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Output Distortions and the Choice of Legal Form of Organization

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Output Distortions and the Choice of Legal Form of Organization

Abstract

We study the distortions to aggregate output created by the differential tax treatment of corporations and pass through entities. We develop an industry equilibrium model in which the legal form of organization is an endogenous choice for firms facing trade off between tax treatment of business income, access to external capital, and the evolution of productivity over time. We match this model to features of the US economy. We find that, relative to the benchmark economy, revenue-neutral tax reform in which legal forms receive the same tax treatments leads to 1.25% increase in the aggregate output.

JEL-Codes: E620, H320, H250.

Keywords: output distortions, legal form of organization, pass through.

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1 Introduction

The share of pass-through businesses in total number of businesses in the U.S. has increased substantially since 1986 (Carroll and Joulfaian, 1997; Cooper et al., 2016; Dyrda and Pugsley, 2018; Gordon and Slemrod, 1998; Slemrod, 1996; Smith et al., 2019a,b; Yagan, 2015). They now constitute over 95 percent of tax returns and 38.8 percent of business tax receipts.¹ The Tax Reform Act of 1986 made pass-through legal forms more attractive choices for many businesses in terms of their tax treatment, by reducing the individual tax rate below the combined corporate and divided tax rates. If firms choose a pass-through organizational form just for tax savings reasons, it is important to understand how this choice affects various margins of the economy, such as employment (Chen et al., 2018), corporate sector labor share (Smith et al., 2019b) and output growth. This is because differences between legal forms of organization extend beyond the tax treatment and include limitations on financial structure and availability of external capital. This, in turn, may induce differences in investment decisions, financing decisions and dividend policies. In this paper we show that choosing a legal form of organization based only on the tax treatment, rather than economic features of an organizational form, creates inefficiencies, leads to misallocation of capital and as a consequence lower aggregate output for the economy.

In a pass-through business the profit is passed entirely to the owners and taxed according to the individual income tax code. In a C corporation, profit is taxed first at the business level according to the corporate income tax code. Later, when dividends are paid out or a shareholder realizes capital gain, the shareholder pays dividend income tax or capital gains tax based on individual income tax code. Since 1986 the individual income tax rate has been lower than the combined corporate and dividend/ capital gains tax rates making the tax burden on pass-through entities lower. However, C corporations are the only legal forms of organization that have access to the stock market to generate capital. In turn, capital of pass-through businesses is mainly financed thorough their owners' equity. This increases their cost of capital relative to C corporations (see Chen et al. (2018)). These features give rise to a trade-off between two choices (1) having access to the public equity market, but facing double taxation (for C corporations) or (2) having no access to public equity, but facing only single taxation of profits (for pass-through entities).

In this paper we develop an industry equilibrium model to capture this trade-off. In our model, the change in the legal form of organization leads to changes in the demand for

¹See <https://www.irs.gov/statistics/soi-tax-stats-integrated-business-data>

capital, which in turn influence savings and investments. The key contribution of this paper is to provide a role for the choice of legal form of organization as a mechanism through which changes in the tax legislation distort the capital allocation and aggregate output in the economy. Using this framework we quantify the distortionary effects of the current tax structure on the choice of legal form of organization, the subsequent impact on the allocation of capital among businesses, and the associated effects on real aggregate output of the US economy.

The novelty of our paper is that the legal form of organization is an endogenous choice for firms. The majority of theoretical contributions analysing the distortions created by the corporate tax system take the choice of legal organization form as given. We follow more recent contributions where the legal organization form is endogenous and affects risk sharing and employment (Chen et al., 2018; Kotlikoff and Miao, 2013). Our paper offers broader implications by considering the effects on aggregate output. The production takes place in two types of firms: C-type firms which represent C corporations; and P-type firms which represent pass-through entities. These two types are different with respect to their cost of capital, fixed costs of production, and the potential evolution of their productivity over time. Also, they receive different tax treatments. Profit of a C-type firm is being taxed at the entity level and, once distributed to shareholders, the after-tax profit is taxed based on the individual income tax code. For a P-type firm, the profit passed to the shareholders is subject to individual income tax.

In each period, firms that are heterogeneous with respect to their productivity, decide if they want to stay for the next period or if they want to exit. If they choose to stay, they can choose their type for the next period, taking into consideration all the differences among a C-type and a P-type firm. At the beginning of each period, there is an unlimited supply of new firms that can enter the market by paying entry cost. Once they enter, they realize their productivity level for that period and then they can choose their type. This makes it feasible for the model to incorporate the trade-off between choosing to be a C corporation versus a pass-through entity that generates firm dynamics. This also makes the model consistent with empirical evidence documented by Dyrda and Pugsley (2018), who show that the significant increase in pass-through entities is coming from two sources: a secular increase in the share of new businesses which chooses the legal form of a pass-through entity, offsetting nearly perfectly a decline in the share formed as traditional corporations; and second, an increase in the share of corporations converting to pass-through entities since the tax reform of 1986.

We calibrate the model to match the salient features of the U.S. economy and use it to

evaluate two types of experiments. First, we study the aggregate impact of both types of firms receiving a uniform tax treatment, thus eliminating the tax advantages of the different legal forms of organization. We use three modified tax structures, keeping the calibrated parameters fixed and the government tax revenue constant at its benchmark level. Second, we consider the effects of the Tax Cuts and Jobs Act (TCJA) reform of 2017 on aggregate output of the US economy.

In the first tax structure, we consider taxing both C-type and P-type firms only at the corporate level, keeping labor income taxation the same as in the benchmark economy. We show that the aggregate output of the economy increases by 1.25% compared to the benchmark economy, which is the result of reallocating capital to more productive firms. In the second tax structure, we consider taxing both types of firms at the corporate level with a single tax rate levied on both labor and business incomes. To keep the reform revenue neutral, the imposed labor income tax is higher than in the benchmark and business income tax lower than in the first tax reform. We show that this increases aggregate output by 2% relative to the benchmark case. For the third tax structure, we consider eliminating business tax altogether and taxing dividends from both types of firms at the same rate as labor income which is effectively a uniform consumption tax regime. We find that the aggregate output increases by 10.8%. This result highlights the distortionary effect of taxing capital income.

