defecting from the constitution in the first place.

## IX. APPENDIX

### A. Appendix 1

In this part of the appendix we show that the citizens correctly forecast the share of tax evaders if the noise in the observation vanishes. From (20) we know that the share of agents who receive a signal short of their threshold or - equivalently - the amount of evaders is  $W(\tilde{\xi}|k) = \int_{x=k-\varepsilon}^{x=k+\varepsilon} \frac{1}{2\varepsilon} \int_{\xi=x}^{\xi=\infty} f(\xi) d\xi dx$  if the true value is  $k$ . Now  $\Omega(\tilde{\xi}|x) = \int_{k=x-\varepsilon}^{k=x+\varepsilon} \frac{1}{2\varepsilon} W(\tilde{\xi}|k) dk$  is the expected share of tax evaders if the observation is  $x$ . Taking the limit for vanishing  $\varepsilon$  gives  $\lim_{\varepsilon \rightarrow 0} \Omega(\tilde{\xi}|x) = W(\tilde{\xi}|k)$ .

### B. Appendix 2

In this appendix we do comparative statics involving the government's instruments. By (5) and (6), the system determining the detection effort for a given  $k_t$  is

$$r = C_r^{-1}(\theta)$$

$$r = \theta^{-1}(\tau, \tilde{P}, \tau^c)$$

The correspondence  $r = C_r^{-1}(\theta)$  has  $r(\theta = 0) = 0$  and is increasing in  $\theta$  and the correspondence  $r = \theta^{-1}(\tau, \tilde{P}, \tau^c)$  is decreasing in  $\theta$ . Consider  $\tau = 0$  and  $\theta^{-1}(0, \tilde{P}, \tau^c)$ . For  $P \equiv 0$  all  $i$  we have  $\theta^{-1} \leq \underline{\eta}$  and  $r(\theta) = 0$ . But because  $\theta^{-1}$  shifts upwards with an increase in



$\tau$ , there is some  $\underline{\tau}$  such that  $\theta^{-1}(\underline{\tau}, 0, \tau^c) = 0 = r(\theta)$ . Therefore, in the absence of further restrictions, for tax rates  $\tau > \underline{\tau}$ . As pointed out in the text, taxes are never lower because this would be inefficient.

Levels of taxation which are too high may yield  $\theta > \bar{\theta}$  so  $r$  is not enforced. Therefore we claim in the text that the derivative exists for all efficient levels of taxation.

In order to derive (7) in the text we totally differentiate the system with respect to  $r$ :

$$C_{rr}^{-1}(\theta) - \theta_{\tau}^{-1} \frac{d\tau}{dr} - \theta_r^{-1} = 0.$$

Using  $\theta_{\tau}^{-1} = -\theta_r^{-1}$  and rearranging we get (7).

In order to get  $\frac{d\tau}{d\tilde{P}}$  we totally differentiate with respect to  $\tilde{P}$ :

$$C_{r\theta}^{-1} \theta_{\tilde{P}} - \theta_{\tau}^{-1} \frac{d\tau}{d\tilde{P}} - \theta_{\tilde{P}}^{-1} = 0$$

where  $\theta_{\tilde{P}} > 0$  and  $\theta_{\tilde{P}}^{-1} > 0$  and  $C_{r\theta}^{-1} > 0$ . Using  $C_{r\theta}^{-1} \theta_{\tilde{P}} = C_{rr}^{-1} \theta_{\tilde{P}}^{-1}$  and rearranging gives

$$\frac{d\tau}{d\tilde{P}} = \frac{(C_{rr}^{-1} - 1) \theta_{\tilde{P}}^{-1}}{\theta_{\tau}^{-1}} < 0 \text{ if } C_{rr}^{-1} < 1.$$

