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Understanding International Long-Term Interest Rate Comovement*

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Abstract

Long-term interest rates of small open economies correlate strongly with the US long-term rate. Can central banks in those countries decouple from the US? An estimated DSGE model for the UK (vis-à-vis the US) establishes three structural empirical results. (1) Comovement arises due to nominal fluctuations, not through real rates or term premia. (2) The cause of comovement is the central bank of the small open economy accommodating foreign inflation trends, rather than systematically curbing them. (3) Small open economies may find themselves much more affected by changes in US inflation trends than the US itself.

JEL Classification: E43, E44, F30, F44, F65, G15
Keywords: DSGE Model, Small Open Economy, Yield Curve, Long-Term Interest Rates, Term Premia, Comovement

1 Introduction

Long-term nominal interest rates strongly comove across countries. Figure 1 plots 10-year (zero-coupon) interest rates for 10 industrialized countries (obtained from the Wright (2011) database). The high degree of synchronization of yields across countries jumps out. This comovement poses a major policy challenge. If a country's long-term interest rate is mostly determined by foreign factors, that country's central bank may have little leverage over its domestic long-term interest rate. If the central bank does not influence long-term interest rates, this breaks the core mechanism of monetary policy transmission.

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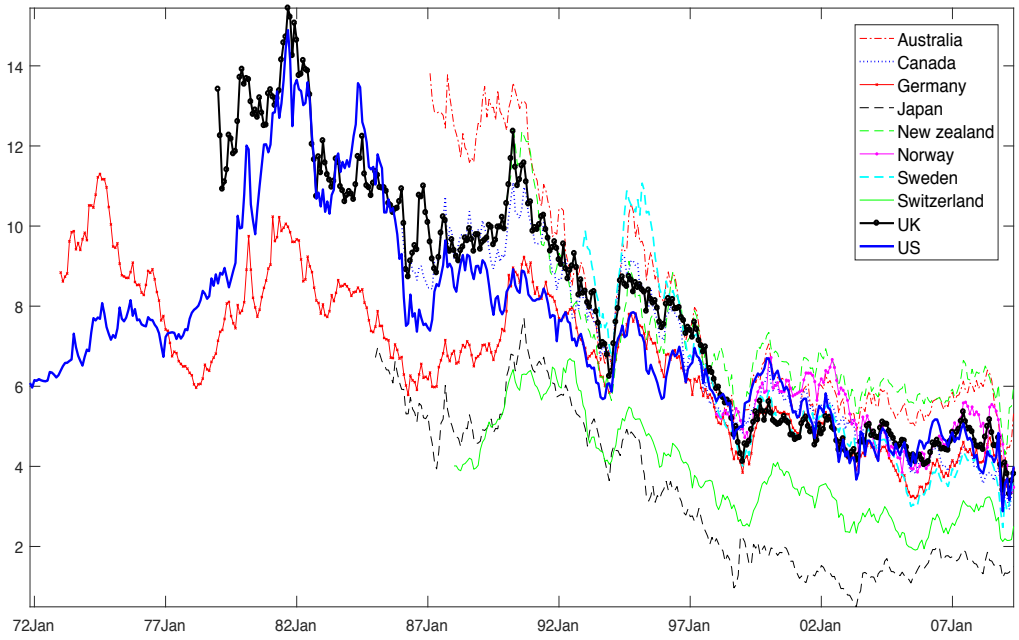
Figure 1: Long-Term Interest Rate Comovement (Wright (2011) data)



The policy challenge proves to be very real. For instance, on August 2013 the Bank of England (BoE) launched a forward guidance unconventional policy program: flattening the expected path of policy rates aiming to reduce long-term interest rates, in view of ultimately boosting demand and closing the output gap. Economists and policy makers expressed concerns about the ability of the UK Monetary Policy Committee (MPC) to "decouple" the long end of the UK yield curve from the US long-term interest rate given that they are so highly correlated (0.93, Table 1).

The case of the UK is indicative of a wider phenomenon; the strong correlation with US long-term interest rates is a stylized fact across a large number of small open economies (the average correlation is 0.85, Table 1).[1] Hence, multiple industrialized economies face the exact same policy challenge. If external factors determine a country's long-term interest rates, domestic (un)conventional policies geared to combat the Great Recession may prove futile. Currently, as the US is rebounding from the Great Recession faster than many other countries, this raises the concern that as the Fed starts increasing its policy interest rate substantially, the induced US long rate increase will push long rates abroad up as well, potentially jeopardizing the fragile global economic recovery. In their communication, several "small" inflation targeting countries' central banks maintain they have control over domestic policy by affecting the long end of the yield curve, but such is not apparent from the data. Increasingly, this leads to central banks missing targets, difficult policy communication, and the adoption of unrealistic

[1] See also Figure 1 of Dahlquist and Hasseltoft (2013) and Jotikasthira et al. (2015).

assumptions in policy assessments, forecasts and scenario analysis.[2]

Table 1: Long-Term Interest Rate Comovement

	Correlation with US 10 year rate
Australia	0.93
Canada	0.97
Switzerland	0.84
Germany	0.74
Japan	0.89
Norway	0.63
New Zealand	0.89
Sweden	0.85
United Kingdom	0.93
Average	0.85

Notes: Calculations based on Wright (2011) data.

While the empirical presence of long-term interest rate comovement is well-known, the phenomenon is poorly understood. Academic research has not provided much help on the issue. The cause for this lack of guidance lies in the absence of a structural framework that enables understanding what causes the comovement. The current term-structure literature, by estimating factor and affine term structure models (see Diebold et al. (2008), Dahlquist and Hasseltoft (2013) and Jotikasthira et al. (2015) among others), attributes fluctuations in interest rates to world 'level' and/or 'slope' factors or term-premia. These are reduced-form concepts and, consequently, are consistent with multiple structural interpretations. They fail to help policy makers around the world understand whether they actually can decouple domestic long-term interest rates from the US.

Our work sets out to understand international long-term interest rate comovement in a structural manner. We estimate a small open economy model (SOE) for the UK (with the US as the foreign economy) that enables to structurally identify the sources of long-term interest rate comovement. The model encompasses various potential sources of comovement: nominal factors, real rates or term premia.

Our analysis reveals that international long-term interest rate comovement arises because changing foreign (US) long-term inflation trends are not sufficiently offset by the domestic (UK) central bank. To understand the mechanism at work, consider an increase in long-term US inflation expectations, or in the US inflation target. The effect on the US is standard (e.g. Cogley et al. (2010)): the US real rate is temporarily reduced, since inflation expectations jump up while the policy rate increases only gradually. The low real interest rate generates a temporary boost in US activity. Since the US real interest rate does not change in the long run, the Fed's policy interest rate eventually increases to a permanently higher level. Because the policy interest rate is permanently higher, the US long-term interest rate increases (through the expectations hypothesis channel).

So how does this rise in US inflation expectations affect the UK? As apparent from our estimates and from actual BoE communication, the BoE is in fact complacent when it comes to foreign inflation

[2]For an example of the significant policy impact of failing to understand the drivers of foreign interest rates, see Svensson's discussion of the importance of global long-term interest rate forecasts in actual policy making (e.g. Riksbank (2010), Svensson (2011)). The lack of a proper model for foreign long-term interest rates is a frequently voiced concern by multiple countries' central bankers (e.g. conference on "Central Bank Macro Modeling", Norges Bank, Oslo; September 2017).

trends. Because it does not fight the permanently higher foreign inflation, inflation expectations in the UK rise accordingly. The BoE accommodates those higher inflation expectations: while it permanently increases the nominal policy interest rate, it does so insufficiently to prevent a reduction in the UK real interest rate, which engenders a UK economic expansion. The fact that the UK policy rate rises permanently causes UK long-term interest rates to increase. The higher US inflation expectations thus cause an increase in both the US and the UK long-term interest rate, i.e. long-term interest comovement arises.

Our findings have an immediate and strong policy implication. In principle, the BoE could *systematically* respond strongly to the foreign inflation: by increasing the policy rate temporarily, it then succeeds in increasing the real rate. This generates a temporary UK recession, causing lower domestic inflation. The reduced domestic inflation counteracts the higher imported inflation, and thereby undoes any meaningful impact on UK inflation. Because a temporary policy response is sufficient, UK long-term interest rates hardly move. In such a situation there is no international long-term interest rate comovement. Our analysis thus suggests that a central bank faced with the policy challenge posed by international interest rate comovement can fully regain control, since its own complacency to foreign inflation trends is the root of the problem.

Our analysis relates to and extends various results in the literature. The first key result is that international comovement is the result of nominal factors. While, in principle, changing real rates or term premia could underlie international interest rate comovement, they fail to account for it quantitatively. The model estimates reveal the comovement between US and UK long-term interest rates is primarily driven by long-term changes in US inflation expectations. There is ample literature studying the occurrence, causes and consequences of changing inflation trends in a US setting. The high sensitivity of long-term interest rates to macroeconomic news in the US is well established (e.g. Gürkaynak et al. (2005)), as is the specific role for time-varying perceptions of long-term inflation (e.g. De Graeve et al. (2009)). What is novel is that this excess sensitivity is crucially important for other countries. This constitutes our second key result: it is possible (and in fact likely) that small open economies react more to changes in long-term US inflation expectations than the US itself does. In addition to measuring this phenomenon, the advantage of our structural approach is that it is readily understood. The small open economy's high sensitivity occurs because the complacent response of the domestic central bank causes its economy to respond endogenously, which amplifies the fluctuations coming from abroad.

We show that the two key components of the transmission channel identified by the estimated SOE DSGE model are easily validated externally. On the one hand, using simple univariate regressions as well as VARs, we find that changes in long-term US inflation expectations affect UK inflation even more than they do US inflation. On the other hand, actual central bank communication from the BoE explicitly states that it disregards foreign inflation trends. Both findings are apparent without any reliance on the DSGE model structure, yet completely conform our structural interpretation of international comovement.

While changing inflation trends generate international comovement, they are not the sole drivers of long-term interest rates. Our model encompasses and quantitatively assesses several key determinants of long-term rates. Let us consider these in turn, and discuss why they do not explain international comovement.

On the one hand, changes in *real rates* are a potential source of fluctuations in nominal long-term interest rates. As such they can, in principle, generate positive comovement across countries. Henriksen et al. (2013), though not focusing on long rates, establish that possibility in a calibrated model with (internationally) correlated productivity shocks. When confronted with the data, however, we find the role for changing real rates in explaining comovement is limited: while they provide correlation in the right direction, their variance is far too small. Our findings are entirely consistent with Wright (2011), who finds that (national) nominal long-term interest rates bear little relation to real fluctuations. While there may be a role for excess sensitivity in real rates in recent years (as shown in e.g. Beechey and Wright (2009)), it is harder to make a case for real rates explaining the low-frequency movements seen in long-term interest rates around the globe during the 70's and 80's.[3] Because their role in explaining historical domestic long-term nominal interest rates is limited, changing real rates do not historically account for international comovement in the data.

On the other hand, a large body of research attributes fluctuations in long-term interest rates to fluctuations in *term premia* and the present paper is no exception. In fact, the term premia estimated in our structural macro model bear a close resemblance to those estimated in reduced form (macro-) finance models (e.g. Adrian et al., 2013). Moreover, consistent with recent evidence of Krishnamurthy and Vissing-Jorgensen (2012), Gourinchas and Jeanne (2012) and Rey (2016), our evidence points to a significant impact of "safety" on long-term yields. As in Del Negro et al. (2017), we find that the Savings Glut reduced US long-term yields by approximately 90bp over the last two decades. Adding to the above evidence, our estimates suggest that while safety considerations may have reduced US yields, they implied upward pressure on foreign yields: as international investors seek the safety of US Treasuries, they substitute away from other sovereigns. Thus, while the Savings Glut can contribute to understanding the level of US and UK long-term interest rates, it does not generally explain the (positive) international comovement of long-term interest rates.

Various macro and finance metrics give credence to the model's empirical performance and thus its plausibility. First, when data simulated by the estimated DSGE model is used to estimate reduced-form factor type models, the variance decomposition is very similar to those reported by the aforementioned term structure studies. In other words, our structural model is entirely consistent with reduced form findings in the literature. Second, the estimated DSGE model delivers term-premium estimates quite similar to the estimates produced by affine term-structure models. Third, in terms of the model's macroeconomic performance, not only is its in-sample fit impressive, the model also correctly identifies the contribution of foreign shocks to the domestic economy (a severe problem identified by Justiniano and Preston (2010a) that threatens the usefulness of SOE DSGE models more generally). As a result, the model also replicates the correlation patterns between output and consumption across countries (e.g. Kose et al. (2003)), which is another contentious issue within the open macro DSGE literature (e.g. Kollmann (1996, 2001)).

The paper starts by laying out the theoretical foundations of the estimated model. In essence, our model is a standard SOE model, extended to allow for a meaningful empirical analysis of long-term interest rates determined by nominal fluctuations, real interest rate changes and term premia. Section 3 describes the data used for estimation and details the estimation procedure. Section 4 assesses the model's performance: in terms of fitting the data, consistency with reduced form models of

[3] This argument is further corroborated by the evidence in Antolin-Diaz et al. (2017) and Del Negro et al. (2017), who show that changing real rates are a more recent phenomenon.

international comovement, and agreement with earlier studies of the US and UK yield curves in the literature that do not focus on international comovement. Having established broad success in all these dimensions, we then analyse comovement from a structural perspective in Section 5. We first quantify the contribution of the different sources of comovement. After identifying foreign inflation trends as the primary culprit, we explain how they transmit to the domestic economy. We then validate the transmission mechanism with information outside the model, and draw policy implications. We describe a battery of robustness checks in Section 6.

2 Theoretical Model

The baseline model we build on is used in a large number of SOE studies, the features and predictions of which are well understood (see Gali and Monacelli (2005), Justiniano and Preston (2010a), Mumtaz and Theodoridis (2015), Adolfson et al. (2007), and Christiano et al. (2011) among others). We extend the SOE model with a term structure of interest rates. The model is set up such that fluctuations in nominal yields can derive from standard business cycle fluctuations, changing real and nominal low-frequency trends, as well as term premia. The former is immediate by supplementing the SOE model with the expectations hypothesis. We allow both economies to be subject to low-frequency changes in the growth rate of productivity as well as changes in the inflation target in either economy. Regarding term premia, we augment the SOE model with two additional assets – domestic and foreign long-term government debt – and include financial intermediaries which operate in the government bond markets. This opens up the possibility that liquidity and risk premia affect the yield curve.

The model is described in full detail in an online Appendix. The description below focuses on the domestic small open economy (UK). It trades (goods and assets) with the large foreign economy (US) that is very similar in structure, except for the fact that it is not influenced by fluctuations in the small open economy. The model consists of four types of agents: firms, households, financial intermediaries and the government.

2.1 Firms

As is common in SOE models there are three sectors: (i) domestic producers, (ii) importers and (iii) exporters. Each sector consists of two sets of firms: a continuum of intermediate monopolistic goods producers and a continuum of final goods producers who operate under perfect competition. A fraction of the monopolistic suppliers set prices based on a Calvo (1983) pricing scheme, while those firms that "miss" the random signal to re-optimise profits set prices based on a backward indexation rule (i.e. $p_t^j = \pi_{t-1}^{j,\kappa_j} \left(\bar{\pi}_t^c\right)^{1-\kappa_j} p_{t-1}^j$, where $\bar{\pi}_t^c$ is the UK CPI inflation target). This setup gives rise to the following (resp. domestic, import and export) Phillips curve equations:

$$\begin{aligned} \hat{\pi}_t - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\beta\kappa_y} E_t\left(\hat{\pi}_{t+1} - \hat{\bar{\pi}}_{t+1}^c\right) + \frac{\kappa_y}{1+\beta\kappa_y}\left(\hat{\pi}_{t-1} - \hat{\bar{\pi}}_t^c\right) - \frac{\beta\kappa_y}{1+\beta\kappa_y}\left(\hat{\bar{\pi}}_t^c - \hat{\bar{\pi}}_{t+1}^c\right) \\ &+ \frac{\left(1-\xi_y\right)\left(1-\beta\xi_y\right)}{\xi_y\left(1+\beta\kappa_y\right)}\left[\hat{w}_t - \left(\hat{y}_t - \hat{h}_t\right) - \hat{\tilde{p}}_t\right] \end{aligned} \tag{1}$$

$$\hat{\pi}_t^m - \hat{\bar{\pi}}_t^c = \frac{\beta}{1+\beta\kappa_m} E_t\left(\hat{\pi}_{t+1}^m - \hat{\bar{\pi}}_{t+1}^c\right) + \frac{\kappa_m}{1+\beta\kappa_m}\left(\hat{\pi}_{t-1}^m - \hat{\bar{\pi}}_t^c\right) - \frac{\beta\kappa_m}{1+\beta\kappa_m}\left(\hat{\bar{\pi}}_t^c - \hat{\bar{\pi}}_{t+1}^c\right) + \frac{(1-\xi_m)(1-\beta\xi_m)}{\xi_m(1+\beta\kappa_m)}\left(\hat{q}_t - \hat{\tilde{p}}_t^m\right) \quad (2)$$

$$\hat{\pi}_t^x - \hat{\bar{\pi}}_t^c = \frac{\beta}{1+\beta\kappa_x} E_t\left(\hat{\pi}_{t+1}^x - \hat{\bar{\pi}}_{t+1}^c\right) + \frac{\kappa_x}{1+\beta\kappa_x}\left(\hat{\pi}_{t-1}^x - \hat{\bar{\pi}}_t^c\right) - \frac{\beta\kappa_x}{1+\beta\kappa_x}\left(\hat{\bar{\pi}}_t^c - \hat{\bar{\pi}}_{t+1}^c\right) + \frac{(1-\xi_x)(1-\beta\xi_x)}{\xi_x(1+\beta\kappa_x)}\left(\hat{\tilde{p}}_t - \hat{q}_t - \hat{\tilde{p}}_t^x\right). \quad (3)$$

Equation (1) indicates that domestically generated inflation ($\hat{\pi}_t$) is a function of the current and expected unit labour cost defined as the difference between household's real wage ($\hat{w}_t$) and labour productivity $\left(\hat{y}_t - \hat{h}_t\right)$ adjusted for the domestic relative price $\left(\hat{\tilde{p}}_t = \hat{\tilde{p}}_{t-1} + \hat{\pi}_t - \hat{\bar{\pi}}_t^c\right)$. Output is produced using a Cobb-Douglas production function $\hat{y}_t = (1-\phi)\hat{h}_t + \hat{z}_t$, while $\hat{\bar{\pi}}_t^c - 0.999\hat{\bar{\pi}}_{t-1}^c = \rho_{\bar{\pi}}\left(\hat{\bar{\pi}}_{t-1}^c - 0.999\hat{\bar{\pi}}_{t-2}^c\right) + \sigma_{\bar{\pi}}\omega_{\bar{\pi},t}$ is the time-varying UK inflation target.[4] Equation (2) illustrates that import price inflation ($\hat{\pi}_t^m$) is a function of the current and expected deviations from the law of one price $\left(\hat{q}_t - \hat{\tilde{p}}_t^m\right)$, where $\hat{q}_t$ is the real exchange rate and $\hat{\tilde{p}}_{t-1}^m + \hat{\pi}_t^m - \hat{\pi}_t^c$ is the relative price of imports. Similarly, expression (3) describes exporters' pricing decisions, where $\hat{\tilde{p}}_t^x = \hat{\tilde{p}}_{t-1}^x + \hat{\pi}_t^x - \hat{\pi}_t^{c,*}$ is the relative price of exports. The notation uses *-superscripts to indicate variables in the foreign economy.

2.2 Domestic Financial Intermediaries

Financial intermediaries operate in between households and government bond markets in a perfectly competitive environment. These financial firms issue deposits to households that pay a gross interest rate r_t^h, and the proceeds from these deposits are used to purchase a portfolio of short- and long-term government bonds (paying interest r_t^S and r_t^L, respectively), as well as long-term debt issued by the foreign government (paying interest $r_t^{L,*}$). Similar to Andres et al. (2004), Chen et al. (2012), Harrison (2012) and De Graeve and Theodoridis (2016), we follow Woodford (2001) in the treatment of long-term bonds: government bonds are modelled as perpetuities that cost $p_{L,t}$ at time t and pay an exponentially decaying coupon κ^s at time $t+s+1$ where $0 < \kappa \leq 1$. As explained in Woodford (2001) and Chen et al. (2012) the advantage of this formulation is that the period t price of a bond issued s periods ago, $p_{L-s,t}$, can be expressed as a function of the coupon and the current price $p_{L,t}$:

$$p_{L-s,t} = \kappa^s p_{L,t}. \quad (4)$$

This relation allows expressing the intermediary balance sheet equations and the government budget constraints (below) in a parsimonious form. Furthermore, for simplicity, we rule out the existence of a secondary market for long-term bonds, implying that agents who invest in long-term debt must hold it until maturity. Finally, we assume that all government bonds issued are purchased by these firms.[5] The intermediary's balance sheet consists of holdings of three types of assets: short-term domestic

[4] We follow the convention in the literature and model the inflation target as a near non-stationary process, as in De Graeve et al. (2009), Cogley et al. (2010) and Del Negro et al. (2015). Note that this persistence can also be estimated, which in the present model essentially delivers a virtually identical value (0.998). The parameter $\rho_{\bar{\pi}}$ captures a potentially smoother target process than a pure random walk.

[5] Andres et al. (2004) and Chen et al. (2012) provide detailed discussions of the advantages of these assumptions.

bonds ($b^S_{\kappa,t}$), long-term domestic bonds ($b^L_{\kappa,t}$) and foreign long-term bonds ($b^{L,*}_{\kappa,t}$):

$$b^h_{\kappa,t} = \frac{b^S_{\kappa,t}}{\varepsilon^{b^S}_t} + \frac{p_{L,t} b^L_{\kappa,t}}{\varepsilon^{b^L}_t} + \frac{q_t \varrho p^*_{L,t} b^{L,*}_{\kappa,t}}{\varepsilon^{b^L}_t}$$

or

$$b^h_{\kappa,t} = \frac{b^S_{\kappa,t}}{\varepsilon^{b^S}_t} + \frac{\bar{b}^L_{\kappa,t}}{\varepsilon^{b^L}_t} + \frac{\bar{b}^{L,D,*}_{\kappa,t}}{\varepsilon^{b^L}_t}. \tag{5}$$

$p_{L,t}$ and $p^*_{L,t}$ are the prices of domestic and foreign long-term bonds ($p_{S,t}$ is set to unity), given by

$$\begin{aligned} p_{L,t} &= \frac{1}{r^L_t - \kappa} \\ p^*_{L,t} &= \frac{1}{r^{L,*}_t - \kappa^*}. \end{aligned}$$

The term ϱ reflects the fraction of the foreign long-term debt held by the domestic financial sector. The real exchange rate q_t converts foreign bond purchases ($\varrho p^*_{L,t} b^{L,*}_{\kappa,t}$) into domestic currency. Motivated by the work of Smets and Wouters (2007) we assume the balance sheet equation is subject to two 'financial' shocks: a short- and a long-term risk premium shock denoted by $\varepsilon^{b^S}_t$ and $\varepsilon^{\bar{b}^L}_t$, respectively. The intermediary's profit function is given by

$$\begin{aligned} \xi_{\kappa,t} = & \underbrace{b^h_{\kappa,t} + \frac{r^S_{t-1}}{\pi^c_t} b^S_{\kappa,t-1} + \frac{r^L_t}{\pi^c_t} p_{L,t} b^L_{\kappa,t-1} + \varrho \frac{r^{L,*}_t}{\pi^{c,*}_t} p^*_{L,t} b^{L,*}_{\kappa,t-1}}_{\text{revenues}} \\ & \underbrace{- \frac{b^S_{\kappa,t}}{\varepsilon^{b^S}_t} - \frac{p_{L,t} b^L_{\kappa,t}}{\varepsilon^{\bar{b}^L}_t} - \frac{q_t \varrho p^*_{L,t} b^{L,*}_{\kappa,t}}{\varepsilon^{\bar{b}^L}_t} - \frac{r^h_{t-1}}{\pi^c_t} b^h_{\kappa,t-1} - \frac{x}{2} \left[lq_{\kappa,t-1} - \vartheta lq_{t-2} - (1-\vartheta)\, lq \right]^2 \frac{Z_{t-1}}{\pi^c_t}}_{\text{expenditures}} \end{aligned}$$

where the prices of domestic and foreign consumption goods are denoted by π^c_t and $\pi^{c,*}_t$, respectively. The profits reflect real returns on government debt holdings less new bond purchases and interest payments on household deposits. An additional expenditure for the intermediary is an adjustment cost, $\frac{x}{2}\left[lq_{\kappa,t} - \vartheta lq_{t-1} - (1-\vartheta)\, lq \right]^2$, where $lq_{\kappa,t} = \frac{b^S_{\kappa,t}}{\bar{b}^L_{\kappa,t} + \bar{b}^{L,D,*}_{\kappa,t}}$ is the ratio of short- to long-term bonds. The parameter ϑ captures the cost induced by changes in intermediaries' current liquidity position relative to the previous period's liquidity ratio and x is a parameter that controls the overall size of the liquidity friction. This financial friction indicates that it is costly for intermediaries when bond holdings deviate from their steady-state values. The specification nests both static and dynamic adjustment costs.[6] Substituting the balance sheet equation into the (one-period ahead) profit function gives

$$\begin{aligned} E_t \xi_{\kappa,t+1} = & \frac{r^S_t}{E_t \pi^c_{t+1}} b^S_{\kappa,t} + E_t \left\{ \frac{r^L_{t+1}}{\pi^c_{t+1}} \frac{p_{L,t+1}}{p_{L,t}} \right\} \bar{b}^L_{\kappa,t} + E_t \left\{ \frac{r^{L,*}_{t+1}}{\pi^{c,*}_{t+1}} \frac{p^*_{L,t+1}}{p^*_{L,t}} \frac{q_{t+1}}{q_t} \right\} \bar{b}^{L,*}_{\kappa,t} \\ & - E_t \left\{ \frac{r^h_t}{\pi^c_{t+1}} \right\} b^h_{\kappa,t} - \frac{x}{2} \left(\delta \frac{b^S_{\kappa,t-1}}{\bar{b}^L_{\kappa,t-1} + \bar{b}^{L,D,*}_{\kappa,t-1}} - 1 \right)^2 \frac{1}{E_t \pi^c_{t+1}}. \end{aligned}$$

[6] De Graeve and Theodoridis (2016) document the virtues of these adjustment costs.

2.3 Interest Rates and Term Premia

Profit maximisation with respect to domestic short-term debt, and domestic and foreign long-term debt, subject to the balance sheet condition delivers expressions for the effective interest rate faced by households ($\hat{r}_t^h$), the long-term interest rate and the exchange rate.[7]

Short-Term Household Interest Rate

$$\hat{r}_t^h = \hat{r}_t^S + \chi\left(\widehat{lq}_t - \vartheta\widehat{lq}_{t-1}\right) + \hat{\varepsilon}_t^{b^S} \tag{6}$$

where $\chi = \frac{x\beta}{b^S}$,

$$\widehat{lq}_t = \frac{\bar{b}^L}{\bar{b}^L + \bar{b}^{L,D,*}}\hat{\bar{b}}_t^L + \frac{\bar{b}^{L,D,*}}{\bar{b}^L + \bar{b}^{L,D,*}}\hat{\bar{b}}_t^{L,D,*} + \frac{1}{\bar{b}^L + \bar{b}^{L,D,*}}\hat{\varsigma}_t - \hat{b}_t^S \tag{7}$$

summarises the liquidity friction and $\hat{\varsigma}_t$ is a persistent wedge between the domestic and foreign stochastic trend

$$\begin{aligned}\hat{\gamma}_t^* &= \hat{\gamma}_t + \Delta\hat{\varsigma}_t \\ \hat{\varsigma}_t &= \rho_\varsigma\hat{\varsigma}_{t-1} + \sigma_\varsigma\omega_{\varsigma,t}.\end{aligned}$$

The model incorporates non-stationary technology shocks ($\hat{\gamma}_t^*, \hat{\gamma}_t$) to allow for the possibility of international comovement through comovement in equilibrium real interest rates.

Domestic Long-term Interest Rate

$$E_t\hat{r}_{t+1}^L + E_t\Delta\hat{p}_{L,t+1} = \hat{r}_t^h - \hat{\varepsilon}_t^{\bar{b}^L} + \frac{\chi}{\delta}\left(\widehat{lq}_t - \vartheta\widehat{lq}_{t-1}\right) \tag{8}$$

Equations (6) and (8) make clear that interest rates in the model have an additional liquidity component relative to more standard DSGE models. The household interest rate $\hat{r}_t^h$ is influenced by both the short and the long rate.

As shown in the online Appendix, the domestic and foreign long-term interest rates can be decomposed into two primary components: a component that reflects expectations of future policy rates, and a term premium. The term premium in the model can be further decomposed into risk premium and liquidity premium components. The risk premium component is exogenous, while the liquidity premium is endogenous (a result of the portfolio adjustment cost). The domestic long-term interest rate is given by

$$\begin{aligned}\hat{r}_t^L = &\underbrace{(1-\beta\kappa)\sum_{i=0}^{\infty}(\beta\kappa)^i E_t\hat{r}_{t+i}^S}_{\text{Policy Rate Expectations }\left(\widehat{PE}_t\right)} + \underbrace{(1-\beta\kappa)\sum_{i=0}^{\infty}(\beta\kappa)^i\left(E_t\hat{\varepsilon}_{t+i}^{b^S} - E_t\hat{\varepsilon}_{t+i}^{b^L}\right)}_{\text{Risk Premium }\left(\widehat{RP}_t\right)} \\ &+ \underbrace{\frac{(1-\beta\kappa)\chi(1+\delta)}{\delta}\sum_{i=0}^{\infty}(\beta\kappa)^i E_t\left(\widehat{lq}_{t+i} - \vartheta\widehat{lq}_{t+i-1}\right)}_{\text{Liquidity Premium }\left(\widehat{LP}_t\right)}\end{aligned} \tag{9}$$

[7] The detailed derivations can be found in the online Appendix.

or more succinctly

$$\hat{r}_t^L = \widehat{PE}_t + \underbrace{\widehat{RP}_t + \widehat{LP}_t}_{\text{Term Premium } \left(\widehat{TP}_t\right)} \tag{10}$$

where $\hat{r}_t^L$ is the long-term interest rate expressed relative to its steady-state value, and $\beta\kappa$ is the subjective discount factor. These decompositions allow us to *i*) understand the channels through which the strong correlation between US and UK long-term interest rates arise (through synchronised changes in either policy expectations or term premia) and *ii*) derive DSGE term premium estimates, that can be compared to estimates from no-arbitrage term structure models. Similarly, the decomposition of the foreign long-term interest rate is given by

$$\begin{aligned}\hat{r}_t^{L,*} &= \underbrace{(1-\beta\kappa^*)\sum_{i=0}^{\infty}(\beta\kappa^*)^i E_t\hat{r}_{t+i}^{S,*}}_{\text{Policy Rate Expectations } \left(\widehat{PE}_t^*\right)} + \underbrace{(1-\beta\kappa^*)\sum_{i=0}^{\infty}(\beta\kappa^*)^i\left(E_t\hat{\varepsilon}_{t+i}^{b^S,*} - E_t\hat{\varepsilon}_{t+i}^{b^L,*}\right)}_{\text{Risk Premium } \left(\widehat{RP}_t^*\right)} \\ &+ \underbrace{\frac{(1-\beta\kappa)\chi^*(1+\delta^*)}{\delta^*}\sum_{i=0}^{\infty}(\beta\kappa)^i E_t\left(\widehat{lq}_{t+i}^* - \vartheta\widehat{lq}_{t+i-1}^*\right)}_{\text{Liquidity Premium } \left(\widehat{LP}_t^*\right)}\end{aligned} \tag{11}$$

or

$$\hat{r}_t^{L,*} = \widehat{PE}_t^* + \underbrace{\widehat{RP}_t^* + \widehat{LP}_t^*}_{\text{Term Premium } \left(\widehat{TP}_t^*\right)}. \tag{12}$$

It is perhaps worthwhile to illustrate how these expressions are related to earlier studies. For instance, in equation (6) if the liquidity friction is set to zero $(x = 0)$, then the effective interest rated faced by the households $\hat{r}_t^h = \hat{r}_t^S + \hat{\varepsilon}_t^{b^S}$ is the one used by Smets and Wouters (2007). In this case, the long-term interest rate $\left(\hat{r}_t^L\right)$ is fully determined by the policy rate and the two exogenous asset shocks. Furthermore, if both short- and long-term assets are subject to the same asset price shock $\left(\hat{\varepsilon}_t^{b^S} = \hat{\varepsilon}_t^{b^L}\right)$ in the intermediary's balance sheet equation (5), then the risk premium component of the long-term interest rate drops from its definition (9). In that case, the long rate only reflects expectations of policy rate as in most DSGE models (Christiano et al. (2005), Smets and Wouters (2007), Justiniano et al. (2010)).

Furthermore, long-term interest rates can be decomposed in terms of expectations about the real rate in the economy and the inflation, namely

$$\begin{aligned}\hat{r}_t^L &= \underbrace{(1-\beta\kappa)\sum_{i=0}^{\infty}(\beta\kappa)^i E_t\left(\hat{r}_{t+i}^S - \pi_{t+1+i}\right)}_{\text{Real Rate Expectations } \left(\widehat{RRE}_t\right)} + \underbrace{(1-\beta\kappa)\sum_{i=0}^{\infty}(\beta\kappa)^i E_t\pi_{t+1+i}}_{\text{Inflation Expectations } \left(\widehat{IE}_t\right)} + \underbrace{(1-\beta\kappa)\sum_{i=0}^{\infty}(\beta\kappa)^i\left(E_t\hat{\varepsilon}_{t+i}^{b^S} - E_t\hat{\varepsilon}_{t+i}^{b^L}\right)}_{\text{Risk Premium } \left(\widehat{RP}_t\right)} \\ &+ \underbrace{\frac{(1-\beta\kappa)\chi(1+\delta)}{\delta}\sum_{i=0}^{\infty}(\beta\kappa)^i E_t\left(\widehat{lq}_{t+i} - \vartheta\widehat{lq}_{t+i-1}\right)}_{\text{Liquidity Premium } \left(\widehat{LP}_t\right)}\end{aligned} \tag{13}$$

or

$$\hat{r}_t^L = \widehat{RRE}_t + \widehat{IE}_t + \widehat{TP}_t. \tag{14}$$

2.4 Exchange Rate

Standard SOE models formulate the UIP condition in terms of expected policy rates and an exogenous exchange rate risk premium shock (see Adolfson et al. (2007) and Burgess et al. (2013) among others). The latter is added to the model to capture variations of the exchange rate in the data that cannot be accounted by the real interest rate differentials. Interestingly, in our setup this "wedge" arises endogenously, as a function of asset specific risk shocks and liquidity spreads. The real exchange rate ($\hat{q}_t$, the relative price of US goods in terms of UK goods) is determined by:

$$\begin{aligned} E_t\Delta\hat{q}_{t+1} &= \underbrace{\hat{r}_t^S - E_t\pi_{t+1}^c - \left\{\hat{r}_t^{S,*} - E_t\pi_{t+1}^{c,*}\right\}}_{\text{Real Interest Rate Differentials}} + \\ &\underbrace{\frac{\chi\left(1+\delta\right)}{\delta}\left(\widehat{lq}_t - \vartheta\widehat{lq}_{t-1}\right) + \hat{\varepsilon}_t^{b^S} - \hat{\varepsilon}_t^{\bar{b}^L} - \left\{\frac{\chi^*\left(1+\delta^*\right)}{\delta^*}\left(\widehat{lq}_t^* - \vartheta\widehat{lq}_{t-1}^*\right) + \hat{\varepsilon}_t^{b^{S,*}} - \hat{\varepsilon}_t^{\bar{b}^{L^*}}\right\}}_{\text{Endogenous UIP Deviations}} \end{aligned} \tag{15}$$

We later show that the estimated model fits data for the real exchange rate remarkably well (Figure 2 shows one-step-ahead Kalman Filter projections against the data), which supports the formulation of our model.

2.5 Households

The economy is populated by a continuum of households that enjoy consumption and leisure. The maximisation of their utility function subject to their budget constraint gives rise to the following (aggregate) equilibrium conditions

$$\frac{\gamma^{\sigma_C} - \beta b}{\gamma^{\sigma_C}}\hat{\lambda}_t = -\frac{\sigma_C}{1-\frac{b}{\gamma}}\left\{\left(1+\frac{\beta b^2}{\gamma^{1+\sigma_C}}\right)\hat{c}_t - \frac{b}{\gamma}\left(\hat{c}_{t-1} - \hat{\gamma}_t\right) - \frac{\beta b}{\gamma^{\sigma_C}}\left(E_t\hat{c}_{t+1} + E_t\hat{\gamma}_{t+1}\right)\right\} \tag{16}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} - \sigma_C E_t\hat{\gamma}_{t+1} + r_t^h - E_t\hat{\pi}_{t+1}. \tag{17}$$

Equation (16) relates the aggregate consumption ($\hat{c}_t$) to marginal utility ($\hat{\lambda}_t$), while the Euler equation (17) illustrates how consumption is allocated across time optimally. Furthermore, consumption is a CES aggregator of the domestically produced $\left(\hat{c}_t^d\right)$ and imported consumption goods $\left(\hat{c}_t^m\right)$. The demand functions

$$\hat{c}_t^d = -\eta_c\hat{\tilde{p}}_t + \hat{c}_t \tag{18}$$

$$\hat{c}_t^m = -\eta_c\hat{\tilde{p}}_t^m + \hat{c}_t \tag{19}$$

show how these components are related to overall demanded quantity and their relative prices. Similarly, the CPI is a CES aggregator of domestic and import prices

$$\hat{\pi}_t^c - \hat{\tilde{p}}_{t-1} = \left(1-\alpha\right)\hat{\pi}_t + \alpha\tilde{p}^m\left(1-\eta_c\right)\left(\hat{\pi}_t^m + \hat{\tilde{p}}_{t-1}^m - \hat{\tilde{p}}_{t-1}\right). \tag{20}$$

It is further assumed that each household supplies a differentiated labour service to the production

sector. This feature makes them monopoly supplier of their labour (in the short run) and allows them to set their wage (Erceg et al. (2000)). Similar to price-setting, a fraction of households receives a random signal and set their wage optimally, while the remaining fraction uses a backward looking indexation scheme. The combination gives rise to the following labour supply curve

$$\begin{aligned}\left[\Delta\hat{w}_t+\hat{\gamma}_t+\hat{\pi}_t-\kappa_w\hat{\pi}_{t-1}+(1-\kappa_w)\,\bar{\pi}_t\right] &= \beta\gamma^{1-\sigma_c}\left[\Delta\hat{w}_{t+1}+\hat{\gamma}_{t+1}+\hat{\pi}_{t+1}-\kappa_w\hat{\pi}_t-(1-\kappa_w)\,\bar{\pi}_{+1}\right]\\ &\quad+\frac{\left(1-\beta\xi_w\gamma^{1-\sigma_c}\right)(1-\xi_w)}{\frac{[(\varphi+1)\lambda_w-1]\xi_w}{[\lambda_w-1]}}\left\{\varphi\hat{h}_t-\hat{\lambda}_t-\hat{w}_t\right\}+\hat{\hat{\lambda}}_{w,t}\end{aligned} \tag{21}$$

where $\hat{\hat{\lambda}}_{w,t}=\rho_w\hat{\hat{\lambda}}_{w,t-1}+\sigma_w\omega_{w,t}-\psi_w\sigma_w\omega_{w,t-1}$ captures exogenous variations of households' monopoly power.

