Ohlendorf, Susanne

Working Paper
Expectation Damages, Divisible Contracts, and Bilateral Investment

Bonn econ discussion papers, No. 2006,25

Provided in Cooperation with:
Bonn Graduate School of Economics (BGSE), University of Bonn

Suggested Citation: Ohlendorf, Susanne (2006) : Expectation Damages, Divisible Contracts, and Bilateral Investment, Bonn econ discussion papers, No. 2006,25

This Version is available at:
http://hdl.handle.net/10419/22970

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Expectation Damages, Divisible Contracts, and Bilateral Investment

by

Susanne Ohlendorf

December 2006
The Bonn Graduate School of Economics is sponsored by the

Deutsche Post World Net

MAIL EXPRESS LOGISTICS FINANCE
Expectation Damages, Divisible Contracts, and Bilateral Investment

Susanne Ohlendorf∗

December 19, 2006

Abstract

We consider a bilateral trade setting with costless renegotiation and investment by both the buyer and the seller. Whether first best levels of relationship-specific investment can be induced by a simple contract depends on available breach remedies. As demonstrated by Edlin and Reichelstein (1996), a contract specifying an up-front transfer, a quantity and a per-unit price is in general not able to achieve the first best when the breach remedy is expectation damages. We show that this result is due to the linearity of the cost function in their counterexample, and does not extend beyond the linear case. If marginal cost is increasing, then at intermediate prices both parties face the risk of breaching, and the first best becomes attainable.

JEL classification: K12, D86, L14

Keywords: expectation damages, breach remedies, renegotiation, hold-up

1 Introduction

There are many explanations for the fact that real-world contracts are surprisingly simple given the complexity of the environment. For example, when economic agents can rely on renegotiation or default legal rules, they may already be able to reach efficient outcomes with a non-contingent contract. In an important article, Edlin and Reichelstein

∗Bonn Graduate School of Econonomics, University of Bonn, Adenauerallee 24-26, 53113 Bonn, Germany; email: susanne.ohlendorf@uni-bonn.de. For valuable comments and discussions I am especially grateful to Urs Schweizer and Patrick Schmitz, furthermore I would like to thank Bastian Goldluecke, Stefan Reichelstein, Andreas Roiter, Alexander Stremitzer, and seminar participants at Bonn University.
(1996, henceforth ER) explore what can be achieved with contracts that specify an up-front transfer, a quantity and a per-unit price, when renegotiation is costless and the breach remedy is either specific performance or expectation damages. They find that a continuous quantity in the contract is a powerful tool to adjust incentives. When only one side invests, a per-unit price, a quantity and an up-front transfer are sufficient to reach the first best. When both sides to the contract invest, it depends on the breach remedy whether the first best can be attained. Specific performance induces a symmetry that allows simple contracts to obtain the first best for a particular class of payoff functions. Using a deterministic and linear cost function, ER show that it is not possible to achieve the first best with expectation damages, at least not for all types of payoff functions.

In the present paper, we show that this inefficiency result does not extend beyond the linear case. In ER’s counterexample, once the seller’s investment is sunk, only one party will breach the contract. This no longer holds if the cost function is strictly convex, so that at intermediate prices both parties face the risk of breaching. The symmetry between the two parties is restored and the first best becomes available with a simple contract. For the linear case, the effect of an intermediate price can be achieved by a lottery over a high and a low price.

We obtain our results for the same framework as ER. A seller and a buyer, both of whom are risk-neutral, invest in preparation of future trade. Investments are specific to the relationship, but not contractible, and a party’s investment does not directly affect the other party’s payoff, only indirectly via the optimal quantity, which is higher the more the parties invest. From an ex ante perspective, optimal trade is subject to uncertainty, which is resolved only after the parties make their investment decisions.

When relevant events are left out of the contract, efficient trade can turn out different from what is stipulated. This leaves room for renegotiation, and may lead to a hold-up problem as identified by Williamson (1975, 1985): parties invest too little because they anticipate losing part of the investment’s return in subsequent renegotiations. The

\[\text{\small 1} \text{This is called } \textit{selfish} \text{ investment in Che and Chung (1999) who study } \textit{cooperative} \text{ investments which have a direct effect on the other party’s payoff. Assuming that ex ante investments are hidden actions, they find that the expectation damage rule does very poorly in the context of cooperative investment. In contrast, Schweizer (2006) shows that the first best can always be reached in a setting where the investment can be contracted upon and both sides can claim damages.}\]
impact of this effect depends on the contractual obligations and the consequences if one party does not adhere to the contract. Under the regime of specific performance, each party would get the right to make the other fulfill the contractual obligation. The usual remedy for breach of contract, however, is the expectation damage rule. This rule states that unilateral breach is possible, but the victim of breach has to be made whole by being reimbursed for loss of profit.

The expectation damage rule has repeatedly been a subject of research in the law and economics literature. In early papers, such as Shavell (1980), different remedies for breach of contract are studied with contracts over a single unit. The remedies are compared with respect to the efficiency of the induced breach decision and reliance expenditures. Typically, in this literature only one party faces uncertainty and will be the one to breach, and only one of the parties has to take an investment decision. The expectation damage rule is found to cause efficient breach, the importance of which is diminished by introducing costless renegotiation (Rogerson 1984, Shavell 1984). Furthermore, the breached against party is fully insured against breach and therefore over-invests. The breaching party’s incentives are undistorted, since it receives all gains from the efficient breach decision.

It is recognized by Edlin (1996) that efficient breach and reliance by one party always result if the contract specifies the best possible outcome for the other party. Moreover, when contracts are divisible, the up-front transfer and per-unit price can be used to endogenize the identity of the breaching party. Therefore, when only one party incurs reliance expenditures, the first best can be reached with a contract that is always breached by the investing party.

In their article, ER also provide an efficiency result for one-sided investment and make use of divisibility of contracts. In contrast to Edlin (1996), the per-unit price is used to make sure that the investing party never breaches. Anticipating that the contracted quantity will sometimes be higher than the optimal quantity, this party invests too much. In other contingencies the optimal quantity is higher than the contracted quantity and can only be reached by renegotiation, creating incentives to underinvest. Since the

---

2 Reliance is the legal term for investment in a contract relationship.
3 A contract is called divisible if it consists of several items and the price to be paid is apportioned to each item. A divisible contract can be broken into its component parts, such that each unit together with the per-unit price are treated as separate contracts that can be fulfilled or breached independently.
contracted quantity can be adjusted continuously, it can be used to balance the two effects. If only one party invests, this balancing turns out to be possible for both damage rules. For the bilateral investment case, however, ER show that a non-contingent contract can implement the first best only under the legal rule of specific performance. For the case of expectation damages, they give an example of cost and valuation functions for which no simple price/quantity contract exists that aligns the incentives of both parties.