We then use our model to evaluate the effects of the Tax Cuts and Jobs Act (TCJA) on aggregate output of the US economy. The new flat 21% corporate tax rate for C corporations may induce some pass through entities to convert to C corporations. We show that with a tax structure that mimics features of the TCJA reform more firms choose to organize as C-type and the output share of the C-type increases. Also, because of the lower corporate income tax rate, firms increase their production and the aggregate output of the US economy increases by 7.9% compared to the benchmark case. However, this increase in output comes at the cost of almost 10% decline in the government tax revenue. This highlights the trade-offs of this reform.

The paper closest to ours is Dyrda and Pugsley (2018). They investigate the effects of changes in the dynamics of legal forms of businesses on income inequality. They propose a heterogeneous agent equilibrium model with workers, entrepreneurs and endogenous choice of legal form of organization and quantify the contribution of tax reforms through the business reorganization channel on the evolution of income inequality of workers and entrepreneurs.

More broadly, this paper also contributes to a strand of empirical literature evaluating the effects of taxes on the choice of legal forms of organization. Carroll and Joulfaian (1997)

show that higher corporate-noncorporate tax rate differentials increase the likelihood that a firm will convert from C to S corporation status, where the tax savings are the greatest for profitable firms. schlagenhauf (2019) shows that Kansas business tax cuts in 2012-2016 have encouraged adoption of pass-through status over C-corporation status and this reduced output, capital formation and employment growth in Kansas. Smith et al. (2019b) show that the rise of pass-through entities in the US is associated with a decline in the corporate sector labor share. Using evidence from the UK, Tazhitdinova (2020) shows that different tax liabilities across different organization forms affect business entry and income shifting margins, while Elschner (2013) shows that availability of tax incentives affects incorporation decisions of firms.

2 Legal Forms of Organization of U.S. Businesses

Businesses in the United States can operate in a variety of organizational forms. The choice of legal form of organization usually reflects the need for capital, flexibility and owners protection from the liabilities that the business takes on. Further, the legal form of organization determines the federal level tax burden imposed on the business. The main legal forms of organization in the United states are: sole proprietorship; general partnership; limited partnership; limited liability company; S corporation; and C corporation. In the discussion in this section we focus on characteristics of legal form of organization relevant to our model. In particular, we focus on the differences between pass-through entities and C corporations in terms of their tax treatment and access to external capital. For more detailed discussion of the pass-through entities and their characteristics based of tax returns and census data see (Dyrda and Pugsley, 2018; Smith et al., 2019a,b).

C corporations are subject to corporate income tax at both federal and state levels, any earnings distributed to shareholders as dividends or capital gains are subject to a second level of taxation at personal income tax rates. All other forms of organization are pass-through entities for tax purposes. This means that their income passes through to shareholders so that it is subject to a single level of taxation, at the personal level². Second, C corporations have access to the public equity market, while pass-through entities have to rely on their owner personal funds to finance investments. This is likely to make capital more costly for pass-through entities.³

²Some states, most notably California and New York, recognize the pass-through nature of S- corporations but still impose a tax at the entity level.

³Smith et al. (2019b) show that S-corporations pay a lot of dividends. This suggests that there may

In this paper, we focus on differences between pass-through entities and c-corporations and we do not distinguish between different forms of pass-through entities. However, it is important to note that the main difference between various types of pass-through entities is the liability protection offered. Sole-proprietorship, general partnership and limited partnership do not provide any liability protection for the owner. Limited liability company (LLC) is a hybrid between partnership and S-corporation and enjoys limited liability. S-corporations and C-corporations are different in that these are separate legal entities from its owner. All owners of a corporation enjoy limited liability.

Since 1980, the overall number of tax returns reporting business income has grown from 13 millions to 32.7 millions. The number of C corporation returns have declined by 25 percent, while the total number of corporate returns (C corporation plus S corporations) increased. In Figures ?? and ?? we show the significant changes in the distribution of legal forms of organizations in the US over the past thirty years. The share of C corporations in total number of entities dropped from 16.6 percent in 1980 to 4.9 percent in 2012, while the share of business receipts of C corporations in total business receipts decreased from 86.2 percent to 61.2 over the same period. Over the same period, the share of organizational forms that provide liability protection among all U.S. businesses stay almost constant. Within these forms, the share of pass-through entities increased significantly in term of number of businesses as well as their share of business receipts. Evidence from Figure ?? suggests that the secular change in the roles of organizational forms and economic activities is coming from the sharp decline in the share of C corporations. While C corporations reported 74 percent of net income less deficit in 1980, by 2012 that share had declined to 35 percent. The shares for partnerships, mainly limited liability companies, and S corporations grew over the same period.

The unprecedented expansion of pass-through entities in the US can be linked to legal changes that took place since Tax Reform Act of 1986 (hereafter TRA86). TRA86 reduced the top individual tax rate below the top combined corporate and dividend tax rate. Hence, C corporations, are facing a higher tax rate following 1986 (Figure ??). According to evidence from Dyrda and Pugsley (2018) using micro level data, there was a spike in the conversion rate of C corporations to pass-through entities during that period. Recent tax reforms, notably the Tax Cuts and Jobs Act of 2017 have reversed these trends with the corporate tax rate being cut from 35 percent to 21 percent. Since 2017 the combined corporate and

be some S-corporations for which financing constraints may be small. In our context because pass-through firms consist of sole proprietorships, general partnerships, limited partnerships, limited liability companies in addition to S-corporations, fraction of firms for which financial constraints may not matter is very small.

dividend tax rates are lower than top personal income tax rate.

3 Model

To understand the mechanism linking the choice of legal form of organization and output distortions, we develop a model of firm dynamics featuring endogenous choice of legal forms of organization. Time is discrete and infinite. The economy consists of a representative household, a unit measure of heterogeneous firms, and a government. We focus on a steady-state analysis of the model. First, we describe decisions of each type of agent in the model, then we define the stationary equilibrium.

3.1 Preferences

The economy is populated with a unit measure of identical infinitely-lived households, who value the path of consumption according to the following utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

where c_t is the consumption in period t and $0 < \beta < 1$ is the time discount factor. Households are endowed with one unit of time in each period, which they supply to the market inelastically.

