

Miyagiwa, Kazuyuki; Ono, Yoshiyasu

**Working Paper**

## Immigration and secular stagnation

ISER Discussion Paper, No. 1054

**Provided in Cooperation with:**

The Institute of Social and Economic Research (ISER), Osaka University

*Suggested Citation:* Miyagiwa, Kazuyuki; Ono, Yoshiyasu (2019) : Immigration and secular stagnation, ISER Discussion Paper, No. 1054, Osaka University, Institute of Social and Economic Research (ISER), Osaka

This Version is available at:

<https://hdl.handle.net/10419/230461>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

Discussion Paper No. 1054

**IMMIGRATION  
AND SECULAR STAGNATION**

Kaz Miyagiwa  
Yoshiyasu Ono

Revised June 2019  
May 2019

The Institute of Social and Economic Research  
Osaka University  
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

# Immigration and Secular Stagnation\*

by

Kaz Miyagiwa\*\* and Yoshiyasu Ono†

May 9, 2019

Revised June 4, 2019

## Abstract

We examine the effect of immigration on the host country in the dynamic model that can deal with secular unemployment. Immigration has contrasting effects, depending on the economic state of the host country. If there is unemployment, immigration worsens unemployment and decreases consumption by native residents whereas if full employment prevails, immigration has the effect of boosting consumption while keeping full employment. However, an influx of too many immigrants can turn the host country into stagnation. We also find that immigrants' remittances are harmful to the host country under full employment but beneficial under secular stagnation.

JEL: F16, F22, F41

Keywords: Immigration, secular stagnation, unemployment, aggregate demand deficiency, remittances

---

\*The authors thank seminar participants at ISER, Osaka University, and Wayne State University, for their valuable comments. Ono acknowledges financial support from the Joint Usage/Research Center at ISER, Osaka University, and the Grants-in-Aid for Scientific Research (#15H05728), the Japan Society for the Promotion of Science (JSPS).

\*\*Department of Economics, Florida International University, Miami, FL33199, U.S.A. E-mail: kmiyagiwa@gmail.com

†Institute of Social and Economic Research, Osaka University, 6-1, Mihogaoka, Ibaraki, Osaka 567-0047, JAPAN. E-mail: ono@iser.osaka-u.ac.jp.

## 1. Introduction

In recent years immigration has once again become a contentious issue in the United States and other developed economies. It is estimated that more than one million immigrants have entered the U.S. every year since 2000.<sup>1</sup> The situation is not different in the E.U. In 2017, for example, 2.4 million immigrants entered the EU from non-EU countries.<sup>2</sup> Canada, Australia, and New Zealand have faced similar situations in recent years. Even Japan is no longer completely immune to illegal immigration.

Motivated by such resurgences of immigration, we present a new model of immigration from the perspectives of host countries. Although there already exists extensive literature investigating the impact of immigration, most of this, to be reviewed below, has focused on the “real side” of the economy, applying standard microeconomic tools. Departing from this tradition, this paper presents the dynamic model in which the real and the monetary side of the economy play key roles. The attractive feature of this model is that we can explain both full employment and unemployment from the agent’s intertemporal welfare-maximizing behavior without introducing additional constraints.

In the next two sections we present the benchmark model of a small open country (ignoring immigration). This benchmark model is an extension of the closed-country models of Ono (1994, 2001), Ono and Ishida (2014) and Michau (2018) to an international setting, where the host country is open to the world capital and equity markets. All agents are infinitely-lived and maximize intertemporal utility with respect to consumption of the aggregate good and real money balances, given the initial endowments of internationally traded assets.

The model shares two salient features with Ono (1994, 2001). One is that the marginal utility of real balances is bounded away from zero. The other is that, although the labor market

---

<sup>1</sup> <https://cis.org/Report/Record-445-Million-Immigrants-2017>

<sup>2</sup> [https://ec.europa.eu/eurostat/statistics.../Migration\\_and\\_migrant\\_population\\_statistics](https://ec.europa.eu/eurostat/statistics.../Migration_and_migrant_population_statistics)

adjusts according to the Walrasian mechanism, adjustment is not instantaneous. It is worth emphasizing however that it is not sluggish wage adjustment but bounded marginal utility of real balances that causes chronic unemployment in our model. Since the nominal wage and price keep falling as long as there is unemployment, if expanding real money balances can drive their marginal utility so low as to make consumption more valuable than hoarding money, then consumers start spending, which boosts aggregate demand, creates jobs and stimulates more spending – a process that continues until full employment is restored. However, if the marginal utility of real balances is bounded away from zero, expanding real balances can fail to initiate spending, and as a consequence the above propitious process is stymied, condemning the economy to secular stagnation.

The benchmark model yields the following results. The country enjoys full employment if it holds internationally traded equities below some threshold level, to be specified later. Otherwise, it suffers from chronic unemployment.

We then turn to our main question: how an influx of immigrants can affect the host country. Immigrants are assumed to have the same preferences as host-country natives but differ in two respects. First, immigrants enter the host country with a given number of internationally traded assets but no host-country currency. Thus, immigrants exchange part of the international assets they hold for host-country currency to satisfy their demands for real balances. Second, immigrants remit part of their earnings to families and relatives back home, whereas natives have no such obligations.

Our key findings can now be stated. (1) If natives are fully employed before immigration, then after immigration they not only remain fully employed but consume a greater quantity of the good. This result however is predicated on immigrants being neither too many nor too rich (in the sense defined later). An influx of immigrants who are too many or too rich can turn the host country from full employment into stagnation. (2) If the host country suffers

unemployment initially, an influx of immigrants always reduces consumption and worsens unemployment. (3) Immigrants' remittances also have contrasting effects, depending on the economic state of the host country. If full employment prevails, remittances lower consumption whereas if there is unemployment, they boost both consumption and employment.

We now review the relevant literature. There exists an extensive volume of literature, both formal and descriptive, investigating various aspects of immigration and immigration policies. To save space, we restrict our review to formal studies only. Early literature has treated immigration as a case of international factor mobility within standard factor-endowment trade models.<sup>3</sup> Although this approach has yielded many valuable insights, subsequent research has come to emphasize the distinction between immigration and capital movements. Pioneering in this line of research, Ethier (1985) has focused on temporary migration – motivated by guest-worker programs administered in West Germany and elsewhere at that time. However, since then most temporary immigrants in Europe have opted to stay permanently in their host countries. Also, the majority of today's immigrants appear to be permanent settlers rather than temporary job-seekers. In this paper, therefore, we study the effect of permanent immigration.

More recent work on immigration has turned attention to the presence of unemployment in host countries, investigating how immigrants and host-country immigration policies can affect employment of the native labor force.<sup>4</sup> To model unemployment, this strand of research has typically adopted search-theoretic approaches, where unemployment arises as an equilibrium phenomenon.<sup>5</sup> For example, Liu (2010), Chassamboulli and Palivos (2014) and Battisti et al.

---

<sup>3</sup> See e.g., Berry and Soligo (1969), Dixit and Norman (1980) and Markusen (1983).

<sup>4</sup> Ethier (1986) is the first to have highlighted host-country unemployment in his study of illegal immigration. Subsequent work on illegal immigration, with and without unemployment, includes Bonds and Chen (1987), Djajić (1997), Carter (1999), Woodland and Yoshida (2006), Liu (2010), Mangin and Zenou (2016), and Miyagiwa and Sato (2019), among others.

<sup>5</sup> Pissarides (2000) is the standard reference for equilibrium unemployment.

