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A Real-Business-Cycle model with search-and-matching frictions and efficiency (“fair”) wages

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Abstract

In this paper we combine two important ingredients - search and matching frictions and “fair wages,” and investigate their combined quantitative importance in explaining fluctuations in Bulgarian labor markets. Overall, the calibrated real-business-cycle model for Bulgaria after the introduction of the currency board, when featuring both search-and-matching frictions, and reciprocity in labor relations, performs better vis-a-vis data, and in addition dominates the model with search frictions only (Vasilev 2016a), and setups with efficiency wages alone (Vasilev 2017a,b). The search and matching mechanism captures the volatility of unemployment, while the efficiency wage mechanism fits better the cyclicalities of the wage rate.

Keywords: general equilibrium, reciprocity, gift exchange, efficiency wages, unemployment, Bulgaria

JEL Classification Codes: E24, E32, J41

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1 Introduction and Motivation

In this paper, we combine two important mechanisms that were separately shown to be quantitatively important for business cycle fluctuations in Bulgaria, namely search frictions in the labor market, e.g. Vasilev (2016a), and the setup with efficiency wages of the "gift exchange" type as in Vasilev (2017a). Each of the two setups had pros and cons. More specifically, the model with search and matching frictions generates volatility in unemployment as in data, while the setup with reciprocity in labor relations generates cyclical behavior in wages as in data.

The first novelty is that, in contrast to Malholmson and Mavroeidis (2007), who combine search frictions with efficiency wages of no-shirking type a la Shapiro and Stiglitz (1984), here we use efficiency wages from the "gift exchange" literature. Introduction of reciprocity considerations is an important real rigidity in the model, and as demonstrated in Vasilev (2017a) for the case of Bulgaria, that real rigidity amplifies the propagation of exogenous shocks. In addition, Malholmson and Mavroeidis (2007) also argue that "adding efficiency wages to matching frictions may be a better way to fit the data than simply searching for an alternative wage formulation." This has to do with the fact that the wage rate, determined as an outcome from Nash bargaining does not fit data well, a fact acknowledged by Shimer (2010) himself. Therefore, instead of searching for an ad hoc way of modelling the determination of the wage rate, we take a more structural approach and introduce the reciprocity mechanism in the model. Note that the efficiency wage rate in the augmented setup will be shown to depend on the degree of market tightness, which will show in the firm's optimality conditions.

Combining the two mechanisms also allows the model to distinguish between the different types of unemployment. For example, Michailat (2012) argues that in recessions matching frictions are of little importance for labor market performance. What is more important, as in Michailat (2012) turns out to be the rationing unemployment, or the unemployment due to the shortage of jobs in the absence of matching frictions.¹ Our paper also adds to the literature by extending Martin and Wang (2018), by adding physical capital and government sector, and then carefully simulating the dynamic model and compare it vis-a-vis

¹Note that efficiency wages are above-equilibrium rates, which leads to involuntary unemployment.

data, while Martin and Wang (2018) only focus on the steady-state. Thus, we meet some of the limitations acknowledges in Martin and Wang (2018). In addition, we setup the problem from explicit micro-foundations, and explicitly show the aggregation.²

The paper proceeds to evaluate the quantitative importance of search and matching frictions and efficiency wages in the case of the Bulgarian business cycle after the introduction of the currency board arrangement, which, when complemented with other reforms, brought aggregate stability to the economy. Those real rigidities in the labor markets introduce history dependence in the employment status, which makes employment, unemployment, and output more persistent. Such real rigidities in the labor market could be thus regarded as a qualitatively important propagation mechanism that can replicate data behavior, especially along the labor market dimension.

In the model, the efficiency wage channel is quantitatively stronger, and the efficiency wage unemployment makes the majority of the unemployment rate, and that component is immune of business cycle fluctuations. In other words, informational frictions connected to effort observation are more important quantitatively than search and marching frictions. This result is important from policy perspectives, as the model suggests that active labor market policies aiming to solve the effort problem should be the priority, while policies aiming at lowering search and matching frictions are of little help in Bulgaria.

Overall, the search and matching model, and augmented with reciprocity in labor relations, generates persistence in output and both employment and unemployment, and is able to respond to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996), who argue that the RBC model does not have a strong internal propagation mechanism besides the strong persistence in the TFP process. Finally, the dynamic correlation between vacancies and unemployment in Bulgaria is also well-captured by the model.

²Last, but not least, Martin and Wang (2018) they refer to Shapiro and Stiglitz (1984) in the discussion of efficiency wages, while the efficiency wages they use in the setup are more in the gift-exchange spirit of Akerloff (1982) and Akerloff and Yellen (1990), and introduced in general-equilibrium in Vasilev (2017a).

