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**INNOVATION OFFSHORING  
WITH FULLY ENDOGENOUS GROWTH**

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May 2019

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# Innovation Offshoring with Fully Endogenous Growth

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May 2019

## Abstract

In recent years firms have started to offshore their innovation activities to emerging economies. This paper investigates the implications of innovation offshoring for productivity growth in a two-country framework that features a tension between access to technical knowledge and low-cost high-skilled labor in the innovation location decision. Industry and innovation tend to concentrate in the asset-wealthy country when trade costs are relatively high. A positive relationship between innovation costs and industry concentration then ensures that improved international knowledge diffusion coincides with an increase in net offshoring flows in innovation from the asset-wealthy country to the asset-poor country, and potentially with faster productivity growth.

*JEL Classifications:* F12, F43, R11

*Keywords:* innovation offshoring, international knowledge diffusion, endogenous productivity growth, process innovation, industry location, international trade

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# 1 Introduction

Although research and development (R&D) has traditionally been concentrated in advanced countries, in recent years firms have begun to offshore their innovation activities towards emerging economies. For example, despite the steady position of the U.S. as the largest producer of R&D services in terms of gross domestic expenditure (NSF 2016), over the past decade it has experienced growing trade deficits in R&D services with a number of emerging economies. In particular, U.S. trade deficits with China and India grew at average annual rates of 22% and 17% over the period from 2006 through 2015 to become US\$2.4 billion and US\$3.4 billion in 2016 (BEA 2018). More generally, the main hosts for R&D offshoring outside the U.S. and the European Union are now Brazil, China, India, Russia, Singapore, and Taiwan (Hausmann et al. 2007; Puga and Trefler 2010; Santos-Paulino et al. 2014). The rise in the offshoring of R&D services has attracted the attention of policymakers who are concerned about the implications for economic growth and who hope to attract firms offering the high wages associated with R&D employment (UNCTAD 2005).

Beginning to address these concerns, there is an emerging empirical literature investigating the links between R&D offshoring, innovation performance, and economic growth. At the firm level, Nieto and Rodríguez (2011) and Bertrand and Mol (2013) find that offshoring R&D leads to a higher propensity for the introduction of new products. Similarly, Rodríguez and Nieto (2016) document a positive relationship between innovation offshoring and sales growth. At the aggregate level, D’Agostino et al. (2013) show that OECD regions with firms that offshore innovation to emerging economies have more patent applications, and Castellani and Pieri (2013) report that European regions with a greater number of outward oriented R&D investment projects exhibit higher rates of labor productivity growth. While the empirical literature suggests a positive relationship between R&D offshoring and innovation-based economic growth, to the best of our knowledge this relationship has not been formally modeled within an endogenous growth framework.

This paper develops an endogenous market structure and endogenous growth framework

(Peretto 1996; Aghion and Howitt 1998; Peretto and Connolly 2007; Etro 2009) to study the relationship between offshoring patterns in innovation and manufacturing and long-run productivity growth. In particular, we extend the two-country trade model of Davis and Hashimoto (2015) to include an occupational choice for skill-differentiated workers between low-skilled employment in production and high-skilled employment in R&D. Firms create new differentiated products for supply to a monopolistically competitive market, while investing in process innovation to reduce future production costs. Geographic patterns of industry and R&D activity are determined endogenously, as the free movement of investment allows firms to shift product development, production, and process innovation separately between countries with the aim of minimizing costs (Martin and Ottaviano 1999, 2001).

The model captures two key factors that have been emphasized in the business literature when considering the attractiveness of a location for R&D: access to technical knowledge and the supply of low-cost high-skilled labor (see, for example, Chung and Yeaple 2008; Manning et al. 2008; Lewin et al. 2009; Demirbag and Glaister 2010). On the one hand, knowledge spillovers from production to innovation are local in nature, leading to higher labor productivity in R&D in the country hosting the greater share of industry. On the other hand, the concentration of industry generates greater demand for high-skilled labor in innovation, pushing up high-skilled wages. These factors generate a tension in the firm-level location decision for R&D activity.

At the aggregate level, market size is determined endogenously through a circular causality between production and innovation location patterns that is generated by two mechanisms. The first is a *knowledge spillover* effect whereby increases in a country's investment income and labor income expand the national market, attracting manufacturing and strengthening knowledge spillovers. Increased R&D activity in turn raises labor income, expands market size, and further attracts manufacturing. The second mechanism is a *wage* effect which regulates how fast high-skilled wages can rise as the market expands without inducing firms to relocate innovation internationally in search of lower cost high-skilled labor. We

show that the wage effect dominates and thus economic activity concentrates in the asset-wealthy country for high trade costs, while the knowledge spillover effect dominates and economic activity therefore concentrates in the asset-poor country for low trade costs.

We then study the direction of net offshoring flows in production and innovation at the macro-level using the difference between the value of world output owned by a country and the value of output located in the country. There are three cases. First, for high trade costs, the market of the asset-wealthy country is not large enough to support the production and innovation of all domestically-owned firms, and net offshoring flows towards the asset-poor country. Second, for intermediate trade costs, the larger market of the asset-wealthy country attracts the production and innovation activities of all domestically-owned firms, and the asset-wealthy country therefore receives net offshoring inflows from the asset-poor country. Third, for low trade costs, net offshoring flows from the asset-wealthy country towards the larger labor-income-based market of the asset-poor country.

Long-run growth is driven by firm-level investment in process innovation, which is in turn closely linked with industry location patterns both directly through changes in innovation costs and indirectly through the inverse relationship between firm scale and the number of firms in the market. In particular, increased industry concentration leads to higher innovation costs, as rising high-skilled wages more than offset the benefit of improved knowledge spillovers, with two effects on market entry. The first is a *product development* effect whereby the level of market entry falls as the cost of creating new product designs rises. The second is a *process innovation* effect with rising innovation costs reducing firm-level employment in process innovation, raising firm-level profits, and inducing a greater level of market entry. Combining these effects, we find that there is a negative relationship between industry concentration and productivity growth when the process innovation effect dominates, and a convex relationship when the product development effect dominates. Although there is no general consensus in the empirical literature on the relationship between industry concentration and economic growth, the negative relationship that arises for most cases

within our framework is consistent with several of the empirical results of Bosker (2007) and Gardiner et al. (2011).

Focusing on the case for which the asset-wealthy country has the larger market, we study the effects of greater economic integration. First, a decrease in trade costs increases the concentration of industry and R&D in the asset-wealthy country, causing net offshoring flows out of the asset-wealthy country to fall. Consequently, the unit cost of process innovation rises, potentially dampening productivity growth. Second, an improvement in knowledge diffusion increases net offshoring flows out of the asset-wealthy country as firms shift innovation to the asset-poor country to take advantage of lower high-skilled wages. As a result, the increased dispersion of production and innovation away from the asset-wealthy country leads to lower innovation costs and potentially a higher rate of productivity growth. In this manner, our framework yields a positive relationship between net innovation offshoring flows and productivity growth, which is consistent with the results of the empirical literature introduced above (Castellani and Pieri 2013; D'Agostino et al. 2013).

There is a well-developed international trade literature studying the determinants and effects of multinational production patterns within quantitative general equilibrium frameworks (Burstein and Monge-Naranjo 2009; Ramondo 2009; Ramondo and Rodriguez-Claire 2013; Grumpert et al. 2017; Tintelnot 2017). Importantly, Ekholm and Hakkala (2007) introduce a two-country model which allows for the geographic separation of R&D and high-tech production, with imperfect technology spillovers and trade costs generating agglomeration forces in each activity. A home market effect in the high-tech industry and competing demands for skilled labor across sectors generate higher skilled wages in the large country, potentially inducing firms to offshore innovation to the small country with its cheaper skilled labor. More recently, Arkolakis et al. (2018) construct a multi-country model with heterogeneous production technologies and skilled differentiated workers, and show that home market effects interact with comparative advantage to produce a pattern of specialization with some countries specializing in innovation while others specialize in

production. While these frameworks allow for a study of the welfare gains associated with multinational production patterns, they do not consider the implications for long-run growth.

Our analysis builds on antecedents in the endogenous growth literature that extend the variety-expansion model of Grossman and Helpman (1991) to consider the effects of manufacturing offshoring on innovation-based growth (Martin and Ottaviano 1999, 2001; Gao 2005, 2007; Naghavi and Ottaviano 2009a, 2009b).<sup>1</sup> In this literature, it is common for innovation to agglomerate fully in the advanced country either by assumption or because of localized knowledge spillovers. As a consequence, these models are not viable for the study of how R&D offshoring from developed countries to emerging economies affects economic growth. Davis (2013) investigates the trade-off between localized knowledge spillovers and high-skilled wage costs in the R&D location choice for firms by including an occupational choice for skill-differentiated workers, but the link between industry location and economic growth is cut, leaving no relationship between offshoring patterns and economic growth. The key contribution of our paper is the introduction of a framework that allows for a theoretical study of the positive link between R&D offshoring and innovation-based economic growth that has been documented in the empirical literature.

The paper proceeds as follows. Section 2 introduces the model. Section 3 characterizes industry and innovation location patterns and derives the direction of offshoring flows. We then outline the effects of economic integration on market entry and productivity growth, and study the link between net offshoring flows and long-run growth. Section 4 concludes.

## 2 The Model

We propose a model of international trade with two countries indexed by  $i$  and  $j$ . The countries potentially employ labor in four activities: traditional production, manufacturing, process innovation, and product development. Firms are free to separate their production

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<sup>1</sup>McGrattan and Prescott (2009) extend the neoclassical growth model to include investment in technology capital and show that openness to foreign direct investment promotes improvements in productivity.



and innovation activities geographically. Labor is the sole factor of production with symmetric endowments across countries. There is no international migration, but labor is mobile between sectors with national supplies of low-skilled and high-skilled labor determined endogenously through an occupational choice made by skill-differentiated workers.

## 2.1 Household Preferences

Dynastic households in each country select optimal expenditure-saving paths to maximize utility over an infinite time horizon. The lifetime utility of the representative household in country  $i$  is

$$U_i(t) = \int_t^\infty e^{-\rho(\tau-t)} (\alpha \ln C_{Xi}(\tau) + (1 - \alpha) \ln C_{Yi}(\tau)) d\tau, \quad (1)$$

where the consumptions of a composite of manufacturing goods and a traditional good at time  $t$  are  $C_{Xi}(t)$  and  $C_{Yi}(t)$ , the subjective discount rate is  $\rho$ , and  $\alpha \in (0, 1)$ . The manufacturing composite is

$$C_{Xi}(t) = \left( \int_0^{N_i(t)} x_{ii}(\omega, t)^{\frac{\sigma-1}{\sigma}} d\omega + \int_0^{N_j(t)} x_{ij}(\omega, t)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \quad (2)$$

where  $x_{ii}(t)$  and  $x_{ij}(t)$  are the country  $i$  household demands for each of the  $N_i(t)$  and  $N_j(t)$  manufacturing varieties produced in countries  $i$  and  $j$ , and  $\sigma > 1$  is the elasticity of substitution. Lifetime utility is maximized subject to a national flow budget constraint:

$$\dot{B}_i(t) = r(t)B_i(t) + I_i(t) - E_i(t), \quad (3)$$

where  $B_i(t)$  is asset wealth,  $I_i(t)$  is labor income,  $E_i(t)$  is household expenditure,  $r(t)$  is the interest rate, and a dot over a variable denotes time differentiation. The solution to the household's utility maximization problem is the optimal expenditure-saving path described by the Euler condition:  $\dot{E}_i(t)/E_i(t) = \dot{E}_j(t)/E_j(t) = r(t) - \rho$ , where with equal access to

an international financial market, the households of both countries face a common interest rate and have common motions for expenditure.

