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The Demographic Transition in a Unified Growth Model

of the English Economy

James Foreman-Peck^{*} and Peng Zhou[†]

Abstract

A dynamic stochastic unified growth model is estimated from English economy data for almost a millennium. At the core of the (seven) overlapping generations, rational expectations structure is household choice about target number and quality of children. The trends of births, deaths, population and, the real wage, are closely matched by the estimated model. In the 19th century English fertility transition, the model shows how the generalized child price relative to the child quality price rose. The rising opportunity cost of education was as decisive for the transition as the parental shift to child quality.

Key Words: Economic Development, Demography, Unified Growth, Overlapping Generations, English Economy

JEL Classification: O11, J11, N13

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Unified growth theory (UGT) is an evolving system that accounts for Malthusian stagnation, the escape from economic stagnation, the demographic transition, and the arrival of modern economic growth (Galor and Weil, 2000). In total it provides an analytical framework for understanding the divergence in national incomes per capita in the last two centuries or more and for demographic transitions, the focus of this paper.

The transition for Becker (1981) is driven by rising income triggering substitution between quantity and quality of children. By contrast unified growth theories have modeled endogenous transitions as consequences of *technological progress* that alters the quality/education-fertility trade-off, or of mortality decline (see Galor (2005) for a survey and Doepke (2005) for a model driven by mortality decline). In De la Croix and Licandro (2012) the trade-off is extended to the number of children and adult human capital. The driver in Cervellatti and Sunde (2015) is the agent's choice of skilled or unskilled human capital and the number and quality of children. Net fertility declines as skilled capital accumulates because skilled workers have fewer children than unskilled. From a large-scale empirical exercise Murtin (2013) attributes the fertility transition everywhere to primary schooling, rather than to income or health, while Doepke (2004) in a unified growth model offers a compelling case that policies to reduce or eliminate child labor are much more powerful than subsidizing education.

Following Galor and Weil (2000) the model proposed here assumes an exogenous or parametric character of technological progress in the long run, but an endogenous *pace*³. Human capital accumulation is the endogenous key driver, consistent with the findings of Madsen and Murtin (2017). Higher human capital investment helps generating new scientific knowledge needed for technological progress but does not change the sequence of technologies.

Two fundamental mechanisms determine human capital accumulation. First, a natural selection mechanism leads to evolution of preferences—negative population growth (especially the Black Death) selects for removal the "less fit" portion of the population distribution (Galor and Moav, 2002). Different surviving utility functions result in different decisions about child bearing and rearing, as well as consumption. The direction of evolution points to a more flexible trade-off between child quantity and quality, so human capital accumulation is faster after big mortality events. Second, a rational optimization mechanism leads to changes of decisions—a higher "price" of children

³ However, unlike them we do not assign a positive role to population growth in technical progress because Crafts and Mills (2009) for England find no evidence for it.

means a lower quantity demanded. Here, the prices are defined in general terms including both monetary and time costs of raising and educating a child (both formally and informally).

The paper's theoretical contribution is to recognize that for very long-term economic growth, it is necessary that deep parameters have time-varying values without changing the deep structure. The present model achieves this by explicitly building in an evolutionary feature *endogenously* to more general assumption about preference responsiveness. The evolutionary path is a continuous spectrum of steady states, *not* transitional dynamics. Consequently, the model is empirically powerful, with a generic structural component and a specific auxiliary component, not simple calibration.

The unified growth modeling for England here assumes an exogenous role for mortality (as also identified by Bar and Leukhina (2010) for example). Generation-specific mortality rates show how effects differ between life phases. The intensity and frequency of mortality crises (shocks) diminish with the success of Western European quarantine regulations from the early 18th century (Chesnais, 1992 p141). Such a decline in mortality was exogenous to the English economy even though it may have been endogenous to Western Europe as a whole⁴. Greater child survival triggered by falling mortality is realized more rapidly than effects operating through human capital accumulation; greater child quality must take longer to be translated into higher wages than the stronger demand for children's numbers takes to trigger an increase in population.

The model offers a slightly less ambitious interpretation of economic history than current UGT; the escape from the Malthusian trap was not inevitable in England, it was triggered by a demographic catastrophe. After the very high mortality shock of the 14th and 15th centuries, interest rates and skill premia did not return to their previous levels despite population growth and increasing land scarcity. The explanation is that new, non-Malthusian equilibria were attained, as lower mortality induced changes in desired child quality and greater savings (Van Zanden, 2009 p162). This pattern, with a high age at female first marriage/birth and female childlessness, is a historically contingent feature of the model (Hajnal, 1961).

The link between demography and economy allows the model to show that in the long term increasing productivity from human capital accumulation raises the demand for children, boosting population. Eventually this technical progress associates rising population and real wage growth. The present paper is concerned especially with the next

⁴ This may be why we were unable to accommodate the endogenous mortality (as proposed for instance by De la Croix and Licandro (2012), Voigtlander and Voth (2013a) and Cervelatti and Sunde (2015)) in the model for the English data.

stage, the fertility transition. Economic historians of England continue to find the transition somewhat enigmatic (Baines, 1994; Wood, 1992). By contrast econometric analyses (Crafts, 1984; Tzannatos and Symons, 1989) present exogenous changes in generalized English child price and quality as explanations, but their identification is less strong than in the present model⁵. Here we derive explicitly from the model these generalized prices and explain their movements.

Potential exogenous contributors to the fertility transition are assessed by simulating the model, having set their values to the 1850 levels. Two variables affecting surviving child costs are critical. One of the most important variables is mortality decline, lowering the "price" and raising target family size. The other is an offsetting spread of family-financed schooling (measured with the wider series e.g. Lindert (2004 Table 5.1)). This raises child "price" and lowers both target family size and crude birth rate. Greater schooling implies falling child labor opportunities (which are more difficult to measure), another contributor to the reversal of intergenerational transfers. In contrast to target family size, crude birth rate is not affected by mortality, but similarly, urbanization and male wage premium play a smaller role in the transition.

Section 1 sets out the historical facts of the demographic transition in England, section 2 discusses the model structure, section 3 presents the properties of a restricted version of the model and section 4 gives the results.

1 The English Demographic Transition

Fertility measured by Crude Birth Rate (CBR) appears to have been on a falling trend in England throughout most of the 19th century – though the data are not entirely reliable before the mid-century (Woods, 1992 Table 4). Gross Reproduction Rates (GRR)⁶ peaked after the Napoleonic Wars and declined thereafter (Woods, 1992 Figure 2). More reliable data is available from 1841⁷. These show that CBR fell in England and Wales from the 35 births per 1000 population in 1871 to 24.3 in 1911 (and to a low of 14.4 in 1933) (Mitchell, 1962 pp29-30). Proximate causes of this decline were the rise in female first marriage age from 25.13 in 1871 to 26.25 in 1911 and rising childlessness (or celibacy): the proportion of married women aged 15-45 fell from about 50 percent

⁵ Identification is problematic because, as in our model, at the aggregate level generalized prices are endogenous.

⁶ The gross reproduction rate is the average number of daughters a woman would have if she survived all of her childbearing years, which is roughly to the age of 45, subject to the age-specific fertility rate and sex ratio at birth throughout that period.

⁷ Registration of births was not virtually complete until well into the 1860s. Glass (1951 Table 13) calculated that births in the 1840s were underestimated by about 8 percent, 4 percent in the 1850s and 2 percent in the 1860s.

to 48 percent (calculated from Mitchell (1962))⁸. As a proportion of the population, women in this age group increased from around 23 percent to almost 25 percent). Together with the fall in CBR this implies a 33% fall in births per married woman aged 15-44.

Assuming people have a completed family size in mind, the falling mortality rate would give some idea as to whether this target changed very much⁹. Death rates in the 0-4 age range fell by 35% between 1871-1911 and by more for older children (though their rates were much lower than those of the younger). But identities are not behavioral relations. So, it is consistent with a death rate identity to show that death rate fell because birth rate declined but this would not be consistent with the model we develop. The ultimate causes of the CBR transition are the changes in generalized price of children which are driven by processes reflecting the 'natural' path of technical progress.

Such processes could include relative (to male) female wages. In manufacturing industry this ratio did not increase from an 1886 benchmark, but there is some evidence that female domestic service wage rates rose relative to manufacturing (Layton, 1908), as did those of female post office clerical workers (Routh, 1954). The opportunity cost of children, implicit in the generalized price, need not simply be paid work for females outside the home; it includes alternatives in the period before marriage and leisure in the marital home. Hence, although only 10.5% of married women were in paid employment in 1911, this does not rule out rising opportunity cost in a Becker model as an explanation for the English transition (cf. Baines, 1994).

Increases in the direct cost of childbearing include the costs of schooling as well as accommodation, care, food and clothing. When child labor was widespread the intergenerational transfer may have gone from children to parents. From 1833 legislation was passed (but not always enforced) about the age at which children could work (at 10 they could begin, with half time schooling from 10 to 14). As legislation and practice reduced child labor, the transfer increasingly went the other way. These are likely to be largely exogenous to the growth process and therefore to vary between economies, leading to different fertility experiences, supposing that they are a dominant influence. Crafts (1984) finds that rising relative child costs were an important contributor to declining English fertility. But he does not directly consider schooling costs, instead employing price indices to measure aspects of child costs.

⁸ The illegitimacy rate was low and falling.

⁹ The CBR identity is, where *B* is births, *P* is population, *M* is married women aged 15-44, *W* is women aged 15 to 44: $\frac{B}{P} = {\binom{B}{M}} \times {\binom{M}{W}} \times {\binom{W}{P}}$. The CDR identity is: $\frac{D}{P} = {\binom{B}{P}} - {\binom{N}{P}}$ where *N* is number of children and *D* is deaths.

A common way of measuring English schooling costs (e.g. Tzannitos and Symons, 1989; Galor, 2005) is to use only attendance at inspected schools i.e. those in receipt of some government funding. This very much under-estimates schooling for most of the 19th century; Lindert's (2004) estimates of schooling by decade¹⁰ shows in 1850 almost eight times the enrolments in total, as attendance in inspected schools. Schooling was compulsory from 1880 for 5-10 year olds and the leaving age was raised to 11 in 1893 (Curtis 1961). Most public elementary schools were free from 1891, but this was after the fertility decline began. In 1899 the school leaving age was raised to 12.

Information, ideology and ideological change could play a role in fertility decline, creating a willingness to adopt more effective contraception (Crafts, 1984; Bhattacharya and Chakraborty, 2017). Ostry and Frank (2010) and Guinnane (2011) dismiss innovations in contraception as drivers of fertility decline because they were insufficiently widespread or cheap enough to have a substantial effect.

However, as CBR decline began, the 1877 Bradlaugh-Besant obscenity trial publicized the idea of birth control. As opposed to a previous average circulation of about 700 copies a year of the text at issue, Knowlton's *Fruits of Philosophy*¹¹ (1832), between March and June 1877 125,000 copies were sold (Banks and Banks, 1954). The impact should not be measured by increased sales only, for newspaper reports of the trial reached people who would never have bought a "dubious" pamphlet. On the other hand, it may be that motivation, not means, mattered for the change in fertility behavior (Perkin, 1989 p235).

