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Sub-National Optimal Budget Allocation and Borrowing under Soft Budget Constraint

by

Ernesto Crivelli

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Sub-National Optimal Budget Allocation and Borrowing under Soft Budget Constraint

Ernesto Crivelli†

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Abstract: Local governments have borrowed largely from the banking system to finance their deficits instead of responding to the rigors of bond markets. This paper analyzes how sub-national governments optimally reallocate the provision of public goods and decide on borrowing, in a model where the banking system faces a soft budget constraint. In contrast with recent literature, sub-national governments allocate a higher (lower) than optimal amount of resources to consumption public goods (infrastructure investment) and overborrow if they expect the banking system to be bailed out. Controls on sub-national borrowing like the golden rule seem to be inefficient to avoid excessive indebtedness at state level. (JEL: H7, R5)

Key words: fiscal federalism; soft budget constraints; local public investment; sub-national borrowing

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1 Introduction

The problem of bailouts to lower level governments in financial distress is of great concern. By promising financial aid in times of difficulties, bailouts weaken the budget constraint of local governments who anticipate the reaction of the central government and engage in opportunistic behavior: higher deficits and increasing indebtedness is expected. Even if the central authority could commit not to directly bail out local governments, they could still indirectly benefit from the willingness of the central government to rescue financial institutions. The main contribution of this paper to the literature of soft budget constraints is to show that local governments allocate a higher (lower) than optimal amount of resources to consumption public goods (infrastructure investment) and overborrow if they expect the banking system to be bailout by the central authority.

State governments have indeed borrowed largely from the banking system to finance their deficits instead of responding to the rigors of bond markets. The existence of public owned banks or indirect control of commercial banks could certainly affect the degree of commitment if local governments expect the central authority (or the central bank) to foot the bill to avoid an overall financial crisis. Recent episodes of rescue operations in the sphere of lower-level governments confirm this hypothesis. In Argentina\(^1\), the central government has often used extraordinary resources to face fiscal and financial crises at provincial level since the return of democracy in 1983. State-owned banks were the most important source of credit to sub-national governments until 1995, with over 20 provincial banks. In 1990, they provided more than 60 percent of the credit needs of provincial gov-

\(^1\)See for example Nicollini et al (2002)
ernments at low or zero interest rates and the central bank lent massive amounts through rediscounts to prevent the collapse of several provincial banks, due to poor loan recovery and overstaffing\(^2\) (Webb, 2003). During the period 1983-1995, current expenditures grew from 55 percent to 85 percent of total own resources at provincial level. Brazil is in nature a very similar case. There were also several financial arrangements with banks owned by states themselves that resulted in the states being their biggest clients. Until 1996, it was with no doubt the traditional source of credit to subnational governments with 25 out of 27 states having their own financial institutions. In 1993, the central government assumed the obligations of state and municipal governments with a federal financial institution (Caixa Econômica Federal -CEF). Again in 1997 the federal government was forced to step in assuming local government obligations\(^3\). The most important case is that of the Bank of the State of Sao Paulo -BANESPA. Although repeated bailouts, more than 92 percent of local debt with this bank was by 1999 owned either directly by the federal government or by its financial institutions (Dillinger and Webb, 1998).

Developed countries with recent episodes of sub-national bailouts include Italy and Germany\(^4\). In Italy, regions cannot borrow to finance current expenditure and provinces and municipalities are allowed to borrow from banks, the Deposit and Loan Fund and other public banking institutions to finance investment projects

\(^2\)In 1993, the federal government offered to finance the privatization of provincial banks but prior to 1995 only four small provinces accepted the offer. The success of this and other fiscal reforms accelerated after the Mexico crisis in 1995.

\(^3\)In 1996 however, the federal government offered the state governments long term loans to privatize, liquidate or transform their banks.

only\textsuperscript{5}. However, the Local Health Unit and transportation companies\textsuperscript{6} have been authorized to borrow from commercial banks to pay suppliers’ bills. As in previous years, the regions were allowed in 1993 to borrow from the Deposit and Loan Fund to repay outstanding arrears. This time however, resulted in higher current spending but outstanding arrears were not reduced (Emiliani et al, 1997). In Germany, the central government assumed the debts of the two smallest state governments, Bremen and Saarland. Despite constantly increasing dependence on transfers, both continued to increase current spending, run large deficits, and rely heavily on debt to fund current expenditures throughout the 1980s and 1990s. Also in this case, sub-national governments rely primarily on direct bank loans to finance their deficits and the states indirectly control a network of commercial banks\textsuperscript{7} (Rodden, 2003).

