Kirstein, Roland; Kirstein, Annette

Working Paper

Inefficient Intra-Firm Incentives Can Stabilize Cartels in Cournot Oligopolies.

CSLE Discussion Paper, No. 2004-09

Provided in Cooperation with:
Saarland University, CSLE - Center for the Study of Law and Economics


This Version is available at:
http://hdl.handle.net/10419/23069

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Inefficient Intra-Firm Incentives Can Stabilize Cartels in Cournot Oligopolies.

Roland Kirstein* Annette Kirstein**

Center for the Study of Law and Economics
Discussion Paper 2004-09
September 2004

Abstract

The need for intra-firm incentive schemes allows remodeling the Cournot duopoly in wages (rather than in output levels). In both versions of the Cournot model, a cartel agreement is unstable. The new formulation, however, allows us to demonstrate that a collective wage agreement on minimum wages can stabilize the cartel solution. Beyond its relevance for strategic management, this result has a policy implication: competition authorities should observe collective wage agreements for their potential collusive effect on product markets. Moreover, the model may provide a new explanation why firms in reality pay lower than efficient variable wages and higher fixed wages than predicted by contract theory.

JEL classification: C72, C78, D43, J33, J50, K31, L41
Encyclopedia of Law and Economics: 5550, 5300, 0550
Keywords: Principal-agent theory, piece rate, fixed wage, collective wage agreements, Nash bargaining solution.

*Corresponding author. Center for the Study of Law and Economics (CSLE), University of Saarland, D-66041 Saarbrücken, Germany (email: rol@mx.uni-saarland.de). We are grateful to Max Albert, Siegfried Berninghaus, Hans Gerhard, Veronika Nolte, Clemens Puppe, Roseanna Ravsin, Dieter Schmidtchen and Birgit Will for helpful comments. Remaining errors, however, remain our responsibility.

**Sonderforschungsbereich 504 “Bounded Rationality”, University Mannheim, and Institut für Wirtschaftstheorie and Operations Research, University of Karlsruhe, D-76128 Karlsruhe, Germany, (email: Kirstein@wiwi.uni-karlsruhe.de).
## Contents

1 Introduction ........................................... 3

2 Related literature ...................................... 4

3 The model ............................................... 7
   3.1 Setup ............................................. 7
   3.2 Duopoly wages .................................... 8
      3.2.1 The agents’ choices ......................... 8
      3.2.2 Contract offers in the duopoly .......... 9
   3.3 Cartel wages and the instability problem ..... 10
      3.3.1 Optimal piece rates in the cartel ....... 10
      3.3.2 Incentives to deviate from a collective wage agreement 12
   3.4 The collective wage agreement ................. 15

4 Conclusion ............................................. 16
1 Introduction

This paper analyzes how cartels can be stabilized by collective wage agreements that introduce inefficient intra-firm incentives. Cartel agreements in Cournot oligopolies suffer from an inherent instability: no cartel solution is a Nash equilibrium. Thus, each competitor has an incentive to choose an output greater than his cartel quota. Moreover, cartel agreements are in general not legally enforceable.\(^1\)

This analysis rests on the assumption that production takes place in the absence of intra-firm conflicts. In our paper, we acknowledge that intra-firm conflicts which result from delegation and moral hazard problems may exist in oligopolistic firms. These conflicts are modeled as principal-agent problems. The principal is unable to choose the firm’s output directly, but can influence it by choosing appropriate variable payments (“piece rates”) and fixed wages. Thus, the firm owners face two problems: the instability of a cartel, and the intra-firm conflict with their respective agents. We demonstrate that these two problems do not reinforce each other. To the contrary, the principals can stabilize their cartel by providing inefficient intra-firm incentives.

In a world with risk neutral agents and risk neutral principals, efficient intra-firm incentives can easily be achieved when the agent is assigned the position of the residual claimant. If the principal has complete bargaining power, he can attain the complete cooperation rent via the fixed wage. A contract that provides a piece rate smaller than the efficient one for the risk neutral agent implements lower than efficient effort. The firm’s output is, c.p., smaller than under a first-best contract.\(^2\)

Consider a two-stage interaction: during the first stage, the Cournot competitors close an agreement about intra-firm incentives. In the second stage, outputs are produced and sold. The firms can, during the first stage, calibrate the intra-firm incentives inefficiently and establish the cartel solution as a Nash equilibrium in the second stage. However, such an agreement is not a subgame perfect equilibrium in the two stage game. Each firm had an incentive to deviate by offering efficient intra-firm incentives to its own agents, even if all other firms did obey the agreement. With respect to this instability problem, there is no difference between an indirect cartel agreement in wages and the direct cartel in output quotas.