(2018) have utilized such an approach to examine how an influx of immigrants affects natives' wages and welfare, while Ortega (2000), Miyagiwa and Sato (2019) have studied the effect of host country immigration policy under endogenous immigration flows. The present paper is a contribution to this line of research. Departing from these precursory studies, however, the present work is unique in that it deals with involuntary unemployment instead of frictional unemployment.<sup>6</sup>

The remainder of this paper is organized in 6 sections. The next section describes the general environment of the model. Section 3 presents the benchmark model of a small open economy and studies its properties in the absence of immigration. Section 4 details how the benchmark model is adapted in the presence of immigration. Section 5 studies the effect of immigration when the host country enjoys full employment prior to immigration. Section 6 extends the analysis to the case of secular stagnation. Section 7 concludes.

## 2. Environment

Consider a small open host country in a continuous infinite-time horizon. The country produces the aggregate product with labor and capital according to the neoclassical production function  $F(L(t), K(t))$ , where  $K(t)$  and  $L(t)$  are quantities of capital and labor used at time  $t$ , respectively. (The time index  $t$  is suppressed below, unless ambiguities arise.) Since the function  $F$  exhibits constant returns to scale, we can rewrite it as

$$F(L, K) = f(n)K,$$

where

$$n \equiv L/K.$$

---

<sup>6</sup> See Ono (2010) for a formal analysis of Japan's economic slump since the 1990s.

The host country is open to the world capital market and takes the world equity rate  $r$  as given. Capital moves freely across borders to instantaneously adjust the host country's capital stock so as to keep the domestic real equity rate locked at  $r$ .

Perfect competition prevails everywhere. We ignore investment, to keep things simple. Then firms carry no state variables and they maximize momentary profits  $f(n)K - wn - rK$  at each instant, taking  $r$  and  $w$  (the real wage) as given. The first-order conditions are:

$$f'(n) = w, \quad (1)$$

$$f(n) - nf'(n) = r. \quad (2)$$

Under diminishing returns to factors, these equations uniquely determine  $n$  and  $w$ , given  $r$ . Note that the equilibrium  $n$  and  $w$  are independent of time.

We use the subscripts  $h$  and  $i$  to denote host-country natives and immigrants, respectively. All individuals  $j(= h, i)$  are endowed with one unit of labor. They derive no utility from leisure, preferring to supply their entire labor endowments to the labor market. However, they may not be able to do so due to a demand shortage. To incorporate such possibilities into our model, let  $\sigma$  denote the realized rate of employment, with range

$$0 \leq \sigma \leq 1.$$

The typical individual's realized real labor income is given by  $\sigma w$ . When  $\sigma = 1$  we say there is full employment; otherwise, unemployment prevails.

All individuals have identical preferences, deriving momentary utility  $u(c_j(t)) + v(m_j(t))$  from consuming  $c_j(t)$  units of the aggregate good and holding real money balances  $m_j(t)$  at time  $t$ . The subutility functions are assumed to satisfy



*Assumption 1:* (a) For all  $c_j \geq 0$ ,  $u(c_j)$  is strictly increasing, strictly concave and twice continuously differentiable, and satisfies the Inada conditions; i.e.,  $\lim_{c_j \rightarrow +0} u'(c_j) = \infty$  and

$$\lim_{c_j \rightarrow \infty} u'(c_j) = 0.$$

(b) For all  $m_j \geq 0$ ,  $v(m_j)$  is twice continuously differentiable with positive first derivatives and weakly concave. Specifically, there is  $\bar{m} > 0$  such that  $v'(m_j)$  is strictly decreasing for all  $m_j < \bar{m}$  and  $v'(m_j) = \beta > 0$  for all  $m_j \geq \bar{m}$ .

It is to be demonstrated in the next section that the presence of the lower bound  $\beta > 0$  on  $v'(m_j)$  is crucial for the existence of unemployment as discussed by Ono (1994, 2001) and Illing et al. (2018).

The representative individual maximizes the utility functional

$$\int_0^{\infty} (u(c_j) + v(m_j)) \exp(-\rho t) dt, \quad (3)$$

where  $\rho$  denotes the subjective discount rate, subject to the stock budget constraint and the flow budget constraint:

$$a_j = m_j + b_j, \quad (4)$$

$$\dot{a}_j = w\sigma + (ra_j - Rm_j) - c_j - \tau_j. \quad (5)$$

(4) shows that agent  $j$  can hold his real assets  $a_j$  in two forms: real money balances and real equities (or bonds). The latter are traded in the world markets and yield the real return  $r$  per unit (henceforth they are simply referred to as “bonds” and denoted by  $b_j$ .) By contrast, real money balances, denoted by  $m_j$ , are neither internationally traded nor yield any interest. Thus, the agent holding  $a_j$  receives the real interest income equal to  $ra_j - Rm_j = rb_j - \pi m_j$ , where  $R$  is the nominal interest rate and  $\pi \equiv R - r$  is the rate of inflation (deflation if negative). (5) describes how real asset holdings change over time (the “dot” over the variable denotes its time

derivative; e.g.,  $\dot{a}_j \equiv da_j/dt$ ). The first term on the right is the agent's labor income. The second term is the real interest income as shown above. These income terms increase the value of his assets, while consumption  $c_j$  and remittances  $\tau_j (> 0)$  diminish it.

The Hamiltonian of the utility maximization problem is given by

$$H = u(c_j) + v(m_j) + \lambda(w\sigma + ra_j - Rm_j - c_j - \tau_j),$$

where  $\lambda$  is the co-state variable. The first-order conditions are

$$\lambda = u'(c_j),$$

$$\lambda R = v'(m_j),$$

$$\dot{\lambda} = (\rho - r)\lambda,$$

which combine to yield the optimality condition that  $c_j$  and  $m_j$  must fulfill at each instant:

$$\rho + \pi + \eta_j \frac{\dot{c}_j}{c_j} = R = \frac{v'(m_j)}{u'(c_j)}, \quad (6)$$

where  $\eta_j \equiv -u''c_j/u' > 0$ . Condition (6) has the intuitive explanation. The left-hand side represents the intertemporal marginal rate of substitution in consumption, i.e., the desire to consume (now instead of later). If this desire is less than the nominal interest rate  $R$ , the individual would decrease consumption. On the other hand, the right-hand side of (6) measures the intratemporal marginal rate of substitution between real balances and consumption, i.e., the desire to hold real balances or the "liquidity premium." If this desire is greater than  $R$ , the individual would sell bonds to increase real balances. Thus, (6) guarantees that no one has the incentive to change his consumption level and real money balances. In addition to (6), the optimal  $c_j$  and  $a_j$  must fulfill the transversality condition:

$$\lim_{t \rightarrow \infty} u'(c_j(t))a_j(t) \exp(-\rho t) = 0. \quad (7)$$

Turning to the money market, we assume that the host-country monetary authority keeps the country's money supply fixed at  $M^S$  (This assumption is slightly modified when discussing the effect of immigration.) Equilibrium in the money market requires

$$\frac{M^s}{P} = m, \quad (8)$$

where  $m$  denotes the economy-wide real balances and  $P$  is the nominal price. As we assume perfect flexibility of  $P$ , time differentiation of (8) yields

$$\dot{m} = -\pi m. \quad (9)$$

The small host country takes the international nominal price  $P^I$  of the aggregate product as given. Since the product is traded freely in the international market, the exchange rate  $\epsilon$  adjusts instantaneously to satisfy

$$P = \epsilon P^I. \quad (10)$$

Further, we make this assumption, common in all standard small-open economy models:

*Assumption 2:  $\rho = r$ .*

This assumption allows us to use  $\rho$  and  $r$  interchangeably. Setting  $\rho = r (= R - \pi)$  in (6) yields

$$\frac{\dot{c}_j}{c_j} = 0,$$

implying that under assumption 2  $c_j$  is constant over time. This enables us to rewrite (6) as

$$\rho + \pi = R = \frac{v'(m_j)}{u'(c_j)} \quad \text{for } j = i, h. \quad (11)$$