This paper considers a two-tier hierarchy model with the central government at the top and \(N\) identical state governments at the bottom. State governments decide on the allocation of own resources into public consumption goods or infrastructure investment and borrow either from the banking system or directly from the bond market. However, sub-national borrowing must not exceed infrastructure investment outlays (golden rule). As in Qian and Roland (1998), local governments compete with each other for foreign capital. By increasing infrastructure investment, they induce foreign capital to be allocated in their region.

\textsuperscript{5}In the 1970s there were only few controls on local government borrowing. In 1975, about 50 percent of sub-national deficit was financed by banks and special credit institutions and around 42 percent by the Deposit and Loan Fund (Emiliani et al, 1997).

\textsuperscript{6}In 1992, ordinary regions in Italy spent over 80 percent of their total resources on health services and public transport. However, only 3% of their revenues came from own taxes, and 96% were central government transfers and grants (Bordignon, 2000)

\textsuperscript{7}However, all governments operate under the constraint that annual borrowing must not exceed the outlays for investment purposes (Seitz, 1999).
To model total infrastructure investment and borrowing decisions, a variable scale set-up application of Hölmstrom and Tirole (1997) is introduced. Borrowing capacity for a given state government is determined by its own infrastructure investment budget (which plays the role of collateral). A moral hazard problem arises at this level: the state government faces incentives to reduce the probability of success of the investment project and thereby to enjoy a private benefit. To address this moral hazard problem, three parties are included in the financial contract: state government’s own infrastructure budget, and two sources of external funds (borrowing possibilities): the banking system, who monitors the state government and investors (bond market). Together the three agents determine the optimal size of infrastructure investment spending.

In the model, the central government bailouts only the banking system. By doing this, the central government distorts intermediaries’ optimal decisions on monitoring. It is assumed that banks ask for a lower equilibrium rate of return in order to monitor investment projects if they expect to be bailed out in case the project fails. Bailouts generate a signalling effect on the bond markets and state’s borrowing possibilities increase. Although state governments do not receive a direct bailout from the center, the optimal allocation of resources is indirectly influenced by the expectation of bailouts. Local governments allocate a higher than optimal amount of resources in consumption public goods and overborrow in order to finance infrastructure investment.

Related literature: This paper builds on a number of literatures on bailouts and soft budget constraints. Technically, it is most related to Qian and Roland (1998). They emphasize the importance of competition among local governments for foreign
capital to explain the optimal allocation of local public goods to interpret China’s recent experience of transition to a market economy. In their model however, the central government bails out directly state governments by increasing their availability of consumption public goods and there is no debt. In contrast to my results, competition for grants makes local governments to allocate less (more) than optimal resources in consumption public goods (infrastructure investment). My analysis also complements Goodspeed (2002). State governments find it optimal to overborrow if they expect to be directly bailed out by the center. However, the model do not differentiate between banking or market borrowing and there are no controls or restrictions on sub-national borrowing. Other recent literature on soft budget constraints include Wildasin (1997), Inman (2001) and Sanguinetti and Tommasi (2002). See also Kornai (1986), who introduces the discussion on soft budget constraints in the study of state-owned enterprises, and Maskin (1999) for a survey.

The reminder of the paper is organized as follows. Section II introduces the basic set-up of the model. Section III presents the solution both under hard and soft budget constraint. Section IV summarizes and concludes.

2 The Basic Set-up

The interaction of the agents in the model is as follows: Local governments compete between each other in a simultaneous game for the allocation of foreign capital
and decide how state’s budget is assigned to infrastructure investment $A_i$ and to consumption public goods $z_i$. The Central Government, on the other hand decides on the optimal allocation of bailouts to the banking system, given the optimal decisions made by the state governments. The game between state governments and the central authority is thus of a sequential nature, and perfect information among the different tiers of government is assumed.

Following Qian and Roland (1998), there is a number of private firms in the economy which activities in region $i$ are described by a production function $f(K_i, I_i)$, where $K_i$ represents foreign capital and $I_i$ represents total public infrastructure investment. $f(.)$ is increasing and strictly concave in both arguments $f'_K(K_i, I_i) > 0$, $f'_I(K_i, I_i) > 0$, $f''_{KK}(K_i, I_i) < 0$, $f''_{II}(K_i, I_i) < 0$. Further, public infrastructure investment raises the marginal productivity of foreign capital $f''_{KI}(K_i, I_i) > 0$. For analytical simplicity, it is assumed that the total amount of foreign capital in the country is fixed ($\sum K_i = K$) and foreign capital is perfectly mobile across states.

