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Improving Public Good Supply and Income Equality: Facing a Trade-Off

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Abstract

It is explored in this paper how – depending on the agents' preferences – an unequal income distribution may lead to a higher public good supply in a non-cooperative Nash equilibrium than in a cooperative Lindahl equilibrium that arises from a balanced income distribution. The degree of inequality that is needed for producing this result may be moderate what in particular is shown through an example with CES preferences.

JEL-Codes: D310, H410, Q540.

Keywords: public good, Nash equilibrium, Lindahl equilibrium, income distribution.

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1. Introduction

When public goods are provided non-cooperatively redistribution of income among agents has consequences that, at least at first sight, are unexpected and surprising. On the hand, in a standard public good economy redistribution of income among agents that actively contribute to the public good will neither affect the levels of public supply nor the levels of private consumption of the contributors – which is the famous Warr neutrality (see Warr, 1983, and Cornes and Sandler, 1996). On the other hand, redistribution of income can in some situations – especially when there are non-contributing agents in the Nash equilibrium (see Bergstrom, Blume and Varian, 1986, Itaya, de Meza and Myles, 1997, and Cornes and Sandler, 2000) or when productivity differentials between the agents exist (see Cornes, 1993, or Buchholz and Konrad, 1995) – lead to increases in total public good supply and even to a Pareto improvement in the Nash equilibrium.

In this paper, we will add to this literature another not obvious outcome by showing how less equality of the income distribution can lead to a higher level of public good supply in the non-cooperative Nash equilibrium as compared to the cooperative Lindahl equilibrium. Hence, a trade-off between distributional equity and a high public good supply may arise. Our results moreover help to deepen the understanding of the factors that may cause an “overprovision anomaly” in public good economies, which means that some Nash equilibria exhibit a higher public good supply than some Pareto optimal allocations. In Buchholz and Peters (2001) this phenomenon has been described by simply comparing extremely skewed income distributions for which the ensuing Nash equilibria are Pareto optimal themselves. While it has been shown there that the income distribution matters a lot for the causation of the overprovision anomaly it was not explored whether the anomaly is triggered by more or less income inequality. In this paper we will deal with this topic, in particular showing that – unlike in Buchholz and Peters (2001) but similar to Kleinberg and Ma (2020) – specific assumptions on the agents’ preferences have to be taken into account.

The paper will be organized as follows: After exposing the framework of the analysis in Section 2, Section 3 presents our main result, i.e. how – due to a specific property of the underlying preferences – public good supply in the Nash equilibrium after an inequality increasing income shift may become higher than in the Lindahl equilibrium that results for the income distribution before the shift. The income redistribution that may produce this outcome need not be too extreme, which is shown by an example with CES preferences in Section 4. Section 5 concludes by providing an interpretation of the theoretical results.

2. The framework

There are n agents $i = 1, \dots, n$ that have utility functions $u^i(x_i, G)$ where x_i is agent i 's private consumption and G is public good supply. Each utility function is twice differentiable and strictly monotone increasing in both variables, and both goods are strictly non-inferior for each agent. The public good is produced by a summation technology for which the marginal rate of transformation mrt between the public good and the private good (i.e. the technically given price of the public good in terms of the private good) is equal to one for each agent.

If agent i is endowed with income w_i , which is measured in units of the private good, her standalone level of public good supply is $G_i(w_i) = \arg \max_{G \geq 0} u^i(w_i - G, G)$. If total income

$W := \sum_{i=1}^n w_i$ were completely assigned to some agent i she would choose $\bar{G}_i := G_i(W)$ as public

good supply. Such “dictator allocations” with public good supply $\bar{G}_i$ clearly are Nash equilibria. We will now confront them with the Lindahl equilibria $(x_1^L, \dots, x_n^L, G^L)$, which under our assumptions exist and are unique for any given income distribution $(w_1, \dots, w_n)$. In a Lindahl equilibrium each agent i will choose public good supply G^L as a hypothetical price-taker being confronted with an adequately chosen personalized public good price p_i^L , which clearly differs from the technically given $mrt = 1$. This means that $G^L = \arg \max_G u^i(w_i - p_i^L G, G)$ holds for each agent $i = 1, \dots, n$. Lindahl equilibria are Pareto optimal (see, e.g., Cornes and Sandler, 1996, pp. 201-204) and have, for a long time, been considered as the most prominent efficient solution in a public good economy. Hence they traditionally serve as an outstanding candidate for the outcome of cooperative public good provision.