The rest of the paper is organized as follows. Section 2 describes the model setup. Setup 3 outlines the model parameterization and the calibration strategy employed. Section 4 presents the steady state results. Section 5 discusses the impulse responses, compares simulated to empirical moments and evaluates the model’s overall goodness-of-fit. Section 6 concludes.

1.1 Households

There is a continuum of identical one-member households distributed on the $[0, 1]$ interval and indexed by i . Each household i derives utility out of consumption and leisure. As in Vasilev (2017a), household i ’s expected discounted total utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t^i + \ln(1 - h_t^i) - h_t^i \left[\frac{(e_t^i)^2}{2} - \mathfrak{R}(e_t^i, \cdot) \right] \right\}, \quad (1)$$

where E_0 is the expectation operator conditional on the information available to household i at time 0, $0 < \beta < 1$ denotes the discount factor, c_t^i is consumption of household i in period t , h_t^i is the fraction of time available to household i that is spent working, and e_t^i is the level of effort exerted. The total time endowment available to each household i is normalized to unity, thus leisure, $l_t = 1 - h_t$ is implicitly expressed as time off work.

Next, as in Danthine and Kurmann (2010), and Vasilev (207a), $\mathfrak{R}(e_t^i, \cdot)$ utility term is included to capture that workers may derive utility out of ”reciprocal behavior towards their employer.” As long as $\mathfrak{R}_e(e_t^i, \cdot) > 0$, household i would be willing to reward a wage that is perceived to be above the competitive one (even in the absence of any direct material gain resulting from such an action) with a level of effort above the required minimum (say, zero).³

As in Hansen (1985) and Rogerson (1988) household i ’s labor supply is assumed to be indivisible, i.e. $h_t^i \in \{0, \bar{h}\}, \forall t$. In equilibrium, only a fraction n_t would be selected by the firm to work a full shift in each period t . In order to Pareto-improve the consumption bundle

³Note that this way of modelling such a bilateral ”gift exchange” between a worker and employer is consistent with earlier works, e.g., Akerlof (1982) and Rabin (1993).

received by both workers and non-workers, a lottery market can be included to provide insurance against unemployment (i.e., not being selected for work) in certain period. Such an arrangement would achieve full insurance (efficient risk sharing), so everyone would receive the same consumption independent of the employment status. If we assume that all households pool their resources together and maximize aggregate welfare, the resulting discounted utility function becomes⁴

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + n_t \left[\ln(1 - \bar{h}) - \frac{(e_t)^2}{2} + \mathfrak{R}(e_t, \cdot) \right] \right\}. \quad (2)$$

Each household starts with $k_0^i = k_0$ initial capital, which is equal to the aggregate capital in period 0. Aggregate capital stock then evolves as follows:

$$k_{t+1} = i_t + (1 - \delta)k_t \quad (3)$$

where $0 < \delta < 1$ denotes the depreciation rate on capital. The before-tax rental rate on capital is r_t , and in addition the households have legal claim on all the firm's profit (π_t).

In addition to capital income, households receive labor income as well. The hourly wage rate in the economy is w_t , so the total before-tax labor income generated in each period is $w_t n_t \bar{h}$. In addition, the labor market is characterized by two-sided search frictions, and aggregate employment will be assumed to evolve according to the following law of motion:⁵

$$n_{t+1} = (1 - \psi)n_t + p_t(1 - n_t), \quad (4)$$

where $0 \leq \psi \leq 1$ denotes the transition rate from employment to unemployment, and $p_t \geq 0$ denotes the probability of a match in period t , which depends on the tightness of the labor market. Households take the probability p_t at which the aggregate search effort produces a match as given. The aggregate household's budget constraint is then

$$(1 + \tau^c)c_t + k_{t+1} - (1 - \delta)k_t = (1 - \tau^y)w_t n_t \bar{h} + (1 - \tau^y)r_t k_t + \pi_t + g_t^t, \quad (5)$$

where τ^c is the consumption tax rate, τ^y is the common (labor and capital) income tax rate, and g_t^t are aggregate government transfers. The problem now is to maximize aggregate

⁴For the detailed derivation of this result, see Vasilev (2018).