Finally, we adopt the conventional Walrasian wage adjustment mechanism in the labor market. More specifically, when there is unemployment, the nominal wage  $W$  declines over time according to

$$\frac{\dot{W}}{W} = \alpha(\sigma - 1),$$

where the parameter  $\alpha (> 0)$  represents the speed of adjustment.<sup>7</sup> When there is full employment upward wage adjustment is assumed to occur instantaneously. Full employment or not, equations (1) and (2), together with assumption 2, imply that  $P$  and  $W$  move in tandem to keep the real wage constant ( $w = W/P$ ). Given that the money supply  $M^s$  does not expand over time, we can summarize the above price adjustment process as follows:<sup>8</sup>

$$\begin{aligned}\pi &= \alpha(\sigma - 1) < 0 \quad \text{for } \sigma < 1, \\ \pi &= 0 \quad \text{for } \sigma = 1.\end{aligned}\tag{12}$$

### 3. The model without immigration (benchmark)

This section presents the model without immigration. To keep the analysis simple, we normalize the native population to one. With this normalization, the budget constraints (4) and (5) apply to the whole country, with  $m = m_h$  ( $h$  stands for natives). Substituting from (4) and applying (9) enables us to rewrite the flow budget constraint (5) as

$$\dot{b}_h = \rho b_h + w\sigma - c_h.\tag{13}$$

We prove, in the appendix, that the host-country economy is always in a steady state so that its current account is always balanced. Therefore, if  $b_h^0$  denotes the native's initial bond holdings, taken as exogenous, we have  $b_h = b_h^0$ . Substituting this in (13) and letting  $\dot{b}_h = 0$  yields

$$c_h = w\sigma + \rho b_h^0.\tag{14}$$

The right-hand side represents the native's total income, comprising the wage income and the interest income from his bond holdings. (14) shows that the native consumes all his income.

---

<sup>7</sup> Ono and Ishida (2014) present the microfoundations of wage adjustment mechanism that converges to such adjustment.

<sup>8</sup> Schmitt-Grohé and Uribe (2016, 2017) also assume a similar wage adjustment mechanism.

### 3.1. The benchmark model with full employment

If the host country has full employment ( $\sigma = 1$ ), the benchmark model can be solved recursively. First, setting  $\sigma = 1$  in (14) pins down the consumption level:

$$c_h = c_h^F \equiv w + \rho b_h^0.^9$$

Next, since the nominal price is constant under full employment, setting  $\pi = 0$  in (11) yields

$$\rho = \frac{v'(m_h^F)}{u'(c_h^F)}. \quad (15)$$

Substituting the above  $c_h^F (\equiv w + \rho b_h^0)$  into (15) determines  $m_h^F (= m$  under the normalization of the native population). Then, the money market-clearing condition (8) determines the nominal price  $P$ , given the money supply  $M^s$ . The exchange rate then adjusts to satisfy (10).

Having solved the benchmark model, we ask under what conditions the host country has full employment. To that end, define  $\bar{c}$  by

$$\rho = \frac{\beta}{u'(\bar{c})}. \quad (16)$$

Combining (15) and (16) yields

$$\rho = \frac{v'(m_h)}{u'(c_h^F)} = \frac{\beta}{u'(\bar{c})}.$$

Since  $v'(m_h) \geq \beta$  by assumption 1(b), the above equation implies that

$$c_h^F \equiv w + \rho b_h^0 \leq \bar{c}. \quad (17)$$

That is, consumption is bounded from above by  $\bar{c}$ . In the next subsection we show that the converse of this result holds. Thus, condition (17) is both necessary and sufficient for the existence of an equilibrium with full employment. Moreover, this equilibrium is unique.

**Proposition 1:** Under assumptions 1 and 2 the model admits a unique equilibrium with full employment if and only if  $c_h^F (\equiv w + \rho b_h^0) \leq \bar{c}$ .

---

<sup>9</sup> We assume  $b_h^0 > -w/\rho$  to ensure  $c_h^F > 0$ .

Since the native spends all his income on consumption, if he is endowed with a large sum of bonds, his total income ( $w + \rho b_h^0$ ) can exceed the limit  $\bar{c}$  in violation of condition (17). In such a case, there cannot be an equilibrium with full employment by proposition 1. We thus explore the possibility of an equilibrium with unemployment.

### 3.2. The benchmark model with unemployment

We first characterize an equilibrium with unemployment ( $\sigma < 1$ ). Since  $\pi = \alpha(\sigma - 1) < 0$  by (12), the nominal price  $P$  continuously falls, increasing the real balances  $m = M^s/P$  beyond the threshold level  $\bar{m}$  such that  $v'(m_h) = \beta$  holds. Therefore the optimality condition (11) becomes

$$\rho + \alpha(\sigma - 1) = \frac{\beta}{u'(c_h)}. \quad (18)$$

To keep the left-hand side of (18) positive for any  $\sigma \in [0,1]$ , we assume

$$\rho > \alpha;$$

that is, the speed of wage adjustment is not too fast.

Since the current account (13) must always be balanced, equation (14) holds despite deflation. Solving (14) and (18) simultaneously, we can determine the equilibrium consumption level and employment rate. In figure 1 we depict (18) by the monotone-increasing curve and (14) by the straight line.<sup>10</sup> The intersection point A gives us the equilibrium values, denoted by  $c_h^*$  and  $\sigma^*$ .

---

<sup>10</sup> The graph of (18) is strictly upward-sloping but not necessarily concave as drawn in figure 1.

We next investigate the conditions for the existence of equilibrium with unemployment. From figure 1 there is unemployment ( $0 < \sigma^* < 1$ ) if the straight line is strictly above the curve at  $\sigma = 1$  and strictly below it at  $\sigma = 0$ .<sup>11</sup> The first condition is satisfied if

$$c_h^F \equiv w + \rho b_h^0 > \bar{c} = u'^{-1}(\beta/\rho), \quad (19)$$

while the second implies

$$\rho b_h^0 < u'^{-1}(\beta/(\rho - \alpha)). \quad (20)$$

Inverting condition (19) gives us  $\rho < \beta/u'(c_h^F)$ . This inequality says that, under full employment, the desire to consume,  $\rho$ , is less than the desire to hold real balances,  $\beta/u'(c_h^F)$ . Thus, when (19) holds there cannot be full employment.<sup>12</sup> On the other hand, condition (20) is more technical and rules out the possibility of zero employment.

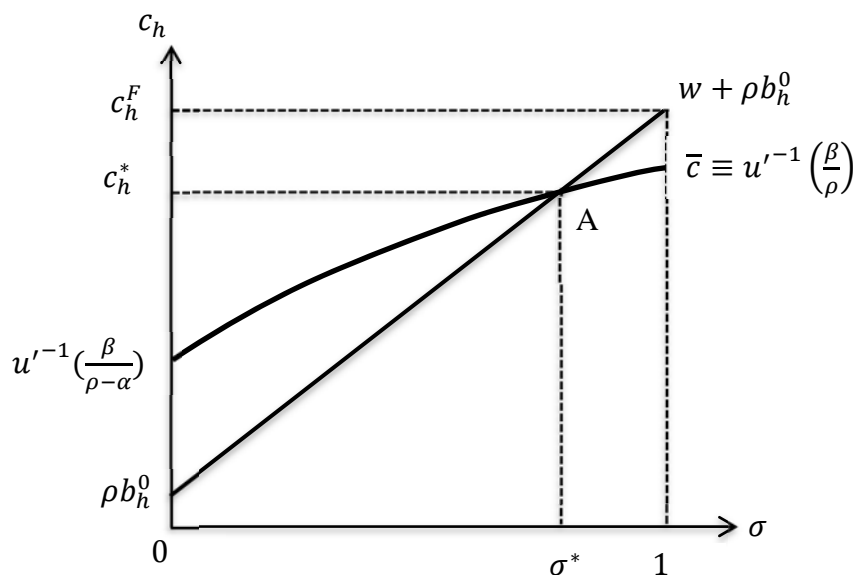


Figure 1:  
Equilibrium with unemployment (benchmark)

<sup>11</sup> Figure 1 depicts the case in which  $b_h^0 > 0$ . That is just for the sake of presentation. If  $b_h^0 < 0$ , the straight line cuts the  $\sigma$ -axis at  $\sigma > 0$ . This however does not affect our analysis, given that condition (20) holds.