The core problem of the paper is to show quantitatively the impact of these possible contributors to the fall in CBR and in target family size and explain how they fit in to UGT.

2 The Model

A theoretically meaningful and empirically measurable model of the interaction between population and the economy must allow for fertility choice and differential mortality chances of life stages. The traditional two period life cycle¹² implies at least a 30year "generation" duration, which would require transforming the annual data to 30-

¹⁰ <u>http://economics.ucdavis.edu/people/fzlinder/peter-linderts-webpage/data-and-estimates/lindert-data-for-cup-book/App._T._A1__primary_enrol.xls/view</u>

¹¹ <u>http://onlinebooks.library.upenn.edu/webbin/gutbook/lookup?num=38185</u>

¹² Following Galor and Weil (2000), Lagerlof's (2006) calibration is illustrative. This exercise simplifies life to two generations. Consequently, there is no infant mortality rate, only one mortality rate in the adult period. Each period is 20 years, so the full adult life is only 40 years. Population begins falling in generation 40 (equivalent to 1870) and stops growing in generation 45 (equivalent to 1970 because a period = 20 years) which is at odds with the data.

year averages, resulting in a considerable loss of information. On the other hand, a more refined generation structure such as a period length of 5 years or even one year would result in colossal computation burden. Here, we adopt a 15-year period to be consistent with the conventional definition of childhood; the representative agent of each generation can live up to 105 years old (7 periods), although facing the risk of premature death. A full life includes childhood, adulthood and elderhood, with adulthood being further divided into three periods in line with the different choices and constraints facing the adult.

- **Phase 1** Period 0 (0~15), childhood: no decision is made, but human capital is formed then by parental choices;
- **Phase 2** (16~60), adulthood:
 - Period 1 (16~30), early adulthood: working, mating and family planning;
 - Period 2 (31~45), middle adulthood or parenthood: working and childcare;
 - Period 3 (46~60), late adulthood: working;
- **Phase 3** Period 4-6 (61~105), elderhood: no decision is made, but care of elders is taken by the work force.

The model consists of parameters (both time-varying and fixed), endogenous variables and exogenous variables (random shocks and those in auxiliary regressions), which are linked by three key mechanisms: (1) Natural Selection extended from Galor and Moav (2002), (2) Individual Decision-Making in the neoclassical paradigm and (3) Aggregate Interactions such as Malthusian checks and marriage search-matching.

2.1 Natural Selection

(Sexless) agents face a risk of dying at the *beginning* of each period with generationspecific mortality rates *m*0, *m*1, *m*2, *m*3 and *m*4. All mortality rates surged during the late Middle Age due to a series of famines and plagues. This high mortality in the 14th century opened a new era in English history. The resulting scarcity of labor led to the breakdown of feudal system, which cleared institutional obstacles for economic growth. The frailest childhood generation with the lowest quality were hit the most, leading to evolution of preferences over quality and quantity by extinction and heredity. For the 14th century De Witte and Wood (2008) find that the Black Death was selective with respect to weakness. Almost 400 years later, in the crisis of 1727-1730, Healey (2008) shows similar selectivity; there was a close connection between poverty and mortality.

We take from Galor and Moav (2002) the fundamental insight that the distribution of preferences evolves over time through natural selection; that is by inheritance through surviving major mortality events. We assume the only heterogeneity in preferences

within a generation is the elasticity of substitution (*s*), which governs the substitutability among utility inputs. The initial probability density function of *s* is defined over the interval 0 and 1. *s* follows a uniform distribution $f_t(s)$ bounded between $[\underline{s}_t, \overline{s}_t]$, which evolves over time *t*.

To operationalize the assumption of natural selection, we assume that ordinary mortality shocks do not change the lower bound \underline{s}_t . However, we allow that major mortality shocks (such as the Black Death) truncate the lower end of the distribution proportionately. Adaptability is the key to evolutionary survival. In periods of higher mortality, the "price" of a surviving child is higher. Those that can more easily substitute child "quality" for child numbers – have a higher elasticity of substitution between numbers and quality – will be more likely to survive because they are more adaptable. They can more readily choose the lower price options. In contrast, those with inflexible preferences are less likely to survive harsh times because of their reluctance to trade quantity for quality.

We distinguish between these two types of mortality events by zero population growth, i.e. when the percentage change of population (g_{Pt}) is negative it is counted as a major mortality event. To reflect this argument, we assume that any population shrinkage is accounted for by those with the lowest elasticity of substitution (adaptability) when major mortality events occur. Therefore, the mean elasticity of substitution evolves towards 1 in an irreversible fashion, as the lower bound <u>s</u> is cut off proportionately in the following manner:

(N1)
$$s_t \equiv \mathbf{E}[s] = \int_{\underline{S}t}^1 f_t(s) s ds = \frac{\overline{s} + \underline{S}_t}{2}$$

(N2) $\frac{\underline{S}_t - \underline{S}_{t-1}}{\overline{s} - \underline{S}_{t-1}} = -g_{P,t-1}$ if $g_{P,t-1} < 0$; $= 0$ if $g_{P,t-1} \ge 0$

As shown in **Figure 1**, the mean elasticity of substitution starts at $s_0 = 0.5$ (the mean of the original distribution defined over 0 and 1), jumps above 0.8 during the Black Death, and finally stays stable around 0.9 before the Industrial Revolution. The implied density function of *s* does not change much after 1800.



Figure 1 Evolution of Elasticity of Substitution and Major Mortality Events

Notes: The upper panel is the evolution of the mean elasticity of substitution. The lower panel is the growth of population, with indicators for some major mortality events.

2.2 Individual Decisions

This component incorporates rational expectations and optimization into individual decision-making in demography and economy. Under the given (generalized) prices, the representative household of each generation and producer maximize their objective functions (with n the number of surviving children, q their quality relative to the parent generation and z other consumption) subject to constraints.

The representative (sexless) agent born in period¹³ t - 1 (period 0) makes decisions in period t (period 1), under given prices π_n, π_q, π_z , with CES utility (in view of the evolving substitution elasticity):

$$\max_{n_t, q_t, z_t} \mathbf{E}[U(n_t, q_t, z_t)] \equiv \left[\alpha^{\frac{1}{s_t}} \cdot n_t^{\frac{s_t - 1}{s_t}} + \beta^{\frac{1}{s_t}} \cdot q_t^{\frac{s_t - 1}{s_t}} + \gamma^{\frac{1}{s_t}} \cdot \left(\frac{z_t}{z_{t-1}}\right)^{\frac{s_t - 1}{s_t}}\right]^{\frac{s_t}{s_t - 1}}, \text{ subject to:}$$

¹³ A period is named by the end of that period, e.g. period t is the interval [t - 1, t]. The time subscript of a variable indicates when it is determined, not when it takes effect, e.g. z_t is the consumption determined in period t, but it affects periods t, t + 1, t + 2.

(H1)
$$\ln A_t = a_0 + a_A \ln A_{t-1} + a_b b_t + \epsilon_t^A$$
, where $b_t \equiv \frac{n_t}{(1 - m0_t)(1 - m1_{t+1})}$

(H1) is a time and social convention constraint (Hajnal, 1961; Voigtlander and Voth, 2013b). The age of first-time mother (A_t) follows an autoregression and is negatively affected by the total births per married woman b_t (rather than target live births n_t). When b_t rises (either due to a higher demand for number of children or due to a higher child mortality rate), A_t is about to drop because the highest average mother's age at the final birth is assumed to be fixed (at 45 years old). The target number of surviving children is defined as children surviving up to 30 years old for the reason of eldercare. That is why both m0 and m1 are considered.

(H2)
$$z_t \equiv m2_t \times z1_t + (1 - m2_t)m3_{t+1} \times z2_t + (1 - m2_t)(1 - m3_{t+1}) \times z3_t$$

(H2a) $\pi_{zt}z1_t + ADR_t \times z1_t = w_t$, where ADR_t is the 60+ dependency ratio

(H2b)
$$\sum_{i=0}^{1} \left(\pi_{z,t+i} z 2_t + ADR_{t+i} \pi_{z,t+i} z 2_t \right) + \frac{1}{2} \pi_{n,t+1} b_t + \frac{1}{2} \pi_{q,t+1} q_t b_t = \sum_{i=0}^{1} w_{t+i}$$

(H2c) $\sum_{i=0}^{2} \left(\pi_{z,t+i} z 3_t + ADR_{t+i} \pi_{z,t+i} z 3_t \right) + \frac{1}{2} \pi_{n,t+1} b_t + \frac{1}{2} \pi_{q,t+1} q_t b_t = \sum_{i=0}^{2} w_{t+i}$

The second constraint (H2) defines the expected consumption flow z_t as a probabilityweighted average of the consumption flows under three cases. These cases are the three different optimal consumption flows $(z1_t, z2_t, z3_t)$ depending on whether the agent expects their life to end prematurely in life period 1, 2 or 3. The possibilities imply three possible budget constraints (H2a)-(H2c). The consumption flows in the three states differ in the number of periods of expenditure and income as well as in whether child quantity and quality should be considered—if the agent dies before period 2, then they would not have the opportunity to worry about children. In addition to childcare, the working generations also have eldercare responsibilities. The burden of caring for all the surviving retired generations (those who are in their periods 4-6) is shared among all the working generations (those who are in their periods 1, 2 and 3), and this burden is measured by the 60+ dependency ratio (*ADR*). The retired generations are assumed to consume the same amount at the same price as the working generations themselves, so *ADR* acts like a consumption tax, such as might be imposed to finance the operation of the 1601 Poor Law.

The production side of the economy assumes competitive output and input markets. Y_t is the aggregate output, L_t is the working generations (labor force) during period t, H_t is the human capital in period t. Human capital here is broadly defined, including any productive resources that is produced by human, such as physical capital, knowledge capital, health capital, institutional and political capital. \overline{F} is the natural capital such as

land and natural resources¹⁴, which can be normalized to 1 so that Y_t is interpreted as aggregate output per unit natural capital. A difference between human and natural capital is whether the resources can be *reproduced* by humans. The representative production unit's (farm's or firm's) problem is:

$$\max \Pi_t = Y_t - w_t L_t$$
, subject to:

(F1) Production Function: $Y_t = \exp(\epsilon_t^Y) L_t^{\theta_1} H_t^{\theta_2} \overline{F}^{1-\theta_1-\theta_2}$, where $\epsilon_t^Y \sim N(0, \sigma_Y^2)$

If we divide (F1) by total population stock P_{t-1} on both hand sides, we have per capita production function, where Y_t/P_{t-1} , L_t/P_{t-1} and H_t/P_{t-1} are output per capita, labor force ratio and human capital per capita. Defining the growth rate of human capital per capita as $\hat{H}_t \equiv H_t/H_{t-1}$, analogously to the Solow neoclassical growth model, output per capita will grow at the rate of \hat{H}^{θ_2} along the balanced growth path.