### C. Appendix 3

Uniqueness of  $\hat{k}$  in (21) can be established as follows:  $W$  decreases monotonically in  $k_t$  for any given  $\tilde{\xi}$  and  $\phi$  strictly increases in  $k_t$ . If  $\xi(i) \in [\underline{k} - \varepsilon, \bar{k} + \varepsilon]$  (which we establish in the next section) then  $W(\tilde{\xi}|\underline{k}) \geq \phi(\underline{k})$  and  $W(\tilde{\xi}|\bar{k}) \leq \phi(\bar{k})$  and there exists a unique point  $\hat{k}_t \in [\underline{k}, \bar{k}]$  which satisfies

$$\hat{k}_t = W^{-1}(\phi(\hat{k}_t)|\tilde{\xi}). \quad (26)$$

#### D. Appendix 4

In this appendix we show that iterative elimination of dominated strategies yields an unique equilibrium point  $k^*$ . Before we do this define the cumulative distribution function  $F(\xi)$ .

Then an equivalent representation of (20) is

$$W(\tilde{\xi}|k) = \int_{x=k-\varepsilon}^{x=k+\varepsilon} \frac{1}{2\varepsilon}(1 - F(x))dx. \quad (27)$$

Because  $(1 - F(x))$  gives  $\Pr(\xi \geq x|x)$  it is immediate, that in the case of homogenous agents where all  $\xi$  coincide in equilibrium, the term in parentheses simply equals 1. As the distribution  $G(i)$  is continuous, the distribution of critical values,  $P^*(i)$  is continuous as well and  $F(\xi)$  inherits continuity from  $P^*(i)$ .

First, rule out that a positive measure of agents do not attack when receiving a message  $x^i = \underline{k} - \varepsilon$ . Consider agent  $i$  and assume that for all other agents,  $\xi_0^{-i(b)} < \underline{k} - \varepsilon$  where the index  $(b)$  indicates that we iterate from below. Then,  $\Gamma(\tilde{\xi}^{-i}|k) = 0$  for  $k \leq \underline{k}$  and  $\phi(k) = 0$  for  $k \leq \underline{k}$ . Therefore, the probability of a successful attack is  $P^i = 1$  and all agents attack when they receive the message  $x^i = \underline{k} - \varepsilon$ . Let  $\xi_{t+1}^{i(b)}$  be the solution to

$$\xi_{t+1}^{i(b)} = \text{Min}\{x | \int_{x-\varepsilon}^{k_t^{(b)}} \frac{1}{2\varepsilon} dk = P^{i*}\}$$

where  $k_t^{(b)} = \Gamma^{-1}(\phi(k_t^{(b)})|\xi_t^{(b)})$ . As we have seen,  $k_0^{(b)} = \underline{k}$ .

We have an increasing sequence with  $\xi_{t+1}^{i(b)} \geq \xi_t^{i(b)}$  for all  $i$  implying that  $F(\xi_{t+1}^{(b)}) \leq F(\xi_t^{(b)})$  for all  $\xi_t^{(b)}$ . Therefore,  $W$  increases weakly and  $k_{t+1}^{(b)} \geq \underline{k}_t^{(b)}$ . This in turn determines  $\xi_{t+2}^{(b)}$ .

We define the equilibrium point  $k^*$  which simultaneously solves (6) for  $k^*$  and equations of the form  $\xi^{i*} = \text{Min}\{x | \int_{x-\varepsilon}^{k^*} \frac{1}{2\varepsilon} dk = P^{i*}$  for all  $i$ .<sup>15</sup> It is immediate that the relationship in

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<sup>15</sup>Pech (2002) shows that this equilibrium point is, indeed, unique.

each sequence is strict for  $k_t^{(b)} < k^*$  and we have  $\lim_{t \rightarrow \infty} k_t^{(b)} = k^*$ .

Similarly, there is a decreasing sequence  $\xi_{t+1}^{i(a)} \leq \xi_t^{i(a)}$  for all  $i$  with (a) indicating that we iterate from above. This sequence starts with  $\xi_0^{-i(a)} > \bar{k} + \varepsilon$ , satisfies

$$\xi_{t+1}^{i(a)} = \text{Max}\{x \mid \int_{x-\varepsilon}^{k_t^{(a)}} \frac{1}{2\varepsilon} dk = P^{i*}\}$$

and is strictly decreasing for  $k_t^{(a)} > k^*$ .