<sup>12</sup> This proves that (17) is also sufficient for the existence of an equilibrium with full employment, as alluded to in the paragraph leading to proposition 1.

The above discussion suggests that both conditions (19) and (20) are necessary for the existence of an equilibrium with unemployment. They are also sufficient because, if they both hold, an appeal to the intermediate-value theorem proves the existence of an equilibrium with  $0 < \sigma < 1$ .

Moreover, if the equilibrium in figure 1 is unique, the straight line must be steeper than the curve at the intersection point. A little algebra expresses this condition as

$$\Omega \equiv w + \left(\frac{\alpha}{\beta}\right) \frac{(u')^2}{u''} > 0. \quad (21)$$

For the remainder of our analysis we assume the equilibrium to be unique so condition (21) holds in its neighborhood.

Finally, we show that the transversality condition (7) is satisfied even though real balances  $m_h$  keep expanding. Since the price falls at the rate  $\pi (= \alpha(\sigma - 1))$ , we can write  $m_h(t) = m_h(0) \exp(-\pi t)$ . Furthermore, from (18) we have

$$\pi = \alpha(\sigma - 1) = \frac{\beta}{u'(c_h)} - \rho (< 0).$$

Substituting these into the first expression below yields

$$\lim_{t \rightarrow \infty} u'(c_h) m_h(t) \exp(-\rho t) = \lim_{t \rightarrow \infty} u'(c_h) m_h(0) \exp\left(-\frac{\beta}{u'(c_h)} t\right) = 0.$$

Because  $b_h$  stays constant at  $b_h^0$ , the above implies that

$$\lim_{t \rightarrow \infty} u'(c_h) a_h(t) \exp(-\rho t) = \lim_{t \rightarrow \infty} u'(c_h) (m_h(t) + b_h^0) \exp(-\rho t) = 0,$$

proving that the transversality condition (7) is satisfied.

The next proposition summarizes our findings of this subsection so far.

**Proposition 2:** Under assumptions 1 and 2, there is an equilibrium with unemployment if and only if  $b_h^0$  satisfies conditions (19) and (20). If it is unique, condition (21) holds in its neighborhood.



To understand what causes unemployment, suppose that as  $m_h \rightarrow \infty$ ,  $v'(m_h)$  approaches zero instead of  $\beta > 0$ . Then the optimality condition (18) is replaced by

$$\rho + \alpha(\sigma - 1) = \frac{v'(m_h)}{u'(c_h)}. \quad (22)$$

As expanding real balances drives  $v'(m_h)$  down toward zero,  $c_h$  must keep increasing to hold condition (22). This continuous rise in consumption creates jobs, raising the employment rate  $\sigma$  until full employment is achieved and the nominal price halts its descent. This shows that unemployment cannot occur without the boundedness of marginal utility of real balances away from zero.

We now offer an intuitive explanation of Proposition 2. Suppose that the native's income  $w + \rho b_h^0$  happens to equal  $\bar{c}$ . Then the line and the curve in figure 1 meet only at  $\sigma = 1$ , with the native consuming  $\bar{c} = w + \rho b_h^0 \equiv c_h^F$  and holding real balances  $\bar{m}$ . There is full employment supported by the optimality condition:

$$\rho = \frac{v'(\bar{m})}{u'(c_h^F)} = \frac{\beta}{u'(\bar{c})}.$$

If the native has a greater income ( $w + \rho b_h^0 > \bar{c}$ ), however, full employment cannot be maintained. To show this, recall that consumption cannot exceed the limit  $\bar{c}$ ; cf. (17). Consumption cannot be equal to  $\bar{c}$ , either, because if so the native's spending falls short of his income by  $w + \rho b_h^0 - \bar{c} > 0$ . If he spends this difference to buy foreign assets, the host country runs a perpetual current account surplus, calling for appreciation of the host-country currency. Currency appreciation however makes host-country firms less competitive compared with foreign firms, reducing employment and hence the income. Actually, because the foreign and home goods are assumed homogeneous, currency appreciation does not materialize. Instead, adjustment occurs through changes in the employment rate  $\sigma$ . To balance the current account,  $\sigma$  must fall enough to bring down the native's income equal to his actual consumption

level; i.e.,  $w\sigma + \rho b_h^0 = c_h$ . This new consumption  $c_h$  is less than  $\bar{c}$  by (16) and (18) because  $\sigma < 1$ .

The above discussion implies that  $\rho b_h^0 = \bar{c} - w$  is the native's maximum endowment of bonds consistent with full employment. If he holds fewer bonds, there is full employment by proposition 1. If he holds more bonds, there is unemployment by proposition 2. We record this result in

**Corollary 1:** (a) If  $\rho b_h^0 \leq \bar{c} - w$ , the host country has full employment.

(b) If  $\rho b_h^0 > \bar{c} - w$ , the host country suffers from unemployment.

#### 4. Immigration: an overview

We now extend the benchmark model to study the effect of immigration. To that end we assume the following. At some time ( $t = t_0$ ), a given number, say,  $x_i$  of immigrants enter the host country (the subscript  $i$  denotes immigrants). Immigrants are endowed with one unit of labor and maximize the utility functional given in (3). As mentioned in the introduction, however, immigrants differ in two respects. First, the typical immigrant arrives with  $b_i^0$  units of internationally-traded bonds but with no host-country currency. Thus, upon entry into the host country, the immigrant sells bonds (or borrow against their future incomes) in exchange for local money to satisfy his demand for real balances. If each immigrant acquires  $m_i$  units of the host-country currency from the monetary authority, the latter holds international bonds totaling  $x_i m_i$ . We assume that the monetary authority rebates the interest earnings from these bond holdings evenly to all natives.<sup>13</sup> At the end of the day, therefore, it is as if each native has

---

<sup>13</sup> Because natives' income increases, they too want to increase money holdings. To meet this money demand, the monetary authority purchases bonds with new money, further increasing the country's money stock. Simultaneously, it also rebates the interest earnings on the newly acquired bonds to natives. Such adjustment is

increased his bond holdings to  $b_h^0 + x_i m_i$  (while each immigrant's bond holdings has fallen to  $b_i^0 - m_i$ ).

The second way immigrants differ from natives is with respect to remittances. It is assumed that each immigrant remits  $\tau_i (\geq 0)$  units of the aggregate good back home whereas natives make no such remittances ( $\tau_h = 0$ ). We take  $\tau_i$  as given and investigate its effect.

With the above changes, we can write the native's flow asset constraint at  $t > 0$  as

$$\dot{a}_h = \rho(b_h^0 + x_i m_i) - \pi m_h + w\sigma - c_h, \quad (23)$$

and the immigrant's as

$$\dot{a}_i = \rho(b_i^0 - m_i) - \pi m_i + w\sigma - c_i - \tau_i. \quad (24)$$

Adding up these equations over all the host-country residents (i.e., natives and immigrants combined) gives us the aggregate flow budget constraint:

$$\dot{a}(= \dot{b} + \dot{m}) = \rho(b_h^0 + x_i b_i^0) - \pi m + w\sigma(1 + x_i) - (c_h + x_i c_i) - x_i \tau_i,$$

where  $m \equiv m_h + x_i m_i$  (the variables without subscripts henceforth denote the aggregated values). Substituting from (9) and rearranging terms, we can rewrite the above constraint as

$$\dot{b} = \rho(b_h^0 + x_i b_i^0) + w\sigma(1 + x_i) - (c_h + x_i c_i) - x_i \tau_i (= 0). \quad (25)$$

As shown in the appendix, the right-hand side of (25) equals zero because the host country's current account must always be balanced ( $\dot{b} = 0$ ).

