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Imperfect Decision-Making and the Tax Payer Puzzle

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Abstract

Empirical studies show that the actual tax evasion rate is smaller than rational choice models predict. In addition to this, tax payers do not respond on parameter changes as predicted by these models. Some authors tried to explain these observations by assuming “tax morale”.

We propose a different way to explain tax payers’ behavior, which is based on a theory of imperfect decision-making. Facing a complex decision problem, their computational limits keep tax payers from reaching the optimal solution.

JEL-Classification: H 26, K 42
Encyclopedia of Law and Economics: 0710, 6020
Keywords: tax compliance, bounded rationality, imperfect decision-making, detection skill, rule-governed behavior.
1. Introduction

Tax payers’ behavior still creates a puzzle for economists. The situation is aptly described in the introduction of a widely cited paper, titled "Why Do People Pay Taxes?":

"In recent years economists have devoted increasing attention to the study of individual tax evasion. Despite these efforts, however, our understanding of the reasons behind individual tax compliance behavior remains limited. In fact, the puzzle of tax compliance is that most people continue to pay their taxes."

According to the rational choice model in the tradition of Becker (1968), risk-neutral tax payers would be expected to evade taxes on a higher scale than empirically observable: "A purely economic analysis of the evasion gamble implies that most individuals would evade if they are 'rational', because it is unlikely that cheaters will be caught and penalized."2 According to Alm/McClelland/Schulz, slightly more than one percent of tax returns lead to an audit, and the punishment for deceitful evasion is only 75 percent of the tax evaded.

Furthermore, there is overwhelming empirical evidence that changes in the probability of detection or the level of punishment do not have the impact on tax payers’ behavior as the rational choice theory predicts3. Rising penalties or higher detection probabilities do not significantly reduce tax evasion. Taking these observations into account, the theory of strictly rational decision-making seems to be unable to explain the tax payers’ actual behavior. Referring to empirical results, Baldry (1987, 376) for instance concludes: "...these results suggest that it is perhaps the theory which needs revision."4 Graetz/Wilde (1985, 357) even claim that "most of the theoretical work to date is not particularly useful either for policy analysis nor empirical study.” Alm/Sanchez/deJuan (1995, 15) add that the traditional approach is a "reasonable starting point, but not a good ending point”.

In order to find a solution to this puzzle, some authors proposed to introduce the notion of tax morale. Vast amounts of econometric studies try to suggest a negative correlation between tax morale and tax evasion5. Disutility caused by shame or the feeling

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1Alm/McClelland/Schulz (1992, 21); see also Alm/Sanchez/deJuan (1995, 1). It is not our purpose to argue whether this Tax Payers Puzzle actually exists, we take it as given.
3See Pommerehne/Frey (1993, 6) and Roth/Scholz/Witte (1989).
4Similar Alm/McClelland/Schulz (1989, 19): "It is apparent that a new theory of compliance is needed, one that can deal with those factors that are either ignored or dealt with unsatisfactorily by expected utility theory. Such a theory must incorporate the major elements that enter individual’s compliance decision: the fear of detection and punishment, the overweighting of low probabilities, and the recognition of government services. It should also recognize another element that is widely believed to affect compliance: social norms.”
of guilt increases the opportunity costs of tax evasion. It is not only a feeling of duty to obey the law that is said to encourage tax payers, but also the insight that taxes are necessary to finance public services. Laboratory experiments were done to support these hypotheses. This branch of the literature associates obeying tax morale with a loss in money income or wealth. An individual accepts such a loss either because she is programmed to obey the law, or she attempts to avoid disutilities concerned with tax evasion. To maximize a utility function that depends on income as well as morale means nothing but to calculate another optimum instead of the one that is derived from a utility function that only depends on income. The modified optimization problem then takes into account the marginal rate of substitution between morale and income. However, introducing tax morale by manipulating the utility function such that it fits with the observation is quite unsatisfactory from the methodological point of view. In accordance with Becker/Stigler (1977, 89) such an approach should be refuted, since it gives "the appearance of considerate judgement, yet really have only been ad hoc arguments that disguise analytical failures." Becker/Stigler object to attempts to explain changes in behavior by changes in utility functions. Another attempt to explain the puzzle within the framework of perfect rationality would be to introduce the assumption that tax payers are risk averse. Tax morale as well as risk aversion lead to modifications of the maximization problem - either by introducing new explanatory variables to the utility function or by changing its properties.

A more fruitful approach would be to assume additional constraints, such as exogenous rules, that have not been considered by the classical models. The introduction of rules yet requires an explanation why decision-makers obey rather than simply break them. Other constraints that might be relevant for the analysis of the puzzle might be based on the complexity of the tax payer’s decision situation. E. g., Slemrod (1989, 178) points out that much research on the extent of noncompliance exists, but only recently research has been focused on the relationship between complexity, compliance costs, and tax evasion. Some authors recently made use of bounded rationality approaches, e.g. by drawing on psychological insights, in particular the anomalies and paradoxes which have frequently been shown in laboratory experiments. Alm/Mc Clelland/Schulz (1989), (1992), for instance, employ the well tested hypothesis that people systematically overestimate small probabilities. Consequently, the predicted tax evasion is below the optimum.