### 2.1 The Government’s objective function

The model assumes welfare maximizing governments. State government in region $i$ maximizes $W_i$ and the central government maximizes $W = \sum W_i$. The objective function of state government in region $i$ becomes

$$ W_i = x(K_i, I_i) + u(z_i) $$

(1)

where $x(K_i, I_i) = f(K_i, I_i) - K_i f_K(K_i, I_i)$ is an increasing and concave function in its arguments. The government is not concerned with returns to foreign capital
Additionaly, the total utility derived from local consumption public goods is defined by $u(z_i)$, which is assumed to be concave in $z_i$.

### 2.2 State Infrastructure Investment and Borrowing

This section introduces an application of Hölmstrom and Tirole’s (1997) model of financial intermediation with variable investment scale adapted to explain infrastructure investment ($I_i$) and borrowing decisions of state governments. The basic set-up is modified to allow the banking system being bailed out by the central government.$^9$

Three types of agents interact in the model: the real sector, intermediaries (banking system) and investors (bond market). All parties are assumed risk neutral. The real sector is represented by the state governments who allocate their own resources (state’s budget constraint) to infrastructure investment $A_i$ and consumption public goods $z_i$. However, infrastructure investment projects in each state $i$ can be undertaken at any scale $I_i$. It is assumed that $A_i < I_i$, thus the government needs at least $I_i - A_i$ in external funds to be able to invest$^{10}$. In the model, it is state’s infrastructure investment budget $A_i$ the asset that can be pledged as collateral in the financial contract, and therefore determines local debt capacity.

All benefits and costs of the investment project are proportional to $I_i$. The project generates a verifiable, financial return equaling either 0 (failure) or $R_i I_i$ (success). Thus, the investment technology has constant returns of scale.

There is a moral hazard problem: a good investment project has a probability

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$^9$In contrast with other literature, state governments are not bailed out directly by the central authority. See for example: Qian and Roland (1998), Goodspeed (2002).

$^{10}$We assume that local governments operate under "golden rule": borrowing must not exceed total infrastructure investment, that is $I_i = A_i + D_i$. This control on sub-national borrowing $D_i$, is of extended use among federations. See Ter-Minassian (1997).
of success of $P_H$ and no private benefit. Moreover, there are two possible bad projects (with probability of success $P_L < P_H$), one with private benefit $B.I_i$ and one with private benefit $b.I_i$, where $B > b > 0$. Therefore, in the absence of proper incentives or outside monitoring, the infrastructure investment minister will choose the high private benefit project ($B$-project) irrespectively of the financial contract, thus reducing the probability of success of the project\textsuperscript{11}.

External funds are provided by two type of agents: banks and investors. The banking system consist of many intermediaries. The function of intermediaries is to monitor state governments and thereby alleviate the moral hazard problem\textsuperscript{12}. In practice, it is assumed that the monitor invest own capital in the project it monitors in amount $I_m$ and can prevent the state government from undertaking the $B$-Project. This reduces the government’s opportunity cost of being diligent from $B.I_i$ to $b.I_i$. However, monitoring is privately costly: the intermediary has to pay a non verifiable amount $c.I_i > 0$ in order to eliminate the $B$-project. Monitoring is a partial substitute for collateral. State governments have access to the bond markets only after the monitor has taken a large enough financial interest in the project such that, the investors can be assured that the government will behave diligently. Individual (uninformed) investors are assumed small and provide external funds in amount $I_u$. They demand an expected rate of return $\delta$.

There are three parties to the financial contract: the state government, the intermediary, and the uninformed investors, where $A + I_m + I_u = I_i$. An optimal three-party contract takes the form: in case the project fails, no one is

\textsuperscript{11}Furthermore, in the relevant rage of return on investor capital, denoted by $\delta$, only the good project is economically viable; that is, $P_H.RI_i - \delta.I_i > 0 > P_L.RI_i - \delta.I_i + B.I_i$

\textsuperscript{12}In practice, monitoring of local governments takes many forms: supervision of public accounts, growth and budget deficit goals, viability of infrastructure investment projects.
paid anything; in case the project succeeds, the payoff \( R.I_i \) is divided up so that
\( R_g + R_u + R_m = R.I_i \), where \( R_g, R_u \) and \( R_m \) denote the local government, the
investors and the intermediary’s share of the benefits respectively.

