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IRTG 1792 Discussion Paper 2018-033



# Optimal contracts under competition when uncertainty from adverse selection and moral hazard are present

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# Optimal contracts under competition when uncertainty from adverse selection and moral hazard are present

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## Abstract

In a continuous-time setting where a risk-averse agent controls the drift of an output process driven by a Brownian motion, optimal contracts are linear in the terminal output; this result is well-known in a setting with moral hazard and – under stronger assumptions – adverse selection. We show that this result continues to hold when in addition reservation utilities are type-dependent. This type of problem occurs in the study of optimal compensation problems involving competing principals.

Keywords: Principal-agent modelling; contract design; stochastic process; stochastic control

## 1 Introduction

The purpose of this note is to show that linear payoffs arise as optimal contracts offered by a principal to agents in a setting with moral hazard, private information and competition for agents. The design of compensation schemes in a principal-agent relationship typically involves a bonus component to incentivize the agent (e.g. employee, worker) to act in the principal's (employer's) interest. The complicated, often highly non-linear form of contracts suggested by economic models is seldom met in reality where bonus schemes encountered are simple and frequently linear in the output generated by the agent. One explanation for this gap between theory and practice lies in the fact that linear compensation schemes are robust in the richer, more diffuse real environment, whereas the optimal contract in a highly stylised economic model fails at the slightest change of model assumptions or parameters.

Against this backdrop, Holmstrom and Milgrom (1987) show that the optimal contract for an agent controlling the drift of an output process driven by a Brownian motion, when the principal observes only the output process, is a linear function of the terminal output. The principal's inability to directly observe the action of the agent is a *moral hazard problem*, implying that the agent's effort cannot be contractually agreed, but instead the agent must be incentivized to exert effort. Restricting the principal's observability to the terminal output, Sung (2005) extends this to a setting where agents have different capabilities that the principal cannot observe or otherwise directly infer (*private information*). In this *adverse selection problem*, it is optimal to offer a menu of linear contracts designed for the different agent types, incentivizing each agent to choose the contract designed for their type (*screening*).

Screening typically provides agent types with different utilities. As a consequence, if there are several principals competing for agents, reservation utilities for agents will be type-dependent (as opposed to the case, for example, where the outside option is not to work at all). This setting is found in the recent literature on competition, e.g. Jullien (2000); Bénabou and Tirole (2016);

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Banner *et al.* (2016). We prove that the linearity of contracts in the Holmstrom-Milgrom model carries over to the case of moral hazard, private information and competition. The proofs use techniques from stochastic control theory, see e.g. Fleming and Rishel (1975).

## 2 The model

The setting is similar to Holmstrom and Milgrom (1987), Schaettler and Sung (1993) and Sung (2005). An agent employed by a principal exerts (costly) effort, which in turn increases her output. The principal observes only the output, consisting of the agent's effort level and random noise (*moral hazard*). Hence, the effort level cannot be contracted and must be incentivized by a bonus scheme. With agents of different capabilities that are unobservable by the principal (*private information*), the principal may find it optimal to offer different contracts specifically designed for each type, and must take care that each contract appeals most to the type that it is designed for. Finally, with several principals competing for agents, reservation utilities are derived from the outside option of being employed by another principal. Just as the utilities offered to agents of different capabilities differ, so do their reservation utilities. These must be taken into account by a principal wishing to employ agents of all types. The principal is hence faced with an optimization problem taking into account random noise, unobservable agent types and type-dependent reservation utilities. This type of problem can be solved using martingale theory to derive a Hamilton-Jacobi-Bellman equation, which in turn leads to constant controls that do not depend on time. First, we formalize the setting and then give the principal's optimization problem in the next section.

A risk-neutral principal employs a risk-averse agent whose preferences are expressed by CARA utility with parameter  $\rho$ , i.e.,  $U(x) = 1 - e^{-\rho x}$ . The agent's capabilities  $\theta \in \{\theta_L, \theta_H\}$  are private information. The probabilities  $\alpha$ , resp.  $1 - \alpha$ , of meeting an  $H$ -type, resp.  $L$ -type, agent are known to the principal.