3.2 Technology

In this economy production can take place in two types of firm, one type is a C corporation, denoted by C-type, and the other type is a pass-through firm, denoted by P-type. Both types use capital and labor as inputs and use the same production technology to produce a homogeneous output:

$$f(s, k, n) = s^{1-\gamma} (k^\theta n^{1-\theta})^\gamma \tag{1}$$

where k is capital, n is labor, and s is the productivity shock. Both types of firms are subject to an idiosyncratic productivity shock at each period. A C-type firm pays tax at the corporate level. Any distribution of the after-tax profit among shareholders is also subject to an individual level tax. Therefore it pays tax twice while a P-type firm passes all profits or losses to the owners and they pay income tax.

Apart from receiving different tax treatments, these two types vary along three dimensions:

- Fixed cost of production,

A C-type firm faces a higher fixed cost of production compared to a P-type firm. The overhead cost in C-corporations are higher, e.g. the cost of using specific accounting system, keeping records of the all the board meetings, and tax preparation.

- Rental rate of capital,

While C corporations have access to the public equity market which provides an elastic supply of external equity, a pass-through entity can only rely on its owners personal funds to use either as equity or as a collateral for issuing debt. This feature is captured in the model by introducing a wedge over rental rate of capital for pass-through entities. Let R be the rental rate of capital in the market, then the R^p is the rate at which pass-through entities can raise capital,

$$R^p = R(1 + \zeta) \tag{2}$$

where ζ is the wedge that pass-through entities face. This captures the idea that it is harder and therefore, more expensive for pass-through entities to generate external capital.

- Evolution of the productivity shock over time,

The final difference between the two types of firms is with respect to their productivity paths. Productivity evolves according to an exogenous $AR(1)$ process with an innovation that is independent across firms. The standard deviation for the innovation process is bigger for C-type relative to P-type firms. C corporation is the only legal form that has access to the public equity market, which makes them more likely to pursue projects with higher levels of risk, mainly because they have better opportunities to diversify the risk compared to pass-through entities.

3.3 Government

In this model government collects revenue by taxing labor income, dividend income, taxing C-type firms at the corporate level, and taxing P-type firm at their owners income level.

3.4 Timing

The timing of the events within a period is as follows:

Incumbent Firms, at the beginning of each period, an incumbent firm pays the fixed cost of production C^F , then the productivity shock for the period, s_t is realized and the firm decides on its current period capital and labor demand. At the end of the period the firm decides between exiting the market and staying for the next period. If the firm exits, it disappears from the model and receives zero profits in all future period. If it stays, it can choose its type for the next period; it can either continue with the same type or switch to the new type.

New Entrants, at each period there is a continuum of ex-ante identical potential entrants. New firms have to pay a one-time entry cost of C_e . Once this cost has been paid, the new entrant draws a productivity shock s_t from a distribution η . These draws are independently and identically distributed across entering firms. Given s_t , an entrant chooses its type and its problem is equivalent to an incumbent firm with the same type that has the productivity shock s_t .

Households, at the beginning of each period, households rent their labor endowments and capital stocks to the firms. We assume that households own equal shares in all firms and at the end of each period they receive the labor income, capital income and profit of firms. They choose their consumption and savings at the end of the period.

3.5 Decision Problems

This section describes decision problems for both types of firms and the household.

3.5.1 Firms' Decision Problem

The state of each firm in any period, can be described by a pair (Δ, s) , where Δ is the type of firm in that period, $\Delta \in \{C, P\}$ and s is the corresponding period productivity shock. While productivity evolves exogenously, firms choose their type endogenously. At the end of each period, after observing current productivity, s , a firm decides on its type for the next period, upon staying in the economy. In this model the value of each firm is determined by the present value of the stream of after-tax profits that are collected by the owners.

C-type Incumbent Firm: for a C-type firm with state (C, s) that is subject to the corporate income tax and the dividend tax, the after-tax profit in each period is determined in the following way:

$$\Pi(C, s) = \max_{k,n} \left[\left([f(s, k, n) - wn - wC_F^C - \delta k][1 - \tau^c] - Rk \right) (1 - \tau^d) \right] \quad (3)$$

where w is the wage rate, R is the rental rate of capital, τ^c is the corporate tax rate and τ^d is the dividend tax rate. The value function of a C-type incumbent firm with state (C, s) is denoted by $V(C, s)$ which is given by:

$$V(C, s) = \left[\Pi(C, s) + \beta \max_{C,P,exit} \left\{ \int V(C, s') Q^C(s, ds'), \int V(P, s') Q^P(s, ds') - wC_S^C, G \right\} \right] \quad (4)$$

Here $Q^C(s, s')$ is the transition function for the Markov process of shock s^C , and $Q^P(s, s')$ is the transition function for the Markov process of shock s^P . G is the exit value. C_S^C is the switching cost that the c-corporation has to pay, if it chooses to switch type to a pass-through entity for the next period.

P-type Incumbent Firm: for a P-type incumbent firm with state (P, s) , that is subject only to individual tax, the after-tax profit in each period is determined in the following way:

$$\Pi(P, s) = \max_{k,n} \left[(f(s, k, n) - wn - wC_F^P - \delta k)(1 - \tau^i) - R^P k \right] \quad (5)$$

where, τ^i is the individual tax rate. The value function of a P-type incumbent firm with state (P, s) is denoted by $V(P, s)$ which is given by

$$V(P, s) = \left[\Pi(P, s) + \beta \max_{C,P,exit} \left\{ \int V(P, s') Q^P(s, ds'), \int V(C, s') Q^C(s, ds') - wC_S^P, G \right\} \right] \quad (6)$$

Here, C_S^P is the switching cost for a pass-through entity.

An Entrant Firm, the value of entering gross of entry cost can be computed by:

$$V^e = \int_s \max\{V(C, s), V(P, s)\} \eta(ds) \quad (7)$$

As value functions are increasing in the level of productivity shock, we can show that there exists a threshold value of shock, \bar{s} , such that for a productivity shock above that value, a new entrant chooses to be a C-type firm and for a productivity shock below that level, a new entrant chooses to be a P-type firm.

The firm's decision problem produces four decision rules: the optimal choice of capital $k(\Delta, s)$, the optimal choice of labor $n(\Delta, s)$, stay or exit decision $\mathbb{1}_{exit}$ and the decision to switch the type $\mathbb{1}_{switch}$ ($\mathbb{1}$ is an indicator function).