In the following, agent i 's marginal rate of substitution between the public and the private good (i.e. her marginal willingness to pay for the public good) at some point (x_i, G) will be

denoted by $m_i(x_i, G) = \frac{\partial u^i / \partial G}{\partial u^i / \partial x_i}(x_i, G)$. We assume that $m_i(0, G) = \lim_{x_i \rightarrow 0} m_i(x_i, G) = 0$ for any G

. In the dictator allocation of agent i we clearly have $m_i(W - \bar{G}_i, \bar{G}_i) = 1 = mrt$, and in a Lindahl

equilibrium $m_i(x_i^L, G^L) = p_i^L$ for all $i = 1, \dots, n$. The Samuelson condition which applies for any interior Pareto optimal public good allocation thus gives $\sum_{i=1}^n p_i^L = 1$.

We will now deal with situations where at least for some agent $i = 1, \dots, n$ the following assumption on her preferences¹ holds:

(A1) The functions $\phi_G^i(x_i) = \frac{x_i}{m_i(x_i, G)}$ are decreasing in x_i for any $G > 0$.

If the utility function of agent i is additively separable, i.e. if $u^i(x_i, G) = f_i(x_i) + g_i(G)$ holds (with $f_i'(x_i) > 0$, $g_i'(x_i) > 0$, $f_i''(x_i) \leq 0$ and $g_i''(x_i) \leq 0$), we get $\phi_G^i(x_i) = \frac{x_i}{f_i'(x_i)}$ so that condition **A1** boils down to $-\frac{f_i''(x_i)x_i}{f_i'(x_i)} > 1$. In this special case condition **A1** is completely independent from the level of public good supply, i.e. from $g_i(G)$.

3. The basic result: The role of preferences

Let $\widehat{\Omega}^L$ be the sub-group of agents, for which $\bar{p}_i^L \geq \frac{1}{n}$ holds. Based on **A1** we then get the following result:

Proposition 1: Let $(\widehat{x}_1^L, \dots, \widehat{x}_n^L, \widehat{G}^L)$ be the Lindahl equilibrium with personalized public good prices $(\bar{p}_1^L, \dots, \bar{p}_n^L)$ that is obtained for an equal income distribution $(\frac{W}{n}, \dots, \frac{W}{n})$ among the n agents. If there is an agent $k \in \widehat{\Omega}^L$ whose preferences meet the condition **A1** then $\widehat{G}^L < \bar{G}_k$ holds.

Proof: In a x_k - G -diagram (see Figure 1) agent k 's standalone allocation is given by the point $A_k = (W - \bar{G}_k, \bar{G}_k)$, which lies on the budget line with slope -1 that connects $(W, 0)$ and $(0, W)$. We now draw another budget line with slope $-\frac{1}{n}$, which starts at point $(\frac{W}{n}, 0)$ on the

¹ This condition is equivalent to complementarity between the public and the private good given that the public good serves as the numéraire (see Buchholz and Rübbelke, 2018). If $m(0, G) = 0$ it is also implied by convexity of $m(x_i, G)$ as a function of x_i , which is a central assumption on preferences in Kleinberg and Ma (2020).

x_k -axis and which intersects the G -axis at $(0, W)$, and then look at the point $P_k = (\tilde{x}_k, \bar{G}_k) = (\frac{W - \bar{G}_k}{n}, \bar{G}_k)$ on this line.

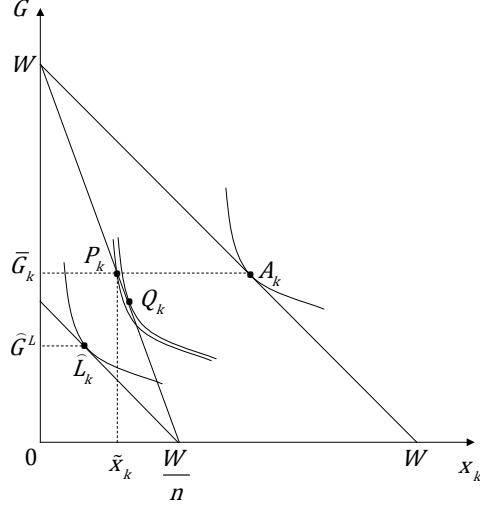


Figure 1: Comparing Lindahl equilibria with dictator allocations

As $\tilde{x}_k < W - \bar{G}_k$ condition **A1** gives

$$(1) \quad \frac{\tilde{x}_k}{m_k(\tilde{x}_k, \bar{G}_k)} > \frac{W - \bar{G}_k}{m_k(W - \bar{G}_k, \bar{G}_k)} = \frac{W - \bar{G}_k}{1},$$

which implies $m_k(\tilde{x}_k, \bar{G}_k) < \frac{\tilde{x}_k}{W - \bar{G}_k} = \frac{1}{n}$. Agent k 's optimum Q_k on the budget line connecting