It is assumed further, that intermediaries (the banking system) can be bailed
out by the central government. The bailout amounts a fraction \( 0 \leq \phi_i \leq 1 \) of
the intermediary’s share of the benefits \( R_m \) and is offered to intermediaries who
monitors and co-finance infrastructure investment projects in state \( i \) only in case
the project fails, which happens with probability \( (1 - P_H) \). Intermediaries will
ask for a lower equilibrium rate of return \( \beta \) in order to monitor if they expect to
be bailed out in case the project fails.

A state government with infrastructure investment budget \( A_i \) will choose its
overall level of investment \( I_i \), its own capital contribution \( A \), and the variables \( R_f \)
, \( R_m \), \( R_u \), \( I_m \), \( I_u \), to solve the program:

\[
max \ U(A_i) = P_H R I_i - P_H R_m - P_H R_u + \delta (A_i - A) \tag{2}
\]

subject to:

\( (i) \quad A \leq A_i \)
\( (ii) \quad A + I_m + I_u \geq I_i \)
\( (iii) \quad R_g \geq bI_i / \Delta p \)
\( (iv) \quad R_m, \Delta p \geq cI_i - (1 - P_H) \phi_i R_m \)
\( (v) \quad R_m [P_H + (1 - P_H) \phi_i] \geq \beta I_m \)
\( (vi) \quad P_H R_u \geq \delta I_u \)
\( (vii) \quad R_g + R_m + R_u \leq R I_i \)

Condition \( (iii) \) is the state government’s incentive constraint and reflects the
fact that the infrastructure investment minister in state $i$ is left to choose between the good project and the low private benefit project (the $b$-project), where $\Delta p = P_H - P_L$. Condition (iv) is the incentive constraint imposed by the intermediary. He will monitor the project if the share of the benefits compensate the costs of monitoring. If the project fails, he receives $\phi_i R_m$, where $0 \leq \phi_i \leq 1$. If $\phi_i > 0$, the monitor asks for a lower share of the benefits $R_m$ in order to monitor the investment project. Condition (v) simply defines the rate of return on intermediary capital $\beta$ given the possibility of bailouts. Condition (vi) is the incentive constraint imposed by the uninformed investors, where $\delta$ is the expected rate of return (opportunity cost of the funds invested). They will supply the balance $I_u = I_i - A - I_m$, whenever this amount is positive, as it is defined in Condition (ii). Condition (vii) state how payoff is divided between the parties in the financial contract.

Dividing all equations in the program by the state’s infrastructure investment budget $A_i$, yields a program in which all choice variables are scaled by $A_i$ and all the parameters are independent of $A_i$. This feature greatly simplifies the analysis since $A_i$ could differ from state to state or from one period to the other. In equilibrium, all constraints bind: the state government invests all its infrastructure investment budget and it is paid just enough to be diligent; the intermediary is paid just enough to have an incentive to monitor and it is required to invest up to the point where its return on capital is $\beta$; and the uninformed investors contribute up to the point where the expected return on the capital invested equals the market return $\delta$. This way, the state government maximizes the leverage and return on its own funds. To find the maximum level of investment, substitute equalities (i) and (iii)-(vii) into (ii) to get:
\[ A_i + \frac{I_i \cdot c \cdot P_H}{\beta[\Delta p + (1 - P_H)\phi_i]} + \frac{I_i \cdot P_H}{\delta} \left\{ R - \left[ \frac{b}{\Delta p} + \frac{c}{\Delta p + (1 - P_H)\phi_i} \right] \right\} \geq I_i \]

The highest sustainable level of investment is

\[ I(A_i, \delta, \beta, \phi_i) = \frac{A_i}{L(\delta, \beta, \phi_i)} \]

where

\[ L_i(\delta, \beta, \phi_i) = 1 - \frac{c \cdot P_H}{\beta[\Delta p + (1 - P_H)\phi_i]} - \frac{P_H}{\delta} \left\{ R - \left[ \frac{b}{\Delta p} + \frac{c}{\Delta p + (1 - P_H)\phi_i} \right] \right\} \]

\( L(\delta, \beta, \phi_i) \) represents the amount of own state government’s infrastructure investment budget needed to undertake an investment of unit size \( (I_i = 1) \). Clearly, \( L(\delta, \beta, \phi_i) < 1 \), the state government can lever its own infrastructure investment budget; this leverage effect is higher, the lower is \( L(\delta, \beta, \phi_i) \). In equilibrium, rates of return must also be such that \( L(\delta, \beta, \phi_i) > 0 \), else the state government would want to invest without limit\(^{13}\).