2.6 Government, Monetary Policy & Resource Constraint

The government issues short and long-term debt and collects lump-sum taxes $\left(\hat{T}_t\right)$ to finance government consumption $(\hat{g}_t)$. Equation (22) represents the government budget constraint. The share of output consumed by the government follows an AR(1) process (23), while taxes are decided on the basis of a debt targeting rule that ensures that the amount of debt in the economy is eventually repaid (24).

$$\begin{aligned}\hat{b}_t^S+\delta\bar{b}_t^L &= \frac{1}{\beta}\left(\hat{b}_{t-1}^S+\delta\bar{b}_{t-1}^L-(1+\delta)\,\hat{\gamma}_t\right)-\frac{T}{b^S}\hat{T}_t\\ &\quad+\frac{1}{\beta}\left\{\hat{r}_{t-1}^S-\hat{\pi}_t^c+\delta\left(\hat{r}_t^L-\hat{\pi}_t^c+\hat{p}_{L,t}-\hat{p}_{L,t-1}\right)\right\}+\frac{\tilde{p}gy}{b^S}\left(\hat{\tilde{p}}_t+\hat{g}_t+\hat{y}_t\right)\end{aligned} \tag{22}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1}+\sigma_g\omega_{g,t} \tag{23}$$

$$\frac{T}{b^S}\hat{T}_t = \theta\left(\hat{b}_{t-1}^S+\delta\bar{b}_{t-1}^L-(1+\delta)\,\hat{\gamma}_t\right). \tag{24}$$

The central bank sets the policy rate based on a Taylor rule:

$$\hat{r}_t^S-\hat{\hat{\pi}}_t^c=\phi_R\left(\hat{r}_{t-1}^S-\hat{\hat{\pi}}_{t-1}^c\right)+(1-\phi_R)\left[\phi_\pi\left(\hat{\pi}_t^c-\hat{\hat{\pi}}_t^c\right)+\phi_y\hat{y}_t\right]+\sigma_R\omega_t^R. \tag{25}$$

Finally, equation (26) is the resource constraint:

$$\hat{y}_t=\frac{c^d}{(1-g)\,y}\hat{c}_t^d+\frac{c^x}{(1-g)\,y}\hat{c}_t^x+\hat{\hat{g}}_t. \tag{26}$$

2.7 Exports

In our model, the evolution of foreign debt is determined in the foreign economy. We derive an expression for exports $(\hat{c}_t^x)$ that ensures consistency between the domestic economy's debt and the evolution of foreign long-term debt given imports (in the online Appendix). In other words, exports are used to close the model. This is another attractive feature of this model, as the export demand function is typically assumed in these types of models, and not derived from agents' optimising behavior (see Justiniano and Preston (2010a), Mumtaz and Theodoridis (2015), Adolfson et al. (2007), and

Christiano et al. (2011) among others). We can derive the following expression

$$\begin{aligned}\frac{c^x}{\bar{b}^{L,D,*}}\left(\hat{q}_t+\hat{\tilde{p}}_t^x+\hat{c}_t^x\right) &= \frac{c^m}{\bar{b}^{L,D,*}}\left(\hat{q}_t+\hat{c}_t^m\right)+\hat{\tilde{b}}_t^{L,D,*}+\hat{\omega}_t-\varepsilon_t^{\bar{b}^L}\\ &\quad -\frac{1}{\beta}\left\{\hat{r}_t^{L,*}-\hat{\pi}_t^{c,*}+\Delta\hat{p}_{L,t}^*+\Delta\hat{q}_{t+1}+\hat{\tilde{b}}_{t-1}^{L,D,*}-\gamma_{t-1}+\hat{\varsigma}_t\right\} \qquad (27)\end{aligned}$$

where exports can be viewed as what the domestic economy needs to pay to foreign agents for imports and foreign assets after the capital returns on investing in foreign assets are subtracted. In other words, exports act as residuals in the net foreign asset position accumulation equation.

3 Estimation

3.1 Data

The model is estimated using data for both the US (the foreign economy) and the UK (the small open economy). The observable variables are, for both countries: real GDP per capita (year on year growth), real wage (year on year growth), CPI inflation (year on year growth), policy rate, 10-year zero-coupon government bond yield. Additionally, we observe the bilateral real exchange rate and UK import price inflation (year on year growth). The estimation sample runs from $1976Q1$ to $2016Q4$. We obtain data for the US from the Federal Reserve Economic Data source, maintained by the Federal Reserve Bank of St. Louis, and data for the UK from the Bank of England database (see Appendix D for more details).

3.2 Calibrated Parameters

Table 2 summarises the parameters that are calibrated prior to the estimation of the model. We follow the literature and set the discount factor equal to 0.995, which implies a steady-state value of the (annual) real interest rate of around 2% (for both countries). We assume log utility preferences ($\sigma_C^* = \sigma_C = 1$), a common choice in the literature (see Justiniano et al. (2010), Chen et al. (2012), Christiano et al. (2014) among others). Similar to Christiano et al. (2005) and Christiano et al. (2014), the Frisch elasticity and steady-state wage markup are set equal to 1 ($\sigma_L^* = \sigma_L = 1$) and 1.05 ($\lambda_w^* = \lambda_w = 1.05$), respectively. The government spending to GDP ratio – $g^* = 0.18$ and $g = 0.17$ – is calibrated to match the US and UK steady-state values of the government spending to GDP ratio in the data (see Smets and Wouters (2007) and Burgess et al. (2013)). The capital share in the production function ($\phi = \phi^*$) and the capital depreciation rate ($\delta_K = \delta_{K^*}$) are set equal to 0.36 and 0.025 respectively (as in Christiano et al. (2005) and Jermann and Quadrini (2012)). Similar to Rudebusch and Swanson (2012), we set the steady-state values of output, hours and TFP equal to 1, 1/3 and 1, respectively for both countries. Finally, the value of the US short-term debt-to-GDP ratio $\left(\frac{b^{S,*}}{y^*} = 0.65\right)$ and the ratio between long- and short-term debt δ_B^* match the data average of these measures (see De Graeve and Theodoridis (2016)).

Table 2: Calibrated Parameters

Mnemonic	Description	Value
$100(\beta^{-1}-1)$	Time Discount Factor	0.50
σ_C	Inverse Intertemporal Substitution	1.00
σ_L	Inverse Labour Supply Elasticity	1.00
λ_w	Steady-State Wages Markup	1.05
ϕ	Capital Share	0.36
δ_K	Capital Depreciation Rate	0.03
h	Steady-State Hours	0.33
g	Steady-State Government Spending to GDP Ratio	0.17
$\frac{b^S}{y}$	Steady-State Short-Term Debt to Output Ratio	0.65
σ_C^*	Foreign Inverse Intertemporal Substitution	1.00
σ_L^*	Foreign Inverse Labour Supply Elasticity	1.00
λ_{w^*}	Foreign Steady-State Foreign Wages Markup	1.05
ϕ^*	Foreign Capital Share	0.36
δ_K^*	Foreign Capital Depreciation Rate	0.03
δ_B^*	Foreign Long- to Short-Term Debt Ratio	1.00
h^*	Steady-State Foreign Hours	0.33
g^*	Steady-State Foreign Government Spending to GDP Ratio	0.18
$\frac{b^{S,*}}{y^*}$	Steady-State Short-Term Debt to Output Ratio	0.65

3.3 Prior Distributions

Tables 3 and 4 summarise the prior density function of the estimated parameters (which we refer to as primitive priors $\pi(\varpi)$). The moments of these distributions are set in line with those used in the literature (see Smets and Wouters (2007) and Adolfson et al. (2007)) and we, therefore, do not discuss them in great detail here. We focus attention on those parameters that are less common in the literature, starting with the liquidity adjustment cost ($\chi = \chi^*$). The prior mean of the financial friction is 5 and its standard deviation equal to 1. Although its prior mean is higher than the number in Chen et al. (2012), the two parameters are not exactly comparable.[8] Furthermore, the substantial prior dispersion around the mean ensures the strength of the liquidity friction can easily deviate away from the prior mean. The prior distribution for the maturity adjustment costs ($\vartheta^* = \vartheta$) is 0.75 and its standard deviation equal to 0.1. The prior mean of the lump sum tax response coefficient to total debt ($\theta^* = \theta$) is equal to 0.025, while the prior standard deviation is 0.015. This specification aims to induce a slow repayment of total debt consistently with what is observed in the data. Finally, the prior mean of the domestic long- and foreign long- to short-term debt ratio ($\delta_{B,D}$ and $\delta_{B,F}$) has been set equal to 0.5, so the sum of the two ratios delivers the same degree of liquidity in two economies.

The above priors are for individual parameters and are typically specified independent of one another. We follow Del Negro and Schorfheide (2008), Liu et al. (2013) and Christiano et al. (2011), among others, and form our priors "endogenously". This requires additional priors that reflect our beliefs regarding selected data moments, which are described in Table 5. The endogenous priors allow one to step away from the (occasionally unrealistic) indepent-prior assumption. This approach formalises the decisions most researchers make when deciding the prior moments of the estimated structural parameters. We briefly outline the main idea here, though interested readers are advised to explore

[8] As in De Graeve and Theodoridis (2016), the parameter $\tilde{x}$ controls not only the the elasticity of the term spread to bond supply but also its impact on the real economy. In Chen et al. (2012) the size of the latter effect depends on the degree of the market segmentation.

Table 3: Description of the Foreign Economy Estimated Parameter & Prior Moments

Mnemonic	Description	Density	Mean	STD
λ_{y^*}	Steady-State Foreign Prices Markup	Normal	1.20	0.05
χ^*	Liquidity Adjustment Cost	Normal	5.00	1.00
ϑ^*	Dynamic Liquidity Adjustment Cost	Beta	0.75	0.10
h^*	Consumption Habit	Beta	0.70	0.09
κ_y^*	Price Indexation	Beta	0.50	0.15
ξ_y^*	Price Reset Probability	Beta	0.50	0.09
κ_w^*	Wage Indexation	Beta	0.50	0.09
ξ_w^*	Wage Reset Probability	Beta	0.50	0.10
$\phi_{r^S}^*$	Policy Rate Smoothing	Beta	0.75	0.10
ϕ_π^*	Policy Reaction to Inflation	Normal	1.50	0.10
ϕ_y^*	Policy Reaction to Output	Normal	0.12	0.05
θ^*	Lump Sum Tax Response	Normal	0.03	0.01
ρ_γ^*	Persistence of Non Stationary Productivity Shock	Beta	0.50	0.20
$\rho_{B^S}^*$	Persistence of Short-Term Debt Risk Premium Shock	Beta	0.50	0.20
$\rho_{\bar{B}^L}^*$	Persistence of Long-Term Debt Risk Premium Shock	Beta	0.50	0.20
ρ_p^*	Persistence of Price Markup Shock	Beta	0.50	0.20
ρ_w^*	Persistence of Wage Markup Shock	Beta	0.50	0.20
$\rho_{\bar{\pi}}^*$	Persistence of Inflation Target Shock	Beta	0.50	0.20
$100\sigma_\gamma^*$	Uncertainty of Non Stationary Productivity Shock	Inv-Gamma	0.10	2.00
$100\sigma_{B^S}^*$	Uncertainty of Short-Term Debt Risk Premium Shock	Inv-Gamma	0.10	2.00
$100\sigma_{\bar{B}^L}^*$	Uncertainty of Long-Term Debt Risk Premium Shock	Inv-Gamma	0.10	2.00
$100\sigma_p^*$	Uncertainty of Price Markup Shock	Inv-Gamma	0.10	2.00
$100\sigma_w^*$	Uncertainty of Wage Markup Shock	Inv-Gamma	0.10	2.00
$100\sigma_{\bar{\pi}}^*$	Uncertainty of Inflation Target Shock	Inv-Gamma	0.05	2.00
$100\sigma_{r^S}^*$	Uncertainty of Policy Rate Shock	Inv-Gamma	0.10	2.00

Notes: STD denotes the prior standard deviation.

the preceding references.

Let $\mathcal{M}(\varpi)$ denote a vector of DSGE model-implied data moments (expressed as a function of the structural parameters vector) and $\widehat{\mathcal{M}}$ its empirical counterpart. Let us further assume that two vectors of moments are the same up to a vector of measurement errors $\mathcal{V}$

$$\widehat{\mathcal{M}} = \mathcal{M}(\varpi) + \mathcal{V}. \tag{28}$$

Then, as explained in Del Negro and Schorfheide (2008), a conditional distribution that reflects the beliefs about the above moment conditions can be obtained by combining the conditional density of (28), $\mathcal{L}\left(\mathcal{M}(\varpi)|\widehat{\mathcal{M}}\right)$, Bayes theorem, and the primitive prior distribution of the structural parameter vector:

$$p\left(\varpi|\widehat{\mathcal{M}}\right) \propto \mathcal{L}\left(\mathcal{M}(\varpi)|\widehat{\mathcal{M}}\right)\pi(\varpi). \tag{29}$$

There are several advantages of using this type of prior. For instance, as we can infer from (29), structural parameters are no longer treated as independent, as is typically assumed in the DSGE literature. Furthermore, shock processes are unobserved variables which makes it difficult to justify beliefs regarding the persistence and the volatility of these exogenous processes. In this set-up this is not a problem, since these prior moments adjust endogenously to "match" the selected data moments.

Finally, the empirical application in Del Negro and Schorfheide (2008) suggests that these priors can be helpful when DSGE parameters are not well identified.

3.4 Posterior Estimation

The posterior distribution of the DSGE parameter vector is approximated using the steps described in An and Schorfheide (2007). Specifically, after finding the posterior mode, we center the model parameters around the mode and run a Metropolis-Hastings sampler to obtain an estimate of the covariance matrix of the parameter vector; this step delivers also the scale of the covariance matrix that ensures an acceptance rate between 20% and 33%. Next, we use the posterior mode and the scaled inverse Hessian matrix (from the previous step) to simulate four chains of 75000 draws. We discard the first 150000, and from the remaining 150000 we keep 1200 draws.

3.5 Parameter Estimates

Tables 6 and 7 report the posterior moments of the estimated structural parameters, for the US and the UK respectively. The vast majority of the estimates are in line with those reported in previous studies, and we keep the discussion of the parameter estimates brief as a result.

US Estimates: Both the price and wage Phillips curves are flatter ($\kappa_y^* = 0.31$, $\xi_y^* = 0.92$, $\kappa_w^* = 0.42$ and $\xi_w^* = 0.73$) than those estimated by Smets and Wouters (2007). This is consistent with the findings of Del Negro et al. (2015), who argue that in a model with financial frictions, price and wage setters need to be more "forward looking" (relative to those in Smets and Wouters) in order for the model to be able to reconcile why inflation did not "fall of the cliff" during the Great Recession when demand collapsed. The degree of consumption smoothing ($h^* = 0.70$) is similar to estimates reported in the literature (Smets and Wouters (2007), Justiniano et al. (2010)). The debt reaction coefficient ($\theta^* = 0.02$) indicates that the government debt is repaid very gradually. The monetary reaction coefficients $\left(\phi_{rS}^* = 0.83, \phi_\pi^* = 1.74 \text{ and } \phi_y^* = 0.04\right)$ are again in the spectrum of estimates reported by other researchers. Finally, the liquidity adjustment cost parameter is less than the half of its prior mean ($\chi^* = 2.05$), while the data seems to favour a dynamic adjustment liquidity cost parameter close to 1 ($\vartheta^* = 0.94$).

UK Estimates: Domestic prices $\left(\kappa_y = 0.28 \text{ and } \xi_y = 0.91\right)$ and real wages ($\kappa_w = 0.77$ and $\xi_w = 0.73$) in the UK display similar degree of inertia as in the US. On the other hand, the import price Philips curve appears to be steeper ($\kappa_m = 0.25$ and $\xi_m = 0.50$) than the one for domestic prices suggesting a faster pass-through from the real exchange rate to import prices than from the labour share to domestic prices. Unlike import prices, export prices display less sensitivity to real exchange rate movements ($\kappa_x = 0.25$ and $\xi_x = 0.87$), consistent with the literature that wants exporting firms to be "large" (size) firms, which use markups to absorb variations in the exchange rate (see Gopinath and Itskhoki (2010) and Amiti et al. (2014)). The degree of UK consumption smoothing is similar to the US ($h = 0.81$). The import price elasticity is 1.45 and close to its prior mean. Fiscal authorities do not seem to respond aggressively to debt variations from its steady state ($\theta = 0.01$). Interestingly, the parameter measuring the UK financial liquidity friction ($\chi = 5.10$) is significantly

higher than its US counterpart, while the opposite is true for the dynamic liquidity adjustment cost ($\vartheta = 0.25$). The data favours a larger steady state share of domestic than foreign long-term debt ($\delta_{B,D} = 0.46$, $\delta_{B,F} = 0.18$). Although the UK policy reaction coefficient to inflation is much smaller than the US one $\left(\phi_{rS} = 0.90,\ \phi_{\pi} = 1.01 \text{ and } \phi_{y} = 0.11\right)$, the long run policy response is approximately the same across the two countries $\left(\frac{1.01}{1-0.9} \approx \frac{1.78}{1-0.83}\right)$. In other words, UK authorities respond less aggressively to changes in CPI inflation but these policy changes last for longer.

Table 4: Description of the Domestic Economy Estimated Parameter & Prior Moments

Mnemonic	Description	Density	Mean	STD
λ_y	Steady-State Domestic Prices Markup	Normal	1.20	0.05
λ_m	Steady-State Import Prices Markup	Normal	1.20	0.05
λ_x	Steady-State Export Prices Markup	Normal	1.20	0.05
η_C	Import Price Elasticity	Normal	1.50	0.15
$\delta_{B,D}$	Domestic Long- to Short-Term Debt Ratio	Normal	0.50	0.05
$\delta_{B,F}$	Foreign Long- to Short-Term Debt Ratio	Normal	0.50	0.05
χ	Liquidity Adjustment Cost	Normal	5.00	1.00
ϑ	Dynamic Liquidity Adjustment Cost	Beta	0.75	0.10
h	Consumption Habit	Beta	0.75	0.10
κ_y	Domestic Price Indexation	Beta	0.50	0.15
ξ_y	Domestic Price Reset Probability	Beta	0.50	0.09
κ_m	Import Price Indexation	Beta	0.50	0.15
ξ_m	Import Price Reset Probability	Beta	0.50	0.09
κ_w	Wage Indexation	Beta	0.50	0.15
ξ_w	Wage Reset Probability	Beta	0.50	0.09
κ_x	Export Price Indexation	Beta	0.50	0.15
ξ_x	Export Price Reset Probability	Beta	0.50	0.09
ϕ_{r^S}	Policy Rate Smoothing	Beta	0.75	0.10
ϕ_π	Policy Reaction to Inflation	Normal	1.50	0.09
ϕ_y	Policy Reaction to Output	Normal	0.12	0.05
θ	Lump Sum Tax Response	Normal	0.03	0.01
ρ_ω	Persistence of Stationary Productivity Shock	Beta	0.50	0.20
ρ_{B^S}	Persistence of Short-Term Debt Risk Premium Shock	Beta	0.50	0.20
$\rho_{\bar{B}^L}$	Persistence of Long-Term Debt Risk Premium Shock	Beta	0.50	0.20
ρ_p	Persistence of Price Markup Shock	Beta	0.50	0.20
ρ_w	Persistence of Wage Markup Shock	Beta	0.50	0.20
ρ_m	Persistence of Import Pice Markup Shock	Beta	0.50	0.20
$\rho_{\bar{\pi}}$	Persistence of Inflation Target Shock	Beta	0.50	0.20
$100\sigma_\omega$	Uncertainty of Productivity Shock	Inv-Gamma	0.10	2.00
$100\sigma_{B^S}$	Uncertainty of Short-Term Debt Risk Premium Shock	Inv-Gamma	0.10	2.00
$100\sigma_{\bar{B}^L}$	Uncertainty of Long-Term Debt Risk Premium Shock	Inv-Gamma	0.10	2.00
$100\sigma_p$	Uncertainty of Price Markup Shock	Inv-Gamma	0.10	2.00
$100\sigma_w$	Uncertainty of Wage Markup Shock	Inv-Gamma	0.10	2.00
$100\sigma_m$	Uncertainty of Import Pice Markup Shock	Inv-Gamma	0.10	2.00
$100\sigma_{\bar{\pi}}$	Uncertainty of Inflation Target Shock	Inv-Gamma	0.05	2.00
$100\sigma_{r^S}$	Uncertainty of Policy Rate Shock	Inv-Gamma	0.10	2.00

Notes: STD denotes the prior standard deviation.

Table 5: Prior Distribution of Selected Moments

Mnemonic	Description	Density	Mean	STD
$\rho_{r^L,r^{L,*}}$	Correlation between Domestic and Foreign Long-Term Interest Rate	Normal	0.93	0.01
ρ_{r^S,r^L}	Correlation between Short- and Long-Term Domestic Interest Rate	Normal	0.90	0.01
$\rho_{r^S,r^{S,*}}$	Correlation between Domestic and Foreign Short-Term Interest Rate	Normal	0.84	0.01
ρ_{y,y^*}	Correlation between Domestic and Foreign GDP	Normal	0.65	0.01
ρ_{π,π^*}	Correlation between Domestic and Foreign Inflation	Normal	0.85	0.01
ρ_{c,c^*}	Correlation between Domestic and Foreign Consumption	Normal	0.62	0.01
σ_{r^S}	Standard Deviation of Domestic Short-Term Interest Rate	Normal	4.50	0.10
σ_{r^L}	Standard Deviation of Domestic Long-Term Interest Rate	Normal	3.63	0.10
$\sigma^*_{r^S}$	Standard Deviation of Foreign Short-Term Interest Rate	Normal	4.02	0.10
$\sigma^*_{r^L}$	Standard Deviation of Foreign Long-Term Interest Rate	Normal	3.12	0.10
σ_y	Standard Deviation of Domestic GDP	Normal	2.19	0.10
σ_{y^*}	Standard Deviation of Foreign GDP	Normal	2.09	0.10
σ_π	Standard Deviation of Domestic Inflation	Normal	4.07	0.10
σ^*_π	Standard Deviation of Foreign Inflation	Normal	2.04	0.10
σ_q	Standard Deviation of Real Exchange Rate	Normal	11.22	0.10
σ_w	Standard Deviation of Domestic Wage	Normal	1.53	0.10
σ^*_w	Standard Deviation of Foreign Wage	Normal	1.47	0.10

Notes: STD denotes the prior standard deviation.

Table 6: Foreign Parameter Estimates

Mnemonic	Description	Mode	Mean	5^{th}	95^{th}
λ_{y^*}	Steady-State Foreign Prices Markup	1.11	1.10	1.07	1.13
χ^*	Liquidity Adjustment Cost	2.05	1.98	1.85	2.12
ϑ^*	Dynamic Liquidity Adjustment Cost	0.94	0.94	0.93	0.95
h^*	Consumption Habit	0.70	0.70	0.68	0.72
κ^*_y	Price Indexation	0.31	0.23	0.21	0.26
ξ^*_y	Price Reset Probability	0.92	0.92	0.92	0.92
κ^*_w	Wage Indexation	0.42	0.41	0.39	0.44
ξ^*_w	Wage Reset Probability	0.73	0.74	0.70	0.76
$\phi^*_{r^S}$	Policy Rate Smoothing	0.83	0.84	0.82	0.86
ϕ^*_π	Policy Reaction to Inflation	1.78	1.81	1.79	1.83
ϕ^*_y	Policy Reaction to Output	0.04	0.03	0.03	0.04
θ^*	Lump Sum Tax Response	0.02	0.01	0.01	0.02
ρ^*_γ	Persistence of Non Stationary Productivity Shock	0.09	0.05	0.01	0.08
$\rho^*_{B^S}$	Persistence of Short-Term Debt Risk Premium Shock	0.91	0.93	0.92	0.95
$\rho^*_{\bar{B}^L}$	Persistence of Long-Term Debt Risk Premium Shock	0.89	0.89	0.87	0.90
ρ^*_p	Persistence of Price Markup Shock	0.02	0.03	0.01	0.06
ρ^*_w	Persistence of Wage Markup Shock	0.90	0.91	0.89	0.93
$\rho^*_{\bar{\pi}}$	Persistence of Inflation Target Shock	0.21	0.21	0.16	0.24
$100\sigma^*_\gamma$	Uncertainty of Non Stationary Productivity Shock	0.97	1.02	0.97	1.07
$100\sigma^*_{B^S}$	Uncertainty of Short-Term Debt Risk Premium Shock	0.18	0.14	0.10	0.17
$100\sigma^*_{\bar{B}^L}$	Uncertainty of Long-Term Debt Risk Premium Shock	0.43	0.44	0.38	0.50
$100\sigma^*_p$	Uncertainty of Price Markup Shock	0.12	0.12	0.11	0.13
$100\sigma^*_w$	Uncertainty of Wage Markup Shock	0.71	0.70	0.66	0.74
$100\sigma^*_{\bar{\pi}}$	Uncertainty of Inflation Target Shock	0.02	0.02	0.02	0.02
$100\sigma^*_{r^S}$	Uncertainty of Policy Rate Shock	0.19	0.18	0.17	0.19

Notes: The column Mode refers to the argument of the posterior kernel minimisation, while the columns Mean, 5^{th} and 95^{th} to the mean, 0.05 and 0.95 percentiles of the posterior distribution of the structural parameter vector.

Table 7: Domestic Parameter Estimates

Mnemonic	Description	Mode	Mean	5^{th}	95^{th}
λ_y	Steady-State Domestic Prices Markup	1.13	1.15	1.13	1.16
λ_m	Steady-State Import Prices Markup	1.24	1.24	1.23	1.25
λ_x	Steady-State Export Prices Markup	1.05	1.05	1.05	1.06
η_C	Import Price Elasticity	1.45	1.47	1.42	1.52
$\delta_{B,D}$	Domestic Long- to Short-Term Debt Ratio	0.46	0.47	0.46	0.48
$\delta_{B,F}$	Foreign Long- to Short-Term Debt Ratio	0.18	0.19	0.18	0.19
χ	Liquidity Adjustment Cost	5.10	4.94	4.57	5.26
ϑ	Dynamic Liquidity Adjustment Cost	0.25	0.25	0.22	0.28
h	Consumption Habit	0.81	0.81	0.79	0.82
κ_y	Domestic Price Indexation	0.28	0.24	0.20	0.27
ξ_y	Domestic Price Reset Probability	0.91	0.92	0.91	0.92
κ_m	Import Price Indexation	0.25	0.22	0.20	0.24
ξ_m	Import Price Reset Probability	0.50	0.51	0.50	0.52
κ_w	Wage Indexation	0.77	0.78	0.74	0.80
ξ_w	Wage Reset Probability	0.73	0.75	0.73	0.77
κ_x	Export Price Indexation	0.27	0.26	0.23	0.30
ξ_x	Export Price Reset Probability	0.87	0.89	0.87	0.92
ϕ_{r^S}	Policy Rate Smoothing	0.90	0.90	0.89	0.91
ϕ_π	Policy Reaction to Inflation	1.01	1.01	1.00	1.01
ϕ_y	Policy Reaction to Output	0.11	0.09	0.08	0.10
θ	Lump Sum Tax Response	0.01	0.01	0.01	0.01
ρ_ω	Persistence of Stationary Productivity Shock	0.16	0.15	0.11	0.19
ρ_{B^S}	Persistence of Short-Term Debt Risk Premium Shock	0.87	0.87	0.86	0.88
$\rho_{\bar{B}^L}$	Persistence of Long-Term Debt Risk Premium Shock	0.83	0.83	0.80	0.85
ρ_p	Persistence of Price Markup Shock	0.01	0.02	0.01	0.03
ρ_w	Persistence of Wage Markup Shock	0.93	0.93	0.91	0.95
ρ_m	Persistence of Import Pice Markup Shock	0.77	0.77	0.75	0.80
$\rho_{\bar{\pi}}$	Persistence of Inflation Target Shock	0.04	0.03	0.01	0.04
$100\sigma_\omega$	Uncertainty of Productivity Shock	2.50	2.56	2.44	2.68
$100\sigma_{B^S}$	Uncertainty of Short-Term Debt Risk Premium Shock	0.47	0.46	0.42	0.50
$100\sigma_{\bar{B}^L}$	Uncertainty of Long-Term Debt Risk Premium Shock	0.65	0.65	0.59	0.72
$100\sigma_p$	Uncertainty of Price Markup Shock	0.52	0.54	0.51	0.58
$100\sigma_w$	Uncertainty of Wage Markup Shock	0.53	0.54	0.50	0.57
$100\sigma_m$	Uncertainty of Import Pice Markup Shock	1.67	1.62	1.48	1.76
$100\sigma_{\bar{\pi}}$	Uncertainty of Inflation Target Shock	0.01	0.01	0.01	0.01
$100\sigma_{r^S}$	Uncertainty of Policy Rate Shock	0.15	0.15	0.14	0.15

Notes: The column Mode refers to the argument of the posterior kernel minimisation, while the columns Mean, 5^{th} and 95^{th} to the mean, 0.05 and 0.95 percentiles of the posterior distribution of the structural parameter vector.

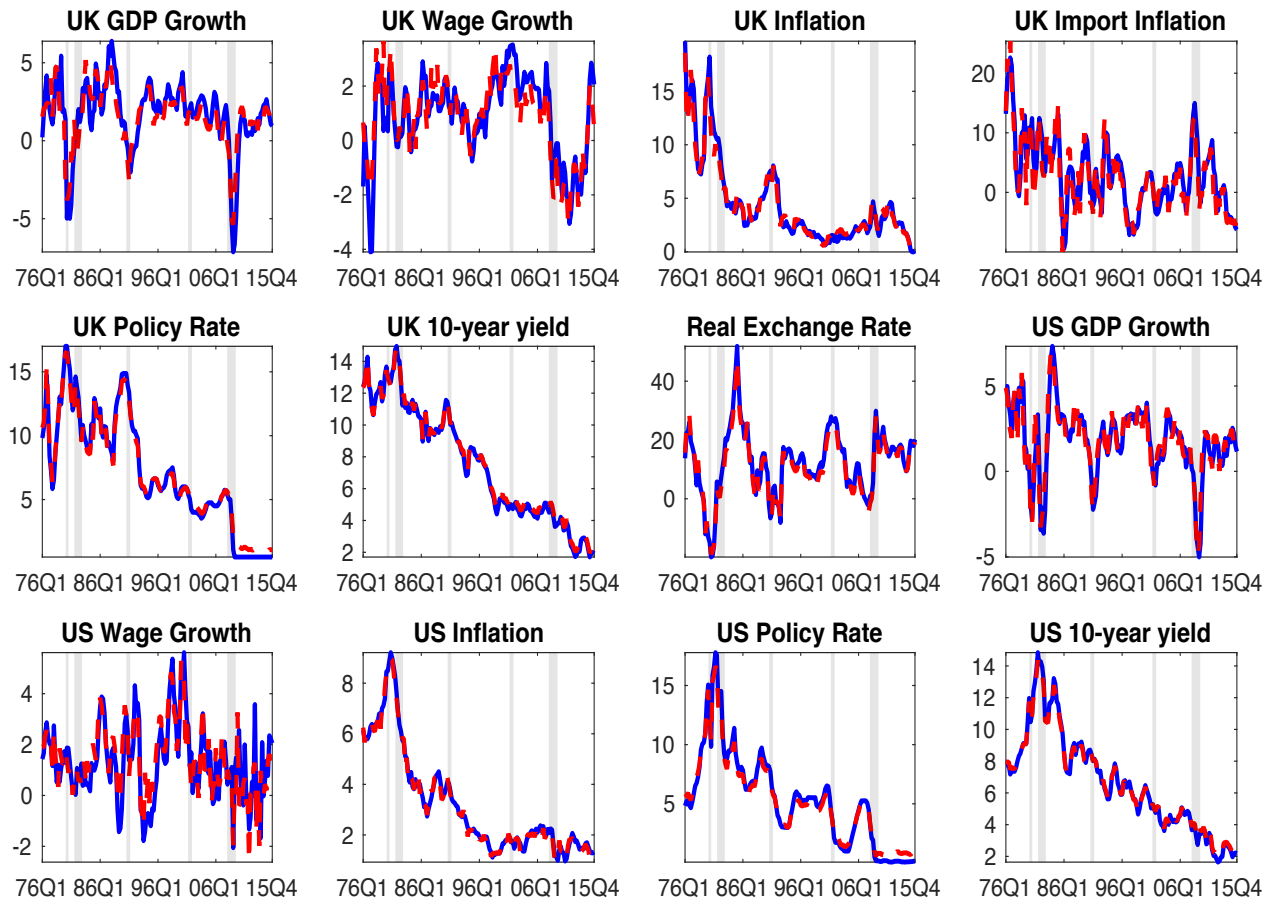
4 Model validation

Prior to analysing comovement, we evaluate the model's empirical performance from various perspectives. We first illustrate the model's superior fit compared to the prototypical small open economy DSGE models. We then validate the model's estimate of term premia in both countries by comparing them to off-model information. Finally, we show the structural model implies reduced form estimates in line with those of the reduced form literature on international long-term interest rate comovement. We describe these successes fairly succinctly, as in themselves they do not constitute the core question of the paper. They do provide support for the model's interpretation of international comovement which we turn to at length in Section 5.

4.1 Model Fit and Moment Estimates

Following Adolfson et al. (2007), Figure 2 shows the one-step-ahead Kalman filter (in sample) model predictions (red dashed line) against the observed data series (blue solid line). Overall, the model is able to match the full range of real economy and financial data remarkably well. This holds through all phases of the business cycle.

Figure 2: DSGE Model Fit



Notes: Observed data (solid line), one-step-ahead predictions (dashed line). Shaded areas denote US NBER recessions.

Table 8 shows a selection of key moments from the estimated model. Reassuringly, the model matches

the correlation between the US and UK long-term interest rates, and the correlation between short- and long-term interest rates. Specific for long-term interest rate comovement, it is worth stressing the present model matches *both* correlations and standard deviations of interest rates. Kulish and Rees (2011) emphasize their success in replicating correlations, but completely miss the variance of long-term interest rates (see e.g. their Table 5). As shown in De Graeve et al. (2009), attempts to explain fluctuations at the long end of the yield curve on the basis of standard short-lived business cycle shocks (as Kulish and Rees (2011) do) is bound to fall short of capturing the substantial long-term interest rate volatility observed in the data.

A promising feature of Table 8 is the ability of the model to match the observed correlation between US and UK GDP as well as consumption. As discussed in Kollmann (1996, 2001), these correlations are characteristics of the data that open economy DSGE studies have had difficulty in replicating. As shown in Kollmann (2001), sticky prices, sticky wages, and incomplete asset markets (which are all features of our model) can help these theoretical models to move closer to matching the correlation in the data. In our case, however, these moments are replicated almost exactly, which we interpret this as further support for the model structure proposed here.

Finally, it is worth emphasizing the model's ability to replicate the standard deviations of the estimated series, even the exchange rate. At least in part, this result arises from the use of endogenous priors. It is well known – though not discussed widely – that 'purely' Bayesian estimated DSGE models can imply moments that are far off those observed in the data (see Del Negro and Schorfheide (2008), Christiano et al. (2011)). The present model matches *both* the historical data (Figure 2) and its unconditional moments (Table 8).

Table 8: Posterior Distribution of Selected Moments

Mnemonic	Description	Data	Mode	5^{th}	95^{th}
$\rho_{r^L,r^{L,*}}$	Correlation between Domestic and Foreign Long-Term Interest Rate	0.93	0.93	0.93	0.94
ρ_{r^S,r^L}	Correlation between Short- and Long-Term Domestic Interest Rate	0.90	0.89	0.89	0.90
$\rho_{r^S,r^{S,*}}$	Correlation between Domestic and Foreign Short-Term Interest Rate	0.84	0.79	0.79	0.80
ρ_{y,y^*}	Correlation between Domestic and Foreign GDP	0.65	0.65	0.64	0.66
ρ_{π,π^*}	Correlation between Domestic and Foreign Inflation	0.85	0.83	0.83	0.84
ρ_{c,c^*}	Correlation between Domestic and Foreign Consumption	0.62	0.61	0.60	0.62
σ_{r^S}	Standard Deviation of Domestic Short-Term Interest Rate	4.50	4.67	4.62	4.72
σ_{r^L}	Standard Deviation of Domestic Long-Term Interest Rate	3.63	3.48	3.43	3.53
$\sigma^*_{r^S}$	Standard Deviation of Foreign Short-Term Interest Rate	4.02	3.56	3.50	3.62
$\sigma^*_{r^L}$	Standard Deviation of Foreign Long-Term Interest Rate	3.12	2.68	2.61	2.73
σ_y	Standard Deviation of Domestic GDP	2.19	2.15	2.08	2.22
σ_{y^*}	Standard Deviation of Foreign GDP	2.09	2.12	2.03	2.18
σ_π	Standard Deviation of Domestic Inflation	4.07	4.69	4.64	4.74
σ^*_π	Standard Deviation of Foreign Inflation	2.04	3.00	2.95	3.05
σ_q	Standard Deviation of Real Exchange Rate	11.22	11.36	11.28	11.43
σ_w	Standard Deviation of Domestic Wage	1.53	1.71	1.67	1.76
σ^*_w	Standard Deviation of Foreign Wage	1.47	1.63	1.57	1.68

Notes: The 5^{th} and 95^{th} columns refer to 0.05 and 0.95 percentiles of the posterior distribution centred around the posterior mode moments.