$(\frac{W}{n}, 0)$ and $(0, W)$, where public good supply is $\tilde{G}_k := \arg \max_G u_k(\frac{W - G}{n}, G)$, therefore must

lie below P_k so that $\tilde{G}_k < \bar{G}_k$. Now consider agent k 's Lindahl position $\hat{L}_k = (\hat{x}_k^L, \hat{G}_k^L)$, which

lies on the budget line with slope $-\frac{1}{\hat{p}_k}$ passing through $(\frac{W}{n}, 0)$. As $\hat{p}_k \geq \frac{1}{n}$ has been assumed

this budget line is flatter (or at least not steeper) than the line between $(\frac{W}{n}, 0)$ and $(0, W)$.

Hence, as the public good is non-inferior and thus a non-Giffen good, agent k 's Lindahl point

$\widehat{L}_k$ is situated below Q_k , i.e. $\widehat{G}_L \leq \widetilde{G}_k$. Combining $\widetilde{G}_k < \overline{G}_k$ and $\widehat{G}^L \leq \widetilde{G}_k$ yields $\widehat{G}^L < \overline{G}_k$ and thus the assertion. QED

From the Samuelson condition $\sum_{i=1}^n \widehat{p}_i^L = 1$ it follows that $\widehat{\Omega}^L$ cannot be empty. Therefore, it is an immediate consequence of Proposition 1 that when all agents have preferences that satisfy condition **A1** there exist Nash equilibria in which public good supply is higher than in the Lindahl equilibrium arising from a balanced income distribution.²

In Proposition 1 the symmetric income distribution has been confronted with a quite extreme income distribution in which one agent holds the entire amount of aggregate income while all other agents have no income and thus cannot have any positive private consumption. An additional argument, however, shows that the result of Proposition 1 may also hold for Nash equilibria that are based on a much less skewed income distribution.

Proposition 2: If the assumptions of Proposition 1 are fulfilled then there is a critical income level $\underline{w}_k < W$ for agent k so that Lindahl public good supply $\widehat{G}^L$ is smaller than public good supply in all Nash equilibria that arise for income distributions $(w_1, \dots, w_n)$ which have $w_k > \underline{w}_k$.

Proof: Agent k 's standalone public good supply $G_k(w_k)$ is continuous and – given non-inferiority – strictly monotone increasing in w_k . Since under the given assumptions we have $\overline{G}_k = G_k(W) > \widehat{G}^L$ by Proposition 1 and, clearly, $\lim_{w_k \rightarrow 0} G_k(w_k) = 0$ the intermediate value theorem yields that there exists a $\underline{w}_k < W$ for which $G_k(\underline{w}_k) = \widehat{G}^L$. From monotonicity of $G_k(w_k)$ we then get $G_k(w_k) > G_k(\underline{w}_k) = \widehat{G}^L$ for all $w_k > \underline{w}_k$. The assertion of Proposition 2 now follows because – as a general result – for all income distributions in which agent k 's income is w_k public good supply in the associated Nash equilibrium can never fall below agent k 's standalone public good supply $G_k(w_k)$. Otherwise, normality of the public good would imply that agent k 's private consumption in such a Nash equilibrium would be lower than in her

² If the functions $\phi_G^i(x_i) = \frac{x_i}{m_i(x_i, G)}$ are not decreasing but *increasing* in x_i it can be shown in an analogous way as in the proof of Proposition 1 that the Lindahl equilibrium arising from a symmetric income distribution is *higher* than in a standalone allocation of some agent. See Proposition 2 in Kleinberg and Ma (2020) on this where **A1** is included in the assumption that $m_i(x_i, G)$ is a concave function of x_i .

standalone allocation and her public good contribution thus would be higher than in her standalone allocation, i.e. higher than $G_k(w_k)$. This, however, is not compatible with having a lower public good supply in such a Nash equilibrium. QED

By a standard continuity argument, the results of Propositions 1 and 2 also hold for Lindahl equilibria, which arise from income distributions that are sufficiently close to the completely symmetric income distribution.

