ECONSTOR
Make Your Publications Visible.

A Service of

ZBW Leibniz-Informationszentrum Wirtschaft
Leibniz Information Centre for Economics

Kovac, Eugen; Mylovanov, Tymofiy

Working Paper

Stochastic Mechanisms in Settings without Monetary Transfers: Regular Case

Bonn Econ Discussion Papers, No. 23/2006

Provided in Cooperation with:
Bonn Graduate School of Economics (BGSE), University of Bonn

Suggested Citation: Kovac, Eugen; Mylovanov, Tymofiy (2006) : Stochastic Mechanisms in Settings without Monetary Transfers: Regular Case, Bonn Econ Discussion Papers, No. 23/2006, University of Bonn, Bonn Graduate School of Economics (BGSE), Bonn

This Version is available at:
https://hdl.handle.net/10419/22968

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

WWW.ECONSTOR.EU

Mitglied der Leibniz-Gemeinschaft

Bonn Econ Discussion Papers

Discussion Paper 23/2006

Stochastic Mechanisms in Settings without Monetary Transfers: Regular Case

by

Eugen Kovac, Tymofiy Mylovanov

December 2006

Bonn Graduate School of Economics
Department of Economics
University of Bonn
Adenauerallee 24 - 42
D-53113 Bonn

The Bonn Graduate School of Economics is
sponsored by the

Deutsche Post World Net

MAIL EXPRESS LOGISTICS FINANCE

Stochastic Mechanisms in Settings without Monetary Transfers: Regular Case*

Eugen Kováč† Tymofiy Mylovanov‡

December 11, 2006

Abstract

We study relative performance of stochastic and deterministic mechanisms in a principal-agent model with hidden information and no monetary transfers. We present an example in which stochastic mechanisms perform strictly better than deterministic ones and can implement any outcome *arbitrarily close* to the first-best. Nevertheless, under the common assumption of quadratic payoffs and a certain regularity condition on the distribution of private information and the agent's bias, the optimal mechanism is deterministic. We provide an explicit characterization of this mechanism.

JEL codes: D78, D82, L22, M54.

Keywords: optimal delegation, cheap talk, principal-agent relationship, no monetary transfers, stochastic mechanisms.

*We thank Georg Noldeke and Larry Samuelson for helpful comments. We are grateful to Ganguly and Ray for making their survey on cheap talk available to us. Financial support from the Deutsche Forschungsgemeinschaft through the project SFB/TR 15, Projektbereich A, is greatly appreciated.

†Department of Economics, University of Bonn and CERGE-EI, Charles University, Prague; e-mail: eugen.kovac@uni-bonn.de.

‡Department of Economics, University of Bonn, and EERC-EROC and KEI, Kiev; e-mail: mylovanov@uni-bonn.de.

1 Introduction

The literature on optimal mechanisms in the principal-agent model with hidden information, no monetary transfers, and single-peaked preferences has restricted attention to deterministic mechanisms (Alonso and Matouschek [3], Holmström [17], [18], Martimort and Semenov [30], and Melumad and Shibano [31]). This may contain some loss of generality since stochastic mechanisms can outperform deterministic ones.[1] Nevertheless, very little is known about relative performance of stochastic and deterministic mechanisms in this setting. The purpose of our paper is to address this question.

In order to illustrate a potential power of stochastic allocations, in Section 3 we provide an example in which stochastic mechanisms perform strictly better than deterministic ones (Proposition 2) and can implement any outcome *arbitrarily close* to the first-best (Proposition 1). In this example, the parties' payoffs have different degrees of curvature: the agent has a quadratic loss function, whereas the principal has an absolute value loss function. This allows the principal to use variance to improve agent's incentives without imposing any cost on herself.

Our main results, however, are obtained for the environment with quadratic preferences of both parties. This is the setting most frequently studied in the literature.[2] Proposition 3 in Section 4 shows that under a certain regularity condition an optimal *stochastic* mechanism is, in fact, deterministic; it explicitly characterizes this mechanism. The regularity condition in this proposition is satisfied in most applications and is similar to the regularity condition in the optimal auction in Myerson [34] that requires virtual valuation to be monotone.

The characterization of the optimal mechanism in Proposition 3 is closely related to the existing results for deterministic mechanisms: Under the regularity condition, this proposition implies Propositions 2–6 in Alonso and Matouschek [3] (henceforth, AM), Propositions 2–3 in Martimort and Semenov [30] (henceforth, MS), and Proposition 3 in Melumad and Shibano [31].[3] Hence, our results complement the existing literature by showing that the optimal deterministic mechanisms are also optimal on the *entire* set of incentive-compatible mechanisms, including the ones that result in stochastic allocations.

In a related paper, Goltsman and Pavlov [15] study optimal communication rules

[1] For example, this is the case in the standard principal-agent model with monetary transfers (Stiglitz [36], Arnott and Stiglitz [6], and Strausz [37]).

[2] The setting with quadratic preferences is the leading example in Crawford and Sobel [10]. It has applied in models in political science, finance, monetary policy, design of organizations. Quadratic preferences have recently been used (i) as the main framework in Alonso [1], Alonso, Dessein and Matouschek [2], Alonso and Matouschek [4], Ambrus and Takahashi [5], Dessein and Santos [12], Ganguly and Ray [14], Goltsman and Pavlov [15], Kraehmer [23], Krishna and Morgan [24], [26], Li [28], Li [29], Morgan and Stocken [32], Morris [33], Ottaviani and Squintani [35], and Vidal [39], and (ii) to obtain more specific results in Blume and Board [8], Chakraborty and Harbaugh [9], Dessein [11], Gordon [16], Ivanov [19], Kartik, Ottaviani and Squintani [21], Kawamura [22], Krishna and Morgan [25], [27], and Szalay [38]. For a survey of the earlier literature see Ganguly and Ray [13].

[3] AM and Melumad and Shibano [31] provide results for the case in which our regularity condition does not hold. Moreover, the environment in AM is more general than in this paper because their results do not require quadratic preferences of the agent.

that transform messages from the agent into recommendations for the principal. They also consider a benchmark case, in which the principal can commit to a stochastic mechanism, and demonstrate a result similar to Proposition 3 in this paper. Our results have been obtained independently and our methods of proof are different. Furthermore, the results in Goltsman and Pavlov [15] are obtained for the setting with a uniform distribution of private information and a constant bias of the agent. Proposition 3 in this paper allows for a significantly broader set of distributions and conflict of preferences.

A growing literature studies multiple extensions of cheap talk communication (Crawford and Sobel [10]): For instance, Krishna and Morgan [26] consider two rounds of communication. Ganguly and Ray [14], Goltsman and Pavlov [15], and Krishna and Morgan [26] analyze communication through a mediator. Finally, in Blume and Board [8] and Kawamura [22] there is exogenous noise added to the messages of the agent. This literature identifies equilibria that are Pareto superior to the equilibria in Crawford and Sobel [10]. In these equilibria, the players' behavior induces a lottery over decisions. By contrast, the equilibrium allocation in Crawford and Sobel [10] is deterministic. This raises a question whether optimal stochastic allocations can outperform optimal deterministic ones if the principal could commit to a mechanism. Proposition 3 in this paper and Theorem 1 in Goltsman and Pavlov [15] answer this question negatively.

The technical approach in this paper is different from the rest of the literature on optimal mechanisms in the settings with single-peaked preferences. AM, for example, derive the optimal deterministic mechanism by considering effects of adding and removing decisions available to the agent in a mechanism. As they observe, this is equivalent to a difficult optimization problem over the power set of available decisions. We do not know how to extend their method to stochastic allocations considered in this paper. In a setting with single-peaked preferences and monetary transfers, Krishna and Morgan [27] characterize the optimal deterministic mechanism using optimal control. Their method is applicable to our setting. It would require, however, a technical assumption that an optimal allocation is piecewise differentiable. The arguments in this paper are simpler; they do not deal with power sets, do not require piecewise differentiability of an allocation, and avoid differential equations. Nevertheless, we have to restrict attention to quadratic payoffs.

On the other hand, our approach is similar to the one in the optimal auction literature (e.g., Myerson [34]). For instance, a byproduct of our proof is a characterization of incentive-compatible allocations, analogous to the one in the literature on mechanism design with monetary transfers.[4] Nevertheless, there is a problem of incorporating constraint of non-negative variance; this difficulty is absent in auction models. We resolve this problem by expressing the principal's payoff in terms of a derivative of a function, whose value can be interpreted analogously to the virtual valuation. The regularity condition requires this function to be monotone, which in turn guarantees that the optimal allocation is deterministic.[5]

[4]The necessary and sufficient conditions for incentive compatibility of deterministic allocations in the setting without transfers are given in MS and Melumad and Shibano [31].

[5]Our regularity condition is connected with conditions used in AM and MS. It is also related to

The remainder of the paper is organized as follows: Section 2 introduces the model. Section 3 presents the example. Section 4 derives the main results. Section 5 concludes. The proofs omitted in the main text are presented in the appendix.

2 Environment

There is a principal (she) and an agent (he). The agent has one-dimensional private information $\omega \in \mathbb{R}$ called a state of the world. The principal's prior beliefs about ω are represented by a cumulative distribution function $F(\omega)$ with support $\Omega = [0, 1]$ and atomless density $f(\omega)$ that is positive and absolutely continuous on Ω. The parties must make a decision $p \in \mathbb{R}$. There are no outside options. The agent has a quadratic loss function, $u_a(p, \omega) = -(p - \omega)^2$. The principal's payoff is $u_p(p, \omega) = u\big(p - z(\omega)\big)$, where $u : \mathbb{R} \to \mathbb{R}$ is a single-peaked function and $z : \Omega \to \mathbb{R}$ is an absolutely continuous function. The value $z(\omega)$ represents principal's ideal decision and the difference $b(\omega) = \omega - z(\omega)$ represents the agent's bias. We will consider two versions of the principal's payoff: In Section 3 we assume that the principal has an absolute value loss function, $u_p(p, \omega) = -|p - z(\omega)|$. In Section 4 we consider a principal with a quadratic loss function, $u_p(p, \omega) = -[p - z(\omega)]^2$.