⁵In this model we do not distinguish between unemployed and out of the labor force.

utility subject to the aggregate budget constraint. The first-order optimality conditions are as follows:

$$c_t : \frac{1}{c_t} = \Lambda_t(1 + \tau^c) \quad (6)$$

$$e_t : e_t = \mathfrak{R}_e(e_t, \cdot) \quad (7)$$

$$n_{t+1} : -\ln(1 - \bar{h}) + \frac{(e_{t+1})^2}{2} - \mathfrak{R}(e_{t+1}, \cdot) = \Lambda_{t+1}(1 - \tau^y)w_{t+1}\bar{h} \quad (8)$$

$$k_{t+1} : \Lambda_t = \beta E_t \Lambda_{t+1} [1 + (1 - \tau)r_{t+1} - \delta] \quad (9)$$

$$TVC : \lim_{t \rightarrow \infty} \Lambda_t k_{t+1} = 0, \quad (10)$$

where Λ_t is the Lagrangian multiplier attached to the household's budget constraint. The first optimality condition equates the marginal utility of consumption to the marginal utility of wealth. The second condition is called the Effort Condition (EC), or Solow (1979) condition. The third condition describes the optimal employment decision. We can think of it as determining the labor supply. However, in this case choosing employment is a dynamic problem, as the value of a match extends to more than one period. Therefore, the aggregate household will also balance the intertemporal costs and benefits at the margin. The net benefit to the household is the discounted payoff from working, i.e., the after-tax income, plus the utility value of the gift consideration, minus the disutility from working. The fourth is the so called Euler equation, which describes the optimal allocation of capital in any two adjacent periods. The last condition, the Transversality condition (TVC), is a boundary condition that needs to be imposed to eliminate explosive solutions.

1.2 Reciprocity

As in Vasilev (2017a) and Danthine and Kurmann (2010), we represent the reciprocity $\mathfrak{R}(e_t, \cdot)$ term in the household's utility function as a product of the mutual "gifts" of an employed household and the representative firm:

$$\mathfrak{R}(e_t, \cdot) = d(e_t)g(w_t), \quad (11)$$

where $d(e_t)$ denote the gift of the employed household towards the firm, expressed in terms of effort exerted, and $g(w_t, \cdot)$ is the counter-gift of the firm to the worker in terms of the wage rate paid. Both terms are assumed to be concave in their respective arguments, i.e.

$d_e(e_t) > 0, d_{ee}(e_t) < 0$ and $g_w(w_t) > 0, g_{ww}(w_t) < 0$. Hence, when a worker receives a wage offer that is perceived as generous (e.g. a wage above the competitive rate), i.e. $g(w_t) > 0$, the household's utility increases if there is a reciprocal gift expressed in terms of higher effort, $d(e_t) > 0$.⁶ In defining the two gifts, both are expressed as deviations from some expected norm ("reference level"), and to simplify the analysis, we will normalize the minimum acceptable effort level to be $e_{min} = 0$. The worker's gift then can be expressed as:

$$d(e_t) = e_t^\alpha, \quad (12)$$

with $0 < \alpha < 1$. Next, the firm's gift is as follows:⁷

$$g(w_t) = \left[\frac{w_t - \bar{w}_t}{\bar{w}_t} \right] \zeta, \quad (13)$$

where $\zeta > 0$ is the curvature parameter, and \bar{w}_t defines some reference wage level. In other words, the gift is positive, if the wage received exceeds some reference norm. Next, plugging this expression into the optimal effort condition

$$e_t = \mathfrak{R}_e(e_t, \cdot) = d_e(e_t)g(w_t), \quad (14)$$

where the last equality follows from the assumption that $g(w_t)$ did not vary with e_t . Rearranging terms, the effort supply function is

$$e_t = b \left[\frac{w_t - \bar{w}_t}{\bar{w}_t} \right]^\sigma, \quad (15)$$

where $b = e = \alpha^{\frac{1}{2-\alpha}} > 0$, $\sigma = \frac{\zeta}{2-\alpha} > 0$ is the curvature of the effort function, and \bar{w}_t denotes some reference wage. Next, we can express the wage rate as a time-varying mark-up (and a function of effort) above the reference wage

$$w_t = \bar{w}_t \left[1 + \frac{\sigma e_t}{b} \right], \quad (16)$$

but since effort level is an endogenous variable, we need to pin it down from the firm problem, in order to ultimately express the efficiency wage rule as a function of model parameters.

⁶From the perspective of an atomistic worker, the wage rate is taken as given, that is why $d_w(e_t) = 0$. In addition, employed households do not take into consideration the effect of their (individual) effort on the firm's output, and hence on the gift made by the firm to the worker, i.e., $g_e(w_t) = 0$ from the perspective of an employed household.

⁷The approach follows Summers (1988) and Martin and Wang (2018).

1.3 Firm

There is a representative firm in the setup using a Cobb-Douglas production function,⁸ which uses both capital and labor

$$Y_t = A_t k_t^{1-\alpha} (e_t n_t \bar{h})^\alpha, \quad (17)$$

where $0 < 1 - \alpha < 1$ measures capital share. With search externalities, α is no longer the labor share. Still, the production function features constant returns to scale. The firm chooses how much capital to rent, how many to employ, and how many vacancies to advertise. Firm's problem now becomes dynamic due to the value of the match, and the fact that if a vacancy is filled, then the firm can economize on advertising the position. The advertising cost incurred equals aV_t , $a > 0$. Those are considered as part of production costs, and thus will be deducted from the firm's profit. The firm takes the aggregate law of employment as a constraint when maximizing its discounted profit:

$$n_{t+1} = (1 - \psi)n_t + q_t v_t \quad (18)$$

The firm takes the endogenous probability that a vacancy is filled, $\{q_t\}$, as given.