The two optimization problems imply marginal conditions: for the household, the *expected* marginal rate of substitution among n, q and z is equal to the price ratios; for the producer, the marginal product of labor is equal to the real wage (w) (Appendix I).

2.3 Aggregate Interactions

The aggregate-level variables are defined from accounting identities (\equiv) or from the individual-level variables associated with each other behaviorally (=).

The law of motion for the total population (P_t : total population stock at time t) is:

$$(A1) P_t \equiv P_{t-1} - D_t + B_t$$

Total deaths (D_t : death flow in period t) are the sum of premature and natural deaths, so $CDR_t \equiv \frac{D_t}{P_{t-1}}$ is the crude death rate:

 $(A2) D_t \equiv m0_t \times B_t + m1_t \times (1 - m0_{t-1})B_{t-1} + m2_t \times (1 - m1_{t-1})(1 - m0_{t-2})B_{t-2} + m3_t \times (1 - m2_{t-1})(1 - m1_{t-2})(1 - m0_{t-3})B_{t-3} + m4_t \times (1 - m3_{t-1})(1 - m2_{t-2})(1 - m1_{t-3})(1 - m0_{t-4})B_{t-4} + (1 - m4_{t-2})(1 - m3_{t-3})(1 - m2_{t-4})(1 - m1_{t-5})(1 - m0_{t-6})B_{t-6}$

¹⁴ As do Galor and Moav (2002), we assume that there are no property rights over \overline{F} , so the return to \overline{F} is 0. That is to say, the amount of \overline{F} available for everyone and for the whole country is the same, or in other words, \overline{F} is non-rival and non-excludable (public goods). This is equivalent to excluding the land-lords from our model. Marx and Engels abhorred: "in extant society, private property has been abolished for nine-tenths of the population; it exists only because these nine-tenths have none of it." (Lindert, 1986 p1128).

Total births (B_t : birth flow in period t) depend on the population of fertile females (15-45) and the total number of children (b_t) determined in the household's problem, so $CBR_t \equiv \frac{B_t}{P_{t-1}}$ is the crude birth rate. To accommodate the fact that childbearing age is concentrated in the second half of period 2 and the first half of period 3, we divide the fertile population, ($G1_t + G2_t$), by 2.

(A3)
$$B_t \equiv (1 - \mu_t) \times \frac{(G_{1t} + G_{2t})}{2} \times b_t$$
, where μ_t is the childlessness/celibacy rate.

In the equations above, Gi_t denotes the generational population stock in their period *i* surviving at the end of period *t*:

(A4)
$$G1_t \equiv (1 - m1_t)(1 - m0_{t-1}) \times B_{t-1}$$

(A5) $G2_t \equiv (1 - m2_t) \times G1_{t-1}$
(A6) $G3_t \equiv (1 - m3_t) \times G2_{t-1}$
(A7) $G4_t \equiv (1 - m4_t) \times G3_{t-1}$
(A8) $ADR_t \equiv \frac{G4_t + G4_{t-1} + G4_{t-2}}{l_t}$ is the dependency rate of the 60+ age group.

Turning to the production side, where Q_t is generational human capital measuring the average human capital of the generation born in period t, the labor force and the average human capital of the labor force in period t are:

$$(A9) L_t \equiv G1_t + G2_t + G3_t$$

(A10)
$$H_{t-1} \equiv \frac{G1_t}{L_t} Q_{t-1} + \frac{G2_t}{L_t} Q_{t-2} + \frac{G3_t}{L_t} Q_{t-3}$$

In addition to the accounting identities (A1)-(A10), we describe the aggregate determination of births, deaths, marriages and human capital under the headings preventive check, positive check, search-matching theory and human capital accumulation.

[**Preventive Check: Birth**] The Malthusian preventive check can be interpreted as effects through the price determination mechanisms. When mortality rates rise in the 14th century, the effective price of a surviving child increases, leading to a relative rise in child quality, though the absolute levels of both quantity and quality drop, due to complementarity in preferences¹⁵. With the end of a high mortality shock in the mid-15th

¹⁵ This implies that the elasticity of substitution (*s*) is always smaller than 1. In a static version of the model, we proved that there is no converging solution when s > 1. That is, the complementarity always dominates substitutability in the preferences over current and future generation and over 'bearing' and 'caring'. The reason is that when s > 1 the substitution effect is so strong that child quantity will easily fall below 1, leading to an unsustainable population shrinkage.

century, marriage age (or more precisely, the first-time mother's age A_t) rises, to limit births, as implied by equation (H1).

We assume "generalized" prices¹⁶ (A11 and A12) that include time costs (t_n, t_q) as well as monetary costs (p_n, p_q) incurred by these activities: consumption is the numeraire), for child quantity $(\pi_n \equiv p_n + w \times t_n)$ and for child quality $(\pi_q \equiv p_q + w \times t_q)$. So higher wages mean higher child price and quality, because of greater opportunity costs, other things equal.

(A11)
$$\pi_{nt} = \exp(\epsilon_t^{\pi n}) \overline{\Phi}_{nt} w_t$$
, where $\epsilon_t^{\pi n} \sim N(0, \sigma_n^2)$.

(A12)
$$\pi_{qt} = \exp(\epsilon_t^{qn}) \overline{\Phi}_{qt} w_t$$
, where $\epsilon_t^{\pi q} \sim N(0, \sigma_q^2)$.

The coefficients $\overline{\Phi}_{nt}$ and $\overline{\Phi}_{qt}$ are time-varying. With the help of exogenous historical data, we will specify auxiliary regressions (R1) and (R2) in 4.3 below to explain the fluctuations in $\overline{\Phi}_{nt}$ and $\overline{\Phi}_{qt}$.

[Positive Check: Death] Mortality rates are specific to each generation and each period. The improvement of life expectancy in the last two centuries is mainly attributed proximately to a secular decline in m0 (0~15) and m4 (60+). The substantial changes in $m1 \sim m3$ had little overall effect except in war time, because the mortality levels were so much lower. Greater life expectancy can raise the returns to investment in human capital because there is a longer period over which the benefits accrue. Eventually, accumulation can trigger an acceleration of technical progress (Boucekkine et al., 2003; Lagerlof, 2003; Cervellati and Sunde, 2005). Bar and Leukhina (2010) identify a link between gains in adult mortality and productivity growth through the longer survival of implicit knowledge. De La Croix and Licandro (2012) hypothesize that increasing longevity drove falling fertility because of a parental trade-off between their own human capital investment and time spent rearing children. In the present model the mortality impact on child numbers is independent of the impact on child quality when s = 1, discussed in Section 3.

[Search-Matching: Marriage] The proportion μ_t (including both never-married and the infertile) follows an autoregression with search and matching costs (Keeley, 1977; Choo and Sow, 2006) depending on marriage age and wage growth:

(A13)
$$\mu_t = \tau_0 + \tau_\mu \times \mu_{t-1} + \tau_A \times \ln A_t + \tau_w \times g_{wt} + \epsilon_t^{\mu}$$
, where $\epsilon_t^{\mu} \sim N(0, \sigma_{\mu}^2)$

¹⁶ Note that π_{zt} can be normalised to 1 only if z does not cost any time for consumption, i.e. $t_z = 0$. Mortality is not included in these generalised prices.

The later people marry, the higher the proportion of unmatched individuals because more people are searching for partners. Moreover, a marriage is more likely to be childless if delayed to a later age. The effect of the wage (τ_w) is ambiguous because the model does not explicitly distinguish male and female. According to the neo-local hypothesis, a higher wage means a greater chance of getting married and a lower μ_t . However, if the rise in wage is mainly due to the rise in female wage, it implies a higher opportunity cost of early marriage and a higher μ_t . We leave the sign to be pinned down by the data empirically.

[Human Capital Accumulation] Generational human capital Q_t is determined in period t and takes effect in period t + 1. The parents' influence is $(Q_{t-2}q_{t-1})$: the target quality of children formed by "family education" ¹⁷. There is also a "nonfamily education" effect from the average human capital of the existing labor force H_t . Formal schooling and apprentice training are still "family education" if fully financed, and the returns are fully captured, by the family. "Nonfamily" education is an externality or spill-over effect such as caused by tax-financed education and urbanization. The contribution weight of nonfamily education (an externality) is ε , and there is a human capital productivity shock ε_t^Q to capture the efficiency of knowledge transmission.

(A14)
$$Q_t = \exp(\epsilon_t^Q) H_t^{\varepsilon} (Q_{t-2}q_{t-1})^{1-\varepsilon}$$
, where $\epsilon_t^Q \sim N(0, \sigma_Q^2)$

2.4 Shock Structure

If we wish to use all the observables to estimate the model (there are six in total P, W, B, D, A and μ) in principle we need six shocks. However, P and W are the most reliable data and they span the whole sample period. To minimize the distortion due to data uncertainty, we only use P and W as observables, so only two shocks are needed. The two most fundamental — price shocks to π_n and π_q equations (A11, A12) — are utilized. The other four observables (B, D, A and μ) are used to evaluate the model predictions.

Lee (1993) maintains that exogenous shocks were principally responsible for the approximately 250-year European demographic cycle. The 1348 Black Death shock clearly originated elsewhere than England and wreaked simultaneous havoc elsewhere as well. Exogenous Western European quarantine regulations from the early 18th century subsequently reduced the impact of plague in England (Chesnais, 1992 p141). A substantial part of the 19th century decline in mortality was due to advances in public

¹⁷ q is defined as the ratio of children's to parents' human capital. It is therefore multiplied by the parents' generational human capital to convert the bracketed expression to an absolute value of familyoriginating human capital.

health, but these benefits took decades to be fully experienced (Szreter, 1988; Colgrove, 2002;).

The effects of epidemic diseases such as bubonic plague, typhus and smallpox are included in the mortality variable. Weather-induced shocks to agricultural productivity cause changes in prices and quantities and affect wages in Voigtlander and Voth's (2006) model. Runs of poor harvests (such as the Great European Famine of 1315-17) and livestock disease constitute a negative productivity shock. In the current run of the model, these mortality and productivity shocks are incorporated in the two generalized price shocks ($\epsilon^{\pi n}$ and $\epsilon^{\pi q}$) in (A11) and (A12).

