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ORIGINAL ARTICLE

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Equilibrium opacity in ultimatum-offer bargaining

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Abstract

We consider ultimatum bargaining between a seller and a buyer of an asset. They know each other's valuation of the asset. Both can defer their decisions to delegates. These delegates have opaque preferences. Seller and buyer choose the opacity of their delegate. For the seller's delegate this choice is restricted to a random reservation price drawn from the set of symmetric two-point distributions around the seller's true reservation price. The opacity choice of the buyer's delegate is restricted to a random willingness-to-pay drawn from the set of symmetric two-point distribution around the buyer's true willingness-to-pay. We characterize the set of pure-strategy equilibria in their delegation choices. Multiple equilibria arise. Except for two corner solutions, both players will exploit the strategy of opacity. A large set of efficient equilibria exist. For these, opacity choices do not reduce the probability of transacting, but benefit the buyer compared with the no-delegation equilibrium. We also study the robustness of the results with respect to the player's ability to also resort to a tougher delegate in addition to the opacity choice.

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1 | INTRODUCTION

Delegation in the context of ultimatum bargaining is a well-studied issue. Suppose two players negotiate about a deal. Each player attributes a given value to completing the deal. If the players know each other's valuations, the offer-making player can propose a deal that makes the responder just indifferent whether to accept and reject, thereby absorbing the complete surplus. As Schelling (1960, p. 29) pointed out, to escape from this outcome, the responding player might want to instruct a delegate to reject all offers that do not attribute the bulk of the surplus to the responder. Making the delegate tougher by suitable instructive delegation can ideally relocate all surplus to the responding player. However, as Katz (1991), Fershtman and Kalai (1997), and Koçkesen and Ok (2004) highlight, to be credible, observability of the delegation contract is quintessential. If the instruction given cannot be observed, it is weakly dominant to instruct a delegate to accept offers that coincide with the delegator's own preferences.

This credibility issue motivates our analysis. We consider a dimension of a delegate's decision making that may overcome this credibility issue: Delegation to decision makers whose values are opaque and who are genuinely unpredictable in their decisions and where this unpredictability is common knowledge for both players. Such opacity can have multiple institutional causes that make the type and degree of opacity a matter of choice. First, in negotiations between nations incomplete information emerges from idiosyncratic elements in the frame of mind of the politicians who are delegated to attend the negotiations and represent their nations at the negotiation table. The representatives' values may depend on factors, such as career concerns (Fingleton & Raith, 2005) and professional and personal ties to the decision-maker's multiple supporters. These individual political costs are a source of opacity. Only the actual representatives know their own values.[1] Typically both nations appoint delegates; each of them might select a predictable representative who has a long track record, or a 'black horse' without a track record, or even one who is known for being volatile and unpredictable.

Another means to choose between more or less opacity is by design of a group decision mechanism. Think about a large group that has to collectively accept or reject an ultimatum offer by a majority vote. Suppose each member of the group has his or her own reservation price. Let these be drawn from a symmetric random distribution. The median voter position is decisive. For a large group the median voter's reservation price is in a very narrow range around the average reservation price. In comparison, if the group randomly appoints a small subcommittee or one of its members to make the decision, then the decision is based on a reservation price that has the same mean, but a much larger variance than in the majoritarian decision of the large group.

Furthermore, in some cases even the identity of the negotiator may remain opaque. A good example for this type of opacity is the significant market for ransom negotiations in cases of kidnapping (K&R).[2] For professional negotiators, in particular K&R insurance companies, it is crucial that bargaining takes place under asymmetric information for the kidnappers. In her analysis of the governance in the K&R insurance market, Shortland (2016) stresses that kidnappers must be left in the dark about the entity in charge of paying the ransom (e.g., family,

[1]For an application of this type of information asymmetry to climate negotiations, see Konrad and Thum (2014, 2018). Because we want to abstract from strategic delegation to a specific type of negotiator, we assume that the political cost of the negotiator is also stochastic from the parent party's perspective. Due to delegation, the agreement concluded by the representative is binding for the parent party.

[2]The total turnover in the criminal market for kidnappings is estimated to be at least £1 billion/annum (McAvoy & Randall, 2010).

firm, insurer, and government) and its financial capabilities. If a professional negotiator is hired by the insurer, he or she always acts in the background. Hence, it is not the tough negotiator whose fallback position differs from the fallback position of the victim's family but the ambiguity about the type of negotiator that matters here.

These examples illustrate: Players on each side of an ultimatum offer might use an opaque delegate to position them better for the actual bargaining game. Equilibrium requires that the opacity choices are mutually optimal replies. Can the offer-maker (seller) benefit from choosing a delegate with opaque preferences? Can the responder (buyer) gain from sending an opaque delegate? And what are the mutually optimal replies in terms of choices of opaque delegates? We characterize the equilibrium set for this problem and analyze its efficiency properties for a restricted class of possible random distributions. For two-sided opacity choices we find a multiplicity of equilibrium outcomes. A subset of these equilibria is efficient. These efficient equilibria differ in how they allocate the maximum surplus between the players.[3] A further set of equilibria is characterized by inefficient outcomes.

The analysis contributes to the literature on strategic delegation. As discussed, a seminal starting point of this analysis is Schelling (1960). Jones (1989) and Burtraw (1992) discuss delegates' toughness in the context of a Nash bargaining approach where the two parties select tough negotiators in a first stage to represent them in the negotiations in the second stage.[4] Kennan and Wilson (1993) offer an early literature review.[5] Ellingsen and Miettinen (2008) and Li (2011) study bilateral bargaining with firm commitments to being tough and analyze when commitment tactics cause an impasse. Ellingsen and Miettinen (2014) consider bilateral bargaining if commitments decay stochastically. Alonso and Matouschek (2008) study delegation when the delegate is better informed than the delegator, but has different genuine objectives. Optimal delegation then responds to the trade-off between a more informed decision and a systematic distortion from the principal's interests. The question is when and how a task or decision should be delegated given this trade-off. The information asymmetry is given in such a setup and not the object of choice. In our framework, the opacity of the delegate is the main choice variable. This also distinguishes our approach from the discussion of opacity in the context of political negotiations, where it typically refers to other types of information problems. For instance, the term 'transparency' is occasionally used in connection with the bargaining process itself rather than the preferences of the representative or his/her constituency.[6] Sometimes transparency also refers to the observability of offers made.[7]

[3]This might be surprising, as the opacity of the delegate's preferences can create a challenge for the efficiency of ultimatum bargaining: It confronts the proposer with a problem of incomplete information about the responder type. As is well known, such incomplete information may lead to inefficient bargaining outcomes (Chatterjee & Samuelson, 1983; Myerson & Satterthwaite, 1983).