4.2 Term Premium Estimates

As described in Section 2, one can easily derive term premium estimates directly from our model: $\widehat{TP}_t^* = r_t^{L,*} - \frac{1}{n}\sum_{i=0}^{n-1} E_t r_{t+i}^*$. We here compare our structural model estimates with estimates from the macro-finance literature. There is large body of research dedicated to the estimation of term premia, where the majority of studies use reduced-form no-arbitrage term structure models (see Dai and Singleton (2000), Duffee (2002) and Kim and Wright (2005) among others). These estimates reflect the difference between the observed bond yield, and the average expected short rate over the life of the long-term bond. For our comparisons, we use term premia estimates obtained using the methodology developed in Adrian et al. (2013), for the US and UK.[9] The US estimate is obtained from the Federal Reserve of New York website, and the UK estimate is taken from Malik and Meldrum (2016), who use the same approach to derive UK term premium estimates from $1997Q1$ onwards.[10]

Figures 3 and 4 show the term premium fluctuations for both the reduced-form model (solid lines) and the DSGE model (dashed lines). Despite the very different nature of the models underlying these estimates, they both pick up similar cyclical variation in term premia. This holds for both the US and the UK. The correlation between the time-series and DSGE estimate is 0.65 and 0.6 for US for UK, respectively. For both the US and the UK, the term premium estimates rise prior to recessions, and they stay elevated for a few years after the economy has returned to positive GDP growth. This is consistent with the conventional theory and empirical evidence suggesting that term premia are countercyclical (e.g. Rudebusch and Swanson (2012)).

4.3 Reduced Form Models of Comovement

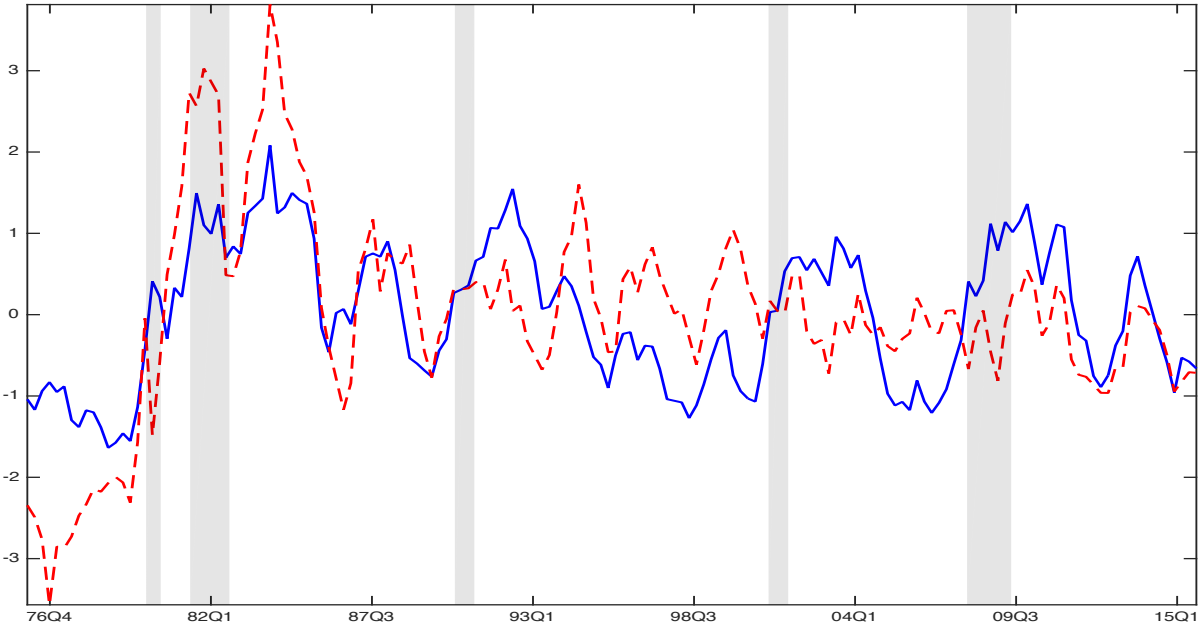
The empirical literature on international interest rate comovement describes international interest rates across the maturity spectrum as a function of level and slope factors, both of which can be local or global. This is the case in Affine Term Structure Models (e.g. Diebold et al. (2008); Jotikasthira et al. (2015)) and in dynamic factor models (e.g. Abbritti et al. (2018)) of international interest rates. The first column block of Table 9 replicates the core underlying result documented in these studies, applied to our dataset. Specifically, we run a dynamic FAVAR model on the interest rates (with maturities 3 months, 5 year and 10 years) of both the US and the UK.[11] The Table shows that world interest rates can be captured largely by a world level factor (affecting yields of all maturities in both countries). Country-specific factors explain only a small fraction of the remaining short-maturity variance (i.e. a local slope factor). While these models are void of economic structure, it is worthwhile asking whether our structural model is consistent with their reduced form results. To that end, we simulate pseudo-data from the estimated DSGE model's posterior and estimate the same reduced form model 500 times. The second column block reports the median estimated variance decompositions across replications. As in the data, the model explains the bulk of variance in interest rates by a global

[9] The estimation of term structure models is a computationally challenging task, as it requires numerical Maximum Likelihood optimization techniques to be applied to large scale models. Recently, Adrian et al. (2013) proposed a methodology/model where the Maximum Likelihood optimization problem can be replaced by a three-step ordinary least squares estimator. The authors illustrate that their model performs equally well (if not better) relative to models that have been widely used in the literature.

[10] A slight difference is that the reduced-form term premium estimates reflect the difference between bond yields and average (equally-weighted) policy rate expectations over the life of the bond, while the DSGE term premia are based on unequally weighted expectations. That is, as shown in equation (9), longer-term expectations are more heavily discounted relative to the near-term (by a factor of $\beta\kappa$).

[11] The detailed specification of the factor model is provided in appendix A.

Figure 3: US DSGE versus Adrian et al. (2013) US Term Premium Estimate



Notes: The DSGE Model US Kalman Smoothed Term Premium $\left(\widehat{TP}_t^* = \widehat{RP}_t^* + \widehat{LP}_t^*\right)$ estimate (dashed line) against the Adrian et al. (2013) Affine Term-Structure Model Term Premium estimate (solid line). Shaded areas denote NBER US recession periods. The correlation coefficient between the two estimates is 0.65.

level factor. Country-specific slope factors absorb the remaining variation, although their variance (at the median) is slightly larger for the simulated samples than it is in the data. The sampling variance (indicated by the 5-95th percentile bands) indicates that the factor model estimated on the data falls well within the range covered by the DSGE model's posterior distribution. This shows that the DSGE model (which has a very specific structure) is consistent with reduced form models in the literature. Of course, the present model has the advantage that there is a clear interpretation of the different factors underlying the reduced form correlations. It can thus structurally explain comovement, in a manner which earlier reduced form models cannot.

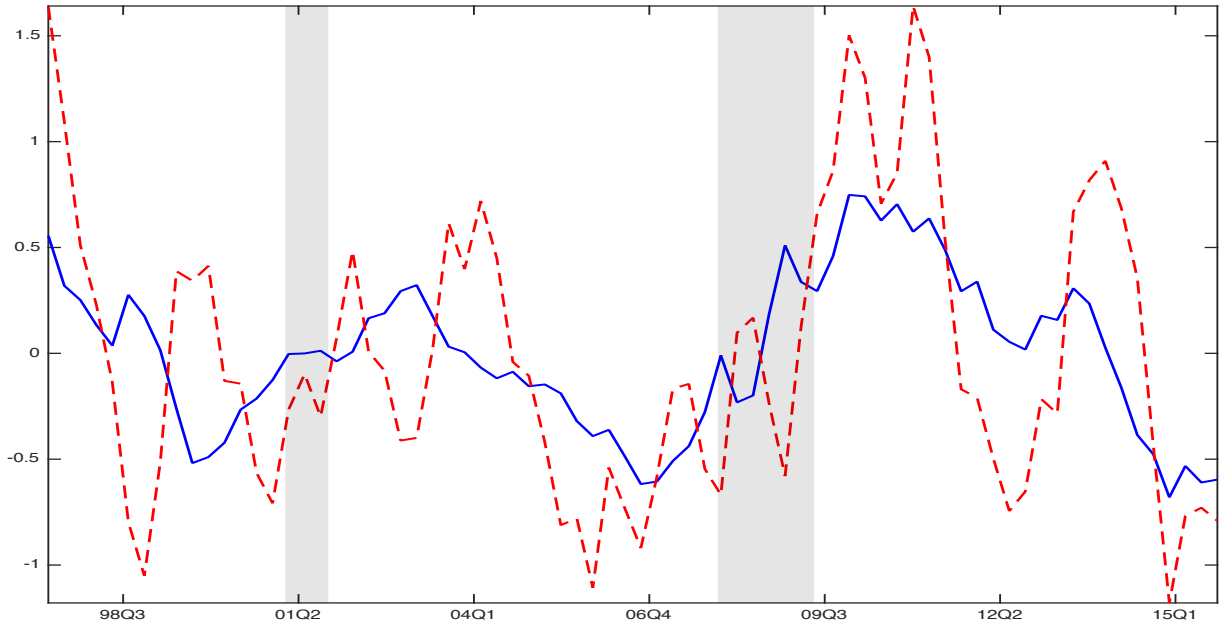
Table 9: Reduced-Form Analysis: Variance Contributions

Variables				**Simulations**								
	Data			**Median**			5^{th}			95^{th}		
	World	US	UK	World	US	UK	World	US	UK	World	US	UK
US 3 month	66.35	33.65	0.00	42.69	57.31	0.00	10.68	9.90	0.00	90.10	89.32	0.00
US 5 year	84.49	15.51	0.00	65.72	34.28	0.00	19.63	2.00	0.00	98.00	80.37	0.00
US 10 year	90.42	9.58	0.00	76.28	23.72	0.00	29.27	1.77	0.00	98.23	70.73	0.00
UK 3 month	85.94	0.00	14.06	78.05	0.00	21.95	21.25	0.00	1.09	98.91	0.00	78.75
UK 5 year	98.99	0.00	1.01	83.32	0.00	16.68	16.34	0.00	1.04	98.96	0.00	83.66
UK 10 year	99.27	0.00	0.73	91.21	0.00	8.79	17.60	0.00	0.56	99.44	0.00	82.40

Notes: The estimated DSGE model is used to simulate 500 pseudo datasets of length equal to our sample's time span. The factor model is estimated for each pseudo dataset. The table shows the distribution across these estimates, in terms of the variance contribution of world, US and UK factors.

Summing up, the validation exercises performed all provide support for the estimated SOE model. The fit is impressive compared to extant international DSGE models, the estimated dynamics of term

Figure 4: UK DSGE versus Adrian et al. (2013) UK Term Premium Estimate ($1997Q1 - 2015Q4$)



Notes: The DSGE Model US Kalman Smoothed Term Premium $\left(\widehat{TP}_t^* = \widehat{RP}_t^* + \widehat{LP}_t^*\right)$ estimate (solid line) against the Adrian et al. (2013) Affine Term-Structure Model Term Premium estimate (dashed line). Shaded areas denote NBER US recession periods. The correlation coefficient between the two estimates is 0.60.

premia very much in line with single country estimates for both the US and the UK. In addition, the model is fully consistent with reduced form model estimates of international long-term interest rate comovement. These empirical successes lend credence to the model's structural interpretation of comovement, to which we turn next.

5 Understanding Comovement

5.1 Decomposing Comovement

What causes comovement? Table 10 decomposes international long rate comovement conditional on each structural shock. Any explanation for comovement needs to satisfy two conditions: generate a realistically high positive correlation between *and* a sufficiently high variance of long-term interest rates in both countries. The first column shows the conditional correlation between long-term interest rates in the US and the UK induced by each shock. The second and third columns show the standard deviation of long rates in both countries, again conditional on each structural shock.

Four shocks produce positive correlations that are in the vicinity of the high correlation observed in the data (productivity, ST debt risk premium, price markup, inflation target). However, high conditional correlation is not sufficient to explain international comovement. Consider, for instance, the non-stationary productivity shock. While it causes a very high and realistic correlation between long rates, it does not generate much variance in long rates in either country. Temporary US monetary policy shocks do spill over to the UK, but are not hugely important in explaining long-term yields overall (see e.g. De Graeve et al. (2009)), or the macroeconomy for that matter (see e.g. Smets

and Wouters (2007)). Conversely, consider the LT debt risk premium shock. This shock is able to generate quantitatively significant variance in both US and UK long-term interest rates, yet it causes a correlation of the wrong sign. These shocks capture, for instance, an increase in the convenience yield or safety premium attached to US bonds. As such, they imply a substitution away from non-US bonds, and thereby a negative correlation between US and UK rates. The last row of the table shows that the only shock that captures comovement in the data is the inflation-target shock: it generates a very high positive correlation between the US and the UK *and* a variance of both countries' long rates that is substantial relative to that observed in the data.

Table 10: Decomposing Comovement

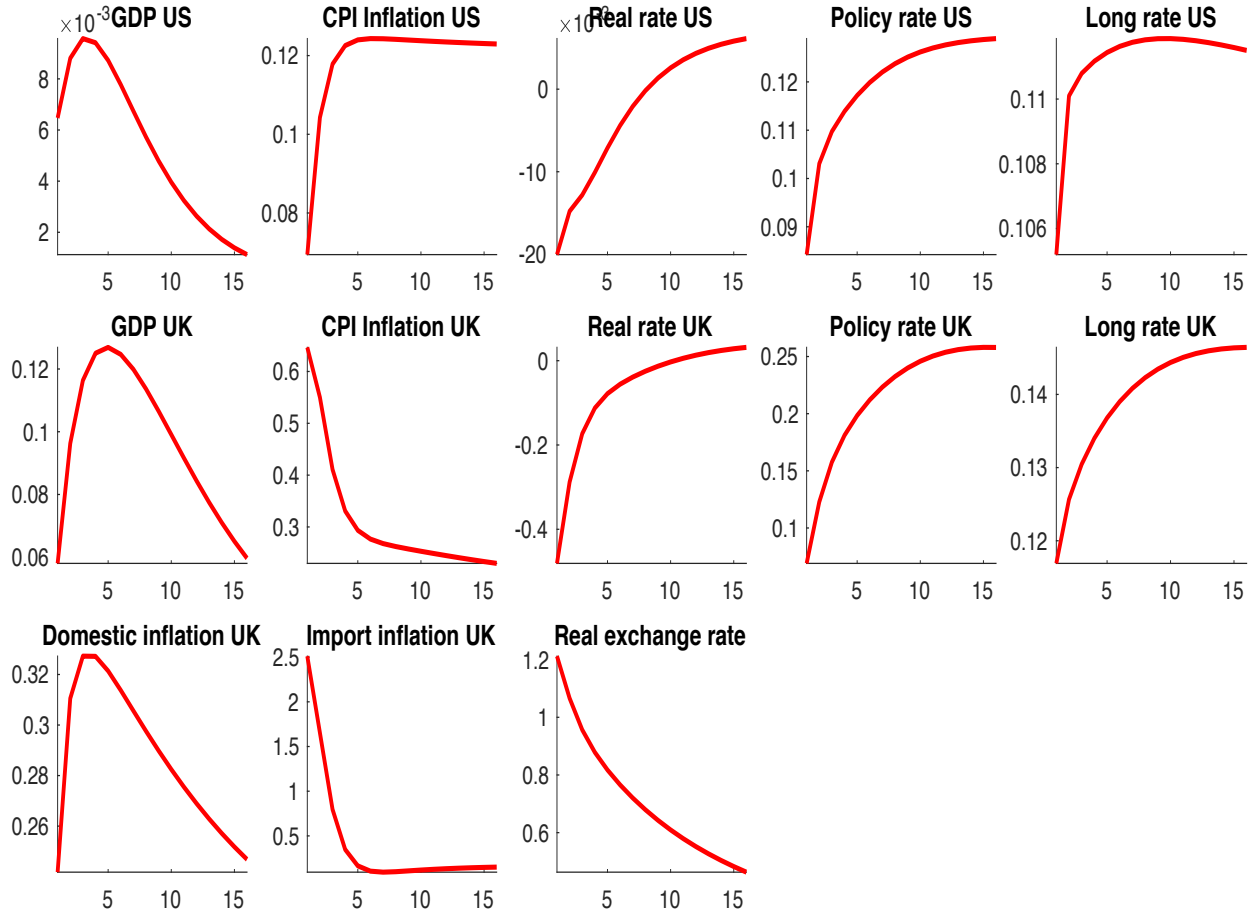
	$\rho\left(r^L, r^{L*}\right)$	$\sigma\left(r^{L*}\right)$	$\sigma\left(r^L\right)$
Data	0.93	3.12	3.63
US Productivity	0.98	0.12	0.13
US Monetary policy	0.67	0.16	0.06
US LT debt risk premium	-0.32	1.16	0.25
US ST debt risk premium	0.84	0.15	0.06
US Wage markup	-0.06	0.10	0.02
US Price markup	0.92	0.02	0.01
US Inflation target	1.00	1.41	2.33

Notes: The table suppresses UK shocks since these cannot generate variance in the US due to the small open economy nature of the UK in the model. In other words, conditional on UK shocks $\sigma(r^{L*}) = \rho(r^L, r^{L*}) = 0$. In the $\sigma(r^L)$-column the contributions do not sum to the total data variance because of the additional variance caused by UK-specific shocks (not reported).

Since inflation-target shocks are the only ones able to empirically explain comovement, let us now analyse their transmission. Figure 5 shows the impulse response functions to an increase in the US inflation target. US inflation expectations increase on impact and rise to a permanently higher level, which gradually feeds into US CPI inflation. The Fed gradually increases the nominal rate to reflect the higher target. Because the policy rate rises to a permanently higher level the US long-term interest rate rises on impact. In the long run, however, the US real rate returns to its initial level. In the short term, the real rate is temporarily reduced, causing a temporary expansion in the US. These effects are standard, and match those in e.g. Smets and Wouters (2005), De Graeve et al. (2009) and Cogley et al. (2010).

Now let us turn to the impact on the UK. The UK experiences a very strong boom in GDP and a substantial increase in inflation, both much higher than what is witnessed in the US. Two forces combine to make this happen. On the one hand, the BoE does not respond strongly to inflation, as apparent from the estimated coefficient on inflation ($\phi_\pi = 1.01$). On the other hand, nothing the UK does changes the fact that US inflation is permanently higher. Combined, they cause inflation expectations in the UK to rise permanently, and the central bank response to it is insufficient to offset that. As a result, the UK real interest rate drops substantially: the real rate drops by a factor much larger than in the US. Consequently, the UK experiences a large boom for two reasons: directly, as a result of the low UK real rate, and indirectly, as the real exchange rate depreciates substantially (the latter a reflection of the prolonged low UK real rate relative to the US). Despite the BoE increasing the policy rate substantially and permanently, agents in the model understand it does not suffice to offset the higher imported inflation and its pass-through into domestic inflation. The permanent increase in the policy rate naturally causes the UK long-term interest rate to increase. Consequently,

Figure 5: US Inflation Target Shock



in response to a US inflation target increase, the US and UK long-term rate both go up. In other words, international long-term interest rate comovement arises.

5.2 The Forces Driving Comovement: Counterfactuals

Taken together, the estimated model suggests that changing inflation trends in the US: 1) generate long-term interest rate comovement, 2) cause the UK to respond more strongly than the US, and 3) that both are caused by the complacency of the BoE toward low frequency movements in foreign inflation.

We here first develop the intuition behind that result, making reference only to core components of traditional international macro models. We then show by means of counterfactuals that this intuition carries over to the full empirical model.

5.2.1 Intuition

International macro models typically consist of the following three relations:

(i) the definition of the **real exchange rate**: $Q_t = \frac{S_t P_t^*}{P_t}$, where S is the nominal exchange rate

(amount of domestic currency per dollar), and P (resp. P^*) the price level in the domestic economy (resp. US). In log differences:

$$\Delta q_t = \Delta s_t + \pi_t^* - \pi_t. \tag{30}$$

(ii) **PPP:** Purchasing power parity (PPP) states that prices of baskets of goods, expressed in a common currency, are equalized across countries. PPP is embedded in the majority of international macro models. Let us here assume PPP holds in its weakest form, i.e. relative PPP holds in the long run:

$$\lim_{t\to\infty} \Delta q_t = 0. \tag{31}$$

In words, in the short term the real exchange rate can flexibly adjust, but it cannot grow indefinitely.[12]

(iii) **UIP:** The third key component of international macro models is uncovered interest rate parity (UIP). The UIP condition states that the domestic currency is expected to appreciate relative to the dollar if the foreign economy (the US) has higher nominal interest rates:

$$E_t \Delta s_{t+1} = r_t^S - r_t^{S,*}. \tag{32}$$

The combination of these three conditions has the following implication: in the long run, short term real interest rates are equalized across countries (e.g. Obstfeld (1986)).[13] Let us simplify notation by dropping time scripts and simply write that, in the long run:

$$r^S - \pi = r^{S,*} - \pi^*. \tag{33}$$

Now assume that the short rate in each country is determined by a central bank policy rule which responds to inflation:[14]

$$r^S = \phi_\pi \pi \tag{34}$$

$$r^{S,*} = \phi_\pi^* \pi^*. \tag{35}$$

Then, from equation (33), it is immediate that:

$$\pi = \frac{\phi_\pi^* - 1}{\phi_\pi - 1} \pi^*. \tag{36}$$

Equation (36) has two implications. First, shocks that change US long term inflation (π^*) will generally transmit to the UK (π). In other words, if there are permanent shocks to US inflation, then (relative) PPP implies that UK inflation will inherit such unit root behaviour. Note that equation (36) only holds in the long term, as PPP need not hold in the short run. As a result, in response to short term fluctuations, the real exchange rate provides an additional channel through which international

[12] Relative PPP is weaker than absolute PPP in that the latter requires the real exchange rate to return to a given level, whereas the former only requires it ultimately settles (see, e.g. Sarno and Taylor (2002)).

[13] This follows from shifting the log-differenced real exchange rate one period forward, taking expectations, substituting UIP, taking the limit and imposing PPP holds in the long run.

[14] Richer policy rules (e.g. including a smoothing term, an inflation target, or a response to the output gap) complicate notation, but do not alter the substantive conclusion of this section.

adjustment can occur, and inflation rates in the two countries need not comove as strongly as suggested by the long term condition (36).

Second, equation (36) shows that the response of the UK may well be larger than that of the US. Specifically, this will occur if the systematic inflation-response of the BoE is small relative to that of the Fed, causing $\frac{\phi_\pi^*-1}{\phi_\pi-1} > 1$.

5.2.2 Counterfactuals

The intuition from the above core equations of international macro models extends to our more elaborate empirical model. This becomes apparent from considering the following counterfactuals.

On the one hand, Equation (36) suggests that the degree to which US inflation trends spill over to the UK depends on the central bank responsiveness (ϕ_π) of each country. Specifically, equation (36) suggests that the excessively large response of UK inflation (and consequently the long rate) compared to the US found in Figure (5) may be due to the fact that the BoE is less responsive to inflation compared to the Fed. In a first counterfactual (Figure 6, dotted line) we consider the effect of assuming the BoE responds very strongly to inflation ($\phi_\pi = 5$). Of course, this changes nothing for the US IRF, an immediate reflection of it being "large". For the UK, however, the real interest rate now persistently rises. This causes a UK recession, which helps curb the impact of the foreign target change on UK CPI inflation. Note that the policy rate change is far smaller than in the baseline case. In other words, because agents understand the BoE responds strongly to inflation (and incorporate that into their expectations), the policy rate must rise only by a moderate amount to induce a restrictive monetary policy stance (i.e. a rise in the UK real rate). The temporary moderate response of the policy rate implies the long-term interest rate hardly changes in this counterfactual.[15] As a result, a *systematic* strong anti-inflationary stance of the UK suffices to undo long-term interest rate comovement.

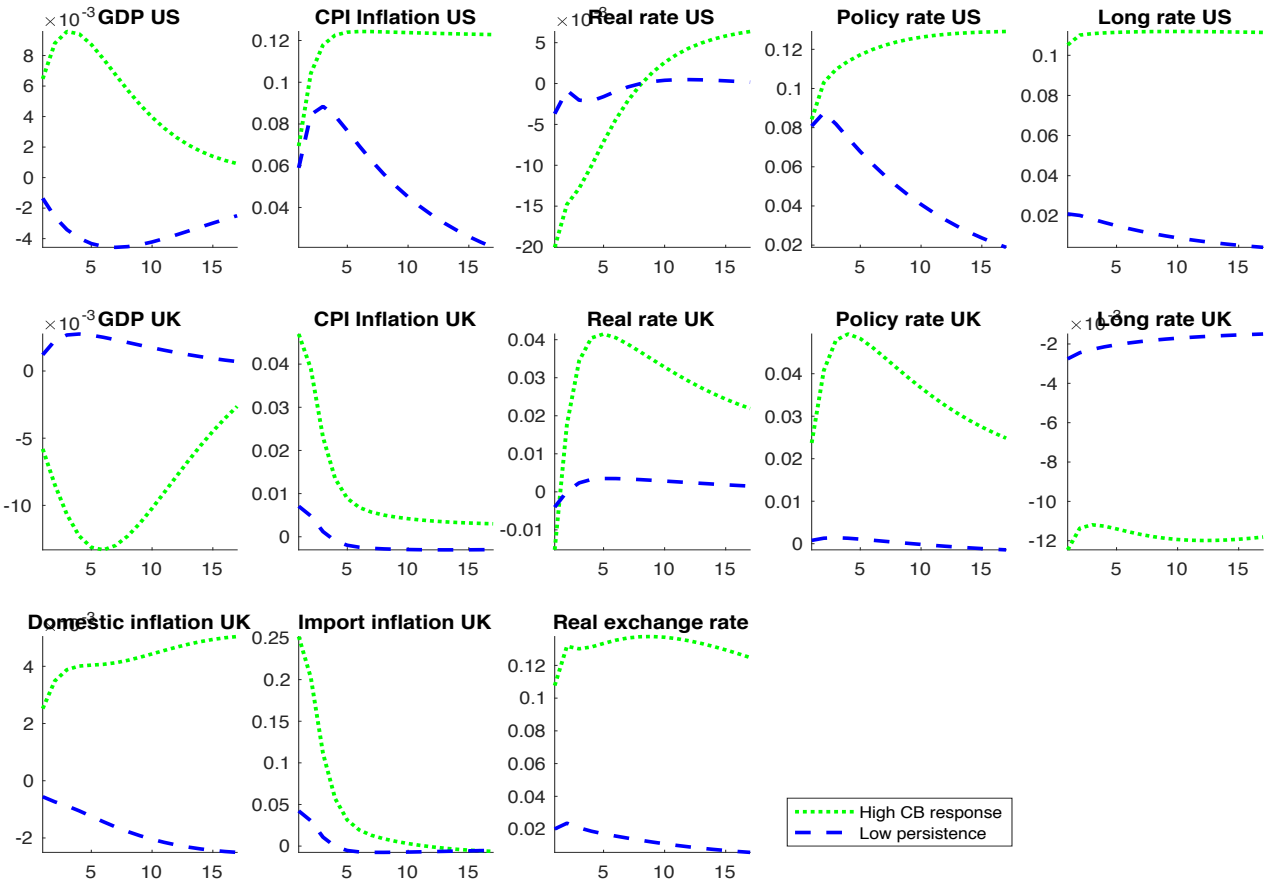
On the other hand, equation (36) only obtains in the long run. In the short run the real exchange rate is not constrained by PPP and this flexibility can help insulate the UK from changes in US inflation. In a second counterfactual (Figure 6, dashed line), we consider a case where the shock to the US inflation target is only temporary (persistence = 0.9). In this case, the shock induces a relatively small response of the UK compared to the US. The short-term nature of the shock implies the exchange rate can largely absorb the shock, which insulates the UK from significant real effects: the US inflation rate rises, while UK inflation is only marginally affected. Because the shock is temporary and because the cross-country comovement in inflation is far reduced, this counterfactual also eliminates long term interest rate comovement.

5.3 The Forces Driving Comovement: External Validation

The two counterfactual experiments document the two key structural features behind international comovement. We here argue both factors are strongly supported by the data, even without recourse to our specific DSGE model.

[15] The small negative impact on the long-term rate comes about due to the endogenous response of the liquidity component of long rates.

Figure 6: US Inflation Target Shock Under Counterfactual Calibrations



The first factor is the significance of US inflation-target shocks. There is ample evidence in academic literatures focusing on the US alone that argue changing inflation targets are an important macroeconomic phenomenon. The presence of unit-root-like behaviour in US inflation is well documented: Stock and Watson (1999, 2007), among others, document the success of reduced form models of inflation that allow for a changing trend component in inflation. For structural interpretations of such a trend in terms of changing inflation targets, see e.g. Smets and Wouters (2003, 2005), Cogley et al. (2010), or Del Negro et al. (2015). Moreover, Kozicki and Tinsley (2001) De Graeve et al. (2009) and Doh (2012) connect changing inflation trends to the US term structure of interest rates. That said, identifying inflation-target changes as a source of international comovement goes a step further: it suggests that US target changes (strongly) affect the UK. Here too, off-model support is broadly in line with the features of the model. Firstly, there is ample research documenting that there is global comovement in inflation. Examples of empirical models documenting that include Ciccarelli and Mojon (2010) and Mumtaz and Surico (2012). The evidence provided in this literature is typically reduced form in nature and thus difficult to tie to structural developments. The transmission of inflation trends identified in the DSGE model provides a potential structural explanation for that phenomenon. Secondly, as a matter of more directly tying global inflation to changes in US inflation targets, Table 11 shows coefficient estimates from regressions of both US and UK inflation on the longest available measure of long-term US inflation expectations.[16] Two features stand out. First, the UK responds

[16]The proxy for long-run inflation expectations is a spliced survey based measure of long horizon PCE inflation expectations used in the FRB-US model and available from the Federal Reserve Board webpage. It measures inflation

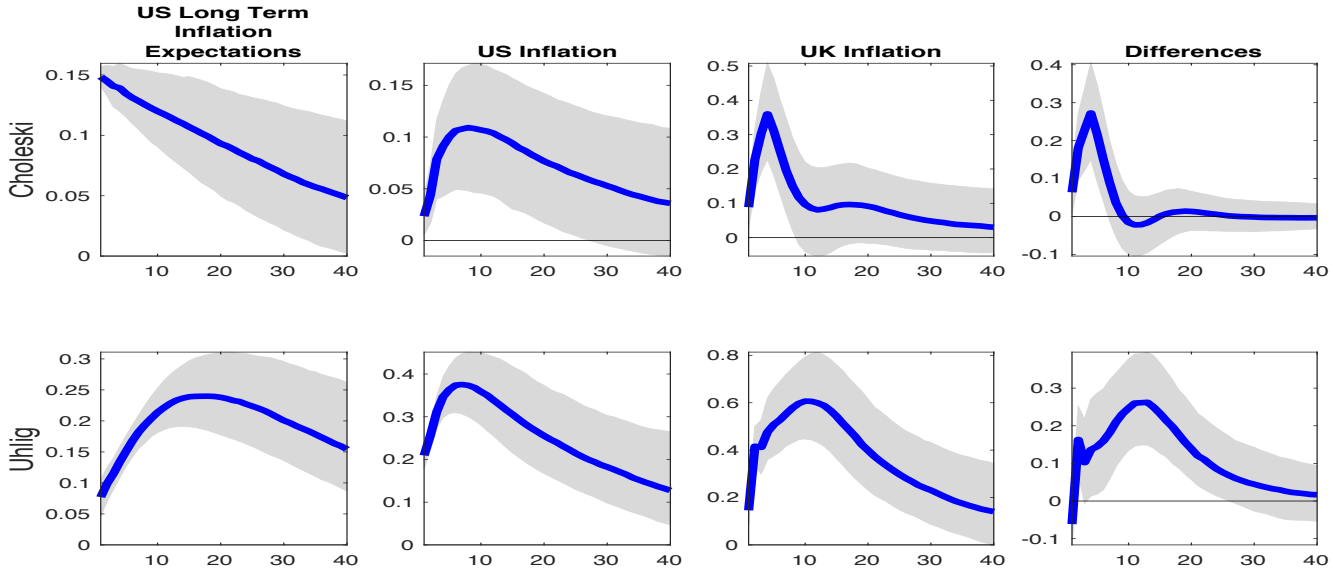
Table 11: The Effect of US Long-Term Inflation Expectations on US and UK Inflation

	Coefficient on US LT expectations ($\hat{\beta}$)	
Regression:	$i = US$	$i = UK$
$\pi_t^i = c + \beta E_t(\pi_{t+40}^{US}) + \varepsilon_t$	1.13	1.75
	(28.86)	(12.70)
$\pi_t^i = c + \beta \Delta E_t(\pi_{t+40}^{US}) + \varepsilon_t$	1.44	2.73
	(1.51)	(1.49)
$\pi_t^i = c + \rho \pi_{t-1}^i + \beta E_t(\pi_{t+40}^{US}) + \varepsilon_t$	-0.04	0.15
	(-1.25)	(2.30)
$\pi_t^i = c + \rho \pi_{t-1}^i + \beta \Delta E_t(\pi_{t+40}^{US}) + \varepsilon_t$	0.32	1.01
	(2.64)	(2.33)

Notes: t-stats in parenthesis. Long-term inflation expectations from Federal Reserve Board. Sample: 1975Q1-2015Q4.

significantly to a change in US expectations. Second, quantitatively, the UK responds by more than the US. These two findings hold across all specifications in the table. The same findings arise in more elaborate econometric models, too. Figure 7 shows the impulse responses from a similar exercise in a simple BVAR (which captures the dynamics more rigorously) estimated on $(E_t(\pi_{t+40}^{US}), \pi_t^{US}, \pi_t^{UK})$. The figure plots the IRF to an unexpected increase in $E_t(\pi_{t+40}^{US})$ for two different identification strategies. The strong response of the UK (also relative to the US) emerges once again. In light of the above literature and evidence, the potential significance of the international transmission of inflation-target shocks does not seem a far stretch.

Figure 7: BVAR



Notes: IRF from a BVAR on $(E_t(\pi_{t+40}^{US}), \pi_t^{US}, \pi_t^{UK})$ under two different identification schemes. Choleski: shock to $E_t(\pi_{t+40}^{US})$ with $E_t(\pi_{t+40}^{US})$ ordered first. Uhlig: shock that maximizes FEVD of $E_t(\pi_{t+40}^{US})$ at horizon t to $t+40$. Solid line: median response. Shaded area: 16th-84th percentile bands. IRFs in the fourth column measure the response of $\pi_{t+h}^{UK} - \pi_{t+h}^{US}$. Sample: 1975Q1-2015Q4.

expectations at a long horizon, up to 10 years.

The second crucial factor for international transmission in our model is the low response of the UK central bank to foreign inflation trends. While there is perhaps less abundant literature available on the issue, the evidence on the BoE's complacency with respect to foreign inflation trends is both direct and readily available. Since it's adoption of Inflation Targeting, the BoE is formally required to publicly motivate any substantial deviation (>1%-point) from its official target. In every such instance, the BoE Governor writes a letter to the Chancellor of the Exchequer.[17] Virtually every deviation is at least partly and most often largely attributed to foreign developments. The BoE almost invariably argues it "looks through" these foreign influences and focuses on medium term domestic inflation, suggesting the latter is still on target. It is noteworthy that most of the letters are not isolated events, but typically appear in cycles of deviations from target that persist for substantial periods of time: at the time of writing the past two cycles took approximately two years of incessant (same-signed) deviations. We interpret both the complacency towards foreign inflation *and* the persistence of deviations from target to be further evidence that supports the transmission mechanism uncovered by the estimated DSGE model.

5.4 Policy Implication

The structural features underlying comovement hint at a strong policy implication. The first key component is the highly persistent nature of the inflationary forces coming from abroad. There are several possible interpretations of US long-term inflation trends. These range from US inflation-target changes in the strict sense (e.g. Smets and Wouters (2005); De Graeve et al. (2009), Cogley et al. (2010)), to changing inflation perceptions of that trend by the public (see the learning literature, e.g., Dewachter and Lyrio (2008)), to changing trends of imported inflation (e.g., food, oil, or energy prices more broadly, e.g. Catão and Chang (2015)). Whatever the interpretation, however, the UK is likely to have to take these trends as given. On its own, it cannot tackle foreign inflation at its root. This leaves the UK with the option of fighting the consequences of foreign inflation trends. Hence, if the UK wishes to decouple its long-term interest rates from the US, it can do so by *systematically* responding strongly to persistent foreign disturbances.[18]

6 Robustness

We illustrate in the online appendix that the DSGE estimation results are robust to various model extensions. These extensions are: (*i*) Making US long-term inflation expectations observed to the econometrician in estimating the DSGE model. (*ii*) Estimating the model on a sample ending in $2007Q4$, to reduce concerns about the zero lower bound affecting our results. (*iii*) Increasing the persistence of $\varepsilon_t^{\bar{b}^L}$ and $\varepsilon_t^{\bar{b}^{L,*}}$ to 0.995, to give term premia more chance to pick up low frequency movements in interest rates. (*iv*) Increasing the persistence of the productivity growth shocks, to give real rates more chance to pick up low frequency movements in interest rates. Almost invariably, we find that the dominant source of long-term interest rate comovement turns out to be the US inflation-target shock, and the BoE's accommodation of its consequences.

[17] These letters are available online at www.gov.uk/government/collections/inflationary-targets.

[18] An alternative policy option would consist of adjusting the UK inflation target to foreign developments, but that violates the independent setting of the BoE mandate and is clearly counterfactual in a historical context.

Appendices B and C make two additional points. First, in appendix B, we document that the model's interpretation of US long-term interest rate dynamics is entirely in line with that in the literature focusing on the US alone. In an influential paper Del Negro et al. (2017) illustrate convincingly that the fall of the US long-term interest is caused by the excess – world – demand for 'safe' assets and their relatively limited supply. To be precise, the US is perhaps the only supplier of safe assets (see Gourinchas and Jeanne (2012) and Rey (2016)) and agents are willing to pay a convenience premium to hold these assets, thus causing their returns to fall (Krishnamurthy and Vissing-Jorgensen (2012)). Our estimates suggest that this channel contributes on average 90 bps to the decline of the US long-term interest seen from $1998Q1$ onwards. This number is very similar to the estimates reported by Del Negro et al. (2017) and Krishnamurthy and Vissing-Jorgensen (2012). As regards fluctuations during the 1970s and 80s, the majority of low frequency dynamics in US long-term yields are a reflection of changes in the US inflation target, which is consistent with estimates of Dewachter and Lyrio (2008), De Graeve et al. (2009) and Doh (2012), among others.