At each moment in time, households allocate constant expenditure shares to the manufacturing composite and the traditional good:  $P_{X_i}(t)C_{X_i}(t) = \alpha E_i(t)$  and  $P_{Y_i}(t)C_{Y_i}(t) = (1 - \alpha)E_i(t)$ , where  $P_{X_i}(t)$  is the price index over manufacturing goods and  $P_{Y_i}(t)$  is the traditional good price. In particular, the price index over manufacturing goods is

$$P_{X_i}(t) = \left( \int_0^{N_i(t)} p_{X_i}(t, \omega)^{1-\sigma} d\omega + \int_0^{N_j(t)} (\zeta p_{X_j}(t, \omega))^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}, \quad (4)$$

where  $p_{X_i}(t)$  and  $p_{X_j}(t)$  are the prices associated with manufacturing varieties produced in countries  $i$  and  $j$ . Iceberg costs are incurred on international trade in manufacturing goods with a shipment of  $\zeta > 1$  units required for every unit sold in an export market. Viewing (4) as the unit expenditure function over manufacturing goods, Shephard's Lemma yields the country  $i$  demands for varieties produced in countries  $i$  and  $j$ :

$$x_{ii}(t) = \alpha p_{X_i}(t)^{-\sigma} P_{X_i}(t)^{\sigma-1} E_i(t), \quad x_{ij}(t) = \alpha (\zeta p_{X_j}(t))^{-\sigma} P_{X_i}(t)^{\sigma-1} E_i(t). \quad (5)$$

## 2.2 Occupational Choice

Each country has a population of workers  $Z$  with heterogeneous skill levels  $z$  that follow a continuous uniform distribution with support  $[0, 1]$ . At each moment in time, workers choose between low-skilled employment in production and high-skilled employment in innovation. Workers employed in production in country  $i$  each supply one unit of low-skilled labor, regardless of skill level, earning the low-skilled wage rate  $w_{Li}(t)$ . In contrast, workers employed in innovation supply high-skilled labor according to their skill levels. Specifically, a worker with skill level  $z$  supplies  $z$  units of high-skilled labor, and earns income  $zw_{Hi}(t)$ , where  $w_{Hi}(t)$  is the high-skilled wage rate.

Competitive national labor markets ensure that all firms offer the same low-skilled and

high-skilled wages. Thus, with active production and innovation sectors, each country has a marginal worker who can earn equal incomes from low-skilled and high-skilled employment, and is indifferent between employment type. The skill level of this marginal worker equals the relative wage rate  $z = w_{Li}(t)/w_{Hi}(t)$ . We separate the labor force into those workers  $z \in [0, 1/w_i(t)]$  who choose employment in production, and those workers  $z \in [1/w_i(t), 1]$  who select employment in innovation, where  $w_i(t) \equiv w_{Hi}(t)/w_{Li}(t)$  denotes the relative high-skilled wage rate in country  $i$ .

The occupational choice results in the following effective low-skilled and high-skilled labor supplies for country  $i$ :

$$L_i(t) = \frac{Z}{w_i(t)}, \quad H_i(t) = \frac{(w_i(t)^2 - 1)Z}{2w_i(t)^2}. \quad (6)$$

As such, expected national labor income, conditional on employment levels, becomes  $I_i(t) \equiv w_{Li}(t)L_i(t) + w_{Hi}(t)H_i(t) = w_{Li}(t)(1 + w_i(t)^2)Z/(2w_i(t))$ .

### 2.3 Traditional Production

Traditional firms employ one unit of low-skilled labor with a constant returns to scale technology to produce one unit of output for supply to an international market characterized by free trade. Setting the traditional good as the model numeraire, we suppose that demand is sufficiently large that both countries continue to produce traditional goods at all moments in time. The traditional good price and low-skilled wage rates then equal one ( $P_{Yi}(t) = P_{Yj}(t) = w_{Li}(t) = w_{Lj}(t) = 1$ ), and the combined demands of households in countries  $i$  and  $j$  determine the world demand for low-skilled labor in traditional production:

$$L_{Yi}(t) + L_{Yj}(t) = (1 - \alpha)E(t), \quad (7)$$

with  $E(t) \equiv E_i(t) + E_j(t)$ .

## 2.4 Manufacturing

The manufacturing sector is monopolistically competitive (Dixit and Stiglitz 1977), with each firm producing a single unique product in one location for supply to both domestic and export markets. Following Peretto and Connolly (2007), the technology of a firm with production located in country  $i$  is

$$x_i(t) = \theta(t)^\gamma (l_{Xi}(t) - \psi), \quad (8)$$

where  $x_i(t)$  is firm-level output,  $l_{Xi}(t)$  is low-skilled employment in production,  $\theta(t)$  is firm-level productivity,  $\psi > 0$  is a fixed operating cost, and  $\gamma \in (0, 1)$  is the productivity elasticity of output. Although each firm employs a unique production technology, in order to simplify the analysis, we assume that initial productivity levels are symmetric across firms ( $\theta(t) \equiv \theta_i(t) = \theta_j(t)$ ) operating in both countries.

Firms maximize operating profit ( $\pi_i(t) = p_{Xi}(t)x_i(t) - l_{Xi}(t)$ ) by setting price equal to a constant markup over unit cost:  $p_{Xi}(t) = p_{Xj}(t) = \sigma\theta(t)^{-\gamma}/(\sigma - 1)$ . Equating supply with the combined demands from households in both countries (5),  $x_i(t) = x_{ii}(t) + \zeta x_{ij}(t)$ , the optimal operating profit of a firm with production located in country  $i$  is

$$\pi_i(t) = \frac{\alpha p_{Xi}(t)^{1-\sigma}}{\sigma} \left( \frac{E_i(t)}{P_{Xi}(t)^{1-\sigma}} + \frac{\varphi E_j(t)}{P_{Xj}(t)^{1-\sigma}} \right) - \psi, \quad (9)$$

where  $\varphi \equiv \zeta^{1-\sigma} \in (0, 1)$  describes the freeness of trade between countries with  $\varphi = 0$  indicating prohibitively high trade costs and  $\varphi = 1$  indicating perfectly free trade.

Firms are free to shift production between countries, with the objective of maximizing operating profit (Martin and Ottaviano 1999, 2001). Therefore, when there are active manufacturing sectors in both countries, operating profit is the same for all firms, regardless of the location of production; that is,  $\pi_i(t) = \pi_j(t)$ .<sup>2</sup> Combining the price indices (4) with

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<sup>2</sup>In order to keep the model tractable, we assume that there are no additional fixed costs associated with the separation of production and innovation activities across countries. See Tintelnot (2017) for a quantitative general equilibrium framework that examines how the tension between trade costs and the fixed costs of

operating profit (9), and recalling that the prices of differentiated varieties are equal across firms and locations, we solve for the equilibrium share of firms with production located in country  $i$  as follows:

$$s_{Xi}(t) \equiv \frac{N_i(t)}{N(t)} = \frac{E_i(t) - \varphi E_j(t)}{(1 - \varphi)E(t)}, \quad (10)$$

where  $N(t) \equiv N_i(t) + N_j(t)$ . Note that a common motion for household expenditures ( $\dot{E}_i(t)/E_i(t) = \dot{E}_j(t)/E_j(t)$ ) results in constant production shares over time ( $\dot{s}_{Xi}(t) = 0$ ). Substituting the manufacturing shares of each country back into operating profit yields firm-level employment in production for all locations:

$$l_{Xi}(t) = l_{Xj}(t) = \frac{\alpha(\sigma - 1)E(t)}{\sigma N(t)} + \psi, \quad (11)$$

where we have used the production function (8).

## 2.5 Process Innovation

Firms invest in process innovation with the aim of lowering unit production costs. In particular, the evolution of productivity for a firm with process innovation located in country  $i$  follows

$$\dot{\theta}_i(t) = k_i(t)\theta_i(t)h_{Ii}(t), \quad (12)$$

where  $h_{Ii}(t)$  is firm-level high-skilled employment and  $k_i(t)\theta_i(t)$  is labor productivity in process innovation. Following the in-house process innovation literature (Smulders and van de Klundert 1995; Peretto 1996; Peretto and Connolly 2007), firm-level R&D exhibits an intertemporal knowledge spillover through which the technical knowledge created by current innovation activity improves the labor productivity of future innovation efforts. Specifi-

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establishing foreign production generates various multinational production patterns in a multi-country model.

cally, the productivity coefficient  $\theta_i(t)$  represents the current stock of technical knowledge, and  $k_i(t)$  determines the strength of intertemporal knowledge spillovers from the stock of knowledge into current innovation activity. As the stock of technical knowledge expands over time, innovation costs fall, creating the potential for perpetual productivity growth in long-run equilibrium.

There is a large body of empirical research documenting both the localized nature of knowledge spillovers and the international scope for knowledge diffusion (Bottazzi and Peri 2007; Mancusi 2008; Coe et al. 2009; Ang and Madsen 2013).<sup>3</sup> Adapting the setup of Baldwin and Forslid (2000), we capture the geographic nature of knowledge diffusion with the following specification for the strength of knowledge spillovers from production into process innovation located in country  $i$ :

$$k_i(t) = s_{Xi}(t) + \delta s_{Xj}(t), \quad (13)$$

where the localized nature of knowledge spillovers is regulated by the degree of knowledge diffusion  $\delta \in (0, 1)$ . Under this specification, the labor productivity of high-skilled workers in process innovation is determined as the weighted-average productivity of the observable stock of knowledge, with a stronger weighting for production technologies employed in proximity to the innovation department of the firm.

Total per-period profit is  $\Pi_i(t) = \pi_j(t) - w_i(t)h_{Ii}(t)$ , with the high-skilled wage rate now denoted by  $w_i(t) = w_{H_i}(t)$ . Firm value equals the presented discounted value of expected profits:

$$V_i(t) = \int_t^\infty e^{-\int_t^\tau (r(\tau') + \lambda) d\tau'} \Pi_i(\tau) d\tau, \quad (14)$$

where  $\lambda > 0$  is an instantaneous default rate which indicates the probability that a firm-specific shock forces the firm to exit the market (Baldwin 1999).

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<sup>3</sup>See Keller (2004) for a survey of the various channels through which knowledge spillovers arise.