[4]See also Sengul, Gimeno, and Dial (2012) and Caparrós (2016).

[5]Abreu and Gul (2000) consider reputation in an alternating-offer bargaining framework with rational and 'obstinate' types. Also in a dynamic game, Wolitzky (2012) considers reputation issues when one player's type is endogenous: The player can choose to publicly announce a bargaining posture and is committed to this choice, with some probability. The heterogeneity of player types makes their work related to our question.

[6]For instance, Perry and Samuelson (1994) analyze open- versus closed-door negotiations. Under closed-door bargaining, the constituency (principal) can only accept or reject the final agreement that was negotiated by a representative (agent). With open-door bargaining, the constituency can also terminate negotiations at an intermediate stage when the agent makes or receives offers.

[7]In Stasavage (2004) and Fingleton and Raith (2005), with closed-door bargaining the constituency is only informed about the outcome; under open-door bargaining, the constituency can also observe both the proposal and the identity of the proposer. To distinguish from this literature, we consider 'opacity' which refers to uncertainty about the preferences of the responder in an ultimatum-bargaining game.

Our analysis is related to the literature on information design as reviewed by Bergemann and Morris (2019). Bayesian persuasion as described by Kamenica and Gentzkow (2011) is a most prominent subclass of such problems.[8] The common feature with our analysis is that information design can influence the probability distribution which is the basis for the decision-maker's choice of action subject to an expected-value constraint, whereas in our context the delegating offer-maker and responder can choose the random distribution of valuations for their delegates, also constrained by a choice set that is a given class of distributions. Preference uncertainty and its design is studied in two contributions that are particularly close to our framework. These contributions consider monopoly offers made to a buyer whose maximum willingness-to-pay is uncertain. This preference uncertainty turns the seller's problem into one of a monopolist facing uncertain demand. More specifically, Roesler and Balász (2017) consider a buyer who has a true valuation of the object, which is unknown. A signal leads to updating of beliefs. It molds the shape of the distribution of the true valuation and thereby establishes the shape of the random demand function. They ask: What distribution of signals maximizes the buyer's expected surplus, given how the specific demand uncertainty created shapes the seller's offer? Second, in the seller–buyer game by Condorelli and Szentes (2020) the buyer might take a (potentially costly) action that molds the buyer's true valuation for the object. This causes the buyer's valuation to become random. They ask what is the optimal shape of the random distribution of the valuation for the buyer, if the seller knows this distribution and chooses the payoff-maximizing offer price in view of this distribution. Both problems yield very similar optimal distributions for the buyer's valuation. In both cases the buyer's random valuation can be interpreted as a random demand function, and the optimal random demand function turns out to be isoelastic.

Our framework shifts the buyer–seller interaction from the true buyer and seller to their delegates. The delegating buyer and seller can choose the distribution of valuations of their delegates from some given set of distributions. If only the buyer delegates but not the seller, or if the seller chooses a "trivial" delegate who is a clone of the seller himself/herself, then the delegation problem reduces to one-sided delegation. Its solution has essentially the same structure as the ones in Roesler and Balász (2017) and Condorelli and Szentes (2020). More importantly, we also look at the two-sided delegation problem. Both the seller and the buyer might choose and generate opacity about the objectives of their delegates. The seller need not necessarily want to delegate to a clone of himself/herself. We find an equilibrium in which such trivial delegation by the seller is part of the equilibrium delegation strategies. However, the set of equilibria with two-sided delegation is much larger. There are equilibria when the seller also generates opacity about the preferences of his/her delegate, and equilibria with this property split the surplus more favorably for the seller.

2 | THE MODEL

Consider the following ultimatum-bargaining problem with two-sided delegation. There are two players: player A can buy an asset from player B. The asset has zero value for B and a value

[8]In the generic application, an information designer has an interest in the choice made by a decision maker. Recent contributions include Boleslavsky and Cotton (2018) who analyze how multiple agents might influence an organization's decision making and Camara and Eguia (2017) who study how an agenda setter might arrange proposal packages as an information design problem.

for A which is normalized to 1. Player B delegates the offer-making and player A delegates the decision whether to accept this offer. The set of possible actions by player A is a set of feasible two-point distributions of the valuation which A's delegate attributes to the asset: $v \in \{1-\alpha, 1+\alpha\}$ with equal probabilities 1/2. Player A's action is fully described by the parameter α. A distribution with a higher α corresponds to a mean-preserving spread of the delegate's valuation around A's true valuation. This is a simple but convenient way to study a class of distributions that describe mean-preserving spreads and ensure that the delegate's willingness-to-pay is the same, and not smaller than that of A in expectation. The mean-preserving property distinguishes the problem from the choice of a delegate who simply has a lower valuation than player A. It allows us to focus on the option to make the type of the delegate opaque but not tougher.[9] Summarizing, we can describe A's strategy space as $\alpha \in [0, 1]$, and for all choices of α it holds that

$$E(v) = \frac{1}{2}(1-\alpha) + \frac{1}{2}(1+\alpha) = 1. \tag{1}$$

Player B simultaneously chooses a delegate who is characterized by the parameter $\beta \in [0, 1]$, where $w \in \{-\beta, \beta\}$ is a random variable that determines the reservation value which a delegate of type β attributes to the asset. The delegate's reservation value is also a draw from a two-point distribution, where $w \in \{-\beta, \beta\}$ emerge with equal probabilities 1/2. The parameter β, hence, fully characterizes B's choice of distribution and B's strategy space is $\beta \in [0, 1]$. For all choices of β it holds that

$$E(w) = -\frac{1}{2}\beta + \frac{1}{2}\beta = 0. \tag{2}$$

These choices determine the distributions of delegated values for two agents D_A and D_B. These agents negotiate in an ultimatum-offer game. Delegate D_A knows his own valuation v and knows β, and delegate D_B knows her own valuation w and knows α. Delegate D_B makes an ultimatum offer to D_A, which consists of an asking price y. D_A accepts or rejects this offer. By definition, D_B's payoff is y if D_A accepts, and equal to w if D_A rejects the offer. Similarly, D_A's payoff is $v - y$ if the offer is accepted and 0 if the offer is not accepted. This implements the standard equilibrium of an ultimatum-offer game among D_A and D_B with incomplete information. The perfect equilibrium offer in the continuation game among the delegates is characterized by a demand price y which is accepted by the responder D_A if $y \leq v$ and rejected otherwise. Players A and B are risk neutral[10]: Payoff for player A is $1 - y$ if D_B's offer is a demand price of y and is accepted by D_A and zero otherwise. Payoff for player B is y if D_B's offer is a demand price of y and is accepted by D_A. The choice of a gross value of 1 for A in case of acceptance and of zero in case of rejection are normalizations. Similarly, payoff for player B is zero otherwise.