Second, the estimated DSGE model is an extension of standard SOE models. One concern is that some of the non-standard features drive our conclusions. Let us list the less standard parts: 1) endogenous term premia, 2) using exports to close the model, and 3) estimation using endogenous priors. We emphasise that while each additional feature may not be common to the SOE literature, they all have ample backing in other fields (see, e.g., respectively, Vayanos and Vila (2009); Schmitt-Grohe and Uribe (2003); Del Negro and Schorfheide (2008)). The first feature is required to enable evaluating the role of term premia in international comovement. The second and third features enable the model to match the data, where other SOE DSGE models typically fail. Hence, the non-standard components are necessary to quantitatively model comovement. That said, none of them matter for our *qualitative* conclusion that comovement is caused by the SOE central bank's lackluster response to foreign inflation trends. To show that, appendix C calibrates a version of the Justiniano and Preston (2010b) model (which arguably is the standard empirical SOE model, but -as Justiniano and Preston argue- in many ways fails to capture open economy dynamics). The model features a complacent response of the central bank in the wake of foreign inflation trends. All the qualitative features of the transmission mechanism laid out in the paper go through in that model, too. Specifically, long-term interest rate comovement arises due to changing foreign inflation trends, and the SOE responds stronger than the foreign economy. We infer that the added features of the present model, while necessary to match the international macro and financial data, are not essential for the structural mechanism to explain comovement.

7 Conclusion

Our analysis uncovers a structural explanation of international comovement in long-term interest rates. Comovement arises as a result of foreign inflation trends, which small open economy central banks choose not to offset. Both components are established facts: the first much documented in the academic literature, the latter readily apparent in actual central bank behaviour. The structural analysis shows that a central bank wanting to decouple from US interest rates can do so, but that this requires a change to a strong systematic policy response against foreign inflation trends.

The analysis in the paper considers the UK vis-à-vis the US. On the one hand, we focus on one particular SOE, the UK. Yet features of monetary policy conduct are very similar in other SOE's.

To name but one, Sweden is characterized by a similar monetary policy conduct, also featuring a complacency with regards to foreign inflation trends and long-term interest rate fluctuations. On the other hand, we consider the US as the foreign large economy. The US is a dominant (goods and asset) trade partner for the typical SOE, but clearly not the only one. Yet the documented features underlying comovement are not specific to the US being the source. Our policy implications stand, irrespective of whether the foreign economy captures the US alone or the world as a whole: the SOE cannot affect inflationary trends outside its borders, and thus must offset them if it aims to decouple its long-term interest rates. Therefore, while our quantitative results need not generalize to other country pairs, there is good reason to expect the qualitative conclusions apply more broadly. This is no doubt a fruitful avenue for future research. Another way forward lies in identifying the exact sources behind the foreign inflation trends, whether they originate in the US or more globally, and whether that matters for the central bank's response.

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A Structural versus Reduced-Form Analysis

Table 9 in the paper reports variance decompositions based on estimates of a reduced form factor model. We here detail the specification of said model. Consider the following Factor Augmented VAR model (FAVAR), where the 3 $(r^{S,*}, r^{S})$ month, 5 year $(r^{5L,*}, r^{5L})$ and 10 year $(r^{L,*}, r^{L})$ government debt interest rates for the US and the UK are related to world and country specific factors:

$$
\begin{aligned}
\hat{r}_t^{S,*} &= \lambda_{r^{S,*}}^{W,L} L_t^W + \lambda_{r^{S,*}}^{US,L} L_t^{US} + \varpi_t^{r^{S,*}} \\
\hat{r}_t^{L,*} &= \lambda_{r^{L,*}}^{W,L} L_t^W + \lambda_{r^{L,*}}^{W,S} S_t^W + \lambda_{r^{L,*}}^{US,L} L_t^{US} + \lambda_{r^{L,*}}^{US,S} S_t^{US} + \varpi_t^{r^{L,*}} \\
\hat{r}_t^{5L,*} &= \lambda_{r^{5L,*}}^{W,L} L_t^W + \lambda_{r^{5L,*}}^{W,S} S_t^W + \lambda_{r^{5L,*}}^{US,L} L_t^{US} + \lambda_{r^{5L,*}}^{US,S} S_t^{US} + \varpi_t^{r^{5L,*}} \\
\hat{r}_t^{S} &= \lambda_{r^{S}}^{W,L} L_t^W + \lambda_{r^{S}}^{UK,L} L_t^{UK} + \varpi_t^{r^{S}} \\
\hat{r}_t^{L} &= \lambda_{r^{L}}^{W,L} L_t^W + \lambda_{r^{L}}^{W,S} S_t^W + \lambda_{r^{L}}^{UK,L} L_t^{UK} + \lambda_{r^{L}}^{UK,S} S_t^{UK} + \varpi_t^{r^{L}} \\
\hat{r}_t^{5L} &= \lambda_{r^{5L}}^{W,L} L_t^W + \lambda_{r^{5L}}^{W,S} S_t^W + \lambda_{r^{5L}}^{UK,L} L_t^{US} + \lambda_{r^{5L}}^{UK,S} S_t^{US} + \varpi_t^{r^{5L}} \\
L_t^W &= \phi_{L,L}^W L_{t-1}^W + \phi_{L,S}^W S_{t-1}^W + \sigma_L^W v_t^{L^W} \\
S_t^W &= \phi_{S,L}^W L_{t-1}^W + \phi_{S,S}^W S_{t-1}^W + \sigma_S^W v_t^{S^W} \\
L_t^{US} &= \phi_{L^{US}}^{L^W} L_{t-1}^W + \phi_{L^{US}}^{S^W} S_{t-1}^W + \phi_{L^{US}}^{L^{US}} L_{t-1}^{US} + \phi_{L^{US}}^{S^{US}} S_{t-1}^{US} + \sigma_L^{US} v_t^{L^{US}} \\
S_t^{US} &= \phi_{S^{US}}^{L^W} L_{t-1}^W + \phi_{S^{US}}^{S^W} S_{t-1}^W + \phi_{S^{US}}^{L^{US}} L_{t-1}^{US} + \phi_{S^{US}}^{S^{US}} S_{t-1}^{US} + \sigma_S^{US} v_t^{L^{US}} \\
L_t^{UK} &= \phi_{L^{UK}}^{L^W} L_{t-1}^W + \phi_{L^{UK}}^{S^W} S_{t-1}^W + \phi_{L^{UK}}^{L^{US}} L_{t-1}^{UK} + \phi_{L^{UK}}^{S^{US}} S_{t-1}^{UK} + \sigma_L^{UK} v_t^{L^{UK}} \\
S_t^{UK} &= \phi_{S^{UK}}^{L^W} L_{t-1}^W + \phi_{S^{UK}}^{S^W} S_{t-1}^W + \phi_{S^{UK}}^{L^{US}} L_{t-1}^{UK} + \phi_{S^{UK}}^{S^{US}} S_{t-1}^{UK} + \sigma_S^{UK} v_t^{L^{UK}}
\end{aligned}
$$

The 3 month interest rate loads on a world (L_t^W) and a country specific $(L_t^j$, where $j = US, UK)$ level factor. Longer maturity interest rates also load on world (S_t^W) and country specific (S_t^j) slope factors. The errors in the measurement equation reflect idiosyncratic factors. The dynamics of all factors are described by a restricted VAR: US- and UK-specific factors cannot have an effect on world factors, while world factors can impact US and UK factors with a lag.

B Historical Decompositions of US and UK Long-Term Interest Rates

Figure 8 plots the US long-term interest rate over our sample period and decomposes its evolution into several structural components. While a full analysis is beyond the scope of the present paper, we highlight a few observations that show the model's interpretation is in broad agreement with studies analysing the long-term interest rate from a national perspective.

First, inflation-target shocks play a big role in capturing the 1980's disinflation, in line with the evidence in Dewachter and Lyrio (2006, 2008), De Graeve et al. (2009) and Doh (2012). This holds for both the US and the UK. While the term premium explains some of the volatility in both countries during this episode, the role for changing real rates in this period is quantitatively limited.

Second, the decompositions suggest that a fall in the term premium contributes on average 90 bps to the decline of the US long-term interest seen from $1998Q1$ onwards (Figure 10). This number is very similar to the estimates reported by Del Negro et al. (2017) and Krishnamurthy and Vissing-Jorgensen (2012), who interpret it as an increase in the convenience yield.

Third, since the Great Recession the role for nominal fluctuations in the term structure is noticeable once more. This is consistent with the findings of De Graeve and Queijo von Heideken (2015), who suggest inflation-target shocks pick up two factors. On the one hand, they can reflect the combined role of forward guidance and QE in long-term rates: i.e. inflation-target shocks pick up persistent deviations of the policy rate from the historical policy rule. On the other hand, there is a potential role for low-frequency movements in inflation related to fiscal policy.

Fourth, the last decade also witnessed a reduction in the real interest rate, consistent with the evidence of Antolin-Diaz et al. (2017). Its quantitative contribution to nominal yields is limited compared to the inflation and term premium components. The real rate does play a more significant role for the UK than for the US in the later period. This is consistent with the protracted slowdown of growth in UK after the crisis (productivity puzzle, Figure 9).

Figure 8: US Long-Term Interest Rate Historical Decomposition

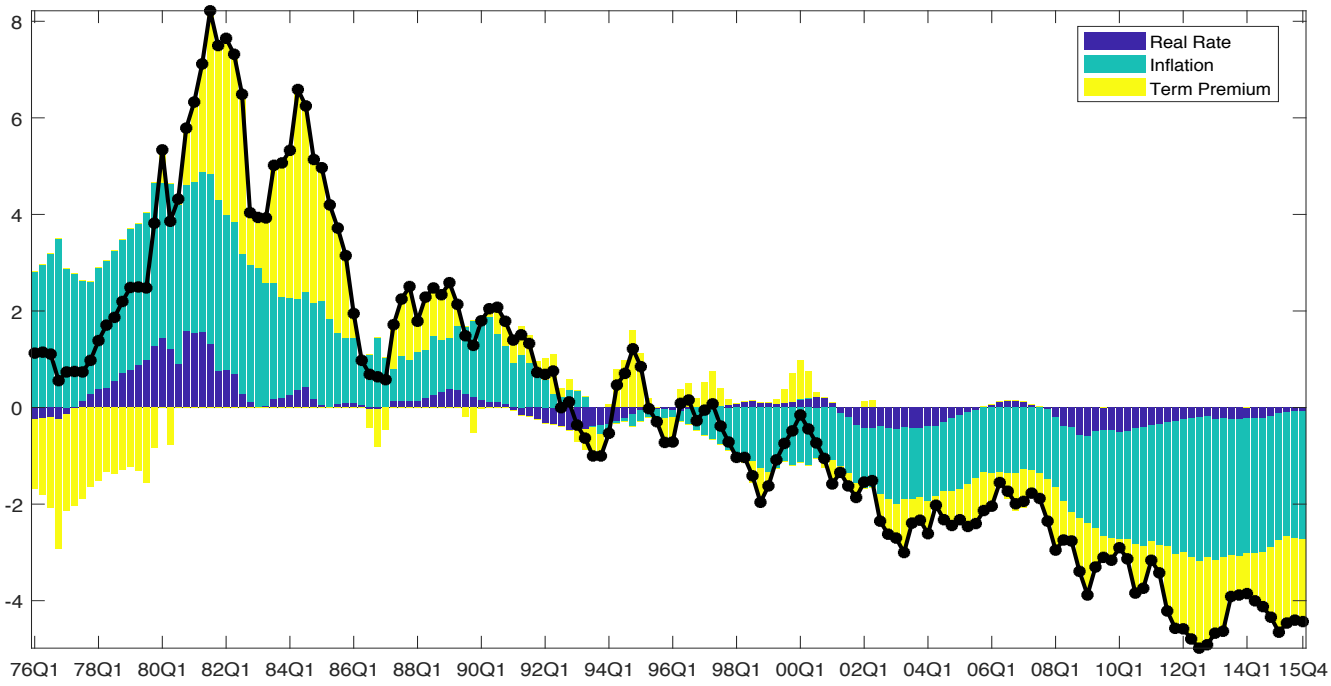


Figure 9: UK Long-Term Interest Rate Historical Decomposition

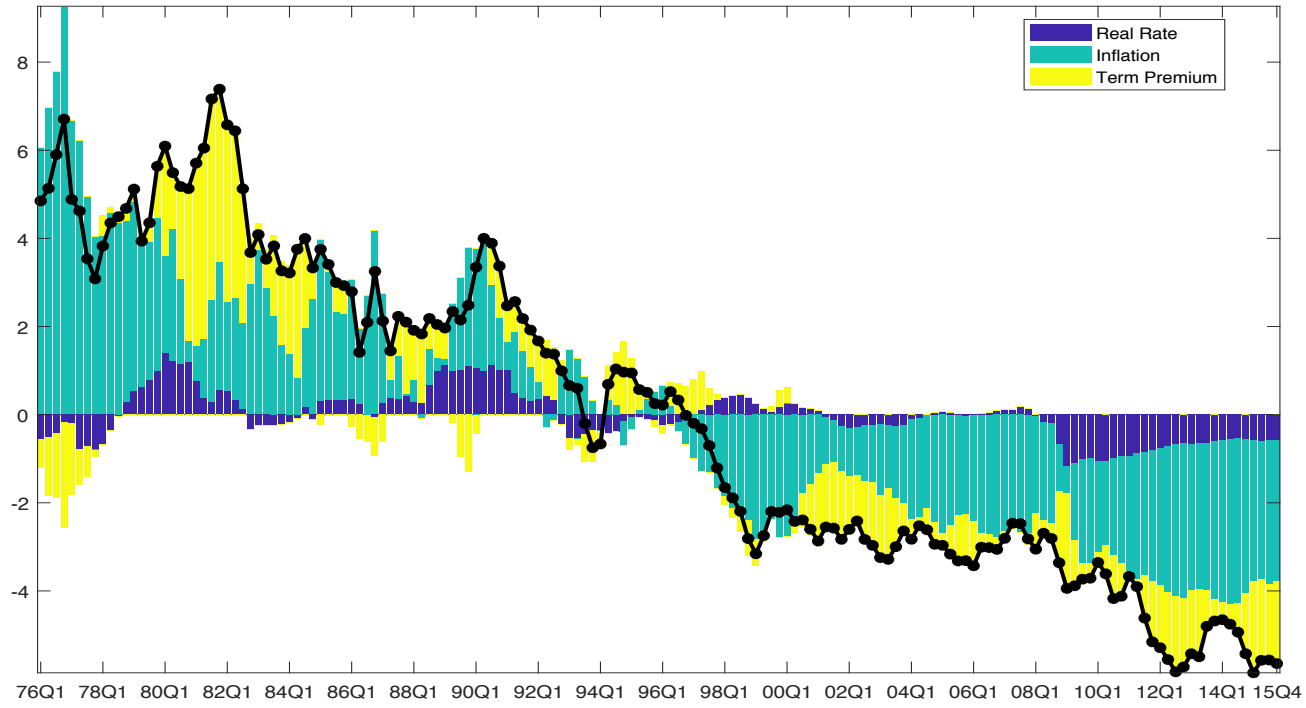
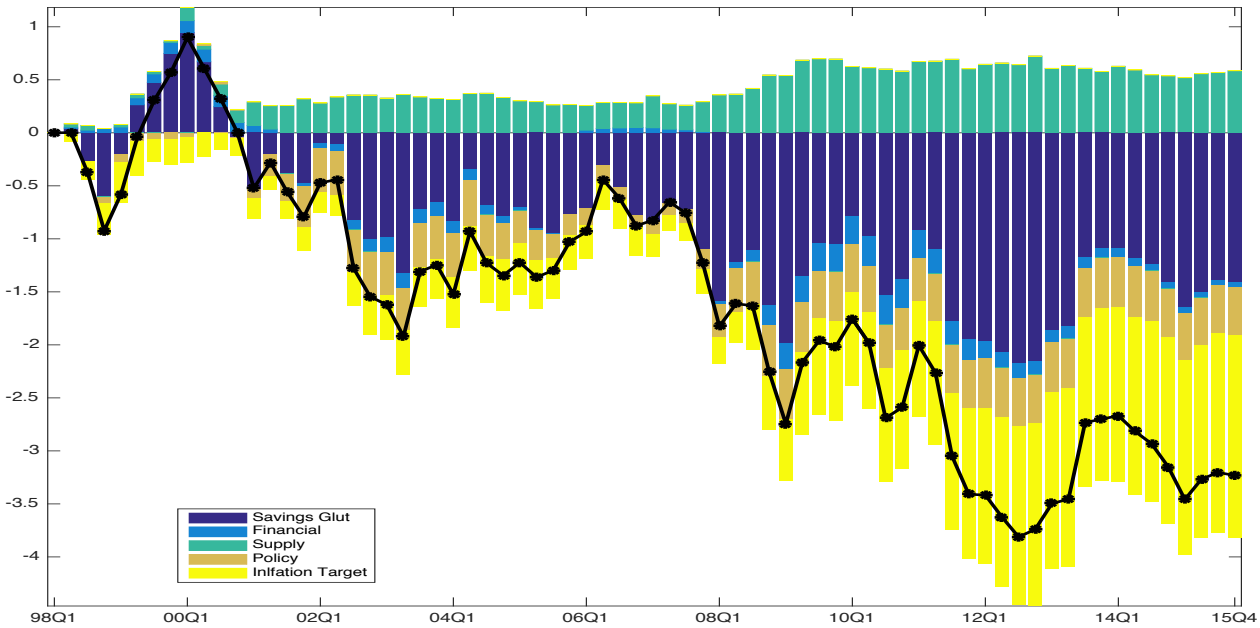


Figure 10: US Long-Term Interest Rate Historical Decomposition Relative to 1998$Q1$

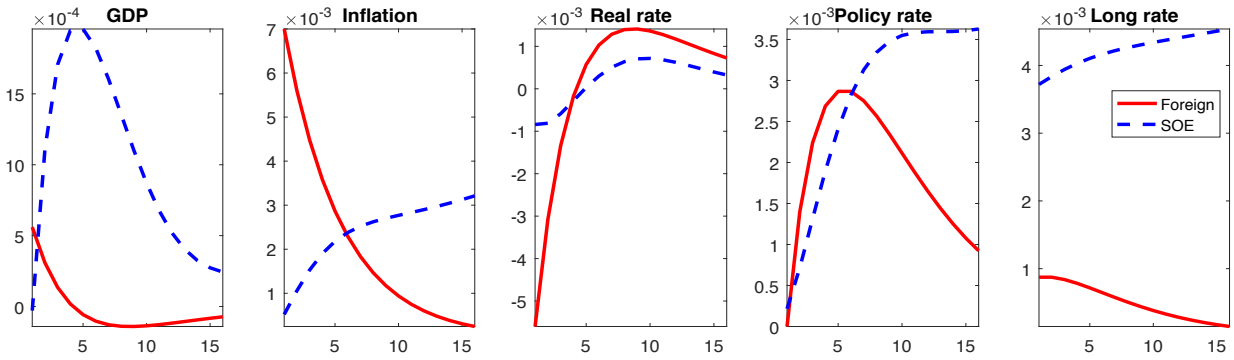


Notes: The historical decomposition of US long-term interest rate is expressed as deviation from 1998$Q1$. Supply includes: the non-stationary TFP, wage and price markup shocks, Financial includes: the short-term risk premium shock, Policy includes: the monetary policy shock.

C The Qualitative (Un)importance of Non-Standard Features in the DSGE Model

This appendix documents impulse responses derived from a simplified version of the model of Justiniano and Preston (2010b), with two key features 1) the presence of a low SOE response coefficient to inflation, and 2) persistent foreign inflation shocks. The model is in no way meant to match the data - a feature Justiniano and Preston (2010b) argue it cannot. Qualitatively, however, the IRFs below document transmission channels that mimic those in the paper: output and inflation persistently rise and eventually do so by more in the SOE than in the foreign block. The SOE's complacency requires the policy rate to increase by a lot (since output and GDP respond so strongly), and eventually also by more than in the foreign economy. This persistent policy response causes the SOE's long-term interest rate to move up, thus generating long-term interest rate comovement. These IRF suggest that the features particular to the estimated model in the paper (e.g. the way in which the model is closed, or the presence of endogenous term premia) are not central to the structural channel underlying comovement.

Figure 11: A Typical SOE DSGE Model



D Data

The list below shows ONS and FRED codes for the series used to construct our dataset.

- UK Real GDP: ONS Code ABMI
- UK Population: ONS Code MGSL
- UK CPI: Bank of England Database
- UK Policy Rate: ONS Code ABEDR
- UK 10 Year Yield: Bank of England Database (quarterly average)
- UK Nominal Wages: ONS Code KAB7 (quarterly average)
- US Nominal GDP: FRED Code GDPC96

- US Population: FRED Code LF+LH
- US CPI: FRED Code JCXFE
- US GDP Deflator: FRED Code GDPDEF
- US Policy Rate: FRED Code FEDFUNDS
- US 10 Year Yield: Bank of England Database (quarterly average)
- US Nominal Wages: FRED Code LXNFC
- US/UK Nominal Exchange Rate: Bank of England Database

NOT FOR PUBLICATION - Online Appendix: Understanding International Long-Term Interest Rate Comovement

July 13, 2018

1 Theoretical Model

1.1 Domestic Economy

1.1.1 Firms

Three types of firms operate in the domestic economy. The intermediate monopolistically competitive domestic firms use labour supplied by households to produce a differentiated good that is sold to a final goods producer who employs a continuum of these differentiated goods in her constant elasticity of substitution (CES) production function to deliver the final good. The monopolistically competitive importing firms use a costless technology and turn a homogenous good – bought in the world market – into a differentiated good, which is then sold to the domestic consumers. The exporting monopolistically competitive firms use a similar 'brand naming' technology and transform the domestic final good into a differentiated product that is sold to foreign households.

Domestic Firms This sector consists of three firms, the 'labour packer' who hires labour from households and transforms it into a homogenous input good – h_t^d, a continuum of monopolistically competitive firms that buys h_t^d and produces an intermediate $y_{i,t}$ and the final good producer who combines all these intermediate products into a single good consumed by households. The final goods producer's CES production function is given by

$$y_t^d = \left[\int_0^1 y_{i,t}^{\frac{1}{\lambda_y}} di\right]^{\lambda_y} \tag{1}$$

where

$$\lambda_y = \left(1 - \rho_{\lambda_y}\right)\lambda_y + \rho_{\lambda_y}\lambda_{y,t-1} + \sigma_{\lambda_y}\omega_{\lambda_y,t} \tag{2}$$

denotes the time-varying mark-up in the domestic goods market. The final goods producer's demand curve for $y_{i,t}$ arises from the profit maximisation problem

$$\max_{y_{i,t}} \left\{ p_t \left[\int_0^1 y_{i,t}^{\frac{1}{\lambda_y}} di\right]^{\lambda_{y,t}} - \int_0^1 p_{i,t} y_{i,t} \right\} :$$

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t}\right)^{-\frac{\lambda_y}{\lambda_{y,t-1}}} y_t^d. \tag{3}$$

The final goods price index is obtained by combining 1 and 3

$$p_t = \left[\int_0^1 p_{i,t}^{\frac{1}{1-\lambda_y}} di\right]^{1-\lambda_y}. \tag{4}$$

Intermediate goods producers use the following production function

$$y_{i,t} = Z_t \left(h_{i,t}^d\right)^{1-\phi} k^{\phi} \tag{5}$$

where

$$z_t = \ln\left(\frac{Z_t}{Z_{t-1}}\right) = (1-\rho_z)\, z + \rho_z z_{t-1} + \sigma_z \omega_{z,t} \tag{6}$$

is a stationary exogenous technological process and $h_{i,t}^d$ is the amount of homogeneous labour rented by the firm i^{ih}. The intermediate firm select $h_{i,t}^d$ in order to minimise its production cost

$$\min_{h_{i,t}^d} p_t^c w_t h_{i,t}^d + mc_t p_t \left[y_{i,t} - z_t \left(h_{i,t}^d\right)^{1-\alpha} k^{\alpha}\right]. \tag{7}$$

The real marginal cost for the intermediate firms is given by the first order condition of (7) with respect to $h_{i,t}^d$:

$$mc_t = \frac{p_t^c}{p_t} \frac{w_t h_{i,t}^d}{(1-\phi)\, y_{i,t}} = \frac{w_t h_{i,t}^d}{\tilde{p}_t\, (1-\phi)\, y_{i,t}} \tag{8}$$

where w_t is the real wage and $\tilde{p}_t = \frac{p_t}{p_t^c}$ is the domestic relative price.

A fraction $\left(1-\xi_y\right)$ of intermediate firms receive a random signal and are allowed to optimally reset their prices – $p_{i,t}^{new}$. The proportion ξ_y of firms that cannot reoptimise prices will set p_t based on backward-looking rule

$$p_t = (\pi_{t-1})^{\kappa_y}\, \bar{\pi}_t^{1-\kappa_y} p_{t-1} \tag{9}$$

where $\pi_t = \frac{p_t}{p_{t-1}}$ is the gross inflation and κ_y is the indexation parameter. The pricing problem of firm i is then

$$\max_{p_{i,t}^{new}} E_t \sum_{j=0}^{\infty} \left(\beta \xi_y\right)^j \frac{\lambda_{t+j}}{\lambda_t} \left\{\left(\prod_{s=1}^{j} \pi_{t+s-1}^{\kappa_y} \bar{\pi}_t^{1-\kappa_y} \frac{p_{i,t}^{new}}{p_{t+j}} - mc_{t+j}\right) y_{i,t+j}\right\} \tag{10}$$

subject to

$$y_{i,t+j} = \left(\prod_{s=1}^{j} \pi_{t+s-1}^{\kappa_y} \breve{\pi}_{t+s}^{1-\kappa_y} \frac{p_{i,t}^{new}}{p_{t+j}}\right)^{-\frac{\lambda_{y,t}}{\lambda_y - 1}} y_{t+j}^d. \tag{11}$$

The first-order condition is expressed as system of difference equations

$$f_{1,t} = \lambda_t mc_t y_t^d + \beta\xi_y E_t \left(\frac{\pi_t^{\kappa_y} \bar{\pi}_{t+1}^{1-\kappa_y}}{\pi_{t+1}} \right)^{-\frac{\lambda_y}{\lambda_y - 1}} f_{1,t+1} \tag{12}$$

$$f_{2,t} = \lambda_t \bar{\pi}_t y_t^d + \beta\xi_d E_t \left(\frac{\pi_t^{\kappa_y} \bar{\pi}_{t+1}^{1-\kappa_y}}{\pi_{t+1}} \right)^{-\frac{1}{\lambda_y - 1}} \left(\frac{\breve{\pi}_t}{\breve{\pi}_{t+1}} \right) f_{2,t+1} \tag{13}$$

$$0 = \lambda_y f_{1,t} - f_{2,t} \tag{14}$$

$$1 = \xi_y \left(\frac{\pi_{t-1}^{\kappa_y} \bar{\pi}_{t+1}^{1-\kappa_y}}{\pi_t} \right)^{-\frac{1}{\lambda_y - 1}} + \left(1 - \xi_y\right) \breve{\pi}_t^{-\frac{1}{\lambda_y - 1}} \tag{15}$$

where $\breve{\pi}_t \equiv \frac{p_t^{new}}{p_t}$.

Market clearing condition in the domestic sector:

$$y_t = \int_0^1 y_{i,t} di = \int_0^1 \left(\frac{p_{i,t}}{p_t} \right)^{-\frac{\lambda_y}{\lambda_y - 1}} di y_t^d = \upsilon_t^p y_t^d \tag{16}$$

where $\upsilon_t^p = \int_0^1 \left(\frac{p_{i,t}}{p_t} \right)^{-\frac{\lambda_{d,t}}{\lambda_{d,t}-1}} d_i$ is the price dispersion term and it is given by

$$\upsilon_t^p = \xi_y \left(\frac{\pi_{t-1}^{\kappa_y} \bar{\pi}_t^{1-\kappa_y}}{\pi_t} \right)^{-\frac{\lambda_y}{\lambda_y - 1}} \upsilon_{t-1}^p + \left(1 - \xi_y\right) \breve{\pi}_t^{-\frac{\lambda_{y,t}}{\lambda_y - 1}}. \tag{17}$$

Importing firms The import sector consists of a continuum of monopolistically competitive firms that buy a homogenous good c_t^m in the world market at price p_t^*. These firms have access to a costless technology and transform the homogenous good into a differentiated product – $c_{i,t}^m$ – consumed by domestic households. Similar to Adolfson et al. (2007) and Burgess et al. (2013) we assume local currency pricing in order to allow for incomplete exchange rate pass-through to the import prices. To be precise, the importing firms follow the Calvo price-setting scheme, meaning that a fraction – $1-\xi_m$ – of them is allowed to reset their price optimally – $p_{m,t}^{new}$ – only when they receive a random price change signal, while those firms that missed this signal can only index their prices by past inflation – $p_{m,t} = \pi_{m,t-1}^{\kappa_m} p_{m,t-1}$. The pricing problem of the firm becomes

$$\max_{p_{i,t}^{m,new}} E_t \sum_{j=0}^{\infty} \left(\beta\xi_m\right)^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \left(\prod_{s=1}^{j} \left(\pi_{t+s-1}^m\right)^{\kappa_m} \bar{\pi}_{t+s}^{1-\kappa_m} \frac{p_{i,t}^{m,new}}{p_{t+j}^m} - mc_t^m \right) c_{i,t}^m \right\} \tag{18}$$

where $mc_t^m \equiv \frac{s_t p_t^*}{p_t^m}$ is the real marginal cost of the importing firm and s_t is the nominal exchange rate.

The final import good is a composite of a continuum of these differentiated imported goods and it is given by the following CES production function

$$c_t^{m,d} = \left[\int_0^1 \left(c_{i,t}^m \right)^{\frac{1}{\lambda_m}} di \right]^{\lambda_m}. \tag{19}$$

Taking p_t^m and $p_{i,t}^m$ as given the final import goods producer's demand curve for $C_{i,t}^m$ can be derived

form the profit maximisation problem

$$\max_{c_{i,t}^m} \left\{ p_t^m \left[\int_0^1 \left(c_{i,t}^m \right)^{\frac{1}{\lambda_m}} di \right]^{\lambda_m} - \int_0^1 p_{i,t}^m c_{i,t}^m \right\} :$$

$$c_{i,t}^m = \left(\frac{p_{i,t}^m}{p_t^m} \right)^{-\frac{\lambda_m}{\lambda_m - 1}} c_t^{m,d}. \tag{20}$$

Finally, total amount of imported goods is obtained by integrating over all differentiated imported goods

$$c_t^m = \int_0^1 c_{i,t}^m di. \tag{21}$$

$p_{i,t}^{m,new}$ is derived by maximising (18) subject to

$$c_{i,t+j}^m = \left(\prod_{s=1}^{j} \left(\pi_{t+s-1}^m \right)^{\kappa_m} \bar{\pi}_{t+s}^{1-\kappa_m} \frac{p_{i.t}^{m,new}}{p_{t+j}^m} \right)^{-\frac{\lambda_m}{\lambda_m - 1}} c_{t+j}^m \tag{22}$$

and the first-order condition – expressed as a system of first-order difference equations – is

$$g_{1,t} = \lambda_t m c_t^m c_{t+j}^m + \beta \xi_m E_t \left(\frac{\left(\pi_t^m \right)^{\kappa_m} \bar{\pi}_{t+1}^{1-\kappa_m}}{\pi_{t+1}^m} \right)^{-\frac{\lambda_m}{\lambda_m - 1}} g_{1,t+1} \tag{23}$$

$$g_{2,t} = \lambda_t \bar{\pi}_t^m c_{t+j}^m + \beta \xi_m E_t \left(\frac{\left(\pi_t^m \right)^{\kappa_m} \bar{\pi}_{t+1}^{1-\kappa_m}}{\pi_{t+1}^m} \right)^{-\frac{1}{\lambda_m - 1}} \left(\frac{\breve{\pi}_t^m}{\breve{\pi}_{t+1}^m} \right) g_{2,t+1} \tag{24}$$

$$0 = \lambda_{m,t} g_{1,t} - g_{2,t} \tag{25}$$

$$1 = \xi_m \left(\frac{\left(\pi_{t-1}^m \right)^{\kappa_m} \bar{\pi}_t^{1-\kappa_m}}{\pi_t^m} \right)^{-\frac{1}{\lambda_m - 1}} + (1 - \xi_m) \left(\breve{\pi}_t^m \right)^{-\frac{1}{\lambda_m - 1}} \tag{26}$$

where $\breve{\pi}_t^m = \frac{p_t^{m,new}}{p_t^m}$.

The market clearing condition in the import sector is

$$c_t^m = \int_0^1 c_{i,t}^m di = \upsilon_t^m c_t^{m,d} \tag{27}$$

where $\upsilon_t^m = \int_0^1 \left(\frac{p_{i,t}^m}{p_t^m} \right)^{-\frac{\lambda_m}{\lambda_m - 1}} d_i$ is the import price dispersion term with its law of motion

$$\upsilon_t^m = \xi_m \left(\frac{\left(\pi_{t-1}^m \right)^{\kappa_m} \bar{\pi}_t^{1-\kappa_m}}{\pi_t^m} \right)^{-\frac{\lambda_m}{\lambda_m - 1}} \upsilon_{t-1}^m + (1 - \xi_m) \left(\breve{\pi}_t^m \right)^{-\frac{\lambda_m}{\lambda_m - 1}} \tag{28}$$

Exporting firms There is a continuum of exporting firms indexed by i on the unit interval. Each firm i buys a homogeneous final domestic good in the domestic market c_t^x and differentiates it by using a costless branding technology. They sell the differentiated goods to the rest of the world. Foreign

households' demand schedule is given by

$$c_{i,t}^{x} = \left(\frac{p_{i,t}^{x}}{p_{t}^{x}} \right)^{-\frac{\lambda_x}{\lambda_x - 1}} c_{t}^{x,d} \tag{29}$$

where $\frac{\lambda_x}{\lambda_x - 1}$ denotes the elasticity of substitution between differentiated exporting goods. The export price index is

$$p_{t}^{x} = \left[\int_{0}^{1} \left(p_{i,t}^{x} \right)^{\frac{1}{1-\lambda_x}} di \right]^{1-\lambda_x}. \tag{30}$$

The total amount of exported goods is obtained by integrating over all goods

$$c_{t}^{x} = \int_{0}^{1} c_{i,t}^{x} di. \tag{31}$$

Under Calvo pricing and indexation $p_{i,t}^{x,new}$ is derived by maximising

$$\max_{p_{i,t}^{x,new}} E_t \sum_{j=0}^{\infty} (\beta \xi_x)^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \left(\prod_{s=1}^{j} \left(\pi_{t+s-1}^{x} \right)^{\kappa_x} \bar{\pi}_{t+s}^{1-\kappa_x} \frac{p_{i,t}^{x,new}}{p_{t+j}^{x}} - mc_{t}^{x} \right) c_{i,t}^{x} \right\} \tag{32}$$

subject to

$$c_{i,t+j}^{x} = \left(\prod_{s=1}^{j} \left(\pi_{t+s-1}^{x} \right)^{\kappa_x} \bar{\pi}_{t+s}^{1-\kappa_x} \frac{p_{i.t}^{x,new}}{p_{t+j}^{x}} \right)^{-\frac{\lambda_x}{\lambda_x - 1}} c_{t+j}^{x} \tag{33}$$

where $mc_{t}^{x} = \frac{p_t}{s_t p_{t}^{x}}$. The first-order condition can be expressed as

$$u_{1,t} = \lambda_t mc_{t}^{x} c_{t+j}^{x} + \beta \xi_x E_t \left(\frac{\left(\pi_{t}^{x} \right)^{\kappa_x} \bar{\pi}_{t+1}^{1-\kappa_x}}{\pi_{t+1}^{x}} \right)^{-\frac{\lambda_x}{\lambda_x - 1}} u_{1,t+1} \tag{34}$$

$$u_{2,t} = \lambda_t \bar{\pi}_{t}^{x} c_{t+j}^{x} + \beta \xi_x E_t \left(\frac{\left(\pi_{t}^{x} \right)^{\kappa_x} \bar{\pi}_{t+1}^{1-\kappa_x}}{\pi_{t+1}^{x}} \right)^{-\frac{1}{\lambda_x - 1}} \left(\frac{\breve{\pi}_{t}^{x}}{\breve{\pi}_{t+1}^{x}} \right) u_{2,t+1} \tag{35}$$

$$0 = \lambda_{x,t} u_{1,t} - u_{2,t} \tag{36}$$

$$1 = \xi_x \left(\frac{\left(\pi_{t-1}^{x} \right)^{\kappa_x} \bar{\pi}_{t}^{1-\kappa_x}}{\pi_{t}^{x}} \right)^{-\frac{1}{\lambda_x - 1}} + (1 - \xi_x) \left(\breve{\pi}_{t}^{x} \right)^{-\frac{1}{\lambda_x - 1}} \tag{37}$$

where $\breve{\pi}_{t}^{x} = \frac{p_{t}^{x,new}}{p_{t}^{x}}$.