2.1 Allocations

Let $\mathcal{P}$ be the set of probability distributions on $\mathbb{R}$ with a finite variance. An *allocation* M is a (Borel measurable) function $M : \Omega \to \mathcal{P}$ that maps the agent's information into a lottery over decisions. An allocation M is deterministic if for every $\omega \in \Omega$ the lottery $M(\omega)$ implements one decision with certainty. Let $\mathcal{M}$ denote the set of all allocations.

An allocation has two interpretations. First, it can describe the outcome of interaction of the agent and the principal in some game. Second, it can describe a decision problem for the agent in which he chooses a report $\omega \in \Omega$ and obtains a lottery $M(\omega)$ over p. If this interpretation is used, we call M a (direct) *mechanism* and let $\mathbb{E}^{M(\omega)}$ denote the expectation operator associated with this lottery.

A function $r : \Omega \to \Omega$ that maps the agent's information into a report is an *equilibrium* in a direct mechanism M if it maximizes the agent's expected payoff, i.e.,

$$r(\omega) \in \arg\max_{s \in \Omega} \mathbb{E}^{M(s)} u_a(p, \omega) \quad \text{for all } \omega \in \Omega.$$

An allocation M is *incentive-compatible* if truth-telling, i.e., $r(\omega) = \omega$ for all $\omega \in \Omega$, is an equilibrium in the mechanism M. By the Revelation Principle we can restrict attention to incentive-compatible allocations.

Consider an allocation M and let $\mu^M(\omega) = \mathbb{E}^{M(\omega)} p$ and $\tau^M(\omega) = \mathrm{Var}^{M(\omega)} p$ denote the expected decision and the variance of the lottery $M(\omega)$. Allocation M is deterministic if and only if $\tau^M(\omega) = 0$ for all $\omega \in [0, 1]$. Since the agent's loss is quadratic,

the sufficient condition for the optimality of deterministic mechanisms in the principal-agent problem with monetary transfers in Strausz [37]. We discuss the relationship between our results and the existing literature in detail in Section 5.

his payoff in a state ω from a report ω' in the mechanism M can be expressed using μ^M and τ^M:

$$U_a^M(\omega, \omega') = \mathbb{E}^{M(\omega')} u_a(p, \omega) = -[\mu^M(\omega') - \omega]^2 - \tau^M(\omega'). \tag{1}$$

In addition, let $V_a^M(\omega) = U_a^M(\omega, \omega)$ denote the agent's expected payoff from truth-telling if the state is ω. The following lemma provides a characterization of incentive-compatible allocations in terms of (μ^M, τ^M).

Lemma 1. *An allocation M is incentive-compatible if and only if:*

(IC_1) *μ^M is non-decreasing,*

(IC_2) *for all $\omega \in \Omega$:*

$$\tau^M(\omega) = -V_a^M(0) - [\mu^M(\omega) - \omega]^2 - 2\int_0^\omega [\mu^M(s) - s]\,\mathrm{d}s,$$

(IC_3) *$\tau^M(\omega) \geq 0$ for all $\omega \in \Omega$.*

Proof. See Appendix A.

An allocation M is *optimal* in $\mathcal{M}$ if it is a solution of the following program

$$\begin{aligned} (\mathrm{E}) \qquad & \max_{M \in \mathcal{M}} \mathbb{E} u_p(p, \omega) \\ & \text{s.t.} \quad (\mathrm{IC}_1), (\mathrm{IC}_2), \text{and } (\mathrm{IC}_3), \end{aligned}$$

where $\mathbb{E}$ denotes the expectation operator associated with the cumulative distribution function F. An optimal allocation maximizes the principal's ex-ante payoff on the set of incentive-compatible allocations. As illustrated by Proposition 1 in the next section, an optimal allocation might fail to exist.

3 Absolute value loss function and constant bias

Consider a principal with an absolute value loss function $u_p(p, \omega) = -|p - (\omega - b)|$, where $b > 0$. The principal's ex-ante payoff is maximized by the first-best allocation that implements $p = \omega - b$ for almost all $\omega \in \Omega$. This allocation, however, is not incentive-compatible: In the direct mechanism corresponding to this allocation, the agent's payoff is maximized by the report $\omega' = \min\{\omega + b,\, 1\}$ for almost all $\omega \in [0, 1]$.

In this setting, the variance of the lottery does not have any effect on the principal's payoff if all decisions in a lottery are higher than the principal's preferred decision $p = \omega - b$. This is not true for the agent. Consider two lotteries, one with an average decision close to the agent's preferred decision, $p_a = \omega$, and the other with an average decision close to the principal's preferred decision, $p_p = \omega - b$. If the variance of the first lottery is relatively high, then the agent prefers the second lottery. This

suggests that the principal can use variance to implement decisions closer to her most preferred alternatives. Proposition 1 shows that there exist stochastic incentive-compatible allocations in which the principal obtains a payoff arbitrarily close to the first-best payoff of zero. In these allocations, the agent selects a lottery that with high probability implements the principal's preferred decision; he avoids lotteries with more attractive decisions because they are associated with higher variance. In order to state this proposition, consider some $\varepsilon > 0$ and an allocation M such that

$$\mu^M(\omega) = \omega - b + \varepsilon, \qquad \tau^M(\omega) = 2(b-\varepsilon)\omega, \quad \text{and} \quad \operatorname{supp} M(\omega) \subseteq [\omega - b, \infty), \quad (2)$$

for all $\omega \in \Omega$.[6]

Proposition 1. *For any $\varepsilon \in (0, b)$, the allocation M satisfying (2) is incentive-compatible and yields the principal's ex-ante payoff $-\varepsilon$.*

Proof. It is straightforward to verify that M satisfies (IC_1)–(IC_3) and hence is incentive-compatible. Because the support of $M(\omega)$ belongs to $[\omega - b, \infty)$, the principal's expected payoff from this allocation equals to the expected value of $\mathbb{E}\left(\omega - b - \mu^M(\omega)\right) = -\varepsilon$. □

Thus, the upper bound of the payoffs that can be achieved by stochastic allocations is zero. By contrast, the upper bound of the payoffs that can be achieved by deterministic allocations is negative.

Proposition 2. *The upper bound of the principal's ex-ante payoff on the set of incentive-compatible deterministic allocations is negative.*

Proof. See Appendix A.

4 Quadratic payoffs

In this section we consider a principal with a quadratic loss function $u_p(p, \omega) = -[p - z(\omega)]^2$. The same argument as in the previous section implies that there is no incentive-compatible allocation that implements the first-best. This section derives the optimal stochastic mechanism under a regularity condition (Assumption 1). We proceed in three steps. First, we observe that without loss of generality one can concentrate on allocations that are continuous at 0 and 1 (Lemma 2). Second, we show that the optimal allocation is constant for a set of low values and a set of high values of ω (Lemmata 3 and 4). Finally, in Proposition 3 we characterize the optimal mechanism by applying integration by parts twice to the objective function in program (E).

[6]Such an allocation exists. For instance, an allocation that implements the decision $p = \omega - b$ with probability $q = 2(b-\varepsilon)\omega/[\varepsilon^2 + 2(b-\varepsilon)\omega]$ and the decision $p = \omega - b + [\varepsilon^2 + 2(b-\varepsilon)\omega]/\varepsilon$ with probability $1 - q$ for all $\omega \in [0, 1]$ satisfies (2).

Let M be an incentive-compatible allocation. Because the principal's loss is quadratic, her payoff given the allocation in a state ω can be expressed as

$$U_p^M(\omega) = \mathbb{E}^{M(\omega)} u_p(p, \omega) = -[\mu^M(\omega) - z(\omega)]^2 - \tau^M(\omega),$$

where the expectation is taken over p. After the substitution of the value of $\tau^M(\omega)$ from (IC_2) and taking the expectation over ω, we obtain that the principal's ex-ante expected payoff from allocation M is

$$V_p^M = \mathbb{E} U_p^M(\omega) = 2 \int_0^1 \mu^M(\omega) g(\omega) \, \mathrm{d}\omega + h^M(0), \tag{3}$$

where

$$g(\omega) = \left(\frac{1 - F(\omega)}{f(\omega)} + z(\omega) - \omega \right) f(\omega) = 1 - F(\omega) + [z(\omega) - \omega] f(\omega), \tag{4}$$

$$h^M(\omega) = V_a^M(\omega) + \omega^2 - \mathbb{E}[z(\omega')]^2, \tag{5}$$

and the expectation $\mathbb{E}[z(\omega')]^2$ is taken over ω'. The function g defined in (4) is absolutely continuous. We also impose a *regularity assumption*.

Assumption 1. *If $0 \leq g(\omega) \leq 1$, then g is decreasing in ω.*[7]

This assumption is satisfied, for example, if the distribution function F has increasing hazard rate $f(\omega)/[1 - F(\omega)]$ and the agent's bias $b(\omega) = \omega - z(\omega)$ is positive and non-decreasing. Furthermore, Assumption 1 is satisfied if $z(\omega) = \omega$, because in this case $g(\omega) = 1 - F(\omega) \in [0, 1]$. Therefore, as we will show in Proposition 4, it is also satisfied if the preferences of the agent and the principal are sufficiently close.