[9]The one-sided delegation problem of A can also be solved for a general set of distributions but the two-sided problem in which both A and B choose their optimal delegates becomes cumbersome with general distributions. We refrain from presenting the analysis for the general one-sided case here as a related information design problem is solved formally in Condorelli and Szentes (2020).

[10]Risk aversion has been discussed in a previous version of the paper; the analysis is available upon request.

3 | ONE-SIDED DELEGATION

To approach the solution of the game of opacity choices by A and B who anticipate the continuation game between D_A and D_B, consider first the case $\beta \equiv 0$. This case can be seen as a one-sided opacity choice by A, whereas B has to choose no delegate or a delegate who is identical to B himself/herself. The offer-maker D_B knows that the responder has a willingness-to-pay of v that is a random draw but does not know the realization of this random draw. The delegated value can be of only two types, characterized by $\{v_{\mathrm{L}}, v_{\mathrm{H}}\} = \{1 - \alpha, 1 + \alpha\}$ with $\alpha \in [0, 1]$ and with the respective probabilities $p_{\mathrm{H}} = 1 - p_{\mathrm{L}} = \frac{1}{2}$.

A choice of $\alpha^* = \frac{1}{3}$, leading to

$$v_{\mathrm{L}}^* = \frac{2}{3}, \quad v_{\mathrm{H}}^* = \frac{4}{3}, \tag{3}$$

maximizes the payoff for two-point distributions. The payoff for A is $1 - y^* = \frac{1}{3}$. To see why this equilibrium result holds, consider the choice of B. Only two requested amounts y are economically reasonable: $y = 1 - \alpha$ or $y = 1 + \alpha$. Any request higher than $(1 + \alpha)$ is rejected with certainty. Any request $y \in (1 - \alpha, 1 + \alpha)$ is accepted with the same probability $\frac{1}{2}$ as $y = (1 + \alpha)$, so B's payoff from $y = (1 + \alpha)$ dominates any of the requests from the open interval $(1 - \alpha, 1 + \alpha)$. A request $y = 1 - \alpha$ is accepted with probability 1, so the higher probability might compensate for the lower payment in the case of acceptance. And for demands smaller than $y = 1 - \alpha$ again the dominance argument applies: $y = 1 - \alpha$ is accepted with the same probability as any $y < 1 - \alpha$, but yields a higher payment to B. A delegated value that is drawn from the symmetric distribution with possible outcomes $(1 - \alpha)$ and $(1 + \alpha)$ gives the offer-maker an expected payoff of $\frac{1}{2}(1 + \alpha)$ if the offer is $y = (1 + \alpha)$, and a payoff of $(1 - \alpha)$ if the offer is $y = 1 - \alpha$. B (weakly) prefers $y = (1 - \alpha)$ for all α with

$$(1 - \alpha) \geq \frac{1}{2}(1 + \alpha), \tag{4}$$

that is, for all $\alpha \in [0, \frac{1}{3}]$. Among these A prefers the largest possible α. This leads to (3). Strictly speaking, B is indifferent between $v_{\mathrm{L}}^* = \frac{2}{3}$ and $v_{\mathrm{H}}^* = \frac{4}{3}$. The choice of v_{L}^* implements efficiency, in this case, and we assume that B chooses $v_{\mathrm{L}}^* = \frac{2}{3}$. More generally, and throughout the analysis we assume efficient tie-breaking: If several y yield the same payoff for B, then B chooses the smallest y from this set. The proposer might consider a different choice in case of indifference. Intuitively, this might be due to (lexicographically) secondary interests, such as spiteful behavioral preferences or reputational concerns that are not part of the model here. The assumption of efficient tie-breaking is reminiscent of benevolent tie-breaking in the standard ultimatum game, where the responder accepts an offer that makes the responder at least indifferent to rejecting.[11] Finally, alternatives to the assumption of

[11]Benevolent tie-breaking in the ultimatum-bargaining analysis would be subject to the same possible objections about secondary motives, such as spite or reputation, which are not part of the model, but it is standardly assumed for reasons that apply similarly to our assumption.

efficient tie-breaking are technically less convenient but do not hint at qualitatively different results.[12]

Let us compare the intuition for why opacity is beneficial with the original intuition for instructive delegation by Schelling (1960). The responder A benefits if A can replace its own decision making by a tougher decision maker or decision mechanism. Ideally the responder could extract the full surplus, but this solution may meet credibility problems, as discussed. The constraint that expected toughness cannot be increased takes up this concern. What the responder can still do is to replace own decision making by an opaque decision maker: by someone who is unpredictable, and tough or soft with some probability. If A chooses opaqueness, then A can choose how tough the tough type of delegate should be, taking into account that a tougher tough type implies a softer soft type. Making the tough type tougher increases the payoff of the responder, but only if the offer-maker still chooses a request low enough to be acceptable to the tough type. An increase in the spread between the tough type and the soft type makes it less and less attractive for the offer-maker to choose such a low request. It becomes more and more attractive to choose a high request, even though only the soft type accepts it. The responder chooses the largest spread that still induces the offer-maker to make the low request.

4 | BILATERAL DELEGATION

Suppose now that A chooses $\alpha \in [0, 1]$ and B chooses $\beta \in [0, 1]$. These simultaneous choices determine the distributions of the delegated valuations of D_A and D_B. Delegate D_A's valuation of the deal is a random draw $v \in \{1 - \alpha, 1 + \alpha\}$ with equal probabilities 1/2. Similarly, delegate D_B's reserve price is a random draw from $w \in \{-\beta, \beta\}$ with equal probabilities 1/2. Let the draws of v and w be mutually stochastically independent. The following holds:

Proposition 1. *The set of subgame perfect equilibrium choices of opacity consists of pairs* $(\alpha, \beta) = \left(\frac{1-\beta}{3}, \beta\right)$ *in the range* $\beta \in \left[0, \frac{3}{5}\right]$ *and* $(\alpha, \beta) = \left(\frac{1+\beta}{3}, \beta\right)$ *in the range* $\beta \in \left[\frac{3}{5}, 1\right]$.

