

Pele, Daniel Traian; Mazurencu-Marinescu-Pele, Miruna

**Working Paper**

## Cryptocurrencies, Metcalfe's law and LPPL models

IRTG 1792 Discussion Paper, No. 2018-056

**Provided in Cooperation with:**

Humboldt University Berlin, International Research Training Group 1792 "High Dimensional Nonstationary Time Series"

*Suggested Citation:* Pele, Daniel Traian; Mazurencu-Marinescu-Pele, Miruna (2018) : Cryptocurrencies, Metcalfe's law and LPPL models, IRTG 1792 Discussion Paper, No. 2018-056, Humboldt-Universität zu Berlin, International Research Training Group 1792 "High Dimensional Nonstationary Time Series", Berlin

This Version is available at:

<https://hdl.handle.net/10419/230767>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

IRTG 1792 Discussion Paper 2018-056

# Cryptocurrencies, Metcalf's law and LPPL models

Daniel Traian Pele  
Miruna Mazurencu-Marinescu-Pele



\* Bucharest University of Economic Studies, Romania

This research was supported by the Deutsche  
Forschungsgemeinschaft through the  
International Research Training Group 1792  
"High Dimensional Nonstationary Time Series".

<http://irtg1792.hu-berlin.de>  
ISSN 2568-5619



International Research Training Group 1792

# Cryptocurrencies, Metcalfe's law and LPPL models

Daniel Traian Pele<sup>a</sup>, Miruna Mazurencu-Marinescu-Pele<sup>b</sup>

## Abstract

In this paper we investigate the statistical properties of cryptocurrencies by using alpha-stable distributions. We also study the benefits of the Metcalfe's law (the value of a network is proportional to the square of the number of connected users of the system) for the evaluation of cryptocurrencies. As the results showed a potential for herding behaviour, we used LPPL models to capture the behaviour of cryptocurrencies exchange rates during an endogenous bubble and to predict the most probable time of the regime switching.

**JEL Classification Codes:** C22, C32, C51, C53, C58, E41, E42, E47, E51, G1, G17

**Keywords:** cryptocurrency, Bitcoin, CRIX, Log-Periodic Power Law, Metcalfe's law, stable distribution

Authors Details:

<sup>a</sup> Corresponding author; Bucharest University of Economic Studies, Bucharest, Department of Statistics and Econometrics, Piata Romana, nr.6, Sector 1, 010371, Bucharest, Romania , email: danpele@ase.ro.

<sup>b</sup> Bucharest University of Economic Studies, Bucharest, Department of Statistics and Econometrics, Piata Romana, nr.6, Sector 1, 010371, Bucharest, Romania , email: miruna@ase.ro.

## 1. Introduction

After 2008, when the pseudonymous Satoshi Nakamoto developed the Bitcoin (Nakamoto, 2008), an explosion of other cryptocurrencies begun, based on the blockchain technology.

According to one of the major websites dealing with cryptocurrencies<sup>1</sup>, at the beginning of September 2018 the total market capitalization was around 180 billion USD, making cryptocurrencies market one of the most important in the global assets market.

This new class of assets became interesting not only for traders, but also for market regulators and academics.

For instance, in 2018, the European Supervisory Authorities for securities, banking and insurance and pensions, released a statement warning, claiming that the "VCs (virtual currencies) such as bitcoin, are subject to extreme price volatility and have shown clear signs of a pricing bubble and consumers buying VCs should be aware that there is a high risk that they will lose a large amount, or even all, of the money invested"<sup>2</sup>.

From the academic side, there are a lot of papers dealing with the subject of cryptocurrencies, especially in terms of their statistical properties and the modelling of risk. For the purpose of this paper, we will refer only to the most recent papers dealing with three areas regarding the

---

<sup>1</sup> <https://coinmarketcap.com/>

<sup>2</sup> [https://www.esma.europa.eu/sites/default/files/library/esma50-164-1284\\_joint\\_esas\\_warning\\_on\\_virtual\\_currenciesl.pdf](https://www.esma.europa.eu/sites/default/files/library/esma50-164-1284_joint_esas_warning_on_virtual_currenciesl.pdf)

cryptocurrencies market: statistical properties of returns, valuation of cryptocurrencies and log-periodic power laws applied to cryptocurrencies.

From the point of view of their statistical properties, Hu et al. (2018) realized a survey dealing with some stylized facts about the cryptocurrencies market, showing that the time series of returns are characterized by large values of kurtosis and volatility.

Zhang et al. (2018) highlighted some statistical properties of the cryptocurrencies return: the presence of heavy tails, strong volatility clustering and leverage effects and the existence of a power-law correlation between price and volume.

Chen et al. (2017) applied statistical methods (ARIMA, GARCH and EGARCH models) to the CRIX indices family, allowing them to observe the volatility clustering phenomenon and the presence of fat tails.

Another analysis of the CRIX index (Chen et al. (2017)) deals with a pricing model of derivatives for CRIX index and Bitcoin options, by using an affine jump diffusion model, SVCJ (Stochastic Volatility with Correlated Jumps) model. An important conclusion arising from this paper is that the jumps presented in the cryptocurrencies prices are an essential component.

From the point of view of valuation, there are several papers dealing with the Metcalfe's law, who states that a network's value is proportional to the square of the number of its users.

Wheatley et al. (2018) are estimating the Metcalfe's law for BITCOIN, proving the existence of a log-linear relationship between the market capitalization and a proxy the number of users (the number of unique addresses).

Peterson (2017) also used the Metcalfe's law as a Model for Bitcoin's value, by estimating a model of supply (number of bitcoins) and demand (number of bitcoin wallets) and concluding that the Metcalfe's law is a very good fit for Bitcoin's price.

If the Metcalfe's law is valid for cryptocurrencies, then a significant correlation between the number of users and the market price should be present. If the correlation is also a causality (in one way or another), then there may be the room for the occurrence of some herding behaviour: if the market is driven by expected future price increases, then more and more players will enter the market, causing the price to develop a bubble ending eventually in a crash.

For example, the Bitcoin market has experienced several crashes during his lifetime, the first one being in 2012, due to a Ponzi fraud involving Bitcoin. Another crash occurred in 2014, when Mt. Gox, a bitcoin exchange handling over 70% of all Bitcoin transactions worldwide, closed its website and exchange service, and filed for bankruptcy protection from creditors; the value of Bitcoin then dropped by 50 percent in two days.

The most recent collapse, at the end of 2017, occurred after South Korean regulators threatened to shut down cryptocurrency exchanges.

LPPL (Log-Periodic Power Law) models are widely used to describe the behaviour of stock prices during an endogenous bubble and to predict the most probable time of the regime switching (see Sornette (2000) and Sornette (2003)), as the aggregated behaviour of the investors is reflected in a log-periodic evolution of the trading price before the crash.

Fry (2015) and MacDonell (2014) both used the LPPL models to test the presence of a bubble in Bitcoin prices before the price crash of December 2013 and they concluded that LPPL models are a valuable tool for understanding the bubble behaviour in digital currencies.

Wheatley et al. (2018) used also a variant of LPPL model to estimate the most probable time of the crash for the 2017 Bitcoin bubble.

For this paper we are focusing on applying three major statistical methods for studying the behaviour of cryptocurrencies market.

First, we are using the alpha-stable distributions to emphasize the heavy-tail property of the distribution of cryptocurrencies daily logreturns.

Second, we employ the generalized Metcalfe's law for the most important cryptocurrency, the Bitcoin, for understanding the relationship between the Bitcoin's price and the number of network users, deriving from there a potential herding behaviour.

Third, we are using LPPL model to fit the bubble dynamics for one major cryptocurrencies index, CRIX, showing the value of log-periodic power laws in anticipating the regime switching.

The rest of the paper is organized as follows: Section 2 details the methodology; Section 3 presents the dataset and the empirical results and Section 4 concludes.

## 2. Methodology

The methodology used in this paper has three layers: first, we study the statistical properties of the daily logreturns of the selected cryptocurrencies and we estimate the parameters of alpha-stable distributions, in order to derive their propensity for large scale deviations.

Second, we investigate the validity of the Metcalfe's law for the most popular cryptocurrency, Bitcoin, showing the existence of a potential for herding behaviour.

Third, we apply the Log-Periodic Power Law models (Sornette, 2000) to identify the bubble regime in Bitcoin prices and in the evolution of the CRyptocurrency Index.

### 2.1. Stable distributions

A random variable  $X$  follows an alpha-stable distribution<sup>3</sup> with parameters  $(\alpha, \beta, \gamma, \delta)$  (Nolan, 2011) if exists  $\gamma > 0$ ,  $\delta \in \mathbb{R}$ , such as  $X$  and  $\gamma Z + \delta$  have the same distribution, where  $Z$  is a random variable with the characteristic function

$$\phi(t) = \mathbf{E}[e^{itZ}] = \begin{cases} \exp(-|t|^\alpha [1 - i\beta \tan(\frac{\pi\alpha}{2}) \text{sign}(t)]), & \alpha \neq 1 \\ \exp(-|t|[1 + i\beta t \frac{2}{\pi} \text{sign}(t)(\ln(|t|))]), & \alpha = 1 \end{cases}. \quad (1)$$

A random variable  $X$  follows an alpha-stable distribution  $S(\alpha, \beta, \gamma, \delta; 0)$  if his characteristic function has the form (Nolan 2011):

$$\varphi(t) = \mathbf{E}[e^{itX}] = \begin{cases} \exp(-\gamma^\alpha |t|^\alpha [1 + i\beta \tan(\frac{\pi\alpha}{2}) \text{sign}(t)(|\gamma t|^{1-\alpha} - 1)] + i\delta t), & \alpha \neq 1 \\ \exp(-\gamma |t|[1 + i\beta t \frac{2}{\pi} \text{sign}(t)(\ln(|\gamma t|)) + i\delta t]), & \alpha = 1 \end{cases}. \quad (2)$$

---

<sup>3</sup> Or, simply stated, a stable distribution.

In the above notations  $\alpha \in (0,2]$  is the stability index, controlling for probability in the tails (for Gaussian distribution  $\alpha = 2$ ),  $\beta \in [-1,1]$  is the skewness parameter,  $\gamma \in (0, \infty)$  is the scale parameter and  $\delta \in \mathbb{R}$  is the location parameter.

The behaviour of stable distributions is driven by the values of stability index  $\alpha$ : small values are associated to higher probabilities in the tails of the distribution.