A type- $k$  agent, by exerting effort  $\mu = (\mu_t)_{t \geq 0}$ , which is assumed to be bounded, controls the drift of an output process with dynamics

$$dZ_t = \mu_t \theta_k dt + \sigma dW_t, \quad t \geq 0,$$

with  $\sigma > 0$  and where  $W = (W_t)_{t \geq 0}$  is a Brownian motion independent of  $\theta$ . The agent observes the Brownian motion  $W$ , so that  $\mu$  is adapted to the filtration generated by the Brownian motion. The agent's effort is subject to an instantaneous cost  $c(\mu_t)$ ,  $t \geq 0$ , where  $c(0) = 0$ ,  $c$  is strictly increasing, convex and continuously differentiable. In addition, we require  $c''' \geq 0$  for Proposition 2. We shall assume that  $\theta_H > \theta_L$ , expressing that a type- $H$  agent generates a higher drift at equal effort cost than a type- $L$  agent.

The principal observes the output process  $Z = (Z_t)_{t \geq 0}$ , but neither  $W$  nor  $\mu$ , so the compensation, realised at time 1, can be contingent on  $(Z_t)_{0 \leq t \leq 1}$  only. At time 0, the principal offers contracts, consisting of sharing rules  $\{S((Z_t)_{t \in [0,1]}, \theta_k), k \in \{H, L\}\}$ , where  $S(\cdot, \theta_k)$  denotes the sharing rule *designed* for a type- $k$  agent. An agent choosing contract  $S(\cdot, \theta_k)$  at time 0, receives at time 1

$$S((Z_t)_{0 \leq t \leq 1}, \theta_k) - \int_0^1 c(\mu_t) dt,$$

while the principal receives

$$Z_1 - S((Z_t)_{0 \leq t \leq 1}, \theta_k).$$

### 3 The principal's problem

Denote by  $S(\cdot, \theta_m)$  the contract designed for an  $m$ -type agent. An agent of type  $k \in \{H, L\}$ , when choosing contract  $S(\cdot, \theta_m)$  exerts effort  $\mu^{k,m} = (\mu_t^{k,m})_{t \geq 0}$  and derives certainty equivalent  $w_{k,m}$  at time 1. Whenever  $k = m$ , we write  $\mu^k$  and  $w_k$ . The principal's problem is as follows:

**Problem 1.** Choose controls  $\{\mu^H, \mu^L\}$  and a menu of contracts  $\{S(\cdot, \theta_H), S(\cdot, \theta_L)\}$  maximising

$$\alpha \mathbb{E} [Z_1^H - S((Z_t^H)_{0 \leq t \leq 1}, \theta_H)] + (1 - \alpha) \mathbb{E} [Z_1^L - S((Z_t^L)_{0 \leq t \leq 1}, \theta_L)],$$

subject to

$$(1) \quad dZ_t^k = \mu_t^k \theta_k dt + \sigma dW_t, \quad k \in \{H, L\},$$

$$(2) \quad \mu_t^{k,m} \in \operatorname{argmax}_{(\mu_t)_{0 \leq t \leq 1}} \mathbb{E} \left[ U \left( S((Z_t)_{0 \leq t \leq 1}, \theta_m) - \int_0^1 c(\mu_t) dt \right) \right], \text{ where } dZ_t = \mu_t \theta_k dt + \sigma dW_t, \text{ and } k, m \in \{H, L\},$$

$$(3) \quad \mathbb{E} \left[ U \left( S((Z_t^k)_{0 \leq t \leq 1}, \theta_k) - \int_0^1 c(\mu_t^k) dt \right) \right] \geq \mathbb{E} \left[ U \left( S((Z_t^{k,m})_{0 \leq t \leq 1}, \theta_m) - \int_0^1 c(\mu_t^{m,k}) dt \right) \right], \text{ where } dZ_t^{k,m} = \mu_t^{k,m} \theta_k dt + \sigma dW_t \text{ and } m, k \in \{H, L\}, \quad (\text{ICC})$$

$$(4) \quad \mathbb{E} \left[ U \left( S((Z_t^k)_{0 \leq t \leq 1}, \theta_k) - \int_0^1 c(\mu_t^k) dt \right) \right] \geq U(w_k), \text{ where } k \in \{H, L\}, \quad (\text{PC}).$$

The first constraint defines the dynamics of the output processes of each type when choosing the contract designed for her. The second constraint expresses that agents maximise their expected utility. The third constraint, the incentive compatibility constraint (ICC), makes each agent optimally choose the contract designed from her. Finally, the fourth constraint, the participation constraint (PC), ensures that an agent contracts with the principal instead of choosing her outside option with certainty equivalent  $w_k$  (the general results do not change if it were unprofitable to attract a particular agent type).