Proof. For given choices α and β, nature decides the types v and w as stochastically independent draws from the sets $\{1 - \alpha, 1 + \alpha\}$ and $\{-\beta, \beta\}$. The delegate D_B chooses y. All types of D_A accept all $y \leq 1 - \alpha$, all $y \in (1 - \alpha, 1 + \alpha]$ are accepted if $v = 1 + \alpha$, that is, with probability 1/2, all $y > 1 + \alpha$ are rejected by both types of D_A, that is, with probability 1. Only two values of y are candidates for a maximum of the objective function of D_B: $y = 1 - \alpha$ that leads to sure acceptance, and $y = 1 + \alpha$ that leads to acceptance with probability 1/2. Any other y is dominated either by $y = 1 - \alpha$, by $y = 1 + \alpha$ or by both.

If $w = \beta \geq 0$, then D_B chooses $y = 1 - \alpha$ if and only if $1 - \alpha \geq \frac{1}{2}(1 + \alpha) + \frac{1}{2}\beta$, that is, for $\beta \leq 1 - 3\alpha$. If $w = -\beta \leq 0$, then D_B chooses $y = 1 - \alpha$ if $1 - \alpha \geq \frac{1}{2}(1 + \alpha) - \frac{1}{2}\beta$, that is, for $\beta \geq 3\alpha - 1$. Figure 1 shows the square of possible combinations of α and β. The request $y = 1 - \alpha$ is always made for (α, β)-combinations in Region I, which is on

[12]One could resort to standard epsilon-equilibrium arguments to address the problem. Suppose that B were to break the indifference between $y = 1 - \alpha$ and $y = 1 + \alpha$ in favor of $y = 1 + \alpha$. Then, $v_L^* + \varepsilon$ and $v_H^* - \varepsilon$ for small but positive ε values constitute delegated valuations that fulfill $E(v) = 1$ and make B prefer $\tilde{v}_L$ strictly to $\tilde{v}_H$. This choice gives A a payoff that is arbitrarily close to the payoff in our efficient tie-breaking equilibrium for $\lim_{\varepsilon \to 0}$.

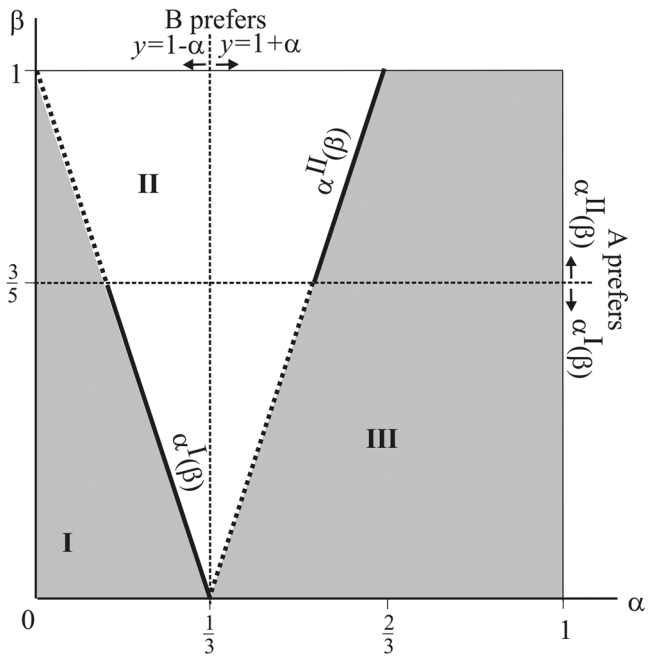

FIGURE 1 The graph shows the set of possible combinations of the delegates' valuations characterized by α and β. The equilibrium combinations are the solid lines

and to the left of the downward sloping line $\beta = 1 - 3\alpha$. This request is chosen in half of the cases for combinations in Region II, which is the region between the downward sloping line and the upward sloping line $\beta = 3\alpha - 1$, and never chosen in Region III, that is, for combinations to the right of the upward sloping line.

Given these probabilities for a choice $y = 1 - \alpha$, we can discuss the optimal replies of A to any given β and optimal replies of B to any given α and then consider the intersection or overlap of these optimal replies.

Consider A. The payoff of A is α in Region I, it is $\frac{1}{2}\alpha - \frac{1}{2}\frac{1}{2}\alpha = \frac{1}{4}\alpha$ in Region II, and equal to $-\frac{1}{2}\alpha$ in Region III. Player A's payoff in Region I is maximized for the largest α in this region for a given β. This choice yields payoff

$$\pi_A^{\mathrm{I}}(\beta) = \alpha^{\mathrm{I}}(\beta) = \frac{1}{3}(1 - \beta).$$

Region III is clearly not attractive for A. The expected payoff there is negative. The payoff in Region II is $\frac{1}{4}\alpha$, and is maximized for the right border of this region and yields payoff

$$\pi_A^{\mathrm{II}}(\beta) = \frac{1}{4}\alpha^{\mathrm{II}}(\beta) = \frac{1}{4}\left(\frac{1}{3}(1 + \beta)\right).$$

Note that $\pi_A^{\mathrm{I}} > \pi_A^{\mathrm{II}}$ in the range $\beta < \frac{3}{5}$, $\pi_A^{\mathrm{I}} = \pi_A^{\mathrm{II}}$ for $\beta = \frac{3}{5}$, and $\pi_A^{\mathrm{I}} < \pi_A^{\mathrm{II}}$ in the range $\beta > \frac{3}{5}$. Hence, A's optimal reply is

$$\alpha^*(\beta) = \begin{cases} \frac{1}{3}(1 - \beta) & \text{for } \beta \in \left[0, \frac{3}{5}\right], \\ \frac{1}{3}(1 + \beta) & \text{for } \beta \in \left(\frac{3}{5}, 1\right]. \end{cases}$$