There are several methods for estimating the parameters of the stable distributions (see Appendix A):

- McCulloch method (1986), based on the quintiles of the empirical distribution;
- Regression based methods (Kogon and Williams 1998), using an iterative estimation process; this algorithm is implemented as a SAS macro in Pele (2014) and can be used to obtain estimates for the parameters of stable distributions (see Appendix A).

## 2.2. Metcalfe's law

In the 1980s, Robert Metcalfe, the co-inventor of Ethernet, stated what was called later the Metcalfe's law (Gilder 1993): the value of a network is proportional to the square of the size the number of connected users.

Metcalfe's law was validated in various contexts, by using social network data: Zhang et al. (2015) proved the validity of the law for Facebook and Tencen (Chinese social network). Other researchers (Madureira et al., 2013, Van Hove, 2014, 2016, Metcalfe, 2013) have shown the validity of the law, mostly regarding internet networks.

Peterson (2017) showed that the Metcalfe's law can be used to explain the evolution of BITCOIN transaction price, by using factors relating to supply (number of bitcoins) and demand (number of wallets).

In this paper we are using the Metcalfe's law following Wheatley et al. (2018):

$$C_t = e^\alpha u_t^\beta \tag{3}$$

where:

- $C_t$  is the market capitalization at time  $t$ ;
- $u_t$  is the number of unique addresses at time  $t$ ;
- $\beta=2$ .

## 2.3. Log-periodic power laws (LPPL)

Sornette (2000) compares seismic activity to the evolution of speculative bubbles, and deduces the evolution law for stock prices before and during the crash, which is seen as a critical time.

According to the field theory (Goldenfeld, 1992), an imitative process can be described through its hazard rate  $h(t)$ :  $\frac{dh}{dt} = Ch^\delta$ , where  $C>0$ , and  $\delta+1>1$  is the average number of interactions.

Then  $h(t) = \left(\frac{h_0}{t_c-t}\right)^\alpha$ , with  $\alpha = \frac{1}{\delta-1}$  and  $t_c$  being the critical time, so the price dynamics prior to the crash should be  $\ln \frac{p(t)}{p(0)} = k \int_{t_0}^t h(u) du$ .

As the crash probability should be compensated by larger price changes, prior to the stock market crash (Blanchard, 1979), the hazard rate could be expressed via the Ising model:  $h(t) \approx B_0(t_c - t)^{-\alpha} + B_1(t_c - t)^{-\alpha} \cos[\omega \ln(t_c - t) + \psi']$ .

Thus, the trading price before the crash follows a log-periodic power law:

$$E[\log p(t)] = A + B(t_c - t)^\beta \{1 + C \cos[\omega \ln(t_c - t) + \phi]\}, \quad (4)$$

where  $p(t)$  is the price at moment  $t$ ,  $t_c$  is the critical time (the most probable moment of the crash), and  $\beta, B_0, B_1, \omega, \phi$  are the parameters of the model which give its log-periodic feature.

In order to have a proper specification of the model, there are several constraints applied to the parameters:

- $A > 0$  - usually this is the price at the critical time  $t_c$ ;
- $B < 0$  ;
- $C \neq 0, |C| < 1$  - this parameter controls the magnitude of oscillations around the exponential trend;
- $0 < \beta < 1$  - controls the growth rate of the magnitude and is the most important feature capturing the imminence of a regime switching, as its value is close to zero;
- $\omega \in (0, \infty)$  - controls for the amplitude of oscillations;
- $\phi \in [0, 2\pi]$  - a phase parameter.

Johansen, Ledoit and Sornette (2000) have applied these models to successfully predict famous crashes like the one in October 1987 and for the Brazilian market, Cajueiro, Tabak and Werneck (2009) have applied these models to predict the catastrophic behaviour of the price series of 21 stocks. The Financial Crisis Observatory (ETH - Zurich) has released during the past few years predictions about the bubble behaviour of different assets and they have succeeded to predict two famous events of this type: Oil Bubble - 2008 and Chinese Index Bubble - 2009.

Fantazzini and Geraskin (2013) provide an extensive review of theoretical background behind the LPPL models, estimation methods and various applications, pointing out that although the literature on this subject is heterogeneous, LPPL fit for asset bubbles could be a useful tool in predicting the catastrophic behaviour of capital markets as a whole.

Moreover, even using such a model, the prediction of critical time is not very accurate, Kurz-Kim (2012) shows that LPPL models could be used as an early warning mechanism of regime switching in case of a stock market.

As the industry of cryptocurrencies has grown exponentially over the past several years, there are many applications of financial models to the study of this new markets.

MacDonell (2014) used the LPPL model to forecast the Bitcoin price crash that took place on December 4, 2013, showing how the model can be a valuable tool for detecting bubble behaviour in digital currencies.

Malhotra *et al.* (2013) investigated the evolution of Bitcoin exchange rates in 2013-2014, showing evidence of super-exponential growth in Bitcoin exchange rates.

Fantazzini *et al.* (2016) also applied the LPPL modelling to Bitcoin exchange rates, finding evidence of explosive behaviour in the bitcoin-USD exchange rates during August - October 2012 and November, 2013 - February, 2014.

### 3. Empirical results

#### 3.1. Dataset

The dataset consists of daily cryptocurrency data (transaction count, on-chain transaction volume, value of created coins, price, market capitalization and exchange volume)<sup>4</sup>. One market index was also used for the analysis: Cryptocurrency Index<sup>5</sup> as a reference for the cryptocurrencies market (Trimborn and Härdle, 2018).

Table 1. Description of the dataset

No.	Symbol	Cryptocurrency/ Index	Number of daily observations	Start date	End date
1	ANT	Aragon	502	5/19/2017	10/2/2018
2	BTC	Bitcoin	1983	4/29/2013	10/2/2018
3	DASH	Dash	1691	2/15/2014	10/2/2018
4	DCR	Decred	965	2/11/2016	10/2/2018
5	DGB	Digibyte	1699	2/7/2014	10/2/2018
6	DOGE	Dogecoin	1752	12/16/2013	10/2/2018
7	ETC	Ethereum Classic	800	7/25/2016	10/2/2018
8	ETH	Ethereum	1152	8/8/2015	10/2/2018
9	GNO	Gnosis	519	5/2/2017	10/2/2018
10	GNT	Golem	683	11/19/2016	10/2/2018
11	GOLD	GoldCoin	2122	12/11/2012	10/2/2018
12	ICN	Iconomi	732	10/1/2016	10/2/2018
13	LSK	Lisk	909	4/7/2016	10/2/2018
14	LTC	Litecoin	1983	4/29/2013	10/2/2018
15	MAID	MaidSafeCoin	1618	4/29/2014	10/2/2018
16	NEO	NEO	753	9/10/2016	10/2/2018
17	PIVX	PIVX	962	2/14/2016	10/2/2018
18	REP	Augur	1071	10/28/2015	10/2/2018
19	USDT	Theter	590	2/20/2017	10/2/2018
20	VTC	Vertcoin	1716	1/21/2014	10/2/2018
21	WAVES	Waves	846	6/3/2016	9/26/2018
22	XEM	NEM	1280	4/2/2015	10/2/2018
23	XLM	Stellar	1519	8/6/2014	10/2/2018
24	XMR	Monero	1595	5/22/2014	10/2/2018
25	XRP	Ripple	1835	8/5/2013	8/13/2018
26	XVG	Verge	1438	10/26/2014	10/2/2018
27	ZEC	ZCash	703	10/30/2016	10/2/2018
28	CRIX	CRyptocurrency IndeX	1524	8/1/2014	10/2/2018

<sup>4</sup> The source for these data is <https://coinmarketcap.com>.

<sup>5</sup> The CRyptocurrency IndeX is a benchmark for the crypto market. The CRIX is realtime computed by the Ladislaus von Bortkiewicz Chair of Statistics at Humboldt University Berlin, Germany.



The dataset used in this paper deals only with cryptocurrencies for which at least 2 years of daily transaction data (at least 500 daily observations) were available at the moment of the data collection (October 2<sup>nd</sup>, 2018). For the purpose of data analysis, the statistical software SAS 9.3 was used.

### 3.2. Estimating the parameters of an alpha - stable distribution for cryptocurrencies daily logreturns

In order to fit the stable-distribution to the selected time series of daily logreturns  $r_t = \log(P_t) - \log(P_{t-1})$ , a SAS macro (Pele 2014) was applied, the results being presented below.

Table 2. Parameters of the estimated alpha - stable distributions

No.	Symbol	$\alpha$	95% half-width	$\beta$	95% half-width	$\delta$	95% half-width	$\gamma$	95% half-width
1	ANT	1.825	0.063	-0.066	0.015	0.081	0.029	0.047	0.037
2	BTC	1.468	0.100	0.169	0.033	0.211	0.025	0.017	0.074
3	DASH	1.494	0.073	-0.391	0.023	-0.195	0.142	0.030	0.053
4	DCR	1.645	0.084	-0.528	0.158	-0.107	0.397	0.038	0.055
5	DGB	1.620	0.056	-0.245	0.040	0.064	0.103	0.046	0.037
6	DOGE	1.306	0.087	-0.338	0.050	-0.714	0.462	0.024	0.073
7	ETC	1.501	0.089	-0.459	0.054	-0.164	0.255	0.031	0.064
8	ETH	1.589	0.099	-0.457	0.077	0.081	0.273	0.030	0.067
9	GNO	1.733	0.060	-0.030	0.065	-0.077	0.105	0.045	0.037
10	GNT	1.772	0.061	-0.167	0.074	0.439	0.133	0.049	0.037
11	GOLD	1.543	0.089	0.080	0.054	-0.048	0.100	0.003	0.063
12	ICN	1.669	0.060	-0.167	0.085	0.204	0.164	0.052	0.039
13	LSK	1.361	0.025	-0.302	0.068	-0.099	0.220	0.042	0.020
14	LTC	1.336	0.081	-0.202	0.039	-0.384	0.252	0.020	0.066
15	MAID	1.789	0.048	0.028	0.031	-0.079	0.043	0.038	0.029
16	NEO	1.525	0.050	-0.417	0.045	0.058	0.146	0.045	0.035
17	PIVX	1.630	0.076	-0.242	0.038	0.062	0.109	0.054	0.050
18	REP	1.573	0.055	-0.125	0.045	0.076	0.109	0.037	0.037
19	USDT	0.509	0.306	0.111	0.126	-0.003	0.034	0.001	0.680
20	VTC	1.549	0.043	-0.325	0.030	-0.078	0.097	0.045	0.030
21	WAVES	1.716	0.053	-0.015	0.027	0.128	0.046	0.042	0.033
22	XEM	1.631	0.048	-0.208	0.053	0.053	0.117	0.040	0.032
23	XLM	1.515	0.072	-0.279	0.055	-0.072	0.180	0.032	0.052
24	XMR	1.701	0.068	-0.169	0.007	0.167	0.034	0.036	0.043
25	XRP	1.323	0.072	-0.306	0.023	-0.534	0.243	0.023	0.059
26	XVG	1.551	0.105	-0.177	0.076	-0.082	0.226	0.073	0.074
27	ZEC	1.575	0.037	-0.082	0.037	0.107	0.077	0.039	0.025
<b>28</b>	<b>CRIX</b>	<b>1.490</b>	<b>0.109</b>	<b>0.254</b>	<b>0.103</b>	<b>0.427</b>	<b>0.169</b>	<b>0.015</b>	<b>0.080</b>