In the present paper, the effects of an intermediate price in the contract are explored in greater detail. The main result is that the inefficiency of simple price/quantity contracts found in ER for expectation damages and bilateral investment does not extend beyond the case of a linear cost function. When payoff functions are strictly concave, price matters. The probability of the event “seller breaches” varies with price, and this provides us with an additional instrument to fine-tune both parties’ incentives to invest. A contract that specifies an up-front payment, a quantity and a per-unit price suffices to obtain the first best.\(^4\) Besides, we find that an optimal contract could also take the form of a fixed quantity and a lottery over a very high and a very low price. This resembles price adjustment clauses, which make the price conditional on exogenously determined circumstances. This kind of contract can also solve ER’s counterexample, as does a simple option contract.

The remainder of the paper is organized as follows: Section 2 introduces the model, while in Section 3 the ex post consequences of expectation damages with divisible contracts are discussed. The main result is presented in Section 4, and Section 5 then treats the linear case and option contracts. Concluding remarks can be found in Section 6. Most proofs are relegated to the appendix.

\(^4\) Other means to obtain the first best in this framework include renegotiation design as in Aghion et al. (1994), where parties can assign full bargaining power to one party. An exogenously given bargaining game can have the same effect and helps to achieve the first best in Nöldeke and Schmidt (1995) with option contracts. Furthermore, since investments are selfish, parties could achieve the first best with a contract to trade the optimal quantity at a price which is calculated from ex-post information in an incentive-compatible way (see Rogerson (1992), also footnote 9 in Hart and Moore (1988).)
2 Model Description

Two risk-neutral parties, a seller and a buyer, plan to trade several units of a good at some future date. The quantity of traded goods is denoted by \( q \) and could also be interpreted as duration of the business relationship.\(^5\) For simplicity, it is modeled as a continuous variable, \( q \in \mathbb{R}_{\geq 0} \). A contract consists of an up-front payment \( T \), a quantity \( \bar{q} \) to be traded, and a per-unit price \( \bar{p} \). The up-front payment \( T \) is a transfer that is independent of the traded quantity and legal remedies. It can be adjusted to divide the gains from trade after price and quantity were chosen to maximize joint surplus.

The sequence of events is illustrated in Fig. 1. A contract is signed at date 1 to protect the parties’ relationship-specific investment which both seller and buyer incur at date 2 to increase the value of trade. The cost of their investments is denoted by \( \sigma \in [0, \sigma_{\text{max}}] \) and \( \beta \in [0, \beta_{\text{max}}] \), respectively. Since the assets they invest in have little value outside the relationship, the parties are locked in to the relationship in order to capture the investment’s returns. The investment levels are not contractible, in fact investment decisions may not even be observable.

The exact shape of the cost and valuation functions becomes commonly known at date 3, when all uncertainty is resolved. This is modeled as a move of nature: after the parties have invested, the state of the world \( \theta \in \Theta \) is realized according to some distribution function \( F \). The state space \( \Theta \) is assumed to be a compact subset of \( \mathbb{R}^n \), which includes the most common examples of a finite set and the \([0,1]\)-interval.

The consequences of breach are determined by the standard breach remedy, which is assumed to be expectation damages. This is true for Anglo-American law, especially

\(^5\)For a dynamic analysis of the interpretation as duration of the relationship, see Guriev and Kvasov (2005).
since we assume that courts can readily assess the damages incurred. Alternatively, the contract could explicitly specify this breach remedy. Damages under this rule are the nonnegative amount that is needed to put the breached against party in as good a position as if the contract had been performed. A more detailed description of the consequences of breach follows in Section 3.

At date 4 both seller and buyer decide whether they want to breach to a quantity lower than the one specified in the contract. The payoff as determined by the legal consequences constitutes the disagreement point in subsequent renegotiations. The outcome of the negotiations is assumed to be the (generalized) Nash bargaining solution, where \( \gamma \in [0,1] \) denotes the bargaining power of the seller. That is, the parties share the additional gain from efficient trade so that the seller gets a fraction of \( \gamma \) and the buyer a fraction of \( 1 - \gamma \).

Cost and valuation thus depend on ex ante investment, the move of nature and the traded quantity. The seller’s investment decreases his marginal cost and the buyer’s investment increases her marginal benefit of the good. No discounting takes place. We use the following notation and assumptions, which are basically the same as in ER, with the main difference that the cost function is strictly convex.

\[
C(\sigma, \theta, q) \quad \text{is the cost of producing amount } q \text{ when the seller spent } \sigma \text{ in investment and contingency } \theta \text{ occurred. The cost function is increasing and strictly convex in } q, \text{ and } \quad (\sigma, q) \mapsto C(\sigma, \theta, q) \text{ is twice continuously differentiable for all } \theta \in \Theta, \text{ with } C_{\sigma q} \leq 0.
\]

The functions \( C \) and \( C_{\sigma} \) are assumed to be continuous in \( \theta \).

\[
V(\beta, \theta, q) \quad \text{is the benefit of the buyer when she invested an amount } \beta \text{ and receives amount } q \text{ in contingency } \theta. \text{ This valuation function is increasing and strictly concave in } q \text{ for all } \theta, \text{ and } (\beta, q) \mapsto V(\beta, \theta, q) \text{ is twice continuously differentiable with } V_{\beta q} \geq 0.
\]

Moreover, \( V \) and \( V_\beta \) are continuous in \( \theta \).

**First Best**

Since the contingency realizes before production begins, ex post it is optimal to trade the quantity that maximizes

\[
W(\beta, \sigma, \theta, q) := V(\beta, \theta, q) - C(\sigma, \theta, q).
\]
The two parties can generate the maximal surplus when trading

\[ Q^*(\beta, \sigma, \theta) := \arg\max_{q \in \mathbb{R} \geq 0} W(\beta, \sigma, \theta, q). \]

First best investment levels maximize

\[
\int W(\beta, \sigma, \theta, Q^*(\beta, \sigma, \theta))dF(\theta) - \sigma - \beta.
\]

Maximizing investment levels, denoted by \((\beta^*, \sigma^*)\), are assumed to exist uniquely in the interior of \([0, \sigma_{\text{max}}] \times [0, \beta_{\text{max}}]\).