2.5 Stationarization and Steady States

The above system is non-stationary because of growth in human capital and population. But standard numerical methods for solving this dynamic equation system require stationarity. n_t , q_t , A_t , μ_t are stationary by definition, so for them no change is necessary. The non-stationary endogenous variables can be categorized into three groups in terms of their balanced growth path rates, or of their deflators. Where a hat "^" indicates a stationarized variable:

Deflated by
$$P_{t-1}$$
: $\hat{B}_t \equiv \frac{B_t}{P_{t-1}} \equiv CBR_t$, $\hat{D}_t \equiv CDR_t$, $\hat{G}i_t$, \hat{L}_t , $\hat{P}_t \equiv \frac{P_t}{P_{t-1}} = 1 + g_{Pt}$
Deflated by H_t : $\hat{Q}_t \equiv \frac{Q_t}{H_t}$, $\hat{H}_t \equiv \frac{H_t}{H_{t-1}} = 1 + g_{Ht}$
Deflated by $H_t^{\theta_2}$: $\hat{Y}_t \equiv \frac{Y_t}{H_t^{\theta_2}}$, \hat{w}_t , $\hat{\pi}_{nt}$, $\hat{\pi}_{qt}$, \hat{z}_t , $\hat{z}1_t$, $\hat{z}2_t$, $\hat{z}3_t$

The model is solved by a perturbation method in the DSGE literature, involving loglinearization of the original nonlinear equations around the steady state. We first obtain the steady state for each period separately and then add on the complementary functions to capture the deviation from the steady state.

We only focus on steady states in the neighborhood of the observations, so the uniqueness of the steady state in each period is guaranteed. This also marks a difference between our model and that of Galor and Weil (2000). The latter has two equilibria (two solutions) from a single parameterization, with one being a Malthusian regime and the other a modern growth regime. In contrast, our model explains history assuming a unique steady state in each (15-year) period, and a series of evolving processes lead to multiple steady states over time.

To obtain these time-varying steady states, we make use of the moving averages of two key observables after stationarization, population growth (\hat{P}) and wage growth (\hat{W}), to

recursively calculate the steady states of other endogenous variables. We have 25 equations for the 25 endogenous variables discussed. If two of them (\hat{P}, \hat{W}) are already known, it leaves two extra degrees of freedom. We have two unknown time-varying parameters, i.e. $\overline{\Phi}_{nt}, \overline{\Phi}_{qt}$, enabling the identification condition to be met—25 equations for 25 unknowns.

3 Model Properties

Unlike many unified growth calibrated models this has a CES utility function – given the evolution of $s \le 1$; the approach precludes closed form solutions. Nonetheless, it is helpful for understanding the properties of the model at first to restrict the elasticity of substitution to one in the utility function. During the demographic transition, we have shown in **Figure 1** that *s* has evolved close to 1. This allows the derivation of several quasi-reduced form relation by combining a subsets of the equilibrium conditions (detailed derivation is in Appendix I).

Child quality is the key to economic growth ([1]).

$$q^{D} = \frac{\beta}{\alpha - \beta} \frac{\hat{\pi}_{n}}{\hat{\pi}_{q}}, \text{ if } \alpha > \beta.$$
^[1]

The cross-elasticity with $\hat{\pi}_n$ turns out to be quite important. Mortality has no effect on child quality but equation [1] is only the demand side. From the supply side ([2]), however, mortality does have an effect via the channel of family education (in the square brackets).

$$q^{S} = \left[\frac{1}{1 + \frac{(1-m2)}{\bar{p}} + \frac{(1-m2)(1-m3)}{\bar{p}^{2}}} + \frac{\frac{(1-m2)}{\bar{p}}}{1 + \frac{(1-m2)(1-m3)}{\bar{p}^{2}} + \frac{1}{\bar{p}}} + \frac{\frac{(1-m2)(1-m3)}{\bar{p}^{2}}}{1 + \frac{(1-m2)(1-m3)}{\bar{p}} + \frac{1}{\bar{p}^{2}}} \frac{1}{\bar{p}^{2}}\right]^{\frac{\varepsilon}{\varepsilon - 1}} \hat{H}^{2-\varepsilon}$$
[2]

This process determining the supply of q is unrelated to $\hat{\pi}_q$ but positively related to the per-capita human capital growth rate (\hat{H}) , which also defines the technological progress growth rate (\hat{H}^{θ_2}) —see the production function equation (F1).

In the special case where there is no external effect of nonfamily education ($\varepsilon = 0$). q^S is then a simple quadratic function of the overall human capital growth rate: $q = \hat{H}^2$. In other words, the overall human capital growth only comes from family education. It is quadratic because there are two generations between the parents and their children. As the externality from nonfamily education increases, perhaps due to an expanding role of the state, child quality increases (for given past human capital), because by assumption $\varepsilon < 1$ to ensure constant returns to scale. In equation [2], as exogenous adult mortality m2 and m3 fall, or longevity (1 - m2, 1 - m3) increases, investment in children's "quality" expands, because the transmitters of the nonfamily externality have greater experience. However, a fall in child mortality (m0 and m1) has no impact on this investment¹⁸.

Define an effective relative price of n (with respect to the price of z, the numeraire) as:

$$\widehat{\Pi}_n \equiv \widehat{\pi}_n \frac{\left(\frac{(1-m_2)m_3}{2(1+ADR)} + \frac{(1-m_2)(1-m_3)}{3(1+ADR)}\right)}{(1-m_0)(1-m_1)}$$
[3]

In equation [3], there are different demographic effects of m2 and m3 in the numerator, stemming from people being liable either to die in childcare period 2 or in post-childcare period 3. Childcare cost is incurred only in period 2, while eldercare is shared by entire work force (through periods 1 to 3 if workers survive). An agent who dies in period 2 must spend on both childcare and eldercare *for two periods*, while one who dies in period 3 must have spent on both childcare for one period and eldercare *for three periods*. Therefore, a drop in m2 raises the possibility of incurring childcare cost which raises the price of n relative to consumption ($\widehat{\Pi}_n$). In contrast, if m3 drops, there is no effect on childcare cost, but expenditure on z increases with the chance of another period's extra eldercare. In general equilibrium, ADR will rise with falling m2/m3, which will dampen the positive effect of m2 and reinforce the negative effect of m3. Falls in m0 and m1 reduce the effective price of children because of higher survival rates.

Define the expected disposable income as:

$$\widehat{\omega} \equiv \frac{m^2}{1+ADR} \times \widehat{w} + \frac{(1-m^2)m^3}{2(1+ADR)} \times \left(\widehat{w} + \widehat{w}\widehat{H}^{\theta_2}\right) + \frac{(1-m^2)(1-m^3)}{3(1+ADR)} \times \left(\widehat{w} + \widehat{w}\widehat{H}^{\theta_2} + \widehat{w}\widehat{H}^{2\theta_2}\right),$$

Therefore, the solution for the demand for n can be written as:

$$n^{D} = \frac{\alpha - \beta}{\alpha + \gamma} \frac{\widehat{\omega}}{\widehat{\Pi}_{n} \widehat{H}^{\theta_{2}}}$$

$$\tag{4}$$

If there is no economic growth, $\hat{H} = 1$. If there *is* economic growth, then *n* falls, unless there are other changes such as declines in child mortality or ADR lowering child price

¹⁸ The counterparts here of the equation (4), child quality, in Cervelatti and Sunde (2015) are our \hat{H}^{θ_2} in the production function, corresponding to their 1 + g (growth of total factor productivity) and our $\hat{\pi}_q$, corresponding to their \tilde{r} (voluntary extra time cost of child). We do not impose their fixed cost of children \underline{r} , while they ignore the cross effect from $\hat{\pi}_n$ i.e. that a higher child price increases child quality in our model.

(eventual demographic transition). But higher wages, normally a consequence of economic growth, mean *higher* n through $\hat{\omega}$.

When m0 and m1 rise, there is an equal negative effect on n, but no effect on b through equation [3] in equation [4]. Higher child mortality reduces target family size n but increases the birth rate necessary to achieve that target, so b does not change. There are two *direct* effects of m2 and m3 on n: (1) the income effect through the numerator $(\hat{\omega})$, and (2) the substitution effect through the denominator $(\hat{\Pi}_n)$ just discussed¹⁹.

The sign of mortality rate m2 in the n demand equation ([4]) is ambiguous. When m2 rises, the life-time expected wage drops, leading to a negative income effect. At the same time, a rise in m2 also means a relatively cheaper price of n, leading to a positive substitution effect. By contrast, when m3 rises, the effect on $\widehat{\Pi}_n$ is positive, so the substitution effect reinforces the income effect.

On the supply side of n, m^2 and childlessness rate are influential. A drop in m^2 means a greater chance of bearing childcare and eldercare costs, reducing the supply of n. Using equations (A3)-(A5) another quasi-reduced form for n can be derived.

$$n^{S} = \frac{2}{(1-\mu)\left(\frac{1}{\hat{p}} + \frac{1-m^{2}}{\hat{p}^{2}}\right)}$$
[5]

This supply function of n^{S} ([5]) is fixed independently of $\hat{\pi}_{n}$. Combining these supply and demand functions [1], [2], [4] and [5] will determine the generalized prices and quantities in equilibrium as analyzed in the next section.

Equilibrium n and q will determine respectively the future labour force (L) and human capital (H), the two factor inputs of the production function (F1). Economic growth therefore alters when these two underlying variables change along the evolving steady state path.

4 Results

The model is initially calibrated based on 2SLS estimates of a subset of model equations wherever data are available. Because of the evolutionary path of s_t , the steady state of the model in each period is solved with these calibrated parameters. Next, a global optimization algorithm is applied to search the parameter space for the best set of values to minimize the squared gap between the model predictions and data observations. The

¹⁹ There are also *indirect* effects of m2 and m3 through wage and human capital, which can only be obtained after solving for the general equilibrium.

estimated model is then simulated under different settings to identify the contributions of model mechanisms to the demographic transition and long-run economic growth in England.

4.1 Empirical Performance

In **Table 1** the calibration column includes the parameter values either from 2SLS estimates (the first seven) or from guestimates (the last five), while the estimation column includes the final estimates starting from these initial values. The first three parameters are for the first-time mother's age equation (H1). The negative coefficient indicates by how much a fall in target births raises A. The next four coefficients are for the childlessness μ equation (A13). The second parameter τ_{μ} indicates that the final estimate has childlessness negatively autocorrelated, and the third τ_A that a higher marriage age raises the childlessness rate equally. The fourth coefficient τ_w indicates that faster wage growth boosts childlessness. The human capital elasticity of output is high (θ_2) compared to unskilled labor (θ_1), leaving only about 0.15 for fixed inputs such as land. ε of 0.4 indicates that human capital spillovers accounted for three fifths of privately born investment in human capital (A14).

Symbol	Parameter	Calibration	Estimation
a_0	intercept of A equation	1.965	2.132
a_A	coefficient of lagged A	0.425	0.384
a_b	coefficient of b	-0.042	-0.055
$ au_0$	intercept of μ equation	-2.939	-2.728
$ au_{\mu}$	coefficient of lagged μ	0.290	-0.975
$ au_A$	coefficient of A	0.931	0.895
$ au_w$	coefficient of wage growth	0.128	0.463
α	utility weight of <i>n</i>	0.300	0.305
β	utility weight of q	0.300	0.227
θ_1	income share of L	0.400	0.146
θ_2	income share of <i>H</i>	0.400	0.703
Е	nonfamily education externality	0.400	0.393

Table 1 Calibrated and Estimated Structural Parameters

In **Figure 2** the evolving steady states of population and earnings growth capture the broad data movements over 800 years. The population decline during the 14th century is an exception because steady state population growth cannot be negative. The method of solving the model is unrelated to the structure of the model. To solve the model, using population and earnings as the inputs to the model, we recursively derive the other endogenous variables. The remaining four panels can be thought of as a form of "out of sample" predictions of these endogenous variables. The fall in the CBR in the 19th

century is captured quite well, as is the decline in CDR²⁰. Predicted and actual mother's age at first child and childless rate both rise in the period of fertility decline. As endogenous variables, their effects on CBR, outlined above, are taken into account when the responses to exogenous variables are considered.