The comparison between cooperative and non-cooperative allocations in Propositions 1 and 2 is based on the implicit assumption that the cooperation leading to the Lindahl equilibrium is actually working. Yet, it is to be feared that cooperation is not stable and might collapse – especially when a large number of agents is involved.³ Then the agents might end up in the Nash equilibrium arising from the balanced income distribution in which not only public good supply is lower than $\widehat{G}_L$ (and thus in $\overline{G}_k$) but also utility of all agents may be lower than in the Nash equilibria arising from skewed income distributions as described in Propositions 1 and 2.⁴

It will now be shown by an example with CES utility functions that the threshold $\underline{w}_k$ described by Proposition 2 may be much lower than total income W . This furthermore implies that the degree of inequality, as measured by the Gini coefficient, that is required to make public good supply in the Nash equilibrium larger than that in the Lindahl equilibrium needs not be too high. This makes the results in Proposition 1 and 2 more relevant.

4. An example

In this example we assume that all agents $i = 1, \dots, n$ have a CES utility function, i.e.

$$(2) \quad u^i(x_i, G) = \frac{1}{1-\eta_i} (x_i^{1-\eta_i} + G^{1-\eta_i}),$$

holds for all $i = 1, \dots, n$ where $\eta_i > 0$ and $\eta_i \neq 1$. (For $\eta_i = 1$ we set $u^i(x_i, G) = \ln x_i + \ln G$). Con-

sequently, $m_i(x_i, G) = \frac{G^{-\eta_i}}{x_i^{-\eta_i}} = \frac{x_i^{\eta_i}}{G^{\eta_i}}$ so that $\varphi_G^i(x_i) = \frac{x_i}{m_i(x_i, G)} = x_i^{1-\eta_i} G^{\eta_i}$. Hence, condition **A1**

³ Stability of Lindahl equilibria is considered in various different contexts, see, e.g. Danziger (1976) and Ythier (1998).

⁴ That for the utility profiles in the Nash equilibria arising from various income distributions some Pareto ranking is possible has been the insight in Cornes and Sandler (2000) who in particular have shown that in this respect corner solutions are generally doing better than interior ones.

is fulfilled for agent i if and only if $\eta_i > 1$. Therefore, if all agents $i = 1, \dots, n$ have CES preferences with $\eta_i > 1$ Proposition 1 and Proposition 2 apply, i.e. there exist Nash equilibria, which have higher a public good supply than the Lindahl equilibria arising from a balanced income distribution. Moreover, it can be seen that the group $\hat{\Omega}^L$ consists of the agents with the smallest η_i : Since $\hat{G}^L$ maximizes $u^i(\frac{W}{n} - p_i^L G, G) = (\frac{W}{n} - p_i^L G)^{1-\eta_i} + G^{1-\eta_i}$ it is characterized by the first-order condition $-p_i^L (\frac{W}{n} - p_i^L \hat{G}^L)^{-\eta_i} + (\hat{G}^L)^{-\eta_i} = 0$, which gives

$$(3) \quad \hat{G}_L = \frac{W}{n((p_i^L)^{\frac{1}{\eta_i}} + p_i^L)}$$

for all agents $i = 1, \dots, n$. Since the right-hand side of (3) is decreasing in p_i^L and – as $p_i^L < 1$ – also in η_i , a smaller η_i must be accompanied by a higher p_i^L to achieve equality of both sides of (3). This shows that the income distribution must be skewed in favor of the agents with a low preference parameter η_i to get a higher public good supply than in the Lindahl equilibrium arising from a balanced income distribution.

In order to determine specific values for the threshold level $\underline{w}_k$ that is defined in Proposition 2 we now assume that all n agents have the same CES utility function with an identical preference parameter $\eta > 1$. Then $p_i^L = \frac{1}{n}$ holds for all $i = 1, \dots, n$ so that (3) gives

$$(4) \quad \hat{G}^L = \frac{W}{1 + n^{\frac{\eta-1}{\eta}}}$$

It directly follows from (4) that in this case $\hat{G}^L$ is decreasing in n and η .