3.5.2 Households' Decision problem

In this economy households rent the labor and capital to the firms. They are also the owners of the firms and receive their profits. The state of households can be described with the capital K and the firm ownership given by a measure x over firm types. The decision problem of a household with state (K, x) can be written as

$$H(K, x) = \max_{c, k'} \{u(c) + \beta H(K', x')\} \quad (8)$$

s.t.

$$\begin{aligned} c + M C_e + K' &\leq w(1 - \tau^i) + \\ &(1 + R) \left(\int k(C, s) x(C, ds) + \int_{\bar{s}} k(C, s) \eta(ds) + \right. \\ &\quad \left. \int k(P, s) x(P, ds) + \int_{\bar{s}} k(P, s) \eta(ds) \right) + \\ &\quad \int \Pi(P, s) x(P, ds) + \int_{\bar{s}} \Pi(P, s) \eta(ds) + \\ &\quad \int \Pi(C, s) x(C, ds) + \int_{\bar{s}} \Pi(C, s) \eta(ds) + Tr \end{aligned} \quad (9)$$

$$\begin{aligned}
x'(C, \mathcal{S}) &= \int (1 - \mathbb{1}_{switch}(C, s))(1 - \mathbb{1}_{exit}(C, s))Q^C(s, \mathcal{S})x(C, ds) & (10) \\
&\quad \int \mathbb{1}_{switch}(P, s)(1 - \mathbb{1}_{exit}(P, z))Q^C(s, \mathcal{S})x(P, ds) \\
&\quad + M \int_{\bar{s}} (1 - \mathbb{1}_{switch}(C, s))(1 - \mathbb{1}_{exit}(C, s))Q^C(s, \mathcal{S})\eta(ds) \\
&\quad + M \int_{\bar{s}} \mathbb{1}_{switch}(P, s)(1 - \mathbb{1}_{exit}(P, z))Q^C(s, \mathcal{S})\eta(ds)
\end{aligned}$$

$$\begin{aligned}
x'(P, \mathcal{S}) &= \int (1 - \mathbb{1}_{switch}(P, s))(1 - \mathbb{1}_{exit}(P, s))Q^P(s, \mathcal{S})x(P, ds) & (11) \\
&\quad + \int \mathbb{1}_{switch}(C, s)(1 - \mathbb{1}_{exit}(C, z))Q^P(s, \mathcal{S})x(C, ds) \\
&\quad + M \int_{\bar{s}} (1 - \mathbb{1}_{switch}(P, s))(1 - \mathbb{1}_{exit}(P, s))Q^P(s, \mathcal{S})\eta(ds) \\
&\quad + M \int_{\bar{s}} \mathbb{1}_{switch}(C, s)(1 - \mathbb{1}_{exit}(C, z))Q^P(s, \mathcal{S})\eta(ds)
\end{aligned}$$

where Tr is the transfer to the household.⁴ Equations (10) and (11) give the next period ownership of firms x' . The first integral in equation (10), represents the incumbent firms of type C , who stayed and did not switch type in period t , the second integral represents the incumbent firms of type P , who stayed but switched to other type in period t . The third line, shows the new entrants who choose C type for the current period and keep their type for the next period as well. And finally the last line, represents the mass of new entrants who choose type P in the current period but decide to switch to C -type for the next period. Equation (11) can be interpreted in the similar way.

3.6 Equilibrium

Given the model specified above, a stationary equilibrium is defined in the following way. At steady state equilibrium, the aggregate state of the economy (K^*, x^*) and equilibrium

⁴In the equilibrium, the transfer Tr is equal to

$$Tr = (R^p - R) \left(\int k(P, s)x(P, ds) + \int_{\bar{s}} k(P, s)\eta(ds) \right) \quad (12)$$

prices w^* and R^* are constant over time. Firms solve their problem taking equilibrium prices as given and generate decision rules $n^*(\Delta, s)$, $k^*(\Delta, s)$, $\mathbb{1}_{exit}(\Delta, s)$ and $\mathbb{1}_{switch}(\Delta, s)$. Households solve their decision problem, taking firms' decision rules, equilibrium prices and transfers as given, and choose $K' = K^*$ and $x' = x^*$ for the next period. Market clears for goods, capital and labor services and government budget constraint holds. Appendix A provides a formal definition of the equilibrium, and Appendix B outlines the algorithm for solving the model.

4 Parametrization

In this section, we detail the parametrization of the model and motivate the choices of targets. We choose parameters such that the model is consistent with firm dynamic features of the U.S. economy over the period 2008-2010.⁵

Model Specification, model period is one year. Households' period utility function has the log utility form, $u(c) = \log(c)$. The log productivity shock, s , follows an $AR(1)$ process with the persistency of ρ and standard deviation of σ . The shock process differs by firm's type, persistency parameter is common between the two but the standard deviation of the innovation is different.

$$\log(s') = \rho \log(s) + \epsilon_{\Delta} \text{ where } \epsilon_{\Delta} \sim N(0, \sigma_{\Delta}^2)$$

We use the method developed in Tauchen (1986) to construct a first-order Markov process approximation. Distribution of productivity shocks for new firms, $\eta(s)$, is a composite lognormal-Pareto distribution which takes a log-normal density up to a threshold and a Pareto density thereafter. We provide a detailed derivation of this composite distribution in Appendix C.

The depreciation rate, δ , is the total depreciation of private fixed assets by corporate firms, partnerships, and sole proprietorships (NIPA Fixed Asset Table 6.4 lines 2, 6, and 7) divided by the total private fixed assets of corporate firms, partnerships, and sole proprietorships (NIPA Fixed Asset Table 6.1 lines 2, 6, and 7). The estimated annual depreciation rate is 7.1 percent. The value of parameter γ which controls returns to scale is set to 0.8 which is within the bounds used in literature (Guner et al. (2008)). Parameter θ , which controls the capital share, is set such that the model is consistent with the capital share of output in

⁵We choose this period to have the maximum set up moments to use as targets in our calibration.

the data. Tax rates are set to their statutory values over the period 2008 to 2010, i.e. the corporate tax rate is 35%, the dividend tax rate is 15% and the top marginal income tax rate for individuals is 35%. We use County Business Pattern data to capture moments of firm-size distribution for each organizational form.

Panel A in Table 1 reports parameters that are set independently. The remaining parameters are calibrated jointly and reported in panel B. Table ?? provides a description of data moments that are used as targets and compares them with moments generated by the model. Calibrated model fits the data well, as it matches the size distribution of firms and the output share of C-corporation.