In contrast to Vasilev (2016), and as in Vasilev (2017a), the firm will also have to set wages.

The first-order optimality conditions are as follows:

$$k_t : (1 - \alpha) \frac{y_t}{k_t} = r_t \quad (19)$$

$$n_{t+1} : \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[\alpha \frac{y_{t+1}}{n_{t+1}} - w_{t+1} \bar{h} + \frac{a(1 - \psi)}{q_{t+1}} \right] = \frac{a}{q_t} \quad (20)$$

$$w_t : \alpha \frac{y_t}{e_t} \frac{\partial e_t}{\partial w_t} = n_t \bar{h} \quad (21)$$

The first one is the usual optimality condition for capital, saying that the input is rewarded its marginal product. The optimality condition for labor is different from the one in standard RBC-search models. In the labor-search literature, the second optimality condition is also referred to as the job creation condition (JCC).⁹ On the right-hand side is the effective

⁸The assumption of constant returns to scale is a useful one, as it allows to think of the stand-in form as an aggregation of single-vacancy outlets.

⁹Vacancies are already chosen optimally, and the optimality condition was plugged in the JCC equation.

cost of a vacancy, which is the product of the advertising cost per opened vacancy, a , and the expected time on average that this vacancy stays unfilled, $1/q_t$. The expression on the left-hand side is the expected discounted benefit from a vacancy: when filled, the return to the firm is the difference between the marginal product of labor plus the indirect effect working through the effort channel, less the wage, plus the saved cost on not advertising a vacancy, weighted by the probability of the match not being destroyed. The last equation describes how efficiency wages are set, i.e. how the firm chooses a wage rate to inspire the worker to supply optimum effort.

Next, multiplying last equation by w_t yields:

$$\alpha \frac{y_t}{e_t} \frac{\partial e_t}{\partial w_t} w_t = w_t n_t \bar{h} \quad (22)$$

or, in elasticity form:¹⁰

$$\alpha y_t \epsilon(e_t, w_t) = w_t n_t \bar{h} \quad (23)$$

Next, dividing the equation above by employment, and shifting it one period forward yields

$$\alpha \frac{y_{t+1}}{n_{t+1}} \epsilon(e_{t+1}, w_{t+1}) = w_{t+1} \bar{h}. \quad (24)$$

Substituting the wage bill expression into the firm's optimality condition for labor produces

$$\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[\alpha \frac{y_{t+1}}{n_{t+1}} [1 - \epsilon(e_{t+1}, w_{t+1})] + \frac{a(1-\psi)}{q_{t+1}} \right] = \frac{a}{q_t} \quad (25)$$

or, in effort elasticity formulation

$$\epsilon(e_{t+1}, w_{t+1}) = \frac{w_t e'(w_t)}{e} = 1 - \frac{E_t \frac{\Lambda_t}{\beta \Lambda_{t+1}} \frac{a}{q_t} - E_t \frac{a(1-\psi)}{q_{t+1}}}{\alpha E_t \frac{y_{t+1}}{n_{t+1}}} \quad (26)$$

Note that from the effort function it follows

$$e'(w_t) = \frac{b\sigma}{\bar{w}_t} \left[\frac{w_t - \bar{w}_t}{\bar{w}_t} \right]^{\sigma-1} \quad (27)$$

Multiply the expression above by w_t/e_t to obtain

$$\frac{w_t e'(w_t)}{e} = \sigma \frac{w_t}{w_t - \bar{w}_t} \quad (28)$$

¹⁰Note that with search frictions spending on wages does not equal the labor share, so the elasticity of effort with respect to the wage rate, $\epsilon(e_t, w_t) \neq 1$.