Due to the symmetry of CES preferences for each agent i with income w_i the standalone public good supply of agent $i = 1, \dots, n$ with income w_i is $G_i(w_i) = \frac{w_i}{2}$. The threshold level $\underline{w}_k$ therefore is defined by the condition

$$(5) \quad \hat{G}_L = \frac{W}{1 + n^{\frac{\eta-1}{\eta}}} = \frac{\underline{w}_k}{2} = G_k(\underline{w}_k),$$

which gives

$$(6) \quad \underline{\alpha}(n) = \frac{w_k}{W} = \frac{2}{1 + n^{\frac{\eta-1}{\eta}}}.$$

$\underline{\alpha}(n)$ indicates the share in total income an agent has to have to get the result of Proposition

2. To make this more concrete take $\eta = 2$. Then $\underline{\alpha}(n) = \frac{2}{1 + n^{\frac{1}{2}}}$, which goes to zero if n ap-

proaches infinity. The following table visualizes how $\underline{\alpha}(n)$ depends on the population size n in this case:

n	4	9	16	25	36	49	64	81	100	...	225
$\underline{\alpha}(n)$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{7}$	$\frac{1}{4}$	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{2}{11}$	...	$\frac{1}{8}$

Table 1: Dependence of $\underline{\alpha}(n)$ on n

This numerical example confirms that an extremely skewed distribution of income is not required to yield the result of Proposition 2. Returning to the general case of CES utility functions with an arbitrary $\eta > 1$ this insight is reinforced by looking at the Gini coefficient $\Gamma(n)$ of income distributions in which one agent k has income w_k (and the income share $\underline{\alpha}(n)$) while the rest of income is equally distributed among the remaining $n-1$ agents. (Among all income distributions in which agent k has income w_k this is the income distribution with the smallest Gini coefficient.) As can be observed from Figure 2, this Gini coefficient is calculated as

$$(7) \quad \Gamma(n) = 1 - 2\left(\frac{(n-1)(1-\underline{\alpha}(n))}{2n} + \frac{1-\underline{\alpha}(n)}{n} + \frac{\underline{\alpha}(n)}{2n}\right) = \underline{\alpha}(n) - \frac{1}{n} = \frac{1}{1 + n^{\frac{\eta-1}{\eta}}} - \frac{1}{n}.$$

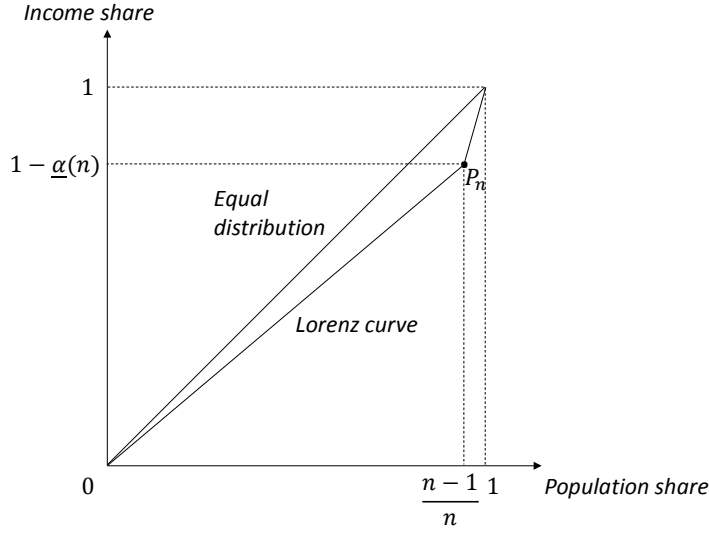


Figure 2: The Lorenz curve for a skewed income distribution

Since $\lim_{n \rightarrow \infty} \Gamma(n) = 0$, it can be concluded that for any $\eta > 1$ there exists – for a sufficiently large population size n – an income distribution with an arbitrarily small Gini coefficient for which the result in Proposition 2 applies. In Figure 2 this is reflected by the fact that the point $P_n = (\frac{n-1}{n}, 1 - \underline{\alpha}(n))$ converges to the 45°-line when n goes to infinity.