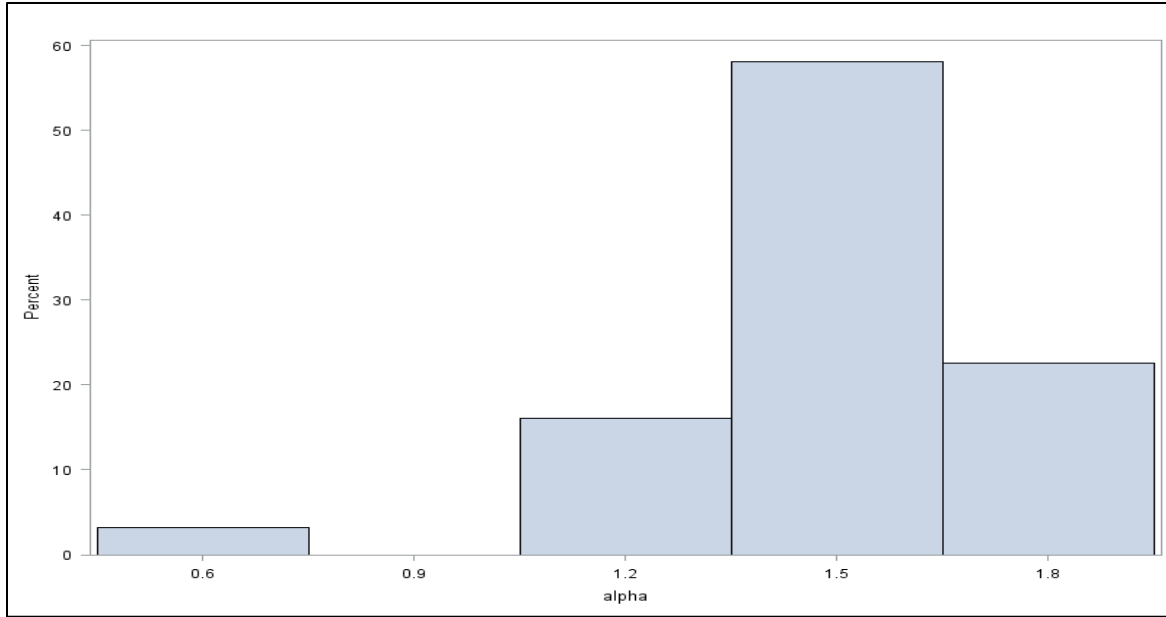


Figure 1. Distribution of the stability index  $\alpha$  for logreturns distribution of selected assets

As depicted in Table 2 and Figure 1, in most of the cases, all the analysed the cryptocurrencies exhibits large departures from normality, the values of the stability index  $\alpha$  being significantly lower than 2, the value corresponding to the Gaussian distribution.

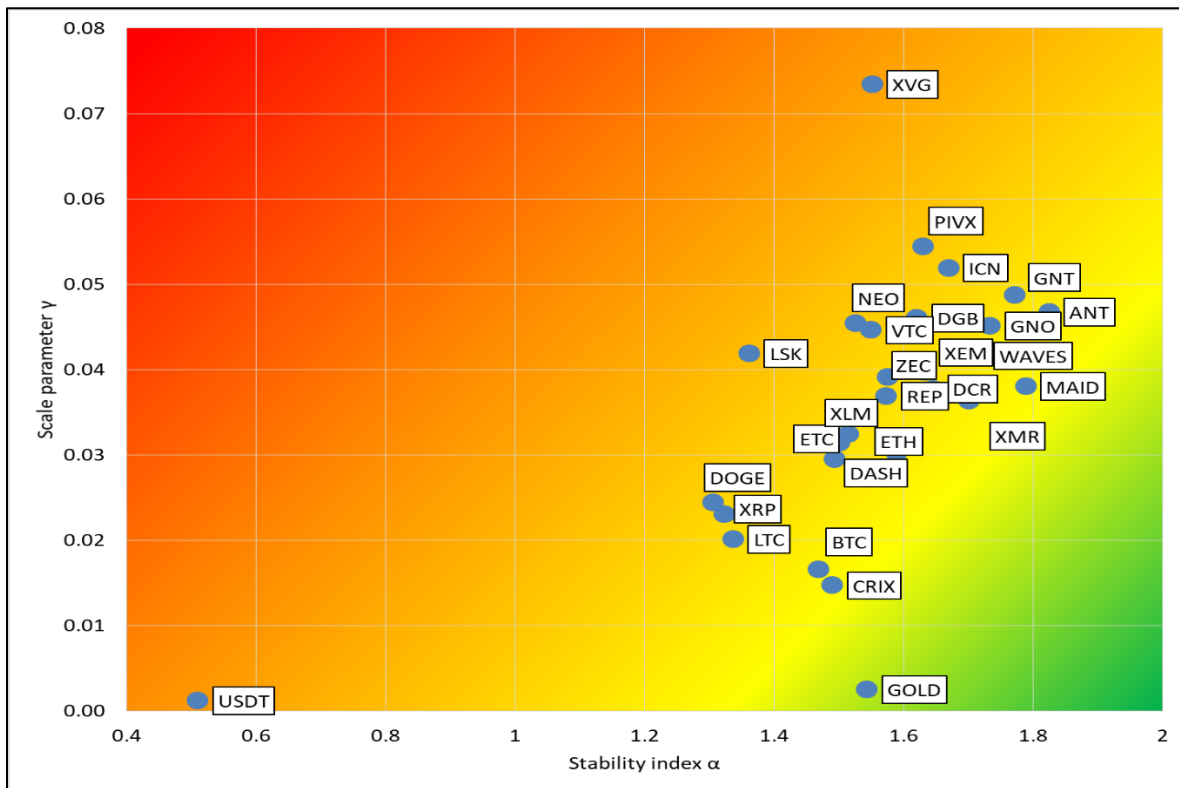
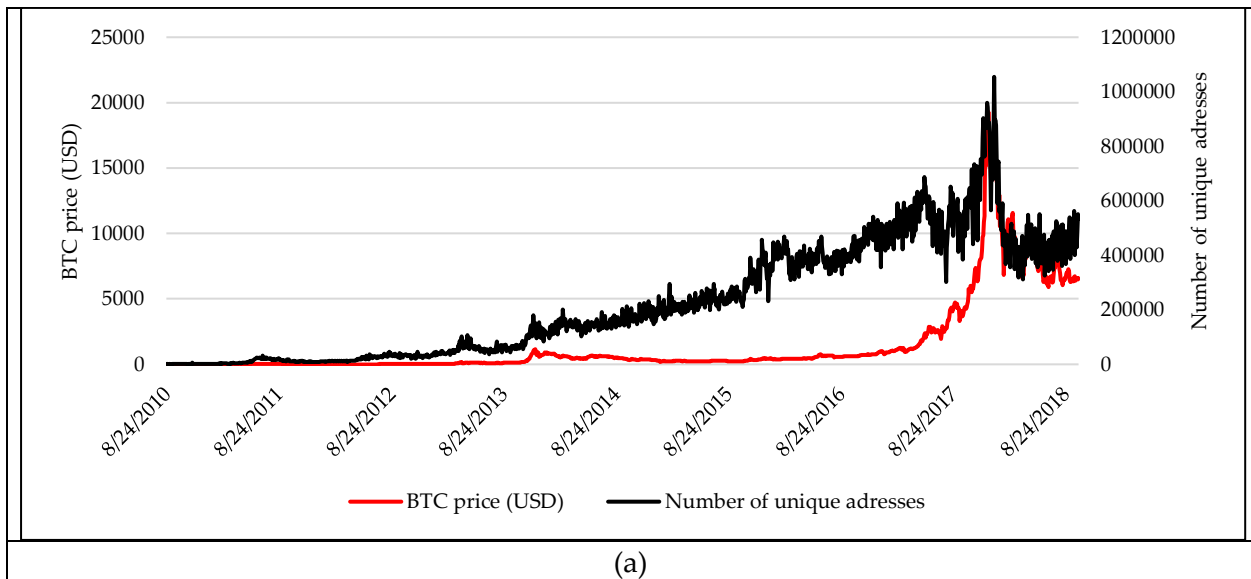


Figure 2. Heatmap of scale parameter  $\gamma$  versus stability index  $\alpha$  for selected assets

The Figure 2 shows the correlation between the scale parameters  $\gamma$  (the equivalent of the volatility in the classical approach) and the stability index  $\alpha$ , controlling for the tail probability. Based on this correspondence, we are able to cluster the selected cryptocurrencies based on their propensity to heavy-tailness and the likelihood of high volatility. For example, the cryptocurrency Theter (USDT) has the lowest stability index  $\alpha$  (large departure from normality), but the scale parameter is low, so USDT is placed in the orange zone. The closest to the normal distribution is Aragon (ANT), yet his scale parameter is around the sample average, so it is placed in the yellow zone.

### 3.3. Metcalfe's law

In order to evaluate the applicability of the Metcalfe's law for cryptocurrencies, we limit ourselves to the most known and traded cryptocurrency, the BITCOIN, also due to the availability of transaction and network data<sup>6</sup>.