Assumption 1 is somewhat similar to the regularity assumption in the optimal auction setting in Myerson [34] that requires $(1 - F(\omega))/f(\omega) + z(\omega) - \omega$ to be decreasing. In Myerson's setting ω is the valuation of the buyer and $z(\omega)$ is a revision effect function (Myerson [34], p. 60) that captures the effect of ω on the payoffs of other players. Finally, Assumption 1 is related to some of conditions used in AM and MS; this will be discussed in more detail in Section 5.

Let $\mathcal{M}^c$ denote the set of all incentive-compatible allocations M where both $\mu^M(\omega)$ and $\tau^M(\omega)$ are continuous from the right at $\omega = 0$ and continuous from the left at $\omega = 1$. In what follows, we restrict our analysis to allocations in $\mathcal{M}^c$. The next lemma shows that this is without loss of generality.[8]

[7] We say that function g is *decreasing in point* ω if there exists some open neighborhood O of ω such that for all $\omega' \in \Omega \cap O$: If $\omega' < \omega$, then $g(\omega') < g(\omega)$, and if $\omega' > \omega$, then $g(\omega') > g(\omega)$. An alternative (stronger) definition would be to require the function g to be decreasing on some neighborhood of this point.

[8] More precisely, we may consider an equivalence relation on the set of all incentive-compatible allocations $\mathcal{M}$. We say that two allocations M_1 and M_2 are *equivalent* if $\mu^{M_1}(\omega) = \mu^{M_2}(\omega)$ and $\tau^{M_1}(\omega) = \tau^{M_2}(\omega)$ for all $\omega \in (0, 1)$. Lemma 2 then claims that every equivalence class contains an allocation $M^c \in \mathcal{M}^c$. Therefore, we may identify each equivalence class with an allocation from $\mathcal{M}^c$. Next, if we know the set of optimal allocations in $\mathcal{M}^c$, then the set of all optimal incentive-compatible allocations can be found by perturbing $\mu^M(0)$, $\tau^M(0)$, $\mu^M(1)$ and $\tau^M(1)$ such that conditions (22) and (21) in the proof of Lemma 2 hold.

Lemma 2. *Let M be an incentive-compatible allocation. Then there exists an incentive-compatible allocation M^c such that*

(i) $V_p^{M^c} = V_p^M$ *and* $V_a^{M^c}(\omega) = V_a^M(\omega)$ *for all* $\omega \in [0,1]$,

(ii) $M(\omega) = M^c(\omega)$ *for all* $\omega \in (0,1)$,

(iii) $\mu^{M^c}(\omega)$ *and* $\tau^{M^c}(\omega)$ *are continuous at* 0 *and* 1.

Proof. See Appendix A.

Now consider an allocation $M \in \mathcal{M}^c$. The principal's ex-ante payoff from this allocation is given by (3). By incentive compatibility, μ^M is *non-decreasing*. It follows that the principal will be (weakly) better off given an allocation M' with $\mu^{M'}$ that is constant whenever $g(\omega)$ is negative. In order to determine the optimal intervals on which $\mu^{M'}$ is constant let

$$G(\omega) = \int_0^\omega g(s)\,\mathrm{d}s = \mathbb{E}[z(\omega') \mid \omega' \leq \omega]\,F(\omega) + \omega[1 - F(\omega)]. \tag{6}$$

This function is clearly continuous. A possible shape of function G is illustrated on Figure 1.

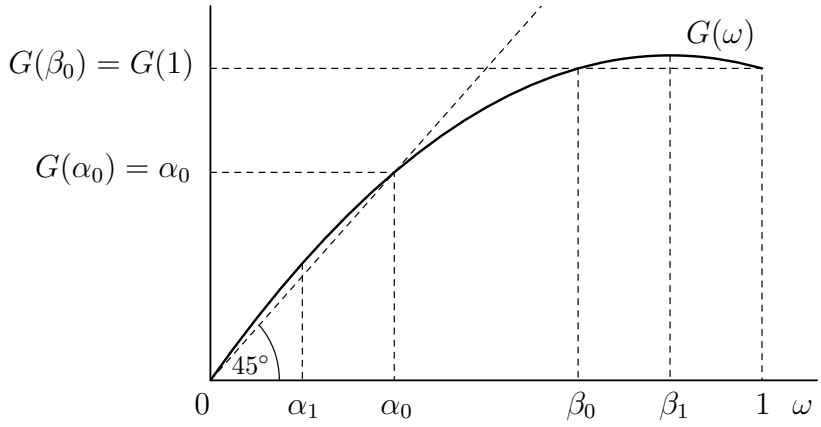

Figure 1: Shape of function G

Let us now assume that the allocation $M \in \mathcal{M}^c$ has $\mu^M(\omega)$ constant on $(\tilde{\omega}, 1]$. The principal's expected payoff from this allocation equals

$$V_p^M = 2\int_0^{\tilde{\omega}} \mu^M(\omega) g(\omega)\,\mathrm{d}\omega + 2\mu^M(\tilde{\omega})[G(1) - G(\tilde{\omega})] + h^M(0), \tag{7}$$

where $h^M(0)$ is defined in (5) and does not depend on $\tilde{\omega}$. Next, consider the effect of an infinitesimal decrease in $\tilde{\omega}$ on the principal's payoff, in the case when $G(1) - G(\tilde{\omega}) < 0$. The first term of (7) decreases at the rate $2\mu^M(\tilde{\omega})g(\tilde{\omega})$. There are two effects on the second term. First, there is an increase at the same rate due to the change in $G(\tilde{\omega})$,

which cancels out with the previous effect. Second, there is an increase due to a decrease in $\mu^M(\tilde{\omega})$. Hence, the principal's payoff improves. This suggests that it is optimal to set $\tilde{\omega}$ to the value for which the last effect is zero, given by

$$\beta_0 = \inf\{\omega \in [0,1] : G(\omega) \geq G(1)\}. \tag{8}$$

Because M is an arbitrary allocation in $\mathcal{M}^c$, in the optimal allocation μ^M is constant on $(\beta_0, 1]$.

Similarly, it might be optimal to have μ^M constant on $[0, \tilde{\omega})$ for some $\tilde{\omega} \in [0,1]$. For this allocation using (IC_2) we obtain that

$$V_p^M = 2\int_{\tilde{\omega}}^1 \mu^M(\omega) g(\omega)\,\mathrm{d}\omega + 2\mu^M(\tilde{\omega})[G(\tilde{\omega}) - \tilde{\omega}] + h^M(\tilde{\omega}). \tag{9}$$

An argument similar to the one above suggests that it is optimal to set $\tilde{\omega}$ to

$$\alpha_0 = \sup\{\omega \in [0,1] : G(\omega) \geq \omega\}. \tag{10}$$

The conditions $G(\omega) \geq G(1)$ in (8) and $G(\omega) \geq \omega$ in (10) are equivalent to $\mathbb{E}[z(\omega') \mid \omega' \geq \omega] \leq \omega$ and $\mathbb{E}[z(\omega') \mid \omega' \leq \omega] \geq \omega$ respectively. In Appendix B, we prove several facts, (F_1)–(F_9), about properties of α_0, β_0 and function g.

In order to prove that α_0 and β_0 are indeed the optimal values of cutoffs in (7) and (9), let M be an allocation in $\mathcal{M}^c$ and define a new allocation

$$\overline{M}(\omega) := \begin{cases} M(\alpha_0), & \text{if } 0 \leq \omega < \alpha_0; \\ M(\omega), & \text{if } \alpha_0 \leq \omega \leq \beta_0; \\ M(\beta_0), & \text{if } \beta_0 < \omega \leq 1. \end{cases} \tag{11}$$

The values of μ in these two allocations are depicted in Figure 2. Observe that the value of α_0 and β_0 are independent of a specific allocation M and depend only on the principal's prior beliefs and the agent's bias. The following Lemma establishes incentive compatibility of $\overline{M}$.

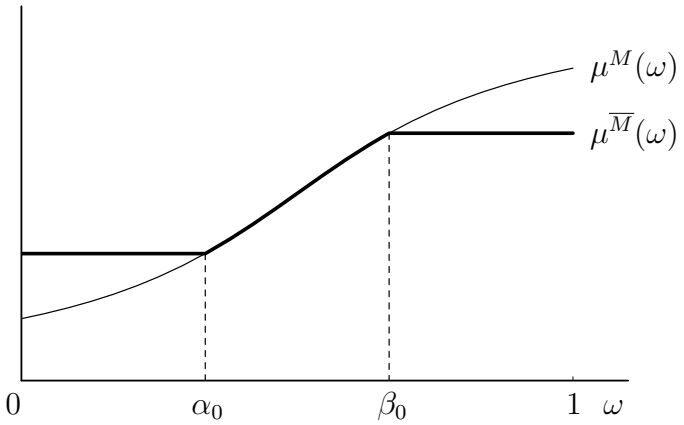

Figure 2: Expected decisions in allocations M and $\overline{M}$

Lemma 3. *Allocation $\overline{M}$ is incentive-compatible.*

Proof. See Appendix A.

Our next result, Lemma 4, demonstrates that the principal prefers $\overline{M}$ to M.

Lemma 4. *If M is an allocation in $\mathcal{M}^c$, then $V_p^{\overline{M}} \geq V_p^M$. If, in addition, $\mu^M(0) < \mu^M(\alpha_0)$ or $\mu^M(\beta_0) < \mu^M(1)$, then $V_p^{\overline{M}} > V_p^M$.*

Proof. See Appendix A.