The market clearing condition in the export sector is given

$$c_{t}^{x} = \upsilon_{t}^{x} c_{t}^{x,d} \tag{38}$$

where $\upsilon_{t}^{x} = \int_{0}^{1} \left(\frac{p_{i,t}^{x}}{p_{t}^{x}} \right)^{-\frac{\lambda_x}{\lambda_x - 1}} d_i$ is the export price dispersion term with

$$\upsilon_{t}^{x} = \xi_x \left(\frac{\left(\pi_{t-1}^{x} \right)^{\kappa_x} \bar{\pi}_{t}^{1-\kappa_x}}{\pi_{t}^{x}} \right)^{-\frac{1}{\lambda_x - 1}} \upsilon_{t-1}^{x} + (1 - \xi_x) \left(\breve{\pi}_{t}^{x} \right)^{-\frac{1}{\lambda_x - 1}}. \tag{39}$$

1.1.2 Households

The domestic economy is populated by a continuum of households that attain utility from consumption – $c_{\varkappa,t+j}$ – and leisure – $h_{\varkappa,t+j}$. Household preferences are separable

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ d_{t+j} \log\left(c_{\varkappa,t+j} - b c_{\varkappa,t+j-1}\right) - \psi_t \frac{h_{\varkappa,t+j}^{1+\varphi}}{1+\varphi} d\varkappa \right\} \tag{40}$$

subject to two shock processes

$$d_t = (1-\rho_d)\, d + \rho_d d_{t-1} + \sigma_d \omega_{d,t} \tag{41}$$

$$\psi_t = \left(1-\rho_\psi\right) \psi + \rho_\psi \psi_{t-1} + \sigma_\psi \omega_{\psi,t}. \tag{42}$$

β is the discount factor, φ the inverse of the Frisch elasticity, σ_c the inverse of intertemporal elasticity of substitution and b the habit formation parameter. Aggregate consumption is a function of domestically produced – c_t^d – and imported – c_t^m – goods:

$$c_{\varkappa,t} = \left[(1-\alpha)^{\frac{1}{\eta_c}} \left(c_{\varkappa,t}^{dp,d}\right)^{\frac{\eta_c-1}{\eta_c}} + \alpha^{\frac{1}{\eta_c}} \left(c_{\varkappa,t}^{m,d}\right)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}. \tag{43}$$

The elasticity of substitution between domestic and foreign goods in given by η_c and α measures the 'trade openness'. The maximisation of (43) subject to the budget constraint $p_t^c c_{\varkappa,t} = p_t c_{\varkappa,t}^{dp,d} + p_t^m c_{\varkappa,t}^{m,d}$ delivers the following demand functions

$$c_{\varkappa,t}^{dp} = (1-\alpha)\, \tilde{p}_t^{-\eta_c} c_{\varkappa,t} \tag{44}$$

$$c_{\varkappa,t}^{m} = \alpha\, (\tilde{p}_t^m)^{-\eta_c}\, c_{\varkappa,t} \tag{45}$$

where $\tilde{p}_t \equiv \frac{p_t}{p_t^c}$ and $\tilde{p}_t^m \equiv \frac{p_t^m}{p_t^c}$ is the relative domestic consumption and import price, respectively. Plugging (44) and (45) into the budget constraint we obtain the definition of the consumer price index – CPI

$$(p_t^c)^{1-\eta_c} = (1-\alpha)\, p_t^{1-\eta_c} + \alpha\, (p_t^m)^{1-\eta_c} \tag{46}$$

alternatively

$$1 = (1-\alpha)\, \tilde{p}_t^{1-\eta_c} + \alpha\, (\tilde{p}_t^m)^{1-\eta_c} \tag{47}$$

The following relation links CPI inflation ($\pi_t^c = \frac{p_t^c}{p_{t-1}^c}$) with home produced inflation (π_t), imported inflation (π_t^m) and the terms of trade ($\mathcal{T}_t$):

$$\left(\frac{\pi_t^c}{\tilde{p}_{t-1}}\right)^{1-\eta_c} = (1-\alpha)\, \pi_t^{1-\eta_c} + \alpha\, (\pi_t^m \mathcal{T}_{t-1})^{1-\eta_c} \tag{48}$$

Households' real budget constraint is given by

$$D_{\varkappa,t}^h + c_{\varkappa,t} + \Upsilon_{\varkappa,t} = \frac{r_{t-1}^h}{\pi_t^c} D_{\varkappa,t-1}^h + w_{\varkappa,t} h_{\varkappa,t} + F_t - T_t \tag{49}$$

The household $\varkappa$ uses its labour income – $w_{\varkappa,t}h_{\varkappa,t}$, gross interest rates earned on financial intermediary deposits – $\frac{r^h_{t-1}}{\pi^c_t}D^h_{\varkappa,t-1}$, government transfers – T_t – and profits – F_t – to finance consumption and new purchases of financial assets – $c_{\varkappa,t} + D_{\varkappa,t} + \Upsilon_{\varkappa,t}$– where $\Upsilon_{\varkappa,t}$ denotes the whole set of state state-contingent securities traded among households and help them to hedge themselves against the idiosyncractic risk they face as they cannot always adjust wages optimally. The household maximises (40) with respect to $c_{\varkappa,t}$, and $D^h_{\varkappa,t}$ subject to (49):

$$\frac{d_t}{c_{\varkappa,t} - bc_{\varkappa,t-1}} - E_t\frac{\beta b d_{t+1}}{c_{\varkappa,t+1} - hc_{\varkappa,t}} = \lambda_{\varkappa,t} \tag{50}$$

$$\lambda_{\varkappa,t} = \beta E_t\left\{\lambda_{\varkappa,t+1}\frac{r^h_t}{\pi^c_{t+1}}\right\}. \tag{51}$$

1.1.3 Financial intermediary

The financial intermediary firm issues deposits to households paying a gross interest rate r^h_t. The firm then purchases a portfolio of short and long term government issued bonds paying interest r^S_t and r^L_t as well as a small fraction of long term debt issued by the foreign government $\left(r^{L,*}_t\right)$.

Similar to Andres et al. (2004), Chen et al. (2012), Harrison (2012) and **?** we follow the formulation in Woodford (2001) such that long-term bonds are perpetuities that cost $p_{L,t}$ at time t and pay an exponentially decaying coupon κ^s at time $t+s+1$ where $0 < \kappa \leq 1$. As it is explained in Woodford (2001) and Chen et al. (2012) the advantage of this formulation is that the price in period t of a bond issued s periods ago $p_{L-s,t}$ is a function of the coupon the current price $p_{L,t}$

$$p_{L-s,t} = \kappa^s p_{L,t} \tag{52}$$

This relation allows to express the balance sheet equation and government budget constraint (below) in a familiar form that it is easy to work with (see the discussion in Chen et al. (2012)). Furthermore, for simplification, we rule out the possibility of a secondary market for long-term bonds, meaning that agents who buy long-term debt must hold it until maturity.[1] Finally, for simplicity we assume that all government bonds issued are purchased by this firm.

The intermediary's balance sheet is given

$$b^h_{\varkappa,t} = \frac{b^S_{\varkappa,t}}{\varepsilon^{b^S}_t} + \frac{p_{L,t}b^L_{\varkappa,t}}{\varepsilon^{\bar{b}^L}_t} + \frac{q_t\varrho_t p^*_{L,t}b^{L,*}_{\varkappa,t}}{\varepsilon^{\bar{b}^L}_t}$$

or

$$b^h_{\varkappa,t} = \frac{b^S_{\varkappa,t}}{\varepsilon^{b^S}_t} + \frac{\bar{b}^L_{\varkappa,t}}{\varepsilon^{\bar{b}^L}_t} + \frac{\bar{b}^{L,D,*}_{\varkappa,t}}{\varepsilon^{\bar{b}^L}_t} \tag{53}$$

[1] See the discussion in Andres et al. (2004) for the advantages of that assumption.

where

$$p_{L,t} = \frac{1}{r_t^L - \kappa}$$
$$p_{L,t}^* = \frac{1}{r_t^{L,*} - \kappa^*}$$

and the term ϱ_t reflects the fraction of the foreign long-term debt held by the domestic financial intermediary. Motivated by the work of Smets and Wouters (2007) we assume the balance sheet equation is subject to two 'financial' shocks: a short and a long-term risk premium shocks dented by $\varepsilon_t^{b^S}$, $\varepsilon_t^{\bar{b}^L}$ and $\varepsilon_t^{\bar{b}^{L,*}}$, respectively.

The intermediary's profit function is then given by

$$\xi_t = \underbrace{b_{\varkappa,t}^h + \frac{r_{t-1}^S}{\pi_t^c} b_{\varkappa,t-1}^S + \frac{r_t^L}{\pi_t} p_{L,t} b_{\varkappa,t-1}^L + \varrho \frac{r_t^{L,*}}{\pi_t^*} p_{L,t}^* b_{\varkappa,t-1}^{L,*}}_{\text{revenues}}$$
$$\underbrace{- \frac{b_{\varkappa,t}^S}{\varepsilon_t^{b^S}} - \frac{p_{L,t} b_{\varkappa,t}^L}{\varepsilon_t^{\bar{b}^L}} - \frac{q_t \varrho p_{L,t}^* b_{\varkappa,t}^{L,*}}{\varepsilon_t^{\bar{b}^L}} - \frac{r_{t-1}^h}{\pi_t^c} b_{\varkappa,t-1}^h - \frac{x}{2} \left[lq_{\varkappa,t-1} - \vartheta lq_{t-2} - (1-\vartheta)\, lq \right]^2 \frac{Z_{t-1}}{\pi_t^c}}_{\text{expenditures}}$$

where

$$lq_{\varkappa,t-1} = \frac{b_{\varkappa,t-1}^S}{\bar{b}_{\varkappa,t-1}^L + \bar{b}_{\varkappa,t-1}^{D,*}}.$$

Intermediary profits are subject to two adjustment costs. The fist one captures the idea that altering the foreign debt held by domestic intermediaries to GDP ratio is costly.[2] The second term reflects the situation where although intermediaries prefer to hold more long than short-term debt that decreases 'liquidity'.

Using the balance sheet equation the profit function becomes

$$E_t \xi_{t+1} = \frac{r_t^S}{\pi_{t+1}^c} b_{\varkappa,t}^S + E_t \left\{ \frac{r_{t+1}^L}{\pi_{t+1}^c} \frac{p_{L,t+1}}{p_{L,t}} \right\} \bar{b}_{\varkappa,t}^L + E_t \left\{ \frac{r_{t+1}^{L,*}}{\pi_{t+1}^*} \frac{p_{L,t+1}^*}{p_{L,t}^*} \frac{q_{t+1}}{q_t} \right\} \bar{b}_{\varkappa,t}^{L,D,*}$$
$$- E_t \left\{ \frac{r_t^h}{\pi_{t+1}^c} \right\} b_{\varkappa,t}^h - \frac{x}{2} \left[lq_{\varkappa,t} - \vartheta lq_{t-1} - (1-\vartheta)\, lq \right]^2 \frac{Z_t}{E_t \pi_{t+1}}.$$

Profit maximisation with respect to short, domestic and foreign long-term debt and subject to the balance sheet condition delivers an expression for the effective interest rate faced by the household, the long-term interest rate and the exchange rate:

Short-term debt

$$0 = \frac{r_t^S}{E_t \pi_{t+1}^c} - \frac{r_t^h}{\varepsilon_t^{b^S} E_t \pi_{t+1}^c} - x \left[\delta_{\varkappa,t}^B - \vartheta \delta_{t-1}^B - (1-\vartheta)\, \delta^B \right] \frac{Z_t}{\bar{b}_{\varkappa,t}^L + \bar{b}_{\varkappa,t}^{D,*}} \frac{1}{E_t \pi_{t+1}^c}$$
$$\frac{r_t^h}{\varepsilon_t^{b^S}} = r_t^S - x \left[lq_{\varkappa,t} - \vartheta lq_{t-1} - (1-\vartheta)\, lq \right] \frac{Z_t}{\bar{b}_{\varkappa,t}^L + \bar{b}_{\varkappa,t}^{D,*}} \tag{54}$$

[2] In our model this term is not required to make the net foreign asset position of the model stationary (see the discussion in Schmitt-Grohe and Uribe (2003)) as we properly model long-term debt in the foreign economy.

Long-term domestic debt

$$0 = E_t\left\{\frac{r^L_{t+1}}{\pi_{t+1}}\frac{p_{L,t+1}}{p_{L,t}}\right\} - \frac{r^h_t}{\varepsilon^{\bar{b}^L}_t E_t\pi^c_{t+1}} + x\left[lq_{\varkappa,t} - \vartheta lq_{t-1} - (1-\vartheta)\,lq\right]\frac{Z_t b^S_{\varkappa,t}}{\left(\bar{b}^L_{\varkappa,t} + \bar{b}^{D,*}_{\varkappa,t}\right)^2}\frac{1}{E_t\pi^c_{t+1}}$$

$$E_t\left\{r^L_{t+1}\frac{p_{L,t+1}}{p_{L,t}}\right\} = \frac{r^h_t}{\varepsilon^{\bar{b}^L}_t} - x\left[lq_{\varkappa,t} - \vartheta lq_{t-1} - (1-\vartheta)\,lq\right]\frac{Z_t b^S_{\varkappa,t}}{\left(\bar{b}^L_{\varkappa,t} + \bar{b}^{D,*}_{\varkappa,t}\right)^2} \tag{55}$$

Long-term foreign debt

$$0 = E_t\left\{\frac{r^{L,*}_{t+1}}{\pi^*_{t+1}}\frac{p^*_{L,t+1}}{p^*_{L,t}}\frac{q_{t+1}}{q_t}\right\} - \frac{r^h_t}{\varepsilon^{\bar{b}^L}_t E_t\pi^c_{t+1}} + x\left[lq_{\varkappa,t} - \vartheta lq_{t-1} - (1-\vartheta)\,lq\right]\frac{b^S_{\varkappa,t}Z_t}{\left(\bar{b}^L_{\varkappa,t} + \bar{b}^{D,*}_{\varkappa,t}\right)^2 E_t\pi^c_{t+1}}$$

$$E_t\left\{\frac{r^{L,*}_{t+1}}{\pi^*_{t+1}}\frac{p^*_{L,t+1}}{p^*_{L,t}}\frac{q_{t+1}}{q_t}\right\} = \frac{r^h_t}{\varepsilon^{\bar{b}^L}_t E_t\pi^c_{t+1}} - x\left[lq_{\varkappa,t} - \vartheta lq_{t-1} - (1-\vartheta)\,lq\right]\frac{b^S_{\varkappa,t}Z_t}{\left(\bar{b}^L_{\varkappa,t} + \bar{b}^{D,*}_{\varkappa,t}\right)^2} \tag{56}$$

It is important to stress here the Uncover Interest Rate Parity (UIP) condition (56) is defined in terms of long and not short-term real interest rate differentials as it is typically done in the SoE literature (see Adolfson et al. (2007), Christiano et al. (2011) and Burgess et al. (2013) among others). This is because *i*) we restrict households to invest in domestic and/or foreign assets directly and *ii*) intermediaries can hold only foreign long and not short-term debt. The first restriction allows domestic households in our model to own multiple assets and we are able to pin down their prices without employing complicated portfolio asset solution techniques. The model can be linearised, estimated and, furthermore, to use regime-switching techniques (similar to Liu et al. (2011)) to study how changes in the domestic or/and foreign macroeconomic uncertainty affect asset prices and what this means for policy makers. The second assumption restricts the number of the UIP conditions to one and makes the domestic long-term interest rate a direct function of the foreign long-term debt and rates via the term-premia.

1.1.4 Wages

We follow Erceg et al. (2000) and assume that each monopolistically competitive household supplies a differentiated labour service to the production section. They set their nominal wage and supply any amount of labour demanded by the firms at that wage rate. For convenience, we assume that there exists a representative firm that combines households' labour inputs into a homogeneous input good - h^d_t - using a CES production function

$$h^d_t = \left[\int_0^1 h^{\frac{1}{\lambda_w}}_{\varkappa,t}\,d\varkappa\right]^{\lambda_w} \tag{57}$$

where

$$\lambda_{w,t} = \left(1-\rho_{\lambda_w}\right)\lambda_w + \rho_{\lambda w}\lambda_{w,t-1} + \sigma_{\lambda_w}\omega_{\lambda_w,t} \tag{58}$$

denotes the time-varying mark-up in the labour market. Taking w_t and $w_{\varkappa,t}$ as given, the aggregator's demand for the labour hours of household $\varkappa$ results from profit maximisation

$$\max_{h_{\varkappa,t}} \left\{ w_t \left[\int_0^1 h_{\varkappa,t}^{\frac{1}{\lambda_w}} d\varkappa \right]^{\lambda_w} - \int_0^1 w_{\varkappa,t} h_{\varkappa,t} \right\} :$$

$$h_{\varkappa,t} = \left(\frac{w_{\varkappa,t}}{w_t} \right)^{-\frac{\lambda_{w,t}}{\lambda_w - 1}} h_t^d \tag{59}$$

The aggregate wage arises from the profit condition and the demand curve

$$w_t = \left[\int_0^1 w_{\varkappa,t}^{\frac{1}{1-\lambda_w}} d\varkappa \right]^{1-\lambda_w} \tag{60}$$

In each period, a fraction – $1-\xi_w$ – of households receive a random signal and they are allowed to reset wages optimally – w_t^{new}. All other households can only partially index their wages by past inflation. The problem of setting wages can be described as follows

$$\max_{w_t^{new}} E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \left\{ -\psi_{t+j} \frac{h_{\varkappa,t+j}^{1+\varphi}}{1+\varphi} + \lambda_{t+j} \prod_{s=1}^{j} \frac{\left(\pi_{t+s-1}^c\right)^{\kappa_w} \bar{\pi}_{t+s}^{1-\kappa_w}}{\pi_{t+s}^c} w_{\kappa,t} h_{\varkappa,t} \right\} \tag{61}$$

subject to

$$h_{\varkappa,t} = \left(\prod_{s=1}^{j} \frac{\left(\pi_{t+s-1}^c\right)^{\kappa_w} \bar{\pi}_{t+s}^{1-\kappa_w}}{\pi_{t+s}^c} \frac{w_{\varkappa,t}}{w_t} \right)^{-\frac{\lambda_w}{\lambda_w - 1}} h_t^d. \tag{62}$$

The first order is summarised by the following recursive equations

$$v_{1,t} = \lambda_t \frac{1}{\lambda_w} \left(w_t^{new}\right)^{\frac{1}{1-\lambda_w}} w_t^{\frac{\lambda_w}{\lambda_w - 1}} h_t^d + \beta \xi_w E_t \left(\frac{\left(\pi_t^c\right)^{\varkappa_w} \bar{\pi}_{t+1}^{1-\kappa_w}}{\pi_{t+1}^c} \right)^{\frac{1}{1-\lambda_w}} \left(\frac{w_{t+1}^{new}}{w_t^{new}} \right)^{\frac{1}{\lambda_w - 1}} v_{1,t+1} \tag{63}$$

$$v_{1,t} = \lambda_t \psi_t \left(\frac{w_t}{w_t^{new}} \right)^{\frac{(1+\varphi)\lambda_w}{\lambda_{w,t}-1}} \left(h_t^d \right)^{1+\varphi} + \beta \xi_w E_t \left(\frac{\left(\pi_t^c\right)^{\kappa_w} \bar{\pi}_{t+1}^{1-\kappa_w}}{\pi_{t+1}^c} \right)^{\frac{(1+\varphi)\lambda_{w,t+1}}{1-\lambda_w}} \left(\frac{w_{t+1}^{new}}{w_t^{new}} \right)^{\frac{(1+\varphi)\lambda_w}{\lambda_w - 1}} v_{1,t+1} \tag{64}$$

$$w_t^{\frac{1}{1-\lambda_w}} = \xi_w \left(\frac{\left(\pi_{t-1}^c\right)^{\kappa_w} \bar{\pi}_t^{1-\kappa_w}}{\pi_t^c} \right)^{\frac{1}{1-\lambda_w}} w_{t-1}^{\frac{1}{1-\lambda_w}} + (1-\xi_w) \left(w_t^{new}\right)^{\frac{1}{1-\lambda_{w,t}}} \tag{65}$$

The market clearing condition in the labour market is

$$h_t = \int_0^1 h_{\varkappa,t} d\varkappa = v_t^w h_t^d \tag{66}$$

where $v_t^w = \int_0^1 \left(\frac{w_{i,t}}{w_t} \right)^{-\frac{\lambda_w}{\lambda_w - 1}} d_i$ is the wage dispersion term and its evolution is described

$$v_t^w = \xi_w \left(\frac{\left(\pi_{t-1}^c\right)^{\kappa_w} \bar{\pi}_t^{1-\kappa_w}}{\pi_t^c} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{w_{t-1}}{w_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} v_{t-1}^w + (1-\xi_w) \left(\frac{w_t^{new}}{w_t} \right)^{\frac{\lambda_w}{1-\lambda_w}}. \tag{67}$$

1.1.5 Government

The government's budget constraint adjusted for long-term debt is given by

$$\begin{aligned}\frac{b_t^S}{\varepsilon_t^{b^S}}+\frac{p_{L,t}b_t^L}{\varepsilon_t^{\bar{b}^L}} &= \frac{r_{t-1}^S}{\pi_t^c}b_{t-1}^S+\frac{r_t^L}{\pi_t^c}\frac{p_{L,t}}{p_{L,t-1}}p_{L,t-1}b_{t-1}^L+\tilde{p}_tG_t-T_t\\ \frac{b_t^S}{\varepsilon_t^{b^S}}+\frac{\bar{b}_t^L}{\varepsilon_t^{\bar{b}^L}} &= \frac{r_{t-1}^S}{\pi_t^c}b_{t-1}^S+\frac{r_t^L}{\pi_t^c}\frac{p_{L,t}}{p_{L,t-1}}\bar{b}_{t-1}^L+\tilde{p}_tG_t-T_t\end{aligned}$$

where the left hand side is the total (short plus long-term) debt issued by the government at time t, while the right hand side reflects the total deficit at time t. Similar to Chen et al. (2012) we assume that the government controls the supply of long-term bonds and this is fixed

$$\frac{b_t^L}{b^L}=1. \tag{68}$$

Government consumption is a fraction (g_t) of GDP

$$G_t = g_ty_t \tag{69}$$

$$\frac{g_t}{g} = \left(\frac{g_t}{g}\right)^{\rho_g}e^{\sigma_g\omega g,t}. \tag{70}$$

Finally, transfers are adjusted according to the following rule

$$T_t=\Phi\left(b_t^S,\bar{b}_t^L;\theta\right). \tag{71}$$

1.1.6 Monetary policy

The monetary authority sets the short-term interest rate according to a Taylor rule

$$\frac{r_t}{r\bar{\pi}_t}=\left(\frac{r_{t-1}}{r\bar{\pi}_{t-1}}\right)^{\phi_R}\left(\frac{\pi_t^c}{\bar{\pi}_t}\right)^{(1-\phi_R)\phi_\pi}\left(\frac{y_t}{y}\right)^{(1-\phi_R)\phi_y}e^{\sigma_R\omega_{R,t}}. \tag{72}$$

In other words, the policymaker adjusts the nominal interest rate in response to its lag value, to inflation deviations from the target – $\pi=1$ – and to output deviations from the long-run equilibrium – y.

1.1.7 Exports

As in our model the evolution of the foreign debt is determined in the foreign economy, we use this section to derive an expression for exports that ensures consistency between the domestic economy's debt and the evolution of the foreign long-term debt given imports. In other words, exports become the residual in the net foreign asset position accumulation equation. To derive the latter expression we start from the clearing conditions in the markets for domestic intermediate goods. Notice that there is full indexation in the steady-state. This implies that price and wage dispersion are bounded to one in the steady-state and zero for all t when the model is log-linearised. This feature is used in this section in order for the derivations to be simplified. Namely

$$y_t=\int y_{i,t}=\int c_{i,t}^{dp}+\int c_{i,t}^{x}=c_t^{dp}+c_t^x$$

and households' integrated budget constraint

$$D_t^h + c_t = \frac{r_{t-1}^h}{\pi_t^c} D_{t-1}^h + w_t h_t + F_t - T_t.$$

Furthermore, the resource constraint implies that

$$y_t^d = c_t^{dp,d} + c_t^{x,d} + G_t + i. \tag{73}$$

Substituting (44) into (73) we obtain

$$(1-\alpha)\,\tilde{p}_t^{-\eta_c} c_t + c_t^{x,d} + G_t = y_t \tag{74}$$

Profits are given by

$$\begin{aligned} F_t &= c_t - \tilde{p}_t c_t^{dp,d} - \tilde{p}_t^m c_t^{m,d} + \frac{1}{p_t^c}\int_0^1 p_{i,t}^m c_{i,t}^m - \frac{s_t p_t^{c,*}}{p_t^c} c_t^m + \frac{s_t p_t^{c,*}}{p_t^c}\frac{p_t^x}{p_t^{c,*}} c_t^x - \frac{s_t}{p_t^c}\int_0^1 p_{i,t}^x c_{i,t}^x \\ &+ \tilde{p}_t y_t^d - \tilde{p}_t \int_0^1 w_{i,t} l_{i,t} di + \frac{r_{t-1}^S}{\pi_t^c} b_{\varkappa,t-1}^S + \frac{r_t^L}{\pi_t^c}\frac{p_{L,t}}{p_{L,t-1}} \bar{b}_{\varkappa,t-1}^L + \frac{r_t^{L,*}}{\pi_t^*}\frac{p_{L,t}^*}{p_{L,t-1}^*}\frac{q_t}{q_{t-1}} \bar{b}_{\varkappa,t-1}^{L,*} \\ &- \frac{r_{t-1}^h}{\pi_t^c} b_{t-1}^h - \frac{x}{2}\left[lq_t - \vartheta lq_{t-1} - (1-\vartheta)\, lq\right]^2 \frac{1}{\pi_t^c}. \end{aligned}$$

Next we use government's budget constraint to obtain an expression for $F_t - T_t$

$$\begin{aligned} F_t - T_t &= c_t - q_t c_t^m + q_t \tilde{p}_t^x c_t^x - \tilde{p}_t w_t h_t + \frac{r_t^{L,*}}{\pi_t^*}\frac{p_{L,t}^*}{p_{L,t-1}^*}\frac{q_t}{q_{t-1}} \bar{b}_{\varkappa,t-1}^{L,*} \\ &- \frac{r_{t-1}^h}{\pi_t^c} b_{t-1}^h - \frac{x}{2}\left[lq_t - \vartheta lq_{t-1} - (1-\vartheta)\, lq\right]^2 \frac{1}{\pi_t^c} + \left(\frac{b_t^S}{\varepsilon_t^{b^S}} + \frac{\bar{b}_t^L}{\varepsilon_t^{\bar{b}^L}}\right). \end{aligned}$$

We substitute the latter expression, the definition of the balance sheet equation and labour market condition into household's integrated budget constraint

$$\begin{aligned} D_t^h + c_t &= \frac{r_{t-1}^h}{\pi_t^c} D_{t-1}^h + \tilde{p}_t w_t h_t + c_t - q_t c_t^m + q_t \tilde{p}_t^x c_t^x - \tilde{p}_t w_t h_t \\ &+ \frac{r_t^{L,*}}{\pi_t^*}\frac{p_{L,t}^*}{p_{L,t-1}^*}\frac{q_t}{q_{t-1}} \bar{b}_{\varkappa,t-1}^{L,*} - \frac{r_{t-1}^h}{\pi_t^c} b_{t-1}^h - \frac{x}{2}\left[lq_t - \vartheta lq_{t-1} - (1-\vartheta)\, lq\right]^2 \frac{1}{\pi_t^c} + \left(\frac{b_t^S}{\varepsilon_t^{b^S}} + \frac{\bar{b}_t^L}{\varepsilon_t^{\bar{b}^L}}\right) \end{aligned}$$

and this leads to

$$q_t \tilde{p}_t^x c_t^x = q_t c_t^m + \frac{\bar{b}_t^{L,*}}{\varepsilon_t^{\bar{b}^L}} - \frac{r_t^{L,*}}{\pi_t^*}\frac{p_{L,t}^*}{p_{L,t-1}^*}\frac{q_t}{q_{t-1}} \bar{b}_{t-1}^{L,*} + \frac{x}{2}\left[lq_t - \vartheta lq_{t-1} - (1-\vartheta)\, lq\right]^2 \frac{Z_{t-1}}{\pi_t^c} \tag{75}$$

1.2 Foreign economy

The foreign agents' decision problems are very similar to those discussed in the previous section: the foreign economy is subject to the same functional forms, shocks, and frictions. However, the foreign economy is "large" relative to the domestic economy. As a result, the system governing the foreign economy does not typically include variables of the domestic economy. To avoid repetition we here just list the first-order conditions required for the solution of the model. We keep the same notation as in the domestic economy and we add a star symbol – $*$ – to separate the foreign from the domestic variables.

1.2.1 Supply

The production of the intermediate goods producer is given by

$$y_{i,t}^{*}=Z_{t}^{*}\left(h_{i,t}^{d,*}\right)^{1-\phi}\left(k^{*}\right)^{\phi} \tag{76}$$

$$z_{t}^{*}=\left(1-\rho_{z^{*}}\right)z^{*}+\rho_{z^{*}}z_{t-1}^{*}+\sigma_{z^{*}}\omega_{z^{*},t}. \tag{77}$$

The marginal cost is given

$$mc_{t}^{*}=\frac{w_{t}^{*}h_{i,t}^{d,*}}{\left(1-\phi\right)y_{i,t}^{*}} \tag{78}$$

while the following equations describe firm's pricing first-order conditions

$$f_{1,t}^{*} = \lambda_{t}^{*}mc_{t}^{*}y_{t}^{d,*}+\beta\xi_{d}^{*}E_{t}\left(\frac{\left(\pi_{t}^{c,*}\right)^{\kappa_{y}^{*}}\left(\bar{\pi}_{t+1}^{*}\right)^{1-\kappa_{y}^{*}}}{\pi_{t+1}^{c,*}}\right)^{-\frac{\lambda_{y}^{*}}{\lambda_{y,t-1}^{*}}}f_{1,t+1}^{*} \tag{79}$$

$$f_{2,t}^{*} = \lambda_{t}^{*}\bar{\pi}_{t}^{*}y_{t}^{d,*}+\beta\xi_{d}^{*}E_{t}\left(\frac{\left(\pi_{t}^{c,*}\right)^{\kappa_{y}^{*}}\left(\bar{\pi}_{t+1}^{*}\right)^{1-\kappa_{y}^{*}}}{\pi_{t+1}^{c,*}}\right)^{-\frac{1}{\lambda_{y}^{*}-1}}\left(\frac{\breve{\pi}_{t}^{*}}{\breve{\pi}_{t+1}^{*}}\right)f_{2,t+1}^{*} \tag{80}$$

$$0 = \lambda_{y}^{*}f_{1,t}^{*}-f_{2,t}^{*} \tag{81}$$

$$1 = \xi_{d}^{*}\left(\frac{\left(\pi_{t-1}^{c,*}\right)^{\kappa_{y}^{*}}\left(\bar{\pi}_{t}^{*}\right)^{1-\kappa_{y}^{*}}}{\pi_{t}^{c,*}}\right)^{-\frac{1}{\lambda_{y}^{*}-1}}+\left(1-\xi_{d}^{*}\right)\left(\breve{\pi}_{t}^{*}\right)^{-\frac{1}{\lambda_{y}^{*}-1}} \tag{82}$$

where $\breve{\pi}_{t}^{*}\equiv\frac{p_{t}^{new,*}}{p_{t}^{*}}$.

1.2.2 Households

Foreign household's utility maximisation first-order conditions with respect to $c_{t}^{*}$and D_{t}^{*} subject to its budget constraint the consumption Euler condition

$$\frac{d_{t}^{*}}{c_{t}^{*}-bc_{t-1}^{*}}-E_{t}\frac{\beta bd_{t+1}^{*}}{c_{t+1}^{*}-hc_{t}^{*}}=\lambda_{t}^{*} \tag{83}$$

$$\lambda_{t}^{*}=\beta E_{t}\left\{\lambda_{t+1}^{*}\frac{r_{t}^{h,*}}{\pi_{t+1}^{c,*}}\right\}. \tag{84}$$

Agents' utility function is subject to a discount factor and labour supply shock.

$$d_{t}^{*}=\left(1-\rho_{d^{*}}\right)d^{*}+\rho_{d^{*}}d_{t-1}^{*}+\sigma_{d^{*}}\omega_{d^{*},t}. \tag{85}$$

$$\psi_{t}^{*}=\left(1-\rho_{\psi^{*}}\right)\psi+\rho_{\psi^{*}}\psi_{t-1}^{*}+\sigma_{\psi^{*}}\omega_{\psi^{*},t} \tag{86}$$

1.2.3 Financial intermediary

The financial intermediary firm issues deposits households paying a gross interest rate $r_{t}^{h,*}$. The firm then purchases a portfolio of short and long term government issued bonds paying interest r_{t} and r_{t}^{L}. For simplicity we assume that all government bonds issued are purchased by this firm. The

intermediary's balance sheet is

$$b_{\varkappa,t}^{h,*} = \frac{b_{\varkappa,t}^{S,*}}{\varepsilon_t^{b^{S,*}}} + \frac{p_{L,t}^* b_{\varkappa,t}^{L,*}}{\varepsilon_t^{\bar{b}^{L,*}}} \tag{87}$$

where

$$p_{L,t}^* = \frac{1}{r_t^{L,*} - \varkappa}.$$

Intermediary's profit function is given by

$$\begin{aligned}\xi_{t+1}^* &= \frac{r_t^{S,*}}{E_t \pi_{t+1}^{c,*}} b_{\varkappa,t}^{S,*} + E_t \left\{ \frac{r_{t+1}^{L,*}}{\pi_{t+1}^{c,*}} \frac{p_{L,t+1}^*}{p_{L,t}^*} \right\} \bar{b}_{\varkappa,t}^{L,*} - \frac{r_t^{h,*}}{E_t \pi_{t+1}^{c,*}} b_{\varkappa,t}^{h,*} \\ &\quad - \frac{x^*}{2} \left[lq_{\varkappa,t-1}^* - \vartheta lq_{t-2}^* - (1-\vartheta)\, lq^* \right]^2 \frac{Z_{t-1}^*}{E_t \pi_{t+1}^{c,*}}.\end{aligned}$$

Profit maximisation subject to the balance sheet condition gives

$$\frac{r_t^{h,*}}{\varepsilon_t^{b^{S,*}}} = r_t^{S,*} - x^* \left[lq_{\varkappa,t-1}^* - \vartheta lq_{t-2}^* - (1-\vartheta)\, lq^* \right] \frac{Z_t^*}{\bar{b}_{\varkappa,t}^{L,*}} \tag{88}$$

$$E_t \left\{ r_{t+1}^{L,*} \frac{p_{L,t+1}^*}{p_{L,t}^*} \right\} = \frac{r_t^{h,*}}{\varepsilon_t^{\bar{b}^{L,*}}} - x^* \left[lq_{\varkappa,t-1}^* - \vartheta lq_{t-2}^* - (1-\vartheta)\, lq^* \right] \frac{\delta^* b_{\varkappa,t}^{S,*} Z_t^*}{\left(\bar{b}_{\varkappa,t}^{L,*} \right)^2}. \tag{89}$$

1.2.4 Wages

The evolution of wages in the foreign economy is described by

$$\begin{aligned}v_{1,t}^* &= \frac{1}{\lambda_w^*} \left(w_t^{new,*} \right)^{\frac{1}{1-\lambda_w^*}} \lambda_t^* \left(w_t^* \right)^{\frac{\lambda_w^*}{\lambda_w^* - 1}} h_t^{d,*} + \\ &\quad \beta \xi_w^* E_t \left(\frac{\left(\pi_t^{c,*} \right)^{\kappa_w} \left(\bar{\pi}_{t+1}^{c,*} \right)^{1-\kappa_w}}{\pi_{t+1}^{c,*}} \right)^{\frac{1}{1-\lambda_w^*}} \left(\frac{w_{t+1}^{new,*}}{w_t^{new,*}} \right)^{\frac{1}{\lambda_{w,t}^* - 1}} v_{1,t+1}^*\end{aligned} \tag{90}$$

$$\begin{aligned}v_{1,t}^* &= \psi_t^* \left(\frac{w_t^*}{w_t^{new,*}} \right)^{\frac{(1+\varphi)\lambda_{w,t}^*}{\lambda_w^* - 1}} \left(h_t^{d,*} \right)^{1+\varphi} + \\ &\quad \beta \xi_w^* E_t \left(\frac{\left(\pi_t^{c,*} \right)^{\kappa_w} \left(\bar{\pi}_{t+1}^{c,*} \right)^{1-\kappa_w}}{\pi_{t+1}^{c,*}} \right)^{\frac{(1+\varphi)\lambda_{w,t}^*}{1-\lambda_w^*}} \left(\frac{w_{t+1}^{new,*}}{w_t^{new,*}} \right)^{\frac{(1+\varphi)\lambda_{w,t}^*}{\lambda_w^* - 1}} v_{1,t+1}^*\end{aligned} \tag{91}$$

$$\left(w_t^* \right)^{\frac{1}{1-\lambda_w^*}} = \xi_w^* \left(\frac{\left(\pi_{t-1}^{c,*} \right)^{\kappa_w} \left(\bar{\pi}_t^{c,*} \right)^{1-\kappa_w}}{\pi_t^{c,*}} \right)^{\frac{1}{1-\lambda_w^*}} \left(w_{t-1}^* \right)^{\frac{1}{1-\lambda_w^*}} + \left(1 - \xi_w^* \right) \left(w_t^{new,*} \right)^{\frac{1}{1-\lambda_w^*}}. \tag{92}$$

1.2.5 Monetary policy

Foreign monetary policy authorities follow a similar with the domestic economy Taylor type rule

$$\frac{r_t^*}{r\bar{\pi}_t^*} = \left(\frac{r_{t-1}^*}{r\bar{\pi}_{t-1}^*}\right)^{\phi_{R^*}} \left(\frac{\pi_t^{c,*}}{\bar{\pi}_t^*}\right)^{(1-\phi_{R^*})\phi_{\pi^*}} \left(\frac{y_t^{d,*}}{y^*}\right)^{(1-\phi_{R^*})\phi_{Y^*}} e^{\sigma_{R^*}R^*,t} \tag{93}$$

1.2.6 Government

The government's budget constraint adjusted for long-term debt is given by

$$b_t^{S,*} + \bar{b}_t^{L,*} = \frac{r_{t-1}^*}{\pi_t^{c,*}} b_{t-1}^{S,*} + \frac{r_t^{L,*} p_{L,t}^*}{\pi_t^{c,*} p_{L,t-1}^*} \bar{b}_{t-1}^{L,*} + G_t^* - T_t^* \tag{94}$$

where the left hand side is the total (short plus long-term) debt issued by the government at time t, while the right hand side reflects the total deficit at time t.