5 Findings

In Panel A in Figure (2) we compare the distribution of C-type and P-type firms from our model at each productivity level. For lower levels of productivity, more firms choose to organize as P-types. As productivity increases, more firms choose to organize as C-types. However, with very high levels of productivity, P-type structure is more prevalent. More productive firms demand more capital (and labor) and by choosing to be P-type, these firms face a higher cost of capital. However, the higher tax burden associated with C-type creates a large disadvantage at higher levels of productivity. This means that despite the higher cost of capital, P-type is the optimal form of organization for these firms. This highlights the negative impact of higher tax burden on C corporations. In what follows, we use our model to quantify the distortionary effect of different tax treatment of capital income on the aggregate output of the economy.

5.1 Discussion

In this section, we do two types of experiments. First, we use the benchmark model to study the aggregate impact of three modified tax structures, keeping the calibrated parameters fixed and the government tax revenue constant at its level in the benchmark economy. Those experiments change the nature of the tax base compared to the benchmark tax system, with both types of firms receiving a uniform tax treatment. Thus, we eliminate the tax advantages of one type over the other. The goal of these exercises is to highlight the quantitative importance of each type of tax we have in the model and to understand the mechanism. Second, we show what happens in our framework when we introduce a reform similar to the Tax Cut and Jobs Act of 2017.

5.1.1 Uniform business income taxation

In this exercise, the benchmark tax structure is replaced with a tax system in which both C-type and P-type firms receive the same tax treatment. The dividend tax for C-type firms is abolished and both types pay business income tax at the entity level, at the same rate τ_1 , which is chosen such that tax revenue is the same as in the benchmark economy. Also, labor income is taxed at the same rate as in the benchmark economy.

Equalizing tax treatment of all businesses increases the aggregate output of the economy by 1.25 percent (column 3 in Table 2). With this tax regime the most productive firms, which were P-type in the benchmark case, choose to be a C-type (see Panel B in Figure 2). Therefore the output share of C-type increases from 62% to 84% relative to the benchmark case. The calculated tax rate τ_1 is 0.37, which is higher than the tax rate levied on both types in the benchmark economy (0.35). In spite of this higher tax rate, the aggregate output increases. Since, both types of firms are subject to business income tax, we still have the distortion on capital accumulation, similar to the benchmark tax system. However, here the tax structure does not favor a particular organization type. Therefore, the increase in output is the result of reallocation of capital across types. Unifying tax treatment across all firms affects their choice of legal form of organization, for which the trade-off is between the fixed cost of production and the cost of capital.

5.1.2 Uniform tax regime

The tax structure in this exercise is similar to the previous one, except that both labor income and business income are taxed at the same rate τ_2 , which is chosen to keep the tax revenue constant. The calculated tax rate τ_2 is 0.36, which is lower compared to the calculated tax rate in uniform business income taxation ($\tau_1 = 0.37$). Hence, the aggregate output is higher. However, the tax rate on labor income, which is τ_2 is higher than the labor income tax in the benchmark economy (0.35). Thus, we decrease tax rate on business income at the expense of increasing the tax rate of labor income, which explains the larger increase in total output relative to the first exercise (column 4 in Table 2). Also, similar to the previous exercise, the most productive firms choose to be C-type.

5.1.3 Uniform consumption tax regime

Here, we eliminate taxation at the entity level, i.e. the tax base is comprised of dividend from both types of firms and labor income. Dividends from both types of firms are taxed at

the same rate as labor income, τ_3 , which is set to keep tax revenue constant. Note that this tax code is effectively a consumption tax system.

The most noticeable change relative to other reforms is a very large increase in the aggregate output. Replacing the benchmark tax system with this uniform dividend and labor income tax code, increases output by 10.8% (column 5 in Table 2). Comparing this tax reform with previous reforms highlights the distortionary effect of capital income taxation on capital accumulation. The large increase in output is the result of capital accumulation and capital reallocation. Abolishing the capital income taxation removes the distortion on capital accumulation, while equalizing the tax treatment of both types of firms removes only the distortion on capital allocation. Although the calculated tax rate in this experiment is highest among the three reforms (0.43), the increase in the output is larger than in the first two cases, highlighting the distortionary effect of taxing capital income.

5.1.4 A reform similar to "Tax Cut and Jobs Act" reform of 2017

In this exercise we replace the benchmark tax structure with the one that mimics features of TCJA reform. We keep the corporate tax rate at 21%, the top income tax rate, which affects the P-type entities and labor income in our model, at 35% and dividend tax rate at 15%. With the lower tax rate on C corporations, many businesses find it optimal to choose C-type and as a result we see the output share for this form increase by 60 percent compared to the benchmark economy. This increase output by 7.9 percent, because of the lower tax burden on corporations. Note that, the government revenue from taxation decreases by 9.1 percent compared to the benchmark economy, reflecting the main cost of this reform.

6 Conclusion

In this paper we argue that choosing a legal form of organization based on the tax treatment of businesses distorts the capital allocation and affects the size of the aggregate output. We develop a model of firm dynamics in which the legal form of organization is an endogenous choice for businesses that are heterogeneous with respect to their productivity. The model captures the trade-off between tax treatments of each form of organization and the access to capital. We calibrate the model to be consistent with the firm dynamic characteristics of the U.S. businesses as well as the contributing share of each legal form to total output. Using the calibrated model and taking into consideration the general equilibrium effect under the assumption of revenue neutrality, we find that unifying the tax treatment

across all legal form of businesses increases aggregate output by 1.25 percent in the long run. The key mechanism for this finding is that removing the tax distortions affecting the choice of legal form of organization reallocates capital towards more productive firms and improves the aggregate output.