Let $\overline{\mathcal{M}}$ be the set of all incentive-compatible allocations $M \in \mathcal{M}^c$ with μ^M constant on $[0, \alpha_0)$ and constant on $(\beta_0, 1]$. Lemmata 3 and 4 imply that we may restrict attention to allocations in $\overline{\mathcal{M}}$.

Corollary 1. *An allocation is optimal if and only if it maximizes the principal's payoff among allocations from $\overline{\mathcal{M}}$.*

This corollary immediately implies that in an optimal allocation $\mu^M(\omega)$ is constant if $\alpha_0 > \beta_0$. Let now $\alpha_0 \leq \beta_0$. The payoff in an allocation from $\overline{\mathcal{M}}$ equals

$$V_p^M = 2 \int_{\alpha_0}^{\beta_0} \mu^M(\omega) g(\omega) \, \mathrm{d}\omega + h^M(\alpha_0). \tag{12}$$

It follows from the facts (F_1) and (F_6)–(F_8) proven in Appendix B that $g(\omega) \in (0, 1)$ for all $\omega \in (\alpha_0, \beta_0)$. Therefore, g is decreasing on (α_0, β_0) by Assumption 1. As the next proposition shows, this implies that an optimal allocation exists and is deterministic. Furthermore, this allocation is unique in $\mathcal{M}^c$. The implemented decision in this allocation is continuous in ω and takes the minimax form $\mu^M(\omega) = \min\{\max\{\alpha_0, \omega\}, \beta_0\}$. This allocation is well-known to be optimal among *deterministic* allocations (Propositions 2–5 in AM, Proposition 3 in MS, and Proposition 3 in Melumad and Shibano [31]. It is also known to be optimal among stochastic allocations in the special case of a uniform distribution and a constant bias (Theorem 1 in Goltsman and Pavlov [15]).

Proposition 3. *An optimal allocation in $\mathcal{M}^c$ exists and is unique. If $\alpha_0 < \beta_0$, then the optimal allocation M from $\mathcal{M}^c$ is deterministic. It implements the decision*

$$\mu^M(\omega) = \begin{cases} \alpha_0, & \text{if } 0 \leq \omega < \alpha_0; \\ \omega, & \text{if } \alpha_0 \leq \omega \leq \beta_0; \\ \beta_0, & \text{if } \beta_0 < \omega \leq 1. \end{cases} \tag{13}$$

If $\alpha_0 \geq \beta_0$, then the optimal allocation in $\mathcal{M}^c$ is deterministic and is independent of ω. It implements the decision $\mu^M(\omega) = \mathbb{E}z(\omega')$ for all $\omega \in [0, 1]$.

Proof. Case $\alpha_0 < \beta_0$. Let M be an allocation in $\overline{\mathcal{M}}$. From (12), the principal's payoff from M is

$$V_p^M = 2\int_{\alpha_0}^{\beta_0} [\mu^M(\omega) - \omega] g(\omega)\,\mathrm{d}\omega + V_a^M(\alpha_0) + C, \tag{14}$$

where

$$C = \alpha_0^2 - \mathbb{E}[z(\omega')]^2 + 2\int_{\alpha_0}^{\beta_0} \omega g(\omega)\,\mathrm{d}\omega \tag{15}$$

is a constant independent of a particular allocation M. Since function g is absolutely continuous, it is differentiable almost everywhere and we can use integration by parts:

$$\begin{aligned} &2\int_{\alpha_0}^{\beta_0} [\mu^M(\omega) - \omega] g(\omega)\,\mathrm{d}\omega = \\ &= 2g(\beta_0)\int_{\alpha_0}^{\beta_0} [\mu^M(s) - s]\,\mathrm{d}s - 2\int_{\alpha_0}^{\beta_0} g'(\omega)\left(\int_{\alpha_0}^{\omega} [\mu^M(s) - s]\,\mathrm{d}s\right)\mathrm{d}\omega = \\ &\overset{(\mathrm{IC}_2)}{=} g(\beta_0)V_a^M(\beta_0) - g(\alpha_0)V_a^M(\alpha_0) - \int_{\alpha_0}^{\beta_0} g'(\omega)\, V_a^M(\omega)\,\mathrm{d}\omega. \end{aligned}$$

The substitution of this expression into (14) gives

$$V_p^M = g(\beta_0)V_a^M(\beta_0) + [1 - g(\alpha_0)]V_a^M(\alpha_0) - \int_{\alpha_0}^{\beta_0} g'(\omega)\, V_a^M(\omega)\,\mathrm{d}\omega + C. \tag{16}$$

Now recall that $V_a^M(\omega) \le 0$ for all $\omega \in [0,1]$, where equality holds if and only if $\mu^M(\omega) = \omega$ and $\tau^M(\omega) = 0$. In addition, as follows from the discussion preceding this proposition, function g is decreasing on (α_0, β_0). Therefore, $g'(\omega) < 0$ almost everywhere on (α_0, β_0). This and the fact that $g(\alpha_0) < 1$ and $g(\beta_0) > 0$ imply that the first three terms of the right hand side of (16) are non-positive. Therefore, we obtain

$$V_p^M \le C \text{ for any } M \in \overline{\mathcal{M}},$$

where equality holds if and only if

$$\begin{gathered} \mu^M(\omega) = \omega, \qquad \tau^M(\omega) = 0, \\ \text{for } \omega = \alpha_0,\ \omega = \beta_0, \text{ and for almost all } \omega \in (\alpha_0, \beta_0). \end{gathered} \tag{17}$$

It follows that M is optimal if and only if it satisfies (17).

We can now prove the statement of the proposition. First, the allocation given by (13) satisfies (17). It also satisfies (IC_1)–(IC_3) and is, therefore, incentive-compatible. Thus, it is optimal.

Conversely, consider an allocation $M \in \overline{\mathcal{M}}$ that satisfies (17). We will show that it satisfies (13). The monotonicity condition (IC_1) implies that $\mu^M(\omega) = \omega$ for all $\omega \in [\alpha_0, \beta_0]$. The constraint ($\mathrm{IC}_2$) together with continuity imply that $\tau^M(\omega) = 0$ for all $\omega \in [\alpha_0, \beta_0]$. It remains to show that $\mu^M(\omega) = \alpha_0$ for all $\omega \in [0, \alpha_0)$ and $\mu^M(\omega) = \beta_0$ for all $\omega \in (\beta_0, 1]$. Because $M \in \overline{\mathcal{M}}$, the value of μ^M is constant on

$[0, \alpha_0)$ and on $(\beta_0, 1]$. Let k_1 and k_2 denote these constants respectively. Then for any $\omega \in [0, \alpha_0)$, we obtain from (IC_2) that

$$V_a^M(\alpha_0) - V_a^M(\omega) = \omega^2 - \alpha_0^2 + 2\int_\omega^{\alpha_0} \mu^M(s)\,\mathrm{d}s.$$

Since $V_a^M(\alpha_0) = 0$, this reduces to $\tau^M(\omega) = -(k_1 - \alpha_0)^2$, which implies that $\tau^M(\omega) = 0$ and $k_1 = \alpha_0$. Similarly, for $\omega \in (\beta_0, 1]$, we have

$$V_a^M(\omega) - V_a^M(\beta_0) = \beta_0^2 - \omega^2 + 2\int_{\beta_0}^{\omega} \mu^M(s)\,\mathrm{d}s,$$

which reduces to $\tau^M(\omega) = -(k_2 - \beta_0)^2$. Hence, $\tau^M(\omega) = 0$ and $k_2 = \beta_0$.

Case $\alpha_0 \geq \beta_0$. If either $\alpha_0 > \beta_0$ or $\alpha_0 = 1$ or $\beta_0 = 0$, then any allocation $M \in \overline{\mathcal{M}}$ has $\mu^M \equiv k$ constant on $[0, 1]$. The principal's payoff from such an allocation is

$$V_p^M = -[k - \mathbb{E}z(\omega')]^2 + [\mathbb{E}z(\omega')]^2 - \mathbb{E}[z(\omega')]^2 - \tau^M(0).$$

It is maximized on the set $\overline{\mathcal{M}}$ if and only if

$$k = \mathbb{E}z(\omega') \quad \text{and} \quad \tau^M(0) = 0.$$

The remainder of the argument is analogous to the case $\alpha_0 < \beta_0$.

Finally, if $\alpha_0 = \beta_0 \in (0, 1)$, then $\mathbb{E}z(\omega') = G(1) = G(\beta_0) = G(\alpha_0) = \alpha_0$. The principal's expected payoff reduces to

$$V_p^M = h^M(\alpha_0) = V_a^M(\alpha_0) + \alpha_0^2 - \mathbb{E}[z(\omega')]^2.$$

It is maximized by $V_a^M(\alpha_0) = 0$ or, equivalently, by $\mu^M(\alpha_0) = \alpha_0 = \mathbb{E}z(\omega')$ and $\tau^M(\alpha_0) = 0$. The reminder of the argument is analogous to the case $\alpha_0 < \beta_0$. □

Proposition 3 shows that there is a unique optimal allocation in $\mathcal{M}^c$. If $\alpha_0 \geq \beta_0$, this allocation gives the principal the ex-ante payoff of $-\operatorname{Var} z(\omega')$. In this case, the conflict of preferences between the parties is so severe that it is optimal for the principal to disregard the agent and make a decision based on her prior beliefs.

If $\alpha_0 < \beta_0$, the optimal allocation gives the principal the payoff of C as given by (15). In this allocation, the implemented decision depends on the agent's information. It is equal to the agent's most preferred decision if $\omega \in (\alpha_0, \beta_0)$ and is independent of ω otherwise. The following corollaries describe the conditions under which $\alpha_0 = 0$ and $\beta_0 = 1$. They are the counterpart of Proposition 3–5 in AM for the case of deterministic mechanisms.