\(^{1}\)The shadow of the future may induce cartel agreements as Nash equilibria. In this paper, we focus on one-shot games. In repeated games, however, the non-trivial problem of equilibrium selection arises because the cartel supergame has an infinite number of Pareto-efficient Nash equilibria.

\(^{2}\)A similar result would be achieved if a risk-averse agent is employed instead of a risk neutral one: only second-best solutions are attainable, because a trade-off between risk allocation and incentives cannot be circumvented. A third variation of intra-firm inefficiency would be an organization with many agents: if the compensation for these agents is characterized by a budget-balanced sharing rule, this induces inefficient intra-firm incentives, see Holmstrom (1982).
Yet, there is a simple and legal way to solve the instability problem of the indirect cartel agreement. All Cournot competitors may agree upon an industry wide collective wage agreement with a labor union in order to establish minimum wages and thereby introduce inefficient intra-firm incentives. The important difference between a direct cartel agreement and a collective wage agreement is that the latter can be legally enforced. If the firms have full bargaining power and can make a take-it or leave-it offer, then the indirect cartel agreement enables them to divide the full monopoly profit among them. Even if the Union demands a share of the cartel profit for its members, the Nash bargaining solution would guarantee each competitor a share that is at least as large as its profit under the Cournot solution.

Section 2 presents the related literature. The model we set up in section 3 acknowledges the presence of intra-firm conflicts within each firm. These conflicts have to be solved by incentive contracts. Therefore, we re-model the Cournot duopoly as a game in wages rather than in outputs in section 3.1. We derive the agents' reactions to contract offers, the duopolists' reaction curves in piece rates, and the decentralized Cournot solution in section 3.2. In section 3.3, we demonstrate the monopolistic cartel agreement in wages and its instability. Furthermore, we prove that an enforceable collective agreement on minimum wages may stabilize the cartel solution. Finally, we demonstrate the symmetric Nash bargaining solution between labor union and employers association (in section 3.4). In section 4, we draw conclusions.

2 Related literature

Papers on the principal-agent problem, as well as on oligopoly theory, are too numerous to count. Surprisingly, the number of papers that simultaneously model Cournot competition between firms and the existence of intra-firm conflicts between owners and managers is rather small. In a recent AER paper, RAITH (2003, 1425) has described the two unresolved questions concerning managerial incentives: how they are related to product market competition, and to risk. According to RAITH, in both of these fields further research is required to resolve differences between theory and empirical evidence.

The most prominent idea in the literature on the relation between product market competition and managerial incentives is that the former may serve as a device to discipline managers, and thereby contribute to the solution of the latter. This idea has already been brought about by BERLE/MEANS (1932) and LEIBENSTEIN (1966). HART (1983) has rigorously derived conditions under which increased product market competition can reduce “organizational slack”. The driving force in his model is the information about his agent a principal can gain from observing the competitors. This is also the core idea in NALEBUFF/STIGLITZ (1983). HERMALIN (1992) has extended this approach and

---

In Gal-Or (1995), it is the payment schedule that has an informational (and a strategic)
demonstrated that competition can provide incentives for managers to work harder even if inference from market outcomes is excluded.

The empirical research on the relation between managerial incentives, competition and firm’s performance is, to say the least, mixed. One example of an empirical study is the paper by Nygaard/Myrveit (2000), in which the authors have examined several types of contractual relations in Norway. Gaver/Gaver/Battistel (1992) have not found significant stock market reactions to the introduction of performance payment schemes for top managers. Nickell (1996) acknowledges that there are some theoretical reasons for the idea that product market competition improves corporate performance, but they are “not overwhelming” and the empirical evidence is weak.

Arruñada/Gonzáles (1997) have analyzed the impact of competition on team production. They have set up a dynamic model which describes the how competition between teams influences the effort decisions. The driving force in their model is the ability of honest team members to punish cheaters by leaving the team. The mutual impact of competition and intra-team incentives is the subject of the experimental paper by Bornstein/Gneezy (2002). However, the intra-firm conflict in their paper is represented by two types of coordination games to be played between the firm members. The members of the respective teams play either a “chicken” game or a “coordination” game, while the teams compete in a Bertrand market. However, in their model the intra-firm incentives are exogenously given and constitute the type of the firm. In our model, both market behavior and choice of intra-firm contracts are endogenous.