### E. Appendix 3

In this appendix, we show how a prior  $k_{t-1}$  affects the equilibrium value  $k^*$  at the next stage.

(a) If  $x^0 \in (k_{t-1} - \kappa + \varepsilon, k_{t-1} + \kappa - \varepsilon)$  we have  $w(k|x^0, k_{t-1}) = w(k|x^0) = \frac{1}{2\varepsilon}$  for  $k \in (x^0 - \varepsilon, x^0 + \varepsilon)$ . In this case, set  $X = [x^0 - \varepsilon, x^0 + \varepsilon]$  of  $k_t$  which is possible from the observation  $x^0$  is covered by the set  $K = [k_{t-1} - \kappa, k_{t-1} + \kappa]$  of  $k_t$  which is possible from the prior  $k_{t-1}$ , so the prior does not affect the probability distribution over  $k$ .

(b) If  $x^i \in (k_{t-1} - \kappa, k_{t-1} - \kappa + \varepsilon)$  we have  $w(k|x^i, k_{t-1}) = \frac{1}{2\varepsilon - D}$  for  $k \in (\bar{x} - (\varepsilon - D/2), \bar{x} + (\varepsilon - D/2))$ . In this case it is clear that  $k_t$  is in the intersection  $X \cap K$ . The length of the interval for the posterior distribution of  $k_t$  is shortened by the absolute difference  $D = |X| - |X \cap K|$  where  $D = |k_{t-1} - x^i + \varepsilon - \kappa|$ . The updated expected value for  $k_t$  is  $\bar{x} = \frac{k_{t-1} - \kappa + x^i + \varepsilon}{2}$ . (22) becomes

$$\psi^i(W(k, \tilde{\xi}^{-i}), \phi(k)|x^i, k_{t-1}) = \int_{\bar{x} - (\varepsilon - D/2)}^{\hat{k}} \frac{1}{2\varepsilon - D} dk.$$

Consider the case with homogeneous agents, i.e. where  $\xi^i = \xi^{-i}$ . Using the identity  $k_{t-1} = x^i - \varepsilon + \kappa + D$  for  $x^i < k_{t-1}$  in the expression for  $\bar{x}$ , and solving the integral we find

$$\psi(W, \xi|x^i, k_{t-1}) = \frac{\xi - x^i}{2\varepsilon - D} + (1 - W)\left(\frac{2\varepsilon}{2\varepsilon - D}\right) - \frac{D + \kappa - \varepsilon}{2\varepsilon - D}.$$

Solving for the critical observation  $x^i = \xi$ ,

$$\psi^*(W, \xi | \xi, k_{t-1}) = (1 - W) \left( \frac{2\varepsilon}{2\varepsilon - D} \right) - \frac{D + \kappa - \varepsilon}{2\varepsilon - D}.$$

Recalling that  $\frac{\partial D}{\partial \xi} < 0$ , this expression is not monotonic in  $\xi$ . However, the range where case b) applies, i.e. where  $x^i \in (k_{t-1} - \kappa, k_{t-1} - \kappa + \varepsilon)$  vanishes if  $\varepsilon \rightarrow 0$ .<sup>16</sup>

c) The case  $x^i \in (k_{t-1} + \kappa - \varepsilon, k_{t-1} - \kappa + \varepsilon)$  is treated in a similar way.

#### AFFILIATIONS

K.J. Bernhard Neumärker, Department of Economics and Statistics, Saarland University, and Department of Economics, Ruhr-University Bochum.

Gerald Pech, May Wong-Smith fellow, CRIEFF, University of St Andrews.

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<sup>16</sup>A similar restriction is used in Morris/Shin (1999).

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