This optimal reply is depicted by the bold parts of the functions $\beta = 1 - 3\alpha$ and $\beta = 3\alpha - 1$ in Figure 1. Turn now to B's optimal reply to possible choices of α. Player B prefers an offer $y = (1 - \alpha)$ (which is always accepted) to an offer $y = (1 + \alpha)$ (which is accepted only with probability $\frac{1}{2}$) if $\alpha < \frac{1}{3}$, is indifferent if $\alpha = \frac{1}{3}$ and prefers $y = (1 + \alpha)$ if $\alpha > \frac{1}{3}$. The optimal reply β to a given α is set-valued: any $\beta \in [0, 1 - 3\alpha]$ yields a combination of (α, β) which induces $y = 1 - \alpha$ and payoff $\pi_B = 1 - \alpha$ for B in the range $\alpha \in \left[0, \frac{1}{3}\right]$. The optimal reply β is also set-valued in the range $\alpha \in \left[\frac{1}{3}, 1\right]$. In this range each $\beta \in [0, \min\{(-1 + 3\alpha), 1\}]$ leads to implementation of $y = 1 + \alpha$. In Figure 1 this optimal reply correspondence of B is given by the shaded areas below the curves $\beta = 1 - 3\alpha$ and $\beta = 3\alpha - 1$. Equilibrium combinations of α and β in pure strategies are the intersections of the optimal reply function $\alpha(\beta)$ of player A with the optimal reply correspondence of player B. These combinations are the pairs $(\alpha, \beta) = \left(\frac{1-\beta}{3}, \beta\right)$ in the range $\beta \in \left[0, \frac{3}{5}\right]$ and $(\alpha, \beta) = \left(\frac{1+\beta}{3}, \beta\right)$ for $\beta \in \left[\frac{3}{5}, 1\right]$. □

Proposition 1 is a characterization of the complete set of equilibria of opacity choices if the choice sets are the sets of symmetric two-point distributions. There are multiple pairs of mutually optimal choices of opacity. For low degrees of opacity, B's and A's opacity choices are substitutes. The more opaque D_B, the less opaque is D_A that is chosen as an optimal reply in the equilibrium. Any of the equilibrium outcomes is efficient in the lower range of (α, β): a deal is reached with probability 1. The equilibria are not Pareto ranked. The share in the aggregate rent that goes to A is a monotonic function of A's opacity parameter α. The equilibrium $(\alpha, \beta) = \left(\frac{1}{3}, 0\right)$ corresponds to the equilibrium in Section 3 with one-sided delegation. At the other extreme, the equilibrium $(\alpha, \beta) = \left(\frac{1}{5}, \frac{3}{5}\right)$ gives lower payoff to A and highest payoff to B. There are also equilibria (α, β) with $\alpha \in \left(\frac{1}{3}, \frac{2}{3}\right)$ and $\beta(\alpha) = \frac{\alpha}{3} - 1$ in which both players choose high degrees of opacity, and in which a deal is made only with probability $\frac{1}{2}$. These equilibria are inefficient and waste rent with a probability of $\frac{1}{4}$. For each of these equilibria there exists at least one equilibrium on the downward sloping locus of equilibrium combinations (α, β) that gives both players a higher payoff.[13]

To select among the efficient equilibria let us consider iterated elimination of weakly dominated strategies. Strategies $\alpha \in \left(\frac{2}{3}, 1\right]$ are strictly dominated, for instance, by $\alpha = \frac{1}{3}$, as $-\frac{1}{4}\alpha < \frac{1}{4}\alpha$. For the remaining range $\alpha \in \left[0, \frac{2}{3}\right]$ strategies $\beta > 0$ are weakly dominated by $\beta = 0$. We have shown already in the proof of Proposition 1 that B's optimal reply to $\alpha \in \left[0, \frac{2}{3}\right] \setminus \left\{\frac{1}{3}\right\}$ is set-valued and includes $\beta = 0$. Moreover, β's unique optimal reply to $\alpha = \frac{1}{3}$ is $\beta = 0$. Hence, all strategies $\beta > 0$

[13]To confirm this we show that the equilibrium with $\alpha = \frac{1}{3}$ and $\beta = 0$ Pareto dominates these inefficient equilibria. The efficient equilibrium has payoffs $\pi_A^{\mathrm{I}} = \frac{1}{3}$ and $\pi_B^{\mathrm{I}} = \frac{2}{3}$. In the inefficient equilibria, expected payoffs are $\pi_A^{\mathrm{II}} = \frac{1}{4}.\frac{1}{3}(1 + \beta)$ and $\pi_B^{\mathrm{II}} = \frac{3}{4} - \frac{1}{4}.\frac{1}{3}(1 + \beta)$. None of the inefficient equilibria gives A a higher payoff than $\left(\frac{1}{3}, 0\right)$ as $\pi_A^{\mathrm{I}} = \frac{1}{3} > \pi_A^{\mathrm{II}}(\beta = 1) = \frac{1}{4}.\frac{2}{3}$. Also B is never better off with an inefficient equilibrium: $\pi_B^{\mathrm{I}} = \frac{2}{3} > \pi_B^{\mathrm{II}}\left(\beta = \frac{3}{5}\right) = \frac{3}{4} - \frac{1}{4}.\frac{1}{3}\left(1 + \frac{3}{5}\right) = \frac{37}{60}$.

are weakly dominated by $\beta = 0$. Therefore, $(\alpha, \beta) = \left(\frac{1}{3}, 0\right)$ is the only equilibrium that survives iterated elimination of weakly dominated strategies. One should note, however, that sequential delegation choices would bring back the equilibrium $(\alpha, \beta) = \left(\frac{1}{5}, \frac{3}{5}\right)$ if B moves first.

The introduction alluded to the credibility problem of a responder in a bargaining context to instruct an agent to be tougher than the responder himself: If the offer-maker cannot observe the actual contract, all strategies that make the delegate tougher are weakly dominated by instructing the agent to accept precisely all prices that the responder would accept himself (Fershtman & Kalai, 1997). The same logic applies in our context, when the responder A instructs an agent by choosing α. This can be illustrated using the example of unilateral delegation. In this case, the responder A would want B to believe that the delegate has an α of $1/3$, making B demand $y^* = 2/3$. But if A secretly agrees with the delegate to an $\alpha = 0$, then this does not harm compared with any other α if $y^* = 2/3$, and A is at least as well off or even really better off if B accidentally makes some choices other than y^*. The delegation choice $\alpha = 1/3$ is thus weakly dominated, and the same applies to all other positive α. As a contractual choice, $\alpha > 0$ is, hence, not credible, if α is not observable. It is therefore important to recall that we assume that player A does not instruct the delegate to behave according to α, but makes an observable choice among the set of delegates, some of which are known to be more predictable, others are known to be less predictable. We believe that the personal idiosyncrasies that the representatives of a state in international negotiations inevitably have bring precisely this unpredictability (to both sides) into the negotiations. This ultimately strengthens the side that would be in the strategically weaker responder role if A and B themselves had deterministic and known preferences.