Another branch of the existing literature in this field is concerned with the macroeconomic effects of imperfect product market competition, see Amable/Gatti (2002), and with its impact on manager employment, see Amable/Gatti (2001), Fee/Hadlock (2000), and Kühn (1994). The impact of profit-sharing on employment has been analyzed by Weitzman (1985), Stewart (1989) and Hart (1990).

Some other papers deal with isolated aspects of the interplay between intra-firm incentives and competition: Glazer/Israel (1990) have demonstrated that management compensation schemes can serve as a signaling mechanism on the product market. Toulemonde (1999) has observed that wages may deter potential competitors from market entry. Richards (1983) points out that wage-spillovers should be taken into account when analyzing a market which is characterized by a dominant firm and some smaller competitors. Goering/Harikumar (1999) describe how managers’ incentives to invest in long- and short-run projects are afflicted by competition. Aggarwal/Samwick (1999) come to the result that strategic interaction between the firms accounts for the empirical lack of compensation schemes that are based upon relative performance.
A paper which is close to ours is Sklivas (1987).\(^5\) The author asks whether firms in an oligopoly actually choose intra-firm incentives so as to maximize profits. His results differ substantially from ours: regarding quantity competition, he derives that the existence of intra-firm conflicts implement market outcomes that are more competitive than those in the Cournot model without intra-firm conflicts. This result is driven by his assumption that contracts cannot be made contingent on quantity outcomes. In his model, the agent’s payment depends on a linear combination of the firm’s profit and returns.

The same assumption has been made by Fershtmann/Judd (1987). The most striking result of these two papers is that, in their framework, it is not optimal for principals in Cournot competition to offer their agents incentive contracts which are contingent on profit maximization only. The optimal contract, from the principals point of view, offer rewards which are contingent on a linear combination of profit and returns. Both papers, however, do not make explicit the incentive mechanism, and they neglect its impact on the firms’ costs. In our paper, we explicitly model the incentive problem and include the agents’ wages into the firms’ cost functions. Furthermore, both papers overlook the collusive role a collective wage agreement can play, which is the subject of our model.

A further difference between these papers and ours is the nature of the intra-firm contracts. Their contracts offer the respective agent a linear combination of a share of the return and a share of the profit. Such a compensation scheme perhaps reflects the situation of a top manager. As a consequence, the agents act strategically interdependent. In our model, the principals offer their agents a combination of fixed wage and piece rate. This compensation scheme is rather adequate to describe the situation of a worker. The agents in our model, therefore, act independently from each other. Strategic interaction only occurs between the principals and the respective agents, between the principals in the product market, and between the principals and the labor union.

Another paper which appears to be close to ours at the first glance is Bensaïd/Gary-Bobo (1991). In their model, however, profit-sharing contract explicitly plays no role with respect to intra-firm incentives. Effort costs within the firm are assumed to be zero. Therefore, profit-sharing contracts are only an alternative to fixed wages for satisfying the participation constraint. They focus on the role of profit-sharing as a commitment device that may establish or stabilize strategic alliances. As in our model, the resulting game between the Cournot oligopolists has a prisoners’ dilemma structure: even if it would be beneficial to the industry as a whole to use fixed wage contracts only, each oligopolist’s best reply is to use profit-sharing. If, however, each oligopolist makes use of profit sharing, then each accrues a lower profit than under the fixed wage scheme.

The title of Haucap/Pauly/Wey (2001) could have been our title as well, but their model highlights a different anti-competitive aspect of collective wage setting than our model does. They start with two types of firms in one industry,

\(^{5}\) Reitman (1993) has chosen the same approach to analyze the impact of stock-options as managerial compensation.
one type produces with a labor-intensive technology, the other operates capital-intensive. The latter type might find it beneficial to agree upon high wages in a collective wage agreement, and to lobby for legally enforced inclusion of all firms into this agreement. This raises rivals’ costs and increases the own market share. In our model, the firms are homogeneous, and it is the wage structure (piece rate vs. fixed wage) that plays the crucial role.

We explicitly take the intra-firm conflict between owner and agent into account and model the effort decision acknowledging effort costs. Thus, our model provides a link between wage scheme and the respective firm’s output. The prisoner’s dilemma in our model does not reflect suboptimal behavior in oligopoly, but incentives to deviate from a cartel agreement.

3 The model

3.1 Setup

Consider a market with two firms, labeled 1 and 2. They use labor as the only input factor. The amount of labor employed by firm \( i \) is denoted \( e_i; i \in \{1; 2\} \).