Each firm sets its level of high-skilled employment in process innovation to maximize firm value (14) subject to (12). We solve this optimization problem with the following current-value Hamiltonian function:  $F_i(t) = \Pi_i(t) + p_{Ii}(t)k_i(t)\theta_i(t)h_{Ii}(t)$ , where  $p_{Ii}(t)$  can be conceived as the cost of a unit mass of new process innovations developed by a firm in country  $i$  over the time interval  $dt$ . The first order conditions for optimization provide the following static and dynamic efficiency conditions:

$$p_{Ii}(t) = \frac{w_i(t)}{k_i(t)\theta(t)}, \quad p_{Ii}(t)(r(t) + \lambda) - \dot{p}_{Ii}(t) = \frac{\partial \pi_j(t)}{\partial \theta(t)} = \frac{\alpha(\sigma - 1)\gamma E(t)}{\sigma \theta(t)N(t)}, \quad (15)$$

where we have used (9) and (11). Note that under monopolistic competition, each firm perceives the price indices ( $P_{Xi}(t)$ ) and knowledge spillovers ( $k_i(t)\theta(t)$ ) as constant when maximizing firm value, given its atomistic market share.<sup>4</sup>

Firms are free to shift their innovation activities between countries, ensuring a common cost for new process innovations when there is innovation located in both countries; that is,  $p_{Ii}(t) = p_{Ij}(t)$ . We use this condition to obtain the country  $i$  production share that equates the cost of process innovations across countries:

$$s_{Xi}(t) = \frac{w_i(t) - \delta w_j(t)}{(1 - \delta)(w_i(t) + w_j(t))}, \quad (16)$$

where we have used (13) and (15). Note that constant production shares ( $\dot{s}_{Xi}(t) = 0$ ) lead to common motions for high-skilled wages ( $\dot{w}_i(t)/w_i(t) = \dot{w}_j(t)/w_j(t)$ ). In addition, similar to the framework of Ekholm and Hakkala (2007), the degree of knowledge diffusion regulates the range of relative high-skilled wages over which dispersed location patterns are feasible; that is,  $w_i(t)/w_j(t) \in (\delta, 1/\delta)$  is required for  $s_{Xi}(t) \in (0, 1)$ .

In order to emphasize the role of high-skilled wages and knowledge spillovers, we set the regional cost component of unit innovation costs as  $c_i(t) \equiv w_i(t)/k_i(t)$  such that the

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<sup>4</sup>Using (11), (12), and (15), steady-state firm-level employment in process innovation becomes  $h_{Ii} = (l_{Xi} - \psi)/w_i - (\rho + \lambda)/k_i$ . Accordingly, firm-level investment in process innovation ( $h_{Ii}$ ) increases with the scale of production, as in Rubini (2014) and Piguillem and Rubini (2019).

price of new innovations is  $p_{Ii}(t) = c_i(t)/\theta(t)$  in country  $i$ . Then, substituting (16) back into (13) yields the regional component of the unit cost for process innovation as

$$c(t) = \frac{w_i(t)}{k_i(t)} = \frac{w_j(t)}{k_j(t)} = \frac{w_i(t) + w_j(t)}{1 + \delta}. \quad (17)$$

This unit cost component captures two key factors in the location decision for R&D that have been emphasized in the literature: the observable stock of technical knowledge and the cost of employing high-skilled labor (Chung and Yeaple 2008; Manning et al. 2008; Lewin et al. 2009; Demirbag and Glaister 2010).

## 2.6 Product Development

New firms employ high-skilled labor in product development as they prepare to enter the manufacturing industry. The creation of a new product design requires  $\alpha E(t)/(N_i(t) + \delta N_j(t))$  units of high-skilled labor in country  $i$ , where the numerator captures the overall size of the market for manufacturing varieties, generating a positive relationship between entry costs and market size (Etro 2004, Peretto and Connelly 2007, Peretto and Valente 2015). The denominator captures the benefit of knowledge spillovers into the product development process, where we assume that the national stock of knowledge associated with product design is measured by the number of product varieties currently being produced in the country. Then, because knowledge spillovers diminish with distance,  $\delta \in (0, 1)$ , there are greater knowledge spillovers into product development in the country with the larger share of production (Baldwin et al. 2001).

Free entry into the manufacturing industry drives firm value (14) down to the cost of designing a new product, and when both countries have active R&D sectors, the cost of product development equalizes across locations. Using (14), we therefore have

$$V_i(t) = V_j(t) = \frac{\alpha E(t)}{k_i(t)N(t)}w_i(t) = \frac{\alpha E(t)}{k_j(t)N(t)}w_j(t) = \frac{\alpha E(t)}{N(t)}c(t). \quad (18)$$



The time derivative of (14) yields the following no-arbitrage condition for investment in a new product design (Grossman and Helpman 1991):

$$(r(t) + \lambda)V_i(t) - \dot{V}_i(t) = \pi_i(t) - w_j(t)h_{Ij}(t). \quad (19)$$

Together the investment conditions (15) and (19) imply that  $c(t) = w_i(t)/k_i(t) = w_j(t)/k_j(t)$ ,  $w_i(t)h_{Ii}(t) = w_j(t)h_{Ij}(t)$ , and  $k_i(t)h_{Ii}(t) = k_j(t)h_{Ij}(t)$ , when both countries host innovation activity.

Aggregating the high-skilled labor employed in product development across countries, and subtracting the number of firms ( $\lambda N(t)$ ) that default at each moment of time yields a differential equation for the evolution of market entry and exit:

$$\frac{\dot{N}(t)}{N(t)} = \frac{k_i(t)H_{Ni}(t) + k_j(t)H_{Nj}(t)}{\alpha E(t)} - \lambda, \quad (20)$$

where  $H_{Ni}(t)$  and  $H_{Nj}(t)$  are the levels of high-skilled labor employed in the product development sectors of each country.

## 2.7 Labor Market Clearing

In this section, we derive the market clearing conditions associated with low-skilled and high-skilled labor. First, combining the low-skilled labor supply (6) with the labor demands from the traditional sector (7) and the manufacturing sector (11) yields the world market clearing condition for low-skilled labor as follows:

$$L(t) \equiv L_i(t) + L_j(t) = \mu E(t) + \psi N(t), \quad (21)$$

with  $\mu \equiv (\sigma - \alpha)/\sigma \in (0, 1)$ .

Second, utilizing the high-skilled labor supply (6) with the labor demands from process innovation (12) and product development (20) we obtain the world market clearing condition

for high-skilled labor as

$$w_i(t)H_i(t) + w_j(t)H_j(t) = \left( N(t)\frac{\dot{\theta}(t)}{\theta(t)} + \alpha E(t)\frac{\dot{N}(t)}{N(t)} + \alpha E(t)\lambda \right) c(t), \quad (22)$$

where we have used  $c(t) = w_i(t)/k_i(t) = w_j(t)/k_j(t)$  and  $w_i(t)h_{Ii}(t) = w_j(t)h_{Ij}(t)$ .

Finally, denoting the number of firms with process innovation located in country  $i$  by  $M_i(t)$ , we show that the total value of country  $i$ 's R&D output (process innovation and product development combined) is equal to high-skilled labor income; that is,  $M_i(t)p_{Ii}(t)\dot{\theta}(t) + V_i(t)\dot{N}_i(t) = M_i(t)w_i(t)h_{Ii}(t) + w_i(t)H_{Ni}(t) = w_i(t)H_i(t)$ . As such, we derive country  $i$ 's share of the total value of world R&D output as follows:

$$s_{Ii}(t) \equiv \frac{w_i(t)H_i(t)}{w_i(t)H_i(t) + w_j(t)H_j(t)}. \quad (23)$$

Thus, we find that national shares of innovation activity are determined proportionately with shares of high-skilled labor income.

### 3 Long-run Location and Trade Patterns

We derive the long-run industry and innovation location patterns consistent with equilibrium in the investment and labor markets. To simplify the analysis, we focus on the level of market entry relative to overall market size for the manufacturing sector:  $n \equiv N/(\alpha E)$ , where we henceforth suppress time arguments to simplify notation. Steady-state equilibrium is characterized by constant values for household expenditure ( $\dot{E} = \dot{E}_i = \dot{E}_j = 0$ ), high-skilled wages ( $\dot{w}_i = \dot{w}_j = 0$ ), market entry ( $\dot{n} = 0$ ), and asset wealth ( $\dot{B} = \dot{B}_i = \dot{B}_j = 0$ ). The number of new product designs introduced each period matches the number of firms exiting the market, ensuring a constant steady-state number of incumbent firms ( $\dot{N} = 0$ ). We set country  $i$  as the asset wealthy country and country  $j$  as the asset poor country to simplify the exposition; that is, we assume  $B_i > B_j$ .

### 3.1 National Labor Allocations

Long-run equilibrium is fully characterized through two conditions that determine high-skilled wages and hence the division of national labor forces into low-skilled and high-skilled labor. The first condition is an *investment* locus that describes the high-skilled wage combinations for which investment levels in process innovation and product development are optimal. Denoting total asset wealth by  $B \equiv B_i + B_j$  and total labor income by  $I = I_i + I_j$ , we have  $B = NV_i = \alpha Ec$  from (18), and  $\dot{B}/B = r + (I - E)/B$  from (3). Combining these expressions with (15), (18), (19), and (21), we obtain the investment locus as

$$\frac{\nu - (\rho + \lambda)c}{\psi - (\rho + \lambda)c} = \frac{(1 - \alpha\rho c)L}{\alpha\psi I} - \frac{\mu}{\alpha\psi}, \quad (24)$$

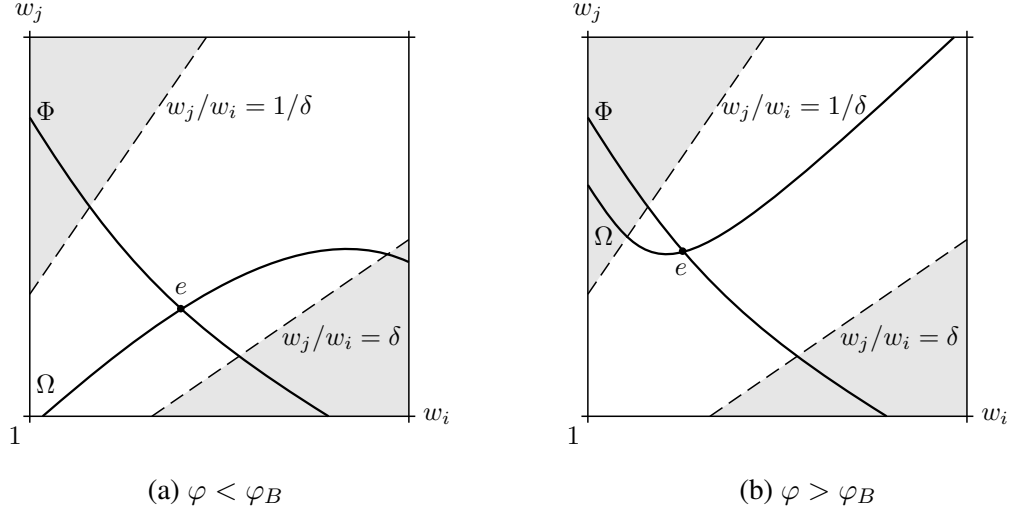
where we have used  $\dot{E} = \dot{w}_i = \dot{w}_j = 0$ , and the marginal profit associated with an increase in the size of the firm's market segment provides a measure of firm-level market power:  $\nu \equiv \partial\Pi/\partial(1/n) = (1 - (\sigma - 1)\gamma)/\sigma \in (0, 1)$ .<sup>5</sup> Noting that  $1 > \alpha\rho c$  is necessary for a positive level of expenditure given world labor income,  $E = I/(1 - \alpha\rho c)$ , the investment locus has a strictly negative slope in  $(w_i, w_j)$  space, as depicted by the  $\Phi$ -curve in Figure 1.