3 Divisible Contracts

The contract and the legal consequences define a game between seller and buyer, which we can solve by backward induction. In this section we analyze the ex post situation, when cost and valuation functions are realized and observable by both players and the court.

The interpretations of \(q\) as quantity or time of the relationship make it possible that the contract specifies a price per unit or per unit of time, like a rent or a salary. This makes the contract divisible: when one party breaches the contract on some units, the parties’ obligation with respect to the other units still holds. \(^6\)

The expectation damage rule postulates that unilateral breach is possible, but that the victim of breach has to be reimbursed for loss of profit. That is, damages equal the amount that is necessary to make the victim as well off as if the breaching party had performed. Besides, it is understood that damages can never be negative, i.e. even if one party’s breach turns out to be advantageous for the other, it is not possible to sue for a reward.

Breach decisions are made before production starts, which is crucial for the analysis. While the buyer would never reject goods (which have no value for the seller) once they have been produced, she can go into anticipatory breach, announcing her breach decision beforehand. In that case, the seller has a duty to mitigate: he can only recover the profit margin of the canceled goods, but not their cost of production.

\(^6\)This is automatically given with a contract over time, where service was accepted every day prior to breach. If one party ends the relationship prematurely, money is still owed for services rendered.
The duty to mitigate leads to symmetric positions of the buyer and the seller in the subgame that they play at stage 4. In this stylized breach game, the buyer chooses a quantity \( q_B \leq \bar{q} \) and the seller a quantity \( q_S \leq \bar{q} \). They trade the lesser quantity \( q_{\min} = \min\{q_S, q_B\} \), and whoever chose it has to pay damages to the other. If their dispute ever ends up in front of a court, the court will calculate the damages from the announced quantities. The order of announcements does not matter for damages.

Since contracts are divisible, partial fulfillment of the contract by one side implies that the other party’s obligation is adjusted proportionately. Absent renegotiations the quantity \( q_{\min} \) will be traded at a price of \( \bar{p} q_{\min} \). The payoff of the two parties before damages are paid is

\[
S(\sigma, \theta, q_{\min}) := \bar{p} q_{\min} - C(\sigma, \theta, q_{\min})
\]

for the seller and

\[
B(\beta, \theta, q_{\min}) := V(\beta, \theta, q_{\min}) - \bar{p} q_{\min}
\]

for the buyer. Damages are calculated as follows: If \( q_{\min} = q_S \), the seller has to pay

\[
D_S(q_S, q_B) = \max(B(\beta, \theta, q_B) - B(\beta, \theta, q_S), 0)
\]

to the buyer, and if \( q_{\min} = q_B \) the buyer pays

\[
D_B(q_S, q_B) = \max(S(\sigma, \theta, q_S) - S(\sigma, \theta, q_B), 0)
\]

to the seller.\(^7\)

For the analysis of the resulting game, we also need the variables

\[
\hat{Q}_S := \arg\max_{q \leq \bar{q}} S(\sigma, \theta, q)
\]

and

\[
\hat{Q}_B := \arg\max_{q \leq \bar{q}} B(\beta, \theta, q).
\]

\(^7\)We assume that damages are evaluated ex post. A different possibility is to use the hypothetic cost and valuation functions \( V(\beta^*, \theta, q) \) and \( C(\sigma^*, \theta, q) \). Schweizer (2005) calls the expectation damage measure that uses optimal investments the \textit{efficient expectation damage measure}. It can be justified by the postulate that damages have to be reasonably foreseeable (Cooter 1985). Also, Leitzel (1989) finds that this damage formula Pareto-dominates other breach remedies. It gets rid of the overreliance effect and has therefore better efficiency properties. This would-have-been damage is however too hard for the court to estimate in a case where the investment decisions could be private information. Yet another possibility is that the court ignores the higher breach quantity and calculates damages always with respect to the contracted quantity.
Since we assumed that $-C$ and $V$ are strictly concave in $q$, so are $S$ and $B$, and the quantities $\hat{Q}_S$ and $\hat{Q}_B$ are well-defined. See Fig. 2 for an illustration of these values in a diagram which depicts marginal cost and marginal valuation. The quantities $\hat{Q}_S$ and $\hat{Q}_B$ can be interpreted as supply and demand at the contract price, while

$$P^* := C_q(\sigma, \theta, Q^*) = V_q(\beta, \theta, Q^*)$$

would be the equilibrium price. Note, however, that $\hat{Q}_S$ and $\hat{Q}_B$ are constrained by the contracted quantity.

The figure already provides an intuition for the outcome of the breach game. In the case that is illustrated there, no damage is done by breach on the units above $\hat{Q}_S$, and no damages have to be paid. For breach on all other units, the buyer has to pay the difference between $\bar{p}$ and the “supply curve” to the seller, while she gains the difference between $\bar{p}$ and the “demand curve”. These differences are equal at $Q^*$. Hence, as is shown in the following Lemma, the efficient breach property of expectation damages still holds whenever the contracted quantity is higher than the efficient quantity.

**Lemma 1.** Given $\beta, \sigma$ and $\theta$, the subgame that is played by the parties after $\theta$ is realized leads to the following payoffs:

(i) If $Q^* \leq \hat{Q}_S$, the buyer breaches to $Q^*$. The seller’s payoff is $S(\sigma, \theta, \hat{Q}_S)$, the buyer’s is $W(\beta, \sigma, \theta, Q^*) - S(\sigma, \theta, \hat{Q}_S)$.

(ii) If $Q^* \leq \hat{Q}_B$, the seller breaches to $Q^*$. The buyer’s payoff is $B(\beta, \theta, \hat{Q}_B)$, the seller’s is $W(\beta, \sigma, \theta, Q^*) - B(\beta, \theta, \hat{Q}_B)$. 

Figure 2: This figure shows that at $Q^*$, marginal damages equal the seller’s marginal loss.
(iii) If $Q^* > \bar{q}$ the parties renegotiate to $Q^*$ and share the renegotiation surplus

$$\Delta(\beta, \sigma, \theta, \bar{q}) := W(\beta, \sigma, \theta, Q^*) - W(\beta, \sigma, \theta, \bar{q}).$$

Their payoffs are $S(\sigma, \theta, \bar{q}) + \gamma \Delta(\beta, \sigma, \theta, \bar{q})$ and $B(\beta, \theta, \bar{q}) + (1 - \gamma)\Delta(\beta, \sigma, \theta, \bar{q})$, respectively.