Corollary 2. *The optimal allocation M in $\mathcal{M}^c$ implements $\mu^M(\omega) = \max\{\alpha_0, \omega\}$ for all $\omega \in [0, 1]$ if and only if $z(1) \geq 1$.*

Corollary 3. *The optimal allocation M in $\mathcal{M}^c$ implements $\mu^M(\omega) = \min\{\omega, \beta_0\}$ for all $\omega \in [0, 1]$ if and only if $z(0) \leq 0$.*

Corollary 4. *The optimal allocation M in $\mathcal{M}^c$ implements $\mu^M(\omega) = \omega$ for all $\omega \in [0,1]$ if and only if $z(0) \leq 0$ and $z(1) \geq 1$.*

All corollaries follow directly from Proposition 3 and facts (F_6)–(F_8) proven in Appendix B.

The next proposition demonstrates that Assumption 1 is satisfied if the parties' preferences are sufficiently aligned. It also provides comparative statics results for α_0 and β_0. In order to state the proposition, consider an absolutely continuous function $\tilde{z} : [0,1] \to \mathbb{R}$. Now let us analyze the principal's maximization problem (E) under the assumption that her optimal ideal decision is $z^\lambda(\omega) = \lambda\tilde{z}(\omega) + (1-\lambda)\omega$, where $\lambda \in [0,1]$.[9] In this case, $g^\lambda(\omega) = 1 - F(\omega) + \lambda[\tilde{z}(\omega) - \omega]f(\omega)$.

Proposition 4. *If both functions f and $\tilde{z}$ are differentiable and, furthermore, have bounded derivatives on $[0,1]$, then:*

(i) *There exists some $\bar{\lambda} > 0$ such that g^λ satisfies Assumption 1 for all $\lambda < \bar{\lambda}$.*

(ii) *If $\tilde{z}(0) > 0$ and $0 < \lambda < \bar{\lambda}$, then α_0^λ is increasing in λ.*

(iii) *If $\tilde{z}(1) < 1$ and $0 < \lambda < \bar{\lambda}$, then β_0^λ is decreasing in λ.*

(iv) *If $\lambda \to 0$, then $\alpha_0^\lambda \to 0$ and $\beta_0^\lambda \to 0$.*

Proof. See Appendix A.

For the case of deterministic mechanisms, the result in part (i) of this proposition has been obtained in Proposition 6 in AM.

5 Related literature

We conclude the paper with a discussion of the related literature. The first part of this section connects our results with results in AM, MS, and Strausz [37]. The second part of this section compares our approach with the approach in Krishna and Morgan [27] who study optimal mechanisms in the setting with single-peaked preferences and monetary transfers.

AM analyze optimal deterministic mechanisms for the environment in which the principal's preferences are quadratic while the agent's preferences are described by a symmetric single-peaked payoff function. If we additionally impose that the preferences of the agent are quadratic and Assumption 1 is satisfied, then Proposition 3 implies Propositions 2–6 in AM. Following AM, define the effective backward bias $T(\omega) = \omega - G(\omega)$ and the effective forward bias $S(\omega) = G(\omega) - G(1)$. Observe that Assumption 1 can be equivalently stated as the condition that $T(\omega)$ is convex if $T'(\omega) \geq 0$ and $S'(\omega) \geq 0$.

[9] For this problem we will modify our notation by adding the superscript λ.

Let us now consider Proposition 2 in AM. It states that the optimal deterministic allocation is independent of ω if and only if

$$\text{there is no } \omega \in (0,1) \text{ such that } T(\omega) > 0 \text{ and } S(\omega) < 0. \tag{18}$$

This condition implies that $\alpha_0 \geq \beta_0$. Furthermore, if Assumption 1 is satisfied, then $\alpha_0 \geq \beta_0$ if and only if (18) is satisfied (this follows from fact (F_5) proven in Appendix B). Thus, under Assumption 1, the statement of Proposition 2 in AM coincides with the second part of Proposition 3 in our paper and therefore holds also for stochastic mechanisms.

Proposition 3–5 in AM provide conditions under which the optimal deterministic mechanism is continuous and characterize this mechanism. Under Assumption 1, the statements of these propositions follow from the first part of Proposition 3 and are given in Corollaries 2-4. This is straightforward to check for Proposition 3 and 4 in AM. Proposition 5 in AM states that the allocation satisfying $\mu^M(\omega) = \omega$ is optimal if and only if

$$\begin{aligned}
&\max_{\omega} z(\omega) \geq 1 \text{ and } \min_{\omega} z(\omega) \leq 0,\\
&T(\omega) \text{ and } S(\omega) \text{ are increasing, and}\\
&T(\omega) \text{ is convex on } [0,1].
\end{aligned}$$

Under Assumption 1, these conditions are necessary and sufficient for $\alpha_0 = 0$ and $\beta_0 = 1$. To establish sufficiency, observe that

$$T(\omega) = F(\omega)\left(\omega - [z(\omega') \mid \omega' \leq \omega]\right)$$

and hence $T(0) = 0$. Furthermore, $\alpha_0 = \sup\{\omega \in [0,1] : T(\omega) \leq 0\}$ by definition. Hence, if $T(\omega)$ is increasing, then $\alpha_0 = 0$. A symmetric argument demonstrates that $\beta_0 = 1$ if $S(\omega)$ is increasing. The necessity of these conditions follows from facts (F_7) and (F_8) proven in Appendix B that imply that either $\alpha_0 > 0$ or $\beta_0 < 1$ if $\max_\omega z(\omega) \geq 1$ or $\min_\omega z(\omega) \leq 0$ is not satisfied.

Proposition 6 in AM demonstrates that the optimal deterministic allocation is given by (13) if preferences of the principal and the agent are sufficiently similar. This result corresponds to part (i) of Proposition 4 in our paper.

MS consider a setting with a constant bias $\omega - z(\omega) = -\delta < 0$ for all $\omega \in [0,1]$. In this setting, $z(1) > 1$ and, therefore, $\beta_0 = \beta_1 = 1$ by fact (F_6) proven in Appendix B. Proposition 2 in MS demonstrates that in the optimal deterministic allocation $\mu^M(\omega)$ is continuous if $f(\omega) - \delta f'(\omega) \geq 0$ for almost all ω. Observe that this condition is equivalent to requiring $g(\omega)$ to be non-increasing almost everywhere and hence is similar to Assumption 1. Under the additional assumption that F is strictly log-concave, Proposition 3 in MS shows that if $\alpha_0 < 1$ the optimal deterministic allocation satisfies $\mu^M(\omega) = \max\{\omega, \alpha_0\}$, and that if $\alpha_0 \geq 1$ the optimal deterministic allocation is independent from ω. Proposition 3 in this paper extends their result to stochastic mechanisms and distribution functions that are not log-concave.

The result that Assumption 1 is sufficient for the optimality of deterministic mechanism is analogous to the result in Strausz [37] for the principal-agent model with

monetary transfers. Strausz demonstrates that if an optimal deterministic mechanism includes no bunching, then this mechanism is also optimal among stochastic mechanisms. In that environment, the bunching does not occur if a monotonicity constraint similar to (IC_1) can be relaxed. In our setting, Assumption 1 guarantees that (IC_1) can be ignored for $\omega \in (\alpha_0, \beta_0)$.

Section 5 in Krishna and Morgan [27] (henceforth, KM) studies optimal deterministic allocations in the setting with monetary transfers, single-peaked payoff functions, and a constant bias. They describe qualitative properties of the optimal allocation and explicitly characterize it for the case of quadratic preferences and a uniform distribution. The formal structure of our models are closely related. In their model, the principal's and agent's payoffs are given by

$$u_p(\mu, \omega) - \tau \quad \text{and} \quad u_a(\mu, \omega, b) + \tau,$$

where u_p, u_a are single-peaked, ω is the agent's private information, b is the agent's bias, μ is the implemented decision, and τ is a positive transfer from the principal to the agent. In Section 4 in our paper, the principal's and agent's payoffs are given by

$$u_p(\mu, z(\omega)) - \tau \quad \text{and} \quad u_a(\mu, \omega) - \tau,$$

where u_p, u_a are quadratic, ω is the agent's private information, $z(\omega)$ is the most preferred alternative of the principal, μ is the expected implemented decision, and τ is the variance of the implemented decision. Hence, in our model τ is a cost imposed on both players, whereas in KM τ is a (positive) payment from the principal to the agent. KM demonstrate that payments to the agent may improve the principal's expected payoff. By contrast, Proposition 3 in this paper shows that under Assumption 1 and quadratic preferences the principal cannot improve her expected payoff if costs are imposed on both players.[10]

A Proofs omitted in the text

Proof of Lemma 1. Let M be an incentive-compatible allocation. Select any $\omega, \omega' \in \Omega$. By incentive compatibility,

$$\begin{aligned} -[\mu^M(\omega) - \omega]^2 - \tau^M(\omega) &\geq -[\mu^M(\omega') - \omega]^2 - \tau^M(\omega'), \\ -[\mu^M(\omega') - \omega']^2 - \tau^M(\omega') &\geq -[\mu^M(\omega) - \omega']^2 - \tau^M(\omega). \end{aligned}$$

Adding the above inequalities gives

$$[\mu^M(\omega) - \mu^M(\omega')]\,(\omega - \omega') \geq 0,$$

which implies (IC_1). Because μ^M is non-decreasing on Ω, the derivative of the agent's payoff with respect to ω,

$$\frac{\partial U_a^M(\omega, \omega')}{\partial \omega} = 2[\mu^M(\omega') - \omega]$$

[10] It is known, however, that if the principal cannot commit to a mechanism, imposing costs only on the agent may improve the payoffs of both players (Austen-Smith and Banks [7] and Kartik [20]).

is uniformly bounded.[11] Therefore, the integral form envelope theorem (Milgrom, 2004, Theorem 3.1) implies

$$U_a^M(\omega,\omega) = U_a^M(0,0) + \int_0^\omega \frac{\partial U_a^M(s,s')}{\partial s}|_{s'=s}\,\mathrm{d}s \quad \text{for all } \omega \in \Omega. \tag{19}$$

We obtain (IC_2) by substituting (1) with $\omega' = \omega$ into (19). Finally, condition (IC_3) means that variance of $M(\omega)$ must be non-negative.