From now on, it is assumed that the interest rate of the economy \( \delta \) (or market rate of return) is represented by a function \( h \) that depends positively on the aggregate stock of local debt \( D_T \),

\[ \delta = h[D_T] = h\left[ \sum \frac{1 - L_i(\delta, \beta, \phi_i)}{L_i(\delta, \beta, \phi_i)} A_i \right] \]

\(^{13}\)See that \( \beta \) must be high enough to make the intermediary prefer monitoring to investing its capital in the open market, where it would earn a rate of return \( \delta \). The minimum acceptable rate of return \( \beta \) is now determined by the condition, \( R_m[PH + (1 - P_H)\phi_i] - c = \delta \).

\(^{14}\)The stock of debt in state \( i \) amounts \( D_i = I_i - A_i = \frac{A_i}{L_i(\delta, \beta, \phi_i)} - A_i = \frac{1 - L_i(\delta, \beta, \phi_i)}{L_i(\delta, \beta, \phi_i)} A_i \)
where \( h' = \frac{\partial \delta}{\partial D_T} > 0 \) and \( h'' = \frac{\partial^2 \delta}{\partial^2 D_T} > 0 \)

Equations (4) and (5) above determine a system of equations from which it follows,

**Lemma 1** Assume the Central Authority cannot commit to a hard budget constraint \( (\phi_i > 0) \), leverage increases in state government \( i \), and it falls for all \( j \neq i \) with the amount of bailouts offered to the banking system in state \( i \), that is \( \frac{\partial L_i(\delta, \beta, \phi_i)}{\partial \phi_i} < 0 \) and \( \frac{\partial L_j(\delta, \beta, \phi_i)}{\partial \phi_i} > 0 \). Further, for a given level of \( A_i, A_j \), the stock of debt in the economy \( D_T \) increases and thus \( \frac{\partial \delta}{\partial \phi_i} > 0 \).

**Proof.** Appendix A

The intuition goes as follows: If banks financing investment projects in state \( i \) expect to be bailed out \( (\phi_i > 0) \), they ask for a lower share of the benefits in order to monitor the investment project as the expected amount of bailouts increases, and thereby leverage increases \( \frac{\partial L_i}{\partial \phi_i} < 0 \). State government \( i \) is able to increase infrastructure investment \( I_i \) by increasing borrowing. On the other hand, the interest rate \( \delta \) increases given the positive effect of the bailout on the total stock of debt \( D_T \). State governments that have not benefited from a bailout to the banking system \( j \neq i \) suffer from capital squeeze since leverage falls \( \frac{\partial L_j}{\partial \phi_i} > 0 \) given the higher interest rate \( \delta \).

### 2.3 Budget Constraint

In the basic model, there is no distortionary taxation neither at state nor at central government level. State governments decide how to assign state revenue to public
consumption goods \((z_i)\) and infrastructure investment \((A_i)\),

\[
z_i + A_i = T_i = \frac{T}{N} \tag{6}
\]

where \(T_i\) is the amount of block transfers that state \(i\) receives from the central government. Equivalently \(T_i = \frac{T}{N}\) where \(T\) is the total amount of central government funds to be assigned among \(N\) equally populated state governments.

The central government budget constraint,

\[
\sum T_i + \sum (1 - P_H) \phi_i R_m = T + B \tag{7}
\]

where \(\sum T_i\) is the amount of block transfers distributed among the \(N\) state governments and \(\sum (1 - P_H) \phi_i R_m\) is the amount of bailouts to the banking system, where \(R_m\) is the intermediary’s share of the benefits in the investment project he co-finances in state \(i\) and \(0 \leq \phi_i \leq 1\) is the fraction of the benefits the central authority recognizes the intermediary in case the project fails (with probability \(1 - P_H\)). Block transfers and bailouts are financed with non distortionary lump-sum taxes \(T + B\) and it is assumed for simplicity that bailouts do not modify the amount of federal grants assigned among state governments, such that \(\sum T_i = T\) and \(\sum (1 - P_H) \phi_i R_m = B\)

\[^{15}\text{The absence of distortionary taxation simplifies very much the analysis and helps to focus on the problem of local budget allocation and borrowing. Recall however, that in only a very limited number of cases, state governments have real control on tax rates or even tax bases. See for example Panizza (1999).}\]
3 Solution

Timing of the game: First local governments make optimal decisions on budget allocation \((z_i, A_i)\) and borrowing, taking into account the central government’s optimal allocation of bailouts to the banking system. In a second step, the Central authority decides on the allocation of bailouts, given the optimal decisions of local governments. The game between state governments and the central authority is solved by backward induction. Therefore, the optimal allocation of bailouts by the central government is decided in first place.