Figure 2 Comparison of Key Variables between the Model and the Data

Notes: The data sources can be found in Appendix II.

The discrepancy between the model predictions and the collapse of first-time mother's age in the late 15th century may reflect problems with the baseline data (here a small sample of Inquisition Post-mortems, Russell (1948)) rather than shortcomings of the model. That is, the simulated series here may be a better guide to history than the available "data". Similarly, with the childless rate which apparently shoots up in the 17th century and collapses in the 18th century. A jump in clandestine marriage (and therefore overestimation of childlessness) may have been a contributor to this statistical oddity (Schofield, 1985).

²⁰ CDR (crude death rate) depends on the overlapping generational structure of the model as well as exogenous mortality rates. The tendency for simulated CDR to be too low might be attributable to the omission of emigration from the model.

4.2 Explaining the Path of Generalized Prices

The fundamental mechanism for economic and demographic growth is the change in the ratio between $\hat{\pi}_n$ and $\hat{\pi}_q$. The time path of $\hat{\pi}_n$ and $\hat{\pi}_q$ (Figure 3) can be derived using the structural model equations and the observed variables (population growth \hat{P} , wage growth \hat{W} and mortality rates m). To simplify the analysis, we utilize the demand and supply functions for n and q in the special case s = 1, but the qualitative conclusions hold in general case.

Figure 3 Implied Unobserved Endogenous Variables: Generalized Prices



The demand and supply functions of n and q look qualitatively similar but have different elasticities as shown in **Figure 4**. We denote the initial equilibrium as 0, and there are three possible shifts 1, 2 and 3 to explain the historical time paths of $\hat{\pi}_n$ and $\hat{\pi}_q$. The comparative static signs of the demand and supply schedules ([1], [2], [4] and [5]) derived in section 3 are re-summarized as follows:

$$q^{D}\left(\hat{\pi}_{q}(-),\hat{\pi}_{n}(+)\right)$$
[1]A

$$q^{S}\left(\hat{P}(-),m(-)\right)$$
[2]A

$$n^{D}\left(\hat{\pi}_{n}(-),\widehat{W}(+),m(-)\right)$$
[4]A

$$n^{S}\left(\hat{P}(+), m(+)\right)$$
[5]A

As the high mortality rates of the 14th century recedes, population begins to rise and wages to fall, the demand for and supply of n ([4]A and [5]A) shifts to the left in **Figure** 4, so $\hat{\pi}_n$ falls (point 1 in **Figure 4**). The fall in $\hat{\pi}_n$ leads to a reduction in q^D ([1]A). At the same time, the decline in population growth shifts n^S ([5]A) to the left, leading to a rise in $\hat{\pi}_q$ (point 1).

After 1550, population growth slows (partly due to mortality no longer declining) and shifts n^D ([4]A) to the right, resulting in a higher $\hat{\pi}_n$, which in turn moves q^D ([1]A) to the right. Population growth slow-down also shifts q^S ([2]A) to the right. The combination brings a lower $\hat{\pi}_q$ (point 2 in **Figure 4**).

The falling ratio of $\hat{\pi}_n/\hat{\pi}_q$ ($\hat{\pi}_q$ rises and $\hat{\pi}_n$ falls) in the 18th century accounts for the accelerated population growth during the early Industrial Revolution. Overall though $\hat{\pi}_q$ dropped remarkably from 1550 onwards, driving the rise in the $\hat{\pi}_n/\hat{\pi}_q$ ratio and the slow acceleration of economic growth.

After 1850 technological progress raised the generalized child price $\hat{\pi}_n$, reducing the (crude) birth rate and target family size. In this final phase, the post-1850 demographic transition, our model's explanation is that generalized child price rises strongly because mortality declines and wage/income growth increases. The generalized price of child quality does not rise as much because the supply of human capital expands. This shift in relative price (of quantity against quality) lowers target family size (a move from point 0 to point 3 in **Figure 4**).



Figure 4 Graphical Analysis of Equilibrium Changes: Generalized Prices

4.3 Explaining the Shocks to Generalized Prices

The structural model proposed is generic to all economic conditions, but countries may experience different factors driving the changes of generalized prices. To account for this specific heterogeneity, we use auxiliary regressions to capture the detail of the transition in the English case. From (A11) and (A12) of the structural model, the ratio Φ_{nt}/Φ_{qt} is equal to relative prices $\hat{\pi}_{nt}/\hat{\pi}_{qt}$. We propose two auxiliary regression models to explain these two time-varying parameters Φ_{nt} and Φ_{qt} .

In UGT, technological progress is exogenous in the sense that there is a hierarchy of knowledge and a fixed path (not pace) of technical advancement. Along this fixed path, there are some accompanying processes to embody the exogeneity of technological progress. To explain the changes in Φ_{nt} and Φ_{qt} , we identify the following candidate processes, which are exogenous to the structural model:

- School enrolment (SCH), driven by increasing technological sophistication.
- Inspected school enrolment (\widetilde{SCH}) , similar to SCH, but inspected school enrolment usually reflects effective and high-quality education.
- Male wage premium (*WP*), mainly caused by structural transformation and its impact on the role of women in the service sector.
- Female literacy (*FL*), perhaps mainly caused by also by structural transformation.
- Urbanization (*URB*), mainly caused by rising productivity and transportation and communication technologies improvements.

The two auxiliary regressions (R1 and R2) estimate the impact of this period- and country-specific technical progress on the two shocks to $\hat{\pi}_{nt}$ and $\hat{\pi}_{at}$:

(R1) $\ln \Phi_{nt} = \phi_{n0} + \phi_{n1}SCH + \phi_{n2}WP + \phi_{n3}URB + \epsilon_t^{\pi n}$

(R2)
$$\ln \Phi_{qt} = \phi_{q0} + \phi_{q1}\widetilde{SCH} + \phi_{q2}FL + \phi_{q3}URB + \epsilon_t^{\pi q}$$

Column (1) of **Table 2** indicates that the strongest effect on the relative generalized child price (Φ_n or the ratio $\frac{\hat{\pi}_n}{\hat{w}}$) is from school attendance (*SCH*), confirmed by the simulations below. A higher school enrolment implies a smaller child labor income, as well as greater direct costs, so it increases the effective price of child. The male wage premium (*WP*) implies that higher relative female wages raise the generalized child price because of the higher opportunity cost of childcare. There is a positive (but statistically

insignificant) effect of urbanization (URB^{21}), reflecting that higher mortality and rents, and greater opportunities of city life raise the cost and price of children²².

	(1)	(2)	(3)
Dependent Var.	$\ln \Phi_n$	$\ln \Phi_q$	$\ln \Phi_q$
SCH	0.847***	-	-
ĨСН	-	-0.767**	-0.052
WP	-0.150**	-	-
FL	-	-0.155	-0.744*
URB	0.113	1.125*	0.503
Constant	-2.251***	-1.478***	-1.180***
Sample Period	1086:2016	1086:2016	1400:2016
Sample Size	63	63	42
\mathbb{R}^2	0.797	0.076	0.204
ADF Test P-value	0.0299	0.4723	0.0198

 Table 2 Estimation Results of Auxiliary Regressions

Notes: The significance levels (* 10%, ** 5%, *** 1%) are based on one-sided tests because we have explicit hypotheses on the signs of the regressors. The null hypothesis of the ADF tests is that the residual of the regression follows an I(1) process with no drift and no trend.

If we use the full sample to estimate the $\ln \Phi_q$ equation (column (2) of **Table 2**), then female literacy (*FL*) has an insignificant effect. However, this is mainly due to the poor quality of the data on female literacy before 1400. If we restrict our sample to 1400+ (column (3)), then the effect of *FL* on $\ln \Phi_q$ is significant and negative. The ADF tests show that the auxiliary regressors in columns (1) and (3) are co-integrated with the dependent variables. The exception is column (2). As argued earlier, the subsample estimates of column (3) are more credible.

4.4 Simulations

First, we evaluate the importance of the relative prices of n and q to the demographic transition in the late 19th century. Setting Φ_n and Φ_q at 1850 levels is equivalent to fixing the price ratio between n and q, because wage (\hat{w}) in both cancels out according to (A11) and (A12). In this case, a demographic transition no longer takes place and CBR stays above 65% (**Figure 5**). Furthermore, **Figure 5** also shows that changes in Φ_n are the main contributor to the transition, while the effect of Φ_q is insignificant.

²¹ Wrigley and Schofield (1981) see the high mortality of towns curtailing population growth in the nineteenth century. Lucas (1988) and Duranton and Puga (2014) find cities to be a cause not a result of economic growth.

²² Other variables tested but found insignificant were birth control technology (based on illegitimacy and a user survey, female literacy and domestic appliances (based on electricity connections).





Notes: The model predictions are based on the steady states solved under the estimated parameters. The two time-varying parameters Φ_n and/or Φ_q are then fixed at the 1850 level to simulate the consequent CBR to see the effect of prices. The CBR here are defined in line with the data, i.e. 15-year birth flow divided by the beginning-of-period population, which is higher than 15 multiplied by the annual CBR due to an expanding population base.

To explore the detailed story behind the English demographic transition, we can fix the exogenous processes in the auxiliary regressions and simulate the structural model to see how much these processes contribute to the fertility decline. If mortality was set at the (high) 1850 level in the simulation of **Figure 6**, the target number of children (*n*) subsequently would have been lower, because of the greater number of births necessary and therefore the higher surviving child costs. This is what n^D predicts ([4]A). Whereas if schooling had been fixed at (the low) 1850 levels, Φ_n and therefore $\hat{\pi}_n$ would have been lower according to the auxiliary regression, so the target number of children would have been much higher (**Figure 6**). Mortality and schooling effects partly offset each other so the simulated target number of children tracks the original steady states, driven in the transition period by the rising price of children relative to their quality. In addition, changes in the male wage premium and urbanization also contribute to a higher opportunity cost of *n*.

Figure 6 Simulations of *n* based on Auxiliary Equations



Figure 7 shows that the simulated CBRs under various ways of fixing auxiliary processes does not decline substantially in the late 19th century. The conventional demographic transition story is that mortality falls and then births (CBR) fall with a lag. Had mortality remained at 1850 levels, along with the wage premium, urbanization and schooling, crude birth rate would have risen. But on its own lower mortality did not contribute to the decline of CBR because the higher target family size offsets the smaller number of births necessary to achieve a target. The single factor contributing most to CBR decline was schooling/child labor. Mortality decline would have raised target family size substantially had it not been for the rise in opportunity cost of schooling (driven by technology), though the wage premium and urbanization also made a substantial contribution to the fall in the family target.