The production functions are \( Y_i(e_i) = e_i \) where \( Y_i \) is the amount of output produced by firm \( i \). Providing the effort level \( e_i \) causes costs \( c_i(e_i) = e_i^2 \). The total amount produced in the market is denoted as \( Y \), with \( Y = Y_1 + Y_2 \). The consumers’ inverse demand is \( p = a - Y \) with \( a > 0 \), where \( p \) represents the price charged by the firms.

We will refer to the firm owners as principals \( P_1 \) and \( P_2 \), and to the respective input providers as agents \( A_1 \) and \( A_2 \). No other firms or agents may enter the game. We assume all players to be risk neutral. Intra-firm incentive schemes contingent on effort are excluded by the assumption that effort is not verifiable.\(^6\)

We limit our view to fixed wages and piece rates. If a contract offer is rejected, then the respective agent receives his outside option \( u \geq 0 \).\(^7\)

The two firms have, in principle, three ways to interact with each other:

- they can choose their strategies without coordination. We analyze this duopoly game in section 3.2.
- They can try to coordinate their behavior without making use of an institutional framework to stabilize this coordination. The resulting cartel game is analyzed in section 3.3. We first derive the piece rate a cartel would set in order to maximize its joint profit, and we compute the additional profit generated by the cartel solution. Our analysis shows that

\(^6\)The actual output may depend on effort and a random variable, e.g., \( Y_i = e_i + \eta_i \) with \( E(\eta_i) = 0 \) and \( \sigma^2(\eta_i) > 0 \). If players are risk neutral, and limited liability problems can be neglected (as it is assumed here), then each random variable can be replaced by its expected value.

\(^7\)For technical reasons we assume \( u < a^2/5 \), only to guarantee that duopolists may find it interesting at all to engage in this market.
this cartel solution is not an equilibrium, but can be stabilized by an enforceable minimum wage agreement.

• Finally, the firms can negotiate with a labor union about a collective minimum wage agreement in the first place. We analyze the third game in section 3.4. Without such an agreement, the players enter the duopoly game which is, hence, a subgame of the third game and constitutes the threat points of the bargaining parties. With an agreement, the parties enjoy the benefits of the cartel solution derived in the second game. However, as the negotiations about the collective wage agreement involve the labor union, the cartel has to share the additional profit with its employees when applying the Nash bargaining solution to the first stage of the game.

We assume collective wage agreements to be perfectly enforceable: it is, therefore, impossible to deviate downwards if such an agreement prevails (the collective wage agreement imposes minimum wages). However, each employer is free to offer a higher compensation parameter to his agent.

3.2 Duopoly wages

In this section, we analyze uncoordinated behavior between the two firms. The interaction takes place in three stages.

• The two principals $P_i, i \in \{1; 2\}$ offer contracts $(F_i^D, w_i^D)$ to their respective agent $A_i$. The index $D$ indicates the duopoly situation.

• Each agent $A_i$ decides whether to accept the contract offer of his principal. If he rejects it, he receives his outside option $u$ and the respective principal produces nothing.

• If, on the other hand, agent $A_i$ accepts the contract offer by his principal $P_i$, he chooses his effort $e_i$. The output $Y_i(e_i)$ is produced, the market price for the good is determined, the total output is sold, and payoffs are paid.

The Cournot solution in wages is derived by backwards induction. First, we determine the optimal reaction of the respective agent to his principal’s contract offer. The effort choice of the respective agent determines the firm’s output. Then we analyze the wage setting game between the two firms (each of them anticipating its agent’s reaction).

3.2.1 The agents’ choices

In the last stage, each of the agents $A_i$ faces the following maximization problem: $e_i^* = \arg \max F_i + w_i Y_i(e_i) - c_i(e_i)$. With $Y_i = e_i$ and $c_i = e_i^2$, the first-order
condition for an internal maximum is \( w_i = 2e_i \). Hence, the optimal effort reaction of each agent to a wage offer by his respective principal \( w_i \) is

\[
e_i^*(w_i) = \frac{w_i}{2}.
\]

In the third stage, each agent decides whether to accept the respective contract. Anticipating his own later effort reaction, agent \( i \) expects, when accepting the contract, a payoff that amounts to \( F_i + w_iY_i[e_i^*(w_i)] - c_i[e_i^*(w_i)] \). Using the production function and effort cost, this equals \( F_i - w_i^2/4 \). Agent \( A_i \) accepts if this payoff exceeds his outside option \( u \). Therefore,

\[
F_i(w_i) = u - \frac{w_i^2}{4} \tag{2}
\]

is the minimal fixed wage that obeys the participation constraint of agent \( A_i \).