The second condition is a *share* locus that indicates the high-skilled wage combinations which are consistent with national production shares that equalize operating profits, process innovation costs, and product development costs across countries. We use  $\dot{B} = \rho B + I - E = 0$  together with the household flow budget constraint (3) to obtain  $E_i = I_i + b_i\alpha\rho cI/(1 - \alpha\rho c)$ , where  $b_i \equiv B_i/B$  is country  $i$ 's share of asset wealth. Substituting household expenditure into (10) and equating the result with (16) then yields the share locus

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<sup>5</sup>Substitution of (8), (11), (12), and (15) into  $\Pi_i = \pi_j - w_i h_{Ii}$  gives per-period profit as  $\Pi = (1 - (\sigma - 1)\gamma)/(\sigma n) + (\rho + \lambda)c - \psi$ .

Figure 1: Equilibrium Labor Allocations



These stylized labor allocation patterns can be reproduced numerically using  $\alpha = 0.85$ ,  $\sigma = 3.5$ ,  $\gamma = 0.35$ ,  $\rho = 0.01$ ,  $\lambda = 0.01$ ,  $\psi = 0.2$ ,  $b = 0.65$ ,  $\delta = 0.8$ ,  $Z = 1$ , and  $\varphi_B = 0.96$ , for both panels, and with  $\varphi = 0.6$  for Panel (a) and  $\varphi = 0.97$  for Panel (b).

as follows (see Appendix B):

$$\frac{(b_i - \varphi b_j)\alpha\rho c}{1 - \varphi} + \frac{(I_i - \varphi I_j)(1 - \alpha\rho c)}{(1 - \varphi)I} = \frac{w_i - \delta w_j}{(1 - \delta)(w_i + w_j)}. \quad (25)$$

This expression has lower and upper hyperbolic branches, as illustrated by the  $\Omega$ -curves in Figures 1a and 1b, with asymptotes implicitly defined by  $w_i = w_j$  and  $1 - \delta\varphi + (\delta - \varphi)w_i w_j + \alpha\rho(1 - \delta)(1 + \varphi)(b_j w_i w_j - b_i)c = 0$ , and a vertical transverse axis for  $b_i > b_j$ .<sup>6</sup>

Long-run equilibrium is determined at point  $e$  where the investment locus and the share locus intersect. We investigate the local dynamics around this steady state and obtain the following stability conditions for a long-run equilibrium with positive high-skilled employment levels in both countries.

**Lemma 1** *A long-run equilibrium with dispersed industry and innovation is saddle-path*

<sup>6</sup>Using the expressions introduced in Appendix B, the share locus can be rewritten as follows:

$$\frac{(1 - \delta)(1 + \varphi)(b_i - b_j)(1 + w_i w_j)w_j \alpha \rho c}{(1 - \delta\varphi + (\delta - \varphi)w_i w_j + (1 - \delta)(1 + \varphi)(b_j w_i w_j - b_i)\alpha\rho c)(w_i - w_j)} = 1.$$

The denominator describes the asymptotes that arise for  $b_i > b_j$ .

stable for  $\psi > (\rho + \lambda)c$ .

Proof: See Appendix A.

We focus our analysis on long-run equilibria that satisfy the stability condition outlined in Lemma 1. In addition, we consider cases for which the share locus (25) crosses the investment locus (24) once from below to ensure the uniqueness of equilibrium.

### 3.2 Production and Innovation Location Patterns

In this section, we study the long-run location patterns that arise for different levels of trade costs. In particular, while each firm selects independent, and potentially different, locations for product development, production, and process innovation, we show that aggregate location patterns are linked through a circular causality driven by two mechanisms: a *knowledge spillover* effect and a *wage* effect.

The knowledge spillover effect is described by the LHS of the share locus (25), with increases in a country's share of investment income (first term) or labor income (second term) expanding the national market, attracting manufacturing, and strengthening knowledge spillovers from production to innovation. In turn, stronger knowledge spillovers invite more innovation, raising labor income as high-skilled employment increases and further expanding the national market. In this manner, national shares of production and innovation are linked with market size through a home market effect (Krugman 1980), and thus the strength of the knowledge spillover effect is increasing in the freeness of trade.

The wage effect is captured by the RHS of the share locus (25), and regulates the rate at which a country's high-skilled wages can rise as its market expands without inducing firms to relocate their innovation activities internationally to reduce the cost of employing high-skilled labor. The wage effect is increasing in the degree of knowledge diffusion: greater knowledge diffusion facilitates a faster rise in high-skilled wages as a country's market expands.

The tension between market expansion and rising high-skilled wages generates two po-

tential cases for location patterns. We identify these patterns by observing the direction of the labor market adjustments that occur after an increase in country  $i$ 's share of investment income ( $b_i$ ). In particular, through the share locus (25), we show that an increase in  $b_i$  raises country  $i$ 's shares of production ( $s_{Xi}$ ) and innovation ( $s_{Ii}$ ) when  $\varphi < \varphi_B$ , but lowers them when  $\varphi > \varphi_B$ , where

$$\varphi_B \equiv \frac{1 + \delta w_i w_j - (1 - \delta)(b_i - w_i w_j b_j) \alpha \rho c}{\delta + w_i w_j + (1 - \delta)(b_i - w_i w_j b_j) \alpha \rho c},$$

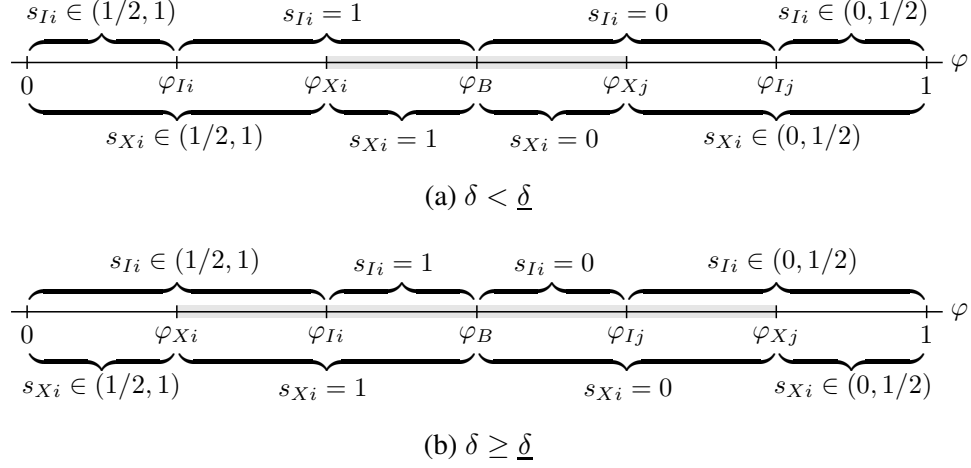
with positive values for both the numerator and denominator since  $1 > \alpha \rho c$ .

First, when  $\varphi < \varphi_B$ , the wage effect dominates the knowledge spillover effect, and the increase in  $b_i$  initially raises  $s_{Xi}$  causing relative innovation costs to fall for country  $i$ ; that is,  $c_i < c_j$ . Firms then relocate innovation and production to country  $i$ , with  $w_i/w_j$  rising at a faster rate than  $k_i/k_j$  until innovation costs are again equal. In this first case, the asset wealthy country  $i$  has a larger market with greater high-skilled employment, and larger shares of production ( $s_{Xi} > 1/2$ ) and innovation ( $s_{Ii} > 1/2$ ).

Second, when  $\varphi > \varphi_B$ , the knowledge spillover effect dominates, and the initial shift in location patterns resulting from an increase in  $b_i$  leads to higher innovation costs for country  $i$ ; that is,  $c_i > c_j$ . Accordingly, firms shift innovation and production to country  $j$  with  $k_i/k_j$  falling at a faster rate than  $w_i/w_j$  until innovation costs are again equal across countries. In this second case, the asset-poor country  $j$  has a larger market with greater high-skilled employment, and higher production ( $s_{Xi} < 1/2$ ) and innovation shares ( $s_{Ii} < 1/2$ ).

Changes in trade costs have opposing effects on the location patterns described for the two cases outlined above. For the first case ( $\varphi < \varphi_B$ ) shown in Figure 1a, a decrease in trade costs moves point  $e$  rightward along the investment locus, with country  $i$ 's shares of production and innovation expanding as the knowledge spillover effect is magnified. Production concentrates fully in asset-wealthy country  $i$  where the investment locus crosses the  $w_j/w_i = \delta$  dashed line ( $s_{Xi} = 1$ ). We denote the threshold for the freeness of trade asso-

Figure 2: Trade Costs and Location Patterns



ciated with this point by  $\varphi_{X_i}$ . Similarly, innovation concentrates fully in country  $i$  at the intersection of the investment locus and the horizontal axis ( $s_{I_i} = 1$ ). The threshold for the freeness of trade linked with this point is  $\varphi_{I_i}$ .

In contrast, in the second case illustrated for  $\varphi > \varphi_B$  in Figure 1b, an increase in trade costs moves point  $e$  leftward along the investment locus until either production is fully concentrated in asset-poor country  $j$  where the investment locus crosses the  $w_j/w_i = 1/\delta$  dashed line ( $s_{X_i} = 0$ ), or innovation is fully concentrated in country  $j$  at the intersection of the investment locus with the vertical axis ( $s_{I_i} = 0$ ). We use  $\varphi_{X_j}$  and  $\varphi_{I_j}$  to denote the threshold values for the freeness of trade associated with the full concentration of production and innovation in country  $j$ .

The long-run location patterns associated with various trade costs levels are summarized in the following proposition:

**Proposition 1** (i) For  $\varphi < \varphi_B$ , the asset-wealthy country has larger shares of industry and innovation, with the full concentration of industry for  $\varphi \in (\varphi_{X_i}, \varphi_B)$  and the full concentration of innovation for  $\varphi \in (\varphi_{I_i}, \varphi_B)$ . (ii) For  $\varphi > \varphi_B$ , the asset-poor country has larger shares of industry and innovation, with the full concentration of industry for  $\varphi \in (\varphi_B, \varphi_{X_j})$  and the full concentration of innovation for  $\varphi \in (\varphi_B, \varphi_{I_j})$ . There exists a threshold  $\underline{\delta}$  such

that  $\varphi_{Ii} < \varphi_{Xi}$  and  $\varphi_{Ij} > \varphi_{Xj}$  for  $\delta < \underline{\delta}$ , but  $\varphi_{Ii} \geq \varphi_{Xi}$  and  $\varphi_{Ij} \leq \varphi_{Xj}$  for  $\delta \geq \underline{\delta}$ .

Proof: See Appendix C.