Figure 7 Simulations of CBR based on Auxiliary Equations

5 Conclusion

The structure of our unified growth model for England follows Galor and Moav (2002) in its evolutionary natural selection but differs in its greater historical specificity. A distinctive response to catastrophic mortality sets off the process that eventually gives rise to the break out from the Malthusian epoch, but there was no necessity for the particular response. Mortality crises and high mortality levels eventually diminished so that a new stage of development began. Around 1780 the model shows the economy entering a third stage in which first population and then real wages grow secularly, the Industrial Revolution. Economic growth lags behind demographic growth. As the intensity and frequency of mortality crises diminish, more children survive and are planned immediately. The consequence of their higher quality, greater human capital, takes longer to work through the economy.

In the next stage, English fertility decline, the driving force was the rising ratio between the generalized child price and child quality price. Generalized child price climbs strongly because mortality falls and human capital growth increases. Rising human capital accumulation held the increase in child quality price below that of child numbers. One response to this price change was an increasing proportion of women remaining unmarried. We find that falling mortality had little effect on CBR and actually raised target family size (as the Becker model predicts). Fewer births were necessary for a given completed family size. The rising opportunity cost of children was triggered by increasing school attendance and the reduced opportunity for child labor. This in turn can be interpreted both as substitution of quality for quantity and as a reaction to technical change that placed an increasing premium on human capital – as in Galor (2012). Without this change, target family size would have *increased* substantially after 1850s-1860s.

The model structure identified is consistent with one of the two principal explanations of UGT – and is broadly consistent with much of the literature on demographic transitions, though is subject to the obvious caveats about the correct or appropriate measurement of the variables. It has been common to underestimate the strength of the rise in English schooling in the early nineteenth century because it was not provided or monitored by the state.

Despite the complexity of the 25-equation model, it is still a simplification, not taking into account changes in labor force participation or in migration, or other spillovers from the rest of the world – with the exception of the assumed exogeneity of mortality. Inability to measure child labor means that we have been unable to distinguish between this effect on the transition and that of schooling. We can only account for changing values and information such as might have been triggered by the publicity of the Bradlaugh-Besant Trial, by the shocks to the generalized child price, but these are only a small proportion of the total fertility decline.

Transitions have occurred in all high income countries, but at different times, different speeds and apparently at different stages of development. This model has implications for other countries, such as those placing a de facto tax on the number of children per family (as in East Asia), which boost investment in child quality and human capital. Optimal child number therefore falls, and more resources are spent on quality. Such unique national experiences in policy and cultural environment can be incorporated in auxiliary regressions to extend the generic model.

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Appendix I: Mathematical Derivation of Propositions

There are in total 25 endogenous variables \mathbf{y}_t , of which 6 are observable (but only two are finally used for estimation: population and wage). There are 6 exogenous shocks \mathbf{u}_t (but only two are finally kept: $\epsilon_t^{\pi n}$ and $\epsilon_t^{\pi q}$). The parameter vector $\boldsymbol{\theta}$ includes: (1) fixed parameters which are either calibrated or estimated $\overline{\boldsymbol{\theta}}$; (2) time-varying parameters which are exogenously evolving $\boldsymbol{\theta}_t$ such as s_t , WP_t , FL_t , SCH_t , URB_t . Given that some parameters are changing over time, the steady state of the stationarised model is also changing if one time-varying parameter changes.

List of Stationarised Equations

1.
$$\ln A_{t} = a_{0} + a_{A} \ln A_{t-1} + a_{b}b_{t} + \epsilon_{t}^{A}, \text{ where } \epsilon_{t}^{A} \sim N(0, \sigma_{A}^{2})$$
2.
$$b_{t} \equiv \frac{n_{t}}{n_{t}} \frac{1}{(1-m_{0})(1-m_{1})}$$
3.
$$\hat{z}_{t} \equiv m_{2} \times \hat{z}_{1t} + (1-m_{2})m_{3} \times \hat{z}_{2t} + (1-m_{2})(1-m_{3}) \times \hat{z}_{3t}$$
4.
$$\hat{z}_{1t} + ADR_{t} \times \hat{z}_{1t} = \hat{w}_{t}$$
5.
$$\hat{z}_{2t} \sum_{l=0}^{1} (1 + ADR_{t+i}) + b_{t}(\hat{n}_{n,t+1} + \hat{n}_{q,t+1}q_{t})\hat{H}_{t+1}^{\theta_{2}} = \hat{w}_{t} + \hat{w}_{t+1}\hat{H}_{t}^{\theta_{2}} +$$

$$\hat{w}_{t+2}\hat{H}_{t+2}^{\theta_{2}}\hat{H}_{t+1}^{\theta_{2}}$$
7.
$$\frac{\hat{z}_{t-1}}{\hat{h}_{t+1}^{\theta_{1}}\hat{R}_{t}^{\theta_{2}}} \left(\frac{a\hat{z}_{t}}{p_{t}^{\frac{1}{2}-1}q_{t}} \right)^{\frac{1}{s_{t}}} = (1-m_{2}t)m_{3}t \frac{\hat{\pi}_{n,t+1} + \hat{\pi}_{q,t+1}q_{t}}{(1-m_{0})(1-m_{1}t)} + (1-m_{2}t)(1-m_{2}t)(1-m_{3}t) \frac{\hat{\pi}_{n,t+1} + \hat{\pi}_{q,t+1}q_{t}}{\hat{u}_{t-1}^{\frac{1}{2}-1}(1+ADR_{t+i})}$$
8.
$$\frac{\hat{z}_{t-1}}{\hat{h}_{t+1}^{\theta_{1}}\hat{R}_{t}^{\theta_{2}}} \left(\frac{\hat{\beta}_{2t}}{p_{t}^{\frac{1}{2}-1}q_{t}} \right)^{\frac{1}{s_{t}}} = (1-m_{2}t)m_{3}t \frac{\hat{\pi}_{q,t+1}b_{t}}{\hat{\Sigma}_{t=0}^{1}(1+ADR_{t+i})} + (1-m_{2}t)(1-m_{3}t) \frac{\hat{\pi}_{n,t+1} + \hat{\pi}_{q,t+1}q_{t}}{\hat{\Sigma}_{t=0}^{1}(1+ADR_{t+i})}$$
9.
$$\hat{y}_{t} = \exp(\epsilon_{Y}^{Y})\hat{L}_{t}^{\theta_{1}}, \text{ where } \epsilon_{t}^{Y} \sim N(0, \sigma_{Y}^{2})$$
10.
$$\hat{w}_{t} = \frac{\theta_{1}\hat{Y}_{t}}{\hat{L}_{t}} \frac{\hat{h}_{t}}{\hat{h}_{t-1}}\hat{R}_{t}}$$
12.
$$\hat{b}_{t} \equiv m_{0}t \times \hat{b}_{t} + m_{1}t \times (1-m_{0}t_{-1})\frac{\hat{\beta}_{t-1}}{\hat{\beta}_{t-1}} + m_{2}t \times (1-m_{1}t_{-1})(1-m_{0}t_{-2})\frac{\hat{\beta}_{t-3}}{\hat{\beta}_{t-1}}\hat{\beta}_{t-3}} + m_{4}t \times (1-m_{3}t_{-1})(1-m_{2}t_{-2})(1-m_{1}t_{-3})(1-m_{0}t_{-4})\frac{\hat{\beta}_{t-3}}{\hat{\beta}_{t-1}\hat{\beta}_{t-3}\hat{\beta}_{t-4}} + (1-m_{4}t_{-2})(1-m_{3}t_{-3})(1-m_{2}t_{-4})(1-m_{1}t_{-5})(1-m_{0}t_{-6})\frac{\hat{\beta}_{t-6}}{\hat{\beta}_{t-1}\hat{\beta}_{t-4}\hat{\beta}_{t-4}\hat{\beta}_{t-5}\hat{\beta}_{t-4}\hat{\beta}_{t-4}\hat{\beta}_{t-5}\hat{\beta}_{t-4} + (1-m_{1}t_{-5})(1-m_{0}t_{-6})\frac{\hat{\beta}_{t-1}\hat{\beta}_{t-3}\hat{\beta}_{t-4}\hat{\beta}_{t-4}\hat{\beta}_{t-5}\hat{\beta}_{t-6}} \equiv CDR_{t}$$
13.
$$\hat{\beta}_{t} = (1-m_{1}t)(1-m_{0}t_{-1})\frac{\hat{\beta}_{t-1}}{\hat{\beta}_{t-1}}}$$

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$$15. \ \hat{G}2_{t} \equiv (1 - m2_{t}) \frac{\hat{G}1_{t-1}}{\hat{P}_{t-1}} \\
16. \ \hat{G}3_{t} \equiv (1 - m3_{t}) \frac{\hat{G}2_{t-1}}{\hat{P}_{t-1}} \\
17. \ \hat{G}4_{t} \equiv (1 - m4_{t}) \frac{\hat{G}3_{t-1}}{\hat{P}_{t-1}} \\
18. \ ADR_{t} \equiv \frac{\hat{G}4_{t} + \frac{\hat{G}4_{t-1}}{\hat{P}_{t-1} + \frac{\hat{G}4_{t-2}}{\hat{P}_{t-1}\hat{P}_{t-2}}}{\hat{L}_{t}} \\
19. \ \hat{L}_{t} \equiv \hat{G}1_{t} + \hat{G}2_{t} + \hat{G}3_{t} \\
20. \ \hat{H}_{t} \equiv \frac{\hat{G}1_{t}}{\hat{L}_{t}} \hat{Q}_{t-1} + \frac{\hat{G}2_{t}}{\hat{L}_{t}} \frac{\hat{Q}_{t-2}}{\hat{H}_{t-1}} + \frac{\hat{G}3_{t}}{\hat{L}_{t}} \frac{\hat{Q}_{t-3}}{\hat{H}_{t-1}\hat{H}_{t-2}} \\
21. \ \hat{\pi}_{nt} = \exp(\epsilon_{t}^{\pi n}) \overline{\Phi}_{nt} \hat{w}_{t} \\
22. \ \hat{\pi}_{qt} = \exp(\epsilon_{t}^{qn}) \overline{\Phi}_{qt} \hat{w}_{t} \\
23. \ \mu_{t} = \tau_{0} + \tau_{\mu} \times \mu_{t-1} + \tau_{A} \times \ln A_{t} + \tau_{w} \times (\hat{W}_{t} - 1) + \epsilon_{t}^{\mu}, \text{ where } \epsilon_{t}^{\mu} \sim N(0, \sigma_{\mu}^{2}) \\
24. \ \hat{Q}_{t} = \exp(\epsilon_{t}^{Q}) \left(\frac{\hat{Q}_{t-2}}{\hat{H}_{t}\hat{H}_{t-1}} q_{t-1}\right)^{1-\varepsilon}, \text{ where } \epsilon_{t}^{Q} \sim N(0, \sigma_{Q}^{2}) \\
25. \ \hat{W}_{t} \equiv \frac{\hat{w}_{t}\hat{H}_{t}^{\theta_{2}}}{\hat{w}_{t-1}} \\
\end{cases}$$

To completely solve this (stationarised) dynamic equation system, we can use the perturbation method, i.e. obtain the steady state for each period, and then add on the complementary functions to capture the deviation from the steady state. Alternatively, we can derive some propositions analytically based on the quasi-final forms to shed light on the properties of the model.