### 3.2.2 Contract offers in the duopoly

Using equations (1) and (2), the choice problem of each principal in stage 2 of our game can be reduced to one in piece rates \( w_i \) and \( w_j \). In the subgame without a collective wage agreement, both principals do not have to obey legal constraints when choosing their piece rate offer. They anticipate that at least a fixed wage of \( F_i = u - (w_i^2/4) \) is required to make an offer \( w_i \) acceptable for their respective agent. Moreover, they anticipate that this offer will implement \( e_i = w_i/2 \) as their agent’s effort choice. Thus, the profit function of principal \( i \) in the Cournot duopoly subgame is \( \Pi_i^D(w_i, w_j) = [p(e_i(w_i) + e_j(w_j)) - c_i[e_i^*(w_i)] - F_i \) with \( i, j \in \{1; 2\} \) and \( i \neq j \). This yield function can, by making use of the results derived above, be rewritten as

\[
\Pi_i^D(w_i, w_j) = \left[ a - \frac{6w_i + w_j}{2} - w_i \right] \frac{w_i}{2} - \left( u - \frac{w_i^2}{4} \right).
\]

Each firm chooses its piece rate in order to maximize its profit. The first-order condition for firm \( i \), given an internal solution exists, is \( a/2 - w_i - w_j/4 = 0 \). Hence, the optimal reaction of firm \( i \) to the other firm’s choice \( w_j \) is

\[
w_i^D(w_j) = \frac{2a - w_j}{4}. \tag{3}
\]

Substituting \( w_j(w_i) \) into equation (3) yields the Cournot duopoly solution \( w_i^D = w_j^D = 2a/5 \). The corresponding minimal fixed wages are \( F_i^D = u - a^2/25 \).

By offering \((F_i^D, w_i^D)\), both firms implement an individual output of \( e_i^D = a/5 \). The market output, thus, amounts to \( Y^D = 2a/5 \), and the market price is \( p^D = 3a/5 \). Each firm’s profit then accrues to \( \Pi_i^D = 2a^2/25 - u \). The agents receive their outside option \( u \).

In equilibrium, the return of firm \( i \) is computed as \( R_i^D = (a - e_i^D - e_j^D)e_i^D \). The marginal return then is \( MR_i^D = a - e_j^D - 2e_i^D \). Anticipating both agents’
reactions, the marginal return can also be expressed in wages, namely \( MR_i^D = a - w_i - w_j / 2 \). Substitution of the equilibrium piece rates yields \( MR_i^D = 2a/5 \). In equilibrium, both firms therefore choose piece rates equal to their marginal returns. Thus, the piece rate is set efficiently, seen from the individual’s perspective.

### 3.3 Cartel wages and the instability problem

In this section we analyze the two firms’ attempt to coordinate their behavior without first entering into the institutional framework of a legally enforceable collective wage agreement. We assume that the two firms may agree upon piece rates and fixed wages before making their individual offers to their agents. However, such an agreement is not binding (we introduce enforceable agreements in the next section).

The agents’ effort choices are governed by the same reaction functions as derived above, see equation (1), which is anticipated by the cartel members. Hence, we can limit the analysis to the firms’ choices. We proceed in three steps:

- first, we derive what piece rates a cartel would set in order to generate the maximum profit.
- Then we show that this cartel solution is not an equilibrium.
- Finally we demonstrate that an enforceable minimum wage agreement would stabilize the cartel solution.

#### 3.3.1 Optimal piece rates in the cartel

A monopolist who produces in two production sites with increasing marginal costs has a profit function:

\[
\Pi^C = [p(e_i(w_i) + e_j(w_j)) - w_i]e_i - F_i + [p(e_i(w_i) + e_j(w_j)) - w_j]e_j - F_j.
\]

The anticipated reactions of the two agents employed by the cartel are the same as derived above. Therefore, we can simplify the cartel’s profit to

\[
\Pi^C = \frac{a(w_i + w_j) - w_i^2 - w_j^2 - w_i w_j}{2} - 2u.
\]

The first-order conditions for an internal solution are \((a - 2w_i - w_j)/2 = 0\) and \((a - 2w_j - w_i)/2 = 0\). A cartel planner has to choose \((w_i, w_j)\) such that these conditions are simultaneously fulfilled. Imagine the planner considers a certain value of \(w_j\); his optimal “reaction” \(w_i\) is described by

\[
w_i^C(w_j) = \frac{a - w_j}{2}.
\]
The planner’s equilibrium is characterized by the intersection of the planning curves \( w_C^i(w_j) \) and \( w_C^j(w_i) \). Compare the planning curve to the reaction curve of a Cournot duopolist (see equation 3 above) who picks his piece rate in a decentralized choice. It is obvious that both sets of curves (in a \( w_i-w_j \)-diagram) have the same intercept, namely \( a/2 \), but different slopes: the cartel planning curves are steeper than the Cournot reaction curves. The relation between these two sets of curves is depicted in figure 1.