Equate the two expression for the elasticity of effort with respect to the wage to obtain

$$1 - \frac{E_t \frac{\Lambda_t}{\beta \Lambda_{t+1}} \frac{a}{q_t} - E_t \frac{a(1-\psi)}{q_{t+1}}}{\alpha E_t \frac{y_{t+1}}{n_{t+1}}} = \sigma \frac{w_t}{w_t - \bar{w}_t} \quad (29)$$

The wage rate is then

$$w_t = \bar{w}_t \left[1 + \frac{\sigma}{\left(1 - \frac{E_t \frac{\Lambda_t}{\beta \Lambda_{t+1}} \frac{a}{q_t} - E_t \frac{a(1-\psi)}{q_{t+1}}}{\alpha E_t \frac{y_{t+1}}{n_{t+1}}} \right) - \sigma} \right] \quad (30)$$

The mark up over the reference wage contains both gift considerations and/or the importance of the efficiency wage motive (parameter σ) and search frictions (expression in the brackets linked to the value of the match).¹¹ In the absence of advertising costs ($a = 0$), the model collapses to a standard efficiency wage setup. Alternatively, if the elasticity of effort with respect to the wage is zero (no efficiency wage), we are back to the pure search model, as in Vasilev (2018). Finally, when both frictions are off, we are back to the standard neoclassical model.

1.4 Matching technology

Aggregate job matches are assumed to be generated by the following production function:

$$m_t = \chi v_t^{1-\gamma} (1 - n_t)^\gamma, \quad (31)$$

where $\chi > 0$ is a scale parameter, $0 \leq \gamma \leq 1$ measures the elasticity of job matches with respect to unemployment, and V_t is the number of vacancies available in period t . In addition, this type of modeling matches as described above implies endogenous probabilities for the transition from unemployment to employment, defined as

$$p(\theta_t) = \frac{m_t}{(1 - n_t)}, \quad (32)$$

where

$$\theta_t = \frac{v_t}{1 - n_t} = \frac{v_t}{u_t} \quad (33)$$

¹¹This result is very similar to the setup in Martin and Wang (2018).

represents the tightness of the labor market. More specifically, when the market is tight, the probability of finding a job (and filling a position) will be low. Thus, the job-finding rate can be expressed as a function of θ , or

$$p(\theta_t) = \frac{m_t}{1 - n_t} = \theta_t^{1-\gamma}. \quad (34)$$

That is, the probability of making a transition from being unemployed to becoming employed decreases with the congestion caused by either increase in unemployment or the search effort. Lastly,

$$q(\theta_t) = \frac{m_t}{v_t} = \left(\frac{1}{\theta_t}\right)^\gamma \quad (35)$$

is the transition probability from an unfilled vacancy to a filled one. It is increasing in the search effort, the amount of vacancies and unemployment, and decreasing in market tightness.¹²

1.5 Equilibrium Unemployment

In order to analyze unemployment further, we will assume that the reference wage is defined as in Summers (1988)

$$\bar{w}_t = (1 - \phi u_t) w_t^a, \quad (36)$$

where w_t^a denotes the alternative wage offered by other firms, and $0 < \phi < 1$ denotes the importance of unemployment in determining a worker's outside option.¹³ Next, we will impose that in equilibrium $w_t = w_t^a$, hence

$$\bar{w}_t = (1 - \phi u_t) w_t. \quad (37)$$

In turn, it follows that effort is also a function of unemployment

$$e(u_t) = b \left[\frac{\phi u_t}{1 - \phi u_t} \right]^\sigma \quad (38)$$

¹²Alternatively, the inverse of the transition probability from unemployment to employment, $1/q$, can be interpreted as the expected duration of a vacancy.

¹³Thus, this parameter could be taken as a proxy for the relevance of overall macroeconomic conditions for wage determination. In data, the parameter ϕ will be set equal to the generosity of unemployment benefits. However, since ϕu is quantitatively small, this parameter is not driving the results.

Next, from the firm problem

$$w_t = \bar{w}_t \left[1 + \frac{\sigma}{\left(1 - \frac{E_t \frac{\Lambda_t}{\beta \Lambda_{t+1}} \frac{a}{q_t} - E_t \frac{a(1-\psi)}{q_{t+1}}}{\alpha E_t \frac{y_{t+1}}{n_{t+1}}} \right) - \sigma} \right] \quad (39)$$

substitute for \bar{w}_t to obtain

$$1 = [1 - \phi u_t] \left[1 + \frac{\sigma}{\left(1 - \frac{E_t \frac{\Lambda_t}{\beta \Lambda_{t+1}} \frac{a}{q_t} - E_t \frac{a(1-\psi)}{q_{t+1}}}{\alpha E_t \frac{y_{t+1}}{n_{t+1}}} \right) - \sigma} \right] \quad (40)$$

Evaluate in the steady state

$$1 = [1 - \phi u] \left[1 + \frac{\sigma}{\left(1 - \frac{\frac{1}{\beta} \frac{a}{q} - \frac{a(1-\psi)}{q}}{\alpha \frac{y}{n}} \right) - \sigma} \right] \quad (41)$$

In the absence of advertising costs ($a = 0$):

$$u(a = 0) = u^{ew} = \frac{\sigma}{\phi}, \quad (42)$$

where the expression above denotes the steady-state unemployment rate due to informational frictions, such as unobservability of effort, and inability to pre-contract a certain level of effort in the face of an absent (efficient) commitment device. If we treat the informational problems as being structural for the economy, then this would be the structural part of unemployment. Note that in this model labor is homogeneous, so each household is perfectly qualified to exercise any task. In other words, there is no skill mismatch.