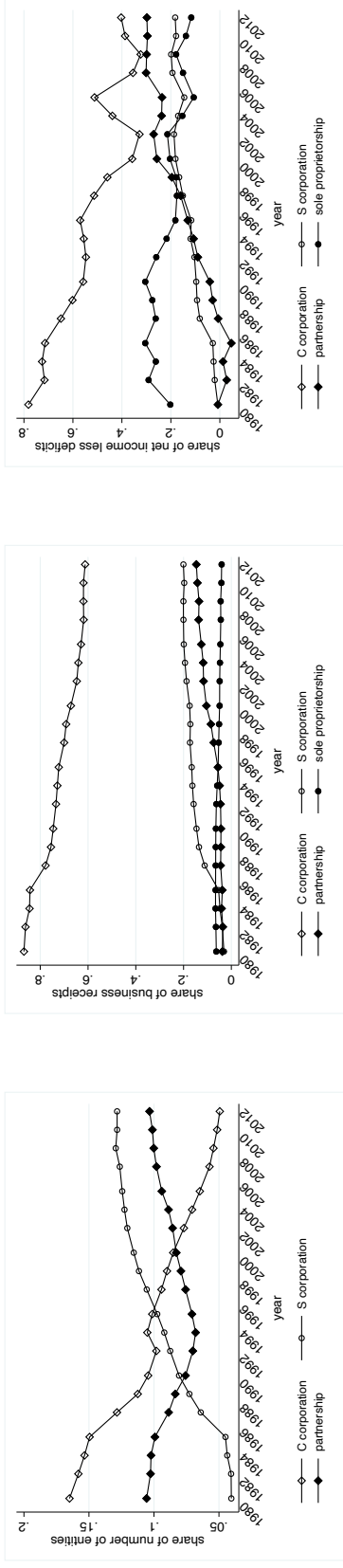
From a policy perspective, the reform that generates the largest output increase, while still remaining to be revenue neutral is the consumption tax that removes the corporate tax altogether. While such a reform might be optimal for generating large growth of the economy, it may be politically challenging. This is because the revenue neutrality requires the tax rates to be set at much higher level than they currently are.

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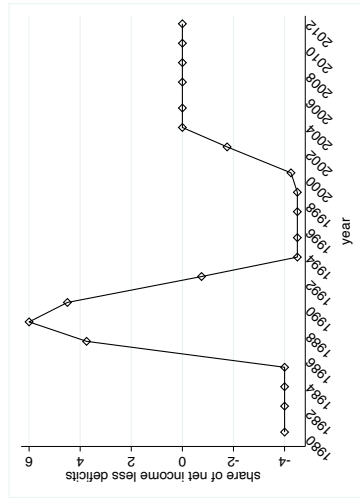
Figure 1: Pass-throughs vs C-corporations.



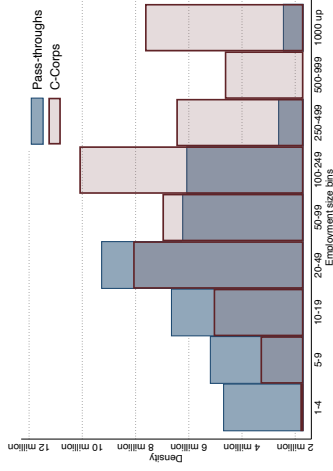
a Share of U.S. Businesses Organized in each of the three Basic Legal Form of Organization. Sole proprietorship excluded.

b Share of Each Legal Form of Organization in Total U.S. Business Receipts

c Share of Legal forms of Organization in Total Net Income Less Deficits



d Difference Between the Top Corporate Tax Rate and the Top Individual Tax Rate

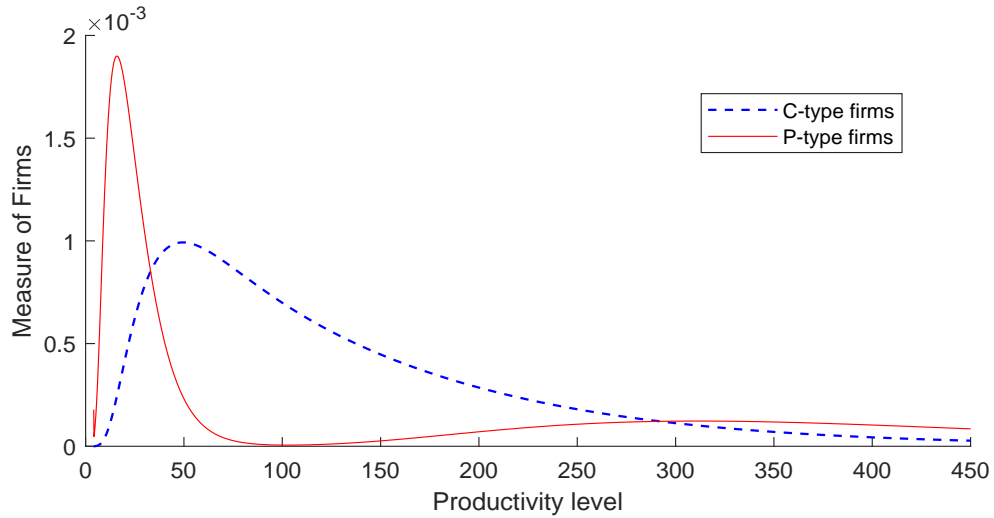


e Distribution of the number of establishments by type, 2008 - 2010 average.

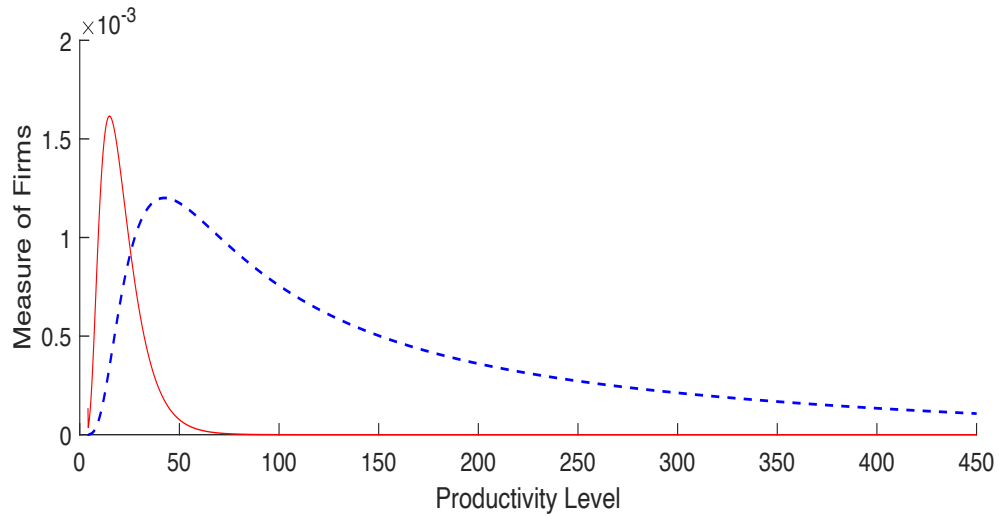
f Distribution of number of employees by type, 2008 - 2010 average.