Our paper also models tax payers as boundedly rational, yet it follows an entirely different approach, which is based on Heiner’s theory of imperfect decision-making. It is assumed that the relative value of two available options depends on which state of nature occurs. The decision-maker receives some signal about the true state. Interpreting this signal means to form an internal message. Having formed an internal message he carries out one of the available actions. In creating this message, an imperfect decision-maker

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6 See Pommerehne/Weck-Hanneman (1992) or Pommerehne/Hart/Frey (1996) on the influence of political participation on tax compliance

7 See Brandstätter (1994, 213).

8 As Schmidtchen (1994, 194, No. 4) points out.

9 See Kahneman/Tverski (1979).

is likely to commit errors. This is the reason why this theory has been labelled as imperfect decision-making.

In this context, the decision between actions can equivalently be described as a decision between reaction strategies, i.e. plans how to react on the internal messages. These reaction strategies might be contingent on the internal message\(^{11}\) or non-contingent\(^{12}\). To choose an non-contingent reaction strategy leads to rule-governed behavior: The decision-maker then follows a rule of thumb rather than analyzes his environment; by excluding one of the available options from further consideration, he chooses to systematically disregard information, even if it is provided by the nature for free\(^{13}\).

Obviously, the concept of imperfect decision-making is closely related to Simon’s principle of bounded rationality:

"The capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problems whose solution is required for objectively rational behavior in the real world..."\(^{14}\)

Simon assumes that human behavior is intendedly rational, but limited in attention and computational capability\(^{15}\). The boundedly rational decision-maker can be seen as an agent who tries to perform expected-utility calculations and to update in a Bayesian way, but is constrained by his limited abilities.

We show that the decision to follow a rule is an expected-utility maximizing choice if a decision-maker faces a competence-difficulty gap ("C-D-gap"). A C-D-gap arises if the individual competence to analyze a decision problem is below a tolerance limit which depends on the difficulty of the situation. His competence then turns out to be insufficient to cope with the complexity he faces. In contrast to the traditional approach, which models tax evasion decision as a simple lottery, we presume that tax evasion is a highly complex decision problem that confronts decision-makers with Knightian uncertainty, i.e. the inability to fully comprehend the state space without making mistakes. According to our model, a C-D-gap will keep a taxpayer of limited competence from carrying out the optimal evasion rate. Hence, our model provides an explanation for the taxpayer puzzle. On the other hand, our model predicts that taxpayers of perfect detection skill would always choose a contingent reaction strategy and carry out the optimal evasion rate.

According to Simon, bounded rationality provides room for a genuine theory of organization. We follow his insights in the attempt to show that bounded rationality also provides room for a genuine economic theory of rule-governed behavior. Furthermore, the new approach to the taxpayer puzzle we present in this paper allows for a surprising

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\(^{11}\)Such as "if I observe message one, I carry out option a; having observed message two, I choose option b".

\(^{12}\)E.g. "whatever the message is, I choose a".

\(^{13}\)See Heiner (1983).

\(^{14}\)Simon (1957, 198).

\(^{15}\)See Carroll (1989).
interpretation: Increasing the complexity of the tax law system might lead to higher compliance by tax payers, even though this sounds counterintuitive at first glance.

The second section outlines a theory of imperfect decision making and rule-governed behavior. In the third section, we apply the results of this theory to the situation of a tax payer who has to decide whether to increase his tax evasion rate or to stick to the rate he is used to. The fourth section summarizes and briefly discusses the results.

2. A Theory of Imperfect Decision Making

2.1 The setup of the theory

The theory we present in this section relates imperfect decision-making to rule-governed behavior. Consider an option set \( A = \{c; d\} \). The payoffs for the decision-maker depend on the true state of the world, which is unknown to him. The set of possible states is denoted as \( S \), which is partitioned into disjoint, non-empty subsets \( S_l \) and \( S_g \). A state from the subset \( S_l \) occurs with probability \( 1 - \pi \), hence \( \pi \) is the probability of \( S_g \). The decision-maker is assumed to maximize the expected monetary payoff.

The payoffs related to combinations of actions and true states of the world are defined such that the option \( c \) always brings 0, whereas \( d \) brings either a gain \( g > 0 \) (in case of \( S_g \)), or a loss \( l < 0 \) (in case of \( S_l \)). The payoff function is visualized in Table 1.