<sup>6</sup> <https://www.blockchain.com>

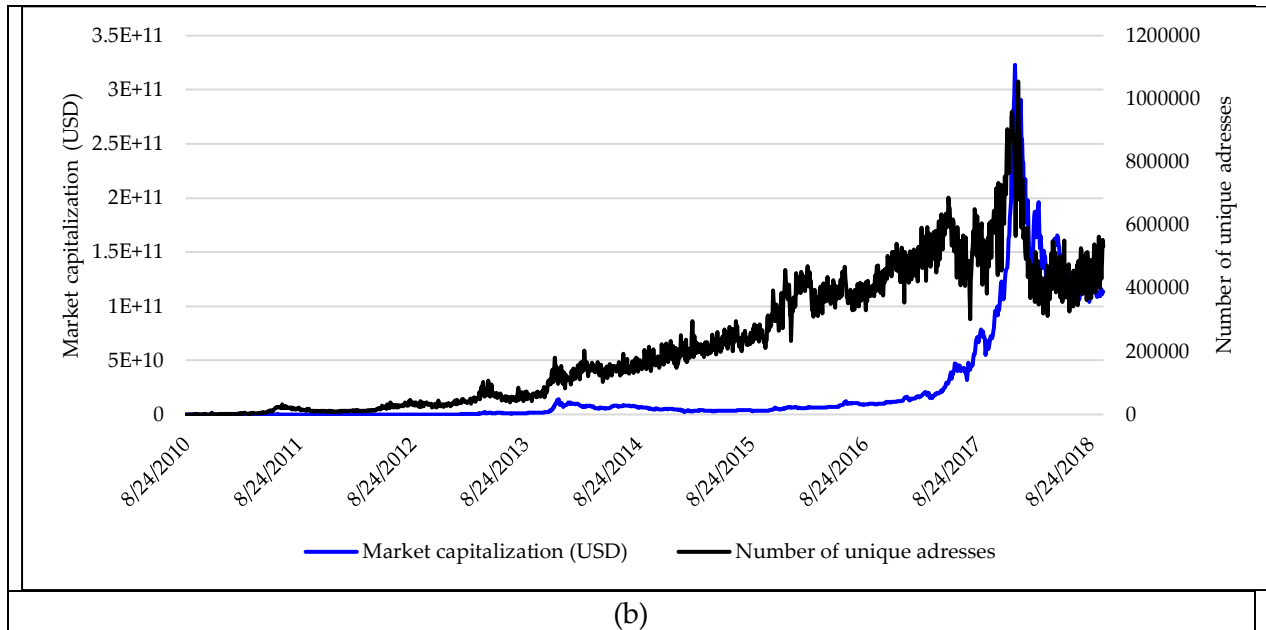


Figure 3. (a) Bitcoin average price (USD) vs. Number of unique addresses. (b) Bitcoin market capitalization (USD) vs. Number of unique addresses.

As stated in the original formulation of the Metcalfe’s law, the value of the network should be proportional to the squared number of network users; however, in the case of cryptocurrencies, the actual number of user is unknown and we need to use a proxy, the number of unique addresses.

Unique addresses in the Bitcoin ecosystem are payment addresses that have a non-zero balance; this metric can be used as a proxy for the number of network users, although we cannot state that the number of users is equal to the number of unique addresses. The number of unique addresses is not constant over time: when fees are high, investors leave their cryptocurrencies in multiple addresses, because a consolidation into a single address will require a high cost. When fees are low, investors can consolidate their funds into a single address.

As the Bitcoin network grows, the number of unique addresses will also grow over time, but when the market is going down, less unique addresses are in use because also the number of transactions reduces.

We are estimating the generalized Metcalfe’s law, which is a log-linearization of the equation (3):

$$\log C_t = \alpha + \beta \log u_t + \varepsilon_t. \quad (5)$$

where:

- $C_t$  is the Bitcoin’s market capitalization at time  $t$ ;
- $u_t$  is the number of unique Bitcoin addresses at time  $t$ .

The estimation results for the equation (5) are reported below, using daily data for the period 2010/08/24 – 2018/10/05.

Table 3. Estimation results for the equation (5)

Parameter	Estimated value	Std. Error	t-Statistic	Prob.
$\alpha$	1.856	0.146	12.715	0.000
$\beta$	1.696	0.013	134.256	0.000
R-squared	0.924			

Although the slope of the equation (5) is  $\beta=1.696$ , below the theoretical value of 2, the model has a high explanatory power ( $R^2 = 0.924$ ), supporting the validity of the Metcalfe's law for Bitcoin.

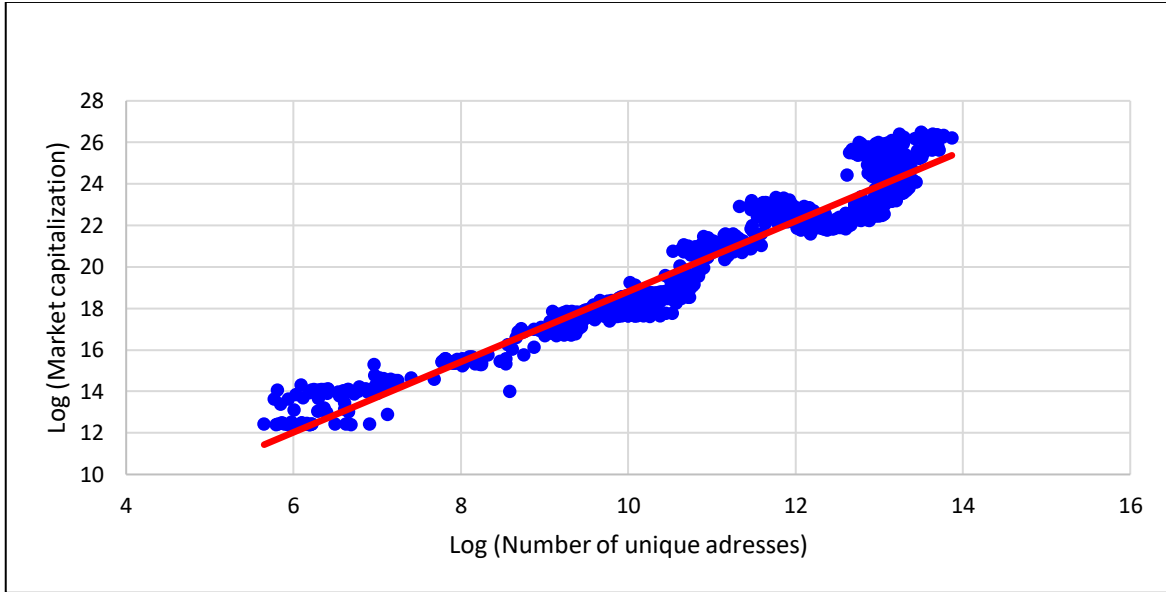


Figure 4. Generalised Metcalfe's law for Bitcoin's market capitalization

From the validity of the Metcalfe's law for Bitcoin one derive the existence of a possible herding effect: as an increase of the number of users is reflected in an increase of the market capitalization, this may be explained by the fact that there is a mimetic effect among users, making the price to have an ascendant trend.

One insight into this direction can be found by estimating the generalized Metcalfe's law for Bitcoin's price:

$$\log P_t = \alpha + \beta \log u_t + \varepsilon_t . \quad (6)$$

Table 4. Estimation results for the equation (6), using daily data for the period 2010/08/24 – 2018/10/05.

Parameter	Estimated value	Std. Error	t-Statistic	Prob.
$\alpha$	-12.040	0.143	-83.915	0.000
$\beta$	1.489	0.012	119.921	0.000
$R^2 = 0.906$				

The results of the estimation shown that there is strong log-linear relationship between the Bitcoin's market price and the number of unique addresses, as a proxy for the number of Bitcoin's

network users; moreover, what the estimated results tells us is that price increase may be a direct effect of the increasing network size, through a possible mimetic behaviour.

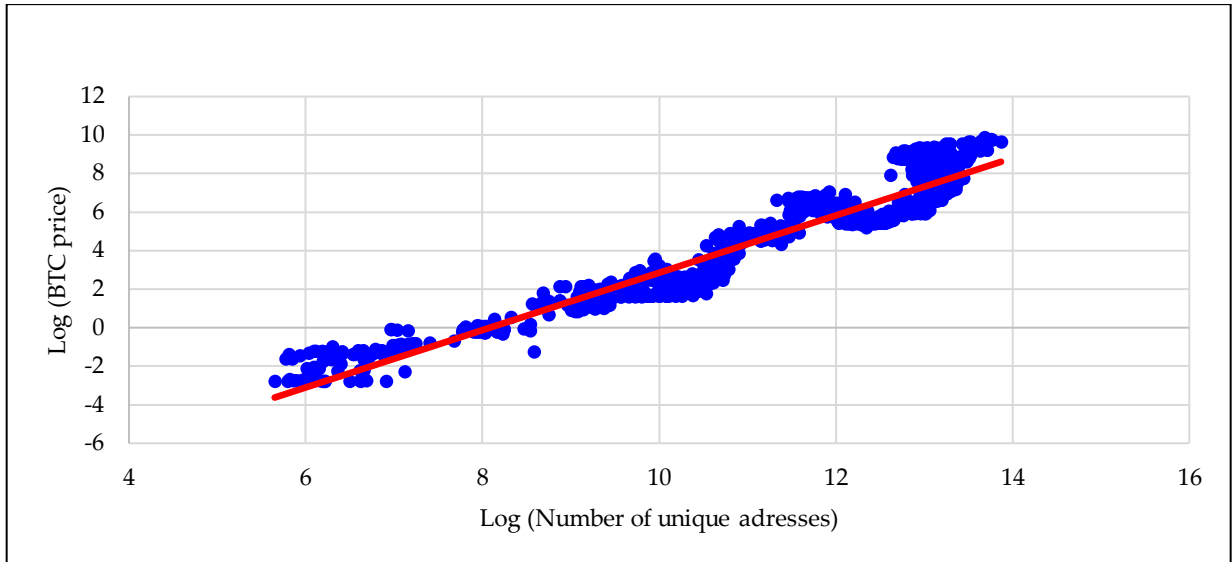


Figure 5. Generalised Metcalfe’s law for Bitcoin’s price

Going deeper with the analysis, we also performed a Granger causality test in order to detect the existence of the causal links between the Bitcoin’s price and the number of unique addresses. As the two time series are not stationary and not of them are integrated  $I(1)$ , in order to test for Granger causality, the Toda-Yamamoto (1995) procedure was applied, following the steps below:

- Test the two time-series to determine their order of integration.
- Let the  $m=1$  the maximum order of integration for the group of the two time-series.
- Estimate a VAR model in level.
- Determine the appropriate maximum lag length ( $p$ ) for the variables in the VAR, using the AIC, criterion.
- Check and correct for serial correlation in the residuals.
- Test for cointegration of the two time series.
- Estimate the VAR( $p+m$ ) model and test the Granger causality using the Block Exogeneity Wald Test.