## 5. Full employment

Suppose that we have full employment in a post-immigration equilibrium. Since  $\pi = 0$  under full employment, the optimal consumption and real balances satisfy

$$\rho = \frac{v'(m_h)}{u'(c_h)} = \frac{v'(m_i)}{u'(c_i)} \left( = \frac{\beta}{u'(\bar{c})} \right), \quad (26)$$

---

instantaneously completed the moment the host country takes immigrants in. Thereafter, the money stocks remains constant.

where the last equality follows (16). Since  $v'(m_j) \geq \beta$ , (26) implies that  $c_j \leq \bar{c}$ . As in the benchmark model, the consumption levels are bounded from above by  $\bar{c}$ . With the nominal price constant, the individual asset holdings do not change over time, either. Thus, we set  $\dot{a}_j = 0$  in the flow budget constraints (23) and (24) to obtain

$$c_h = \rho(b_h^0 + x_i m_i) + w, \quad (27)$$

$$c_i = \rho(b_i^0 - m_i) + w - \tau_i. \quad (28)$$

Equations (26), (27) and (28) can be solved for determine the equilibrium levels of  $c_j$  and  $m_j$ , which we denote by  $\tilde{c}_j$  and  $\tilde{m}_j$  ( $j = h, i$ ).

(26) is consistent with two types of equilibria, depending on whether the immigrant's real balances exceed  $\bar{m}$ . We begin with the case in which  $\tilde{m}_i \leq \bar{m}$ . Then (26) can be arranged to yield

$$\tilde{m}_i = v'^{-1}(\rho u'(\tilde{c}_i)) \equiv \varphi(\tilde{c}_i) \quad \text{for } \tilde{m}_i \leq \bar{m}; \quad \varphi'(\cdot) > 0, \quad \varphi(\bar{c}) = \bar{m}. \quad (29)$$

Substituting for  $\tilde{m}_i$  from (29) into (28) yields

$$\tilde{c}_i + \rho\varphi(\tilde{c}_i) = (\rho b_i^0 - \tau_i) + w. \quad (30)$$

Given the monotonicity of  $\varphi(c_i)$ , (30) determines a unique  $\tilde{c}_i$ , which clearly depends on the immigrant's labor income  $w$ , interest income  $\rho b_i^0$  and remittances  $\tau_i$ . We then substitute  $\tilde{c}_i$  into (29) to determine the real balances  $\tilde{m}_i = \varphi(\tilde{c}_i)$ . Since  $\varphi' > 0$  by (29), (30) implies

$$\rho b_i^0 - \tau_i \uparrow \Rightarrow \tilde{c}_i \uparrow, \quad \tilde{m}_i \uparrow \quad \text{for } \tilde{m}_i \leq \bar{m}. \quad (31)$$

Let us call the term  $(\rho b_i^0 - \tau_i)$  the immigrant's "net worth." Then by (31), when the immigrant has a greater net worth, he consumes more and demands more real balances. It is straightforward to show that there is a unique net worth given by

$$NW^0 \equiv \rho b_i^0 - \tau_i = (\bar{c} - w) + \rho \bar{m},$$

such that the immigrant consumes  $\tilde{c}_i = \bar{c}$  and holds  $\tilde{m}_i = \bar{m}$ . Then, (31) holds only for  $\rho b_i^0 - \tau_i \leq NW^0$ .

If the immigrant is so rich that his net worth exceeds  $NW^0$ , we have a second type of equilibrium, in which the immigrant consumes exactly  $\bar{c}$  while holding  $\tilde{m}_i > \bar{m}$ . Since marginal utility of real balances is bounded by  $\beta$ , consuming in excess of  $\bar{c}$  puts the desire to hold real balances above the desire to consume. Thus, the immigrant sells bonds for money, which reduces his interest income. In the end, his income is brought down to equal  $\bar{c}$  such that

$$\bar{c} = \rho(b_i^0 - m_i) + w - \tau_i.$$

This last equation determines the immigrant's money balances:

$$\tilde{m}_i = \rho b_i^0 - \tau_i + w - \bar{c} > \bar{m}. \quad (32)$$

It is evident from the last two equations that

$$\rho b_i^0 - \tau_i \uparrow \Rightarrow \tilde{c}_i = \bar{c}, \tilde{m}_i \uparrow \text{ for } \tilde{m}_i > \bar{m}. \quad (33)$$

(31) and (33) demonstrate that an increase in the immigrant's net worth always raises his real balances  $\tilde{m}_i$ , whether  $\tilde{m}_i$  exceeds  $\bar{m}$  or not.

In section 3 we showed that when natives are rich enough to hold real balances above  $\bar{m}$ , full employment fails to exist (proposition 2). In contrast, when the immigrant holds  $\tilde{m}_i > \bar{m}$ , full employment still prevails. This difference can be understood as follows. As already explicated, immigrants can sell as many bonds as needed to bring their income down to equal  $\bar{c}$ . By contrast, natives lack such latitude because, if they sell bonds to reduce their income, the monetary authority would give them back the interest income on the acquired bonds. Thus, their income remains unchanged whether they sell bonds or not. As detailed in section 3, their income is reduced only by a fall in the employment rate.

The preceding discussion showed that full employment persists after immigration. However, this claim is subject to the qualification that the monetary authority does not acquire too many bonds from immigrants, i.e.,  $x_i \tilde{m}_i$  should not be too large. Otherwise, the monetary authority would rebate so much interest income to natives that their total income could top the

consumption limit  $\bar{c}$ , violating the condition for full employment. Since  $\tilde{c}_h \leq \bar{c}$ , (27) gives us the following result:

**Proposition 3:** There exists an equilibrium with full employment after immigration only if

$$\rho b_h^0 + w \leq \bar{c} - \rho x_i \tilde{m}_i, \quad (34)$$

where  $\tilde{m}_i$  is determined by (29) and (30) or by (32).

We complete the analysis by calculating for the equilibrium consumption and real money balances for natives. Plugging the  $\tilde{m}_i$  given above into (26) and (27) yields:

$$\tilde{m}_h = v'^{-1}(\rho u'(\tilde{c}_h)) \equiv \varphi(\tilde{c}_h),$$

$$\tilde{c}_h = \rho(b_h^0 + x_i \tilde{m}_i) + w (\leq \bar{c}).$$

The second equation says that  $\tilde{c}_h$  increases with immigrants' real balances  $\tilde{m}_i$ . This is evident because natives receive more interest income when immigrants exchange more bonds for money. Moreover, the immigrant with a greater net worth ( $\rho b_i^0 - \tau_i$ ) holds more money; cf. (31) and (33). These facts establish

**Proposition 4:** Suppose that (34) holds so that there is full employment before and after immigration. Then:

- (a) Immigration increases natives' consumption.
- (b) The richer are immigrants (the more bonds they hold before immigration), the greater is natives' consumption.
- (c) The greater remittances immigrants send home, the smaller is natives' consumption.

Proposition 4 has the intuitive explanation. Immigrants exchange international bonds for local currency. While the host-country currency is worthless in international transactions, the

acquisition of internationally traded bonds allows natives to import and consume more. Because immigrants convert more international bonds when richer, natives' consumption rises with immigrant's net worth, provided that condition (34) holds.