Recall the model assumes that state governments compete with each other for foreign capital. By increasing their own infrastructure investment budget \(A_i\) and thereby total infrastructure investment \(I_i = \frac{A_i}{L_i(\delta, \beta, \phi_i)}\), each locality induce foreign capital \(K_i\) to be allocated there. Given the choices of any other region \(j (j \neq i)\), the levels of \(A_i\) and \(z_i\) chosen by state government \(i\), are then implicitly given by maximization problem (8) subject to the budget constraint defined in equation (6)

\[
\max_{A_i, z_i} W_i = x(K_i, \frac{A_i}{L_i(\delta, \beta, \phi_i^*)}) + u(z_i) \tag{8}
\]

where \(\phi_i^*\) will be determined by the central government and foreign capital \(K_i\), under the assumption of perfect mobility, is allocated to equalize the marginal returns \(x_k(K_i, I_i) = x_k(K_j, I_j)\), for all \(i\) and \(j\). Together with \(\Sigma K_i = K\), determines foreign capital allocation for any given \((I_1, ..., I_N)\): \(K_1(I_1, ..., I_N), ..., K_N(I_1, ..., I_N)\). Recall \(\frac{\partial K_i}{\partial I_i} > 0\) and \(\frac{\partial K_i}{\partial I_j} < 0\) for all \(j \neq i\).

Given the choices of local governments \((A^*, z^*)\), the allocation of bailouts \(\phi_i\), \(i = 1, ..., N\), by the central government is implicitly given by maximization problem
subject to the budget constraint defined in equation (7)

$$\max_{\phi_i} \sum [x(K_i, \frac{A^*_i}{L_i(\delta, \beta, \phi_i)}) + u(z^*_i)]$$  \hspace{1cm} (9)$$

The central government’s allocation of bailouts to the banking system has no direct impact on the provision of consumption public goods \((z_1, ..., z_N)\). The allocation of bailouts must satisfy the first-order condition given in (10)\(^{16}\)

$$\frac{\partial x}{\partial I_i} (k_i, I_i) + \frac{\partial x}{\partial K_i} \frac{\partial K_i}{\partial I_i} (I_1, ..., I_N) = \frac{\partial x}{\partial I_j} (k_j, I_j) + \frac{\partial x}{\partial K_j} \frac{\partial K_j}{\partial I_j} (I_1, ..., I_N) \hspace{1cm} (10)$$

for all \(i, j\)

### 3.1 Hard Budget Constraint

As a benchmark solution, let first assume that the Central Government can commit to a hard budget constraint, so that the banking system receives no bailouts at all \((\phi_i = 0)\). Given the choices of other state governments on infrastructure investment \((A_j, \text{ for all } j \neq i)\), and the hard budget constraint assumption \((\phi_i = 0)\), the levels \(A^HBC_i\) and \(z^HBC_i\) satisfy the following first-order condition of maximization problem (8),

$$\left(\frac{\partial x}{\partial I_i} + \frac{\partial x}{\partial K_i} \frac{\partial K_i}{\partial I_i}\right) \left(\frac{\partial I_i}{\partial A_i} + \frac{\partial I_i}{\partial L_i} \frac{\partial \delta}{\partial A_i}\right) = u'(z_i) \hspace{1cm} (11)$$

Given the optimal levels \(A^HBC_i\) and \(z^HBC_i\), it is possible to define optimal values for total infrastructure investment \(I^HBC_i\); the stock of debt in state \(i\), \(D^HBC_i = \frac{1-L^HBC_i(\delta, \beta)}{L^HBC_i(\delta, \beta)} A^HBC_i\); the total stock of debt \(D^HBC_T = \sum D^HBC_i\); and the

\(^{16}\)One can easily prove that condition (10) simplifies to \(\frac{\partial x}{\partial I_i}(K_i, I_i) = \frac{\partial x}{\partial I_j}(K_j, I_j)\) if \(x(K, I)\) takes the form of a Cobb-Douglas function.
interest rate in the economy $\delta^{HBC} = h[\sum \frac{1-L_i^{HBC}(\delta,\beta)}{L_i^{HBC}(\delta,\beta)} A_i^{HBC}].$

### 3.2 Soft Budget Constraint

The Central Government’s optimal allocation of bailouts to the banks who monitor and co-finance infrastructure investment in state $i$ is now implicitly defined by the first-order condition (10).