Table 5. VAR Granger Causality/Block Exogeneity Wald Tests

Included observations: 1468				
Dependent variable: LOG_P				
Excluded	Chi-sq	df	Prob.	
LOG_U	22.06608	15	0.1061	
All	22.06608	15	0.1061	
Dependent variable: LOG_U				
Excluded	Chi-sq	df	Prob.	
LOG_P	121.1914	15	0.0000	
All	121.1914	15	0.0000	

Note: the optimum number of lags (15) was chosen based on the lag length criteria from VAR specification.

Based on the Granger causality tests, one can deduce the existence of a unidirectional causal relationship from the Bitcoin's prices to the size of the network, expressed as the number of unique addresses.

The temporal dependency can be captured via a Vector Autoregressive (VAR (p)) model, of the following form:  $Y_t = A_1 Y_{t-1} + \dots + A_p Y_{t-p} + \varepsilon_t$ , where  $Y_t = (\ln P_t, \ln u_t)'$ .

Table 6. VAR (5) estimates

	LOG_U	LOG_P
LOG_U(-1)	0.3490*** (-0.0261)	0.0133 (-0.0147)
LOG_U(-2)	0.1560*** (-0.0272)	-0.0397*** (-0.0153)
LOG_U(-3)	0.2442*** (-0.0267)	0.0113 (-0.0151)
LOG_U(-4)	0.1651*** -0.0272	-0.0095 -0.0153
LOG_U(-5)	0.0645*** (-0.0260)	0.0237 (-0.0147)
LOG_P(-1)	0.1849*** (-0.0468)	0.9258*** (-0.0264)
LOG_P(-2)	-0.1326*** (-0.0632)	0.0847*** (-0.0356)
LOG_P(-3)	-0.0542 (-0.0634)	0.0657 (-0.0357)
LOG_P(-4)	0.2577*** (-0.0632)	-0.1004*** (-0.0356)
LOG_P(-5)	-0.2461*** (-0.0467)	0.0223 (-0.0263)
C	0.2024*** (-0.0643)	0.0265 (-0.0362)
Adj. R-squared	0.9924	0.9990
Sum sq. resids	37.5591	11.9292
S.E. equation	0.1600	0.0902
F-statistic	19224.4400	149859.1000

Note: Standard errors in (); \*\*\* denotes significance at 99% confidence level.

One can note from the above table with the VAR estimation results that the past realizations of the Bitcoin's price can be used to forecast the future realizations of the network size. For example, if at time  $t-1$  the Bitcoin's price increase by 1%, at time  $t$  one can expect a 0.189% increase of the number of unique addresses.

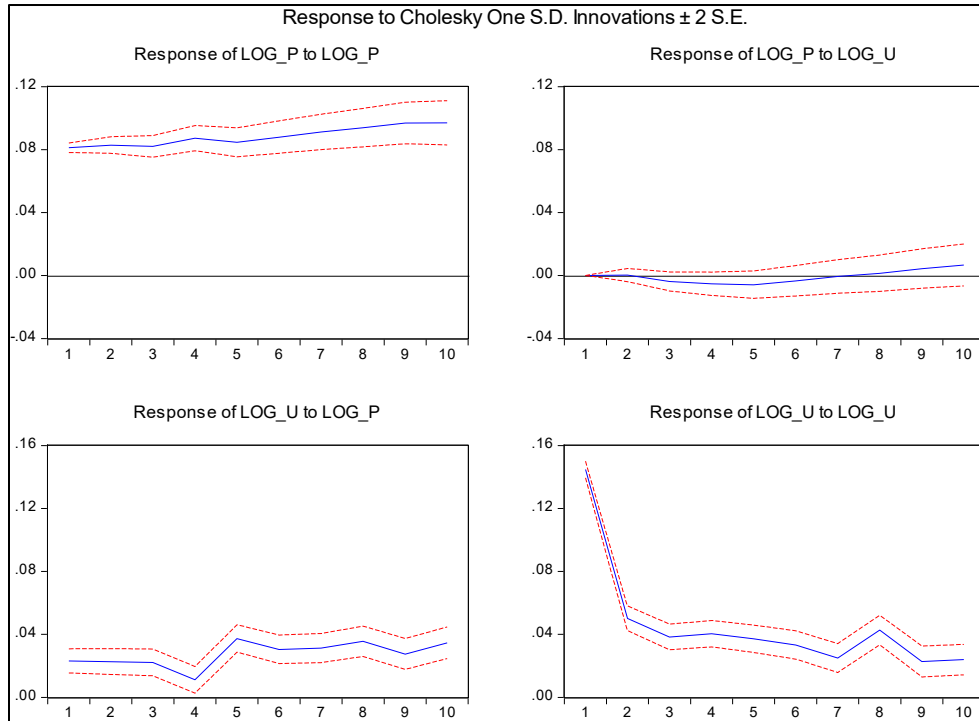


Figure 6. Impulse Response Function for the estimated VAR model

Moreover, the behaviour of the impulse response function offers an indication that a shock from the Bitcoin's price have a positive effect on the network size, and the effect is permanent and significantly different from zero

One can infer from this analysis that the expected price increase is a driver for more investors to join the Bitcoin network, which may lead in the end to a super-exponential price growth, due to a herding behaviour of investors.

### 3.4. LPPL models

In order to estimate capture the bubble regime and to estimate the most probable time of the crash, the algorithm from Pele (2012), using price gyrations and peak detection was applied.

#### 3.4.1. Numerical results for Bitcoin

In case of Bitcoin, the regime switching was recorded in December 2017, the exchange rate hitting a local maxima on December 19<sup>th</sup>, 2017. The initial sample for fitting LPPL model in the case of Bitcoin for predicting the phase transition from December 2017 was 1 Jan 2016 – 30 Nov 2017 (700 daily observations).

Starting from the last observation in the initial sample, we extended the sample using a rolling window with fixed lower limit, so we estimated at every step the LPPL model for  $t \in [1, T+k]$ ,  $k=1 \dots 17$ :



$$E[\ln p(t)] = A_k + B_k (t_{c;k} - t)^\beta \{1 + C_k \cos[\omega_k \ln(t_{c;k} - t)^\beta + \phi_k]\}. \quad (7)$$

Table 6. The best fit for Bitcoin's LPPL model

Obs	Start date	End date	A	B	C	$\beta$	$\omega$	$\phi$	RMSE	AdjRSq	$t_c$	Date of crash
711	01 Jan 2016	11 Dec 2017	9.768	-0.161	-0.062	0.494	3.863	6.280	0.148	0.975	712	12 Dec 2017
701	01 Jan 2016	01 Dec 2017	9.328	-0.080	0.085	0.588	3.472	5.585	0.152	0.971	702	02 Dec 2017
706	01 Jan 2016	06 Dec 2017	9.489	-0.104	0.076	0.552	3.588	4.830	0.157	0.970	707	07 Dec 2017

As a result of the estimation, three models were kept, with the best Root Minimum Squared Error (RMSE). The model with the minimum RMSE anticipated on December 11<sup>th</sup> 2017 an imminent crash for the next day.

The other two selected models offers close predictions, for December 2<sup>nd</sup> 2017 and December 7<sup>th</sup> 2017.

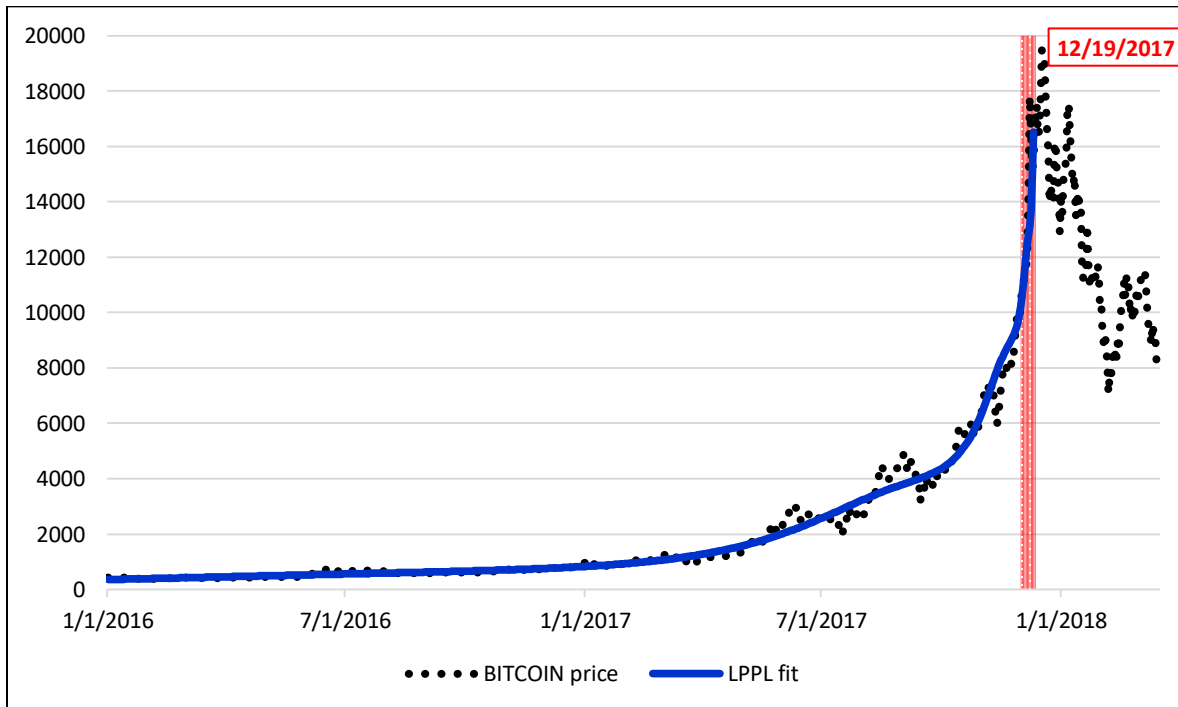


Figure 7. LPPL fit for BTC (model with the minimum RMSE)

### 3.2. Numerical results for CRIX Index

The local maxima for the CRIX index was recorded on January 7<sup>th</sup> 2018, this being the moment of the regime switching.

The initial sample for fitting LPPL model in the case of CRIX index for predicting the phase transition from January 2018 was 1 Jan 2016 – 15 Dec 2017 (716 daily observations).