[Proposition 1] Demand function for child quality

The most important two equations in the system are the two marginal conditions (7) and (8).

$$7. \quad \frac{\hat{z}_{t-1}}{\hat{H}_{t+1}^{\theta_2} \hat{H}_t^{\theta_2}} \left(\frac{\alpha \hat{z}_t}{\gamma_{t+1}^{\hat{t}_{t-1}} \hat{H}_t^{\theta_2}} \right)^{\frac{1}{s_t}} = (1 - m2_t) m3_t \frac{\frac{\hat{n}_{n,t+1} + \hat{n}_{q,t+1} q_t}{(1 - m0_t)(1 - m1_t)}}{\sum_{l=0}^{1} (1 + ADR_{t+l})} + (1 - m2_t)(1 - m3_t) \frac{\frac{\hat{n}_{n,t+1} + \hat{n}_{q,t+1} q_t}{(1 - m0_t)(1 - m1_t)}}{\sum_{l=0}^{2} (1 + ADR_{t+l})}$$

$$8. \quad \frac{\hat{z}_{t-1}}{\hat{H}_{t+1}^{\theta_2} \hat{H}_t^{\theta_2}} \left(\frac{\beta \hat{z}_t}{\gamma_{t+1}^{\hat{t}_{t-1}} q_t}}{N_t^{\theta_2} q_t} \right)^{\frac{1}{s_t}} = (1 - m2_t) m3_t \frac{\hat{\pi}_{q,t+1} b_t}{\sum_{l=0}^{1} (1 + ADR_{t+l})} + (1 - m2_t)(1 - m3_t) \frac{\hat{\pi}_{q,t+1} b_t}{\sum_{l=0}^{2} (1 + ADR_{t+l})}$$

If we focus on the steady state, all the shocks are set to zero and time subscripts can be ignored. Multiply (7) by n and (8) by q, then divide the two sides of the two equations (also make use of the definition of b in equation 2), we have:

$$\left(\frac{\alpha}{\beta}\frac{q}{n}\right)^{\frac{1}{s}}\left(\frac{n}{q}\right) = \frac{(1-m2)m3\frac{\hat{\pi}_{n}b+\hat{\pi}_{q}qb}{2(1+ADR)} + (1-m2)(1-m3)\frac{\hat{\pi}_{n}b+\hat{\pi}_{q}qb}{3(1+ADR)}}{(1-m2)m3\frac{\hat{\pi}_{q}qb}{2(1+ADR)} + (1-m2)(1-m3)\frac{\hat{\pi}_{q}qb}{3(1+ADR)}}$$

Factor out the common terms on the denominator and the numerator, resulting in:

$$\left(\frac{\alpha}{\beta}\right)^{\frac{1}{s}} \left(\frac{q}{n}\right)^{\frac{1-s}{s}} = 1 + \frac{\widehat{\pi}_n}{\widehat{\pi}_q} \frac{1}{q} \to \frac{q}{n} \propto \frac{\widehat{\pi}_n}{\widehat{\pi}_q}$$

This is a very informative equation, suggesting that the ratio of quality and quantity is inversely dependent of the ratio between the costs of quality and quantity. In the special case of Cobb-Douglas where s = 1 as assumed by many papers, it results in a quadratic equation and an analytical solution is available:

$$q = \frac{\beta}{\alpha - \beta} \frac{\hat{\pi}_n}{\hat{\pi}_q}$$
, if $s = 1$ and $\alpha > \beta$.

Mortality has no effect on child quality (but this is only on the demand side. See proposition 2 for the supply side where it does have an effect).

[Proposition 2] Supply function of child quality

Make use of equation (24) in steady state:

24.
$$\hat{Q} = \left(\frac{\hat{Q}}{\hat{H}^2}q\right)^{1-\epsilon}$$

Use definitions of \hat{G} and \hat{L} (equations 14-16 and 19) to express \hat{Q} in terms of \hat{H} :

$$\hat{Q} = \frac{\hat{H}}{\frac{(1-m0)(1-m1)}{\hat{p}} + \frac{(1-m0)(1-m1)(1-m2)}{\hat{p}^2} + \frac{(1-m0)(1-m1)(1-m2)(1-m3)}{\hat{p}^3}}, \text{ where:}$$

$$CCC \equiv \frac{(1-ff6)(1-m1)}{\hat{p}} + \frac{(1-ff6)(1-m1)(1-m1)(1-m2)}{\hat{p}^2} + \frac{(1-ff6)(1-m1)(1-m2)(1-m3)}{\hat{p}^3},$$

Combine this expression of \hat{Q} with equation (24), we can solve for q:

$$q = \left(\frac{1}{1 + \frac{(1-m_2)}{\hat{p}} + \frac{(1-m_2)(1-m_3)}{\hat{p}^2}} + \frac{\frac{(1-m_2)}{\hat{p}}}{1 + \frac{(1-m_2)}{\hat{p}} + \frac{(1-m_2)(1-m_3)}{\hat{p}^2}} \frac{1}{\hat{H}} + \frac{\frac{(1-m_2)(1-m_3)}{\hat{p}^2}}{1 + \frac{(1-m_2)}{\hat{p}} + \frac{(1-m_2)(1-m_3)}{\hat{p}^2}} \frac{1}{\hat{H}^2}\right)^{\frac{\varepsilon}{\varepsilon-1}} \hat{H}^{2-\varepsilon} [X]$$

This is another mechanism determining q. It is positively related to the overall human capital growth rate (\hat{H}) , which also defines the technological progress growth rate (\hat{H}^{θ_2}) —see the original production function equation (9). There are two reinforcing sources of this positive effect:

• The first term describes the contribution from family education;

• The second term describes the contribution from nonfamily education.

To see this, consider the special case where there is no external effect of nonfamily education ($\varepsilon = 0$). Now q is a simple quadratic function of the overall human capital growth rate: $q = \hat{H}^2$. In other words, the overall human capital growth only comes from family education. It is quadratic because there are two generations away between the parents and their children.

[Corollary 1] q rises when m2 drops.

To simplify symbols, we focus on the terms in the bracket of equation [X] and define:

$$a \equiv \frac{1-m^2}{\hat{p}} \in (0,1), b \equiv \frac{1-m^3}{\hat{p}} \in (0,1), \text{ and } x \equiv \frac{1}{\hat{H}} \in (0,1)$$

So it becomes:

$$\blacksquare \equiv \frac{1}{1+a+ab} + \frac{ax}{1+a+ab} + \frac{abx^2}{1+a+ab}$$

Take partial derivative of this term with respect to *a*:

$$\frac{\partial \bullet}{\partial a} = -\frac{1+b}{(1+a+ab)^2} - \frac{ax(1+b)}{(1+a+ab)^2} + \frac{x}{1+a+ab} - \frac{abx^2(1+b)}{(1+a+ab)^2} + \frac{bx^2}{1+a+ab}$$
$$= \frac{-(1+ax+abx^2)(1+b)+(1+a+ab)(x+bx^2)}{(1+a+ab)^2}$$

We know that 0 < x < 1, so:

$$1 + ax + abx^2 < 1 + a + ab$$

Therefore, the derivative is negative:

$$\frac{\partial \bullet}{\partial a} < \frac{-(1+ax+abx^2)(1+b)+(1+ax+abx^2)(x+bx^2)}{(1+a+ab)^2} = -\frac{(1+ax+abx^2)(1-x)(1+b(1+x))}{(1+a+ab)^2} < 0$$

To summarise, when m2 drops, $a \equiv \frac{1-m^2}{\hat{p}}$ rises, \blacksquare drops, $\blacksquare \frac{\varepsilon}{\varepsilon-1}$ rises (because $\frac{\varepsilon}{\varepsilon-1} < 0$), so q rises.

[Corollary 2] q rises when m3 drops.

Now take derivative of \blacksquare with respect to *b*:

$$\frac{\partial \bullet}{\partial b} = -\frac{a}{(1+a+ab)^2} - \frac{a^2x}{(1+a+ab)^2} - \frac{a^2bx^2}{(1+a+ab)^2} + \frac{ax^2}{1+a+ab}$$
$$= \frac{-a(1+ax+abx^2) + ax^2(1+a+ab)}{(1+a+ab)^2}$$

Similarly, we know that

$$1 + ax + abx^2 < 1 + a + ab$$

The derivative is also negative:

$$\frac{\partial \bullet}{\partial b} < \frac{-a(1+ax+abx^2)+ax^2(1+ax+abx^2)}{(1+a+ab)^2} = -\frac{a(1+ax+abx^2)(1-x^2)}{(1+a+ab)^2} < 0$$

To summarise, when m3 drops, $b \equiv \frac{1-m_3}{\hat{p}}$ rises, \blacksquare drops, $\blacksquare \frac{\varepsilon}{\varepsilon-1}$ rises (because $\frac{\varepsilon}{\varepsilon-1} < 0$), so q rises.

[Proposition 3] Demand function for child number

Let's define the expected disposable income as:

$$\widehat{\omega} \equiv \frac{m^2}{1+ADR} \times \widehat{w} + \frac{(1-m^2)m^3}{2(1+ADR)} \times \left(\widehat{w} + \widehat{w}\widehat{H}^{\theta_2}\right) + \frac{(1-m^2)(1-m^3)}{3(1+ADR)} \times \left(\widehat{w} + \widehat{w}\widehat{H}^{\theta_2} + \widehat{w}\widehat{H}^{2\theta_2}\right)$$

Then combining equation (3)-(6) gives:

$$b(\hat{\pi}_n + \hat{\pi}_q q) = \frac{\hat{\omega} - \hat{z}}{\hat{H}^{\theta_2} \left[\frac{(1-m_2)m_3}{2(1+ADR)} + \frac{(1-m_2)(1-m_3)}{3(1+ADR)} \right]}$$

In the special case of s = 1, substitute the above into equation (7), we can solve for \hat{z} :

$$\hat{z} = \frac{\gamma}{\alpha + \gamma} \widehat{\omega}$$

Combine with equation (8) and [Proposition 1], we can solve for n (q is already solved in [Proposition 1]):

$$n = (1 - m0)(1 - m1)\frac{\alpha - \beta}{\alpha + \gamma} \frac{\hat{\omega}}{\hat{\pi}_n \left(\frac{(1 - m2)m3}{2(1 + ADR)} + \frac{(1 - m2)(1 - m3)}{3(1 + ADR)}\right)\hat{H}^{\theta_2}}$$

Note that crude birth per mother or per married woman is defined as $b = \frac{n}{(1-m0)(1-m1)}$, so there is no effect of child mortality rates on *b*.