**Lemma 2** Given the choices of other state governments $(A_j, \text{for all } j \neq i)$, the Central Government’s optimal allocation of bailouts in state $i$, negatively depends upon the optimal decision on infrastructure investment budget in state $i$. That is $\frac{\partial \phi_i}{\partial A_i} < 0.$

**Proof.** For given values of $A_j$ (for all $j \neq i$), it follows from condition (10) and the results stated in Lemma 1. Recall $I_i = A_i L_i(\delta,\beta,\phi_i)$ and $x''_I = \frac{\partial^2 x}{\partial I^2} < 0$.

Given the central government’s optimal allocation of bailouts to the banking system given by condition (10) and the decisions of other state governments $j \neq i$, the levels $A_i^{SBC}$ and $z_i^{SBC}$ satisfy the following first-order condition of maximization problem (8),

$$
\frac{\partial x}{\partial I_i} + \frac{\partial x}{\partial K_i} \frac{\partial I_i}{\partial A_i} + \frac{\partial I_i}{\partial L_i} \frac{\partial L_i}{\partial \delta} \frac{\partial \delta}{\partial A_i} + \frac{\partial x}{\partial I_i} + \frac{\partial x}{\partial K_i} \frac{\partial I_i}{\partial K_i} \frac{\partial I_i}{\partial L_i} \frac{\partial L_i}{\partial \phi_i} \frac{\partial \phi_i}{\partial A_i} = u'(z_i) \quad (12)
$$

Comparison of first-order condition (11) with condition (12) makes it possible to show how the soft budget constraint faced by the banking system in state $i$ generates an allocative distortion at state level,

**Lemma 3** The soft budget constraint faced by the banking system in state $i$ ($\phi_i > 0$), generates an allocative distortion with too much expenditure in consumption.
public goods \( (z_i^{SBC} > z_i^{HBC}) \) and a lower than optimal own infrastructure investment budget \( (A_i^{SBC} < A_i^{HBC}) \).

**Proof.** It follows from the results of Lemma 1 \( (\frac{\partial L_i}{\partial \phi_i} < 0) \) and Lemma 2 \( (\frac{\partial \phi_i}{\partial A_i} < 0) \) and the assumption that marginal utility from consumption public goods is decreasing \( (u''(z_i) = \frac{\partial^2 u(z_i)}{\partial z_i^2} < 0) \). ■

From the results stated in Lemma 1 and Lemma 3, state \( i \) optimally reduces its own infrastructure investment budget \( A_i \) in the presence of bailouts, but on the other hand, state \( i \) is able to increase leverage on its own infrastructure investment budget \( (L_i \text{ falls}) \). Let now investigate what happens with the stock of debt both in state \( i \) and the total stock, given these two competing effects.

**Proposition 1** Bailouts to the banking system in state \( i \) \( (\phi_i > 0) \) generate an increase in the total stock of debt \( (D_T^{SBC} > D_T^{HBC}) \) and in particular, in the stock of debt in state \( i \) \( (D_i^{SBC} > D_i^{HBC}) \), thus bailouts generate an increase in the interest rate \( (\delta^{SBC} > \delta^{HBC}) \).

**Proof.** The proof is by contradiction. Assume that \( D_i^{SBC} < D_i^{HBC} \) and thus \( \delta^{SBC} < \delta^{HBC} \). If this is the case, and for given values of \( A_j \) for all \( j \neq i \), \( I_j^{SBC} > I_j^{HBC} \) because \( L_j(\delta^{SBC}) < L_j(\delta^{HBC}) \) and \( D_j^{SBC} > D_j^{HBC} \) for all \( j \neq i \).\(^{17}\)

Now for condition (10) to hold it has to be true that also \( I_i^{SBC} > I_i^{HBC} \) what is only possible with \( D_i^{SBC} > D_i^{HBC} \) given that \( A_i^{SBC} < A_i^{HBC} \) and therefore it must be that \( D_T^{SBC} > D_T^{HBC} \). Given the last result, assume now that \( D_i^{SBC} < D_i^{HBC} \) but \( D_T^{SBC} > D_T^{HBC} \) and thus \( \delta^{SBC} > \delta^{HBC} \). It follows immediately that \( I_j^{SBC} < I_j^{HBC} \)

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\(^{17}\)Recall total infrastructure investment \( I_i \) is financed either with own state’s budget \( A_i \) or debt: \( I_i = A_i + D_i \) for all \( i \).
and $D_j^{SBC} < D_j^{HBC}$ for all $j \neq i$. and therefore it must be that $D_i^{SBC} > D_i^{HBC}$ because $D_T^{SBC} > D_T^{HBC}$. ■

The analysis above allows me to draw the following conclusion,

**Corollary 1** From the comparison of the results under Hard and Soft Budget Constraints, it follows that: $z_i^{SBC} > z_i^{HBC}$ ; $A_i^{SBC} < A_i^{HBC}$ ; $D_i^{SBC} > D_i^{HBC}$ ; $D_T^{SBC} > D_T^{HBC}$ ; $\delta^{SBC} > \delta^{HBC}$ ; $I_i^{SBC} < I_i^{HBC}$\(^{18}\).