**Proof.** In order to solve the breach game we distinguish several cases. We start with the case that $Q^* \leq \bar{q}$ and $P^* \leq \bar{p}$, which is equivalent to $\hat{Q}_S \geq Q^*$. Moreover, in this case it holds that $\hat{Q}_B \leq Q^*$. Whenever the buyer turns out to be the one to breach, the seller gets $S(\sigma, \theta, Q_S)$, while whenever the seller turns out to be the one to breach, he gets this payoff minus possible damages. Therefore, the highest possible payoff that the seller can achieve in this game is $S(\sigma, \theta, \hat{Q}_S)$. Anouncing production of $\hat{Q}_S$ ensures him this payoff: If $q_B \leq \hat{Q}_S$, the buyer has to compensate him such that he is left with this payoff, or, if $q_B > \hat{Q}_S$, the seller has to pay damages $D_S(\hat{Q}_S, q_B)$ to the buyer, but since $B$ is decreasing for $q > \hat{Q}_B$, these are zero. Therefore, choosing $\hat{Q}_S$ is a dominant strategy for the seller.

The buyer chooses either the quantity that solves

$$\max_{q_B \geq \hat{Q}_S} B(\beta, \theta, q_B),$$

which due to concavity of $B$ is $\hat{Q}_S$, or she chooses

$$\arg\max_{q_B < \hat{Q}_S} B(\beta, \theta, q_B) - D_B(\hat{Q}_S, q_B)$$

which is $Q^*$, since the objective function is equal to $W(\beta, \sigma, \theta, q_B) - S(\sigma, \theta, \hat{Q}_S)$ in this range of quantities. A comparison between these two quantities yields

$$W(\beta, \sigma, \theta, Q^*) - S(\sigma, \theta, \hat{Q}_S) \geq B(\beta, \theta, \hat{Q}_S).$$

Hence, the buyer’s best response is to breach to $Q^*$, the seller gets $S(\sigma, \theta, \hat{Q}_S)$, and the buyer ends up with the rest of the social surplus.

If, on the other hand, $\hat{Q}_B \geq Q^*$, then symmetrically breach to $\hat{Q}_B$ is a dominant strategy for the buyer, and the seller’s best response is breach to $Q^*$. Since breach is already efficient, no renegotiation occurs if $Q^* \leq \bar{q}$.

The last case to consider is $Q^* > \bar{Q}_B$ and $Q^* > \hat{Q}_S$, which is equivalent to $Q^* > \bar{q}$. In that case, neither buyer nor seller chooses to breach. Breach can only be downward,
and for $q < Q^*$ damages exceed the gain from breach. The only possibility to trade more is to renegotiate. The amount the parties additionally gain by renegotiating is divided according to bargaining power, where $\gamma$ denotes the bargaining power of the seller. The disagreement point in the negotiations is trade of $\bar{q}$, since for $Q^* > \bar{q}$ it always holds that either $S(\bar{q}) \geq S(0)$ or $B(\bar{q}) = B(0)$, meaning that one of the parties always has an interest to sue for performance.

4 Optimal Contracts

Putting together possible ex post payoffs as described in Lemma 1 we obtain the following expression for the seller’s expected payoff, which depends on his reliance decision $\sigma$ and the buyer’s decision $\beta$:

$$s(\beta, \sigma) = \int_{[Q^* > \bar{q}]} (S(\sigma, \theta, \bar{q}) + \gamma \Delta(\beta, \sigma, \theta, \bar{q})) \, dF + \int_{[\bar{Q}_S \geq Q^*]} S(\sigma, \theta, \bar{Q}_S) \, dF$$

$$+ \int_{[\bar{Q}_B \geq Q^*]} (W(\beta, \sigma, Q^*) - B(\beta, \theta, \bar{Q}_B)) \, dF - \sigma.$$

The buyer’s expected payoff, denoted by $b(\beta, \sigma)$, is analogous to the seller’s expected payoff. The payoff functions are easiest to analyze for extreme contracts, for which at efficient investment at least one of the events ”renegotiation”, ”buyer breaches” and ”seller breaches” never occurs. We define $q_H = \max_\theta Q^*(\beta^*, \sigma^*, \theta)$, $q_L = \min_\theta Q^*(\beta^*, \sigma^*, \theta)$, $p_L = \min_\theta P^*(\beta^*, \sigma^*, \theta)$ and $p_H = \max_\theta P^*(\beta^*, \sigma^*, \theta)$.

Moreover, let

$$\sigma_S(p, q) := \arg\max_{\sigma} s(\beta^*, \sigma)$$

denote the seller’s best response to $\beta^*$ and

$$\beta_B(p, q) := \arg\max_{\beta} b(\beta, \sigma^*)$$

the buyer’s best response to $\sigma^*$ if the contract specifies a quantity $q$ and a price $p$.

**Lemma 2.** For all $p \in [p_L, p_H]$, it holds that

(i) $\sigma_S(p, q_L) \leq \sigma^*$ and $\sigma_S(p, q_H) \geq \sigma^*$

(ii) $\beta_B(p, q_L) \leq \beta^*$ and $\beta_B(p, q_H) \geq \beta^*$
(iii) $\sigma_S(p_L, q_H) = \sigma^*$ and $\beta_B(p_H, q_H) = \beta^*$.

Proof. see the appendix.

The intuition is that, given efficient investment, a contracted quantity as low as $q_L$ means that the contract will always be renegotiated to a higher quantity. In the renegotiations, for each invested dollar a party only receives a fraction of the generated surplus, therefore both underinvest (hold-up effect). A high contracted quantity $q_H$ means renegotiation never occurs, and being sometimes the non-breaching party induces that party to prepare for trade of a high quantity. Both parties overinvest, except for the case of a very high or very low price, in which one party always breaches and invests efficiently, anticipating its efficient breach decision.

Since except for very special cases the set of extreme contracts contains no candidates for an optimal contract, we need a continuity assumption to infer from the behaviour of best responses for extreme contracts to the behaviour for contracts over an intermediate price and quantity. Following the analysis of the one-sided investment case in ER, we assume

Assumption 1. The correspondences $(q, p) \mapsto \sigma_S(p, q)$ and $(q, p) \mapsto \beta_B(p, q)$ have a continuous selection.