We can define an effective price of n (with respect to the price of z, the numeraire):

$$\widehat{\Pi}_n \equiv \widehat{\pi}_n \frac{\binom{(1-m2)m3}{2(1+ADR)} + \frac{(1-m2)(1-m3)}{3(1+ADR)}}{(1-m0)(1-m1)}$$

The terms in the numerator adjust for probability of premature death in childcare and burden in eldercare and infant mortalities.

• The probability of a person dying before she has any child in her period 2 is m2 and the childcare expense is simply 0. The consumption price needs to consider the resources spent on eldercare during her period 1 only, so the effective price of consumption during period 1 is 1(1 + ADR).

- If the person dies after she has children in her period 2 but before period 3, the probability is (1 m2)m3 and the childcare price is π̂_n. The consumption price needs to consider the pension paid for eldercare during her periods 1 and 2, so the effective price of consumption during periods 1 and 2 is 2(1 + ADR).
- If the person dies after period 3, the probability is (1 − m2)(1 − m3) and the childcare price is also π̂_n. The consumption price needs to consider the pension paid for eldercare during her periods 1, 2 and 3, so the effective price of consumption during periods 1, 2 and 3 is 3(1 + ADR).

Therefore, the solution of n can also be rewritten as:

$$n = \frac{\alpha - \beta}{\alpha + \gamma} \frac{\widehat{\omega}}{\widehat{\Pi}_n \widehat{H}^{\theta_2}}$$

If there is no economic growth $\hat{H} = 1$, then this solution is similar to equation (1) in Foreman-Peck (2011) Appendix. If there is economic growth, then n falls unless there are other changes such as falls in child mortality or ADR lowering child price (eventual demographic transition).

When m0 and m1 rise, there is equal negative effect on n, but no effect on b. There are two *direct* effects of m2 and m3 on n: (1) the **income effect** through the numerator ($\hat{\omega}$), and (2) the **substitution effect** through the denominator ($\hat{\Pi}_n$). (NB: There are also *in-direct* effects of m2 and m3 through wage and human capital, which can only be obtained after solving for the general equilibrium.)

Let's start with m2. We can prove that the wealth effect of m2 is negative, i.e. $\frac{\partial \hat{\omega}}{\partial m^2} \leq 0$:

$$\frac{\partial \hat{\omega}}{\partial m^2} = \frac{1}{1 + ADR} \times \hat{w} + \frac{-m^3}{2(1 + ADR)} \times \left(\hat{w} + \hat{w}\hat{H}^{\theta_2}\right) + \frac{-(1 - m^3)}{3(1 + ADR)} \times \left(\hat{w} + \hat{w}\hat{H}^{\theta_2} + \hat{w}\hat{H}^{2\theta_2}\right)$$

In balanced growth path, human capital has a non-negative growth rate, so $\hat{H} \ge 1$, therefore, the above is:

$$\frac{\partial \widehat{\omega}}{\partial m_2} \le \frac{1}{1 + ADR} \times \widehat{w} + \frac{-m_3}{2(1 + ADR)} \times (\widehat{w} + \widehat{w}) + \frac{-(1 - m_3)}{3(1 + ADR)} \times (\widehat{w} + \widehat{w} + \widehat{w}) = 0$$

Therefore, the total effect of m2 on n is ambiguous:

$$\frac{\partial n}{\partial m_2} = \frac{\partial n}{\partial \hat{\omega}} \frac{\partial \hat{\omega}}{\partial m_2} + \frac{\partial n}{\partial \hat{\Pi}_n} \frac{\partial \hat{\Pi}_n}{\partial m_2} \gtrless 0$$

The intuition behind this ambiguous effect is that when mortality rate m2 rises, the lifetime expected wage drops, leading to a negative income effect. At the same time, a rise in m2 also means a relatively cheaper price of n, leading to a positive substitution effect.

Now turn to the effect of m3. We can prove that the wealth effect of m3 is also negative, i.e. $\frac{\partial \hat{\omega}}{\partial m_3} \leq 0$.

$$\frac{\partial \widehat{\omega}}{\partial m_3} = \frac{1 - m_2}{2(1 + ADR)} \times \left(\widehat{w} + \widehat{w}\widehat{H}^{\theta_2}\right) + \frac{-(1 - m_2)}{3(1 + ADR)} \times \left(\widehat{w} + \widehat{w}\widehat{H}^{\theta_2} + \widehat{w}\widehat{H}^{2\theta_2}\right)$$

In balanced growth path, human capital has a non-negative growth rate, so $\hat{H} \ge 1$, therefore, the above is:

$$\frac{\partial \hat{\omega}}{\partial m_3} = \frac{\hat{w}(1-m_2)}{6(1+ADR)} \left(1 + \hat{H}^{\theta_2}\right) \left(1 - \hat{H}^{\theta_2}\right) \le 0$$

Therefore, the total effect of m3 on n is negative:

$$\frac{\partial n}{\partial m^3} = \frac{\partial n}{\partial \hat{\omega}} \frac{\partial \hat{\omega}}{\partial m^3} + \frac{\partial n}{\partial \hat{\Pi}_n} \frac{\partial \hat{\Pi}_n}{\partial m^3} < 0$$

The difference from m2 is that when m3 rises, the effect on $\widehat{\Pi}_n$ is positive, so the substitution effect reinforces the income effect. (includes) The terms in the bracket adjust for probablity of premature death in childcare and burden in eldercare. So the bracketed term reflects that the probability is (1 - m2)m3 of the person dying after she has children in her period 2 but before period 3, and the childcare price is $\widehat{\pi}_n$. The consumption price needs to include the pension paid for eldercare during her periods 1 and 2, so the effective price of consumption during periods 1 and 2 is 2(1 + ADR).

The probability the person dies in period 3 is (1 - m2)(1 - m3) and the childcare price is also $\hat{\pi}_n$. The consumption price needs to consider the pension paid for eldercare during her periods 1, 2 and 3, so the effective price of consumption during periods 1, 2 and 3 is 3(1 + ADR). In addition, the survival chances of the children (denominator) affects the generalised child price.

Appendix II: Data Sources and Definitions

Demographic data are based on Wrigley and Schofield (1981) and Broadberry et al (2015), wage data are based on Clark (2013) and Allen (2007), and steady state mortality rates are calibrated using Wrigley and Schofield (1989, p714).

[Labour Market]

- The annual *real wage* (of a full-time worker) is from the MeasuringWorth website with details explained in Clark (2005), covering the period of 1209-2016.
- The *female nominal wage* combines a variety of sources in the following way:
 - 1260-1850, based on HW (2015) nominal daily wage, assume 10 hours a day, 30% single women doing annual contract, and 70% married women doing casual contract. The percentages are based on the age structure of 15-25 and 26-60 of female population at the time.
 - 1824-1900, based on Layton (1908), assume 40% weight for domestic services and 60% weight for industrial jobs.
 - 1875-1938, based on Crafts (1984), which is already an average of domestic services and industrial jobs. This series is spliced with the Layton series.
 - 1938-1968, based on BLS (1971), monthly average is used for annualization.
 - 1970-2016, based on NES (1970-1996) and ASHE (1997-2016).

These series are combined by averaging the overlapped and interpolating the missing observations.

- The *male nominal wage* is from the MeasuringWorth website with details explained in Clark (2005), covering the period of 1209-2016.
- The *wage premium* (*WP*) is based on the two series above.

[Population]

- The *population* series combines the following sources:
 - o 1086-1679, based on Broadberry et al (2015).
 - o 1680-1841, based on Wrigley (1997).
 - 1842-2016, based on Bank of England's "A millennium of macroeconomic data"
- The *birth* series is combined by splicing WS (1981) series (1539-1870) with HMD/Registrar-General official series (1841-2014).

- The *death* series is constructed in the same way as the birth.
- The *mortality rates* are based on the following sources:
 - 1841-2014, based on HMD/Registrar General official series, age-specific mortality rates.
 - o 1841-2014, based on HMD/official series, life expectancy at birth.
 - o 1541-1871, based on WS (1981, Table 3.1), life expectancy.
 - levels, based on WS (1981, Table A14.5), mortality rates and life expectancy.
 - Before 1541, based on Russell (1948) and Hatcher et al. (2006)

Step 1: Find the two closest neighbouring levels from Table A14.5 by comparing with each observation of Table 3.1 in terms of life expectancy during 1541-1871 (WS, 1981) and calculate the weights (relative distances) of the two levels to each observation.

Step 2: Calculate the mortality rates for each year by the weighted average of the corresponding two levels" mortality rates.

Step 3: The resulting mortality rates are spliced with the HMD/official mortality rates based on the overlapped observations.

- The *first-time mother's age* is based on the following sources:
 - 1617-2011, based on Schofield (1985) and ONS (marriage Table 7). This is the female age at first marriage.
 - o 1938-2013, based on ONS (birth Table 4b). This is first-mother's age.

Step 1: Calculate the average gap between ages at first marriage and at first birth based on the common sample years.

Step 2: Extend the first-time mother's age back to 1617 by adding back the gap. Step 3: The missing values are interpolated.

- The *childless rate* (or celibacy rate) is based on the following sources:
 - 1596-2001, based on WS (1989) and ONS (childless Table), the childless rate.
 - o 1838-1998, based on Mitchell (2003), the number of marriages.
 - o 1841-2014, based on HMD/official series, the fertile population.

Step 1: Calculate the marriage rate between 16 and 45. Multiply the number of marriages by the population series to get the total number of marriages; then divide it by the fertile population series.

Step 2: Estimate the relationship between childless rate and marriage rate by OLS regression on the ground that these two variables are closely and negatively related.

Step 3: Interpolate and extrapolate the childless rate for the missing observations.

[Technological Progress]

- The *school enrolment rate* is based on the following sources:
 - 1830-1930, based on Lindert (2004), the number of pupils of all schools.
 - 1840-1930, based on Mitchell (1962, p41), the age structure of population.
 - 1300-1900, based on De Pleijt (2015), the literacy rates of male and female.
 - 1754-1844, based on Schofield (1968), the literacy rates of male and female.

Step 1: Calculate the school enrolment rate by dividing Lindert series by the total number of school-age population (Mitchell). The school leaving age became 15 in 1947, so the rate is fixed at 100% from 1950. The resulting decadal series is then linearly interpolated.

Step 2: Calculate the overall literacy rate by averaging the De Pleijt and Schofield series after linear interpolation/extrapolation.

Step 3: Within the common sample, calculate the average ratio between school enrolment rate and literacy rate in 15 years. This ratio is then used to extend the school enrolment rate to 1300.