Note: Sources: IRS, SOI Tax Stats CBP Census, and author's calculation. In Panel A we show how the percentage of U.S. businesses organized in each of the three main legal forms of organization has evolved over time. We exclude sole proprietorship for which the share was around 75% throughout the sample period. In Panel B we show the evolution of the share of each legal form of organization in the total U.S. business receipts. In Panel C we show how the total net income less deficits of all U.S. businesses is divided among all organizational forms. In Panel D we show the top corporate income tax rate minus the top individual income tax rate. Hence, for periods in which the corporate tax rate is higher than the individual tax rate the graph shows positive values. In Panels E and F we show distribution of number of establishments and employees by firm size respectively.

Figure 2: Productivity distributions



a Benchmark distributions



b Uniform business tax distributions.

Note: In this figure we plot the distributions of firms across productivity levels generated by the model. C-type firms are represented by the blue dashed line and P-type firms are represented by the red solid line. In Panel A we plot the distributions for the benchmark case and in Panel B we plot the distributions for the uniform business tax case.

Table 1: Calibrated Parameters.

Panel A: Parameters Calibrated Independently		
Parameter	Value	
δ	0.07	Depreciation rate (NIPA tables)
θ	0.406	Importance of capital
γ	0.8	Return to scale
τ^c	0.35	Corporate tax rate
τ^d	0.15	Dividend tax rate
τ^i	0.35	Individual income tax rate
Panel B: Parameters Calibrated Jointly		
β	0.946	Discount rate, target K/Y
ρ	0.969	Persistency of productivity shock, target firm-size distribution
σ_C	0.462	Std. deviation of productivity shock for C-type, targeting size distribution of C-corporations
σ_P	0.361	Std. deviation of productivity shock for P-type, targeting size distribution of pass-thorough entities
C_F^C	1.5	Fixed cost of production, C-type *, targeting average size of C-corporations
C_F^P	0.75	Fixed cost of production, P-type *, targeting average size of Pass-thorough entities
C_e	5.2	Fixed entry cost *, targeting employment share of new entrants
ζ	0.17	Markup on capital rent for P-type, targeting relative output share of each type
C_{switch}^C	0.26	Cost of switching from C-type to P-type *, targeting the share of C-corporations that switch
θ_{eta}	100	Threshold parameter for entrant's productivity dist. η
α_η	0.80	Tail parameter for η , targeting moments of firm-size distribution
σ_η	1.23	Std. deviation for η

Note: *, these cost are in wage unit

Table 2: Effects of Changing the Tax Structure

	Benchmark	Uniform Business Tax	Uniform Tax	Consumption Tax	TCJA
Calculated Tax Rate, τ	.	0.37	0.36	0.43	.
Capital Output Ratio	1.79	1.78	1.81	2.28	2.07
Output	2.4	2.43	2.45	2.66	2.59
		(+1.25%)	(+2%)	(+10.8%)	(+7.9%)
Output share of C-type	62	85	84	89	97
Average size of C-type	1.61	1.63	1.68	1.47	1.12
Average size of P-type	0.61	0.31	0.32	0.29	0.21

Note: This table reports how the aggregate variables change with changes in the tax structure. Uniform Business Tax is a system in which both types of firms are taxed symmetrically at the entity level, there is no dividend tax and the labor income is taxed at the same rate as in the benchmark economy. Uniform Tax is a uniform business income and labor income tax system. Compared to the uniform business tax, the only difference is that both labor income and business income are taxed at the same rate. Consumption tax is a uniform dividend and labor income tax system, where both types of business taxation are abolished and the dividend from both types is taxed at the same rate as the labor income. Note that the first three reforms are revenue neutral, so the calculated tax rate reported in the first row of the table, is the one required to keep the government revenue constant. TCJA is a reform similar to "Tax Cut and Jobs Act" reform of 2017 in which the corporate tax rate is 21% and the top income tax rate and dividend tax rates are the same as in the benchmark. Note that this reform is not revenue neutral and leads to 9.1% drop in tax revenues.

Appendices

Appendix A: Equilibrium definition

In this appendix I define the equilibrium for the economy at the steady state.

In the model economy, firms are heterogeneous with respect to their types ($\Delta \in \{C, P\}$), and productivity level ($s \in S$). For aggregating, I need to define a measure $x(\Delta, s)$, which is the mass of firms with state (Δ, s) . This measures define the distribution over firm's state space.

Definition of Equilibrium. A steady state equilibrium is a collection of decision rules $n^*(\Delta, s)$, $k^*(\Delta, s)$, $\mathbb{1}_{exit}(\Delta, s)$ and $\mathbb{1}_{switch}(\Delta, s)$, $K'(K, x)$ and $x'(K, x)$, factor prices w and R , transfer Tr^* and aggregate capital K , government consumption G , such that:

- $n^*(\Delta, s)$, $k^*(\Delta, s)$, $\mathbb{1}_{exit}(\Delta, s)$ and $\mathbb{1}_{switch}(\Delta, s)$ are optimal decision rules.
- Taking firms decisions, prices, M^* and \bar{s} as given, households solve for $K' = K^*$, $x' = x^*$
- Market clearing conditions are satisfied:

$$K^* = \int k^*(\Delta, s)x^*(d\Delta \times ds) + M^* \int k^*(\Delta, s)\eta(ds) \quad (13)$$

$$1 = \int n^*(\Delta, s)x^*(d\Delta \times ds) + M^* \int n^*(\Delta, s)\eta(ds) \quad (14)$$

$$c^* + I^* + M^*C_e + G^* + C_e = \int f(s, k^*(\Delta, s), n^*(\Delta, s))x^*(d\Delta \times ds) + M^* \int f(s, k^*(\Delta, s), n^*(\Delta, s))\eta(ds) \quad (15)$$