Notice that (34) implies (17) and hence full employment exists in a post-immigration state only if there is full employment before immigration. However, this converse does not hold. If the native is rich enough to consume close to  $\bar{c}$  before immigration, his post-immigration income may exceed this consumption limit due to the additional interest income  $\rho x_i \tilde{m}_i$  he receives from the monetary authority. Even if the native is not as rich, an influx of too many or too rich immigrants can make  $\rho x_i \tilde{m}_i$  so large to produce the same effect. In such cases, if (34) is violated, we can only have unemployment after immigration, the topic to which we turn in the next section.

## 6. Unemployment

We begin this section by characterizing a post-immigration equilibrium with unemployment. In the presence of unemployment ( $\sigma < 1$ ), the nominal wage and price keep falling according to (12) so that  $\pi < 0$ . The implication is that natives' real balances exceed the threshold  $\bar{m}$ , implying the optimality condition:

$$\rho + \alpha(\sigma - 1) = \frac{\beta}{u'(c_h)} = \frac{v'(m_i)}{u'(c_i)} \left( < \frac{\beta}{u'(\bar{c})} \right). \quad (35)$$

The inequality, due to (16), implies that  $c_j \leq \bar{c}$  for  $j = h, i$ . Given the fact that the price is continuously falling, (35) suggests two possible scenarios, depending on the size of the immigrant's real money balances  $m_i$ . In one scenario the immigrant holds  $m_i < \bar{m}$ , while in the other  $m_i$  keeps increasing beyond  $\bar{m}$ .

Consider the case in which  $m_i < \bar{m}$ . Then,  $m_i$  stays constant, implying that the immigrant's asset holding is not expanding despite deflation. Thus we set  $\dot{a}_i = 0$  in (24) to obtain

$$\rho b_i^0 - \tau_i + \sigma w - c_i - [\rho + \alpha(\sigma - 1)]m_i = 0. \quad (36)$$

Equations (35) and (36) determine the immigrant's optimal real balances and consumption in terms of  $\sigma$  and his net worth  $\rho b_i^0 - \tau_i$ ; algebra shows that

$$m_i = m_i(\sigma; \rho b_i^0 - \tau_i), \quad \frac{\partial m_i}{\partial (\rho b_i^0 - \tau_i)} > 0 \quad \text{for } m_i < \bar{m}, \quad (37)$$

$$c_i = c_i(\sigma; \rho b_i^0 - \tau_i), \quad \frac{\partial c_i}{\partial (\rho b_i^0 - \tau_i)} > 0 \quad \text{for } c_i < u'^{-1}\left(\frac{\beta}{\rho + \alpha(\sigma - 1)}\right). \quad (38)$$

The host-country's current account also remains balanced despite deflation, validating (25). We can substitute from (36), (37) and (38) into (25) to obtain, after arranging,

$$c_h = w\sigma + \rho b_h^0 + x_i[\rho + \alpha(\sigma - 1)]m_i(\sigma; \rho b_i^0 - \tau_i). \quad (39)$$

This equation and the optimality condition from (35)

$$\rho + \alpha(\sigma - 1) = \frac{\beta}{u'(c_h)}, \quad (40)$$

can be solved simultaneously for the native's consumption  $\hat{c}_h$  and the unemployment rate  $\hat{\sigma}$ . In figure 2, the broken curve through point B represents (39) while the solid curve depicts equation (40). The intersection point B specifies  $\hat{c}_h$  and  $\hat{\sigma}$ . Note that (40) is identical to (18) in section 3. Substituting the  $\hat{\sigma}$  into (37) and (38) gives us the consumption  $\hat{c}_i$  and real balances  $\hat{m}_i$  for the immigrant.

Having solved the model, we can appeal to figure 2 to derive the conditions guaranteeing the existence of an equilibrium with unemployment. To ensure that  $\hat{\sigma} < 1$ , the broken curve must take a greater value at  $\sigma = 1$  than the solid curve does. This requirement is fulfilled if

$$c_h|_{\sigma=1} \equiv w + \rho \left( b_h^0 + x_i m_i(1; \rho b_i^0 - \tau_i) \right) > \bar{c},$$

where  $c_h|_{\sigma=1}$  is the value of  $c_h$  in (39) for  $\sigma = 1$ . Since  $x_i m_i > 0$ , this condition necessarily



holds if

$$c_h^F \equiv \rho b_h^0 + w > \bar{c}.$$

i.e., the host country has unemployment before immigration; cf. corollary 1. Further, to have  $\hat{\sigma}$  in  $(0, 1)$  requires that

$$c_h|_{\sigma=0} \equiv \rho b_h^0 + (\rho - \alpha)x_i m_i(0; \rho b_i^0 - \tau_i) < u'^{-1}\left(\frac{\beta}{\rho - \alpha}\right),$$

i.e., the broken curve takes a smaller value than the solid curve at  $\sigma = 0$ .<sup>14</sup>

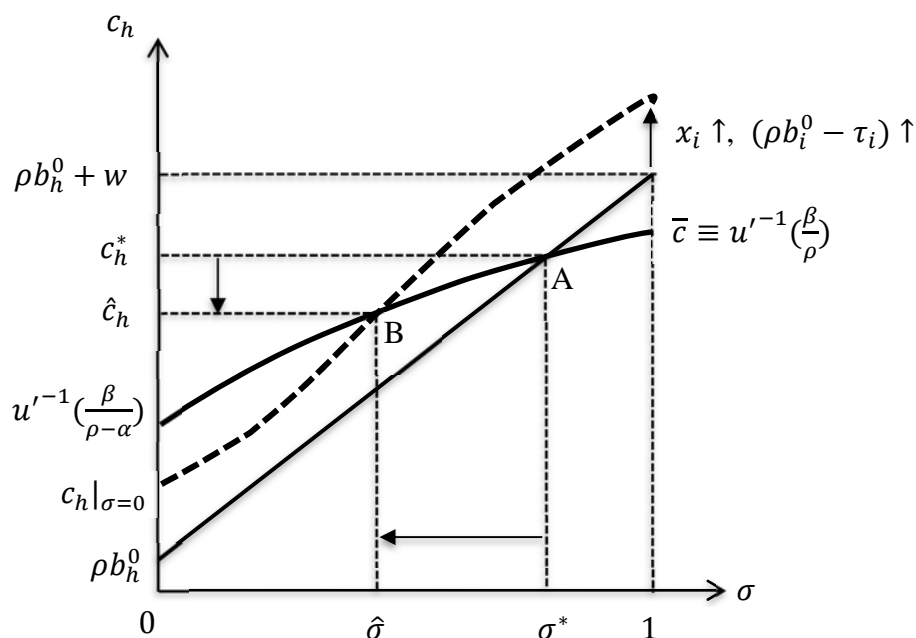


Figure 2:  
Post- and pre-immigration equilibria with unemployment  
(scenario 1)

It is easy to check from (37) and (39) that an increase in the immigrant's net worth  $\rho b_i^0 - \tau_i$  shifts the dotted curve up in figure 2. Since (40) is unaffected, the solid curve through A

<sup>14</sup> Given condition (20), this condition holds if  $x_i$  is not too large.

remains intact. Hence, we conclude that both  $\hat{\sigma}$  and  $\hat{c}_h$  decrease as the immigrant's net worth increases.

Manipulation of (39) and (40) yields

$$dc_h = (w + \alpha x_i m_i) d\sigma + \frac{\beta x_i}{u'} dm_i,$$

$$\alpha d\sigma = -\frac{\beta u''}{(u')^2} dc_h.$$

Combining these equations and using the definition of  $\Omega (> 0)$  given in (21), we find that

$$\left(\frac{\beta x_i}{u'}\right) \frac{dm_i}{d(\rho b_i^0 - \tau_i)} = -(\Omega + \alpha x_i m_i) \frac{d\sigma}{d(\rho b_i^0 - \tau_i)} > 0,$$

where the inequality follows from  $\frac{d\hat{\sigma}}{d(\rho b_i^0 - \tau_i)} < 0$ .<sup>15</sup> Thus, the immigrant's real balances  $m_i$  increases with his net worth  $(\rho b_i^0 - \tau_i)$ , reaching  $\bar{m}$  at some cutoff net worth. If the immigrant's net worth grows still, he must hold  $m_i > \bar{m}$ . In such a case, (37) no longer applies and we turn to the second scenario.