Now assume that (IC_1)–(IC_3) are satisfied. By substituting (IC_2) with $\omega = \omega'$ into (1), we obtain

$$U_a^M(\omega,\omega') = U_a^M(0,0) - \omega^2 + 2\mu^M(\omega')(\omega-\omega') + 2\int_0^{\omega'} \mu^M(s)\,\mathrm{d}s \quad \text{for all } \omega,\omega' \in \Omega,$$

Therefore,

$$U_a^M(\omega,\omega) - U_a^M(\omega,\omega') = 2\int_{\omega'}^{\omega} [\mu^M(s) - \mu^M(\omega')]\,\mathrm{d}s \quad \text{for all } \omega,\omega' \in \Omega, \tag{20}$$

By monotonicity of μ^M the right hand side of (20) is non-negative. □

Proof of Proposition 2. Let M be an incentive-compatible deterministic allocation. Define $\bar\omega = \min\{b,1\}$ and consider a function $\tilde\varepsilon : \mathbb{R}\times[0,\bar\omega] \to \mathbb{R}$ such that

$$\tilde\varepsilon(p,\omega) = -\int_0^\omega |p-(s-b)|f(s)\,\mathrm{d}s - \int_\omega^{\bar\omega}(b-s)f(s)\,\mathrm{d}s$$

for all $p \in \mathbb{R}$ and all $\omega \in [0,\bar\omega]$.

By (IC_1), the function μ^M is non-decreasing and the limit $m = \lim_{\omega\to 0+}\mu^M(\omega)$ exists. If $m \geq 0$, it follows from (IC_1) that $\mu^M(\omega) \geq 0$ for all $(0,\bar\omega]$. In this case, $u_p\big(\mu^M(\omega),\omega\big) = -|\mu^M(\omega)-(\omega-b)| \leq b-\omega$ for all $\omega \in [0,\bar\omega]$. Therefore, the principal's expected payoff is bounded from above by $-\int_0^{\bar\omega}(b-s)f(s)\,\mathrm{d}s = \tilde\varepsilon(0,0)$.

Let $m < 0$. Then, $\mu^M(\omega) < 0$ for some $\omega \in (0,\bar\omega]$. Now define

$$\omega^* = \sup\{\omega \in [0,\bar\omega] : \mu^M(\omega) < 0\}.$$

Clearly, $\omega^* > 0$. Furthermore, incentive compatibility implies that $\mu^M(\omega) = m$ for all $\omega \in (0,\omega^*)$.[12] Finally, if $\omega^* < \bar\omega$, then $\mu^M(\omega) \geq 0$ for all $\omega \in (\omega^*,\bar\omega]$. Therefore, the principal's expected payoff is bounded from above by

$$-\int_0^{\omega^*} |m-(s-b)|f(s)\,\mathrm{d}s - \int_{\omega^*}^{\bar\omega}(b-s)f(s)\,\mathrm{d}s = \tilde\varepsilon(m,\omega^*).$$

[11] The lower bound is $2[\mu^M(0)-1]$ and the upper bound is $2\mu^M(1)$.

[12] Assume otherwise. Then there exists ω', $0 \leq \omega' < \omega$, such that $\mu^M(\omega') < \mu^M(\omega) < 0$. This implies $\mu^M(\omega') - \omega' < \mu^M(\omega) - \omega' < 0$ and, hence, $U_a(\omega',\omega') < U_a(\omega',\omega)$ in contradiction with incentive compatibility.

The function $\tilde{\varepsilon}$ is continuous and *negative* on $\mathbb{R}\times[0,\bar{\omega}]$. Let $\bar{\varepsilon}$ denote its maximum on the (compact) set $[-b,0]\times[0,\bar{\omega}]$. Clearly, $\bar{\varepsilon}<0$. Furthermore, $\tilde{\varepsilon}(p,\omega)<\tilde{\varepsilon}(-b,\omega)\leq\bar{\varepsilon}$ for all $p<-b$ and all $\omega\in[0,\bar{\omega}]$. Therefore, $\bar{\varepsilon}<0$ is the minimum of $\tilde{\varepsilon}$ on $(-\infty,0]\times[0,\bar{\omega}]$ and is an upper bound on the principal's expected payoff of the set of deterministic incentive-compatible allocations. □

Proof of Lemma 2. We adopt the standard notation, where superscript "+" at function's argument denotes the limit from the right and superscript "−" denotes the limit from the left. For example, $\mu^{M'}(0^+)=\lim_{\omega\to 0^+}\mu^{M'}(\omega)$.

Let M' be an allocation that satisfies (IC_1)–(IC_3) for all $\omega\in(0,1)$. By monotonicity and continuity of the integral in (IC_2), the limits $\mu^{M'}(0^+)$, $\tau^{M'}(0^+)$, $\mu^{M'}(1^-)$, and $\tau^{M'}(1^-)$ exist. Moreover, (IC_2) implies that

$$-[\mu^{M'}(1^-)-1]^2-\tau^{M'}(1^-)=-[\mu^{M'}(1)-1]^2-\tau^{M'}(1).$$

This together with (IC_3) gives

$$\tau^{M'}(1)=\tau^{M'}(1^-)+[\mu^{M'}(1)-\mu^{M'}(1^-)]\,[2-\mu^{M'}(1^-)-\mu^{M'}(1)]\geq 0. \tag{21}$$

Conversely, it is straightforward to establish that (21) implies that (IC_1)–(IC_3) hold for $\omega=1$. Similarly we can show that (IC_1)–(IC_3) hold for $\omega=0$ if and only if

$$\tau^{M'}(0)=\tau^{M'}(0^+)+[\mu^{M'}(0^+)-\mu^{M'}(0)]\,[\mu^{M'}(0^+)+\mu^{M'}(0)]\geq 0. \tag{22}$$

Now let M be an incentive-compatible allocation and M^c be an allocation that satisfies

$$\begin{array}{lll}\mu^{M^c}(\omega)=\mu^M(\omega) & \text{and} \quad \tau^{M^c}(\omega)=\tau^M(\omega) & \text{for all } \omega\in(0,1),\\ \mu^{M^c}(0)=\mu^M(0^+), & \tau^{M^c}(0)=\tau^M(0^+), & \\ \tau^{M^c}(1)=\tau^M(1^-), & \mu^{M^c}(1)=\mu^M(1^-). & \end{array}$$

Allocation M^c satisfies conditions (21) and (22) and is, therefore, incentive-compatible. Furthermore, it satisfies conditions (i)–(iii) by construction. □

Proof of Lemma 3. Because $M\in\mathcal{M}^c$, this allocation is incentive-compatible. By construction the allocation $\overline{M}$ satisfies (IC_1) and (IC_3). In order to verify that $\overline{M}$ satisfies (IC_2) we rewrite it as

$$-[\mu^M(\omega)]^2-\tau^M(\omega)+[\mu^M(0)]^2+\tau^M(0)=2\int_0^\omega\mu^M(s)\,\mathrm{d}s-2\omega\mu^M(\omega). \tag{IC_2'}$$

First, let $\omega\leq\alpha_0$. In this case $\overline{M}$ satisfies (IC_2) because case both sides of (IC_2') are equal to zero. Second, let $\alpha_0<\omega\leq\beta_0$. Subtracting ($\mathrm{IC}_2'$) for allocation M with the state ω and the state $\omega=\alpha_0$ and using $\overline{M}(0)=\overline{M}(\alpha_0)$, we obtain that ($\mathrm{IC}_2'$) is satisfied for $\overline{M}$. Finally, if $\omega>\beta_0$, (IC_2') for allocation $\overline{M}$ is equivalent to (IC_2') for allocation M for $\omega=\beta_0$ and hence is satisfied. □

Proof of Lemma 4. The difference of the principal's payoffs from M and $\overline{M}$ is

$$V_p^M - V_p^{\overline{M}} = 2\int_{\beta_0}^{1} [\mu^M(\omega) - \mu^M(\beta_0)]\, g(\omega)\, \mathrm{d}\omega + \\ + 2\int_0^{\alpha_0} [\mu^M(\omega) - \mu^M(\alpha_0)]\, g(\omega)\, \mathrm{d}\omega + V_a^M(0) - V_a^{\overline{M}}(0). \tag{23}$$

We may rewrite the first integral as

$$\int_{\beta_0}^{\beta_1} [\mu^M(\omega) - \mu^M(\beta_0)]\, g(\omega)\, \mathrm{d}\omega + \int_{\beta_1}^{1} [\mu^M(\omega) - \mu^M(\beta_0)]\, g(\omega)\, \mathrm{d}\omega, \tag{24}$$

where β_1 is defined in Appendix B and $\beta_0 \leq \beta_1$ follows from facts (F_6) and (F_8) proven in Appendix B. By incentive compatibility, $\mu^M(\omega)$ is non-decreasing. Furthermore, g is positive on $[\beta_0, \beta_1)$ and negative on $(\beta_1, 1]$ by fact (F_1) proven in Appendix B. Therefore, the first integral in (24) is lower than or equal to $\int_{\beta_0}^{\beta_1} [\mu^M(\beta_1) - \mu^M(\beta_0)]\, g(\omega)\, \mathrm{d}\omega$, and the second integral in (24) is lower than or equal to $\int_{\beta_1}^{1} [\mu^M(\beta_1) - \mu^M(\beta_0)]\, g(\omega)\, \mathrm{d}\omega$. Moreover, if $\mu^M(\beta_0) < \mu^M(1)$, then $\beta_0 < 1$. In this case, at least one of these inequalities is strict by continuity of μ^M at 1. We have,

$$\int_{\beta_0}^{1} [\mu^M(\omega) - \mu^M(\beta_0)]\, g(\omega)\, \mathrm{d}\omega \leq [\mu^M(\beta_1) - \mu^M(\beta_0)]\, [G(1) - G(\beta_0)] \leq 0,$$

where the first inequality is strict inequality if $\mu^M(\beta_0) < \mu^M(1)$.