### 4 The agent's choice of drift

We consider the agent's problem when faced with a menu of contracts. The following result is slightly adapted from Holmstrom and Milgrom (1987).

**Theorem 1.** The adapted stochastic process  $(\mu_t)_{0 \leq t \leq 1}$  is implemented with certainty equivalent  $w$  by a type- $k$  agent by a sharing rule  $S((Z_t)_{0 \leq t \leq 1}, \theta_k)$  only if

$$S((Z_t)_{0 \leq t \leq 1}, \theta_k) = w + \int_0^1 c(\mu_t) dt + \int_0^1 \frac{c'(\mu_t)}{\theta_k} dZ_t - \int_0^1 c'(\mu_t) \mu_t dt + \frac{\rho}{2} \int_0^1 \left( \frac{c'(\mu_t)}{\theta_k} \right)^2 \sigma dt. \quad (1)$$

*Proof.* See Theorem 6 of Holmstrom and Milgrom (1987) and Corollary 4.1 of Schaettler and Sung (1993).  $\square$

The first two terms provide a certainty equivalent of  $w$  and a direct compensation of the effort cost should the agent choose to exert effort  $(\mu_t)_{0 \leq t \leq 1}$ . The third term incentivizes the agent to choose effort level  $\mu$ . The last two terms compensate the agent for the mean and risk of the output process, i.e., they correspond to the certainty equivalent of the third term.

**Proposition 1.** *A type- $k$  agent, when choosing the contract designed for the  $m$ -type agent, derives expected utility*

$$\mathbb{E} \left[ U \left( w_m + \int_0^1 c'(\mu_t^{k,m}) \mu_t^{k,m} - c(\mu_t^{k,m}) - (c'(\mu_t^m) \mu_t^m - c(\mu_t^m)) dt \right) \right], \quad (2)$$

where  $\mu^{k,m}$  denotes the  $k$ -type agent's optimal control, which solves

$$c'(\mu_t^{k,m}) = \frac{\theta_k}{\theta_m} c'(\mu_t^m). \quad (3)$$

Furthermore, the  $H$ -type agent exerts greater effort and derives greater utility from a given contract than the  $L$ -type agent.

*Proof.* Let  $S((Z_t)_{0 \leq t \leq 1}, \theta_m)$  be the contract designed for an  $m$ -type agent offering  $w_m$  and implementing  $\mu^m$ . A type- $k$  agent implementing  $\mu$ , that is,  $dZ_t = \mu_t \theta_k dt + \sigma dW_t$ ,  $t \geq 0$ , derives expected utility

$$\mathbb{E} \left[ U \left( w_m + \int_0^1 c(\mu_t^m) - c(\mu_t) dt + \int_0^1 \frac{c'(\mu_t^m)}{\theta_m} dZ_t - \int_0^1 c'(\mu_t^m) \mu_t^m - \frac{\rho}{2} \left( \frac{c'(\mu_t^m)}{\theta_m} \right)^2 \sigma^2 dt \right) \right],$$

which can be written as  $\mathbb{E}[U(X_1^\mu)]$  with state process

$$\begin{aligned} X_u^\mu := w_m u + \int_0^u c(\mu_t^m) - c(\mu_t) dt + \int_0^u \frac{c'(\mu_t^m)}{\theta_m} \sigma dW_t \\ + \int_0^u c'(\mu_t^m) \mu_t \frac{\theta_k}{\theta_m} - c'(\mu_t^m) \mu_t^m + \frac{\rho}{2} \left( \frac{c'(\mu_t^m)}{\theta_m} \right)^2 \sigma^2 dt. \end{aligned}$$

By the Itô formula,

$$dX_t^\mu = \left[ w_m + c(\mu_t^m) - c(\mu_t) + c'(\mu_t^m) \left( \mu_t \frac{\theta_k}{\theta_m} - \mu_t^m \right) + \frac{\rho}{2} \left( \frac{c'(\mu_t^m)}{\theta_m} \right)^2 \sigma^2 \right] dt + \frac{c'(\mu_t^m)}{\theta_m} \sigma dW_t.$$