<table>
<thead>
<tr>
<th>True state in ( S_g )</th>
<th>( S_l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob. distribution</td>
<td>( \pi )</td>
</tr>
<tr>
<td>Actual choice is ( d )</td>
<td>( g &gt; 0 )</td>
</tr>
<tr>
<td>Actual choice is ( c )</td>
<td>0</td>
</tr>
</tbody>
</table>

We assume that option \( c \) is the customary behavior, the option the decision-maker is used to carry out. To deviate from this custom by choosing \( d \) would provide a higher degree of uncertainty to him. Hence, the payoff distribution of option \( d \) has greater variance than the one of \( c \).

Now we assume that nature sends a signal whether the true state is \( S_g \) or \( S_l \). This signal is perfectly correlated with the true state, yet the decision-maker knows that he

\[ ^{16} \text{The presentation in this section follows the ideas of HEINER (1983), (1985), (1986, 99ff.) and HEINER/SCHMIDTCHEN (1995). GREEN/SWETS (1966) and SWETS (1988) provide good introductions into the ”Signal Detection Theory” HEINER refers to.} \]

\[ ^{17} \text{The following proposition could also be derived under the assumption that the payoffs from option } c \text{ depend on the true state, as shown in HEINER (1986, 99 ff.).} \]
is only imperfectly able to interpret it. To interpret an external signal means to create an internal message. With $\hat{S}_i$, we denote the internal message that the true state is $S_i$, $i \in A$. An imperfect decision-maker is likely to interpret the signal wrongly, e.g. his interpretation of the signal $S_l$ might lead to the internal message $\hat{S}_g$ instead of $\hat{S}_l$. We define the probability of *rightly* forming the message $\hat{S}_g$ as

$$r = \text{prob}\{\hat{S}_g | S_g\}$$

and the probability of *wrongly* doing so as

$$w = \text{prob}\{\hat{S}_g | S_l\}$$

When creating the internal message, two errors are possible. The decision-maker expects himself to commit these errors either with probability $w$ or with $1 - r$. Thus, the parameters $r$ and $w$ provide a measure for the tax payer’s ability to analyze his environment. We call this ability detection skill:

- If $(r - w) = 1$, then the decision-maker’s detection skill is called perfect
- $(r - w) \in ]0, 1[$ holds for an decision maker of imperfect detection skill
- the case of $(r - w) = 0$ we call zero detection skill

The parameters $r$, $w$ and $\pi$ can be derived from experiments, even if the decision-makers themselves are not fully aware of them. Imperfect decision-making can be described as a deviation of the subjective from the objective partition of the state set $S$, as it is shown in figure 1.

The box symbolizes the state set $S$, with the dashed line indicating the objective partition $\{S_l; S_g\}$. The decision maker’s subjective partition $\{\hat{S}_l; \hat{S}_g\}$, denoted by the strong line, deviates from the objective one if the decision-maker is imperfect. The entries in the four subsets of $S$ show the joint probabilities of the four possible combinations of states and internal messages.

### 2.2 Intended but limited rationality

The following figure 2 summarizes the imperfect decision-maker’s situation. First, nature $N$ chooses its true state and sends a perfectly correlated signal. This is indicated

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18 To assume imperfectly correlated signals the decision maker interprets perfectly would lead to a quite similar model. However, there is an important difference between this approach and the one we have chosen: Our theory allows to compare the performance of agents that receive the same signals, yet differ in their imperfection.


Figure 1: Subjective and objective partitions of $S$

by the upper left box in figure 2. The decision-maker $E$ interprets the signal by creating an internal message. He commits an error with probability $1 - r$ or $w$ respectively. Having observed an internal message, $E$ finds himself in one out of two informations sets (indicated by the dashed lines in figure 2) and forms posterior probabilities concerning the true state. At each of his decision nodes, the decision-maker can choose between the actions $d$ and $c$.

Even if the decision-maker’s detection skill is only imperfect, he is assumed to intend acting rationally. Thus, having observed an internal message, he updates his beliefs according to the Bayes rule\textsuperscript{21}. We define the posterior beliefs as $\mu = \text{prob}\{S_g | \hat{S}_g\}$ and $\nu = \text{prob}\{S_l | \hat{S}_l\}$. Substituting $r, w, \pi$ into the Bayes formula then leads to the following expressions for $\mu$ and $\nu$:

\begin{align*}
\frac{r\pi}{r\pi + w(1 - \pi)} &= \mu \\
\frac{(1 - w)(1 - \pi)}{(1 - w)(1 - \pi) + (1 - r)\pi} &= \nu
\end{align*}

$(1 - \mu), (1 - \nu)$ can be derived from these equations respectively. Defining $T := \frac{-l(1 - \pi)}{g\pi}$,

\textsuperscript{21}See, e.g., Hirschleifer/Riley (1992, 175)
we come to the central proposition of the theory of imperfect decision-making\footnote{See Heiner (1983) and Schmidtchen/Neunzig (1997).}: 

**Proposition 2.1:** Given values of $r, w$ and $T$, the decision-maker prefers option $d$ over $c$ if, and only if

- he observes the internal message $\hat{S}_g$ and $\frac{r}{w} > T$ holds, or
- he observes the internal message $\hat{S}_l$ and $\frac{1-r}{1-w} > T$ holds.