Starting from the last observation in the initial sample, we extended the sample by using a rolling window with fixed lower limit, so we estimated at every step the LPPL model for  $t \in [1, T+k]$ ,  $k=1 \dots 20$ :

$$E[\ln p(t)] = A_k + B_k (t_{c;k} - t)^\beta \{1 + C_k \cos[\omega_k \ln(t_{c;k} - t)^{\beta_k} + \varphi_k]\}. \quad (8)$$

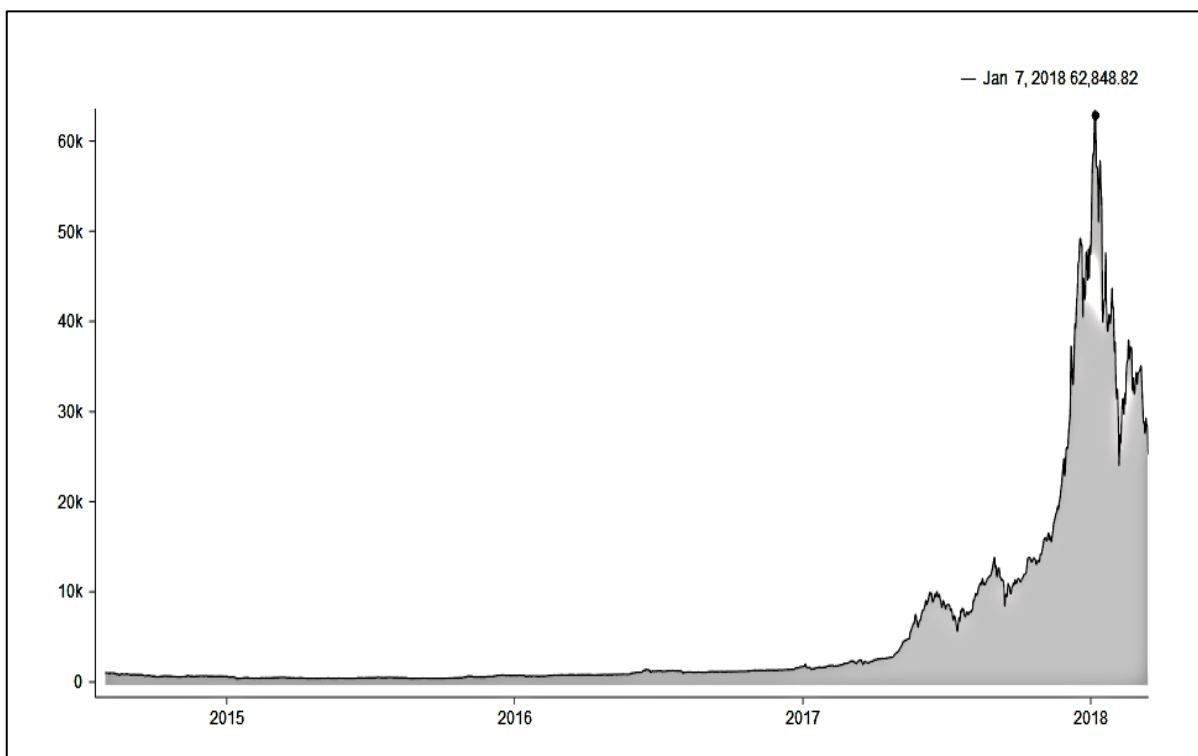


Figure 8. CRIX Index

Table 7. The best fit for CRIX's LPPL model

Obs	A	B	C	$t_c$	$\beta$	$\omega$	$\phi$	Start date	End date	RMSE	AdjRSq	Date of crash
729	12.393	-0.627	-0.007	737	0.344	-10361.290	67211.290	01 Jan 2016	30 Dec 2017	0.2406	0.9578	07 Jan 2018
732	12.373	-0.603	-0.008	739	0.349	-9103.550	58656.450	01 Jan 2016	02 Jan 2018	0.2407	0.9587	10 Jan 2018
727	12.383	-0.631	0.006	736	0.342	-5960.180	38870.710	01 Jan 2016	28 Dec 2017	0.2408	0.9571	06 Jan 2018

The best fit for the CRIX index was given by the model estimated for the period January 1<sup>st</sup> 2016 – December 30<sup>th</sup> 2017, for which the estimated critical time was exactly the date of local maximum, January 7<sup>th</sup> 2018.

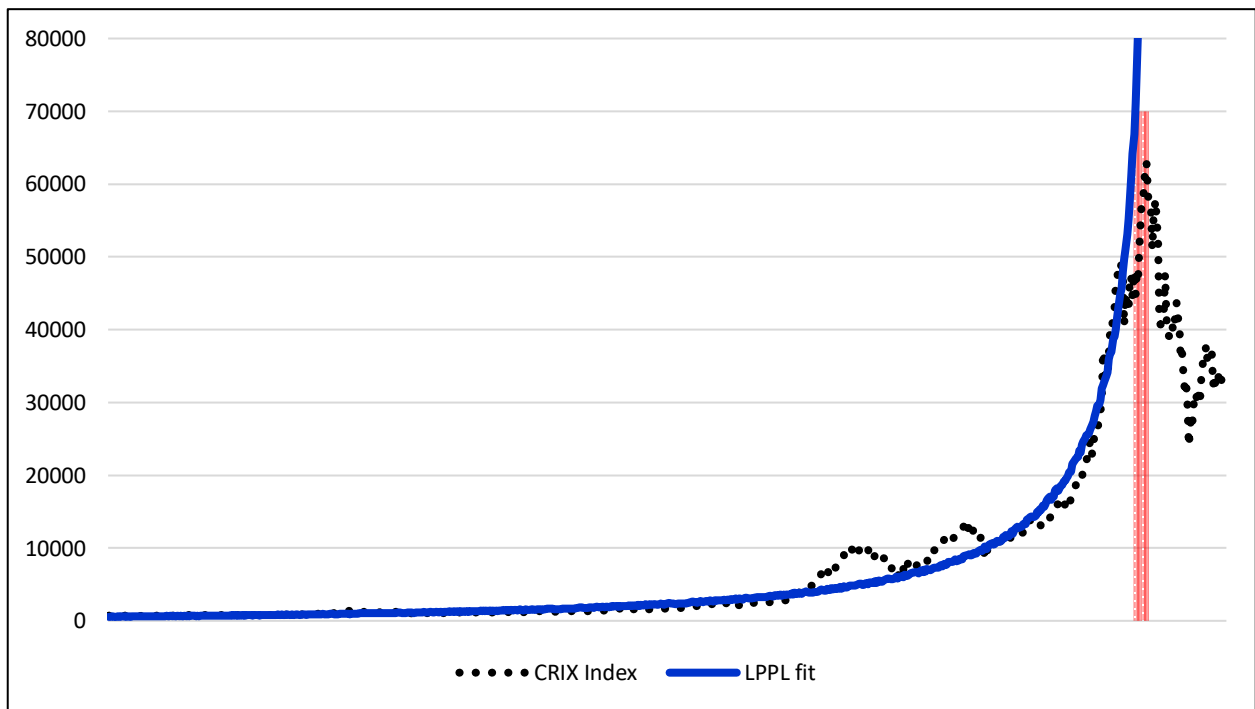


Figure 9. LPPL fit for CRIX Index (the model with the minimum RMSE)

## Conclusions

Our paper deals with a new class of assets, digital currencies or cryptocurrencies, from the point of view of their statistical properties. One of the main findings is that daily cryptocurrencies logreturns exhibits large departures from normality, leaving room for high uncertainty levels, as shown the estimated stability indexes of stable distributions.

Moreover, by analysing Bitcoin related data, we prove the validity of the Metcalfe's law, linking both the market capitalization and the exchange rates to the network size. As there is a strong correlation between the size of the network and the market price of cryptocurrencies, this may be a sign for a mimetic behaviour of investors, who enter the market driven by high expected currency rates, which may lead the market into a super-exponential bubble regime.

LPPL models could be useful in estimating the most probable time of the regime switching for an endogenous cryptocurrency bubble.

Analysing the behaviour of the Bitcoin's price and the CRIX index, we have proven that LPPL models can be a useful tool in recognizing and mapping out the behaviour of a developing bubble.

This is a validation of the predictive power of LPPL models in detecting the imitative behaviour of investors in the cryptocurrencies market, our results being useful both from a theoretical point of view and from a business perspective.

From a theoretical point of view, the analysis provided strong evidence that the herding behaviour could be detected also in cryptocurrencies markets, this being a sign that LPPL models have a great potential for universal applications. From a business perspective, such an instrument could be used as a risk management tool, supporting the investment decisions in order to minimize risk and to benefit from market evolutions. The LPPL models could be used as an early warning tool for detecting the development of a bubble regime and also to predict the critical time of the regime switching.

A recommendation for risk management arising from these results is to implement an iterative estimation method for LPPL models, allowing to periodically asses the likelihood of a phase transition in the cryptocurrency market.

Yet, the research in this direction needs to be furtherly honed, as this type of models have also some weaknesses, like the over-parameterization or the serious constraint of the LPPL model that during a bubble the trading price cannot decrease, which may be a questionable assumption.

Also, for LPPL models, a serious risk may be overfitting and the evidence that such models cannot always predict but often can only retrodict.



## Acknowledgements

We would like to express our thanks to the anonymous referees and the Editor whose very careful and detailed comments have helped to improve this paper considerably. Any remaining errors are our responsibility.

Also we would like to express our gratefulness to Prof. dr. Wolfgang Karl Härdle, Ladislaus von Bortkiewicz Chair of Statistics, Humboldt-Universität zu Berlin for his support in conducting our research.