Since it is this condition that is failed if the cost function is linear, it would be desirable to have a characterization of the cost and valuation functions for which it holds. Sufficient conditions can be found, as for example the following

Assumption 2. Let $(\sigma, q) \mapsto C(\sigma, \theta, q)$ be strictly convex and $(\beta, q) \mapsto V(\beta, \theta, q)$ be strictly concave for all $\theta \in \Theta$.

This assumption already implies some of the assumptions that were introduced in Section 2, and it also holds that

Lemma 3. Assumption 2 implies Assumption 1.

Proof. see the appendix.

Continuity of best responses indeed ensures existence of an optimal contract, as is illustrated in Fig. 3. This figure provides the intuition for the main result.
Figure 3: This figure shows the space of price/quantity contracts. For low and high quantity contracts it is indicated whether the best response to efficient investment is underinvestment, overinvestment or efficient investment. The contracts on the paths have the property that the best response is equal to first best investment.

**Proposition 1.** Given Assumption 1, there exists a non-contingent contract \((T, \bar{q}, \bar{p})\) such that the first best investment levels \((\beta^*, \sigma^*)\) constitute a Nash equilibrium of the induced game.

**Proof.** For all \(p \in [p_L, p_H]\), define
\[
\bar{q}_S(p) := \max\{q \in [0, q_H] : \sigma_S(q, p) = \sigma^*\}
\]
and
\[
\bar{q}_B(p) := \max\{q \in [0, q_H] : \beta_B(q, p) = \beta^*\}.
\]
From Lemma 2, Assumption 1 and the intermediate value theorem it immediately follows that these functions are well-defined and continuous. Lemma 2 tells us that \(\bar{q}_S(p_L) = q_H \geq \bar{q}_B(p_L)\) and \(\bar{q}_B(p_H) = q_H \geq \bar{q}_S(p_H)\). Applying the intermediate value theorem again yields existence of a \(\bar{p}\) such that \(q_S(\bar{p}) = q_B(\bar{p}) =: \bar{q}\). This contract \((\bar{q}, \bar{p})\) thus leads to \(\beta^*\) as a best response to \(\sigma^*\) and \(\sigma^*\) as a best response to \(\beta^*\).

In order to find the optimal contract given a particular problem we can use the first order conditions of the parties’ maximization problem. For example, the derivatives of the expected payoff functions take a particularly simple form if cost and valuation functions belong to the following class of functions:

**Assumption 3.**
\[
C(\sigma, \theta, q) = C_1(\sigma)q + C_2(\theta, q) + C_3(\sigma, \theta)
\]
\[
V(\beta, \theta, q) = V_1(\beta)q + V_2(\theta, q) + V_3(\beta, \theta),
\]
This functional form is assumed by ER in order to show their possibility result for specific performance and bilateral investment.

**Corollary 1.** If Assumption 3 holds, the optimal contract is characterized by the following equations:

\[
\int_{[Q_S \geq Q^*]} (Q_S - Q^*) dF = (1 - \gamma) \int_{[Q^* > \bar{q}]} (Q^* - \bar{q}) dF
\]

\[
\int_{[Q_B \geq Q^*]} (Q_B - Q^*) dF = \gamma \int_{[Q^* > \bar{q}]} (Q^* - \bar{q}) dF,
\]

where all quantities are evaluated at \( \sigma = \sigma^*, \beta = \beta^* \), \( p = \bar{p} \) and \( q = \bar{q} \).

*Proof.* see the appendix.

To illustrate, consider the following example of only two states of the world, \( \Theta = \{\theta_L, \theta_H\} \) such that

\[
Q^*(\beta^*, \sigma^*, \theta_H) \geq Q^*(\beta^*, \sigma^*, \theta_L).
\]

We stay with the functional form as needed to apply the corollary. The first order conditions translate to (since clearly \( \bar{q} \) and \( \bar{p} \) will assume intermediate values)

\[
\hat{Q}_S(\sigma^*, \theta_L, \bar{p}) - Q^*(\beta^*, \sigma^*, \theta_L) = (1 - \gamma)(Q^*(\beta^*, \sigma^*, \theta_H) - \bar{q})
\]

\[
\bar{q} - Q^*(\beta^*, \sigma^*, \theta_H) = \gamma(Q^*(\beta^*, \sigma^*, \theta_H) - \bar{q}).
\]

Consequently, the optimal contract in this example is

\[
\bar{q} = Q^*(\beta^*, \sigma^*, \theta_H) \text{ and } \bar{p} = P^*(\beta^*, \sigma^*, \theta_L).
\]

**5 The Linear Case**

ER’s inefficiency result

ER show that for \( \gamma \in (0, 1) \) and the following form of functions

\[
V(\beta, \theta, q) = V_1(\beta)q + V_2(\theta, q)
\]

\[
C(\sigma, \theta, q) = C_1(\sigma)q
\]

there exists no contract \((T, \bar{q}, \bar{p})\) that can achieve the first best when the valuation function has positive variance. The case of a linear and deterministic cost function is special because once investment decisions have been made, there can only be breach
by one side. When the probability of breach is zero for one party, there is nothing to countervail the hold-up effect for that party, since the overinvestment effect only occurs for the non-breaching party. In order to balance both incentives, both the events “buyer breaches” and “seller breaches” must occur with positive probability. Intermediate prices which make both breach do not exist in this case: For the low and high price as defined in the previous section it holds that $p_L = p_H = C_1(\sigma^*)$. However, the effect of price controlling who breaches can be also achieved by a stochastic price (with an intermediate expected value).

**Price Adjustment Clauses**

Instead of an intermediate price $\tilde{p}$, another way to achieve breach of both parties is to write a lottery over a very high and a very low price in the contract. Let $p_{\min}$ be a price such that the buyer will never breach the contract, and let $p_{\max}$ be such that the seller will never breach, i.e.

$$p_{\min} \leq P^*(\beta, \sigma, \theta) \quad \text{and} \quad p_{\max} \geq P^*(\beta, \sigma, \theta) \quad \text{for all } \sigma, \beta, \theta.$$ 

The contract can specify a lottery over $p_{\min}$ with probability $\lambda$ and $p_{\max}$ with probability $1 - \lambda$, or equivalently, the parties can choose an event that occurs with probability $\lambda$ independently of cost and valuation functions, and assign the low price $p_{\min}$ to the event and $p_{\max}$ to its complement. This resembles so-called *price escalator clauses* or *price adjustment clauses*, which parties can use to share the risk of breach.\(^8\)

**Proposition 2.** *Given that best responses are continuous in $q$ and $\lambda$, there is always a $q \in [0, q_H]$ and a $\lambda \in [0, 1]$, such that a contract over $q$ and a lottery over $p_{\min}$ with probability $\lambda$ and $p_{\max}$ with probability $1 - \lambda$ induces the first best.*

Since this result does not require continuity of best responses in price, it can also be applied to linear cost or valuation functions\(^9\). The optimal contract illustrates quite well how the performance of expectation damages depends on who will breach the contract.