Similar to Chen et al. (2012) we assume that the government controls the supply of long-term bonds and it is fixed

$$\frac{b_t^{L,*}}{b^{L,*}} = 1. \tag{95}$$

Finally, we assume adjust its primary surplus based on debt targeting rule

$$T_t^* = \Phi\left(b_t^{S,*}, \bar{b}_t^{L,*}; \theta^*\right) \tag{96}$$

$$G_t^* = g_t^* y_t^* \tag{97}$$

$$\frac{g_t^*}{g^*} = \left(\frac{g_t^*}{g^*}\right)^{\rho_g} e^{\sigma_{g^*}\omega_{g^*,t}}. \tag{98}$$

1.2.7 Market clearing conditions

$$y_t^{d,*} = c_t^* + G_t^* + i^* + \frac{x^*}{2}\left[lq_t^* - \vartheta lq_{t-1}^* - (1-\vartheta)\, lq^*\right]^2 Z_t^* \tag{99}$$

where $\upsilon_t^{p,*}$ is the price dispersion term

$$\upsilon_t^{p,*} = \xi_d^* \left(\frac{\left(\pi_{t-1}^{c,*}\right)^{\kappa_y}\left(\pi^{c,*}\right)^{1-\kappa_y}}{\pi_t^{c,*}}\right)^{-\frac{\lambda_y^*}{\lambda_y^*-1}} \upsilon_{t-1}^{p,*} + (1-\xi_d)\left(\bar{\pi}_t^*\right)^{-\frac{\lambda_{y,t}^*}{\lambda_y^*-1}} \tag{100}$$

$$h_t^{d,*} = \frac{h_t^*}{\upsilon_t^{w,*}} \tag{101}$$

where $\upsilon_t^{w,*}$ is the wage dispersion term

$$\upsilon_t^{w,*} = \xi_w^* \left(\frac{\left(\pi_{t-1}^{c,*}\right)^{\kappa_w}\left(\pi^{c,*}\right)^{1-\kappa_w}}{\pi_t^{c,*}}\right)^{\frac{\lambda_w^*}{1-\lambda_w^*}} \left(\frac{w_{t-1}^*}{w_t^*}\right)^{\frac{\lambda_w^*}{1-\lambda_w^*}} + (1-\xi_w^*)\left(\frac{w_t^{new,*}}{w_t^*}\right)^{\frac{\lambda_w^*}{1-\lambda_w^*}}. \tag{102}$$

1.3 Stationary Equations

$$\frac{d_t}{\left(c_t - b\frac{c_{t-1}}{\gamma_t}\right)^{\sigma_c}} - E_t\frac{\beta b d_{t+1}}{\left(c_{t+1}\gamma_{t+1} - hc_t\right)^{\sigma_c}} = \lambda_t \tag{103}$$

$$\lambda_t = \beta E_t\left\{\lambda_{t+1}\frac{r_t^h}{\gamma_{t+1}^{\sigma_C}\pi_{t+1}^c}\right\} \tag{104}$$

$$\frac{r_t^h}{\varepsilon_t^{b^S}} = r_t^S - x\left[lq_t - \vartheta lq_{t-1} - (1-\vartheta)\,lq\right]\frac{1}{\bar{b}_t^L + \bar{b}_t^{D,*}} \tag{105}$$

$$E_t\left\{r_{t+1}^L\frac{p_{L,t+1}}{p_{L,t}}\right\} = \frac{r_t^h}{\varepsilon_t^{\bar{b}^L}} - x\left[lq_t - \vartheta lq_{t-1} - (1-\vartheta)\,lq\right]\frac{b_t^S}{\left(\bar{b}_t^L + \bar{b}_t^{D,*}\right)^2} \tag{106}$$

$$E_t\left\{\frac{r_{t+1}^{L,*}}{\pi_{t+1}^*}\frac{p_{L,t+1}^*}{p_{L,t}^*}\frac{q_{t+1}}{q_t}\right\} = \frac{r_t^h}{\varepsilon_t^{\bar{b}^L}E_t\pi_{t+1}^c} - x\left[lq_t - \vartheta lq_{t-1} - (1-\vartheta)\,lq\right]\frac{b_t^S}{\left(\bar{b}_t^L + \bar{b}_t^{D,*}\right)^2 E_t\pi_{t+1}^c} \tag{107}$$

$$v_{1,t} = \lambda_t\frac{1}{\lambda_w}\left(w_t^{new}\right)^{\frac{1}{1-\lambda_w}}w_t^{\frac{\lambda_w}{\lambda_w-1}}h_t^d + \beta\xi_w E_t\left(\frac{\left(\pi_t^c\right)^{\kappa_w}\bar{\pi}_{t+1}^{1-\kappa_w}}{\pi_{t+1}^c}\right)^{\frac{1}{1-\lambda_w}}\left(\frac{w_{t+1}^{new}\gamma_{t+1}}{w_t^{new}}\right)^{\frac{1}{\lambda_w-1}}v_{1,t+1} \tag{108}$$

$$v_{1,t} = \psi_t\left(\frac{w_t}{w_t^{new}}\right)^{\frac{(1+\varphi)\lambda_w}{\lambda_w-1}}\left(h_t^d\right)^{1+\varphi} + \beta\xi_w E_t\left(\frac{\left(\pi_t^c\right)^{\kappa_w}\bar{\pi}_{t+1}^{1-\kappa_w}}{\pi_{t+1}^c}\right)^{\frac{(1+\varphi)\lambda_w+1}{1-\lambda_w}}\left(\frac{w_{t+1}^{new}\gamma_{t+1}}{w_t^{new}}\right)^{\frac{(1+\varphi)\lambda_w}{\lambda_w-1}}v_{1,t+1} \tag{109}$$

$$w_t^{\frac{1}{1-\lambda_w}} = \xi_w\left(\frac{\left(\pi_{t-1}^c\right)^{\kappa_w}\bar{\pi}_t^{1-\kappa_w}}{\pi_t^c}\right)^{\frac{1}{1-\lambda_w}}\left(\frac{w_{t-1}}{\gamma_t}\right)^{\frac{1}{1-\lambda_w}} + (1-\xi_w)\left(w_t^{new}\right)^{\frac{1}{1-\lambda_w}} \tag{110}$$

$$\frac{b_t^S}{\varepsilon_t^{b^S}} + \frac{\bar{b}_t^L}{\varepsilon_t^{\bar{b}^L}} + T_t = \frac{r_{t-1}^S}{\pi_t^c}\frac{b_{t-1}^S}{\gamma_t} + \frac{r_t^L}{\pi_t^c}\frac{p_{L,t}}{p_{L,t-1}}\frac{\bar{b}_{t-1}^L}{\gamma_t} + \tilde{p}_t G_t \tag{111}$$

$$q_t\tilde{p}_t^x c_t^x = q_t c_t^m + \frac{\bar{b}_t^{D,*}}{\varepsilon_t^{\bar{b}^L}} - \frac{r_t^{L,*}}{\pi_t^*}\frac{p_{L,t}^*}{p_{L,t-1}^*}\frac{q_t}{q_{t-1}}\frac{\bar{b}_{t-1}^{D,*}}{\gamma_t^*} + \frac{x}{2}\left[lq_t - \vartheta lq_{t-1} - (1-\vartheta)\,lq\right]^2\frac{1}{\pi_t^c} \tag{112}$$

$$f_{1,t} = \lambda_t mc_t y_t^d + \beta\xi_y E_t\left(\frac{\pi_t^{\kappa_y}\bar{\pi}_{t+1}^{1-\kappa_y}}{\pi_{t+1}}\right)^{-\frac{\lambda_y}{\lambda_y-1}}f_{1,t+1}\gamma_{t+1}^{1-\sigma_C} \tag{113}$$

$$f_{2,t} = \lambda_t\bar{\pi}_t y_t^d + \beta\xi_d E_t\left(\frac{\pi_t^{\kappa_y}\bar{\pi}_{t+1}^{1-\kappa_y}}{\pi_{t+1}}\right)^{-\frac{1}{\lambda_y-1}}\left(\frac{\bar{\pi}_t}{\bar{\pi}_{t+1}}\right)f_{2,t+1}\gamma_{t+1}^{1-\sigma_C} \tag{114}$$

$$0 = \lambda_y f_{1,t} - f_{2,t} \tag{115}$$

$$1 = \xi_y\left(\frac{\pi_{t-1}^{\kappa_y}\bar{\pi}_{t+1}^{1-\kappa_y}}{\pi_t}\right)^{-\frac{1}{\lambda_y-1}} + \left(1-\xi_y\right)\bar{\pi}_t^{-\frac{1}{\lambda_y-1}} \tag{116}$$

$$g_{1,t} = \lambda_t mc_t^m c_{t+j}^m + \beta\xi_m E_t \left(\frac{(\pi_t^m)^{\kappa_m} \bar{\pi}_{t+1}^{1-\kappa_m}}{\pi_{t+1}^m} \right)^{-\frac{\lambda_m}{\lambda_m - 1}} g_{1,t+1}\gamma_{t+1}^{1-\sigma_C} \tag{117}$$

$$g_{2,t} = \lambda_t \bar{\pi}_t^m c_{t+j}^m + \beta\xi_m E_t \left(\frac{(\pi_t^m)^{\kappa_m} \bar{\pi}_{t+1}^{1-\kappa_m}}{\pi_{t+1}^m} \right)^{-\frac{1}{\lambda_m - 1}} \left(\frac{\bar{\pi}_t^m}{\bar{\pi}_{t+1}^m} \right) g_{2,t+1}\gamma_{t+1}^{1-\sigma_C} \tag{118}$$

$$0 = \lambda_{m,t} g_{1,t} - g_{2,t} \tag{119}$$

$$1 = \xi_m \left(\frac{\left(\pi_{t-1}^m\right)^{\kappa_m} \bar{\pi}_t^{1-\kappa_m}}{\pi_t^m} \right)^{-\frac{1}{\lambda_m - 1}} + (1 - \xi_m) \left(\bar{\pi}_t^m\right)^{-\frac{1}{\lambda_m - 1}} \tag{120}$$

$$u_{1,t} = \lambda_t mc_t^x c_{t+j}^x + \beta\xi_x E_t \left(\frac{(\pi_t^x)^{\kappa_x} \bar{\pi}_{t+1}^{1-\kappa_x}}{\pi_{t+1}^x} \right)^{-\frac{\lambda_x}{\lambda_x - 1}} u_{1,t+1}\gamma_{t+1}^{1-\sigma_C} \tag{121}$$

$$u_{2,t} = \lambda_t \bar{\pi}_t^x c_{t+j}^x + \beta\xi_x E_t \left(\frac{(\pi_t^x)^{\kappa_x} \bar{\pi}_{t+1}^{1-\kappa_x}}{\pi_{t+1}^x} \right)^{-\frac{1}{\lambda_x - 1}} \left(\frac{\bar{\pi}_t^x}{\bar{\pi}_{t+1}^x} \right) u_{2,t+1}\gamma_{t+1}^{1-\sigma_C} \tag{122}$$

$$0 = \lambda_{x,t} u_{1,t} - u_{2,t} \tag{123}$$

$$1 = \xi_x \left(\frac{\left(\pi_{t-1}^x\right)^{\kappa_x} \bar{\pi}_t^{1-\kappa_x}}{\pi_t^x} \right)^{-\frac{1}{\lambda_x - 1}} + (1 - \xi_x) \left(\bar{\pi}_t^x\right)^{-\frac{1}{\lambda_x - 1}} \tag{124}$$

1.4 Steady states

1.4.1 Domestic economy

We assume that

$$\pi^c = \pi = \pi^m = 1 \tag{125}$$

Household's Euler equation implies

$$r^h = r^S = r^L = \frac{\gamma^{\sigma_C}\pi}{\beta} \tag{126}$$

Now, from importing and exporting firms we know

$$\begin{aligned} mc^m &\equiv \frac{sp^*}{p^m} = \frac{sp^*}{p^c}\frac{p^c}{p^m} = \frac{q}{\tilde{p}^m} = \frac{1}{\lambda_m} \\ \tilde{p}^m &= \lambda_m \end{aligned} \tag{127}$$

$$\begin{aligned} mc^x &= \frac{p}{sp^x} = \frac{p^c}{sp^*}\frac{p^*}{p^x}\frac{p}{p^c} = \frac{\tilde{p}}{q\tilde{p}^x} = \frac{1}{\lambda_x} \\ \frac{\tilde{p}}{q} &= \frac{\tilde{p}^x}{\lambda_x} \end{aligned} \tag{128}$$

We also know

$$\begin{aligned} 1 &= (1 - \alpha)\,\tilde{p}^{1-\eta_c} + \alpha\,(\tilde{p}^m)^{1-\eta_c} \\ 1 &= (1 - \alpha) \left(\frac{q\tilde{p}^x}{\lambda_x} \right)^{1-\eta_c} + \alpha\,(q\lambda_m)^{1-\eta_c} \end{aligned}$$

Suppose we set $q = 1$ and $\tilde{p}^x = 1$ then

$$\alpha = \frac{1 - \frac{1}{\lambda_x^{1-\eta_c}}}{\lambda_m^{1-\eta_c} - \frac{1}{\lambda_x^{1-\eta_c}}} \tag{129}$$

$$\tilde{p} = \frac{1}{\lambda_x} \tag{130}$$

$$\frac{i}{k} = \delta_K \tag{131}$$

$$r^k = r - (1 - \delta_K)$$

$$k = \frac{\phi}{1-\phi}\frac{w}{r^k}h$$

$$\begin{aligned} \frac{wh}{\tilde{p}\,(1-\phi)\,Ah^{1-\phi}k^{\phi}} &= \frac{1}{\lambda_y} \\ \frac{wh}{\tilde{p}\,(1-\phi)\,Ah^{1-\phi}\left(\frac{\phi}{1-\phi}\frac{h}{r^k}\right)^{\phi}w^{\phi}} &= \frac{1}{\lambda_y} \end{aligned}$$

$$w = \left[\frac{\tilde{p}\,(1-\phi)\,Ah^{1-\phi}\left(\frac{\phi}{1-\phi}\frac{h}{r^k}\right)^{\phi}}{h\lambda_y}\right]^{\frac{1}{1-\phi}} \tag{132}$$

$$k = \frac{\phi}{1-\phi}\frac{w}{r^k}h \tag{133}$$

For a given value of g consumption is

$$y = c^{dp} + c^x + gy + i$$

Using the demand for domestic consumption expression

$$y = (1-\alpha)\,\tilde{p}^{-\eta_c}c + c^x + gy + i$$

Futhermore, from the current account equation and imports demand equation

$$c^x = \frac{\tilde{p}^M c^m + \bar{b}^{L,D,*}\left(1 - \frac{(\gamma^*)^{\sigma_C - 1}}{\beta}\right)}{\tilde{p}^X}$$

$$y = (1-\alpha)\,\tilde{p}^{-\eta_c}c + \frac{\tilde{p}^M\alpha\,(\tilde{p}^m)^{-\eta_c}\,c}{\tilde{p}^X} + \frac{\bar{b}^{L,D,*}\left(1 - \frac{(\gamma^*)^{\sigma_C - 1}}{\beta}\right)}{\tilde{p}^X} + gy + i$$

$$c = \frac{y - \frac{\bar{b}^{L,D,*}\left(1 - \frac{(\gamma^*)^{\sigma_C - 1}}{\beta}\right)}{\tilde{p}^X} - gy - i}{(1-\alpha)\,\tilde{p}^{-\eta_c} + \frac{\tilde{p}^M}{\tilde{p}^X}\alpha\,(\tilde{p}^m)^{-\eta_c}}$$

Price of long-term debt

$$p_L = \frac{1}{r^L - \kappa} \tag{134}$$

From domestic firms we know

$$\begin{aligned} mc &= \frac{wh}{\tilde{p}\,(1-\phi)\,y} = \frac{1}{\lambda_y} \\ w &= \frac{\tilde{p}\,(1-\phi)\,y}{\lambda_y h} \end{aligned} \tag{135}$$

From the marginal utility expression

$$\frac{\gamma^{\sigma_C} - \beta b}{([\gamma - b]\,c)^{\sigma_c}} = \lambda \tag{136}$$

$$f_1 = \frac{\lambda mcy}{1 - \beta\xi_y\gamma^{1-\sigma_C}} \tag{137}$$

$$f_2 = \frac{\lambda y}{1 - \beta\xi_d\gamma^{1-\sigma_C}} \tag{138}$$

$$mc = \frac{1}{\lambda_y} \tag{139}$$

1.4.2 Foreign economy

$$\pi^{c,*} = 1 \tag{140}$$

$$mc^* = \frac{1}{\lambda_d^*} \tag{141}$$

$$w^* = \frac{z^*}{\lambda_d^*} \tag{142}$$

$$\lambda^* = \frac{1}{(c^*)^{\sigma_c}} \frac{(1-\beta b)\,d}{(1-b)^{\sigma_c}} \tag{143}$$

$$h^* = \left[\frac{w\frac{\lambda}{\lambda_w}}{d\psi}\right]^{\frac{1}{\varphi}} \tag{144}$$

$$\begin{aligned} c^* + g^* z^* h^{d,*} &= z^* h^{d,*} \\ c^* &= (1-g^*)\,z^* h^{d,*} \end{aligned}$$

1.5 Linearised Equations

1.5.1 Domestic Economy

$$\begin{aligned} \hat{\pi}_t - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\beta\kappa_y} E_t\left(\hat{\pi}_{t+1} - \hat{\bar{\pi}}_{t+1}^c\right) + \frac{\kappa_y}{1+\beta\kappa_y}\left(\hat{\pi}_{t-1} - \hat{\bar{\pi}}_t^c\right) - \frac{\beta\kappa_y}{1+\beta\kappa_y}\left(\hat{\bar{\pi}}_t^c - \hat{\bar{\pi}}_{t+1}^c\right) \\ &\quad + \frac{\left(1-\xi_y\right)\left(1-\beta\xi_y\right)}{\xi_y\left(1+\beta\kappa_y\right)}\left[\hat{w}_t - \left(\hat{y}_t - \hat{h}_t\right) - \hat{\tilde{p}}_t\right] \end{aligned} \tag{145}$$

$$\begin{aligned} \hat{\pi}_t^m - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\beta\kappa_m} E_t\left(\hat{\pi}_{t+1}^m - \hat{\bar{\pi}}_{t+1}^c\right) + \frac{\kappa_m}{1+\beta\kappa_m}\left(\hat{\pi}_{t-1}^m - \hat{\bar{\pi}}_t^c\right) - \frac{\beta\kappa_m}{1+\beta\kappa_m}\left(\hat{\bar{\pi}}_t^c - \hat{\bar{\pi}}_{t+1}^c\right) \\ &\quad + \frac{(1-\xi_m)\,(1-\beta\xi_m)}{\xi_m\,(1+\beta\kappa_m)}\left(\hat{q}_t - \hat{\tilde{p}}_t^m\right) \end{aligned} \tag{146}$$

$$\hat{\pi}_t^x - \hat{\bar{\pi}}_t^c = \frac{\beta}{1+\beta\kappa_x} E_t\left(\hat{\pi}_{t+1}^x - \hat{\bar{\pi}}_{t+1}^c\right) + \frac{\kappa_x}{1+\beta\kappa_x}\left(\hat{\pi}_{t-1}^x - \hat{\bar{\pi}}_t^c\right) - \frac{\beta\kappa_x}{1+\beta\kappa_x}\left(\hat{\bar{\pi}}_t^c - \hat{\bar{\pi}}_{t+1}^c\right) + \frac{(1-\xi_x)(1-\beta\xi_x)}{\xi_x(1+\beta\kappa_x)}\left(\hat{\bar{p}}_t - \hat{q}_t - \hat{\bar{p}}_t^x\right). \quad (147)$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} - \sigma_C E_t\hat{\gamma}_{t+1} + r_t^h - E_t\hat{\pi}_{t+1} \quad (148)$$

$$\frac{\gamma^{\sigma_C} - \beta b}{\gamma^{\sigma_C}}\hat{\lambda}_t = \left[\hat{d}_t - \beta\gamma^{1-\sigma_C}E_t\hat{d}_{t+1}\right] - \frac{\sigma_C}{1-\frac{b}{\gamma}}\left\{\left(1 + \frac{\beta b^2}{\gamma^{1+\sigma_C}}\right)\hat{c}_t - \frac{b}{\gamma}\left(\hat{c}_{t-1} - \hat{\gamma}_t\right) - \frac{\beta b}{\gamma^{\sigma_C}}\left(E_t\hat{c}_{t+1} + E_t\hat{\gamma}_{t+1}\right)\right\} \quad (149)$$

$$\hat{v}_{1,t} = \left(1 - \beta\xi_w\gamma^{\frac{(1-\sigma_C)(\lambda_w-1)+1}{\lambda_w-1}}\right)\left(\hat{\lambda}_t - \frac{1}{\lambda_w-1}\hat{w}_t^{new} + \frac{\lambda_w}{\lambda_w-1}\hat{w}_t + \hat{h}_t\right) + \beta\xi_w\gamma^{\frac{(1-\sigma_C)(\lambda_w-1)+1}{\lambda_w-1}}\left\{\begin{array}{c}\hat{v}_{1,t+1} + (1-\sigma_C)\hat{\gamma}_{t+1} \\ +\frac{1}{\lambda_w-1}\left[\hat{\pi}_{t+1}^c - \kappa_w\hat{\pi}_t^c - (1-\kappa_w)\hat{\bar{\pi}}_{t+1}^{1-\kappa_w} + \Delta\hat{w}_{t+1}^{new} + \hat{\gamma}_{t+1}\right]\end{array}\right\} \quad (150)$$

$$\hat{v}_{1,t} = \left(1 - \beta\xi_w\gamma^{\frac{(1-\sigma_C)(\lambda_w-1)+(1+\varphi)\lambda_w}{\lambda_w-1}}\right)\left[\frac{\lambda_w(1+\varphi)}{\lambda_w-1}(\hat{w}_t - \hat{w}_t^{new}) + (1+\varphi)\hat{h}_t\right] + \beta\xi_w\gamma^{\frac{(1-\sigma_C)(\lambda_w-1)+(1+\varphi)\lambda_w}{\lambda_w-1}}\left\{\begin{array}{c}\hat{v}_{1,t+1} + (1-\sigma_C)\hat{\gamma}_{t+1} \\ +\frac{(1+\varphi)\lambda_w}{\lambda_w-1}\left[\hat{\pi}_{t+1}^c - \kappa_w\hat{\pi}_t^c - (1-\kappa_w)\hat{\bar{\pi}}_{t+1}^{1-\kappa_w} + \Delta\hat{w}_{t+1}^{new} + \hat{\gamma}_{t+1}\right]\end{array}\right\} \quad (151)$$

$$\hat{w}_t^{new} - \hat{w}_t = \frac{\xi_w\gamma^{-\frac{1}{1-\lambda_w}}}{1-\xi_w}\left(\hat{\pi}_t^c - \kappa_w\hat{\pi}_{t-1}^c - (1-\kappa_w)\hat{\bar{\pi}}_t^{1-\kappa_w} + \Delta\hat{w}_t + \hat{\gamma}_t\right) \quad (152)$$

$$\hat{w}_t^{obs} = \hat{w}_t + \hat{\bar{\lambda}}_{w,t}$$

$$\hat{\pi}_t^c - \hat{\bar{p}}_{t-1} = (1-\alpha)\hat{\pi}_t + \alpha\tilde{p}^m(1-\eta_c)\left(\hat{\pi}_t^m + \hat{\bar{p}}_{t-1}^m - \hat{\bar{p}}_{t-1}\right) \quad (153)$$

$$\hat{y}_t = (1-\phi)\hat{h}_t + \hat{z}_t \quad (154)$$

$$\hat{y}_t = \frac{c^d}{(1-g)y}\hat{c}_t^d + \frac{c^x}{(1-g)y}\hat{c}_t^x + \hat{\bar{g}}_t \quad (155)$$

$$\hat{\bar{p}}_t^m = \hat{\bar{p}}_{t-1}^m + \hat{\pi}_t^m - \hat{\pi}_t^c \quad (156)$$

$$\hat{\bar{p}}_t = \hat{\bar{p}}_{t-1} + \hat{\pi}_t - \hat{\pi}_t^c \quad (157)$$

$$\hat{\bar{p}}_t^x = \hat{\bar{p}}_{t-1}^x + \hat{\pi}_t^x - \hat{\pi}_t^{c,*} \quad (158)$$

$$\hat{r}_t^h = \hat{r}_t^S + \chi\hat{v}_t + \hat{\varepsilon}_t^{b^S} \quad (159)$$

where $\chi = \frac{x\beta}{b^S}$ and $\hat{v}_t = \frac{\bar{b}^L}{\bar{b}^L+\bar{b}^{L,D,*}}\hat{\bar{b}}_t^L + \frac{\bar{b}^{L,D,*}}{\bar{b}^L+\bar{b}^{L,D,*}}\hat{\bar{b}}_t^{L,D,*} + \frac{1}{\bar{b}^L+\bar{b}^{L,D,*}}\hat{\omega}_t - \hat{b}_t^S$

$$E_t\hat{r}_{t+1}^L + E_t\Delta\hat{p}_{L,t+1} = \hat{r}_t^h - \hat{\varepsilon}_t^{\bar{b}^L} + \frac{\chi}{\delta}\hat{v}_t \quad (160)$$

$$E_t\Delta\hat{q}_{t+1} = \hat{r}_t^S - E_t\pi_{t+1}^c + \frac{\chi(1+\delta)}{\delta}\hat{v}_t + \hat{\varepsilon}_t^{b^S} - \hat{\varepsilon}_t^{\bar{b}^L} - \left\{E_t\hat{r}_{t+1}^{L,*} - E_t\pi_{t+1}^{c,*} + E_t\Delta\hat{p}_{L,t+1}^*\right\} \quad (161)$$

$$\hat{b}_t^S + \delta \bar{b}_t^L = \frac{1}{\beta}\left(\hat{b}_{t-1}^S + \delta \bar{b}_{t-1}^L - (1+\delta)\,\hat{\gamma}_t\right) - \frac{T}{b^S}\hat{T}_t + \frac{1}{\beta}\left\{\hat{r}_{t-1}^S - \hat{\pi}_t^c + \delta\left(\hat{r}_t^L - \hat{\pi}_t^c + \hat{p}_{L,t} - \hat{p}_{L,t-1}\right)\right\} + \frac{\tilde{p}gy}{b^S}\left(\hat{\tilde{p}}_t + \hat{g}_t + \hat{y}_t\right) \tag{162}$$

$$\frac{T}{b^S}\hat{T}_t = \theta\left(\hat{b}_{t-1}^S + \delta \bar{b}_{t-1}^L - (1+\delta)\,\hat{\gamma}_t\right) \tag{163}$$

$$r_t^S = \phi_R r_{t-1}^S + (1-\phi_R)\left(\phi_\pi \pi_t^c + \phi_y y_t\right) + \sigma_R \omega_t^R \tag{164}$$

$$\frac{c^x}{\bar{b}^{L,D,*}}\left(\hat{q}_t + \hat{\tilde{p}}_t^x + \hat{c}_t^x\right) = \frac{c^m}{\bar{b}^{L,D,*}}\left(\hat{q}_t + \hat{c}_t^m\right) + \hat{\tilde{b}}_t^{L,D,*} + \hat{\omega}_t - \varepsilon_t^{\bar{b}^L} - \frac{1}{\beta}\left\{\hat{r}_t^{L,*} - \hat{\pi}_t^{c,*} + \Delta\hat{p}_{L,t}^* + \Delta\hat{q}_{t+1} + \hat{\tilde{b}}_{t-1}^{L,D,*} - \gamma_{t-1} + \hat{\omega}_t\right\} \tag{165}$$

$$\hat{c}_t^m = -\eta_c \hat{\tilde{p}}_t^m + \hat{c}_t \tag{166}$$

$$\hat{c}_t^d = -\eta_c \hat{\tilde{p}}_t + \hat{c}_t \tag{167}$$

$$\hat{\gamma}_t = \hat{\gamma}_t^* - \Delta\hat{\varsigma}_t \tag{168}$$

1.5.2 Foreign Economy

$$\hat{\pi}_t^{c,*} - \hat{\bar{\pi}}_t^* = \frac{\beta}{1+\beta\kappa_y^*}E_t\left(\hat{\pi}_{t+1}^{c,*} - \hat{\bar{\pi}}_{t+1}^*\right) + \frac{\kappa_y^*}{1+\beta\kappa_y^*}\left(\hat{\pi}_{t-1}^{c,*} - \hat{\bar{\pi}}_{t-1}^*\right) - \frac{\beta^*\kappa_y^*}{1+\beta^*\kappa_y^*}\left(\hat{\bar{\pi}}_t^{c,*} - \hat{\bar{\pi}}_{t+1}^{c,*}\right) + \frac{\left(1-\xi_y^*\right)\left(1-\beta\xi_y^*\right)}{\xi_y^*\left(1+\beta\kappa_y^*\right)}\left[\hat{w}_t^* - \left(\hat{y}_t^* - \hat{h}_t^*\right)\right] + \hat{\tilde{\lambda}}_{y,t}^* \tag{169}$$

$$\hat{\lambda}_t^* = \frac{1}{1-\beta b^*}\left\{\left(\hat{d}_t^* - \beta b^* \hat{d}_{t+1}^*\right) - \frac{\sigma_C^*}{1-b^*}\left[\left(1+\beta b^{*2}\right)\hat{c}_t^* - \beta b^* E_t\hat{c}_{t+1}^* + b^*\hat{c}_{t-1}^*\right]\right\} \tag{170}$$

$$\hat{\lambda}_t^* = E_t\hat{\lambda}_{t+1}^* + r_t^{h,*} - E_t\hat{\pi}_{t+1}^{c,*} \tag{171}$$

$$\hat{v}_{1,t}^* = \left(1-\beta\xi_w^*\gamma^{\frac{(1-\sigma_C^*)(\lambda_w^*-1)+1}{\lambda_w-1}}\right)\left(\hat{\lambda}_t^* - \frac{1}{\lambda_w^*-1}\hat{w}_t^{new,*} + \frac{\lambda_w^*}{\lambda_w^*-1}\hat{w}_t^* + \hat{h}_t^*\right) + \beta\xi_w^*\gamma^{\frac{(1-\sigma_C^*)(\lambda_w^*-1)+1}{\lambda_w^*-1}}\left\{\begin{array}{c}\hat{v}_{1,t+1}^* + (1-\sigma_C^*)\,\hat{\gamma}_{t+1}^* \\ +\frac{1}{\lambda_{w-1}^*}\left[\hat{\pi}_{t+1}^{c,*} - \kappa_w\hat{\pi}_t^{c,*} - (1-\kappa_w^*)\,\hat{\bar{\pi}}_{t+1}^{1-\kappa_w,*} + \Delta\hat{w}_{t+1}^{new,*} + \hat{\gamma}_{t+1}^*\right]\end{array}\right\} \tag{172}$$

$$\hat{v}_{1,t}^* = \left(1-\beta\xi_w^*\gamma^{\frac{(1-\sigma_C)(\lambda_w^*-1)+(1+\varphi^*)\lambda_w^*}{\lambda_w-1}}\right)\left[\frac{\lambda_w^*\left(1+\varphi^*\right)}{\lambda_w-1}\left(\hat{w}_t^* - \hat{w}_t^{new,*}\right) + \left(1+\varphi^*\right)\hat{h}_t^*\right] + \beta\xi_w^*\gamma^{\frac{(1-\sigma_C^*)(\lambda_w^*-1)+(1+\varphi^*)\lambda_w^*}{\lambda_w-1}}\left\{\begin{array}{c}\hat{v}_{1,t+1}^* + (1-\sigma_C^*)\,\hat{\gamma}_{t+1}^* \\ +\frac{(1+\varphi^*)\lambda_w^*}{\lambda_w-1}\left[\hat{\pi}_{t+1}^{c,*} - \kappa_w^*\hat{\pi}_t^{c,*} - (1-\kappa_w^*)\,\hat{\bar{\pi}}_{t+1}^{1-\kappa_w,*} + \Delta\hat{w}_{t+1}^{new,*} + \hat{\gamma}_{t+1}^*\right]\end{array}\right\} \tag{173}$$

$$\hat{w}_t^{new,*} - \hat{w}_t^* = \frac{\xi_w^*\gamma^{-\frac{1}{1-\lambda_w^*}}}{1-\xi_w^*}\left(\hat{\pi}_t^{c,*} - \kappa_w^*\hat{\pi}_{t-1}^{c,*} - (1-\kappa_w^*)\,\bar{\pi}_t^{1-\kappa_w,*} + \Delta\hat{w}_t^* + \hat{\gamma}_t^*\right)$$

$$\hat{w}_t^{obs} = \hat{w}_t + \hat{\tilde{\lambda}}_{w,t}$$

$$\hat{y}_t^* = \left(1-\phi^*\right)\hat{h}_t^* + \hat{z}_t^* \tag{174}$$

$$\hat{y}_t^* = \frac{c^*}{\left(1-g^*\right)y^*}\hat{c}_t^* + \hat{\tilde{g}}_t^* \tag{175}$$

$$\hat{r}_t^{h,*} = \hat{r}_t^{S,*} + \chi^* \hat{v}_t^* + \hat{\varepsilon}_t^{b^{S,*}} \tag{176}$$

where $\chi^* = \frac{x^*\beta}{b^{S,*}}$ and $\hat{v}_t^* = \hat{\bar{b}}_t^{L,*} - \hat{b}_t^{S,*}$

$$E_t \hat{r}_{t+1}^{L,*} + E_t \Delta \hat{p}_{L,t+1}^* = \hat{r}_t^{h,*} - \hat{\varepsilon}_t^{\bar{b}^{L,*}} + \frac{\chi^*}{\delta^*} \hat{v}_t^* \tag{177}$$

$$\begin{aligned} \hat{b}_t^{S,*} + \delta^* \bar{b}_t^{L,*} &= \frac{1}{\beta}\left(\hat{b}_{t-1}^{S,*} + \delta^* \bar{b}_{t-1}^{L,*}\right) - \frac{T^*}{b^{S,*}} \hat{T}_t^* \\ &+ \frac{1}{\beta}\left\{\hat{r}_{t-1}^{S,*} - \hat{\pi}_t^{c,*} + \delta^*\left(\hat{r}_t^{L,*} - \hat{\pi}_t^{c,*} + \Delta \hat{p}_{L,t}^*\right)\right\} + \frac{g^* y^*}{b^{S,*}}\left(\hat{g}_t^* + \hat{y}_t^*\right) \end{aligned} \tag{178}$$

$$\frac{T^*}{b^{S,*}} \hat{T}_t^* = \theta^*\left(\hat{b}_{t-1}^{S,*} + \delta^* \bar{b}_{t-1}^{L}\right) \tag{179}$$

1.6 Term Structure Decomposition

Long-Term Interest Rates

$$\hat{r}_{t+1}^L + \hat{p}_{L,t+1} - \hat{p}_{L,t} = \hat{r}_t^h - \hat{\varepsilon}_t^{\bar{b}^L} + \frac{\chi}{\delta}\left(\widehat{lq}_t - \vartheta \widehat{lq}_{t-1}\right)$$

Household Effective Rate

$$\hat{r}_t^h = \hat{r}_t^S + \chi \hat{v}_t + \hat{\varepsilon}_t^{b^S}$$

Long-Term Asset Price

$$\hat{p}_{L,t} = -\frac{1}{1-\beta\kappa} \hat{r}_t^L$$

Substituting the last two expression into first equation

$$\hat{r}_{t+1}^L - \frac{1}{1-\beta\kappa} \hat{r}_{t+1}^L + \frac{1}{1-\beta\kappa} \hat{r}_t^L = \hat{r}_t^S + \chi\left(\widehat{lq}_t - \vartheta \widehat{lq}_{t-1}\right) + \frac{\chi}{\delta}\left(\widehat{lq}_t - \vartheta \widehat{lq}_{t-1}\right) + \hat{\varepsilon}_t^{b^S} - \hat{\varepsilon}_t^{\bar{b}^L} \tag{180}$$

or

$$\hat{r}_t^L = \beta\kappa \hat{r}_{t+1}^L + (1-\beta\kappa)\,\hat{r}_t^S + (1-\beta\kappa)\left(\hat{\varepsilon}_t^{b^S} - \hat{\varepsilon}_t^{\bar{b}^L}\right) + \frac{(1-\beta\kappa)\,\chi\,(1+\delta)}{\delta}\left(\widehat{lq}_t - \vartheta \widehat{lq}_{t-1}\right) \tag{181}$$

Iterating the last equation forwards we obtain

$$\begin{aligned} \hat{r}_t^L &= (1-\beta\kappa)\sum_{i=0}^{\infty} (\beta\kappa)^i\, E_t \hat{r}_{t+i}^S + (1-\beta\kappa)\sum_{i=0}^{\infty} (\beta\kappa)^i \left(E_t \hat{\varepsilon}_{t+i}^{b^S} - E_t \hat{\varepsilon}_{t+i}^{\bar{b}^L}\right) \\ &+ \frac{(1-\beta\kappa)\,\chi\,(1+\delta)}{\delta}\sum_{i=0}^{\infty} (\beta\kappa)^i\, E_t^L\left(\widehat{lq}_t - \vartheta \widehat{lq}_{t-1}\right) \end{aligned} \tag{182}$$

2 Further robustness analysis

In this section we evaluate numerous extensions and alternatives to the benchmark model. The following table provides an overview of these by means of Log Marginal Likelihoods, the next subsections provide more detailed results for each model.

Table 1: Marginal Likelihood Comparison

Model	Log Marginal Likelihood
Benchmark	-3075.9
Domestic Inflation Taylor Rule	-3077.5
Non-Stationary Long-Term Risk Premia	-3707.9
Non-Stationary Productivity Growth	-3722.8

Note that two of the subsections evaluate the performance of the model for a different set of observables: one exercise evaluates adding inflation expectations as observable, another truncates the sample to the pre-Great Recession period. As a result, their marginal likelihood is not directly comparable to that of the benchmark model.

2.1 Observed US Long-Term Inflation Expectations

Although measures for US long-term inflation expectations are available (Del Negro et al. (2015), Del Negro et al. (2017), Mumtaz and Theodoridis (2018)), this is not the same for the UK economy; the available UK data series usually start quite recently (well after 1992). To avoid working with a relatively short time span dataset, or working with asymmetric observables across countries, the benchmark estimation does not include long-term inflation expectations into the vector of observable variables. In this section we investigate whether this influences our results.

All the results presented in this section suggest categorically that our approach of not including the long-term inflation expectations into the econometrician's dataset has almost no effect on the results.