## References

1. Cajueiro, D. O., B. M. Tabak, & F. K. Werneck (2009). Can We Predict Crashes? The Case of the Brazilian Stock Market, *Physica A*, 388(8), 1603-1609.
2. Chen C.Y.H., Härdle W.K., Hou A.J., Wang W. (2018). Pricing Cryptocurrency options: the case of CRIX and Bitcoin. *J Financial Econometrics*, RR 20180601.
3. Chen S., Chen C.Y.H., Härdle W.K., Lee T.M., Ong B. (2017). A first econometric analysis of the CRIX family, in *Handbook of Blockchain, Digital Finance and Inclusion, Vol 1, Cryptocurrency, FinTech, InsurTech, and Regulation*, David LEE Kuo Chuen Robert Deng, eds. ISBN: 9780128104415, Academic Press, Elsevier.
4. Fantazzini, D. & Geraskin, P. (2013). Everything You Always Wanted to Know About Log Periodic Power Laws for Bubble Modelling But Were Afraid to Ask, *European Journal of Finance* 19 (5) (2013) 366–391.
5. Fantazzini, D., Nigmatullin, E., Sukhanovskaya, V., Ivliev, S. (2016). Everything you always wanted to know about Bitcoin modelling but were afraid to ask. Draft (in English), Moscow School of Economics, published (in Russian) in *Applied Econometrics* (2016), vol. 44, 5-24 as part I and *Applied Econometrics*, vol. 45, pp. 5-28 as part 2.
6. Goldenfeld, N. (1992). *Lectures on Phase Transitions and the Renormalization Group* (Frontiers in Physics). Addison-Wesley, Boston.
7. Hu, A, Parlour, C.A. Rajan, U. (2018). Cryptocurrencies: Stylized facts on a new investible instrument. Working Paper, available online at <http://webuser.bus.umich.edu/urajan/research/crypto.pdf>
8. Johansen, A., Ledoit, O., Sornette, D. (2000). Crashes as critical points, *International Journal of Theoretical and Applied Finance*, 3, 219-255.
9. Kogon, S.M., Williams, D.B. (1998). Characteristic Function Based Estimation of Stable Distribution Parameters, in *A Practical Guide to Heavy Tails: Statistical Techniques and Applications*, R.J.Adler, R.E.Feldman, M.Taqqu eds., Birkhauser, Boston, 311-335.
10. Kurz-Kim, J.R. (2012). Early warning indicator for financial crashes using the log periodic power law, *Applied Economics Letters*, 19:15, 1465-1469.
11. MacDonell, A. (2014). Popping the Bitcoin Bubble: An application of log-periodic power law modeling to digital currency. University of Notre Dame working paper, available online at [https://economics.nd.edu/assets/134206/mac\\_donell\\_popping\\_the\\_bitcoin\\_bubble\\_an\\_application\\_of\\_log\\_periodic\\_power\\_law\\_modeling\\_to\\_digital\\_currency.pdf](https://economics.nd.edu/assets/134206/mac_donell_popping_the_bitcoin_bubble_an_application_of_log_periodic_power_law_modeling_to_digital_currency.pdf)
12. Madureira, A., Den Hartog, F., Bouwman, H. and Baken, N. (2013). Empirical validation of Metcalfe's law: how internet usage patterns have changed over time. *Information Economics and Policy*, Vol. 25, No. 4, pp.246–256.
13. Malhotra A., Maloo, M. (2014). Bitcoin—is it a Bubble? Evidence from Unit Root Tests. Available at SSRN: <http://ssrn.com/abstract=247637> .
14. McCulloch, J. H. (1986). Simple consistent estimators of stable distribution parameters. *Commun. Statist.: Simul.* 15:1109-1136.

15. Metcalfe, B. (2013). Metcalfe's Law after 40 Years of Ethernet. *Computer*, vol. 46, no. 12, pp. 26-31, 2013.
16. Nakamoto, S. (2008). Bitcoin: A peer-to-peer electronic cash system. Consulted, 1:2012, 2008. Nolan, J. P. (2011). *Stable Distributions - Models for Heavy Tailed Data*. Boston: Birkhauser. Unfinished manuscript, Chapter 1, online at <http://fs2.american.edu/jpnolan/www/stable/chap1.pdf>.
17. Pele, D. T. (2012). An LPPL algorithm for estimating the critical time of a stock market bubble, *Journal of Social and Economic Statistics*, 2(1): 14-22. Available online at [http://jses.ase.ro/downloads/current\\_issue/122.pele.pdf](http://jses.ase.ro/downloads/current_issue/122.pele.pdf)
18. Pele, D. T. (2014). A SAS Approach for Estimating the Parameters of an Alpha-stable Distribution. *Procedia Economics and Finance* 10, 68-77, 2014. Available online at <http://www.sciencedirect.com/science/article/pii/S2212567114002792>.
19. Peterson, T. (2018). Metcalfe's Law as a Model for Bitcoin's Value. *Alternative Investment Analyst Review*, Q2 2018, Vol. 7, No. 2, 9-18. Available at SSRN: <https://ssrn.com/abstract=3078248> or <http://dx.doi.org/10.2139/ssrn.3078248> .
20. Toda, H.Y. & Yamamoto. (1995). Statistical inference in Vector Autoregressions with possibly integrated processes. *Journal of Econometrics*, 66, 225-250.
21. Trimborn, S., Härdle, W.K. (2018). CRIX an Index for Cryptocurrencies, *Empirical Finance*, <https://doi.org/10.1016/j.jempfin.2018.08.004>
22. Van Hove, L. (2014). Metcalfe's law: not so wrong after all. *NETNOMICS: Economic Research and Electronic Networking*, Vol. 15, No. 1, pp.1-8.
23. Van Hove, L. (2016). Metcalfe's law and network quality: an extension of Zhang et al. *Journal of Computer Science and Technology*, Vol. 31, No. 1, pp.117-123.
24. Wheatley, S., Sornette, D., Huber, T., Reppen, M. & Gantner, R. N. (2018). Are bitcoin bubbles predictable? Combining a generalized Metcalfe's law and the LPPLS model. *Swiss Finance Inst. Res. Pap. No. 18-22*. (2018).
25. Zhang, W., Wang P., Li, X. Shen, D. (2018). Some stylized facts of the cryptocurrency market. *Applied Economics*, 50:55, 5950-5965, DOI:10.1080/00036846.2018.1488076.
26. Zhang, X.Z., Liu, J.J. , Xu, Z.W. (2015). Tencent and Facebook data validate Metcalfe's law. *Journal of Computer Science and Technology*, Vol. 30, No. 2, pp.246-251.
27.  [https://github.com/QuantLet/STF-ToDo/tree/master/11/stab\\_reg\\_kw](https://github.com/QuantLet/STF-ToDo/tree/master/11/stab_reg_kw)
28.  [https://github.com/QuantLet/STF-ToDo/tree/master/Qualifying/mc\\_culloch](https://github.com/QuantLet/STF-ToDo/tree/master/Qualifying/mc_culloch)
29. <https://www.blockchain.com>
30. <https://coinmarketcap.com/>
31. <http://thecrix.de/>
32. [https://www.esma.europa.eu/sites/default/files/library/esma50-164-1284\\_joint\\_esas\\_warning\\_on\\_virtual\\_currencies.pdf](https://www.esma.europa.eu/sites/default/files/library/esma50-164-1284_joint_esas_warning_on_virtual_currencies.pdf)

## Appendix A – Estimating the parameters of an alpha-stable distribution

### A.1. Estimating the parameters of an alpha-stable distribution using McCulloch method

McCulloch method (1986) involves the following steps for estimating the parameters of a  $S(\alpha, \beta, \gamma, \delta; 0)$  random variable:

- estimate  $\alpha$  and  $\beta$ , using the quintiles of the empirical distribution (for more details, see Racheva-Iotova, 2010);


- define  $v_\alpha = \frac{x_{0.95} - x_{0.05}}{x_{0.75} - x_{0.25}}$  and  $v_\beta = \frac{x_{0.95} + x_{0.05} - 2x_{0.25}}{x_{0.95} - x_{0.05}}$ , where  $x_p$  is the  $p$ -quintile of the empirical

distribution, having thus  $v_\alpha = \phi_1(\alpha, \beta)$  and  $v_\beta = \phi_2(\alpha, \beta)$  or, by inversion,  $\alpha = \psi_1(v_\alpha, v_\beta)$  and  $\beta = \psi_2(v_\alpha, v_\beta)$ .

More,  $\alpha = \psi_1(v_\alpha, v_\beta) = \psi_1(v_\alpha, -v_\beta)$  and  $\beta = \psi_2(v_\alpha, v_\beta) = -\psi_2(v_\alpha, -v_\beta)$ .

The functions  $\psi_1(\cdot)$  and  $\psi_2(\cdot)$  are tabulated for different values of  $v_a$  and  $v_b$ , so the estimates of  $\alpha$  and  $\beta$  can be obtained using a bi-linear interpolation.

In a quite similar manner, the location parameter  $\delta$  and the scale parameter  $\gamma$  can be estimated using the corresponding tabulated functions and the previous estimations for  $\alpha$  and  $\beta$ .

The code used in this paper for estimating the parameters of an alpha-stable distribution using McCulloch method can be found as the quantlet  **mc\_culloch** on the website [www.quantlet.de](http://www.quantlet.de).

### A.2. Estimating parameters of an alpha-stable distribution using the Kogon-Williams method

In order to estimate the parameters of a stable distribution in parameterisation S1, the following algorithm can be applied (following Kogon and Williams, 1998 and Pele, 2014):


Step 1. Use the initial estimates  $\alpha_0, \beta_0, \gamma_0, \delta_0$  from McCulloch method and normalize the

sample:  $x_j \rightarrow \frac{x_j - \delta_0}{\gamma_0}$ ;

Step 2. Estimate the regression model  $y_k = b + \alpha_1 w_k + \varepsilon_k$ , with  $k = 0, \dots, 9$ ,  $y_k = \ln[-\text{Re}[\ln(\hat{\phi}(u_k))]]$ ,  $w_k = \ln |u_k|$ ,  $u_k = 0.1 + 0.1k$ ,  $k = 0, \dots, 9$ , and  $\hat{\phi}(\cdot)$  is the empirical characteristic function of the normalized sample. If  $\hat{b}$  and  $\hat{\alpha}_1$  are the estimates of the regression model, then the estimate of the scale parameter is  $\hat{\gamma}_1 = \exp(\hat{b} / \hat{\alpha}_1)$ .

Step 3. Estimate the regression model  $z_k = \delta_{11} + \beta_1 v_k + \eta_k$ , with  $k = 0, \dots, 9$ ,  $z_k = \text{Im}[\ln(\hat{\phi}(u_k))]$ ,  $w_k = \hat{\gamma}_1 u_k (|\hat{\gamma}_1 u_k|^{\hat{\alpha}_1 - 1} - 1) \tan(\pi \hat{\alpha}_1 / 2)$ ,  $u_k = 0.1 + 0.1k$ ,  $k = 0, \dots, 9$ .

Step 4. The final estimates are the following:  $(\alpha_1, \beta_1, \gamma_1, \delta_1) = (\hat{\alpha}_1, \hat{\beta}_1, \hat{\gamma}_1, \hat{\delta}_{11} - \hat{\gamma}_1 \hat{\beta}_1 \tan(\pi \hat{\alpha}_1 / 2))$ .

The code used in this paper for estimating the parameters of an alpha-stable distribution using Kogon-Williams method can be found as the quantlet  **stab\_reg\_kw** on the website [www.quantlet.de](http://www.quantlet.de).