Figure 2 summarizes the location patterns associated with various ranges for the free-ness of trade, with the shaded sections matching with the shaded areas in Figure 1 where production is fully concentrated and the regional component of innovation costs is not equalized across countries ( $c_i \neq c_j$ ). The degree of knowledge diffusion determines whether the full concentration of manufacturing coincides with the full concentration of innovation. As shown in Figure 2a, if  $\delta < \underline{\delta}$ , the knowledge spillover advantage of the larger national market discourages firms from locating innovation in the smaller country, and innovation concentrates fully before manufacturing; that is,  $\varphi_{Xi} > \varphi_{Ii}$  and  $\varphi_{Xj} < \varphi_{Ij}$ . As depicted in Figure 2b, however, if  $\delta > \underline{\delta}$ , a high degree of knowledge diffusion reduces the localized benefits of knowledge spillovers, allowing the smaller country to attract innovation with its low-cost high-skilled labor, even with manufacturing fully concentrated in the larger country; that is,  $\varphi_{Xi} < \varphi_{Ii}$  and  $\varphi_{Xj} > \varphi_{Ij}$ . In Appendix C, we show that a sufficient improvement in the degree of knowledge diffusion reduces high-skilled employment in country  $i$ , but expands it in country  $j$ , decreasing country  $i$ 's shares of manufacturing and innovation.

### 3.3 Net Offshoring Patterns

Net offshoring flows in the manufacturing sector are measured at the macro-level using the difference between the value of world output owned by country  $i$  and the value of production located in country  $i$  at each moment in time:

$$S_X \equiv (b_i N - N_i) p_{Xi} x_i = (b_i - s_{Xi}) \alpha E, \quad (26)$$

where we have used (8), (10), and (11). Net offshoring flows depend on nationals shares of asset wealth and production ( $b_i - s_{Xi}$ ), and overall market size ( $\alpha E$ ). Thus, net offshoring flows from the asset-wealthy country  $i$  to the asset-poor country  $j$  when  $S_X > 0$ , and from



country  $j$  to country  $i$  when  $S_X < 0$ . Characterizing the direction of net offshoring patterns in production using the freeness of trade, we obtain the following:

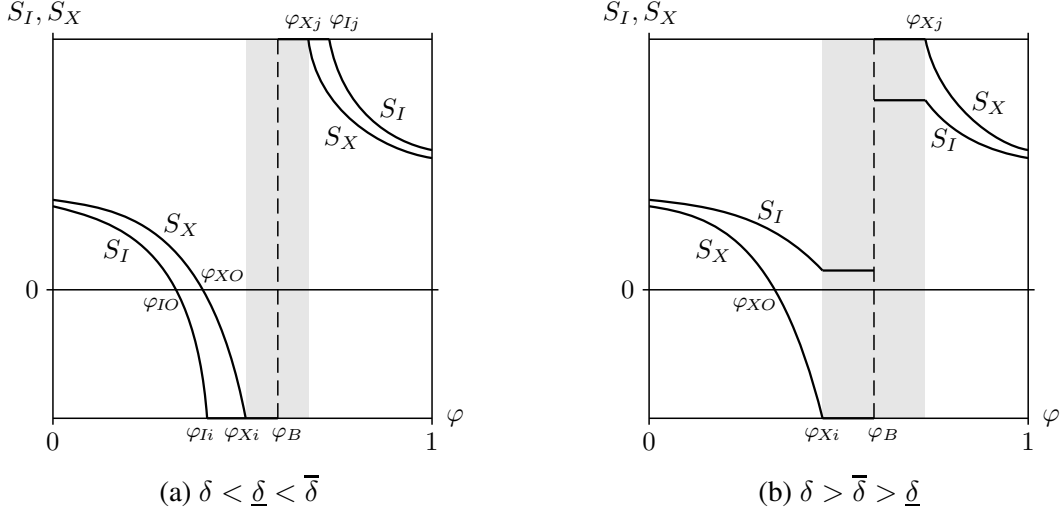
**Proposition 2** *Net offshoring in manufacturing flows from the asset-wealthy country to the asset-poor country for  $\varphi \notin (\varphi_{XO}, \varphi_B)$ , and from the asset-poor country to the asset-wealthy country for  $\varphi \in (\varphi_{XO}, \varphi_B)$ , where  $\varphi_{XO} \in (0, \varphi_X)$ .*

Proof: See Appendix D.

Net offshoring flows for the manufacturing sector are illustrated by the  $S_X$  curve in Figure 3, where the vertical axis shows  $S_X$  over the range  $\varphi \in (0, 1)$  measured on the horizontal axis. After all fall in trade costs, country  $i$ 's share of production increases ( $ds_{Xi}/d\varphi > 0$ ), while total expenditure increases ( $dE/d\varphi > 0$ ) for  $\varphi < \varphi_B$ , but decreases ( $dE/d\varphi < 0$ ) for  $\varphi > \varphi_B$ . In Appendix D, we show that the  $S_X$  curve crosses the horizontal axis once with a negative slope for  $\varphi < \varphi_B$ , and that it has a strictly negative slope for  $\varphi > \varphi_B$ . There are therefore three potential cases for the direction of net offshoring. Starting from a high level of trade costs over the range  $\varphi \in (0, \varphi_{XO})$ , the market of asset-wealthy country  $i$  is not sufficiently large to support the production of all country  $i$  owned firms, resulting in net offshoring flows from country  $i$  to country  $j$  ( $S_X > 0$ ). For a mid-level of trade costs  $\varphi \in (\varphi_{XO}, \varphi_B)$ , the country  $i$  market is sufficiently large to attract the production of a larger share of firms, including the production of firms with country  $j$  owners, and net offshoring therefore flows from country  $j$  to country  $i$  ( $S_X < 0$ ). For a low level of trade costs over the range  $\varphi \in (\varphi_B, 1)$ , the larger market of country  $j$  attracts the greatest share of production, with a share of country  $i$  owned firms also locating production in country  $j$ , thereby generating net offshoring flows from country  $i$  to country  $j$  ( $S_X > 0$ ).

Net offshoring flows in innovation are similarly calculated at the macro-level using the difference between the value of world R&D output owned by country  $i$  and the value of R&D output produced in country  $i$  at each moment in time. Recalling that the total value of country  $i$ 's R&D output is equal to high-skilled labor income, we obtain net offshoring

Figure 3: Equilibrium Offshoring Patterns



These stylized labor allocation patterns can be reproduced numerically using  $\alpha = 0.85$ ,  $\sigma = 3.5$ ,  $\gamma = 0.35$ ,  $\rho = 0.01$ ,  $\lambda = 0.01$ ,  $\psi = 0.2$ ,  $b = 0.65$ , and  $Z = 1$  for both panels, and with  $\delta = 0.4$  and  $\varphi_B = 0.85$  for Panel (a), and  $\delta = 0.8$  and  $\varphi_B = 0.96$  for Panel (b).

flows for country  $i$  as follows:

$$S_I \equiv b_i(w_i H_i + w_j H_j) - w_i H_i = (b_i - s_{Ii})(w_i H_i + w_j H_j), \quad (27)$$

where we have used (23). Net offshoring flows are determined by national shares of asset wealth and innovation activity  $(b_i - s_{Ii})$  and the aggregate scale of innovation activity  $(w_i H_i + w_j H_j)$ . Thus, net offshoring in innovation flows from the asset-wealthy country  $i$  to the asset-poor country  $j$  when  $S_I > 0$ , and from country  $j$  to country  $i$  when  $S_I < 0$ . We use the freeness of trade to characterize net offshoring patterns in innovation as follows:

**Proposition 3** *When  $\delta < \bar{\delta}$ , net offshoring in innovation flows from the asset-wealthy country to the asset-poor country for  $\varphi \notin (\varphi_{IO}, \varphi_B)$  and from the asset-poor country to the asset-wealthy country for  $\varphi \in (\varphi_{IO}, \varphi_B)$ , where  $\underline{\delta} < \bar{\delta}$  and  $\varphi_{IO} \in (0, \varphi_I)$ . When  $\delta > \bar{\delta}$ , net offshoring in innovation always flows from the asset-wealthy country to the asset-poor country.*

Proof: See Appendix D.

The pattern of innovation offshoring depends on the degree of knowledge diffusion. As shown in Figure 3a, where net offshoring flows for innovation are measured on the vertical axis ( $S_I$ ), when knowledge diffusion is relatively low ( $\delta < \bar{\delta}$ ), there are three cases. The first case occurs over the range  $\varphi \in (0, \varphi_{IO})$  where country  $i$ 's greater share of industry provides it with a knowledge spillover advantage that allows it to attract a larger share of innovation. The knowledge spillover advantage is not sufficient, however, to prevent a share of country  $i$  owned firms from locating innovation in country  $j$  with the aim of taking advantage of lower high-skilled wages. As a result, net offshoring in innovation flows from country  $i$  to country  $j$  ( $S_I > 0$ ). The second case occurs for intermediate trade costs  $\varphi \in (\varphi_{IO}, \varphi_B)$  where country  $i$ 's share of industry generates a knowledge spillover advantage that is great enough to ensure that all country  $i$  owned firms, and a share of country  $j$  owned firms, locate innovation in the country  $i$  country, despite higher high-skilled wages. In this case net offshoring flows from country  $j$  to country  $i$  ( $S_I < 0$ ). The third case arises for low trade costs  $\varphi \in (\varphi_B, 1)$  when country  $j$  has a larger share of industry, and all country  $j$  firms and a share of country  $i$  firms locate innovation in country  $j$  to take advantage of greater knowledge spillovers, generating net offshoring flows from country  $i$  to country  $j$  ( $S_I > 0$ ).

In general, marginal improvements in knowledge diffusion have ambiguous effects on national labor allocations. In Appendix C, we show, however, that for  $\varphi < \varphi_B$ , we have  $s_{Xi} > 1/2$ , and  $s_{Ii} > 1/2$ , and a sufficient increase in the degree of knowledge diffusion reduces country  $i$ 's shares of production and innovation causing the  $S_X$  and  $S_I$  curves to shift upwards in Figure 3. In particular, as shown in Figure 3b, for sufficiently high degree of knowledge diffusion, the knowledge spillover advantage associated with industry concentration is relatively weak, creating an incentive for firms to focus on minimizing the cost of employing high-skilled labor when choosing where to locate innovation. As such, innovation never concentrates fully in one country. Indeed, with perfect knowledge diffusion ( $\delta = 1$ ), exactly half of all innovation activity is located in each country:

$S_I = (b_i - 1/2)(w_i H_i + w_j H_j) > 0$ . Therefore, in the case for  $\delta > \bar{\delta}$ , net offshoring in innovation always flows from the asset-wealthy country  $i$  to the asset-poor country  $j$  ( $S_I > 0$ ).

### 3.4 Market Entry and Productivity Growth

We now consider how changes in location patterns affect long-run market entry and productivity growth. Combining (9), (11), (12), (15), and (19) yields the level of market entry

$$n = \frac{\nu - (\rho + \lambda)c}{\psi - (\rho + \lambda)c}, \quad (28)$$

as a function of the regional unit innovation cost ( $c$ ), where  $\nu > (\rho + \lambda)c$  is necessary for a positive level of market entry, as  $\psi > (\rho + \lambda)c$  is required for the saddle-path stability of long-run equilibrium (Lemma 1). Using (28), we obtain the following lemma.

**Lemma 2** *The level of market entry is increasing in the regional component of unit innovation costs ( $c$ ) for  $\nu > \psi$ , and decreasing in  $c$  for  $\nu < \psi$ .*

Proof: See Appendix E.