5 | EXTENSION: MORE TOUGH OR MORE OPAQUE?

Schelling (1960) proposed to send a tough delegate to the negotiations to secure rents for one's own side. The toughness of a negotiator is reflected in a lower willingness-to-pay for the object. Within our model, the question arises how the Schelling strategy interacts with the strategy of opacity we have described in the main section. Is it still worthwhile to create opacity when you can also send a tougher negotiator to the negotiating table? Does the degree of opacity increase or decrease when the toughness of a negotiator increases? To answer these questions, we extend our basic model. To keep the extension of the model simple and transparent, we assume that only player A sends a tougher negotiator to the negotiating table. The delegate's willingness-to-pay is either $a + \alpha$ or $a - \alpha$ with $a \in \left[\frac{3}{5}, 1\right]$.[14] Hence, the delegate's willingness-to-pay a is on average lower than the sender's. We treat the average willingness-to-pay not as a decision variable but as a parameter for player A. This allows us to analyze in a convenient manner whether player A, who sends a tough delegate, additionally uses opacity ($\alpha > 0$) as a strategic instrument. Since the derivation of the perfect equilibrium is analogous to the basic model, the formal analysis is delegated to the appendix. We will discuss here only the result based on Figure 2. As in Figure 1, three regions emerge: In Region I, player B's delegate always demands $a - \alpha$ and this offer is always accepted by A's delegate. In Region II, the high demand $a + \alpha$ and the low demand $a - \alpha$

[14]We restrict the analysis to a 'moderate' Schelling strategy. No further insights can be gained from having a very tough delegate.

are made with probability $\frac{1}{2}$ each; the high demand is only accepted with probability $\frac{1}{2}$. In Region III, player B's delegate always demands $a + \alpha$, which is only accepted with probability $\frac{1}{2}$. In Region I, player A maximizes the expected payoff if he or she chooses the right border ($\alpha^{\mathrm{I}}(\beta) = \frac{a-\beta}{3}$); the expected profit is $\pi_A^{\mathrm{I}} = \frac{3-2a-\beta}{3}$. In Region II, the expected payoff is also maximized at the right border ($\alpha^{\mathrm{II}}(\beta) = \frac{a+\beta}{3}$) and amounts to $\pi_A^{\mathrm{II}} = \frac{9-8a+\beta}{12}$. A comparison of the two expected payoffs shows that—similar to the basic model—A's optimal reply is

$$\alpha^*(\beta) = \begin{cases} \frac{1}{3}(a-\beta) & \text{for } \beta \in \left[0, \frac{3}{5}\right], \\ \frac{1}{3}(a+\beta) & \text{for } \beta \in \left(\frac{3}{5}, 1\right]. \end{cases}$$

This optimal reply is depicted by the solid black lines in Figure 2. For low values of β ($\beta < \frac{3}{5}$), player A prefers the cautious variant with certain negotiation success and chooses $\alpha^{\mathrm{I}}(\beta)$. For $\beta > \frac{3}{5}$, the reaction function is $\alpha^{\mathrm{II}}(\beta)$. The optimal reply correspondence of B is depicted again by the shaded areas.

The comparison of Figures 1 and 2 shows that only the curves of the equilibria have shifted to the left. A comparison of the two figures also makes it clear that toughness and opacity are substitutes for player A. Since $a < 1$ causes the equilibrium curves to shift to the left, using the Schelling strategy results in a lower equilibrium value α for each given β. If player A's elegate becomes tougher, the optimal degree of dispersion shrinks; opacity can thus be lower.

Again, each of the inefficient equilibria is Pareto dominated by at least one of the efficient equilibria. Take again the efficient equilibrium $(\alpha, \beta) = (\frac{a}{3}, 0)$ for the Pareto comparison. Payoffs amount to $\pi_A^{\mathrm{I}}(\beta = 0) = 1 - a + \alpha^{\mathrm{I}}(\beta = 0) = 1 - \frac{2}{3}a$ and $\pi_B^{\mathrm{I}} = a - \alpha^{\mathrm{I}}(\beta = 0) = \frac{2}{3}a$. The

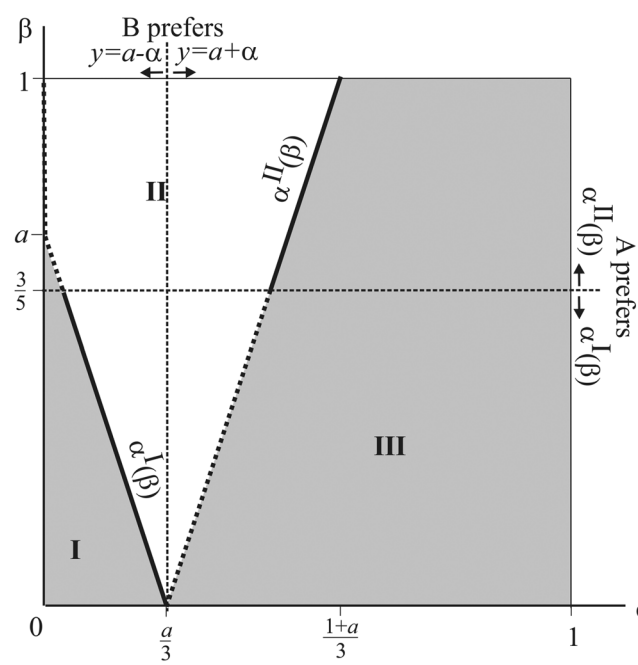

FIGURE 2 The graph shows the set of possible combinations of the delegates' valuations characterized by α and β, if the responder can make the delegate not only more opaque, but also somewhat tougher. The equilibrium combinations are the solid lines

inefficient equilibria on $\alpha^{\mathrm{II}} \in \left(\frac{a}{3}, \frac{1+a}{3}\right)$ yield expected payoffs of $\pi_A^{\mathrm{II}} = \frac{3}{4}(1-a) + \frac{1}{4}\alpha^{\mathrm{II}}$ and $\pi_B^{\mathrm{II}} = \frac{3}{4}a - \frac{1}{4}\alpha^{\mathrm{II}}$ for A and B, respectively. For A, the best equilibrium among the inefficient ones is $\left(\frac{1+a}{3}, 1\right)$. However, the payoff in this equilibrium is still lower than the payoff in the efficient equilibrium: $\pi_A^{\mathrm{I}}(\beta = 0) = 1 - \frac{2}{3}a > \pi_A^{\mathrm{II}}(\beta = 1) = \frac{10-8a}{12}$. For B, the best equilibrium among the inefficient ones is $\left(\frac{1}{3}\left(a + \frac{3}{5}\right), \frac{3}{5}\right)$. Also player B is worse off with the best inefficient equilibrium compared with the efficient equilibrium $\left(\frac{a}{3}, 0\right)$: $\pi_B^{\mathrm{I}}(\beta = 0) = \frac{2}{3}a > \pi_B^{\mathrm{II}}\left(\beta = \frac{3}{5}\right) = \frac{8}{12}a - \frac{1}{20}$. Hence, the efficient equilibria Pareto dominate the inefficient equilibria. Among the efficient equilibria, which all exhibit a total payoff of 1, player A prefers the equilibrium $\left(\frac{a}{3}, 0\right)$, while player B prefers $\left(\frac{5a-3}{15}, 0\right)$.