Figure 1: Cournot reaction and cartel planning

The profit maximizing piece rates for both production sites of the cartel is \( w_C^i = w_C^j = a/3 \). The corresponding minimum fixed wage that induces the agents to accept their contract offer is, for both firms, \( F_C^i = u - a^2/36 \). Hence, \( F_C^i > F_D^i \).

A contract offer \( (F_C^i, w_C^i) \) induces efforts \( e_C^i = e_C^j = a/6 \). Thus, the cartel produces \( Y_C = a/3 \) as its total output. The market price then is \( p_C = 2a/3 \), and the individual profits of the two cartel members amount to \( \Pi_C^i = 43a^2/450 - u \). As members of the cartel, the two firms collect higher profits than the Cournot
duopolists: $\Pi^C_i > \Pi^D_i$. In the cartel optimum, each firms’ revenues are computed as

$$MR^C_i = a - \frac{w^C_i}{2} - w^C_i = a/2.$$  

Obviously, the cartel solution is characterized by $MR^C_i > w^C_i$. The marginal revenue of each firm exceeds the piece rate offered to its agent. Individually efficient would be a piece rate equal to the marginal revenue. In this sense, the cartel would agree upon wages that create inefficient intra-firm incentives. Table 1 compares the results of the decentralized Cournot model and the centralized cartel planning.

<table>
<thead>
<tr>
<th></th>
<th>Cournot duopoly</th>
<th>cartel</th>
<th>comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_i$</td>
<td>$2a/5$</td>
<td>$a/3$</td>
<td>$w^D_i &gt; w^C_i$</td>
</tr>
<tr>
<td>$u - a^2/25$</td>
<td>$u - a^2/36$</td>
<td>$F^C_i &gt; F^D_i$</td>
<td></td>
</tr>
<tr>
<td>$e_i$</td>
<td>$a/5$</td>
<td>$a/6$</td>
<td>$e^D_i &gt; e^C_i$</td>
</tr>
<tr>
<td>$Y$</td>
<td>$2a/5$</td>
<td>$a/3$</td>
<td>$Y^D &gt; Y^C$</td>
</tr>
<tr>
<td>$p$</td>
<td>$3a/5$</td>
<td>$2a/3$</td>
<td>$p^D &lt; p^C$</td>
</tr>
<tr>
<td>$\Pi_i$</td>
<td>$2a^2/5 - u$</td>
<td>$43a^2/450 - u$</td>
<td>$\Pi^C_i = \Pi^D_i = 7a^2/450 &gt; 0$</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>$4a^2/25 - 2u$</td>
<td>$43a^2/225 - 2u$</td>
<td>$MR^C_i &gt; w^C_i$</td>
</tr>
<tr>
<td>$MR_i(e_i)$</td>
<td>$2a/5$</td>
<td>$a/2$</td>
<td>$MR^D_i = w^D_i$; $MR^C_i &gt; w^C_i$</td>
</tr>
</tbody>
</table>

3.3.2 Incentives to deviate from a collective wage agreement

We have demonstrated three results so far:

- the Cournot duopoly game in outputs can be restated as a game in wages if the firms face an intra-firm incentive problem. The piece rate determines the respective firm’s (expected) output.
- In the duopoly situation, the firms under consideration would choose a piece rate $w^D_i = 2a/5$, while the cartel would pay a smaller piece rate, namely $w = a/3$, and thereby reduce the collective output.
- Even though the cartel members are required to pay a higher fixed wage to their agents than in the Cournot oligopoly, cartelization would increase the group profit by $\Pi^C - \Pi^D = 7a^2/450 > 0$.

The desirability of the cartel solution, however, does not imply that it is stable. As it is shown in figure 1, the optimal cartel choice does not lie on the Cournot reaction curves. Moreover, the cartel piece rate is smaller than each cartel member’s marginal revenue. Hence, both cartel members have an
incentive to deviate upwards from the cartel agreement in the second stage of the game.