Define  $J(t, x; \mu) = \mathbb{E}[U(X_1^\mu) | X_t^\mu = x]$ , which is once (twice) continuously differentiable in  $t(x)$  (applying Dominated Convergence for differentiating inside the expectation operator), so that  $J(0, x; \mu) = \mathbb{E}[U(X_1^\mu)]$  corresponds to the objective function. Because  $J$  is a martingale, the following PDE holds:

$$J_t + \frac{1}{2} \left( \frac{c'(\mu_t^m)}{\theta_m} \right)^2 \sigma^2 J_{xx} + \left[ w_m + c(\mu_t^m) - c(\mu_t) + c'(\mu_t^m) \left( \mu_t \frac{\theta_k}{\theta_m} - \mu_t^m \right) + \frac{\rho}{2} \left( \frac{c'(\mu_t^m)}{\theta_m} \right)^2 \sigma^2 \right] J_x = 0,$$

where  $J_t, J_x, J_{xx}$  denote the respective partial first- and second-order derivatives. Setting  $V(t, x) = \sup_\mu J(t, x; \mu)$ , the Hamilton-Jacobi-Bellman PDE is

$$V_t + \frac{1}{2} \sigma^2 V_{xx} + \sup_\mu \left[ c'(\mu_t^m) \frac{\theta_k}{\theta_m} \mu_t - c(\mu_t) \right] V_x + \left[ c(\mu_t^m) - c'(\mu_t^m) \mu_t^m + \frac{\rho}{2} \left( \frac{c'(\mu_t^m)}{\theta_m} \right)^2 \sigma^2 \right] V_x = 0,$$

with boundary condition  $V(1, x) = U(x)$ . The agent's Hamiltonian is given by

$$\mathcal{H} = -c(\mu_t) + c'(\mu_t^m) \mu_t \frac{\theta_k}{\theta_m},$$

leading to the optimal effort choice  $\mu_t^{k,m}$  fulfilling the FOC (3).

If  $k = H$  and  $m = L$ , the FOC expresses that the  $H$ -type agent exerts greater marginal effort on the  $L$ -type's contract than the  $L$ -type. Moreover, because of the convexity of  $c$ , it follows that  $\mu_t^{H,L} > \mu_t^L$ . Conversely, if  $k = L$  and  $m = H$ , then  $\mu_t^{L,H} < \mu_t^H$ .

The  $k$ -type agent's expected utility is given by (2). Because  $c$  is strictly increasing and convex, the mean value theorem implies  $c'(x)x > c(x)$ , and

$$\frac{d}{dx} [c'(x)x - c(x)] = c''(x)x > 0, \quad x > 0.$$

Therefore, when  $m = H$  and  $k = L$ , the integral is strictly positive, and the  $H$ -type agent derives a greater utility from the  $L$ -type's contract than the  $L$ -type. Conversely, if  $m = L$  and  $k = H$ , the  $L$ -type derives a smaller utility from the  $H$ -type's contract than the  $H$ -type.  $\square$

The classical result is obtained that if both types' reservation utilities are equal, then the  $H$ -type has an incentive to imitate if second-best contracts were offered (i.e., contracts with moral hazard only), in which case the contract designed for the  $L$ -type needs to be distorted to prevent the  $H$ -type from imitating (e.g. Salanié (2005)). If reservation utilities are type-dependent, then the situation may be reversed, and the  $H$ -type's contract may need to be distorted to prevent the  $L$ -type from imitating.

## 5 Optimal contracts

Turning to Problem 1, we restrict the analysis to the case where the  $H$ -type has an imitation incentive; the case when the difference of the types' reservation utilities is sufficiently large for the  $L$ -type to have an imitation incentive is treated in a similar way. We omit the proof of the following well-known results when the  $H$ -type has an imitation incentive (e.g. Salanié (2005)): The  $L$ -type's ( $PC$ ) is binding (constraint (4) in Problem 1; to attract the  $L$ -type) and the  $H$ -type's ( $ICC$ ) is binding (constraint (3) in Problem 1; to prevent the high type from imitating), while the  $L$ -type's ( $ICC$ ) is non-binding. The  $H$ -type's contract features the second-best (constant) drift rate  $\mu^{H,*}$  (there is no reason to deviate from the optimum), while the effort level  $\mu^{L,*}$  in the contract for the  $L$ -type is distorted to prevent the  $H$ -type from imitating.