The second scenario occurs only if the immigrant is rich enough to hold real balances greater than  $\bar{m}$  so that  $v'(m_i) = \beta$ . Substituting this marginal utility turns (35) into

$$\rho + \alpha(\sigma - 1) = \frac{\beta}{u'(c)} \quad \text{with } c_h = c_i = c, \quad (41)$$

that is, in the second scenario the immigrant and the native consume exactly the same amount. Setting  $c_h = c_i = c$  in (25) enables us to rewrite the host-country current account constraint as

$$\dot{b} = \rho b_h^0 + x_i(\rho b_i^0 - \tau_i) + (1 + x_i)(w\sigma - c) = 0. \quad (42)$$

The middle expression vanishes as shown because the current account must be in balance, (42) can be arranged to yield

$$c = c_h = w\sigma + \frac{\rho b_h^0 + x_i(\rho b_i^0 - \tau_i)}{1 + x_i}. \quad (43)$$

---

<sup>15</sup> This is readily verified in figure 2.

(41) and (43) jointly determine the equilibrium values of  $c$  and  $\sigma$ , which we denote by  $\hat{c}(= \hat{c}_h = \hat{c}_i)$  and  $\hat{\sigma}$ .

In figure 3, the broken line through point C represents equation (43) whereas the solid curve traces equation (41). The intersection at C indicates the equilibrium values,  $\hat{c}_h(= \hat{c}_i)$  and  $\hat{\sigma}$ . To have  $0 < \hat{\sigma} < 1$  requires that

$$c_{|\sigma=1} = w + \frac{\rho b_h^0 + x_i(\rho b_i^0 - \tau_i)}{1+x_i} > u'^{-1}\left(\frac{\beta}{\rho}\right) = \bar{c}, \quad (44a)$$

$$c_{|\sigma=0} = \frac{\rho b_h^0 + x_i(\rho b_i^0 - \tau_i)}{1+x_i} < u'^{-1}\left(\frac{\beta}{\rho-\alpha}\right). \quad (44b)$$

These conditions ensure that the broken line lies above the curve near  $\sigma = 1$  and below it near  $\sigma = 0$  as in figure 3, which guarantees the existence of an equilibrium with unemployment.

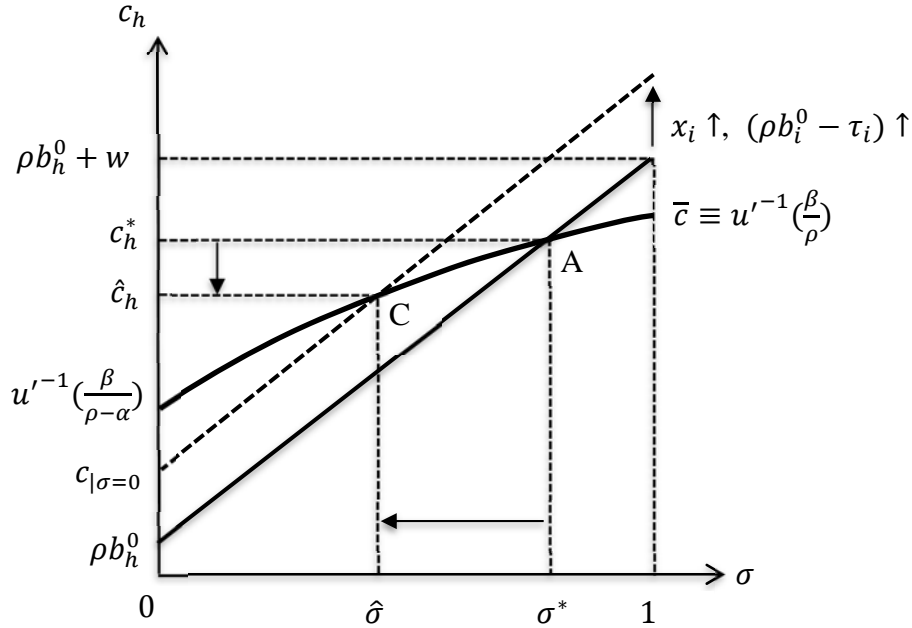


Figure 3:  
Pre- and post-immigration equilibria exhibiting  
unemployment (scenario 2)

To understand the two conditions in (44), notice that the immigrant consumes  $c_i = c_h = w\sigma + \rho b_h^0$ , where the second equality is due to (14). On the other hand, the immigrant sells enough bonds to acquire real balances greater than  $\bar{m}$  so his income is at most equal to  $\rho(b_i^0 - \bar{m}) - \tau_i + w\sigma$ . Since individuals consume their entire incomes, we have

$$w\sigma + \rho b_h^0 \leq \rho(b_i^0 - \bar{m}) - \tau_i + w\sigma,$$

which simplifies to

$$\rho b_h^0 < \rho b_i^0 - \tau_i. \quad (45)$$

(45) implies that  $c_{|\sigma=1}$  in (44) increases with  $x_i$ . Further, since we assume unemployment before immigration, (19) and (20) hold. Therefore, (44a) necessarily holds while (44b) holds for a sufficiently small  $x_i$ .

Although we have seen two types of equilibria with unemployment, in both cases immigration worsens the unemployment and reduces natives' consumption level. To see this, note that as  $x_i \rightarrow 0$ , both equations (39) and (43) converge to equation (14). In term of figure 2, this convergence is equivalent to the broken curve approaching the solid line.<sup>16</sup> Similarly in figure 3, because (45) holds, the broken line converges from above to the solid line. Since point A marks the pre-immigration values  $c_h^*$  and  $\sigma^*$ , taken from figure 1, the above convergence results imply that when the host country has unemployment initially, an influx of immigrants moves the equilibrium from A to B or C, thereby decreasing the employment rate and natives' consumption level.

It is easy to see that the immigrant's net worth also has similar effects in both types of equilibria. An increase in  $(\rho b_i^0 - \tau_i)$  shifts up the broken curve in figure 2 and the broken line in figure 3.<sup>17</sup> Thus, in both cases, the employment rate and the native's consumption fall when

---

<sup>16</sup> This guarantees the existence of a unique equilibrium for sufficiently small  $x_i$ .

<sup>17</sup> This is evident from (43).

the immigrant's net worth increases. The next proposition records these results, which stand in sharp contrast seriatim with those in proposition 4.

**Proposition 5:** Suppose that (17) is violated so that the host country suffers from unemployment before and after immigration. Then:

- (a) Immigration always decreases the rate of employment and the native's consumption.
- (b) The richer are immigrants (in terms of international bond holdings), the lower are the employment rate and the consumption level by natives.
- (c) The more remittances immigrants make, the higher are the employment rate and the consumption level by natives.

## 7. Concluding remarks

In this paper we develop a dynamic model of a small open country, where agents maximize life-time welfare over consumption of the aggregate good and real balances they hold. The model has two salient features: lower boundedness of marginal utility of real balances and sluggish nominal wage adjustment. We find the following. (1) In the absence of immigration, the host country has full employment if it holds a quantity of international interest-earning assets below some threshold level. Otherwise, it has unemployment. (2) If the host country has full employment, an influx of immigrants boosts the native's consumption level, provided that immigrants are neither too rich nor too numerous. An influx of too rich or too many immigrants can give rise to unemployment, however. (3) If the host country suffers from unemployment, immigration always worsens the unemployment rate and reduces the native's consumption. (4) Remittances by immigrants reduce natives' consumption when the host country has full employment. By contrast, when the host country has unemployment, remittances increase natives' consumption as well as their employment.