- Law of motion of distributions is consistent with firm decision rules:

$$\begin{aligned}
x'(C, \mathcal{S}) &= \int (1 - \mathbb{1}_{switch}(C, s))(1 - \mathbb{1}_{exit}(C, s))Q^C(s, \mathcal{S})x(C, ds) \\
&\quad \int \mathbb{1}_{switch}(P, s)(1 - \mathbb{1}_{exit}(P, z))Q^P(s, \mathcal{S})x(P, ds) \\
&\quad + M \int_{\bar{s}} (1 - \mathbb{1}_{switch}(C, s))(1 - \mathbb{1}_{exit}(C, s))Q^C(s, \mathcal{S})\eta(ds) \\
&\quad + M \int_{\bar{s}} \mathbb{1}_{switch}(P, s)(1 - \mathbb{1}_{exit}(P, z))Q^P(s, \mathcal{S})\eta(ds)
\end{aligned} \tag{16}$$

$$\begin{aligned}
x'(P, \mathcal{S}) &= \int (1 - \mathbb{1}_{switch}(P, s))(1 - \mathbb{1}_{exit}(P, s))Q^P(s, \mathcal{S})x(P, ds) \\
&\quad + \int \mathbb{1}_{switch}(C, s)(1 - \mathbb{1}_{exit}(C, z))Q^C(s, \mathcal{S})x(C, ds) \\
&\quad + M \int_{\bar{s}} (1 - \mathbb{1}_{switch}(P, s))(1 - \mathbb{1}_{exit}(P, s))Q^P(s, \mathcal{S})\eta(ds) \\
&\quad + M \int_{\bar{s}} \mathbb{1}_{switch}(C, s)(1 - \mathbb{1}_{exit}(C, z))Q^C(s, \mathcal{S})\eta(ds)
\end{aligned} \tag{17}$$

- Government budget balanced:

$$\begin{aligned}
G^* &= \tau^c \left[\int (f(s, k^*(C, s), n^*(C, s)) - wn^*(C, s) - wC_F^C - \delta k^*(C, s))x(C, ds) \right. \\
&\quad \left. + \int_{\bar{z}} (f(s, k^*(C, s), n^*(C, s)) - wn^*(C, s) - wC_F^C - \delta k^*(C, s))\eta(ds) \right] \\
&\quad + \tau^d \left[\int (f(s, k^*(C, s), n^*(C, s)) - wn^*(C, s) - wC_F^C - \delta k^*(C, s))(1 - \tau^c) \right. \\
&\quad \left. - Rk^*(C, s))x(C, ds) \right. \\
&\quad \left. \int_{\bar{s}} (f(s, k^*(C, s), n^*(C, s)) - wn^*(C, s) - wC_F^C - \delta k^*(C, s))(1 - \tau^c) \right. \\
&\quad \left. - Rk^*(C, s))\eta(ds) \right] + \\
&\quad \tau^i \left[\int (f(s, k^*(P, s), n^*(P, s)) - wn^*(P, s) - wC_F^P - \delta k^*(P, s))x(P, ds) + \right. \\
&\quad \left. \int_{\bar{s}} (f(s, k^*(P, s), n^*(P, s)) - wn^*(P, s) - wC_F^P - \delta k^*(P, s))\eta(ds) + w \right]
\end{aligned} \tag{18}$$

Appendix B: Algorithm for solving the model

1. Set $R = \frac{1}{\beta} - 1$ and $R^P = (1 + \zeta)R$
2. Guess w
3. Taking Prices (r, R^P, w) as given,
 - (a) Solve firms' problems: $k(\Delta, s), n(\Delta, s), \Pi(\Delta, s), V(\Delta, s), \mathbb{1}_{exit}(\Delta, s), \mathbb{1}_{switch}(\Delta, 1)$
 - (b) Calculate V^e using $V(\Delta, s)$
 - (c) Check the free entry condition: $V^e = c_e$, and update the w accordingly and start over from step 2 until the free entry condition is satisfied.
4. Set the mass of entrant $M \equiv 1$, solve for stationary distribution of firms x_0 , using $\mathbb{1}_{exit}(\Delta, s), \mathbb{1}_{switch}(\Delta, s)$
5. Use market clearing condition for labor:

$$\underbrace{1}_{\text{Labor supply}} = M \cdot \underbrace{L_d}_{\text{Labor demand from aggregating } n(\Delta, s)}$$

to find the equilibrium level of M .

6. Set $x(\Delta, s) = M \cdot x_0(\Delta, s)$
7. Use the feasibility condition and Government revenue G to find C, Y and K

$$Y = \int f(s, k(\Delta, s), n(\Delta, s)) dx + \int f(s, k(\Delta, s), n(\Delta, s)) \eta(ds)$$

$$C + I + M \cdot c_e + G + \text{Fixed Costs} = Y$$

where I is the investment ($I = \delta K$ in steady state)

Appendix C: Deriving the Lognormal-Pareto distribution

Cooray and Ananda (2005) were among the first to develop a single composite lognormal-Pareto model which takes a log-normal density up to an unknown threshold and a two-parameter Pareto density thereafter. However, their model has a priori known mixing weights which is very restrictive features, i.e. the continuity and differentiability conditions make a case such that the model says exactly $\sim 39.2\%$ of the observations are from a lognormal model truncated at θ , which is always 64.514^{th} percentile of the underlying lognormal model, and the rest of observations are above θ and in accordance with a certain parameter restricted Pareto model. Scollnik(2007) Scollnik (2007) addresses this issue and suggests a lognormal-Pareto mixture model with threshold θ but with an unrestricted mixing weights. This is the model I'm using in this paper.

Let \mathbf{X} be a random variable with the probability density function

$$f(x) = \begin{cases} r \frac{1}{\Phi\left(\frac{\ln(\theta) - \mu}{\sigma}\right)} f_1(x) & \text{if } 0 < x \leq \theta \\ (1 - r) f_2(x) & \text{if } \theta \leq x < \infty \end{cases}$$

where Φ is the cumulative distribution function of the standard normal distribution, r is the mixing weight, and $f_1(x)$ and $f_2(x)$ are the lognormal and Pareto densities given by the following equations:

$$f_1(x) = \frac{(1\pi)^{-1/2}}{x\sigma} \exp\left[-\frac{1}{2}\left(\frac{\ln(x) - \mu}{\sigma}\right)^2\right], \quad x > 0$$

$$f_2(x) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, \quad x > \theta$$

where θ, μ, σ and α are unknown parameters of the density function.