The principal thus solves

$$\sup_{\mu^H, \mu^L, S(\cdot, \theta_H), S(\cdot, \theta_L)} \mathbb{E} [\alpha(Z_1^H - S((Z_t^H)_{0 \leq t \leq 1}, \theta^H)) + (1 - \alpha)(Z_1^L - S((Z_t^L)_{0 \leq t \leq 1}, \theta^L))],$$

subject to

$$(1) \quad \mathbb{E} \left[ U \left( S((Z_t^L)_{0 \leq t \leq 1}, \theta^L) - \int_0^1 c(\mu_t^L) dt \right) \right] = U(w_L); \quad (PCL)$$

$$(2) \quad \mathbb{E} \left[ U \left( S((Z_t^{H,L})_{0 \leq t \leq 1}, \theta^L) - \int_0^t c(\mu_t^{H,L}) dt \right) \right] = \mathbb{E} [U(S((Z_t^H)_{0 \leq t \leq 1}, \theta^H) - c(\mu^{H,*}))]$$

$$= \mathbb{E} \left[ 1 - \exp \left( - \underbrace{\rho \left( w_L + \int_0^1 c'(\mu_t^{H,L}) \mu_t^{H,L} - c(\mu_t^{H,L}) - [c'(\mu_t^L) \mu_t^L - c(\mu_t^L)] dt \right)}_{=: u_1(\mu^L)} \right) \right],$$

$$\text{with } \mu_t^{H,L} \text{ given by Equation (3);} \quad (ICCH)$$

$$(3) \quad \mathbb{E} [U(S((Z_t^H)_{0 \leq t \leq 1}, \theta^H) - c(\mu^{H,*}))] \geq \mathbb{E}[U(w_H)]; \quad (PCH)$$

**Proposition 2.** *Under adverse selection, moral hazard and when reservation utilities are type-dependent, the optimal effort levels  $\mu^{H,*}$ ,  $\mu^{L,*}$  in the contracts designed for the  $H$ -type, resp.  $L$ -type agent are constant, and optimal contracts are linear in the terminal outputs  $Z_1^H$  and  $Z_1^L$ .*

In the proof it is shown that if (PCH) is non-binding, then  $\mu^{L,*}$  satisfies

$$c'(\mu^{L,*}) = \left( \theta_L - \frac{\alpha}{1-\alpha} \left\{ c''(\mu^{H,L,*}) \mu^{H,L,*} \frac{\partial}{\partial \mu^{L,*}} \mu^{H,L,*} - c''(\mu^{L,*}) \mu^{L,*} \right\} \right) \left( 1 + \frac{\rho \sigma^2}{\theta_L^2} c''(\mu^{L,*}) \right)^{-1}, \quad (4)$$

By the assumption that  $c''' \geq 0$  it follows directly that the optimal control is smaller than the second best optimal control without adverse selection (which is obtained when  $\alpha = 0$ ). If (PCH) is binding, then  $\mu^{L,*}$  satisfies

$$w_H - w_L = c'(\mu^{H,L,*}) \mu^{H,L,*} - c(\mu^{H,L,*}) - [c'(\mu^{L,*}) \mu^{L,*} - c(\mu^{L,*})]. \quad (5)$$

*Proof.* First, assume that (PCH) is non-binding (this case arises when the difference between reservation utilities is small or zero). From (ICCH), where the right-hand side depends only on the effort level incentivized by the principal, it follows that the  $H$ -type's certainty equivalent is  $-\frac{\ln \mathbb{E}[e^{-\rho u_1(\mu^L)}]}{\rho}$ .