Table 2: Foreign Parameter Estimates

Mnemonic	Description	Benchmark	Non-stationary Risk Shocks
λ_{y^*}	Steady-State Foreign Prices Markup	1.11	1.12
χ^*	Liquidity Adjustment Cost	2.05	2.38
ϑ^*	Dynamic Liquidity Adjustment Cost	0.94	0.94
h^*	Consumption Habit	0.70	0.66
κ_y^*	Price Indexation	0.31	0.20
ξ_y^*	Price Reset Probability	0.92	0.92
κ_w^*	Wage Indexation	0.42	0.38
ξ_w^*	Wage Reset Probability	0.73	0.80
$\phi_{r^S}^*$	Policy Rate Smoothing	0.83	0.81
ϕ_π^*	Policy Reaction to Inflation	1.78	1.21
ϕ_y^*	Policy Reaction to Output	0.04	0.03
θ^*	Lump Sum Tax Response	0.02	0.02
ρ_γ^*	Persistence of Non Stationary Productivity Shock	0.09	0.06
$\rho_{B^S}^*$	Persistence of Short-Term Debt Risk Premium Shock	0.91	0.97
$\rho_{\bar{B}^L}^*$	Persistence of Long-Term Debt Risk Premium Shock	0.89	0.88
ρ_p^*	Persistence of Price Markup Shock	0.02	0.50
ρ_w^*	Persistence of Wage Markup Shock	0.90	0.91
$\rho_{\bar{\pi}}^*$	Persistence of Inflation Target Shock	0.21	0.22
$100\sigma_\gamma^*$	Uncertainty of Non Stationary Productivity Shock	0.97	1.06
$100\sigma_{B^S}^*$	Uncertainty of Short-Term Debt Risk Premium Shock	0.18	0.06
$100\sigma_{\bar{B}^L}^*$	Uncertainty of Long-Term Debt Risk Premium Shock	0.43	0.50
$100\sigma_p^*$	Uncertainty of Price Markup Shock	0.12	0.10
$100\sigma_w^*$	Uncertainty of Wage Markup Shock	0.71	0.76
$100\sigma_{\bar{\pi}}^*$	Uncertainty of Inflation Target Shock	0.02	0.01
$100\sigma_{r^S}^*$	Uncertainty of Policy Rate Shock	0.19	0.19

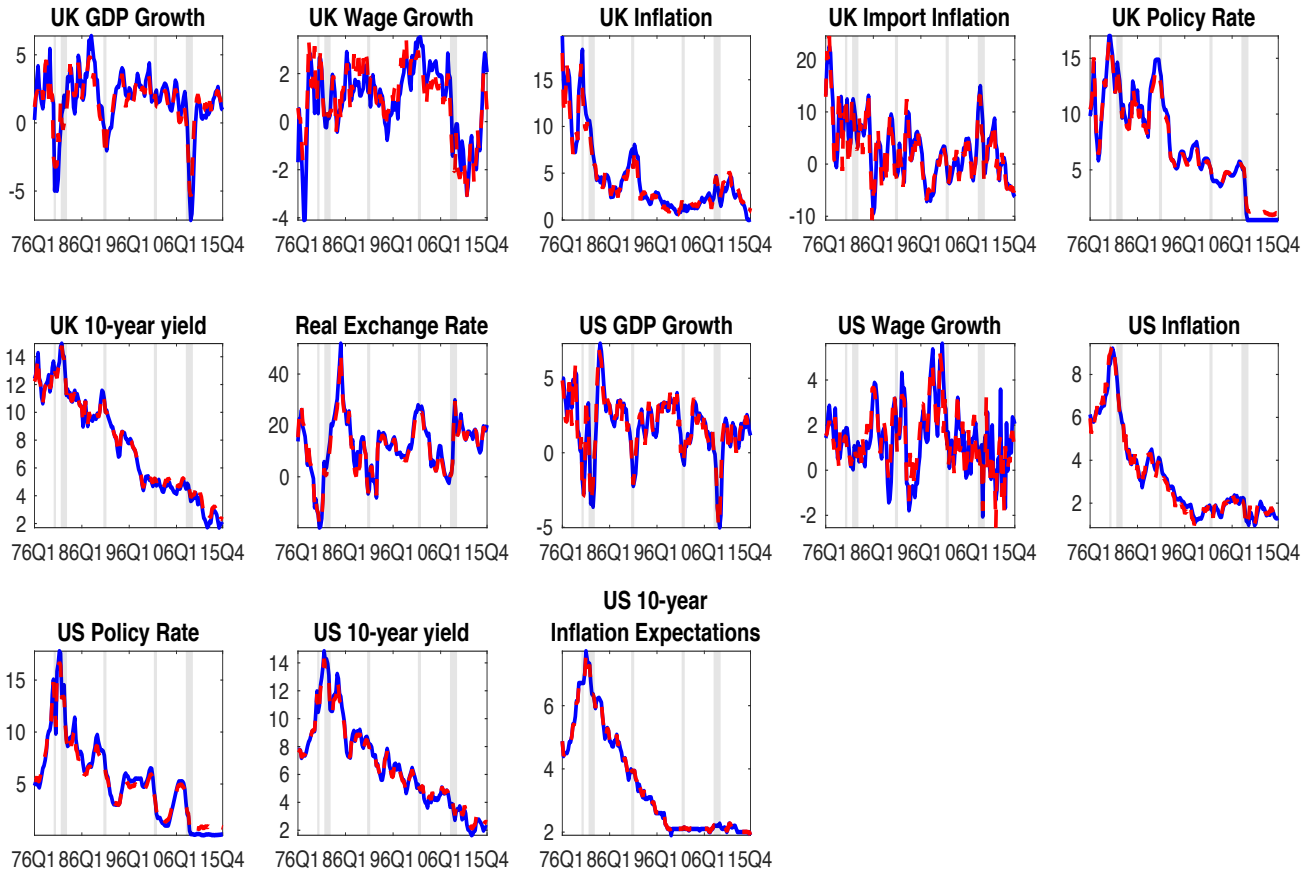
Table 3: Domestic Parameter Estimates

Mnemonic	Description	Benchmark	Non-stationary Risk Shocks
λ_y	Steady-State Domestic Prices Markup	1.13	1.15
λ_m	Steady-State Import Prices Markup	1.24	1.25
λ_x	Steady-State Export Prices Markup	1.05	1.05
η_C	Import Price Elasticity	1.45	1.41
$\delta_{B,D}$	Domestic Long- to Short-Term Debt Ratio	0.46	0.62
$\delta_{B,F}$	Foreign Long- to Short-Term Debt Ratio	0.18	0.18
χ	Liquidity Adjustment Cost	5.10	4.66
ϑ	Dynamic Liquidity Adjustment Cost	0.25	0.46
h	Consumption Habit	0.81	0.85
κ_y	Domestic Price Indexation	0.28	0.20
ξ_y	Domestic Price Reset Probability	0.91	0.91
κ_m	Import Price Indexation	0.25	0.20
ξ_m	Import Price Reset Probability	0.50	0.50
κ_w	Wage Indexation	0.77	0.68
ξ_w	Wage Reset Probability	0.73	0.75
κ_x	Export Price Indexation	0.27	0.28
ξ_x	Export Price Reset Probability	0.87	0.87
ϕ_{r^S}	Policy Rate Smoothing	0.90	0.90
ϕ_π	Policy Reaction to Inflation	1.01	1.04
ϕ_y	Policy Reaction to Output	0.11	0.05
θ	Lump Sum Tax Response	0.01	0.01
ρ_ω	Persistence of Stationary Productivity Shock	0.16	0.34
ρ_{B^S}	Persistence of Short-Term Debt Risk Premium Shock	0.87	0.88
$\rho_{\bar{B}^L}$	Persistence of Long-Term Debt Risk Premium Shock	0.83	0.91
ρ_p	Persistence of Price Markup Shock	0.01	0.05
ρ_w	Persistence of Wage Markup Shock	0.93	0.93
ρ_m	Persistence of Import Pice Markup Shock	0.77	0.83
$\rho_{\bar{\pi}}$	Persistence of Inflation Target Shock	0.04	0.05
$100\sigma_\omega$	Uncertainty of Productivity Shock	2.50	2.37
$100\sigma_{B^S}$	Uncertainty of Short-Term Debt Risk Premium Shock	0.47	0.46
$100\sigma_{\bar{B}^L}$	Uncertainty of Long-Term Debt Risk Premium Shock	0.65	0.40
$100\sigma_p$	Uncertainty of Price Markup Shock	0.52	0.60
$100\sigma_w$	Uncertainty of Wage Markup Shock	0.53	0.56
$100\sigma_m$	Uncertainty of Import Pice Markup Shock	1.67	1.73
$100\sigma_{\bar{\pi}}$	Uncertainty of Inflation Target Shock	0.01	0.01
$100\sigma_{r^S}$	Uncertainty of Policy Rate Shock	0.15	0.16

Table 4: Posterior Distribution of Selected Moments

Mnemonic	Description	Data	Benchmark	Non Stationary Risk Shocks
$\rho_{r^L,r^{L,*}}$	Correlation between Domestic and Foreign Long-Term Interest Rate	0.93	0.93	0.91
ρ_{r^S,r^L}	Correlation between Short- and Long-Term Domestic Interest Rate	0.90	0.89	0.87
$\rho_{r^S,r^{S,*}}$	Correlation between Domestic and Foreign Short-Term Interest Rate	0.84	0.80	0.75
ρ_{y,y^*}	Correlation between Domestic and Foreign GDP	0.65	0.65	0.66
ρ_{π,π^*}	Correlation between Domestic and Foreign Inflation	0.85	0.83	0.77
ρ_{c,c^*}	Correlation between Domestic and Foreign Consumption	0.62	0.61	0.60
σ_{r^S}	Standard Deviation of Domestic Short-Term Interest Rate	4.50	4.69	5.00
σ_{r^L}	Standard Deviation of Domestic Long-Term Interest Rate	3.63	3.45	3.54
$\sigma^*_{r^S}$	Standard Deviation of Foreign Short-Term Interest Rate	4.02	3.54	3.91
$\sigma^*_{r^L}$	Standard Deviation of Foreign Long-Term Interest Rate	3.12	2.65	2.69
σ_y	Standard Deviation of Domestic GDP	2.19	2.12	2.09
σ_{y^*}	Standard Deviation of Foreign GDP	2.09	2.11	2.23
σ_π	Standard Deviation of Domestic Inflation	4.07	4.70	4.93
σ^*_π	Standard Deviation of Foreign Inflation	2.04	2.98	3.37
σ_q	Standard Deviation of Real Exchange Rate	11.22	11.36	13.12
σ_w	Standard Deviation of Domestic Wage	1.53	1.73	1.86
σ^*_w	Standard Deviation of Foreign Wage	1.47	1.64	1.75

Figure 1: DSGE Model Fit



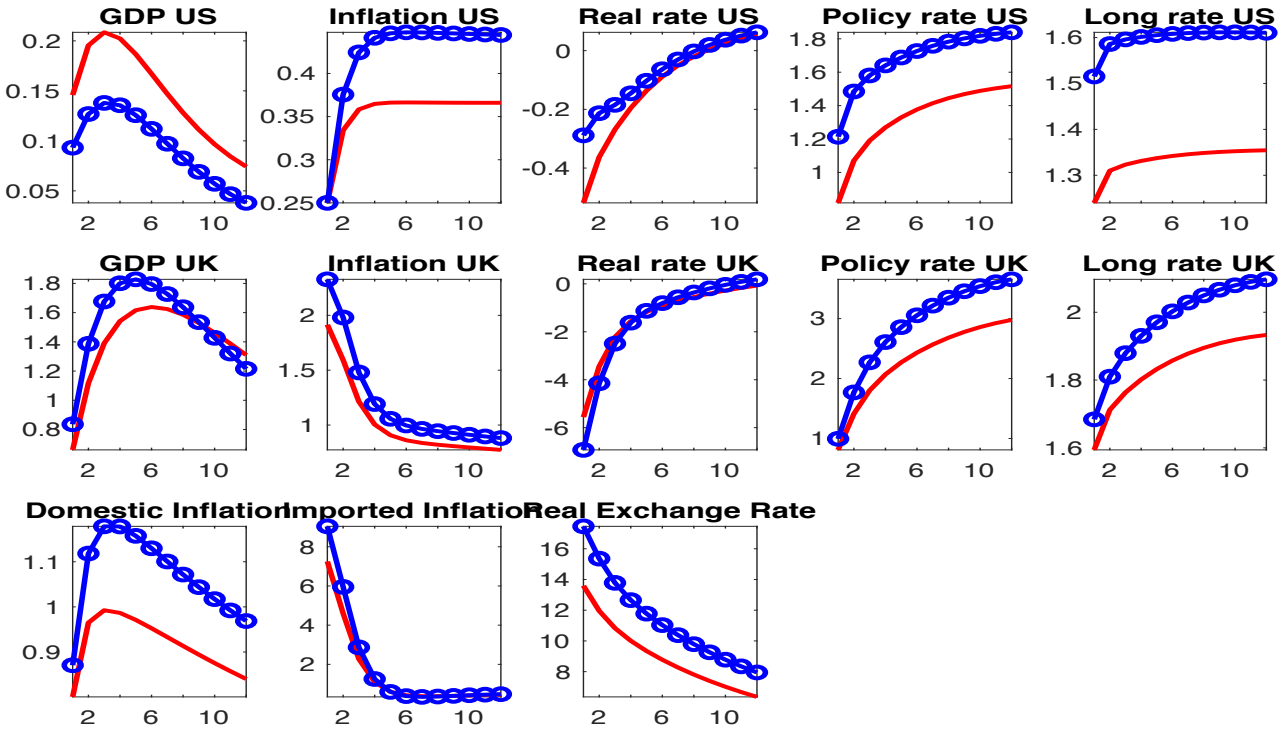
Notes: Observed data shown in the blue lines, model Kalman Filter one step ahead predictions ($E_{t-1}x_t$). Shaded areas denote NBER US recession periods. shown in red-dashed lines.

Table 5: Decomposing Comovement

	$\rho\left(r^L, r^{L*}\right)$	$\sigma\left(r^{L*}\right)$	$\sigma\left(r^L\right)$
Data	0.93	3.12	3.63
US Productivity	1.00	0.19	0.15
US Monetary policy	0.76	0.18	0.04
US LT debt risk premium	-0.56	1.21	0.49
US ST debt risk premium	0.95	0.47	0.12
US Wage markup	0.16	0.06	0.02
US Price markup	0.90	0.08	0.03
US Inflation target	1.00	0.92	1.59

Notes: The table suppresses UK shocks since these cannot generate variance in the US due to the small open economy nature of the UK in the model. In other words, conditional on UK shocks $\sigma(r^{L*}) = \rho(r^L, r^{L*}) = 0$. In the $\sigma(r^L)$ -column the contributions do not sum to the total data variance because of the additional variance caused by UK-specific shocks (not reported).

Figure 2: US Inflation Target Shock



Notes: The shock has been normalised to increase the US annual inflation by 1*pps*. The blue circle-dashed line illustrates agents' optimal responses in the benchmark economy, while the solid red line in the economy where US long-term inflation expectations are observed

2.2 UK Authorities Target Domestic Inflation

In this section we investigate whether our results are sensitive to the inflation measure targeted by domestic monetary authorities. In the benchmark case, UK policymakers stabilise CPI inflation ($\hat{\pi}_t^c$) as it is implied by Bank of England's Remit (see the discussion in Burgess et al. (2013)). However, as discussed in the main text, it has been admitted many a time by the BoE's governors that they are "looking through" temporary price shocks caused either by the exchange rate and/or oil prices. We therefor re-estimate the model with a domestic (UK) policy rule defined in terms of domestic inflation ($\hat{\pi}_t$) as

$$\hat{r}_t^S - \hat{\bar{\pi}}_t^c = \phi_R \left(\hat{r}_{t-1}^S - \hat{\bar{\pi}}_{t-1}^c\right) + (1-\phi_R)\left[\phi_\pi \left(\hat{\pi}_t - \hat{\bar{\pi}}_t^c\right) + \phi_y \hat{y}_t\right] + \sigma_R \omega_t^R.$$

The model performs slightly worse than the benchmark model, as can be seen from the slight deterioration in terms of marginal likelihood (Table 1). While some of the point estimates of parameters and impulse responses change slightly quantitatively, the qualitative message is still very much the same. Even if the BoE is assumed to target domestic inflation rather than CPI, changes in foreign inflation trends are the main driver of comovement, and the transmission of these shocks is very similar to that found in the benchmark model.

Table 6: Foreign Parameter Estimates

Mnemonic	Description	Benchmark	Non-stationary Risk Shocks
λ_{y^*}	Steady-State Foreign Prices Markup	1.11	1.14
χ^*	Liquidity Adjustment Cost	2.05	3.57
ϑ^*	Dynamic Liquidity Adjustment Cost	0.94	0.97
h^*	Consumption Habit	0.70	0.65
κ_y^*	Price Indexation	0.31	0.20
ξ_y^*	Price Reset Probability	0.92	0.92
κ_w^*	Wage Indexation	0.42	0.45
ξ_w^*	Wage Reset Probability	0.73	0.74
$\phi_{r^S}^*$	Policy Rate Smoothing	0.83	0.81
ϕ_π^*	Policy Reaction to Inflation	1.78	1.31
ϕ_y^*	Policy Reaction to Output	0.04	0.03
θ^*	Lump Sum Tax Response	0.02	0.04
ρ_γ^*	Persistence of Non Stationary Productivity Shock	0.09	0.13
$\rho_{B^S}^*$	Persistence of Short-Term Debt Risk Premium Shock	0.91	0.95
$\rho_{\bar{B}^L}^*$	Persistence of Long-Term Debt Risk Premium Shock	0.89	0.90
ρ_p^*	Persistence of Price Markup Shock	0.02	0.04
ρ_w^*	Persistence of Wage Markup Shock	0.90	0.91
$\rho_{\bar{\pi}}^*$	Persistence of Inflation Target Shock	0.21	0.24
$100\sigma_\gamma^*$	Uncertainty of Non Stationary Productivity Shock	0.97	0.99
$100\sigma_{B^S}^*$	Uncertainty of Short-Term Debt Risk Premium Shock	0.18	0.09
$100\sigma_{\bar{B}^L}^*$	Uncertainty of Long-Term Debt Risk Premium Shock	0.43	0.43
$100\sigma_p^*$	Uncertainty of Price Markup Shock	0.12	0.12
$100\sigma_w^*$	Uncertainty of Wage Markup Shock	0.71	0.76
$100\sigma_{\bar{\pi}}^*$	Uncertainty of Inflation Target Shock	0.02	0.02
$100\sigma_{r^S}^*$	Uncertainty of Policy Rate Shock	0.19	0.19

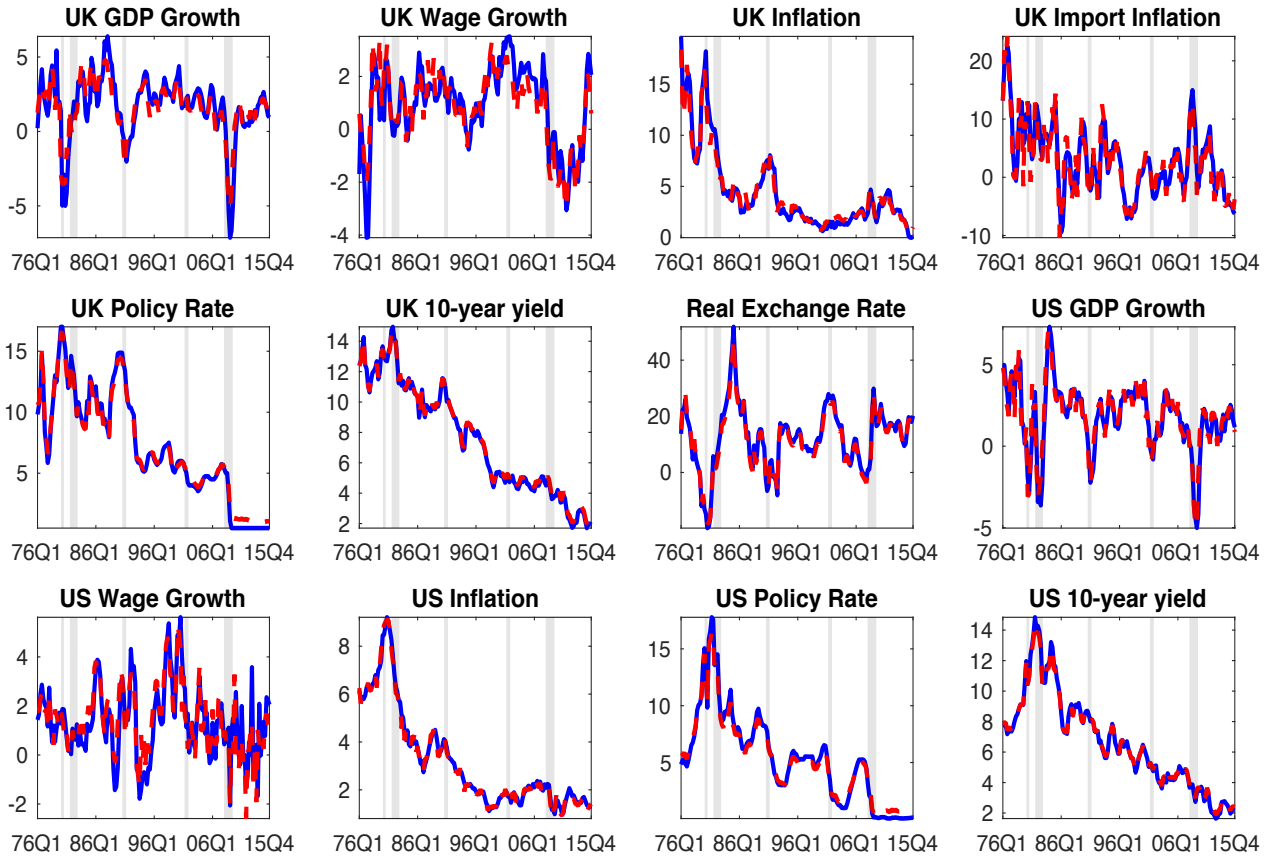
Table 7: Domestic Parameter Estimates

Mnemonic	Description	Benchmark	Non-stationary Risk Shocks
λ_y	Steady-State Domestic Prices Markup	1.13	1.17
λ_m	Steady-State Import Prices Markup	1.24	1.25
λ_x	Steady-State Export Prices Markup	1.05	1.05
η_C	Import Price Elasticity	1.45	1.49
$\delta_{B,D}$	Domestic Long- to Short-Term Debt Ratio	0.46	0.61
$\delta_{B,F}$	Foreign Long- to Short-Term Debt Ratio	0.18	0.18
χ	Liquidity Adjustment Cost	5.10	5.01
ϑ	Dynamic Liquidity Adjustment Cost	0.25	0.42
h	Consumption Habit	0.81	0.85
κ_y	Domestic Price Indexation	0.28	0.20
ξ_y	Domestic Price Reset Probability	0.91	0.92
κ_m	Import Price Indexation	0.25	0.20
ξ_m	Import Price Reset Probability	0.50	0.50
κ_w	Wage Indexation	0.77	0.70
ξ_w	Wage Reset Probability	0.73	0.77
κ_x	Export Price Indexation	0.27	0.27
ξ_x	Export Price Reset Probability	0.87	0.86
ϕ_{r^S}	Policy Rate Smoothing	0.90	0.89
ϕ_π	Policy Reaction to Inflation	1.01	1.02
ϕ_y	Policy Reaction to Output	0.11	0.03
θ	Lump Sum Tax Response	0.01	0.01
ρ_ω	Persistence of Stationary Productivity Shock	0.16	0.37
ρ_{B^S}	Persistence of Short-Term Debt Risk Premium Shock	0.87	0.88
$\rho_{\bar{B}^L}$	Persistence of Long-Term Debt Risk Premium Shock	0.83	0.87
ρ_p	Persistence of Price Markup Shock	0.01	0.04
ρ_w	Persistence of Wage Markup Shock	0.93	0.94
ρ_m	Persistence of Import Pice Markup Shock	0.77	0.81
$\rho_{\bar{\pi}}$	Persistence of Inflation Target Shock	0.04	0.06
$100\sigma_\omega$	Uncertainty of Productivity Shock	2.50	2.30
$100\sigma_{B^S}$	Uncertainty of Short-Term Debt Risk Premium Shock	0.47	0.46
$100\sigma_{\bar{B}^L}$	Uncertainty of Long-Term Debt Risk Premium Shock	0.65	0.52
$100\sigma_p$	Uncertainty of Price Markup Shock	0.52	0.60
$100\sigma_w$	Uncertainty of Wage Markup Shock	0.53	0.56
$100\sigma_m$	Uncertainty of Import Pice Markup Shock	1.67	1.79
$100\sigma_{\bar{\pi}}$	Uncertainty of Inflation Target Shock	0.01	0.01
$100\sigma_{r^S}$	Uncertainty of Policy Rate Shock	0.15	0.17

Table 8: Posterior Distribution of Selected Moments

Mnemonic	Description	Data	Benchmark	Non Stationary Risk Shocks
$\rho_{r^L,r^{L,*}}$	Correlation between Domestic and Foreign Long-Term Interest Rate	0.93	0.93	0.92
ρ_{r^S,r^L}	Correlation between Short- and Long-Term Domestic Interest Rate	0.90	0.89	0.88
$\rho_{r^S,r^{S,*}}$	Correlation between Domestic and Foreign Short-Term Interest Rate	0.84	0.80	0.76
ρ_{y,y^*}	Correlation between Domestic and Foreign GDP	0.65	0.65	0.65
ρ_{π,π^*}	Correlation between Domestic and Foreign Inflation	0.85	0.83	0.80
ρ_{c,c^*}	Correlation between Domestic and Foreign Consumption	0.62	0.61	0.60
σ_{r^S}	Standard Deviation of Domestic Short-Term Interest Rate	4.50	4.69	4.94
σ_{r^L}	Standard Deviation of Domestic Long-Term Interest Rate	3.63	3.45	3.66
$\sigma^*_{r^S}$	Standard Deviation of Foreign Short-Term Interest Rate	4.02	3.54	3.65
$\sigma^*_{r^L}$	Standard Deviation of Foreign Long-Term Interest Rate	3.12	2.65	2.77
σ_y	Standard Deviation of Domestic GDP	2.19	2.12	2.06
σ_{y^*}	Standard Deviation of Foreign GDP	2.09	2.11	2.24
σ_π	Standard Deviation of Domestic Inflation	4.07	4.70	4.97
σ^*_π	Standard Deviation of Foreign Inflation	2.04	2.98	3.16
σ_q	Standard Deviation of Real Exchange Rate	11.22	11.36	13.15
σ_w	Standard Deviation of Domestic Wage	1.53	1.73	1.83
σ^*_w	Standard Deviation of Foreign Wage	1.47	1.64	1.73

Figure 3: DSGE Model Fit



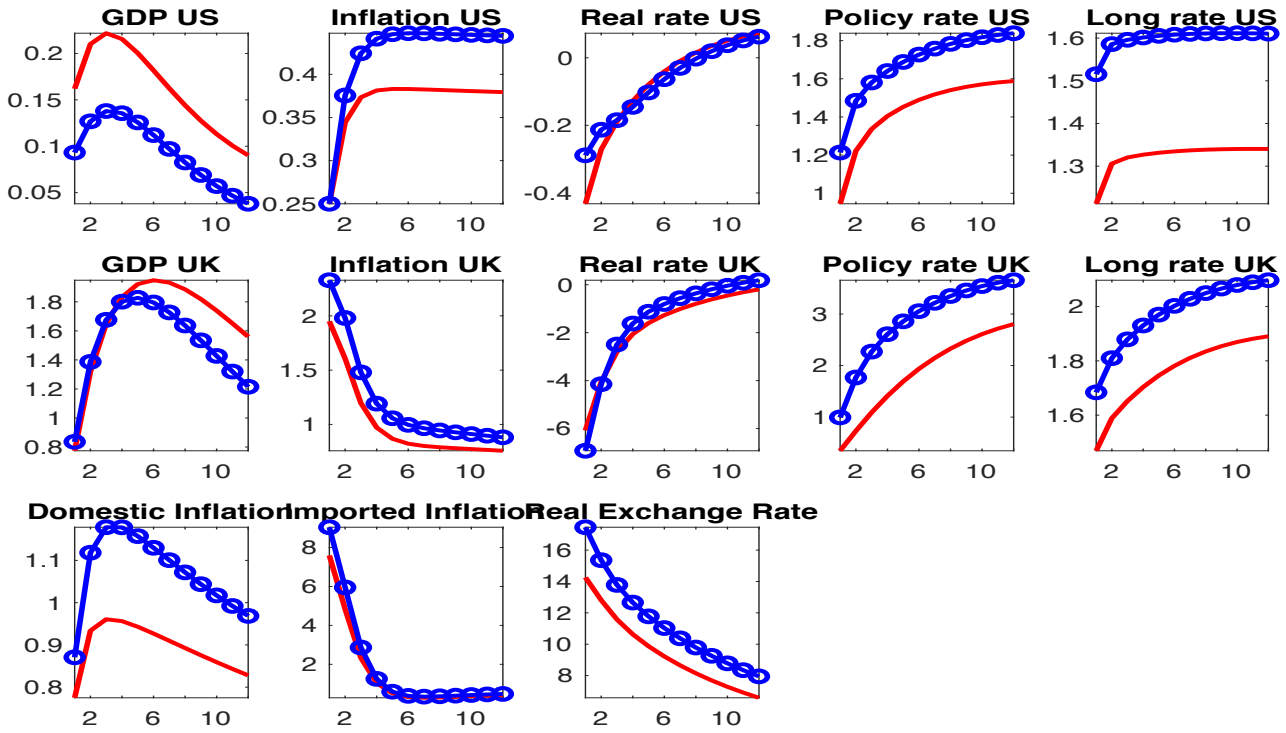
Notes: Observed data shown in the blue lines, model Kalman Filter one step ahead predictions ($E_{t-1}x_t$). Shaded areas denote NBER US recession periods. shown in red-dashed lines.

Table 9: Decomposing Comovement

	$\rho\left(r^L, r^{L*}\right)$	$\sigma\left(r^{L*}\right)$	$\sigma\left(r^L\right)$
Data	0.93	3.12	3.63
US Productivity	0.99	0.11	0.09
US Monetary policy	0.20	0.16	0.01
US LT debt risk premium	-0.53	1.25	0.14
US ST debt risk premium	0.65	0.14	0.02
US Wage markup	0.36	0.09	0.00
US Price markup	0.68	0.01	0.00
US Inflation target	1.00	1.34	2.21

Notes: The table suppresses UK shocks since these cannot generate variance in the US due to the small open economy nature of the UK in the model. In other words, conditional on UK shocks $\sigma(r^{L*}) = \rho(r^L, r^{L*}) = 0$. In the $\sigma(r^L)$ -column the contributions do not sum to the total data variance because of the additional variance caused by UK-specific shocks (not reported).

Figure 4: US Inflation Target Shock



Notes: The shock has been normalised to increase the US annual inflation by 1*pps*. The blue circle-dashed line illustrates agents' optimal responses in the benchmark economy, while the solid red line in the economy where UK monetary authorities target domestic (instead of CPI) inflation

2.3 Estimation Sample Truncated to $2007Q4$

The benchmark version of the model is estimated using data between $1976Q1$ and $2015Q4$. We are pleased to see the ability of the model to 'fit' the recession and the 'weak' recovery period. However, researchers may question that the period post $2008Q1$ may drives the main results. To assure ourselves that this is not the case as we have identified an economic channel that exists in all times, we truncate the estimation sample to $2007Q4$. The results presented in this section are almost unchanged relative to those obtained using the full sample.

Table 10: Foreign Parameter Estimates

Mnemonic	Description	Benchmark	Non-stationary Risk Shocks
λ_{y^*}	Steady-State Foreign Prices Markup	1.11	1.15
χ^*	Liquidity Adjustment Cost	2.05	1.89
ϑ^*	Dynamic Liquidity Adjustment Cost	0.94	0.95
h^*	Consumption Habit	0.70	0.67
κ_y^*	Price Indexation	0.31	0.25
ξ_y^*	Price Reset Probability	0.92	0.92
κ_w^*	Wage Indexation	0.42	0.41
ξ_w^*	Wage Reset Probability	0.73	0.75
$\phi_{r^S}^*$	Policy Rate Smoothing	0.83	0.79
ϕ_π^*	Policy Reaction to Inflation	1.78	1.67
ϕ_y^*	Policy Reaction to Output	0.04	0.03
θ^*	Lump Sum Tax Response	0.02	0.01
ρ_γ^*	Persistence of Non Stationary Productivity Shock	0.09	0.07
$\rho_{B^S}^*$	Persistence of Short-Term Debt Risk Premium Shock	0.91	0.92
$\rho_{\bar{B}^L}^*$	Persistence of Long-Term Debt Risk Premium Shock	0.89	0.86
ρ_p^*	Persistence of Price Markup Shock	0.02	0.03
ρ_w^*	Persistence of Wage Markup Shock	0.90	0.92
$\rho_{\bar{\pi}}^*$	Persistence of Inflation Target Shock	0.21	0.23
$100\sigma_\gamma^*$	Uncertainty of Non Stationary Productivity Shock	0.97	1.00
$100\sigma_{B^S}^*$	Uncertainty of Short-Term Debt Risk Premium Shock	0.18	0.16
$100\sigma_{\bar{B}^L}^*$	Uncertainty of Long-Term Debt Risk Premium Shock	0.43	0.63
$100\sigma_p^*$	Uncertainty of Price Markup Shock	0.12	0.12
$100\sigma_w^*$	Uncertainty of Wage Markup Shock	0.71	0.66
$100\sigma_{\bar{\pi}}^*$	Uncertainty of Inflation Target Shock	0.02	0.02
$100\sigma_{r^S}^*$	Uncertainty of Policy Rate Shock	0.19	0.19

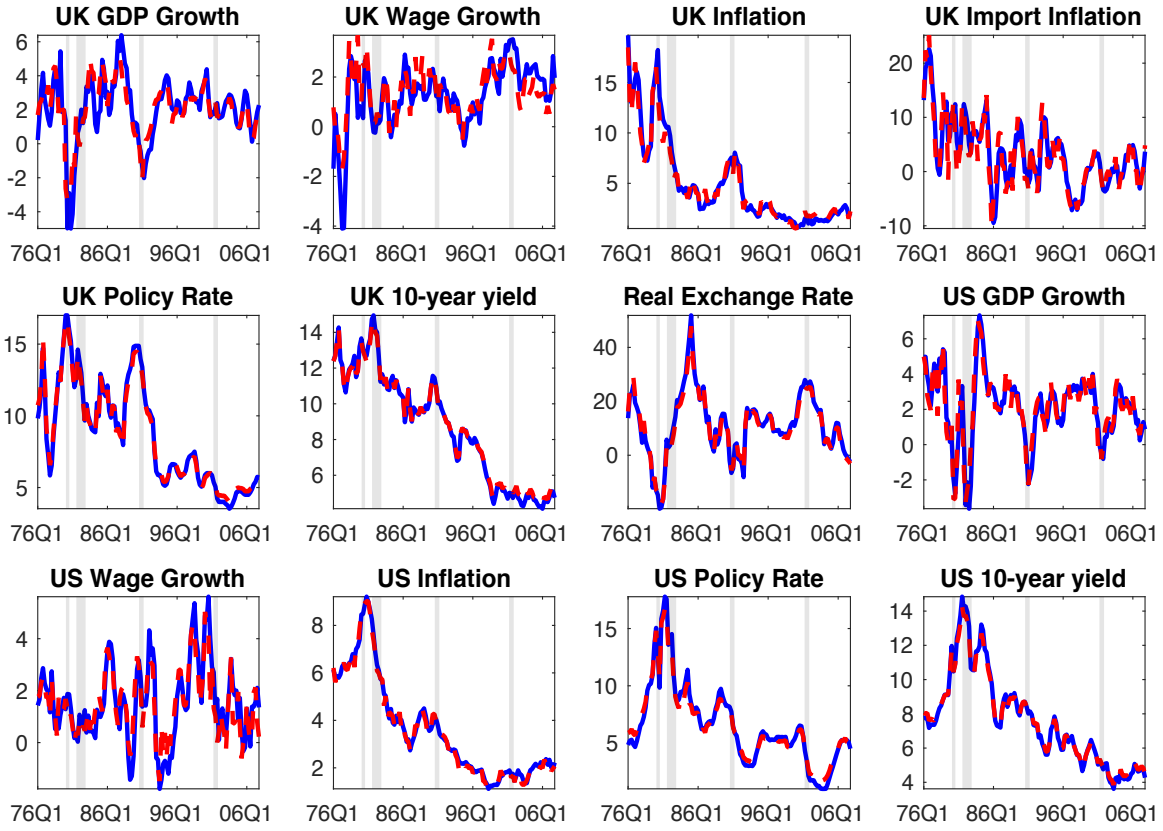
Table 11: Domestic Parameter Estimates

Mnemonic	Description	Benchmark	Non-stationary Risk Shocks
λ_y	Steady-State Domestic Prices Markup	1.13	1.13
λ_m	Steady-State Import Prices Markup	1.24	1.28
λ_x	Steady-State Export Prices Markup	1.05	1.05
η_C	Import Price Elasticity	1.45	1.42
$\delta_{B,D}$	Domestic Long- to Short-Term Debt Ratio	0.46	0.53
$\delta_{B,F}$	Foreign Long- to Short-Term Debt Ratio	0.18	0.18
χ	Liquidity Adjustment Cost	5.10	5.16
ϑ	Dynamic Liquidity Adjustment Cost	0.25	0.49
h	Consumption Habit	0.81	0.84
κ_y	Domestic Price Indexation	0.28	0.23
ξ_y	Domestic Price Reset Probability	0.91	0.90
κ_m	Import Price Indexation	0.25	0.25
ξ_m	Import Price Reset Probability	0.50	0.50
κ_w	Wage Indexation	0.77	0.74
ξ_w	Wage Reset Probability	0.73	0.75
κ_x	Export Price Indexation	0.27	0.30
ξ_x	Export Price Reset Probability	0.87	0.87
ϕ_{r^S}	Policy Rate Smoothing	0.90	0.90
ϕ_π	Policy Reaction to Inflation	1.01	1.01
ϕ_y	Policy Reaction to Output	0.11	0.09
θ	Lump Sum Tax Response	0.01	0.01
ρ_ω	Persistence of Stationary Productivity Shock	0.16	0.25
ρ_{B^S}	Persistence of Short-Term Debt Risk Premium Shock	0.87	0.87
$\rho_{\bar{B}^L}$	Persistence of Long-Term Debt Risk Premium Shock	0.83	0.85
ρ_p	Persistence of Price Markup Shock	0.01	0.01
ρ_w	Persistence of Wage Markup Shock	0.93	0.92
ρ_m	Persistence of Import Pice Markup Shock	0.77	0.84
$\rho_{\bar{\pi}}$	Persistence of Inflation Target Shock	0.04	0.05
$100\sigma_\omega$	Uncertainty of Productivity Shock	2.50	2.34
$100\sigma_{B^S}$	Uncertainty of Short-Term Debt Risk Premium Shock	0.47	0.49
$100\sigma_{\bar{B}^L}$	Uncertainty of Long-Term Debt Risk Premium Shock	0.65	0.57
$100\sigma_p$	Uncertainty of Price Markup Shock	0.52	0.60
$100\sigma_w$	Uncertainty of Wage Markup Shock	0.53	0.61
$100\sigma_m$	Uncertainty of Import Pice Markup Shock	1.67	1.61
$100\sigma_{\bar{\pi}}$	Uncertainty of Inflation Target Shock	0.01	0.01
$100\sigma_{r^S}$	Uncertainty of Policy Rate Shock	0.15	0.17