In steady-state, and in the absence of search frictions ($a = 0$), $u = u^{ew}$. In the presence of advertising/hiring costs (in steady state), $u > u^{ew}$, or $u^{fr} = u - u^{ew}$ will denote the (steady-state) unemployment due to the search frictions (frictional unemployment). In other words, when $a > 0$, we need to solve a quadratic expression for the unemployment rate, take the positive root (which has to be a fraction), and solve for the frictional unemployment as the difference from total and structural unemployment: $u^{fr} = u - u^{ew}$. It turns out that for a wide variety of parameter values, the majority of steady-state unemployment rate is due to the structural component, with the frictional unemployment making a very small share.

For example, out of 5.3% unemployment rate in 2018, 5.1% is due to efficiency wage considerations, and only 0.2% due to the presence of search and matching frictions.¹⁴ Therefore, the unemployment is due to the too high wages, and the fact that households want to work for that rate, but there are no jobs available for them.

1.6 Government

The government will be assumed to be running a balanced budget in every period. The government collects revenue from levying taxes on consumption, on capital and labor income, and then spends on government consumption and transfers, which are returned lump-sum to the households:

$$\tau^c c_t + \tau^y [r_t k_t + w_t n_t h] = g_t^c + g_t^t, \quad (43)$$

where g_t^c are government purchases. Government spending share will be set equal to its long-run average, so the level will be varying with output. Government transfers will be residually determined and will always adjust to make sure the budget is balanced.

1.7 Stochastic process

To close the model, it will be assumed that total factor productivity (TFP) process $\{A_t\}_{t=0}^\infty$ is stochastic, and follows an AR(1) dynamics

$$A_{t+1} = (1 - \rho_a)A_0 + \rho_a A_t + \epsilon_t^a, \quad (44)$$

where $A_0 = A$ is the steady-state level of TFP, parameter ρ_a measures the persistence of the process, and $\epsilon_t \sim N(0, \sigma_a^2)$ are the unexpected innovations to the TFP, which are i.i.d. normal with zero mean and standard deviation σ_a .

1.8 Decentralized Dynamic Equilibrium with Search Frictions in the Labor market and Efficiency ("Fair") Wages

Given the process followed by total factor productivity $\{A_t\}_{t=0}^\infty$, the two tax rates $\{\tau^c, \tau^y\}$, initial capital endowments stock k_0 , hours worked per household \bar{h} , the decentralized dynamic

¹⁴Our results are thus quantitatively stronger than those in Martin and Wang (2018).

equilibrium with search frictions and efficiency wages is a list of sequences $\{c_t, i_t, k_t, n_t, e_t\}_{t=0}^{\infty}$ for each household i , input levels $\{k_t, n_t, e_t\}$ chosen by the firm in each time period t , aggregate labor market characteristics $\{m_t, u_t, v_t, p_t, q_t, \theta_t\}_{t=0}^{\infty}$, a sequence of government purchases and transfers $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, and input prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that (i) each household i maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes dynamic profit by choosing employment in the face of search frictions, and by setting an efficiency wage to satisfy the workers' incentive compatibility constraint and to induce an optimal effort level; (iii) government budget is balanced in each period; (iv) all markets clear.

2 Data and model calibration

When modelling business cycle fluctuations in Bulgaria, we will focus on the period after the introduction of the currency board (1999-2018). Data on output, consumption and investment was collected from National Statistical Institute (2019), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2019). The calibration strategy described in this section follows Vasilev (2015c). First, as in Vasilev (2016c), the average income tax rate was set to its (average effective) rate $\tau^y = 0.100$. Consumption tax rates is also set to its value in data, $\tau^c = 0.2$. The depreciation rate of physical capital in Bulgaria, $\delta = 0.05$, is taken from Vasilev (2015a). The discount factor, $\beta = 0.942$, is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 3.491$, in the steady-state Euler equation. The labor share parameter, $\alpha = 0.429$, was obtained as the average value of labor income in aggregate output over the period 1999-2014.¹⁵ Next, steady state employment rate in Bulgaria is set to $n = 0.533$, as in Vasilev (2016a). Following Vasilev (2015b), $\bar{h} = 1/3$. Steady-state output will be normalized to unity, which produces $A = 0.605$. For Bulgaria, Burda (1997) estimates $m/n = 0.009$ for Bulgaria, which yields a separation rate of $\psi = 0.009$. The degree of generosity of unemployment insurance, $\phi = 0.6$ was set as in data. As in Martin and Wang (2018), the curvature of the effort function is set to $\sigma = 0.025$. Following Vasilev (2016), we set per vacancy cost $a = 0.1$.¹⁶ Finally, as

¹⁵This value is slightly higher as compared to other studies on developed economies, due to the overaccumulation of physical capital during Communism.