**Proof:** In both of the cases, the expected payoff from choosing $c$ is zero. We first show that $d$ is preferred to $c$ if $\hat{S}_g$ and $\frac{r}{w} > T$. Having observed $\hat{S}_g$, the posterior beliefs of the decision-maker are $(\mu, 1-\mu)$. In this case, the expected payoff from $d$ is $\mu g + (1-\mu)l$. Substituting the expression (3) for $\mu$, this is greater than zero if, and only if $r \pi g + w(1-\pi)l > 0$. This is equivalent to $r/w > T$, q.e.d.

If the internal message is $\hat{S}_l$, the updated beliefs are $(\nu, 1-\nu)$. The expected payoff from $d$ is positive if, and only if, $(1-\nu)g + \nu l > 0$ which is equivalent to $(1-r)\pi g + (1-w)(1-\pi)l > 0$, q.e.d.
2.3 Reaction strategies

A reaction strategy is a combination of actions \((j,k)\) with \(j, k \in A\), where \(j\) is the plan how to react on the internal message \(\hat{S}_g\) and \(k\) how to react on \(\hat{S}_l\). The above decision-tree contains four reaction strategies: \((dd)\), \((cc)\), \((dc)\) and \((cd)\). The first two ones we call non-contingent, the latter ones are contingent reaction strategies. With these definitions, Table 2 states a corollary to the proposition 2.1.

<table>
<thead>
<tr>
<th>(T &gt; 1)</th>
<th>(T = 1)</th>
<th>(T &lt; 1)</th>
<th>Optimal strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r &gt; w T)</td>
<td>(r &gt; w)</td>
<td>(r &gt; 1 - (1 - w) T)</td>
<td>(dc)</td>
</tr>
<tr>
<td>(w T &gt; r &gt; 1 - (1 - w) T)</td>
<td>(r = w)</td>
<td>(\emptyset)</td>
<td>(cc)</td>
</tr>
<tr>
<td>(\emptyset)</td>
<td>(r = w)</td>
<td>(w T &gt; r &gt; 1 - (1 - w) T)</td>
<td>(dd)</td>
</tr>
<tr>
<td>(r &lt; 1 - (1 - w) T)</td>
<td>(r &lt; w)</td>
<td>(r &lt; w T)</td>
<td>(cd)</td>
</tr>
</tbody>
</table>

The first row of Table 1 distinguishes three cases with respect to the value of \(T\). If, e.g., the absolute expected loss from choosing \(d\) exceeds the expected gain, then \(T > 1\). The entries in the columns state the conditions under which the reaction strategy indicated in the last cell of each row would be the expected payoff maximizing choice for the imperfect decision-maker. This relation between detection skill, denoted by the parameters \(r\) and \(w\), and the tolerance limit \(T\) is visualized in the following Figure 3. The main diagonal in the medium box covers the case of \(r = w\) and \(T = 1\), where the decision maker is indifferent between his non-contingent strategies \((dd)\) and \((cc)\).

According to the theory of imperfect decision making, the ratio \(r/w\) is a measure of a decision-maker’s detection skill or competence, and \(T\) is a tolerance limit for this competence\(^{23}\). With these definitions, we can state another result.

\(^{23}\text{Heiner (1983)}\)
Proposition 2.2: An imperfect decision-maker of positive detection skill always chooses the contingent reaction strategy (dc), if his competence exceeds the tolerance limit.

Proof: For the cases of $T = 1$ and $T > 1$, the proposition can immediately be derived from table 2. If $T < 1$, then (dc) is optimal if $r > 1 - (1 - w)T$, which is equivalent to

$$\frac{r}{w} > 1 - \frac{(1 - w)T}{w} = \frac{1 - T}{w} + T$$

Due to $T < 1$, the right hand side exceeds $T$: q.e.d.

If the competence exceeds the tolerance limit, the decision-maker finds it optimal not to disregard the signals nature sends. He then takes into account the internal message created by his evaluation when choosing the action to be carried out (which is either $c$ or $d$). If, on the other hand, his competence does not exceed the tolerance limit, we call this a competence-difficulty-gap or C-D-gap\(^{24}\). In this case, the decision problem is too complex for the ability of the decision-maker to analyze it. His optimal behavior is to choose one of the non-contingent reaction strategies.

Proposition 2.3: An imperfect decision-maker of zero detection skill will never choose a contingent reaction strategy. The non-contingent strategy (cd) will only be employed by decision-makers of negative detection skill.

Proof: Both of the statements can easily be seen in figure 3. Zero detection skill means $r/w = 1$, hence the $r - w$-combination is on the main diagonal in the unit box. In case of $T > 1$, (cc) is optimal, in case of $T < 1$, (dd) is optimal. With $T = 1$, the decision-maker is indifferent between (dd) and (cc). (cd) is only optimal if $r < w$: q.e.d.