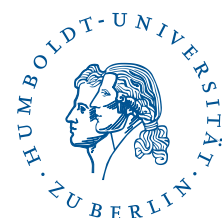
# IRTG 1792 Discussion Paper Series 2018

For a complete list of Discussion Papers published, please visit [irtg1792.hu-berlin.de](http://irtg1792.hu-berlin.de).

- 001 "Data Driven Value-at-Risk Forecasting using a SVR-GARCH-KDE Hybrid" by Marius Lux, Wolfgang Karl Härdle and Stefan Lessmann, January 2018.
- 002 "Nonparametric Variable Selection and Its Application to Additive Models" by Zheng-Hui Feng, Lu Lin, Ruo-Qing Zhu and Li-Xing Zhu, January 2018.
- 003 "Systemic Risk in Global Volatility Spillover Networks: Evidence from Option-implied Volatility Indices " by Zihui Yang and Yinggang Zhou, January 2018.
- 004 "Pricing Cryptocurrency options: the case of CRIX and Bitcoin" by Cathy YH Chen, Wolfgang Karl Härdle, Ai Jun Hou and Weining Wang, January 2018.
- 005 "Testing for bubbles in cryptocurrencies with time-varying volatility" by Christian M. Hafner, January 2018.
- 006 "A Note on Cryptocurrencies and Currency Competition" by Anna Almosova, January 2018.
- 007 "Knowing me, knowing you: inventor mobility and the formation of technology-oriented alliances" by Stefan Wagner and Martin C. Goossen, February 2018.
- 008 "A Monetary Model of Blockchain" by Anna Almosova, February 2018.
- 009 "Deregulated day-ahead electricity markets in Southeast Europe: Price forecasting and comparative structural analysis" by Antanina Hryshchuk, Stefan Lessmann, February 2018.
- 010 "How Sensitive are Tail-related Risk Measures in a Contamination Neighbourhood?" by Wolfgang Karl Härdle, Chengxiu Ling, February 2018.
- 011 "How to Measure a Performance of a Collaborative Research Centre" by Alona Zharova, Janine Tellingner-Rice, Wolfgang Karl Härdle, February 2018.
- 012 "Targeting customers for profit: An ensemble learning framework to support marketing decision making" by Stefan Lessmann, Kristof Coussement, Koen W. De Bock, Johannes Haupt, February 2018.
- 013 "Improving Crime Count Forecasts Using Twitter and Taxi Data" by Lara Vomfell, Wolfgang Karl Härdle, Stefan Lessmann, February 2018.
- 014 "Price Discovery on Bitcoin Markets" by Paolo Pagnottoni, Dirk G. Baur, Thomas Dimpfl, March 2018.
- 015 "Bitcoin is not the New Gold - A Comparison of Volatility, Correlation, and Portfolio Performance" by Tony Klein, Hien Pham Thu, Thomas Walther, March 2018.
- 016 "Time-varying Limit Order Book Networks" by Wolfgang Karl Härdle, Shi Chen, Chong Liang, Melanie Schienle, April 2018.
- 017 "Regularization Approach for Network Modeling of German EnergyMarket" by Shi Chen, Wolfgang Karl Härdle, Brenda López Cabrera, May 2018.
- 018 "Adaptive Nonparametric Clustering" by Kirill Efimov, Larisa Adamyan, Vladimir Spokoiny, May 2018.
- 019 "Lasso, knockoff and Gaussian covariates: a comparison" by Laurie Davies, May 2018.

**IRTG 1792, Spandauer Straße 1, D-10178 Berlin**  
**<http://irtg1792.hu-berlin.de>**

This research was supported by the Deutsche  
Forschungsgemeinschaft through the IRTG 1792.





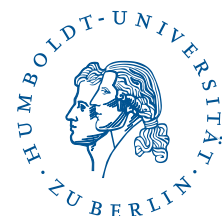
# IRTG 1792 Discussion Paper Series 2018

For a complete list of Discussion Papers published, please visit [irtg1792.hu-berlin.de](http://irtg1792.hu-berlin.de).

- 020 "A Regime Shift Model with Nonparametric Switching Mechanism" by Haiqiang Chen, Yingxing Li, Ming Lin and Yanli Zhu, May 2018.
- 021 "LASSO-Driven Inference in Time and Space" by Victor Chernozhukov, Wolfgang K. Härdle, Chen Huang, Weining Wang, June 2018.
- 022 " Learning from Errors: The case of monetary and fiscal policy regimes" by Andreas Tryphonides, June 2018.
- 023 "Textual Sentiment, Option Characteristics, and Stock Return Predictability" by Cathy Yi-Hsuan Chen, Matthias R. Fengler, Wolfgang Karl Härdle, Yanchu Liu, June 2018.
- 024 "Bootstrap Confidence Sets For Spectral Projectors Of Sample Covariance" by A. Naumov, V. Spokoiny, V. Ulyanov, June 2018.
- 025 "Construction of Non-asymptotic Confidence Sets in 2 -Wasserstein Space" by Johannes Ebert, Vladimir Spokoiny, Alexandra Suvorikova, June 2018.
- 026 "Large ball probabilities, Gaussian comparison and anti-concentration" by Friedrich Götze, Alexey Naumov, Vladimir Spokoiny, Vladimir Ulyanov, June 2018.
- 027 "Bayesian inference for spectral projectors of covariance matrix" by Igor Silin, Vladimir Spokoiny, June 2018.
- 028 "Toolbox: Gaussian comparison on Euclidian balls" by Andzhey Koziuk, Vladimir Spokoiny, June 2018.
- 029 "Pointwise adaptation via stagewise aggregation of local estimates for multiclass classification" by Nikita Puchkin, Vladimir Spokoiny, June 2018.
- 030 "Gaussian Process Forecast with multidimensional distributional entries" by Francois Bachoc, Alexandra Suvorikova, Jean-Michel Loubes, Vladimir Spokoiny, June 2018.
- 031 "Instrumental variables regression" by Andzhey Koziuk, Vladimir Spokoiny, June 2018.
- 032 "Understanding Latent Group Structure of Cryptocurrencies Market: A Dynamic Network Perspective" by Li Guo, Yubo Tao and Wolfgang Karl Härdle, July 2018.
- 033 "Optimal contracts under competition when uncertainty from adverse selection and moral hazard are present" by Natalie Packham, August 2018.
- 034 "A factor-model approach for correlation scenarios and correlation stress-testing" by Natalie Packham and Fabian Woebbecking, August 2018.
- 035 "Correlation Under Stress In Normal Variance Mixture Models" by Michael Kalkbrener and Natalie Packham, August 2018.
- 036 "Model risk of contingent claims" by Nils Detering and Natalie Packham, August 2018.
- 037 "Default probabilities and default correlations under stress" by Natalie Packham, Michael Kalkbrener and Ludger Overbeck, August 2018.
- 038 "Tail-Risk Protection Trading Strategies" by Natalie Packham, Jochen Papenbrock, Peter Schwendner and Fabian Woebbecking, August 2018.

**IRTG 1792, Spandauer Straße 1, D-10178 Berlin**  
**<http://irtg1792.hu-berlin.de>**

This research was supported by the Deutsche  
Forschungsgemeinschaft through the IRTG 1792.



# IRTG 1792 Discussion Paper Series 2018

For a complete list of Discussion Papers published, please visit [irtg1792.hu-berlin.de](http://irtg1792.hu-berlin.de).

- 039 "Penalized Adaptive Forecasting with Large Information Sets and Structural Changes" by Lenka Zbonakova, Xinjue Li and Wolfgang Karl Härdle, August 2018.
- 040 "Complete Convergence and Complete Moment Convergence for Maximal Weighted Sums of Extended Negatively Dependent Random Variables" by Ji Gao YAN, August 2018.
- 041 "On complete convergence in Marcinkiewicz-Zygmund type SLLN for random variables" by Anna Kuczmaszewska and Ji Gao YAN, August 2018.
- 042 "On Complete Convergence in Marcinkiewicz-Zygmund Type SLLN for END Random Variables and its Applications" by Ji Gao YAN, August 2018.
- 043 "Textual Sentiment and Sector specific reaction" by Elisabeth Bommers, Cathy Yi-Hsuan Chen and Wolfgang Karl Härdle, September 2018.
- 044 "Understanding Cryptocurrencies" by Wolfgang Karl Härdle, Campbell R. Harvey, Raphael C. G. Reule, September 2018.
- 045 "Predicative Ability of Similarity-based Futures Trading Strategies" by Hsin-Yu Chiu, Mi-Hsiu Chiang, Wei-Yu Kuo, September 2018.
- 046 "Forecasting the Term Structure of Option Implied Volatility: The Power of an Adaptive Method" by Ying Chen, Qian Han, Linlin Niu, September 2018.
- 047 "Inferences for a Partially Varying Coefficient Model With Endogenous Regressors" by Zongwu Cai, Ying Fang, Ming Lin, Jia Su, October 2018.
- 048 "A Regime Shift Model with Nonparametric Switching Mechanism" by Haiqiang Chen, Yingxing Li, Ming Lin, Yanli Zhu, October 2018.
- 049 "Strict Stationarity Testing and GLAD Estimation of Double Autoregressive Models" by Shaojun Guo, Dong Li, Muye Li, October 2018.
- 050 "Variable selection and direction estimation for single-index models via DC-TGDR method" by Wei Zhong, Xi Liu, Shuangge Ma, October 2018.
- 051 "Property Investment and Rental Rate under Housing Price Uncertainty: A Real Options Approach" by Honglin Wang, Fan Yu, Yinggang Zhou, October 2018.
- 052 "Nonparametric Additive Instrumental Variable Estimator: A Group Shrinkage Estimation Perspective" by Qingliang Fan, Wei Zhong, October 2018.
- 053 "The impact of temperature on gaming productivity: evidence from online games" by Xiaojia Bao, Qingliang Fan, October 2018.
- 054 "Topic Modeling for Analyzing Open-Ended Survey Responses" by Andra-Selina Pietsch, Stefan Lessmann, October 2018.
- 055 "Estimation of the discontinuous leverage effect: Evidence from the NASDAQ order book" by Markus Bibinger, Christopher Neely, Lars Winkelmann, October 2018.
- 056 "Cryptocurrencies, Metcalfe's law and LPPL models" by Daniel Traian Pele, Miruna Mazurencu-Marinescu-Pele, October 2018.

**IRTG 1792, Spandauer Straße 1, D-10178 Berlin**  
**<http://irtg1792.hu-berlin.de>**

This research was supported by the Deutsche  
Forschungsgemeinschaft through the IRTG 1792.