---

\(^8\)Usually, one would think of price adjustment clauses as insurance against events that are correlated with either cost and valuation. Such a clause can also help to balance incentives, but the point is much simpler to make for the independent case.

\(^9\)A sufficient condition corresponding to Assumption 2 would be that $W(\beta, \sigma, \theta, q)$ is strictly concave in $(\sigma, q)$ and $(\beta, q)$, hence one of the payoff functions can be linear.
Contract \((T, \bar{p})\) is signed

Investment \(\sigma, \beta\) is chosen

\(\theta\) is realized

Buyer's purchase decision \(q_B\)

Seller's breach decision \(q_S\)

Renegotiation and trade

**Figure 4:** Timeline of the model with option contracts.

This is especially true for the case of the payoff functions defined in Assumption 3, for which the contract takes a very intuitive form.

**Proposition 3.** When Assumption 3 holds, a contract over \(\bar{q} = \mathbb{E}[Q^\ast(\beta^\ast, \sigma^\ast)]\) and a lottery over \(p_{min}\) with probability \(\gamma\) (the seller's bargaining power) and \(p_{max}\) with probability \(1 - \gamma\) induces the first best.

**Proof.** see the appendix.

It is worthwhile to compare this result to ER's result for specific performance. They show that with specific performance, the same quantity, with this form of payoff functions, is able to induce first best investment of both buyer and seller. Since price does not matter for incentives, this means that they are able to align two incentives with one instrument. It should therefore be clear that the proof is of a different type than the proof of the one-sided case, which balances incentives using continuity of best responses. In fact, the result is valid under much weaker assumptions, given that the payoff functions have this symmetric form. While this symmetry can be directly exploited when the breach remedy is specific performance, in the case of expectation damages the symmetry must be artificially restored by a lottery over the contract price.

**Option contracts**

Are there other simple contracts that can reach the first best in the linear case? It is not hard to see that the deterministic linear case can be solved with an option contract, but it turns out that in general simple option contracts together with expectation damages perform poorly. We define an option contract to specify an upfront payment and a per-unit price \(\bar{p}\). The sequence of events induced by an option contract is illustrated in Fig. 4. At date 4 the buyer can order any quantity she wants at price \(\bar{p}\). At date 5, the seller can decide whether he wants to breach. The outcome can also be renegotiated.
This game is easy to analyze given what we already know from Section 3. At date 4, the buyer orders the quantity \( \hat{Q}_B \) which ensures her the maximal payoff of \( B(\beta, \theta, \hat{Q}_B) \) plus a possible gain from renegotiation. The seller will deliver \( Q^* \) if \( Q^* \leq \hat{Q}_B \) and \( \hat{Q}_B \) otherwise. The buyer will never breach, which provides an intuition for why expectation damages perform poorly with a buyer-option contract. Besides, there is only one instrument, price, to fine-tune both incentives to invest, which will work only in special cases.

**Proposition 4.** An option contract together with expectation damages can only implement the first best if either

(i) \( \gamma = 1 \), in which case \( \bar{p} \) is chosen such that \( \int V_3(\beta^*, \theta, \hat{Q}_B(\beta^*, \theta))dF = 0 \) at \( p = \bar{p} \), or

(ii) \( Q^*(\sigma^*, \beta^*, \theta) = \hat{Q}_B(\sigma^*, \beta^*, \theta) \) for almost all \( \theta \). With a constant per-unit price \( \bar{p} \) and positive variance of \( Q^* \), this is true if and only if \( C(\sigma, \theta, q) = C_1(\sigma)q \) and \( \bar{p} = C_1(\sigma^*) \).

Proof. see the appendix.

6 Conclusion

We have shown that in the framework of Edlin and Reichelstein (1996), in the case of expectation damages with bilateral investment, the first best can be restored with a divisible contract, consisting of an up-front transfer, a per-unit price and a quantity. Consequently, also in this case simple contracts can arise because they are already optimal.

Nevertheless, there is a general truth behind ER’s inefficiency example: the expectation damage rule treats the breaching party and the party suffering from breach asymmetrically. If there is the possibility of renegotiation, the only contract that overcomes the hold-up problem of the breaching party specifies such a high quantity that the non-breaching party is completely insured and overinvests. This effect is also responsible for the poor performance of expectation damages with option contracts.

In contrast, the symmetry between the parties can be restored when they both face the risk of breaching. When contracts are divisible, this can be achieved with a lottery
over a high and a low per-unit price, and with an intermediate deterministic price if the cost function is strictly concave. In both cases, the probability of breaching for each party varies with price, and price and quantity are sufficient to fine-tune both sides’ incentives to invest. Therefore, the asymmetry induced by expectation damages only matters in environments that do not allow divisible contracts, where how much it matters remains to be studied.

Which breach remedy performs best seems to be highly dependent on the circumstances. As a legal remedy, the expectation damage rule has the advantage that it leads to the efficient ex post decision even if renegotiation breaks down. It is inferior to specific performance with respect to informational requirements, since to assess the damages, courts have to evaluate cost or valuation functions. The contractual obligation is a lot easier to determine than damages, but on the other hand expectation damages make it easier to monitor whether (and how) the breaching party carries out the decision of the court. Since both remedies have their advantages, this decision should probably better be left to the contracting parties.