Using Theorem 1, the conditions imply

$$\begin{aligned} S((Z_t^L)_{0 \leq t \leq 1}, \theta_L) \\ = w_L + \int_0^1 c(\mu_t^L) dt + \int_0^1 \frac{c'(\mu_t^L)}{\theta_L} dZ_t^L - \int_0^1 c'(\mu_t^L) \mu_t^L dt + \frac{\rho}{2} \int_0^1 \left( \frac{c'(\mu_t^L)}{\theta_L} \right)^2 \sigma^2 dt \end{aligned} \quad (6)$$

and

$$\begin{aligned} S((Z_t^H)_{0 \leq t \leq 1}, \theta_H) \\ = \frac{-\ln \mathbb{E}[e^{-\rho u_1(\mu^L)}]}{\rho} + c(\mu^{H,*}) + \frac{c'(\mu^{H,*})}{\theta_H} Z_1^H - c'(\mu^{H,*}) \mu^{H,*} + \frac{\rho}{2} \left( \frac{c'(\mu^{H,*})}{\theta_H} \right)^2 \sigma^2. \end{aligned} \quad (7)$$

Setting

$$\begin{aligned} X_t^{\mu^L} := & \alpha \left( Z_t^H - \left\{ -\frac{\ln \mathbb{E}[e^{-\rho u_t(\mu^L)}]}{\rho} + c(\mu^{H,*})t + \frac{\rho}{2} \left( \frac{c'(\mu^{H,*})}{\theta_H} \right)^2 \sigma^2 t + \frac{c'(\mu^{H,*})}{\theta_H} \sigma W_t \right\} \right) \\ & + (1-\alpha) \left( Z_t^L - \left\{ w_L t + \int_0^t c(\mu_u^L) du + \frac{\rho}{2} \int_0^t \left( \frac{c'(\mu_u^L)}{\theta_L} \right)^2 \sigma^2 du + \int_0^t \frac{c'(\mu_u^L)}{\theta_L} \sigma dW_t \right\} \right), \end{aligned}$$

with  $dZ_t^L = \mu_t^L \theta_L dt + \sigma dW_t$  and  $dZ_t^H = \mu^{H,*} \theta_H dt + \sigma dW_t$ , the principal's problem is

$$\sup_{\mu^L} \mathbb{E}[X_1^{\mu^L}].$$

The dynamics of  $X^{\mu^L}$  are

$$\begin{aligned} dX_t^{\mu^L} = & \alpha \left( \left\{ \mu^{H,*} \theta_H + \frac{\partial}{\partial t} \frac{\ln \mathbb{E}[e^{-\rho u_t(\mu^L)}]}{\rho} - c(\mu^{H,*}) - \frac{\rho}{2} \left( \frac{c'(\mu^{H,*})}{\theta_H} \right)^2 \sigma^2 \right\} dt \right. \\ & \left. + \left( 1 - \frac{c'(\mu^{H,*})}{\theta_H} \right) \sigma dW_t \right) \\ & + (1-\alpha) \left( \left\{ \mu_t^L \theta_L - w_L - c(\mu_t^L) - \frac{\rho}{2} \left( \frac{c'(\mu_t^L)}{\theta_L} \right)^2 \sigma^2 \right\} dt + \left( 1 - \frac{c'(\mu_t^L)}{\theta_L} \right) \sigma dW_t \right), \end{aligned}$$



and the principal's Hamiltonian is

$$\mathcal{H} = \alpha \frac{\partial \ln \mathbb{E}[\mathbf{e}^{-\rho u_t(\mu^L)}]}{\partial t} \frac{1}{\rho} + (1 - \alpha) \left\{ \mu_t^L \theta_L - c(\mu_t^L) - \frac{\rho}{2} \left( \frac{c'(\mu_t^L)}{\theta_L} \right)^2 \sigma^2 \right\},$$

with

$$\frac{\partial \ln \mathbb{E}[\mathbf{e}^{-\rho u_t(\mu^L)}]}{\partial t} \frac{1}{\rho} = - \frac{\mathbb{E} \left[ \mathbf{e}^{-\rho u_t(\mu^L)} \left\{ w_L + c'(\mu_t^{H,L}) \mu_t^{H,L} - c(\mu_t^{H,L}) - [c'(\mu_t^L) \mu_t^L - c(\mu_t^L)] \right\} \right]}{\mathbb{E}[\mathbf{e}^{-\rho u_t(\mu^L)}]}. \quad (8)$$