Following Davis and Hashimoto (2015), a rise in the regional component of unit innovation costs has two effects on the level of market entry. The first is a *product development* effect under which higher innovation costs dampen investment in product development, lowering the level of market entry. The second is a *process innovation* effect whereby higher innovation costs reduce the optimal level of investment in process innovation, leading to higher per-period profits, inducing investment in product development, and putting upward pressure on the level of market entry. The overall balance of these effects depends on the level of market power ( $\nu$ ) relative to the per-period fixed cost ( $\psi$ ). When  $\nu < \psi$ , the product development effect dominates and market entry falls. Alternatively, when  $\nu > \psi$ , the process innovation effect dominates and market entry rises.

The long-run rate of productivity growth is derived as a function of the regional component of unit innovation costs ( $c$ ) from (12), (15), and (28):

$$g \equiv \frac{\dot{\theta}}{\theta} = \frac{(\sigma - 1)\gamma(\psi - (\rho + \lambda)c)}{\sigma(\nu - (\rho + \lambda)c)c} - \rho - \lambda. \quad (29)$$

Note that long-run productivity growth is not biased by a scale effect as changes in overall population size ( $2Z$ ) are fully absorbed by adjustments in the number of firms in the market ( $N$ ), leaving the level of market entry ( $n$ ) and the rate of productivity growth unchanged.<sup>7</sup> We use (29) to obtain the following lemma.

**Lemma 3** *The productivity growth rate is decreasing in the regional unit innovation cost ( $c$ ) for  $\nu > \psi$ , and convex in  $c$  with a minimum at  $\underline{c} = (\psi - \sqrt{\psi(\psi - \nu)})/(\rho + \lambda)$  for  $\nu < \psi$ . Proof: See Appendix E.*

An increase in the regional component of unit innovation costs ( $c$ ) affects long-run productivity growth through two mechanisms (Davis and Hashimoto 2015). The first is the direct negative effect of higher innovation costs on investment in process innovation. The second is the effect of adjustments in the level of market entry. From Lemma 2, when  $\nu > \psi$  the level of market entry increases and the overall result is a slower rate of productivity growth. Alternatively, when  $\nu < \psi$  productivity growth decelerates if  $c < \underline{c}$  as the negative process innovation effect dominates, but productivity growth accelerates if  $c > \underline{c}$  as the positive effect of lower market entry dominates.

Improved economic integration resulting from either a fall in trade costs or a rise in knowledge diffusion affects market entry and productivity growth through adjustments in the regional component of innovation costs. Beginning with an investigation of the effects of adjustments in trade costs, we obtain the following result.

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<sup>7</sup>There is now a large empirical literature concluding that there is no significant relationship between economic growth and population size (Dinopoulos and Thompson 1999; Barro and Sala-i-Martin 2004; and Laincz and Peretto 2006). The framework presented in this paper corrects for scale effects by focusing on the innovation associated with the production technologies of individual product lines, rather than considering R&D at the national level.

**Proposition 4** *If  $\varphi < \varphi_B$ , a fall in trade costs expands market entry and slows productivity growth for  $\nu > \psi$ , but reduces market entry and initially slows ( $c < \underline{c}$ ) but then hastens ( $c > \underline{c}$ ) productivity growth for  $\nu < \psi$ . Alternatively, if  $\varphi > \varphi_B$ , a fall in trade costs reduces market entry and hastens productivity growth for  $\nu > \psi$ , but expands market entry and initially slows ( $c > \underline{c}$ ) but then hastens ( $c < \underline{c}$ ) productivity growth for  $\nu < \psi$ .*

Proof: See Appendix E.

Greater economic integration through a reduction in trade costs affects market entry and productivity growth through adjustments in national shares of production ( $s_{Xi}$ ). Importantly, an increase in the concentration of manufacturing raises the regional component of innovation costs ( $dc/ds_{Xi} > 0$ ) because the negative effect of increased high-skilled wages always dominates the positive effect of improved knowledge spillovers.<sup>8</sup> Thus, as we have seen in Proposition 1, a rise in the freeness of trade increases the concentration of manufacturing in country  $i$  for  $\varphi < \varphi_B$ , raising innovation costs, but decreases the concentration of manufacturing in country  $j$ , lowering innovation costs for  $\varphi > \varphi_B$ . Combining the effects of changes in trade costs on innovation costs with the results of Lemmas 2 and 3 generates the cases outlined in Proposition 4.

Investigating the effects of an adjustment in the degree of knowledge diffusion, we obtain the following proposition.

**Proposition 5** *When  $\varphi < \varphi_B$ , a rise in the degree of knowledge diffusion reduces market entry and hastens productivity growth for  $\nu > \psi$ , but expands market entry and initially slows ( $c > \underline{c}$ ) but then hastens ( $c < \underline{c}$ ) productivity growth for  $\nu < \psi$ . In contrast, when  $\varphi > \varphi_B$ , a rise in the degree of knowledge diffusion has ambiguous effects on both market entry and productivity growth.*

Proof: See Appendix E.

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<sup>8</sup>In general the empirical literature reports mixed results for the effect of industry concentration on economic growth. See Gardiner et al. (2011) for a literature survey and for evidence supporting a negative relationship between a number of measures of industry concentration and GDP growth.

As discussed in the previous section, the relationship between the degree of knowledge diffusion and national labor market allocations is generally ambiguous. However, when  $\varphi < \varphi_B$  and production and innovation are concentrated in the asset-wealthy country  $i$ , a sufficient rise in the degree of knowledge diffusion decreases the concentration of industry causing innovation costs to fall ( $dc/ds_{Xi} > 0$ ). As a result, for  $\nu > \psi$  the level of market entry falls and productivity growth accelerates. For  $\nu < \psi$ , however, market entry rises, and productivity growth either decelerates or accelerates depending on whether the positive process innovation effect or the negative market entry effect dominates, according to the current level of regional innovation costs. When  $\varphi > \varphi_B$ , the effect of a rise in the degree of knowledge diffusion on national labor allocations, and thus on innovation costs, is generally ambiguous.

### 3.5 Innovation Offshoring and Long-run Productivity Growth

In this final section we tie together the results of the previous sections with the aim of clarify the relationships between net offshoring flows and the rate of productivity growth. Focusing on the case for which the asset-wealthy country has greater shares of production and innovation ( $\varphi < \varphi_B$ ), we summarize the effects of improved economic integration in the following proposition.

**Proposition 6** *When  $\varphi < \varphi_B$ , improved economic integration, through a fall in trade costs or a rise in the degree of knowledge diffusion, generates a positive relationship between net offshoring flows in innovation from the asset-wealthy country to the asset-poor country and the long-run rate of productivity growth both for  $\nu > \psi$  and for  $\nu < \psi$  with  $c < \underline{c}$ .*

As we have seen, a dispersed location pattern for industry and innovation reduces innovation costs ( $dc/ds_{Xi} > 0$ ) as the positive effect of lower high-skilled wages dominates the negative effect of reduced knowledge spillovers. In addition, because innovation activity shifts out of the asset-wealthy country, less concentrated location patterns also coincide with

Table 1: Improvements of Knowledge Diffusion

$\nu - \psi$	$\delta$	$s_{Xi}$	$s_{Ii}$	$S_{Xi}$	$S_{Ii}$	$c$	$n$	$g$
0.1	0.05	0.513	0.81	0.468	-0.005	1.981	1.471	0.014
0.1	0.1	0.512	0.703	0.469	0.004	1.818	1.464	0.017
0.1	0.15	0.511	0.638	0.469	0.011	1.68	1.459	0.021
-0.1	0.05	0.514	0.707	0.467	0.005	2.026	0.574	0.059
-0.1	0.1	0.513	0.639	0.468	0.015	1.862	0.576	0.069
-0.1	0.15	0.512	0.596	0.469	0.023	1.722	0.578	0.079

Base parameters values are  $\alpha = 0.95$ ,  $\gamma = 0.19$ ,  $\rho = 0.02$ ,  $\psi = 0.25$ ,  $b = 0.75$ ,  $\varphi = 0.15$ , and  $Z = 1$ .  
In the upper half of the table,  $\sigma = 2.203$  and  $\lambda = 0.001$ , and in the lower half  $\sigma = 3.5$  and  $\lambda = 0.0125$ .

greater net offshoring flows in innovation from the asset-wealthy country to the asset-poor country. Therefore, turning to improvements in economic integration, a fall in trade costs lowers net offshoring flows out of the asset-wealthy country and raises innovation costs, while a rise in the degree of knowledge diffusion increases net offshoring flows out of the asset-wealthy country and lowers innovation costs. As such, economic integration generates a positive relationship between net innovation offshoring flows out of the asset-wealthy country and long-run productivity growth when  $\nu > \psi$  and when  $\nu < \psi$  with  $c < \underline{c}$ , following from Proposition 5. Table 1 provides numerical examples for the effects of improved knowledge diffusion on net offshoring flows in manufacturing ( $S_X$ ) and innovation ( $S_I$ ), the level of market entry ( $n$ ), and the rate of productivity growth ( $g$ ). The examples suggest that improved knowledge diffusion may be a key factor in the recent pattern of innovation offshoring from advanced to emerging economies, and that this pattern may have a positive influence on long-run productivity growth.

## 4 Concluding Remarks

This paper has introduced a two-country model of trade to examine the relationship between net offshoring patterns in innovation and manufacturing and fully endogenous productivity growth. Central to the model, monopolistically competitive firms invest in process innovation that lowers production costs and drives aggregate productivity growth. The occupational choice of skill-differentiated workers into low-skilled employment in production



and high-skilled employment in innovation determines national labor allocations, while the free movement of investment allows firms to shift their production and innovation activities freely and separately between countries. These two mechanisms create a tension between accessing the technical knowledge contained in production processes and sourcing low-cost high-skilled labor as firms independently select the optimal locations for production and innovation. A key feature of the model is a positive relationship between the unit cost of process innovation and the geographic concentration of industry and innovation as the benefit of greater knowledge spillovers is offset by the cost of rising high-skilled wages.

We characterize location patterns according to the level of trade costs. Specifically, while high trade costs lead to a larger market and the concentration of production and innovation in the asset-wealthy country, when trade costs are low production and innovation concentrate in proximity to the larger market of the asset-poor country. Given these location patterns, we use the level of trade costs to identify three cases for the directions of net offshoring in innovation and manufacturing. For high trade costs, although the asset-wealthy country has greater shares industry and innovation, the domestic market is not sufficiently large to attract the production and innovation activities of all firms with domestic owners, and net offshoring thus flows towards the asset-poor country. For intermediate trade costs, however, net offshoring flows from the asset-poor country towards the larger market of the asset-wealthy country. Finally, for low trade costs, net offshoring flows towards the asset-poor country as it maintains greater concentrations of industry and innovation.

Focusing on the case for which the asset-wealthy country has greater shares of industry and innovation activity, we investigate the effects of an improvement in knowledge diffusion between countries, and find that net offshoring flows in innovation and manufacturing from the asset-wealthy country to the asset-poor country increase as firms offshore innovation to the asset-poor country to take advantage of lower wages for high-skilled workers. The resulting increased dispersion of industry and innovation activity away from the asset-wealthy country results in a lower unit cost for process innovation and therefore potentially

accelerates productivity growth.