The reaction strategy (cd) means to act just contrary to the internal message. If a decision-maker of negative detection skill chooses (cd), this simply corrects his erroneous interpretation of signals. Thus we can disregard the strategy (cd) and limit our view to decision-makers with non-negative detection skill.

Choosing a non-contingent reaction strategy leads to "rule-governed behavior": The decision-maker systematically disregards information provided by nature and limits his behavioral program to only one of the two available options, excluding the other one from it\(^{25}\). Hence, this model explains why agents who try to decide rationally (but are only limitedly able to do so) sometimes choose to follow rules instead of evaluating their

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\(^{24}\)Heiner (1983).

\(^{25}\)Heiner points out that Simon has shown in his work on bounded rationality how decision-makers systematically exclude information, i.e. available options, from their consideration to solve complex decision problems; see Heiner (1983, 564) with some references.
environment. This notion allows to describe imperfect decision-making as if it were taking place in two steps:

- **The constitutional choice**: In the first step, the decision-maker chooses a reaction strategy. The decision for a non-contingent reactions strategy leads to rule-governed behavior.

- **The post-constitutional choice**: In the second step, given the internal choice, having received a signal and interpreted it, he carries out an option according to the chosen reaction strategy.

If it is optimal for a decision-maker to choose one of the non-contingent reaction strategies, then his behavior is predictable with certainty. To express this in terms of probability, having chosen \( dd \) means that the probability of carrying out action \( d \) is one, whereas the choice of \( cc \) leads to a zero probability of carrying out \( d \). On the other hand, if a decision-maker chooses the contingent strategy \( dc \), then his actual behavior depends on the parameters \( r, w, \) and \( \pi \). Therefore, this theory only allows to derive a probability distribution on which action will actually be carried out. Let \( h \) denote the probability that a decision-maker of positive detection skill who has chosen the reaction strategy \( dc \) carries out the action \( d \), then

\[
h = r\pi + w(1 - \pi) = \pi(r - w) + w \tag{5}
\]

The option \( c \) will carried out with probability \( 1 - h \). Obviously, \( h = \pi \) holds for a perfect decision-maker, which means that he only carries out \( d \) if it is beneficial to do so. Modifications of the parameters \( \pi, l \) and \( g \) will have an influence on the tolerance limit \( T \) as well as on \( h \). However, marginal modifications are likely to have no influence on the action being carried out. To give an example, if \( T > r/w \) holds in the first place, then even after a slight increase of \( g \) the decreased \( T \) might still be greater than \( r/w \). In this case, a decision-maker of positive detection skill would still choose \( cc \). Thus, even though the parameter shift has made the option \( d \) more attractive, he keeps excluding the option \( d \) and follows rule-governed behavior. As long as \( r/w < T \) holds, an increase in \( T \) would have no influence on the imperfect decision-makers.

On the other hand, a small change in the parameters might induce the decision-maker to switch from a non-contingent reaction strategy to a contingent one, which would cause a dramatic change in his observable behavior. This non-marginality explains why changes

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26The terminology is adapted from Buchanan, who points out “... the necessity of distinguishing two stages of social interaction, one which involves the selection of rules and one which involves action within these rules.” (1975, X). Our model applies this concept to individual decision-making, which can be analyzed as a game between an agent on the constitutional stage against the same agent on the post-constitutional stage. In his later work, Buchanan describes a similar idea, yet rather vaguely: “Somehow and for some reason, the individual has chosen the set of prospects from which future choices are to be made.” (1994, 125). Compare this concept to Selten (1990, 652), who mentions a predecision that is supposed to activate the levels of decision-making, as described by his three level theory in Selten (1978).
in parameters do not always have the effect that is predicted by the rational choice theory\textsuperscript{27}.

3. Imperfect tax evasion

3.1 Complexity and rule-governed behavior

Most economic models describe tax systems by a simple taxation function. In reality, however, taxation is based on a very complex system of laws, comments, and judicature\textsuperscript{28}. Facing such a complex environment, tax payers are likely to commit errors when making their decision whether to follow the law or to deviate from it. To deviate might be beneficial, but it also might turn out to be harmful. If a C-D-gap occurs, rule-governed behavior can be the optimal choice for an imperfect decision-maker who attempts to act rationally, even though he has only limited ability to evaluate his environment.

In this chapter, we extend this basic theory that was outlined in the previous section. Consider a tax payer who is used to evade taxes by a certain rate of her income. Let us now assume that the income of this tax payer is increased. She then has to decide whether to report at least a part of this increase, if she wants to stick to the evasion rate she is used to, or not to report it, which would lead to an increased evasion rate.