Equation (8) describes the change in certainty equivalent offered to the  $H$ -type agent, including the information rent to prevent her from imitating. This is  $\mathcal{F}_0$ -measurable, i.e., fixed at time 0. By the principle of optimality, the optimal change in certainty equivalent does not depend on any particular time  $t$ ; likewise the optimal control does not depend on the particular time  $t$ , so that Equation (8) is constant. This is fulfilled for a deterministic and constant control. A constant optimal control is necessary as well, as the Hamiltonian is optimised by a deterministic choice of  $\mu_t^L$ , which is constant by the principle of optimality. Hence, (8) becomes

$$\frac{\partial \ln \mathbb{E}[\mathbf{e}^{-\rho u_t(\mu^L)}]}{\partial t} \frac{1}{\rho} = - \{ w_L + c'(\mu^{H,L}) \mu^{H,L} - c(\mu^{H,L}) - [c'(\mu^L) \mu^L - c(\mu^L)] \},$$

and the Hamiltonian simplifies to

$$\mathcal{H} = -\alpha \left\{ \underbrace{c'(\mu^{H,L})}_{=c'(\mu^L)\theta_H/\theta_L} \mu^{H,L} - c(\mu^{H,L}) - [c'(\mu^L) \mu^L - c(\mu^L)] \right\} + (1 - \alpha) \left\{ \mu^L \theta_L - c(\mu^L) - \frac{\rho}{2} \left( \frac{c'(\mu^L)}{\theta_L} \right)^2 \sigma^2 \right\}.$$

The optimum is determined via

$$\frac{\partial}{\partial \mu^L} \mathcal{H} = -\alpha \left\{ \frac{\theta_H}{\theta_L} c''(\mu^L) \mu^{H,L} - \frac{\partial}{\partial \mu^L} \mu^{H,L} - c''(\mu^L) \mu^L \right\} + (1 - \alpha) \left\{ \theta_L - c'(\mu^L) - \frac{\rho \sigma^2}{\theta_L^2} c'(\mu^L) c''(\mu^L) \right\},$$

which is zero if  $\mu^{L,*}$  fulfills Equation (4), requiring this is nonnegative. It is easily verified that this is a minimum by  $c''' \geq 0$ . The sharing rules (6) and (7) depend only on  $Z_1$  instead of  $(Z_t)_{0 \leq t \leq 1}$  and are linear in  $Z_1$ .

If ( $PCH$ ) is binding, then

$$\mathbb{E}[\mathbf{e}^{-\rho u_1(\mu^L)}] = \mathbf{e}^{-\rho w_H},$$

and

$$d\mathbb{E}[\mathbf{e}^{-\rho u_t(\mu^L)}] = d\mathbf{e}^{-\rho w_H t} = -\rho w_H \mathbf{e}^{-\rho w_H t} dt.$$

The left-hand side is therefore an expectation of an exponential accruing at a constant rate. Since  $u_1(\mu^L)$  is comprised of a constant and an integral with respect to time, by the principle of optimality, the exponent itself must be constant. Hence  $\mu^{L,*}$  is constant. Furthermore,  $\mu^{L,*}$  solves the binding participation constraint, which can be expressed as in Equation (5).  $\square$

## References

- C. E. Banner, E. Feess, and N. Packham. Incentive schemes, private information and the double-edged role of competition for agents. Working Paper, June 2016.
- R. Bénabou and J. Tirole. Bonus culture: Competitive pay, screening, and multitasking. *Journal of Political Economy*, 124(2):305–370, 2016.
- W. H. Fleming and R. W. Rishel. *Deterministic and Stochastic Optimal Control*. Springer, 1975.
- B. Holmstrom and P. Milgrom. Aggregation and linearity in the provision of intertemporal incentives. *Econometrica*, 55:303–328, 1987.
- B. Jullien. Participation constraints in adverse selection models. *Journal of Economic Theory*, 93(1):1–47, 2000.
- B. Salanié. *The Economics of Contracts*. MIT Press, 2nd edition, 2005.
- H. Schaettler and J. Sung. The first-order approach to the continuous-time principal–agent problem with exponential utility. *Journal of Economic Theory*, 61(2):331–371, 1993.
- J. Sung. Optimal contracts under adverse selection and moral hazard: a continuous-time approach. *Review of Financial Studies*, 18(3):1021–1073, 2005.

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