Appendix

Proof of Lemma 2. The steps of the proof are exercised in detail only for the seller’s payoff function, the result for the buyer can then be derived in a similar way. In a first step, we calculate the derivative of the seller’s expected profit. For this, note that as a direct application of the envelope theorem we get that for all \( \theta \in \Theta \):

\[
\frac{\partial}{\partial \sigma} W(\beta, \sigma, \theta, Q^*(\beta, \sigma, \theta)) = -C_\sigma(\sigma, \theta, Q^*(\beta, \sigma, \theta))
\]

and, from the envelope theorem for constrained maximization,

\[
\frac{\partial}{\partial \sigma} S(\sigma, \theta, \hat{Q}_S(\sigma, \theta)) = -C_\sigma(\sigma, \theta, \hat{Q}_S(\sigma, \theta)).
\]

Next, to calculate the derivative \( \frac{\partial}{\partial \sigma} s(\sigma) \), we hold \( \theta \) constant such that the function under the integral in \( s(\sigma) \) is the piecewise defined function

\[
\sigma \mapsto \begin{cases} 
(1 - \gamma)S(\sigma, \theta, \hat{q}) + \gamma W(\beta, \sigma, \theta, Q^*) - \gamma B(\beta, \theta, \hat{q}) & \text{if } Q^* > \hat{q} \\
S(\sigma, \theta, \hat{Q}_S) & \text{if } Q^* \leq \hat{Q}_S \\
W(\beta, \sigma, \theta, Q^*) - B(\beta, \theta, \hat{Q}_B) & \text{if } Q^* \leq \hat{Q}_B
\end{cases}
\]
and show that it is differentiable with derivative

\[
\begin{align*}
\sigma \mapsto & \begin{cases} 
-\gamma C_\sigma(\sigma, \theta, Q^*) - (1 - \gamma) C_\sigma(\sigma, \theta, \bar{q}) & \text{if } Q^* > \bar{q} \\
-C_\sigma(\sigma, \theta, \hat{Q}_S) & \text{if } Q^* \leq \hat{Q}_S \\
-C_\sigma(\sigma, \theta, Q^*) & \text{if } Q^* \leq \hat{Q}_B
\end{cases}
\end{align*}
\]

Due to continuity of \(\sigma \mapsto Q^*(\beta, \sigma, \theta)\) and \(\sigma \mapsto \hat{Q}_S(\sigma, \theta)\) this is true for all \(\sigma\) but maybe those with \(Q^*(\beta, \sigma, \theta) = \bar{q}\) or \(Q^*(\beta, \sigma, \theta) = \hat{Q}_S(\sigma, \theta, \bar{p})\). At these points the function could have a kink. Instead, however, the pieces of the function are joined smoothly. This is deduced from continuity of the piecewise defined derivative, which is straightforward to check. Due to integrability of \(C_\sigma\) we can interchange integration and differentiation, hence the derivative of \(s(\sigma, \beta)\) is

\[
\frac{\partial}{\partial \sigma} s(\sigma, \beta) = -\int_{[Q^* > \bar{q}]} (\gamma C_\sigma(\sigma, \theta, Q^*) + (1 - \gamma) C_\sigma(\sigma, \theta, \bar{q})) dF - 1 \\
- \int_{[\bar{Q}_S \geq Q^*]} C_\sigma(\sigma, \theta, \hat{Q}_S) dF - \int_{[\hat{Q}_B \geq Q^*]} C_\sigma(\sigma, \theta, \hat{Q}_B) dF
\]

Because we already know that expected joint surplus is uniquely maximized at \(\sigma^*\), we will study the function

\[
\tilde{s}(\sigma) := s(\beta^*, \sigma) - \left( \int W(\beta^*, \sigma, \theta, Q^*(\beta^*, \sigma, \theta)) dF - \sigma \right),
\]

which has derivative

\[
\tilde{s}'(\sigma) = -(1 - \gamma) \int_{[Q^* > \bar{q}]} \Delta_\sigma(\beta^*, \sigma, \theta, \bar{q}) dF - \int_{[\bar{Q}_S \geq Q^*]} \Delta_\sigma(\beta^*, \sigma, \theta, \hat{Q}_S) dF.
\]

By exploiting \(C_{\sigma q} \leq 0\), it is straightforward to see that

\[
\Delta_\sigma(\beta^*, \sigma, \theta, q) = -C_\sigma(\sigma, \theta, Q^*(\beta^*, \sigma, \theta)) + C_\sigma(\sigma, \theta, q)
\]

is weakly decreasing in \(q\), and that the first term in \(\tilde{s}'(\sigma)\) is negative and the second is positive (if they do not vanish).

Now, in order to prove the lemma, consider first \(q = q_L\). In this case,

\[
\tilde{s}'(\sigma) = -(1 - \gamma) \int \Delta_\sigma(\beta^*, \sigma, \theta, q_L) dF \leq 0,
\]
i.e. \( s \) is a monotonically decreasing function. Therefore, all \( \sigma > \sigma^* \) lead to a lower payoff than \( \sigma^* \), hence \( \sigma_S(q_L, p) \leq \sigma^* \). For a contract over \( q_H \), since \( Q^* \) is nondecreasing in \( \sigma \), the first term in \( s' \) vanishes for \( \sigma < \sigma^* \), i.e. \( s \) is an increasing function. Therefore, at \( q_H \) all \( \sigma < \sigma^* \) are dominated by \( \sigma^* \), and \( \sigma_S(q_H, p) \geq \sigma^* \).

Finally, consider \( q_H \) and a low price. By definition of \( p_L \), at \( p = p_L \) it holds that \( \hat{Q}_S(\sigma^*, \theta) \leq Q^*(\sigma^*, \beta^*, \theta) \) for all \( \theta \in \Theta \). It is straightforward to verify that \( \hat{Q}_S(\sigma, \theta) \leq Q^*(\sigma, \beta^*, \theta) \) still holds for \( \sigma \geq \sigma^* \). Therefore, the function \( \hat{s} \) is weakly decreasing for \( \sigma \geq \sigma^* \), hence \( \sigma_S(q_H,p_L) = \sigma^* \). For the buyer, the corresponding claims follow from the assumption that \( V_{\beta q} \geq 0 \).

Proof of Lemma 3. Again, we look only at the seller’s best response. Since \( s \) is continuous in \( q \), \( p \) and \( \sigma \) (which is straightforward to check), according to Berge’s theorem, the argmax correspondence \( \sigma_S(q,p) \) is upper hemicontinuous. Since upper hemicontinuity coincides with continuity if the correspondences are functions, for Assumption 1 to hold it suffices that the function \( \sigma \mapsto s(\sigma, \beta^*) \) has a unique maximizer for all \( q \) and \( p \). In the following, we therefore show that \( s \) is strictly concave, given that Assumption 2 holds.

This is done by showing that the derivative (see equation 1 in the proof of Lemma 2) is strictly decreasing. The integrand in this derivative is continuous, and we will show that it is piecewise decreasing in \( \sigma \), hence decreasing everywhere (which implies strict concavity of \( s \)).