**Proof.** It follows immediately from the conclusions stated in Lemma 1, Lemma 2, Lemma 3 and Proposition 1. ■

## 4 Concluding Remarks

A model is developed to explain fiscal responsibility at state level. Lower-level governments engage in opportunistic behavior if they expect the central authority will bail out the banking system in case investment projects fail. As a result, state governments distort the optimal allocation of public goods: they expend a higher (lower) than optimal amount of resources in consumption public goods (infrastructure investment) and overborrow from both, the banking system and the bond markets.

The model helps explain most of the recent experiences of bailouts and fiscal indiscipline at state level. In several countries, especially but not exclusively federations, deficits and debt have emerged and risen over time at sub-national level

\(^{18}\)An interesting case arises when $h' = \frac{\partial s}{\partial D_T} = 0$ so that the interest rate does not depend upon the total stock of local debt. It follows that, $I_i^{SBC} = I_i^{HBC}$ but still $D_i^{SBC} > D_i^{HBC}$ ; $D_T^{SBC} > D_T^{HBC}$. See that state $i$ suffers no capital squeeze at all notwithstanding that $A_i^{SBC} < A_i^{HBC}$. This is possible because bailouts increase leverage on own infrastructure investment budget $A_i$. Therefore, local investment is under soft budget constraints financed with a higher ratio of debt to own resources.
Moreover, the fact that state governments rely heavily on banking borrowing, subject to soft budget constraints, seems to be the case not only in developing but also in developed countries. As a result, lower-level governments have overspent usually in areas of consumption public goods like health, personnel and social security. However, in most of the cases some kind of control on sub-national borrowing exists. The analysis above shows namely that golden rule type controls are not efficient to avoid overborrowing.

5 Appendix A

Proof of Lemma 1

$L_i(\delta, \beta, \phi_i)$ and $\delta = h[\sum \frac{1-L_i(\delta,\beta,\phi_i)}{L_i(\delta,\beta,\phi_i)} A_i]$ determine the following system of equations, which determine the impact of the bailout on leverage $\frac{\partial L_i}{\partial \phi_i}, \frac{\partial L_j}{\partial \phi_i}$ for all $j \neq i$ and on the interest rate of the economy $\frac{\partial \delta}{\partial \phi_i}$.

\[
\frac{\partial \delta}{\partial \phi_i} = -\frac{h'}{L^2} \sum_k A_k \frac{\partial L_k}{\partial \phi_i} \quad (13)
\]

\[
\frac{\partial L_i}{\partial \phi_i} = -\frac{D}{\delta E^2} + \frac{B}{\delta^2 E} \frac{\partial \delta}{\partial \phi_i} \quad (14)
\]

\[
\frac{\partial L_j}{\partial \phi_i} = \frac{B}{\delta^2 E} \frac{\partial \delta}{\partial \phi_i} \quad \forall j \neq i \quad (15)
\]

where we have defined

\[
R_u = R - \frac{h}{N_0} + \frac{C}{P}
\]
\[ R_u P_H [\Delta p + (1 - P_H) \phi_i] + c P_L = B \]
\[ [\Delta p + (1 - P_H) \phi_i] = E \]
\[ c(1 - P_H) \Delta p = D \]

Solving the system,

\[ \frac{\partial \delta}{\partial \phi_i} = \frac{h' A_i D}{L_i^2 \delta^2 E^2 + h' N A_i E B} > 0 \] (16)

\[ \frac{\partial L_i}{\partial \phi_i} = \frac{-D[L_i^2 \delta^2 E^2 + h'(N - 1) A_i B]}{\delta^2 E^2[L_i^2 \delta^2 E + h' N A_i B]} < 0 \] (17)

\[ \frac{\partial L_i}{\partial \phi_i} = \frac{B}{\delta^2 E} \frac{\partial \delta}{\partial \phi_i} > 0 \] (18)

References