Table 12: Posterior Distribution of Selected Moments

Mnemonic	Description	Data	Benchmark	Non Stationary Risk Shocks
$\rho_{r^L,r^{L,*}}$	Correlation between Domestic and Foreign Long-Term Interest Rate	0.93	0.93	0.92
ρ_{r^S,r^L}	Correlation between Short- and Long-Term Domestic Interest Rate	0.90	0.89	0.88
$\rho_{r^S,r^{S,*}}$	Correlation between Domestic and Foreign Short-Term Interest Rate	0.84	0.80	0.76
ρ_{y,y^*}	Correlation between Domestic and Foreign GDP	0.65	0.65	0.65
ρ_{π,π^*}	Correlation between Domestic and Foreign Inflation	0.85	0.83	0.80
ρ_{c,c^*}	Correlation between Domestic and Foreign Consumption	0.62	0.61	0.61
σ_{r^S}	Standard Deviation of Domestic Short-Term Interest Rate	4.50	4.69	4.99
σ_{r^L}	Standard Deviation of Domestic Long-Term Interest Rate	3.63	3.45	3.62
$\sigma^*_{r^S}$	Standard Deviation of Foreign Short-Term Interest Rate	4.02	3.54	3.60
$\sigma^*_{r^L}$	Standard Deviation of Foreign Long-Term Interest Rate	3.12	2.65	2.74
σ_y	Standard Deviation of Domestic GDP	2.19	2.12	2.03
σ_{y^*}	Standard Deviation of Foreign GDP	2.09	2.11	2.20
σ_π	Standard Deviation of Domestic Inflation	4.07	4.70	4.99
σ^*_π	Standard Deviation of Foreign Inflation	2.04	2.98	3.06
σ_q	Standard Deviation of Real Exchange Rate	11.22	11.36	12.89
σ_w	Standard Deviation of Domestic Wage	1.53	1.73	1.85
σ^*_w	Standard Deviation of Foreign Wage	1.47	1.64	1.55

Figure 5: DSGE Model Fit



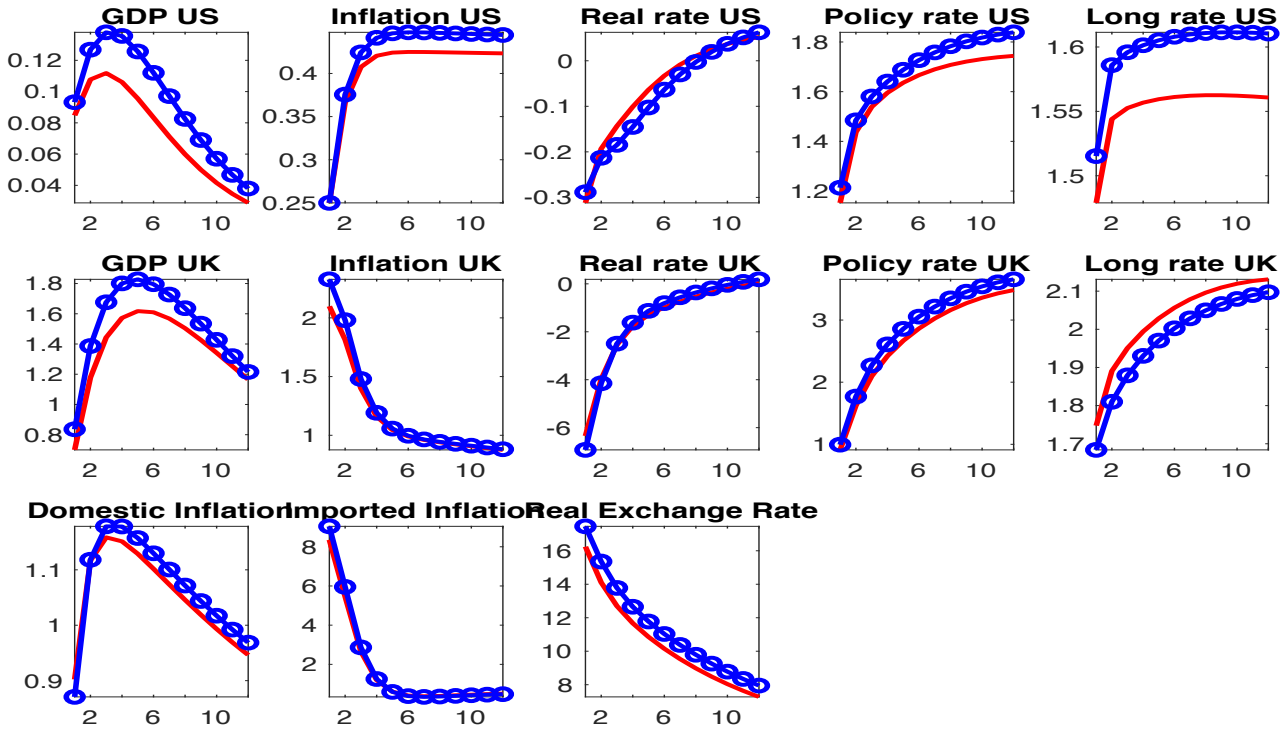
Notes: Observed data shown in the blue lines, model Kalman Filter one step ahead predictions ($E_{t-1}x_t$). Shaded areas denote NBER US recession periods. shown in red-dashed lines.

Table 13: Decomposing Comovement

	$\rho\left(r^L, r^{L*}\right)$	$\sigma\left(r^{L*}\right)$	$\sigma\left(r^L\right)$
Data	0.93	3.12	3.63
US Productivity	0.99	0.08	0.10
US Monetary policy	0.53	0.12	0.04
US LT debt risk premium	-0.18	1.05	0.33
US ST debt risk premium	0.96	0.10	0.03
US Wage markup	-0.23	0.09	0.02
US Price markup	0.92	0.02	0.00
US Inflation target	1.00	1.23	2.08

Notes: The table suppresses UK shocks since these cannot generate variance in the US due to the small open economy nature of the UK in the model. In other words, conditional on UK shocks $\sigma(r^{L*}) = \rho(r^L, r^{L*}) = 0$. In the $\sigma(r^L)$ -column the contributions do not sum to the total data variance because of the additional variance caused by UK-specific shocks (not reported).

Figure 6: US Inflation Target Shock



Notes: The shock has been normalised to increase the US annual inflation by 1*pps*. The blue circle-dashed line illustrates agents' optimal responses in the benchmark economy, while the solid red line the same responses but the estimation sample is truncated to 2007$Q4$

2.4 Non-Stationary Savings Glut Shock

In the benchmark version of the model the persistence of the Savings Glut shock process ($\varepsilon_t^{\bar{b}^{L,*}}$) is estimated. While its estimate is quite high at 0.9, it is significantly smaller than the calibrated value used in Del Negro et al. (2017). In this section we investigate whether our approach to estimate – instead of calibrating – this parameter influences our main result and the Savings Glut shock is actually the true driver of the strong correlation of the two asses prices across countries.

The data fit of the model is now clearly worse. The marginal likelihood deteriorates strongly compared to the benchmark model (Table 1). This can also be seen in more detail from the fact that a number of model predictions start showing more significant deviations from data outcomes (Figure 7). Moreover, the model now exhibits signs of difficulty replicating yield curve moments (Table 16). Comovement is still mainly captured by US inflation target shocks, whose transmission remains largely the same as in the benchmark model.

Table 14: Foreign Parameter Estimates

Mnemonic	Description	Benchmark	Non-stationary Risk Shocks
λ_{y^*}	Steady-State Foreign Prices Markup	1.11	1.05
χ^*	Liquidity Adjustment Cost	2.05	6.21
ϑ^*	Dynamic Liquidity Adjustment Cost	0.94	0.97
h^*	Consumption Habit	0.70	0.77
κ_y^*	Price Indexation	0.31	0.20
ξ_y^*	Price Reset Probability	0.92	0.92
κ_w^*	Wage Indexation	0.42	0.51
ξ_w^*	Wage Reset Probability	0.73	0.56
$\phi_{r^S}^*$	Policy Rate Smoothing	0.83	0.86
ϕ_π^*	Policy Reaction to Inflation	1.78	1.50
ϕ_y^*	Policy Reaction to Output	0.04	0.07
θ^*	Lump Sum Tax Response	0.02	0.02
ρ_γ^*	Persistence of Non Stationary Productivity Shock	0.09	0.04
$\rho_{B^S}^*$	Persistence of Short-Term Debt Risk Premium Shock	0.91	0.56
ρ_p^*	Persistence of Price Markup Shock	0.02	0.53
ρ_w^*	Persistence of Wage Markup Shock	0.90	0.94
$\rho_{\bar{\pi}}^*$	Persistence of Inflation Target Shock	0.21	0.34
$100\sigma_\gamma^*$	Uncertainty of Non Stationary Productivity Shock	0.97	1.26
$100\sigma_{B^S}^*$	Uncertainty of Short-Term Debt Risk Premium Shock	0.18	2.09
$100\sigma_{\bar{B}^L}^*$	Uncertainty of Long-Term Debt Risk Premium Shock	0.43	0.01
$100\sigma_p^*$	Uncertainty of Price Markup Shock	0.12	0.09
$100\sigma_w^*$	Uncertainty of Wage Markup Shock	0.71	0.75
$100\sigma_{\bar{\pi}}^*$	Uncertainty of Inflation Target Shock	0.02	0.02
$100\sigma_{r^S}^*$	Uncertainty of Policy Rate Shock	0.19	0.19

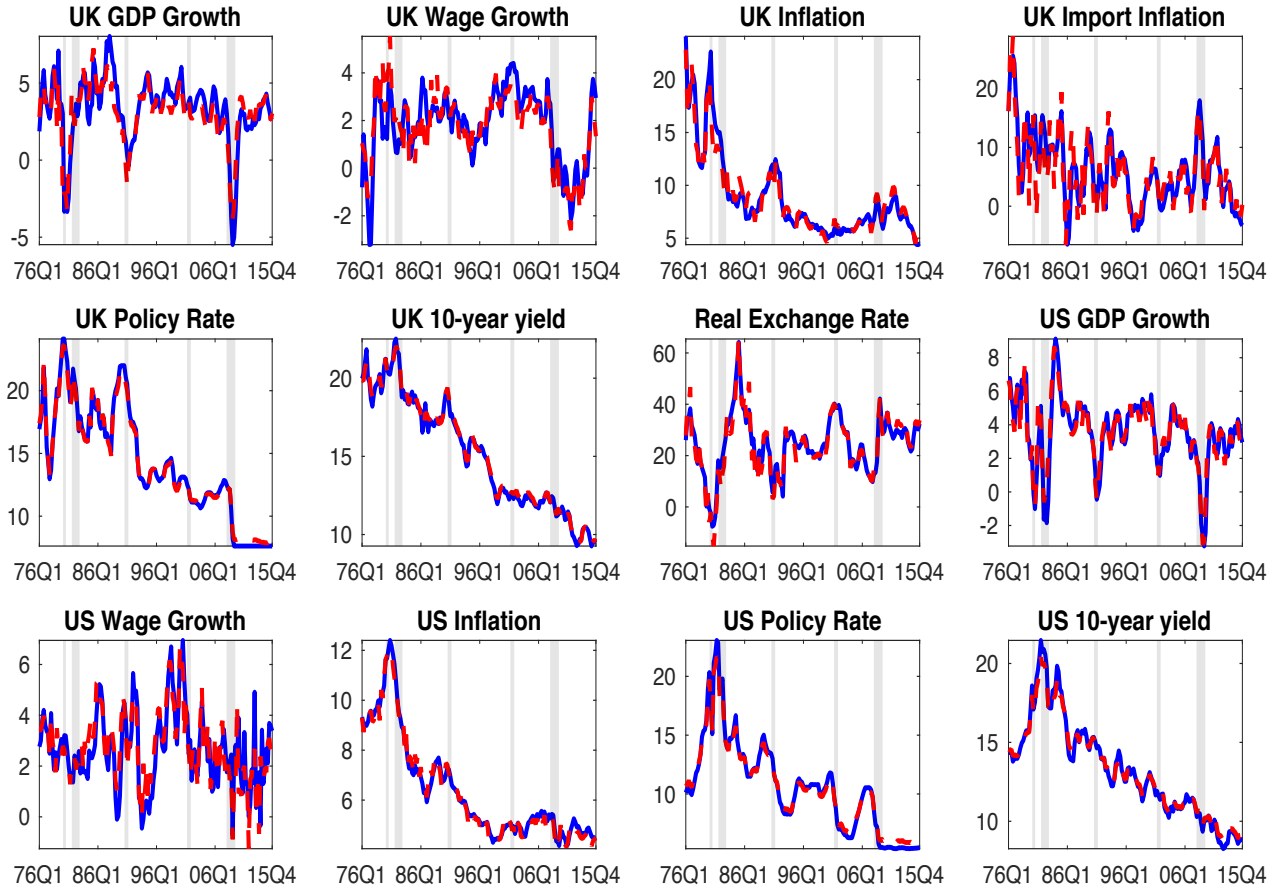
Table 15: Domestic Parameter Estimates

Mnemonic	Description	Benchmark	Non-stationary Risk Shocks
λ_y	Steady-State Domestic Prices Markup	1.13	1.05
λ_m	Steady-State Import Prices Markup	1.24	1.21
λ_x	Steady-State Export Prices Markup	1.05	1.05
η_C	Import Price Elasticity	1.45	1.10
$\delta_{B,D}$	Domestic Long- to Short-Term Debt Ratio	0.46	0.49
$\delta_{B,F}$	Foreign Long- to Short-Term Debt Ratio	0.18	0.18
χ	Liquidity Adjustment Cost	5.10	6.16
ϑ	Dynamic Liquidity Adjustment Cost	0.25	0.25
h	Consumption Habit	0.81	0.87
κ_y	Domestic Price Indexation	0.28	0.20
ξ_y	Domestic Price Reset Probability	0.91	0.92
κ_m	Import Price Indexation	0.25	0.20
ξ_m	Import Price Reset Probability	0.50	0.50
κ_w	Wage Indexation	0.77	0.69
ξ_w	Wage Reset Probability	0.73	0.85
κ_x	Export Price Indexation	0.27	0.22
ξ_x	Export Price Reset Probability	0.87	0.92
ϕ_{r^S}	Policy Rate Smoothing	0.90	0.95
ϕ_π	Policy Reaction to Inflation	1.01	1.00
ϕ_y	Policy Reaction to Output	0.11	0.07
θ	Lump Sum Tax Response	0.01	0.01
ρ_ω	Persistence of Stationary Productivity Shock	0.16	0.75
ρ_{B^S}	Persistence of Short-Term Debt Risk Premium Shock	0.87	0.03
ρ_p	Persistence of Price Markup Shock	0.01	0.08
ρ_w	Persistence of Wage Markup Shock	0.93	0.96
ρ_m	Persistence of Import Pice Markup Shock	0.77	0.85
$\rho_{\bar{\pi}}$	Persistence of Inflation Target Shock	0.04	0.04
$100\sigma_\omega$	Uncertainty of Productivity Shock	2.50	2.27
$100\sigma_{B^S}$	Uncertainty of Short-Term Debt Risk Premium Shock	0.47	3.00
$100\sigma_{\bar{B}^L}$	Uncertainty of Long-Term Debt Risk Premium Shock	0.65	0.01
$100\sigma_p$	Uncertainty of Price Markup Shock	0.52	0.66
$100\sigma_w$	Uncertainty of Wage Markup Shock	0.53	0.60
$100\sigma_m$	Uncertainty of Import Pice Markup Shock	1.67	1.69
$100\sigma_{\bar{\pi}}$	Uncertainty of Inflation Target Shock	0.01	0.01
$100\sigma_{r^S}$	Uncertainty of Policy Rate Shock	0.15	0.16

Table 16: Posterior Distribution of Selected Moments

Mnemonic	Description	Data	Benchmark	Non Stationary Risk Shocks
$\rho_{r^L,r^{L,*}}$	Correlation between Domestic and Foreign Long-Term Interest Rate	0.93	0.93	0.85
ρ_{r^S,r^L}	Correlation between Short- and Long-Term Domestic Interest Rate	0.90	0.89	0.90
$\rho_{r^S,r^{S,*}}$	Correlation between Domestic and Foreign Short-Term Interest Rate	0.84	0.80	0.74
ρ_{y,y^*}	Correlation between Domestic and Foreign GDP	0.65	0.65	0.63
ρ_{π,π^*}	Correlation between Domestic and Foreign Inflation	0.85	0.83	0.75
ρ_{c,c^*}	Correlation between Domestic and Foreign Consumption	0.62	0.61	0.64
σ_{r^S}	Standard Deviation of Domestic Short-Term Interest Rate	4.50	4.69	5.42
σ_{r^L}	Standard Deviation of Domestic Long-Term Interest Rate	3.63	3.45	3.91
$\sigma^*_{r^S}$	Standard Deviation of Foreign Short-Term Interest Rate	4.02	3.54	3.80
$\sigma^*_{r^L}$	Standard Deviation of Foreign Long-Term Interest Rate	3.12	2.65	2.69
σ_y	Standard Deviation of Domestic GDP	2.19	2.12	2.38
σ_{y^*}	Standard Deviation of Foreign GDP	2.09	2.11	2.17
σ_π	Standard Deviation of Domestic Inflation	4.07	4.70	5.45
σ^*_π	Standard Deviation of Foreign Inflation	2.04	2.98	3.24
σ_q	Standard Deviation of Real Exchange Rate	11.22	11.36	12.71
σ_w	Standard Deviation of Domestic Wage	1.53	1.73	1.91
σ^*_w	Standard Deviation of Foreign Wage	1.47	1.64	2.03

Figure 7: DSGE Model Fit



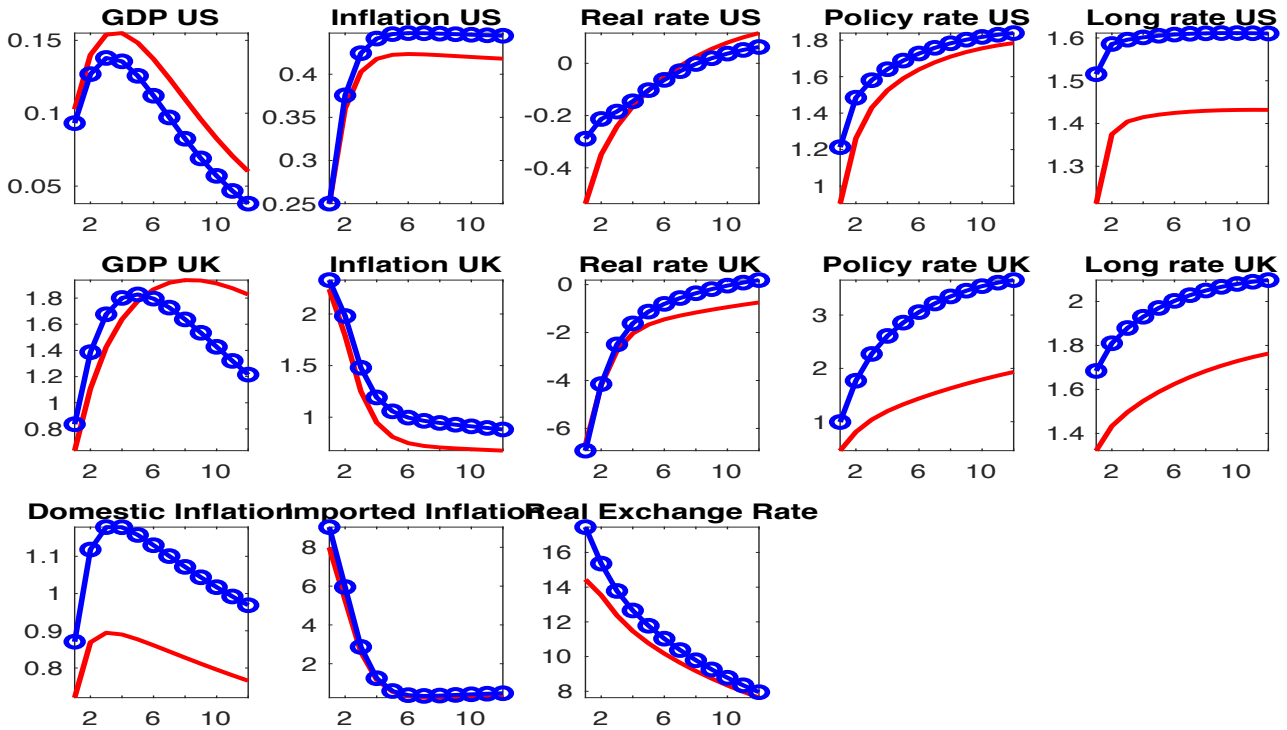
Notes: Observed data shown in the blue lines, model Kalman Filter one step ahead predictions ($E_{t-1}x_t$). Shaded areas denote NBER US recession periods. shown in red-dashed lines.

Table 17: Decomposing Comovement

	$\rho\left(r^L, r^{L*}\right)$	$\sigma\left(r^{L*}\right)$	$\sigma\left(r^L\right)$
Data	0.93	3.12	3.63
US Productivity	0.86	0.08	0.17
US Monetary policy	0.90	0.15	0.06
US LT debt risk premium	-1.00	0.33	0.96
US ST debt risk premium	0.76	0.57	0.09
US Wage markup	0.85	0.16	0.01
US Price markup	0.96	0.09	0.02
US Inflation target	1.00	1.73	3.03

Notes: The table suppresses UK shocks since these cannot generate variance in the US due to the small open economy nature of the UK in the model. In other words, conditional on UK shocks $\sigma(r^{L*}) = \rho(r^L, r^{L*}) = 0$. In the $\sigma(r^L)$ -column the contributions do not sum to the total data variance because of the additional variance caused by UK-specific shocks (not reported).

Figure 8: US Inflation Target Shock



Notes: The shock has been normalised to increase the US annual inflation by 1*pps*. The blue circle-dashed line illustrates agents' optimal responses in the benchmark economy, while the solid red line in the economy where the persistence of the Savings Glut and domestic long-term debt risk premium shocks have been calibrated to 0.995

2.5 Non-Stationary Productivity Growth Shock

In the final exercise, we allow the productivity growth process to be a non-stationary one. This is another shock that can have a long lasting effect on the long-term interest rate. The analysis of Antolin-Diaz et al. (2017) provides the empirical justification of treating TFP growth as a non-stationary process. It is perhaps important to remind the reader that the two (home and foreign) productivity processes are co-integrated for the model to display a balanced growth path. Furthermore, this is an efficient shock that does not create strong inflationary pressure. Due to these two properties the estimation uses this shock to explain the correlation between the two long-term rates but also to explain the Great Moderation period (strong growth without significant inflation pressures). Because it is picking up these phenomena, there is no more role left for inflation-target shocks in terms of explaining comovement. Yet the deterioration of the marginal likelihood is so strong compared to the benchmark model (Table 1), that the data seem to strongly reject this specification.

Table 18: Foreign Parameter Estimates

Mnemonic	Description	Benchmark	Non-stationary Productivity Growth Shock
λ_{y^*}	Steady-State Foreign Prices Markup	1.11	1.33
χ^*	Liquidity Adjustment Cost	2.05	4.25
ϑ^*	Dynamic Liquidity Adjustment Cost	0.94	1.00
h^*	Consumption Habit	0.70	0.65
κ_y^*	Price Indexation	0.31	0.20
ξ_y^*	Price Reset Probability	0.92	0.92
κ_w^*	Wage Indexation	0.42	0.20
ξ_w^*	Wage Reset Probability	0.73	0.92
$\phi_{r^S}^*$	Policy Rate Smoothing	0.83	0.76
ϕ_π^*	Policy Reaction to Inflation	1.78	1.33
ϕ_y^*	Policy Reaction to Output	0.04	0.04
θ^*	Lump Sum Tax Response	0.02	0.05
$\rho_{B^S}^*$	Persistence of Short-Term Debt Risk Premium Shock	0.91	0.91
$\rho_{\bar{B}^L}^*$	Persistence of Long-Term Debt Risk Premium Shock	0.89	0.76
ρ_p^*	Persistence of Price Markup Shock	0.02	0.37
ρ_w^*	Persistence of Wage Markup Shock	0.90	0.97
$\rho_{\bar{\pi}}^*$	Persistence of Inflation Target Shock	0.21	0.04
$100\sigma_\gamma^*$	Uncertainty of Non Stationary Productivity Shock	0.97	0.02
$100\sigma_{B^S}^*$	Uncertainty of Short-Term Debt Risk Premium Shock	0.18	0.20
$100\sigma_{\bar{B}^L}^*$	Uncertainty of Long-Term Debt Risk Premium Shock	0.43	0.97
$100\sigma_p^*$	Uncertainty of Price Markup Shock	0.12	0.10
$100\sigma_w^*$	Uncertainty of Wage Markup Shock	0.71	0.68
$100\sigma_{\bar{\pi}}^*$	Uncertainty of Inflation Target Shock	0.02	0.01
$100\sigma_{r^S}^*$	Uncertainty of Policy Rate Shock	0.19	0.20

Table 19: Domestic Parameter Estimates

Mnemonic	Description	Benchmark	Non-stationary Productivity Growth Shock
λ_y	Steady-State Domestic Prices Markup	1.13	1.10
λ_m	Steady-State Import Prices Markup	1.24	1.22
λ_x	Steady-State Export Prices Markup	1.05	1.07
η_C	Import Price Elasticity	1.45	0.89
$\delta_{B,D}$	Domestic Long- to Short-Term Debt Ratio	0.46	0.57
$\delta_{B,F}$	Foreign Long- to Short-Term Debt Ratio	0.18	0.34
χ	Liquidity Adjustment Cost	5.10	5.25
ϑ	Dynamic Liquidity Adjustment Cost	0.25	1.00
h	Consumption Habit	0.81	0.87
κ_y	Domestic Price Indexation	0.28	0.20
ξ_y	Domestic Price Reset Probability	0.91	0.89
κ_m	Import Price Indexation	0.25	0.20
ξ_m	Import Price Reset Probability	0.50	0.89
κ_w	Wage Indexation	0.77	0.92
ξ_w	Wage Reset Probability	0.73	0.92
κ_x	Export Price Indexation	0.27	0.92
ξ_x	Export Price Reset Probability	0.87	0.92
ϕ_{r^S}	Policy Rate Smoothing	0.90	0.86
ϕ_π	Policy Reaction to Inflation	1.01	1.01
ϕ_y	Policy Reaction to Output	0.11	0.15
θ	Lump Sum Tax Response	0.01	0.02
ρ_ω	Persistence of Stationary Productivity Shock	0.16	0.78
ρ_{B^S}	Persistence of Short-Term Debt Risk Premium Shock	0.87	0.81
$\rho_{\bar{B}^L}$	Persistence of Long-Term Debt Risk Premium Shock	0.83	0.98
ρ_p	Persistence of Price Markup Shock	0.01	0.01
ρ_w	Persistence of Wage Markup Shock	0.93	0.94
ρ_m	Persistence of Import Pice Markup Shock	0.77	0.12
$\rho_{\bar{\pi}}$	Persistence of Inflation Target Shock	0.04	0.06
$100\sigma_\omega$	Uncertainty of Productivity Shock	2.50	1.65
$100\sigma_{B^S}$	Uncertainty of Short-Term Debt Risk Premium Shock	0.47	0.65
$100\sigma_{\bar{B}^L}$	Uncertainty of Long-Term Debt Risk Premium Shock	0.65	0.04
$100\sigma_p$	Uncertainty of Price Markup Shock	0.52	0.69
$100\sigma_w$	Uncertainty of Wage Markup Shock	0.53	0.63
$100\sigma_m$	Uncertainty of Import Pice Markup Shock	1.67	1.30
$100\sigma_{\bar{\pi}}$	Uncertainty of Inflation Target Shock	0.01	0.01
$100\sigma_{r^S}$	Uncertainty of Policy Rate Shock	0.15	0.19

Table 20: Posterior Distribution of Selected Moments

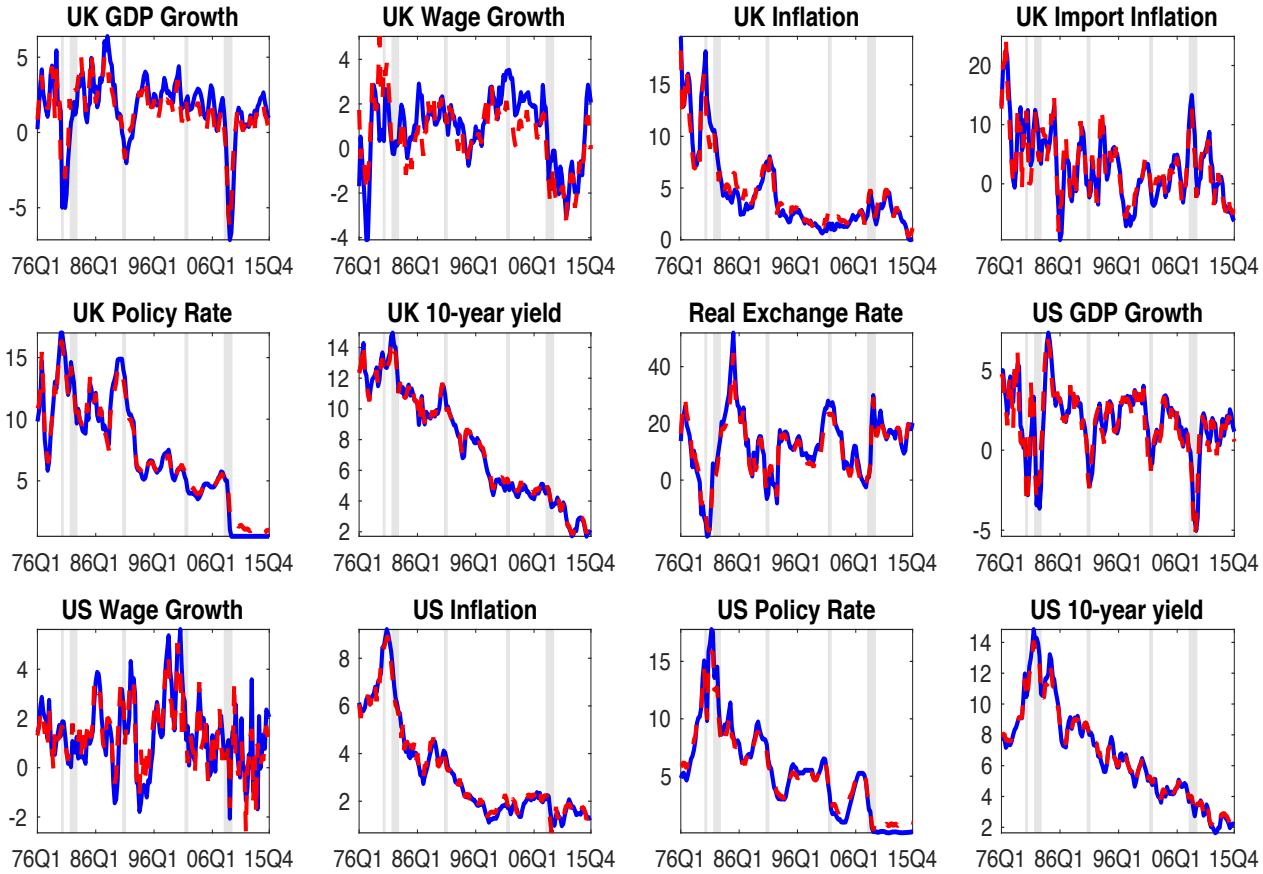
Mnemonic	Description	Data	Benchmark	Non Stationary Productivity Growth Shock
$\rho_{r^L,r^{L,*}}$	Correlation between Domestic and Foreign Long-Term Interest Rate	0.93	0.93	0.95
ρ_{r^S,r^L}	Correlation between Short- and Long-Term Domestic Interest Rate	0.90	0.89	0.91
$\rho_{r^S,r^{S,*}}$	Correlation between Domestic and Foreign Short-Term Interest Rate	0.84	0.80	0.81
ρ_{y,y^*}	Correlation between Domestic and Foreign GDP	0.65	0.65	0.58
ρ_{π,π^*}	Correlation between Domestic and Foreign Inflation	0.85	0.83	0.70
ρ_{c,c^*}	Correlation between Domestic and Foreign Consumption	0.62	0.61	0.58
σ_{r^S}	Standard Deviation of Domestic Short-Term Interest Rate	4.50	4.69	5.44
σ_{r^L}	Standard Deviation of Domestic Long-Term Interest Rate	3.63	3.45	4.55
$\sigma^*_{r^S}$	Standard Deviation of Foreign Short-Term Interest Rate	4.02	3.54	4.14
$\sigma^*_{r^L}$	Standard Deviation of Foreign Long-Term Interest Rate	3.12	2.65	3.74
σ_y	Standard Deviation of Domestic GDP	2.19	2.12	2.05
σ_{y^*}	Standard Deviation of Foreign GDP	2.09	2.11	2.21
σ_π	Standard Deviation of Domestic Inflation	4.07	4.70	4.24
σ^*_π	Standard Deviation of Foreign Inflation	2.04	2.98	2.45
σ_q	Standard Deviation of Real Exchange Rate	11.22	11.36	13.94
σ_w	Standard Deviation of Domestic Wage	1.53	1.73	2.22
σ^*_w	Standard Deviation of Foreign Wage	1.47	1.64	2.06

Table 21: Decomposing Comovement

	$\rho\left(r^L, r^{L*}\right)$	$\sigma\left(r^{L*}\right)$	$\sigma\left(r^L\right)$
Data	0.93	3.12	3.63
US Productivity	1.00	1.16	2.31
US Monetary policy	0.93	0.14	0.03
US LT debt risk premium	0.63	0.90	0.11
US ST debt risk premium	0.95	0.31	0.13
US Wage markup	0.72	0.17	0.03
US Price markup	0.94	0.09	0.02
US Inflation target	-0.95	0.35	0.02

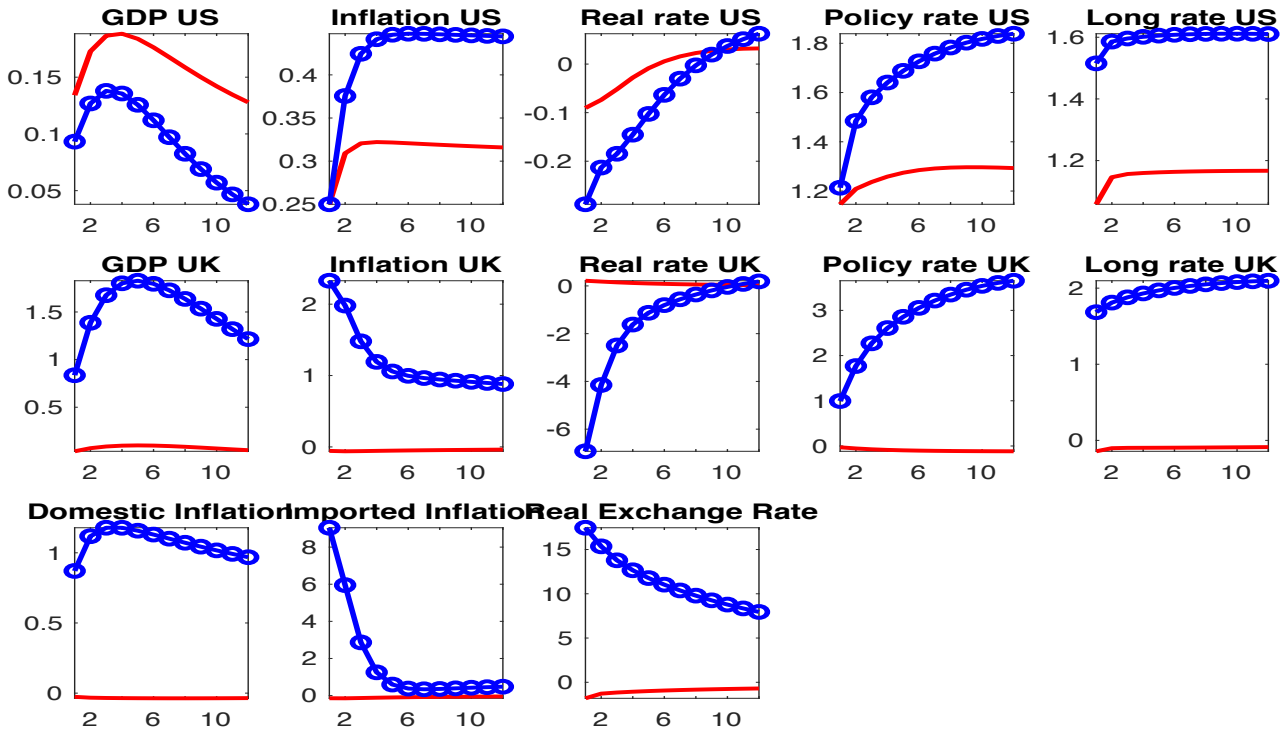
Notes: The table suppresses UK shocks since these cannot generate variance in the US due to the small open economy nature of the UK in the model. In other words, conditional on UK shocks $\sigma(r^{L*}) = \rho(r^L, r^{L*}) = 0$. In the $\sigma(r^L)$ -column the contributions do not sum to the total data variance because of the additional variance caused by UK-specific shocks (not reported).

Figure 9: DSGE Model Fit



Notes: Observed data shown in the blue lines, model Kalman Filter one step ahead predictions ($E_{t-1}x_t$). Shaded areas denote NBER US recession periods. shown in red-dashed lines.

Figure 10: US Inflation Target Shock



Notes: The shock has been normalised to increase the US annual inflation by 1*pps*. The blue circle-dashed line illustrates agents' optimal responses in the benchmark economy, while the solid red line in the economy where the persistence of the persistence of the productivity growth process have been calibrate to 0.999

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