Several extensions manifest themselves. First, although we assumed native and immigrant workers homogeneous, some studies have explored the implications of skill differences between them.<sup>18</sup> If there is a single aggregate good, one straightforward way to introduce the inferiority of immigrant labor into our model is as follows. Assume that the immigrant possesses only a fraction of (effective) labor compared with the native so that the immigrant earns a lower wage than the native. In this setting, we expect our results to remain unaffected qualitatively. Second, our model can be applied to study the effect of emigration on the source country. More challenging is an extension to the case of two large countries and labor movement between them. This necessarily introduces interdependence both on the real and the monetary side of the two economies.<sup>19</sup> We hope to address these issues in our future research.

---

<sup>18</sup> See, e.g., Liu (2010), Chassambouilli and Palivos (2014) and Battisti et al. (2018), who have used two-sector models.

<sup>19</sup> Ono (2014, 2018) have studied policy interdependence between large countries without consideration of immigration issues.

## Appendix: Stability

The stability around the full-employment steady state is standard so we focus on the case with unemployment. As mentioned in deriving (11),  $c_j$  stays constant over time in all cases. Having this property in mind, we first examine the benchmark model with unemployment. From (18) we have

$$\rho + \alpha(\sigma - 1) = \frac{\beta}{u'(c_h)} \Rightarrow c_h = c_h(\sigma). \quad (\text{A1})$$

Substituting this  $c_h$  to (13) gives us

$$\begin{aligned} \dot{b}_h &= \rho b_h + w\sigma - c_h(\sigma), \\ \frac{\partial \dot{b}_h}{\partial \sigma} &= w + \left(\frac{\alpha}{\beta}\right) \frac{(u')^2}{u''} \equiv \Omega > 0, \end{aligned}$$

where  $\Omega$  is given in (21). These two equations indicate that the dynamics of  $b_h$  is unstable, implying that  $\sigma$  and  $c_h(\sigma)$  immediately jump to the levels that make  $\dot{b}_h = 0$  and stay there, keeping  $b_h$  at the initial level  $b_h^0$ .

Turning next to the post-immigration steady state with unemployment, consider the second scenario from the text, in which both natives and immigrants hold real balances above  $\bar{m}$ ; i.e.,  $v'(m_j) = \beta$  for  $j = h, i$ . In this case, the post-immigration dynamics is given by (42) and the analysis goes through as above, mutatis mutandis, with  $b_h$  being replaced by  $b_h + x_i(b_i - \tau_i/\rho)$ . Consider next the first scenario, in which  $m_h > \bar{m}$  while  $m_i < \bar{m}$ . In this case,  $c_h(\sigma)$  is given by (A1) while (35) yields

$$\frac{v'(m_i)}{u'(c_i)} - \alpha(\sigma - 1) = \rho \Rightarrow m_i = m_i(c_i, \sigma).$$

Applying these  $c_h(\sigma)$  and  $m_i(c_i, \sigma)$  to the dynamics of the immigrant's asset holdings in (24) and the current account in (25), we obtain

$$\begin{aligned} \dot{a}_i &= \rho b_i^0 - \tau_i + w\sigma - [\rho + \alpha(\sigma - 1)]m_i(c_i, \sigma) - c_i, \\ \dot{b} &= \rho(b_h^0 + x_i b_i^0) + w\sigma(1 + x_i) - (c_h(\sigma) + x_i c_i) - x_i \tau_i, \end{aligned}$$

where  $\sigma$  and  $c_i$  stay constant over time. If they jump so that  $\dot{a}_i$  and/or  $\dot{b}$  are non-zero, either the feasibility condition or the non-Ponzi game condition is violated. Thus, they initially jump to the levels that make  $\dot{a}_i$  and  $\dot{b}$  zero and stay invariant thereafter, i.e., (35) and (36) hold.



## References

- Battisti, M., G. Felbermayr, G. Peri and P. Poutvaara, 2018, Immigration, search and redistribution: a quantitative assessment of native welfare, *Journal of the European Economic Association* 16, 1137-1188.
- Bond, E. W., and T. J. Chen, 1987, The welfare effects of illegal immigration, *Journal of International Economics* 23, 315-328.
- Berry, R. A., and R. Soligo, 1969, Some welfare aspects of international migration, *Journal of Political Economy* 77, 778-794.
- Carter, T. J., 1999, Illegal immigration in an efficiency wage model, *Journal of International Economics* 49, 385-401.
- Chassamboulli, A., and T. Palivos, 2014, A search-theoretic approach to the effects of immigration on labor market outcomes, *International Economic Review* 55, 111-129.
- Dixit, A., and V. Norman, 1980, *Theory of International Trade*, London: Cambridge University Press.
- Djajić, S., 1997, Illegal immigration and resource allocation, *International Economic Review* 38, 97-117.
- Ethier, W. J., 1985, International trade and labor migration, *American Economic Review* 75, 691-707.
- Ethier, W. J., 1986, Illegal immigration: the host-country problem, *American Economic Review* 76, 56-71.
- Giordani, P. E., and M. Ruta, 2013, Coordination failures in immigration policy, *Journal of International Economics* 89, 55-67.
- Illing, G., Y. Ono, and M. Schlegl, 2018, Credit booms, debt overhang and secular stagnation, *European Economic Review* 108, 78-104.
- Liu, X., 2010, On the macroeconomic and welfare effects of illegal immigration, *Journal of*

- Economic Dynamics and Control* 34, 2547-2567.
- Mangin, S., and Y. Zenou, 2016, Illegal migration and policy enforcement, *Economics Letters* 148, 83-86.
- Markusen, J. R., 1983, Factor movements and commodity trade as complements, *Journal of International Economics* 14, 341-356.
- Michau, J.B., 2018, Secular stagnation: theory and remedies, *Journal of Economic Theory* 176, 552-618.
- Miyagiwa, K., and Y. Sato, 2019, Illegal immigration, unemployment, and multiple destinations, *Journal of Regional Science* 59, 118-144.
- Ono, Y., 1994, *Money, Interest, and Stagnation - Dynamic Theory and Keynes's Economics*, Oxford: Clarendon Press.
- Ono, Y., 2001, A reinterpretation of chapter 17 of Keynes's General Theory: effective demand shortage under dynamic optimization, *International Economic Review* 42, 207-236.
- Ono, Y., 2010, Japan's long-run stagnation and economic policies, in B. Bateman, T. Hirai, M.C. Marcuzzo, eds., *The Return to Keynes*, Cambridge, Mass.: Harvard University Press, 32-50.
- Ono, Y., 2014, International economic interdependence and exchange-rate adjustment under persistent stagnation, *Japanese Economic Review* 65, 70-92.
- Ono, Y., 2018, Macroeconomic interdependence between a stagnant and a fully employed country, *Japanese Economic Review* 69, 450-477.
- Ono, Y., and J. Ishida, 2014, On persistent demand shortages: a behavioral approach, *Japanese Economic Review* 65, 42-69.
- Ortega, J., 2000, Pareto-improving immigration in an economy with equilibrium unemployment, *Economic Journal* 110, 92-112.
- Pissarides, C. A., 2000, *Equilibrium Unemployment Theory*, Cambridge, Mass.: MIT Press.

- Schmitt-Grohé, S., and M. Uribe, 2016, Downward nominal wage rigidity, currency pegs, and involuntary unemployment, *Journal of Political Economy* 124, 1466-1514.
- Schmitt-Grohé, S., and M. Uribe, 2017, Liquidity traps and jobless recoveries, *American Economic Journal: Macroeconomics* 9, 165-204.
- Wooldland, A. D., and C. Yoshida, 2006, Risk preferences, immigration policy and illegal immigration, *Journal of Development Economics* 81, 500-513.