## Appendix A: Saddle-Path Stability

This appendix derives the necessary conditions for the saddle-path stability of long-run equilibrium. For this purpose, we reduce the dynamic system to two differential equations in  $w_i$  and  $n$ , and use the fact that common motions for high-skilled wages ( $\dot{w}_i/w_i = \dot{w}_j/w_j$ ) ensure a constant international high-skilled wage differential:  $w_j/w_i = k_i/k_j \equiv \kappa$ .

First, combining (6) and (21) with  $B = \alpha Ec$ ,  $\dot{B} = rB + I - E$ , and  $\dot{E} = (r - \rho)E$  yields the following differential equation for high-skilled wages:

$$\frac{\dot{w}_i}{w_i} = \frac{(n - n_I)\psi I}{cL}, \quad n_I = \frac{(1 - \alpha\rho c)L}{\alpha\psi I} - \frac{\mu}{\alpha\psi}, \quad (\text{A1})$$

where  $n_I$  is the steady-state level of market entry consistent with optimal investment in process innovation.

Next, substituting (15), (18), and (21) into (19) leads to the following differential equation for the level of market entry:

$$\frac{\dot{n}}{n} = \left( (n_N - n) \frac{\psi - (\rho + \lambda)c}{c} + 2(1 - n) \frac{\dot{w}_i}{w_i} \right) \frac{(\mu + \alpha\psi n)}{(\mu + \alpha\psi n^2)}, \quad n_N = \frac{\nu - (\rho + \lambda)c}{\psi - (\rho + \lambda)c}, \quad (\text{A2})$$

where  $n_N$  is the steady-state level of market entry consistent with optimal investment in product development. Setting  $n_I = n_N$  gives the steady-state investment locus (24).

We evaluate the local dynamics around the steady-state described by (24) and (25) using a Taylor expansion of (A1) and (A2) with  $w_j = \kappa w_i$ . The determinant of the Jacobian matrix ( $J_1$ ) for the linearized system is

$$|J_1| = - \frac{(\psi - (\rho + \lambda)c)(\mu + \alpha\psi n)\psi n w_i I}{(\mu + \alpha\psi n^2)c^2 L} \frac{\partial \Phi}{\partial w_j} \left( \frac{w_j}{w_j} - \frac{dw_j}{dw_i} \Big|_{\Phi=0} \right).$$

As the high-skilled wage rate ( $w_i$ ) is a control variable and the level of market entry ( $n$ ) is a

state variable, we require one positive and one negative eigenvalue for saddle-path stability. Accordingly, we consider long-run equilibria that satisfy  $|J_1| < 0$ . In Appendix C, we show that  $\partial\Phi/\partial w_j > 0$  and  $dw_j/dw_i|_{\Phi=0} < 0$ . Thus,  $\psi > (\rho + \lambda)c$  is a sufficient condition for saddle-path stability as outlined in Lemma 1.

## Appendix B: Derivation of the Share Locus

First, we use  $\dot{E} = (r - \rho)E$ ,  $\dot{B}_i = rB_i + I_i - E_i$  and  $\dot{B} = rB + I - E$  to obtain  $E_i = I_i - b_i(I - E)$ . Then, substituting  $E_i$  into (10) and equating with (16) we have

$$\frac{(b_i - \varphi b_j)}{1 - \varphi} \left( 1 - \frac{(\mu + \alpha\psi n)I}{L} \right) + \frac{(I_i - \varphi I_j)(\mu + \alpha\psi n)}{(1 - \varphi)L} = \frac{w_i - \delta w_j}{(1 - \delta)(w_i + w_j)}, \quad (\text{B1})$$

where we have assumed common growth rates for asset wealth ( $\dot{B}_i/B_i = \dot{B}_j/B_j$ ), ensuring that  $\dot{b}_i = 0$  at all moments in time. Substituting  $n_L$  from (A1) into (B1) yields the steady-state share locus (25).

## Appendix C: Proposition 1

Using  $L = (w_i + w_j)Z/(w_i w_j)$  and  $I = (w_i + w_j)(1 + w_i w_j)Z/(2w_i w_j)$ , the investment locus (24) and the share locus (25) are rewritten as

$$\Phi = \frac{\nu - (\rho + \lambda)c}{\psi - (\rho + \lambda)c} - \frac{2(1 - \alpha\rho c)}{\alpha\psi(1 + w_i w_j)} + \frac{\mu}{\alpha\psi} = 0, \quad (\text{C1})$$

$$\Omega = \frac{w_i}{w_j} - \frac{1 - \delta\varphi + (\delta - \varphi)w_i w_j + (1 - \delta)(1 + \varphi)(b_i w_i w_j - b_j)\alpha\rho c}{1 - \delta\varphi + (\delta - \varphi)w_i w_j + (1 - \delta)(1 + \varphi)(b_j w_i w_j - b_i)\alpha\rho c} = 0. \quad (\text{C2})$$

The slopes of (24) and (25) are then  $(dw_j/dw_i)|_{\Phi} = -(\partial\Phi/\partial w_j)/(\partial\Phi/\partial w_i) < 0$  and  $(dw_j/dw_i)|_{\Omega} = -(\partial\Omega/\partial w_i)/(\partial\Omega/\partial w_j)$ , where

$$\begin{aligned}\frac{\partial\Phi}{\partial w_i} &= \frac{(\nu - \psi)(\rho + \lambda)}{(1 + \delta)(\psi - (\rho + \lambda)c)^2} + \frac{2(1 - \alpha\rho c)w_j}{\alpha\psi(1 + w_iw_j)^2} + \frac{2\rho}{(1 + \delta)\psi(1 + w_iw_j)} > 0, \\ \frac{\partial\Phi}{\partial w_j} &= \frac{(\nu - \psi)(\rho + \lambda)}{(1 + \delta)(\psi - (\rho + \lambda)c)^2} + \frac{2(1 - \alpha\rho c)w_i}{\alpha\psi(1 + w_iw_j)^2} + \frac{2\rho}{(1 + \delta)\psi(1 + w_iw_j)} > 0, \\ \frac{\partial\Omega}{\partial w_i} &= \frac{1}{w_j} - \frac{(1 - \delta\phi + (\delta - \varphi)w_iw_j)(1 + w_iw_j) + (1 - \delta^2)(1 + \varphi)(1 - \alpha\rho c)w_jc}{\alpha\rho(1 - \delta^2)(1 + \varphi)(b_i - b_j)(1 + w_iw_j)^2w_j^2(w_i - w_j)^{-2}c^2}, \\ \frac{\partial\Omega}{\partial w_j} &= -\frac{w_i}{w_j^2} - \frac{(1 - \delta\phi + (\delta - \varphi)w_iw_j)(1 + w_iw_j) + (1 - \delta^2)(1 + \varphi)(1 - \alpha\rho c)w_ic}{\alpha\rho(1 - \delta^2)(1 + \varphi)(b_i - b_j)(1 + w_iw_j)^2w_j^2(w_i - w_j)^{-2}c^2},\end{aligned}$$

and we have used the share locus:  $(w_i - w_j)/w_j = \alpha\rho(1 - \delta)(1 + \varphi)(b_i - b_j)(1 + w_iw_j)c / (1 - \delta\varphi + (\delta - \varphi)w_iw_j + \alpha\rho(1 - \delta)(1 + \varphi)(b_jw_iw_j - b_i)c)$ . Because  $1 > \alpha\rho c$ , we have  $\partial\Phi/\partial w_i > 0$  and  $\partial\Phi/\partial w_j > 0$  for  $\nu > \psi$ . Furthermore, since  $\partial^2\Phi/(\partial w_i\partial\psi) < 0$ ,  $\partial^2\Phi/(\partial w_j\partial\psi) < 0$ , and  $\lim_{\psi \rightarrow \infty} \partial\Phi/\partial w_i = \lim_{\psi \rightarrow \infty} \partial\Phi/\partial w_j = 0$ , we have  $\partial\Phi/\partial w_i > 0$  and  $\partial\Phi/\partial w_j > 0$  for all values of  $\psi$ . The investment locus (24) thus has a negative slope. The slope of the share locus (25) may be positive or negative. We limit the analysis to cases for which the share locus cuts the investment locus once from below to ensure the uniqueness of equilibrium.

We now derive the threshold value  $\varphi_B$ . Using (C1) and (C2), we obtain

$$\frac{dw_i}{db_i} = \frac{1}{|J_2|} \frac{\partial\Phi}{\partial w_j} \frac{\partial\Omega}{\partial b_i}, \quad \frac{dw_j}{db_i} = -\frac{1}{|J_2|} \frac{\partial\Phi}{\partial w_i} \frac{\partial\Omega}{\partial b_i},$$

where  $|J_2| = (\partial\Omega/\partial w_j)(\partial\Phi/\partial w_j)((dw_j/dw_i)|_{\Omega=0} - (dw_j/dw_i)|_{\Phi=0}) < 0$  under the assumption of uniqueness for long-run equilibrium, and

$$\frac{\partial\Omega}{\partial b_i} = -\frac{(1 - \delta)(1 + \varphi)(w_i + w_j)(1 + w_iw_j)\alpha\rho c}{(1 - \delta\varphi + (\delta - \varphi)w_iw_j + (1 - \delta)(1 + \varphi)(b_jw_iw_j - b_i)\alpha\rho c)w_j}.$$

Setting the denominator of this partial derivative to zero and solving for  $\varphi_B$ , we have  $\partial\Omega/\partial b_i < 0$ ,  $dw_i/db_i > 0$ , and  $dw_j/db_i < 0$  when  $\varphi < \varphi_B$ , but  $\partial\Omega/\partial b_i > 0$ ,  $dw_i/db_i < 0$ ,

and  $dw_j/db_i > 0$  when  $\varphi > \varphi_B$ . From (16) we have  $\partial s_{Xi}/\partial w_i > 0$  and  $\partial s_{Xi}/\partial w_j < 0$ , and from (23) we have  $\partial s_{Ii}/\partial w_i > 0$  and  $\partial s_{Ii}/\partial w_j < 0$ . Thus,  $w_i > w_j$ ,  $s_{Xi} > 1/2$ , and  $s_{Ii} > 1/2$  for  $\varphi < \varphi_B$ , and  $w_i < w_j$ ,  $s_{Xi} < 1/2$ , and  $s_{Ii} < 1/2$  for  $\varphi > \varphi_B$ .

Next, we derive the threshold values  $\varphi_{Xi}$ ,  $\varphi_{Ii}$ ,  $\varphi_{Xj}$ , and  $\varphi_{Ij}$ . From (C1) and (C2),

$$\frac{dw_i}{d\varphi} = \frac{1}{|J_2|} \frac{\partial \Phi}{\partial w_j} \frac{\partial \Omega}{\partial \varphi} > 0, \quad \frac{dw_j}{d\varphi} = -\frac{1}{|J_2|} \frac{\partial \Phi}{\partial w_i} \frac{\partial \Omega}{\partial \varphi} < 0, \quad (C3)$$

where  $\partial \Omega / \partial \varphi = -(1 + \delta)(w_i - w_j)^2 / ((1 - \delta)(1 + \varphi)^2(b_i - b_j)w_j^2 \alpha \rho c) < 0$ . Therefore,  $ds_{Xi}/d\varphi > 0$  and  $ds_{Ii}/d\varphi > 0$  ensure the existence of the following threshold values:  $\varphi_{Xi} \in (0, \varphi_B)$  for  $s_{Xi} = 1$ ,  $\varphi_{Ii} \in (0, \varphi_B)$  for  $s_{Ii} = 1$ ,  $\varphi_{Xj} \in (\varphi_B, 1)$  for  $s_{Xi} = 0$ , and  $\varphi_{Ij} \in (\varphi_B, 1)$  for  $s_{Ii} = 0$ .