3.2 The model

Our theory of imperfect tax evasion explains the observable tax payers’ behavior as an imperfect decision between tax evasion rates. With $\alpha$, we denote the evasion rate the tax payer is used to (the starting point), whereas $\beta$ is an upward deviation from this rate. We will show that the higher this starting point, the more likely is a C-D-gap to occur. For tax payers with limited competence, we derive three areas of evasion rates:

- In the lowest area, the tax payer would always increase his evasion rate.
- In the intermediate area, the tax payer would decide to let this decision depend on the signal he receives, and which he is able to interpret only imperfectly.
- In the highest area, the tax payer will never increase his evasion rate.

According to the rational choice model of tax evasion, there exists a unique optimal tax evasion rate, denoted as $\alpha^*$, if the expected net income function $X(\alpha)$ (i.e. the sum of

\textsuperscript{27}See Schmidtchen (1994).

\textsuperscript{28}The German Income Tax Law alone consists of more than 140 Paragraphs that easily fill more than 200 pages, left aside the administrative instructions on this law a careful tax-payer also should keep in mind.
non-taxed income, taxed income, and expected punishment) is continuous and concave in the evasion rate\textsuperscript{29}.

It would be optimal for her to choose the optimal evasion rate $\alpha^*$. However, the legal rules that govern the tax system are complex. Kaplow (1994) points out that a certain degree of complexity cannot be avoided, if ability to pay is supposed to be the base of taxation\textsuperscript{30}.

The allowed deductions from the total income, the probability of detection, or the marginal punishment might be unknown or not precisely determinable\textsuperscript{31}. In tax systems with progressive tax rates, even the marginal tax rate is difficult to derive: "Taxpayers are only partially informed about their own objective circumstances, including the requirements and costs of compliance and the risks and rewards of noncompliance."\textsuperscript{32} Complexity hence may prove to be an obstacle for the tax-payer when trying to exactly compute her optimal tax evasion rate $\alpha^*$. Therefore, we assume that a decision-maker sees the true (but unknown to her) $\alpha^*$ as a random variable with a distribution function

$$F_{\alpha^*}(\alpha) := \text{prob}\{\alpha^* \leq \alpha\}$$

which is non-decreasing in $\alpha \in [0, 1]$, with $F_{\alpha^*}(0) = 0$ and $F_{\alpha^*}(1) = 1$.

Let us denote the tax payer’s starting point as $\alpha \in [0, 1]$, which is the evasion rate that was chosen in the former period\textsuperscript{33}. Not to report a small increase in her income would lead to an evasion rate $\beta$ which exceeds $\alpha$. In this highly simplified and stylized situation, the option set of the tax payer is $\{\alpha; \beta\}$.

To carry out the option $\beta$ instead of $\alpha$ might lead to an increased expected income or to a loss, depending on whether the starting point $\alpha$ is below the optimum $\alpha^*$ or not\textsuperscript{34}.

We denote the effect of an increased evasion rate as

\textsuperscript{29}Consider e.g. an expected income function such as

$$X(\alpha) = (1 - q)(1 - t(1 - \alpha)) + q[1 - t - P(\alpha)]$$

where $q$ is the probability of detection, if evasion takes place, $t$ is the marginal tax rate, and $P(\alpha)$ is the punishment, with $P(\alpha) > 0 < P_{\alpha^*}$. See Allingham/Sandmo (1972) on such an approach. Pommerehne/Weck-Hannemann (1992), Alm/McClelland/Schulz (1989, 7), Rubinstein/Frey (1995, 1) and Graetz/Wilde (1985) did analyze the influence of risk-aversion on the tax-payer’s decision.

\textsuperscript{30}See also Moser (1994). Slemrod (1989, 157) distinguishes four aspects of complexity: the predictability of the tax law, its enforceability, the costs of compliance due to its difficulty, and its manipulability, i.e. the tax payer’s possibilities to evade. In this paper we do not take into account enforcement problems, since we focus on the three other aspects concerning the tax payer’s situation.

\textsuperscript{31}See also Slemrod (1989, 179).

\textsuperscript{32}Roth/Weitz-Witte (1989, 9).

\textsuperscript{33}Another starting point to consider deviations from would be the tax payer’s observations of other peoples’ behavior, see Alm/McClelland/Schulz (1989, 27). Sen (1994, 388) also proposes to take into account the decision-maker’s tendency to imitate others: “This type of reasoning ... points to the need to bring in the influence of established norms of behavior in explaining what actions will or will not be chosen.”

\textsuperscript{34}Let us, for simplicity, assume that ($\beta - \alpha$) is small enough to hold the following: $\alpha < \alpha^* \Rightarrow \beta < \alpha^*$. 

14
\[
d X(\alpha) := X(\alpha) - X(\alpha) = \begin{cases} 
 l(\alpha) < 0 & : \alpha \geq \alpha^* \\
 g(\alpha) \geq 0 & : \alpha < \alpha^* 
\end{cases}
\]

This definition allows to state the following proposition:

**Proposition 3.1:** Given a starting point \( \alpha \) and an upward deviation \( \beta \) with \((\beta - \alpha) \to 0\), then the first derivative of \( dX(\alpha) \) is negative.