In this section we demonstrate that a collective agreement about minimum wages forms an effective obstacle against such deviation, even though it allows for upwards deviations. A cartel member who tries to offer a higher piece rate simultaneously wishes to pay a lower fixed wage which, however, is prohibited by the collective wage agreement. We demonstrate that no incentive exists to increase the piece rate without decreasing the fixed wage.

Figure 2 illustrates the intra-firm incentives. First of all, the figure shows the participation constraint of agent $A_i$. The participation constraint consists of $F_i-w_i$ combinations which leave the agent with an (expected) payoff equal to $u$. It is a downward sloped curve with intercept $u$ at the $F_i$-axis. The agent prefers $F_i-w_i$-combinations above the participation constraint, as indicated by the tiny arrow.

Furthermore, figure 2 shows the cartel solution (point C) and the duopoly solution (point D). In addition to this, the area to the north-east of C depicts the compensation parameter combinations the firms are allowed to offer under a collective minimum wage agreement (this area is called “permitted deviations from C”).

The last component of figure 2 is the iso-profit curve of firm $P_i$ that represents its individual profit level in the cartel solution. Denote this profit level as $\Pi^C_i$. In general, a firm’s iso-profit curve in a $F_i-w_i$-diagram for the profit level $\Pi_i$ is given by

$$F_i = \frac{2a - w_j}{4} w_i - \frac{3}{4} w_i^2 - \Pi_i.$$  

The cartel situation is characterized by $w_j = a/3$ and $\Pi_i = \Pi^C_i$, and the iso-profit curve of $P_i$ can be simplified to

$$F_i = \frac{5a}{12} w_i - \frac{3}{4} w_i^2 - \Pi^C_i.$$  

The first derivative of this iso-profit curve with respect to $w_i$ is

$$\frac{\partial F_i}{\partial w_i} = \frac{5a}{12} - \frac{3}{2} w_i.$$  

If the other firm sets the cartel wage $w_j = a/3$, then the iso-profit curves of $P_i$ have their maximum at $\bar{w}_i = 5a/18$. Note that $\bar{w}_i < w^C_i$. For $w_i > 5a/18$, the iso-profit curve representing $\Pi^C_i$ has negative slope, but is flatter than the agent’s participation constraint, and the two curves intersect in point C, the optimal cartel combination.

Principal $P_i$ prefers wage combinations below his iso-profit curves, as the tiny arrow indicates. Hence, the iso-profit curve and the participation constraint open up a “lens” between them which contains wage combinations that are bilaterally beneficial, compared to the collective wage agreement, for the
principal and his agent. Even though the collective wage agreement maximizes their joint profit, both firms have an incentive to deviate and generate a higher individual profit, combined with a negative externality that burdens the other firm.

A unilateral deviation is only attractive if it consists of an increased piece rate and a lowered fixed wage - a move towards south-east in figure 2. The collective minimum wage agreement, however, only allows the principals to deviate towards north-east. The only intersection between the lens and permitted deviations is the point C itself. The collective wage agreement, therefore, effectively implements a “fixed wage brake” against the temptation to deviate.
3.4 The collective wage agreement

In the previous sections we have demonstrated that a cartel agreement with $F^C_i$ and $w^C_i$ attains a monopoly solution and, thereby, maximizes the firms’ value. Furthermore, it is stable if downwards deviations are effectively prohibited. Hence, it generates an agreement rent between the parties of the collective wage agreement. In this section, we derive the symmetric Nash-bargaining solution.\(^8\) The game now consists of four stages:

1. The employers’ association representing firms $P_1$ and $P_2$ bargains with the labor union (which represents workers $A_1$ and $A_2$) over a collective wage agreement. Such an agreement consists of a minimum fixed wage, denoted $F^B$, and a minimum piece rate $w^B$ (the index $B$ denotes that these contract parameters are the result of a bargaining process). This stage may end with or without an agreement.

2. The two principals separately offer “their” agent a contract $(F_i, w_i)$. If no agreement was concluded during the first stage, the players enter the Cournot duopoly game analyzed in section 3.2. The outcome of this game, therefore, constitutes the threat points of the bargaining parties. If, on the other hand, a collective wage agreement $(F^B, w^B)$ was closed, then each individual offer must obey the constraints $F_i \geq F^B$ and $w_i \geq w^B$.

3. Each agent chooses whether to accept the offer made by “his” principal. If an agent rejects his principal’s offer, he earns his outside option $u_i \geq 0$, and the respective principal produces nothing.