We now derive an upper bound for the second part of (23). Observe that $V_a^M(\alpha_0) = V_a^{\overline{M}}(\alpha_0)$ by construction of $\overline{M}$. Thus,

$$V_a^M(0) - V_a^{\overline{M}}(0) \;=\; [V_a^M(0) - V_a^M(\alpha_0)] - [V_a^{\overline{M}}(0) - V_a^{\overline{M}}(\alpha_0)] = \\ \overset{(\mathrm{IC}_2)}{=} -2\int_0^{\alpha_0} [\mu^M(\omega) - \mu^M(\alpha_0)]\, \mathrm{d}\omega.$$

This gives

$$\int_0^{\alpha_0} [\mu^M(\omega) - \mu^M(\alpha_0)]\, g(\omega)\, \mathrm{d}\omega + V_a^M(0) - V_a^{\overline{M}}(0) = \\ = \int_0^{\alpha_0} [\mu^M(\omega) - \mu^M(\alpha_0)]\, [g(\omega) - 1]\, \mathrm{d}\omega \leq \\ \leq \int_0^{\alpha_0} [\mu^M(\alpha_1) - \mu^M(\alpha_0)]\, [g(\omega) - 1]\, \mathrm{d}\omega = [\mu^M(\alpha_1) - \mu^M(\alpha_0)]\, [G(\alpha_0) - \alpha_0] \leq 0,$$

where the inequality in the second line follows from an argument analogous to the one above. This inequality is strict if $\mu^M(0) < \mu^M(\alpha_0)$. We obtain that $V_p^M - V_p^{\overline{M}} \leq 0$ with the strict inequality if either $\mu^M(0) < \mu^M(\alpha_0)$ or $\mu^M(\beta_0) < \mu^M(1)$. □

Proof of Proposition 4. (i) We will prove a stronger statement that there exists $\bar{\lambda} > 0$ such that $\frac{\mathrm{d}}{\mathrm{d}\omega} g^\lambda(\omega) < 0$ for all $\lambda < \bar{\lambda}$ and $\omega \in [0,1]$. Let m denote the minimum of function f on $[0,1]$. It exists and is positive. By the assumption $|\tilde{z}'(\omega)| \leq K_1$ and $|f'(\omega)| \leq K_2$ for all $\omega \in [0,1]$ and some $K_1, K_2 > 0$. Next, function g^λ is differentiable and

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\omega} g^\lambda(\omega) &= -f(\omega) + \lambda\big[\big(\tilde{z}'(\omega) - 1\big) f(\omega) + \big(\tilde{z}(\omega) - \omega\big) f'(\omega)\big] \leq \\ &\leq -m + \lambda[|K_1 - 1| f(\omega) + |\tilde{z}(\omega) - \omega| K_2].\end{aligned}$$

The function $(K_1 - 1) f(\omega) + |\tilde{z}(\omega) - \omega| K_2$ is continuous on $[0,1]$ and, hence, is bounded; let $K_3 > 0$ be its upper bound. Then, $\frac{\mathrm{d}}{\mathrm{d}\omega} g^\lambda(\omega) < -\frac{1}{2} m + \lambda K_3$. Setting $\bar{\lambda} = \min\{1, m/(2K_3)\}$ completes the proof.

(ii) If $\tilde{z}(0) > 0$, then $z^\lambda(0) > 0$ for all $\lambda \in [0,1]$. Therefore, $\alpha_0^\lambda > 0$ by (F_7) and (F_8) proven in Appendix B. Furthermore, α_0^λ solves $G^\lambda(\omega) = \omega$ by definition. For $\omega > 0$, this equation can be rewritten as

$$H(\omega, \lambda) = 0, \qquad \text{where} \qquad H(\omega, \lambda) = \frac{\int_0^\omega F(s)\,\mathrm{d}s}{\int_0^\omega [\tilde{z}(s) - s] f(s)\,\mathrm{d}s} - \lambda. \tag{25}$$

For $\omega = 0$ we define $H(0, \lambda) = -\lambda$. (This extension is continuous.[13]) Part (i) and (F_5) proven in Appendix B imply that (25) has a unique solution for $\lambda < \bar{\lambda}$. Next,

$$\frac{\partial}{\partial \omega} H(\omega, \lambda) = \frac{F(\omega)}{\int_0^\omega [\tilde{z}(s) - s] f(s)\,\mathrm{d}s} - \frac{[\tilde{z}(\omega) - \omega] f(\omega) \int_0^\omega F(s)\,\mathrm{d}s}{\big(\int_0^\omega [\tilde{z}(s) - s] f(s)\,\mathrm{d}s\big)^2}. \tag{26}$$

After substitution $\omega = \alpha_0^\lambda$ and using that $H(\alpha_0^\lambda, \lambda) = 0$, we obtain

$$\frac{\partial}{\partial \omega} H(\omega, \lambda)\big|_{\omega = \alpha_0^\lambda} = \frac{1 - g^\lambda(\alpha_0^\lambda)}{\int_0^{\alpha_0^\lambda} [\tilde{z}(s) - s] f(s)\,\mathrm{d}s}.$$

If $0 < \lambda < \bar{\lambda}$, the denominator equals $\frac{1}{\lambda} \int_0^{\alpha_0^\lambda} F(s)\,\mathrm{d}s > 0$. The numerator is positive by (F_8) proven in Appendix B. Using the *Implicit function theorem* we obtain

$$\frac{\mathrm{d}\alpha_0^\lambda}{\mathrm{d}\lambda} = -\frac{\frac{\partial}{\partial \lambda} H(\omega, \lambda)\big|_{\omega = \alpha_0^\lambda}}{\frac{\partial}{\partial \omega} H(\omega, \lambda)\big|_{\omega = \alpha_0^\lambda}} > 0.$$

(iii) The proof is analogous to part (ii).

(iv) Since $\lambda = 0$ implies $\alpha_0^\lambda = 0$, it remains to show that α_0^λ is continuous in $\lambda = 0$. Applying *L'Hospital rule* to (26), it is straightforward to verify that $\frac{\partial}{\partial \omega} H(\omega, \lambda)\big|_{\substack{\lambda = 0 \\ \omega = 0}} = 1/[2\tilde{z}(0)] \neq 0$. The remainder of the argument follows from the *Implicit function theorem.* □

[13] Using *L'Hospital rule* we obtain $\lim_{\omega \to 0} H(\omega, \lambda) = \lim_{\omega \to 0} F(\omega) / \big[\big(\tilde{z}(\omega) - \omega\big) f(\omega)\big] - \lambda = -\lambda$.

B Additional proofs

Let

$$\beta_1 = \max\big(\{\omega \in [0,1) : g(\omega) \geq 0\} \cup \{0\}\big),$$
$$\alpha_1 = \min\big(\{\omega \in (0,1] : g(\omega) \leq 1\} \cup \{1\}\big).$$

There are several useful facts about $\alpha_0, \beta_0, \alpha_1$, and β_1.

(F_1) If $\omega \in [0, \beta_1)$, then $g(\omega) > 0$, and if $\omega \in (\beta_1, 1]$, then $g(\omega) < 0$. Similarly, if $\omega \in [0, \alpha_1)$, then $g(\omega) > 1$, and if $\omega \in (\alpha_1, 1]$, then $g(\omega) < 1$.

(F_2) $\beta_1 = 1$ is equivalent to $g(1) = [z(1) - 1]f(1) \geq 0$, or $z(1) \geq 1$. Similarly, $\alpha_1 = 0$ is equivalent to $g(0) = 1 + z(0)f(0) \leq 1$, or $z(0) \leq 0$.

(F_3) $\beta_1 = 0$ is equivalent to $g(\omega) < 0$ for all $\omega \in (0, 1]$. Similarly, $\alpha_1 = 1$ is equivalent to $g(\omega) > 1$ for all $\omega \in [0, 1)$.

(F_4) If $0 < \beta_1 < 1$, then $g(\beta_1) = 0$. Similarly, if $0 < \alpha_1 < 1$, then due to continuity $g(\alpha_1) = 1$. Moreover, in both these cases $\alpha_1 < \beta_1$.

(F_5) If $\omega \in [0, \beta_0)$, then $G(\omega) < G(1)$, and if $\omega \in (\beta_0, 1)$, then $G(\omega) > G(1)$. Similarly, if $\omega \in (0, \alpha_0)$, then $G(\omega) > \omega$, and if $\omega \in (\alpha_0, 1]$, then $G(\omega) < \omega$.

(F_6) If $z(1) \geq 1$, then $\beta_0 = 1$ and $\beta_1 = 1$. Similarly, if $z(0) \leq 0$, then $\alpha_0 = 0$ and $\alpha_1 = 0$.