**Proof:** \( l_\alpha, g_\alpha < 0 \) follows from the concavity of \( X(\alpha) \): q.e.d.

Starting at \( \alpha \), the possible gain \( g > 0 \) from carrying out the deviation \( \beta \) is decreasing with an increase in the starting point \( \alpha \). The same holds true for the possible loss \( l < 0 \). During the first stage of the two-step decision process as outlined in section 2, the taxpayer decides whether to include the upward deviation into her behavioral program or not. If the taxpayer includes \( \beta \) into her behavioral program, she might repeat this consideration for further deviations, yet starting from \( \beta \). The tolerance limit for any possible value of \( \alpha^* \) and any starting point \( \alpha \) under consideration is the ratio of the absolute expected loss and the expected gain:

\[
T(\alpha) := \frac{-l(\alpha)F_{\alpha^*}(\alpha)}{g(\alpha)[1 - F_{\alpha^*}(\alpha)]}
\]

Let us define the absolute expected loss as \( L := -l(\alpha)F_{\alpha^*}(\alpha) \) and the absolute expected gain as \( G := g(\alpha)[1 - F_{\alpha^*}(\alpha)] \), then the tolerance limit can simply be written as \( T = L/G \), and \( L, G > 0, L_\alpha \geq 0 \geq G_\alpha \) holds. We assume that nature not only decides on the true value of \( \alpha^* \) and therefore on whether a deviation \( \alpha \) is harmful or beneficial. Nature also sends perfect signals, which however the taxpayer is only imperfectly able to transform into an internal message. Let us denote the state \( \alpha^* > \alpha_i \) as \( S_G \) and the corresponding internal message as \( \hat{S}_G \). For the state \( \alpha^* \leq \alpha_i \) we define the signal \( S_L \) and the internal message \( \hat{S}_L \) respectively. Then we can define the taxpayer’s detection skill parameters as

\[
r := \text{prob}\{\hat{S}_G|S_G\}
\]

\[
w := \text{prob}\{\hat{S}_G|S_L\}
\]

The parameters \( r \) and \( w \) are assumed as exogenously fixed. According to the proposition 2.1, a taxpayer prefers the deviation \( \beta \) over the starting point \( \alpha \) if, and only if

- \( \frac{r}{w} > T(\alpha) \) holds and she observes \( \hat{S}_G \)
- \( \frac{1 - r}{1 - w} > T(\alpha) \) holds and she observes \( \hat{S}_L \)
The first derivative of the tolerance limit $T$ has an important property, as stated in the following proposition:\footnote{In this basic model, we don’t need to derive the second derivative of $T$.}

**Proposition 3.2:** The first derivative of the tolerance limit is non-negative.

**Proof:** From $T = L/G$ follows $T_\alpha = \frac{L_{\alpha}G - LG_{\alpha}}{G^2}$, with $G_\alpha \leq 0 \leq L_\alpha$ and $L, G > 0$. Hence, $T_\alpha \geq 0$ follows directly: q.e.d.

### 3.3 Optimal reaction strategies

In this model, reaction strategies are pairs of actions where the first entry denotes the reaction on the message $\hat{S}_G$ and the second stands for the reaction on $\hat{S}_L$. E.g., choosing the non-contingent reaction strategy $(\alpha\alpha)$ means never to deviate from $\alpha$, regardless of the internal message. The tax payer chooses rule-governed behavior. On the other hand, the contingent strategy $(\beta\alpha)$ describes the tax payer’s attempt to evaluate her interpretation of nature’s signal. The following figure 4 makes use of the propositions 2.1 and 3.2. It visualizes the relation between the tolerance limit (which is increasing in $\alpha$), the optimal reaction strategies, and the parameters $r$ and $w$ for the case of a tax payer with positive, but imperfect detection skill ($1 > r > w > 0$).

If the tolerance limit is very small, namely if $T < (1 - r)/(1 - w)$ holds, then the tax payer will always increase his evasion rate. If $T$ is higher than $r/w$, then a competence-difficulty gap occurs: The limited ability to evaluate the environment is not sufficient anymore to cope with the complexity of the situation. Let us, for given values of $r$ and $w$, define
• an upper threshold value \( \alpha_h := T^{-1}(\frac{T}{w}) \)
• and a lower threshold value as \( \alpha_h := T^{-1}(\frac{1-r}{1-w}) \).