¹⁶Alternatively, the scale parameter for advertising a could be set to match average vacancy duration, which equals the ratio of posted vacancies/newly-created job matches (v/m). Again, the size of advertising

in Vasilev (2016b), the moments of the total factor productivity process were obtained from running an AR(1) regression on the detrended Solow residuals. Table 1 below summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters

Parameter	Value	Description	Method
β	0.942	Discount factor	Calibrated
α	0.429	Labor Share	Data average
δ	0.050	Depreciation rate on physical capital	Data average
\bar{h}	0.333	Share of time spent working	Calibrated
n	0.533	Employment rate	Data average
σ	0.025	Curvature of effort function	Calibrated
γ	0.400	Elasticity of job matches with respect to search effort	Set
ψ	0.009	Transition rate from employment to unemployment	Data average
ϕ	0.600	Generosity of unemployment insurance	Data average
a	0.100	Per-unit advertising costs	Set
τ^c	0.200	Consumption tax rate	Data average
τ^y	0.100	Average tax rate on income	Data average
ρ_a	0.701	AR(1) parameter, total factor productivity	Estimated
σ_a	0.044	st.dev, total factor productivity	Estimated

3 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are reported in Table 2 on the next page. The steady-state level of output was normalized to unity (hence the level of technology A differs from unity), which greatly simplified the computations, and allows the steady-state to be solved by hand. Next, the model matches consumption-to-output ratio by construction; The investment and government purchases costs are very small as a share in output, and the size of the scale parameters is of little importance when it comes to the model dynamics over the business cycle.

ratios are also closely approximated. The shares of income are also identical to those in data, which follows directly from the constant-returns to scale featured by the aggregate production function. The after-tax return, where $\tilde{r} = (1 - \tau^y)r - \delta$ is also relatively well-captured by the model.

Table 2: Data Averages and Long-run Solution

Variable	Description	Data	Model
y	Steady-state output	N/A	1.000
c/y	Consumption-to-output ratio	0.674	0.674
i/y	Investment-to-output ratio	0.201	0.175
g^c/y	Government cons-to-output ratio	0.159	0.151
k/y	Capital-to-output ratio	3.491	3.491
wnh/y	Labor income-to-output ratio	0.571	0.571
rk/y	Capital income-to-output ratio	0.429	0.429
h	Share of time spent working	0.333	0.333
n	Employment rate	0.533	0.533
e	Effort level	N/A	0.542
A	Scale parameter of the production function	N/A	2.030
\tilde{r}	After-tax net return on capital	0.056	0.061

4 Simulation and moment-matching

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. We then simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 below summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at quarterly frequency. To minimize the sample error, the simulated

moments are averaged out over the computer-generated draws. The model matches quite well the absolute volatility of output. However, the model overestimates the variability in consumption, and investment. Still, the model is qualitatively consistent with the stylized fact that consumption generally varies less than output, while investment is more volatile than output. By construction, government spending in the model varies as much as in data.

Table 3: Business Cycle Moments

	Data	Model
σ_y	0.05	0.05
σ_c/σ_y	0.55	0.73
σ_i/σ_y	1.77	2.94
σ_g/σ_y	1.21	1.00
σ_n/σ_y	0.63	0.19
σ_w/σ_y	0.83	0.80
σ_u/σ_y	3.22	4.02
$corr(c, y)$	0.85	0.81
$corr(i, y)$	0.61	0.83
$corr(g, y)$	0.31	1.00
$corr(n, y)$	0.49	0.49
$corr(w, y)$	-0.01	-0.16
$corr(u, y)$	-0.47	-0.50
$corr(n, w)$	-0.14	-0.91

With respect to the labor market variables, the variability of employment predicted by the model is much less than in data, while the variability of unemployment is relatively closer to the volatility exhibited in data. The variability of wages is well-matched.

Next, in terms of contemporaneous correlations, the model slightly over-predicts the procyclicality of some of the main aggregate variables e.g. investment, and government consumption. This, however, is a common limitation of this class of models. However, along the labor market dimension, the contemporaneous correlation of employment with output, and unemployment with output, is relatively well-matched. With wages, the model cor-

rectly predicts mild counter-cyclicality. Overall, the model with both search frictions and efficiency wages shows promise to explain better labor markets dynamics in Bulgaria than a pure search-and-matching model, e.g. Vasilev (2016a), or a pure efficiency-wage model, e.g. Vasilev (2017a,b).