To finish the treatment of the example we now explore what happens if the Lindahl equilibrium arising from a balanced income distribution is not stable, but cooperation breaks down and all agents fall back to non-cooperative Nash behavior. Assume, e.g., $\eta = 2$ and $n = 16$ and let a symmetric income distribution be given. Then public good supply $\hat{G}^N$ and each agent's private consumption $\hat{x}^N$ in the corresponding interior Nash equilibrium are

$\hat{G}^N = \hat{x}^N = \frac{W}{n+1} = \frac{W}{17}$ ⁵, which is smaller than $\hat{G}^L = \frac{W}{5}$ according to (4). In the Nash equilibrium

utility of each agent is $\hat{u}^N = -((\frac{W}{17})^{-1} + (\frac{W}{17})^{-1}) = -\frac{34}{W}$.

⁵ This allocation is a Nash equilibrium as it is feasible given total endowment W and $m_i(\bar{x}_N, \hat{G}_N) = \frac{\hat{G}_N^{-2}}{\hat{x}_N^{-2}} = 1 = mrt$ holds for all agents $i = 1, \dots, n$.

For a comparison of utilities we now look at the Nash equilibrium that results when one agent k has the income $\underline{w}_k = \frac{2}{1+16^{\frac{1}{2}}}W = \frac{2}{5}W$ while the rest of the total endowment is equally distributed among the remaining $n-1$ agents each of whom thus has the income $w_i = \frac{1}{15} \cdot \frac{3W}{5} = \frac{W}{25}$. In her standalone allocation agent k then provides $G_k(\underline{w}_k) = \hat{G}^L = \frac{W}{5}$. But being confronted with this public good supply each agent $i \neq k$ has no incentive to make a contribution to the public good⁶ so that agent k 's standalone allocation becomes the (corner) Nash equilibrium in this case. Then each agent $i \neq k$ attains the position $(\frac{W}{25}, \frac{W}{5})$ and has utility $-\left((\frac{W}{25})^{-1} + (\frac{W}{5})^{-1}\right) = -\frac{25+5}{W} = -\frac{30}{W} > -\frac{34}{W} = \hat{u}^N$. Utility of agent k is $-\left((\frac{W}{5})^{-1} + (\frac{W}{5})^{-1}\right) = -\frac{10}{W}$ and thus even larger. This shows that in the special case the Nash equilibrium arising from the skewed income distribution is Pareto superior to the Nash equilibrium arising from the balanced income distribution, which might be attained if cooperation fails and the Lindahl equilibrium breaks down.

5. Interpretation

Proposition 1 and Proposition 2 show how under certain assumptions on preferences a less equal distribution of income may – as a special case for the overprovision anomaly – lead to a higher public good supply in a non-cooperative Nash equilibrium than in a cooperative Lindahl equilibrium. As the example with CES preferences shows not even too much income inequality is needed to produce this outcome. Hence, if increasing the level of public good supply is considered to be of major importance more income inequality may be beneficial: Without cooperation a more unequal income distribution may bring about a better outcome in terms of public good supply than cooperation based on a more equal distribution of income so “diversity may

⁶ This follows as $m_i(\frac{W}{25}, \frac{W}{5}) = \frac{(\frac{W}{5})^{-2}}{(\frac{W}{25})^{-2}} = \frac{5^2}{25^2} \cdot \frac{1}{25} < 1 = mrt$ holds for each agent $i \neq k$ so that marginal willingness

to pay for the public good is smaller than the marginal rate of transformation between the public and the private good.

pay”⁷ also with respect to the income distribution. Cooperation between autonomous agents, moreover, is not self-enforcing. Rather it is prone to deviation and thus instability, which is deemed a major obstacle for overcoming underprovision of global public goods through collective action between autonomous states.⁸ This means that under certain circumstances more income inequality could serve as a substitute for an inherently fragile cooperative agreement. If cooperation breaks down public good supply would be further reduced – and then all agents could become even be worse-off than in the Nash equilibrium based on a more skewed income distribution. Anticipating that cooperative agreements might fail agents therefore might – as a remedy against the instability of collective action – unanimously and voluntarily prefer a more unequal income distribution.

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⁷ This phrase has been coined by Finus and McGinty (2019) concerning stability of public good agreements where diversity refers to the asymmetry of the benefits from and the costs of providing the public good between the agents involved.

⁸ See the seminal contributions of Barrett (1990) and Finus (2001). For a recent discussion of the stability problems in the context of international cooperation on global public goods as climate protection see, e.g., Finus and Caparrós (2017) and Buchholz and Sandler (2020).

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