(F_7) If $\mathbb{E}z(\omega') \leq 0$, then $\beta_0 = 0$. Similarly, if $\mathbb{E}z(\omega') \geq 1$, then $\alpha_0 = 1$.

(F_8) If $z(1) < 1$ and $\mathbb{E}z(\omega') > 0$, then $\beta_0 \in (0, 1)$. Moreover, in this case $\beta_0 < \beta_1$ or equivalently $g(\beta_0) > 0$, and $G(\beta_0) = G(1)$ or equivalently $\mathbb{E}[z(\omega') \mid \omega' \geq \beta_0] = \beta_0$. Similarly, if $z(0) > 0$ and $\mathbb{E}z(\omega') < 1$, then $\alpha_0 \in (0, 1)$. Moreover, in this case $\alpha_1 < \alpha_0$ or equivalently $g(\alpha_0) < 1$, and $G(\alpha_0) = \alpha_0$ or equivalently $\mathbb{E}[z(\omega') \mid \omega' \leq \alpha_0] = \alpha_0$.

(F_9) $\alpha_0 \geq \beta_0$ if and only if $\mathbb{E}[z(\omega') \mid \omega' \geq \omega] \leq \omega \leq \mathbb{E}[z(\omega') \mid \omega' \leq \omega]$ for some $\omega \in [0, 1]$.

Proofs of (F_1)–(F_9). We will prove (F_1)–(F_8) for β_1 and β_0. The proofs for α_1 and α_0 are analogous.

(F_1) The second part follows directly from the definition of β_1. The first part clearly holds if $\beta_1 = 0$. Let $\beta_1 > 0$. By continuity of g, we have $g(\beta_1) \geq 0$. Then Assumption 1 implies that there is $\omega' \in (0, \beta_1)$ such that $g(\omega) > 0$ for all $\omega \in [\omega', \beta_1)$. Assume now (by contradiction) that the set $\{\omega \in [0, \omega'] : g(\omega) \leq 0\}$ is non-empty. This set is bounded and closed. Therefore, it is compact and has a maximal element ω''. By continuity of g, we have $g(\omega'') = 0$. Then $\omega'' < \omega'$ and g is decreasing at ω'' by Assumption 1. This is a contradiction with maximality of ω''.

(F_2) Since $f(1) > 0$, then $g(1) \geq 0$ is equivalent to $z(1) \geq 1$. The equivalence between $\beta_1 = 1$ and $g(1) \geq 0$ follows from (F_1).

(F_3) The equivalence follows from (F_1).

(F_4) This statement follows from (F_1).

(F_5) The first inequality follows from definition of β_0. Clearly, $G(\beta_0) \geq G(1)$. Let $G(\omega') \leq G(1)$ for some $\omega' \in (\beta_0, 1)$. Then by the *Lagrange mean value theorem* there exists $\omega'' \in (\beta_0, \omega')$ such that $g(\omega'') = [G(\omega') - G(\beta_0)]/(\omega' - \beta_0) \leq 0$. Then ($F_1$) implies that G is decreasing on $(\omega'', 1]$, which is a contradiction with $G(\omega') \leq G(1)$.

(F_6) By (F_2), the inequality $z(1) \geq 1$ implies $\beta_1 = 1$. Then, by (F_1), function G is increasing on the whole interval $[0, 1]$. Therefore, $G(\omega) < G(1)$ for all $\omega \in [0, 1)$, which implies $\beta_0 = 1$.

(F_7) This follows from the fact that $G(0) = 0$, $G(1) = \mathbb{E}z(\omega')$, and the definition of β_0.

(F_8) By (F_2), we have that $\beta_1 < 1$ if $z(1) < 1$. Since $\mathbb{E}z(\omega') < 1$, then $\beta_0 > 0$ and $G(\omega) < G(\beta_0)$ for all $\omega \in [0, \beta_0)$ by (F_5). Therefore, G is non-decreasing at β_0, which implies $g(\beta_0) \geq 0$. Thus, $\beta_0 \leq \beta_1 < 1$. The equality $G(\beta_0) = G(1)$ follows from continuity. It remains to show that $\beta_0 < \beta_1$. If $\beta_0 = \beta_1$, then G is decreasing on $(\beta_0, 1]$ by (F_1). This is a contradiction with $G(\beta_0) \geq G(1)$.

(F_9) By the definitions of α_0 and β_0, we have that $\alpha_0 \geq \beta_0$ if and only if $G(\omega) \geq \max\{G(1), \omega\}$. This is equivalent to $\mathbb{E}[z(\omega') \mid \omega' \geq \omega] \leq \omega \leq \mathbb{E}[z(\omega') \mid \omega' \leq \omega]$.

□

References

[1] Ricardo Alonso, *Shared control and strategic communication*, Mimeo, November 2006.

[2] Ricardo Alonso, Wouter Dessein, and Niko Matouschek, *When does coordination require centralization?*, Mimeo, July 2006.

[3] Ricardo Alonso and Niko Matouschek, *Optimal delegation*, Mimeo, Northwestern University, January 2006.

[4] ———, *Relational delegation*, Mimeo, Northwestern University, July 2006.

[5] Attila Ambrus and Satoru Takahashi, *Multi-sender cheap talk with restricted state space*, Mimeo, July 2006.

[6] Richard J. Arnott and Joseph E. Stiglitz, *Randomization with asymmetric information*, RAND Journal of Economics **19** (1988), 344–62.

[7] David Austen-Smith and Jeffrey S. Banks, *Cheap talk and burned money*, Journal of Economic Theory **91** (2000), 1–16.

[8] Andreas Blume and Oliver Board, *Noisy talk*, Mimeo, July 2006.

[9] Archishman Chakraborty and Rick Harbaugh, *Comparative Cheap Talk*, Journal of Economic Theory (forthcoming).

[10] Vincent P. Crawford and Joel Sobel, *Strategic information transmission*, Econometrica **50** (1982), 1431–51.

[11] Wouter Dessein, *Authority and communication in organizations*, Review of Economic Studies **69** (2002), 811–38.

[12] Wouter Dessein and Tano Santos, *Adaptive organizations*, Journal of Political Economy **114** (2006), no. 5, 956–995.

[13] Chirantan Ganguly and Indrajit Ray, *Cheap talk: Basic models and new developments*, Mimeo, February 2006.

[14] ———, *On mediated equilibria of cheap-talk games*, Mimeo, August 2006.

[15] Maria Goltsman and Gregory Pavlov, *Mediated cheap talk*, Mimeo, October 2006.

[16] Sidartha Gordon, *Informative cheap talk equilibria as fixed points*, Mimeo, October 2006.

[17] Bengt Holmström, *On incentives and control in organizations*, PhD thesis, Stanford University, 1977.

[18] ———, *On the theory of delegation*, Bayesian Models in Economic Theory (M. Boyer and R. E. Kihlstrom, eds.), North-Holland, 1984, pp. 115–41.

[19] Maxim Ivanov, *Informational control and organizational design*, Mimeo, October 2006.

[20] Navin Kartik, *A note on cheap talk and burned money*, Journal of Economic Theory (forthcoming).

[21] Navin Kartik, Marco Ottaviani, and Francesco Squintani, *Credulity, lies, and costly talk*, Journal of Economic Theory (forthcoming).

[22] Kohei Kawamura, *Constrained communication with multiple agents: Anonymity, equal treatment, and public good provision*, Mimeo, October 2006.

[23] Daniel Kraehmer, *Message-contingent delegation*, Journal of Economic Behavior and Organization (forthcoming).

[24] Vijay Krishna and John Morgan, *Asymmetric information and legislative rules: Some amendments*, American Political Science Review **95** (2001), 435–52.

[25] ———, *A model of expertise*, Quarterly Journal of Economics **116** (2001), no. 2, 747–775.

[26] ———, *The art of conversation: eliciting information from experts through multi-stage communication*, Journal fo Economic Theory **117** (2004), 147–179.

[27] ———, *Contracting for information under imperfect commitment*, Mimeo, February 2006.

[28] Ming Li, *To disclose or not to disclose: Cheap talk with uncertain biases*, Mimeo, August 2004.

[29] Tao Li, *The messenger game: strategic information transmission through legislative committees*, Mimeo, May 2006.

[30] David Martimort and Aggey Semenov, *Continuity in mechanism design without transfers*, Economics Letters **93** (2006), no. 2, 182–189.

[31] Nahum D. Melumad and Toshiyuki Shibano, *Communication in settings with no transfers*, RAND Journal of Economics **22** (1991), 173–98.

[32] John Morgan and Phillip C. Stocken, *An analysis of stock recommendations*, The RAND Journal of Economics **34** (2003), no. 1, 183–203.

[33] Stephen Morris, *Political Correctness*, Journal of Political Economy **109** (2001), no. 2, 231–265.

[34] Roger B. Myerson, *Optimal auction design*, Mathematics of Operations Research **6** (1981), no. 1, 58–73.

[35] Marco Ottaviani and Francesco Squintani, *Naive audience and communication bias*, International Journal of Game Theory (forthcoming).

[36] Joseph E. Stiglitz, *Pareto efficient and optimal taxation and the new welfare economics*, Handbook of Public Economics (A. Auerbach and M. Feldstein, eds.), vol. II, North-Holland, Amsterdam, 1987, p. 9911042.

[37] Roland Strausz, *Deterministic versus stochastic mechanisms in principal-agent models*, Journal of Economic Theory **128** (2006), 306–14.

[38] Dezso Szalay, *The economics of extreme options and clear advice*, Review of Economic Studies **72** (2005), 1173–98.

[39] Jordi Blanes i Vidal, *Credibility and strategic communication: Theory and evidence from securities analysts*, Mimeo, May 2006.