4. If $A_1$ has accepted the contract, he chooses his effort $e_i$. $Y_i(e_i)$ is produced, the market price for the good is determined, the produced amounts of the good are sold, and payoffs are paid to all the agents and the principals.

We have already demonstrated in section 3.3 that a stable cartel agreement increases the joint payoff. According to the Nash bargaining solution, this agreement rent is distributed between the negotiating parties via a fixed wage that may exceed the minimal fixed wage in the cartel situation: $F^B \geq F^C_i$. The piece rate, on the other hand, is not subject to negotiations, since only $w^B = w^C_i$ maximizes the firms’ joint profit, which is a prerequisite to satisfy the axiom of Pareto-superiority.

$$(F^B - F^C_i) \geq 0$$

is the additional fixed wage each firm pays under the collective wage agreement. The union’s share of the agreement rent, therefore, amounts to $2(F^B - F^C_i)$. The employers association’s share is computed as follows: each firm receives the cartel profit minus the additional rent, and gives up the disagreement payoff, namely the duopoly profit. Hence, the employers’ share is

\(^8\)Alternative concepts to this model of collective wage negotiations would be the “monopoly labor union” or the “right-to-manage” approach, see the survey in Espinosa/Rhee (1989). For the main results of our paper, the solution concept applied to the bargaining stage is immaterial.
Therefore, the symmetric Nash bargaining solution can be derived as

\[
\hat{F}_B = \text{arg max } 4(\Pi^C_i - (F^B - F^C_i)) - \Pi^D_i)\pi^B - F^C_i).
\]

The first-order condition for an interior solution is \(\Pi^C_i - \Pi^D_i + 2(F^C_i - F^B) = 0\), and the optimum is attained at

\[
\hat{F}_B = \frac{\Pi^C_i - \Pi^D_i}{2} + F^C_i.
\]

Substitution of the results derived above (see table 1) yields \(F^B = u - a^2/50\), which exceeds \(F^C_i\). The individual profit of each firm is smaller than in a cartel without the collective wage agreement (\(\Pi^B_i < \Pi^C_i\)), but exceeds the individual duopoly profit: \(\Pi^B_i > \Pi^D_i\).

Figure 3 demonstrates the situation under a collective wage agreement \((F^B, w^B)\), represented by point B. As the collective wage agreement consists of a higher fixed wage, the derivation area is shifted upwards, compared to the cartel solution. The lower profit level corresponds with a higher iso-profit curve (for \(w_j = a/3\)). Since the agent’s participation constraint remains unmodified, the lens becomes greater. Nevertheless, as figure 3 shows, the only intersection between lens and the permitted deviations area is the point B itself. Thus, there is no bilaterally beneficial deviation which is permitted under the collective wage agreement.

4 Conclusion

We have set up a model that combines a Cournot duopoly with intra-firm conflicts in the context of a simple moral hazard model. We have derived the Cournot equilibrium in wages. We also have derived the wages an enforceable cartel (with two production sites and convex marginal costs) would choose. The cartel would choose lower variable wages, in order to reduce output, and pay a higher fixed wage.

The cartel wage structure, however, does not constitute a subgame perfect equilibrium. Insofar, the wage cartel does not differ from the production quota cartel. But there is one important difference between these two settings: while the quota cartel is illegal, a collective wage agreement can legally be enforced. The distribution of the cartel rent between the two firms and a labor union according to the Nash bargaining solution requires a side payment to the benefit of the union (or its members). This side payment may take the form of a higher fixed wage.

The principal-agent model we have employed here is rather simple, yet sufficient to derive the basic insights. There are many options to enrich the model.
Figure 3: Intra-firm incentives with collective wage agreement

E.g., we could introduce risk-aversion on the side of the agent. However, this would only reinforce the derived results.

The insights of this paper are relevant for strategic management considerations. They also may contribute a solution to the “fixed wage puzzle”: while economic theory strongly favors variable payment, fixed wages are omnipresent in the real world. According to our results, firms in an oligopoly situation have an incentive to pay higher fixed and lower variable wages than isolated firms. Such an incentive structure would appear inefficient when the analysis neglects the strategic competition on the product market.

Moreover, the results of this paper are relevant for economic policy, and in particular for regulation of competition. Cartel authorities should not only look at direct cartel agreements when trying to identify illegal collusive behavior. Collective wage agreements may also be suspicious, in particular if they provide intra-firm incentives that appear inefficient at the first glance.
References