6 | CONCLUSION

In ultimatum bargaining, if the offer-maker and the responder can delegate their decisions to agents whose actual decision rules are opaque, the responder may benefit from this opacity. Opacity prevents the responder from being exploited by the offer-making side. Multiple equilibria exist that differ in how the offer-maker and the responder split the surplus. Also, some of these equilibria preserve the efficiency of the complete information outcome that emerges without delegation, some other equilibria do not.

Part of the existing literature has focused on the delegation to tough negotiators to secure some rents for the responding party and discussed credibility issues of such commitment. We have shown that the protection of rents can also be achieved through the genuine opacity of the delegate's valuation. Our results are related to the literature studying the choice of randomness of true valuation for a buyer who faces a monopolist supplier (Condorelli & Szentes, 2020; Roesler & Balász, 2017). The one-sided delegation solution yields a similar optimal information design in these two setups. Two major differences emerge. First, in our context the delegating responder delegates decision making to an opaque delegate, but keeps the same, commonly known valuation. Second, we consider simultaneous delegation choices by both players who bargain. This is relevant, as it leads to a whole set of equilibria of which the one-sided delegation equilibrium remains an element.

From a practical point of view, the beneficial effects of opacity point to a trade-off that emerges in negotiations. Suppose an organization or person is about to enter into negotiations and can delegate negotiations to an expert, who at the same time has to make decisions about possible negotiation offers. While experienced experts may know the tricks of the trade, they are also more transparent in terms of their policy stance. Sending dark horses to the negotiation table makes it more difficult for the other side to pin the responder down to its reservation utility. The design of the selection process, which determines the identity of the expert, and the decision rules within a delegation, which is mandated with the negotiation, provide opportunities for a choice of opacity.

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REFERENCES

Abreu, D., & Gul, F. (2000). Bargaining and reputation. *Econometrica*, *68*(1), 85–117.

Alonso, R., & Matouschek, N. (2008). Optimal delegation. *Review of Economic Studies*, *75*(1), 259–293.

Bergemann, D., & Morris, S. (2019). Information design: A unified perspective. *Journal of Economic Literature*, *57*(1), 44–95.

Boleslavsky, R., & Cotton, C. (2018). Limited capacity in project selection: Competition through evidence production. *Economic Theory*, *65*(2), 385–421.

Burtraw, D. (1992). Strategic delegation in bargaining. *Economics Letters*, *38*(2), 181–185.

Camara, O., & Eguia, J. X. (2017). Slicing and bundling. *Journal of Politics*, *79*(4), 1460–1464.

Caparrós, A. (2016). Bargaining and international environmental agreements. *Environmental and Resource Economics*, *65*(1), 5–31.

Chatterjee, K., & Samuelson, W. (1983). Bargaining under incomplete information. *Operations Research*, *31*(5), 835–851.

Condorelli, D., & Szentes, B. (2020). Information design in the hold-up problem. *Journal of Political Economy*, *128*(2), 681–709.

Ellingsen, T., & Miettinen, T. (2008). Commitment and conflict in bilateral bargaining. *American Economic Review*, *98*(4), 1629–1635.

Ellingsen, T., & Miettinen, T. (2014). Tough negotiations: Bilateral bargaining with durable commitments. *Games and Economic Behavior*, *87*, 353–366.

Fershtman, C., & Kalai, E. (1997). Unobserved delegation. *International Economic Review*, *38*(4), 763–774.

Fingleton, J., & Raith, M. (2005). Career concerns of bargainers. *Journal of Law, Economics, and Organization*, *21*(1), 179–204.

Jones, S. R. G. (1989). Have your lawyer call my lawyer, bilateral delegation in bargaining situations. *Journal of Economic Behavior and Organization*, *11*(2), 159–174.

Kamenica, E., & Gentzkow, M. (2011). Bayesian persuasion. *American Economic Review*, *101*(6), 2590–2615.

Katz, M. L. (1991). Game-playing agents: Unobservable contracts as precommitments. *RAND Journal of Economics*, *22*(3), 307–328.

Kennan, J., & Wilson, R. (1993). Bargaining with private information. *Journal of Economic Literature*, *31*(1), 45–104.

Koçkesen, L., & Ok, E. A. (2004). Strategic delegation by unobservable incentive contracts. *Review of Economic Studies*, *71*(4), 397–424.

Konrad, K. A., & Thum, M. (2014). Climate policy negotiations with incomplete information. *Economica*, *81*(332), 244–256.

Konrad, K. A., & Thum, M. (2018). Does a clean development mechanism facilitate international environmental agreements? *Environmental and Resource Economics*, *69*(4), 837–851.

Li, D. Z. (2011). Commitment and compromise in bargaining. *Journal of Economic Behavior and Organization*, *77*(2), 203–211.

McAvoy, E., & Randall, D. (2010). The £1 billion hostage trade—How kidnapping became a global industry. *The Independent*, October 16

Myerson, R. B., & Satterthwaite, M. A. (1983). Efficient mechanisms for bilateral trading. *Journal of Economic Theory*, *29*(2), 265–281.

Perry, M., & Samuelson, L. (1994). Open- versus closed-door negotiations. *RAND Journal of Economics*, *25*, 348–359.

Roesler, A.-K., & Balász, S. (2017). Buyer optimal learning and monopoly pricing. *American Economic Review*, *107*(7), 2072–2080.

Schelling, T. C. (1960). *The strategy of conflict* (1980 ed.). Cambridge, MA: Harvard University Press.

Sengul, M., Gimeno, J., & Dial, Y. (2012). Strategic delegation: A review, theoretical integration, and research agenda. *Journal of Management*, *38*(1), 375–414.