The effects of changes in the degree of knowledge diffusion for large values of  $\delta$  are obtained from (C1) and (C2) as follows:

$$\begin{aligned} \frac{dw_i}{d\delta} &= \frac{1}{|J_2|} \left( \frac{2(1 - \alpha \rho c)w_i}{\alpha \psi(1 + w_i w_j)^2} \frac{\partial \Omega}{\partial \delta} - \left( \frac{1}{c} \frac{\partial \Omega}{\partial \delta} + \frac{\partial \Omega}{\partial w_j} \right) \frac{\partial \Phi}{\partial \delta} \right) < 0, \\ \frac{dw_j}{d\delta} &= -\frac{1}{|J_2|} \left( \frac{2(1 - \alpha \rho c)w_i}{\alpha \psi(1 + w_i w_j)^2} \frac{\partial \Omega}{\partial \delta} - \left( \frac{1}{c} \frac{\partial \Omega}{\partial \delta} + \frac{\partial \Omega}{\partial w_i} \right) \frac{\partial \Phi}{\partial \delta} \right) > 0, \end{aligned}$$

where

$$\begin{aligned} \frac{\partial \Phi}{\partial \delta} &= -\frac{(\nu - \psi)(\rho + \lambda)c}{(1 + \delta)(\psi - (\rho + \lambda)c)^2} - \frac{2\rho c}{(1 + \delta)\psi(1 + w_i w_j)} < 0, \\ \frac{\partial \Omega}{\partial \delta} &= \frac{2(1 - \delta\varphi + (\delta - \varphi)w_i w_j) + (1 - \delta^2)(w_i w_j - \varphi)}{\alpha \rho(1 - \delta)^2(1 + \delta)(1 + \varphi)(b_i - b_j)(1 + w_i w_j)w_j^2(w_i - w_j)^{-2}c} > 0, \\ \frac{1}{c} \frac{\partial \Omega}{\partial \delta} + \frac{\partial \Omega}{\partial w_i} &= \frac{(1 - \varphi)(w_i - w_j)^2}{(1 - \delta)^2(1 + \varphi)(b_i - b_j)w_j^2 \alpha \rho c^2} - \frac{(1 - \alpha \rho c)(w_i - w_j)^2}{(b_i - b_j)(1 + w_i w_j)^2 w_j \alpha \rho c} + \frac{1}{w_j}, \\ \frac{1}{c} \frac{\partial \Omega}{\partial \delta} + \frac{\partial \Omega}{\partial w_i} &= \frac{(1 - \varphi)(w_i - w_j)^2}{(1 - \delta)^2(1 + \varphi)(b_i - b_j)w_j^2 \alpha \rho c^2} - \frac{(1 - \alpha \rho c)w_i(w_i - w_j)^2}{(b_i - b_j)(1 + w_i w_j)^2 w_j^2 \alpha \rho c} - \frac{w_i}{w_j^2}, \end{aligned}$$

and  $2(1 - \delta\varphi + (\delta - \varphi)w_i w_j) + (1 - \delta^2)(w_i w_j - \varphi) > 0$  for all  $\delta, \varphi \in (0, 1)$ . Focusing on the horizontal axis in Figure 1, since  $dw_i/d\delta|_{\Phi=0; w_j=1} = -(\partial \Phi / \partial \delta) / (\partial \Phi / \partial w_i) > 0$ , a rise in  $\delta$  shifts the investment locus to the right, while the  $w_j/w_i = \delta$  dashed line shifts to

the left. Thus, we can define the threshold value  $\delta = \underline{\delta}$  at which  $\varphi_{Xi} = \varphi_{Ii}$ , and we have  $\varphi_{Xi} > \varphi_{Ii}$  for  $\delta < \underline{\delta}$  and  $\varphi_{Xi} \leq \varphi_{Ii}$  for  $\delta \geq \underline{\delta}$ . Similarly, given the symmetric nature of the investment locus, we have  $\varphi_{Xj} < \varphi_{Ij}$  for  $\delta < \underline{\delta}$  and  $\varphi_{Xj} \geq \varphi_{Ij}$  for  $\delta \geq \underline{\delta}$ . This completes the derivation of the thresholds values shown in Figure 2. Finally, for sufficiently large values of  $\delta$ ,  $dw_i/d\delta < 0$  and  $dw_j/d\delta > 0$  imply  $ds_{Xi}/d\delta < 0$  and  $ds_{Ii}/d\delta < 0$ .

#### Appendix D: Propositions 2 and 3

First, from (26) and  $E = I/(1 - \alpha\rho c)$  we have

$$\frac{dS_X}{d\varphi} = -\frac{ds_{Xi}}{d\varphi}\alpha E + \frac{\alpha(b_i - s_{Xi})}{(1 - \alpha\rho c)} \left( \frac{dI}{d\varphi} + \alpha\rho E \frac{dc}{d\varphi} \right),$$

where the first term on the righthand side is negative, while the second term is positive for  $\varphi < \varphi_B$  and negative for  $\varphi > \varphi_B$ . Since  $dS_X/d\varphi < 0$  at  $b_i - s_{Xi} = 0$ , however, the  $S_X$  curve has a single point and a negative slope at  $S_X = 0$ . Thus, we derive the threshold value  $\varphi_{XO} \in (0, \varphi_B)$ , for which  $S_X = 0$ . As  $w_j/w_i < 1$  for  $\varphi < \varphi_B$ , evaluating (C2) at  $\varphi = 0$  yields

$$1 > \frac{w_j}{w_i} \Big|_{\Omega=0; \varphi=0} = \frac{1 + \delta w_i w_j + (1 - \delta)(b_j w_i w_j - b_i)\alpha\rho c}{1 + \delta w_i w_j + (1 - \delta)(b_i w_i w_j - b_j)\alpha\rho c} > \frac{b_j + \delta b_i}{b_i + \delta b_j} > \delta,$$

for  $b_i > b_j$ . We have  $S_X \geq 0$  for  $w_j/w_i \geq (b_j + \delta b_i)/(b_i + \delta b_j)$ , and  $S_X \equiv -(1 - b_i)\alpha E < 0$  for  $w_j/w_i = \delta$ . Thus, since  $d(w_j/w_i)/d\varphi < 0$  from (C3), there exists a threshold value  $\varphi_{XO} \in (0, \varphi_B)$ , with  $S_X < 0$  for  $\varphi \in (\varphi_{XO}, \varphi_B)$  and  $S_X > 0$  for  $\varphi \notin (\varphi_{XO}, \varphi_B)$ .

Second, from (27) we have

$$\frac{dS_I}{d\varphi} = b_i \frac{I_j}{w_j} \frac{dw_j}{d\varphi} - b_j \frac{I_i}{w_i} \frac{dw_i}{d\varphi} < 0.$$

We derive the threshold value  $\varphi_{IO} \in (0, \varphi_B)$ , for which  $S_I = 0$ . From (27), we have  $S_I = 0$  when  $b_i/b_j = w_i H_i/(w_j H_j)$ , which requires  $w_j/w_i < 1$  for  $b_i > b_j$ . This condition has

a positive slope,  $dw_j/dw_i = w_j^2(1+w_i)b_j/(w_i(1+w_j^2)b_i) > 0$ , and thus intersects the investment locus once in  $(w_i, w_j)$  space at  $\varphi_{IO} \in (0, \varphi_B)$ . Then, because  $d(w_j/w_i)/d\varphi < 0$  from (C3), we have  $S_I < 0$  for  $\varphi \in (\varphi_{IO}, \varphi_B)$  and  $S_I > 0$  for  $\varphi \notin (\varphi_{IO}, \varphi_B)$ .

Third, because  $dw_i/d\delta < 0$  and  $dw_j/d\delta > 0$  for large values of  $\delta$ , sufficient increases in  $\delta$  shift the  $S_X$  and  $S_I$  curves upwards in Figure 3, as  $ds_{Xi}/d\delta < 0$  and  $ds_{Ii}/d\delta < 0$ .

## Appendix E: Propositions 4 and 5

First, the effects of changes in regional unit innovation costs on market entry and productivity growth are

$$\frac{dn}{dc} = \frac{(\nu - \psi)(\rho + \lambda)}{(\psi - (\rho + \lambda)c)^2}, \quad \frac{dg}{dc} = -\frac{g + \rho + \lambda}{c} - \frac{(g + \rho + \lambda)(\rho + \lambda)(\nu - \psi)}{(v - (\rho + \lambda)c)(\psi - (\rho + \lambda)c)},$$

with  $d^2g/dc^2 = (2(g + \rho + \lambda)/c)(1/c + (\nu - \psi)\psi(\rho + \lambda)/((\nu - (\rho + \lambda)c)(\psi - (\rho + \lambda)c)^2)) > 0$ .

Thus, as outlined in Lemma 2, we have  $dn/dc > 0$  when  $\nu > \psi$ , and  $dn/dc < 0$  when  $\nu < \psi$ . And, as summarized in Lemma 3, we have  $dg/dc < 0$  when  $\nu > \psi$ , but  $dg/dc > 0$  for  $c < \underline{c}$  and  $dg/dc \leq 0$  for  $c \geq \underline{c}$  when  $\nu < \psi$ , where  $\underline{c} = (\psi - \sqrt{(\psi - \nu)\psi})/(\rho + \lambda)$ . Lastly, the following are used with  $|J_2| < 0$  to obtain Propositions 4 and 5:

$$\begin{aligned} \frac{dc}{d\varphi} &= -\frac{2(1 - \alpha\rho c)(w_i - w_j)^3}{\alpha^2\psi\rho c(1 - \delta)(1 + \varphi)^2(b_i - b_j)(1 + w_iw_j)^2w_j^2|J_2|}, \\ \frac{dc}{d\delta} &= \frac{2(1 - \alpha\rho c)}{\alpha\psi(1 + \delta)(1 + w_iw_j)^2|J_2|} \left[ c \left( w_i \frac{\partial\Omega}{\partial w_i} - w_j \frac{\partial\Omega}{\partial w_i} \right) + (w_i - w_j) \frac{\partial\Omega}{\partial\delta} \right], \end{aligned}$$

where  $w_i\partial\Omega/\partial w_i - w_j\partial\Omega/\partial w_i = 2w_i/w_j + (1 - \alpha\rho c)(w_i - w_j)^3/((b_i - b_j)(1 + w_iw_j)^2w_j^2\alpha\rho c)$  and  $|J_2| < 0$ . Hence, for  $\varphi < \varphi_B$  we have  $dc/d\varphi > 0$  and  $dc/d\delta < 0$ , and for  $\varphi > \varphi_B$  we have  $dc/d\varphi < 0$  and  $dc/d\delta \leq 0$ . Combining these results yields Propositions 4 and 5.

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