Derivatives of the pieces are calculated in the following. First, the derivative of \( \sigma \mapsto C_\sigma(\sigma, \theta, Q^*) \), which is the second derivative of \( \sigma \mapsto W(\sigma, \theta, Q^*) \), equals

\[
\sigma \mapsto W_{\sigma \sigma}(Q^*) - \frac{W_{\sigma q}(Q^*)W_{q \sigma}(Q^*)}{W_{qq}(Q^*)}.
\]

Second, the derivative of \( \sigma \mapsto C_\sigma(\sigma, \theta, \hat{Q}_S) \) is

\[
\sigma \mapsto -C_{\sigma \sigma}(\sigma, \theta, \hat{Q}_S) + \frac{C_{\sigma q}^2(\sigma, \theta, Q^*)}{C_{qq}(\sigma, \theta, Q^*)}.
\]

Since \( -C \) and \( W \) are strictly concave in \((\sigma, q)\), the determinants of the respective Hessian matrices are negative. Therefore, the two derivatives above as well as \( -C_{\sigma \sigma}(\sigma, \theta, \hat{q}) \) are negative.

Proof of Corollary 1. The derivative of \( s(\beta, \sigma) \) with respect to \( \sigma \), as calculated in the
proof of Lemma 2 (Equation 1), evaluated at \( \sigma^* \), is denoted by \( s_\sigma(q, \bar{p}) \) and amounts to

\[
s_\sigma(q, \bar{p}) = (1 - \gamma) \int_{\{Q^* > \bar{q}\}} \Delta_\sigma(q) dF - \int_{\{\bar{Q}_S(\bar{p}) \geq Q^*\}} \Delta_\sigma(\bar{Q}_S(\bar{p})) dF.
\]

It follows that \( \bar{q}_S(\bar{p}) \in \{ s_\sigma(q, \bar{p}) = 0 \} \). This set is an interval, since \( s_\sigma(q, \bar{p}) \) is monotonically increasing in \( \bar{p} \) and \( \bar{q} \). If Assumption 2 holds, it is a singleton. The corollary follows since for the kind of functions as defined in Assumption 3 it holds that

\[
\Delta_\sigma(q) = -C_1'(\sigma^*)(Q^* - q) \quad \text{and} \quad \Delta_\beta(q) = V_1'(\beta^*)(Q^* - q).
\]

\[ \square \]

**Proof of Proposition 2.** When the price is \( p_{min} \), the buyer makes a profit on each unit. Therefore, the seller breaches to \( Q^* \). When price is \( p_{max} \), the situation is reversed. Expected payoff is analogous to, but simpler than the case with an intermediate price and looks as follows (again only for the seller):

\[
s(\sigma, \beta) = \int W(\beta, \sigma, \theta, Q^*) dF - \sigma - \int B(\beta, \theta, \bar{q}) dF - (1 - \gamma) \int_{\{Q^* > \bar{q}\}} \Delta(\beta, \sigma, \theta, \bar{q}) dF - (1 - \lambda) \int_{\{Q^* < \bar{q}\}} \Delta(\beta, \sigma, \theta, \bar{q}) dF
\]

with \( p = \lambda p_{min} + (1 - \lambda) p_{max} \). The result can be proved following the same steps as the proof of Proposition 1, the role of the price now played by \( \lambda \).

\[ \square \]

**Proof of Proposition 3.** We prove this result independently of previous results in this paper, in order to highlight the connection to the result for specific performance in ER. For specific performance, the seller’s expected payoff function looks as follows:

\[
s(\beta^*, \sigma) = \bar{p}\bar{q} + (1 - \gamma) \left( \int -C(\sigma, \theta, \bar{q}) dF - \sigma \right) + \gamma \left( \int W(\beta, \sigma, \theta, Q^*) dF - \sigma \right) - \gamma \int V(\beta, \theta, \bar{q}) dF.
\]

It is straightforward to verify that for \( \lambda = \gamma \), expected payoff functions with expectation damages as stated in equation 2 reduce to the same function, with \( \bar{p} = \gamma p_{min} + (1 - \gamma) p_{max} \). Next, consider the defining equation of \( \sigma^* \), which is that for all other \( \sigma \)

\[
\int W(\sigma^*, \beta^*, \theta, Q^*(\sigma, \beta^*, \theta)) dF - \sigma^* \geq \int W(\sigma, \beta^*, \theta, Q^*(\sigma, \beta^*, \theta)) dF - \sigma.
\]

Furthermore, from the definition of \( Q^* \) we know that

\[
W(\sigma, \beta^*, \theta, Q^*(\sigma, \beta^*, \theta)) \geq W(\sigma, \beta^*, \theta, Q^*(\sigma^*, \beta^*, \theta)) \quad \text{for all} \ \sigma, \theta.
\]

21
From these two equations, it follows that

$$\sigma^* \in \arg\max_{\sigma} \int -C(\sigma, \beta^*, \theta, Q^*(\sigma^*, \beta^*, \theta))dF - \sigma$$

Since we assumed the special payoff functions defined in Assumption 3 it follows that

$$\sigma^* \in \arg\max_{\sigma} \int -C(\sigma, \beta^*, \theta, \bar{q})dF - \sigma.$$  

Hence, when $\beta = \beta^*$, all terms in the seller’s payoff function are maximized at $\sigma^*$, and it is straightforward to show that the same holds symmetrically for the buyer. It is clear from the proof that this result holds very generally, for arbitrary investment decisions.

Proof of Proposition 4. The derivative of the seller’s payoff function, evaluated at $\sigma^*$, is

$$-(1 - \gamma) \int_{[Q^* \geq \hat{Q}_B]} \Delta_\sigma(\sigma^*, \theta, \hat{Q}_B)dF.$$  

Therefore, a necessary condition for first best investment levels is $\gamma = 1$ or $Q^*(\sigma^*, \beta^*) \leq \hat{Q}_B(\beta^*)$ almost surely. In case of $\gamma = 1$, choose $\bar{p}$ such that

$$\int V_{\beta}(\beta^*, \theta, \hat{Q}_B)dF = 1$$

at $p = \bar{p}$. Then choice of $\beta^*$ is a dominant strategy for the buyer, and $\sigma^*$ is the seller’s best response. If $Q^*(\sigma^*, \beta^*) \leq \hat{Q}_B(\beta^*)$ a.s., the buyer will overinvest except if $Q^*(\sigma^*, \beta^*) = \hat{Q}_B(\beta^*)$ a.s., which would lead to investments $\sigma^*$ and $\beta^*$ and efficient trade without renegotiation. However, for this to hold the price function must equal the cost function, which therefore has to be deterministic and linear.

References