Shortland, A. (2016). Governing kidnap for ransom: Lloyd's as a 'private regime'. *Governance*, *30*(2), 283–299.

Stasavage, D. (2004). Open-door or closed-door? Transparency in domestic and international bargaining. *International Organization*, *58*(4), 667–703.

Wolitzky, A. (2012). Reputational bargaining with minimal knowledge of rationality. *Econometrica*, *80*(5), 2047–2087.

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APPENDIX A

We determine the equilibrium choices, when player A's delegate has an average willingness-to-pay of $a \in \left[\frac{3}{5}, 1\right]$. The set of subgame perfect equilibrium choices of opacity consists of pairs $(\alpha, \beta) = \left(\frac{a-\beta}{3}, \beta\right)$ in the range $\beta \in \left[0, \frac{3}{5}\right]$ and $(\alpha, \beta) = \left(\frac{a+\beta}{3}, \beta\right)$ in the range $\beta \in \left[\frac{3}{5}, 1\right]$.

For given choices α and β, nature decides the types v and w as stochastically independent draws from the sets $\{a - \alpha, a + \alpha\}$ and $\{-\beta, \beta\}$. The delegate D_B chooses y. All types of D_A accept all $y \leq a - \alpha$, all $y \in (a - \alpha, a + \alpha]$ are accepted if $v = a + \alpha$, that is, with probability $\frac{1}{2}$, all $y > a + \alpha$ are rejected by both types of D_A, that is, with probability 1. The two candidates for a maximum of the objective function of D_B are: $y = a - \alpha$ that leads to sure acceptance, and $y = a + \alpha$ that leads to acceptance with probability $\frac{1}{2}$.

If $w = \beta \geq 0$, then D_B chooses $y = a - \alpha$ if and only if $a - \alpha \geq \frac{1}{2}(a + \alpha) + \frac{1}{2}\beta$, that is, for $\beta \leq a - 3\alpha$. If $w = -\beta \leq 0$, then D_B chooses $y = a - \alpha$ if $a - \alpha \geq \frac{1}{2}(a + \alpha) - \frac{1}{2}\beta$, that is, for $\beta \geq 3\alpha - a$.

We now determine the optimal replies of A to any given β. A's payoff is $1 - a + \alpha$ in Region I (see Figure 2), it is $\frac{1}{4}(3(1 - a) + \alpha)$ in Region II, and equal to $\frac{1}{2}(1 - a - \alpha)$ in Region III. Player A's payoff in Region I is maximized for the largest α in this region for a given β, which is $\alpha^{\mathrm{I}}(\beta) = \max\left\{0, \frac{a-\beta}{3}\right\}$. This choice yields payoff

$$\pi_A^{\mathrm{I}}(\beta) = 1 - a + \alpha^{\mathrm{I}}(\beta) = \begin{cases} 1 - \dfrac{2}{3}a - \dfrac{1}{3}\beta & \text{for } \beta < a, \\ 1 - a & \text{for } \beta \geq a. \end{cases}$$

The payoff in Region II is $\frac{1}{4}(3(1-a)+\alpha)$. It is maximized for the right border of this region and yields payoff

$$\pi_A^{\mathrm{II}}(\beta) = \frac{1}{4}(3(1-a)+\alpha^{\mathrm{II}}(\beta)) = \frac{3}{4} - \frac{2}{3}a + \frac{1}{12}\beta.$$

Region III is not attractive for A. The expected payoff there is $\frac{1}{2}(1-a-\alpha)$, which is maximized for the smallest possible α in this region, that is, $\pi_A^{\mathrm{III}}(\beta) = \frac{1}{2} - \frac{2}{3}a - \frac{1}{6}\beta$ for $\alpha^{\mathrm{III}}(\beta) = \frac{1}{3}\beta + \frac{1}{3}a$. This is clearly smaller than $\pi_A^{\mathrm{II}}(\beta)$.

For $\beta < a$, a comparison of profits yields $\pi_A^{\mathrm{I}}(\beta) \gtrless \pi_A^{\mathrm{II}}(\beta) \Leftrightarrow \frac{3}{5} \gtrless \beta$. Hence, $\alpha^{\mathrm{I}}(\beta)$ is preferred for $0 \le \beta < \frac{3}{5}$ and $\alpha^{\mathrm{II}}(\beta)$ for $\frac{3}{5} \le \beta < a$. For $\beta \ge a$, a similar comparison leads to $\pi_A^{\mathrm{I}}(\beta) \gtrless \pi_A^{\mathrm{II}}(\beta) \Leftrightarrow \frac{1}{4} \gtrless \frac{1}{3}a + \frac{1}{12}\beta$. Due to $a > \frac{3}{5}$, in this range, A always prefers $\alpha^{\mathrm{II}}(\beta)$. This yields the following optimal reply of A:

$$\alpha^*(\beta) = \begin{cases} \frac{1}{3}(a-\beta) & \text{for } \beta \in \left[0, \frac{3}{5}\right], \\ \frac{1}{3}(a+\beta) & \text{for } \beta \in \left(\frac{3}{5}, 1\right]. \end{cases}$$

We turn now to B's optimal reply to possible choices of α. Player B prefers an offer $y = (a-\alpha)$ that is always accepted to an offer $y = (a+\alpha)$ that is accepted only with probability $\frac{1}{2}$ if $\alpha \le \frac{a}{3}$, is indifferent if $\alpha = \frac{a}{3}$ and prefers $y = (a+\alpha)$ if $\alpha > \frac{a}{3}$. The optimal reply β to a given α is set-valued: any $\beta \in [0, a-3\alpha]$ yields a combination of (α, β) which induces $y = a - \alpha$ and payoff $\pi_B = a - \alpha$ for B in the range $\alpha \in [0, \frac{a}{3}]$. The optimal reply β is also set-valued in the range $\alpha \in [\frac{a}{3}, 1]$. In this range each $\beta \in [0, \min\{(-a+3\alpha), 1\}]$ leads to implementation of $y = a + \alpha$.

Equilibrium combinations of α and β (in pure strategies) are the intersections of the optimal reply function $\alpha(\beta)$ of player A with the optimal reply correspondence of player B. These combinations are the pairs $(\alpha, \beta) = \left(\frac{a-\beta}{3}, \beta\right)$ in the range $\beta \in \left[0, \frac{3}{5}\right]$ and $(\alpha, \beta) = \left(\frac{a+\beta}{3}, \beta\right)$ for $\beta \in \left[\frac{3}{5}, 1\right]$.