In the next subsection, we investigate the dynamic correlation between labor market variables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

4.1 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients empirical ACFs and CCFs at different leads and lags are presented in Table 4 against the simulated ACFs and CCFs. Following Canova (2007), this comparison is used as a goodness-of-fit measure. As seen from Table 4 on the next page, the model compares well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model. Overall, the model with both search frictions and reciprocity in labor relations generates too high persistence in output and both employment and unemployment, but is able to respond to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996), who argue that the RBC class of models do not have a strong internal propagation mechanism besides the strong persistence in the TFP process.

Next, as seen from Table 5 on the next page, over the business cycle, in data labor productivity leads employment. The model with fair wages, however, cannot account for this fact. Therefore, the effect between employment and labor productivity is only a contemporaneous one. Still, the model with search frictions and reciprocity in labor relations is a clear improvement over the perfectly-competitive labor market paradigm used in Vasilev (2009), but performs slightly worse than the pure model with efficiency wages as in Vasilev

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(u_t, u_{t-k})$	1.000	0.765	0.552	0.553
Model	$corr(u_t, u_{t-k})$	1.000	0.951	0.892	0.825
	(s.e.)	(0.000)	(0.030)	(0.057)	(0.081)
Data	$corr(n_t, n_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(n_t, n_{t-k})$	1.000	0.951	0.892	0.825
	(s.e.)	(0.000)	(0.030)	(0.057)	(0.081)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$	1.000	0.942	0.875	0.800
	(s.e.)	(0.000)	(0.033)	(0.063)	(0.089)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$	1.000	0.955	0.901	0.838
	(s.e.)	(0.000)	(0.027)	(0.052)	(0.075)
Data	$corr(c_t, c_{t-k})$	1.000	0.971	0.952	0.913
Model	$corr(c_t, c_{t-k})$	1.000	0.955	0.901	0.838
	(s.e.)	(0.000)	(0.027)	(0.052)	(0.075)
Data	$corr(i_t, i_{t-k})$	1.000	0.810	0.722	0.594
Model	$corr(i_t, i_{t-k})$	1.000	0.927	0.843	0.752
	(s.e.)	(0.000)	(0.039)	(0.072)	(0.101)
Data	$corr(w_t, w_{t-k})$	1.000	0.760	0.783	0.554
Model	$corr(w_t, w_{t-k})$	1.000	0.939	0.869	0.791
	(s.e.)	(0.000)	(0.036)	(0.067)	(0.094)

(2017a,b), and much worse than the pure search and matching setup with wage determined as an efficient outcome of Nash bargaining in Vasilev (2016a).

Table 5: Dynamic correlations for Bulgarian data and the model economy

		k						
Method	Statistic	-3	-2	-1	0	1	2	3
Data	$corr(n_t, (y/n)_{t-k})$	-0.342	-0.363	-0.187	-0.144	0.475	0.470	0.346
Model	$corr(n_t, (y/n)_{t-k})$	0.080	0.080	0.080	0.212	-0.037	-0.050	-0.055
	(s.e.)	(0.340)	(0.298)	(0.247)	(0.296)	(0.197)	(0.236)	(0.236)
Data	$corr(n_t, w_{t-k})$	0.355	0.452	0.447	0.328	-0.040	-0.390	-0.57
Model	$corr(n_t, w_{t-k})$	-0.029	-0.040	-0.055	-0.967	-0.326	-0.239	-0.171
	(s.e.)	(0.327)	(0.283)	(0.231)	(0.073)	(0.242)	(0.281)	(0.318)

5 Conclusions

In this paper we combine two ingredients - search frictions in the labor market, and informational problems connected to the observability of workers' effort, and investigate their quantitative importance in explaining fluctuations in Bulgarian labor markets. In contrast to Vasilev (2017b), who introduces efficiency wages of no-shirking type a la Shapiro and Stiglitz (1984), here we incorporated reciprocity arrangement in labor relations and wage determination a la "gift exchange" as in Vasilev (2017a), and Danthine and Kurmann (2010) in particular, into a relatively standard RBC model with government sector. We calibrated the model to Bulgarian data after the introduction of the currency board arrangement. Overall, the calibrated model with both search friction in the labor market, and gift exchange mechanism, performs well vis-a-vis data when it comes to relative volatilities of time series, auto- and cross-correlation functions, and in addition dominates both the market-clearing labor market framework featured in the standard RBC model, e.g Vasilev (2009), and the models with search frictions only (Vasilev 2016a), and setups with efficiency wages alone (Vasilev 2017a,b). This is because the search mechanism helps to capture the volatility of unemployment, while the efficiency wage schedule improves the behavior of the wage rate over the business cycle in the model.

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