An imperfect decision-making tax payer will never increase her evasion rate above the upper threshold value \( \alpha_h \). If the tax payer starts her considerations from a point \( \alpha \) that is definitely lower than \( \alpha^* \), then she will never carry out the optimal evasion rate, because for an imperfect decision-maker holds \( \alpha_h < \alpha^* \). The reason ist straightforward: Approaching \( \alpha^* \), \( G \) shrinks to zero, hence \( T \) grows toward infinity. Therefore, a decision-maker of finite \( r/w \) faces \( \alpha_h \) as a stop value before having reached \( \alpha^* \).

Thus the model provides an explanation for the tax payer puzzle: If an external observer knows the actual value of \( \alpha^* \) and the parameter settings are such that \( \alpha_h < \alpha^* \), then the model predicts that an imperfect decision-making tax payer will choose less than the optimal evasion rate.

In the intermediate case, where \( r/w > T > (1-r)/(1-w) \) holds, the tax payer chooses her contingent reaction strategy (\( \beta\alpha \)). Observing the internal message \( S_G \) induces her to increase her evasion rate, whereas \( S_L \) leads her to stick with the lower rate. In this case, the model only allows for a stochastic prediction of the tax payer’s observable behavior, since the probability of the action that actually gets carried out depends on the parameters \( r \) and \( w \) and the distribution \( F_{\alpha^*}(\alpha) \).

The model also provides an explanation for the empirical observations that tax payers’ observable behavior does not respond to changes in the tax policy parameters \( t, q \) and \( P_\alpha \) the way the rational choice model predicts. E. g., to decrease the expected punishment \( qP_\alpha \) might have no influence on the observable behavior. Of course, a lower expected punishment increases \( \alpha^* \) would decrease the value of the probability distribution \( F_{\alpha^*}(\alpha) \) for each \( \alpha < 1 \), leading to a higher probability that an upward deviation is beneficial. This would decrease the tolerance limit \( T(\alpha) \), which causes a higher \( \alpha_h \). However, this parameter change might not have any effect on an imperfect tax payer’s behavior. Only the choice of some marginal tax payers might be influenced by an increase in \( \alpha_h \), given that this increase makes the contingent reaction strategy optimal just for them.

Our model of imperfect tax evasion therefore provides an explanation for the empirical observation that parameter shifts have no (or only a small) impact on tax payers’ behavior. Due to the stochastical predictions, an empirical test of this model would require to solve some difficulties. However, at least our main result definitely is a falsifiable prediction: An imperfect decision-making tax payer will never carry out an evasion rate above \( \alpha_h \).

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36Recall that for a perfect decision-maker, \( r/w \) is infinite, hence this stop value will necessarily be greater than any \( \alpha^* \).

37With respect to changes of the parameter \( t \), it can be argued in the similar way.

38Slemrod (1989, 178) describes the need to find valid proxies for complexity as the main obstacle.
4. Conclusion

An explanation for the tax payer puzzle - people pay more taxes than the rational choice theory predicts - should not simply state that the suboptimal tax evasion rate is due to morality. People have the option to deviate from moral rules as well as from tax laws. But their computational limits (competence) and the complexity of the decision problem (difficulty) can keep them from reaching the optimal solution. Taking this into account, the model of imperfect decision-making explains the puzzle for tax payers who decide whether to deviate from a low evasion rate.

The lower the rate a tax payer starts to consider deviating from, the more likely a deviation is attractive to him. However, as the considered tax evasion rate increases, the occurrence of a competence-difficulty-gap that motivates the imperfect decision-maker not to consider further deviations becomes more likely. This gap can stop an imperfect decision-making tax payer before he reaches the optimal evasion rate as derived by perfect choice models.

One might consider some objections against crucial assumptions of our model, e.g. our neglect of learning effects: If the tax payer repeatedly decides on his evasion rate, this might increase his competence. Hence the values of \( r \) and \( w \) should be endogenized in a multi-period model. Another sensible modification of the model would be to introduce a probability of detection \( q \) that increases with the deviation rate. The latter would lead to a faster growing tolerance limit \( T \).

Imperfect tax evasion as outlined here is only a basic model. Some further modifications should be taken into account when following the signal-detection theory on which Heiner has based his theory of imperfect decision-making. According to this theory, the values of \( r \) and \( w \) are not independent of the potential losses or gains that might be caused by a deviation, as we assumed here. Hence the predicted stop value for the consideration of deviations, \( \alpha_h \), would be different from the one we showed in our model. However, we are convinced that modifications like these could easily be taken into account and would not lead to qualitatively different results.

The complexity of tax laws is not just a matter of justice, which might require detailed tax laws, but also an impact on tax compliance. The more complex the tax system, the more likely a C-D-gap that keeps a tax payer from increasing her evasion. Of course, this proposition only holds true if tax authorities and courts have sufficient detection skill\(^{39}\). Since the creation of detection skill is at least in part a matter of experience, it would not be surprising if the specialists in the authorities had an advantage compared to single tax payers.

\(^{39}\text{Krause (1996) discusses the interaction of imperfect tax payers with an imperfect tax authority. See also Slemrod (1989, 176).}\)
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